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8237
2020

April 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 8237*

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Abstract

This paper analyzes sin goods consumption when individuals exhibit present-focused preferences. It considers three types of present focus: present-bias with varying degrees of naiveté, Gul-Pesendorfer preferences, and a dual-self approach. We investigate the incentives to deviate from healthy consumption (the extensive margin). In the first model, the extensive margin of consumption is independent of the degree of present-bias and naiveté. Likewise, in the latter frameworks, the strength of temptation and the cost of self-control do not affect the extensive margin. Hence, present-focused preferences affect the intensive margin of sin goods consumption, but not the extensive margin.

JEL-Codes: D110, D150, D600, D910, I120.

Keywords: present-bias, self-control, temptation, dual-self, sin goods.

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April 10, 2020

1 Introduction

Cigarettes, alcohol, unhealthy foods, and drugs bring immediate gratification and later health costs. Thus, they are labeled as sin, or temptation, goods. Economists have developed several classes of models to understand the behavior of sin goods consumers. Ericson and Laibson (2018) group these models into a category called "present-focused preferences". By definition, "present-focused preferences exist if agents are more likely in the present to choose an action that generates immediate experienced utility, than they would be if all the consequences of the actions in their choice set were delayed by the same amount of time" (Ericson and Laibson, 2018).

In this article, we define the extensive margin of sin goods consumption as a deviation from the health-maximizing consumption level. That is, we explicitly account for the consumers' decision to start or abstain from sin goods consumption. We analyze how a present focus impacts the extensive margin. The main insight from our paper is that for the most widely used frameworks, present-focused preferences do *not* affect the extensive margin of sin goods consumption. They affect only the intensive margin, i.e., the degree of deviation. Hence, present-focused preferences cannot explain *why* people consume sin goods; they only determine the intensity of sin goods consumption, given that an individual has decided to consume sin goods.

This insight is derived within a framework of an individual who chooses the optimal consumption path of a temptation good. There exists a consumption level that is health-maximizing, and we refer to it as the healthy consumption level. This healthy level may be zero. For example, in the case of cigarettes, there exist health risks of even light and intermittent smoking (Schane et al., 2010). However, the healthy level may also be positive. In the case of added sugar, Ramne et al. (2018) find a U-shaped relationship between consumption and all-cause mortality. They find the lowest mortality risk at added sugar consumption between 7.5% and 10% of the total energy intake.

We analyze the consumption choice in three different present focus frameworks. First, we use the present-bias model developed by Strotz (1956), Phelps and Pollak (1968), and Laibson (1997), and allow the consumer to be either a sophisticated or a naive quasi-hyperbolic discounter. In the second case, we consider the temptation model of Gul and Pesendorfer (2001, 2004, 2005), where the individual is capable of costly self-control.

Lastly, we study the consumption decisions in the dual-self model of Fudenberg and Levine (2006) with a myopic short-run self and a patient long-run self who can exert costly self-control.

In each framework, we find that the decision to deviate (in the steady state) from healthy consumption on the extensive margin is independent of the degree of present focus of preferences. In the quasi-hyperbolic discounting model, both the degree of present-bias and naiveté do not influence the extensive margin. Instead, an individual deviates from healthy consumption if and only if the instantaneous utility-maximizing sin good level differs from the healthy level. This is a rational reason for deviating at the extensive margin and is not related to the degree of present-bias or naiveté. In the model of Gul and Pesendorfer (2001), the consumer deviates from healthy consumption if and only if the commitment utility-maximizing consumption differs from the healthy level. The cost of self-control and the strength of temptation do not affect the extensive margin. Lastly, in the dual-self framework, the short- and long-run selves agree on the decision on the extensive margin, i.e., on whether and in which direction to deviate from healthy consumption. They disagree only on the degree of deviation (the intensive margin).

The intuition behind these results is the following. In the absence of a present focus, the consumer faces a trade-off between maximizing her instantaneous utility and minimizing the long-run health costs of consumption. If and only if the instantaneous utility-maximizing consumption level is above the healthy level, does the individual overconsume. When the individual exhibits present-focused preferences, the fundamental trade-off remains unaffected. The only change is the degree of deviation away from healthy consumption. However, both time-inconsistent (i.e., present-biased) and time-consistent (e.g., Gul-Pesendorfer) present-focused preferences cannot affect the decision on the extensive margin.

In our main analysis, the sin good under consideration is not addictive. To show that this assumption is without loss of generality, we extend, in Section 5, the present-bias model to consider an addictive sin good. We prove that our main result remains unchanged, i.e., neither the degree of present-bias not the degree of naiveté affect the extensive margin.

This paper is related to the literature on present-focused preferences. According to the aforementioned definition of Ericson and Laibson (2018), individuals with such preferences are *more likely* to choose actions that generate immediate gratification in the present. When the action generating immediate gratification is the consumption of a sin good, our results show that the definition may only hold for the intensive margin of consumption but not the extensive margin.

All three frameworks that we analyze predict demand for commitment (Ericson and Laibson, 2018). However, we rarely observe such demand (Laibson, 2015), and even when it exists, the willingness to pay for commitment is low (Laibson, 2018). Less than 15% of experimental participants accept a commitment mechanism for smoking cessation (Giné et al., 2010; Halpern et al., 2015), preventive health care (Bai et al., 2017), or gym attendance (Royer et al., 2015). Bhattacharya et al. (2015) find a slightly higher demand for exercise pre-commitment contracts among the users of the commitment contract site stikK.com. In other health domains, we observe more demand for commitment: between one third and one half of experimental subjects choose commitment for sobriety (Schilbach, 2019) and healthy food (Schwartz et al., 2014; Sadoff et al., 2015; Toussaert, 2019), while Alan and Ertac (2015) find strong demand for commitment among chocolate-eating children (69% take-up rate).

Studying procrastination, Laibson (2015) identifies four drivers of weak demand for commitment: naivete, high cost of commitment, uncertainty about the opportunity cost of time, high cost of delay. Our results contribute to the literature by providing a new explanation for the weak demand for commitment in the health domain. Commitment mechanisms focus on achieving healthy behavior such as, e.g., smoking cessation, (alcohol and drug) sobriety, healthy weight. However, our result that the extensive margin of sin good consumption is independent of present focus implies that an individual who overconsumes a sin good would not find it optimal to choose the healthy consumption in the absence of present-focused preferences. Hence, a commitment device not only has the usual benefit of preventing the utility loss due to present focus, but also the additional cost of causing a utility loss due to implementing healthy consumption. This cost arises because the individual would like to choose an unhealthy consumption in the absence of present focus. The individual, therefore, demands a commitment device only if the utility loss due to present-focused preferences is larger than the utility loss from healthy consumption.

The rest of the paper proceeds as follows. In Section 2, we present the quasi-hyperbolic discounting model. In Sections 3 and 4, we analyze the unitary-self and dual-self models with temptation and self-control, respectively. Section 5 extends the model, while Section 6 concludes.

2 Quasi-Hyperbolic Discounting

2.1 The Model

Consider a representative infinitely-lived individual. Time periods evolve discretely. In period $t \in \{0, 1, \ldots\}$, the representative individual consumes x_t units of a sin good and a bundle of other goods whose quantity is denoted by z_t. Following Becker and Murphy (1988) and Gruber and Köszegi (2001), past consumption affects current utility through accumulation of a consumption stock. Let s_t be the stock of past consumption in period t. It follows the equation of motion

$$s_t = x_{t-1} + (1-d)s_{t-1}, \tag{1}$$

where $d \in]0, 1]$ denotes the decay of the stock between two consecutive periods.

Instantaneous utility of the individual is $u(x_t, z_t, s_t) = w(x_t, z_t) - c(s_t)$, where $w(\cdot)$ is consumption utility and satisfies $w_i(\cdot) > 0 > w_{ii}(\cdot)$ for $i = x, z$ and $w_{xx}w_{zz} - w_{xz}^2 > 0$. The term $c(s_t)$ represents the health costs of past consumption.[1] We define a healthy stock of past consumption, s^H, as the stock for which the marginal health costs equal zero, i.e.,

$$c'(s^H) = 0. \tag{2}$$

The corresponding healthy consumption level, x^H, is the steady state consumption associated with a healthy steady state stock, i.e., $x^H = ds^H$ from Equation (1).

The healthy consumption stock s^H may be either positive or zero, depending on the sin good's type. In the case of $s^H > 0$, the health costs $c(s_t)$ are assumed to be

[1]The assumed utility function does not consider habits in consumption. In Section 5, we extend the analysis to include habits and show that the main results continue to hold.

U-shaped around s^H. If the current stock is above the healthy level, then the marginal health costs are positive: $c'(s_t) > 0$ for $s_t > s^H$. If the current consumption stock is below the healthy level, then the marginal costs are negative: $c'(s_t) < 0$ for $s_t < s^H$. In the case of $s^H = 0$, the current consumption stock s_t cannot be below the healthy stock s^H and the health costs function is increasing for all $s_t > 0$; that is, $c'(s_t) > 0$ for all $s_t > s^H = 0$. Independent of $s^H > 0$ or $s^H = 0$, we require $c''(\cdot) \geq 0$, which guarantees that the consumption choices of the individual are well-behaved.

For most sin goods, we have $s^H = 0$. Examples are cigarettes or drugs. A consumer with zero consumption in the past faces no health costs from smoking or using drugs for the first time. However, further consumption creates health costs. Put differently, there is no healthy consumption level of cigarettes or drugs that is strictly positive (Schane et al. (2010) review the evidence that even intermittent smoking is associated with health risks). In contrast, however, if the sin good under consideration is unhealthy food, the stock of past consumption can approximately be measured by the individual's body mass index (BMI). The healthy stock s^H then represents the healthy BMI level between $22.5 - 25.0$ kg/m^2, which means $s^H > 0$. In this case, $c'(s^H) = 0$ indicates that an individual with BMI in the healthy range does not face positive health costs by slightly increasing her BMI. A meta-analysis of more than 200 studies finds the hazard ratio for mortality to be a U-shaped function of the body-mass index with a minimum at the healthy BMI (Global BMI Mortality Collaboration, 2016). In the related case of unhealthy nutrients, $s^H > 0$ is also possible. There is evidence of a U-shaped relationship between added sugar consumption and mortality risk. Using Swedish data, Ramne et al. (2018) find that all-cause, cardiovascular, and cancer mortality are U-shaped functions of added sugar consumption. The lowest mortality risk is found at added sugar intake between 7.5% and 10% of the total energy intake. Health costs may arise at very low sugar consumption because sugar is an ingredient of some "healthy foods" such as yogurt (Erickson and Slavin, 2015). Moreover, it enhances food safety by preventing high growth of some microorganisms (Erickson and Slavin, 2015). Finally, in the case of alcohol, there is empirical evidence that moderate consumption may improve cardiovascular health (Cawley and Ruhm, 2011). Hence, $s^H > 0$ may also hold for the sin good alcohol.

The individual may exhibit present-bias, i.e., it seeks immediate gratification, which is inconsistent with its long term preferences. Present-bias is modelled using quasi-hyperbolic discounting, following Laibson (1997). The lifetime utility of the individual in period t is given by

$$U_t = u(x_t, z_t, s_t) + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u(x_\tau, z_\tau, s_\tau), \tag{3}$$

where $\delta \in]0,1]$ denotes the degree of exponential discounting and $\beta \in]0,1]$ is the rate of quasi-hyperbolic discounting. If $\beta = 1$, then there is no present-bias and the preferences are time-consistent. To the contrary, $\beta < 1$ denotes the desire for immediate gratification and time-inconsistency, as the discount factor between any two consecutive future periods (δ) is larger than the discount factor between the current and next period ($\beta\delta$). For $\beta < 1$, the individual has no power of self-control. Throughout this section, we refer interchangeably to individuals with $\beta < 1$ either as individuals with self-control problems or as present-biased individuals.

Both goods are produced at a constant marginal cost under perfect competition. We normalize the price of z_t to one, and the relative price of x_t is p_t. Thus, the time t budget constraint of the individual is

$$p_t x_t + z_t = e, \tag{4}$$

where e denotes the exogenously given income of the individual in period t. Each period the individual chooses x_t and z_t to maximize the lifetime utility (3) under consideration of the equation of motion (1) and the budget constraint (4).

If the individual exhibits present-bias, then the optimal consumption path depends on whether and to what extent the individual expects its future selves to behave time-inconsistently, i.e., how sophisticated the individual is. We follow O'Donoghue and Rabin (2001) and assume that an individual with discount rate β expects its future selves to have a taste for immediate gratification $\hat{\beta} \in [\beta, 1]$. If $\hat{\beta} = \beta < 1$, then the individual is said to be sophisticated, i.e., it anticipates its future self-control problems correctly. An individual is naive if it is characterized by $\beta < 1$ and $\hat{\beta} = 1$ because this individual is not aware of the present-bias of its future selves. Partial naiveté is present when $\beta < \hat{\beta} < 1$.[2]

[2]This form of modeling the degree of sophistication of individuals with self-control problems is standard in the literature. See, e.g., Gruber and Köszegi (2001, 2004) for application to cigarette consumption and

To distinguish the different types of individuals in the remaining analysis, we index the variables using a superscript $i = s, n$, where s denotes a sophisticated individual and n denotes full or partial naiveté.

2.2 Optimal Consumption and the Extensive Margin

The representative individual of type i maximizes the perceived lifetime utility at time t, given by Equation (3). Note that an individual of type i consumes x_t^i units of the sin good in period t and expects to be a sophisticate individual with present-bias $\hat{\beta}$ from period $t+1$ onwards. We denote the expected period $t+1$ consumption of a type i individual as $x_{t+1}^s(\hat{\beta})$. A sophisticate individual correctly predicts to consume $x_{t+1}^s(\beta)$ in period $t+1$, while a naive individual incorrectly expects to consume the quantity that a sophisticate with present-bias $\hat{\beta}$ would optimally choose.

To simplify the notation in the following analysis, we introduce the instantaneous utility function $\omega(x_t) \equiv w(x_t, e - p_t x_t)$. In Appendix A, we derive the Euler equation of an individual of type i for $i = s, n$. In the case of positive consumption of the sin good, the Euler equation is given by[3]

$$\omega_x(x_t^i) \quad = \quad \frac{\beta\delta}{\hat{\beta}} \left\{ \omega_x(x_{t+1}^s(\hat{\beta})) \left[1 - d + (1 - \hat{\beta}) \frac{\partial x_{t+1}^s(\hat{\beta})}{\partial s_{t+1}^i} \right] + \hat{\beta} c'(s_{t+1}^i) \right\}, \tag{5}$$

where $\omega_x(x_t^i)$ denotes the *net* marginal utility of sin good consumption in period t and is given by $\omega_x(x_t^i) = w_x(\cdot) - p_t w_z(\cdot)$. The term $\omega_x(x_{t+1}^s(\hat{\beta}))$ is defined analogously. Equation (5) has the following interpretation. Along the optimal path, the individual cannot increase its utility by a marginal increase in consumption in period t, followed by a reduction in period $t+1$, such that the consumption stock in period $t+2$ remains unaffected. The term on the left-hand side of (5) gives the marginal utility that a consumer derives of consuming one more unit of the sin good in period t. An additional unit of consumption in period t also increases the stock in period $t+1$, s_{t+1}^i. The marginal health effect of the change in s_{t+1}^i is captured by the last term on the right-hand side of (5). The change in

Diamond and Köszegi (2003) in the context of quasi-hyperbolic discounting and retirement.

[3] We report the Euler equation in case of positive consumption in (5) to simplify the exposition of our results. The complete Euler equation that takes into account the possibility of $x_t^i = 0$ and $x_{t+1}^s(\hat{\beta}) = 0$ is given by Equation (A.17) in Appendix A.

the stock also affects the optimal consumption in period $t+1$, $x^s_{t+1}(\hat{\beta})$. If the individual expects to be time-inconsistent, that is, if $\hat{\beta} < 1$, this effect also impacts the next period utility (captured by the second term in brackets in (5)). Finally, to undo the consumption stock effects of the period t change in consumption, the individual must lower period $t+1$ consumption by $1-d$ units. The utility effect of this change is captured by the first term in brackets in (5).

The intensive margin of consumption of sophisticate and naive present-biased consumers is discussed in detail by Gruber and Köszegi (2000).[4] Present-bias increases consumption by understating the future health costs. Sophisticates may either consume more or less compared to naifs, depending on the relative sizes of several effects. On the one hand, sophisticates may consume more than naifs owing to a pessimism effect; that is, because (i) they are pessimistic about their future self-control and (ii) high future consumption increases present consumption due to complementarity between x_t and x_{t+1}. On the other hand, sophisticates may consume less than naifs in order to lower the future consumption stock, s_{t+1}. A sophisticate does so in order to (i) incentivize its future selves to consume less (because consumption x_{t+1} is increasing in the stock s_{t+1} in Gruber and Köszegi (2000)) and (ii) to lower the damage that future selves could do (a damage control effect).

In contrast to Gruber and Köszegi (2000, 2001) and the subsequent literature, we analyze the conditions under which the steady state consumption deviates from its healthy level. Hence, we focus on the steady state extensive margin of sin good consumption. To do so, we first define a "desired" consumption level x^F. It is the amount of sin good consumption that maximizes the individual's instantaneous utility when the price is at its steady state level. Let a variable with a bar denote its steady state value. For a given price $\overline{p}$, we define $\omega_x(x^F) = 0$. To understand the intuition behind this equation, rewrite

[4]There are several differences between the model in this section and Gruber and Köszegi (2000). On the one hand, this section abstracts away from addiction, while Gruber and Köszegi (2000) assume the sin good is addictive (modeled by $u_{xs} > 0$). Even though addictiveness makes the marginal utility of the consumption stock ambiguous, Gruber and Köszegi (2000) assume it to be everywhere negative. Thus, in their model, the healthy stock is zero. On the other hand, this model is more general by allowing for a positive healthy stock, $s^H > 0$. Moreover, Gruber and Köszegi (2000) assume $\hat{\beta} = \{\beta, 1\}$, while we allow for $\hat{\beta} \in [\beta, 1]$. We also model addiction in an extension in Section 5.

it as

$$\frac{w_x(x^F, e - \overline{p}x^F)}{w_z(x^F, e - \overline{p}x^F)} \;=\; \overline{p}. \tag{6}$$

The left-hand side of (6) gives the marginal rate of substitution between the sin good and the bundle of other goods consumption, while the right-hand side gives the relative price of the sin good. Furthermore, if $\omega_x(0) \leq 0$, we define $x^F = 0$.

Note that x^F and x^H are the same for both types because x^F is determined by the instantaneous utility function and the relative price, while x^H is determined by the level of the healthy stock and the equation of motion. Both are not influenced by β or $\hat{\beta}$. We derive the following results regarding the steady state consumption $\overline{x}^i$ for $i = s, n$.

Proposition 1. *Suppose that $x^H > 0$. Then, there exist three possible steady states for the consumer of type $i = s, n$:*

(a) If $x^F > x^H$, then $x^H < \overline{x}^i < x^F$. The condition $x^F > x^H$ is necessary and sufficient for overconsumption: $\overline{x}^i > x^H$.

(b) If $x^F < x^H$, then $x^H > \overline{x}^i \geq x^F$. The condition $x^F < x^H$ is necessary and sufficient for underconsumption: $\overline{x}^i < x^H$.

(c) If $x^F = x^H$, then $x^H = \overline{x}^i = x^F$. The condition $x^F = x^H$ is necessary and sufficient for healthy consumption $\overline{x}^i = x^H$.

If $x^H = 0$, then only cases (a) and (c) exist.

Proof: See Appendix B. □

According to Proposition 1, it is sufficient to know the relation between the "desired" level of consumption and the healthy consumption to determine whether an individual deviates from x^H on the extensive margin. Thus, the decision on the extensive margin is not influenced by present-bias (neither by β nor by $\hat{\beta}$). Present-bias only affects the extent of the deviation (the intensive margin), but not whether the individual deviates from healthy consumption. The intuition behind this insight is the following. The individual has two goals: (i) maximization of instantaneous utility and (ii) minimization of the health problems. It achieves the first goal when x is equal to x^F and the second when x is equal to x^H. In the optimum, the individual consumes between these two amounts. Thus, β and $\hat{\beta}$ do not influence the extensive margin of consumption.

3 Temptation

The previous section considers the (β, δ) – model of a time-inconsistent individual, who has no self-control. In this section, we extend the analysis to consider individuals with temptation problems who are capable of self-control. In so doing, we follow the approach of the so-called Gul-Pesendorfer preferences (Gul and Pesendorfer, 2001, 2004, 2005).

Gul and Pesendorfer define preferences over sets of lotteries (or consumption sets) using two distinct utility functions (Gul and Pesendorfer, 2001). The first one describes commitment utility (u) and gives the utility in the absence of temptation. The second one describes temptation utility (v) and ranks consumption sets according to temptation. The preferences over a choice from a given set are defined as follows: given a consumption set A, the individual solves $\max_{x\in A}[u(x)+v(x)] - \max_{\widetilde{x}\in A} v(\widetilde{x})$, where x and $\widetilde{x}$ are the actual and most tempting consumption levels, respectively. By choosing actual consumption in order to maximize $u+v$, the consumer compromises between commitment and temptation utility. Moreover, the term $\max_{\widetilde{x}\in A} v(\widetilde{x}) - v(x)$ represents the cost of self-control, i.e., the cost of not choosing the most tempting consumption.

We analyze self-control within the Gul-Pesendorfer framework by specifying utility recursively as in Krusell et al. (2010). In this framework, the consumer is time-consistent and there is just one type. Thus, we drop the superscript i introduced in the previous section. Denote the actual consumption decisions as x_t and z_t and the corresponding actual stock of past consumption as s_t. The (hypothetical) temptation consumption choices in period t are $\widetilde{x}_t$ and $\widetilde{z}_t$. Given an actual stock s_t, the hypothetical stock in period $t+1$, if the individual succumbs to temptation in t, is $\widetilde{s}_{t+1}$, and is determined by

$$\widetilde{s}_{t+1} = \widetilde{x}_t + (1-d)s_t. \tag{7}$$

We represent the preferences recursively as

$$W(s_t) \quad = \quad \max_{x_t, z_t} \left\{ u(x_t, z_t, s_t) + \delta W(s_{t+1}) + V(x_t, z_t, s_t, s_{t+1}) - \max_{\widetilde{x}_t, \widetilde{z}_t} \left\{ V(\widetilde{x}_t, \widetilde{z}_t, s_t, \widetilde{s}_{t+1}) \right\} \right\}, \tag{8}$$

where $W(s_t)$ is the value function representing the self-control preferences of the individual in period t, $\delta \in]0,1]$ is a time discount factor, $u(x_t, z_t, s_t) = w(x_t, z_t) - c(s_t)$ is defined as in Section 2, and $V(\cdot)$ is the temptation function

$$V(x_t, z_t, s_t, s_{t+1}) = \gamma \left[u(x_t, z_t, s_t) + \beta \delta W(s_{t+1}) \right], \quad \text{with} \quad \gamma > 0,\ \beta \in]0,1[. \tag{9}$$

The temptation value function $V(\cdot)$ differs from the value function $W(s_t)$ in its discount factor $\beta\delta < \delta$. Moreover, $\gamma > 0$ gives the weight of temptation in overall utility. Thus, the term γ captures the cost of self-control, while $1 - \beta$ represents the strength of temptation (Amador et al., 2006).

The temptation function $V(\widetilde{x}_t, \widetilde{z}_t, s_t, \widetilde{s}_{t+1})$ is given by

$$V(\widetilde{x}_t, \widetilde{z}_t, s_t, \widetilde{s}_{t+1}) = \gamma \left[u(\widetilde{x}_t, \widetilde{z}_t, s_t) + \beta \delta W(\widetilde{s}_{t+1}) \right]. \tag{10}$$

It depends both on the hypothetical temptation choices $\widetilde{x}_t, \widetilde{z}_t$ (and the associated future stock $\widetilde{s}_{t+1}$) and the realized current stock s_t, because s_t is pre-determined from period $t-1$. We insert Equations (9) and (10) in (8), which gives

$$\begin{aligned} W(s_t) \quad = \quad \max_{x_t, z_t} & \left\{ (1+\gamma) u(x_t, z_t, s_t) + \delta(1+\beta\gamma) W(s_{t+1}) \right. \\ & \left. -\gamma \max_{\widetilde{x}_t, \widetilde{z}_t} \left\{ u(\widetilde{x}_t, \widetilde{z}_t, s_t) + \beta\delta W(\widetilde{s}_{t+1}) \right\} \right\}. \end{aligned} \tag{11}$$

Krusell et al. (2010) show in a consumption-savings model that in the case of CRRA utility, as $\gamma \to \infty$, the preferences represented by Equation (11) converge to the quasi-hyperbolic (β, δ) – model.

The optimal consumption decisions are described by two Euler equations: one for the actual choices and one for the hypothetical temptation choices. In Appendix C, we derive the following Euler equation for realized consumption:[5]

$$\omega_x(x_t) \;=\; \frac{\delta(1+\beta\gamma)}{1+\gamma}\left\{(1-d)\omega_x(x_{t+1})+c'(s_{t+1})+\gamma(1-d)\Big[\omega_x(x_{t+1})-\omega_x(\widetilde{x}_{t+1})\Big]\right\}, \tag{12}$$

where we use the same notation as in Section 2 with $\omega(x_t) \equiv w(x_t, e - p_t x_t)$. There are two differences to the Euler equation from Section 2. First, there is no term containing $\partial x^s_{t+1}/\partial s^i_{t+1}$ because the individual is time-consistent. Second, the term $\gamma(1-d)\left[\omega_x(x_{t+1})-\omega_x(\widetilde{x}_{t+1})\right]$ captures the cost of self-control at the margin. The Euler equation describing the optimal hypothetical temptation consumption reads

$$\omega_x(\widetilde{x}_t) \;=\; \beta\delta\left\{(1-d)\omega_x(x^h_{t+1})+c'(\widetilde{s}_{t+1})+\gamma(1-d)\Big[\omega_x(x^h_{t+1})-\omega_x(\widetilde{x}^h_{t+1})\Big]\right\}, \tag{13}$$

where x^h_{t+1} and $\widetilde{x}^h_{t+1}$ represent the actual and temptation choices in period $t+1$ in the hypothetical situation, where the individual succumbs to temptation in period t.

Our main result from Section 2 is that the decision on the extensive margin is independent of β and $\hat{\beta}$. The individual deviates from the healthy consumption level x^H in the steady state if and only if x^H differs from the "desired" consumption x^F. We now show that this result continues to hold in the presence of self-control. Define x^H and x^F as in Section 2. We can then derive the following result.

Proposition 2. *Suppose that $x^H > 0$. Then, the steady state actual consumption $\overline{x}$ fulfills the following properties:*

(a) If and only if $x^F > x^H$, the individual overconsumes in steady state: $\overline{x} > x^H$.

(b) If any only if $x^F < x^H$, the individual underconsumes in steady state: $\overline{x} < x^H$.

(c) If and only if $x^F = x^H$, the individual consumes healthy in steady state: $\overline{x} = x^H$.

If $x^H = 0$, then only cases (a) and (c) exist.

[5] Similarly to the previous section, we present the Euler equation in the case of positive actual and temptation consumption levels in (12) in order to simplify the exposition of the model. The complete Euler equation that takes into account the possibility of a corner solution with zero consumption is given by Equation (C.29) in Appendix C. Similarly, (13) describes the Euler equation for temptation consumption in the case of positive consumption. The temptation Euler equation that takes into account corner cases in given by (C.31) in Appendix C.

Proof: See Appendix D. □

According to Proposition 2, neither the cost of self-control γ nor the degree of temptation $1-\beta$ influence the extensive margin. The intuition behind this insight is straightforward. The individual's commitment utility u contains a trade-off between healthy consumption x^H and "desired" consumption x^F. Hence, the commitment utility-maximizing consumption deviates from x^H if and only if $x^F \neq x^H$. The presence of temptation utility leads to a trade-off between actual and temptation consumption. Because the temptation discount factor $\beta\delta$ is lower than the discount factor δ, temptation consumption is larger than actual consumption if there are positive marginal costs of consumption today ($s_t > s^H$) and smaller than actual consumption if there are negative marginal costs of consumption today ($s_t < s^H$). Therefore, the presence of temptation only determines the degree of deviation from healthy consumption, given that the individual deviates in the absence of temptation, i.e., given $x^F \neq x^H$. If, however, $x^F = x^H$, then the temptation and actual choices are the same in the steady state. In any case, the present focus parameters γ and β do not impact the individual's decision at the extensive margin.

4 A Dual-Self Model

A third type of framework that describes present-focused preferences is the dual-self framework (see, e.g., Thaler and Shefrin, 1981; Bernheim and Rangel, 2004; Benhabib and Bisin, 2005; Fudenberg and Levine, 2006). In contrast to the time-inconsistent multiple-self model where the consumer is a different self in each period t, the dual-self models suppose that two selves co-exist in each period: a short-run and a long-run self. While the short-run self may be myopic (Fudenberg and Levine, 2006) or addicted (Bernheim and Rangel, 2004), the long-run self takes the full lifetime utility into account. Note that one interpretation of the Gul-Pesendorfer preferences is that they also represent a dual-self model, where the short-run self's utility is the temptation utility v, while the commitment utility u describes the long-run self's preferences (Bryan et al., 2010).

We use the framework introduced by Fudenberg and Levine (2006). In this framework, the short-run self has period t preferences $u(\widetilde{x}_t, \widetilde{z}_t, s_t) = w(\widetilde{x}_t, \widetilde{z}_t) - c(s_t)$, where $\widetilde{x}_t$ and $\widetilde{z}_t$ are the consumption levels of x_t and z_t, respectively, that the short-run self would like to consume. Thus, this self is fully myopic and wishes to choose $\widetilde{x}_t, \widetilde{z}_t$ in order to maximize the short-run utility $u(\widetilde{x}_t, \widetilde{z}_t, s_t)$. Using the budget constraint (4) to substitute $\widetilde{z}_t$ by $e - p_t \widetilde{x}_t$, we derive the first-order condition $w_x(\cdot) - p_t w_z(\cdot) \equiv \omega_x(\widetilde{x}_t) \leq 0$ (with strict equality when $\widetilde{x}_t > 0$). Hence, when the price equals the steady state level $\overline{p}$, the short-run self chooses $\widetilde{x} = x^F$ according to Equation (6). The long-run self chooses the actual period t consumption level by maximizing the exponentially discounted sum of utilities

$$U_0 = \sum_{t=0}^{\infty} \delta^t \left[u(x_t, z_t, s_t) - \gamma \left(\max_{\widetilde{x}_t, \widetilde{z}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, z_t, s_t) \right)^a \right], \quad \gamma > 0, \ a \geq 1, \tag{14}$$

where $\gamma \left[\max_{\widetilde{x}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, z_t, s_t) \right]^a$ represents the cost of self-control, and $a \geq 1$ represents the cognitive load of self-control. If a is strictly greater than one, the cost of self-control is nonlinear and its marginal cost is an increasing function. This feature captures the psychological evidence that higher self-control is associated with higher cognitive load (Fudenberg and Levine, 2006).[6]

We derive the Euler equation in Appendix E. In the case of positive consumption levels x_t, x_{t+1}, it is given by[7]

$$\begin{aligned} \delta c'(s_{t+1}) &= \omega_x(x_t) \left\{ 1 + \gamma a \left[\max_{\widetilde{x}_t, \widetilde{z}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, z_t, s_t) \right]^{a-1} \right\} \\ &\quad - \delta(1-d)\omega_x(x_{t+1}) \left\{ 1 + \gamma a \left[\max_{\widetilde{x}_{t+1}, \widetilde{z}_{t+1}} u(\widetilde{x}_{t+1}, \widetilde{z}_{t+1}, s_{t+1}) - u(x_{t+1}, z_{t+1}, s_{t+1}) \right]^{a-1} \right\}. \end{aligned} \tag{15}$$

We derive the following results.

Proposition 3. *Suppose that $x^H > 0$. The steady state consumption $\overline{x}$ is characterized by the following conditions:*

(a) If $x^F > x^H$, then $x^H < \overline{x} < x^F$. The condition $x^F > x^H$ is necessary and sufficient for overconsumption: $\overline{x} > x^H$.

[6] If $a = 1$, then the cost of self-control is linear, and the utility function (14) is a special case of the Krusell et al. (2010) representation of the Gul-Pesendorfer preferences where $\beta = 0$.

[7] When either $x_t = 0$ or $x_{t+1} = 0$, the Euler equation is given by Equation (E.13) in Appendix E.

(b) If $x^F < x^H$, then $x^H > \overline{x} \geq x^F$. The condition $x^F < x^H$ is necessary and sufficient for underconsumption: $\overline{x} < x^H$.

(c) If $x^F = x^H$, then $x^H = \overline{x} = x^F$. The condition $x^F = x^H$ is necessary and sufficient for healthy consumption: $\overline{x} = x^H$.

If $x^H = 0$, then only cases (a) and (c) exist.

Proof: See Appendix F. □

Hence, the deviation from x^H on the extensive margin is independent of the cost of self-control (γ) and the cognitive load of self-control (a) in the dual-self model of Fudenberg and Levine (2006). The intuition behind this result is that the preferences of the long-run and short-run selves are perfectly aligned on the extensive margin. To see this, note that the short-run self would wish to consume x^F in steady state, while the long-run self chooses $\overline{x}$, which is a compromise between x^H and x^F. Hence, the sign of $\overline{x} - x^H$ is identical to the sign of $x^F - x^H$. Thus, the two selves always agree on whether they should over-, underconsume, or consume at a healthy level the temptation good (extensive margin) even though they disagree on how much to deviate, given that they agreed to deviate (intensive margin).

5 Addictive sin goods

In the previous sections, we neglect one of the major characteristics of some sin goods: their addictiveness. Sin goods with zero healthy consumption are usually addictive (e.g., cigarettes, illegal drugs). In this section, we extend the (β, δ)– model of Section 2 to take addiction into account. We follow Becker and Murphy (1988) and Gruber and Köszegi (2001) and assume that utility takes the quadratic form

$$u(x_t, z_t, s_t) \quad = \quad \alpha_x x_t + \frac{\alpha_{xx}}{2} x_t^2 + \alpha_{xs} x_t s_t + \alpha_s s_t + \frac{\alpha_{ss}}{2} s_t^2 + \alpha_z z_t + \frac{\alpha_{zz}}{2} z_t^2, \qquad (16)$$

where α_x, α_z are positive constants, while $\alpha_{xx}, \alpha_{ss}, \alpha_{zz}$ are negative. Moreover, $\alpha_{xs} > 0$ measures the addictiveness of the sin good, as past consumption of addictive goods creates habits that often increase their marginal utility (Becker and Murphy, 1988; Gruber and

Köszegi, 2001). For simplicity, we use the budget constraint to substitute z_t by $e - px_t$ in the utility function and express utility as

$$\begin{aligned}\widehat{u}(x_t, s_t) &\equiv u(x_t, e - p_t x_t, s_t) &(17)\\ &= \alpha_x x_t + \frac{\alpha_{xx}}{2} x_t^2 + \alpha_{xs} x_t s_t + \alpha_s s_t + \frac{\alpha_{ss}}{2} s_t^2 + \alpha_z (e - p_t x_t) + \frac{\alpha_{zz}}{2} (e - p_t x_t)^2.\end{aligned}$$

The marginal utility of the consumption stock s_t is given by

$$\widehat{u}_s(x_t, s_t) = \alpha_s + \alpha_{xs} x_t + \alpha_{ss} s_t. \tag{18}$$

Equation (18) measures both the marginal utility and the marginal health costs of past consumption. We redefine the healthy consumption stock s^H as the stock that makes past sin good consumption harmless at the margin when consumption x_t equals the level associated with s^H in steady state, i.e., when $x_t = x^H \equiv ds^H$. Thus, we define s^H implicitly by $\widehat{u}_s(ds^H, s^H) = 0$. Because most addictive goods are likely to have a zero harmless consumption level, we focus, without loss of generality, on the case $x^H = s^H = 0$ in the remaining analysis. This case emerges if

$$\widehat{u}_s(0, 0) = 0, \tag{19}$$

that is, if $\alpha_s = 0$ according to (18).[8]

Furthermore, we again define the "desired" consumption x^F as well as its respective steady state stock $s^F = x^F/d$ as the consumption level that maximizes the instantaneous utility. Thus, we define x^F by the condition

$$\widehat{u}_x(x^F, s^F) = 0, \tag{20}$$

where $\widehat{u}_x(x_t, s_t) = \alpha_x + \alpha_{xx} x_t + \alpha_{xs} s_t - p_t[\alpha_z + \alpha_{zz}(e - p_t x_t)]$ is the *net* marginal utility of current consumption. Analogously to the previous sections, if $\widehat{u}_x(0, 0) < 0$, then we set $x^F = 0$.

[8]The assumption $\alpha_s = 0$ does not affect qualitatively our results regarding the extensive margin of consumption. A positive healthy consumption stock exists if $\alpha_s > 0$. This case can be analyzed analogously to the case $\alpha_s = 0$.

Finally, as in Section 2, we differentiate between sophisticate and naive individuals, and define them analogously. We again index each type with a superscript i, where $i = s, n$, and solve the model similarly to Section 2. The Euler equation in the case of positive consumption is (see Appendix G for a derivation)[9]

$$\widehat{u}_x\left(x_t^i, s_t^i\right) \;=\; \frac{\beta\delta}{\hat{\beta}}\left\{\widehat{u}_x\left(x_{t+1}^s(\hat{\beta}), s_{t+1}^i\right)\left[1 - d + (1-\hat{\beta})\frac{\partial x_{t+1}^s(\hat{\beta})}{\partial s_{t+1}^i}\right] - \hat{\beta}\widehat{u}_s(x_{t+1}^s(\hat{\beta}), s_{t+1}^i)\right\}. \tag{21}$$

Equation (21) has the same interpretation as the Euler equation (5). However, there are two differences. First, the stock of past consumption influences the net marginal utility of current consumption $\widehat{u}_x(\cdot)$. Second, the current consumption level x_t affects the marginal utility of past consumption $\widehat{u}_s(\cdot)$. While these effects complicate the analysis, it is again possible to prove that the degrees of present-bias and naiveté (β and $\hat{\beta}$) do not influence the decision to consume unhealthy in steady state. In the following proposition, we summarize these results and list all possible cases for steady state consumption.

Proposition 4. *Suppose that the steady state is stable, such that the convergence factor satisfies $ds_{t+1}^i/ds_t^i < 1$, $i = s, n$. If $x^F = x^H = 0$, then the individual consumes healthy: $\overline{x}^i = x^H = x^F$ for $i = s, n$. If $x^F > x^H = 0$, the following cases emerge.*

(I) If $\alpha_{xs} < \min\left\{-\frac{\alpha_{ss}}{d}, -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}\right\}$, then, $x^F > \overline{x}^i > x^H$.

(II) If $\alpha_{xs} \in \left]-\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}, -\frac{\alpha_{ss}}{d}\right[$, then, $x^F > \overline{x}^i = x^H$.

(III) If $\alpha_{xs} \in \left]-\frac{\alpha_{ss}}{d}, -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}\right[$, then, $\overline{x}^i > x^F > x^H$.

(IV) If $\alpha_{xs} = -\frac{\alpha_{ss}}{d} < -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}$, then, $\overline{x}^i = x^F > x^H$.

(V) If $\alpha_{xs} = -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz} < -\frac{\alpha_{ss}}{d}$,

then, $\overline{x}^i = x^H$, if additionally $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] \leq 0$,

or $\overline{x}^i > x^H$, if additionally $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] > 0$.

[9] The complete Euler equation that takes into account the possibility of zero consumption in periods t and $t+1$ is given by Equation (G.5) in Appendix G.

Proof: See Appendix G. □

We discuss first the assumption of a stable steady state ($ds^i_{t+1}/ds^i_t < 1$). It places an upper bound on α_{xs} such that the steady state can be reached from any initial stock $s^i_0 \neq \overline{s}^i$.

The intuition behind the results in Proposition 4 is the following. First, the case $x^F = x^H$ leads to healthy steady state consumption, which is identical to the result from Proposition 1 and has the same interpretation. Second, when $x^F > x^H$, there are five cases, which have the following interpretation. The marginal utility of past consumption, $\beta\delta\widehat{u}_s(\cdot)$, is decreasing in the steady state stock of past consumption if $\alpha_{xs} < -\alpha_{ss}/d$. Moreover, the net marginal utility of current consumption, $\widehat{u}_x(\cdot)$, is decreasing in the steady state stock of past consumption if $\alpha_{xs} < -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}$. If these two conditions are fulfilled, we are in case I. This case is also the situation considered in Proposition 1 in the absence of habits. Therefore, steady state consumption is characterized by $x^F > \overline{x}^i > x^H$, as in Section 2.

In case II, the marginal utility of past consumption is decreasing in the steady state stock ($\alpha_{xs} < -\alpha_{ss}/d$), while the net marginal utility is increasing in $\overline{s}^i$ ($\alpha_{xs} > -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}$). In this case, the steady state consumption is healthy for any "desired" level $x^F \geq 0$. The intuition is that, in this case, habits make the net marginal utility increasing in the consumption stock. Thus, whenever $\overline{s}^i \in]s^H, s^F[$, the individual faces negative marginal utility of past consumption and negative net marginal utility of current consumption, both of which point to the need to decrease consumption. Due to the stability assumption, the sum of marginal utilities is negative also for $\overline{s}^i \geq s^F$. Thus, in the steady state, the individual chooses to consume healthy, even for $x^F > 0$.

Case III is the opposite to case II, meaning that the marginal utility of past consumption is increasing in the steady state stock, while the net marginal utility of current consumption is decreasing in the stock of past consumption. In this case, the consumer consumes unhealthy if $x^F > x^H$, as in case I. However, because the marginal utility of the stock of past consumption is an increasing function (owing to habits), the individual "overshoots" and consumes above the "desired" level x^F.

The two remaining cases in Proposition 4 are special cases. In case IV, the marginal utility of past consumption is independent of $\overline{s}^i$ because the marginal impact of habits exactly compensates the marginal health costs: $\alpha_{xs} = -\alpha_{ss}/d$. For this reason, the individual perceives the sin good as harmless and chooses consumption to maximize instantaneous utility, i.e., $\overline{x}^i = x^F$. In the second special case (case V), the net marginal utility of current consumption $\widehat{u}_x(\cdot)$ is not a function of $\overline{s}^i$ in the steady state. If $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] \leq 0$, then $\widehat{u}_x(\cdot) \leq 0$ for any $\overline{s}^i \geq 0$. Because the sin good is harmful, that is, $\widehat{u}_s(\cdot) < 0$ for any $\overline{s}^i > 0$, the individual consumes healthy, $\overline{x}^i = x^H$. In the second subcase of the case V, we have $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] > 0$, which means that the net marginal utility is everywhere positive, i.e., $\widehat{u}_x(\cdot) > 0$ for any $\overline{s}^i \geq 0$. Therefore, the individual chooses to consume unhealthy in steady state, $\overline{x}^i > x^H$.

Thus, Proposition 4 shows that even when the sin good is addictive, present focus does not affect the extensive margin. Time-consistent and time-inconsistent individuals consume unhealthy quantities of addictive sin goods under the same conditions.

6 Conclusions

In this paper, we analyze the determinants of sin goods consumption when individuals have present-focused preferences. We find that the extensive margin of unhealthy consumption is not affected by present focus. This result holds in the quasi-hyperbolic framework of Laibson (1997), the temptation model of Gul and Pesendorfer (2001), and the dual-self model of Fudenberg and Levine (2006).

Our results have important implications for public policy. Paternalistic policies that correct the internality caused by present-bias should only affect the intensive margin of consumption, but not the extensive margin. In a related paper, we analyze the implications of our results for the optimal paternalistic tax on unhealthy food, when consumers are present-biased (Kalamov and Runkel, 2020). There, we show that the optimal paternalistic tax, which is chosen by a government that maximizes the long-term utility of consumers, corrects only the intensive margin of obesity among obese people but not the prevalence of obesity. Moreover, a tax that implements healthy consumption may be worse than no taxation at all if the consumers' present-bias is not too strong.

Furthermore, a common feature of the three present-focus frameworks, considered in this paper, is the prediction of demand for commitment (Ericson and Laibson, 2018).

A sophisticated individual with $(\beta, \delta)-$ preferences should seek commitment to force its future selves to stick to the long-term utility-maximizing consumption choices. The same is true for individuals with Gul-Pesendorfer and dual-self preferences. Our results can explain the weak demand for commitment contracts in the health domain by the fact that such contracts seek to promote healthy behavior, such as, e.g., sobriety, smoking cessation, and healthy eating habits. Healthy behavior is, however, not optimal for a person engaging in an unhealthy activity irrespective of whether her preferences are present-focused. A present-focused individual must deviate sufficiently from a patient individual to demand such a contract.

Moreover, our analysis points to the need for a commitment mechanism that commits to the consumption that would be optimal in the absence of present focus, instead of healthy consumption. Present-focused individuals would demand it because it would increase their long-term utility. However, one problem with such mechanisms is that their design requires information about the utility functions of individuals, whereas designing a contract that commits to healthy consumption requires no such information.

Additionally, our results can explain the success of nontraditional policies in treating drug addiction. Several European countries and Canada implement supervised injectable heroin (SIH) treatment to treat heroin addicts (EMCDDA, 2012). This treatment is prescribed to patients who do not respond to traditional treatments or rehabilitation and allows the patients to self-administer injectable heroin while being fully supervised. There is strong empirical evidence that, for these patients, SIH is more effective than traditional treatments (Perneger et al., 1998; van den Brink et al., 2003; March et al., 2006; Haasen et al., 2007; Oviedo-Joekes et al., 2009; Strang et al., 2010). If the treated individuals have present-focused preferences, our results give the following explanation for the effectiveness of SIH. By sticking to an SIH treatment, where the quantity administered is strictly controlled, the individuals might achieve a higher long-term utility than in the cases of uncontrolled consumption and zero (healthy) consumption. Hence, individuals would demand it as a commitment device.

References

Alan, S. and Ertac, S. (2015). Patience, self-control and the demand for commitment: Evidence from a large-scale field experiment. *Journal of Economic Behavior & Organization*, 115:111 – 122. Behavioral Economics of Education.

Amador, M., Werning, I., and Angeletos, G.-M. (2006). Commitment vs. Flexibility. *Econometrica*, 74(2):365–396.

Bai, L., Handel, B., Miguel, E., and Rao, G. (2017). Self-Control and Demand for Preventive Health: Evidence from Hypertension in India. NBER Working Papers 23727, National Bureau of Economic Research, Inc.

Becker, G. S. and Murphy, K. M. (1988). A Theory of Rational Addiction. *Journal of Political Economy*, 96(4):675–700.

Benhabib, J. and Bisin, A. (2005). Modeling internal commitment mechanisms and self-control: A neuroeconomics approach to consumption-saving decisions. *Games and Economic Behavior*, 52(2):460 – 492. Special Issue on Neuroeconomics.

Bernheim, B. D. and Rangel, A. (2004). Addiction and cue-triggered decision processes. *American Economic Review*, 94(5):1558–1590.

Bhattacharya, J., Garber, A. M., and Goldhaber-Fiebert, J. D. (2015). Nudges in Exercise Commitment Contracts: A Randomized Trial. NBER Working Papers 21406, National Bureau of Economic Research, Inc.

Bryan, G., Karlan, D., and Nelson, S. (2010). Commitment devices. *Annual Review of Economics*, 2(1):671–698.

Cawley, J. and Ruhm, C. J. (2011). *The Economics of Risky Health Behaviors*, volume 2 of *Handbook of Health Economics*, chapter 0, pages 95–199. Elsevier.

Diamond, P. and Köszegi, B. (2003). Quasi-hyperbolic discounting and retirement. *Journal of Public Economics*, 87(9-10):1839–1872.

EMCDDA (2012). New heroin-assisted treatment. Recent evidence and current practices of supervised injectable heroin treatment in Europe and beyond. European Monitoring Centre for Drugs and Drug Addiction.

Erickson, J. and Slavin, J. (2015). Are restrictive guidelines for added sugars science based? *Nutrition Journal*, 14(124):1–6.

Ericson, K. M. and Laibson, D. (2018). Intertemporal Choice. NBER Working Papers 25358, National Bureau of Economic Research, Inc.

Fudenberg, D. and Levine, D. K. (2006). A dual-self model of impulse control. *American Economic Review*, 96(5):1449–1476.

Giné, X., Karlan, D., and Zinman, J. (2010). Put your money where your butt is: A commitment contract for smoking cessation. *American Economic Journal: Applied Economics*, 2(4):213–35.

Global BMI Mortality Collaboration (2016). Body-mass index and all-cause mortality: individual-participant-data meta-analysis of 239 prospective studies in four continents. *The Lancet*, 388(10046):776 – 786.

Gruber, J. and Köszegi, B. (2000). Is Addiction "Rational"? Theory and Evidence. NBER Working Papers 7507, National Bureau of Economic Research, Inc.

Gruber, J. and Köszegi, B. (2001). Is Addiction "Rational"? Theory and Evidence. *The Quarterly Journal of Economics*, 116(4):1261–1303.

Gruber, J. and Köszegi, B. (2004). Tax incidence when individuals are time-inconsistent: the case of cigarette excise taxes. *Journal of Public Economics*, 88(9-10):1959–1987.

Gul, F. and Pesendorfer, W. (2001). Temptation and Self-Control. *Econometrica*, 69(6):1403–1435.

Gul, F. and Pesendorfer, W. (2004). Self-Control and the Theory of Consumption. *Econometrica*, 72(1):119–158.

Gul, F. and Pesendorfer, W. (2005). The Revealed Preference Theory of Changing Tastes. *Review of Economic Studies*, 72(2):429–448.

Haasen, C., Verthein, U., Degkwitz, P., Berger, J., Krausz, M., and Naber, D. (2007). Heroin-assisted treatment for opioid dependence: Randomised controlled trial. *British Journal of Psychiatry*, 191(1):55–62.

Halpern, S. D., French, B., Small, D. S., Saulsgiver, K., Harhay, M. O., Audrain-McGovern, J., Loewenstein, G., Brennan, T. A., Asch, D. A., and Volpp, K. G. (2015). Randomized trial of four financial-incentive programs for smoking cessation. *New England Journal of Medicine*, 372(22):2108–2117. PMID: 25970009.

Harris, C. and Laibson, D. (2001). Dynamic Choices of Hyperbolic Consumers. *Econometrica*, 69(4):935–957.

Kalamov, Z. Y. and Runkel, M. (2020). Paternalistic Taxation of Unhealthy Food and the Intensive versus Extensive Margin of Obesity. mimeo.

Knapp, K. C. (1983). Steady-state solutions to dynamic optimization models with inequality constraints. *Land Economics*, 59:300 – 304.

Krusell, P., Kuruşçu, B., and Anthony A. Smith Jr. (2010). Temptation and Taxation. *Econometrica*, 78(6):2063–2084.

Laibson, D. (1997). Golden Eggs and Hyperbolic Discounting. *The Quarterly Journal of Economics*, 112(2):443–478.

Laibson, D. (2015). Why don't present-biased agents make commitments? *American Economic Review*, 105(5):267–72.

Laibson, D. (2018). Private paternalism, the commitment puzzle, and model-free equilibrium. *AEA Papers and Proceedings*, 108:1–21.

March, J., Oviedo-Joekes, E., Perea-Milla, E., and Carrasco, F. (2006). Controlled trial of prescribed heroin in the treatment of opioid addiction. *Journal of substance abuse treatment*, 31:203–11.

O'Donoghue, T. and Rabin, M. (2001). Choice and Procrastination. *The Quarterly Journal of Economics*, 116(1):121–160.

Oviedo-Joekes, E., Brissette, S., Marsh, D. C., Lauzon, P., Guh, D., Anis, A., and Schechter, M. T. (2009). Diacetylmorphine versus methadone for the treatment of opioid addiction. *New England Journal of Medicine*, 361(8):777–786. PMID: 19692689.

Perneger, T. V., Giner, F., del Rio, M., and Mino, A. (1998). Randomised trial of heroin maintenance programme for addicts who fail in conventional drug treatments. *BMJ*, 317(7150):13–18.

Phelps, E. S. and Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *The Review of Economics Studies*, 35(2):185–199.

Ramne, S., Alves Dias, J., Gonzalez-Padilla, E., Olsson, K., Lindahl, B., Engström, G., Ericson, U., Johansson, I., and Sonestedt, E. (2018). Association between added sugar intake and mortality is nonlinear and dependent on sugar source in 2 swedish population-based prospective cohorts. *The American Journal of Clinical Nutrition*, 109(2):411–423.

Royer, H., Stehr, M., and Sydnor, J. (2015). Incentives, commitments, and habit formation in exercise: Evidence from a field experiment with workers at a fortune-500 company. *American Economic Journal: Applied Economics*, 7(3):51–84.

Sadoff, S., Samek, A. S., and Sprenger, C. (2015). Dynamic inconsistency in food choice: Experimental evidence from a food desert. Technical Report 2572821, Becker Friedman Institute for Research in Economics Working Paper.

Schane, R. E., Ling, P. M., and Glantz, S. A. (2010). Health Effects of Light and Intermittent Smoking A Review. *Circulation*, 121(13):1518–1522.

Schilbach, F. (2019). Alcohol and Self-Control: A Field Experiment in India. *American Economic Review*, 109(4):1290–1322.

Schwartz, J., Mochon, D., Wyper, L., Maroba, J., Patel, D., and Ariely, D. (2014). Healthier by precommitment. *Psychological Science*, 25(2):538–546. PMID: 24390824.

Strang, J., Metrebian, N., Lintzeris, N., Potts, L., Carnwath, T., Mayet, S., Williams, H., Zador, D., Evers, R., Groshkova, T., Charles, V., Martin, A., and Forzisi, L. (2010). Supervised injectable heroin or injectable methadone versus optimised oral methadone as treatment for chronic heroin addicts in england after persistent failure in orthodox treatment (riott): a randomised trial. *Lancet*, 375:1885–95.

Strotz, R. H. (1956). Myopia and Inconsistency in Dynamic Utility Maximization. *Review of Economic Studies*, 23:165–180.

Thaler, R. H. and Shefrin, H. M. (1981). An economic theory of self-control. *Journal of Political Economy*, 89(2):392–406.

Toussaert, S. (2019). Revealing temptation through menu choice: Evidence from a weight loss challenge. Working paper, Unpublished.

van den Brink, W., Hendriks, V. M., Blanken, P., Koeter, M. W. J., van Zwieten, B. J., and van Ree, J. M. (2003). Medical prescription of heroin to treatment resistant heroin addicts: two randomised controlled trials. *BMJ*, 327(7410):310.

A Derivation of the Euler equation (5)

To derive the Euler equation (5), we follow Harris and Laibson (2001). Define the current-value function of a type i individual as $W^i(s_t^i)$ and the continuation-value function as $V^i(s_t^i)$. The optimal consumption is derived from the solution of the problem

$$W^i(s_t^i) \;=\; \max_{x_t^i, z_t^i} \left\{ u(x_t^i, z_t^i, s_t^i) + \beta\delta V^i(s_{t+1}^i) \right\} \tag{A.1}$$

subject to the constraints $x_t^i \geq 0$ and $e = z_t^i + p_t x_t^i$. We follow Knapp (1983) and solve the maximization problem by defining the Lagrangian

$$L(x_t^i, s_t^i, \lambda_t^i) \;=\; u(x_t^i, e - p_t x_t^i, s_t^i) + \beta\delta V^i(s_{t+1}^i) + \lambda_t^i x_t^i, \tag{A.2}$$

where λ_t^i is the Lagrange multiplier associated with the period t nonnegativity constraint, and we used the budget constraint (4) to replace z_t^i by $e - p_t x_t^i$. Using Equation (1), we derive the following first-order conditions:

$$\frac{\partial L(\cdot)}{\partial x_t^i} \;=\; \omega_x(x_t^i) + \beta\delta {V^i}'(s_{t+1}^i) + \lambda_t^i = 0, \tag{A.3}$$

$$x_t^i \;\geq\; 0, \tag{A.4}$$

$$\lambda_t^i x_t^i \;=\; 0, \tag{A.5}$$

$$\lambda_t^i \;\geq\; 0, \tag{A.6}$$

where $\omega(x_t^i) \equiv w(x_t^i, e - p_t x_t^i)$ and thus $\omega_x(x_t^i) = w_x(x_t^i, e - p_t x_t^i) - p_t w_z(x_t^i, e - p_t x_t^i)$.

The continuation-value function of the self in period t, $V^i(s_{t+1}^i)$, is determined by

$$V^i(s_{t+1}^i) \;=\; u(x_{t+1}^s(\hat{\beta}), z_{t+1}^s(\hat{\beta}), s_{t+1}^i) + \delta V^i(s_{t+2}^i). \tag{A.7}$$

Two comments are necessary. First, Equation (A.7) determines the continuation-value function of the self in period t and, therefore, the individual discounts exponentially at the rate δ between periods $t+1$ and $t+2$ in accordance with the lifetime utility (3). Second, the individual believes that its future selves in periods $t+1, t+2, \ldots$ will have self-control problems $\hat{\beta}$. Thus, it expects to be a sophisticated consumer with present-bias $\hat{\beta}$ from period $t+1$ onwards and to consume $x^s_{t+1}(\hat{\beta})$ in that period. Moreover, the consumer expects to purchase $z^s_{t+1}(\hat{\beta}) = e - p_{t+1}x^s_{t+1}(\hat{\beta})$ of the bundle z. We differentiate the above equation with respect to s^i_{t+1} and derive the following value for $V^{i\prime}(s^i_{t+1})$:

$$V^{i\prime}(s^i_{t+1}) = \omega_x(x^s_{t+1}(\hat{\beta}))\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}} - c'(s^i_{t+1}) + \delta V^{i\prime}(s^i_{t+2})\left[1 - d + \frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right] \quad (A.8)$$

where $\omega_x(x^s_{t+1}(\hat{\beta})) = w_x(x^s_{t+1}(\hat{\beta}), e - p_{t+1}x^s_{t+1}(\hat{\beta})) - p_{t+1}w_z(x^s_{t+1}(\hat{\beta}), e - p_{t+1}x^s_{t+1}(\hat{\beta}))$. The next step in deriving the optimal stream of consumption is to solve the maximization problem that the self in t expects to solve in $t+1$, which is given by

$$W^i(s^i_{t+1}) = \max_{x^s_{t+1}, z^s_{t+1}} \left\{u(x^s_{t+1}, z^s_{t+1}, s^i_{t+1}) + \hat{\beta}\delta V^i(s^i_{t+2})\right\} \quad (A.9)$$

subject to $x^s_{t+1} \geq 0$ and the period $t+1$ budget constraint. The only difference between Equations (A.1) and (A.9) is that the expected self-control problem $\hat{\beta}$ may differ from the actual present-bias β. Defining the Lagrangian analogously to (A.2) and denoting the period $t+1$ Lagrange multiplier as $\lambda^s_{t+1}(\hat{\beta})$, we get the following first-order conditions that the type i individual expects in period $t+1$:

$$\begin{aligned}
\frac{\partial L(\cdot)}{\partial x^s_{t+1}} &= \omega_x(x^s_{t+1}(\hat{\beta})) + \hat{\beta}\delta V^{i\prime}(s^i_{t+2}) + \lambda^s_{t+1}(\hat{\beta}) = 0, && (A.10)\\
x^s_{t+1}(\hat{\beta}) &\geq 0, && (A.11)\\
\lambda^s_{t+1}(\hat{\beta})x^s_{t+1}(\hat{\beta}) &= 0, && (A.12)\\
\lambda^s_{t+1}(\hat{\beta}) &\geq 0. && (A.13)
\end{aligned}$$

We use Equations (A.3) and (A.10) to replace the terms $V^{i\prime}(s^i_{t+1})$ and $V^{i\prime}(s^i_{t+2})$ in (A.8), and derive the Euler equation

$$\begin{aligned}
\omega_x(x^i_t) + \lambda^i_t = \frac{\beta\delta}{\hat{\beta}}&\left\{\omega_x(x^s_{t+1}(\hat{\beta}))\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right] + \hat{\beta}c'(s^i_{t+1})\right.\\
&\left. + \lambda^s_{t+1}(\hat{\beta})\left[1 - d + \frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right]\right\}. \quad (A.14)
\end{aligned}$$

Finally, we simplify (A.14) by proving that

$$\lambda^s_{t+1}(\hat\beta)\frac{\partial x^s_{t+1}(\hat\beta)}{\partial s^i_{t+1}} \;=\; 0. \tag{A.15}$$

To prove (A.15), differentiate (A.12) with respect to s^i_{t+1}:

$$\frac{\partial \lambda^s_{t+1}(\hat\beta)}{\partial s^i_{t+1}} x^s_{t+1}(\hat\beta) + \lambda^s_{t+1}(\hat\beta)\frac{\partial x^s_{t+1}(\hat\beta)}{\partial s^i_{t+1}} \;=\; 0. \tag{A.16}$$

If $\lambda^s_{t+1}(\hat\beta) = 0$, then (A.15) is satisfied. If $\lambda^s_{t+1}(\hat\beta) > 0$, then according to (A.12), $x^s_{t+1}(\hat\beta) = 0$, and thus (A.16) becomes equivalent to (A.15). Hence, (A.15) holds. Using (A.15) to simplify (A.14), we get

$$\omega_x(x^i_t) + \lambda^i_t \;=\; \frac{\beta\delta}{\hat\beta}\left\{\omega_x(x^s_{t+1}(\hat\beta))\left[1 - d + (1-\hat\beta)\frac{\partial x^s_{t+1}(\hat\beta)}{\partial s^i_{t+1}}\right] + \hat\beta c'(s^i_{t+1}) + \lambda^s_{t+1}(\hat\beta)(1-d)\right\}. \tag{A.17}$$

In the case of positive current and expected consumption, $\lambda^i_t = \lambda^s_{t+1}(\hat\beta) = 0$ and (A.17) is identical to (5).

B Proof of Proposition 1

To prove Proposition 1, we start by deriving the term $\partial x^s_{t+1}(\hat\beta)/\partial s^i_{t+1}$, which is implicitly determined by the first-order condition (A.10). If the solution to (A.10) contains $\lambda^s_{t+1}(\hat\beta) > 0$, we have $x^s_{t+1}(\hat\beta) = 0$ and thus $\partial x^s_{t+1}(\hat\beta)/\partial s^i_{t+1} = 0$. If $\lambda^s_{t+1}(\hat\beta) = 0$ and $x^s_{t+1}(\hat\beta) \geq 0$, then totally differentiating (A.10) with respect to $x^s_{t+1}(\hat\beta)$ and s^i_{t+1} gives

$$\frac{\partial x^s_{t+1}(\hat\beta)}{\partial s^i_{t+1}} \;=\; -(1-d)\frac{\hat\beta\delta V^{i\prime\prime}(s^i_{t+2})}{\omega_{xx}(x^s_{t+1}(\hat\beta)) + \hat\beta\delta V^{i\prime\prime}(s^i_{t+2})} \in]-(1-d), 0[, \tag{B.1}$$

where

$$\omega_{xx}(x^s_{t+1}(\hat\beta)) \;\equiv\; w_{xx} - 2p_{t+1}w_{xz} + p^2_{t+1}w_{zz} = \frac{w_{xx}w_{zz} - w^2_{xz} + (w_{xz} - p_{t+1}w_{zz})^2}{w_{zz}} < 0.$$

$$(B.2)$$

Equation (B.2) is negative owing to the strict concavity of $w(x_t, z_t)$ (i.e., $w_{xx}w_{zz} - w_{xz}^2 > 0, w_{zz} < 0$), while (B.1) is in the interval $]-(1-d), 0[$ due to (B.2) and the strict concavity of the continuation-value function ($V^{i\prime\prime} < 0$).[10]

Moreover, for a given constant price $\overline{p}$, the steady state consumption $\overline{x}^i$ and consumption stock $\overline{s}^i$ are determined by Equations (1) and (A.17) and are given by

$$d\overline{s}^i = \overline{x}^i, \tag{B.3}$$

$$0 = \omega_x(\overline{x}^i) + \overline{\lambda}^i - \omega_x(\overline{x}^s(\hat{\beta}))\frac{\beta\delta}{\hat{\beta}}\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right] - \overline{\lambda}^s\frac{\beta\delta}{\hat{\beta}}(1-d) - \beta\delta c'(\overline{s}^i). \tag{B.4}$$

Note that in the case $i = s$, the individual is sophisticated and $\overline{x}^s = \overline{x}^s(\hat{\beta})$. In the case of a naive consumer, $\overline{x}^s(\hat{\beta})$ is additionally determined by the Euler equation of a hypothetical sophisticate individual with present-bias $\hat{\beta}$.

Consider now the case of a positive steady state consumption, $\overline{x}^i > 0, \overline{x}^s(\hat{\beta}) > 0$, and analyze the steady state Euler equation (B.4). In this case, $\overline{\lambda}^i = \overline{\lambda}^s(\hat{\beta}) = 0$. Denote the resulting expression in (B.4) as $\Phi^i(\overline{s}^i)$ and use (B.3) to re-write it as

$$\Phi^i(\overline{s}^i) = \omega_x(d\overline{s}^i) - \omega_x(d\overline{s}^s(\hat{\beta}))\frac{\beta\delta}{\hat{\beta}}\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right] - \beta\delta c'(\overline{s}^i) = 0, \tag{B.5}$$

where we used (B.3) to additionally replace $\overline{x}^s(\hat{\beta})$ by an expression containing the steady state stock consistent with this consumption level: $d\overline{s}^s(\hat{\beta})$. We prove that $\Phi^{i\prime}(\overline{s}^i) < 0$. To do so, evaluate (A.8) and (A.10) in a steady state with positive consumption and use them to simplify (B.5):

$$\Phi^i(\overline{s}^i) = \omega_x(d\overline{s}^i) + \beta\delta V^{i\prime}(\overline{s}^i) = 0. \tag{B.6}$$

[10]A sufficient condition for the solution of the maximization problem (A.1) to maximize utility is that $U(x_t, e - p_t x_t, s_t)$ is strictly concave. In this case, $U_x(x_t, e - p_t x_t, s_t) = 0$ gives a global maximum for a fixed s_t and this maximum is stricty concave in s_t, i.e., $V^{i\prime\prime} < 0$ (Gruber and Köszegi, 2001). Furthermore, $\partial x^s_{t+1}(\hat{\beta})/\partial s^i_{t+1}$ is negative due to the absence of addiction. When the sin good is addictive, then this term may become positive (Gruber and Köszegi, 2001). We consider addiction in Section 5.

The derivative of (B.6) with respect to $\overline{s}^i$ is

$$\Phi^{i\prime}(\overline{s}^i) \;=\; \omega_{xx}(d\overline{s}^i)d + \beta\delta V^{i\prime\prime}(\overline{s}^i) < 0, \tag{B.7}$$

where we already proved $\omega_{xx}(\cdot) < 0$ in (B.2) and $V^{i\prime\prime}(\cdot) < 0$ due to the strict concavity of the utility function (see footnote 10).

We now start with the proof of Proposition 1. From Equation (B.3), we know that $\overline{x}^s = d\overline{s}^s$. Moreover, s^H and s^F are the steady state stock levels associated with healthy and "desired" steady state consumption, respectively, i.e., $s^H = x^H/d$ and $s^F = x^F/d$. Thus, $\overline{s}^s, s^H$, and s^F are defined in equal proportions to $\overline{x}^s, x^H$, and x^F, respectively. Therefore, we can refer interchangeably to the stock and consumption variables in the proof of Proposition 1.

We start by proving parts (a) and (c) for the case $s^H = 0$, and analyze first the sophisticate individual. Evaluate the Euler equation (B.4) when $i = s$, using $\overline{x}^s = d\overline{s}^s$:

$$\omega_x(d\overline{s}^s)\left\{1-\delta\left[1-d+(1-\beta)\frac{\partial x^s_{t+1}}{\partial s^s_{t+1}}\right]\right\} + \overline{\lambda}^s[1-\delta(1-d)] - \beta\delta c'(\overline{s}^s) \;=\; 0. \tag{B.8}$$

Suppose first that $x^F > 0$ and assume $\overline{x}^s = \overline{s}^s = 0$. In this case, $\omega_x(d\overline{s}^s) = \omega_x(0) > 0$ owing to $\omega_{xx}(\cdot) < 0$. Moreover, the term in curvy brackets on the left-hand side of (B.8) is positive due to $\partial x^s_{t+1}/\partial s^s_{t+1} \in]-(1-d), 0]$. Also, $\overline{\lambda}^s \geq 0$ from (A.6). Moreover, $c'(0) = 0$ due to our assumption $s^H = 0$. Thus, the left-hand side of (B.8) is positive. This is a contradiction and we conclude that if $x^F > x^H = 0$, then $\overline{x}^s = 0 = x^H$ is not possible. In the case $\overline{s}^s > 0$, $\overline{\lambda}^s = 0$ from (A.5); and the left-hand side of (B.8) is equivalent to $\Phi^s(\overline{s}^s)$. Moreover, evaluated at $\overline{x}^s = x^F > 0$, the left-hand side is negative due to $\omega_x(x^F) = 0, \overline{\lambda}^s = 0, c'(s^F) > 0$. Due to $\Phi^{s\prime}(\overline{s}^s) < 0$, we conclude that (B.8) can only be satisfied for $\overline{s}^s < s^F$. Analogously, $\overline{x}^s < x^F$. Since we already know that at $\overline{x}^s = 0$ the left-hand side of (B.8) is positive, we conclude that, in the case $x^F > 0 = x^H$, we must have $x^F > \overline{x}^s > 0 = x^H$.

Suppose now that $x^F = 0$ and assume $\overline{x}^s > 0$. In this case $\omega_x(\overline{x}^s) < 0$ owing to $\omega_{xx}(\cdot) < 0$. Additionally, $\overline{\lambda}^s = 0$ from (A.5). Furthermore, $c'(\overline{s}^s) > 0$ owing to $s^H = 0$. Thus, the left-hand side of (B.8) is negative. This is a contradiction and we conclude that in the case of $x^F = x^H = 0$, $\overline{x}^s > 0$ cannot be optimal. On the other hand,

$\overline{x}^s = 0 = x^F = x^H$ satisfies (B.8) with $\overline{\lambda}^s \propto -\omega_x(0) \geq 0$.

From the discussion in the two previous paragraphs, we conclude that $\overline{x}^s = x^H = 0$ if and only if $x^F = x^H = 0$, and $\overline{x}^s > x^H = 0$ if and only if $x^F > x^H = 0$.

We now turn to the case $s^H > 0$ and analyze the sophisticate individual. Consider part (a) of Proposition 1. Suppose $s^F > s^H > 0$ and assume the steady state satisfies $\overline{s}^s \notin]s^H, s^F[$. The case $\overline{s}^s \leq s^H < s^F$ makes the left-hand side of (B.8) positive due to $\omega_x(d\overline{s}^s) > 0, c'(\overline{s}^s) \leq 0$ and $\overline{\lambda}^s \geq 0$. The case $\overline{s}^s \geq s^F > s^H$ makes the left-hand side of (B.8) negative because it implies $\omega_x(d\overline{s}^s) \leq 0, c'(\overline{s}^s) > 0$ and $\overline{\lambda}^s = 0$. Therefore, $\overline{s}^s \notin]s^H, s^F[$ cannot be an equilibrium when $s^F > s^H$. We conclude that if $s^F > s^H > 0$ holds, then $s^F > \overline{s}^s > s^H$. Thus, $s^F > s^H$ is sufficient for overconsumption $\overline{s}^s > s^H$. To prove that $s^F > s^H$ is also necessary for overconsumption, suppose that the opposite is true. That is, suppose $s^F \leq s^H$ and $\overline{s}^s > s^H$. The steady state $\overline{s}^s > s^H \geq s^F$ implies that the left-hand side of (B.8) is negative owing to $c'(\overline{s}^s) > 0, \omega_x(d\overline{s}^s) < 0$ and $\overline{\lambda}^s = 0$. This is a contradiction and we conclude that $s^F > s^H$ is also necessary for overconsumption in steady state.

We now turn to part (b) of Proposition 1 for a sophisticate individual. We use again proof by contradiction. Suppose that $0 \leq s^F < s^H$ and $\overline{s}^s \notin]s^F, s^H[$. We already know from the proof of part (a) that if $s^F \leq s^H$, then $\overline{s}^s > s^H$ contradicts (B.8). Thus, the case $\overline{s} > s^H$ is not possible when $s^F < s^H$. Moreover, $\overline{s} = s^H > s^F \geq 0$ implies $c'(\overline{s}^s) = 0, \overline{\lambda}^s = 0$ and $\omega_x(d\overline{s}^s) < 0$. Thus, $\overline{s} = s^H > s^F \geq 0$ makes the left-hand side of (B.8) negative, which is a contradiction. Consider now the case $\overline{s}^s \leq s^F < s^H$. This case implies $c'(\overline{s}^s) < 0, \overline{\lambda}^s \geq 0$. Moreover, it implies $\omega_x(d\overline{s}^s) \geq 0$ if $\omega_x(ds^F) = 0$; and $\omega_x(d\overline{s}^s) < 0$ if $s^F = 0$ and $\omega_x(0) < 0$. Thus, the left-hand side of (B.8) is positive for all possible cases of s^F, except for the case $s^F = 0$ when $\omega_x(0) < 0$ is sufficiently negative. Thus, there is a contradiction whenever $\omega_x(0) \geq 0$. We conclude that if $s^F < s^H$, then $\overline{s}^s \in]s^F, s^H[$ if $\omega_x(0) \geq 0$. If $\omega_x(0) < 0$, then either $\overline{s}^s = s^F = 0 < s^H$ and

$$\overline{\lambda}^s = \frac{-\omega_x(0)\left\{1 - \delta\left[1 - d + (1-\beta)\frac{\partial x^s_{t+1}}{\partial s^s_{t+1}}\right]\right\} + \beta\delta c'(0)}{1 - \delta(1-d)}, \tag{B.9}$$

if $\omega_x(0)$ is sufficiently negative for the numerator of (B.9) to be nonnegative, or $\overline{s}^s \in]s^F, s^H[$, if the numerator in (B.9) is negative. This proves the sufficiency part in part (b).

To prove the necessary part, assume the opposite holds; that is, $s^F \geq s^H$ and $\overline{s}^s < s^H$. We already know from part (a) of Proposition 1 that if $s^F > s^H$, then $\overline{s}^s < s^H$ is not possible. If $s^F = s^H$, then $\overline{s}^s < s^H$ makes the left-hand side of (B.8) positive (due to $c'(\overline{s}^s) < 0, \omega_x(d\overline{s}^s) > 0$ and $\overline{\lambda}^s \geq 0$). This is a contradiction. We conclude that $s^F < s^H$ is necessary and sufficient for underconsumption by a sophisticate individual: $\overline{s}^s < s^H$.

To prove part (c) of Proposition 1, it is easy to see that, if $s^F = s^H > 0$, then $\overline{s}^s = s^H = s^F$ and $\overline{\lambda}^s = 0$ satisfy (B.8). This proves the sufficiency part. Moreover, from parts (a) and (b) from Proposition 1, we know that $\overline{s}^s = s^H$ is not possible for $s^F \neq s^H$. Thus, $s^F = s^H$ is both necessary and sufficient for healthy consumption by a sophisticate individual. This concludes the proof for sophisticate individuals.

We turn now to the naive individual. In this case, the steady state is determined by

$$\omega_x(d\overline{s}^n) + \overline{\lambda}^n - \overline{\lambda}^s(\hat{\beta})\frac{\beta\delta}{\hat{\beta}}(1-d) - \beta\delta c'(\overline{s}^n) - \omega_x(d\overline{s}^s(\hat{\beta}))\frac{\beta\delta}{\hat{\beta}}\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right] \;=\; 0, \tag{B.10}$$

where $\overline{s}^s(\hat{\beta})$ is determined by Equation (B.8) when $\beta = \hat{\beta}$.

Analogously to the analysis of a sophisticate individual, we first consider the case $x^H = 0$. Suppose $x^F > 0 = x^H$. Then, the analysis of sophisticate individuals tells us that $x^F > \overline{x}^s(\hat{\beta}) = d\overline{s}^s(\hat{\beta}) > 0$ and $\overline{\lambda}^s(\hat{\beta}) = 0$. We use proof by contradiction. Assume that $\overline{s}^n = 0$. In this case, $\omega_x(0) > 0$ (due to $x^F > 0$), $\overline{\lambda}^n \geq 0$, and $c'(0) = 0$ due to $x^H = 0$. Thus, $\omega_x(0) > \omega_x(d\overline{s}^s(\hat{\beta})) > 0$ due to $x^F > \overline{x}^s(\hat{\beta}) = d\overline{s}^s(\hat{\beta}) > 0$. Thus, the left-hand side of (B.10) becomes positive:

$$\overline{\lambda}^n + \omega_x(0) - \omega_x(d\overline{s}^s(\hat{\beta}))\frac{\beta\delta}{\hat{\beta}}\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right] \;>\; 0 \quad \text{for} \quad x^F > 0 = x^H. \tag{B.11}$$

The inequality (B.11) contradicts the Euler equation (B.10). We conclude that if $x^F > 0 = x^H$, then $\overline{s}^n = 0$ cannot be a steady state. Also, the left-hand side of (B.10) is negative for $\overline{s}^n = s^F$ (since it means $\omega_x(d\overline{s}^n) = \overline{\lambda}^n = 0$ and $c'(\overline{s}^n) > 0$). Owing to $\Phi^{n\prime}(\overline{s}^n) < 0$ (from (B.7)), the steady state must satisfy $\overline{s}^n < s^F$. Since, additionally, $\overline{s}^n$ cannot be zero (owing to (B.11)), we conclude that $\overline{s}^n \in]0, s^F[$ solves (B.10).

Consider now the case $x^F = 0 = x^H$. In this case, the analysis of sophisticate individuals tells us that $\overline{x}^s(\hat{\beta}) = d\overline{s}^s(\hat{\beta}) = 0$ and $\overline{\lambda}^s(\hat{\beta}) \geq 0$. We prove $\overline{s}^n = 0$ by contradiction. Assume $\overline{s}^n > 0$. Then, $\omega_x(d\overline{s}^n) < 0, \overline{\lambda}^n = 0$ and $c'(\overline{s}^n) > 0$. Moreover, $\omega_x(d\overline{s}^n) < \omega_x(d\overline{s}^s(\hat{\beta})) \leq 0$ due to $x^F = d\overline{s}^s(\hat{\beta}) = 0$. Thus, the left-hand side of (B.10) is negative, which is a contradiction. We conclude that if $x^F = 0 = x^H$, then $\overline{s}^n > 0$ is not possible. If $\omega_x(0) = 0$, (B.10) is satisfied for $\overline{s}^n = \overline{\lambda}^n = 0$. If $\omega_x(0) < 0$, then $\overline{s}^n = 0, \overline{\lambda}^n > 0$ satisfy (B.10). Hence, $\overline{s}^n = 0$ is the unique solution to (B.10) in the case $x^F = x^H = 0$.

We conclude that if $x^H = 0$, the condition $x^F > x^H$ is both necessary and sufficient for $x^F > \overline{x}^n > 0$ and $x^F = x^H$ is both necessary and sufficient for $\overline{x}^n = 0$.

Consider now the case $s^H > 0$. To prove part (a) from Proposition 1, we analyze the situation where $s^F > s^H > 0$. From our discussion on sophisticates, we know that if $s^F > s^H$, then $\overline{s}^s(\hat{\beta}) \in]s^H, s^F[$. Thus, $\overline{\lambda}^s(\hat{\beta}) = 0$ in (B.10). Assume $\overline{s}^n \notin]s^H, s^F[$. The case $\overline{s}^n \leq s^H$ implies $\overline{\lambda}^n \geq 0, c'(\overline{s}^n) \leq 0$, as well as $\overline{s}^n < \overline{s}^s(\hat{\beta}) < s^F$. The latter inequality implies $\omega_x(d\overline{s}^n) > \omega_x(d\overline{s}^s(\hat{\beta})) > 0$. Thus, the left-hand side of (B.10) is positive, which is a contradiction. The case $\overline{s}^n \geq s^F$ implies $\overline{\lambda}^n = 0, c'(\overline{s}^n) > 0$, and $\omega_x(d\overline{s}^n) \leq 0$. Moreover, we already know that, in this case, $\overline{s}^s(\hat{\beta}) \in]s^H, s^F[$, which implies $\omega_x(d\overline{s}^s(\hat{\beta})) > 0$. Thus, the left-hand side of (B.10) is negative, which is a contradiction. We conclude that if $s^F > s^H$, then $\overline{s}^n \in]s^H, s^F[$. In other words, $s^F > s^H$ is sufficient for a steady state with overconsumption ($\overline{s}^n > s^H$). To prove that it is also necessary, suppose that the opposite holds, i.e., $s^F \leq s^H$ and $\overline{s}^n > s^H$. From our discussion on sophisticate consumers, we know that $s^F \leq s^H$ is necessary and sufficient for $s^F \leq \overline{s}^s(\hat{\beta}) \leq s^H$. Thus, $\overline{s}^n > s^H \geq s^F$ implies $\overline{\lambda}^n = 0, c'(\overline{s}^n) > 0$ and $\overline{s}^n > \overline{s}^s(\hat{\beta}) \geq s^F$. The latter inequality means that $\omega_x(d\overline{s}^n) < \omega_x(d\overline{s}^s(\hat{\beta})) \leq 0$. Thus, the left-hand side of (B.10) is negative which is a contradiction. Therefore, $s^F > s^H$ is both necessary and sufficient for a steady state with overconsumption. This concludes the proof of part (a) for a naive individual.

To prove part (b) from Proposition 1 for a naive individual, assume $0 \leq s^F < s^H$ and $\overline{s}^n \notin]s^F, s^H[$. In the case $0 \leq s^F < s^H$, we know that a sophisticate individual's consumption satisfies $\overline{s}^s(\hat{\beta}) \in [s^F, s^H[$ and $\overline{\lambda}^s(\hat{\beta}) = 0$ (except for the case $\omega_x(0) < 0$, where $\overline{\lambda}^s(\hat{\beta}) > 0$ may emerge). From the previous paragraph, we know that if $s^F \leq s^H$, then $\overline{s}^n > s^H$ cannot be a steady state. Thus, in the case $0 \leq s^F < s^H$, $\overline{s}^n > s^H$ is not possible. Moreover, $\overline{s}^n = s^H > s^F$ would make the left-hand side of (B.10) negative due to $\overline{\lambda}^n = 0, c'(\overline{s}^n) = 0$ and $\omega_x(d\overline{s}^n) < \omega_x(d\overline{s}^s(\hat{\beta})) \leq 0$. The case $\overline{s}^n \leq s^F < s^H$ also contradicts (B.10) if $\omega_x(0) \geq 0$. The reason is that in this case $\overline{\lambda}^s(\hat{\beta}) = 0, \overline{s}^s(\hat{\beta}) \in]s^F, s^H[, c'(\overline{s}^n) < 0, \overline{\lambda}^n \geq 0, \omega_x(d\overline{s}^n) \geq 0 > \omega_x(d\overline{s}^s(\hat{\beta}))$ and the left-hand side of (B.10) is positive. On the other hand, if $\omega_x(0) < 0$, then $\overline{s}^n = s^F = 0 < s^H$ is a possible solution to (B.10). We conclude that if $0 \leq s^F < s^H$, then $s^F \leq \overline{s}^n < s^H$. This also proves that $s^F < s^H$ is sufficient for underconsumption by a naive individual: $\overline{s}^n < s^H$. To prove that $s^F < s^H$ is also necessary for underconsumption, assume that $s^F \geq s^H > 0$ and $\overline{s}^n < s^H$. We already know that when $s^F \geq s^H > 0$, the sophisticate's consumption stock satisfies $s^H \leq \overline{s}^s(\hat{\beta}) \leq s^F$ with $\overline{\lambda}^s(\hat{\beta}) = 0$. Thus, $\overline{s}^n < s^H$ would imply $\overline{s}^n < \overline{s}^s(\hat{\beta}) \leq s^F$ and thus $\omega_x(d\overline{s}^n) > \omega_x(d\overline{s}^s(\hat{\beta})) \geq 0$. Moreover, $\overline{s}^n < s^H$ means $\overline{\lambda}^n \geq 0$ and $c'(\overline{s}^n) < 0$. Thus, the left-hand side of (B.10), in this case, is positive, which is a contradiction. Thus, $s^F < s^H$ is also necessary for underconsumption by a naive individual.

It remains to prove part (c) from Proposition 1 for the naive individual when $s^H > 0$. First, we know from parts (a) and (b) that $x^F \neq x^H$ is incompatible with healthy consumption. Moreover, if $x^F = x^H > 0$, then $\overline{s}^n = \overline{s}^s(\hat{\beta}) = s^H = s^F$ and $\overline{\lambda}^n = \overline{\lambda}^s(\hat{\beta}) = 0$ satisfy both (B.8) and (B.10). Thus, $x^F = x^H$ is both sufficient and necessary for a healthy consumption. This concludes the proof of Proposition 1. □

C Derivation of the Euler Equations (12) and (13)

To derive Equations (12) and (13), we begin by restating the maximization problem:

$$\begin{aligned} W(s_t) &= \max_{x_t, z_t} \Big\{ (1+\gamma) u(x_t, z_t, s_t) + \delta(1+\beta\gamma) W(s_{t+1}) \\ &\quad -\gamma \max_{\widetilde{x}_t, \widetilde{z}_t} \Big\{ u(\widetilde{x}_t, \widetilde{z}_t, s_t) + \beta\delta W(\widetilde{s}_{t+1}) \Big\} \Big\}, \end{aligned} \tag{C.1}$$

subject to

$$x_t \geq 0, \tag{C.2}$$

$$z_t + p_t x_t = e, \tag{C.3}$$

$$\widetilde{z}_t + p_t\widetilde{x}_t = e, \tag{C.4}$$

$$s_{t+1} = x_t + (1-d)s_t, \tag{C.5}$$

$$\widetilde{s}_{t+1} = \widetilde{x}_t + (1-d)s_t. \tag{C.6}$$

Analogously to Appendix A, we define the Lagrangian

$$\begin{aligned} L(x_t, s_t, \lambda_t) &= (1+\gamma)u(x_t, e - p_t x_t, s_t) + \delta(1+\beta\gamma)W(s_{t+1}) \\ &\quad -\gamma \max_{\widetilde{x}_t} \left\{ \left[u(\widetilde{x}_t, e - p_t\widetilde{x}_t, s_t) + \beta\delta W(\widetilde{s}_{t+1}) \right] \right\} + \lambda_t x_t, \end{aligned} \tag{C.7}$$

where λ_t is the Lagrange multiplier. The first-order conditions are

$$\frac{\partial L(\cdot)}{\partial x_t} = (1+\gamma)\omega_x(x_t) + \delta(1+\beta\gamma)W'(s_{t+1}) + \lambda_t = 0, \tag{C.8}$$

$$x_t \geq 0, \tag{C.9}$$

$$\lambda_t x_t = 0, \tag{C.10}$$

$$\lambda_t \geq 0, \tag{C.11}$$

where $\omega_x(x_t) = w_x(x_t, e - p_t x_t) - p_t w_z(x_t, e - p_t x_t)$. The value function $W(s_{t+1})$ solves the maximized Bellman equation

$$\begin{aligned} W(s_{t+1}) &= (1+\gamma)u(x_{t+1}, e - p_{t+1}x_{t+1}, s_{t+1}) + \delta(1+\beta\gamma)W(s_{t+2}) \\ &\quad -\gamma \max_{\widetilde{x}_{t+1}} \left\{ u(\widetilde{x}_{t+1}, e - p_{t+1}\widetilde{x}_{t+1}, s_{t+1}) + \beta\delta W(\widetilde{s}_{t+2}) \right\}, \end{aligned} \tag{C.12}$$

where s_{t+2} and $\widetilde{s}_{t+2}$ are defined analogously to s_{t+1} and $\widetilde{s}_{t+1}$ in Equations (C.5)-(C.6). The derivative of (C.12) with respect to s_{t+1} is given by

$$\begin{aligned} W'(s_{t+1}) &= \left[(1+\gamma)\omega_x(x_{t+1}) + \delta(1+\beta\gamma)W'(s_{t+2})\right] \frac{\partial x_{t+1}}{\partial s_{t+1}} - (1+\gamma)c'(s_{t+1}) \\ &\quad +\delta(1+\beta\gamma)W'(s_{t+2})(1-d) - \gamma\left[\omega_x(\widetilde{x}_{t+1}) + \beta\delta W'(\widetilde{s}_{t+2})\right] \frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}} \\ &\quad +\gamma c'(s_{t+1}) - \gamma\beta\delta W'(\widetilde{s}_{t+2})(1-d). \end{aligned} \tag{C.13}$$

Moreover, by lagging the first-order conditions (C.8)-(C.11) one period, we get

$$(1+\gamma)\omega_x(x_{t+1}) + \delta(1+\beta\gamma)W'(s_{t+2}) + \lambda_{t+1} \;=\; 0, \tag{C.14}$$

together with $x_{t+1} \geq 0, \lambda_{t+1}x_{t+1} = 0, \lambda_{t+1} \geq 0$, where λ_{t+1} is the Lagrange multiplier associated with the choice of x_{t+1}.

We now derive the optimal temptation consumption. It is determined by the maximization problem

$$\max_{\widetilde{x}_t, \widetilde{z}_t} \quad u(\widetilde{x}_t, \widetilde{z}_t, s_t) + \beta\delta W(\widetilde{s}_{t+1}), \tag{C.15}$$

subject to $\widetilde{x}_t \geq 0$ and the period t budget constraint. Moreover, $\widetilde{s}_{t+1}$ is given by Equation (C.6). Define the Lagrangian as

$$L(\widetilde{x}_t, s_t, \widetilde{\lambda}_t) \;=\; u(\widetilde{x}_t, e - p_t\widetilde{x}_t, s_t) + \beta\delta W(\widetilde{s}_{t+1}) + \widetilde{\lambda}_t\widetilde{x}_t, \tag{C.16}$$

where $\widetilde{\lambda}_t$ is the Lagrange multiplier. The first-order conditions are

$$\frac{\partial L(\cdot)}{\partial \widetilde{x}_t} \;=\; \omega_x(\widetilde{x}_t) + \beta\delta W'(\widetilde{s}_{t+1}) + \widetilde{\lambda}_t = 0, \tag{C.17}$$

$$\widetilde{x}_t \;\geq\; 0, \tag{C.18}$$

$$\widetilde{\lambda}_t\widetilde{x}_t \;=\; 0, \tag{C.19}$$

$$\widetilde{\lambda}_t \;\geq\; 0. \tag{C.20}$$

In the hypothetical situation that the consumer succumbs to temptation in period t, the maximized Bellman equation in period $t+1$ takes the form

$$\begin{aligned} W(\widetilde{s}_{t+1}) \;=\; & (1+\gamma)u(x^h_{t+1}, e - p_{t+1}x^h_{t+1}, \widetilde{s}_{t+1}) + \delta(1+\beta\gamma)W(s^h_{t+2}) \\ & -\gamma \max_{\widetilde{x}^h_{t+1}} \Big\{ u(\widetilde{x}^h_{t+1}, e - p_{t+1}\widetilde{x}^h_{t+1}, \widetilde{s}_{t+1}) + \beta\delta W(\widetilde{s}^h_{t+2}) \Big\}, \end{aligned} \tag{C.21}$$

where x^h_{t+1} is the actual consumption choice in period $t+1$ in the hypothetical situation that the individual succumbs to temptation in t, while $\widetilde{x}^h_{t+1}$ is the optimal temptation choice in period $t+1$ in the same situation. The hypothetical stock levels are determined by

$$
\begin{aligned}
s^h_{t+2} &= x^h_{t+1} + (1-d)\widetilde{s}_{t+1}, && \text{(C.22)}\\
\widetilde{s}^h_{t+2} &= \widetilde{x}^h_{t+1} + (1-d)\widetilde{s}_{t+1}. && \text{(C.23)}
\end{aligned}
$$

The derivative of (C.21) with respect to $\widetilde{s}_{t+1}$ is

$$
\begin{aligned}
W'(\widetilde{s}_{t+1}) &= \left[(1+\gamma)\omega_x(x^h_{t+1}) + \delta(1+\beta\gamma)W'(s^h_{t+2})\right]\frac{\partial x^h_{t+1}}{\partial \widetilde{s}_{t+1}} - (1+\gamma)c'(\widetilde{s}_{t+1})\\
&\quad + \delta(1+\beta\gamma)W'(s^h_{t+2})(1-d) - \gamma\left[\omega_x(\widetilde{x}^h_{t+1}) + \beta\delta W'(\widetilde{s}^h_{t+2})\right]\frac{\partial \widetilde{x}^h_{t+1}}{\partial \widetilde{s}_{t+1}}\\
&\quad + \gamma c'(\widetilde{s}_{t+1}) - \gamma\beta\delta W'(\widetilde{s}^h_{t+2})(1-d). && \text{(C.24)}
\end{aligned}
$$

The hypothetical actual consumption x^h_{t+1} is determined analogously to x_{t+1} in Equation (C.14):

$$
(1+\gamma)\omega_x(x^h_{t+1}) + \delta(1+\beta\gamma)W'(s^h_{t+2}) + \lambda^h_{t+1} = 0, \qquad \text{(C.25)}
$$

together with $x^h_{t+1} \geq 0, \lambda^h_{t+1}x^h_{t+1} = 0, \lambda^h_{t+1} \geq 0$, where λ^h_{t+1} is the Lagrange multiplier associated with the choice of x^h_{t+1}.

The hypothetical temptation consumption $\widetilde{x}^h_{t+1}$ is the argument that solves

$$
\max_{\widetilde{x}^h_{t+1}} \quad u(\widetilde{x}^h_{t+1}, e - p_{t+1}\widetilde{x}^h_{t+1}, \widetilde{s}_{t+1}) + \beta\delta W(\widetilde{s}^h_{t+2}), \qquad \text{(C.26)}
$$

subject to $\widetilde{x}^h_{t+1} \geq 0$. The first-order conditions with respect to $\widetilde{x}^h_{t+1}$ are

$$
\omega_x(\widetilde{x}^h_{t+1}) + \beta\delta W'(\widetilde{s}^h_{t+2}) + \widetilde{\lambda}^h_{t+1} = 0, \qquad \text{(C.27)}
$$

together with $\widetilde{x}^h_{t+1} \geq 0, \widetilde{\lambda}^h_{t+1}\widetilde{x}^h_{t+1} = 0, \widetilde{\lambda}^h_{t+1} \geq 0$, where $\widetilde{\lambda}^h_{t+1}$ is the Lagrange multiplier associated with the choice of $\widetilde{x}^h_{t+1}$.

Use now Equations (C.8), (C.14), and (C.17) lagged by one period to substitute for the $W'(\cdot)$ terms in Equation (C.13). The resulting expression is

$$
\begin{aligned}
\omega_x(x_t) + \frac{\lambda_t}{1+\gamma} \quad &= \quad \frac{\delta(1+\beta\gamma)}{1+\gamma}\Bigg\{(1-d)\omega_x(x_{t+1}) + c'(s_{t+1}) + (1-d)(\lambda_{t+1} - \gamma\widetilde{\lambda}_{t+1}) \\
&\qquad +\gamma(1-d)\Big[\omega_x(x_{t+1}) - \omega_x(\widetilde{x}_{t+1})\Big] \\
&\qquad +\lambda_{t+1}\frac{\partial x_{t+1}}{\partial s_{t+1}} - \gamma\widetilde{\lambda}_{t+1}\frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}}\Bigg\}. \qquad \text{(C.28)}
\end{aligned}
$$

Analogously to Appendix A, Equations (A.15) and (A.16), we can prove that $\lambda_{t+1}(\partial x_{t+1}/\partial s_{t+1}) = 0$ and $\widetilde{\lambda}_{t+1}(\partial \widetilde{x}_{t+1}/\partial s_{t+1}) = 0$. Thus, the third row of (C.28) vanishes and it simplifies to

$$
\begin{aligned}
\omega_x(x_t) + \frac{\lambda_t}{1+\gamma} \quad &= \quad \frac{\delta(1+\beta\gamma)}{1+\gamma}\Big\{(1-d)\omega_x(x_{t+1}) + c'(s_{t+1}) + (1-d)(\lambda_{t+1} - \gamma\widetilde{\lambda}_{t+1}) \\
&\qquad +\gamma(1-d)\Big[\omega_x(x_{t+1}) - \omega_x(\widetilde{x}_{t+1})\Big]\Big\}. \qquad \text{(C.29)}
\end{aligned}
$$

In the case of strictly positive actual and temptation consumption levels, $\lambda_t = \lambda_{t+1} = \widetilde{\lambda}_{t+1} = 0$ and (C.29) simplifies to Equation (12).

To find the Euler equation (13), use (C.17), (C.25), and (C.27) to substitute for the $W'(\cdot)$ terms in Equation (C.24). The resulting expression is

$$
\begin{aligned}
\omega_x(\widetilde{x}_t) + \widetilde{\lambda}_t \quad &= \quad \beta\delta\Bigg\{(1-d)\omega_x(x^h_{t+1}) + c'(\widetilde{s}_{t+1}) + (1-d)(\lambda^h_{t+1} - \gamma\widetilde{\lambda}^h_{t+1}) \qquad \text{(C.30)} \\
&\qquad +\gamma(1-d)\Big[\omega_x(x^h_{t+1}) - \omega_x(\widetilde{x}^h_{t+1})\Big] + \lambda^h_{t+1}\frac{\partial x^h_{t+1}}{\partial \widetilde{s}_{t+1}} - \gamma\widetilde{\lambda}^h_{t+1}\frac{\partial \widetilde{x}^h_{t+1}}{\partial \widetilde{s}_{t+1}}\Bigg\}.
\end{aligned}
$$

Similarly to Equation (C.28), we can prove $\lambda^h_{t+1}(\partial x^h_{t+1}/\partial \widetilde{s}_{t+1}) = 0$ and $\widetilde{\lambda}^h_{t+1}(\partial \widetilde{x}^h_{t+1}/\partial \widetilde{s}_{t+1}) = 0$ by following the proof laid out in Appendix A, Equations (A.15), (A.16). Thus, (C.30) simplifies to

$$
\begin{aligned}
\omega_x(\widetilde{x}_t) + \widetilde{\lambda}_t \quad &= \quad \beta\delta\Big\{(1-d)\omega_x(x^h_{t+1}) + c'(\widetilde{s}_{t+1}) + (1-d)(\lambda^h_{t+1} - \gamma\widetilde{\lambda}^h_{t+1}) \\
&\qquad +\gamma(1-d)\Big[\omega_x(x^h_{t+1}) - \omega_x(\widetilde{x}^h_{t+1})\Big]\Big\}. \qquad \text{(C.31)}
\end{aligned}
$$

Finally, in the case of positive consumption levels $\widetilde{x}_t, x^h_{t+1}, \widetilde{x}^h_{t+1}$, we have $\widetilde{\lambda}_t = \lambda^h_{t+1} = \widetilde{\lambda}^h_{t+1} = 0$ and (C.31) simplifies to Equation (13).

D Proof of Proposition 2

Suppose the consumer reaches steady state consumption $\overline{x}$ with a corresponding steady state stock $\overline{s} = \overline{x}/d$. This is only possible if all the hypothetical consumption levels are also constant. Denote by $\overline{\overline{\widetilde{x}}}$ the steady state value of $\widetilde{x}_t$. Furthermore, we denote the steady state Lagrange multipliers associated with $\overline{x}$ and $\overline{\overline{\widetilde{x}}}$ as $\overline{\lambda}, \overline{\overline{\widetilde{\lambda}}}$. We use Equation (C.6) to define the following hypothetical steady state stock level:

$$\overline{\overline{\widetilde{s}}} \;=\; (1-d)\overline{s} + \overline{\overline{\widetilde{x}}}. \tag{D.1}$$

Now, we rewrite Equations (C.8), (C.13), (C.17) in steady state as[11]

$$\begin{aligned}
(1+\gamma)\omega_x(\overline{x}) + \overline{\lambda} \;&=\; -\delta(1+\beta\gamma)W'(\overline{s}), && \text{(D.2)}\\
\gamma\beta\delta(1-d)\left[W'(\overline{\overline{\widetilde{s}}}) - W'(\overline{s})\right] \;&=\; -c'(\overline{s}) - [1-\delta(1-d)]W'(\overline{s}), && \text{(D.3)}\\
\omega_x(\overline{\overline{\widetilde{x}}}) + \overline{\overline{\widetilde{\lambda}}} \;&=\; -\beta\delta W'(\overline{\overline{\widetilde{s}}}), && \text{(D.4)}
\end{aligned}$$

We furthermore rewrite the Euler equation (C.29) in steady state:

$$\begin{aligned}
0 \;=\;& (1+\gamma)\omega_x(\overline{x})\left[1 - \frac{\delta(1-d)(1+\beta\gamma)}{1+\gamma}\right] + \overline{\lambda} + \gamma\delta(1-d)(1+\beta\gamma)\left[\omega_x(\overline{\overline{\widetilde{x}}}) - \omega_x(\overline{x})\right]\\
&+\delta(1-d)(1+\beta\gamma)(\gamma\overline{\overline{\widetilde{\lambda}}} - \overline{\lambda}) - \delta(1+\beta\gamma)c'(\overline{s}). \qquad \text{(D.5)}
\end{aligned}$$

Part (a) of Proposition 2 claims that $x^F > x^H$ is both necessary and sufficient for $\overline{x} > x^H$. We use proof by contradiction. Suppose $x^F > x^H$ and assume $\overline{x} \leq x^H$. These inequalities imply $\overline{x} \leq x^H < x^F$. Moreover, from the definitions of $\overline{s}, s^H$ and s^F, we have $\overline{s} \leq s^H < s^F$. Therefore, $c'(\overline{s}) \leq 0$. Moreover, $\overline{x} < x^F$ implies $\omega_x(\overline{x}) > 0$ (we proved that $\omega_{xx}(\cdot) < 0$ in Equation (B.2) in Appendix B). Using Equation (D.2), $\omega_x(\overline{x}) > 0$, and $\overline{\lambda} \geq 0$ from (C.11), we get $W'(\overline{s}) < 0$. Together $c'(\overline{s}) \leq 0$ and $W'(\overline{s}) < 0$ imply that the right-hand side of (D.3) is positive. Therefore, the left-hand side must also be positive and thus $W'(\overline{\overline{\widetilde{s}}}) > W'(\overline{s})$. Due to the strict concavity of $u(x_t, z_t, s_t)$, the value

[11] When evaluating (C.13) in steady state, we take into account that the terms containing $\partial x_{t+1}/\partial s_{t+1}$ and $\partial\widetilde{x}_{t+1}/\partial s_{t+1}$ are equal to $\lambda_{t+1}(\partial x_{t+1}/\partial s_{t+1})$ and $\widetilde{\lambda}_{t+1}(\partial\widetilde{x}_{t+1}/\partial s_{t+1})$ and are thus equal to zero (see the paragraph after (C.28) for a derivation).

function is concave in the stock of past consumption ($W'' < 0$), and the last inequality implies $\bar{\bar{s}} < \bar{s}$, i.e., $\bar{\bar{x}} < \bar{x}$. Suppose $x^H = 0$. Then, $\bar{x} \leq x^H = 0$ can only be fulfilled at $\bar{x} = 0$ and $\bar{\bar{x}} < \bar{x} = 0$ is a contradiction. Thus, if $x^F > x^H = 0$, we must have $\bar{x} > x^H = 0$. Suppose now $x^H > 0$. In this case $\bar{\bar{x}} < \bar{x}$ and $\bar{x} < x^F$ give $\bar{\bar{x}} < x^F$. Hence, $\omega_x(\bar{\bar{x}}) > \omega_x(\bar{x}) > 0$. Moreover, $\bar{\bar{x}} < \bar{x}$ implies that $\bar{x} > 0$ and thus $\bar{\lambda} = 0$. Together, $c'(\bar{s}) \leq 0, \omega_x(\bar{\bar{x}}) > \omega_x(\bar{x}) > 0, \bar{\lambda} = 0$ and $\bar{\bar{\lambda}} \geq 0$ from (C.20), however, make the right-hand side of (D.5) positive. This is a contradiction. We conclude that if $x^F > x^H \geq 0$, then it must be true that $\bar{x} > x^H$. This proves the sufficiency part.

To show that $x^F > x^H$ is also necessary for $\bar{x} > x^H$, suppose $x^F \leq x^H$ and assume the steady state is characterized by overconsumption: $\bar{x} > x^H$. Thus, $\bar{x} > x^H \geq x^F$ and, analogously, $\bar{s} > s^H \geq s^F$. The inequality $\bar{s} > s^H$ implies $c'(\bar{s}) > 0$. The inequality $\bar{x} > x^F$, together with $x^F \geq 0$ by definition, leads to $\omega_x(\bar{x}) < 0$ and $\bar{\lambda} = 0$. Therefore, according to (D.2), we have $W'(\bar{s}) > 0$. Equation (D.3), $c'(\bar{s}) > 0$, and $W'(\bar{s}) > 0$ together imply $W'(\bar{\bar{s}}) < W'(\bar{s})$. Therefore, $\bar{\bar{s}} > \bar{s}$ due to the concavity of the value function. Consequently, $\bar{\bar{x}} > \bar{x} > x^F \geq 0$ and $\omega_x(\bar{\bar{x}}) < \omega_x(\bar{x})$. Moreover, $\bar{\bar{x}} > 0$ means $\bar{\bar{\lambda}} = 0$. However, $\omega_x(\bar{\bar{x}}) < \omega_x(\bar{x}) < 0, c'(\bar{s}) > 0$ and $\bar{\lambda} = \bar{\bar{\lambda}} = 0$ make the right-hand side of (D.5) negative. This is a contradiction. Therefore, we conclude that $x^F > x^H$ is necessary for a steady state of overconsumption: $\bar{x} > x^H$.

Next, we prove part (b) of Proposition 2, which states that if and only if $x^F < x^H$, is there underconsumption: $\bar{x} < x^H$. First, we prove that $x^F < x^H$ is sufficient for $\bar{x} < x^H$. From the proof of part (a), we already know that if $x^F \leq x^H$, then $\bar{x} > x^H$ is not possible. Thus, if $x^F < x^H$, then $\bar{x} > x^H$ cannot hold. Hence, it remains to prove that if $x^F < x^H$, then $\bar{x} = x^H$ is not possible. We use proof by contradiction. Assume that $x^F < x^H$ and $\bar{x} = x^H > x^F$. Thus, $c'(\bar{s}) = 0$ and $\omega_x(\bar{x}) < 0$. Moreover, $\bar{\lambda} = 0$ due to $\bar{x} > x^F \geq 0$. Thus, $W'(\bar{s}) > 0$ according to (D.2) and $W'(\bar{\bar{s}}) < W'(\bar{s})$ according to (D.3). The last inequality and the concavity of the value function imply $\bar{\bar{s}} > \bar{s}$ and thus $\bar{\bar{x}} > \bar{x}$. The last inequality implies $\omega_x(\bar{\bar{x}}) < \omega_x(\bar{x})$ and $\bar{\bar{\lambda}} = 0$. Together $c'(\bar{s}) = 0, \omega_x(\bar{x}) < 0, \omega_x(\bar{\bar{x}}) < \omega_x(\bar{x})$, and $\bar{\lambda} = \bar{\bar{\lambda}} = 0$ make the right-hand side of (D.5) negative, which is a contradiction. We conclude that if $x^F < x^H$, then $\bar{x} < x^H$ must hold. This proves the sufficiency part.

To prove that $x^F < x^H$ is also necessary for underconsumption ($\bar{x} < x^H$), suppose that the opposite holds, i.e., suppose $x^F \geq x^H$ and $\bar{x} < x^H$. We already know from the proof of part (a) of Proposition 2 that if $x^F > x^H$, then $\overline{x} \leq x^H$ cannot hold. Thus, $x^F > x^H$ and $\overline{x} < x^H$ cannot be simultaneously true. It remains to show that $x^F = x^H$ is incompatible with $\overline{x} < x^H$. Suppose that they are satisfied simultaneously, i.e., $\overline{x} < x^H = x^F$. By the definitions of x^H and x^F, we must have $c'(\overline{s}) < 0$ and $\omega_x(\overline{x}) > 0$. Furthermore, $\overline{\lambda} \geq 0$, such that (D.2) implies $W'(\overline{s}) < 0$. Thus, the right-hand side of (D.3) is positive, which implies $W'(\overline{\overline{s}}) > W'(\overline{s})$. Thus, $\overline{\overline{s}} < \overline{s}$ due to the concavity of the value function. Hence, $\overline{\overline{\lambda}} \geq 0$, while $\overline{\lambda} = 0$ because $\overline{\overline{s}} < \overline{s}$ requires a strictly positive $\overline{s}$. Moreover, $\omega_x(\overline{\overline{x}}) > \omega_x(\overline{x})$ due to the concavity of the function $\omega(x)$. Together $c'(\overline{s}) < 0, \omega_x(\overline{\overline{x}}) > \omega_x(\overline{x}), \omega_x(\overline{x}) > 0, \overline{\lambda} = 0$ and $\overline{\overline{\lambda}} \geq 0$ make the right-hand side of (D.5) positive, which is a contradiction. Hence, $x^F \geq x^H$ is incompatible with $\overline{x} < x^H$. We conclude that $x^F < x^H$ is both necessary and sufficient for underconsumption: $\overline{x} < x^H$. This concludes the proof of part (b) of Proposition 2.

To prove part (c), note first that if $\omega_x(0) \geq 0$, then in the case $x^F = x^H$, $\overline{x} = \overline{\overline{x}} = x^H = x^F$ together with $\overline{\lambda} = \overline{\overline{\lambda}} = 0$ lead to $c'(\overline{s}) = \omega_x(\overline{x}) = \omega_x(\overline{\overline{x}}) = 0$ and satisfy Equations (D.2)-(D.5). If $\omega_x(0) < 0$, then $x^F = 0$ and $x^F = x^H$ is possible only for $x^H = 0$. In this case, $\overline{x} = \overline{\overline{x}} = x^H = x^F = 0$ together with $\overline{\lambda} = -(1+\gamma)w_x(0) > 0, \overline{\overline{\lambda}} = -\omega_x(0) > 0$ satisfy Equations (D.2)-(D.5). Thus, $x^F = x^H$ is sufficient for healthy steady state consumption. Moreover, from the proofs of parts (a) and (b), we know that if $x^F \neq x^H$, then $\overline{x} = x^H$ is not possible. We conclude that if and only if $x^F = x^H$, is steady state consumption healthy. □

E Derivation of the Euler Equation (15)

To derive Equation (15), we begin by defining the value function of the long-run self, $V(s_t)$, as

$$V(s_t) = \max_{x_t, z_t} \left\{ u(x_t, z_t, s_t) - \gamma \left[\max_{\widetilde{x}_t, \widetilde{z}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, z_t, s_t) \right]^a + \delta V(s_{t+1}) \right\}, \tag{E.1}$$

subject to $x_t \geq 0$ and $e = z_t + p_t x_t$. We define the Lagrangian as

$$L(x_t, s_t, \lambda_t) = u(x_t, e - p_t x_t, s_t) - \gamma \left[\max_{\widetilde{x}_t, \widetilde{z}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, e - p_t x_t, s_t) \right]^a$$

$$+\delta V(s_{t+1}) + \lambda_t x_t, \tag{E.2}$$

where λ_t is the Lagrange multiplier. The first-order conditions of the long-run self are

$$\frac{\partial L(\cdot)}{\partial x_t} = \omega_x(x_t)\left\{1 + \gamma a \left[\max_{\widetilde{x}_t, \widetilde{z}_t} u(\widetilde{x}_t, \widetilde{z}_t, s_t) - u(x_t, z_t, s_t)\right]^{a-1}\right\} + \delta V'(s_{t+1}) + \lambda_t = 0, \tag{E.3}$$

$$x_t \geq 0, \tag{E.4}$$

$$\lambda_t x_t = 0, \tag{E.5}$$

$$\lambda_t \geq 0, \tag{E.6}$$

where $\omega_x(x_t) \equiv w_x(x_t, e - p_t x_t) - p_t w_z(x_t, e - p_t x_t)$.[12] The value function $V(s_{t+1})$ solves the maximized Bellman equation

$$\begin{aligned} V(s_{t+1}) = {} & u(x_{t+1}, z_{t+1}, s_{t+1}) - \gamma \left[\max_{\widetilde{x}_{t+1}, \widetilde{z}_{t+1}} u(\widetilde{x}_{t+1}, \widetilde{z}_{t+1}, s_{t+1}) - u(x_{t+1}, z_{t+1}, s_{t+1})\right]^a \\ & + \delta V(s_{t+2}). \end{aligned} \tag{E.7}$$

The derivative of (E.7) with respect to s_{t+1} is given by

$$\begin{aligned} V'(s_{t+1}) = {} & \left\{\omega_x(x_{t+1})\left[1 + \gamma a \left(\max_{\widetilde{x}_{t+1}, \widetilde{z}_{t+1}} u(\widetilde{x}_{t+1}, \widetilde{z}_{t+1}, s_{t+1}) - u(x_{t+1}, z_{t+1}, s_{t+1})\right)^{a-1}\right]\right. \\ & \left. + \delta V'(s_{t+2})\right\}\frac{\partial x_{t+1}}{\partial s_{t+1}} - c'(s_{t+1}) \\ & - \gamma a \left[\max_{\widetilde{x}_{t+1}, \widetilde{z}_{t+1}} u(\widetilde{x}_{t+1}, \widetilde{z}_{t+1}, s_{t+1}) - u(x_{t+1}, z_{t+1}, s_{t+1})\right]^{a-1} [c'(s_{t+1}) - c'(s_{t+1})] \\ & - \gamma a \left[\max_{\widetilde{x}_{t+1}, \widetilde{z}_{t+1}} u(\widetilde{x}_{t+1}, \widetilde{z}_{t+1}, s_{t+1}) - u(x_{t+1}, z_{t+1}, s_{t+1})\right]^{a-1} \omega_x(\widetilde{x}_{t+1})\frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}} \\ & + \delta V'(s_{t+2})(1 - d). \end{aligned} \tag{E.8}$$

[12]To simplify the notation in (E.3) and in the remainder of Appendix E, we express $e - p_t x_t$ back as z_t.

Moreover, by lagging the first-order condition (E.3) one period, we get

$$\begin{aligned} 0 &= \omega_x(x_{t+1})\left\{1+\gamma a\left[\max_{\widetilde{x}_{t+1},\widetilde{z}_{t+1}} u(\widetilde{x}_{t+1},\widetilde{z}_{t+1},s_{t+1})-u(x_{t+1},z_{t+1},s_{t+1})\right]^{a-1}\right\} \\ &\quad +\delta V'(s_{t+2})+\lambda_{t+1}. \end{aligned} \tag{E.9}$$

Equation (E.9) is the first-order condition with respect to x_{t+1}. Moreover, the myopic self maximizes the instantaneous utility $u(\cdot)$ each period and its the first-order condition in period $t+1$ is given by

$$\omega_x(\widetilde{x}_{t+1}) \leq 0, \tag{E.10}$$

together with $\widetilde{x}_{t+1} \geq 0, \widetilde{x}_{t+1}\omega_x(\widetilde{x}_{t+1}) = 0$.

To find the Euler equation for actual consumption (Equation (15)), use (E.3), (E.9), and (E.10) to simplify Equation (E.8). The resulting expression is

$$\begin{aligned} \delta\lambda_{t+1}\frac{\partial x_{t+1}}{\partial s_{t+1}}+\delta c'(s_{t+1}) &= \omega_x(x_t)\left[1+\gamma a\left(\max_{\widetilde{x}_t,\widetilde{z}_t} u(\widetilde{x}_t,\widetilde{z}_t,s_t)-u(x_t,z_t,s_t)\right)^{a-1}\right]+\lambda_t \\ -\delta(1-d)&\left\{\omega_x(x_{t+1})\left[1+\gamma a\left(\max_{\widetilde{x}_{t+1},\widetilde{z}_{t+1}} u(\widetilde{x}_{t+1},\widetilde{z}_{t+1},s_{t+1})-u(x_{t+1},z_{t+1},s_{t+1})\right)^{a-1}\right]+\lambda_{t+1}\right\} \\ -\delta\gamma a&\left[\max_{\widetilde{x}_{t+1},\widetilde{z}_{t+1}} u(\widetilde{x}_{t+1},\widetilde{z}_{t+1},s_{t+1})-u(x_{t+1},z_{t+1},s_{t+1})\right]^{a-1}\omega_x(\widetilde{x}_{t+1})\frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}}. \end{aligned} \tag{E.11}$$

Analogously to Appendix A, Equations (A.15) and (A.16), we can show that $\lambda_{t+1}(\partial x_{t+1}/\partial s_{t+1}) = 0$. Moreover, totally differentiating the first-order condition $\widetilde{x}_{t+1}\omega_x(\widetilde{x}_{t+1}) = 0$ with respect to s_{t+1}, we get

$$\frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}}\omega_x(\widetilde{x}_{t+1})+\widetilde{x}_{t+1}\omega_{xx}(\widetilde{x}_{t+1})\frac{\partial \widetilde{x}_{t+1}}{\partial s_{t+1}} = 0. \tag{E.12}$$

According to (E.12), if (E.10) is fulfilled with equality such that $\omega_x(\widetilde{x}_{t+1}) = 0$, then $\omega_x(\widetilde{x}_{t+1})(\partial\widetilde{x}_{t+1}/\partial s_{t+1}) = 0$. Moreover, if (E.10) is fulfilled with inequality such that $\omega_x(\widetilde{x}_{t+1}) < 0$, then $\widetilde{x}_{t+1} = 0$ and again, according to (E.12), $\omega_x(\widetilde{x}_{t+1})(\partial\widetilde{x}_{t+1}/\partial s_{t+1}) = 0$. Thus, the term in the third row of (E.11) vanishes. This result and $\lambda_{t+1}(\partial x_{t+1}/\partial s_{t+1}) = 0$

together simplify (E.11) to

$$\delta c'(s_{t+1}) = \omega_x(x_t)\left[1+\gamma a\left(\max_{\widetilde{x}_t,\widetilde{z}_t} u(\widetilde{x}_t,\widetilde{z}_t,s_t)-u(x_t,z_t,s_t)\right)^{a-1}\right]+\lambda_t \tag{E.13}$$
$$-\delta(1-d)\left\{\omega_x(x_{t+1})\left[1+\gamma a\left(\max_{\widetilde{x}_{t+1},\widetilde{z}_{t+1}} u(\widetilde{x}_{t+1},\widetilde{z}_{t+1},s_{t+1})-u(x_{t+1},z_{t+1},s_{t+1})\right)^{a-1}\right]+\lambda_{t+1}\right\}.$$

When consumption in periods t and $t+1$ is positive, such that $x_t > 0$ and $x_{t+1} > 0$, the Lagrange multipliers equal zero ($\lambda_t = \lambda_{t+1} = 0$) and (E.13) simplifies to Equation (15).

F Proof of Proposition 3

To prove Proposition 3, evaluate the Euler equation (E.13) in steady state, where $x_t = \overline{x}, z_t = \overline{z}, s_t = \overline{s}$, and $\lambda_t = \overline{\lambda}$. Note furthermore that in steady state $\widetilde{x} = x^F$ and $\widetilde{z} = z^F$, where $z^F = e - \overline{p}x^F$. We have

$$\omega_x(\overline{x})\left[1-\delta(1-d)\right]\left\{1+\gamma a\Big[u(x^F,z^F,\overline{s})-u(\overline{x},\overline{z},\overline{s})\Big]^{a-1}\right\}+\overline{\lambda}\left[1-\delta(1-d)\right]-\delta c'(\overline{s}) = 0. \tag{F.1}$$

Note that the term $u(x^F,z^F,\overline{s})-u(\overline{x},\overline{z},\overline{s})$ is nonnegative because (x^F, z^F) maximize $u(\cdot)$ for a given stock level $\overline{s}$. Thus, Equation (F.1) is qualitatively identical to Equation (B.8) from Appendix B that describes the steady state consumption of a sophisticate consumer with quasi-hyperbolic discounting. Applying the proof of Proposition 1 for a sophisticate individual to Equation (F.1) proves Proposition 3. □

G Proof of Proposition 4

We start with the first-order condition of the self in time period t. The maximization problem of a type i individual in period t is defined analogously to (A.1) in Appendix A, when the instantaneous utility function is given by (17). We define the Lagrangian analogously to (A.2) in Appendix A, denote again the period t Lagrange multiplier as λ_t^i

and derive the following first-order conditions:

$$\begin{aligned}
\frac{\partial L(\cdot)}{\partial x_t^i} &= \widehat{u}_x(x_t^i, s_t^i) + \beta\delta V^{i\prime}((1-d)s_t^i + x_t^i) + \lambda_t^i = 0, && \text{(G.1)}\\
x_t^i &\geq 0, && \text{(G.2)}\\
\lambda_t^i x_t^i &= 0, && \text{(G.3)}\\
\lambda_t^i &\geq 0, && \text{(G.4)}
\end{aligned}$$

where $\widehat{u}_x(x_t^i, s_t^i) = \alpha_x + \alpha_{xx}x_t^i + \alpha_{xs}s_t^i - p_t[\alpha_z + \alpha_{zz}(e - p_t x_t^i)]$. Moreover, the continuation-value function $V^i(s_{t+1}^i)$ is determined analogously to Equation (A.7), while the period $t+1$ maximization problem of the individual of type i is analogous to (A.9). Following the same steps as in Appendix A, we derive the following Euler equation for the individual of type $i = s, n$:

$$\begin{aligned}
\widehat{u}_x\left(x_t^i, s_t^i\right) + \lambda_t^i &= \frac{\beta\delta}{\hat{\beta}}\left\{\widehat{u}_x\left(x_{t+1}^s(\hat{\beta}), s_{t+1}^i\right)\left[1 - d + (1-\hat{\beta})\frac{\partial x_{t+1}^s(\hat{\beta})}{\partial s_{t+1}^i}\right]\right.\\
&\qquad \left. -\hat{\beta}\widehat{u}_s(x_{t+1}^s(\hat{\beta}), s_{t+1}^i) + \lambda_{t+1}^s(\hat{\beta})(1-d)\right\}, \qquad \text{(G.5)}
\end{aligned}$$

where $\widehat{u}_s(x_{t+1}^s(\hat{\beta}), s_{t+1}^i) = \alpha_{xs}x_{t+1}^s(\hat{\beta}) + \alpha_{ss}s_{t+1}^i$ and $\widehat{u}_x(x_{t+1}^s(\hat{\beta}), s_{t+1}^i) = \alpha_x + \alpha_{xx}x_{t+1}^s(\hat{\beta}) + \alpha_{xs}s_{t+1}^i - p_{t+1}[\alpha_z + \alpha_{zz}(e - p_{t+1}x_{t+1}^s(\hat{\beta}))]$. The sin good consumption $x_{t+1}^s(\hat{\beta})$ is defined, identically to Section 2, as the optimal consumption in period $t+1$ of a sophisticate individual with present-bias $\hat{\beta}$.

Next, we consider how changes in the consumption stock s_t^i affect optimal consumption x_t^i. First, if the solution to (G.1)-(G.4) is $x_t^i = 0, \lambda_t^i > 0$, then s_t^i does not affect the optimal consumption: $\partial x_t^i/\partial s_t^i = 0$. Second, if the solution has $x_t^i \geq 0, \lambda_t^i = 0$, then we can totally differentiate (G.1) to derive

$$\frac{\partial x_t^i}{\partial s_t^i} = -\frac{\alpha_{xs} + \beta\delta(1-d)V^{i\prime\prime}}{\alpha_{xx} + p_t^2\alpha_{zz} + \beta\delta V^{i\prime\prime}}. \qquad \text{(G.6)}$$

We now analyze the conditions for steady state stability. The steady state is stable if the convergence factor ds_{t+1}^i/ds_t^i is less than one in absolute value. If $\lambda_t^i > 0$ and thus $\partial x_t^i/\partial s_t^i = 0$, then it follows directly from (1) that the steady state is stable. If $\lambda_t^i = 0$, then we use (1) and (G.6) to get

$$\frac{ds^i_{t+1}}{ds^i_t} = (1-d) + \frac{\partial x^i_t}{\partial s^i_t} = \frac{(1-d)[\alpha_{xx} + p_t^2\alpha_{zz}] - \alpha_{xs}}{\alpha_{xx} + p_t^2\alpha_{zz} + \beta\delta V^{i\prime\prime}} > 0. \tag{G.7}$$

The expression in (G.7) is positive owing to $\alpha_{xx} < 0, \alpha_{zz} < 0, \alpha_{xs} > 0$, and $V^{i\prime\prime} < 0$. The steady state is stable if (G.7) is less than unity. Rearranging $ds^i_{t+1}/ds^i_t < 1$, we get

$$\alpha_{xs} + d[\alpha_{xx} + p_t^2\alpha_{zz}] + \beta\delta V^{i\prime\prime} < 0. \tag{G.8}$$

In the following analysis, we assume that (G.8) is satisfied. Note that it is always satisfied in the absence of habits (i.e., when $\alpha_{xs} = 0$), which was the case in Section 2.

Consider for a moment the situation where both the period t consumption and the expected period $t+1$ consumption are positive and thus $\lambda^i_t = \lambda^s_{t+1}(\hat{\beta}) = 0$. Evaluate the Euler equation in steady state, and denote it as $\Phi^i(\overline{s}^i)$. Analogously to Appendix B (Equations (B.5) and (B.6)), we use $\overline{x}^i = d\overline{s}^i$ and express $\Phi^i(\overline{s}^i)$ in the following way:

$$\begin{aligned}
\Phi^i(\overline{s}^i) &= \widehat{u}_x(d\overline{s}^i, \overline{s}^i) + \beta\delta\widehat{u}_s(d\overline{s}^s(\hat{\beta}), \overline{s}^i) \\
&\quad - \widehat{u}_x(d\overline{s}^s(\hat{\beta}), \overline{s}^i)\frac{\beta\delta}{\hat{\beta}}\left[1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^i_{t+1}}\right] \qquad \text{(G.9)} \\
&= \widehat{u}_x(d\overline{s}^i, \overline{s}^i) + \beta\delta V^{i\prime}(\overline{s}^i) = 0, \qquad \text{(G.10)}
\end{aligned}$$

where

$$\begin{aligned}
\widehat{u}_x(d\overline{s}^i, \overline{s}^i) &= \alpha_x + \alpha_{xx}d\overline{s}^i + \alpha_{xs}\overline{s}^i - \overline{p}\left[\alpha_z + \alpha_{zz}\left(e - \overline{p}d\overline{s}^i\right)\right], \\
\widehat{u}_x(d\overline{s}^s(\hat{\beta}), \overline{s}^i) &= \alpha_x + \alpha_{xx}d\overline{s}^s(\hat{\beta}) + \alpha_{xs}\overline{s}^i - \overline{p}\left\{\alpha_z + \alpha_{zz}\left[e - \overline{p}d\overline{s}^s(\hat{\beta})\right]\right\}, \\
\widehat{u}_s(d\overline{s}^s(\hat{\beta}), \overline{s}^i) &= \alpha_{xs}d\overline{s}^s(\hat{\beta}) + \alpha_{ss}\overline{s}^i.
\end{aligned}$$

Differentiating (G.10) with respect to $\overline{s}^i$, one immediately sees that the stability condition (G.8) implies $\Phi^{i\prime}(\cdot) < 0$:

$$\Phi^{i\prime}(\overline{s}^i) = \alpha_{xs} + d[\alpha_{xx} + \overline{p}^2\alpha_{zz}] + \beta\delta V^{i\prime\prime} < 0. \tag{G.11}$$

Consider now a sophisticate individual. To improve the tractability of the proof, we use the following notation:

$$g(s) = \beta\delta\widehat{u}_s(ds, s), \tag{G.12}$$

$$f(s) = \widehat{u}_x(ds, s)\left[1 - \delta\left(1 - d + (1-\beta)\frac{\partial x^s_{t+1}}{\partial s^s_{t+1}}\right)\right]. \tag{G.13}$$

In the case of a sophisticate individual, the sum of $g(s)$ and $f(s)$, when both are evaluated at $s = \overline{s}^s$, gives $\Phi^s(\overline{s}^s)$; that is, $\Phi^s(\overline{s}^s) = f(\overline{s}^s) + g(\overline{s}^s)$. Furthermore, by the definitions of s^F and s^H, we have $f(s^F) = 0$ and $g(s^H) = 0$. Next, we take the derivatives of $f(s)$ and $g(s)$. In doing so, we take into account Theorem 1 of Gruber and Köszegi (2001), who show that, in the case of quadratic utility, x^s_t is a linear function of s^s_t. Thus, the derivative $\partial x^s_t/\partial s^s_t$ is contant. Therefore, $f(s)$ and $g(s)$ are linear functions of s with the following derivatives:

$$\begin{aligned} f' &\equiv \frac{\partial}{\partial s}\left\{\widehat{u}_x(ds, s)\left[1 - \delta\left(1 - d + (1-\beta)\frac{\partial x^s_{t+1}}{\partial s^s_{t+1}}\right)\right]\right\} \\ &= \left[\alpha_{xx}d + \alpha_{xs} + \overline{p}^2 d\alpha_{zz}\right]\left[1 - \delta\left(1 - d + (1-\beta)\frac{\partial x^s_{t+1}}{\partial s^s_{t+1}}\right)\right], \end{aligned} \tag{G.14}$$

$$g' \equiv \frac{\partial}{\partial s}\left[\beta\delta\widehat{u}_s(ds, s)\right] = \beta\delta\left[\alpha_{xs}d + \alpha_{ss}\right]. \tag{G.15}$$

The signs of (G.14) and (G.15) are, in general, ambiguous. However, according to (G.5) and (G.11), these functions satisfy the following properties:

$$f(\overline{s}^s) + g(\overline{s}^s) + \overline{\lambda}^s\left[1 - \delta(1-d)\right] = 0, \tag{G.16}$$

$$f' + g' \equiv \Phi^{s\prime} < 0. \tag{G.17}$$

Equation (G.16) is the Euler equation of a sophisticate individual, evaluated in the steady state. Moreover, (G.17) gives the derivative of the sophisticate's Euler equation in the case of a positive steady state consumption. It is negative owing to the assumption of a stable steady state. The signs of f' and g' are crucial for the determination of the possible steady states. In the following, we will consider all combinations that satisfy (G.17).

There are five possible combinations of f' and g' that satisfy (G.17). Denote the first of them as Case I and define it as

$$g' < 0 \quad \text{and} \quad f' < 0. \tag{Case I}$$

Case I occurs when habits are relatively weak, such that α_{xs} is not too large. Moreover, $f(s^F) = 0$ and $g(s^H = 0) = 0$ together with $g' < 0, f' < 0$ imply $g(\overline{s}^s) < 0$ for $\overline{s}^s > 0$ and $f(\overline{s}^s) \gtreqless 0 \Leftrightarrow \overline{s}^s \lesseqgtr s^F$. Using (G.14) and (G.15), we can show that Case I is satisfied for

$$\alpha_{xs} < \min\left\{-\frac{\alpha_{ss}}{d}, -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}\right\}. \tag{Case I'}$$

This case is also satisfied by the utility function in Section 2, where the sin good is not addictive (and thus $\alpha_{xs} = 0$). Thus, the Euler equation (G.16) is qualitatively the same as (B.8) from Appendix B. Hence, following the same proof as in Appendix B, we can prove $\overline{x}^s = x^H$ if $x^F = x^H = 0$, as well as part I from Proposition 4.

The second case, labeled as Case II occurs when

$$g' < 0 < f'. \tag{Case II}$$

The difference to Case I is that, under Case II, the net marginal utility of current consumption $\widehat{u}_x(\cdot)$ is increasing in the steady state consumption stock. Thus, $f(s^F) = 0$ and $g(s^H = 0) = 0$ together with $g' < 0 < f'$ imply $g(\overline{s}^s) < 0$ for $\overline{s}^s > 0$ and $f(\overline{s}^s) \lesseqgtr 0 \Leftrightarrow \overline{s}^s \lesseqgtr s^F$. Using (G.14) and (G.15), we show that Case II emerges for

$$\alpha_{sx} \in \left]-\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}, -\frac{\alpha_{ss}}{d}\right[. \tag{Case II'}$$

To analyze Case II, consider first the situation $s^F > 0 = s^H$. Suppose that $\overline{s}^s \in]0, s^F]$. Due to $g' < 0$ and $f' > 0$, we know that, in this case, $g(\overline{s}^s) < 0$ and $f(\overline{s}^s) \leq 0$, respectively. Moreover, $\overline{s}^s > 0$ implies $\overline{\lambda}^s = 0$. Thus, the left-hand side of (G.16) is negative, which is a contradiction. Moreover, from (G.17), we know that $g' + f' < 0$, such that any value $\overline{s}^s > s^F$ would make the left-hand side of (G.16) even more negative. Thus, a positive steady state cannot emerge in this case. Moreover, $\overline{s}^s = 0 = s^H$ gives $g(0) = 0$ and $f(0) < 0$, which satisfies (G.16) for $\overline{\lambda}^s = -f(0)/(1 - \delta(1 - d)) > 0$. Thus, the only solution in this case is $\overline{s}^s = 0$. The second possibility under Case II is $s^F = 0 = s^H$. In this case, $\overline{s}^s = 0 = s^H = s^F = \overline{\lambda}^s$ is a solution to (G.16) if $\widehat{u}_x(0,0) = 0$. If, however, $\widehat{u}_x(0,0) < 0$, then $\overline{s}^s = 0 = s^H = s^F$ together with $\overline{\lambda}^s = -f(0)/(1-\delta(1-d)) > 0$ is the steady state. Moreover, due to $g' + f' < 0$ from (G.17), $g(\overline{s}^s) + f(\overline{s}^s) < 0$ for any $\overline{s}^s > 0$. Thus, the unique solution in the case of $s^F = 0 = s^H$ is $\overline{s}^s = 0$. We conclude that, in Case II, $\overline{s}^s = 0$ for any value of $s^F \geq 0$.

Define Case III as the situation where the following conditions hold:

$$g' > 0 > f'. \tag{Case III}$$

In Case III, $f(s^F) = 0$ and $g(s^H = 0) = 0$ together with $g' > 0 > f'$ imply $g(\overline{s}^s) > 0$ for $\overline{s}^s > 0$ and $f(\overline{s}^s) \gtreqless 0 \Leftrightarrow \overline{s}^s \lesseqgtr s^F$. This case holds when

$$\alpha_{xs} \in \left]-\frac{\alpha_{ss}}{d}, -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}\right[. \tag{Case III'}$$

Suppose now that $s^F > 0$ and assume $\overline{s}^s \in]0, s^F]$. This assumption implies $\overline{\lambda}^s = 0, g(\overline{s}^s) > 0$ (due to $g' > 0$) and $f(\overline{s}^s) \geq 0$ (due to $f' < 0$). Thus, the right-hand side of (G.16) is positive and $\overline{s}^s \in]0, s^F]$ cannot be an equilibrium. Due to $g' + f' < 0$ from (G.17), (G.16) can only be fulfilled for larger values of the steady state consumption stock, i.e., $\overline{s}^s > s^F$. We conclude that if $s^F > 0$ in Case III, then $\overline{s}^s > s^F > 0 = s^H$. Suppose now that $s^F = 0$. If $s^F = 0$ follows from $\widehat{u}_x(0,0) = 0$, then $\overline{s}^s = 0 = s^H = s^F = \overline{\lambda}^s$ satisfy (G.16). If $s^F = 0$ follows from $\widehat{u}_x(0,0) < 0$, then $\overline{s}^s = 0 = s^H = s^F$ together with $\overline{\lambda}^s = -f(0)/[1-\delta(1-d)] > 0$ satisfy (G.16). Moreover, due to $g' + f' < 0$ from (G.17), any positive values of $\overline{s}^s$ violate (G.16). We conclude that in the case $s^F = 0$, the unique solution is $\overline{s}^s = 0 = s^F = s^H$.

Case IV emerges when

$$g' = 0 > f'. \tag{Case IV}$$

It holds when

$$\alpha_{xs} = -\frac{\alpha_{ss}}{d} < -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz}. \tag{Case IV'}$$

In this special case, the positive effect of current consumption on the marginal utility of past consumption exactly compensates the marginal health costs. Because, by definition, $g(0) = 0$, the condition $g' = 0$ implies $g(s) = 0$ for all s. Thus, the Euler equation is given by $f(\overline{s}^s) + \overline{\lambda}^s [1 - \delta(1 - d)] = 0$. Suppose $s^F = 0$. Then, at any positive level of $\overline{s}^s$, the left-hand side of the Euler equation is negative due to $f' < 0$ and $\overline{\lambda}^s = 0$. This is a contradiction and we conclude that if $s^F = 0$, then $\overline{s}^s = s^F = s^H = 0$ is the only solution of the Euler equation (G.16). Moreover, $\overline{\lambda}^s = -f(0)/[1 - \delta(1 - d)] \geq 0$. If $x^F > 0$, then due to $f' < 0$, the left-hand side of the Euler equation (G.16) is positive at $\overline{s}^s = 0$ and is equal to zero at $\overline{s}^s = s^F > 0, \overline{\lambda}^s = 0$. Thus, in Case IV, $\overline{s}^s = s^F$ for any $s^F \geq 0$.

The last case (Case V) is

$$f' = 0 > g'. \tag{Case V}$$

It holds when

$$\alpha_{xs} = -\alpha_{xx}d - \overline{p}^2 d\alpha_{zz} < -\frac{\alpha_{ss}}{d}. \tag{Case V'}$$

The condition $f' = 0$ means the net marginal utility $\widehat{u}_x(\cdot)$ is constant and either $\widehat{u}_x > 0$ or $\widehat{u}_x \leq 0$ for all s. The subcase $\widehat{u}_x \leq 0$ emerges when $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] \leq 0$. Thus, no positive consumption can be a steady state because it would imply $\widehat{u}_x \leq 0, g(\overline{s}^s) < 0$ and $\overline{\lambda}^s = 0$, and violate (G.16). Thus, in this subcase, we have $\overline{s}^s = 0 = s^H$. The subcase $\widehat{u}_x > 0$ exists if $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] > 0$. In this case, $f > 0$ for all s. Since $\overline{\lambda}^s \geq 0$, the only possible steady state that satisfies (G.16) involves $g(\overline{s}^s) < 0$, i.e., $\overline{s}^s > 0 = s^H$.

To analyze the naive individual, we first rewrite its Euler equation in steady state. It is given by

$$\begin{aligned} \widehat{u}_x\left(\overline{x}^n, \overline{s}^n\right) + \overline{\lambda}^n \quad &= \quad \frac{\beta\delta}{\hat{\beta}} \left\{ \widehat{u}_x\left(\overline{x}^s(\hat{\beta}), \overline{s}^n\right) \left[1 - d + (1 - \hat{\beta}) \frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right] \right. \\ &\qquad \left. - \hat{\beta}\widehat{u}_s(\overline{x}^s(\hat{\beta}), \overline{s}^n) + \overline{\lambda}^s(\hat{\beta})(1 - d) \right\}, \end{aligned} \tag{G.18}$$

where $\overline{x}^n = d\overline{s}^n$ from Equation (1). Due to the quadratic form of the utility function, $\partial x^s_{t+1}(\hat{\beta})/\partial s^n_{t+1}$ is constant (Gruber and Köszegi, 2001). Moreover, the marginal utilities are linear and can be reformulated in the following way:

$$\widehat{u}_s(\overline{x}^s(\hat{\beta}),\overline{s}^n) = \widehat{u}_s(\overline{x}^n,\overline{s}^n)+\alpha_{xs}(\overline{x}^s(\hat{\beta})-\overline{x}^n), \tag{G.19}$$

$$\widehat{u}_x(\overline{x}^s(\hat{\beta}),\overline{s}^n) = \widehat{u}_x(\overline{x}^n,\overline{s}^n)+[\alpha_{xx}+\overline{p}^2\alpha_{zz}](\overline{x}^s(\hat{\beta})-\overline{x}^n). \tag{G.20}$$

Using (G.19), (G.20) and $\overline{x}^n = d\overline{s}^n$, Equation (G.18) can be rewritten as

$$\begin{aligned} 0 &= \beta\delta\widehat{u}_s(d\overline{s}^n,\overline{s}^n)+\widehat{u}_x(d\overline{s}^n,\overline{s}^n)\left[1-\frac{\beta\delta}{\hat{\beta}}\left(1-d+(1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right)\right] \\ &\quad +\beta\delta\left[\overline{x}^n-\overline{x}^s(\hat{\beta})\right]\left[\frac{1}{\hat{\beta}}\left(1-d+(1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right)(\alpha_{xx}+\overline{p}^2\alpha_{zz})-\alpha_{xs}\right] \\ &\quad +\overline{\lambda}^n-\frac{\beta\delta}{\hat{\beta}}\overline{\lambda}^s(\hat{\beta})(1-d). \end{aligned} \tag{G.21}$$

The first term on the right-hand side of (G.21) is $g(s)$ from (G.12), evaluated at $s=\overline{s}^n$. The second term is proportional to $f(s)$ from (G.13), when evaluated at $s=\overline{s}^n$. Its derivative with respect to s is also proportional to the derivative f', defined in (G.14). The term in the second row of (G.21) has the opposite sign of $[\overline{x}^n-\overline{x}^s(\hat{\beta})]$ due to $\alpha_{xx}<0, \alpha_{zz}<0$ and $\alpha_{xs}>0$.

Consider now Case I, defined by $g'<0$ and $f'<0$. This case is qualitatively identical to the case without addiction considered in Proposition 1. Hence, we have $\overline{s}^n>s^H$ if $s^F>s^H$ and $\overline{s}^n=s^H$ if $s^F=s^H$ by the proof from Appendix B.

Consider now Case II and analyze first the case $s^H=s^F=0$. From our analysis of the sophisticate individual, we know that it results in $\overline{s}^s(\hat{\beta})=s^H=0$ and $\overline{\lambda}^s=-f(0)/[1-\delta(1-d)]\geq 0$. The solution $\overline{s}^n=s^H=s^F=0$ satisfies (G.21) with

$$\overline{\lambda}^n = \frac{\beta\delta}{\hat{\beta}}\overline{\lambda}^s(\hat{\beta})(1-d)-\widehat{u}_x(0,0)\left[1-\frac{\beta\delta}{\hat{\beta}}\left(1-d+(1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right)\right]\geq 0, \tag{G.22}$$

where $\overline{\lambda}^n$ is strictly greater than zero if $\widehat{u}_x(0,0)<0$ and equal to zero if $\widehat{u}_x(0,0)=0$.

Suppose now that $s^F>s^H$ in Case II. In this situation, we know that $\overline{s}^s(\hat{\beta})=0=s^H<s^F$ and $\overline{\lambda}^s(\hat{\beta})=-f(0)/(1-\delta(1-d))>0$. Assume that $\overline{s}^n\in]s^H,s^F]$. This assumption must, in that case, satisfy $\overline{\lambda}^n = 0$ and

$$
\begin{aligned}
0 \quad > \quad & \beta\delta\widehat{u}_s(d\overline{s}^n, \overline{s}^n) + \widehat{u}_x(d\overline{s}^n, \overline{s}^n)\left[1 - \frac{\beta\delta}{\hat{\beta}}\left(1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right)\right] \\
& + \beta\delta(\overline{x}^n - \overline{x}^s(\hat{\beta}))\left[\frac{1}{\hat{\beta}}\left((1-d) + (1-\hat{\beta})\frac{\partial \overline{x}^s(\hat{\beta})}{\partial \overline{s}^n}\right)(\alpha_{xx} + \overline{p}^2\alpha_{zz}) - \alpha_{xs}\right] \\
& - \frac{\beta\delta}{\hat{\beta}}\overline{\lambda}^s(\hat{\beta})(1-d),
\end{aligned} \tag{G.23}
$$

where the inequality follows from $g' < 0, f' > 0, \overline{x}^s(\hat{\beta}) = 0$, and $\overline{\lambda}^s(\hat{\beta}) > 0$. Thus, according to (G.23), Equation (G.21) is violated and $\overline{s}^n \in]s^H, s^F]$ cannot be a steady state. Moreover, because $g' + f' < 0$ from (G.17), any value of $\overline{s}^n$ above s^F makes the right-hand side of (G.23) even more negative. Hence, the steady state in case II can only be achieved at $\overline{s}^n = s^H = 0$. In this case, $\overline{\lambda}^n$ is again given by (G.22), which is now satisfied as a strict inequality due to $\overline{\lambda}^s(\hat{\beta}) > 0$ and $\widehat{u}_x(0,0) \propto f(0) < 0$.

Consider now Case III. If $s^F = s^H = 0$, we know that $\overline{s}^s(\hat{\beta}) = s^H$ and $\overline{\lambda}^s \geq 0$ from our discussion of a sophisticate individual. Analogously to Case II, one can verify that the unique solution to (G.21) is $\overline{s}^n = s^H = s^F = 0$ with $\overline{\lambda}^n \geq 0$.

Consider now the case $s^F > s^H$ in Case III. We already know that this case results in $\overline{s}^s(\hat{\beta}) > s^F > s^H$ and $\overline{\lambda}^s(\hat{\beta}) = 0$. Assume that $\overline{s}^n \in]s^H, s^F]$. In this case, we derive the following inequality:

$$
\begin{aligned}
0 \quad < \quad & \beta\delta\widehat{u}_s(d\overline{s}^n, \overline{s}^n) + \widehat{u}_x(d\overline{s}^n, \overline{s}^n)\left[1 - \frac{\beta\delta}{\hat{\beta}}\left(1 - d + (1-\hat{\beta})\frac{\partial x^s_{t+1}(\hat{\beta})}{\partial s^n_{t+1}}\right)\right] \\
& + \beta\delta(\overline{x}^n - \overline{x}^s(\hat{\beta}))\left[\frac{1}{\hat{\beta}}\left((1-d) + (1-\hat{\beta})\frac{\partial \overline{x}^s(\hat{\beta})}{\partial \overline{s}^n}\right)(\alpha_{xx} + \overline{p}^2\alpha_{zz}) - \alpha_{xs}\right] + \overline{\lambda}^n,
\end{aligned} \tag{G.24}
$$

where the inequality follows from $g' > 0, f' < 0$ and $\overline{\lambda}^n = 0$. Moreover, if $\overline{s}^n = s^H = 0$, the right-hand side of (G.24) is again positive due to $\overline{\lambda}^n \geq 0$ and $\widehat{u}_x(0,0) > 0$ (since $s^F > 0$ and $f' < 0$). Thus, (G.21) is violated for $\overline{s}^n \in [s^H, s^F]$. Due to $g' + f' < 0$, the positive right-hand side of (G.24) can only become equal to zero for $\overline{s}^n > s^F$. We conclude that, in Case III, $\overline{s}^n > s^F$ if $s^F > s^H$.

Consider now Case IV. We already know that, in this case, we have $\overline{x}^s(\hat{\beta}) = x^F, \overline{\lambda}^s(\hat{\beta}) \geq 0$ for any $s^F \geq 0$. Thus, if $\overline{s}^n > s^F \geq 0$, the right-hand side of (G.21) is negative due to $\widehat{u}_s(\cdot) = g = 0, f(\overline{s}^n) < 0$ and $\overline{\lambda}^n = 0$. Therefore, $\overline{s}^n > s^F \geq 0$ cannot be fufilled in a steady state. Thus, if $s^F = 0$, then $\overline{s}^n = s^F = 0$. If $s^F > 0$, then we must check whether $\overline{s}^n < s^F$ can be optimal. In this case, we already proved $\overline{x}^s(\hat{\beta}) = s^F$ and $\overline{\lambda}^s(\hat{\beta}) = 0$. Therefore, $\overline{s}^n < s^F$ implies that the right-hand side of (G.21) is positive due to $\widehat{u}_s(\cdot) = g = 0, f(\overline{s}^n) > 0$ and $\overline{\lambda}^n \geq 0$. This is a contradiction. We conclude that if $g' = 0$, then $\overline{s}^n = s^F$ for any $s^F \geq 0$.

Finally, consider Case V. In its first subcase, we have $\widehat{u}_x \leq 0$ and $\overline{x}^s(\hat{\beta}) = 0, \overline{\lambda}^s(\hat{\beta}) \geq 0$. It is easy to verify that any strictly positive $\overline{s}^n$ makes the right-hand side of (G.21) negative (due to $g' < 0, \overline{\lambda}^n = 0$) and thus cannot be a steady state. Thus, we conclude that in the first subcase of Case V, we have $\overline{s}^n = s^H = 0$. In the second subcase, we have $\widehat{u}_x > 0$ and $\alpha_x - \overline{p}[\alpha_z + \alpha_{zz}e] > 0$, which implies $\overline{x}^s(\hat{\beta}) > 0$ and $\overline{\lambda}^s(\hat{\beta}) = 0$. If we evaluate (G.21) at $\overline{s}^n = 0$, we get a positive right-hand side due to $\widehat{u}_x > 0, g(0) = 0, \overline{\lambda}^n \geq 0$ together with $\overline{x}^s(\hat{\beta}) > 0$ and $\overline{\lambda}^s(\hat{\beta}) = 0$. We conclude that $\overline{s}^n = 0$ cannot be a solution in the second subcase of Case V. Hence, (G.21) can only be satisfied for some positive $\overline{s}^n$; that is $\overline{s}^n > s^H = 0$. $\square$