CESifo WORKING PAPERS

8422
2020

July 2020

Wage Bargaining and Employment Revisited: Separability and Efficiency in Collective Bargaining

Claus-Jochen Haake, Thorsten Upmann, Papatya Duman

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 8422*

Wage Bargaining and Employment Revisited: Separability and Efficiency in Collective Bargaining

Abstract

We analyse the two-dimensional Nash bargaining solution (NBS) deploying a standard labour market negotiations model (McDonald and Solow, 1981). We show that the two-dimensional bargaining problem can be decomposed into two one-dimensional problems such that the two solutions together replicate the solution of the two-dimensional problem, if the NBS is applied. The axiom of Independence of Irrelevant Alternatives turns out to be crucial for decomposability. Our result has significant implications for actual negotiations, as it allows for the decomposition of a multi-dimensional bargaining problem into simpler problems—and thus helps to facilitate real-world negotiations.

JEL-Codes: J520, J410, C780.

Keywords: labour market negotiations, efficient bargains, Nash bargaining solution, sequential bargaining, restricted bargaining games.

Claus-Jochen Haake
Paderborn University / Germany
CJHaake@wiwi.upb.de

Thorsten Upmann
University of Oldenburg / Germany
Thorsten.Upmann@hifmb.de

Papatya Duman
Paderborn University / Germany
PDuman@wiwi.upb.de

July 2, 2020

1 Introduction

Across the OECD countries, labour market institutions are quite diversely organized. To accommodate economic modelling to this diversity, researchers have established a large variety of labour market models, among which are two classes designed to represent labour market negotiations: the non-cooperative approach based upon Rubinstein's alternating offer bargaining game (Rubinstein, 1982), and the cooperative approach applying familiar solution concepts of the axiomatic bargaining theory. With respect to the latter class, the solution concept almost exclusively applied in labour economics is the Nash (1950) bargaining solution (NBS). The NBS is so universally applied that we may arguably refer to it as *the* bargaining solution applied to labour markets. The only exceptions from this 'rule' we are aware of are Alexander (1992), Alexander and Ledermann (1996), Gerber and Upmann (2006), Amine et al. (2009), Dittrich (2010), Dittrich and Knabe (2013), l'Haridon et al. (2013), Liu et al. (2017) and Amine et al. (2018), all of whom apply the Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975) to model the outcome of labour market negotiations; while Jacquet et al. (2014) apply the egalitarian solution. The rest of the literature deploys the NBS to model labour market negotiations (or use a non-cooperative approach). There is thus a tremendous dominance of the NBS in models of labour market negotiations.

Basically, labour market negotiations can be either on wages exclusively, or on all work-related variables. While employment is left at the discretion of firms in the former approach, the employment level is a matter of negotiation in the second one. Since all variables, *viz.* at least the wage rate and the employment level, are bargained in the latter model, an agreement may be reached leaving no scope for a further mutual benefit. For that reason, combined employment–wage negotiations are referred to as *efficient negotiations*. When this approach is combined with the NBS, we obtain two models: the right-to-manage model where only the wage rate is determined by the NBS (Nash wage bargains), and efficient negotiations where both the wage rate and the employment level are determined according to the NBS (efficient Nash bargains).

From an empirical perspective, the question arises which of both models is more descriptive for actual labour market institutions. In this regard, the empirical evidence for whether labour market equilibria lie on the labour demand curve or on the Pareto curve (or offside of both) is mixed,[1] a finding that hardly comes at a surprise in view of the huge variety of industrial institutional differences. Because there is thus some empirical evidence

[1] For a review of the empirical literature, the interested reader is referred to the valuable overviews of Aidt and Tzannatos (2008), Lawson (2011), and Cahuc et al. (2014, Ch. 7.4) providing valuable surveys of the literature. In particular, a list of articles that find empirical evidence in favour of efficient bargains can be found in Upmann and Müller (2014, p. 339).

in favour of equilibria on the Pareto curve, efficient bargains models appear to be descriptive in those cases. Correspondingly, there is substantial theoretical literature on labour market negotiations that applies the efficient negotiations model: Horn and Svensson (1986), Svejnar (1986), Paz Espinosa and Rhee (1989), Dowrick (1989, 1990), Clark (1990), Bughin (1996), Bayındır-Upmann and Raith (2003, 2005), Kraft (2006), Upmann (2009), Dittrich (2010), Walsh (2012), Eichner and Upmann (2012, 2014), Fanti and Gori (2013), Upmann and Müller (2014), Müller and Upmann (2018) and others document this significance of the efficient bargains model. Still, the larger part of the theoretical literature on labour market negotiations presupposes that labour market equilibria lie on the labour demand curve and thus adopt the simple one-dimensional right-to-manage model.[2]

In view of this overwhelming presence of the NBS in the literature—both the right-to-manage model and the efficient bargains model predominantly apply the NBS—and its proven empirical relevance, it is indispensable to thoroughly study the properties of the NBS within the particular situation of labour market negotiations. In this regard, the articles of McDonald and Solow (1981), Oswald (1985), Binmore et al. (1986) and Creedy and McDonald (1991) are fundamental, but also Alexander and Ledermann (1994, 1996) conceptually contribute in two, yet frequently overlooked, articles to the examination of the properties of the NBS in labour markets. Since then, the focus of conceptual interest in the NBS shifted towards the issue of agenda setting and pattern bargaining in labour markets: Dowrick (1990), Dobson (1994), Bughin (1999), Petrakis and Vlassis (2000), and Creane and Davidson (2011) are examples for this strand of the literature. Those authors investigate the (strategic) choice of the set of variables to negotiate on (agenda setting) and the sequence of issues to be negotiated (pattern bargaining).

In this article, we contribute to both, the fundamentals of the NBS in labour market negotiations, and sequential negotiations on the wage rate and the employment level. Even after about 30 years of work on the NBS in labour markets, the analysis is still incomplete. It thus seems imperative to us to provide a rigorous analysis of that solution concept within the framework of a two-party bargaining situation on the labour market allowing us to shed light on some still unnoticed aspects of the properties of the NBS. First, we decompose the original two-dimensional bargaining problem into two distinct, one-dimensional, and thus simpler, bargaining problems: one problem where the employer and the employees only negotiate on the wage rate, and one where they only negotiate on the employment level. Applying the NBS to each of these one-dimensional bargaining problems yields two parametrised

[2] See, for example, Zhao (1995), Dutt and Sen (1997), Petrakis and Vlassis (2000), Grandner (2001), Strand (2002), Albrecht and Vromen (2002), Flinn (2006), Boeri and Burda (2009), Wehke (2009), Gertler and Trigari (2009), Belan et al. (2010), Krusell et al. (2010), Creane and Davidson (2011), Ranjan (2013), Santoni (2014), Colciago and Rossi (2015), and many others.

solutions, to which we refer as the $w-$ and the L–Nash curve, respectively. The intersection of these two curves marks the NBS of the efficient bargaining model, in which both parties simultaneously negotiate over the wage rate and the employment level (Propositions 1 and 2). This property of decomposability allows us to show that sequential (Proposition 3) and iterative (Proposition 4) one-dimensional negotiation agendas can be designed to arrive at the bargaining outcome of the (static) two-dimensional bargaining problem, and thus bring about the globally efficient bargaining outcome. The results underline the paramount significance of the axiom of *Independence of Irrelevant Alternatives* (IIA axiom). Although, this axiom is constitutive for the NBS, its salient significance apparently has been left unnoticed in the literature on wage bargaining. Bargaining solutions that do not satisfy the IIA axiom, such as the Kalai-Smorodinsky solution, can generically not be decomposed in the way described above and fail to reach an efficient outcome via sequential negotiations. Finally, we present a rigorous and coherent formal treatment of the NBS accompanied with fully-fledged diagrammatic illustration which is still missing, a lack that has occasionally led to flawed figures in the literature. Particularly, the study of corresponding diagrams in the space of physical outcomes (wage and employment) and in utility space underlines the value of applying game theoretic concepts to the underlying economic problem.

2 The Model

We deploy the well-known labour market bargaining model elaborated by McDonald and Solow (1981) and study labour market negotiations between an employer (*viz.* a firm) and a trade union representing the interests of all workers. The firm behaves competitively on the output market and produces its product by means of labour (and some fixed factor). Both parties negotiate on a labour contract specifying the wage rate and the employment level. We subsequently characterize both parties' preferences over possible agreements and the resulting Pareto efficient allocations. In the next section we then formulate the resulting bargaining problem.

Labour supply. The supply side of the labour market consists of a mass N of labourer households. For ease of presentation, we shall subsequently speak of N as the *number* of labourer households or workers. Suppose that workers have identical preferences and productivity, so that they are homogeneous regarding all aspects of interest. An employed worker derives utility $v : \mathbb{R}_+ \to \mathbb{R}_+$ from the received wage w. We assume $v(0) = 0, v' > 0, v'' \leq 0$. An unemployed worker obtains a fixed utility level $\bar{v} \geq 0$ representing the utility from unemployment benefits and leisure time. Suppose that there exists some wage rate

$\bar{w} \geq 0$ with $v(\bar{w}) = \bar{v}$, so that $\bar{w} := v^{-1}(\bar{v})$ is the reservation wage, below which a worker is not willing to work. Consequently, aggregate labour supply equals zero for all $w < \bar{w}$, equals N for all $w > \bar{w}$, and is indeterminate for $w = \bar{w}$, that is:

$$L^s(w) = \begin{cases} N & \text{if} \quad w > \bar{w}, \\ [0, N] & \text{if} \quad w = \bar{w}, \\ 0 & \text{if} \quad w < \bar{w}. \end{cases} \tag{1}$$

Following Oswald (1985), the trade union represents the interests of all workers, so that its utility depends on the negotiated wage w and employment level L (considered as a continuous variable) and is given by $u_1 : \mathbb{R}_+ \times [0, N] \to \mathbb{R}$,

$$u_1(w, L) := Lv(w) + (N - L)v(\bar{w}).$$

The utility of the labour union may thus be viewed as the aggregate utility of its members. Differentiation of u_1 with respect to w and L gives the marginal rate of substitution between wage and employment:

$$\frac{dw}{dL}\bigg|_{u_1 \text{ const.}} = -\frac{\frac{\partial u_1(w,L)}{\partial L}}{\frac{\partial u_1(w,L)}{\partial w}} = -\frac{v(w) - \bar{v}}{Lv'(w)} = -\frac{w}{L}\frac{1}{\sigma(w)}, \tag{2}$$

where $\sigma(w) := (wv'(w))/(v(w) - \bar{v})$ denotes the elasticity of the excess utility (the utility above the reservation utility $\bar{v}$). Since $\sigma(w) > 0$ for all $w > \bar{w}$, the union's indifference curves are downward sloping in the area above the reservation wage $\bar{w}$, and thus for all relevant wage rates. This is illustrated in Figure 1, page 5, where the union's indifference curves are represented by magenta lines. (To have figures that are in line with those in the literature, we display the wage on the vertical and the employment level on the horizontal axis throughout the article, but maintain the order of the variables as defined in the utility functions.) Apparently, the trade union is indifferent between all agreements (w, L) with either $w = \bar{w}$ or $L = 0$, as all of those yield $u_1(\bar{w}, L) = u_1(w, 0) = N\bar{v}$, for all w, L.

Defining the elasticity of the marginal utility by $\varepsilon(w) := \frac{wv''(w)}{v'(w)} \leq 0$, we obtain:

Lemma 1 *For any $w > \bar{w}$, the elasticity of the excess utility σ satisfies*

$$\sigma'(w) \lesseqqgtr 0 \quad \Leftrightarrow \quad \sigma(w) \gtreqqless 1 + \varepsilon(w).$$

The proof of Lemma 1, as well as those of all other proofs, are relegated to Appendix B.

While Lemma 1 applies to any $w < \bar{w}$, we show below that $\sigma(w) > 1$ for any w in the neighbourhood of an equilibrium labour market contract (see Lemma 2).

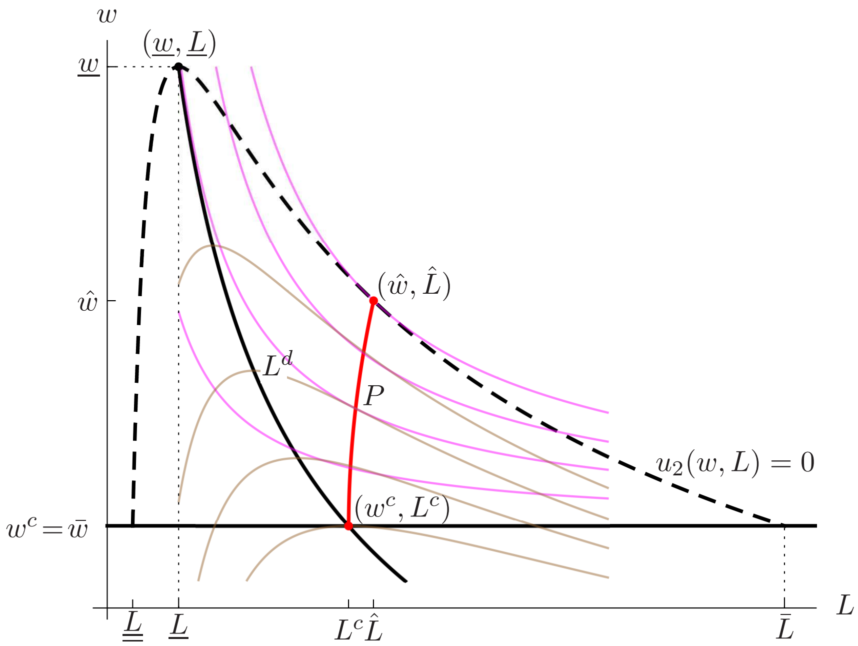


Figure 1: The union's indifference curves (magenta), the reservation wage (horizontal line), the firm's iso-profit lines (brown), the zero profit line (the dashed curve), the labour demand curve (black curve) and the Pareto curve (red).

Labour demand. The firm produces its output by means of a technology featuring decreasing returns to scale, with production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$. Normalizing the output price to unity, the profit obtained by employing L workers at wage rate w is given by $u_2 : \mathbb{R}_+ \times [0, N] \to \mathbb{R}$ such that

$$u_2(w, L) \;:=\; f(L) - wL - \mathbf{1}_{\{L:L>0\}}C,$$

where $\mathbf{1}_A$ denotes the indicator function for event A, and $C \geq 0$ denotes the setup cost incurred upon start of production, *i.e.*, if some positive amount of labour is deployed. (The limit case $C = 0$ is analysed in Appendix A.) Thus, we have $u_2(w, 0) = 0$, but $\lim_{L\searrow 0} u_2(w, L) < 0$ if $C > 0$. In other words, when $C > 0$, there is a minimum number of labour $\underline{\underline{L}}$ to ensure non-negative profits, so $u_2(\bar{w}, \underline{\underline{L}}) = 0$ and $u_2(\bar{w}, L) < 0$ for all $L < \underline{\underline{L}}$. Moreover, we assume that labour supply is large enough (or the reservation wage is high enough) so that not all labourers can be profitably employed.[3]

Assumption 1 *Not all labourers can be employed at non-negative profits:*

$$u_2(\bar{w}, N) = f(N) - N\bar{w} - C < 0,$$

and there exists an employment level $L \in [0, N]$ *with* $u_2(f'(L), L) > 0$.

[3]For most of our results this assumption is not necessary, and it would suffice to assume that N exceeds the largest efficient employment level $\hat{L}$ (see definition on page 7), so that N does not limit the Pareto curve.

Assumption 1 implies that there is some employment level $\bar{L} < N$ such that $u_2(\bar{w}, \bar{L}) = f(\bar{L}) - \bar{L}\bar{w} - C = 0$ and $u_2(\bar{w}, L) < 0$ for all $L > \bar{L}$. Accordingly, any reasonable employment level lies above $\underline{\underline{L}}$ but below $\bar{L}$. Moreover, for each employment level L, let $w^0(L)$ represent the wage level that yields zero profit, *i.e.*, $w^0(L) = \frac{f(L)-C}{L}$.

Differentiating u_2 with respect to w and L, we obtain the slope of the iso-profit curves:

$$\left.\frac{dw}{dL}\right|_{u_2 \text{ const.}} = -\frac{\frac{\partial u_2(w,L)}{\partial L}}{\frac{\partial u_2(w,L)}{\partial w}} = -\frac{w - f'(L)}{L}. \tag{3}$$

Each iso-profit curve is increasing in L until $f'(L) = w$, and decreasing afterwards (displayed by brown lines in Figure 1). Accordingly, for any given wage rate, the profit-maximizing employment level is located where the iso-profit curve is horizontal. Since a minimum number of labour is required to ensure non-negative profits, there is a maximum wage that the firm is willing to pay: the wage rate $\underline{w} > \bar{w}$ defined as the root of $u_2\left(\cdot, (f')^{-1}(\cdot)\right)$. Hence, when employment is unprofitable, labour demand equals zero, and is characterized by $f'(L) = w$ else, so that labour demand is given by

$$L^d(w) = \begin{cases} (f')^{-1}(w) & \text{if} \quad 0 \leq w \leq \underline{w}, \\ 0 & \text{else.} \end{cases} \tag{4}$$

Since L^d is decreasing, the minimum number of labour on the labour demand curve is obtained for the maximal wage rate: $\underline{L} := L^d\left(\underline{w}\right)$. So, the "origin" (or the "upper end") of the labour demand curve is located at $(\underline{w}, \underline{L})$ (see Figure 1). For the limiting cases $f'(0) \to \infty$ and $C = 0$, we have $\underline{w} \to \infty$ (see Appendix A). We will henceforth restrict our attention to $\mathcal{L} := [\underline{L}, \bar{L}]$ and $\mathcal{W} := [\bar{w}, \underline{w}]$ since these intervals cover all reasonable employment levels and wage levels.

Pareto efficient allocations. Using the characterizations of the parties' preferences, we can identify Pareto efficient labour contracts; that is, efficient pairs of a wage rate and an employment level (w, L) (depicted by red lines in Figure 1). These contracts can be obtained by equating the parties' marginal rates of substitution, given by eqs. (2) and (3), yielding:

$$\sigma(w) = \frac{w}{w - f'(L)}. \tag{5}$$

Eq. (5) equates the elasticity of excess utility with the reciprocal of the wage markup above the marginal productivity of labour given by $(w - f'(L))/w$.

Lemma 2 *In the neighbourhood of the Pareto curve, the elasticity of (excess) utility exceeds unity,* i.e., $\sigma(w) > 1$.

It follows from the combination of Lemma 1 and 2 that the elasticity of the excess utility σ is decreasing in the neighbourhood of the Pareto curve.

The competitive labour market equilibrium is obtained by equating labour supply, eq. (1), and labour demand, eq. (4), yielding the pair $(w^c, L^c) = \big(\bar{w}, L^d(\bar{w})\big)$. Clearly, the competitive equilibrium is Pareto efficient as it satisfies eq. (5). While the competitive equilibrium leaves the labour union with its reservation (or status quo) utility level, the employer obtains the maximum feasible profit:

$$u_1^c := u_1(w^c, L^c) = N\bar{v}, \quad u_2^c := u_2(w^c, L^c) = f(L^c) - \bar{w}L^c - C > 0.$$

In other words, u_2^c is the maximal feasible profit when the utility of the labour union is at least $N\bar{v}$. Formally,

$$u_2^* := \max_{w\geq 0,\, 0\leq L\leq N} u_2(w, L) \quad \text{s.t.} \quad u_1(w, L) \geq N\bar{v}\,.$$

It is straightforward to see that the constraint is binding at the solution, and hence, the unique feasible utility allocation most favourable for the firm is induced by the pair $(w^c, L^c) = \big(\bar{w}, L^d(\bar{w})\big)$, yielding $u_2^* = u_2^c$.

Similarly, the individually rational utility level most favourable to the labour union, u_1^*, is achieved by the pair $(\hat{w}, \hat{L})$ which solves

$$u_1^* := \max_{w\geq 0,\, 0\leq L\leq N} u_1(w, L) \quad \text{s.t.} \quad u_2(w, L) \geq 0\,.$$

It is immediate to verify that firms are left with zero profits, *i.e.*, $u_2(\hat{w}, \hat{L}) = f(\hat{L}) - \hat{w}\hat{L} - C = 0$. Moreover, $(\hat{w}, \hat{L})$ must be Pareto efficient, *i.e.*, it must satisfy eq. (5).[4] Hence, $u_1^* = u_1(\hat{w}, \hat{L})$. Consequently, w^c and $\hat{w}$ are the lowest respectively the highest Pareto efficient wage rates, and L^c and $\hat{L}$ are the lowest respectively the highest Pareto efficient employment levels. We therefore define the sets $\mathcal{L}^\circ := [L^c, \hat{L}]$ and $\mathcal{W}^\circ := [w^c, \hat{w}]$, representing the range of all individually rational, Pareto efficient employment and wage levels, respectively. Then, we define the function $\Phi^P : \mathcal{W}^\circ \times \mathcal{L}^\circ \to \mathbb{R}$ by

$$\Phi^P(w, L) := \sigma(w) - \frac{w}{w - f'(L)}.$$

Thus, the roots of Φ^P are the Pareto efficient pairs (w, L). Accordingly, we implicitly define the Pareto curve $P : \mathcal{L}^\circ \to \mathcal{W}^\circ$ by $\Phi^P(P(\cdot), \cdot) = 0$, *i.e.*, $w = P(L)$ is root of $\Phi^P(., L)$.

Lemma 3 *The slope of the Pareto curve P is positive with $P'(L^c) = +\infty$,* i.e.*, the Pareto curve is vertical at its "lower end" $(w^c, L^c) = (\bar{w}, L^d(\bar{w}))$.*

[4]To see this, observe that the constraints $0 \leq L \leq N$ are not binding at $\hat{L}$, because $u_1(w, L^c) > N\bar{v}$ for $w = (f(L^c) - C)/L^c$ and $u_1(w, N) < N\bar{v}$ for $w = (f(N) - C)/N$ since $(f(N) - C)/N < \bar{w}$ by Assumption 1.

3 The Bargaining Model

While in the previous section we characterized the possible combinations of a wage rate and an employment level that may emerge as an outcome of the bargaining problem, we now discuss the utilities (or payoffs) induced by those outcomes. More precisely, we extend our view on the problem by considering the utility allocations resulting from feasible (w, L) agreements. Hence, we formulate an axiomatic bargaining problem which goes back to the seminal work of Nash (1950).

3.1 The abstract bargaining problem

A *two-person bargaining problem* is a pair (S, d), where S is a closed, convex, and comprehensive[5] subset of $\mathbb{R}^2$, $S_d := S \cap (d + \mathbb{R}^2_+)$ is bounded and $d \in S$. The set S is called the *bargaining set* and may be interpreted as the collection of all utility allocations resulting from feasible agreements among the two parties, to which we may refer here, as *players*, in accordance with the game-theoretic terminology. The point $d \in S$, termed *disagreement point* (or *status quo point*), reflects the utility allocation that either becomes effective, when negotiations break down, or is realized during negotiations, depending on the specific interpretation of the model.[6] Since neither player can be forced to sign an unfavourable contract, that is, a contract that brings about a result worse than the disagreement point, only individually rational points $x \in S$, *i.e.*, $x \geq d$, are reasonable agreements as neither player strictly prefers to let negotiations fail. Then, a *bargaining solution* on a set $\mathcal{B}$ of two-person bargaining problems is a mapping $F : \mathcal{B} \to \mathbb{R}^2$ with $F(S, d) \in S$ for all $(S, d) \in \mathcal{B}$. In general, a bargaining solution constitutes, in some reasonable sense, a "fair" way to share common gains (above d). Although there is a large variety of bargaining solutions in the literature, the solution proposed by Nash (1950) is arguably the most prominent one; in particular, in the labour market literature it is almost exclusively used (as documented in the Introduction).

In the Nash bargaining solution the product of excess utilities, *i.e.*, the utilities above d,

[5] A set $T \in \mathbb{R}^2_+$ is comprehensive, if for all $x \in T$, $y \leq x$ implies $y \in T$.

[6] As thoroughly explained by Binmore et al. (1986), the interpretation of d as an alternative labour market contract (*e.g.*, the wage obtained in an alternative firm/industry, unemployment benefits, etc.) is most appropriate within a risk-of-breakdown model where the opportunity for beneficial negotiations may stochastically disappear due to random, unforeseen, exogenous effects; while the interpretation of d as the utility obtained during a dispute (*e.g.* the income during the period of strike or lock-off, etc.) is most appropriate within a time-preference model where impatient parties discount future benefits. Both of these interpretations are compatible with our model.

is maximized. It is immediate that the solution point necessarily has to be Pareto optimal and individually rational. From a descriptive point of view, maximization of the product naturally involves balancing the factors, so that the final utility allocation is located "in the middle" of the Pareto boundary of S. Formally, the *Nash bargaining solution* $F^N(S,d)$ of a bargaining problem (S,d) is the unique point in S maximizing the product of excess utilities above the disagreement point:[7]

$$F^N(S,d) := \underset{(x_1,x_2)\in S_d}{\text{argmax}} \ (x_1 - d_1)(x_2 - d_2). \tag{6}$$

Besides these descriptive arguments supporting the Nash solution, Nash (1950) provides an axiomatic foundation for his solution concept. Apart from the axioms of Pareto optimality, individual rationality, symmetry and scale covariance, the crucial axiom, which will play a key role in our analysis later, is the Independence of Irrelevant Alternatives:

IIA A bargaining solution F satisfies the axiom of the *Independence of Irrelevant Alternatives*, if for any two bargaining problems (S,d) and (T,d) with $S \subseteq T$ and $F(T,d) \in S$ we have $F(S,d) = F(T,d)$.

Thus, suppose there are two distinct bargaining problems with the same disagreement point, (S,d) and (T,d), but with "more" allocation possibilities in T, *i.e.*, $S \subseteq T$. Then, if the solution $F(T,d)$ of the "larger" problem is feasible in the "smaller" problem S, this point should also be the solution for S. In this sense, the allocations in $T \setminus S$ are irrelevant for the solution in S.

In the welfaristic context, one typically abstracts from the fact that there is a set of "physical" outcomes generating the utility possibility set. Arguments for or against a particular solution therefore only take the resulting utilities into account. In an economic framework, as here in the case of labour market negotiations, we are, though, interested in the underlying variables yielding a particular utility allocation. In labour markets, the underlying variables—wages, employment, unemployment, working hours, *etc.*—are arguably of more political and economic concern than are the utilities of the trade union and the employers' federation.

Returning to the labour market model of Section 2, we will generate different specific two-person bargaining problems induced by negotiations on specific economic variables in the following subsection. The solutions of these bargaining problems in the utility space correspond to physical outcomes (w,L) of a wage rate and an employment level. Since any

[7]Since the product $(x_1 - d_1)(x_2 - d_2)$ is strictly quasi-concave and S is assumed to be convex, there is exactly one maximizer in (6), so that $F^N(S,d)$ is well-defined.

(w, L) pair induces a utility allocation $u(w, L)$, we are able to reformulate the maximization problem (6) as a maximization problem over (sets of) (w, L) combinations.

3.2 One-dimensional bargaining problems of the labour market

The parties may either bargain over the wage rate, w, or on the employment level, L, or on both. The latter scenario, where the set of issues of negotiations is two-dimensional, is referred to as the *efficient bargains model*, because when the parties bargain over w and L simultaneously, the induced utility space is the largest possible bargaining set S compatible with the labour market model. Our focus is on disentangling negotiations: We decompose the two-dimensional bargaining problem into two simpler, *i.e.*, one-dimensional problems, in which either quantity, w or L, is fixed while the other is negotiated. With the help of these models, we then demonstrate that the Nash solution of the efficient bargaining problem can be recovered from Nash solutions of these two one-dimensional problems. In particular, the fixed point of the Nash solutions of the two one-dimensional problems coincides with the Nash solution of the two-dimensional problem, and thus with the Nash solution of the efficient bargains model. Given this, we then show that different bargaining agendas (or protocols) lead, in the limit, to the Nash bargaining solution of the efficient bargains model.

The decomposition of the two-dimensional problem into two one-dimensional bargaining problems is not only suitable for an analytic purpose, but is also descriptive. In real-world negotiations, we frequently observe that only the wage is subject to negotiations while the employment level is considered as fixed, *e.g.*, as the result of earlier employment contracts determining workforce, manning rules, job guarantees, *etc.* Conversely, wage agreements negotiated on a central level may restrict parties on a firm level: The employer and the workers' council may be limited to negotiate exclusively on the employment level by means of manning rules, working time regulations, etc.

We now formulate those two types of restricted bargaining problems. For fixed $L \in \mathcal{L}$, define the two-person bargaining problem $(S^L(L), d^L(L))$ by

$$S^L(L) := \{x \in \mathbb{R}^2 \mid \exists w \in \mathcal{W} : x \leq (u_1(w, L), u_2(w, L))\}, \quad d^L(L) := d = (N\bar{v}, 0),$$

and for fixed $w \in \mathcal{W}$, define the bargaining problem $(S^w(w), d^w(w))$ by

$$S^w(w) := \{x \in \mathbb{R}^2 \mid \exists L \in \mathcal{L} : x \leq (u_1(w, L), u_2(w, L))\}, \quad d^w(w) := d = (N\bar{v}, 0).$$

Apparently, the strict Pareto boundary of S^L has to be included in the set $\{x \in \mathbb{R}^2 \mid x = (u_1(w, L), u_2(w, L)), w \in \mathcal{W}\}$. The analogous observation is true for S^w. In each (restricted) bargaining problem, we assume that in case of a breakdown of negotiations there is no (overall) agreement, *i. e.*, no worker is hired, the wage level does not have to be determined and the setup costs C are not effective. Each worker is paid the reservation wage and nothing is produced, implying that the status quo point is $(N\bar{v}, 0) = d$. In this context, when we consider 'fixed wage level' or 'fixed employment level', the fixed variable should be viewed as 'tentatively fixed' and in case of a breakdown of negotiations the 'fixation' becomes immaterial. Since the disagreement point is independent of the fixed quantity, for simplicity, we have $d = d^L(L) = d^w(w)$.

Remark 1 *The disagreement point d is the only individually rational point in the set $S^L(\bar{L})$. All individually rational points in $S^w(\bar{w})$ provide a utility of $N\bar{v}$ for the trade union, and hence no positive excess utility.*

Lemma 4 *(i) For each $L \in \mathrm{Int}(\mathcal{L})$, (S^L, d) is a well-defined two-person bargaining problem. (ii) For each $w \in \mathrm{Int}(\mathcal{W})$, (S^w, d) is a well-defined two-person bargaining problem.*

4 Nash Curves

The following two subsections discuss the classes of restricted bargaining problems and their Nash solutions. The analysis results in defining two Nash curves, depending on which quantity is bargained over. For each of the restricted bargaining problems, we obtain a well-defined Nash curve.

4.1 Fixed wage level and the L–Nash curve

Assume that the wage level is fixed to $w \in \mathrm{Int}(\mathcal{W})$, so that parties only negotiate on employment levels L in $\mathcal{L}$. For formal reasons, we first exclude the boundaries of $\mathcal{W}$, because in either of the two cases, one of the parties is not able to realize a strictly positive excess utility. In analogy to Subsection 3.2, we may rewrite the maximization problem behind the Nash solution as a maximization problem over employment levels in $\mathcal{L}$ and determine the employment level at which the product of excess utilities is maximal. In effect, we determine a physical outcome $(w, L^*(w))$, the utility allocation of which, $u(w, L^*(w)) \in S^w(w)$, is the Nash bargaining solution of the bargaining problem $(S^w(w), d)$. Phrased differently, for fixed $w \in \mathrm{Int}(\mathcal{W})$, employment level $L^*(w)$ is the employment level that is associated with the Nash bargaining solution of the restricted bargaining problem (S^w, d). Formally,

$$L^*(w) \in \operatorname*{argmax}_{L \in \mathcal{L}} (u_1(w, L) - d_1)(u_2(w, L) - d_2), \quad w \in \mathrm{Int}(\mathcal{W}). \tag{7}$$

The function $L^*(\cdot)$ is termed as *L–Nash curve.* The L–Nash curve can be continuously extended to the boundary of its domain. While this is not crucial for the formal analysis, we make use of this fact in the exposition of the L–Nash curve. For instance, the next lemma determines one of its endpoints explicitly. Evaluating the first-order condition of (7), the Nash curve can be expressed by a function $\Phi^L : \mathcal{W} \times \mathcal{L} \to \mathbb{R}$ with

$$\Phi^L(w, L) = w - \frac{1}{2}\left(f'(L) + \frac{f(L) - C}{L}\right). \tag{8}$$

All (w, L) combinations on the L–Nash curve, *i.e.*, $L = L^*(w)$, satisfy $\Phi^L(w, L) = 0$.

Lemma 5 *L^* is downward sloping for each $w \in \mathcal{W}$ and converges to the point $(\underline{w}, \underline{L})$ as w goes to $\underline{w}$.*

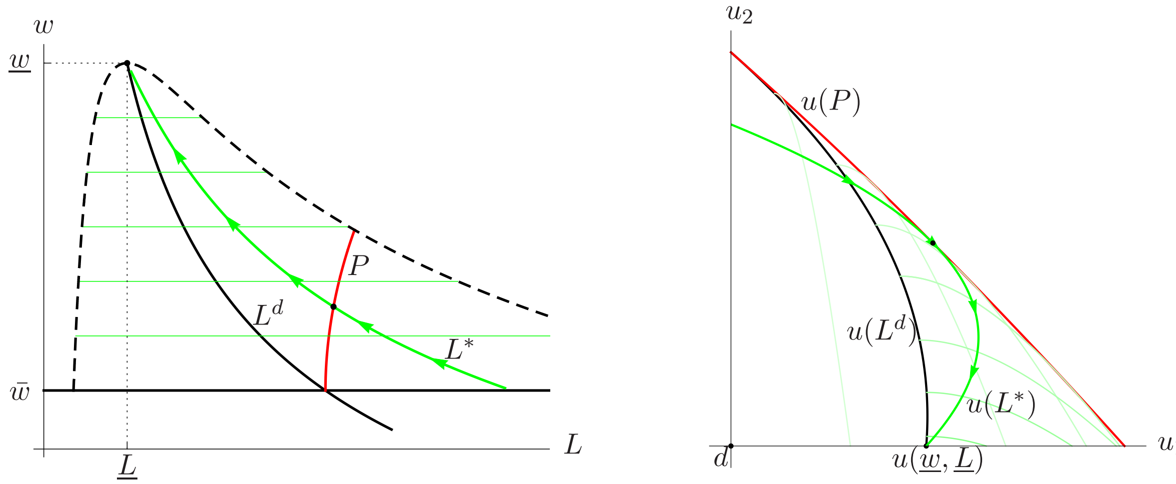
It is an immediate consequence of $\Phi^L(w, L) = 0$, that $w = \frac{1}{2}\left((f(L) - C)/L + f'(L)\right)$. Hence, the L–Nash curve can be viewed as the wage rate being a function of L that is constructed as the average of the marginal and average products. This viewpoint is prevalent in the literature. However, the very construction of the L–Nash curve takes each wage level w to a compromising employment level L, which is given by the NBS of the restricted bargaining problem $(S^w(w), d)$. The L–Nash curve should therefore be seen as given by the function L^*, which depends on the wage level w: It is the employment level that is determined by the wage rate so as to make the wage rate equally weighing the interest of the union (average product of labour) and of the employer (marginal product of labour).

Figure 2a displays the L–Nash curve, the Pareto curve in the w–L space. For given w, indicated by a dashed horizontal line, the corresponding employment level on the L–Nash curve marks the Nash bargaining solution of the restricted bargaining problem. Consequently, Figure 2b shows the images of the L–Nash curve and the Pareto curve in utility space. The direction indicated by the arrows corresponds to increasing w. The thin lines depict Pareto frontiers of restricted bargaining problems. The image of the L–Nash curve is thus the collection of Nash bargaining solution points. The image of the Pareto curve is the Pareto frontier of the efficient bargaining problem and the Nash bargaining solution of it is the common point of the Pareto frontier and the image of the L–Nash curve.

4.2 Fixed employment level and the w–Nash curve

We now analyze the scenario in which the employment level is fixed to $L \in \text{Int}(\mathcal{L})$ and bargaining takes place over the wage rate w only. Formulating the maximization problem behind the Nash solution of $(S^L(L), d)$ as one over wage levels, we obtain

$$w^*(L) \in \underset{w \in \mathcal{W}}{\operatorname{argmax}} (u_1(w, L) - d_1)(u_2(w, L) - d_2). \tag{9}$$



a) The L–Nash curve in the w–L space

b) Image of the L–Nash curve in the utility space

Figure 2: The L–Nash curve L^* (green), the Pareto curve P (red), the labour demand curve L^d (black), and their images in the utility space: Each thin green horizontal line in (a) contains (w, L) combinations with a fixed wage w. Its image, depicted in (b), shows the Pareto optimal points of the corresponding restricted bargaining problem $(S^w(w), d)$.

Our interest is on this function taking each employment level to the corresponding Nash wage. We term the function w^* as the w*–Nash curve*. The w–Nash curve can be continuously extended to the boundary of its domain, which we mainly use for expositional reasons. Since the w–Nash curve is only given implicitly via the first-order condition of (9), we formalize this by defining a function $\Phi^w : \mathcal{W} \times \mathcal{L} \to \mathbb{R}$ such that

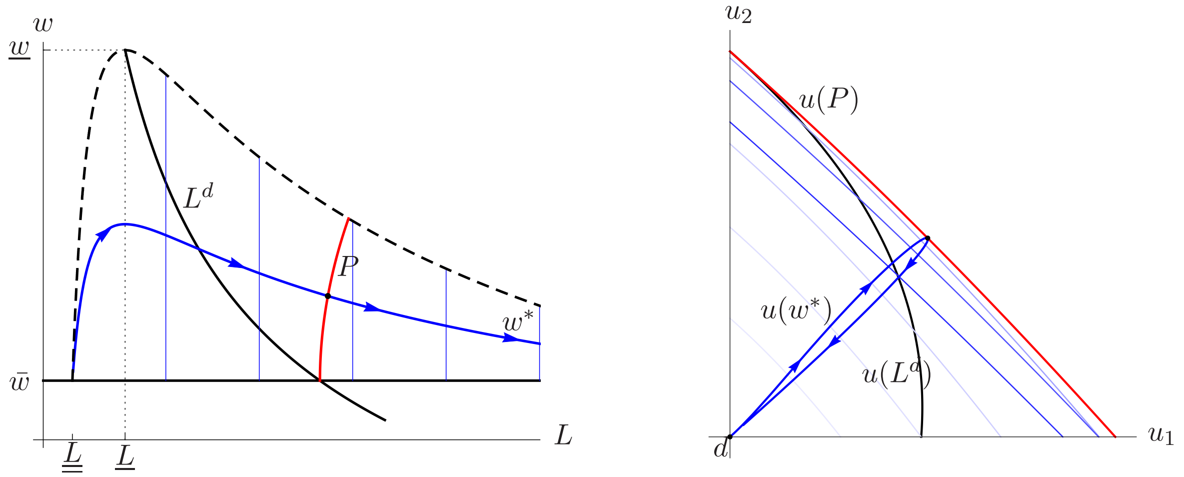
$$\Phi^w(w, L) = \sigma(w) - \frac{wL}{u_2(w, L)}. \tag{10}$$

All (w, L) combinations on the w–Nash curve, *i. e.*, with $w = w^*(L)$, satisfy $\Phi^w(w, L) = 0$.

Figures 3a and 3b parallel Figures 2a and 2b by showing the w–Nash curve and Pareto curve in the w–L space as well as their images in utility space. The arrows correspond to increasing L. The image of the w–Nash curve contains the Nash bargaining solution points of all restricted bargaining problems (given by the thin lines in Figure 3b) as well as the one of the efficient bargaining problem.

Lemma 6 *w^* is downward sloping in the neighbourhood of the Pareto curve and converges the point $(\bar{w}, \bar{L})$ as L goes to $\bar{L}$.*

It follows from Lemma 2 and 6 that in the neighbourhood of the Pareto curve, the wage share $wL/f(L)$ is greater than the profit share $(f(L) - wL - C)/f(L)$. Since various reasonable



a) The w–Nash curve in the w–L space

b) Image of the w–Nash curve in the utility space

Figure 3: The w–Nash curve w^* (blue), the Pareto curve P (red), the labour demand curve L^d (black), and their images in the utility space: Each thin blue vertical line in (a) contains (w, L) combinations with a fixed employment level L. Its image, depicted in (b), shows the Pareto optimal points of the corresponding restricted bargaining problem $(S^L(L), d)$.

utility functions satisfy $\sigma' \leq 0$—for example, the utility functions $w^a, \log(w), \sin(w), 1 - \frac{1}{w+1}, aw$—we henceforth assume that the elasticity of excess utility is non-increasing.

Assumption 2 *The derivative of the elasticity of excess utility is non-positive for all $w \in \text{Int}(\mathcal{W})$, i.e., $\sigma' \leq 0$.*

Corollary 1 *Under Assumption 2, the w–Nash curve is downward sloping for $L \in \mathcal{L}$; hence, its supremum on $\mathcal{L}$ is attained at $\underline{L}$.*

For any $L \in \mathcal{L}$, let $w^0(L)$ denote the maximum individual rational wage level, *i.e.*, the wage at which the firm's profit is zero. Then, using the definition of Φ^w, for any given $L \in \mathcal{L}$, the negotiated wage w^* can be expressed as

$$w^*(L) = w^0(L) \frac{\sigma(w^*(L))}{1 + \sigma(w^*(L))} < w^0(L).$$

The negotiated wage rate can thus be interpreted as a discount of the maximum wage level $w^0(L)$. Consequently, the w–Nash curve is located between the zero profit line and the reservation wage. In particular, it follows from Lemma 2 that in a neighbourhood of the Pareto curve, we have $w^*(L) \geq \frac{1}{2} w^0(L)$.

A note on the terminology in the literature seems to be worthwhile at this instance. In the literature, the L–Nash curve is well-known (*e.g.*, McDonald and Solow, 1981) and is mainly used to calculate the Nash bargaining solution in the efficient bargaining problem by finding the intersection point with the Pareto curve. As far as calculation of the NBS of the efficient bargaining problem is the goal, this procedure achieves it. However, our analysis of the restricted bargaining problems shows that there is more structure behind the NBS. Apparently, the literature so far did not recognize that there are *two* Nash curves, and that it consequently refers to the L–Nash curve as *the* Nash curve. The explicit use of restricted bargaining problems, though, demonstrates the existence of a second Nash curve, the w–Nash curve. Moreover, as we demonstrate in the remainder of the article, the unique intersection point of the two Nash curves (i) constitutes the NBS outcome of the efficient bargaining problem and (ii) is the final outcome of well-specified sequential bargaining agendas.

4.3 Location of the Nash curves

In this subsection we analyse the relative location of the two Nash curves and their intersection. The results will be used in Section 5 to study the outcomes from different bargaining agendas. The next proposition shows that the graphs of the two Nash curves must have a unique intersection point (w^I, L^I). For $L < L^I$, the L–Nash curve is located above the w–Nash curve, while the opposite holds for $L > L^I$.

Proposition 1 *(i) The w–Nash curve w^* and the L–Nash curve L^* have a unique point of intersection (w^I, L^I).*

(ii) This intersection point lies on the Pareto curve.

(iii) For $L < L^I$, the L–Nash curve is located above the w–Nash curve. For $L > L^I$, the L–Nash curve is located below the w–Nash curve.

Figure 4 depicts the results in Proposition 1. It shows the two Nash curves in the w–L space and their monotonicity properties as well as their relative position. Moreover, the unique common point is located on the Pareto curve. The thin curves represent iso-product lines of the Nash product. Due to the technical origin of the Nash curves as solutions of corresponding first-order conditions, the Nash curves intersect the iso-product lines, where the corresponding partial derivative of the Nash product is equal to zero. Phrased differently, the L–Nash curve contains all (w, L) combinations, at which the iso-product line is horizontal; while the w–Nash curve contains those points, at which the iso-product line is vertical.

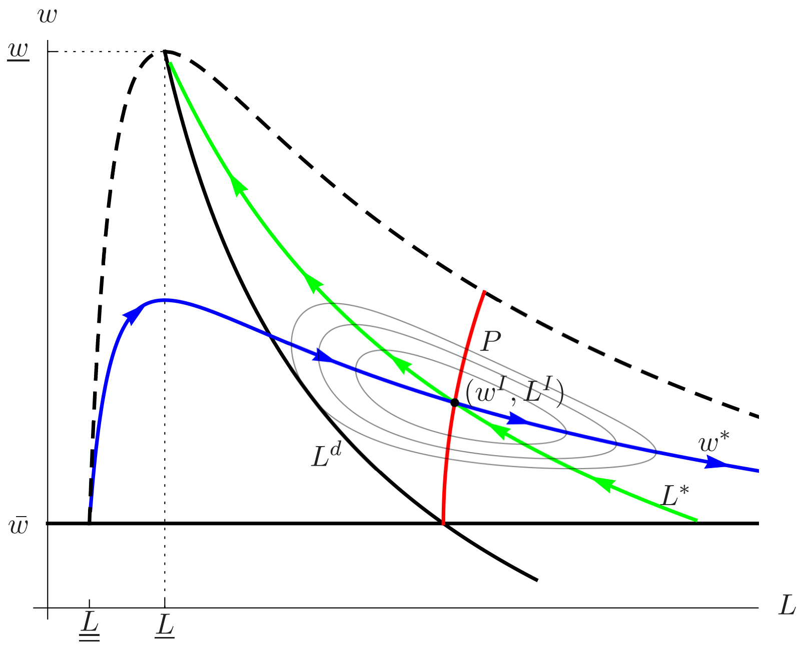


Figure 4: The L–Nash curve (green) and the w–Nash curve (blue) have a common intersection on the Pareto curve (red). The thin ellipsoid–shaped lines depict iso-Nash-product lines, which intersect the L– and the w–Nash curve at the zero and infinite slope, respectively.

4.4 Efficient wage-employment bargaining

Before we have a closer look at different bargaining protocols, we devote this subsection to the scenario in which the trade union and the firm negotiate on the wage and the employment at the same time. In the literature, this scenario is commonly referred to as *efficient bargains*. Accordingly, we define the *efficient bargaining problem* (S^e, d) (*e.g.*, McDonald and Solow, 1981) as

$$S^e := \{x \in \mathbb{R}^2 \,|\, \exists (w, L) \in \mathcal{W} \times \mathcal{L} : x \leq (u_1(w, L), u_2(w, L))\}, \quad d := (N\bar{v}, 0).$$

For each $L \in \mathcal{L}$, the restricted bargaining problem set $S^L(L)$ is included in S^e. Analogously, for each $w \in \mathcal{W}$, $S^w(w) \subseteq S^e$. We will use this fact when applying the IIA axiom to relate the Nash solution of the efficient bargaining problem with the Nash solution of a restricted one. For the efficient bargaining problem (S^e, d), the maximization problem reads as

$$(w^N, L^N) \in \underset{(w,L) \in \mathcal{W} \times \mathcal{L}}{\operatorname{argmax}} \, (u_1(w, L) - d_1)(u_2(w, L) - d_2). \tag{11}$$

The first-order conditions of (11) are precisely those that stem from setting (8) and (10) to zero, respectively. This means that they are precisely the first-order conditions that characterize Nash solutions for restricted bargaining problems. As a result, the maximizer

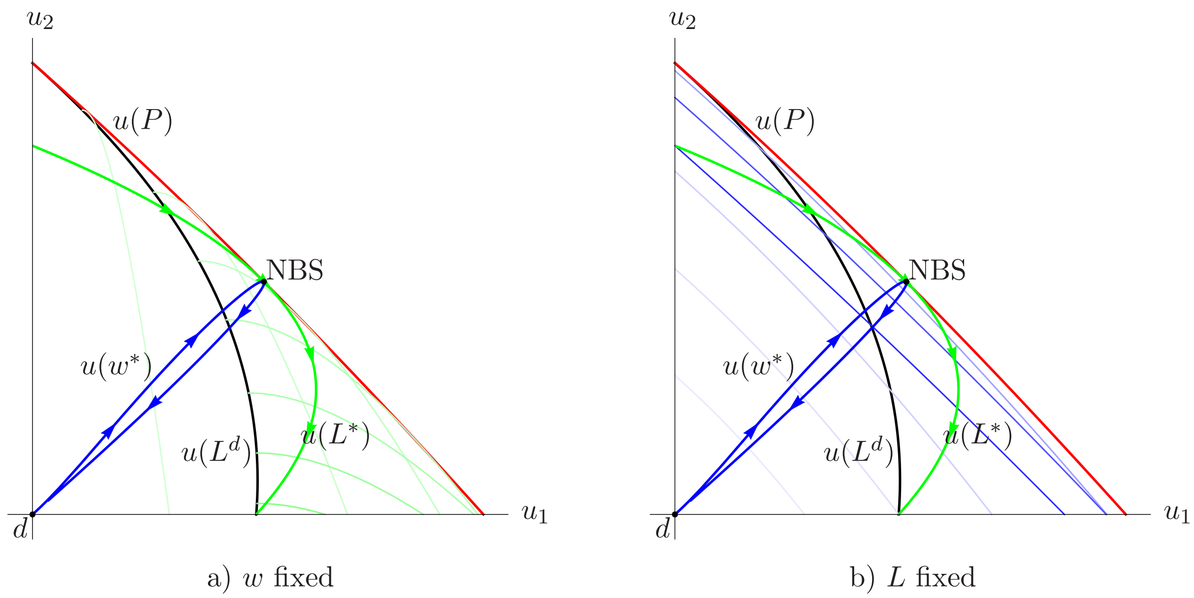


Figure 5: The image of the L–Nash curve (green) in (a) is the collection of NBS of the restricted bargaining problems with fixed w; the w–Nash curve (blue) in (b) is the collection with fixed L. The images of the Nash curves have a common intersection on the image of the Pareto curve (red) at the NBS.

(w^N, L^N) coincides with the intersection of the two Nash curves. We formulate this as a proposition and give a different proof for it using the IIA axiom.

Proposition 2 *The pair (w^N, L^N) that constitutes the Nash bargaining solution of the efficient bargaining problem (in the w–L space) coincides with the point of intersection of the two Nash curves.*

In the proof of Proposition 2 we do not make use of the fact that the two Nash curves intersect on the Pareto curve. Rather, this is a result of the Nash solution satisfying the IIA axiom. This axiom prevents the intersection of the Pareto curve with either Nash curve being different from the solution of the efficient bargaining problem.

Figure 5 illustrates the efficient bargaining problem (S^e, d) as well as restricted ones for some values of L and w. Only if L is fixed to L^N is the Nash solution $F(S^L(L^N), d)$ Pareto efficient in the efficient bargaining problem (S^e, d). In this case, the Nash solutions coincide. The same is true when we fix the wage at w^N and consider the restricted bargaining problem $(S^w(w^N), d)$. The curves denoted by $u(w^*)$ and $u(L^*)$ represent parties' utilities along the Nash curves, *i.e.*, they represent the set of Nash solutions for varying L and w, respectively. Figures 5a and 5b are the counterparts to Figure 4 in the utility space. As before, the arrows attached to the Nash curves indicate their 'directions': Along the arrows of the L–Nash curve, the wage level is increasing; along the w–Nash curve, employment is increasing.

5 Bargaining Agendas

In the efficient bargaining problem, the two parties bargain over two quantities simultaneously. Thus, there is an increase in complexity in comparison to the restricted problems, in which only the wage or the employment level is negotiated, while the other quantity is treated as fixed. In this section we investigate whether the complex bargaining problem on two quantities can be split into a sequence of alternating one-dimensional negotiations on the wage rate and the employment level. More precisely, we introduce two bargaining agendas that iteratively use restricted bargaining problems. Our question is whether the Nash solution of the efficient bargaining problem can be retrieved by iterated application of the Nash solution in restricted problems.

5.1 Two-stage bargaining agenda

In this agenda, we apply the two restricted bargaining problems in which the parties negotiate separately on one of the two quantities, discussed in Section 3, in order to model a sequential two-stage process of one-dimensional bargaining problems. In each stage, one of these restricted problems has to be negotiated. The agreement in Stage 1 fixes either quantity and thus shapes the bargaining problem at Stage 2 over the other quantity. Depending on the order in which the wage and employment are negotiated, we obtain two versions of the agenda.

Agenda Ia (first wage, then employment): In Stage 1, the players agree on a wage, w, and in Stage 2 they agree on an employment level, L. The final outcome is the agreed pair (w, L).

Agenda Ib (first employment, then wage): In Stage 1, the players agree on a wage, L, and in Stage 2 they agree on an employment level, w. The final outcome is the agreed pair (w, L).

A solution concept for this two-stage bargaining problem is certainly connected to the bargaining solution used for a single bargaining problem. For the analysis we employ a cooperative backward induction approach, meaning that it is common knowledge that any bargaining problem is solved using the same commonly accepted bargaining solution, and therefore the outcome of any subsequent bargaining problem can be anticipated. Following our discussion in the previous sections, we restrict our attention to the Nash bargaining solution here; in Section 6 we briefly discuss the differences when other solution concepts, *e.g.*, the Kalai-Smorodinsky solution, are used instead.

Consider Agenda Ia under the assumption that the Nash bargaining solution is applied throughout. Assume further that parties have negotiated on a wage level and agreed on $\tilde{w}$ in Stage 1. Then, for the negotiations on L at Stage 2, the set of feasible utility allocations is given by $S^w(\tilde{w})$. Anticipating that the Nash solution will be applied at Stage 2, the final agreement will be $(\tilde{w}, L^*(\tilde{w}))$, *i.e.*, it is located on the L–Nash curve. Phrased differently, when following Agenda Ia, the parties effectively negotiate on all (w, L) combinations on the L–Nash curve. Analogously, the bargaining problem at Stage 1 over L in Agenda Ib effectively describes negotiations on (w, L) combinations on the w–Nash curve.

Proposition 3 *Assume that in Agenda Ia and Agenda Ib players apply the Nash bargaining solution at Stage 2. Then, there is a well-defined bargaining problem in Stage 1. Application of the Nash solution in Stage 1 yields a solution on the Pareto curve. This solution coincides with the Nash solution of the efficient bargaining problem.*

Another way to look at the results in the two-stage agendas is to inspect Figure 4 and the iso-product lines, *i.e.*, the iso-level curves of the Nash product. In Agenda Ia, the final agreement on wage is reached by selecting the point on the L–Nash curve that maximizes the product of utilities. As depicted in Figure 4, the maximizer (w, L) is the intersection point of the Pareto curve and the L–Nash curve. The IIA axiom guarantees that disentangling the simultaneous negotiation on the two quantities into a two-stage game, still leads to the Nash solution of the efficient bargaining problem. This is because the Nash solution of the efficient bargaining problem is feasible in the Stage 1 bargaining problem (either in Ia or Ib).

5.2 Iterated bargaining agenda

The results in Propositions 3 come at the cost that the determination of the Stage 1 bargaining problem requires the calculation of the Nash solution for any possible $S^w(w)$ or $S^L(L)$, respectively. In order to reduce this type of complexity, we introduce a second agenda in two versions according to which the parties negotiate with infinite horizon, alternating the quantity over which they bargain. Again, we assume that all bargaining problems are solved by using the Nash bargaining solution.

Agenda IIa (iterated separate bargaining w-start): Players alternately bargain over w and L, starting with negotiations on the wage level w.

Agenda IIb (iterated separate bargaining L-start): Players alternately bargain over w and L, starting with negotiations on the employment level L.

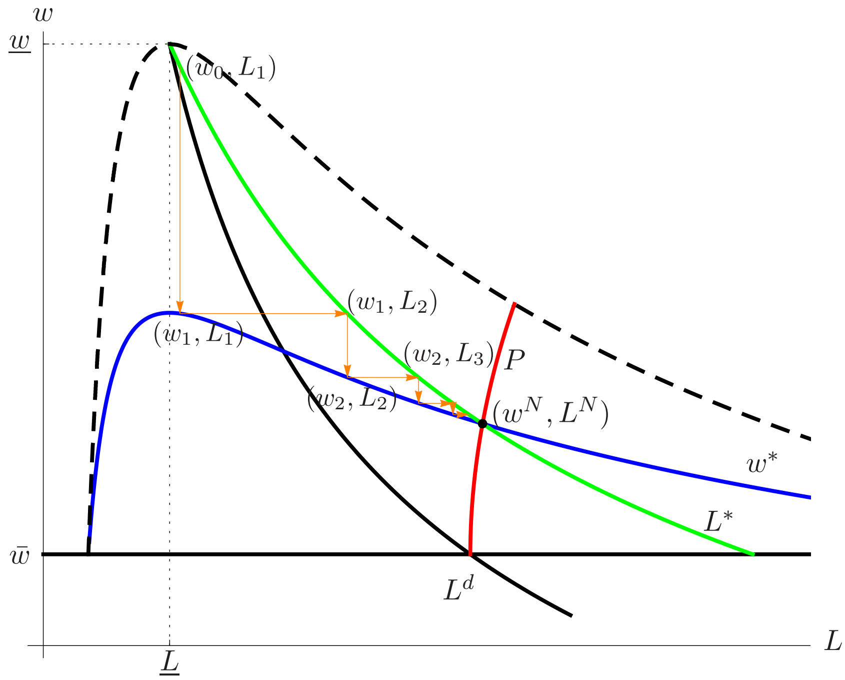


Figure 6: The arrows (orange) represent the dynamics in Agenda IIa. Starting with fixed w_0, the employment and the wage rate are alternately negotiated, which corresponds to switching between the L– and the w–Nash curve towards the limit—the Nash bargaining solution of the efficient bargaining problem.

To illustrate the idea behind Agenda IIa suppose each party sends two agents to form two distinct negotiation teams in two distinct rooms. In the *w–room*, the parties' agents only negotiate on wage, while in the *L–room* only L is negotiated. To start negotiations, fix $w_0 \in \text{Int}(\mathcal{W})$ and announce it to the L–room. Given w_0, there is an agreement on L_1 in the L–room that is reported to the w–room. Given L_1, parties negotiate on wage and agree on w_1. This is reported to the L–room, and so on. Therefore at any iteration of the process a single restricted bargaining problem needs to be solved. The questions are whether the process converges and, if so, to what limit.

Proposition 4 *Assuming that negotiations are resolved with the Nash bargaining solution, in both Agendas IIa and IIb the sequence of intermediate solutions converges to the Nash bargaining outcome of the efficient bargaining problem.*

Figure 6 illustrates the process with starting wage w_0 just below $\underline{w}$. The negotiation result L_1 in the L–room is such that (w_0, L_1) is located on the L–Nash curve. With fixed L_1, the Nash solution wage of the restricted problem $S^L(L_1)$ is w_1, so that (w_1, L_1) lies on the w–Nash curve. In general, for $t \in \mathbb{N}$ the points (w_{t-1}, L_t) are located on the L–Nash curve, while (w_t, L_t) is on the w–Nash curve. The relative position of the two Nash curves as shown in Proposition 1 guarantees the convergence of the sequence to the Nash solution of the efficient bargaining problem.

6 Other bargaining solutions

We are interested in how far our results depend on the choice of the Nash bargaining solution. To this end, we briefly review three alternative solution concepts: the egalitarian solution (Kalai, 1977), the utilitarian solution (Thomson, 1981), and the Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975). It is well-known that the former two satisfy the IIA axiom, while the latter does not.

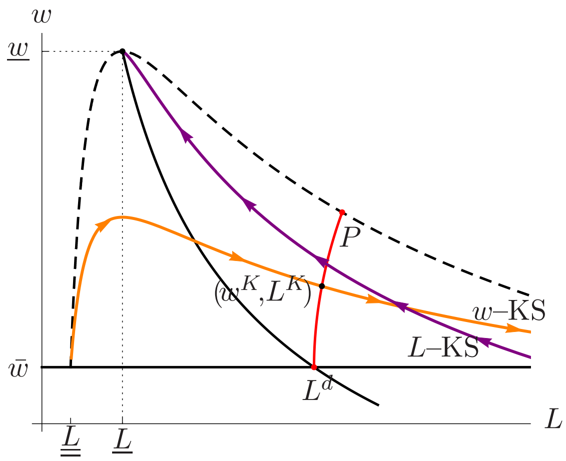
6.1 Egalitarian and utilitarian solutions

For a bargaining problem (S, d), the egalitarian solution F^E selects the weakly Pareto efficient point $F^E(S, d)$ in S, for which $F_1^E(S, d) - d_1 = F_2^E(S, d) - d_2$ holds. The utilitarian solution F^U determines the point $F^U(S, d)$ at which the sum of excess utilities (above d) is maximized. While the egalitarian solution may lead to a weakly Pareto optimal point, the utilitarian solution does not necessarily produce an individually rational result. However, both solutions satisfy the IIA axiom, so that for either solution concept the solution curves for the two one-dimensional bargaining problems intersect on the Pareto curve. Thereby, since the utilitarian solution point is independent of the status quo point, its two solution curves in the w–L space are independent of exogenous quantities such as $\bar{w}, N$, or C. It thus follows that in analogy to our findings on the Nash solution, the repeated bargaining agenda eventually leads to the solution of the efficient bargaining problem. More generally, any bargaining solution that is defined as the maximizer of a monotonic, quasi-concave function over the individual rational utility allocations satisfies Pareto efficiency, individual rationality and the IIA axiom.[8] As a consequence, the corresponding two solution curves have a common intersection on the Pareto curve, which is the solution of the efficient bargaining problem.

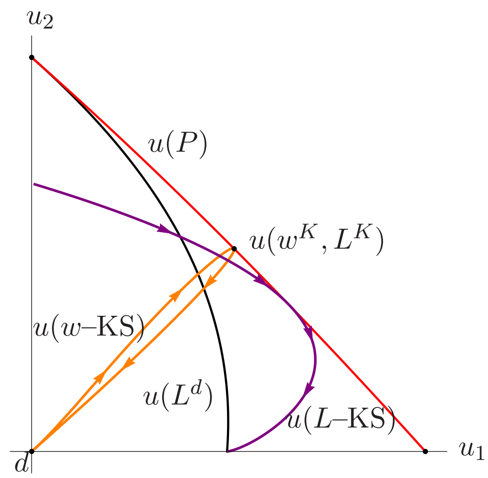
6.2 The Kalai-Smorodinsky solution

Besides the Nash bargaining solution, the solution introduced by Kalai and Smorodinsky (1975) is the second most applied solution in the labour market literature (see references

[8]Monotonicity of the function guarantees Pareto efficiency, individual rationality is trivial, when only individual rational points are under consideration, and IIA results from the fact that a maximizer $\bar{x}$ is also a maximizer over any subset that contains $\bar{x}$.



a) The KS curves in the w–L space



b) The images of the KS curves in the utility space

Figure 7: The solution curves for the KS–solution: the L–KS curve (purple) and the w–KS curve (orange) do not intersect on the Pareto curve (red).

on page 1). To determine the Kalai-Smorodinsky solution (KS solution), we first calculate the maximal possible utilities among all individually rational utility allocations. The KS solution selects the Pareto efficient point in S, where each player receives the same share of their maximal possible excess utility. The original axiomatization of the KS solution rests on the axiomatization of the Nash solution and replacing the IIA axiom by the axiom of *individual monotonicity*. We discuss the lack of IIA along Figure 7, which illustrates the two KS curves in the w–L space (Figure 7a) as well as their images in utility space (Figure 7b). For better comparability, we use the same specification of the model as in the previous sections and as in Figures 4 and 5.

The construction of the L–KS and w–KS curves follows the same route as with the Nash curves: For any fixed w, the corresponding point on the L–KS curve marks the employment level L, so that the utility allocation in (w, L) coincides with the KS solution of the restricted bargaining problem $S^w(w)$, and analogously for the w–KS curve. Figure 7a shows that the relative locations of the L–KS and w–KS curves are as in Proposition 1 for the Nash curves. However, although the two KS curves are qualitatively similar to the Nash curves, the main difference is that the intersection point of the two KS curves in the w–L space is not located on the Pareto curve. Moreover, the utility allocation of the KS solution of the efficient bargaining problem, $u(w^K, L^K)$, is not located on the image of either the w–KS or the L–KS curve. That is, even if the wage (resp. employment) level were fixed to w^K (resp. L^K), the negotiated employment level L (resp. wage level w) in the restricted bargaining problem

$S^w(w^K)$ (resp. $S^L(L^K)$) will not be equal to L^K (resp. w^K), so that the outcome is inefficient.[9] The immediate consequence is that Propositions 3 and 4 do not hold for the KS solution. In particular, Agendas Ia and Ib typically lead to two different inefficient outcomes on the L–KS curve and w–KS curve, respectively. The final outcome of the iterated bargaining agenda is the intersection point of the two KS curves and is therefore not efficient.

Figure 7b displays the situation in utility space. There are two intersection points of the images of KS curves in the utility space. The left intersection point stems from the intersection of the two curves in the w–L space, while the right one is the image of two distinct (w, L) combinations yielding the same utilities, but one is located on the L–KS curve and the other one is located on the w–KS curve. In particular, the former intersection point is the result of the repeated bargaining agenda in utilities and is apparently not Pareto efficient. These considerations again highlight the fundamental significance of the IIA axiom for the results in Propositions 1–4.

7 Conclusion

We present a rigorous analysis of the wage bargaining model first introduced by McDonald and Solow (1981). We formulate the bargaining problem not only within the space of physical outcomes (w, L), but also in the utility space, so that an application of axiomatic bargaining solutions such as the Nash bargaining solution (NBS) is possible. A decomposition of the efficient bargaining problem over the wage and employment level into two (families of) one-dimensional bargaining problems results in the construction of two curves to which we refer as the w–Nash curve and the L–Nash curve. While the former is disregarded in the literature, the prevalent interpretation of the latter is merely technical as parametrised solutions of a first-order condition. However, both of these curves are equipped with an economic interpretation: Each collects outcomes of the Nash bargaining solution applied to restricted (one-dimensional) problems, or represents the generalized bargaining solution of a parametrised one-dimensional bargaining problem.

A stringent analysis of the curves characterizes their (relative) positions in the w–L space. The unique intersection point satisfies stability in the sense that either quantity at the intersection point marks the NBS outcome of the restricted bargaining problem, in which the other quantity is fixed to the level at the intersection and vice versa. It is the strength

[9] Although in Figure 7 it seems that the solution does (w^K, L^K) lie on the w–KS curve, this is not true; only if the Pareto curve is vertical lies (w^K, L^K) on the w–KS curve. (This is also confirmed by numerical calculations.)

of the IIA axiom (and not just bare calculus) that forces the intersection point to coincide with the NBS outcome of the efficient bargaining problem.

The knowledge about structural properties of the two Nash curves leads us to design two dynamic bargaining agendas, according to which either the wage rate or the employment level is negotiated at each stage. Independent of whether each quantity is negotiated by prescient parties once, or quantities are negotiated alternately by myopic agents (with infinite horizon), the final outcome coincides with the intersection point of the two Nash curves—and, as a consequence, is Pareto efficient. Again, the driving force for these results is the IIA axiom, constituting the special features of the NBS.

Although our analysis heavily relies on the NBS, our approach is not limited to this solution concept. Any axiomatic bargaining solution, as is usually defined in utility space, can be transferred to the w–L space in order to (i) mark the solution's outcome itself and (ii) construct two solution curves with the same interpretation as used for the NBS above. However, it is the set of axioms of a bargaining solution that determines to what extent our results hold for a specific solution concept. In particular, the KS solution fails to satisfy the IIA axiom and therefore the common intersection point of the two KS curves is not Pareto efficient, and does not coincide with the KS solution of the efficient bargaining problem.

We consider the following three points as the main insights from our analysis: First, the IIA axiom implants more structure into the NBS than was uncovered in the literature, which enables us to decompose the wage bargaining problem into families of one-dimensional bargaining problems. Second, one has to be careful when applying a particular bargaining solution, since the varying characterizing axioms possibly trigger unwanted results. What leads to an efficient outcome for one solution can cause inefficiencies with another solution. Finally, our analysis gives rise to further investigation of the NBS in other application contexts. The decomposability does not appear to hinge on the fact that parties bargain over two quantities. Instead, when more than two different quantities are subject to negotiation, *e.g.*, when buyer and seller may agree on different terms of trade, a decomposition into restricted bargaining problems over one or simply fewer dimensions seems to be possible and complexity reducing. To what degree our findings hold in such more general models is an interesting open question.

Acknowledgement. This work was partially supported by the German Research Foundation (DFG) within the Collabourative Research Center "On-The-Fly Computing" (SFB 901). HIFMB is a collabouration between the Alfred-Wegener-Institute, Helmholtz-Center for Polar and Marine Research, and the Carl-von-Ossietzky University Oldenburg, initially funded by the Ministry for Science and Culture of Lower Saxony and the Volkswagen Foundation through the "Niedersächsisches Vorab" grant program (grant number ZN3285).

References

Toke S. Aidt and Zafiris Tzannatos. Trade Unions, Collective Bargaining and Macroeconomic Performance: A Review. *Industrial Relations Journal*, 39(4):258–295, 2008. doi: 10.1111/j.1468-2338.2008.00488.x.

James Albrecht and Susan Vromen. A Match Model with Endogenous Skill Requirements. *International Economic Review*, 43(1):283–305, 2002. doi: 10.1111/1468-2354.t01-1-00012.

C. O. Alexander and W. Ledermann. The Constrained Nash Bargaining Solution. *Journal of the Operational Research Society*, 45(8):954–958, 1994. doi: 10.1057/jors.1994.148.

C. O. Alexander and W. Ledermann. Are Nash Bargaining Wage Agreements Unique? An Investigation into Bargaining Sets for Firm-Union Negotiations. *Oxford Economic Papers*, 48(2):232–253, 1996. doi: 10.1093/oxfordjournals.oep.a028567.

Carol Alexander. The Kalai-Smorodinsky Bargaining Solution in Wage Negotiations. *Journal of the Operational Research Society*, 43(8):779–786, 1992. doi: 10.1057/jors.1992.116.

Samir Amine, Pedro Lages dos Santos, Sylvain Baumann, and Fabrice Valognes. Revisiting Nash Wages Negotiations in Matching Models. *Economics Bulletin*, 29(4):3203–3213, 2009.

Samir Amine, Sylvain Baumann, Pedro Lages Dos Santos, et al. Bargaining Solutions and Public Policies in Matching Models. *Economic Studies Journal*, 27(4):3–14, 2018.

Thorsten Bayındır-Upmann and Matthias G. Raith. Should High-Tax Countries Pursue Revenue-Neutral Ecological Tax Reforms? *European Economic Review*, 47(1):41–60, 2003. doi: 10.1016/S0014-2921(01)00193-3.

Thorsten Bayındır-Upmann and Matthias G. Raith. Unemployment and Pollution: Is One Policy Suited for Two Problems? *The Economic Record*, 81(255):378–393, 2005. doi: 10.1111/j.1475-4932.2005.00276.x.

Pascal Belan, Martine Carre, and Stephane Gregoir. Subsidizing Low-Skilled Jobs in a Dual Labor Market. *Labour Economics*, 17(5):776–788, 2010. doi: 10.1016/j.labeco.2010.04.003.

Ken Binmore, Ariel Rubinstein, and Asher Wolinsky. The Nash Bargaining Solution in Economic Modelling. *Rand Journal of Economics*, 17(2):176–188, 1986. doi: 10.2307/2555382.

Tito Boeri and Michael C. Burda. Preferences for Collective versus Individualised Wage Setting. *Economic Journal*, 119(540):1440–1463, 2009. doi: 10.1111/j.1468-0297.2009.02286.x.

Jacques Bughin. Trade Unions and Firms' Product Market Power. *Journal of Industrial Economics*, 44(3):289–307, 1996. doi: 10.2307/2950498.

Jacques Bughin. The Strategic Choice of Union-Oligopoly Bargaining Agenda. *International Journal of Industrial Organization*, 17(7):1029–1040, 1999. doi: 10.1016/S0167-7187(97)00071-4.

Pierre Cahuc, Stéhane Carcillo, and Andr+ Zylberberg. *Labor Economics*. MIT Press, Cambridge, London, 2 edition, 2014. ISBN 978-0-262-02770-0.

Andrew Clark. Efficient Bargains and the McDonald-Solow Conjecture. *Journal of Labor Economics*, 8(4):502–528, 1990. doi: 10.1086/298232.

Andrea Colciago and Lorenza Rossi. Firm Dynamics, Endogenous Markups, and the Labor Share of Income. *Macroeconomic Dynamics*, 19(6):1309–1331, 2015. doi: 10.1017/S1365100513000849.

Anthony Creane and Carl Davidson. The Trade-Offs from Pattern Bargaining with Uncertain Production Costs. *European Economic Review*, 55(2):246–262, 2011. doi: 10.1016/j.euroecorev.2010.05.001.

John Creedy and Ian M. McDonald. Models of Trade Union Behaviour: A Synthesis. *The Economic Record*, 67(4):346–359, 1991. doi: 10.1111/j.1475-4932.1991.tb02564.x.

Marcus Dittrich. Minimum Wages and Unemployment Benefits in a Unionised Economy: A Game-Theoretic Approach. *Annals of Economics and Finance*, 11(2):209–229, 2010.

Marcus Dittrich and Andreas Knabe. Spillover Effects of Minimum Wages under Union Wage Bargaining. *Journal of Institutional and Theoretical Economics*, 169(3):506–518, 2013. doi: 10.1628/093245613X667468.

Paul W. Dobson. Multifirm Unions and the Incentive to Adopt Pattern Bargaining in Oligopoly. *European Economic Review*, 38(1):87–100, 1994. doi: 10.1016/0014-2921(94)90007-8.

Steve Dowrick. Union-Oligopoly Bargaining. *Economic Journal*, 99(398):1123–1142, 1989. doi: 10.2307/2234092.

Steve Dowrick. The Relative Profitability of Nash Bargaining on the Labour Demand Curve or the Contract Curve. *Economics Letters*, 33(2):121–125, 1990. doi: 10.1016/0165-1765(90)90156-U.

Amitava Krishna Dutt and Anindya Sen. Union Bargaining Power, Employment and Output in a Model of Monopolistic Competition with Wage Bargaining. *Journal of Economics (Zeitschrift für Nationalökonomie)*, 65(1):1–17, 1997. doi: 10.1007/BF01239056.

Thomas Eichner and Thorsten Upmann. Labour Markets and Capital Tax Competition. *International Tax and Public Finance*, 19:203–215, 2012. doi: 10.1007/s10797-011-9180-1.

Thomas Eichner and Thorsten Upmann. The (Im)Possibility of Overprovision of Public Goods in Interjurisdicitional Tax Competition. *FinanzArchiv / Public Finance Analysis*, 70(2):218–248, 2014. doi: 10.1628/001522114X681360.

Luciano Fanti and Luca Gori. Efficient Bargaining versus Right to Manage: A Stability Analysis in a Cournot Duopoly with Trade Unions. *Economic Modelling*, 30:205–211, 2013. doi: 10.1016/j.econmod.2012.09.010.

Christopher J. Flinn. Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates. *Econometrica*, 74(4):1013–1062, 2006. doi: 10.1111/j.1468-0262.2006.00693.x.

Anke Gerber and Thorsten Upmann. Bargaining Solutions at Work: Qualitative Differences in Policy Implications. *Mathematical Social Sciences*, 52(2):162–175, 2006. doi: 10.1016/j.mathsocsci.2006.06.002.

Mark Gertler and Antonella Trigari. Unemployment Fluctuations with Staggered Nash Wage Bargaining. *Journal of Political Economy*, 117(1):38–86, 2009. doi: 10.1086/597302.

Thomas Grandner. Unions in Oligopolistic, Vertically Connected Industries. *European Economic Review*, 45(9):1723–1740, 2001. doi: 10.1016/S0014-2921(99)00075-6.

Henrik Horn and Lars E. O. Svensson. Trade Unions and Optimal Labour Contracts. *Economic Journal*, 96(382):323–341, 1986. doi: 10.2307/2233119.

Laurence Jacquet, Etienne Lehmann, and Bruno Van der Linden. Optimal Income Taxation with Kalai Wage Bargaining and Endogenous Participation. *Social Choice and Welfare*, 42(2):381–402, 2014. doi: 10.1007/s00355-013-0736-0.

Ehud Kalai. Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons. *Econometrica*, 45(7):1623–1630, 1977. doi: 10.2307/1913954.

Ehud Kalai and Meir Smorodinsky. Other Solutions to Nash's Bargaining Problem. *Econometrica*, 43(3):513–518, 1975. doi: 10.2307/1914280.

Kornelius Kraft. Wage versus Efficient Bargaining in Oligopoly. *Managerial and Decision Economics*, 27(7):595–604, 2006. doi: 10.1002/mde.1277.

Per Krusell, Toshihiko Mukoyama, and Aysegul Sahin. Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations. *Review of Economic Studies*, 77(4):1477–1507, 2010. doi: 10.1111/j.1467-937X.2010.00700.x.

Nicholas P. Lawson. Is Collective Bargaining Pareto Efficient? A Survey of the Literature. *Journal of Labor Research*, 32(3):282–304, 2011. doi: 10.1007/s12122-011-9112-y.

Olivier l'Haridon, Franck Malherbet, and Sebastien Perez-Duarte. Does Bargaining Matter in the Small Firms Matching Model? *Labour Economics*, 21:42–58, 2013. doi: 10.1016/j.labeco.2012.12.002.

Dehai Liu, Wei Lv, Hongyi Li, and Jiafu Tang. Bargaining model of labor disputes considering social mediation and bounded rationality. *European Journal of Operational Research*, 262(3):1064–1071, 2017. doi: 10.1016/j.ejor.2017.04.003.

Ian M. McDonald and Robert M. Solow. Wage Bargaining and Employment. *American Economic Review*, 71(5):896–908, 1981.

Julia Müller and Thorsten Upmann. Centralised Labour Market Negotiations: Strategic Behaviour Curbs Employment. *Journal of Institutional and Theoretical Economics JITE*, 174(2):278–302, 2018. doi: 10.1628/093245617X14930170168706.

John F. Nash. The Bargaining Problem. *Econometrica*, 18:155–162, 1950. doi: 10.2307/1907266.

Andrew J. Oswald. The Economic Theory of Trade Unions: An Introductory Survey. *Scandinavian Journal of Economics*, 87(2):160–193, 1985. doi: 10.2307/3439820.

Maria Paz Espinosa and Changyong Rhee. Efficient Wage Bargaining as a Repeated Game. *Quarterly Journal of Economics*, 104(3):565–588, 1989. doi: 10.2307/2937811.

Emmanuel Petrakis and Minas Vlassis. Endogenous Scope of Bargaining in a Union-Oligopoly Model: When Will Firms and Unions Bargain Over Employment? *Labour Economics*, 7(3):261–281, 2000. doi: 10.1016/S0927-5371(99)00043-3.

Priya Ranjan. Offshoring, Unemployment, and Wages: The Role of Labor Market Institutions. *Journal of International Economics*, 89(1):172–186, 2013. doi: 10.1016/j.jinteco.2012.06.003.

Ariel Rubinstein. Perfect Equilibrium in a Bargaining Model. *Econometrica*, 50(1):97–110, 1982. doi: 10.2307/1912531.

Michele Santoni. Product Market Integration and Wage Bargaining Institutions. *Labour Economics*, 27:1–15, 2014. doi: 10.1016/j.labeco.2013.12.004.

Jon Strand. Wage Bargaining and Turnover Costs with Heterogenous Labor and Perfect History Screening. *European Economic Review*, 46(7):1209–1227, 2002. doi: 10.1016/S0014-2921(01)00089-7.

Jan Svejnar. Bargaining Power, Fear of Disagreement, and Wage Settlements: Theory and Evidence from U.S. Industry. *Econometrica*, 54(5):1055–1078, 1986. doi: 10.2307/1912322.

William Thomson. Nash's bargaining solution and utilitarian choice rules. *Econometrica*, pages 535–538, 1981. doi: 10.2307/1913329.

Thorsten Upmann. A Positive Analysis of Labor-Market Institutions and Tax Reforms. *International Tax and Public Finance*, 16:621–646, 2009. doi: 10.1007/s10797-008-9079-7.

Thorsten Upmann and Julia Müller. The Structure of Firm-Specific Labour Unions. *Journal of Institutional and Theoretical Economics*, 170(2):336–364, 2014. doi: 10.1628/093245614X13941110635976.

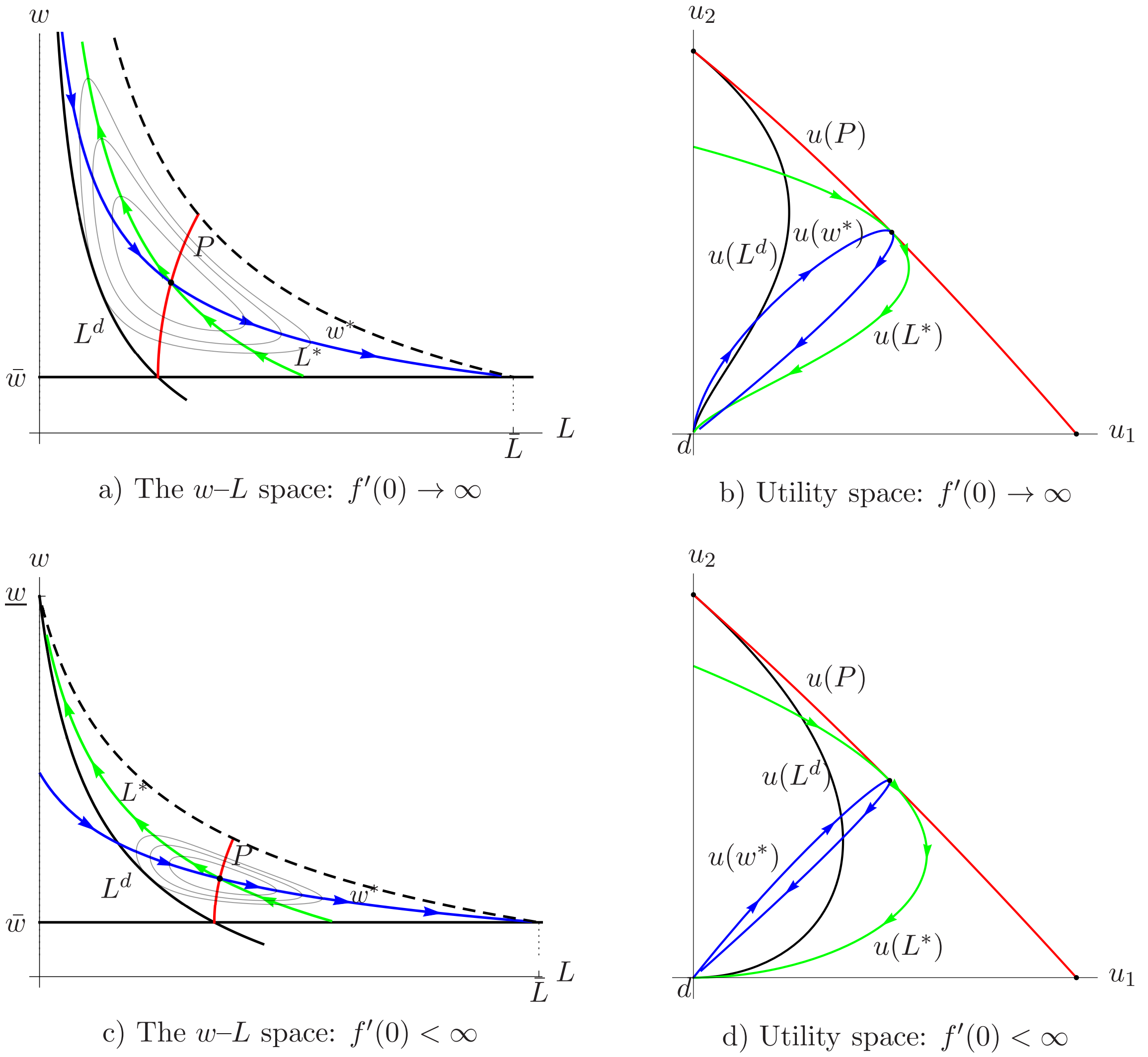
Frank Walsh. Efficiency Wages and Bargaining. *Oxford Economic Papers*, 64(4):635–654, 2012. doi: 10.1093/oep/gpr060.

Sven Wehke. Union wages, hours of work and the effectiveness of partial coordination agreements. *Labour Economics*, 16(1):89–96, 2009. doi: 10.1016/j.labeco.2008.06.001.

Laixun Zhao. Cross-hauling direct foreign investment and unionized oligopoly. *European Economic Review*, 39(6):1237–1253, 1995. doi: 10.1016/0014-2921(94)00038-2.

Appendix A Zero setup cost

We drop the assumption that the firm has to incur setup costs for production, and set $C = 0$, so that the firm's profit function reduces to $u_2(w, L) = f(L) - wL$. Then, we need to distinguish two cases whether f satisfies an Inada condition or not, *i.e.*, whether $f'(0)$ is infinite or finite. Figure 8 illustrates the analogous versions of Figures 1 and 5 for the cases $f'(0) \to \infty$ and $f'(0) < \infty$. Since removing setup costs is accomplished via a monotone transformation of the profit function, the shapes of iso-profit lines do not change



a) The w–L space: $f'(0) \to \infty$

b) Utility space: $f'(0) \to \infty$

c) The w–L space: $f'(0) < \infty$

d) Utility space: $f'(0) < \infty$

Figure 8: Adaptation of Figures 4 and 5 to the case of zero setup costs. In (a) and (b), the Inada condition holds; in (c) and (d), it does not. When $f'(0) \to \infty$, then $\underline{L} \to 0$; else $\underline{L} = 0$.

and therefore the Pareto curve is not affected by setting $C = 0$. It is straightforward that here $\underline{L} = 0$ and that $\underline{w}$ is no longer finite. Consequently, the L–Nash curve, the zero profit line and the labour demand curve go to infinity, when L approaches to $\underline{L} = 0$. By Section 4.2, we know that $w^*(L) = w^0(L)\sigma(w^*(L))/(1+\sigma(w^*(L))) < w^0(L)$ for any employment level L, so that as L approaches 0, the w–Nash curve also tends to infinity. As we see from Figure 8a, the result obtained in the previous sections is still valid: The intersection of the two Nash curves lies on the Pareto curve, and thus represents the Nash solution of efficient bargaining problem. The argumentation via the IIA remains valid.

Figure 8c represents the case in which the marginal product at $L = 0$ is finite. The zero profit line and labour demand curve have a common intersection on the wage axis at

$\underline{w} = f'(0)$. Also, the L–Nash curve intersects with these two curves at $\underline{w}$. In case of the w–Nash curve, when L goes to $\underline{L} = 0$, the w–Nash curve approaches $(0, w)$ with a wage level between $\bar{w}$ and $\underline{w}$ as before. The two Nash curves have exactly one common point, which lies on the Pareto curve and coincides with the Nash solution of the efficient bargaining problem. Hence, all of our results obtained in Sections 2–5 continue to hold in the case of vanishing setup costs. In both cases, *i.e.*, for a finite or an infinite marginal product $f'(0)$, we can retrieve the results from Lemma 5, Corollary 1, and Proposition 1 on monotonicity of the Nash curves and their relative position. As a result, Proposition 4 holds so that the final outcome in the iterated bargaining Agendas IIa and IIb is the Nash bargaining outcome.

Finally, the results for Agendas Ia and Ib, *i.e.*, Proposition 3, remain valid. This can be verified in Figures 8b and 8d. The bargaining problem in Agendas Ia and Ib are formed by the L–Nash curve and the w–Nash curve, respectively. By IIA, the Nash solution of the efficient bargaining problem coincides with the Nash solution in each Stage 1 bargaining problem, which is assumed to be the final outcome in both agendas. that the assumption of positive setup costs in previous sections is convenient as it does not necessitate explicit consideration of an Inada condition; yet, our results are independent of this assumption.

Appendix B Proofs

Proof of Lemma 1. Differentiation of σ yields

$$\sigma'(w) = \frac{(v(w)-\bar{v})\,(wv''(w)+v'(w)) - (v'(w))^2\, w}{(v(w)-\bar{v})^2} = \frac{v'(w)\,(\varepsilon(w) - \sigma(w) + 1)}{v(w) - \bar{v}}, \tag{B.1}$$

from which the result follows immediately, as $v' > 0$ and $w > \bar{w}$ by hypothesis.

Proof of Lemma 2. Since $(w - f'(L))/w < 1$, it follows from eq. (5) that for any Pareto efficient allocation (w, L) we must have $\sigma(w) > 1$.

Proof of Lemma 3. Straightforward calculations show

$$P'(L) = \left.\frac{dw}{dL}\right|_{\Phi^P=0} = -\frac{\frac{\partial \Phi^P(w,L)}{\partial L}}{\frac{\partial \Phi^P(w,L)}{\partial w}} = \frac{wf''(L)}{\sigma'(w)\,(w-f'(L))^2 + f'(L)} = \frac{\sigma(w) f''(L)}{\varepsilon(w)} > 0,$$

where in the last equation we used eqs. (B.1) and (5) along with Lemma 1 and Lemma 2. Hence, P has positive slope, unless it is vertical, which happens if $v'' = 0$. Finally, since

$$\lim_{w \searrow \bar{w}} \sigma(w) = +\infty \quad \Rightarrow \quad \lim_{w \searrow \bar{w}} \frac{\sigma(w) f''(L)}{\varepsilon(w)} = \frac{f''(L^c)}{\varepsilon(\bar{w})} \lim_{w \searrow \bar{w}} \sigma(w) = +\infty,$$

showing that the slope of the Pareto curve is infinite at $(w^c, L^c) = (\bar{w}, L^d(\bar{w}))$.

Proof of Lemma 4.

(i) (S^L, d) *is well-defined for fixed* L. Comprehensiveness of S^L follows from the definition. To show convexity, we show that the slope of the Pareto boundary of S^L decreases with increasing utility of the trade union. The derivatives

$$\frac{\partial u_1(w,L)}{\partial w} = Lv'(w) > 0, \qquad \frac{\partial u_2(w,L)}{\partial w} = -L < 0,$$

imply that for each w the utility allocation $u(w,L)$ is strictly Pareto optimal. The slope of the Pareto boundary at $u(w,L)$ is given by

$$\frac{du_2}{du_1} = \frac{\frac{\partial u_2(w,L)}{\partial w}}{\frac{\partial u_1(w,L)}{\partial w}} = \frac{-1}{v'(w)} < 0,$$

which is decreasing in w, as $v'' < 0$. Therefore, the function describing the strict Pareto boundary of S^L is concave, hence S^L is convex.

The closedness of S^L is a consequence of the continuity of the utility functions. Boundedness follows from the fact that $u_2(w,L)$ is strictly decreasing in w with $\lim_{w\to\underline{w}} u_2(w,L) \leq 0$. Finally, the disagreement point $d = (N\bar{v}, 0)$ is Pareto dominated by $u(\bar{w}, L) \in S$, since $u_1(\bar{w}, L) = N\bar{v}$ and $u_2(\bar{w}, L) \geq 0$. With comprehensiveness, $d \in S^L$.

(ii) (S^w, d) *is well-defined for fixed* w. S^w is closed, comprehensive and the individually rational set is bounded with similar arguments as above.

$$\frac{\partial u_1(w,L)}{\partial L} = v(w) - \bar{v} > 0, \qquad \frac{\partial u_2(w,L)}{\partial L} = f'(L) - w,$$

mean that for $L < f'^{-1}(w) = L^d(w)$ the payoff allocation $u(w,L)$ is not Pareto efficient as an increase in L increases both parties' utilities. Hence, the Pareto boundary of S^w is generated by all L such that $f'(L) < w$. Its slope at $u(w,L)$ is

$$\frac{du_2}{du_1} = \frac{\frac{\partial u_2(w,L)}{\partial L}}{\frac{\partial u_1(w,L)}{\partial L}} = \frac{f'(L) - w}{v(w) - \bar{v}},$$

which is negative for $L > L^d(w)$. Differentiating the last equation w.r.t. L yields $\frac{f''(L)}{v(w)-\bar{v}} < 0$, showing convexity of S^w. Any wage $w > \bar{w}$ guarantees a non-negative utility to the workers; while for $w \leq \underline{w}$, the firm obtains a non-negative profit for all $L \geq \underline{L}$ with $f'(L) = w$. Therefore, the disagreement point d is Pareto dominated and, by comprehensiveness from the definition, it belongs to S^w.

Proof of Lemma 5. Let $M(L)$ denote the maximand in (7). We first show that for each fixed $w > \bar{w}$, $M(L)$ is strictly quasi-concave and therefore admits a unique maximizer. It is sufficient to show strict quasi-concavity of the function Π with

$$\Pi(L) := L(f(L) - w(L) - C) = Lu_2(w, L),$$

since M and Π only differ in the constant factor $v(w) - \bar{v}$.

Next we show that for $L_1, L_2 \in \mathcal{L}$ with $L_1 \neq L_2$ and $\lambda \in (0, 1)$ we have $\Pi(L_\lambda) > \Pi_{min} := \min(\Pi(L_1), \Pi(L_2))$, where $L_\lambda := \lambda L_1 + (1 - \lambda)L_2$. Due to concavity of f, we get

$$\begin{aligned}
\Pi(L_\lambda) &= L_\lambda(f(L_\lambda) - wL_\lambda - C) \\
&\geq L_\lambda(\lambda f(L_1) + (1-\lambda)f(L_2) - wL_\lambda - C) \\
&= L_\lambda(\lambda u_2(w, L_1) + (1-\lambda)u_2(w, L_2)) \\
&= (\lambda L_1 + (1-\lambda)L_2)(\lambda u_2(w, L_1) + (1-\lambda)u_2(w, L_2)) \\
&= \lambda^2 \Pi(L_1) + (1-\lambda)^2 \Pi(L_2) + \lambda(1-\lambda)\left(L_1 u_2(w, L_2) + L_2 u_2(w, L_1)\right) \\
&= \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1}\right)\Pi(L_1) + \left((1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi(L_2) \\
&\geq \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1}\right)\Pi_{min} + \left((1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi_{min} \\
&= \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1} + (1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi_{min} \\
&= \left(1 + (\lambda - \lambda^2)\frac{(L_1 - L_2)^2}{L_1 L_2}\right)\Pi_{min} > \Pi_{min}.
\end{aligned}$$

Thus, Π is a strictly quasi-concave function. This implies the quasi-concavity of M, so that M has unique maximizer. Inserting the specific forms of u_1, u_2, d_1, and d_2, the first-order condition $M'(L) = 0$ is thus sufficient to have the functional form of the L–Nash curve.

$M'(L) = (v(w) - \bar{v})(f(L) - 2wL - C + Lf'(L)) = 0$ implies $w = \frac{1}{2}\left(\frac{f(L)-C}{L} + f'(L)\right)$. To show that the L–Nash curve is downward sloping, we need $w'(L) < 0$:

$$w'(L) = \frac{1}{2L^2}\Big(\underbrace{L^2 f''(L)}_{<0} - \underbrace{[f(L) - f'(L)L - C]}_{\geq 0 \text{ for } L \geq \underline{L}}\Big) < 0.$$

Finally, if $\underline{w}$ is finite, then as w converges to $\underline{w}$ the individually rational part of $S^w(w)$ shrinks to the disagreement point. By definition, $\underline{L}$ is the employment level, and so $u(\underline{w}, \underline{L}) = d$. Hence, $(\underline{w}, \underline{L})$ is the starting point of the L–Nash curve.

Proof of Lemma 6. We first show that the maximand in (9) is strictly concave, yielding a unique maximizer. Inserting the specific forms of u_1, u_2, d_1, and d_2, we view the product as a function M in w:

$$(u_1(w,L)-d_1)\,(u_2(w,L)-d_2) = L(v(w)-\bar{v})(f(L)-wL-C) =: M(w).$$

with second derivative $M''(w) = Lv''(w)u_2(w,L) - 2L^2v'(w) < 0$. The first-order condition $M'(w) = 0$ is thus sufficient and readily reads $\sigma(w) - \frac{wL}{u_2(w,L)} = 0$, so that the w–Nash curve is implicitly defined. Using the implicit function theorem, the slope of the w–Nash curve is given by

$$\frac{dw^*(L)}{dL} = \frac{dw}{dL}\Big|_{\Phi^w(w,L)=0} = -\frac{\frac{\partial\Phi^w(w,L)}{\partial L}}{\frac{\partial\Phi^w(w,L)}{\partial w}} = \frac{w\,(f(L)-C-Lf'(L))}{\sigma'(w)(f(L)-Lw-C)^2 - L(f(L)-C)}.$$

Due to Assumption 1, the numerator is positive. The denominator is negative if $\sigma'(w) \leq 0$ holds. Thus, the w–Nash curve is downward sloping in the neighbourhood of the Pareto curve. Finally, $w^*(L)$ at fixed L has to be located between $\bar{w}$ (zero excess utility for the trade union) and $(f(L)-C)/L)$ (zero profit wage). By definition of $\bar{L}$, $\lim_{L\to\bar{L}} w^0(L) = \bar{w}$.

Proof of Proposition 1. We start with two considerations on the relative positions of starting points of the two curves. By Lemma 5, the L–Nash curve is located between the labour demand curve and the zero profit line, which have a common point in $(\underline{w}, \underline{L})$. Therefore, as L approaches to $\underline{L}$, the corresponding points on the L–Nash curve yield arbitrarily small profits for the firm. However, along the w–Nash curve, profits cannot approach zero for L close to $\underline{L}$, since the wage w is chosen so as to maximize the product of the parties' excess utilities. As a consequence, the w–Nash curve must start below the L–Nash curve at $L = \underline{L}$.

Analogously, for w approaching to $\bar{w}$ the corresponding L is such that (w, L) on the w–curve approaches to $\bar{L}$. Recall that both parties utilities are zero in $(\bar{w}, \bar{L})$. It follows that the endpoint of the L–Nash curve at $\bar{w}$ is given at some employment level $L \leq \bar{L}$. These observations on the starting points and the endpoints together with the monotonicity of both curves (see Lemma 1 and Lemma 5) show that there must be at least one intersection point of the two curves.

Inserting the closed form for the L–Nash curve (8) into eq. (10) immediately yields the condition for Pareto efficient agreements (5). This shows that *any* intersection point must be located on the Pareto curve. By Lemma 3 the Pareto curve is upward sloping, showing that it intersects each Nash curve at most once. Hence, the two Nash curves have a unique intersection, which is located on the Pareto curve. Again, using the location of starting points and endpoints shows (iii).

Proof of Proposition 2. Since the Nash solution is Pareto efficient, (w^N, L^N) must be located on the Pareto curve. As noted above, comparing the efficient bargaining problem with the restricted ones, in which wage or employment is fixed to w^N or L^N, respectively, gives $S^w(w^N) \subseteq S^e$ and $S^L(L^N) \subseteq S^e$. (w^N, L^N) is among the possible agreements from which the bargaining problems $(S^L(L^N), d)$ and $(S^w(w^N), d)$, respectively, are generated. Therefore, $F(S^e, d) = u(w^N, L^N)$ is included in $S^L(L^N)$ and $S^w(w^N)$. Using $S^w(w^N) \subseteq S^e$, $S^L(L^N) \subseteq S^e$ and the IIA axiom for the Nash solution, we obtain $F(S^L(L^N), d) = F(S^e, d^e) = u(w^N, L^N)$ and $F(S^w(w^N), d) = F(S^e, d^e) = u(w^N, L^N)$. It follows by definition of the Nash curves that the pair (w^N, L^N) must be located on both Nash curves.

Proof of Proposition 3. By the construction of two-stage bargaining, the bargaining problem at the first stage is (S, d) with $S = \{x \in \mathbb{R}^2 \,|\, x \leq (u_1(w, L^*(w)), u_2(w, L^*(w))), w \in \mathcal{W}\}$ and $d = (N\bar{v}, 0)$, which is well-defined by Lemma 4. Since the Nash solution is Pareto efficient, the solution point $(w, L^*(w))$ is on the Pareto curve. By the application of the Nash solution in the first stage along with IIA, we have $u(w, L^*(w)) = F(S, d) = F(S^e, d) = u(w^N, L^N)$. Thus, $(w, L^*(w))$ and (w^N, L^N) coincide.

An adaptation of the proof to Agenda Ib is straightforward, so that we omit it. Which variable the negotiation starts with does not have an effect on the result, *i.e.*, the result is independent of the order of negotiation.

Proof of Proposition 4. We show the proposition for Agenda IIa. Fix $w_0 \in \text{Int}(\mathcal{W})$. Let L_1 be such that $a^1 := (w_0, L_1)$ satisfies (8), i.e., (w_0, L_1) is located on the L-Nash curve. Then compute w_1 such that $a^2 := (w_1, L_1)$ satisfies (10), i.e., (w_1, L_1) is located on the w–Nash curve. Proceeding in this way, we construct sequences $(w_t)_t, (L_t)_t$ of wages and employment levels as well as the sequence $(a^k)_k$ of wage/employment combinations such that $a^k = (w_{k/2}, L_{k/2})$, when k is even and $a^k = (w_{(k-1)/2}, L_{(k+1)/2})$ when k is odd. This means that for even k, a^k is located on the w–Nash curve, while a^k lies on the L–Nash curve for odd k. Using Proposition 1, the sequences $(w_t)_t$ and $(L_t)_t$ are monotonic (increasing or decreasing, depending on w_0) and are bounded by w^N and L^N, respectively. It follows that both converge to, say, $\tilde{w}$ and $\tilde{L}$, respectively. Then, a^k is a Cauchy sequence and $(a^k - a^{k+1})_k$ converges to $(\tilde{w}, \tilde{L}) - (\tilde{w}, \tilde{L}) = 0$, which implies that $(\tilde{w}, \tilde{L})$ must be in the intersection of the L–Nash curve and the w–Nash curve. Thus, $(\tilde{w}, \tilde{L}) = (w^N, L^N)$.

The proof for Agenda IIb is immediate. One can view the negotiated wage level of the first round as fixed and can apply the above arguments.