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Policies and Instruments for Self-Enforcing Treaties

Abstract

We characterize the optimal policy and policy instruments for self-enforcing treaties when countries invest in green technology before they pollute. If the discount factor is too small to support the first best, then both emissions and investments will be larger than in the first best, when technology is expensive. When technology is inexpensive, countries must instead limit or tax green investment in order to make future punishment credible. We also uncover a novel advantage of price regulation over quantity regulation, namely that when regulation is sufficiently flexible to permit firms to react to non-compliance in another country, the temptation to defect is reduced. The model is tractable and allows for multiple extensions.

JEL-Codes: D860, F530, H870, Q540.

Keywords: climate change, environmental agreements, green technology, policy instruments, repeated games, compliance, self-enforcing treaties.

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1 Introduction

In the absence of a world government, the fundamental challenge in international cooperation is to achieve actual delivery of public goods. Since coercion is not an option, one must rely on reputation and repeated play in order to motivate other countries to cooperate and to implement the pledges they have made.

It is doubtful, however, whether the prospect of cooperation in the future is sufficient to motivate cooperation today. Any given country will be tempted to increase its emissions or to reduce its abatement effort despite its pledges, especially since defection may not be observed immediately. The temptation is even greater for countries that depend on fossil fuels relative to countries that consume more renewable energy. The level of investment in technology can therefore influence the temptation to raise emissions, as recognized by the IPCC (2014:1178):

> *There is a distinct role for technology policy in climate change mitigation. This role is complementary to the role of policies aimed directly at reducing current GHG emissions.*

This situation raises a number of important questions. When countries decide to both pollute and invest in renewables, which of the two decisions will be distorted when the first best is unachievable? To motivate compliance, is it enough to punish insufficient cooperation, or might it be necessary to also punish excessive cooperation? How does the challenge of motivating compliance influence the optimal choice between quantity regulation (e.g., emission quotas and technology mandates) and price regulation (e.g., emission taxes and investment subsidies)?

To address these questions, we present a model in which countries first invest in technology, before they decide on their emission levels. This two-stage extensive form game is repeated indefinitely and we search for the best pair of policies that can be supported by a reversion to Nash equilibrium (i.e., the Markov-perfect "business-as-usual" equilibrium).

Even though we present a repeated extensive (rather than a normal) form game, a folk theorem holds in our setting. If the discount factor is sufficiently high, we arrive at the standard finding that the first-best outcome can be supported as a subgame-perfect equilibrium (SPE). For smaller discount factors, however, the first best is unattainable and we face one of two alternative types of distortion.

Under the first type of distortion, countries must be allowed to pollute more than the first-best level in order to motivate compliance. A larger emission quota reduces the temptation to defect by allowing countries to pollute more. To (partially) mitigate this inefficiency, it will be optimal to require countries to invest more than they find to be (ex post) optimal, given the negotiated emission quotas. By requiring overinvestment, so that countries will have an overabundance of renewable energy, the temptation to defect by polluting more is weakened. In this case, the optimal second-best policy is one in which both emissions and investments are distorted upwards.

This logic holds whether decisions are regulated by quotas or by prices. In the latter case, the policy is implemented by a standard Pigouvian tax and no investment subsidy, when the discount factor is high enough. Since we have assumed away technological spillovers, investments are first best if just the emission price is optimal. For a smaller discount factor, however, the emission tax must be reduced to discourage defection at the emission stage. In this situation, it will be optimal to subsidize investment in green technology. The harder it is to motivate compliance (i.e., the smaller the discount factor), the smaller the emission price will be, and the larger the optimal investment subsidy.

This first type of distortion arises when the first best requires a relatively large reduction in the consumption of energy, but a relatively small increase in investment in green technology. If, however, investments are relatively inexpensive, while an adjustment in the consumption level is relatively expensive, then the first best requires instead that investments should be the primary solution to climate change. When it is costly to adjust the consumption level of energy, the emission level will be relatively more sensitive to the level of technology developed thus far. This possibility leads to the second case.

Under the second type of distortion, there is a greater temptation to defect at the investment policy stage than at the emission policy stage. This possibility may seem counterintuitive since there is no need to regulate investment in the first best: Firms will invest at the ex post optimal level, given the negotiated emission policy. When emission levels are closely related to the technology level, however, defection at the emission policy stage (alone) is difficult without (also) defecting by investing less in technology. To reduce this temptation, countries should be required to invest *less* at the investment policy stage.

This result is remarkable and worthwhile elaborating on: Given any emission cap, a country would prefer a combination of energy consumption reduction and renewable energy investment. In the best SPE, countries are required to invest less than they find to be optimal, contingent on the negotiated emission level. A country that "defects" by investing more must therefore be punished with a reversion to Nash equilibrium. That is, countries must be punished for "cooperating too much," despite the fact that this investment could, in principle, generate a Pareto improvement.

The intuition for this counterintuitive type of punishment is as follows. If countries were permitted to invest the optimal amount, conditional on the negotiated emission level, then the investing countries would be unable (or unwilling) to raise emissions later on, as a punishment, if a country were to defect. When a country anticipates that there will not be any strong punishment, then the temptation to defect will be great. (A defecting country will then both invest less and pollute more.) A lower level of investment by a particular country is thus desirable, not in order to discourage that country from defecting, but rather to discourage other countries.

The possibility that the compliance constraint binds first at the investment policy stage, rather than the emission policy stage, can occur under quantity regulation, but not under price regulation, since in the latter case the temptation to defect at the investment policy stage is never greater than the temptation to defect at the emission policy stage. To see this, note that if a country deviates by offering lower investment subsidies (or by introducing investment taxes), then, given the lag between the policy announcement and the firms' investment decisions, investors in other countries will also anticipate that cooperation will break down and accordingly they will be less willing to pay for green technology. Therefore, investment is reduced even in the non-defecting countries, thus allowing the non-defecting countries to severely punish the defecting country at the subsequent emission stage.

By comparing the two types of regulations, it is clear that price regulation is preferable to quantity regulation in this setting. The former makes defection less tempting for a country, since investors in other countries will react (by reducing their investments), even if the investment policies in those countries remain unchanged. This reaction is clearly impossible when firms' investment levels are pinned down by technology mandates. Therefore, our analysis uncovers a new argument in favor of price regulation when the main problem facing countries is compliance. The lesson is more nuanced when information is incomplete.

Literature. This paper bridges two important strands of literature. On the one hand, the literature on self-enforcing international environmental agreements has shown that the incentives to free ride can undermine international cooperation in the provision of a global public good, such as climate protection (Barrett, 1994; Dutta and Radner, 2004; and, more recently, Kerr, Lippert, and Lou, 2020), but this literature has ignored the comparison between price and quantity instruments.[1] On the other hand, the price-vs-quantity literature (Weitzman, 1974, and the subsequent literature) examines the performance of the two instruments, but this literature has predominantly focused on static settings and has mostly ignored the compliance problem.[2] We combine features

[1]The literature on international environmental agreements typically examines models in which a country's payoff consists of the costs of private pollution abatement and global emissions, and in which national climate policies take the form of emission caps (see, e.g., Hoel, 1992; and Carraro and Siniscalco, 1993). A few papers, such as those of Eichner and Pethig (2015) and Kornek and Marschinski (2018), have compared the impact of taxes and caps on international environmental agreements, but they have considered a static coalition game and have ignored the impact of instruments on compliance.

[2]Hoel and Karp (2002) and Karp and Traeger (2020) analyze prices versus quantities in a dynamic setting with technology innovation and stock pollution, but do not consider strategic interactions between countries. Endres and Finus (2002) and Mideksa and Weitzman (2019) extend the prices versus quantities framework to a strategic setting, but do not consider repeated interactions and ignore the influence of technology regulation in the design of the optimal climate policy.

of these two strands in a novel framework by investigating how policies and instruments ought to be chosen when the agreement is self-enforcing and compliance is difficult to motivate.

The divergence between quantity and price instruments in self-enforcing environmental agreements crucially relies on the strategic role played by technology regulation in combination with emission regulation. This result links our paper to the literature on optimal climate policy, which dates back to Pigou (1920) and more recent general equilibrium analyses starting with Nordhaus (1994). This literature has shown that optimal environmental regulation must employ both a carbon tax and an investment subsidy, since the use of only the former leads to excessive distortions in the presence of technological externalities (see Golosov et al., 2014, in an exogenous technology setting and Acemoglu et al., 2012, in a setting with direct technical change).[3] While this literature considers a benevolent social planner with full commitment power, we permit multiple countries, each with its own sovereign government that lacks commitment to comply with climate policies. In our multi-country strategic setting, the role of investment regulation is to correct distortions related to the limited enforceability of the global climate treaty rather than those related to the presence of technological externalities. Furthermore, in order to correct these distortions, investment regulation may be forced to penalize rather than incentivize investment in clean technologies.

Technically, our model is closely related to other dynamic games permitting emissions as well as technology investment.[4] The timing of the game and the payoff structure are similar to those in Battaglini and Harstad (2016), for example, in which countries sequentially pollute and invest in green technologies in every period. Departing from their approach, we assume that emission levels are not contractible and focus on self-enforcing agreements sustained as SPEs of a repeated climate policy game.

The paper is most closely related to Harstad, Lancia, and Russo (2019) who emphasize that countries must overinvest in green technology in order to motivate compliance. We extend that contribution in several important directions: First, the emission level is now a continuous variable, rather than a binary one. When the emission level was a binary variable, it was not possible to discuss distortions in the level of emissions, as we do here. Furthermore, in that paper, it was never tempting to defect at the investment policy stage. Therefore, the previous analysis could not detect the possibility that one might

[3]The role of investment subsidies in optimal environmental regulation is discussed by, among others, Fischer and Newell (2008), Grimaud and Rouge (2008), and Gerlagh, Kverndokk, and Rosendahl (2009).

[4]Some papers in this literature have focused on the harmful effects of technology investment on a country's bargaining position in the future (see, e.g., Acemoglu and Rafey, 2019; Beccherle and Tirole, 2011; Harstad, 2012, 2016; and Helm and Schmidt, 2015), while others have analyzed countries' incentives to invest in the presence of technological externalities (see, e.g., Barrett, 2006; de Coninck et al., 2008; Golombek and Hoel, 2005; and Hoel and de Zeeuw, 2010). In contrast, we stress the strategic role of technology investment regulation in providing countries with incentives to comply with an international environmental agreement.

need to punish countries for cooperating (i.e., investing) "too much," which we emphasize as a novel result in the present paper.

Second, the current paper compares the effects of various types of regulations. In the previous paper, we only considered quantity regulation because a model with binary decisions is not appropriate for evaluating small changes in prices. Therefore, we are now able to present the novel result that investment subsidies should be introduced if, and only if, the discount factor is sufficiently small that the Pigouvian emission tax cannot be supported in a SPE.

Finally, we extend the baseline setting to an environment with uncertainty and discuss the implications for imperfect transparency of domestic climate policies under the optimal self-enforcing treaty. This analysis was also absent from the previous paper.

More generally, there is also a related literature in industrial organization on competition and collusion among firms. In Benoit and Krishna (1987) and Davidson and Deneckere (1990), firms may overinvest in production capacity in order to make the punishment harsher if a firm should decide to defect, which is analogous to our result that countries may need to underinvest in green technology in order to be able to punish. However, overinvestment in our paper is instead driven by the necessity to motivate compliance.[5]

In the relational contracting literature, Ramey and Watson (1997) and Halac (2015) develop the idea that technology investments can relax the compliance constraint on individual contributions to a public good; however, their focus is on how an up-front investment by one party affects the hold-up problem. Repeated maintenance investment in the public good are permitted in Halonen-Akatwijuka and Pafilis (2018), but there it is the ownership structure (rather than the technology) that is chosen to mitigate the temptation to free ride. For climate policies, Harstad (2020) shows how current technology investment can motivate future sustainable policies, but the focus there is on a time-inconsistent policymaker, rather than on multiple countries negotiating a self-enforcing agreement.

Outline. The following section presents the model and important benchmark results. Section 3 analyzes quantity regulation, derives the optimal sustainable climate policies, and determines when countries must be punished if they cooperate (i.e., invest) more. Section 4 looks at price regulation and uncovers a new argument in its favor when compliance is challenging. Section 5 presents a generalization under imperfect monitoring of domestic climate policies: The result that price regulation is always better than quantity regulation does not hold when we introduce type I error (in which case, we arrive at a novel trade-off when choosing between the instruments). Furthermore, the result that

[5] Lambson (1994) and Compte, Frédéric, and Rey (2002) consider how asymmetric capacities help to determine whether collusion is self-enforcing.

the compliance constraint cannot bind under price regulation does not necessarily hold when we introduce type II error. Section 6 discusses the importance of the timing of the game, and why sequential decisions motivate compliance better than simultaneous ones. After our concluding section, the Appendix presents the proofs.

2 The Model

2.1 The Stage Game

There are $n \geq 2$ countries, indexed by i or $j \in N \equiv \{1, ..., n\}$. Within each country i, there are many price-taking firms that invest in green technology (such as renewable energy) before they sell the energy to consumers. The consumers in country i consume y_i units of energy, coming from fossil fuels (g_i) and renewables (r_i): $y_i = g_i + r_i$. The variable r_i can also be interpreted as abatement technology. In this case, $g_i = y_i - r_i$ is the actual emission level after an abatement of r_i units.[6]

The benefit from country i's energy consumption is increasing and concave in y_i up to a bliss point $\overline{y}$. Formally,

$$B(y_i) = -\frac{b}{2}(\overline{y} - y_i)^2,$$

where the parameter $b > 0$ measures the importance of energy consumption. The bliss point represents the ideal energy level if there were no concern about pollution. Thus, a country would never produce more than $\overline{y}$ due to the implicit costs of generating or transporting the energy. While actual emissions, namely g_i, are privately beneficial, they contribute to global environmental damage $c\sum_{i\in N} g_i$, where $c > 0$ captures the importance attributed to climate change. The cost of investment in renewable energy technology is given by $K(r_i)$, which we assume to be quadratic in the investment level:

$$K(r_i) = \frac{k}{2}r_i^2,$$

where the parameter $k > 0$ captures the cost of private investment.

Time is discrete and denoted by $t = 1, ..., \infty$. For the sake of brevity, we omit subscript t, whenever this omission is not confusing. A national government's objective is to maximize the present discounted value of its utility:

$$\sum_t \delta^t u(\mathbf{g}_t, r_{it}), \quad \text{with} \quad u(\mathbf{g}_t, r_{it}) \equiv B(y_{it}) - c\sum_{j\in N} g_{jt} - K(r_{it}), \tag{1}$$

where $\delta \in (0, 1)$ is the discount factor common to all countries, and $\mathbf{g}_t = \{g_{1t}, ..., g_{nt}\}$ is

[6] To simplify the analysis, we do not require g_i or r_i to be positive. A negative g_i may be feasible with carbon capture, while r_i may be negative if the abatement technology has other uses apart from energy production.

the vector of emission levels.

Stocks and Technologies. Utility function (1) is equivalent to that used by Battaglini and Harstad (2016), who study coalition formation and Markov-perfect equilibria when countries can commit to emission levels. Although the focus there is completely different, a shared feature with the current paper is that one can easily permit pollution and technology to be stocks that accumulate over time, rather than being outputs that depreciate from one period to the next. If the stock of greenhouse gases is accumulating, we can let c represent the present-discounted cost of emitting another (long-lasting) unit of emission into the atmosphere. The technology r_{it} can also be a stock: the analysis is unchanged if we assume that the marginal cost of adding to the stock is (proportionally) larger when the existing stock is large (this assumption seems natural). We refer readers to Battaglini and Harstad (2016) regarding details on this matter. Other extensions, such as the possibility to allow for "brown" technology, are considered in the Online Appendix.

2.2 Climate Policies

In each period, private firms choose how much to invest in renewable energy technology before the consumers choose the energy consumption level. The sequential timing of the decisions follows naturally from the fact that investment in technology requires time to mature and become operational.[7] The private sector has no incentive to reduce emissions in the absence of public policy intervention.

The national government has the authority to regulate domestic emissions and technology investment decisions by means of climate policy instruments that influence the private sector's behavior. To this aim, the government can use two different sets of policy instruments. First, the government can set binding caps on the levels of emissions and investment in the form of a *quota-based* system. Second, the government can intervene by adopting a *price-based* mechanism in the form of carbon taxes, denoted by $\tau_i \in \mathbb{R}$, and investment subsidies, denoted by $\varsigma_i \in \mathbb{R}$.

We assume that governments cannot commit to future climate policies. Thus, policy instruments are set by governments in each period. Climate investment policy is set just before the private sector's investment in renewable energy technology, while climate emission policy is set just before the consumption of fossil fuel. Climate policies are implemented by all governments simultaneously and are observed by all. The sequential timing of climate policies and the private sector's responses are reported in Figure 1.

[7]Section 6 discusses how the results will change if the timing of the game is changed. Note, however, that the sequential timing follows when there is a minimum length of time, $l \in (0, 1)$, between the investment decision and the point at which the technology becomes operational. If the actual private cost of investment is, say, $\tilde{k}r^2/2$, then its present discounted value, evaluated at the time of the emission, is $kr^2/2$, with $k \equiv \delta^l \tilde{k}$. With this reformulation, we do not need to explicitly discount between the two stages within the same period.

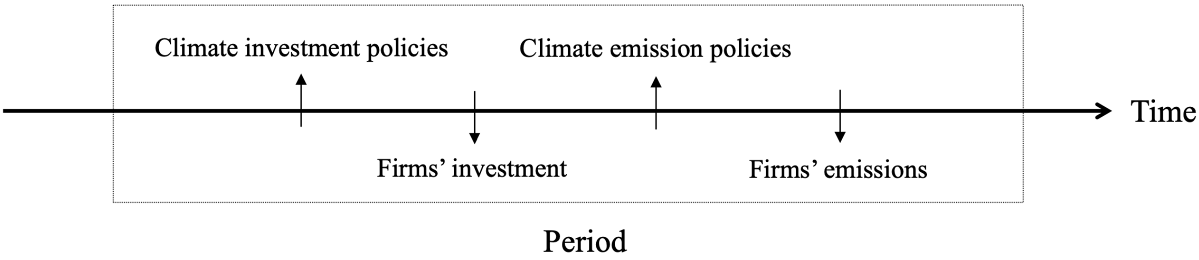

Figure 1: *Timing of the Game.*

Taxes collected and subsidies paid by the national governments do not represent actual costs or revenues from the government's perspective and their only effect is on the private sector's decisions regarding g_i and r_i.

When investors can sell renewable energy technology to the consumers, then, in equilibrium, the marginal cost of investment equals the expected marginal benefit from consuming energy, plus the subsidy, while the expected marginal benefit equals the expected carbon tax:

$$kr_i = \left(\overline{y} - \left(g_i^e + r_i\right)\right) b + \varsigma_i \quad \text{and} \quad \left(\overline{y} - \left(g_i^e + r_i\right)\right) b = \tau_i^e,$$

where the superscript e denotes an expectation. It follows that the investment in renewable energy technology is given by:

$$r\left(\tau_i^e, \varsigma_i\right) = \frac{\tau_i^e + \varsigma_i}{k}. \tag{2}$$

Higher expected carbon taxes or higher investment subsidies increase a firm's investment in abatement technology. After technology investments have been made and carbon taxes have been announced for the current period, consumers choose the actual emission level g_i, which decreases with r_i and τ_i as follows:

$$g\left(\tau_i, r_i\right) = \overline{y} - r_i - \frac{\tau_i}{b}. \tag{3}$$

2.3 Benchmarks and Equilibria

Before analyzing the optimal sustainable climate treaty, we examine how national governments optimally decide on climate policies in two benchmark cases. In the first, each individual country sets investment policy and emission policy non-cooperatively, a case we refer to as business-as-usual (BAU). In the second, countries coordinate among themselves by signing a fully enforceable treaty, which we refer to as first best. For the sake of simplicity, we assume homogeneous countries and focus on symmetric outcomes, in which climate policies and private sector decisions are identical in all countries. Hence, we will hereafter omit the subscript i.

Business-as-Usual. Under BAU, national governments internalize only domestic environmental damage from emissions when maximizing intertemporal utility. If they use a quota-based system, then they choose the following amounts:

$$g^b = \overline{y} - \frac{b+k}{bk}c \quad \text{and} \quad r^b = \frac{c}{k}. \tag{4}$$

Using functions (2) and (3), it is straightforward to verify that the government can equivalently implement (4) by imposing an optimal domestic Pigouvian carbon tax with no investment subsidies, in which case:

$$\tau^b = c \quad \text{and} \quad \varsigma^b = 0,$$

since the private sector anticipates that $\tau^e = \tau^b$ when deciding on technology investment. The optimal domestic utility function is then $u^b \equiv u\left(g^b, r^b\right)$.

First Best. In the first-best case, national governments internalize the global environmental damage from emissions. This case provides a useful benchmark because a self-enforcing environmental agreement which does not legally bind its members cannot do better than the first-best outcome. If governments can set quotas on emissions and investments, then:

$$g^* = \overline{y} - \frac{b+k}{bk}cn \quad \text{and} \quad r^* = \frac{cn}{k}. \tag{5}$$

Note that there is no need to regulate investment in addition, since firms invest in technologies at the efficient level, conditional on emissions:

$$r\left(g_i\right) = \frac{b}{b+k}\left(\overline{y} - g_i\right) = r^* \quad \text{if} \quad g_i = g^*.$$

Using (2) and (3), it is obvious that the government can implement (5) by imposing an optimal global Pigouvian carbon tax alone:

$$\tau^* = cn \quad \text{and} \quad \varsigma^* = 0,$$

since the carbon tax expected by the private sector is $\tau^e = \tau^*$. Clearly, $g^b = g^*$ and $r^b = r^*$ when $n = 1$, while $g^b > g^*$ and $r^b < r^*$ when $n > 1$. The optimal domestic utility function is $u^* \equiv u\left(g^*, r^*\right) > u^b$, since governments do not take into account the global benefits of pollution abatement when deciding upon climate policies in BAU.

Proposition 0. *Quantity regulation and price regulation are welfare-equivalent under BAU and first best.*

Equilibria. In what follows, we describe the subgame-perfect equilibrium (SPE) of a repeated climate policy game played among the governments of the n countries. The game has the structure of a multilateral prisoner's dilemma, in which all countries would be better off in the global first-best climate treaty case than in the BAU case. The stage game of the repeated climate policy game is slightly more complex than in a standard repeated prisoner's dilemma because each period consists of two stages: the investment policy stage and the emission policy stage. Among the sustainable SPEs, we consider those with Nash reversion punishment, in which deviation by any government from a negotiated climate policy triggers permanent reversion to BAU. We will characterize the optimal sustainable climate policy under quantity regulation and under price regulation and will compare the generated levels of welfare.

3 Emission Quotas and Investment Mandates

For a climate policy to be self-enforcing, it must be better to cooperate than to defect. That is, the discounted utility under the optimal policy must be larger than the discounted welfare achieved by deviating and thereafter reverting to BAU. In each period, a national government may be tempted to defect in either the investment policy stage or the emission policy stage. Given the timing of the game, if a government deviates from the equilibrium play at some stage, then such deviation is taken into account already in the subsequent stage by the private sector in that country, as well as by the governments and private sectors of other countries.

Compliance Constraints. A country's intertemporal value function in the case of the negotiated emissions and investment (g, r) is given by:

$$V\left(g, r; \delta\right) = \frac{u\left(g, r\right)}{1-\delta}. \tag{6}$$

At the emission policy stage, investment regulation has already been implemented by every government and the private sector has made its investment decisions accordingly. Moreover, non-deviating governments cannot promptly adjust their emission policies in response to other governments' actions, since policy decisions are made simultaneously. The compliance constraint at the emission policy stage is then given by:

$$V\left(g, r; \delta\right) \geq V^{g}\left(g, r; \delta\right) = u\left(\hat{g}^{d}\left(r\right), r\right) + c\left(n-1\right)\left(\hat{g}^{d}\left(r\right) - g\right) + \frac{\delta u^{b}}{1-\delta}, \tag{7}$$

where $V^{g}\left(g, r; \delta\right)$ is a country's intertemporal value when the national government deviates from the negotiated emission policy by choosing $\hat{g}^{d}\left(r\right)$. The function $\hat{g}^{d}\left(r\right)$ is the optimal domestic consumption of fossil fuel in the *deviating* country (hence, the superscript d) once investments have been made. Formally, $\hat{g}^d(r)$ is equivalent to (3) if $\tau_i = \tau^b$ and the technology investment is at the negotiated level. The larger the investment in renewable energy technology made at the beginning of the period, the smaller $\hat{g}^d(r)$ will be, since the marginal cost of emitting an additional unit of pollution has increased.

At the investment stage, the compliance constraint is given by:

$$V(g, r; \delta) \geq V^r(r; \delta) = c(n-1)\left(g^b - \hat{g}^n(r)\right) + \frac{u^b}{1-\delta}, \tag{8}$$

where $V^r(r;\delta)$ is a country's intertemporal value when the national government deviates from the negotiated investment policy by choosing r^b (and thereafter g^b). Since deviation from either of the two policies will be punished by a permanent reversion to BAU, if a government defects at the investment policy stage, then it will emit $g^b > g$ at the emission policy stage, as well. This strategy leads to the payoff $u^b/(1-\delta)$ for the deviator, plus the one-period gain due to the reduction in emissions by other non-deviating countries induced by their levels of investment r. The function $\hat{g}^n(r)$ is the optimal domestic consumption of fossil fuel in *non-deviating* countries (hence, the superscript n) once the negotiated investment has been made. It is equivalent to (3) when $\tau_i = \tau^b$ for a given r.

The Optimal Quantity Regulation. We now characterize the optimal climate policy, when a government directly controls the private sector's decisions in setting g and r by implementing negotiated quotas on emissions and investment.

Definition 1. *Under quantity regulation, an optimal sustainable climate policy is defined as the pair of policies $(g, r) \in \mathbb{R}^2$ that maximizes the objective function* (6)*, subject to the compliance constraints* (7) *and* (8)*.*

If the discount factor is sufficiently large, then the first best can be sustained as the outcome of a SPE of the repeated climate policy game. For discount factors below some threshold level, long-term consequences become less important and first-best climate policies are no longer sustainable. To find that threshold level, we need to determine whether it is (7) or (8) that binds first as δ becomes smaller. Denote by $\delta^g(g, r)$ and $\delta^r(g, r)$ the levels of δ that solve $V(g, r; \delta) = V^g(g, r; \delta)$ and $V(g, r; \delta) = V^r(r; \delta)$, respectively for any (g, r) and define $\overline{\delta}^g_{quota} \equiv \delta^g(g^*, r^*)$ and $\overline{\delta}^r_{quota} \equiv \delta^r(g^*, r^*)$. The following lemma establishes the condition under which constraint (7) is harder to satisfy than constraint (8) and vice versa.

Lemma 1. If $\frac{k}{b} \geq (<) \frac{1}{2}$, then $\overline{\delta}^r_{quota} = \frac{b-k}{2b} \leq (>) \overline{\delta}^g_{quota} = \frac{k}{b+2k}$.

If the technology is expensive, in that k/b is large, then the role of technology investments is minor and the primary solution to climate change is to reduce the consumption of energy. In this case, it is more tempting to defect at the emission policy stage than at the investment policy stage: $\overline{\delta}^{r}_{quota} \leq \overline{\delta}^{g}_{quota}$. In the opposite case, when k/b is small, then it is costly to change the consumption level. In this case, the emission level is essentially pinned down by the level of technology, and it is thus more tempting to defect at the investment policy stage than at the emission policy stage: $\overline{\delta}^{r}_{quota} > \overline{\delta}^{g}_{quota}$.

The following proposition characterizes the optimal emission quota and investment mandate for any δ.

Proposition 1. *Under quantity regulation, the optimal sustainable climate policy* (g, r) *is characterized by:*

i. *For* $\delta > \max\left\{\overline{\delta}^{g}_{quota}, \overline{\delta}^{r}_{quota}\right\}$, *the outcome if first best:*

$$g = g^* \quad \text{and} \quad r = r^*;$$

ii. *For* $\delta \in (\delta^{r}_{quota}(g, r), \overline{\delta}^{g}_{quota}]$, *which requires* $\frac{k}{b} > \frac{1}{2}$, *both emission and investment levels are higher than the levels that are ex post optimal:*

$$g = g^* + \Lambda_{quota}(\delta) \quad \text{and} \quad r = r^* = r(g) + \frac{b}{b+k}\Lambda_{quota}(\delta), \text{ where}$$
$$r(g) = \frac{b}{b+k}(\overline{y} - g) \text{ and } \Lambda_{quota}(\delta) = \frac{c(n-1)}{b}\left(1 - \delta - \sqrt{\frac{\delta(b + \delta k)}{k}}\right);$$

iii. *For* $\delta \in (\delta^{g}_{quota}(g, r), \overline{\delta}^{r}_{quota}]$, *which requires* $\frac{k}{b} < \frac{1}{2}$, *emission level is higher but the investment level is smaller than the levels that are ex post optimal:*

$$g = g^* + \Xi_{quota}(\delta) \quad \text{and} \quad r = r^* - \Xi_{quota}(\delta), \text{ where}$$
$$\Xi_{quota}(\delta) = \frac{c(n-1)}{k}\left(1 - \delta - \sqrt{\frac{k + \delta^2 b}{b}}\right);$$

iv. *For* $\delta \leq \min\left\{\delta^{g}_{quota}(g, r), \delta^{r}_{quota}(g, r)\right\}$, *both compliance constraints bind and*

$$\{(g, r) \,|V(g, r; \delta) = V^g(g, r; \delta) \text{ and } V(g, r; \delta) = V^r(r; \delta)\}.$$

A Folk Theorem. Part (i) of the proposition verifies that the first-best outcome is achievable when the discount factor is large, as one would expect.

Overinvestments in Green Technology. Part (ii) describes the optimal sustainable climate policy when the discount factor is too small to support the first best, and when k/b is large. In this case, the compliance constraint at the emission policy stage is relevant. Domestic quotas on emissions must be allowed to increase when δ goes below

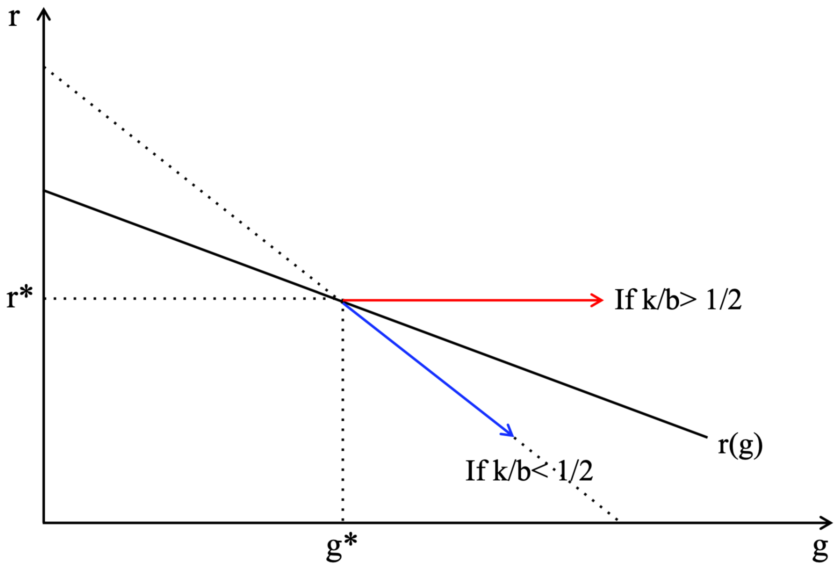

Figure 2: *As δ declines, the distortion follows the horizontal red arrow when $k/b > 1/2$ and the decreasing blue arrow when $k/b < 1/2$.*

$\overline{\delta}^{g}_{quota}$, leading to a distortion measured by $\Lambda_{quota} > 0$. With regard to optimal quotas on investment, we need to consider two countervailing effects. On the one hand, a higher quota on emissions discourages investment in abatement technologies, since investment is a substitute for pollution. On the other hand, governments can strategically dampen the increase in emission quotas by requiring firms to invest more in clean technology up-front. The two effects cancel each other out in the linear-quadratic model, and r remains unchanged at r^* as δ declines. Nevertheless, compared to the conditional efficient level, i.e., $r(g)$, it is clear that investment must be distorted by the amount $(b/(b+k))\,\Lambda_{quota}$, which is decreasing in δ.

Punishing Cooperation. Part (iii) describes the optimal outcome when k/b is small. In this case, emissions are highly responsive to investment in renewable energy technology, such that an upfront increase of r strongly reduces the incentive to pollute. The temptation to deviate from negotiated climate policies is then greater at the investment policy stage than at the emission policy stage. As δ becomes smaller, motivating compliance requires investment to be reduced and, in turn, pollution will be increased by raising the caps on emissions. The associated deviation of investment and emissions from the first-best level is measured by $\Xi_{quota} > 0$. For a fixed level of emissions, promoting investment in clean technologies would have been a Pareto improvement. Nevertheless, more investment cannot be allowed and must be punished by a reversion to BAU. Intuitively, if countries were permitted to invest the optimal amount, conditional on the negotiated emission levels, then non-deviating countries would be unable to raise emissions as a punishment after a country's deviation, which would then increase all countries' temptation to defect at the investment level.

Corollary 1. *When technology is inexpensive, each country invests less than it finds optimal, conditional on the negotiated g, and must be punished if it invests more.*

In Figure 2, the solid line shows that the optimal investment, conditional on the emission quota, $r(g)$, is a downward sloping line. If $k/b > 1/2$ and the discount factor falls, the equilibrium (r, g) follows the horizontal (red) arrow: emissions are larger than the first best levels, while the investment level stays unchanged, implying that countries overinvest relative to the optimal $r(g)$ that is conditional on the larger g. If $k/b < 1/2$ and the discount factor falls, the equilibrium (r, g) follows the declining (blue) arrow: in this case, investments are less than what the countries find to be optimal given the equilibrium g. In other words, further cooperation (i.e., a larger r) must be punished in the optimal sustainable climate treaty.

4 Carbon Taxes and Investment Subsidies

We now turn to the characterization of the optimal climate policy, when governments set carbon taxes and investment subsidies and therefore influence the private sector's responses only indirectly through shifts in marginal costs and benefits.

Definition 2. *Under price regulation, an optimal sustainable climate policy is defined as the pair of policies $(\tau, \varsigma) \in \mathbb{R}^2$ that maximizes the objective function* (6)*, subject to the private sector's response functions* (2) *and* (3) *and the compliance constraints* (7) *and* (8).

Compliance Constraints. In this case, the government is deciding on prices and not on quantities. At the emission policy stage, the compliance constraint is, nevertheless, similar to (7) because the government can select the emission level it prefers by selecting the emission tax accordingly. The compliance constraint at the investment policy stage is slightly different from (8), however, in that the continuation value following defection is $V^r(\widetilde{r}; \delta)$, and not $V^r(r; \delta)$, where $\widetilde{r}$ is the equilibrium investment level in the non-deviating countries when the firms observe a country's deviation from the negotiated investment subsidy. After this observation, all firms will immediately anticipate that emissions will increase and the demand for green technology will decrease. Therefore, firms in non-deviating countries will invest at the level $\widetilde{r} = (\tau^b + \varsigma)/k$, which is obtained from (2) where $\tau^e = \tau^b$ and investment subsidies are set to the negotiated level ς, while firms in the deviating country will adjust their investment decisions to the level r^b. Since $\widetilde{r} < r$, it follows that $\hat{g}^n(\widetilde{r}) > \hat{g}^n(r)$. The temptation to defect at the investment policy stage is thus smaller under price regulation than under quantity regulation.

Lemma 2. *Under price regulation, it is always more tempting to defect at the emission policy stage than at the investment policy stage.*

To see this result, replace $g^b = \overline{y} - r^b - \tau^b/b$ and $\hat{g}^n(\widetilde{r}) = \overline{y} - \widetilde{r} - \tau^b/b$ into (8). The discounted value of deviating at the investment policy stage can be written as:

$$V^r(\varsigma;\delta) = \frac{c(n-1)}{k}\left(\varsigma - \varsigma^b\right) + \frac{u^b}{1-\delta}.$$

Since $\varsigma^* = 0$, it is the case that $V^r(\varsigma^*;\delta) = u^b/(1-\delta)$ and therefore constraint (8) reduces to $u^* > u^b$, which is always satisfied. Hence, there are no gains from deviating at the investment policy stage when implemented policies are first best under price regulation, unlike quantity regulation.

As δ declines, however, it will eventually be tempting to defect at the emission policy stage, since τ^* is not optimal from a national perspective. The threshold level of δ below which constraint (7) becomes binding is obtained by solving $V(\tau^*, \varsigma^*; \delta) = V^g(\tau^*, \varsigma^*; \delta)$ and is equal to $\overline{\delta}^g_{price} = \overline{\delta}^g_{quota}$.

The Optimal Price Regulation. Let $\delta^r(\tau, \varsigma)$ denote the threshold level of δ that solves $V(\tau, \varsigma; \delta) = V^r(\varsigma; \delta)$ for any (τ, ς). The following proposition characterizes the optimal carbon tax and investment subsidy for any δ.

Proposition 2 *Under price regulation, the optimal sustainable climate policy* (τ, ς) *is characterized by*:

i. *For* $\delta > \overline{\delta}^g_{price}$*, the outcome is first best:*

$$\tau = \tau^* \quad \text{and} \quad \varsigma = \varsigma^*;$$

ii. *For* $\delta \in (\delta^r_{price}(\tau, \varsigma), \overline{\delta}^g_{price}]$*, emissions are taxed and investments are subsidized:*

$$\tau = \tau^* - \Lambda_{price}(\delta) \quad \text{and} \quad \varsigma = \Lambda_{price}(\delta)\,, \text{ where } \Lambda_{price}(\delta) = b\Lambda_{quota}(\delta)\,;$$

iii. *For* $\delta \leq \delta^r_{price}(\tau, \varsigma)$*, both compliance constraints bind and*

$$\{(\tau, \varsigma)\,|V(\tau, \varsigma; \delta) = V^g(\tau, \varsigma; \delta) \text{ and } V(\tau, \varsigma; \delta) = V^r(\varsigma; \delta)\}\,.$$

A Folk Theorem. For δ larger than $\overline{\delta}^g_{price}$, it is possible to implement the first-best climate policy (τ^*, ς^*).

Technology Subsidies. As δ becomes smaller, constraint (7) becomes binding and governments are tempted to deviate. To mitigate this temptation, the equilibrium carbon

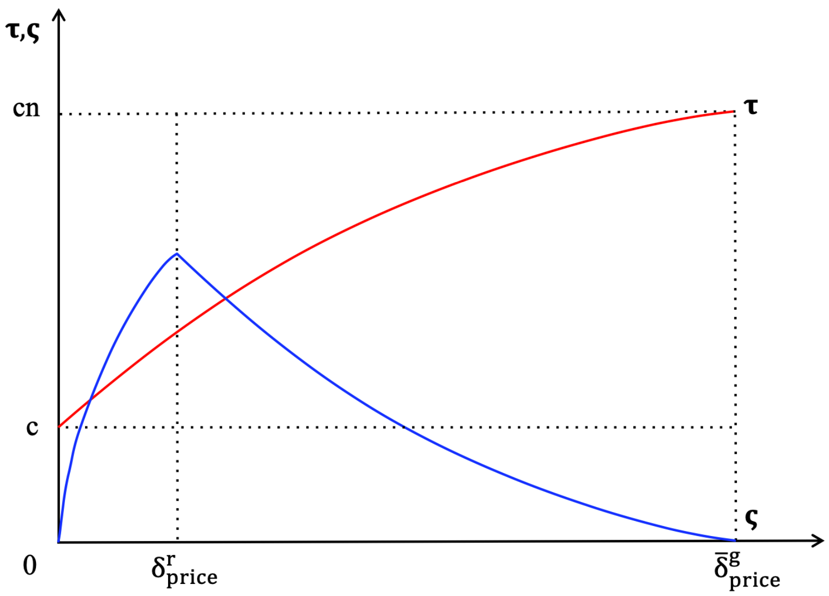

Figure 3: *The blue line represents the optimal investment subsidy and the red line represents the optimal emission tax as functions of the discount factor.*

tax must be lowered. In addition, it becomes optimal to subsidize investment in renewable energy technology by $\varsigma > 0$. This policy reduces the temptation to defect at the emission policy stage, and in turn, the emission level can be reduced by a smaller amount than if investment was unregulated.[8]

Equilibrium climate policies are represented as functions of the discount factor in Figure 3. When the compliance constraint at the emission policy stage starts to bind, the emission tax is reduced below the Pigouvian level, and the optimal investment subsidy is positive. If the discount factor is reduced further, then, eventually, the compliance constraint at the investment policy stage will also bind. For smaller discount factors, governments are required to lower subsidies to a level that approaches zero (when δ approaches zero).

Welfare Comparison. We have characterized the optimal sustainable climate policies under a self-enforcing international environmental agreement when countries regulate their domestic private sectors by means of either quotas or taxes and subsidies. In both cases, we have shown that technology, as well as emissions, needs to be globally regulated when the discount factor is sufficiently small, in order to secure the self-enforceability of the optimal climate agreement. An important implication of the analysis is that the welfare achieved through price instruments is either equal to or higher than that achieved

[8] In the Online Appendix, we introduce the possibility to invest in a "brown" technology in addition to the investment in clean technology. We show that a sustainable climate policy involves a subsidy for clean technology and a tax on brown technology. Furthermore, the distortion from the first-best climate policy is greater when the discount rate is lower and is influenced by the elasticity of substitution between the two types of technologies in a non-linear way.

through quantity instruments when investment regulation plays a strategic role. When $k/b > 1/2$, the optimal allocation implemented by carbon taxes and investment subsidies is equivalent to the optimal allocation implemented by means of quotas, and therefore the two policy instruments achieve the same level of global welfare. When $k/b < 1/2$, however, welfare is higher under price regulation than under quantity regulation. The reason for this is that it is less tempting to deviate with a lower investment subsidy than with an investment quantity, because firms in the non-deviating countries will react (by reducing their own investments) if and only if the investment policy involves a price instrument. Formally, when $\delta \in \left[\overline{\delta}^{g}_{price}, \overline{\delta}^{r}_{quota}\right]$, price instruments are able to sustain the first-best allocation, while quotas fail to enforce a fully cooperative outcome. The welfare loss induced by quotas relative to price instruments increases as δ declines.

Corollary 2. *Any Optimal Sustainable Climate Policy is associated with a higher intertemporal value under price regulation than under quantity regulation.*

5 Technology and Imperfect Transparency

The superiority of price instruments over quotas is difficult to reconcile with the predominance of command-and-control-type regulations as the instrument of environmental policy. Emission reductions and investment in clean technology are frequently specified as targets but are seldom regulated by means of global carbon taxes and investment subsidies.

In the following section, we permit countries' climate policy decisions to be imperfectly observed and show that: (i) motivating compliance may require punishing cooperation also under price regulation; and (ii) the welfare achieved by means of quotas may be higher than that achieved by means of price instruments. Hence, the presence of monitoring imperfections can provide an explanation for the predominance of quantity regulation in practical environmental policy.

To highlight the effect of uncertainty in the simplest way, we assume, as in Maggi (1999), that a country's domestic policy actions can sometimes be misperceived by other countries. Accordingly, we introduce a shock at each stage, which occurs after climate policy decisions have been made, but before private sector decisions have been made. This uncertainty leads to two types of errors: a *type I error* with probability λ, due to a country incorrectly perceiving that another has defected from the negotiated agreement, even if it has not; a *type II error* with probability η, due to the fact that when a country is defecting, the defection goes undetected.[9]

[9] We assume without loss of generality that shocks occurring at the investment policy stage and at the emission policy stage are independent and drawn from the same distribution. Furthermore, shocks do not depend on the type of policy instruments employed. We also assume that the private sector perfectly

For the sake of comparison, we focus on the optimal sustainable climate policy in which deviations are punished by reverting to BAU. Given that the punishment can be triggered by mistake due to uncertainty, however, failures of climate treaties through punishment reversion can be temporary. To show how the presence of uncertainty modifies the optimal sustainable climate treaty, in what follows we separately analyze the case of type I error and the case of type II error. Clearly, the overall effect of uncertainty should be viewed as a combination of the effects of the two types of error.

5.1 Type II Errors

We start by considering the scenario in which $\lambda = 0$ and $\eta > 0$. In this case, undeserved punishments will *not* be triggered along the equilibrium path, since there cannot be any misperceived detection of deviations when countries comply.

Compliance Constraints. In the absence of costly overpunishment, the optimal sustainable climate agreement can be achieved by imposing an infinite duration of punishment, that is, $T = \infty$, which maximizes retaliatory power without reducing welfare. Hence, the intertemporal value of cooperation is given by (6).

The compliance constraint at the emission policy stage is equivalent to constraint (7), in which the continuation value following defection is $(1-\eta)u^b/(1-\delta) + \eta V(g, r; \delta)$ rather than $u^b/(1-\delta)$, since a country's defection from negotiated emission levels goes undetected with probability η.

The compliance constraint at the investment policy stage is equal to:

$$V(g, r; \delta) \geq V^r(g, r; \delta) = c(n-1)(g^b - \hat{g}^n_\eta(r)) + u^b + \delta((1-\eta^2)\frac{u^b}{1-\delta} + \eta^2 V(g, r; \delta)), \quad (9)$$

where $1-\eta^2$ is the probability of detecting a country's deviation from either an emission policy or an investment policy that actually occurred. As in the case without uncertainty, a country that deviates from the investment policy by choosing r^b will also deviate from the emission policy by choosing g^b. If a country defects at the investment policy stage, emissions in each non-deviating country are given by:

$$\hat{g}^n_\eta(r) = \eta g + (1-\eta)\hat{g}^n(r), \quad (10)$$

since non-deviating countries comply with negotiated emission levels with some probability, even after one country deviates from its investment policy. As in the previous section, under price regulation, r in (10) is replaced by $\widetilde{r} = (\tau^b + \varsigma)/k$.

observes its own country's climate policy actions, while it misperceives those of other countries to the same extent as its country's government.

A Folk Theorem. As in the case without uncertainty, the first best can be sustained when the discount factor is higher than a critical threshold, which are explicitly characterized in the following lemma.

Lemma 3. *The compliance constraint binds at the* i-stage, $i \in \{g, r\}$, *for policy instrument* j, $j \in \{quota, price\}$, *when* $\delta < \bar{\delta}_j^i$, *where:*

$$\bar{\delta}_{quota}^g = \frac{k}{(1-\eta)\,b + (2-\eta)\,k} \quad \text{and} \quad \bar{\delta}_{quota}^r = \frac{b - (1-2\eta)\,k}{(2-\eta^2)\,b + \eta\,(2-\eta)\,k},$$

and

$$\bar{\delta}_{price}^g = \bar{\delta}_{quota}^g \quad \text{and} \quad \bar{\delta}_{price}^r = \frac{2\eta - 1}{\eta\,(2-\eta)}.$$

An important departure from the case without uncertainty is that countries may have a stronger temptation to deviate at the investment policy stage than at the emission policy stage under both quantity regulation and price regulation if domestic policies are not sufficiently transparent: that is, $\bar{\delta}_{quota}^r > \bar{\delta}_{quota}^g$ when $\eta > \eta_{quota} \equiv 2 - b/k$ (which requires $k/b < 1/2$) and $\bar{\delta}_{price}^r > \bar{\delta}_{price}^g$ when $\eta > \eta_{price} \equiv (b + 2k)\,/\,(2b + k)$. To see this, notice that the effect of uncertainty is to induce non-deviating countries to respond less promptly to deviations that occur at the investment policy stage. From (10), we can indeed see that $\hat{g}_\eta^n\,(r) < \hat{g}^n\,(r)$. This weaker reaction increases the temptation to defect at the investment policy stage. Hence, satisfying constraint (9) may become harder than satisfying compliance with emission policies also under price regulation if policy transparency is low.

Nevertheless, Lemma 3 shows that $\max\{\bar{\delta}_{quota}^g, \bar{\delta}_{quota}^r\} \geq \max\{\bar{\delta}_{price}^g, \bar{\delta}_{price}^r\}$ for any η. The intuition behind this result is that the non-deviating countries' willingness to raise emissions following a country's defection at the investment policy stage is less under quantity regulation than under price regulation: for a fixed g, $\hat{g}_\eta^n\,(\widetilde{r}) > \hat{g}_\eta^n\,(r)$ since $\hat{g}^n\,(\widetilde{r}) > \hat{g}^n\,(r)$. Hence, even under uncertainty, price regulation is able to sustain the first-best outcome for a wider range of δ than quantity regulation, which makes price instruments superior to quotas.

The Optimal Sustainable Climate Policies. The following proposition characterizes the optimal sustainable climate policies under uncertainty when the global first-best climate treaty cannot be sustained because $\delta < \max\{\overline{\delta}_j^g, \overline{\delta}_j^r\}$.

Proposition 3.

Under price regulation, the optimal sustainable climate policy (τ, ς) *is characterized by:*

P.i. *For* $\delta \in (\delta^r_{price}(\tau,\varsigma), \overline{\delta}^g_{price}]$*, which requires* $\eta < \eta_{price}$*,*

$$\begin{aligned}\tau &= \tau^* - \Lambda^\eta_{price}(\delta) \quad \text{ and } \quad \varsigma = \Lambda^\eta_{price}(\delta), \text{ where} \\ \Lambda^\eta_{price}(\delta) &= \frac{c(n-1)}{1-\eta\delta}\left(1-\delta-\sqrt{\frac{\delta(1-\eta)((1-\eta\delta)b+(1-\eta)\delta k)}{k}}\right);\end{aligned}$$

P.ii. *For* $\delta \in (\delta^g_{price}(\tau,\varsigma), \overline{\delta}^r_{price}]$*, which requires* $\eta > \eta_{price}$*,*

$$\begin{aligned}\tau &= \tau^* - \Xi^\eta_{price}(\delta) \quad \text{ and } \quad \varsigma = -\frac{(1-\eta)(b+k)}{\eta k-(1-\eta)b}\Xi^\eta_{price}(\delta), \text{ where} \\ \Xi^\eta_{price}(\delta) &= \frac{c(n-1)(\eta k-(1-\eta)b)}{(1-\eta^2\delta)k}\left(1-\delta-(1-\eta)\sqrt{\frac{(1+\eta\delta)^2k+(1-\delta)^2b}{\eta^2k+(1-\eta)^2b}}\right).\end{aligned}$$

Under quantity regulation, the optimal sustainable climate policy (g,r) *is characterized by:*

Q.i. *For* $\delta \in (\delta^r_{quota}(g,r), \overline{\delta}^g_{quota}]$*, which requires* $\eta < \eta_{quota}$*,*

$$g = g^* + \Lambda^\eta_{quota}(\delta) \quad \text{ and } \quad r = r^*, \text{ where } \Lambda^\eta_{quota}(\delta) = \frac{1}{b}\Lambda^\eta_{price}(\delta);$$

Q.ii. *For* $\delta \in (\delta^g_{quota}(g,r), \overline{\delta}^r_{quota}]$*, which requires* $\eta > \eta_{quota}$*,*

$$\begin{aligned}g &= g^* + \Xi^\eta_{quota}(\delta) \quad \text{ and } \quad r = r^* - \frac{b}{b+\eta k}\Xi^\eta_{quota}(\delta), \text{ where} \\ \Xi^\eta_{quota}(\delta) &= \frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)bk}\left(1-\delta-\sqrt{\frac{((1-\eta^2)\delta)^2b+((1-\eta)(1+\delta\eta))^2k}{b+\eta^2k}}\right).\end{aligned}$$

Overinvestment in Green Technology. Parts (P.i) and (Q.i) of the proposition show the optimal allocation when uncertainty is sufficiently small that the compliance constraint at the emission policy stage binds first under both types of treaty. The optimal sustained emission and investment levels are the same whether governments use quotas or price instruments, as in the case without uncertainty. As δ becomes smaller, emissions must be allowed to increase in order to motivate compliance, but the investment level stays unchanged at the first-best level. To induce countries to invest more than what they find optimal, conditional on negotiated emissions, quantity regulation requires investment mandates while price regulation requires investment subsidies.

Punishing Cooperation. Parts (P.ii) and (Q.ii) show how the optimal climate regulation must change when uncertainty is high. In this case, the constraint at the investment policy stage binds first when δ decreases under both treaty types. Compliance with the agreement then requires punishing countries that invest too much in green technology in order to dampen the temptation to defect at the investment policy stage. Under price regulation, investments in green technology must therefore be taxed rather than subsidized.

Corollary 3. *When η is large, the optimal sustainable climate treaty requires a tax on investment in green technology rather than a subsidy.*

5.2 Type I Errors

We now consider the scenario in which $\lambda > 0$ and $\eta = 0$. In this case, the punishment *can* occasionally be triggered by mistake along the equilibrium path. Therefore, it may not be optimal to mantain a punishment phase forever. Instead, the duration of the punishment phase, T, should be as short as is necessary to motivate compliance.[10]

Compliance Constraints. A country's intertemporal value of cooperation declines as λ increases:

$$V(g,r;\delta) = u(g,r) - c(n-1)(\hat{g}^n_\lambda(r) - g) + \delta((1-(1-\lambda)^2)V^P(g,r;\delta) + (1-\lambda)^2 V(g,r;\delta)), \tag{11}$$

where $1-(1-\lambda)^2$ is the probability of detecting a country's deviation from either a negotiated emission policy or a negotiated investment policy, even if no deviation actually takes place, and $V^P(g,r;\delta)$ is the discounted value of punishment, given by:

$$V^P(g,r;\delta) = \frac{1-\delta^T}{1-\delta}u^b + \delta^T V(g,r;\delta).$$

Even if no country has deviated at the investment policy stage, all countries can erroneously observe a signal that is consistent with a deviation, which occurs with probability λ, and in response each of them will thereafter emit $\hat{g}^n(r)$. The expected emissions generated by the other countries are therefore:

$$\hat{g}^n_\lambda(r) = (1-\lambda)g + \lambda \hat{g}^n(r), \tag{12}$$

where $r = \widetilde{r}$, under price regulation.

For a country to comply with the emission policy g, conditional on upfront investment r, rather than deviating to $\hat{g}^d(r)$, the following compliance constraint at the emission

[10] Equivalently, we could let the punishment phase start with some probability only when the evidence is consistent with a defection.

policy stage must be satisfied:

$$V(g,r;\delta) \geq V^g(g,r;\delta) = u(\hat{g}^d(r), r) + c(n-1)(\hat{g}^d(r) - \hat{g}^n_\lambda(r)) + \delta V^P(g,r;\delta). \quad (13)$$

The compliance constraint at the investment policy stage is equivalent to (8), in which the continuation value $u^b/(1-\delta)$ is replaced by the larger value $u^b + \delta V^P(g,r;\delta)$.

Impossibility of a Folk Theorem. An important implication of the existence of type I errors is that a folk theorem does not apply, even for a high discount factor. The first-best outcome would be achievable by requiring a long punishment duration, which is, however, not optimal, since retaliation by non-deviating countries takes place along the equilibrium path and, in turn, reduces the achieved welfare. Hence, punishment must be finite when δ is high and its duration must balance the incentives to comply with either negotiated emissions or negotiated investments, depending on the stage at which countries have the strongest temptation to deviate.

Under price regulation, it is always more tempting to defect at the emission policy stage than at the investment policy stage. The absence of type II errors implies that deviations at the investment policy stage are immediately detected and harshly punished by non-deviating governments—thanks to the reduced investment in green technology made by the private sector in each country in anticipation of the cooperation breakdown. Although compliance constraint at the investment policy stage cannot bind first, both compliance constraints can simultaneously bind when uncertainty is high ($\lambda > \lambda^g_{price}$).

Under quantity regulation, the temptation to defect is stronger at the emission policy stage when $\lambda < \lambda^g_{quota} \equiv 1 - (b/2k)^{1/4}$ (which requires that $k/b > 1/2$), while it is more tempting to deviate at the investment policy stage when $\lambda < \lambda^r_{quota} \equiv 1 - (2k/b)^{1/2}$ (which requires that $k/b < 1/2$). It follows that the constraints simultaneously bind when $\lambda > \max\{\lambda^g_{quota}, \lambda^r_{quota}\}$.

All binding constraints can be slackened by increasing T. Hence, when the discount factor decreases, the punishment's duration can be increased in order to restore the incentives to comply. We can then determine the critical levels of δ, above which the most cooperative climate policies can be sustained by the prospect of a punishment of finite length, when the binding constraint is at either the emission policy stage or the investment policy stage. When δ falls below these thresholds, T needs to be infinite.

Lemma 4. *The compliance constraint binds at the* i-stage, $i \in \{g, r\}$, *for policy instrument* j, $j \in \{quota, price\}$, *and the punishment duration is* $T = \infty$, *when* $\delta < \bar{\delta}^i_j$, *where:*

$$\bar{\delta}^g_{quota} = \frac{(1-\lambda)^4 k}{b + 2(1-\lambda)^4 k} \quad \text{and} \quad \bar{\delta}^r_{quota} = \frac{(1-\lambda)^2 b - k}{2(1-\lambda)^2 b},$$

and

$$\bar{\delta}^{g}_{price} = \frac{\left((1-\lambda)\left(\lambda b-(1-\lambda)\,k\right)\right)^{2}}{bk+2\left((1-\lambda)\left(\lambda b-(1-\lambda)\,k\right)\right)^{2}}.$$

The Optimal Sustainable Climate Policies. The following proposition characterizes the optimal sustainable climate policies when T is finite (since δ is larger than the critical discount factors) and when compliance constraints do not simultaneously bind (since λ is small).[11]

Proposition 4.

Under price regulation, the optimal sustainable climate policy (τ, ς) *is characterized by:*

P. *For* $\delta \in [\overline{\delta}^{g}_{price}, 1)$, *which requires* $\lambda < \lambda^{g}_{price}$,

$$\tau = \tau^{*} - \frac{c\,(n-1)\left((1-\lambda)^{2}\lambda b + (1-(1-\lambda)^{3})\,k\right)}{k} \quad \textit{and} \quad \varsigma = \tau^{*} - \tau.$$

Under quantity regulation, the optimal sustainable climate policy (g, r) *is characterized by:*

Q.i. *For* $\delta \in [\overline{\delta}^{g}_{quota}, 1)$, *which requires* $\lambda < \lambda^{g}_{quota}$,

$$g = g^{*} + \frac{c\,(n-1)\,(1-(1-\lambda)^{3})}{b} \quad \textit{and} \quad r = r^{*};$$

Q.ii. *For* $\delta \in [\overline{\delta}^{r}_{quota}, 1)$, *which requires* $\lambda < \lambda^{r}_{quota}$,

$$g = g^{*} + \frac{c(n-1)\,(\lambda k + (1-(1-\lambda)^{2})\,b)}{bk} \quad \textit{and} \quad r = r^{*} - \frac{c(n-1)\,(1-(1-\lambda)^{2})}{k}.$$

The Optimal Price Regulation. Part (P) of the proposition describes the optimal sustainable climate policy when governments use price instruments. Since the compliance constraint at the emission policy stage necessarily binds for any δ, the most cooperative outcome involves a carbon tax which is lower than the optimal global Pigouvian tax, in order to mitigate countries' temptation to deviate from the emission policy, and an investment subsidy in order to induce firms to invest in green technology at the first-best level—which is more than what they find optimal, conditional on negotiated emissions.

The Optimal Quantity Regulation. Parts (Q.i) and (Q.ii) describe the optimal sustainable climate policy when governments use quantity controls. When the compliance

[11] The characterization of the optimal sustainable climate policies when T is infinite is provided in the Online Appendix.

constraint at the emission policy stage binds first, emission caps must allow countries to pollute more than in the first-best case and investment mandates must be set at the first-best level. In contrast, when compliance with negotiated investment is more difficult, more emissions must be permitted, while more investment in green technology must be punished.

Welfare. In the case of type I errors, it is no longer true that price regulation is always better than quantity regulation. In particular, quota controls can achieve higher welfare than price instruments when the compliance constraint at the emission policy stage is the hardest to satisfy under both types of regulations, i.e., $\lambda < \lambda^g_{quota}$, which requires that $k/b > 1/2$.[12]

By inspecting parts (P) and (Q.i.), we can verify that—although investment is at the first-best level under both types of regulations—price regulation permits countries to pollute more than quantity regulation, which directly implies that quotas are preferable over policy instruments in this case. To see this, suppose that under price regulation, emissions are required to be less than the implemented level g, as determined in part (P), i.e, $g' < g$ (for example, g' is equal to the level under quantity regulation), but suppose also that investment is still forced to stay at the first-best level. To implement this allocation, the carbon tax should be set higher, say at $\tau' > \tau$, and the investment subsidy lower, say at $\varsigma' < \varsigma$, since technology is expensive, namely $k/b > 1/2$. Given these policies, firms in non-deviating countries adjust their investment to a level of $\widetilde{r}' = (\tau^b + \varsigma')/k < \widetilde{r} = (\tau^b + \varsigma)/k$ if they mistakenly observe a country deviating at the investment policy stage. This lower level of investment allows governments to punish deviations by means of higher emission levels, i.e., $\hat{g}^n(\widetilde{r}') > \hat{g}^n(\widetilde{r})$, even if deviations have not taken place. Overall, each country faces a larger welfare loss because of unduly strong punishments at the emission policy stage following mistakenly observed deviation at the investment policy stage.

Corollary 4. *When λ is small and technology is expensive, quantity regulation achieves a higher level of welfare than price regulation.*

The superiority of price instruments over quotas, established in Corollary 2, relies on the observation that allowing firms to flexibly adjust their investment decisions to an anticipated reversion to BAU—following another country's actual defection—permits governments to use stronger punishment (than under quantity regulation) and, in turn, to obtain more cooperation. The same logic applies in this case, but since uncertainty implies that the stronger punishment enforceable under price regulation can be triggered

[12]Clearly, when countries must be punished for overinvestment under quota regulation because they are more tempted to deviate at the investment policy stage (which requires that $k/b < 1/2$), while the same countries would not have been punished under price regulation, price instruments are preferable over quotas.

when it should not be, the welfare implication is the opposite of that in the case without uncertainty.

6 Timing

The baseline model relies on the sequential timing of the game, in which governments set emission policies after investment decisions have been made by the private sector. In this section, we demonstrate that when governments set emission and investment policies simultaneously, a lower level of welfare is attained. Hence, allowing for sequential timing of policy decisions is necessary in order to support the most efficient outcome.

To see this, consider a setting in which all governments decide upon climate policies at the beginning of each period and commit to them for the duration of the period. In this case, a government that is tempted to deviate from negotiated climate policies will always deviate from both emission and investment policies. The compliance constraint is then unique and given by:

$$V(g, r; \delta) \geq c(n-1)\left(g^b - g\right) + \frac{1}{1-\delta}u^b. \tag{14}$$

Constraint (14) is similar to constraint (8), such that if governments agree to reduce emissions, then the temptation to deviate from the negotiated agreement increases. However, since emission and investment policies are now implemented simultaneously, g is independent of the investment in renewable energy technology.

Proposition 5. *When governments commit to a climate emission policy and a climate investment policy at the beginning of each period, the optimal sustainable climate policy is characterized as follows*:

i. *Under price regulation, for $\delta > 1/2$, $\tau = \tau^*$ and $\varsigma = 0$, and for $\delta \in [0, 1/2]$,*

$$\tau = \tau^* - \Lambda_s(\delta) \quad \textit{and} \quad \varsigma = 0, \ \textit{where } \Lambda_s(\delta) \equiv (1-2\delta)\, c\,(n-1).$$

ii. *Under quantity regulation, for $\delta > 1/2$, $g = g^*$ and $r = r^*$, and for $\delta \in [0, 1/2]$,*

$$g = g^* + \frac{b+k}{bk}\Lambda_s(\delta) \quad \textit{and} \quad r = r(g) = r^* - \frac{1}{k}\Lambda_s(\delta), \ \textit{where } r(g) = \frac{b}{b+k}(\overline{y} - g).$$

iii. *For any $\delta \in [0, 1)$, quantity regulation and price regulation are welfare equivalent.*

Parts (i) and (ii) of Proposition 5 highlight an important implication of this model. If governments announce climate policies before the private sector's decisions and can commit to them, then it is optimal to decentralize the investment policies and let countries and firms decide on r themselves.

Part (iii) states that quantity and price regulations achieve the same level of welfare, even when the discount factor is so small that the first best cannot be sustained. We can then evaluate the welfare loss from relying solely on emission policies (as in the case in which governments decide on the climate policies simultaneously) instead of combining emission and investment policies (as in the case in which government policy decisions are made sequentially).

Corollary 5. *The optimal sustainable climate policy is associated with a higher intertemporal value when governments decide on investment and emission policies sequentially rather than simultaneously.*

The result is intuitive. When the policy decisions are made sequentially, the scope for free riding is limited to one policy at a time, before the defecting country risks being penalized. If the two policies are simultaneously decided on—or simultaneously observed—then defection becomes more tempting, since countries can free ride on both policy decisions rather than on just one of them.

7 Concluding Remarks

When compliance is challenging and treaties must be self-enforcing, it is crucial that countries be able to observe and detect whether their peers comply or defect. This fundamental insight is the driving force behind the following key results of the analysis: (1) Sequential decisions discourage defection more effectively than simultaneous decisions, since one can then defect under only one (rather than both) policy before being penalized. (2) To strengthen sequentiality, and thus smooth out the important decisions over time, it becomes necessary to overinvest in technology if technology is expensive relative to reducing consumption, but to underinvest if technology is relatively inexpensive. (3) Price regulation is, in the basic model, more effective than quantity regulation because the sequentiality of the decisions is strengthened when investing firms can react after observing the investment policy proposed in other countries. Thus, the driving force behind the three results is, at a deeper level, essentially the same. This fundamental insight also suggests that our results are quite robust and can be expected to apply in many other policy settings.

The most preliminary part of this research agenda is the connection to empirical evidence. Our predictions are testable and we do hope they will motivate empirical investigations. When it comes to global climate change treaties, however, we essentially have the Kyoto Protocol from 1997, and the Paris Agreement from 2015, so the number of data points is limited. In addition, there is reason to doubt that international negotiators have succeeded in arriving at the most efficient treaty. Thus, the world community can still benefit from new findinds on the best ways to cooperate. For these reasons, we are inclined to interpret our results normatively rather than positively.

In that vein, the results describe the least costly distortions that governments must accept when compliance is challenging. Alternatively, these distortions represent the (minimal) costs incurred by society due to the inability to commit to a policy or to immediately observe non-compliance. If policy decisions could be observed more rapidly and precisely, then the distortions characterized by our results would be smaller, exactly as if the discount factor had been increased. With better monitoring technology and greater transparency, the distortions of the emission and investment levels could be reduced, as emphasized above.

Given that the results have important policy implications, it is worth mentioning that the analysis is only a preliminary step toward a better understanding of what constitutes the best self-enforcing treaty. Thus, in order to keep the analysis tractable, we have restricted attention to linear-quadratic utility functions, homogeneous countries, and stationary production functions and have abstracted from fossil fuel trade and extraction, adaptation to climate change, renegotiation, and smarter ways of punishing defections than a simple reversion to the business-as-usual equilibrium. These generalizations need to be incorporated in future research on optimal self-enforcing treaties. Our primary contribution has been to provide a workhorse model that has proven to be sufficiently simple and tractable that it can be extended in several such directions.

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Appendix

Proposition 0 is shown in the text. The proofs of Corollaries 1 and 3 follow from the proofs of Propositions 1 and 3 proved in this Appendix.

Proof of Lemma 1. The proof follows from solving (7) and (8) satisfied with equality with respect to δ, when $g = g^*$ and $r = r^*$. ■

Proof of Proposition 1. Part (i) follows from Lemma 1.

Part (ii) holds when $k/b > 1/2$. In this case, $\overline{\delta}^g_{quota} > \overline{\delta}^r_{quota}$. For smaller δ that are close enough to $\overline{\delta}^g_{quota}$, the optimization problem becomes:

$$V_{quota}(\delta) \equiv \max_{g,r} V(g,r;\delta) = \max_{g,r} \frac{1}{1-\delta}\left(-\frac{b}{2}(\overline{y}-(g+r))^2 - cng - \frac{k}{2}r^2\right), \tag{15}$$

subject to:

$$V(g,r;\delta) \geq V^g(g,r;\delta) = -\frac{b}{2}\left(\overline{y}-\left(\hat{g}^d(r)+r\right)\right)^2 - c\left(\hat{g}^d(r)+(n-1)g\right) - \frac{k}{2}r^2 + \frac{\delta u^b}{1-\delta}, \tag{16}$$

where $\hat{g}^d(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(g,r;\delta) = V^g(g,r;\delta)$ and solving for g, we obtain:

$$g = g^* + \Lambda_{quota}(\delta), \text{ where } \Lambda_{quota}(\delta) = \frac{c(n-1)}{b}\left(1-\delta-\sqrt{\frac{\delta(b+\delta k)}{k}}\right),$$

and $\partial\Lambda(\delta)/\partial\delta < 0$. Hence, $r = r^* = r(g) + (b/(b+k))\Lambda_{quota}(\delta)$, where $r(g) = (b/(b+k))(\overline{y}-g)$.

Part (iii) holds when $k/b < 1/2$. In this case, $\overline{\delta}^g_{quota} < \overline{\delta}^r_{quota}$. For smaller δ that are close enough to $\overline{\delta}^r_{quota}$, the optimization problem consists in maximizing (15), subject to:

$$V(g,r;\delta) \geq V^r(r;\delta) = c(n-1)\left(g^b - \hat{g}^n(r)\right) + \frac{u^b}{1-\delta}, \tag{17}$$

where $g^b = \overline{y} - r^b - c/b$ and $\hat{g}^n(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = \overline{y} - g - cn/b$. Substituting the equilibrium r into $V(g,r;\delta) = V^r(r;\delta)$ and solving for g, we obtain:

$$g = g^* + \Xi_{quota}(\delta), \text{ where } \Xi_{quota}(\delta) = \frac{c(n-1)}{k}\left(1-\delta-\sqrt{\frac{k+\delta^2 b}{b}}\right),$$

and $\partial\Xi_{quota}(\delta)/\partial\delta < 0$. Hence, $r = r^* - \Xi_{quota}(\delta)$.

Part (iv) is obtained by requiring (16) and (17) to be simultaneously satisfied when $\delta \leq \min\left\{\delta^g_{quota}(g,r), \delta^r_{quota}(g,r)\right\}$, where $\delta^r_{quota}(g,r)$ is obtained by solving $V(g,r;\delta) = V^r(r;\delta)$ with respect to δ for the pair (g,r) determined in part (ii) and $\delta^g_{quota}(g,r)$ is obtained by solving $V(g,r;\delta) = V^g(g,r;\delta)$ with respect to δ for the pair (g,r) determined in part (iii). As δ tends to zero, the optimal sustainable climate policy approaches the levels $g = g^b$ and $r = r^b$. ■

Proof of Lemma 2. The result follows from the observation made in the text that compliance constraint at the investment policy stage never binds first under price regulation. ■

Proof of Proposition 2. Part (i) follows from Lemma 2.

To demonstrate part (ii), we solve the following optimization problem, for smaller δ that are close enough to $\overline{\delta}^g_{price}$:

$$\begin{aligned} V_{price}(\delta) &\equiv \max_{\tau,\varsigma} V(\tau,\varsigma;\delta) \\ &= \max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y} - \left(g(\tau,\varsigma) + r(\tau,\varsigma)\right)\right)^2 - cng(\tau,\varsigma) - \frac{k}{2} r(\tau,\varsigma)^2\right), \end{aligned}$$

subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned} V(\tau,\varsigma;\delta) &\geq V^g(\tau,\varsigma;\delta) \\ &= -\frac{b}{2}\left(\overline{y} - \left(\hat{g}^d(\tau,\varsigma) + r(\tau,\varsigma)\right)\right)^2 - c\left(\hat{g}^d(\tau,\varsigma) + (n-1)g(\tau,\varsigma)\right) - \frac{k}{2}(\tau,\varsigma)^2 + \frac{\delta u^b}{1-\delta}, \end{aligned} \tag{18}$$

where $\hat{g}^d(\tau,\varsigma) = \overline{y} - r(\tau,\varsigma) - c/b$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Substituting the equilibrium ς into $V(\tau,\varsigma;\delta) = V^g(\tau,\varsigma;\delta)$ and solving for τ, we obtain:

$$\tau = \tau^* - \Lambda_{price}(\delta), \text{ where } \Lambda_{price}(\delta) = c(n-1)\left(1 - \delta - \sqrt{\frac{\delta(b+\delta k)}{k}}\right),$$

and $\partial\Lambda_{price}(\delta)/\partial\delta < 0$. Hence, $\varsigma = \Lambda_{price}(\delta)$.

Part (iii) can be shown by requiring (18) and the compliance constraint at the investment policy stage,

$$V(\tau,\varsigma;\delta) \geq V^r(\varsigma;\delta) = \frac{c(n-1)}{k}\left(\varsigma - \varsigma^b\right) + \frac{u^b}{1-\delta},$$

to be simultaneously satisfied when $\delta < \delta^r_{price}(\tau,\varsigma)$, where $\delta^r_{price}(\tau,\varsigma)$ is obtained by solving $V(\tau,\varsigma;\delta) = V^r(\varsigma;\delta)$ with respect to δ for the pair (τ,ς) determined in part (ii). When δ tends to zero, the optimal sustainable climate policy approaches the levels $\tau = \tau^b$ and $\varsigma = \varsigma^b$. ■

Proof of Corollary 2. Let $\mathcal{L}_j(\delta) \equiv u^*/(1-\delta) - V_j(\delta)$ for each policy instrument $j \in \{quota, price\}$, where $V_j(\delta)$ is reported in Propositions 1 and 2. Under price regulation, we have:

$$\mathcal{L}_{price}(\delta) = \begin{cases} 0 & if \quad \delta \in [\bar{\delta}^g_{price}, 1) \\ (1/2b)\,\Lambda^2_{price}(\delta) & if \quad \delta \in [\delta^r_{price}(\tau,\varsigma), \bar{\delta}^g_{price}) \end{cases}.$$

Under quantity regulation, we have:

$$\mathcal{L}_{quota}(\delta) = \begin{cases} \begin{array}{lll} 0 & if & \delta \in [\bar{\delta}^g_{quota}, 1) \\ (b/2)\,\Lambda^2_{quota}(\delta) & if & \delta \in [\delta^r_{quota}(g,r), \bar{\delta}^g_{quota}) \end{array}, & \text{for } k > b/2 \\ \\ \begin{array}{lll} 0 & if & \delta \in [\bar{\delta}^r_{quota}, 1) \\ (b/2)\,\Xi^2_{quota}(\delta) & if & \delta \in [\delta^g_{quota}(g,r), \bar{\delta}^r_{quota}) \end{array}, & \text{for } k < b/2 \end{cases},$$

where $\Lambda_{quota}(\delta) = (1/b)\Lambda_{price}(\delta)$. It follows that $\mathcal{L}_{price}(\delta) = \mathcal{L}_{quota}(\delta)$ when $k > b/2$ and $\mathcal{L}_{price}(\delta) < \mathcal{L}_{quota}(\delta)$ when $k < b/2$, since $\bar{\delta}^r_{quota} > \bar{\delta}^g_{price}$ and $\Xi_{quota}(\delta) > \Lambda_{quota}(\delta)$ for $\delta \in [\max\{\delta^r_{price}(\tau,\varsigma), \delta^g_{quota}(g,r)\}, \bar{\delta}^g_{price}]$. ■

Proof of Lemma 3. The proof follows from solving simultaneously the compliance constraint at the emission policy stage and the compliance constraint at the investment policy stage satisfied with equality with respect to δ, when $g = g^*$ and $r = r^*$ under quantity regulation and $\tau = \tau^*$ and $\varsigma = \varsigma^*$ under price regulation. ■

Proof of Proposition 3. First, consider price regulation when $\delta < \max\left\{\bar{\delta}^r_{price}, \bar{\delta}^g_{price}\right\}$. Part (P.i) holds when $\eta < \eta_{price} \equiv (b+2k)/(2b+k)$. In this case, $\bar{\delta}^g_{price} > \bar{\delta}^r_{price}$. For smaller δ that are close enough to $\bar{\delta}^g_{price}$, the optimization problem is:

$$V^\eta_{price}(\delta) \equiv \max_{\tau,\varsigma} V(\tau,\varsigma;\delta) = \max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}(\bar{y} - (g(\tau,\varsigma) + r(\tau,\varsigma)))^2 - cng(\tau,\varsigma) - \frac{k}{2}r(\tau,\varsigma)^2\right), \tag{19}$$

subject to $g(\tau,\varsigma) = \bar{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned} V(\tau,\varsigma;\delta) &\geq V^g(\tau,\varsigma;\delta) \\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}\left(\bar{y} - \left(\hat{g}^d(\tau,\varsigma) + r(\tau,\varsigma)\right)\right)^2 - c\left((n-1)g(\tau,\varsigma) + \hat{g}^d(\tau,\varsigma)\right) - \frac{k}{2}(\tau,\varsigma)^2\right), \\ &\quad + \frac{1}{1-\delta\eta}\frac{\delta(1-\eta)u^b}{1-\delta} \end{aligned}$$

where $\hat{g}^d(\tau,\varsigma) = \bar{y} - r(\tau,\varsigma) - c/b$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Replacing the equilibrium ς into $V(\tau,\varsigma;\delta) = V^g(\tau,\varsigma;\delta)$ and solving for τ, we obtain:

$$\begin{aligned} &\tau = \tau^* - \Lambda^\eta_{price}(\delta,\eta), \text{ where} \\ &\Lambda^\eta_{price}(\delta,\eta) = \frac{c(n-1)}{1-\eta\delta}\left(1 - \delta - \sqrt{\frac{\delta(1-\eta)((1-\eta\delta)b + (1-\eta)\delta k)}{k}}\right), \end{aligned}$$

and $\partial\Lambda^{\eta}_{price}(\delta,\eta)/\partial\delta < 0$ and $\partial\Lambda^{\eta}_{price}(\delta,\eta)/\partial\eta > 0$. Hence, $\varsigma = \Lambda^{\eta}_{price}(\delta,\eta)$.

Part (P.ii) holds when $\eta > \eta_{price}$. In this case, $\overline{\delta}^{g}_{price} < \overline{\delta}^{r}_{price}$. For smaller δ that are close enough to $\overline{\delta}^{r}_{price}$, the optimization problem consists in maximizing (19), subject to:

$$V(\tau,\varsigma;\delta) \geq V^{r}(\tau,\varsigma;\delta) = \frac{c(n-1)}{1-\delta\eta^{2}}\left(g^{b} - \hat{g}^{n}_{\eta}(\widetilde{r})\right) + \frac{u^{b}}{1-\delta},$$

where $\hat{g}^{n}_{\eta}(\widetilde{r}) = \eta g(\tau,\varsigma) + (1-\eta)\hat{g}^{n}(\widetilde{r})$, $\hat{g}^{n}(\widetilde{r}) = \overline{y} - \widetilde{r} - c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for ς, we obtain $\varsigma = -((1-\eta)(k+b)/(\eta k - (1-\eta)b))(\tau^{*}-\tau)$. Replacing the equilibrium ς into $V(\tau,\varsigma;\delta) = V^{r}(\tau,\varsigma;\delta)$ and solving for τ, we obtain:

$$\begin{aligned} &\tau = \tau^{*} - \Xi^{\eta}_{price}(\delta,\eta)\,, \text{ where} \\ &\Xi^{\eta}_{price}(\delta,\eta) = \frac{c(n-1)(\eta k - (1-\eta)b)}{(1-\eta^{2}\delta)k}\left(1-\delta-(1-\eta)\sqrt{\frac{(1+\delta\eta)^{2}k + (1-\delta)^{2}b}{\eta^{2}k + (1-\eta)^{2}b}}\right), \end{aligned}$$

and $\partial\Xi^{\eta}_{price}(\delta,\eta)/\partial\delta < 0$ and $\partial\Xi^{\eta}_{price}(\delta,\eta)/\partial\eta > 0$. Hence,

$$\varsigma = -\frac{(1-\eta)(k+b)}{\eta k - (1-\eta)b}\Xi^{\eta}_{price}(\delta,\eta)\,.$$

Next, consider quantity regulation when $\delta < \max\left\{\overline{\delta}^{r}_{quota}, \overline{\delta}^{g}_{quota}\right\}$. Part (Q.i) holds when $\eta < \eta_{quota} \equiv 2 - b/k$. In this case, $\overline{\delta}^{g}_{quota} > \overline{\delta}^{r}_{quota}$. For smaller δ that are close enough to $\overline{\delta}^{g}_{quota}$, the optimization problem becomes:

$$V^{\eta}_{quota}(\delta) \equiv \max_{g,r} V(g,r;\delta) = \max_{g,r}\frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-(g+r)\right)^{2} - cng - \frac{k}{2}r^{2}\right), \tag{20}$$

subject to:

$$\begin{aligned} V(g,r;\delta) &\geq V^{g}(g,r;\delta) \\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}\left(\overline{y}-\left(\hat{g}^{d}(r)+r\right)\right)^{2} - c\left(\hat{g}^{d}(r)+(n-1)g\right) - \frac{k}{2}r^{2} + \frac{\delta(1-\eta)u^{b}}{1-\delta}\right), \end{aligned}$$

where $\hat{g}^{d}(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^{*}$. Substituting the equilibrium r into $V(g,r;\delta) = V^{g}(g,r;\delta)$ and solving for g, we obtain:

$$g = g^{*} + \Lambda^{\eta}_{quota}(\delta,\eta)\,, \text{ where } \Lambda^{\eta}_{quota}(\delta,\eta) = \frac{1}{b}\Lambda^{\eta}_{price}(\delta,\eta)\,,$$

and $\partial\Lambda^{\eta}_{quota}(\delta,\eta)/\partial\delta < 0$ and $\partial\Lambda^{\eta}_{quota}(\delta,\eta)/\partial\eta > 0$.

Part (Q.ii) holds when $\eta > \eta_{quota}$. In this case, $\overline{\delta}^{g}_{quota} < \overline{\delta}^{r}_{quota}$. For smaller δ that are close enough to $\overline{\delta}^r_{quota}$, the optimization problem consists in maximizing (20), subject to:

$$V(g,r;\delta) \geq V^r(g,r;\delta) = \frac{c(n-1)}{1-\delta\eta^2}\left(g^b - \hat{g}^n_\eta(r)\right) + \frac{u^b}{1-\delta},$$

where $\hat{g}^n_\eta(r) = \eta g + (1-\eta)\hat{g}^n(r)$ and $\hat{g}^n(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = (b/(b+\eta k))(\overline{y}-g) - (1-\eta)cn/(b+\eta k)$. Substituting the equilibrium r into $V(g,r;\delta) = V^r(g,r;\delta)$ and solving for g, we obtain:

$$\begin{aligned} g &= g^* + \Xi^\eta_{quota}(\delta,\eta), \text{ where} \\ \Xi^\eta_{quota}(\delta,\eta) &= \frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)bk}\left(1-\delta-\sqrt{\frac{((1-\eta^2)\delta)^2 b + ((1-\eta)(1+\delta\eta))^2 k}{b+\eta^2 k}}\right), \end{aligned}$$

and $\partial\Xi^\eta_{quota}(\delta,\eta)/\partial\delta < 0$ and $\partial\Xi^\eta_{quota}(\delta,\eta)/\partial\eta > 0$. Hence,

$$r = r^* - \frac{b}{b+\eta k}\Xi^\eta_{quota}(\delta,\eta).$$

■

Proofs of Lemma 4 and Proposition 4. First, consider price regulation. To demonstrate part (P), assume that compliance constraint at the investment policy stage is slack, while (13) binds. In this case, the optimal climate policy is obtained by solving the following maximization problem:

$$\begin{aligned} V^\lambda_{price}(\delta) &\equiv \max_{\tau,\varsigma} V(\tau,\varsigma;\delta) \\ &= \max_{\tau,\varsigma}\frac{1}{1-\delta}\left(u(g(\tau,\varsigma),r(\tau,\varsigma)) - c(n-1)\left(\hat{g}^n_\lambda(\widetilde{r}) - g(\tau,\varsigma)\right) - \frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(\tau,\varsigma)\right), \end{aligned}$$

where $\psi_g(\tau,\varsigma) \equiv (c - b(\overline{y}-(g+r)))^2/2b$, $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$, $r(\tau,\varsigma) = (\varsigma+\tau)/k$, $\hat{g}^n_\lambda(\widetilde{r}) = (1-\lambda)g(\tau,\varsigma) + \lambda(\overline{y} - \widetilde{r} - c/b)$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for τ and ς, we obtain:

$$\tau = \tau^* - \frac{c(n-1)((1-\lambda)^2\lambda b + (1-(1-\lambda)^3)k)}{k} \quad \text{and} \quad \varsigma = \tau^* - \tau, \tag{21}$$

which implements the allocation:

$$g = g^* + c(n-1)\left(\frac{\lambda(1-\lambda)^2}{k} + \frac{1-(1-\lambda)^3}{b}\right) \quad \text{and} \quad r = r^*.$$

Combining $V^P(\tau,\varsigma;\delta) = \left(\left(1-\delta^T\right)/(1-\delta)\right)u^b + \delta^T V(\tau,\varsigma;\delta)$ and $V(\tau,\varsigma;\delta) - V^P(\tau,\varsigma;\delta) =$

$(1/(\delta(1-\lambda)^2))\,\psi_g(\tau,\varsigma)$ and using (21), we obtain:

$$T = [\log(\delta)]^{-1} \log(\Gamma^g_{price}(\delta)) - 1, \tag{22}$$

where $\Gamma^g_{price}(\delta) \equiv ((1-\delta)\,bk)\,/(bk + (1-\lambda)^2\,(\lambda b - (1-\lambda)\,k)^2) + 2\delta - 1$. By solving $\Gamma^g_{price}(\delta) = 0$ with respect to δ, we obtain:

$$\bar{\delta}^g_{price} = \frac{((1-\lambda)\,(\lambda b - (1-\lambda)\,k))^2}{bk + 2\,((1-\lambda)\,(\lambda b - (1-\lambda)\,k))^2}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (21) and the length of punishment is (22), by verifying that:

$$V(\tau,\varsigma;\delta) - V^P(\tau,\varsigma;\delta) \geq \frac{\psi_r(\tau,\varsigma)}{\delta(1-\lambda)^2}, \tag{23}$$

where $\psi_r(\tau,\varsigma) \equiv c\,(n-1)\left(g^b - \hat{g}^n(\widetilde{r})\right) - \left(u\,(g,r) - u^b\right)$, $\hat{g}^n(\widetilde{r}) = \overline{y} - \widetilde{r} - c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Inequality (23) is satisfied when $\lambda \leq \lambda^g_{price}$, where λ^g_{price} is implicitly determined from (23). It follows that both compliance constraints at the emission and investment policy stages simultaneously bind when $\lambda > \lambda^g_{price}$.

Next, consider quantity regulation. To show part (Q.i), assume that compliance constraint at the investment policy stage is slack, while (13) binds. In this case, the optimal climate policy is obtained by solving the following maximization problem:

$$V^{\lambda}_{quota}(\delta) \equiv \max_{g,r} V(g,r;\delta) = \max_{g,r} \frac{1}{1-\delta}\left(u(g,r) - c\,(n-1)\,(\hat{g}^n_{\lambda}(r) - g) - \frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(g,r)\right),$$

where $\psi_g(g,r) \equiv (c - b\,(\overline{y} - (g+r)))^2/2b$ and $\hat{g}^n_{\lambda}(r) = (1-\lambda)\,g + \lambda\,(\overline{y} - r - c/b)$. Solving for g and r, we obtain:

$$g = g^* + \frac{c\,(n-1)\,(1-(1-\lambda)^3)}{b} \quad \text{and} \quad r = r^*. \tag{24}$$

Combining $V^P(g,r;\delta) = \left(\left(1-\delta^T\right)/(1-\delta)\right)u^b + \delta^T V(g,r;\delta)$ and $V(g,r;\delta) - V^P(g,r;\delta) = (1/(\delta(1-\lambda)^2))\psi_g(g,r)$ and using (24), we obtain, we obtain:

$$T = [\log(\delta)]^{-1} \log\left(\Gamma^g_{quota}(\delta)\right) - 1, \tag{25}$$

where $\Gamma^g_{quota}(\delta) \equiv (\delta b - (1-2\delta)\,(1-\lambda)^4 k)/(b + (1-\lambda)^4 k)$. By solving $\Gamma^g_{quota}(\delta)$ with respect to δ, we obtain:

$$\bar{\delta}^g_{quota} = \frac{(1-\lambda)^4\,k}{b + 2\,(1-\lambda)^4\,k}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (24) and the length of punishment is (25). We verify that:

$$V\left(g,r;\delta\right)-V^{P}\left(g,r;\delta\right)>\frac{\psi_{r}\left(g,r\right)}{\delta\left(1-\lambda\right)^{2}},\tag{26}$$

where $\psi_r\left(g,r\right)\equiv c\left(n-1\right)\left(g^{b}-\hat{g}^{n}\left(r\right)\right)-\left(u\left(g,r\right)-u^{b}\right)$ and $\hat{g}^{n}\left(r\right)=\overline{y}-r-c/b$. Inequality (26) is satisfied when $\lambda<\lambda_{quota}^{g}\equiv1-\left(b/2k\right)^{1/4}$, which $k/b>1/2$.

To show part (Q.ii), assume that (13) is slack, while compliance constraint at the investment policy stage binds. In this case, the optimal climate policy is obtained by solving:

$$V_{quota}^{\lambda}\left(\delta\right)\equiv\max_{g,r}V\left(g,r;\delta\right)=\max_{g,r}\frac{1}{1-\delta}\left(u(g,r)-c\left(n-1\right)\left(\hat{g}_{\lambda}^{n}\left(r\right)-g\right)-\frac{p}{q_r-p}\psi_r\left(g,r\right)\right),$$

and it is equal to:

$$g=g^{*}+\frac{c(n-1)((1-(1-\lambda)^{2})b+\lambda k)}{bk}\quad\text{and}\quad r=r^{*}-\frac{c(n-1)(1-(1-\lambda)^{2})}{k}.\tag{27}$$

Using (27), the optimal T is obtained by solving $V\left(g,r;\delta\right)-\left(1/\left(1-\delta\right)\right)u^{b}=(1/(\delta(1-\delta^{T})\left(1-\lambda\right)^{2}))\psi_r\left(g,r\right)$, which is equal to:

$$T=\left[\log(\delta)\right]^{-1}\log\left(\Gamma_{quota}^{r}\left(\delta\right)\right)-1,\tag{28}$$

where $\Gamma_{quota}^{r}\left(\delta\right)\equiv(k-\left(1-\lambda\right)^{2}\left(1-2\delta\right)b)/(\left(1-\lambda\right)^{2}b+k)$. Solving $\Gamma_{quota}^{r}\left(\delta\right)$ with respect to δ, we obtain:

$$\bar{\delta}_{quota}^{r}=\frac{\left(1-\lambda\right)^{2}b-k}{2\left(1-\lambda\right)^{2}b}.$$

We next show under which condition compliance constraint at the emission policy stage is satisfied when climate policies are by (27) and the length of punishment is (28). We verify that:

$$V\left(g,r;\delta\right)-V^{P}\left(g,r;\delta\right)>\frac{\psi_{g}\left(g,r\right)}{\delta\left(1-\lambda\right)^{2}}.\tag{29}$$

Condition (29) is satisfied when $\lambda<\lambda_{quota}^{r}\equiv1-\left(2k/b\right)^{1/2}$, which requires $k/b<1/2$. It follows that both compliance constraint at the emission and investment policy stages simultaneously bind when $\lambda>\max\left\{\lambda_{quota}^{g},\lambda_{quota}^{r}\right\}$. ■

Proof of Corollary 4. When $\lambda<\lambda_{quota}^{g}$, which requires $k/b>1/2$, we obtain $V_{quota}^{\lambda}\left(\delta\right)-V_{price}^{\lambda}\left(\delta\right)=c^{2}(n-1)^{2}\left(1-\lambda\right)^{2}\lambda(2k\left(1-\lambda\right)-b\lambda)/2k^{2}>0$, where $V_{quota}^{\lambda}\left(\delta\right)$ and $V_{price}^{\lambda}\left(\delta\right)$ are reported in Proposition 4. ■

Proof of Proposition 5. To demonstrate part (i), we derive the optimal sustainable climate policy under price regulation. The threshold level of δ, above which the first best can be sustained, is $\bar{\delta} = 1/2$, which is obtained by solving $V\left(\tau^*, \varsigma^*; \bar{\delta}\right) = \hat{V}\left(\tau^*, \varsigma^*; \bar{\delta}\right)$, where $V(\tau, \varsigma; \delta) = (1/(1-\delta))\left(-(b/2)(\overline{y} - (g(\tau, \varsigma) + r(\tau, \varsigma)))^2 - cng(\tau, \varsigma) - (k/2)\, r(\tau, \varsigma)^2\right)$ and $\hat{V}(\tau, \varsigma; \delta) = c(n-1)\left(g^b - g(\tau, \varsigma)\right) + u^b/(1-\delta)$, subject to $g(\tau, \varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau, \varsigma) = (\varsigma + \tau)/k$.

When $\delta \in [0, 1/2)$, the inequality $V(\tau^*, \varsigma^*; \delta) \geq \hat{V}(\tau^*, \varsigma^*; \delta)$ is not satisfied. Hence, the equilibrium policies are obtained by solving the following maximization problem:

$$V_s(\delta) = \max_{\tau, \varsigma} V(\tau, \varsigma; \delta),$$

subject to:

$$V(\tau, \varsigma; \delta) \geq \hat{V}(\tau, \varsigma; \delta). \tag{30}$$

Solving for ς, we obtain $\varsigma = 0$. Substituting the equilibrium ς into (30), we obtain:

$$\tau = \tau^* - \Lambda_s(\delta), \text{ where } \Lambda_s(\delta) \equiv (1 - 2\delta)\, c(n-1).$$

To demonstrate part (ii), we derive the optimal sustainable climate policy under quantity regulation by solving the maximization problem described in part (i) with respect to g and r directly. It is immediate to verify that $g = g^*$ and $r = r^*$ when $\delta > 1/2$, and

$$g = g^* + \frac{b+k}{bk}\Lambda_s(\delta) \text{ and } r = r^* - \frac{1}{k}\Lambda_s(\delta)$$

when $\delta \in [0, 1/2]$.

Finally, to demonstrate part (iii), notice that the equilibrium taxes and subsidies determined in part (i) implement emissions and investment levels equal to those in part (ii). Hence, we conclude that quantity regulation and price regulation achieves the same welfare for any $\delta \in [0, 1)$. ■

Proof of Corollary 5. Let $\mathcal{L}_s(\delta) \equiv u^*/(1-\delta) - V_s(\delta)$, where $V_s(\delta)$ is reported in Proposition 5:

$$\mathcal{L}_s(\delta) = \begin{cases} 0 & if \quad \delta \in [1/2, 1) \\ \left(\frac{b+k}{b}\Lambda_s(\delta) + cn\right)\frac{\Lambda_s(\delta)}{2k} & if \quad \delta \in [0, 1/2) \end{cases},$$

Under sequential climate policy decisions, the first best can be sustained for a wider range of δ than under simultaneous climate policy decisions, since $\max\left\{\bar{\delta}^g_{quota}, \bar{\delta}^r_{quota}\right\} < 1/2$, where threshold levels of δ are reported in Lemma 1. Furthermore, $\mathcal{L}_s(\delta) \geq \mathcal{L}_{quota}(\delta) \geq \mathcal{L}_{price}(\delta)$, where $\mathcal{L}_{quota}(\delta)$ and $\mathcal{L}_{price}(\delta)$ are reported in Corollary 2. By contradiction, suppose that $\mathcal{L}_s(\delta) < \mathcal{L}_{quota}(\delta)$. This implies that $\delta > (1/2)\sqrt{b/(b+k)}$ when $k/b < 1/2$ and $\delta > (b + k + \sqrt{k(b+k)})/4(b+k)$ when $k/b > 1/2$, which cannot be true since in the former case $\delta < \bar{\delta}^r_{quota} < (1/2)\sqrt{b/(b+k)}$, while in the latter case $\delta < \bar{\delta}^g_{quota} <$

$(b + k + \sqrt{k(b + k)})/4(b + k)$. ■

Online Appendix

These appendices present supplementary material referenced in the paper. Appendix A introduces the possibility of investing in brown technologies in addition to green technologies. Appendix B characterizes the optimal sustainable climate policies under uncertainty when only type I errors are present and T is infinity because the discount factor is small.

Appendix A. Multiple Technologies

We have so far assumed that a country can invest only in renewable energy technology, which reduces the marginal value of emitting an additional unit of pollution. In reality, however, energy can be produced from various sources, some of which are "brown,"such as drilling technology, which is beneficial in the extraction and consumption of fossil fuel and therefore complementary to polluting.[13] We can then expand the baseline model by permitting countries to invest in a technology portfolio, which includes clean and renewable technology r_{Ci} and brown technology r_{Bi}. The benefit function of country i is modified as follows:

$$B\left(g_i, r_{Bi}, r_{Ci}\right) = -\frac{b}{2}\left(\overline{y} - \left(g_i + r_{Ci}\right)\right)^2 - \frac{q}{2}\left(g_i - r_{Bi}\right)^2 + z r_{Ci} r_{Bi},$$

with $q > 0$ and $z \in (\underline{z}, \overline{z})$. The quadratic term $-\left(q/2\right)\left(g_i - r_{Bi}\right)^2$ represents the cost of extracting fossil fuel beyond the capacity level r_{Bi}. The capacity to provide fossil fuel is a brown technology since it reduces the cost of providing g_i and thus increases the net benefit from consuming fossil fuel. Brown technologies and clean technologies can be interdependent. The parameter z is positive when energy produced from, for example, solar or wind power is complemented by traditional fossil fuel sources in order to ensure a constant flow of electricity. In contrast, the case of substitute technologies arises when $z \leq 0$. We allow the cost of investment $\left(k_\sigma/2\right) r_{\sigma i}^2$ for each $\sigma \in \{C, B\}$, where $k_\sigma > 0$, to vary across technologies.

Lemma A.1 *When $n > 1$, we have $g^* < g^b$, $r_B^* > (\leq)\, r_B^b$ for $z > (\leq)\, (b + k_C)q/b$ and $r_C^* > (\leq)\, r_C^b$ for $z < (\geq)\, (q + k_B)b/q$.*

Clearly, the first-best level of emissions is lower than the BAU level, since global environmental damage from pollution is internalized only in the first-best case. The first-best levels of investment in technologies, however, can be higher or lower than the BAU levels depending on the type of technology and on the degree of interdependence between types. If technologies are substitutes, namely $z \leq 0$, then there will be more investment

[13] In Harstad, Lancia, and Russo (2019), we studied how technologies of different types affect incentives to comply with emissions when g is a binary variable, and thus we abstracted from climate policy implications.

in clean technologies and less in brown ones in the first-best case relative to the case in which neither emissions nor investment are internationally negotiated. If technologies are complements, namely $z > 0$, then investment in both types of technologies must be either larger (when $z > (b + k_C)q/b$) or smaller (when $z \geq (q + k_B)b/q$) in the first-best case than in BAU.[14]

We have shown that global welfare is higher under a price-based treaty than under a quota-based treaty in the absence of uncertainty. Clearly, the result also holds true in the case of investment in multiple technologies. Therefore, we focus here on the characterization of the optimal sustainable climate policies when national governments use price instruments. As shown in Section 4, it is optimal to implement a global Pigouvian carbon tax equal to $\tau^* = cn$ and investment subsidies for brown and clean technologies equal to $\varsigma_B^* = \varsigma_C^* = 0$, when δ is sufficiently large. As δ becomes smaller, emissions must be allowed to increase in order to provide incentives for governments to comply with the agreement by reducing the emission tax. However, the dampening of the emission tax is partially mitigated by the introduction of subsidies for investment in clean technologies and by taxes imposed on firms investing in dirty technologies. The fiscal distortion generated by the friction of limited enforcement is quantified in the following proposition.

Proposition A.1 *For $\delta \in (\delta^r_{price}(\tau, \varsigma_B, \varsigma_C), \overline{\delta}^g_{price}]$, the optimal sustainable climate policy $(\tau, \varsigma_B, \varsigma_C)$ is characterized by:*

$$\tau^* - \tau = \Lambda_m(\delta, z) > 0,$$

and

$$\varsigma_B - \varsigma_B^* = -(q/(b+q))\Lambda_m(\delta, z) \text{ and } \varsigma_C - \varsigma_C^* = (b/(b+q))\Lambda_m(\delta, z),$$

$\partial \Lambda_m(\delta, z) / \partial \delta < 0$, and

$$\frac{\partial \Lambda_m(\delta, z)}{\partial z} \begin{cases} \geq 0 & if \quad z \leq \min\left\{(b + k_C)\frac{q}{b}, (q + k_B)\frac{b}{q}\right\}, \\ < 0 & if \quad z > \min\left\{(b + k_C)\frac{q}{b}, (q + k_B)\frac{b}{q}\right\}. \end{cases}$$

Proposition A.1 provides new insights by generalizing the results of Proposition 2 to an environment in which countries invest in a technology portfolio. Tax distortions from first-best policies, denoted by $\Lambda_m(\delta, z)$, are greater when the discount rate is lower. Such distortions are also affected by the elasticity of substitution between brown technologies and clean technologies in a non-linear way. In particular, $\Lambda_m(\delta, z)$ is largest when $z = \min\{(b + k_C)q/b, (q + k_B)b/q\}$, since either $r_B^* = r_B^b$ or $r_C^* = r_C^b$. Intuitively, when $r_\sigma^* = r_\sigma^b$ for some σ, it is optimal to implement climate policies so that the domestic private sector

[14]The former occurs when $q/b < \sqrt{(q + k_B) / (b + k_C)}$, that is, when the use of clean technologies is more effective than the use of brown technologies, while the latter occurs in the opposite case.

responds by investing in technology σ at the first-best level, i.e., $r_\sigma(\tau, \varsigma_B, \varsigma_C) = r_\sigma^*$, which restricts the ability of the two complementary investment policies, ς_B and ς_C, to curb fiscal distortions.[15]

Proof of Lemma A.1. The first-best levels of emissions and investment in brown and clean technology are obtained by solving:

$$\max_{g, r_B, r_C} -\frac{b}{2}\left(\overline{y} - (g + r_C)\right)^2 - \frac{q}{2}(g - r_B)^2 + z r_C r_B - cng - \frac{k_B}{2} r_B^2 - \frac{k_C}{2} r_C^2,$$

and are equal to:

$$\begin{aligned} g^* &= \left(\left((q + k_B)k_C + (q - z)\, z\right) \overline{y} b + cn\left(z^2 - (q + k_B)(b + k_C)\right)\right)/\varrho, \\ r_B^* &= \left((z + k_C)\, \overline{y} bq - cn\left((b + k_C)q - bz\right)\right)/\varrho, \text{ and} \\ r_C^* &= \left((z + k_B)\, \overline{y} bq + cn\left((q + k_B)b - qz\right)\right)/\varrho, \end{aligned}$$

where $\varrho \equiv (b + q)\left(k_B k_C - z^2\right) + bq\,(2z + k_B + k_C) > 0$ for $z \in (\underline{z}, \overline{z})$ with

$$\begin{aligned} \underline{z} &\equiv \frac{bq - \sqrt{(bq + (b + q)\, k_B)\,(bq + (b + q)\, k_C)}}{b + q}, \\ \overline{z} &\equiv \frac{bq + \sqrt{(bq + (b + q)\, k_B)\,(bq + (b + q)\, k_C)}}{b + q}. \end{aligned}$$

Since $z < \overline{z}$, then $g^b > g^*$. Furthermore, $r_B^*(n) > (\leq)\, r_B^*(1) = r_B^b$ when $z > (\leq)\, (b + k_C)q/b$ and $r_C^*(n) > (\leq)\, r_C^*(1) = r_C^b$ when $z < (\geq)\, (q + k_B)b/q$. ■

Proof of Proposition A.1. The optimal sustainable climate policy is obtained by solving $\max_\chi V(\chi; \delta)$, where $\chi \equiv (\tau, \varsigma_B, \varsigma_C)$ and $V(\chi; \delta)$ is equal to:

$$-\frac{1}{1 - \delta}\left(\frac{b}{2}\left(\overline{y} - (g + r_C(\chi))\right)^2 + \frac{q}{2}\left(g - r_B(\chi)\right)^2 - z r_C(\chi)\, r_B(\chi) + cng(\chi) + \sum_{\sigma \in \{B, C\}} \frac{k_\sigma}{2} r_\sigma^2(\chi)\right)$$

subject to:

$$\begin{aligned} V(\chi; \delta) \;\; \geq \;\; & V^g(\chi; \delta) = -\frac{b}{2}\left(\overline{y} - \left(\hat{g}^d(\cdot) + r_C(\chi)\right)\right)^2 - \frac{q}{2}\left(\hat{g}^d(\cdot) - r_B(\chi)\right)^2 + z r_B(\chi)\, r_C(\chi) \\ & -c(n - 1)\, g(\chi) - c\hat{g}^d(\cdot) - \sum_{\sigma \in \{B, C\}} \frac{k_\sigma}{2} r_\sigma^2(\chi) + \frac{\delta}{1 - \delta} u^b, \end{aligned}$$

[15] Acemoglu et al. (2016) develop a growth model in which dirty and clean technologies compete in each of many product lines. They find that a shift toward clean technology is possible only when the two energy technologies are not complementary. In contrast, we find that global emissions can be reduced also when technologies are complementary by means of an optimal combination of taxes on dirty technology and subsidies for clean technology, whose respective amounts depend on the degree of complementarity between the two types of technologies.

where $\hat{g}^d(\cdot) = (b\overline{y} - br_C + qr_B - c)/(q+b)$, and

$$V(\chi;\delta) \geq V^r(\chi;\delta) = c(n-1)\left(g^b - \hat{g}^n(\cdot)\right) + \frac{u^b}{1-\delta}, \tag{A.1}$$

where

$$u^b = -\frac{b}{2}\left(\overline{y} - \left(g^b + r_C^b\right)\right)^2 - \frac{q}{2}\left(g^b - r_B^b\right)^2 + zr_B^b r_C^b - cng^b - \sum_{\sigma\in\{B,C\}} \frac{k_\sigma}{2}\left(r_\sigma^b\right)^2(\chi),$$

and $g^b = g^*$, $r_B^b = r_B^*$, and $r_C^b = r_C^*$ for $n = 1$. The optimal private responses $g(\chi)$, $r_B(\chi)$, and $r_C(\chi)$ are obtained by solving:

$$b(\overline{y} - (g + r_C)) - q(g - r_B) - \tau = 0, \tag{A.2}$$

$$q(g - r_B) + zr_C - k_B r_B + \varsigma_B = 0, \tag{A.3}$$

$$b(\overline{y} - (g + r_C)) + zr_B - k_C r_C + \varsigma_C = 0. \tag{A.4}$$

The function $\hat{g}^n(\cdot) = (b\overline{y} - b\widetilde{r}_C + q\widetilde{r}_B - c)/(q+b)$ in (A.1) is the optimal domestic level of emission conditioned on investment $\widetilde{r}_B$ and $\widetilde{r}_C$, which are obtained by solving (A.2)-(A.4) when $\tau = \tau^b$. Replacing g^b and $\hat{g}^n(\cdot)$ into (A.1), we obtain:

$$V(\chi;\delta) \geq V^r(\chi;\delta) = \frac{c(n-1)}{\varrho}\left(b\left((-q+z)\varsigma_B + (q+k_B)\varsigma_C\right) - q\left(k_C\varsigma_B + z\varsigma_C\right)\right) + \frac{u^b}{1-\delta}.$$

Since $\varsigma_B^* = \varsigma_C^* = 0$, we have that $V^r(\chi^*;\delta) = u^b/(1-\delta)$ with $\chi^* \equiv (\tau^*, \varsigma_B^*, \varsigma_C^*)$, which implies that $V(\chi^*;\delta) > V^r(\chi^*;\delta)$ is always satisfied. Let $\delta^g(\chi)$ denote the level of δ that solves $V(\chi;\delta) = V^g(\chi;\delta)$ and $\overline{\delta}^g \equiv \delta^g(\chi^*)$, which is equal to:

$$\overline{\delta}^g = \frac{(b+q)\,k_B k_C + bq\,(k_B + k_C) + 2bqz - (b+q)z^2}{b^2(q+k_B) + q((2k_B+q)\,k_C - 2z^2) + b((2k_C+q)\,q + 2\,((q+k_C)k_B - (z-q)\,z))}.$$

If $\delta > \overline{\delta}^g$, then neither the compliance constraint at the emission policy stage nor the compliance constraint at the investment policy stage binds when taxes and subsidies are set to χ^*. Hence, the optimal sustainable climate policies are $\tau = \tau^*$ and $\varsigma_B = \varsigma_C = 0$. If $\delta \leq \overline{\delta}^g$, then the compliance constraint at the emission policy stage binds when $\chi = \chi^*$. Since $\partial V^g(\chi;\delta)/\partial\varsigma_B = \partial V(\chi;\delta)/\partial\varsigma_B$ and $\partial V^g(\chi;\delta)/\partial\varsigma_C = \partial V(\chi;\delta)/\partial\varsigma_C$, the equilibrium policies $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ are obtained by solving $\partial V(\chi;\delta)/\partial\varsigma_B = 0$ and $\partial V(\chi;\delta)/\partial\varsigma_C = 0$ and are equal to $\varsigma_B(\tau) = (q/(b+q))(\tau - cn)$ and $\varsigma_C(\tau) = (b/(b+q))(cn - \tau)$. Substituting $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ into $V(\chi;\delta) = V^g(\chi;\delta)$ and solving for τ, we obtain:

$$\begin{aligned}
\tau &= \tau^{*} - \Lambda_m(\delta, z), \text{ where} \\
\Lambda_m(\delta, z) &= (n-1)c\left((1-\delta) + \sqrt{\frac{\delta q(k_C(q+k_B\delta) - \delta z^2)}{(b+q)k_Bk_C + qb(k_B+k_C) + 2bqz - (b+q)z^2}}\right) \\
&\quad -(n-1)c\sqrt{\frac{\delta\left((b^2(q+k_B) + bq(q-2z)) + b\delta(qk_C + k_B(q+k_C) - (z-2q)z)\right)}{(b+q)k_Bk_C + qb(k_B+k_C) + 2bqz - (b+q)z^2}},
\end{aligned}$$

where $\Lambda_m(\delta, z) > 0$, $\partial\Lambda_m(\delta, z)/\partial\delta < 0$, and

$$\frac{\partial\Lambda_m(\delta, z)}{\partial z}\begin{cases} \geq 0 & if \quad z \leq \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\}, \\ < 0 & if \quad z > \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\}. \end{cases}$$

The level $\delta^r(\tau, \varsigma_B, \varsigma_C)$ is obtained by solving $V(\chi;\delta) = V^r(\chi;\delta)$ with respect to δ for equilibrium levels $(\tau, \varsigma_B, \varsigma_C)$. ■

Appendix B. Type I Errors when $T = \infty$

In this appendix, we characterize the optimal sustainable climate policy when $\eta = 0$, $\lambda > 0$, and $T = \infty$, because the discount factor is smaller than $\overline{\delta}_j^i$ when constraint at stage $i \in \{g, r\}$ binds under policy instrument $j \in \{quota, price\}$, where $\overline{\delta}_j^i$ is reported in Lemma 4.

First, consider price regulation. When $\lambda < \lambda_{price}^g$, the compliance constraint at the emission policy stage is binding. For $\delta \leq \overline{\delta}_{price}^g$, the optimal climate policy is obtained by $\max_{\tau,\varsigma} V(\tau, \varsigma; \delta)$, where $V(\tau, \varsigma; \delta)$ is given by:

$$\begin{aligned}
&-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}\left(\overline{y} - (g(\tau,\varsigma) + r(\tau,\varsigma))\right)^2 + c\left(g(\tau,\varsigma) + (n-1)\hat{g}_\lambda^n(\widetilde{r})\right) + \frac{k}{2}r(\tau,\varsigma)^2\right) \\
&+\frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},
\end{aligned}$$

subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned}
V(\tau,\varsigma;\delta) &\geq V^g(\tau,\varsigma;\delta) \\
&= -\left(\frac{b}{2}\left(\overline{y} - \left(\hat{g}^d(\tau,\varsigma) + r(\tau,\varsigma)\right)\right)^2 + c\left(\hat{g}^d(\tau,\varsigma) + (n-1)\hat{g}_\lambda^n(\widetilde{r})\right) + \frac{k}{2}r(\tau,\varsigma)^2\right) + \frac{\delta u^b}{1-\delta}.
\end{aligned}$$

where $\hat{g}^d(\tau,\varsigma) = \overline{y} - r(\tau,\varsigma) - c/b$, $\hat{g}_\lambda^n(\widetilde{r}) = (1-\lambda)g(\tau,\varsigma) + \lambda\hat{g}^n(\widetilde{r})$, $\hat{g}^n(\widetilde{r}) = \overline{y} - \widetilde{r} - c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Substituting the equilibrium ς

into $V(\tau, \varsigma; \delta) = V^g(\tau, \varsigma; \delta)$ and solving for τ, we obtain:

$$\begin{aligned}
\tau &= \tau^g - \frac{(1-\delta)c(n-1)(1-\lambda)^2(\lambda b - (1-\lambda)k)}{k} \\
&\quad - \frac{(1-\lambda)c(n-1)}{k}\sqrt{\delta(\delta(1-\lambda)^2((1-\lambda)^2k^2 + \lambda^2b^2) + bk(1-2\lambda(1-\lambda)^3\delta))}
\end{aligned}$$

where $\tau^g = \tau^* - c(n-1)((1-\lambda)^2\lambda b + (1-(1-\lambda)^3)k)/k$.

Next, consider quantity regulation. When $\lambda < \lambda^g_{quota}$, the compliance constraint at the emission policy stage is binding. For $\delta \leq \bar{\delta}^g_{quota}$, the optimal climate policy is obtained by $\max_{g,r} V(g, r; \delta)$, where $V(g, r; \delta)$ is given by:

$$-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}(\overline{y} - (g+r))^2 + c(g + (n-1)\hat{g}^n_\lambda(r)) + \frac{k}{2}r^2\right) + \frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},$$

subject to:

$$\begin{aligned}
V(g, r; \delta) &\geq V^g(g, r; \delta) \\
&= -\left(\frac{b}{2}\left(\overline{y} - \left(\hat{g}^d(r) + r\right)\right)^2 + c\left(\hat{g}^d(r) + (n-1)\hat{g}^n_\lambda(r)\right) + \frac{k}{2}r^2\right) + \frac{\delta u^b}{1-\delta}.
\end{aligned}$$

where $\hat{g}^n_\lambda(r) = (1-\lambda)g + \lambda(\overline{y} - r - c/b)$ and $\hat{g}^d(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(g, r; \delta) = V^g(g, r; \delta)$ and solving for g, we obtain:

$$g = g^g + \frac{c(n-1)(1-\lambda)}{b}\left((1-\delta)(1-\lambda)^2 - \sqrt{\frac{\delta(b + \delta k(1-\lambda)^4)}{k}}\right),$$

where $g^g = g^* + c(n-1)(1-(1-\lambda)^3)/b$.

When $\lambda < \lambda^r_{quota}$, the compliance constraint at the investment policy stage is binding. For $\delta \leq \bar{\delta}^r_{quota}$, the optimal climate policy is obtained by $\max_{g,r} V(g, r; \delta)$, subject to:

$$\begin{aligned}
V(g, r; \delta) &\geq V^r(g, r; \delta) \\
&= -\left(\frac{b}{2}\left(\overline{y} - \left(g^b + r^b\right)\right)^2 + c\left(g^b + (n-1)\hat{g}^n(r)\right) + \frac{k}{2}\left(r^b\right)^2\right) + \frac{\delta u^b}{1-\delta},
\end{aligned}$$

where $\hat{g}^n(r) = \overline{y} - r - c/b$. Solving for r, we obtain:

$$r = (\overline{y} - g) - \frac{c(\lambda + (1-\lambda)n)}{b}.$$

Substituting the equilibrium r into $V\left(g, r; \delta\right) = V^r\left(g, r; \delta\right)$ and solving for g, we obtain:

$$g = g^r + \frac{c(n-1)(1-\lambda)}{k}\left(\left(1-\delta\right)\left(1-\lambda\right) - \sqrt{\frac{(\delta^2(1-\lambda)^2 b + k)}{b}}\right),$$

where $g^r = g^* + c(n-1)((1-(1-\lambda)^2)b + \lambda k)/bk$. Hence,

$$r = r^r - \frac{c(n-1)\left(1-\lambda\right)}{k}\left(\left(1-\delta\right)\left(1-\lambda\right) - \sqrt{\frac{(\delta^2(1-\lambda)^2 b + k)}{b}}\right).$$

where $r^r = r^* - c(n-1)(1-(1-\lambda)^2)/k$.

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