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Poverty and Individual Responsibility

Abstract

Poverty-reducing policies ought to prioritize the "deserving" poor, that is, those who do all that can be reasonably expected from them in their circumstances, but fail to achieve a minimum standard of living. To inform such policies, one needs a theory of justice accommodating norms of individual responsibility. I propose and axiomatically characterize a family of poverty indices that address these issues. Formally, poverty is measured by the sum of specific indices of individual deprivation which (i) keep individuals accountable for their choices, (ii) compare individuals based on the set of attainments they are deprived of, and (iii) prioritize the most deprived individuals. I illustrate the results with Norwegian register data. Among single males, about a third of the income-poor ones are "undeserving," because (based on the estimates) they are unwilling to accept a job that brings them out of poverty.

JEL-Codes: D630, I300.

Keywords: multidimensional poverty, responsibility, fairness.

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1 Introduction

Standard indices of poverty disregard individual responsibility. These indices are unable to distinguish the "deserving" poor—those who are deprived of the opportunity for a good-enough life—from the "undeserving" poor—those who could achieve a higher income, but willingly decide not to. Consequently, these indices are inappropriate to evaluate poverty alleviation policies and guide policy intervention.[1]

In this paper, I propose and axiomatically characterize a novel family of poverty indices that keep individuals accountable for their choices. This contribution prepares the ground for assessing poverty, for analyzing policy implications, and for designing optimal intervention. The goal is to construct a criterion for comparing social situations: the level of poverty ought to reflect the extent to which individuals are deprived, by keeping them responsible for their choices.

The importance of individual responsibility emerges particularly strongly for labor supply choices. For example, an increasing proportion of young men have been shifting from work hours to video gaming and other recreational activities (Aguiar et al., 2017).[2] Their preference for leisure lowers their salaries and pushes some of them into income poverty. Are these individuals lacking the opportunity for a good-enough life? The principles of justice proposed here suggest these individuals are undeserving poor: making them better-off does not decrease the level of poverty.

The key innovation of this paper concerns the role of individuals' choices for measuring poverty. Here, individuals are held accountable for their choices.[3] Individuals' choices are informative about the opportunities they

[1]The term "undeserving" is often misused in political debates. When describing the poor, pejorative stereotyping is frequent, distorts reality, and is often used to argue for one or the other policy. This paper provides a framework to study—both theoretically and empirically—how policies affect poverty in society, differentiating between individuals based on their responsibility and, thus, deservingness.

[2]Another example is the switch to part-time jobs by individuals close to retirement age (Zhao and Burge, 2017).For example, partners of retirees might choose to reduce their labor supply and might see their income decrease below the level of the poverty line.

[3]Not all agree with keeping individuals accountable for their choices. Among those, the

are deprived of and, thus, their deprivation status. Formally, let x_i be the attainment vector of individual i, specifying all relevant outcomes for an individual (income, leisure, etc.). Then, the deprivation of i, $\lambda_i(x_i)$, measures the set of attainments that individual i has no access to, based on the current situation x_i and the attainments that i would want to have access to.

Poverty in society is the sum of a convex transformation of individuals' deprivations, where convexity captures the priority attributed to the most deprived individuals in society. Let f be such a convex transformation, named priority function. Then, poverty is measured by

$$P = \frac{1}{n} \sum_i f\left(\lambda_i\left(x_i\right)\right),$$

where n is the population size.

In a one-dimensional setting, say income, monotonicity of preferences (more income is better than less) rules out different choices. Let y_p be the poverty line. At income y_i, the opportunities an individual is deprived of can be measured by the (relative) income gap: $\lambda_i(y_i) = \frac{y_p - y_i}{y_p}$ if $y_i \leq y_p$ and $\lambda_i(y_i) = 0$ if $y_i > y_p$. Then, the index P simplifies to a standard poverty-gap index

$$P = \frac{1}{n} \sum_i f\left(\max\left[\frac{y_p - y_i}{y_p}, 0\right]\right),$$

which is consistent with the mean income gap—when f is linear—and, more generally, with the Foster-Greer-Thorbecke (FGT) family (Foster et al., 1984)—when f is a power function.

philosopher Philippe van Parijs (1991) argued against restricting welfare programs to the "deserving" poor, claiming that even the Malibu surfers should be fed. As he reports, the Hawaii senator Wadsworth Yee—discussing the newly-introduced residence requirement for welfare support in 1971—stated: "There must be no parasites in paradise." Van Parijs argued that a universal basic income is the policy that best ensures freedom of choice—the greatest opportunities—for all. As will become clear, the results discussed below confirm the importance of opportunities. However, here deprivation will depend on the extent of missing opportunities, rather than on the access to basic opportunities. This difference is key to introduce individual responsibility. Importantly, this paper is not concerned with poverty in developing countries, where poor individuals have virtually no way to exit poverty on their own. As recently suggested by Allen (2017), when people struggle with survival, "necessity displaces desire" (see also Deaton (2016)).

However, economists and philosophers widely agree that individuals' deprivation status cannot be established based solely on income. People differ also in other dimensions, such as health, access to housing, education, liberties, etc. There is little consensus, however, on how to accommodate and combine these dimensions. Clearly, the importance of reporting and summarizing the achievements of the fight against poverty led to a significant number of proposals (see the recent survey by Alkire et al. (2015)). However, more and more authors express concerns about a number of drawbacks of existing indices, which often lead to controversial policy recommendations (see Alkire and Foster (2011b); Ravallion (2011); Thorbecke (2011); Aaberge and Brandolini (2015); Cowell (2015); Duclos and Tiberti (2016); Ravallion (2019)). The family of poverty indices characterized here avoid these drawbacks.

I introduce the following axioms. *Responsibility* requires society to respect how individuals would make choices based on their own preferences. *Continuity* requires small changes in individuals' attainments to lead to small changes in the level of poverty. *Equal-preference transfer* is a multidimensional version of the Pigou–Dalton principle, restricted to individuals with the same preferences: it requires society to prioritize the most deprived individuals. *Separability* imposes the ranking of two alternatives to be independent of the attainments of an individual who is unconcerned by the choice. Finally, *deprivation fairness* introduces a multidimensional equivalent of the poverty line and prevents some types of discrimination.

The empirical illustration with Norwegian register data highlights the importance of keeping individuals responsible for their choices in measuring poverty. I adopt a random-utility model to estimate the preferences of single men of working age without children. I show that income poverty gives a very different view of reality than the index characterized here. In the sample, about 3 percent of individuals earn less than 60 percent of the median income and, thus, are income poor. However, one third of these individuals are not considered "deprived" here. These individuals would not accept a job that brings them out of poverty and constitute the group of *non-deserving poor*.[4]

Interestingly, another 2.4 percent of individuals are not income poor, but should be considered "deprived." These individuals work significantly more than 40h/week and earn little above 60 percent of the median income; according to the estimates, these individuals would be happy to accept a single full-time job, even if it was paying slightly less than 60 percent of the median income, and are thus deprived of the opportunity of a single full-time job that brings them out of income poverty. In modern societies, people who have multiple jobs constitute a significant and increasing share of the labor force.[5] Nevertheless, this phenomenon remains largely unaddressed in economic evaluations and poverty alleviation policies.

Accounting for choices requires a multidimensional framework. Yet, there is no consensus on how to measure multidimensional poverty. The standard framework is to aggregate the multidimensional attainments of each individual into a real value. The social state is thus summarized by a matrix, where each cell specifies the attainment in a specific dimension for a specific individual. It follows that there are two main aggregation methods: over individuals first or over dimensions first.

The first aggregation method—individuals first—is as follows. Define an index of poverty for each dimension separately. Then, composite poverty (or "mashup" index) is an aggregation of these dimension-specific indices of poverty.[6] A well-known example is the Human Development Index (Anand and Sen, 1994). This index is however insensitive to concentration of dimension-

[4]These individuals have been named "non-needy bohemians" by Arneson (1997). He clarifies that "some persons with low incomes will be talented individuals leading rich and satisfying lives, who happen to have aims and values such that they need a lot of leisure and very little cash income to fulfill their most important aims" (Arneson, 1997, p. 345). This category also includes the Malibu surfers discussed by van Parijs (see footnote 3).

[5]As documented by the Bureau of Labor Statistics, people with multiple jobs accounted for 8.3 percent of the employed individuals in the U.S. in 2013 (see www.census.gov). Moreover, 34.8 percent of men and 24.2 percent of women have both the main and the secondary job as full-time employments (defined as working more than 34h/week).

[6]Some authors argue that aggregation across dimensions is *ad hoc* and unnecessary. They instead suggest reporting the entire vector of dimension-specific deprivations (Hicks and Streeten, 1979). Unfortunately, this "dashboard" approach is generally unable to provide a clearcut comparison of social states and, in particular when many dimensions are considered, it may prove difficult to interpret.

specific deprivations among individuals.

The "dual cutoff method" by Alkire and Foster (2011a) addresses this drawback. The first cutoff defines a vector of dimension-specific poverty lines. The second cutoff establishes whether the number of dimension-specific deprivations are sufficient to indentify an individual as (overall) poor. For example, the "intersection approach" considers an individual as poor if she is deprived in all dimensions. Then, multidimensional poverty is the sum of the (possibly weighted and transformed) dimension-specific attainment gaps experienced by the poor individuals. With this proposal, the joint distribution of deprivations now matters. Yet, the joint distribution matters only in identifying the poor individuals: the distribution of dimension-specific attainment gaps among poor individuals remains unaccounted for.[7]

The second aggregation method—over dimensions first—is often labeled the "social welfare approach" to multidimensional poverty (Atkinson, 2003). A "utility-like" function evaluates individuals' attainments, before the aggregation across individuals. Bourguignon and Chakravarty (2003) argue that society ought to choose the utility-like functions for the purpose of evaluating deprivation (see also Duclos et al., 2006).[8] This utility-like function reflects the preferences of society: it identifies when individuals are poor, it identifies how to compare poverty across individuals, and it specifies how much priority to assign to the worst-off individuals.

The present contribution extends the poverty measures of Bourguignon and Chakravarty (2003) to account for individual responsibility. In particular, the characterization singles out poverty contours similar to those assumed by Bourguignon and Chakravarty (2003). However, these contours are not directly adopted for assessing poverty, but rather for assessing the opportunities that individuals are deprived of. A related extension of Bourguignon

[7]Moreover, this proposal leads to an inconsistency between the identification of the poor and the extent of their poverty. Said differently, replacing the attainment gap of a poor individual with the attainment gap of a non-poor might increase the measured level of poverty.

[8]An alternative method is to adopt individuals' cardinal and interpersonally comparable utility functions (Kingdon and Knight, 2006). However, economists and philosophers have convincingly contended that poverty is about lack of access to resources or attainments, rather than happiness (Sen, 1979).

and Chakravarty (2003) is the family of preference-sensitive indices proposed by Decancq et al. (2019) (see also Dimri and Maniquet (2019) for an application). Beyond the characterization, two main differences emerge with the present contribution. First, the criteria in Decancq et al. (2019) establish interpersonal comparability based on *access* to opportunities—as more standard when measuring individuals' well-being (see Fleurbaey and Maniquet (2011); Piacquadio (2017))—while the present criteria do so based on *missing* opportunities. Second, their criteria adopt a weaker axiom of inequality aversion. As a result, their criteria may approve a transfer of attainments from a more deprived individual to a less deprived one as poverty reducing, even if these individuals have the same preferences.

The rest of the paper is organized as follows. Section 2 presents the framework, the axioms, and the main characterization result. Section 3 contains the empirical application of the criteria. Section 4 extends the results to households with possibly different size, composition, and, more generally, needs. I also extend the results to categorical attainments, ensuring the applicability of the criteria to evaluate discrete levels of health, dichotomous indices of access to higher education, etc. Section 5 concludes. The appendix contains all the proofs.

2 Model, axioms, and characterization

2.1 The model

The set of individuals is $N \equiv \{1, ..., n\}$, with $n \geq 3$ and finite. Each individual $i \in N$ is characterized by well-behaved (complete, transitive, continuous, strongly monotonic, and strictly convex) **preferences** R_i defined over the $\ell-$dimensional commodity space $X \equiv \mathbb{R}^{\ell}_{+}$, with ℓ finite. The strict preference and indifference relations induced by R_i are denoted P_i and I_i, respectively. For each i, R_i admits a strictly increasing and concave numerical representation $u_i : X \to \mathbb{R}$ (a utility function) with $\lim_{|x_i|\to\infty} u_i(x_i) = \infty$. Each preference relation is shared by at least two individuals.[9]

A **social state** $x_N \equiv (x_1, ..., x_n)$ assigns an attainments vector x_i to each individual $i \in N$. The set of all possible social states is $X_N \equiv X^n$. A **poverty ranking**, denoted $\trianglerighteq$, is a weak ordering of social states. For each pair of social states $x_N, x'_N \in X_N$, $x_N \trianglerighteq x'_N$ means that x_N is characterized by at least as much deprivation as x'_N. The asymmetric and symmetric relations induced by $\trianglerighteq$ are denoted $\triangleright$ and $\backsimeq$. A **poverty index**, denoted $P : X_N \to \mathbb{R}$, is a numerical representation of the poverty ranking $\trianglerighteq$; that is, $x_N \trianglerighteq x'_N$ holds if and only if $P(x_N) \geq P(x'_N)$.

2.2 The axioms

The first axiom introduces responsibility for choices. If no individual would choose her attainments vector at $x'_N \in X_N$ over her attainments vector at $x_N \in X_N$, then the social state x_N cannot have more poverty than the social state x'_N. In other words, a necessary condition for increasing deprivation in society is that at least one individual be forced to an attainments vector she finds less desirable.[10]

Responsibility: For each pair $x_N, x'_N \in X_N$, $x_i \, R_i \, x'_i$ for each $i \in N$ implies $x'_N \trianglerighteq x_N$.

Next, small changes in the social state do not cause large jumps in the level of poverty.

Continuity: For each $x_N \in X_N$, the set $\{x'_N \in X_N \,| x'_N \trianglerighteq x_N\}$ and the set $\{x'_N \in X_N \,| x_N \trianglerighteq x'_N\}$ are closed.

Next, the poverty ranking of two social states is independent of the attainments vector of an individual who is unconcerned by these alternatives.[11]

[9]This requirement permits the introduction of an axiom that holds only for equal-preference individuals. An alternative is to allow for "clone" economies and introduce a consistency requirement: the evaluation of poverty between two situations should extend when evaluating the same situations for the original economy together with its clones. For a similar axiom, see Fleurbaey and Maniquet (2011).

[10]This axiom corresponds to a weak form of Pareto efficiency.

[11]I adopt the following notation: for each $x_N \in X_N$, each $i \in N$, and each $a_i \in X$, $(a_i, x_{-i}) \in X_N$ denotes the social state that assigns a_i to i and x_j to each $j \in N \setminus \{i\}$.

Separability: For each pair $x_N, x'_N \in X_N$, if there is $i \in N$ such that $x_i = x'_i \equiv a_i$, then for each $b_i \in X$,

$$(a_i, x_{-i}) \trianglerighteq \left(a_i, x'_{-i}\right) \Longleftrightarrow (b_i, x_{-i}) \trianglerighteq \left(b_i, x'_{-i}\right).$$

The next axiom introduces inequality aversion: society should prioritize the worst-off individuals. The diversity of choices across individuals makes the identification of the worst off challenging. Thus, inequality aversion is exclusively imposed on individuals with the same preferences. Formally, the axiom introduces (a weak form of) concavity with respect to the allocations of equal-preference individuals.[12]

Equal-Preference Transfer: For each pair $j, k \in N$ such that $R^* \equiv R_j = R_k$, if there exists a pair $x_N, x'_N \in X_N$ and $\alpha \in \left[0, \frac{1}{2}\right]$ such that:

(i) $x_j - \alpha (x_j - x_k) = x'_j \; R^* \; x'_k = x_k + \alpha (x_j - x_k)$;

(ii) for each $i \in N \backslash \{j, k\}$, $x_i = x'_i$;

then $x_N \trianglerighteq x'_N$.

The last axiom introduces the possibility of poverty and prevents a certain form of discrimination: the deprivation of some individual cannot be considered more important than that of others. First, there exists a subset of (sufficiently large) attainments vectors C such that whether an individual is assigned such attainments or more does not affect poverty. Second, consider a social state where everyone is assigned such a sufficiently large attainments vector, that is,, $x^*_N \in C^n$. Let now an individual $i \in N$ be given a different attainment vector $\bar{x}_i \in X$ so that poverty is larger, that is $\left(\bar{x}_i, x^*_{-i}\right) \rhd x^*_N$. Then, i is discriminated against if, for each attainments vector $x_i \in X$ that she finds equally desirable (i.e., such that $x_i \, I_i \, \bar{x}_i$), there is less poverty at the

[12]This axiom belongs to the family of multidimensional extensions of the Pigou–Dalton principle (see Fleurbaey and Maniquet (2011) and Piacquadio (2017)). Dalton (1920) suggested that a progressive transfer from a richer to a poorer individual (provided the richer/poorer relation is not reversed) leads to a more desirable distribution of income. Here, the allocations compared can be thought of as obtained through a multidimensional progressive transfer.

social state $\left(x_i, x^*_{-i}\right)$ than at any social state $\left(x_j, x^*_{-j}\right) \in X_N$ which assigns a larger attainments vector $x_j > x_i$ to some other individual $j \in N \backslash \{i\}$.[13] In other words, even though individual i's deprivation is always at least as large as that of others, her deprivation would count less and would lead to a lower level of poverty. Such discrimination is prevented here.

Deprivation fairness: There exists a non-empty and closed set $C \subset X$ such that

(i) for each $x^*_N \in C^n$, each $i \in N$, and each $x'_i \in X$ with $x'_i \geq x^*_i$, $\left(0, x^*_{-i}\right) \rhd x^*_N \unrhd \left(x'_i, x^*_{-i}\right)$;

(ii) for each $x^*_N \in C^n$, each $i \in N$, and each $\bar{x}_i \in X$ with $\left(\bar{x}_i, x^*_{-i}\right) \rhd x^*_N$, there exists $x_i \in X$ with $x_i \, I_i \, \bar{x}_i$ such that for each $j \in N \backslash \{i\}$ and each $x_j \in X$, $x_i < x_j$ implies $\left(x_i, x^*_{-i}\right) \rhd \left(x_j, x^*_{-j}\right)$.

2.3 Individual deprivation and poverty

Let the **no-deprivation set** be a non-empty, closed, and convex subset $X_0 \subset X$ such that, if $x \in X_0$ and $x' \geq x$, then $x' \in X$. An individual is regarded as deprived if any bundle from the *no-deprivation set* made her better-off. Thus, the *no-deprivation set* ought to include all sufficiently large attainments that ensure individuals are not deprived.

Next, I introduce the **iso-deprivation contours**. *Iso-deprivation contours* are level curves similar to the "iso-poverty contours" in Bourguignon and Chakravarty (2003) and allow comparisons of deprivation across individuals. However, to account for individual responsibility, these contours are used in combination with individuals' preferences. Let $\succsim$ be a weak order on X that is continuous, strictly antimonotonic on $X \backslash X_0$ (for each $x, x' \in X \backslash X_0$, $x \geq x'$ implies that $x' \succ x$), constant on X_0 (for each $x, x' \in X_0$, $x \sim x'$), and convex (for each $x \in X$, $\{x' \in X \,|x \succ x'\}$ is convex). Then, each iso-deprivation contour is a set of attainments equally ranked by $\succsim$.

[13] Vector inequalities are denoted $\geq$, $>$, and $\gg$.

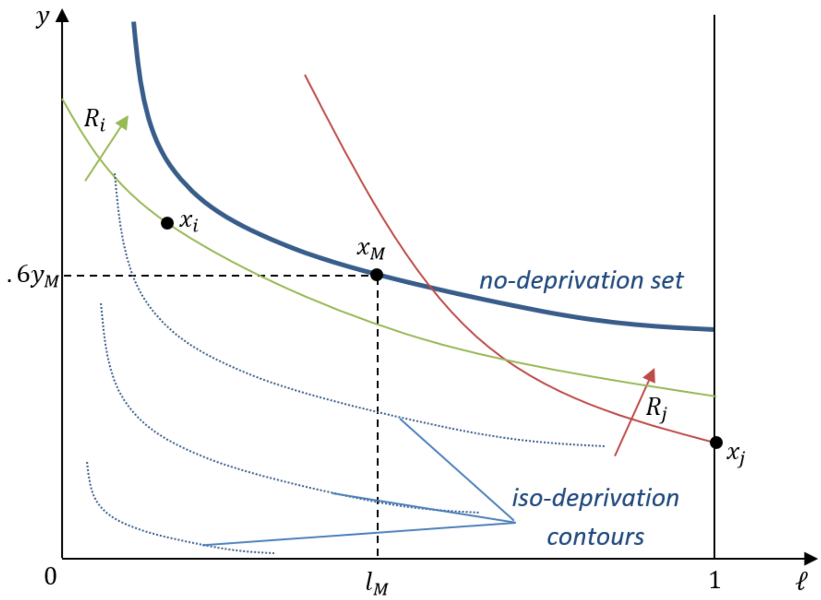

Figure 1: The no-deprivation set and the iso-deprivation contours.

The iso-deprivation contour corresponding to the lowest level of deprivation is the frontier of the *no-deprivation set.* Without loss of generality, this level of deprivation is normalized to 0. The smaller the attainments vector, the closer to the origin is the corresponding iso-poverty contour and the larger is the associated level of deprivation. The iso-deprivation contour corresponding to the highest level of deprivation is the origin (without loss of generality, this is normalized to 1). Let $\lambda : X \to [0,1]$ be a numerical representation of $\succsim$ such that $\lambda(x) = 0$ for $x \in X_0$ and $\lambda(0) = 1$.

To illustrate, consider two individuals, i and j, with attainments $x_i, x_j \in X$. Individual i is considered more deprived than individual j if there exists an *iso-deprivation contour* such that i is better off with any attainments vector from this contour, while j is not. Figure 1 illustrates the construction of the *no-deprivation set* and the *iso-deprivation contours*. Individual i with preferences R_i is not income-poor. Yet, she is considered deprived since she would be better-off with any attainments vector from the no-deprivation set. Individual j with preferences R_j is income-poor. Yet, she is not considered deprived since she would not want to switch from x_j to x_M.

Finally, I introduce the index of individual deprivation. Let $\lambda_i : X \to [0,1]$ be i's **individual deprivation function** for $\succsim$. The individual deprivation functions $\{\lambda_i\}_{i \in I}$ are the unique functions such that *(i)* $\min\{\lambda(x) \,|x\, I_i\, x_i\} = \min\{\lambda(x) \,|x\, I_j\, x_j\}$ implies $\lambda_i(x_i) = \lambda_j(x_j)$ for each $x_i, x_j \in X$ and each $i, j \in I$;[14] *(ii)* λ_i is convex for each $i \in N$; *(iii)* $\{g \circ \lambda_i\}_{i \in I}$ violates condition *(ii)* for each non-convex function g; and *(iv)* $\lambda_i(0) = 0$ and $\lambda_i(x_i) = 1$ for each $x_i \in X_0$ and each $i \in I$. Condition *(i)* establishes interpersonal comparisons of deprivation based on the iso-deprivation curves (identified by λ). Conditions *(ii)* and *(iii)* demand the individual deprivation functions to belong to the class of least convex functions.[15] Finally, condition *(iv)* is a simple normalization.

Let f be a real-valued and convex function, named **priority function**. Then, each member of the family of poverty indices characterized here can be written as follows:

$$P(x_N) \equiv \frac{1}{n} \sum_{i \in N} f(\lambda_i(x_i)) \,. \tag{1}$$

To summarize, the ingredients of (1) are the following:

1. the no-deprivation set X_0 determines that $\lambda_i(x_i) = 0$ if i's attainments vector x_i is large enough ($x_i \in X_0$);
2. the iso-deprivation contours determine the extent of deprivation of each individual in a way that is cardinal and interpersonally comparable: i is at least as deprived as j if and only if $\lambda_i(x_i) \geq \lambda_j(x_j)$;
3. the priority function f defines the priority attributed to the most deprived individuals.

The main result establishes the equivalence between poverty rankings satisfying the introduced axioms and the poverty index in (1).

[14] The existence of a least desirable attainments vector for each iso-deprivation contour is ensured by the continuity of $\succsim$ and the assumption that, for some numerical representation u_i of each individual i's preferences R_i, $\lim_{|x_i| \to \infty} u_i(x_i) = \infty$.

[15] A function ϕ is least convex if any convex function ψ can be written by composing ϕ with a convex function f, that is, $\psi = f \circ \phi$. It is unique up to an increasing affine transformation. See Debreu (1976), Kannai (1977), and Piacquadio (2017).

Theorem 1. *A poverty ranking satisfies responsibility, continuity, separability, deprivation fairness, and equal-preference transfer if and only if it can be represented by a poverty index as (1).*

3 Empirical illustration

3.1 Data

I use the 2016 Norwegian register data. I focus on the universe of Norwegian single men who do not have children and are aged between 22 and 61, consisting of 90,462 individuals.[16] The aim is to reassess income poverty by keeping individuals accountable for their labor supply choices.

The main challenge is the estimation of individuals' preferences over consumption and leisure. I estimate a random utility model, where the utility function of each individual has constant elasticity of substitution. Following Aaberge et al. (1999), I estimate 156 types of individuals, depending on age and education levels. For details of the estimation, see Appendix A.[17]

Together with the observed attainments of individuals, the estimation provides the necessary preference information for keeping individuals responsible for their choices.

3.2 Ethical choices

Three fundamental ethical choices emerge from the characterization result: the no-deprivation set, the iso-deprivation contours, and the priority for the most deprived. The characterization result naturally leaves these choices unspecified to permit applications in different frameworks and to accommodate different ethical views. In this empirical illustration, I explore one set of

[16]This focus on a specific category of individuals significantly reduces the scope for differences in needs, which I disregard here. I propose an extension of the characterization to differences in needs in Section 4.

[17]I leave to future research the empirical assessment of deprivation in Norway using all types of households.

ethical choices and explain their intuition.[18]

The no-deprivation set X_0 is the upper-contour set of a constant elasticity of substitution function. Formally, an income-leisure pair $(y, l) \in X_0$ if

$$[\beta y^{\gamma} + (1-\beta) l^{\gamma}]^{\frac{1}{\gamma}} \geq \nu.$$

The parameters $\beta, \gamma, \nu \in \mathbb{R}$ are uniquely identified by the following ethical choices:

- when an individual works 40 hours per week (the median labor supply in the sample), any income larger than (or equal to) 60 percent of the median income ensures that individual is not deprived;
- when an individual works 20 hours per week (50 percent of the median labor supply), any income larger than (or equal to) 40 percent of the median income ensures that individual is not deprived;
- when an individual works 53 hours per week (33 percent more than the median labor supply), any income larger than (or equal to) the median income ensures that individual is not deprived.

Together, these conditions imply that $\beta = 0.177$, $\gamma = -3$, $\nu = 0.33$ and uniquely identify the no-deprivation set (see Appendix A.2). Figure 2 illustrates this construction. Leisure is measured in hours per week: here, 80 hours of leisure corresponds to 0 hours of work. The dots on the frontier of the no-deprivation set each correspond to the choices illustrated above and, given the functional form, identify the no-deprivation set.

Next, I impose the iso-deprivation contours to be homothetic. Then, the iso-deprivation contour of level $\lambda \in [0, 1]$, denoted $X(\lambda)$, consists of all the

[18] I leave the analysis of alternative ethical choices to future research. Note that the approach and qualitative intuitions are robust to these ethical choices. Instead, the selection of different no-deprivation set, iso-deprivation contours, and priority function generally matter for the quantitative results.

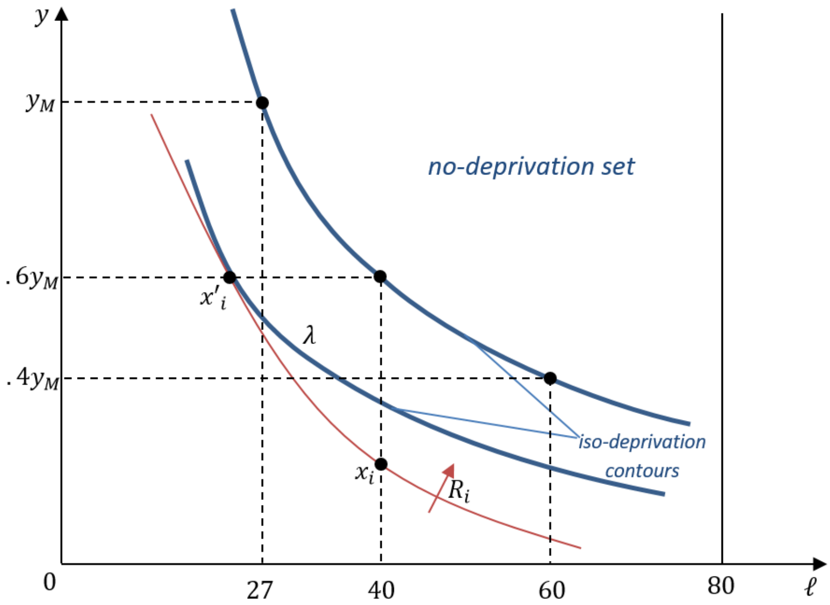

Figure 2: Ethical choices and the individual-specific deprivation function.

attainments vectors $(y, l) \in X$ such that

$$\begin{cases} [\beta y^{\gamma} + (1-\beta)\, l^{\gamma}]^{\frac{1}{\gamma}} = (1-\lambda)\,\nu & if\ \lambda > 0, \\ [\beta y^{\gamma} + (1-\beta)\, l^{\gamma}]^{\frac{1}{\gamma}} \geq \nu & if\ \lambda = 0. \end{cases}$$

The iso-deprivation contours allow constructing the individual-specific deprivation function $\lambda_i : X \to \mathbb{R}$, defined as follows. At the attainments vector (y_i, l_i), i is deprived of level $\lambda_i = \lambda$ if λ is the largest scalar for which i finds $(y_i, l_i)\, R_i\, (y, l)$ for each $(y, l) \in X(\lambda)$. Figure 2 illustrates how the individual-specific deprivation function is constructed. When individual i has the attainment x_i, she is deprived of level λ: i finds x_i equally desirable as x_i', which is the worst bundle for i from the iso-deprivation contour of level λ. Said differently, λ identifies the largest iso-deprivation contour which exclusively contains attainments i would want to switch to.

Finally, the priority function is set to a quadratic power function. Then, there is roughly the same level of poverty whether: individual i has the worst attainment (no income, no leisure), while j is not poor; or i's and j's respective incomes and leisure give them the same deprivation level of .7.[19]

Then, the index of poverty is

$$P \equiv \frac{1}{n} \sum_i [\lambda_i (y_i, l_i)]^2 .$$

3.3 A comparison with income poverty

As standard, set the poverty line at 60 percent of the median after-tax income. In the sample, the median after-tax income is $y_m = 0.333$ (measured in million NOK/year) and the poverty line is $y_p = .2$ (this corresponds to about 22,000 USD/year).

The mean income gap is $P_{mean} \equiv \frac{1}{n} \sum_i \max [y_p - y_i, 0] = .011$. This number tells that if the total income gap were distributed equally in the population, each individual would have an income gap of 1,100 NOK (about 130 USD). The head-count ratio is $P_{head} = .0297$ and tells that $2,692$ of the $90,462$ individuals are income poor, corresponding to 2.97 percent of the population.

The poverty index characterized here is $P \equiv \frac{1}{n} \sum_i [\lambda_i (y_i, l_i)]^2 = .0005$. Importantly, these levels of poverty are not comparable across indices (despite being all normalized between 0 and 1).

Thus, to shed light on the importance of individual responsibility in the measurement of poverty, I look at the extensive margin and discuss the correlation between income poverty and deprivation. The number of deprived individuals is larger (4.4 percent of the population) than the number of income-poor individuals (2.9 percent of the population). Two effects explain this difference.

First, the majority of the deprived individuals (2.4 percent of the population) are not income poor, but since they work significantly more than

[19] Note that no priority to the worse off emerges when this equal deprivation level is .5. The larger this level, the more inequality aversion is introduced. In a one-dimensional setting, a quadratic priority function tells that a situation where two individuals earn 30 percent of the poverty line has the same level of poverty as a situation where one individual earns no income (and the other earns more than the level corresponding to the poverty line).

the number of hours corresponding to the median labor supply—while they would rather not—they are considered deprived. These are the **deserving non-poor**. These individuals work significantly more than 40 hours per week and earn little above the level of income corresponding to the poverty line y_p. The importance of this category reflects the increasing number of people holding multiple jobs in modern societies.

Second, some individuals earn less than 60 percent of the median income, but since they enjoy leisure relatively more than they enjoy income, they should not be considered deprived. These individuals are the **non-deserving poor** who, even if offered, would not accept a single full-time job that would move them out of income poverty. The non-deserving poor constitute about 30 percent of the income poor (0.9 percent of the population). The importance of this category of individuals explains the conditionality and mandatory activation clauses of poverty alleviation policies.

4 Extensions

4.1 Differences in needs

As Tony Atkinson writes (1987, p. 753): "it should be noted that ... families have been assumed to be identical in their needs and the poverty line has been taken as the same for all. In practice, the poverty line is different for families of different size and differing in other respects. There is therefore scope for disagreement not just about the *level* of the poverty line but also about its *structure*." [Emphasis in the original.]

To tackle differences in needs, I introduce the following changes. First, I weaken the axioms that derive their normative appeal from the implicit assumption of equal needs. Second, I introduce information about differences in needs and, correspondingly, axioms that allow incorporating such information in the poverty ranking.

The first change is straightforward. Deprivation fairness and equal-preference transfer should be restricted to individuals with the same needs. By doing so, the axioms lead to a need-specific no-deprivation set and to need-specific iso-deprivation contours. Thus, whenever two individuals are characterized by the same needs, their deprivations are compared through the need-specific iso-deprivation contours. This change leaves open the question of how to compare and aggregate deprivations across individuals with different needs.

Accounting for differences in needs is challenging. The standard answer is to base such comparisons on equivalence scales (see Lewbel (1989) and Ebert and Moyes (2003)). Equivalence scales, however, are constructed by assuming interpersonal comparability of utilities (or equivalently deprivation levels): needs are then implicitly defined by the different quantities of commodities needed to achieve the same level of utility. Here, no information is assumed about how to make interpersonal comparisons of utilities. Instead, such information emerges endogenously from the axioms. Abiding by this approach, I suggest the only additional information available is the set of attainments vectors that are considered sufficient to cover each individual's "basic needs," a multidimensional version of Atkinson's family-specific poverty lines.

Let Θ denote a finite set of types. The population is correspondingly partitioned, that is, there are a finite number of non-empty and disjoint sets of individuals N^θ such that $N \equiv \bigcup_{\theta\in\Theta} N^\theta$. For each $\theta \in \Theta$, N^θ consists of n^θ individuals satisfying the assumptions of Section 2. For each $\theta \in \Theta$ and each $i \in N^\theta$, let the **basic needs** of i be a closed and non-empty set of attainments $B^\theta \subset X$. Let $B \equiv \left(B^\theta\right)_{\theta\in\Theta}$ define the basic needs of each type of individual. For each $\theta \in \Theta$ and each $\alpha > 0$, let $\alpha B^\theta \equiv \left\{x \in X \,\middle|\, x = \alpha b \text{ with } b \in B^\theta\right\}$. With a slight abuse of notation, let $\alpha B^\theta \, P_i \, x_i$ mean that i considers all attainments in αB^θ more desirable than x_i and let $\alpha B^\theta \, \neg P_i \, x_i$ denote its negation, that is, there is at least one alternative in αB^θ that i finds at least as desirable as x_i.

I can now formalize a version of deprivation fairness that accounts for differences in needs. The first condition is unchanged. The second condition is imposed only on individuals belonging to the same type. The third condition is new and accounts for individuals with different needs. Let individual $i \in N^\theta$ find her attainments vector insufficient to cover her basic needs B^θ, that is, $B^\theta \, P_i \, x_i$. In contrast, this is not true for individual $j \in N^{\theta'}$, that is, $B^{\theta'} \neg P_j \, x_i$. Then, i should be considered at least as deprived as j. Importantly, this requirement is also imposed for proportional expansions and contractions of the set of basic needs, that is, when for some $\alpha > 0$, $\alpha B^{\theta} \, P_i \, x_i$ and $\alpha B^{\theta'} \neg P_j \, x_j$, meaning the relation between needs of individuals is preserved when rescaling the attainments.[20]

Needs-adjusted deprivation fairness: For each $\theta \in \Theta$, there exists a non-empty and closed set $C^{\theta} \subset X$ satisfying the following conditions. Let $C_N^{\Theta} \equiv \left\{ x_N \in X^n \,\middle|\, x_i \in C^{\theta} \text{ for each } \theta \in \Theta, i \in N^{\theta} \right\}$. Then:

(i) for each $x_N^* \in C_N^{\Theta}$, each $\theta \in \Theta$, each $i \in N^{\theta}$, and each $x_i' \in X$ with $x_i' \geq x_i^*$, $\left(0, x_{-i}^*\right) \rhd x_N^* \backsimeq \left(x_i', x_{-i}^*\right)$;

(ii) for each $x_N^* \in C_N^{\Theta}$, each $\theta \in \Theta$, each $i \in N^{\theta}$, and each $\bar{x}_i \in X$ with $\left(\bar{x}_i, x_{-i}^*\right) \rhd x_N^*$, there exists $x_i \in X$ with $x_i \, I_i \, \bar{x}_i$ such that for each $j \in N_{\theta} \backslash \{i\}$ and each $x_j \in X$, $x_i < x_j$ implies $\left(x_i, x_{-i}^*\right) \rhd \left(x_j, x_{-j}^*\right)$;

(iii) for each $x_N^* \in C_N^{\Theta}$, each $\alpha > 0$, each pair $\theta, \theta' \in \Theta$, each $i \in N^{\theta}$, each $j \in N^{\theta'}$, and each pair $x_i, x_j \in X$ with $\alpha B^{\theta} \, P_i \, x_i$ and $\alpha B^{\theta'} \neg P_j \, x_j$, then $\left(x_i, x_{-i}^*\right) \trianglerighteq \left(x_j, x_{-j}^*\right)$.

Condition *(iii)* of *needs-adjusted deprivation fairness* introduces differences in needs for measuring individual deprivation. Combined with the other axioms, it forces the no-deprivation sets (one for each type) and the iso-deprivation contours to reflect the basic needs of each individual. In fact, for each type, the iso-deprivation contours are all homothetic and consist of the

[20] This rescaling condition is demanding. It is not obvious that proportional changes of the set of basic needs preserve interpersonal comparability in terms of deprivation of resources. Assume two different families have basic needs of, respectively, 1,000 USD/month and 800 USD/month. Then, in a one-dimensional setting, this proportionality assumption also imposes that if these families earn, respectively, 500 USD/month and 400 USD/month, they are equally poor. In a multidimensional setting, preference diversity complicates this relationship, but the restriction remains demanding. Yet, whenever one can argue against proportionality, more information is available on how deprivation changes with size. This information can then be used to avoid proportionality in the axiom.

convex closure of the set of basic needs. What is left undefined is the size of the type-specific no-deprivation sets, which can be any common proportional expansion (for instance, when looking at relative poverty) or contraction (for instance, when looking at absolute poverty) of the basic needs.

Before stating this result, I present the multidimensional transfer principle. As anticipated, the only difference with *equal-preference transfer* is that transfers are now restricted to individuals with the same preferences and needs.

Transfer among equals: For each $\theta \in \Theta$ and each pair $j, k \in B^{\theta}$ with $R_j = R_k \equiv R_0$, if there exist a pair $x_N, x'_N \in X_N$ and $\alpha \geq 0$ such that:

(i) $x_j - \alpha (x_j - x_k) = x'_j \, R_0 \, x'_k = x_k + \alpha (x_j - x_k)$;

(ii) for each $i \in N \backslash \{j, k\}$, $x_i = x'_i$;

then $x_N \trianglerighteq x'_N$.

Next, I define the deprivation indices that accommodate differences in needs. The type-specific no-deprivation set is defined by $\bar{\alpha} B^{\theta}$ for each $\theta \in \Theta$: it is proportional to the basic needs of each household type. For each individual $i \in N$ of type $\theta \in \Theta$ define:

$$\gamma_i^{\theta}(x_i) = \max \left\{ 0, 1 - \frac{\alpha}{\bar{\alpha}} \left| x_i \, I_i \, x \text{ for some } x \in \alpha B^{\theta} \right. \right\}.$$

Let ϕ be a real-valued, continuous, and increasing function such that: *(i)* $\phi \circ \gamma_i^{\theta}$ is convex for each $\theta \in \Theta$ and each $i \in N^{\theta}$; *(ii)* ϕ is least convex among the functions satisfying *(i);* and, without loss of generality, *(iii)* $\phi(0) = 0$ and $\phi(1) = 1$. For each $\theta \in \Theta$ and each $i \in N^{\theta}$, let the index of individual deprivation be $\lambda_i^{\theta} \equiv \phi \circ \gamma_i^{\theta}$. Finally, let f be the priority function, that is,, a continuous, increasing, and convex function. Then, the poverty index $\bar{P}$ can be defined by setting for each $x_N \in X_N$:

$$\bar{P}(x_N) = \frac{1}{n} \sum_{\theta \in \Theta} \sum_{i \in N^{\theta}} f\left(\lambda_i^{\theta}(x_i)\right). \quad (2)$$

The following result states that *responsibility*, *continuity*, *separability*, and the modified versions of *deprivation fairness* and *equal-preference transfer* characterize the poverty index (2).

Theorem 2. *A poverty ranking satisfies responsibility, continuity, separability, needs-adjusted deprivation fairness, and transfer among equals if and only if it can be represented by a poverty index as (2).*

4.2 Categorical attainments

When measuring poverty, attainments are often categorical variables (see, among others, Alkire and Foster (2011a); Bossert et al. (2013); and Decancq et al. (2019)). I now briefly address an extension of the results where some attainments are categorical.

Let S be the set of all possible combinations of categorical attainments, where the cardinality of S is finite. Define the extended attainment space as $X^+ \equiv X \times S$. Preferences R_i of each individual $i \in N$ are now defined on X^+ and are such that, for each categorical attainment vector $s \in S$, the weak order of attainments in X satisfies the assumptions of Section 2. Moreover, for each $x \in X$, each pair $s, s' \in S$, there exists $x' \in X$ such that $(x, s)\ I_i\ (x', s')$. A social state specifies an attainments vector $x_i^+ \equiv (x_i, s_i) \in X^+$ for each individual $i \in N$. A poverty ranking $\trianglerighteq$ is a weak ordering of social states. A poverty index represents such ranking by a function $D : X_N^+ \to \mathbb{R}$.

I suggest the axioms be changed as follows.

- **Responsibility**$^+$ and **separability**$^+$ are imposed on the extended attainment space X^+. These axioms are independent of the nature of the dimensions and do not pose any difficulties.[21]

- **Continuity**$^+$ and **equal-preference transfer**$^+$ are instead imposed on the space of continuous dimensions X (for each given vector of

[21] As an example, *responsibility*$^+$ now demands that for each pair $x_N^+, \bar{x}_N^+ \in X_N^+$, $x_i^+\ R_i\ \bar{x}_i^+$ for each $i \in N$ implies $\bar{x}_N^+ \trianglerighteq x_N^+$.

categorical attainments). Said differently, for each $s_N \in S^N$, the projection of the poverty ranking on X^N satisfies the *continuity* and *equal-preference transfer* axioms introduced previously.[22]

- **Deprivation fairness**$^+$ is instead imposed for a reference vector of categorical attainments, equal across individuals. That is, there exists a vector of categorical attainments $s_N^* \in S_N$ with $s_i^* = s^*$ for each $i \in N$ that is fixed when imposing the previous version of deprivation fairness. As an example, if health is a categorical variable, s^* might be chosen as the attainment level corresponding to perfect health (as in Fleurbaey and Schokkaert (2009)).[23]

Then, the main result goes through. More precisely, the poverty ranking satisfies the modified axioms if and only if it can be represented by a poverty index

$$P^+\left(x_N^+\right) = \sum_{i \in N} f\left(\tilde{\lambda}_i\left(x_i^+\right)\right), \tag{3}$$

where:

- for a given $s^* \in S$, $\tilde{\lambda}_i\left(x_i^+\right) \geq \tilde{\lambda}_j\left(x_j^+\right)$ if and only if:

$$\min\left\{\lambda\left(x\right) \left| \left(x, s^*\right)\ I_i\, x_i^+\right.\right\} \geq \min\left\{\lambda\left(x\right) \left| \left(x, s^*\right)\ I_j\, x_j^+\right.\right\};$$

- moreover: *(i)* $\tilde{\lambda}_i$ is convex on X for each $i \in N$; *(ii)* for any non-convex function ϕ, $\phi \circ \tilde{\lambda}_i$ is not convex for some $i \in N$*;* and, without loss of generality, *(iii)* $\min_{x_i^+} \tilde{\lambda}_i\left(x_i^+\right) = 0$ and $\max_{x_i^+} \tilde{\lambda}_i\left(x_i^+\right) = 1$ for each $i \in N$.

[22] As an example, *continuity*$^+$ now demands that for each $x_N^+ \equiv (x_N, s_N) \in X_N^+$, the sets $\left\{\bar{x}_N^+ \equiv (\bar{x}_N, \bar{s}_N) \in X_N \left| \bar{x}_N^+ \trianglerighteq x_N^+ \text{ with } s_N = \bar{s}_N\right.\right\}$ and $\left\{\bar{x}_N^+ \equiv (\bar{x}_N, \bar{s}_N) \in X_N \left| x_N^+ \trianglerighteq \bar{x}_N^+ \text{ with } s_N = \bar{s}_N\right.\right\}$ are closed.

[23] Formally, *deprivation fairness*$^+$ now demands that there exist a non-empty and closed set $C \subset X$ and $s_N^* \in S_N$ with $s_i^* = s^*$ for each $i \in N$ such that: (i) for each $x_N^* \in C^n$, each $i \in N$, and each $x_i' \in X$ with $x_i' \geq x_i^*$, $\left(\left(0, s^*\right), \left(x_{-i}^*, s_{-i}^*\right)\right) \triangleright \left(x_N^*, s_N^*\right) \backsimeq \left(\left(x_i', s^*\right), \left(x_{-i}^*, s_{-i}^*\right)\right)$; and ($ii$) for each $x_N^* \in C^n$, each $i \in N$, and each $\bar{x}_i \in X$ with $\left(\left(\bar{x}_i, s^*\right), \left(x_{-i}^*, s_{-i}^*\right)\right) \triangleright \left(x_N^*, s_N^*\right)$, there exists $x_i \in X$ with $\left(x_i, s_i^*\right)\ I_i\ \left(\bar{x}_i, s_i^*\right)$ such that for each $j \in N \backslash \{i\}$ and each $x_j \in X$, $x_i < x_j$ implies $\left(\left(x_i, s^*\right), \left(x_{-i}^*, s_{-i}^*\right)\right) \triangleright \left(\left(x_j, s^*\right), \left(x_{-j}^*, s_{-j}^*\right)\right)$.

- f is the priority function, that is a continuous, increasing, and convex function.

Theorem 3. *A poverty ranking on X_N^+ satisfies responsibility$^+$, continuity$^+$, separability$^+$, deprivation fairness$^+$, and equal-preference transfer$^+$ if and only if it can be represented by a deprivation index as (3).*

To clarify, the no-deprivation set and the iso-deprivation contours continue to be defined on the subspace of continuous dimensions X. In fact, only X has the properties required to define cardinality and interpersonal comparability of deprivations based on individuals' attainments vectors. Yet, this does not prevent the family of deprivation indices from keeping individuals responsible for their choices. The deprivation indices allow comparisons over the entire space of categorical attainments by "preference indifference": whenever all individuals are indifferent between two social states, poverty is unchanged.

5 Conclusions

The indices of poverty and individual deprivation characterized in this paper define a novel method for assessing poverty that keeps individuals responsible for their choices. These indices *(i)* are multidimensional; *(ii)* compare individuals by the set of attainments they are deprived of; *(iii)* are continuous and, thus, robust to measurement errors; *(iv)* decrease when individuals obtain larger attainments and, thus, are monotonic; *(v)* express aversion to inequality among the deprived and, thus, prioritize the poorest (or most deprived) individuals; and *(vi)* decrease when individuals are made better-off and, thus, respect individuals' choices. Moreover, such indices can account for differences in needs and extend to categorical dimensions of deprivation.

The following table summarizes these characteristics of the current proposal in relation to some of the well-known alternatives reviewed in the introduction (for the Foster, Greer, and Thorbecke family, the properties marked with "$\surd^*$" hold only when the index is convex).

	Foster et al. (1984)	Mashup indices (i.e. HDI)	Bourguignon & Chakravarty (2003)	Alkire and Foster (2011)	Current proposal
Multidimensional	✗	✓	✓	✓	✓
Robust to measurement errors	✓*	✗	✓	✗	✓
Monotonic	✓*	✗	✗	✗	✓
Prioritizes the poorest	✓*	✗	✓	✗	✓
Respects choices	–	✗	✗	✗	✓

While overcoming several drawbacks of currently adopted indices, the results of this paper also emphasize that fundamental ethical choices are unavoidable and remain open to debate. These choices include the types of attainments to account for, the substitutability or complementarity across attainments, and the priority to place on the most deprived individuals. Importantly, such ethical choices necessarily depend on each specific application (i.e., absolute or relative poverty, local or global poverty, context), on the data available for the poverty assessment, and on the ethical views that the society embraces.

The results of this paper pave the way for rethinking optimal poverty alleviation programs in settings where individual responsibility matters. A natural starting point is to extend the second-best labor supply setting à la Mirrlees (1971) to differences in choices.[24] Clearly, the lack of information about the types of individuals will not allow targeting the most deserving individuals. However, even if targeting were possible—say with costly monitoring or compulsory activation clauses—it is not necessarily true that targeted policies are socially better than non-targeted policies. The answer ultimately depends on the composition of society (i.e., on the joint distribution of working opportunities and on preferences of individuals), and on ethical choices (i.e., on the index of poverty adopted).

[24]Recent studies addressing optimal poverty policies in a labor supply setting include Kanbur et al. (1994), Chakravarty and Mukherjee (1998), and Pirttilä and Tuomala (2004).

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Appendix A. Empirical exercise: details

A.1 Estimation of preferences

The database is the 2016 Norwegian register data, restricted to single men without children aged between 22 and 61. It consists of 90,462 individuals. For each individual, I use information about his after-tax income, labor supply, age, education, and wage rate.

Each individual $i \in N$ is assumed to have preferences defined over after-tax income y_i, leisure l_i, and unobservable factors q (i.e., non-pecuniary benefits related to the specific job). Income is measured in million NOK. Let h_i denote the number of hours worked per year. The leisure is normalized as follows: $l_i = 1 - \frac{h}{4160}$ ($l_i = 1$ corresponds to 80 hours of work per week). Preferences of individual i can be represented by

$$U_i\left(y, l, z\right) = v_i\left(y, l\right) \varepsilon\left(q\right),$$

which consists of an individual-specific “deterministic” component $v_i\left(y, l\right)$ and a common "stochastic" component $\varepsilon(q)$. Following Aaberge et al. (1999), assume that $\varepsilon(q)$ is extreme-value distributed of type III. Let the deterministic component of preferences be given by

$$\ln v_i(y,l) = \left[\beta_i y^{\zeta} + (1-\beta_i) l^{\zeta}\right]^{\frac{1}{\zeta}} .$$

Each individual is assigned a random opportunity set, consisting of triplets such as (y,l,q). Each individual selects the alternative that maximizes her preferences U_i. Then, the parameters β_i and ζ are identified to maximize the likelihood between the observed distribution of after-tax income and leisure pairs and the estimated ones. The random opportunity sets and the preference parameters β_i are type specific and depend on the observable characteristics of individuals. Preferences depend on age, age squared, and 4 different levels of education. This specification leads to 156 different types of individuals.[25]

Formally, $\beta_i \equiv \frac{\beta_0}{\beta_0+\delta_0+\delta X_i}$ where $\beta_0 = 0.15$ and $\delta_0 = 0.75$ are constants and $\delta \in \mathbb{R}^5$ is a vector collecting the estimates for, in order, the multipliers for age (divided by 100) $\delta_1 = -1.21$, age squared (divided by 100^2) $\delta_2 = 1.88$, secondary school (1 if highest degree) $\delta_3 = -0,09$, college or university lower-degree education $\delta_4 = -0,06$, and university higher degree education $\delta_5 = -0,18$. For simplicity, the elasticity of substitution between consumption and leisure is assumed to be equal across individuals and estimated as $\zeta = -2.55$. All coefficients are highly significant.[26] The interpretation is as follows: individuals' willingness to work (or the relative importance given to consumption) first increases and then decreases with age, achieving a peak at age 32. As for education, the highest willingness to work emerges for individuals with a master/PhD degree; the lowest willingness to work seems to be a trait of individuals without education; finally, individuals with high-school education are slightly more inclined to work than those with college

[25] In the empirical application, $\varepsilon(q)$ is set to unity. I leave to future research any attempt to use the information on the unobservable component estimated for each individual to improve the measurement of individual deprivation.

[26] I report the t-values in parentheses: β_0 (22.0); ζ (−91.4); δ_0 (16.9); δ_1 (−6.0); δ_2 (7.8); δ_3 (−12.0); δ_4 (−6.9); and δ_5 (−16.0).

or university lower-degree education.

A.2 Individual deprivation and poverty

I first recall some definitions. The *no-deprivation set* is

$$X_0 \equiv \left\{ (y,l) \in X \left| [\beta y^\gamma + (1-\beta) l^\gamma]^{\frac{1}{\gamma}} \geq \nu \right. \right\},$$

where β, γ, ν are ethical parameters. The parameters are uniquely identified by the ethical choices discussed in Subsection 3.2. The *iso-deprivation contour* of level $\lambda \in [0,1]$ is the set

$$X(\lambda) \equiv \left\{ (y,l) \in X \left| [\beta y^\gamma + (1-\beta) l^\gamma]^{\frac{1}{\gamma}} = (1-\lambda)\nu \right. \right\}.$$

Then, $X(0) = Fr\{X_0\}$ and $X(1) = \{0\}$.[27] Consider individual $i \in N$ with preferences R_i. Her individual deprivation function $\lambda_i : X \to \mathbb{R}$ is defined by setting for each $(y,l) \in X$,

$$\lambda_i(y,l) \equiv \min \{\lambda \in [0,1] \,|(y,l)\ I_i\ (y',l') \text{ for some } (y',l') \in X(\lambda)\}.$$

By homotheticity of preferences and of *iso-deprivation contours*, the above individual's deprivation function is homogeneous of degree 1 and, thus, is a least convex function. Then, the cardinalizing function ϕ is linear and can be disregarded. Let $V_i \equiv \min_{(y,l)\in X_0} v_i(y,l)$ be the minimum level of well-being of i when she is not deprived. By simple manipulation, the individual deprivation function can be rewritten as

$$\lambda_i(y,l) = \max\left\{0, \frac{V_i - v_i(y,l)}{V_i}\right\}.$$

Poverty is measured by the sum of a convex transformation of individuals' deprivation. Using a power transformation with exponent $\theta \geq 1$ gives the

[27] By monotonicity of preferences, it is irrelevant whether the *iso-deprivation contour* of level 0 is the entire *no-deprivation set* or only its frontier.

following *per capita* index of poverty:

$$D(x_N;\theta) \equiv \frac{1}{n}\sum_i [\lambda_i(y_i,l_i)]^\theta .$$

Appendix B. Proofs

B.1 Proof of Theorem 1

It is straightforward to prove that a poverty ranking represented by (1) satisfies the axioms.

For each $i \in N$, $-\lambda_i$ is a non-decreasing transformation of a representation of preferences R_i: it is a representation of preferences whenever i is deprived (for each x_i such that $X_0 \, R_i \, x_i$); it is constant whenever i is not deprived (for each x_i such that $C \neg P_i \, x_i$). Thus, *responsibility* follows. Since the functions f, ϕ, and, for each $i \in N$, λ_i are continuous, the poverty ranking $\trianglerighteq$ is *continuous*. Since the representation of $\trianglerighteq$ is additive over individuals, *separability* holds. By convexity of f and of $\phi \circ \lambda_i$ for each $i \in N$, *equal-preference transfer* follows.

Since each function λ_i is largest at the 0 attainment vector, first strictly decreasing and then constant, there exists a set C^n (for example, $C = X_0$) such that condition *(i)* of *no-deprivation fairness* holds. Finally, assume condition *(ii)* of *no-deprivation fairness* is violated. Then, there exists a $x_N^* \in C^n$, $i \in N$, and $\bar{x}_i \in X$ with $\left(\bar{x}_i, x_{-i}^*\right) \triangleright x_N^*$, such that for each $x_i \in X$ with $x_i \, I_i \, \bar{x}_i$, each $j \in N \setminus \{i\}$ and each $x_j \in X$ with $x_i < x_j$, it holds that $\left(x_j, x_{-j}^*\right) \trianglerighteq \left(x_i, x_{-i}^*\right)$. By construction of λ_i, the index of deprivation of i at $\bar{x}_i$ is the index λ associated with the smallest lower contour set of $\succsim$ such that i finds each element of this set at least as desirable as $\bar{x}_i$. Let x_i' be (one of) i's least preferred bundles in this set (existence is ensured by continuity of preferences and the assumption that $\lim_{|x_i|\to\infty} u_i(x_i) = \infty$). By continuity of preferences, $x_i' \, I_i \, \bar{x}_i$. Moreover, each $x_j > x_i'$ belongs to a smaller lower contour set of $\succsim$. By construction, the index of deprivation associated with x_j is such that $\lambda_j(x_j) < \lambda_i(\bar{x}_i)$. This inequality contradicts $\left(x_j, x_{-j}^*\right) \trianglerighteq \left(x_i, x_{-i}^*\right)$ and proves condition *(ii)* holds.

I next show the reverse implication. The proof is divided into three steps.

Step 1. *If a poverty ranking* $\trianglerighteq$ *satisfies responsibility, continuity, and separability, then there exists a continuous function* $D : X_N \to \mathbb{R}$*, and, for each* $i \in N$*, a non-increasing real-valued function* g_i *and a representation* $U_i : X \to \mathbb{R}$ *of her preferences* R_i*, such that for each pair* $x_N, x'_N \in X_N$*,* $x_N \trianglerighteq x'_N$ *if and only if*

$$P(x_N) \equiv \sum_{i \in N} g_i \circ U_i(x_i) \geq \sum_{i \in N} g_i \circ U_i(x'_i) \equiv D(x'_N).$$

Proof. By *continuity*, there exists a continuous function $\bar{P} : X_N \to \mathbb{R}$ that represents $\trianglerighteq$. By *responsibility*, there exists a continuous function $\tilde{P} : \mathbb{R}^n \to \mathbb{R}$ non increasing in each argument and, for each $i \in N$, a continuous function $U_i : X \to \mathbb{R}$ representing preferences R_i such that for each $x_N \in X_N$, $\bar{P}(x_N) = \tilde{P}(U_1(x_1), ..., U_n(x_n))$. By *separability* (and the assumption that $|N| \geq 3$), there exists an increasing function $H : \mathbb{R} \to \mathbb{R}$ and, for each $i \in N$, a continuous and non-increasing function $g_i : \mathbb{R} \to \mathbb{R}$ such that $\bar{P}(x) = H\left(\sum_{i \in N} w_i \circ U_i(x_i)\right)$. Let $P : X_N \to \mathbb{R}$ be such that for each $x_N \in X_N$, $P(x_N) \equiv \sum_{i \in N} g_i \circ U_i(x_i)$. Since H is increasing, P also represents the poverty ranking $\trianglerighteq$, proving the result. □

Step 2. *If the poverty ranking* $\trianglerighteq$ *also satisfies deprivation fairness, then there exists a no-deprivation set* $X_0 \subset X$ *, a weak order* $\succsim$ *identifying the iso-deprivation contours, and a strictly increasing and continuous real-valued function* g *such that, for each individual* $i \in N$*,* $g_i \circ U_i = g \circ \gamma_i$*, where* γ_i *is defined by setting* $\gamma_i(x_i) = \min\{\lambda(x) | x\, I_i\, x_i\}$ for each $x_i \in X$*. That is, the poverty ranking* $\trianglerighteq$ *can be represented by* $P = \sum_{i \in N} g \circ \gamma_i$*.*

Proof. Deprivation fairness directly postulates (condition i) the existence of a non-empty and closed set $C \subset X$ such that for each $i \in N$, each $x_N \in X_N$ with $x_i \in C$, and each $a_i \in X$ with $a_i \geq x_i$, $(0, x_{-i}) \triangleright (x_i, x_{-i}) \backsimeq (a_i, x_{-i})$. Thus, $0 \notin C$. Let $\underline{x}_i$ be (one of) the attainments vector(s) that i finds least desirable in C. Its existence follows from each i's preferences admitting a strictly increasing and concave representation $u_i : X \to \mathbb{R}$ such that $\lim_{|x_i|\to\infty} u_i(x_i) = \infty$ and from C being non-empty and closed. Then, using the representation of Step 1, $g_i \circ U_i(0) > g_i \circ U_i(x_i) = g_i \circ U_i(a_i)$ for each $a_i \geq x_i$ with $x_i \in C$. By monotonicity of preferences, $g_i \circ U_i(0) > g_i \circ U_i(\underline{x}_i) = g_i \circ U_i(x_i)$ for each $x_i \, R_i \, \underline{x}_i$.

Let $x^*_N \in C$. I show next that $\left(0, x^*_{-i}\right) \backsimeq \left(0, x^*_{-j}\right)$ for each $i, j \in N$. Assume not. Then, without loss of generality, $\left(0, x^*_{-j}\right) \rhd \left(0, x^*_{-i}\right)$. By continuity of individual preferences and of the poverty ranking, there exists an attainment vector $\varepsilon_j \gg 0$ such that $\left(\varepsilon_j, x^*_{-j}\right) \rhd \left(0, x^*_{-i}\right)$. The existence of such an attainment vector violates *deprivation fairness* (condition *ii*), where $\bar{x}_i = 0$. This result also implies $g_i \circ U_i(0) - g_i \circ U_i(\underline{x}_i) = g_j \circ U_j(0) - g_j \circ U_j\left(\underline{x}_j\right)$ for each $i, j \in N$.

Let $K \equiv [0, g_i \circ U_i(0) - g_i \circ U_i(\underline{x}_i)]$. By *continuity*, for each $k \in K$ and each $i \in N$, there exists an attainments vector $x_i(k)$ such that $g_i \circ U_i(x_i(k)) - g_i \circ U_i(\underline{x}_i) = k$. It follows that $\left(x_i(k), \underline{x}_{-i}\right) \backsimeq \left(x_j(k), \underline{x}_{-j}\right)$ for each $i, j \in N$. For each $i \in N$, let the upper-contour sets at $x_i(k)$ be denoted $UCS_i(k) \equiv \{x_i \in X \, | x_i \, R_i \, x_i(k)\}$. The intersection of these upper-contour sets is $UCS(k) \equiv \bigcap_{i\in N} UCS_i(k)$. Clearly, by definition of upper-contour set, it cannot be that $x_i(k) \; P_i \; x$ for some $x \in UCS(k)$. Assume instead $x \, P_i \, x_i(k)$ for each $x \in UCS(k)$. By continuity of U_i, there exists an attainments vector $x^+_i \in X$ such that, for each $x \in UCS(k)$, $x \, P_i \, x^+_i \, P_i \, x_i(k)$ and $\left(x_j(k), \underline{x}_{-j}\right) \rhd \left(x^+_i, \underline{x}_{-i}\right)$. Since $x^+_i \notin UCS(k)$, for each $x_i \, I_i \, x^+_i$ there exists $j \neq i$ and $x_j > x_i$ with $x_j \, I_j \, x_j(k)$. By transitivity of the poverty ranking, $\left(x_j, \underline{x}_{-j}\right) \rhd \left(x_i, \underline{x}_{-i}\right)$. This is a violation of *deprivation fairness* (condition *ii*). Thus, for each $i \in I$ and each $k \in K$, $x_i(k) \; I_i \, w_i$, where w_i is one of the least desirable attainment vectors for i in $UCS(k)$. Moreover, for each $i, j \in N$ and each $k, k' \in K$, $g_i \circ U_i(x_i(k)) \geq g_j \circ U_j(x_j(k'))$ if and only if $k \geq k'$.

I next construct the weak order $\succsim$ and the corresponding iso-deprivation contours. First, let X_0 be the convex hull of $UCS(0)$. This is non-empty, closed, and, by definition, convex: it is thus a no-deprivation set. For each $k \in K$, let $C(k)$ be the convex hull of $UCS(k)$. Note by concavity of preferences, for each $i \in I$, i's least desirable attainment vectors in $UCS(k)$

are the same as in $C(k)$. Define $\succsim$ by setting, for each $x, x' \in X$, $x \succsim x'$ if and only if $\max[k \in K \,|x \in C(k)] \geq \max[k \in K \,|x' \in C(k)]$. This weak order is continuous, strictly antimonotonic over $X \backslash C$ (i.e., for each $x, x' \in X \backslash C$, $x \geq x'$ implies that $x' \succ x$), constant over C (i.e., for each $x, x' \in C$, $x \sim x'$), and, by convexity of individuals' preferences, has convex lower contours.

Now, let $\lambda : X \to [0,1]$ be a representation of $\succsim$ such that, without loss of generality, $\lambda(x) = 0$ for $x \in X_0$ and $\lambda(0) = 1$.[28] Given λ, γ_i is defined by setting for each $x_i \in X$, $\gamma_i(x_i) = \min\{\lambda(x) \,|x \, I_i \, x_i\}$.

Finally, for each $i, j \in N$ and each $k, k' \in K$, $\gamma_i(x_i(k)) \geq \gamma_j(x_j(k'))$ if and only if $k \geq k'$, which holds if and only if $g_i \circ U_i(x_i(k)) \geq g_j \circ U_j(x_j(k'))$. Thus, there exists a real-valued and increasing function g such that $g_i \circ U_i = g \circ \gamma_i$. □

Step 3. *If the poverty ranking $\trianglerighteq$ also satisfies equal-preference transfer, then there exists a priority function f and a real-valued function ϕ such that (i) $g = f \circ \phi$; (ii) $\lambda \equiv \phi \circ \gamma_i$ is an individual deprivation function. That is, the poverty ranking $\trianglerighteq$ can be represented by $P = \sum_{i \in N} f \circ \lambda_i$.*

Proof. Let $j, k \in N$ be such that $R_j = R_k \equiv R_0$. By Step 2, $\gamma_j = \gamma_k \equiv \gamma_0$. Let $x_N, x'_N \in X^n$ be such that *(i)* $x'_j = x'_k = \frac{x_j + x_k}{2}$; *(ii)* for each $i \in N \backslash \{j,k\}$, $x_i = x'_i$. By *equal-preference transfer*, $x \trianglerighteq x'$. By the previous steps, this is equivalent to

$$g \circ \gamma_0 \left(\frac{x_j + x_k}{2} \right) \leq \frac{g \circ \gamma_0(x_j) + g \circ \gamma_0(x_k)}{2}.$$

Since this holds for each pair of attainments vectors of i and j, $g \circ \gamma_0$ is convex. As the argument holds for each pair of individuals with the same preferences, for each $i \in N$, $g \circ \gamma_i$ is convex. Let ϕ be a normalized least joint convex transformation, that is, it satisfies *(a)* $\phi \circ \gamma_i$ is convex for each individual; *(b)* ϕ is the least convex function to guarantee this; and *(c)* $\phi(0) = 0$ and $\phi(1) = 1$. Its existence follows from Lemma 1 in Piacquadio (2017) by noting that $-\phi$ is least joint concave. Then, there exists a priority function

[28] As an example, let $\bar{\alpha} \equiv \min[\alpha \in \mathbb{R}_+ \,|\alpha \mathbf{1}_\ell \in NDS]$. Clearly, $\bar{\alpha} > 0$. Then, for each $x \in X$, let $\lambda(x) = \frac{\bar{\alpha} - \alpha}{\bar{\alpha}}$ if either $x = \alpha \mathbf{1}_\ell$ or if $x \sim \alpha \mathbf{1}_\ell$.

f (continuous, increasing, and convex) such that $g = f \circ \phi$ and $\trianglerighteq$ can be represented by $P = \frac{1}{n} \sum_{i \in N} f \circ \lambda_i$. $\square$

B.2 Proof of Theorem 2

I show only that the axioms imply the deprivation index (2). I follow the same steps of the proof of Theorem 1.

Step 1 directly extends. Let a poverty ranking $\trianglerighteq$ satisfy *responsibility*, *continuity*, and *separability*. Then, there exists a continuous function $D : X_N \to \mathbb{R}$ and, for each $\theta \in \Theta$ and each $i \in N_\theta$, a non-increasing real-valued function g_i^θ and a representation $U_i^\theta : X \to \mathbb{R}$ of her preferences R_i, such that for each pair $x_N, x'_N \in X_N$, $x_N \trianglerighteq x'_N$ if and only if

$$\bar{P}(x_N) \equiv \frac{1}{n} \sum_{\theta \in \Theta} \sum_{i \in N_\theta} g_i^\theta \circ U_i^\theta (x_i) \geq \frac{1}{n} \sum_{\theta \in \Theta} \sum_{i \in N_\theta} g_i^\theta \circ U_i^\theta (x'_i) \equiv \bar{P}(x'_N).$$

The next step imposes *needs-adjusted deprivation fairness* on the poverty ranking $\trianglerighteq$.

Step 2. *If the poverty ranking $\trianglerighteq$ also satisfies needs-adjusted deprivation fairness, then there exists a constant $\bar{\alpha} > 0$ (identifying the deprivation functions λ_i^θ in Section 4) and a strictly increasing and continuous real-valued function g such that, for each $\theta \in \Theta$ and each $i \in N^\theta$, $g_i^\theta \circ U_i^\theta = g \circ \lambda_i^\theta$. That is, the poverty ranking $\trianglerighteq$ can be represented by $P = \frac{1}{n} \sum_{\theta \in \Theta} \sum_{i \in N^\theta} g \circ \lambda_i^\theta$.*

Proof. The first two conditions of *needs-adjusted deprivation fairness* are equivalent to those of *deprivation fairness*, except the second holds only for equal-type individuals. Thus, repeating Step 2 of the proof of Theorem 1, for each $\theta \in \Theta$, there exists a weak order $\succsim^\theta$ on X (identifying the iso-deprivation contours) and a no-deprivation set X_0^θ such that $\succsim^\theta$ is continuous, strictly antimonotonic over $X \backslash X_0^\theta$ (i.e., for each $x, x' \in X \backslash X_0^\theta$, $x \geq x'$ implies that $x' \succ x$), constant over X_0^θ (i.e., for each $x, x' \in X_0^\theta$, $x \sim x'$), and has convex lower contours. For each $\theta \in \Theta$, let $\phi^\theta : X \to [0, 1]$ be a representation of $\succsim$ such that, without loss of generality, $\phi^\theta(x) = 0$ for $x \in X_0^\theta$ and $\phi^\theta(0) = 1$. For each $\theta \in \Theta$ and each $i \in N^\theta$, let ϕ_i^θ be such that

$\phi_i^\theta(x_i) = \min\left\{\phi^\theta(x) \,|x\, I_i\, x_i\right\}$ and define

$$\alpha_i^\theta \equiv \min\left\{\alpha \in \mathbb{R}_+ \left|\alpha B^\theta\, R_i\, x_i \text{ for some } x_i \in X \text{ with } \phi_i^\theta(x_i) = 0\right.\right\}.$$

Next, I prove there exists $\bar{\alpha}$ such that $\alpha_i^\theta = \bar{\alpha}$ for each $\theta \in \Theta$ and each $i \in N^\theta$. Proceeding by contradiction, assume such an $\bar{\alpha}$ does not exist. Then, there exist $\theta, \theta' \in \Theta$, $i \in N^\theta$, and $j \in N^{\theta'}$ such that $\alpha_i^\theta < \alpha_j^{\theta'}$. For each $\alpha > 0$, let $x_i(\alpha), x_j(\alpha) \in X$ be such that $\alpha B^\theta\, I_i\, x_i(\alpha)$ and $\alpha B^{\theta'}\, I_j\, x_j(\alpha)$. Let $\alpha \in \left(\alpha_i^\theta, \alpha_j^{\theta'}\right)$ and $x_N^* \in X_0^\Theta \equiv \Pi_{\theta\in\Theta} X_0^\theta$. Since $\alpha > \alpha_i^\theta$, $\alpha B^\theta\, P_i\, x_i\left(\alpha_i^\theta\right)$ and $x_i(\alpha)\; P_i\, x_i\left(\alpha_i^\theta\right)$. By definition of ϕ_i^θ, $\phi_i^\theta(x_i(\alpha)) = \phi_i^\theta\left(x_i\left(\alpha_i^\theta\right)\right) = 0$ and, consequently, $\left(x_i(\alpha), x_{-i}^*\right) \sim x_N^*$. Conversely, since $\alpha < \alpha_j^{\theta'}$, $\alpha B^{\theta'} \neg P_j\, x_j\left(\alpha_j^{\theta'}\right)$, $\phi_j^{\theta'}\left(x_j\left(\alpha_j^{\theta'}\right)\right) > \phi_j^{\theta'}(x_j(\alpha)) = 0$, and $\left(x_j(\alpha), x_{-i}^*\right) \rhd x_N^*$. Let $\alpha^- \in \left(\alpha_i^\theta, \alpha\right)$ and $\alpha^+ \in \left(\alpha, \alpha_j^{\theta'}\right)$. By monotonicity of preferences, $\alpha B^\theta\, P_i\, x_i(\alpha^-)$ and $\alpha B^{\theta'} \neg P_j\, x_j(\alpha^+)$. By *efficiency*, $\left(x_j(\alpha^+), x_{-j}^*\right) \rhd \left(x_i(\alpha^-), x_{-i}^*\right)$. This is a contradiction of condition *(iii)* and a proof of the existence of $\bar{\alpha}$.

Next, I show $\left(x_i(\alpha), x_{-i}^*\right) \trianglerighteq \left(x_j(\alpha'), x_{-j}^*\right)$ for each $\alpha, \alpha' \in [0, \bar{\alpha}]$ with $\alpha \leq \alpha'$, each pair $\theta, \theta' \in \Theta$, each $i \in N^\theta$, each $j \in N^{\theta'}$, and each $x_N^* \in X_0^\Theta$. The same argument as above shows that the converse relation, that is, $\left(x_j(\alpha'), x_{-j}^*\right) \rhd \left(x_i(\alpha), x_{-i}^*\right)$, leads to a contradiction of condition *(iii)*. For each $\theta \in \Theta$ and $i \in N^\theta$, define

$$\gamma_i^\theta(x_i) = \max\left\{0, 1 - \frac{\alpha}{\bar{\alpha}} \left|x_i\, I_i\, x \text{ for some } x \in \alpha B^\theta\right.\right\}.$$

Finally, for each pair $\theta, \theta' \in \Theta$, each $i \in N^\theta$, each $j \in N^{\theta'}$, and each $\alpha, \alpha' \in [0, \bar{\alpha}]$, $\gamma_i^\theta(x_i(\alpha)) \geq \gamma_j^{\theta'}(x_j(\alpha'))$ if and only if $\alpha \leq \alpha'$. By Step 1, $\gamma_i^\theta(x_i(\alpha)) \geq \gamma_i^\theta(x_j(\alpha'))$ if and only if $g_i^\theta \circ U_i^\theta(x_i(\alpha)) \geq g_j^\theta \circ U_j^\theta(x_j(\alpha'))$. Thus, there exists a real-valued and increasing function g such that $g_i^\theta \circ U_i^\theta = g \circ \gamma_i^\theta$ for each $i \in N$ and $\trianglerighteq$ can be represented by $D = \sum_{\theta\in\Theta}\sum_{i\in N^\theta} g \circ \gamma_i^\theta$. □

Step 3. *If the poverty ranking $\trianglerighteq$ also satisfies equal-preference transfer, then there exist a priority function f and a least joint convex function ϕ such that $g = f \circ \phi$. That is, the poverty ranking $\trianglerighteq$ can be represented by $D = \sum_{\theta\in\Theta}\sum_{i\in N^\theta} f \circ \phi \circ \gamma_i^\theta$ or, by setting $\lambda_i^\theta = \phi \circ \gamma_i^\theta$, by $D = \sum_{\theta\in\Theta}\sum_{i\in N^\theta} f \circ \lambda_i^\theta$.*

Proof. The proof is equivalent to the corresponding step of Theorem 1 and is omitted. □

B.3 Proof of Theorem 3

Again, the proof that the ranking satisfies the axioms is omitted.

I prove the converse implication. By *deprivation fairness*$^+$, there exists a vector of categorical attainments $s^* \in S$ such that, if each i's attainment is $s_i = s^*$, the same conditions in *deprivation fairness* are imposed. Thus, I first focus on X_N^+ such that $s_N = s_N^*$. All the axioms in Theorem 1 hold on this space. Thus, the poverty ranking $\trianglerighteq$ on X (where X is obtained by letting $s_N = s_N^*$ in X_N^+) can be represented by $P = \frac{1}{n}\sum_{i\in N} f \circ \tilde{\lambda}_i$, where $\tilde{\lambda}_i$ is defined in Section 4. To extend the representation to X_N^+, note that, by assumption, for each $x_N^+ \in X_N^+$ there exists a social state $\bar{x}_N^+ \in X_N^+$ with $s_N = s_N^*$ such that, for each $i \in N$, $(x_i, s_i)\ I_i\ (\bar{x}_i, s^*)$. Then, by *responsibility*$^+$ and *continuity*$^+$, $x_N^+ \simeq \bar{x}_N^+$.

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