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Abstract

This paper analyzes how internal debt financing of multinational firms affects high-tax countries. It uses a dynamic small open economy model and takes into account that internal debt impacts both the multinational firms' investment decisions and the government's tax policy. The government has incentives to redistribute income from firm owners to workers. If the government's redistributive motive is not too strong, internal debt reduces welfare in the short term by decreasing tax revenues. However, debt financing stimulates capital accumulation and exerts a positive long term welfare impact. If the multinational firm additionally manipulates transfer prices, the adverse short term welfare effects may extend to the long term.

JEL-Codes: F230, H250, H700.

Keywords: internal debt, profit shifting, tax havens.

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1 Introduction

Multinational enterprises (MNEs) shift a large proportion of their profits to tax havens. In 2015 more than $600 billion, or 36% of multinationals' worldwide profits, were shifted (Tørsløv et al., 2018). MNEs use internal debt as one of the main channels of international tax planning. It accounts for $25 - 30\%$ of the shifted profits (Heckemeyer and Overesch, 2017; Beer et al., 2019). Hence, in its initiative on base erosion and profit shifting, the OECD calls for, inter alia, measures to address base erosion through the use of internal debt (OECD, 2013, 2015). Moreover, the number of countries applying thin-capitalization rules (i.e., rules that limit interest deductibility) increases over time (Merlo and Wamser, 2015).[1]

Here, I analyze the welfare effects of internal debt in the short and long run. I show that these effects are not necessarily negative. Furthermore, they may be non-monotone, with negative short term and positive long term welfare implications.

This paper builds a dynamic small open economy model. There is one high-tax (non-haven or host) country that hosts a national firm and a subsidiary of a foreign-owned MNE. Workers supply labor that is perfectly mobile between the national and multinational sectors. The MNE invests mobile capital in the host country, and capital adjustment is subject to installation costs. The MNE's headquarter can channel equity financing to its subsidiary as internal debt through a financial center located in a tax haven. The host country government uses a thin-capitalization rule (TCR) to restrict such behavior. Moreover, it chooses a time-invariant corporate tax rate and redistributes income from firm owners to workers.

I show that the short and long term welfare effects of a TCR relaxation are, in general, ambiguous. If the redistributive motive of the government is sufficiently weak, welfare declines unambiguously in the short term and increases in the long term. The intuition is the following. A TCR relaxation stimulates profit shifting and lowers the MNE's cost of capital for a given statutory tax rate. The increase in profit shifting reduces the tax revenues directly, while the cost of capital effect stimulates investment and may increase the optimal tax rate. In the short term, the capital stock adjusts slowly because new capital installation is costly. Hence, welfare declines if the change in the optimal tax rate cannot compensate for the loss of tax revenues (which is the case for a sufficiently weak redistributive motive). In the long term, capital accumulates, which increases welfare to a level that is ultimately higher than its initial level.

Non-monotone welfare effects emerge due to the dynamic nature of the MNEs' responses to TCR reforms, which is supported by the empirical literature. Weichenrieder and Windischbauer (2008) and Buslei and Simmler (2012) analyze the short term ef-

[1] Merlo and Wamser (2015) show that from 1996 to 2012 the number of countries applying thin-capitalization rules increases from 24 to 61.

fects of two reforms in Germany from 2001 and 2008, respectively. Weichenrieder and Windischbauer (2008) look at the impact on the capital stock of subsidiaries of foreign-owned MNEs in Germany two years after the 2001 reform, while Buslei and Simmler (2012) analyze investment of the same type of firms one year after the 2008 reform. Both papers do not identify any significant effects on the capital stock and investment, respectively. Moreover, Harju et al. (2017) measure the real effects of a TCR reform in Finland in 2014 through its impact on output in the two years following the reform. They do not find any significant effects.

However, in accordance with my results, the empirical literature finds significant long term real effects of debt financing. Buettner et al. (2008) analyze the long term impact of TCR on investment using a panel dataset of German multinationals' affiliates in 36 countries. They find statistically and economically significant adverse effects of both the implementation and tightening of TCRs. Moreover, Buettner et al. (2018) find significant negative long term impacts of TCRs on the MNEs' capital stock and the capital stock's tax rate sensitivity in high-tax countries. In addition, De Mooij and Liu (2018) use a panel data of MNEs operating in 34 countries over 2006-14. They find that a TCR introduction doubles the tax rate sensitivity of investment. Furthermore, Blouin et al. (2014) show that TCRs imposed on affiliates of US MNEs lower the overall firm valuation as measured by Tobin's q. Because Tobin's q is a good predictor of investment (Erickson and Whited, 2000; Philippon, 2009), the results of Blouin et al. (2014) also suggest a long term impact of TCR on investment. Furthermore, Suárez Serrato (2019) studies the long term real effects of elimination of tax haven use by US multinationals and finds negative investment and employment effects. While Suárez Serrato (2019) cannot distinguish between different profit shifting channels, his results are consistent with this paper's predictions.

Therefore, my results have important implications for empirical research. They suggest that the timing of the empirical evaluation of policy reforms may be crucial. Weichenrieder and Windischbauer (2008), Buslei and Simmler (2012) and Harju et al. (2017) analyze the short term real effects of three different TCR reforms and do not observe significant effects. However, according to both theory and the empirical results of Buettner et al. (2008, 2018), and De Mooij and Liu (2018), such effects should exist in the long term. Hence, a long run analysis of the above-mentioned reforms might produce different outcomes.

Moreover, the theoretical literature on the welfare implications of internal debt finds conflicting results, which this paper may help reconcile. The two seminal papers are by Hong and Smart (2010) and Haufler and Runkel (2012) and both studies consider static models. First, Hong and Smart (2010) find that (some) internal debt is unambiguously welfare-improving for a small open high-tax country. I show that the results of the static model of Hong and Smart (2010) hold in the long term, but might be reversed in the short term. Second, Haufler and Runkel (2012) find in a two-country model with a fixed capital supply and no redistribution motive by the government that zero internal debt is optimal. In my model, the short term capital stock is fixed due to its adjustment costs. I find that in the absence of a strong redistribution motive, welfare is monotonically decreasing in internal debt in the short term. Thus, a new interpretation of the Haufler and Runkel (2012) result is that it also holds in the short term of a model with elastic capital supply.

Additionally, I consider three extensions of the model. First, I allow the government to set a time-varying statutory tax rate and show that the non-monotone welfare effects persist. Second, I introduce deadweight costs of internal debt. Such costs allow for the derivation of an optimal internal debt. The extension shows that the optimal debt financing balances short term marginal costs to long term marginal benefits of internal debt. Third, I introduce transfer pricing as a second profit shifting channel and allow for substitutability between the two channels. In this case, the negative short term welfare effect might become persistent and carry over to the long term (this result is unambiguous for a sufficiently weak redistributive motive). Hence, when the MNE uses two profit shifting channels, it may be optimal to prohibit internal debt.

Similar to my last extension, Gresik et al. (2015) analyze, in a static model, the interrelation of internal debt and transfer pricing. They find that debt financing may reduce high-tax countries' welfare if transfer pricing is sufficiently aggressive. Here, I extend their analysis to a dynamic setting and substitutability between the profit shifting channels. I show that the same result might emerge over the long term for any degree of transfer pricing.

This paper is related to the literature on the implications of profit shifting for non-haven countries' welfare. All in all, there is no consensus on whether tax havens are good or bad. On the one hand, elimination of tax havens is beneficial to non-haven countries if it improves public good provision (Slemrod and Wilson, 2009; Haufler and Runkel, 2012) or if it removes the secrecy of firm ownership (Weichenrieder and Xu, 2019). On the other hand, the elimination of tax havens may have an ambiguous impact on non-havens' welfare if it intensifies the tax competition among the high-tax countries (Johannesen, 2010) or if it is only partial and lowers competition among the remaining havens (Elsayyad and Konrad, 2012). Some profit shifting may benefit high-tax countries if it raises the optimal tax rates of low-tax jurisdictions (Becker and Fuest, 2012). Moreover, international tax planning may be good for non-havens if MNEs' organizational form responds to tax discrimination (Bucovetsky and Haufler, 2008), if governments respond to tax planning by changing their tax enforcement strategies (Chu, 2014), or in the presence of lobbying by the owners of immobile capital (Chu et al., 2015). Peralta et al. (2006) find possible beneficial welfare effects of profit shifting among non-haven countries when the MNE also makes a choice for the location of its productive subsidiary.[2]

This paper differs from the remaining literature by being the first to develop a dynamic model that differentiates between the short term and long term effects of profit shifting. It is also the first to derive non-monotone welfare effects of profit shifting.

This paper is also related to the recent literature that studies the real effects of profit shifting. Suárez Serrato (2019) finds that elimination of profit shifting to tax havens lowers investment, employment, and wages of affected US multinationals with negative spillover effects to other firms. Alvarez-Martinez et al. (2018) find profit shifting to have positive effects on investment and GDP in the EU, US, and Japan (these effects are, however, insufficient to compensate for the loss in tax revenues). Buettner et al. (2008, 2018) and De Mooij and Liu (2018) find negative effects of internal debt restrictions on investment by MNEs' subsidiaries, while De Mooij and Liu (2020) find similar effects for restrictions on transfer price manipulation. Klemm and Liu (2019) show that profit shifting may stimulate investment in both high- and low-tax countries. The present paper contributes to this literature by linking the real effects of profit shifting to welfare in the short and long term.

Finally, this paper is related to the literature on dynamic tax competition. Wildasin (2003) is the first to show that the government of a dynamic small open economy chooses a positive time-invariant tax on capital, while Wildasin (2011) extends the analysis to two mobile factors of production. Moreover, Köthenbürger and Lockwood (2010) and Becker and Rauscher (2013) analyse the impact of tax competition on economic growth. I contribute to the literature by extending the seminal Wildasin (2003) model to include profit shifting and analyze its welfare implications.

The rest of the paper is structured as follows. Section 2 presents the model. Sections 3 and 4 derive the optimal tax policy and the welfare effects of internal debt, respectively. Section 5 presents the extensions, and Section 6 concludes.

2 The Model

Consider a small open high-tax country. Akin to the model of Hong and Smart (2010), there are two types of infinitely-lived agents in the economy: workers and a representative entrepreneur. The economy produces a single homogeneous good in two sectors: a domestic firm owned by the entrepreneur and a foreign-owned subsidiary of a multinational firm. Workers supply one unit of labor, which is fully mobile between the national and multinational sectors.

The domestic sector produces the homogeneous good using the technology $G(L^d)$,

[2] Furthermore, Desai et al. (2006a) find empirical evidence that high growth firms are more likely to operate in tax havens. Desai et al. (2006b) explain this result theoretically in a model, where tax haven use raises the return on investment.

where L^d denotes the labor input, and labor has positive, but diminishing marginal product. Denote the time-invariant statutory tax rate as τ, the period t labor employed by the national sector as L_t^d and the period t wage rate as w_t.[3] Then, the after-tax profit of the entrepreneurial firm in period t is

$$\pi_t^D \;=\; (1-\tau)(G(L_t^d) - w_t L_t^d). \tag{1}$$

In each period, the entrepreneur maximizes the after-tax profit (1) over the labor input L_t^d, which results in the labor demand equation

$$G_L(L_t^d) = w_t, \tag{2}$$

where the subscript denotes a partial derivative.

I follow Turnovsky and Bianconi (1992) and Wildasin (2003) to model a dynamic version of the MNE's subsidiary considered by Hong and Smart (2010). The multinational firm uses the constant returns to scale technology $F(K, L^m)$, where K is the capital stock, L^m the labor input, and $F(\cdot)$ has positive but diminishing marginal products. The firm has an initial capital stock $K(0) = K_0$. Capital is fully equity financed, either through new equity issues or retained earnings.

The MNE operates a financial center in a tax haven country with a zero corporate tax rate. It can lower the tax liability of its productive subsidiary by channeling a part of the equity financing through the financial center, which in turn provides internal debt to the subsidiary at an exogenous world interest rate r. Following Hong and Smart (2010) and Haufler and Runkel (2012), there are no deadweight costs of using internal debt.[4] Without loss of generality, internal debt is constrained by the government to not exceed an exogenous proportion $b \in [0, 1[$ of the capital stock.[5] In the absence of deadweight costs, the firm would like to use as much internal debt as possible owing to its tax advantage. Hence, the amount of internal debt in period t is bK_t and the interest costs amount to rbK_t. These interest costs also equal the net profit generated by the financial center. Define the net profit of the MNE's subsidiary in period t as its gross profit, $F(K_t, L_t^m) - w_t L_t^m$, minus the interest costs and tax payments. Then, the sum of the subsidiary's and financial center's net profits in period t is equal to

$$\widetilde{\pi}_t^M \;=\; F(K_t, L_t^m) - w_t L_t^m - rbK_t - \tau[F(K_t, L_t^m) - w_t L_t^m - rbK_t]$$

[3] I relax the assumption of a time-invariant tax in Section 5.

[4] Two extensions in Section 5 relax the assumptions of a constant interest rate and no deadweight costs of internal debt, respectively.

[5] There are two types of TCRs: safe harbor rules and earnings stripping rules. A safe harbor rule limits the debt to capital ratio, while an earnings stripping rule allows the deductibility of interest expenses up to a certain proportion of the company's EBITDA. The restriction b on internal debt represents a safe harbor rule, following the modelling choice of Hong and Smart (2010), Haufler and Runkel (2012), Haufler et al. (2018).

$$+rbK_t. \tag{3}$$

The profit $\widetilde{\pi}_t^M$ can either be used to pay dividends D_t or held as retained earnings RE_t to finance new investment.

The MNE augments the capital stock in period t, K_t, at the rate I_t such that the amount of investment is I_tK_t. Assuming, without loss of generality, that capital does not depreciate, the capital stock evolves according to[6]

$$\dot{K}_t \;=\; I_tK_t. \tag{4}$$

Moreover, the firm incurs convex adjustment costs $C(I_t)K_t$, where $\text{sgn}\{C'\} = \text{sgn}\{I\}$, $C'' > 0$ and $C(0) = C'(0) = 0$. Convex adjustment (or installation) costs were initially formalized by Hayashi (1982) and represent the internal costs caused by disruption within the firm due to (dis)investment (House and Shapiro, 2008).[7] Therefore, the total costs of capital adjustment in period t are $(I_t + C(I_t))K_t$. The firm finances these costs through retained earnings RE_t and new equity issues $q_t\dot{E}_t$, where q_t is the price of equity and E_t denotes the stock of existing equity in period t.

Subtraction of the capital adjustment costs from the net profit $\widetilde{\pi}_t^M$ gives the net cash-flow generated by the MNE's subsidiary in period t, π_t^M:

$$\pi_t^M \;=\; F(K_t, L_t^m) - w_tL_t^m - (I_t + C(I_t))K_t - \tau[F(K_t, L_t^m) - w_tL_t^m - rbK_t]. \tag{5}$$

Denote the value of equity in period t as $V_t = q_tE_t$. The objective of the firm is to choose the optimal paths of I_t and L_t^m to maximize V_0, which is equivalent to maximizing (see Appendix A for a derivation)

$$V_0 \;=\; \int_0^\infty \pi_t^M e^{-rt}dt. \tag{6}$$

Thus, the value of the subsidiary in period 0 is the present value of its future net cash-flow, discounted at the interest rate r. Equations (5) and (6) are generalized versions of the equations for net cash-flow and firm value in the model of Wildasin (Wildasin, 2003) in the presence of internal debt.

In Appendix B, I show that the MNE's optimal paths of I_t and L_t^m satisfy the

[6]I assume zero depreciation to remain as close as possible to the static model of Hong and Smart (2010) and make my results comparable to theirs. The inclusion of these costs does not affect the results. Note, furthermore, that a dot indicates a time derivative.

[7]The convex costs are standard in models of small open economies facing perfectly elastic capital supply and are necessary for the existence of non-degenerate dynamics (Turnovsky, 1997).

following equations:

$$\dot{I}_t = \frac{1}{C''}\left[r(1-b\tau) + C(I_t) + C'(I_t)(r - I_t) - F_K(K_t, L_t^m)(1-\tau)\right], \quad (7)$$

$$w_t = F_L(K_t, L_t^m). \quad (8)$$

Equation (8) equates the marginal product of labor to its marginal cost, while (7) determines the optimal change in investment over time, $\dot{I}$. The right-hand side of (7) gives the difference between the costs of new investment and its marginal product. Note that in steady state capital is constant, $\dot{K} = 0$, and, according to (4), $I = 0$. Hence, if a shock increases the marginal costs of investment, the right-hand side of (7) becomes positive. In this situation, the firm disinvests ($I_t < 0$) and to reach steady state, investment must increase to zero, i.e., $\dot{I}_t > 0$.

Finally, the labor market must clear in each period t. Thus, we require

$$L_t^d + L_t^m = 1. \quad (9)$$

The model is in steady state when $\dot{I} = \dot{K} = 0$. Denote steady state variables with a tilde. The steady state is characterized by

$$F_K(\widetilde{K}, \widetilde{L}^m) = \frac{r(1-b\tau)}{1-\tau}, \quad (10a)$$

$$F_L(\widetilde{K}, \widetilde{L}^m) = G_L(\widetilde{L}^d) = \widetilde{w}, \quad (10b)$$

$$\widetilde{L}^m + \widetilde{L}^d = 1, \quad (10c)$$

$$\widetilde{I} = 0. \quad (10d)$$

Equations (10a)-(10c) are identical to the static capital and labor market equilibria of Hong and Smart (2010). In the case of zero adjustment costs, the MNE can change immediately its capital stock and the model is always in steady state. Therefore, the static model is a special case of the model developed here when $C(I) = 0$.

The labor demand equations (2) and (8) together with the labor market clearing condition (9) define the labor inputs L_t^d, L_t^m as well as the wage rate w_t as implicit functions of the capital stock, K_t. Denote these functions as $L_t^m \equiv L_t^m(K_t), L_t^d \equiv L_t^d(K_t), w_t \equiv w_t(K_t)$. Totally differentiating (2), (8) and (9) with respect to L_t^m, L_t^d, w_t, and K_t gives

$$\frac{\partial L_t^m}{\partial K_t} = -\frac{F_{LK}}{F_{LL} + G_{LL}} > 0, \quad \frac{\partial L_t^d}{\partial K_t} = -\frac{\partial L_t^m}{\partial K_t}, \quad \frac{\partial w_t}{\partial K_t} = \frac{G_{LL} F_{LK}}{F_{LL} + G_{LL}} > 0. \quad (11)$$

To interpret (11), note that with a constant returns technology, capital and labor are complements in production, i.e., $F_{LK} > 0$. Thus, an increase in the capital stock makes labor more productive, which raises the demand for labor in the international sector ($\partial L_t^m/\partial K_t > 0$). The wage rate must increase to balance the labor market ($\partial w_t/\partial K_t > 0$), which lowers the demand for labor in the national sector.

Next, I derive the comparative dynamic effects of a change in the tax rate in period 0 on the capital stock in periods t, where $t \geq 0$. Following Wildasin (2003), Appendix C proves the following result:

Lemma 1. *Suppose the government changes the tax rate τ in period 0 and keeps it constant for all future periods. Then, the change in the capital stock in periods $t \geq 0$ is*

$$\frac{\partial K_t}{\partial \tau} = \frac{\partial \widetilde{K}}{\partial \tau}\left(1 - e^{\mu_1 t}\right) < 0, \tag{12}$$

where μ_1 is the speed of convergence to the steady state and is determined by

$$\mu_1 = \frac{r - \sqrt{r^2 - \frac{4(1-\tau)F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})}}}{2} < 0, \tag{13}$$

while $\partial \widetilde{K}/\partial \tau$ is the change in the steady state capital stock, given by

$$\frac{\partial \widetilde{K}}{\partial \tau} = \frac{(F_K - rb)(F_{LL} + G_{LL})}{(1-\tau)F_{KK}G_{LL}} < 0. \tag{14}$$

Proof: See Appendix C. □

According to Equation (12), in the period of the tax change ($t = 0$), the capital stock remains unchanged. The reason is that the capital stock cannot adjust immediately (owing to the adjustment costs). When t becomes large, the exponential term in (12) vanishes and the change in the capital stock approached the negative long term effect, $\partial \widetilde{K}/\partial \tau < 0$. The speed of convergence is $|\mu_1|$. If there are no capital adjustment costs, i.e., $C(\cdot) = 0$, then $\mu_1 \to -\infty$ and adjustment is instantaneous. This is the special case of a static model. The higher the change in the marginal adjustment costs, C'', is, the slower is the rate of adjustment μ_1. Lastly, the comparative dynamic effects on the labor inputs and the wage rate in periods $t \geq 0$ follow from (11) and (12).

3 The Government

Following Hong and Smart (2010), the government's objective is to redistribute income from the entrepreneur to workers. It transfers the tax revenues in a lump-sum way to the workers. The workers do not save and their consumption, X_t^W, equals the total income:

$$X_t^W = w_t + T_t, \tag{15}$$

where $T_t = \tau(G(L_t^d) - w_t L_t^d) + \tau(F(K_t, L_t^m) - w_t L_t^m - rbK_t)$ denotes the tax revenues. The entrepreneur also does not save and its consumption X_t^E is given by

$$X_t^E = \pi_t^D. \tag{16}$$

The government maximizes the welfare function $\Omega_t = X_t^W + \beta X_t^E$ for $\beta \in [0,1]$, where β strictly less than one represents preferences for redistribution of income to workers. Suppose that the government uses the same discount rate r as the multinational firm. Then, it solves

$$\max_{\tau} \quad \int_0^{\infty} \Omega_t e^{-rt} dt, \tag{17}$$

taking into account the impact of taxation on the capital stock $\partial K_t/\partial\tau$, as well as the functions $L_t^m(K_t), L_t^d(K_t)$, and $w_t(K_t)$. I derive the optimal tax rate in Appendix D. Denote this tax rate as τ^*. It is implicitly determined by

$$\begin{aligned} \frac{\mu_1}{r-\mu_1}\tau^*(F_K - rb)\frac{\partial \widetilde{K}}{\partial \tau} &= (1-\beta)\left[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d - \frac{\mu_1}{r-\mu_1}(1-\tau^*)\widetilde{L}^d\frac{\partial \widetilde{w}}{\partial \tau}\right] \\ &\quad + \frac{r}{r-\mu_1}(F_K - rb)\widetilde{K}, \end{aligned} \tag{18}$$

where the term $\partial\widetilde{w}/\partial\tau$ is defined in Equation (D.5) in Appendix D. The left-hand side of (18) gives the marginal costs of an increase in the tax rate, which lowers the MNE's mobile capital stock. The marginal benefits are on the right-hand side of (18). The term in the first row gives the marginal increase in welfare from additional redistribution from the entrepreneur to workers. It is positive for β strictly less than one. The term in the second row of (18) arises due to the dynamic adjustment of the capital stock and is initially derived by Wildasin (2003). The slow adjustment of the capital stock following a tax rate increase creates quasi-rents during the transition period to a new steady state. Since the multinational firm is not owned by the domestic residents, the government has an incentive to tax these rents and distribute them to the workers. This term is greater, the slower the adjustment rate is, i.e., the closer μ_1 is to zero. In the case of an immediate adjustment, $\mu_1 \to -\infty$, there are no quasi-rents, and the term in the second row of (18) vanishes. Moreover, in the latter case, τ^* coincides with the optimal static tax rate from Hong and Smart (2010) (see Appendix E for a proof).

The next section discusses the long and short term effects of a change in profit shifting, as measured by the proportion of internal debt b. It shows that the dynamic economy may behave in a qualitatively different way from the static one.

4 Effects of Internal Debt

I analyze internal debt similarly to Hong and Smart (2010). First, in this section, I examine the effects of a small permanent relaxation of the TCR, $db > 0$, in period 0 on the optimal tax rate τ^*, the long term capital stock $\widetilde{K}$, and welfare. In the next section, I extend the analysis to study the optimal internal debt level.

Consider first how changes in the amount of internal debt affect the optimal tax rate and the long term capital stock. Note that the impact on $\widetilde{K}$ is positive, if the user cost of capital goes down (and vice versa), where the user cost is determined by the right-hand side of Equation (10a). In Appendix F, I derive the following results:

Proposition 1. *Suppose the amount of internal debt b increases by $db > 0$ in period 0. If the economy is static ($\mu_1 \to -\infty$), then*

$$\frac{d\tau^*}{db} > 0, \qquad \frac{d\widetilde{K}}{db} < 0, \quad \textit{if} \quad \tau^* < \frac{1}{2}. \tag{19}$$

Suppose the economy is dynamic with $\mu_1 \in]-\infty, 0[$. Then, there exists a value $\hat{\beta} \in [0, 1[$ such that for $\beta \in [\hat{\beta}, 1]$, the following results emerge:

$$\frac{d\tau^*}{db} > 0, \qquad \frac{d\widetilde{K}}{db} > 0. \tag{20}$$

Proof: See Appendix F. □

The first part of Proposition 2 repeats the results of Hong and Smart (2010). A higher degree of profit shifting raises the optimal statutory tax rate. The effect on the capital stock depends on whether the higher b lowers the user cost by more than the increase in the statutory tax rate raises it. When the economy is static, the latter effect dominates for $\tau^* < 1/2$.

However, as long as the convergence to steady state is not immediate, both effects are, in general, ambiguous. When the redistributive motive of the government is sufficiently weak, i.e., when $\beta \in [\hat{\beta}, 1]$, the overall impact on the user cost of capital is negative. Hence, the capital stock is higher in the new equilibrium. Unfortunately, no unambiguous results can be derived for lower values of β. However, Proposition 1 highlights the qualitative role that the convergence rate plays for the relationship between the amount of internal debt and the capital stock. Only a small deviation from the case of immediate convergence may affect the long term capital stock change qualitatively.

I turn next to the impact of profit shifting on welfare. The focus is on two effects: the short term impact (i.e., the change in Ω_0) and the long term impact (i.e., the change in $\widetilde{\Omega}$). While the relationship between internal debt and welfare is, in general, ambiguous, there are two special cases that lead to unambiguous results. The next proposition summarizes these results.

Proposition 2. *Suppose the economy is initially in a steady state and internal debt b increases by $db > 0$ in period 0.*

(a) If $\mu_1 \to -\infty$, welfare increases, if $\beta < 1$, and remains unchanged, if $\beta = 1$:

$$\frac{d\Omega_0}{db} = \frac{d\widetilde{\Omega}}{db} = (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\frac{\tau^* r}{F_K - br} \geq 0. \tag{21}$$

(b) Suppose the economy is not static, i.e., $\mu_1 \in]-\infty, 0[$. Then, there exists $\underline{\beta} \in [0,1[$ such that for $\beta \in [\underline{\beta}, 1]$, the welfare change is non-monotone with a negative short term and a positive long term change:

$$\frac{d\Omega_0}{db} < 0, \tag{22}$$

$$\frac{d\widetilde{\Omega}}{db} > 0. \tag{23}$$

Proof: See Appendix G. □

Part (a) of Proposition 2 restates the result from the static model (see Hong and Smart, 2010), and coincides with their Proposition 4. However, the impact of b on welfare is, in general, ambiguous. When the economy cannot immediately reach the steady state, the capital stock adjusts only gradually. Moreover, the speed of convergence of the economy affects the response of the tax rate to a change in internal debt, which additionally impacts the transition of the capital stock.

Part (b) of Proposition 2 states that, for a sufficiently weak redistributive motive, there is an unambiguous negative short term effect of more internal debt on welfare, while the long term impact is positive. Hence, welfare responds non-monotonically to an increase in profit shifting. The intuition behind this result is the following. In the short term, the capital stock is fixed, and the only welfare effects come from the direct negative impact of b on the tax revenues and the change in the statutory tax rate. Even though the government finds it optimal to increase its tax rate immediately, this policy cannot compensate for the direct loss of tax revenues in period 0 for a sufficiently high β, and welfare declines. During the transition period, the capital stock increases (see Proposition 1), which raises welfare. In the long term, the positive impact of more investment overcompensates the initial negative welfare change. Hence, in the long term, welfare improves.

Note that the case of a weak redistributive motive is not unrealistic (for the choice of the optimal corporate income tax rate). When $\beta = 1$, welfare Ω_t equals the jurisdiction's national income: $\Omega_t = w_t + T_t + \pi_t^D$. Thus, in this situation, the government maximizes national income. This assumption is common in the literature on corporate taxation (see, e.g., Bond and Samuelson, 1989; Janeba, 1995; Wildasin, 2003). Furthermore, policymakers often motivate corporate tax reforms on inter alia efficiency grounds, instead of redistribution motives. For example, the European Commission's proposal for the implementation of a common consolidated corporate tax base (CCCTB) in the European Union motivates it partially by stating that "It is equally important to also stimulate growth and economic development in the internal market by facilitating cross-border trade and corporate investment" (European Commission, 2016, p. 12).

Even though both cases (a) and (b) of Proposition 2 predict a positive long term welfare change, the intuition behind these results is different. On the one hand, in case (a), the capital stock declines. Nevertheless, welfare increases due to the increase in the statutory tax rate. On the other hand, in case (b), the capital stock increases in the long run and compensates for the loss of tax revenues.

Proposition 2 hints at an ambiguous overall welfare impact of a change in profit shifting. However, the next result shows that the overall welfare impact is nonnegative.

Proposition 3. *Suppose the economy is initially in steady state and internal debt b increases by $db > 0$ in period 0. The overall impact on welfare is unambiguously nonnegative:*

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt \quad = \quad \frac{\tau^*(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)}{F_K - rb} \geq 0. \tag{24}$$

Proof: See Appendix H. □

The intuition behind Proposition 3 is the following. Over the whole planning horizon, the marginal change in the statutory tax rate does not affect welfare (as welfare is maximized over τ). Moreover, at the optimal tax rate, the direct positive effect of internal debt on the capital demand overcompensates the loss in tax revenues due to profit shifting (when there are strict preferences for redistribution). This result is qualitatively identical to the static result of Hong and Smart (2010).

However, Proposition 3 should be interpreted with caution for several reasons. First, it is the only nonrobust result, and crucially depends on several assumptions. It breaks down when one considers either (i) a time-varying tax rate, (ii) deadweight costs of internal debt, or (iii) transfer pricing as a second profit shifting channel. The next section analyzes all three cases as extensions. In the first case, the overall welfare impact depends on whether the tax rate is an increasing or decreasing function of the capital stock. In the second case, overall welfare increases (decreases) in b for small (high) internal debt levels. In the third case, the overall welfare effect is unambiguously *negative* for sufficiently high levels of β and ambiguous, otherwise. Since transfer pricing is the second main profit shifting channel, Proposition 3 is unlikely to hold.

Second, even if Proposition 3 holds, it hides the intertemporal distribution problem that internal debt may cause through a non-monotone welfare change. The severity of this problem depends on how sluggish the adjustment of capital is. The existing empirical evidence points to relatively long-lived adjustment periods. As already discussed in the Introduction, the analyses of three different TCR reforms in Germany and Finland do not find real effects for periods of up to two years (see Weichenrieder and Windischbauer, 2008; Buslei and Simmler, 2012; Harju et al., 2017). However, Buettner et al. (2008, 2018), and De Mooij and Liu (2018) find significant adverse long term effects of TCRs on the investment of MNEs' subsidiaries. Moreover, Suárez Serrato (2019) finds that after the access to a tax haven is prohibited to US multinationals, employment of the exposed firms gradually declines for a period of at least ten years until it settles at a new level (see Figure 8 of Suárez Serrato, 2019).[8] Thus, the short term effects may last at least two years, and full adjustment may take at least a decade. However, additional research is necessary to quantify the adjustment.

5 Extensions

This section presents three extensions of the model. First, I consider a time-varying tax rate and show that all results, except for Proposition 3, remain qualitatively unchanged. Second, I introduce deadweight costs associated with internal debt and derive the optimal internal debt level. Third, I analyze internal debt when the MNE uses transfer pricing and allow the two profit shifting channels to be substitutes. In this case, the negative welfare effects may become more persistent and carry over to the long term.

5.1 Time-varying tax rate

In the previous section, I derive the welfare implications of profit shifting under the assumption of a time-invariant tax rate. In this section, I analyze whether this constraint on the government's policy has a qualitative impact on the previous results. To simplify the analysis, and without loss of generality, I assume here that the adjustment cost function is quadratic, that is, $C(I_t) = 0.5cI_t^2$, where $c > 0$.

Assume the tax rate is time-dependent. Thus, the period t statutory tax is denoted by τ_t. The profit-maximizing first-order conditions of the national and multinational firms are again given by Equations (2), (7), and (8), with the only difference that now the tax rate τ is replaced by τ_t in (7).

To solve for the government's optimal tax policy in this case, one needs to determine how investment I_t depends on the statutory tax rate τ_t and the capital stock K_t. Define

[8] Suárez Serrato (2019) also finds that investment immediately declines following the reform. This is consistent with the theoretical model presented here, as only the capital stock is fixed in period zero. However, investment changes immediately following an increase in internal debt, i.e., $\partial I_0/\partial b \propto \partial \dot{K}_0/\partial b > 0$.

period t investment as the function $I_t \equiv I_t(K_t, \tau_t)$. In Online Appendix OA, I derive the first and second partial derivatives of the function $I_t(\cdot)$.

The govenment maximizes the same objective function as in Section 3. Thus, it solves

$$\max_{\tau_t} \int_0^{\infty} \Omega_t e^{-rt} dt \quad \text{s. t.} \quad \dot{K}_t = I_t K_t, \tag{25}$$

taking into account $I_t = I_t(K_t, \tau_t), L_t^m = L_t^m(K_t), L_t^d = L_t^d(K_t)$, and $w_t = w_t(K_t)$. The optimal policy is summarized in the following proposition:

Proposition 4. *The optimal tax rate τ_t is a function of the capital stock K_t, given by*

$$\tau_t \;=\; \widetilde{\tau} + \alpha \left(K_0 - \widetilde{K} \right) e^{\mu_1 t}, \tag{26}$$

where $\alpha \gtrless 0$ is defined in Equation (OB.17) in Online Appendix OB, while $\widetilde{\tau}$ is the optimal tax rate in steady state and is equal to τ^* from Equation (18).

Proof: See Online Appendix OB. □

According to Proposition 4, the optimal steady state tax is the same as the optimal tax in the model with a constant tax rate. The intuition is that once the economy is in a steady state, the government finds it optimal to levy a constant tax rate, which then coincides with τ^*. However, during the transition, the optimal tax may either be increasing or decreasing in the capital stock K_t (the exact relationship depends on the properties of the production and the adjustment cost functions).

Therefore, the government's response to an increase in profit shifting in the steady state is identical to its reaction in the model with a time-invariant tax rate. Hence, Proposition 1 can be derived analogously by replacing τ^* with $\widetilde{\tau}$. Moreover, the fact that policy responds identically in the long term means that the steady state welfare impact of a change in internal debt is the same as under a constant tax rate (i.e., Equations (21) and (23) from Proposition 2 continue to hold). The short term welfare effect may, however, differ. If $d\widetilde{K}/db > 0$, then the tax rate in the period of the shock, τ_0, increases by less than $\widetilde{\tau}$ if $\alpha > 0$ and by more if $\alpha < 0$ (see Equation (26)). The reverse is true in the case $d\widetilde{K}/db < 0$. In the case of weak redistribution preferences (β sufficiently close to one), the long term capital stock is increasing in internal debt (according to Proposition 1) and, thus, the initial change in the tax rate is less than the long term change if $\alpha > 0$ (and vice versa if $\alpha < 0$). Irrespective of whether the initial tax rate change over- or undershoots the long term change, Online Appendix OC shows that for sufficiently high β, the short term welfare impact is negative under the additional assumption that the steady state is stable. I summarize these and other results in the next proposition, while the formal proof is relegated to Online Appendix OC.

Proposition 5. *In the model with a time-varying tax rate, Proposition 1 holds when one replaces τ^* with $\widetilde{\tau}$. If the steady state is stable, Proposition 2 remains qualitatively unchanged. The overall welfare impact of an increase in internal debt (derived in Proposition 3 in the case of a time-invariant tax rate) can be either positive or negative.*

Proof: See Online Appendix OC. □

Hence, Propositions 1 and 2 are robust to an extension with a time-varying tax rate, while Proposition 3 is not. To understand why it no longer holds, consider the special case $\beta = 1$. In this case, Proposition 3 states that, with a time-invariant tax rate, a change in internal debt does not influence overall welfare. However, with a time-varying tax rate, Equation (OC.14) in Online Appendix OC shows a positive welfare impact if α is negative, and vice versa. The reason is that, in this case, capital accumulates, and $\alpha < 0$ means that the tax rate increases in the short term by more than in the long term. This leads to higher taxation of the quasi-rents during the transition and thus higher welfare. The opposite is true for $\alpha > 0$. Because α has an undetermined sign, the overall welfare impact is undetermined.

5.2 Optimal internal debt

Section 4 characterized the welfare costs and benefits of marginal changes in the internal debt restriction, and showed that the discounted sum of welfare is nondecreasing in internal debt. This result is implausible and depends not only on the assumption of a time-invariant tax rate but also on the assumption of zero deadweight costs of debt financing. Internal debt might, similarly to external debt, be associated with both agency and bankruptcy costs (Hong and Smart, 2010; Gresik et al., 2017). In this subsection, I include such costs and derive the optimal internal debt level.

Suppose again that the statutory tax rate is time-invariant. Following Hong and Smart (2010), the MNE faces deadweight costs associated with a debt-to-capital ratio b specified as $C^B(rb)K$, where $C^{B\prime} > 0, C^{B\prime\prime} > 0$ for $b > 0$ and $C^B(0) = C^{B\prime}(0) = 0$. These costs are, without loss of generality, not tax deductible, such that the net cash flow π_t^M, specified by Equation (5) in Section 2, is given by

$$\pi_t^M \;=\; (1-\tau)[F(K_t, L_t^m) - w_t L_t^m] - (I_t + C(I_t))K_t + \tau r b K_t - C^B(rb)K_t. \quad (27)$$

In the absence of thin-capitalization rules, the MNE would maximize its firm value over b, I_t, and L_t^m. The firm's choice of internal debt $\hat{b}$ is given by $C^{B\prime}(r\hat{b}) = \tau$; that is, where the marginal costs equal the marginal tax benefits. Hence, any TCR above $\hat{b}$ would not be binding. However, Online Appendix OD shows that the government's optimal choice of b lies strictly below $\hat{b}$ and is, thus, binding. Moreover, the optimal internal debt level, denoted by b^*, equates strictly negative short term welfare effects of internal debt to longer term marginal benefits. These results are summarized in the following proposition:

Proposition 6. *Suppose the government maximizes welfare (17) over τ and b. The optimal internal debt level b^* satisfies $0 < b^* < \hat{b}$ for $\beta < 1$ and $b^* = 0$ for $\beta = 1$.*

When the government chooses b^* in time period zero, it faces a strictly positive initial time period $[0, t^*]$ of negative welfare effects

$$\frac{\partial}{\partial b} \int_0^{t^*} \Omega_t e^{-rt} dt < 0. \tag{28}$$

Proof: See Online Appendix OD. □

The first part of Proposition 6 repeats the result of Hong and Smart (2010). The second part, however, shows that the marginal costs of an increase in internal debt occur in the short term, while its marginal benefits emerge later. This result is, furthermore, independent of β. Therefore, a government that sets a positive thin-capitalization rule faces negative short term and positive long term welfare changes irrespective of whether it has strong or weak preferences for redistribution.

5.3 Transfer price manipulation

The second main profit shifting channel is transfer price manipulation. There is recent empirical evidence that internal debt and transfer price manipulation are substitutes (Saunders-Scott, 2015; Nicolay et al., 2017; De Mooij and Liu, 2020). Saunders-Scott (2015) finds that an implementation of a TCR lowers the reported earnings before interest and taxes (EBIT) by 5.1%. Nicolay et al. (2017) find that strict TCRs increase the (absolute value) of the tax rate sensitivity of EBIT by 0.36 percentage points. De Mooij and Liu (2020) show that transfer pricing regulations are effective in countries without TCRs and not significantly effective in countries with some TCR. These observations can be explained, if the MNEs' costs of tax avoidance depend on the total amount of shifted profits (Saunders-Scott, 2015; Nicolay et al., 2017). In this case, a stricter TCR lowers internal debt, which decreases the marginal cost of transfer price manipulation and, thus, incentivizes the use of the second profit shifting channel. This section extends the analysis to consider such effects.

Suppose the MNE distorts the interest rate (transfer price) of internal debt and charges its productive subsidiary an interest rate $\sigma \geq r$.[9] The case $\sigma > r$ denotes transfer price manipulation. It is associated with concealment costs that represent the possibility that the government may detect the transfer price distortion and impose a

[9] Allowing the MNE to distort the transfer price of internal debt is the easiest way to analyze the interdependence of internal debt and transfer price manipulation (Gresik et al., 2015).

fine on the MNE. Define the concealment cost function as $C^{\sigma}((\sigma - r)b)K$, where $C^{\sigma}(\cdot)$ is quadratic with $C^{\sigma\prime} > 0, C^{\sigma\prime\prime} > 0$. Thus, similarly to the cost function $C^{B}(\cdot)$ from Section 5.2, $C^{\sigma}(\cdot)$ represents the cost of interest rate manipulation per unit of capital.[10] Assuming that the concealment costs are not tax deductible, the MNE's net cash flow in period t is

$$\pi_t^M = (1-\tau)[F(K_t, L_t^m) - w_t L_t^m] - (I_t + C(I_t))K_t + \tau\sigma b K_t - C^{\sigma}((\sigma - r)b)K_t \quad (29)$$

In Online Appendix OE, I derive the solution to the MNE's value maximization problem. The optimal interest rate (denoted by $\hat{\sigma}$) and steady state capital are given by

$$\tau = C^{\sigma\prime}((\hat{\sigma} - r)b), \quad (30)$$

$$F_K(\widetilde{K}, \widetilde{L}^m) = \frac{r - \hat{\sigma}\tau b + C^{\sigma}((\hat{\sigma} - r)b)}{1-\tau}. \quad (31)$$

Note that at $\sigma = \hat{\sigma}$, the net cash flow π_t^M is monotonically increasing in b. Thus, the MNE sets its internal debt at the maximal amount possible, as in Section 2. The effects of changes in the policy parameters τ, b on the optimal tranfser price and the steady state capital stock are

$$\frac{\partial \hat{\sigma}}{\partial \tau} = \frac{1}{bC^{\sigma\prime\prime}} > 0, \quad (32a)$$

$$\frac{\partial \hat{\sigma}}{\partial b} = -\frac{\hat{\sigma} - r}{b} < 0, \quad (32b)$$

$$\frac{\partial \widetilde{K}}{\partial \tau} = \frac{(F_K - \hat{\sigma}b)(F_{LL} + G_{LL})}{(1-\tau)F_{KK}G_{LL}} < 0, \quad \text{if} \quad F_K - \hat{\sigma}b > 0, \quad (32c)$$

$$\frac{\partial \widetilde{K}}{\partial b} = -\frac{r\tau(F_{LL} + G_{LL})}{(1-\tau)F_{KK}G_{LL}} > 0. \quad (32d)$$

Equation (32a) states that a higher tax rate incentivizes transfer price manipulation. Equation (32b) shows the substitutability between debt financing and transfer pricing: a more lenient TCR lowers the incentives to shift profits through tranfser pricing. The reason is that it raises the marginal cost of transfer price manipulation. Equation (32c) states that the capital stock is declining in the tax rate if transfer pricing is not too aggressive, i.e., if $F_K - \hat{\sigma}b > 0$. Since there is a consensus in the empirical literature that the statutory tax rate affects negatively MNEs' investment, I assume this condition to

[10] Gresik et al. (2015) define the cost function as $C^{\sigma}(\sigma - r)bK$. Therefore, the optimal transfer price, in their model, is characterized by $\tau = C^{\sigma\prime}(\sigma - r)$ and is independent of b. Since this section focuses on substitutability between b and σ, I consider a concealment cost function that is strictly convex in b. Using instead the function of Gresik et al. (2015) would thus simplify the analysis. However, it would not affect qualitatively the results.

be fulfilled.[11] Lastly, a higher level of internal debt lowers the user cost of capital and increases the steady state capital stock, as shown by (32d).

The solution for the optimal tax rate τ^* and the analysis of the welfare effects of a change in the TCR, db, follow the same steps as in Sections 3 and 4. The following results emerge:

Proposition 7. *Suppose the amount of internal debt increases by $db > 0$ in period 0. Then, there exists a value $\hat{\beta} \in [0, 1[$, such that for $\beta \in [\hat{\beta}, 1]$:*

$$\frac{d\tau^*}{db} \gtrless 0, \tag{33}$$

$$\frac{d\widetilde{K}}{db} > 0, \quad \text{if} \quad \tau^* < \frac{1}{2}, \tag{34}$$

$$\frac{d\Omega_0}{db} < 0, \quad \text{if} \quad \tau^* < \frac{1}{2}, \tag{35}$$

$$\frac{d\widetilde{\Omega}}{db} \gtrless 0. \tag{36}$$

The overall welfare impact is undetermined for small values of β and strictly negative for sufficiently high β:

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt} dt = \frac{\tau^*\left[(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\,\prime\prime}}\right]}{F_K - \sigma b} < 0, \ \text{if } \beta > \underline{\beta}, \tag{37}$$

where $\underline{\beta} < 1$.

Proof: See Online Appendix OE. □

According to Proposition 7, it is sufficient to assume weak redistributive preferences and $\tau^* < 0.5$ for the initial welfare impact to be negative (according to (35)). However, the long term welfare effect is ambiguous. The reason is that the existence of a transfer pricing distortion lowers both the optimal tax rate and the tax rate's response to an increase in debt financing. Moreover, internal debt and transfer pricing are only imperfect substitutes, such that an increase in b lowers tax revenues even after accounting for the change in the transfer price. Thus, the negative short term impact persists while the long term welfare effect becomes ambiguous. According to (37), it is optimal to prohibit internal debt, if the redistributive motive is sufficiently weak ($\beta > \underline{\beta}$). Hence, in the presence of transfer price manipulation, the negative welfare implications of debt financing may extend beyond the short term, despite the substitutability between the profit shifting channels.

[11] The meta-study of Feld and Heckemeyer (2011) estimate a semi-elasticity of FDI with respect to the statutory tax rate equal to -2.49.

Note that Proposition 7 generalizes the results of Gresik et al. (2015), who study, in a static model, the welfare implications of internal debt when the MNE engages in transfer pricing. They derive a negative welfare effect of internal debt when the concealment costs of transfer pricing are sufficiently low. Similarly, Equation (37) in this paper is negative if $C^{\sigma\prime\prime}$ is small enough. In addition, in the dynamic model, a negative welfare effect emerges for any concealment cost level, if there is no redistributive motive, i.e., if $\beta = 1$.

6 Conclusions

This paper addresses the real effects as well as the welfare implications of profit shifting through internal debt. I develop a dynamic model to take explicitly into account that capital is less mobile in the short run compared to the long run. The transitional dynamics may affect both the real effect of internal debt on capital accumulation and the short term welfare impact of debt financing qualitatively. If the redistributive motive of the government in a high-tax country is not too strong, non-monotone welfare effects emerge, with negative short term and positive long term effects. Furthermore, the negative implications of internal debt may carry over to the long term in the presence of transfer price manipulation.

Hence, the results of this paper are of importance to the theoretical work that tries to explain the interrelation between internal debt, capital accumulation, and welfare. I show that the opposite results of the static models of Hong and Smart (2010) and Haufler and Runkel (2012) hold in the long and short term, respectively, of a dynamic small open economy. Thus, this paper reinterprets these results as complementary and not as contrary.

Additionally, my results highlight the importance of the timing of the empirical evaluation of TCR reforms. A reform that affects the deductibility of interest expenses may be evaluated as having either no or negative real effect depending on how much time has passed between the reform and the time of analysis. This conjecture is supported by the empirical literature. While Weichenrieder and Windischbauer (2008), Buslei and Simmler (2012) and Harju et al. (2017) do not find real effects of three different TCR reforms up to two years after these reforms took place, Buettner et al. (2008, 2018) and De Mooij and Liu (2018) find such effects over a longer time horizon.

One limitation of the model is that it considers a single economy setting. One may argue that in the presence of other high-tax countries, the MNEs' global investment might remain unaffected by thin-capitalization rules imposed by a single country. This conjecture is, however, not supported by the existing empirical evidence. Suárez Serrato (2019) finds that elimination of the access of US multinationals to one tax haven lowers their global investment despite some shifting of investment away from the US. Thus, the presence of other high-tax countries may mitigate but not overturn the theoretical results. A dynamic analysis of a multi-country setting, probably similar to the static model of Haufler and Runkel (2012), is thus an important agenda for future research.

Furthermore, I use the framework of Hong and Smart (2010). Future research should conduct a dynamic analysis in other frameworks like, e.g., in a representative agent model. In addition, I do not explicitly model the savings behavior of agents. Including it and endogenizing the interest rate in the model may have important implications. Moreover, this paper views domestic and multinational firms as price-takers. Assuming that the MNE has some market power may change the welfare implications of profit shifting.

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A Derivation of the MNE's maximization problem

The MNE issues new equity E_t in period t with a price q_t on the world capital market. The value of this equity is, thus, $q_t E_t$. Investors purchasing this equity earn next period dividends D_t and capital gains $\dot{q}_t E_t$. They can, however, also invest in other assets in the world market and earn the interest rate r. They are indifferent between investing in the MNE and earning r if

$$\frac{D_t + \dot{q}_t E_t}{q_t E_t} = r. \tag{A.1}$$

Differentiate the value of equity $V_t = q_t E_t$ with respect to time:

$$\dot{V}_t = \dot{q}_t E_t + q_t \dot{E}_t. \tag{A.2}$$

Our objective is to solve (A.2) for the value of equity, V_t. Note that the net profit $\widetilde{\pi}_t^M$ can either be used to pay dividends D_t or be held as retained earnings RE_t, such that

$$\widetilde{\pi}_t^M = D_t + RE_t. \tag{A.3}$$

Furthermore, new investment can be financed either through retained earnings RE_t or new equity issues $q_t \dot{E}_t$. Thus, we have

$$(I_t + C(I_t))K_t = RE_t + q_t \dot{E}_t. \tag{A.4}$$

Use Equations (A.3),(A.4), (3) and (5) to solve for $q_t \dot{E}_t$:

$$q_t \dot{E}_t = (I_t + C(I_t))K_t - RE_t = -\pi_t^M + D_t. \tag{A.5}$$

Inserting (A.5) in (A.2), we get

$$\dot{V}_t = \dot{q}_t E_t - \pi_t^M + D_t. \tag{A.6}$$

Next, we solve (A.1) for $\dot{q}_t E_t$ and insert the resulting expression in (A.6) to get

$$\begin{aligned} \dot{V}_t &= r q_t E_t - \pi_t^M \\ &= r V_t - \pi_t^M. \end{aligned} \tag{A.7}$$

The solution of the differential equation (A.7) is Equation (6).

B Solution of the MNE's optimization problem

The MNE maximizes Equation (6) over I_t and L_t^m subject to the equation of motion (4) and the initial condition $K(0) = K_0$. I use dynamic programming to find the optimum. Define the value function of the maximization problem as $W(K_t)$. The Bellman equation is

$$rW(K_t) = \max_{I_t, L_t^m} \left\{ \pi_t^M + W_K(K_t) I_t K_t \right\}, \tag{B.1}$$

where $W_K(K_t)$ is the derivative of the value function with respect to capital. The first-order conditions are

$$\frac{\partial}{\partial L_t^m} = (1-\tau)(F_L(K_t, L_t) - w_t) = 0, \tag{B.2}$$

$$\frac{\partial}{\partial I_t} = -(1 + C'(I_t))K_t + W_K(K_t)K_t = 0. \tag{B.3}$$

Equation (B.3) gives $W_K(K_t) = 1 + C'(I_t)$. Differentiation of this equation with respect to time results in the expression $W_{KK}(K_t)\dot{K}_t = C''\dot{I}_t$. Moreover, using the Envelope theorem, we can differentiate the maximized Bellman equation with respect to capital to get

$$\begin{aligned} rW_K(K_t) &= \pi_K^M + W_K(K_t)I_t + W_{KK}(K_t)I_tK_t \\ &= F_K - (I_t + C(I_t)) - \tau(F_K - rb) + W_K(K_t)I_t + W_{KK}(K_t)I_tK_t. \end{aligned} \tag{B.4}$$

Solving (B.4) for $W_{KK}(K_t)$ and inserting the resulting expression in $W_{KK}(K_t)\dot{K}_t = C''\dot{I}_t$, we get

$$C''\dot{I}_t = \frac{\dot{K}_t}{I_tK_t}\left[W_K(K_t)(r - I_t) - (1-\tau)F_K + (I_t + C(I_t)) - \tau rb\right]. \tag{B.5}$$

Using Equations (4) and (B.3) to substitute for $\dot{K}_t$ and $W_K(K_t)$ in (B.5) and simplifying, we get Equation (7). Equation (8) follows directly from (B.2).

C Proof of Lemma 1

To perform the comparative dynamic analysis, I follow Wildasin (2003). Suppose that at time 0, the government permanently increases the tax rate by $d\tau > 0$.

First, derive the impact on the steady state capital stock $\widetilde{K}$, given by $\partial\widetilde{K}/\partial\tau$ (Equation (14)). Differentiate totally Equation (10a) with respect to $\widetilde{K}$ and τ, taking into account that $\widetilde{L}^m = L^m(\widetilde{K})$ according to Equation (11). The resulting expression is

$$\left[F_{KK} + F_{KL}\frac{\partial L^m}{\partial \widetilde{K}}\right]\frac{\partial \widetilde{K}}{\partial \tau} = \frac{-r(1-\tau) - r(1-b\tau)(-1)}{(1-\tau)^2},$$

$$\Leftrightarrow \frac{\partial \widetilde{K}}{\partial \tau} = \frac{r(1-b)(F_{LL}+G_{LL})}{(1-\tau)^2 F_{KK}G_{LL}} = \frac{(F_K - rb)(F_{LL}+G_{LL})}{(1-\tau)F_{KK}G_{LL}}. \tag{C.1}$$

Equation (C.1) coincides with (14) in Lemma 1.

To derive $\partial K_t/\partial\tau$, differentiate Equations (7) and (4) with respect to $\tau, K_t, \dot{K}_t, I_t$ and $\dot{I}_t$:

$$C''\frac{\partial \dot{I}_t}{\partial \tau} = -(1-\tau)\left(F_{KK} + F_{KL}\frac{\partial L_t^m}{\partial K_t}\right)\frac{\partial K_t}{\partial \tau} + \left[C''(r - I_t) - C'''\dot{I}_t\right]\frac{\partial I_t}{\partial \tau} + (F_K - rb), \tag{C.2}$$

$$\partial \dot{K}_t = I_t \partial K_t + K_t \partial I_t. \tag{C.3}$$

Suppose that the economy is near steady state with $K_t \approx \widetilde{K}, I_t \approx \widetilde{I} = 0, \dot{I}_t \approx 0$. Then, Equation (C.3) becomes

$$\partial I_t = \frac{\partial \dot{K}_t}{\widetilde{K}}. \tag{C.4}$$

Moreover, we can differentiate Equation (C.4) with respect to time, which gives

$$\partial \dot{I}_t = \frac{\partial \ddot{K}_t}{\widetilde{K}}. \tag{C.5}$$

One can now use Equations (C.4), (C.5) and (11) to simplify (C.2):

$$\frac{\partial \ddot{K}_t}{\partial \tau} - r\frac{\partial \dot{K}_t}{\partial \tau} + \frac{(1-\tau)F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})}\frac{\partial K_t}{\partial \tau} = \frac{\widetilde{K}(F_K - rb)}{C''}. \tag{C.6}$$

Equation (C.6) is a second-order heterogeneous differential equation in $\partial K_t/\partial\tau$.

The particular solution to (C.6) is found by setting $\partial \ddot{K}_t = \partial \dot{K}_t = 0$. Thus, the particular solution is

$$\frac{\partial K_t}{\partial \tau} = \frac{(F_K - rb)(F_{LL}+G_{LL})}{(1-\tau)F_{KK}G_{LL}}. \tag{C.7}$$

To find solution to the homogeneous part (i.e., the left-hand side) of (C.6), we suppose that the solution is of the form $\partial K_t/\partial\tau = Ae^{\mu t}$, where A is an undetermined constant. Under the exponential functional form, we have $\partial \dot{K}_t/\partial\tau = \mu\partial K_t/\partial\tau$ and $\partial \ddot{K}_t/\partial\tau =$

$\mu^2 \partial K_t/\partial\tau$. Hence, the homogeneous part of (C.6) can be rewritten as

$$\mu^2 - r\mu + \frac{(1-\tau)F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})} = 0. \tag{C.8}$$

Equation (C.8) has two solutions for μ, given by

$$\mu_1 = \frac{r - \sqrt{r^2 - \frac{4(1-\tau)F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})}}}{2} < 0, \quad \mu_2 = \frac{r + \sqrt{r^2 - \frac{4(1-\tau)F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})}}}{2} > 0. \tag{C.9}$$

Therefore, Equation (C.8) has one positive and one negative root. The solution to the homogeneous part is, thus,

$$\frac{\partial K_t}{\partial \tau} = A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, \tag{C.10}$$

where A_1 and A_2 are undetermined coefficients. The general solution is the sum of the homogeneous and particular solutions:

$$\frac{\partial K_t}{\partial \tau} = \frac{(F_K - rb)(F_{LL}+G_{LL})}{(1-\tau)F_{KK}G_{LL}} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}. \tag{C.11}$$

Invoking the initial condition $\partial K_0/\partial\tau = 0$ and the terminal condition $\lim_{t\to\infty} \partial K_t/\partial\tau = \partial\widetilde{K}/\partial\tau$, one gets $A_1 = -\partial\widetilde{K}/\partial\tau$ and $A_2 = 0$. This completes the proof of Lemma 1. □

D Derivation of the optimal tax rate (Equation (18))

The government's objective function is

$$\int_0^\infty \Omega_t e^{-rt} dt = \int_0^\infty (X_t^W + \beta X_t^E) e^{-rt} dt \tag{D.1}$$

$$= \int_0^\infty (\tau[F(K_t, L_t^m) - rbK_t] + (1-\tau)w_t L_t^m + G(L_t^d) - (1-\tau)(1-\beta)[G(L_t^d) - w_t L_t^d]) e^{-rt} dt.$$

It maximizes (D.1) subject to $L_t^m = L_t^m(K_t), L_t^d = L_t^d(K_t), w_t = w_t(K_t)$ and Equation (12). The first-order condition is

$$\frac{\partial}{\partial\tau} = \int_0^\infty \Big\{ F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]$$

$$+ \left[\tau(F_K - rb) + (\tau F_L + (1-\tau)w_t)\frac{\partial L_t^m}{\partial K_t} + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right.$$

$$\left. + [G_L - (1-\tau)(1-\beta)(G_L - w_t)]\frac{\partial L_t^d}{\partial K_t}\right] \frac{\partial K_t}{\partial \tau}\Bigg\} e^{-rt}dt = 0. \quad \text{(D.2)}$$

Using the labor demand equations $F_L = w$ and $G_L = w$, we can simplify (D.2):

$$\frac{\partial}{\partial \tau} = \int_0^\infty \Bigg\{ F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]$$

$$+ \left[\tau(F_K - rb) + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right] \frac{\partial K_t}{\partial \tau}\Bigg\} e^{-rt}dt = 0. \quad \text{(D.3)}$$

Following Wildasin (2003), I assume that the economy is near its steady state, such that $K_t \approx \widetilde{K}, L_t^m \approx \widetilde{L}^m, L_t^d \approx \widetilde{L}^d, w_t \approx \widetilde{w}$ and $\partial K_t/\partial\tau = \partial\widetilde{K}/\partial\tau(1 - e^{\mu_1 t})$. Thus, (D.3) becomes

$$\int_0^\infty \Bigg\{ F(\widetilde{K}, \widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d] \quad \text{(D.4)}$$

$$+ \left[\tau(F_K - rb) + (1-\tau)[\widetilde{L}^m + (1-\beta)\widetilde{L}^d]\frac{\partial \widetilde{w}}{\partial \widetilde{K}}\right] \frac{\partial \widetilde{K}}{\partial \tau}(1 - e^{\mu_1 t})\Bigg\} e^{-rt}dt = 0,$$

where $\partial\widetilde{w}/\partial\widetilde{K}$ is the value of $\partial w_t/\partial K_t$, when evaluated at the steady state. Use (11) and (C.1) to define $\partial\widetilde{w}/\partial\tau$ as

$$\frac{\partial \widetilde{w}}{\partial \tau} = \frac{\partial \widetilde{w}}{\partial \widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau} = \frac{(F_K - rb)F_{LK}}{(1-\tau)F_{KK}} < 0. \quad \text{(D.5)}$$

Integrate the left-hand side of (D.4) to get

$$0 = F(\widetilde{K}, \widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d]$$

$$-\frac{\mu_1}{r-\mu_1}\left[\tau(F_K - rb)\frac{\partial \widetilde{K}}{\partial \tau} + (1-\tau)[\widetilde{L}^m + (1-\beta)\widetilde{L}^d]\frac{\partial \widetilde{w}}{\partial \tau}\right]. \quad \text{(D.6)}$$

We can now use the constant returns property of the production function $F(\cdot)$, which allows $F(\cdot)$ to be represented as $F(K, L^m) = F_K K + F_L L^m = F_K K + wL^m$. Thus, (D.6) becomes

$$0 = (F_K - rb)\widetilde{K} + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d]$$

$$-\frac{\mu_1}{r-\mu_1}\left[\tau(F_K - rb)\frac{\partial \widetilde{K}}{\partial \tau} + (1-\tau)[\widetilde{L}^m + (1-\beta)\widetilde{L}^d]\frac{\partial \widetilde{w}}{\partial \tau}\right]. \quad \text{(D.7)}$$

Moreover, the partial derivative F_K is homogeneous of degree zero, which means that $0 \cdot F_K = F_{KK}K + F_{KL}L^m$. Therefore, we get

$$(1-\tau)\widetilde{L}^m \frac{\partial \widetilde{w}}{\partial \tau} = \frac{(1-\tau)\widetilde{L}^m (F_K - rb)F_{LK}}{(1-\tau)F_{KK}} = -(F_K - rb)\widetilde{K}. \tag{D.8}$$

Inserting (D.8) in (D.7), denoting the optimal tax as τ^*, and rearranging gives Equation (18).

E The optimal tax τ^* in the absence of adjustment costs

Here, I prove that the optimal tax in the case $C(I_t) = 0$ and, hence, $\mu_1 \to -\infty$, coincides with the one derived in the static model of Hong and Smart (2010). We take the limit of all terms in Equation (18) for $\mu_1 \to -\infty$, and get

$$(1-\beta)\left[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d + (1-\tau^*)\widetilde{L}^d \frac{\partial \widetilde{w}}{\partial \tau}\right] + \tau^*(F_K - rb)\frac{\partial \widetilde{K}}{\partial \tau} = 0. \tag{E.1}$$

To express (E.1) in a form comparable to the optimal tax rate from Hong and Smart (2010), use their notation and define $\pi \equiv G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d$ and $\rho \equiv r(1-b\tau)/(1-\tau)$, such that in steady state $F_K = \rho$. Now, we make the following rearrangement:

$$\tau^*(F_K - rb) = \tau^*(\rho - rb) = \frac{\tau^* r(1-b)}{(1-\tau)} = \rho - r. \tag{E.2}$$

Equations (E.1), (E.2), and the definition of π together give

$$(1-\beta)\left[\pi + (1-\tau^*)\widetilde{L}^d \frac{\partial \widetilde{w}}{\partial \tau}\right] + (\rho - r)\frac{\partial \widetilde{K}}{\partial \tau} = 0. \tag{E.3}$$

Hong and Smart (2010) take ρ as the strategic variable of the government. Therefore, multiply (E.3) by $\partial\tau/\partial\rho$ to get

$$(1-\beta)\left[\pi\frac{\partial \tau}{\partial \rho} + (1-\tau^*)\widetilde{L}^d \frac{\partial \widetilde{w}}{\partial \rho}\right] + (\rho - r)\frac{\partial \widetilde{K}}{\partial \rho} = 0. \tag{E.4}$$

Equation (E.4) is identical to Equation (16) from Hong and Smart (2010) that determines the equilibrium tax rate in the static model. □

F Proof of Proposition 1

The effects of a change in b on τ^* and $\widetilde{K}$ can be derived from Equations (10a) and (D.6), which determine the steady state capital stock and optimal tax rate, respectively. Using (C.1), (D.8) and the labor market clearing condition (9) to simplify (D.6), we can express (10a) and (D.6) as

$$0 = F_K(\widetilde{K}, \widetilde{L}^m)(1-\tau^*) - r(1-b\tau^*), \tag{F.1}$$

$$\begin{aligned} 0 = {} & F(\widetilde{K}, \widetilde{L}^m) - br\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d] \\ & - \frac{\mu_1}{r-\mu_1}(F_K(\widetilde{K}, \widetilde{L}^m) - rb)\left[\tau^* \frac{\partial \widetilde{K}}{\partial \tau} - \frac{\widetilde{K}(1-\beta \widetilde{L}^d)}{\widetilde{L}^m}\right]. \end{aligned} \tag{F.2}$$

The next step is to totally differentiate (F.1) and (F.2) with respect to $\widetilde{K}, \tau^*$, and b. Note, first, that μ_1 depends on τ and $\widetilde{K}$ in the following way:

$$\frac{d\mu_1}{d\tau} = \frac{-F_{KK}G_{LL}\widetilde{K}}{C''(F_{LL}+G_{LL})\sqrt{r^2 - \frac{4(1-\tau)F_{KK}G_{LL}\widetilde{K}}{c(F_{LL}+G_{LL})}}} = -\frac{\mu_1(r-\mu_1)}{(r-2\mu_1)(1-\tau)}, \tag{F.3}$$

$$\frac{d\mu_1}{d\widetilde{K}} = \frac{(1-\tau)F_{KK}G_{LL}(1+\xi)}{C''(F_{LL}+G_{LL})\sqrt{r^2 - \frac{4(1-\tau)F_{KK}G_{LL}\widetilde{K}}{c(F_{LL}+G_{LL})}}} = \frac{\mu_1(r-\mu_1)(1+\xi)}{(r-2\mu_1)\widetilde{K}}, \tag{F.4}$$

where

$$\xi \equiv \widetilde{K}\frac{\left(F_{KKK} + F_{KKL}\frac{\partial \widetilde{L}^m}{\partial \widetilde{K}}\right)G_{LL}(F_{LL}+G_{LL}) - F_{KK}\left[\frac{\partial \widetilde{L}^m}{\partial \widetilde{K}}(G_{LLL}F_{LL} + G_{LL}F_{LLL}) + G_{LL}F_{LLK}\right]}{F_{KK}G_{LL}(F_{LL}+G_{LL})}. \tag{F.5}$$

Using Equations (F.3) (F.4), as well as (11) and (14), the total differential of (F.1) and (F.2) is

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_{=J} \begin{pmatrix} d\widetilde{K} \\ d\tau^* \end{pmatrix} = \begin{pmatrix} -b_1 \\ -b_2 \end{pmatrix} db, \tag{F.6}$$

where

$$a_{11} \equiv (1-\tau^*)\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}}, \tag{F.7a}$$

$$a_{12} \equiv -(F_K - rb), \tag{F.7b}$$

$$a_{21} \equiv \frac{F_K - rb}{r-\mu_1}\left[\frac{r(1-\tau^*) - \mu_1(1+\tau^*)}{1-\tau^*} - \frac{\mu_1(1-\beta)F_{LL}}{\widetilde{L}^m(F_{LL}+G_{LL})} - \frac{\mu_1\tau^*}{(r-2\mu_1)\widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau}(r+2\mu_1\xi)\right]$$

$$+\frac{\widetilde{K}(1-\beta\widetilde{L}^d)}{(r-\mu_1)\widetilde{L}^m}\left[r\frac{G_{LL}F_{KK}}{F_{LL}+G_{LL}}+\frac{\mu_1(F_K-rb)}{(r-2\mu_1)\widetilde{K}}\left(2(r-\mu_1)+r\xi\right)\right], \quad \text{(F.7c)}$$

$$a_{22}\equiv-\frac{\mu_1(F_K-rb)\left[\frac{\partial\widetilde{K}}{\partial\tau}(r(1-\tau^*)-2\mu_1)+r\frac{\widetilde{K}(1-\beta\widetilde{L}^d)}{\widetilde{L}^m}\right]}{(r-\mu_1)(r-2\mu_1)(1-\tau^*)}, \quad \text{(F.7d)}$$

$$b_1\equiv r\tau^*, \quad \text{(F.7e)}$$

$$b_2\equiv-r\left[\widetilde{K}-\frac{\mu_1}{(r-\mu_1)}\left(2\tau^*\frac{\partial\widetilde{K}}{\partial\tau}-\frac{\widetilde{K}(1-\beta\widetilde{L}^d)}{\widetilde{L}^m}\right)\right]. \quad \text{(F.7f)}$$

The determinant of the matrix J is given by

$$\begin{aligned}|J| &= a_{11}a_{22}-a_{12}a_{21}\\ &= \left\{\frac{\widetilde{K}(1-\beta\widetilde{L}^d)}{\widetilde{L}^m}\left(r(r-3\mu_1)\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}}-\frac{\mu_1(F_K-rb)}{\widetilde{K}}(2(r-\mu_1)+r\xi)\right)\right.\\ &\quad+\frac{(F_K-rb)}{1-\tau^*}\left[(r-2\mu_1)\left(r(1-\tau^*)-\mu_1(2+\tau^*)-\mu_1(1-\tau^*)\frac{(1-\beta)F_{LL}}{\widetilde{L}^m(F_{LL}+G_{LL})}\right)\right.\\ &\quad\left.\left.+\mu_1\tau^*\left(r-\frac{1-\tau^*}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}(r+2\mu_1\xi)\right)\right]\right\}\frac{F_K-rb}{(r-\mu_1)(r-2\mu_1)}>0. \quad \text{(F.8)}\end{aligned}$$

The determinant of J must be positive as required by the second-order condition of the government's maximization problem (i.e., the derivative of (D.6) with respect to τ). Using Cramer's rule, the effects of b on the steady state capital stock and tax rate are:

$$\begin{aligned}\frac{d\widetilde{K}}{db} &= \frac{1}{|J|}\begin{vmatrix}-b_1 & a_{12}\\ -b_2 & a_{22}\end{vmatrix}=\frac{b_2a_{12}-b_1a_{22}}{|J|}\\ &= \frac{r(F_K-rb)}{|J|(r-2\mu_1)(r-\mu_1)(1-\tau^*)}\left\{\mu_1\tau^*\frac{\partial\widetilde{K}}{\partial\tau}\left[2\mu_1(1-2\tau^*)-r(1-\tau^*)\right]\right.\\ &\quad\left.+\widetilde{K}\left[(r-\mu_1)(r-2\mu_1)(1-\tau^*)+\mu_1\frac{(1-\beta\widetilde{L}^d)}{\widetilde{L}^m}(r-2\mu_1(1-\tau^*))\right]\right\}, \quad \text{(F.9)}\\ \frac{d\tau^*}{db} &= \frac{1}{|J|}\begin{vmatrix}a_{11} & -b_1\\ a_{21} & -b_2\end{vmatrix}=\frac{b_1a_{21}-b_2a_{11}}{|J|}\\ &= \frac{r}{|J|(r-\mu_1)}\left\{(F_K-rb)\tau^*\left[r-\frac{\mu_1(3-\tau^*)}{1-\tau^*}-\mu_1\frac{(1-\beta)F_{LL}}{\widetilde{L}^m(F_{LL}+G_{LL})}\right.\right. \quad \text{(F.10)}\\ &\quad\left.-\frac{\mu_1\tau^*(r+2\mu_1\xi)}{(r-2\mu_1)\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\right]+(r-\mu_1)(1-\tau^*)\frac{F_{KK}G_{LL}\widetilde{K}}{F_{LL}+G_{LL}}+\frac{\widetilde{K}(1-\beta\widetilde{L}^d)}{\widetilde{L}^m}.\end{aligned}$$

$$\cdot \left[\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}}(\tau^* r + (1-\tau^*)\mu_1) + \frac{\tau^*\mu_1(F_K - rb)}{(r-2\mu_1)\widetilde{K}}(2(r-\mu_1)+r\xi) \right] \Bigg\} .$$

The expression (F.9) contains only negative terms in its first row and both negative and positive terms in the second row. The change in $\widetilde{K}$ has, thus, an ambiguous sign. The same is true for (F.10): while the first row of (F.10) is positive, the second and third rows are either positive or negative.

To prove the first part of Proposition 1, take the limit of (F.9) when μ_1 approaches $-\infty$:

$$\lim_{\mu_1 \to -\infty} \frac{d\widetilde{K}}{db} = \frac{r(F_K - rb)}{|J|(1-\tau^*)} \left[\tau^* \frac{\partial \widetilde{K}}{\partial \tau}(1-2\tau^*) - \widetilde{K}(1-\beta)\frac{\widetilde{L}^d}{\widetilde{L}^m}(1-\tau^*) \right] < 0, \quad \text{if} \quad \tau^* < \frac{1}{2}, \tag{F.11}$$

The limit of (F.10) when $\mu_1 \to -\infty$ is difficult to sign due to the presence of third derivatives in the term ξ. Therefore, instead of directly evaluating the change in the tax rate, we evaluate it indirectly. Note that the change in the capital stock $\widetilde{K}$ can be split in two effects: a direct effect of b on $\widetilde{K}$ and an indirect effect through the change in the tax rate τ^*:

$$\frac{d\widetilde{K}}{db} = \frac{\partial \widetilde{K}}{\partial \tau}\frac{d\tau^*}{db} + \frac{\partial \widetilde{K}}{\partial b}, \tag{F.12}$$

where the direct effect $\partial \widetilde{K}/\partial b$ is derived by totally differentiating Equation (10a) with respect to $\widetilde{K}$ and b:

$$\frac{\partial K}{\partial b} = -\frac{r\tau^*(F_{LL}+G_{LL})}{(1-\tau^*)F_{KK}G_{LL}} > 0. \tag{F.13}$$

Thus, we can solve for $d\tau^*/db$ from Equation (F.12):

$$\frac{d\tau^*}{db} = \frac{\frac{d\widetilde{K}}{db} - \frac{\partial \widetilde{K}}{\partial b}}{\frac{\partial \widetilde{K}}{\partial \tau}} > 0, \quad \text{if} \quad \mu_1 \to -\infty \quad \text{and} \quad \tau^* < \frac{1}{2}. \tag{F.14}$$

Thus, in a static model, $\tau^* < 1/2$ is sufficient for internal debt to have a positive effect on the optimal tax rate.

To prove the second part of Proposition 1, evaluate (F.9) and (F.10) at $\beta = 1$. Note first that, in this case, the optimal tax rate is determined by

$$\frac{\tau^*}{\widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau} = \frac{r}{\mu_1}, \tag{F.15}$$

where (F.15) is Equation (18), evaluated at $\beta = 1$. Evaluating Equations (F.9) and (F.10) at $\beta = 1$ and using (F.15), one gets

$$\frac{d\widetilde{K}}{db}(\beta = 1) = -\frac{r^2\mu_1\tau^*(F_K - rb)\widetilde{K}}{|J|(r - 2\mu_1)(r - \mu_1)(1 - \tau^*)} > 0, \tag{F.16}$$

$$\begin{aligned}\frac{d\tau^*}{db}(\beta = 1) = \frac{r}{|J|(r - \mu_1)} \Bigg\{ & (F_K - rb)\tau^* \left[r - \frac{\mu_1(3 - \tau^*)}{1 - \tau^*} - \frac{r(r + 2\mu_1\xi)}{(r - 2\mu_1)} \right] \\ & + (r - \mu_1)(1 - \tau^*)\frac{F_{KK}G_{LL}\widetilde{K}}{F_{LL} + G_{LL}} + \widetilde{K}\left[\frac{F_{KK}G_{LL}}{F_{LL} + G_{LL}}(\tau^* r + (1 - \tau^*)\mu_1) \right. \\ & \left. + \frac{\tau^*\mu_1(F_K - rb)}{(r - 2\mu_1)\widetilde{K}}(2(r - \mu_1) + r\xi) \right] \Bigg\}.\end{aligned} \tag{F.17}$$

Equation (F.16) is unambiguously positive. To derive the sign of (F.17), rewrite (F.15) using (C.1):

$$\frac{F_{KK}G_{LL}\widetilde{K}r}{F_{LL} + G_{LL}} = \frac{\tau^*(F_K - br)\mu_1}{(1 - \tau^*)}. \tag{F.18}$$

Inserting (F.18) in (F.17) and simplifying, one gets

$$\frac{d\tau^*}{db}(\beta = 1) = \frac{r(F_K - rb)\tau^*\mu_1\left[\mu_1(3 + \tau^*) - r(2 - \tau^*) - (1 - \tau^*)r\xi\right]}{|J|(r - \mu_1)(r - 2\mu_1)(1 - \tau^*)} \gtreqless 0. \tag{F.19}$$

Furthermore, in the case $\beta = 1$, the determinant $|J|$ becomes

$$|J|(\beta = 1) = \frac{(F_K - rb)^2\mu_1}{(r - \mu_1)(r - 2\mu_1)(1 - \tau^*)}\left[(2 - \tau^*)(\mu_1 - r) - (1 - \tau^*)r\xi\right] > 0. \tag{F.20}$$

The determinant is positive, and, thus, the second-order condition is satisfied, if $\xi > (2 - \tau^*)(\mu_1 - r)/(r(1 - \tau^*))$. Using this condition in the numeration of (F.19), shows that the numerator is positive.

Together, Equations (F.16) and (F.19) prove the second part of Proposition 1 for $\beta = 1$. Since both (F.9) and (F.10) are continuous in β there exist values of β close but not equal to one for which $d\widetilde{K}/db > 0$ and $d\tau^*/db > 0$. Denote the lowest value of β for which these results hold as $\hat{\beta}$. Then, for $\beta \in [\hat{\beta}, 1]$, the steady state capital stock and the optimal tax rate are increasing in the amount of internal debt.

Lastly, note that for all values of β not yet considered, i.e., $\beta \in [0, \hat{\beta}[$, the effects of an increase in b are given by (F.9) and (F.10) and are ambiguous. □

G Proof of Proposition 2

To prove Proposition 2, begin by expressing the steady state welfare as

$$\widetilde{\Omega} = \tau^*[F(\widetilde{K},\widetilde{L}^m) - rb\widetilde{K}] + (1-\tau^*)\widetilde{w}\widetilde{L}^m + G(\widetilde{L}^d) - (1-\tau^*)(1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d]. \tag{G.1}$$

We derive first the effects of a change in b on $\widetilde{\Omega}$. Differentiate the welfare with respect to b, taking into account the effects of b on τ^* and $\widetilde{K}$. The resulting expression is[12]

$$\begin{aligned}\frac{d\widetilde{\Omega}}{db} &= -r\tau^*\widetilde{K} + \left[F(\widetilde{K},\widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\frac{d\tau^*}{db} \\ &\quad + \left[\tau^*(F_K - rb) + (1-\tau^*)\frac{\partial\widetilde{w}}{\partial\widetilde{K}}(\widetilde{L}^m + (1-\beta)\widetilde{L}^d)\right]\frac{d\widetilde{K}}{db}.\end{aligned} \tag{G.2}$$

Using the government's first-order condition (D.6), we can substitute for the term in brackets in the second row of (G.2). Moreover, we can express $F(\cdot)$ as $F = F_K K + F_L L^m$. Equation (G.2) becomes

$$\frac{d\widetilde{\Omega}}{db} = -r\tau^*\widetilde{K} + \left[(F_K - rb)\widetilde{K} + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\left[\frac{d\tau^*}{db} + \frac{r-\mu_1}{\mu_1}\frac{\frac{d\widetilde{K}}{db}}{\frac{\partial\widetilde{K}}{\partial\tau}}\right] \tag{G.3}$$

We can split the effect of b on $\widetilde{K}$ using (F.12) and (F.13). Thus, the welfare change becomes

$$\begin{aligned}\frac{d\widetilde{\Omega}}{db} &= -r\tau^*\widetilde{K} + \left[(F_K - rb)\widetilde{K} + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\left[\frac{d\tau^*}{db} + \frac{r-\mu_1}{\mu_1}\left(\frac{d\tau^*}{db} + \frac{\frac{\partial\widetilde{K}}{\partial b}}{\frac{\partial\widetilde{K}}{\partial\tau}}\right)\right] \\ &= -r\tau^*\widetilde{K} + \left[(F_K - rb)\widetilde{K} + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\left[\frac{r}{\mu_1}\frac{d\tau^*}{db} - \frac{(r-\mu_1)r\tau^*}{\mu_1(F_K - rb)}\right]\end{aligned} \tag{G.4}$$

Further simplification of (G.4) gives:

$$\begin{aligned}\frac{d\widetilde{\Omega}}{db} &= -\frac{r}{\mu_1}\left[r\tau^*\widetilde{K} + \frac{\tau^*(r-\mu_1)}{F_K - rb}(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right. \\ &\quad \left. - \left((F_K - rb)\widetilde{K} + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right)\frac{d\tau^*}{db}\right].\end{aligned} \tag{G.5}$$

[12] Note that the derivatives with respect to the labor inputs cancel out.

The first row of (G.5) is positive, while the second row is negative for $d\tau^*/db > 0$. Hence, the net change in $\widetilde{\Omega}$ is indeterminate.

Focus first on the case $\mu_1 \to -\infty$. Evaluate (G.5) at $\mu_1 \to -\infty$ to get

$$\lim_{\mu_1 \to -\infty} \frac{d\widetilde{\Omega}}{db} = \frac{\tau^* r}{F_K - rb}(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) \geq 0. \tag{G.6}$$

Hence, when the economy is always in steady state, the long term impact of an increase in b on welfare is nonnegative (strictly positive for $\beta < 1$). Moreover, in the case $\mu_1 \to -\infty$, the economy is static and $d\Omega_0 = d\widetilde{\Omega}$. This concludes the proof of part (a) of Proposition 2.

Consider now part (b) of Proposition 2. Evaluate (G.5) at $\beta = 1$:

$$\frac{d\widetilde{\Omega}}{db}(\beta = 1) = -\frac{r\widetilde{K}}{\mu_1}\left[r\tau^* - (F_K - rb)\frac{d\tau^*}{db}(\beta = 1)\right]. \tag{G.7}$$

Use Equations (C.1) and (F.13) to derive the expression

$$\frac{\partial \widetilde{K}}{\partial b} = -\frac{r\tau^*}{(F_K - rb)}\frac{\partial \widetilde{K}}{\partial \tau}. \tag{G.8}$$

Equations (G.8) and (F.12) together gives

$$\frac{d\widetilde{K}}{db} = \frac{\partial \widetilde{K}}{\partial \tau}\frac{d\tau^*}{db} + \frac{\partial \widetilde{K}}{\partial b} = \frac{\partial \widetilde{K}}{\partial \tau}\left[\frac{d\tau^*}{db} - \frac{r\tau^*}{F_K - rb}\right]. \tag{G.9}$$

To simplify (G.7), evaluate (G.9) at $\beta = 1$ and insert it in (G.7):

$$\frac{d\widetilde{\Omega}}{db}(\beta = 1) = \frac{r\widetilde{K}}{\mu_1}\frac{(F_K - rb)\frac{d\widetilde{K}}{db}(\beta = 1)}{\frac{\partial \widetilde{K}}{\partial \tau}} > 0. \tag{G.10}$$

Equation (G.10) proves Equation (23) from Proposition 2. Since welfare is continuous in β, this result holds also for values of β sufficiently close but not equal to one. Define the lowest value of β that satisfies (G.10) as $\underline{\beta}$. Then, (G.10) is satisfied for $\beta \in [\underline{\beta}, 1]$. To derive the short term welfare change, in this case, define short term welfare Ω_0 as

$$\begin{aligned}\Omega_0 &= \tau^*[F(K_0, L_0^m) - rbK_0] + (1-\tau^*)w_0 L_0^m + G(L_0^d) \\ &\quad - (1-\tau^*)(1-\beta)[G(L_0^d) - w_0 L_0^d], \end{aligned} \tag{G.11}$$

where a subscript 0 denotes the initial period where the economy is in steady state (prior to the disturbance) such that $K_0 = \widetilde{K}, L_0^m = \widetilde{L}^m, w_0 = \widetilde{w}, L_0^d = \widetilde{L}^d$. Note that the capital stock cannot change in time period zero, $dK_0/db = 0$, as it is a stock variable. Consequently, the wage rate and the labor demands also remain unchanged at time period 0. Hence, the initial impact on welfare of a change in internal debt in period 0 is

$$\frac{d\Omega_0}{db} = -\tau^* r\widetilde{K} + \left[F(\widetilde{K},\widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\frac{d\tau^*}{db}. \tag{G.12}$$

Evaluate (G.12) at $\beta = 1$ using the the constant returns property $F = F_K K + F_L L^m$:

$$\frac{d\Omega_0}{db}(\beta = 1) = -\tau^* r\widetilde{K} + \left(F_K(\widetilde{K},\widetilde{L}^m) - rb\right)\widetilde{K}\frac{d\tau^*}{db}(\beta = 1). \tag{G.13}$$

Simplifying (G.13) and using (G.7), we get

$$\begin{aligned}\frac{d\Omega_0}{db}(\beta = 1) &= -\tau^* r\widetilde{K} + \left(F_K(\widetilde{K},\widetilde{L}^m) - rb\right)\widetilde{K}\frac{d\tau^*}{db}(\beta = 1)\\ &= -\widetilde{K}\left[r\tau^* - (F_K - rb)\frac{d\tau^*}{db}(\beta = 1)\right]\\ &= \frac{\mu_1}{r}\frac{d\widetilde{\Omega}}{db}(\beta = 1) < 0.\end{aligned} \tag{G.14}$$

Equation (G.14) proves Equation (22) from Proposition 2. Following the same intuition as before, (G.14) holds for $\beta \in [\underline{\beta}, 1]$. This concludes the proof of part (b) from Proposition 2.

Lastly, note that both Equations (G.5) and (G.12) are ambiguous for $\beta < \underline{\beta}$ and $\mu_1 \in]-\infty, 0[$. □

H Proof of Proposition 3

The overall welfare change is given by the effect of b on welfare, as defined in Equation (D.1). The derivative of (D.1) with respect to b is[13]

$$\begin{aligned}\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = \int_0^\infty \Bigg\{ &-\tau^* r K_t + \left[F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]\right]\frac{d\tau^*}{db}\\ &+ \left[\tau^*(F_K - rb) + (1-\tau^*)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\frac{dK_t}{db}\Bigg\} e^{-rt}dt.\end{aligned} \tag{H.1}$$

[13]Note again that the derivatives with respect to the labor inputs cancel out.

The change in the capital stock in period t can be decomposed, using (F.12), in a direct and an indirect effect:

$$\frac{dK_t}{db} = \frac{\partial K_t}{\partial \tau}\frac{d\tau^*}{db} + \frac{\partial K_t}{\partial b}. \tag{H.2}$$

Inserting (H.2) in (H.1) and rearranging, we get

$$\begin{aligned}\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt &= \int_0^\infty \Bigg\{ -\tau^* r K_t + \Bigg[F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d] \\ &\quad + \left[\tau^*(F_K - rb) + (1-\tau^*)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\frac{\partial K_t}{\partial \tau}\Bigg]\frac{d\tau^*}{db} \\ &\quad + \left[\tau^*(F_K - rb) + (1-\tau^*)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\frac{\partial K_t}{\partial b}\Bigg\} e^{-rt}dt. \end{aligned} \tag{H.3}$$

The terms multiplied with $d\tau^*/db$ sum up to the first-order condition with respect to τ and vanish. Thus, (H.3) can be simplified to

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = \int_0^\infty \Bigg\{ -\tau^* r K_t + \left[\tau^*(F_K - rb) + (1-\tau^*)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\frac{\partial K_t}{\partial b}\Bigg\} e^{-rt}dt. \tag{H.4}$$

Suppose that the economy is near steady state with $K_t \approx \widetilde{K}$. Then, following the same steps as in the proof of Lemma 1, one can show that

$$\frac{\partial K_t}{\partial b} = \frac{\partial \widetilde{K}}{\partial b}\left[1 - e^{\mu_1 t}\right], \tag{H.5}$$

where $\partial \widetilde{K}/\partial b$ is defined in Equation (F.13). Insert Equations (11), (F.13), and (H.5) in (H.4), and evaluate the integral for $K_t = \widetilde{K}$. The resulting expression is

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = \frac{\tau^*\widetilde{K}}{r-\mu_1}\left[-r - \mu_1(1-\beta)\frac{\widetilde{L}^d}{\widetilde{L}^m} + \mu_1\frac{\tau^*}{\widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau}\right]. \tag{H.6}$$

Now, we insert Equation (D.5) in (18) and rearrange to get

$$\left[-r - \mu_1(1-\beta)\frac{\widetilde{L}^d}{\widetilde{L}^m} + \mu_1\frac{\tau^*}{\widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau}\right] = \frac{(r-\mu_1)(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)}{(F_K - rb)\widetilde{K}}. \tag{H.7}$$

Together Equations (H.6) and (H.7) give Equation (24) from Proposition 3. □

Online Appendices (not for publication)

OA Properties of the function $I_t(K_t, \tau_t)$

This section derives the first and second partial derivatives of the function $I_t(K_t, \tau_t)$. Start with $\partial I_t/\partial K_t$. First, we take the total differential of (B.3) with respect to I_t and K_t, taking into account that in the case of quadratic adjustment costs $C'' = c$, and rearrange to get

$$\frac{\partial I_t}{\partial K_t} = \frac{W_{KK}(K_t)}{c}. \tag{OA.1}$$

It remains to derive W_{KK}. To do so, differentiate Equation (B.4) with respect to K_t, taking into account that I_t and L_t^m depends on K_t:

$$rW_{KK} = \pi^M_{KK} + 2W_{KK}I_t + W_{KKK}I_tK_t + \left[\pi^M_{KI} + W_K + W_{KK}K\right]\frac{\partial I_t}{\partial K_t}, \tag{OA.2}$$

where

$$\pi^M_{KK} = (1-\tau_t)\left(F_{KK} + F_{KL}\frac{\partial L^m}{\partial K}\right) = (1-\tau_t)\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}}, \tag{OA.3a}$$

$$\pi^M_{KI} = -(1 + cI_t). \tag{OA.3b}$$

Using Equation (B.3) to express W_K, as well as (OA.1), (OA.3a) and (OA.3b), Equation (OA.2) becomes

$$rW_{KK} = (1-\tau_t)\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}} + 2W_{KK}I_t + W_{KKK}I_tK_t + W_{KK}K\frac{W_{KK}}{c}. \tag{OA.4}$$

Equation (OA.4) is quadratic in W_{KK}. To solve it, rewrite it first as

$$W_{KK}^2 + \beta_t W_{KK} + \gamma_t = 0, \tag{OA.5}$$

where

$$\beta_t \equiv \frac{c(2I_t - r)}{K_t}, \tag{OA.6}$$

$$\gamma_t \equiv \frac{c}{K_t}\left[\frac{(1-\tau_t)F_{KK}G_{LL}}{F_{LL}+G_{LL}} + W_{KKK}I_tK_t\right]. \tag{OA.7}$$

Now evaluate (OA.5) around steady state, where $I_t \approx 0, K_t \approx \widetilde{K}$ and $\tau_t \approx \widetilde{\tau}$, and denote the corresponding parameters as $\widetilde{\beta}$ and $\widetilde{\gamma}$. We get

$$\widetilde{\beta} = -\frac{rc}{\widetilde{K}} < 0, \tag{OA.8}$$

$$\widetilde{\gamma} = \frac{(1-\widetilde{\tau})cF_{KK}G_{LL}}{\widetilde{K}(F_{LL}+G_{LL})} < 0. \tag{OA.9}$$

The two solutions to (OA.5) are given by

$$W_{KK} = \frac{c}{2\widetilde{K}}\left[r - \sqrt{r^2 - \frac{4(1-\widetilde{\tau})F_{KK}G_{LL}\widetilde{K}}{c(F_{LL}+G_{LL})}}\right] = \frac{c\mu_1}{\widetilde{K}} < 0, \tag{OA.10}$$

$$W_{KK} = \frac{c}{2\widetilde{K}}\left[r + \sqrt{r^2 - \frac{4(1-\widetilde{\tau})F_{KK}G_{LL}\widetilde{K}}{c(F_{LL}+G_{LL})}}\right] = \frac{c\mu_2}{\widetilde{K}} > 0, \tag{OA.11}$$

where μ_1 and μ_2 are defined in Equation (C.9). Note that the value function $W(K)$ must be concave in the capital stock when the objective function as well as the rate of change of the capital stock $I_t K_t$ are concave in K and I. Since this is satisfied, the solution to W_{KK} is given by the negative root, (OA.10). Thus, (OA.1) and (OA.10) together give

$$\frac{\partial I_t}{\partial K_t} = \frac{\mu_1}{\widetilde{K}} < 0. \tag{OA.12}$$

Next, we derive the second derivative of investment with respect to capital. Differentiation of (OA.1) with respect to K_t gives

$$\frac{\partial^2 I_t}{\partial K_t^2} = \frac{W_{KKK}(K_t)}{c}. \tag{OA.13}$$

It remains to derive W_{KKK}. I follow Kimball (2014). To ease notation, define the right-hand side of (B.1) as H_t, i.e., $H_t \equiv \pi_t^M + W_K(K_t)I_t K_t$.[14] Now, differentiate the Bellman equation $rW(K_t) = H_t$ with respect to capital without invoking the Envelope theorem:

$$rW_K = H_K + H_I \frac{\partial I_t}{\partial K_t}, \tag{OA.14}$$

[14] Kimball (2014) proposes the term "prevalue function" for H_t, as it is the maximization of H_t that yields the value function.

where

$$H_K = \pi_K^M + W_K I_t + W_{KK} I_t K_t, \tag{OA.15a}$$

$$H_I = -(1 + cI_t)K_t + W_K K_t = 0, \tag{OA.15b}$$

where $H_I = 0$ owing to the first-order condition (B.3). Next, differentiate (OA.14) with respect to capital:

$$rW_{KK} = H_{KK} + 2H_{KI}\frac{\partial I_t}{\partial K_t} + H_{II}\left(\frac{\partial I_t}{\partial K_t}\right)^2 + H_I \frac{\partial^2 I_t}{\partial K_t^2}, \tag{OA.16}$$

where

$$H_{KK} = \pi_{KK}^M + 2W_{KK} I_t + W_{KKK} I_t K_t, \tag{OA.17a}$$

$$H_{KI} = \pi_{KI}^M + W_K + W_{KK} K_t, \tag{OA.17b}$$

$$H_{II} = -cK_t, \tag{OA.17c}$$

and π_{KK}^M, π_{KI}^M are defined in (OA.3a), (OA.3b). Lastly, differentiate (OA.16) with respect to K_t:

$$rW_{KKK} = H_{KKK} + 3H_{KKI}\frac{\partial I_t}{\partial K_t} + 3H_{IIK}\left(\frac{\partial I_t}{\partial K_t}\right)^2 + 3\frac{\partial^2 I_t}{\partial K_t^2}\left[H_{KI} + H_{II}\frac{\partial I_t}{\partial K_t}\right] + H_I \frac{\partial^3 I_t}{\partial K_t^3}, \tag{OA.18}$$

where

$$H_{KKK} = \pi_{KKK}^M + 3W_{KKK} I_t + W_{KKKK} I_t K_t, \tag{OA.19a}$$

$$H_{KKI} = 2W_{KK} + W_{KKK} K_t, \tag{OA.19b}$$

$$H_{IIK} = -c, \tag{OA.19c}$$

$$\pi_{KKK}^M = (1 - \tau_t)\frac{F_{KK} G_{LL}}{F_{LL} + G_{LL}} \frac{\xi_t}{K_t}. \tag{OA.19d}$$

Note that ξ_t is the value of ξ, defined in Equation (F.5), evaluated in period t. Note that, according to (B.3), $H_I = 0$. Moreover, Equation (OA.1) gives

$$H_{KI} + H_{II}\frac{\partial I_t}{\partial K_t} = 0.$$

Thus, (OA.18) becomes

$$rW_{KKK} = H_{KKK} + 3H_{KKI}\frac{\partial I_t}{\partial K_t} + 3H_{IIK}\left(\frac{\partial I_t}{\partial K_t}\right)^2. \tag{OA.20}$$

Now, evaluate (OA.20) around steady state, where $I_t \approx 0, K_t \approx \widetilde{K}, \tau_t \approx \widetilde{\tau}$, and using Equations(OA.12), (OA.10), (OA.19a)-(OA.19d). The resulting expression is

$$W_{KKK} = \frac{1}{r-3\mu_1}\left[\frac{(1-\widetilde{\tau})F_{KK}G_{LL}\xi}{\widetilde{K}(F_{LL}+G_{LL})} + 3c\left(\frac{\mu_1}{\widetilde{K}}\right)^2\right]. \tag{OA.21}$$

One way to simplify (OA.21) is to solve (C.8) for the term containing the second derivatives of the production function. Inserting the resulting expression in (OA.21), one gets

$$W_{KKK} = \frac{\mu_1 c}{(r-3\mu_1)\widetilde{K}^2}\left[(r-\mu_1)\xi + 3\mu_1\right]. \tag{OA.22}$$

Thus, (OA.13) and (OA.22) together give

$$\frac{\partial^2 I_t}{\partial K_t^2} = \frac{\mu_1}{(r-3\mu_1)\widetilde{K}^2}\left[(r-\mu_1)\xi + 3\mu_1\right]. \tag{OA.23}$$

Next, use (OA.1) to derive the cross-derivative

$$\frac{\partial^2 I_t}{\partial K_t \partial \tau_t} = \frac{1}{c}\frac{\partial W_{KK}}{\partial \tau_t}, \tag{OA.24}$$

where the effect of the statutory tax rate on the second derivative of the value function is determined by (OA.5). A total differential of (OA.5) gives

$$\frac{\partial W_{KK}}{\partial \tau_t} = -\frac{W_{KK}\frac{\partial \beta_t}{\partial \tau_t} + \frac{\partial \gamma_t}{\partial \tau_t}}{2W_{KK} + \beta_t}. \tag{OA.25}$$

Using Equations (OA.6) and (OA.7), one can derive

$$\frac{\partial \beta_t}{\partial \tau_t} = 2\frac{c}{K_t}\frac{\partial I_t}{\partial \tau_t}, \tag{OA.26}$$

$$\frac{\partial \gamma_t}{\partial \tau_t} = \frac{c}{K_t}\left[-\frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}} + W_{KKK}K_t\frac{\partial I_t}{\partial \tau_t}\right]. \tag{OA.27}$$

In deriving the above expressions, we take into account that the capital stock K_t does not react to changes in τ_t, as it is a stock and cannot change immediately. Using (OA.24)-(OA.27), as well as (OA.8) and (OA.10), we get

$$\frac{\partial^2 I_t}{\partial K_t \partial \tau_t} = \frac{1}{(r-2\mu_1)c}\left[\frac{\partial I_t}{\partial \tau_t}\frac{\mu_1 c}{\widetilde{K}}\left(2 + \frac{(r-\mu_1)\xi + 3\mu_1}{(r-3\mu_1)\widetilde{K}}\right) - \frac{F_{KK}G_{LL}}{F_{LL}+G_{LL}}\right] \tag{OA.28}$$

It remains to derive the first and second derivatives of investment with respect to the tax rate. To do so, differentiate totally (B.3) with respect to I_t and W_K to get

$$\partial I_t = \frac{1}{c}\partial W_K. \tag{OA.29}$$

Then, rewrite (B.4) in the case of a time-varying tax rate as

$$(r - I_t)W_K = F_K - \tau_t(F_K - rb) - (I_t + C(I_t)) + W_{KK} I_t K_t. \tag{OA.30}$$

Then, differentiate (OA.30) with respect to I_t, W_K, W_{KK} and τ_t:

$$(r - I_t)\partial W_K = -(F_K - rb)\partial\tau_t - (1 + cI_t - W_K - W_{KK}K_t)\partial I_t + I_t K_t \partial W_{KK}. \tag{OA.31}$$

We can use (OA.29) and (OA.31) to derive

$$\frac{\partial I_t}{\partial \tau_t} = \frac{-(F_K - rb) + I_t K_t \frac{\partial W_{KK}}{\partial \tau_t}}{rc + 1 - W_K - W_{KK}K_t}. \tag{OA.32}$$

Around steady state, $I_t \approx 0, W_K = 1 + c \cdot 0, W_{KK} = \mu_1 c/\widetilde{K}$. Thus, (OA.32) simplifies to

$$\begin{aligned} \frac{\partial I_t}{\partial \tau_t} &= -\frac{(F_K - rb)}{(r - \mu_1)c} \\ &= -\frac{\mu_1 (F_K - rb)(F_{LL} + G_{LL})}{(1 - \widetilde{\tau})F_{KK} G_{LL} \widetilde{K}} = -\frac{\mu_1}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} < 0, \end{aligned} \tag{OA.33}$$

where the last row was derived by the use of (C.8) to substitute for $(r - \mu_1)c$. Lastly, derive the second derivative of investment with respect to the tax rate, by using (OA.32). It is given by

$$\frac{\partial^2 I_t}{\partial \tau_t^2} = \frac{\frac{\partial I_t}{\partial \tau_t}\left[2K_t \frac{\partial W_{KK}}{\partial \tau_t} + \frac{\partial W_K}{\partial \tau_t}\right]}{rc + 1 - W_K - W_{KK}K_t}. \tag{OA.34}$$

Using Equations (OA.25), (OA.26), (OA.27) and (OA.33), the above equation can be expressed, around a steady state, as

$$\begin{aligned} \frac{\partial^2 I_t}{\partial \tau_t^2} &= -\frac{(F_K - rb)}{(r - \mu_1)^2 (r - 2\mu_1)c} \times \\ &\quad \times \left[\frac{\partial I_t}{\partial \tau_t}\left(r + 2\mu_1 + 2\mu_1 \frac{(r - \mu_1)\xi + 3\mu_1}{(r - 3\mu_1)}\right) - \frac{2\widetilde{K} F_{KK} G_{LL}}{c(F_{LL} + G_{LL})}\right]. \end{aligned} \tag{OA.35}$$

OB Proof of Proposition 4

To derive the optimal tax rate, note first that period t welfare is given by

$$\begin{aligned}\Omega_t &= \tau_t[F(K_t, L_t^m(K_t)) - rbK_t] + (1-\tau_t)w_t(K_t)L_t^m(K_t) + G(L_t^d(K_t)) \\ &\quad -(1-\tau_t)(1-\beta)[G(L_t^d(K_t)) - w_t(K_t)L_t^d(K_t)], \end{aligned} \tag{OB.1}$$

Denote the government's value function as $U(K_t)$. Then, its maximization problem can be written as

$$rU(K_t) = \max_{\tau_t} \left\{\Omega_t + U_K(K_t)I_tK_t\right\}. \tag{OB.2}$$

To simplify the exposition of the proof, use the notation $A(K_t, \tau_t) \equiv I_tK_t$ and $H^G \equiv \Omega_t + U_K(K_t)A(K_t, \tau_t)$, where H^G can be referred to as the prevalue function of the government. To ensure that the maximization probelm is well-behaved, suppose that both Ω_t and $A(K_t, \tau_t)$ are jointly concave in K_t and τ_t. Under this asumption, the value function is also concave, i.e., $U_{KK} < 0$ (Kimball, 2014). The first-order condition of the government is given by

$$\begin{aligned}H_\tau^G &= F(K_t, L_t^m(K_t)) - rbK_t - w_t(K_t)L_t^m(K_t) + (1-\beta)[G(L_t^d(K_t)) - w_t(K_t)L_t^d(K_t)] \\ &\quad +U_K(K_t)A_\tau = 0, \end{aligned} \tag{OB.3}$$

where $A_\tau = (\partial I_t/\partial\tau_t)K_t$. Derive first the steady state tax rate $\widetilde{\tau}$. To find it, one needs first the value of $U_K(K_t)$, evaluated in steady state. It is found by a differentiation of the maximized Bellman equation with respect to capital:

$$rU_K = \Omega_K + U_{KK}A + U_KA_K, \tag{OB.4}$$

where

$$\begin{aligned}\Omega_K &= \tau_t\left(F_K - rb + F_L\frac{\partial L_t^m}{\partial K_t}\right) + (1-\tau_t)w_t\frac{\partial L_t^m}{\partial K_t} + G_L\frac{\partial L_t^d}{\partial K_t} \\ &\quad -(1-\beta)(1-\tau_t)(G_L - w_t)\frac{\partial L_t^d}{\partial K_t} + \frac{\partial w_t}{\partial K_t}(1-\tau_t)(L_t^m + (1-\beta)L_t^d) \\ &= \tau_t(F_K - rb) + \frac{\partial w_t}{\partial K_t}(1-\tau_t)(L_t^m + (1-\beta)L_t^d), \end{aligned} \tag{OB.5}$$

$$A_K = I_t + \frac{\partial I_t}{\partial K_t}K_t. \tag{OB.6}$$

Around steady state, $K_t \approx \widetilde{K}, I_t \approx 0$ and $\partial I_t / \partial K_t$ is given by (OA.12). Thus, one can solve (OB.4) for U_K:

$$U_K(\widetilde{K}) = \frac{\widetilde{\tau}(F_K - rb) + (1-\widetilde{\tau})\frac{\partial \widetilde{w}}{\partial K}(\widetilde{L}^m + (1-\beta)\widetilde{L}^d)}{r - \mu_1}. \tag{OB.7}$$

Moreover, one can evaluate (OB.3) around steady state, using (OA.33) and (OB.7). The resulting expression is (18), where one substitutes τ^* with $\widetilde{\tau}$. Hence, $\widetilde{\tau}$ is equal to τ^* from Section 3.

To derive the path of the optimal tax rate, totally differentiate (OB.3) with respect to time, taking into account that both τ_t and K_t are functions of time. The resulting expression is

$$H^G_{\tau\tau}\dot{\tau}_t + H^G_{\tau K}\dot{K}_t = 0, \tag{OB.8}$$

where

$$H^G_{\tau\tau} = U_K A_{\tau\tau} < 0, \tag{OB.9}$$

$$H^G_{\tau K} = \Omega_{\tau K} + U_{KK}A_\tau + U_K A_{\tau K}, \tag{OB.10}$$

$$\begin{aligned} \Omega_{\tau K} &= F_K - rb + F_L\frac{\partial L^m_t}{\partial K_t} - w_t\frac{\partial L^m_t}{\partial K_t} + (1-\beta)(G_L - w_t)\frac{\partial L^d_t}{\partial K_t} - \frac{\partial w_t}{\partial K_t}(L^m_t + (1-\beta)L^d_t) \\ &= F_K - rb - \frac{\partial w_t}{\partial K_t}(L^m_t + (1-\beta)L^d_t), \end{aligned} \tag{OB.11}$$

$$A_{\tau K} = \frac{\partial I_t}{\partial \tau_t} + \frac{\partial^2 I_t}{\partial \tau_t \partial K_t}K_t. \tag{OB.12}$$

Note that (OB.9) is negative due to our assumption about the concavity of the objective function. One can solve (OB.8) for τ_t near steady state. Note first that near steady state, $\dot{K}_t$ can be approximated (using (4) and (OA.12)) as

$$\begin{aligned} \dot{K}_t = I_t(K_t)K_t &\approx \left[\frac{\partial I_t}{\partial K_t}K_t + I_t\right](K_t - \widetilde{K}) \\ &\approx \mu_1(K_t - \widetilde{K}), \end{aligned} \tag{OB.13}$$

where in the second row of (OB.13) we used the steady state condition , $K_t \approx \widetilde{K}, I_t \approx \widetilde{I} = 0$ and Equation (OA.12). Starting from an initial capital stock K_0, the solution to the differential equation (OB.13) is

$$K_t - \widetilde{K} = (K_0 - \widetilde{K})e^{\mu_1 t}. \tag{OB.14}$$

Thus, one can rewrite (OB.8) as

$$\dot{\tau}_t = -\frac{H^G_{\tau K}}{H^G_{\tau\tau}}\mu_1(K_0 - \widetilde{K})e^{\mu_1 t}. \tag{OB.15}$$

Around a steady state, both $H^G_{\tau K}$ and $H^G_{\tau\tau}$ are constant and can be denoted as $\widetilde{H}^G_{\tau K}, \widetilde{H}^G_{\tau\tau}$. Thus, (OB.15) can be solved by integration (using the terminal condition $\tau_\infty = \widetilde{\tau}$) to get

$$\tau_t = \widetilde{\tau} + \alpha(K_0 - \widetilde{K})e^{\mu_1 t}, \tag{OB.16}$$

where

$$\alpha \equiv -\frac{\widetilde{H}^G_{\tau K}}{\widetilde{H}^G_{\tau\tau}}. \tag{OB.17}$$

Note additionally that the impact of the capital stock in period t on the period t tax rate can be derived from (OB.8), when one multiplies (OB.8) by ∂t and solves for $\partial\tau_t/\partial K_t$:

$$\frac{\partial\tau_t}{\partial K_t} = -\frac{H^G_{\tau K}}{H^G_{\tau\tau}}. \tag{OB.18}$$

Thus, α determines the slope of the function $\tau(K)$ around the steady state, i.e., $\alpha \equiv \partial\widetilde{\tau}/\partial K$. Due to $H^G_{\tau\tau} < 0$, the sign of α is determined by the sign of $H^G_{\tau K}$, which may be either positive or negative (see (OB.10)). Hence, the optimal tax rate may either be an increasing or a decreasing function of the capital stock.

OC Proof of Proposition 5

Proposition 5 states that Proposition 1 holds in the case of a time-varying tax rate when one replaces τ^* by $\widetilde{\tau}$. To prove this, note that τ^* and $\widetilde{\tau}$ coincide (see Proposition 4). Moreover, $\widetilde{K}$ is determined by (10a) in both situations. Hence, Proposition 1 can be proven again using Equations (10a) and (18).

Second, Proposition 5 states that Propositions 2 is qualitatively unchanged. Because $\tau^* = \widetilde{\tau}$, all long term effects of a change in internal debt remain exactly the same as in Proposition 2. Moreover, if the economy is static ($\mu_1 \to -\infty$), the time-varying tax rate model collapses to a static model with a constant tax rate. Therefore, it remains to prove that Equation (22) holds. To derive the initial impact of b on welfare, differentiate Equation (G.11) with respect to b:

$$\frac{d\Omega_0}{db} = -\tau_0 r\widetilde{K} + \left[F(\widetilde{K}, \widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d)\right]\frac{d\tau_0}{db},$$

(OC.1)

where τ_0 is the steady state tax rate $\widetilde{\tau}$ associated with the initial steady state capital stock $K_0 = \widetilde{K}$. Evaluate (OC.1) at $\beta = 1$ and use the the constant returns property $F = F_K K + F_L L^m$:

$$\frac{d\Omega_0}{db}(\beta = 1) = -\widetilde{\tau} r \widetilde{K} + \left(F_K(\widetilde{K}, \widetilde{L}^m) - rb\right) \widetilde{K} \frac{d\tau_0}{db}. \tag{OC.2}$$

The change in the initial tax rate is, according to Equation (26):

$$\frac{d\tau_0}{db} = \frac{d\widetilde{\tau}}{db} - \alpha \frac{d\widetilde{K}}{db}. \tag{OC.3}$$

Together (OC.2) and (OC.3) give

$$\begin{aligned}
\frac{d\Omega_0}{db}(\beta = 1) &= -\widetilde{\tau} r \widetilde{K} + \left(F_K(\widetilde{K}, \widetilde{L}^m) - rb\right) \widetilde{K} \left[\frac{d\widetilde{\tau}}{db} - \alpha \frac{d\widetilde{K}}{db}\right] \\
&= \widetilde{K}(F_K - rb) \left[-\frac{r\widetilde{\tau}}{F_K - rb} + \frac{d\widetilde{\tau}}{db} - \alpha \frac{d\widetilde{K}}{db}\right] \\
&= \frac{\mu_1}{r} \frac{d\widetilde{\Omega}}{db} \left[1 - \frac{\partial \widetilde{K}}{\partial \tau} \frac{\partial \widetilde{\tau}}{\partial K}\right] < 0, \quad \text{if} \quad \frac{\partial \widetilde{K}}{\partial \tau} \frac{\partial \widetilde{\tau}}{\partial K} < 1,
\end{aligned} \tag{OC.4}$$

where I used Equation (G.9) to derive the last row of (OC.4) and replaced α by $\partial\widetilde{\tau}/\partial K$. Note that Equation (10a) determines the steady state capital stock as a function of the tax rate (and, thus, $\partial\widetilde{K}/\partial\tau$), while (OB.3) determines the optimal tax rate as a function of the capital stock (and, thus, $\partial\widetilde{\tau}/\partial K$). The steady state is stable if the product of the slopes of these functions is less than one, i.e., if $\partial\widetilde{K}/\partial\tau(\partial\widetilde{\tau}/\partial K) < 1$. Thus, part *(b)* of Proposition 2 holds in the case of a time-varying tax rate under the assumption of a stable steady state.

The initial negative welfare impact, determined by (OC.4), is more (less) pronounced than in the case of a constant tax rate if $\partial\widetilde{\tau}/\partial K$ is positive (negative).

Lastly, we derive the impact of an increase in internal debt on the overall welfare, similarly to Appendix H. We get

$$\begin{aligned}
\frac{d}{db}\int_0^\infty \Omega_t e^{-rt} dt &= \int_0^\infty \Big\{ -\tau_t r K_t + \left[F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]\right] \frac{d\tau_t}{db} \\
&\quad + \left[\tau_t(F_K - rb) + (1-\tau_t)[L_t^m + (1-\beta)L_t^d] \frac{\partial w_t}{\partial K_t}\right] \frac{dK_t}{db} \Big\} e^{-rt} dt,
\end{aligned} \tag{OC.5}$$

where (OC.5) is analogous to (H.1) in the case of a time-invariant tax rate.

To simplify (OC.5), we make use of the following decompositions. First, note that, using (OB.14), we get

$$\frac{dK_t}{db} = \frac{d\widetilde{K}}{db}(1 - e^{\mu_1 t}). \tag{OC.6}$$

Second, we can decomponse the change in the steady state capital stock according to

$$\frac{d\widetilde{K}}{db} = \frac{\partial \widetilde{K}}{\partial \tau}\frac{d\widetilde{\tau}}{db} + \frac{\partial \widetilde{K}}{\partial b}. \tag{OC.7}$$

Together, (OB.16) and (OC.7) determine the change in the optimal tax rate τ_t as

$$\begin{aligned} \frac{d\tau_t}{db} &= \frac{d\widetilde{\tau}}{db} - \alpha\frac{d\widetilde{K}}{db}e^{\mu_1 t}, \\ &= \frac{d\widetilde{\tau}}{db}\left(1 - \alpha\frac{\partial \widetilde{K}}{\partial \tau}e^{\mu_1 t}\right) - \alpha\frac{\partial \widetilde{K}}{\partial b}e^{\mu_1 t}. \end{aligned} \tag{OC.8}$$

Using (OC.6) and (OC.8), we can represent the right-hand side of (OC.5) as

$$\begin{aligned} \int_0^\infty \Bigg\{ & -\tau_t r K_t + \big[F(K_t, L_t^m) - rbK_t - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]\big] \cdot \qquad \text{(OC.9)} \\ & \cdot \left[\frac{d\widetilde{\tau}}{db}\left(1 - \alpha\frac{\partial \widetilde{K}}{\partial \tau}e^{\mu_1 t}\right) - \alpha\frac{\partial \widetilde{K}}{\partial b}e^{\mu_1 t}\right] \\ & + \left[\tau_t(F_K - rb) + (1-\tau_t)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right](1 - e^{\mu_1 t})\left(\frac{\partial \widetilde{K}}{\partial \tau}\frac{d\widetilde{\tau}}{db} + \frac{\partial \widetilde{K}}{\partial b}\right)\Bigg\} e^{-rt}dt. \end{aligned}$$

We now evaluate (OC.9) around steady state. Note that in steady state (OB.3) and (OA.33) give

$$\begin{aligned} F(\widetilde{K}, \widetilde{L}^m) - rb\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d] &= -U_K(\widetilde{K})\frac{\partial I_t}{\partial \tau_t}\widetilde{K} \\ &= U_K(\widetilde{K})\mu_1\frac{\partial \widetilde{K}}{\partial \tau}. \end{aligned} \tag{OC.10}$$

Moreover, according to (OB.7), we have

$$\widetilde{\tau}(F_K - rb) + (1-\widetilde{\tau})[\widetilde{L}^m + (1-\beta)\widetilde{L}^d]\frac{\partial \widetilde{w}}{\partial K} = (r - \mu_1)U_K(\widetilde{K}). \tag{OC.11}$$

Hence, we can evaluate (OC.9) around steady state where $\tau_t \approx \widetilde{\tau}, K_t \approx \widetilde{K}$, and get

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = \int_0^\infty \Bigg\{ -\widetilde{\tau} r\widetilde{K} + U_K(\widetilde{K})\mu_1 \frac{\partial \widetilde{K}}{\partial \tau}\left[\frac{d\widetilde{\tau}}{db}\left(1-\alpha\frac{\partial\widetilde{K}}{\partial\tau}e^{\mu_1 t}\right) - \alpha\frac{\partial \widetilde{K}}{\partial b}e^{\mu_1 t}\right] + (r-\mu_1)U_K(\widetilde{K})(1-e^{\mu_1 t})\left(\frac{\partial\widetilde{K}}{\partial\tau}\frac{d\widetilde{\tau}}{db}+\frac{\partial\widetilde{K}}{\partial b}\right)\Bigg\} e^{-rt}dt. \quad \text{(OC.12)}$$

Solving the integral on the right-hand side of (OC.12), and using (OC.7) and (G.8) (where in (G.8) we replace τ^* by $\widetilde{\tau}$), we get

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = -\widetilde{\tau}\widetilde{K}\left[1-\frac{\mu_1}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\frac{U_K(\widetilde{K})}{F_K - rb}\right] - \frac{\mu_1\alpha}{r-\mu_1}U_K(\widetilde{K})\frac{\partial\widetilde{K}}{\partial\tau}\frac{d\widetilde{K}}{db}. \quad \text{(OC.13)}$$

We now use (OC.10) to replace $U_K(\widetilde{K})$ in the term in brackets in (OC.13). The resulting expression is

$$\frac{d}{db}\int_0^\infty \Omega_t e^{-rt}dt = \frac{\widetilde{\tau}(1-\beta)[G(\widetilde{L}^d)-\widetilde{w}\widetilde{L}^d]}{F_K - rb} - \frac{\mu_1\alpha}{r-\mu_1}U_K(\widetilde{K})\frac{\partial\widetilde{K}}{\partial\tau}\frac{d\widetilde{K}}{db}. \quad \text{(OC.14)}$$

The first term in (OC.14) is nonnegative and identical to the term in the case of a time-invariant tax rate (see (24)). However, in the case of a time-varying tax rate, the second term emerges, and it depends on α, which determines whether the tax rate is an increasing or a decreasing function of capital, and is ambiguous. For example, in the case $\beta = 1$, the first term vanishes, and the whole expression takes the opposite sign of α. However, it is not possible to sign α, even in the case $\beta = 1$, without making further assumptions regarding the production function $F(\cdot)$ or the adjustment cost function $C(\cdot)$. Hence, we conclude that the overall welfare impact is ambiguous. This completes the proof of Proposition 5. □

OD Proof of Proposition 6

To prove Proposition 6, we first solve the firm's maximization problem, following the same steps as in Appendix B. In solving the problem, we assume that the government's TCR is binding and the firm cannot optimally choose b. While the labor demand choice remains the same as in the main model, the optimal path of investment becomes

$$\dot{I}_t = \frac{1}{C''}\left[r(1-b\tau) + C^B(rb) + C(I_t) + C'(I_t)(r-I_t) - F_K(K_t, L_t^m)(1-\tau)\right]. \quad \text{(OD.1)}$$

Thus, the steady state user cost of capital also changes. Evaluating (OD.1) in steady state, we get

$$F_K(\widetilde{K}, \widetilde{L}^m) \;=\; \frac{r(1-b\tau)+C^B(rb)}{1-\tau}. \tag{OD.2}$$

Equation (OD.2) is the equivalent of (10a) in Section 2. Following the same steps as in Appendix C, one can easily show that Lemma 1 remains unaffected. Thus, both $\partial\widetilde{K}/\partial\tau$ and μ_1 remain unchanged. On the other hand, the impact of b on the steady state capital stock is given by the total differential of (OD.2) with respect to b, taking into account $L^m = L^m(K)$. The resulting expression is

$$\frac{\partial\widetilde{K}}{\partial b} \;=\; \frac{(C^{B\,\prime}-\tau)r(F_{LL}+G_{LL})}{(1-\tau)F_{KK}G_{LL}} > 0. \tag{OD.3}$$

Note that (OD.3) is positive due to $C^{B\,\prime} - \tau < 0$ for all $b < \hat{b}$ (owing to the convexity of the cost function $C^B(\cdot)$).

Welfare is defined analogously to Section 3. The government's maximization problem is

$$\max_{\tau,b} \quad \int_0^\infty \Omega_t e^{-rt} dt. \tag{OD.4}$$

Due to the fact that Lemma 1 remains unchanged, the optimal tax rate is again given by Equation (18), where b is replaced by its optimal value b^*. The first-order condition with respect to b is

$$\frac{\partial}{\partial b} \;=\; \int_0^\infty \left\{ -\tau r K_t + \left[\tau(F_K - rb) + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right] \frac{\partial K_t}{\partial b} \right\} e^{-rt} dt = 0, \tag{OD.5}$$

where $\partial K_t/\partial b = (1-e^{\mu_1 t})\partial\widetilde{K}/\partial b$. We first prove that $0 \le b^* < \hat{b}$. Evaluate the first-order condition (OD.5) at $b = 0$. Note that, in this case, (OD.3) becomes

$$\frac{\partial\widetilde{K}}{\partial b}(b=0) \;=\; \frac{-\tau r(F_{LL}+G_{LL})}{(1-\tau)F_{KK}G_{LL}} = -\frac{\tau r}{F_K}\frac{\partial\widetilde{K}}{\partial\tau} > 0. \tag{OD.6}$$

Moreover, the first-order condition with respect to τ, evaluated at $b = 0$, gives

$$\int_0^\infty \left[F(K_t, L_t^m) - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]\right] e^{-rt} dt$$

$$= -\int_0^\infty \left[\tau F_K + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\frac{\partial K_t}{\partial \tau}e^{-rt}dt. \qquad \text{(OD.7)}$$

Using (OD.6) and (OD.7), we get

$$\begin{aligned}
\frac{\partial}{\partial b}(b=0) &= \int_0^\infty \left\{ -\tau r K_t - \frac{\tau r\left[\tau F_K + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]}{F_K}\frac{\partial K_t}{\partial \tau}\right\} e^{-rt}dt, \\
&= -\tau r \int_0^\infty \left\{ K_t - \frac{F(K_t, L_t^m) - w_t L_t^m + (1-\beta)[G(L_t^d) - w_t L_t^d]}{F_K}\right\} e^{-rt}dt \\
&= \tau r(1-\beta)\int_0^\infty \frac{G(L_t^d) - w_t L_t^d}{F_K}e^{-rt}dt \geq 0, \qquad \text{(OD.8)}
\end{aligned}$$

where in the last equality, we used the expression $F = F_K K + F_L L^m$. Thus, b^* is strictily positive for $\beta < 1$ and zero for $\beta = 1$. Next, evaluate (OD.5) at $b = \hat{b}$. Note that in this case, $C^{B\prime}(\hat{b}) = \tau$ and $\partial \widetilde{K}/\partial b = 0$. Hence, the first-order condition (OD.5) becomes

$$\frac{\partial}{\partial b}(b=\hat{b}) = -\int_0^\infty \tau r K_t e^{-rt}dt < 0. \qquad \text{(OD.9)}$$

Hence, $b^* < \hat{b}$. This proves the first part of Proposition 6. To prove the second part, evaluate the integrand of (OD.5) at $t = 0$, taking into account $\partial K_0/\partial b = 0$:

$$\frac{\partial \Omega_0}{\partial b} = -\tau r K_0 < 0. \qquad \text{(OD.10)}$$

Therefore, there is a negative welfare effect in period 0. Because the change in welfare $\partial\Omega_t/\partial b$ is continuous in time, it is also negative for slightly positive values of t. Denote the largest value of t for which $\partial\Omega_t/\partial b$ is negative as t^*. Then, we must have

$$\int_0^{t^*} \frac{\partial \Omega_t}{\partial b}e^{-rt} < 0. \qquad \text{(OD.11)}$$

□

OE Tranfser price manipulation and proof of Proposition 7

We first solve the firm's maximization problem by following the same steps as in Appendix B. The optimal labor demand, transfer price and path of investment are determined by

$$w = F_L(K_t, L_t^m), \tag{OE.1}$$
$$\tau = C^{\sigma\,\prime}((\hat{\sigma} - r)b), \tag{OE.2}$$
$$\dot{I}_t = \frac{1}{C''}\left[r - \hat{\sigma} b\tau + C^{\sigma}((\hat{\sigma} - r)b) + C(I_t) + C'(I_t)(r - I_t) - F_K(K_t, L_t^m)(1-\tau)\right], \tag{OE.3}$$

where $\hat{\sigma}$ denotes the optimal interest rate. Equations (2), (OE.1), and (9) determine the labor demands L_t^d, L_t^m, and the wage rate w_t as functions of the capital stock, implicitly defined by (11). Evaluating (OE.3) in steady state, we derive the steady state user cost of capital

$$F_K(\widetilde{K}, \widetilde{L}^m) = \frac{r - \hat{\sigma} b\tau + C^{\sigma}((\hat{\sigma} - r)b)}{1-\tau}. \tag{OE.4}$$

Equation (OE.4) is analogous to (10a) in Section 2. Using (11), (OE.2) and (OE.4), we derive the following effects of b and τ on the transfer price $\hat{\sigma}$ and steady state capital stock $\widetilde{K}$:

$$\frac{\partial \hat{\sigma}}{\partial \tau} = \frac{1}{bC^{\sigma\,\prime\prime}} > 0, \tag{OE.5a}$$
$$\frac{\partial \hat{\sigma}}{\partial b} = -\frac{\hat{\sigma} - r}{b} < 0, \tag{OE.5b}$$
$$\begin{aligned}\frac{\partial \widetilde{K}}{\partial \tau} &= \frac{(r - \hat{\sigma} b - C^{\sigma})(F_{LL} + G_{LL})}{(1-\tau)^2 F_{KK} G_{LL}} \\ &= \frac{(F_K - \hat{\sigma} b)(F_{LL} + G_{LL})}{(1-\tau) F_{KK} G_{LL}} < 0, \quad \text{if} \quad F_K - \hat{\sigma} b > 0,\end{aligned} \tag{OE.5c}$$
$$\begin{aligned}\frac{\partial \widetilde{K}}{\partial b} &= \frac{(C^{\sigma\,\prime} \cdot (\hat{\sigma} - r) - \hat{\sigma}\tau)(F_{LL} + G_{LL})}{(1-\tau) F_{KK} G_{LL}} \\ &= -\frac{r\tau(F_{LL} + G_{LL})}{(1-\tau) F_{KK} G_{LL}} > 0.\end{aligned} \tag{OE.5d}$$

Following the same steps as in Appendix C, one can easily show that Lemma 1 remains unaffected, when one replaces $(F_K - r\tau)$ by $(F_K - \hat{\sigma}\tau)$ in $\partial \widetilde{K}/\partial \tau$.

Welfare is defined analogously to Section 3. Period t welfare, Ω_t, is thus defined by

$$\Omega_t = \tau[F(K_t, L_t^m) - \hat{\sigma} b K_t] + (1-\tau) w_t L_t^m + G(L_t^d) - (1-\tau)(1-\beta)[G(L_t^d) - w_t L_t^d]. \tag{OE.6}$$

Hence, the only difference to Section 3 is the change in the interest costs from rbK_t to $\hat{\sigma} b K_t$. The government's maximization problem is

$$\max_{\tau} \int_0^{\infty} \Omega_t e^{-rt} dt. \tag{OE.7}$$

Following the same steps as in Appendix D, we derive the following equation for the optimal tax rate τ^*:

$$\begin{aligned} 0 &= F(\widetilde{K}, \widetilde{L}^m) - \hat{\sigma} b \widetilde{K} - \widetilde{w} \widetilde{L}^m + (1-\beta)[G(\widetilde{L}^d) - \widetilde{w} \widetilde{L}^d] - \frac{\tau^* \widetilde{K}}{C^{\sigma\prime\prime}} \\ & - \frac{\mu_1}{r - \mu_1} (F_K(\widetilde{K}, \widetilde{L}^m) - \hat{\sigma} b) \left[\tau^* \frac{\partial \widetilde{K}}{\partial \tau} - \frac{\widetilde{K}(1 - \beta \widetilde{L}^d)}{\widetilde{L}^m} \right]. \end{aligned} \tag{OE.8}$$

Using Equations (OE.4) and (OE.8), and taking into account (OE.5a), (OE.5b), we can derive the impact of a change in debt financing, db, on τ^* and $\widetilde{K}$. Following the same steps as in Appendix F, we first derive the effects for the case $\beta \in [0, 1]$, and then for the case $\beta = 1$. For brevity, I report only the resulting expressions in the case $\beta = 1$. Note first that in the case $\beta = 1$, the optimal tax rate is determined by

$$\frac{\tau^*}{C^{\sigma\prime\prime}} = \frac{F_K - \hat{\sigma} b}{r - \mu_1} \left[r - \frac{\mu_1 \tau^*}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} \right]. \tag{OE.9}$$

The comparative static effects of a change in b for the case $\beta = 1$ are

$$\begin{aligned} \frac{d\widetilde{K}}{db}(\beta = 1) &= \frac{1}{|J|(r - \mu_1)} \left\{ \frac{(F_K - \hat{\sigma} b) r \widetilde{K}}{(r - 2\mu_1)(1 - \tau^*)} \left[(r(1 - \tau^*) - 2\mu_1(1 - 2\tau^*)) \left(r - \frac{\mu_1 \tau^*}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} \right) \right. \right. \\ & \left. \left. - \mu_1 r \tau^* \right] - \frac{r \tau^* \widetilde{K}}{C^{\sigma\prime\prime}} \left[\mu_1 - 2 \left(r - \frac{\mu_1 \tau^*}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} \right) \right] \right\} > 0, \quad \text{if} \quad \tau^* < \frac{1}{2}, \end{aligned} \tag{OE.10}$$

$$\begin{aligned} \frac{d\tau^*}{db}(\beta = 1) &= \frac{r}{|J|(r - \mu_1)} \left\{ \frac{F_{KK} G_{LL} \widetilde{K}}{F_{LL} + G_{LL}} \left[r - \frac{2\mu_1 \tau^*}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} \right] - \frac{\tau^2}{C^{\sigma\prime\prime}} \right. \\ & \left. + \tau^* (F_K - \hat{\sigma} b) \left[r - \frac{\mu_1 (1 + \tau^*)}{1 - \tau^*} + \frac{\mu_1 \left[2(r - \mu_1) + r\xi - (r + 2\mu_1^{\xi}) \frac{\tau^*}{\widetilde{K}} \frac{\partial \widetilde{K}}{\partial \tau} \right]}{r - 2\mu_1} \right] \right\} \gtrless 0, \end{aligned} \tag{OE.11}$$

where $|J| > 0$ owing to the second-order condition (see Appendix F) and the term $r - (\mu_1\tau^*/\widetilde{K})(\partial\widetilde{K}/\partial\tau) > 0$ owing to (OE.9). Because $d\widetilde{K}/db$ and $d\tau^*/db$ are continuous in β, there exists $\hat{\beta} \in [0,1[$ such that for $\beta \geq \hat{\beta}$ the signs of (OE.10) and (OE.11) continue to hold.

It remains to derive the welfare effects of a change in debt financing. Consider first the long term impact on $\widetilde{\Omega}$. It is derived analogously to Appendix G and equals

$$\begin{aligned}\frac{d\widetilde{\Omega}}{db} &= -\frac{r}{\mu_1}\left\{r\tau^*\widetilde{K} + \frac{\tau^*(r-\mu_1)}{F_K - \hat{\sigma}b}\left[(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right]\right. \\ &\quad \left. - \left[(F_K - \hat{\sigma}b)\widetilde{K} + (1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right]\frac{d\tau^*}{db}\right\}. \qquad \text{(OE.12)}\end{aligned}$$

Evaluate (OE.12) at $\beta = 1$. We get

$$\frac{d\widetilde{\Omega}}{db}(\beta = 1) = -\frac{r}{\mu_1}\left[r\tau^*\widetilde{K} - \frac{\tau^{*2}(r-\mu_1)\widetilde{K}}{(F_K - \hat{\sigma}b)C^{\sigma\prime\prime}} - \left((F_K - \hat{\sigma}b)\widetilde{K} - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right)\frac{d\tau^*}{db}\right]. \qquad \text{(OE.13)}$$

Use now (OE.5c) and (OE.5d) to decompose $d\widetilde{K}/db$:

$$\frac{d\widetilde{K}}{db} = \frac{\partial\widetilde{K}}{\partial\tau}\frac{d\tau^*}{db} + \frac{\partial\widetilde{K}}{\partial b} = \frac{\partial\widetilde{K}}{\partial\tau}\left[-\frac{r\tau^*}{F_K - \hat{\sigma}b} + \frac{d\tau^*}{db}\right]. \qquad \text{(OE.14)}$$

Using (OE.14), we can simplify (OE.13):

$$\frac{d\widetilde{\Omega}}{db}(\beta = 1) = \frac{r\widetilde{K}}{\mu_1}\left[\frac{\frac{d\widetilde{K}}{db}}{\frac{\partial\widetilde{K}}{\partial\tau}}\left((F_K - \hat{\sigma}b) - \frac{\tau^*}{C^{\sigma\prime\prime}}\right) - \frac{\tau^{*2}\mu_1}{(F_K - \hat{\sigma}b)C^{\sigma\prime\prime}}\right]. \qquad \text{(OE.15)}$$

Using further (OE.9), (OE.15) can be simplified to

$$\frac{d\widetilde{\Omega}}{db}(\beta = 1) = -\frac{r\widetilde{K}}{\mu_1}\left[\frac{\frac{d\widetilde{K}}{db}}{\frac{\partial\widetilde{K}}{\partial\tau}}\frac{\mu_1(F_K - \hat{\sigma}b)}{r - \mu_1}\left(1 - \frac{\tau^*}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\right) + \frac{\tau^{*2}\mu_1}{(F_K - \hat{\sigma}b)C^{\sigma\prime\prime}}\right] \text{(OE.16)}$$

In the case $\tau^* < 1/2$, (OE.10) tells us that the first term in brackets in (OE.16) is positive. However, the second term is unambiguously negative. Hence, the change in long term welfare is ambiguous.

Next, derive the short term welfare change. Defining short term welfare analogously to Appendix G, Equation (G.11), one can derive the short term welfare impact as

$$\frac{d\Omega_0}{db} = -\tau^*\widetilde{K}\left[\hat{\sigma} + \frac{\partial\hat{\sigma}}{\partial b}b\right] + \left[F - \hat{\sigma}b\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)(G - \widetilde{w}\widetilde{L}^d) - \tau^*b\widetilde{K}\frac{\partial\hat{\sigma}}{\partial\tau}\right]\frac{d\tau^*}{db}$$

$$= -\tau^* r\widetilde{K} + \left[F - \hat{\sigma} b\widetilde{K} - \widetilde{w}\widetilde{L}^m + (1-\beta)(G - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right]\frac{d\tau^*}{db}. \tag{OE.17}$$

In the case $\beta = 1$, we get

$$\frac{d\Omega_0}{db}(\beta = 1) = \widetilde{K}\left[-\tau^* r + \left((F_K - \hat{\sigma} b) - \frac{\tau^*}{C^{\sigma\prime\prime}}\right)\frac{d\tau^*}{db}\right]. \tag{OE.18}$$

Use (OE.9) to solve for the term multiplied with $d\tau^*/db$ in (OE.18). We get

$$\frac{d\Omega_0}{db}(\beta = 1) = \widetilde{K}\left[-\tau^* r - \frac{\mu_1(F_K - \hat{\sigma} b)}{r - \mu_1}\left(1 - \frac{\tau^*}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\right)\frac{d\tau^*}{db}\right]. \tag{OE.19}$$

Alternatively, (OE.18) can be rewritten using (OE.14), which gives

$$\frac{d\Omega_0}{db}(\beta = 1) = \widetilde{K}\left[(F_K - \hat{\sigma} b)\frac{\frac{d\widetilde{K}}{db}}{\frac{\partial\widetilde{K}}{\partial\tau}} - \frac{\tau^*}{C^{\sigma\prime\prime}}\frac{d\tau^*}{db}\right]. \tag{OE.20}$$

Note that, on the one hand, (OE.19) is negative if $d\tau^*/db < 0$. On the other hand, if $\tau^* < 1/2$, the first term in brackets in (OE.20) is negative. If, additionally, $d\tau^*/db > 0$, the second term is also negative and (OE.20) is overall negative. Hence, irrespective of the sign of $d\tau^*/db$, the condition $\tau^* < 1/2$ is sufficient for a negative short term welfare impact, when $\beta = 1$. Because $d\widetilde{K}/db$ is positive for $\beta \in [\hat{\beta}, 1]$, the same range of values for β leads to a negative short term welfare impact.

It remains to derive the overall welfare impact. It is derived analogously to Appendix H. Noting that over the whole planning horizon, the term $d\tau^*/db$ does not affect welfare (see the derivation of Equation (H.4) in Appendix H), the welfare impact can be written as

$$\frac{\partial}{\partial b}\int_0^\infty \Omega_t e^{-rt}dt = \int_0^\infty \Bigg\{ -\tau r K_t + \left[\tau(F_K - \hat{\sigma} b) + (1-\tau)[L_t^m + (1-\beta)L_t^d]\frac{\partial w_t}{\partial K_t}\right]\cdot$$
$$\cdot\frac{\partial K_t}{\partial b}\Bigg\} e^{-rt}dt. \tag{OE.21}$$

where $\partial K_t/\partial b = (1 - e^{\mu_1 t})\partial\widetilde{K}/\partial b$. Evaluate (OE.21) near steady state, where $K_t \approx \widetilde{K}, w_t \approx \widetilde{w}, L_t^m \approx \widetilde{L}^m, L_t^d \approx \widetilde{L}^d$. After some simplification, we get

$$\frac{\partial}{\partial b}\int_0^\infty \Omega_t e^{-rt}dt = \frac{\tau^*\widetilde{K}}{r - \mu_1}\left[-r - \mu_1(1-\beta)\frac{\widetilde{L}^d}{\widetilde{L}^m} + \mu_1\frac{\tau^*}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\right]. \tag{OE.22}$$

Note now that (OE.8) can be rewritten as

$$-r - \mu_1(1-\beta)\frac{\widetilde{L}^d}{\widetilde{L}^m} + \mu_1\frac{\tau^*}{\widetilde{K}}\frac{\partial \widetilde{K}}{\partial \tau} = \frac{r-\mu_1}{(F_K - \hat{\sigma}b)\widetilde{K}}\left[(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right]. \tag{OE.23}$$

Hence, we can simplify (OE.22) to

$$\frac{\partial}{\partial b}\int_0^\infty \Omega_t e^{-rt}dt = \frac{\tau^*}{F_K - \hat{\sigma}b}\left[(1-\beta)(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}}\right]. \tag{OE.24}$$

If $\beta = 1$, then (OE.24) is unambiguously negative for any value of b. By continuity, this holds also for β sufficiently close to 1. Hence, there exists a value $\underline{\beta} < 1$, defined by implicitly

$$(1-\underline{\beta})(G(\widetilde{L}^d) - \widetilde{w}\widetilde{L}^d) - \frac{\tau^*\widetilde{K}}{C^{\sigma\prime\prime}} = 0, \tag{OE.25}$$

such that for $\beta \in]\underline{\beta}, 1]$, the overall welfare change in negative. If $\underline{\beta} > 0$, then there exist $\beta < \underline{\beta}$ for which welfare is increasing in b. If, on the other hand, $\underline{\beta} < 0$, then there do not exist values of β for which the welfare impact is positive. Hence, the welfare impact is determinate and negative only for sufficiently high values $\beta \in]\underline{\beta}, 1]$. □