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Abstract

We consider a model of monopolistic competition with several heterogeneous sectors and endogenous labor supply. For low (high) values of the labor supply elasticity, we show that there is always a unique equilibrium. For medium values of the labor supply elasticity, the set of equilibria (if non-empty) can comprise of several equilibria, with the number of equilibria bounded by the number of sectors.

JEL-Codes: D430, L130.

Keywords: labor elasticity, multiple equilibria, stability.

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1 Introduction

The monopolistic competition framework is widely employed in many fields of economics.[1] A number of various assumptions are made within this framework: constant and variable elasticities of substitution between varieties, heterogeneous firms and consumers, multiple sectors. At the same time, not much attention has been paid to modeling labor supply in monopolistic competition. This paper tries to fill this gap by considering a simple model of monopolistic competition with a constant elasticity of substitution, homogeneous firms, multiple sectors, and endogenous labor supply. In particular, the goal of our work is to understand better the link between labor supply endogeneity and existence and uniqueness of the equilibrium in the monopolistic competition framework.

In our framework, consumers/workers choose how many units of labor to supply. As a result, there is an interplay of the price index in the economy (which is determined by the number of available varieties and their prices) and the labor supply. A lower price index induces higher labor supply and, vice versa, higher labor supply results in more entry into the market and, therefore, a lower price index. In the paper, we argue that this interplay is important for characterizing equilibrium. In particular, we show that the relative size of the Frisch labor elasticity with respect to the elasticities of substitution within sectors plays a crucial role in determining existence and uniqueness of equilibrium.

We find that, for relatively low (high) values of the labor supply elasticity, there is a unique stable (unstable) equilibrium. We then show that, for medium values of the labor supply elasticity, multiple equilibria are possible. The key thing here is the difference in the elasticities of substitution across sectors, implying that depending on the values of the price index and labor supply, different sectors play a major role in establishing the equilibrium outcome. The findings in the paper are important for qualitative and quantitative policy analysis in the presence of the *multisectoral* monopolistic competition market structure that shows to be crucial for understanding welfare distortions in the economy (see Behrens et al., 2020).

The literature on monopolistic competition with elastic labor supply is rather limited. Bilbiie et al. (2012, 2019) consider a dynamic stochastic general equilibrium model that includes one-sector monopolistic competition and endogenous labor supply. The focus of these papers is the positive and normative analysis of the distortions that arise in such a framework. Colciago (2016) analyzes the optimal labor and dividend income taxation in the one-sector general equilibrium model with oligopolistic competition and free entry. Etro (2018) considers a dynamic general equilibrium model under monopolistic and oligopolistic competition to characterize efficient market structures and to derive optimal tax rules. All these papers focus on a framework with one sector, while the present paper explores the implications of elastic labor supply in a multisectoral framework.

[1] See Thisse and Ushchev (2018) for the literature review.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we analyze the existence and uniqueness of the equilibrium. Section 4 concludes.

2 The Model

We consider a simple multi-sector model of monopolistic competition with homogeneous consumers/workers who endogenously decide how many units of labor to supply.

2.1 Consumption

We assume that consumers have the following utility function:

$$U = \left(\sum_{j=1}^{J} \beta_j \left(\int_0^{N_j^E} q_j^{\rho_j}(i) di \right)^{\frac{\sigma-1}{\rho_j \sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

where $J > 1$ is the number of sectors, $q_j(i)$ is the consumption of a variety i produced in sector j, N_j^E is the number of available varieties in sector j, ℓ is the number of labor units supplied, $\rho_j < 1$ represents the constant elasticity of substitution between varieties within sector j, $\sigma > 1$ is the intersectoral elasticity of substitution such that $1 - \sigma(1 - \rho_j) > 0$ for all j (see Behrens et al., 2020), and $\sum_{j=1}^{J} \beta_j$ is normalized to unity with $\beta_j > 0$. Finally, $\frac{1}{\gamma} > 0$ represents the labor supply elasticity.[2]

The budget constraint is then given by (labor wage is normalized to unity)

$$\sum_{j=1}^{J} \int_0^{N_j^E} p_j(i) q_j(i) di = n\ell,$$

where $p_j(i)$ is the price of a variety produced by a firm in sector j and n is labor productivity of consumers.

The utility function implies that, given the prices and income $n\ell$, demand for variety i in sector j is

$$q_j(i) = n\ell \frac{\beta_j^{\sigma} P_j^{\frac{1}{1-\rho_j}-\sigma} p_j(i)^{1/(\rho_j - 1)}}{\sum_{j=1}^{J} \beta_j^{\sigma} P_j^{1-\sigma}}$$

[2] Adding parameter η representing the magnitude of the income effect (the effect of a rise in nonlabor income on labor income, see Keane, 2011):

$$U = \frac{\left(\sum_{j=1}^{J} \beta_j \left(\int_0^{N_j^E} q_j^{\rho_j}(i) di \right)^{\frac{\sigma-1}{\rho_j \sigma}} \right)^{\frac{\sigma(1-\eta)}{\sigma-1}}}{1-\eta} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

does not change the qualitative implications of the model.

where P_j is the CES price index in sector j given by

$$P_j^{\rho_j/(\rho_j-1)} = \int_0^{N_j^E} p_j(i)^{\rho_j/(\rho_j-1)} di.$$

Taking into account the above expressions, we obtain that

$$U(\ell) = \frac{n\ell}{P} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

where P is the CES price index in the economy given by

$$P = \left(\sum_{j=1}^{J} \beta_j^\sigma P_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{1}$$

Hence, the optimal labor supply is given by $\ell = (n/P)^{1/\gamma}$.

2.2 Market Equilibrium

The total demand for a variety produced by a firm i in sector j is given by

$$Q_j(i) = \frac{n\ell L \beta_j^\sigma P_j^{\frac{1}{1-\rho_j}-\sigma}}{\sum_{j=1}^{J} \beta_j^\sigma P_j^{1-\sigma}} p_j(i)^{1/(\rho_j-1)},$$

where L is the total number of consumers in the economy. Given the isoelastic demand, the optimal price is equal to m_j/ρ_j, where c_j is the marginal cost of production in sector j. The free entry into each sector implies that

$$(p_j(i) - m_j)Q_j(i) - f_j = 0,$$

where f_j is the fixed cost of production in sector j.

Hence, the equilibrium in the model is described by (1), the optimal labor supply condition, and the free entry condition given by

$$C_j \beta_j^\sigma \frac{P_j^{\frac{1}{1-\rho_j}-\sigma}}{P^{1-\sigma}} = \frac{f_j}{n\ell L}, \tag{2}$$

where

$$C_j = m_j^{\rho_j/(\rho_j-1)} \frac{1-\rho_j}{\rho_j^{\rho_j/(\rho_j-1)}}.$$

3 Existence and Uniqueness of Equilibrium

The equilibrium conditions imply that

$$1 = \sum_{j=1}^{J} A_j(L) \left(P^{\rho_j} \ell^{1-\rho_j} \right)^{\frac{(\sigma-1)}{1-\sigma\left(1-\rho_j\right)}}, \tag{3}$$

where

$$A_j(L) = \beta_j^{\frac{\sigma\rho_j}{1-\sigma\left(1-\rho_j\right)}} \left(\frac{f_j}{C_j} \right)^{\frac{1-\sigma}{\frac{1}{1-\rho_j}-\sigma}} (nL)^{\frac{(\sigma-1)\left(1-\rho_j\right)}{1-\sigma\left(1-\rho_j\right)}}.$$

Thus, the equilibrium in the model is determined by the intersection of two curves: the PP curve follows from (3) with a negative relationship between P and ℓ (as $1-\sigma(1-\rho_j) > 0$ for all j); the $\ell\ell$ curve is labor supply curve $\ell = (n/P)^{1/\gamma}$, which also implies a negative relationship between P and ℓ. Notice that if we know P and ℓ, we can find P_j from (2).

If labor supply is inelastic, as usually assumed in monopolistic competition models, there is a unique equilibrium (the equation in (3) has a unique solution with respect to P). Under endogenous labor supply, however, changes in P affect the labor supply, which in turn affects the price index P. This leads to the possibility of multiple equilibria. In the further analysis we distinguish between three cases.

3.1 Low and High Labor Elasticity

Consider the case when the labor supply elasticity, $1/\gamma$, is less than or equal to $\rho_{min}/(1-\rho_{min})$ where ρ_{min} is the minimum ρ_j. In this case, it is straightforward to show that the slope of the PP curve is higher than the slope of the $\ell\ell$ curve (in the (P,ℓ) space): that is, the former crosses the latter from above. As a result, similarly to models with inelastic labor supply, there exists a unique intersection of the curves that defines the equilibrium values of P and ℓ. When $1/\gamma$ is higher than or equal to $\rho_{max}/(1-\rho_{max})$ where ρ_{max} is the maximum ρ_j, the PP curve is flatter than the $\ell\ell$ curve, implying again a unique intersection of the curves.

The above two cases differ in terms of the stability of equilibrium. In the second case, when the labor supply elasticity is high (implying that the PP curve crosses the $\ell\ell$ curve from below in the (P,ℓ) space), the equilibrium is not stable. Indeed, consider a rise in the market size L. Given the labor supply ℓ, this increases the aggregate demand, inducing additional entry and, therefore, reducing the price index P. This in turn increases the labor supply, which decreases the price index further and so on. As a result, when the PP curve is flatter than the $\ell\ell$ curve, the economy converges to the case with infinite labor supply and zero price index. When, however, the PP curve is steeper than the $\ell\ell$ curve, the economy converges to a new equilibrium with finite labor supply and price index.

Based on the above reasoning, we can formulate the following proposition.

Proposition 1. *If $\rho_{min}/(1-\rho_{min}) \geq 1/\gamma$, there exists a stable unique equilibrium. If $1/\gamma \geq \rho_{max}/(1-\rho_{max})$, there is a unique equilibrium, which is not stable.*

Proof. The proof follows from the reasoning above. □

The intuition behind the stability of the equilibrium can be formulated in the following. Consider the indirect utility from consumption taking into account the link between P and ℓ in (3): $n\ell/P(\ell)$. Then, if $1/\gamma \geq \rho_{max}/(1-\rho_{max})$, the labor supply solving $max_\ell\,(n\ell/P(\ell) - \ell^{1+\gamma}/(1+\gamma))$ is equal to infinity: the benefits from higher labor supply are always higher than the costs. This leads to unstable equilibrium. Note that the equilibrium nevertheless exists in this case, as workers take the price index as given. When $\rho_{min}/(1-\rho_{min}) \geq 1/\gamma$, the costs of higher labor supply offset the benefits at some point and, therefore, we have a stable equilibrium. Note also that Proposition 1 implies equilibrium uniqueness in the economy with one sector or with homogeneous multiple sectors.

3.2 Medium Labor Elasticity

The most interesting case arises when the labor elasticity take medium values: $\rho_{max}/(1-\rho_{max}) > 1/\gamma > \rho_{min}/(1-\rho_{min})$. In this case, the PP curve is flatter (steeper) than the $\ell\ell$ curve when P is sufficiently low (high) and ℓ is sufficiently high (low). Why is this case especially interesting? It appears that under the above values of the labor elasticity, multiple equilibria are possible.

This can happen because, under the above assumptions about the value of γ, the PP curve can cross the curve the $\ell\ell$ curve several times. Specifically, if we substitute $\ell = (n/P)^{1/\gamma}$ into the PP curve, we derive

$$1 = \sum_{j=1}^{J} A_j(L) n^{\frac{\left(1-\rho_j\right)(\sigma-1)}{\gamma\left(1-\sigma\left(1-\rho_j\right)\right)}} \left(P^{\gamma\rho_j - 1+\rho_j}\right)^{\frac{(\sigma-1)}{\gamma\left(1-\sigma\left(1-\rho_j\right)\right)}}. \tag{4}$$

Since $\rho_{max}/(1-\rho_{max}) > 1/\gamma > \rho_{min}/(1-\rho_{min})$, $\gamma\rho_j - 1 + \rho_j$ is positive for some j and negative for the others. As a result, it is straightforward to see that, depending on the parameters, the equation in (4) can have multiple solutions. One can formulate the following proposition.

Proposition 2. *If $\rho_{max}/(1-\rho_{max}) > 1/\gamma > \rho_{min}/(1-\rho_{min})$, then there exists a threshold $\bar{L}$ such that 1) if $L > \bar{L}$, there is no equilibrium. 2) if $L < \bar{L}$ there are multiple equilibria. The number of equilibria is at most J.*

Proof. Since $\gamma\rho_j - 1 + \rho_j$ is positive for some j and negative for the others, the right-hand side in (4) has a minimum as a function of P. Let us define $\bar{L}$ as the value of L such that the value of the right-hand side at this minimum is equal to unity. This is possible, as $A_j(L)$ is strictly increasing in L. Thus, the statement in 1) directly follows from the definition of $\bar{L}$. The second statement follows from the following reasoning. The value of the right-hand side in (4) is equal to infinity when P is equal to zero or infinity. This implies that if the right-hand side crosses a unit horizontal line from above (below), it must cross it one more time from below (above). Hence, if $L < \bar{L}$, there are at least two solutions of (4).

To establish that the number of equilibria is at most J, we notice that equation (4) looks very similar to a polynomial. In particular, equation (4) has a form of $\sum_{j=1}^{J} a_j f_j(P) = 0$,where $f_j(P) = P^{\alpha_j}$, $a_j \in \mathbb{R}\backslash 0$, and $\alpha_j \in \mathbb{R}$. A generalization of Descartes' rule of signs (see, e.g., Haukkanen and Tossavainen, 2011) implies then that there can be at most J solutions. □

The intuitive explanation behind multiple equilibria in the model is based on the behavior of the PP curve that has a different slope relative to the slope of the $\ell\ell$ curve depending on the values of P and ℓ and the intersectoral "strategic interplay". Drawing on Proposition 1, sectors with $\rho_j/(1-\rho_j) > 1/\gamma$ will rotate the PP curve clockwise relative to the $\ell\ell$ curve, whereas those with $\rho_j/(1-\rho_j) < 1/\gamma$ anticlockwise. With the relative importance of separate sectors depending on the values of P and ℓ, we can obtain multiple intersections of the PP and $\ell\ell$ curves or none at all.[3] Note that this outcome is not possible in the specification with one sector or homogeneous multiple sectors because of the absence of intersectoral interplay.

Notice also that among multiple equilibria (if this is the case), some equilibria are stable and some are not. In particular, if the right-hand side function in (3) crosses a unit horizontal line from above, we have a stable equilibrium; if it crosses the line from below, the equilibrium is not stable (see Section 3.1 for details).

4 Conclusion

Overall, we study in this paper the existence and uniqueness of the equilibrium in the monopolistic competition framework with several heterogeneous sectors and endogenous labor supply. We show that for low (high) values of labor supply elasticity the equilibrium is unique, while for medium values of the labor elasticity, there is a multiplicity of equilibria.

The estimates of the elasticity of substitution ρ_j and the labor supply elasticity $1/\gamma$ vary a lot across studies. Broda and Weinstein (2006) report $\rho_j/(1-\rho_j) \in [0.2, 21]$ for U.S. data. For UK and French data, Behrens et al. (2020) find that for various sectors $\rho_j \in [0.65, 0.97]$ leading to $\rho_j/(1-\rho_j) \in [1.9, 32]$. The estimates of the labor supply elasticity typically lie in the range $1/\gamma \in [0.3, 1.8]$ (see Saez et al., 2012; Mertens and Montiel Olea, 2018). This implies that empirically more relevant case is likely to be the one with low values of labor supply elasticity leading to a unique equilibrium. However, the estimates of ρ_j depend highly on the level of aggregation. Hence, it is important to take into account the possibility of multiple equilibria

[3] Note that the absence of the equilibrium in the model (when the market size is large enough) can be "resolved" by introducing an upper bound $\bar{\ell}$ on the labor supply. In this case, we derive a corner equilibrium with $\ell = \bar{\ell}$ instead of the outcome with no equilibrium. Multiple equilibria are still possible.

when modeling the market with multi-sector monopolistic competition and endogenous labor supply.

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