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Nature versus Nurture in Social Mobility under Private and Public Education Systems

Abstract

This paper analyzes the roles of innate talent versus family background in shaping intergenerational mobility and social welfare under different education systems. We establish an overlapping-generations model in which the allocation of workforce between a high-paying skilled labor sector and a low-paying unskilled labor sector depends on talent, parental human capital, and educational resources, and the wage rate of skilled workers is governed by their average talent. Our model suggests that under the private education system, income inequality is inversely associated with social mobility, and the steady-state average talent of skilled workers declines as people make greater educational investments on their children. Under the public school system, the allocation of workforce depends more on talent and less on family background. Consequently, both intergenerational mobility and income inequality increase, and social welfare may improve under reasonable conditions. Moreover, if some parents are myopic, public education may yield the highest welfare.

JEL-Codes: H200, H310, H500, O110.

Keywords: innate ability, private education, public education, intergenerational mobility.

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1 Introduction

To achieve future career success, is it better to be born rich or to be born intelligent? The answer may depend on the existing education system. The private school system gives affluent families an edge. A wealthy parent can offer generous funding for her child's schooling to place him to an elite university and then a high-paying job, even if he is not very smart. The further down the economic ladder a parent is, the less likely is it that her child will attain adequate educational resources. Poor children have a slim chance of escaping poverty not because they lack talent but because material disadvantage holds them back. In contrast, the public school system provides all children with an equal learning opportunity, which helps to mitigate the effects of adverse family backgrounds and to highlight the role of innate ability in the determination of educational attainment. Equally gifted children with low-income backgrounds thus stand on the same ground to compete with rich peers, and the gates of upward mobility open for them.

This paper analyzes the roles of individual innate ability ("nature") versus family and school inputs ("nurture") in shaping social mobility and welfare under different educational policies. Intergenerational mobility has received increasing attention in the economics literature. Many empirical studies demonstrate its negative relationship with economic inequality, which is known as the "Great Gatsby curve."[1] While income inequality indicates the pay gap between rich and poor within a single generation, intergenerational mobility reflects the extent to which children's socioeconomic outcomes differ from those of their parents. Reducing inequality and promoting mobility are both important from the welfare perspective, and the poor tend to resent inequality less if their children are more assured of equal opportunity.[2]

Innate ability and educational expenditure are two major determinants of children's outcome (Becker and Tomes 1986, Bernasconi, and Profeta 2012, Abbott, Gallipoli, Meghir, and Violante 2019). People are endowed with different levels of ability, which may influence their labor produc-

[1] See, for example, Corak (2013), Chetty, Hendren, Kline, Saez, and Turner (2014), Jerrim and Macmillan (2015), Goldrick-Rab, Kelchen, Harris, and Benson (2016), and Alesina, Stantcheva, and Teso (2018), and Becker, Kominers, Murphy and Spenkuch (2018).

[2] The concern about mobility has a strong philosophical foundation. In his theory of social justice, Rawls (1971) holds that those who have the same talent and the same willingness to use them should have the same prospects of success regardless of their initial situations. Roemer and Trannoy (2016, p. 1289) address that "*equality of opportunity can be described as seeking to offset differences in outcomes attributable to luck, but not those differences in outcomes for which individuals are responsible.*" Also see Piketty (1995), Fleurbaey (1995, 2008), Alesina and Angeletos (2005), and Saez and Stantcheva (2016).

tivity and hence income (Spence 1973, Stiglitz 1975).[3] The productivity of human capital intensive industries usually relies on the *average talent* of labor force. For example, a software developer's innovation capability is related more to his innate ability than to his early school performance. Bill Gates and Steve Jobs are two examples of gifted innovators who made pioneering contributions to the development of the IT industry and generated positive externalities for other engineers. The theoretical literature, notably Lucas (1988), postulates the spillover effect of human capital and the importance of average human capital in economic development. In this paper, we follow Lucas's (1988) idea but attend to just one aspect of human capital, namely innate ability.

While endowments of innate ability are fairly evenly distributed across the whole population (Papageorge and Thom 2020), this may not be the case for the educational resources received from family and school (as covered in detail in section 2). Such differences lead educational attainment and career success to be weighted in favor of the haves over the have nots. In the U.S., a low-income kindergartner with high test scores for academic talent has only 30% chance of obtaining a college degree and a desirable entry-level job in adulthood, compared to a 70% chance for his rich peer who has low scores (Carnevale, Fasules, Quinn, and Campbell 2019). In addition, 24% of high-potential people born to low-income fathers graduate from college, compared to 63% of those born to high-income fathers, while 27% of low-potential people with high-income fathers graduate from college, which is a greater proportion than that of smartest people from poorest families (Papageorge and Thom 2020). These marked contrasts imply that family background, and relatedly educational expenditure, play a prominent role in shaping a child's outcome.

We build an overlapping generations model with a high-paying skilled labor sector and a low-paying unskilled labor sector. A child receives education, and her probability of becoming a skilled worker is governed by her innate ability, parental human capital, and educational resources. Innate ability is exogenous and random among the population. An adult works for a wage, which can be spent on her private consumption and her only child's education. We assume that the productivity and wage rate of skilled workers depend on their average innate ability to capture the human capital spillovers. Holding educational resources constant, skilled workers have an advantage in educating children than unskilled workers (e.g., Becker, Kominers, Murphy, and Spenkuch 2018).

[3]The literature on the "signaling" and "screening" roles of education in the labor market posits that education can tell the employer about a job candidate's ability, although it contributes little in itself to worker productivity.

We examine how private and public school systems allocate children with different innate abilities and family backgrounds between the skilled and unskilled labor sectors.[4] Under the private school system, rich children receive more educational investments than poor children not only because their parents have a higher income but also because their parents perform more efficiently in using educational resources. For children with equal ability, those from disadvantaged families are less likely to become high-paid skilled workers in future, which hinders intergenerational mobility. In *laissez-faire* equilibrium, low-ability children with skilled parents have a good chance of getting hired in the skilled labor sector, which undermines the productivity of skilled workers and keeps their wage rate low. Our analysis also indicates that income inequality (as measured by the Gini index) is inversely related to social mobility not only in a given period but also along the path of transitional dynamics, which suggests the existence of the "Great Gatsby curve".

Under the public school system, the government imposes a proportional income tax on workers and uses the revenues to provide every child with equal educational expenditure. The allocation of the labor force thus relies more on innate ability and less on family background, which promotes intergenerational mobility. Compared with private education, public education gives talented people an advantage of being employed in the skilled labor sector, leading to an increase in the average ability of skilled workers and hence their productivity and wage rate. The wider pay gap between skilled and unskilled workers makes income inequalities worse. We also derive the condition under which a "top-up system" (i.e., parents make private investments in their children's education in addition to public schooling) is operative in equilibrium.

We then perform a simulation showing the welfare consequences of different educational and redistributive policies. We compare three circumstances, namely the *laissez-faire* case, government intervention in the form of income redistribution ("transfer in cash"), and government intervention in the form of public education ("transfer in kind"). Our simulation finds that transfer in cash leads to a lower income inequality and a mild improvement in mobility, while transfer in kind results in a greater income inequality and a substantial improvement in mobility. Transfer in cash tends to yield a higher social welfare than transfer in kind. Moreover, we study how social welfare will change if some poor parents are myopic and spend little on their children's education.[5] In this case,

[4]Earlier literature that compares private with public education includes Glomm and Ravikumar (1992), Epple and Romano (1998), Fernandez and Rogerson (2003), and de la Croix and Doepke (2004, 2009) among others.

[5]Myopia is a widely recognized topic in public economics literature. See Lindbeck and Persson (2003), Diamond

transfer in kind enhances intergenerational mobility and achieves the highest welfare.

The remainder of this paper is organized as follows. Section 2 presents some empirical evidence that motivates this paper. Section 3 lays out our model setup. Section 4 examines how three different education systems shape the steady state. Section 5 performs a numerical analysis comparing mobility, inequality, and welfare under various policies and extends the analysis to consider the existence of myopic parents. Section 6 concludes.

2 Empirical Motivation

This paper is motivated by some striking cases of rich parents spending an tremendous amount of money on their children's education to send them to prestigious universities. The following is a CNN report from 2012.[6]

> *How much would you pay to get your child into an Ivy League university? For Gerard and Lily Chow, it seems the sky was the limit. In 2007, the Hong Kong couple enlisted Harvard-lecturer-turned-admissions-consultant Mark Zimny to steer their two sons through elite U.S. boarding schools into a top-ranked university preferably Harvard. For a monthly $4000 fee per child, their total education management package included extensive admissions counseling, arranging homestays, private tutoring, and extra-curricular activities, whereby Zimny and his team functioned as parents away from parents for their sons. The Chows later switched to a retainer of $1 million per child.*

The urge to secure children's success has even led to an array of university admissions scandals. For example, in May 2019, it was found that the family of a Stanford student had paid $6.5 million to help her get admitted, including bribing university administrators and sports coaches. The student also confessed in a webcast that her IQ is not quite high and her early academic performance was mediocre.[7]

While rich parents are willing to make enormous outlays to secure an academic certificate for their children, poor parents lack the financial resources to compete and find themselves helpless.

(2004), Cremer, De Donder, Maldonado, and Pestieau (2007), and Cremer and Pestieau (2011) among others.

[6]See "Hong Kong in hot pursuit of Ivy League education." by Alexis Lai, CNN.com, 3 December 2012.

[7]See "Hard work got me into Stanford University, says Chinese student in viral video after parents paid US$6.5 million to get her accepted." by Laurie Chen, *South China Morning Post*, 3 May 2019.

The wide gap in educational resources between rich and poor manifests itself in at least four ways. First, rich parents tend to buy expensive houses in nice neighborhoods with the top-performing school districts. For example, as rising income inequality has translated into a residential sorting effect for American households with children since 1990, rich and poor have become increasingly unlikely to share the same neighborhoods (Owens 2016). Income inequality has also been identified as a major predictor of income segregation between school districts (Owens, Reardon, and Jencks 2016). Neighborhoods influence children's long-term outcome through childhood exposure effects. Chetty, Hendren, and Katz (2016) show that moving to a low-poverty neighborhood before the teenage years improves the likelihood of college attendance and increases earnings. In a study of more than 7 million families that moved across US counties, Chetty and Hendren (2018) show that the neighborhood in which a child grows up considerably shapes his social mobility.

Second, in societies where free public education plays a limited role, parents have to consider sending their children to private schools, which usually charge very high tuition and accommodation fees. In that case, only wealthy children can go to school, or at least, to a high-quality school. Poor children often have to attend low-quality schools, drop out after a few years of schooling, or even do not attend school at all. Skiba et al. (2008) suggest that poor children are more likely to attend schools with a high rate of teacher turnover, fewer experienced teachers, and larger class sizes. Chetty, Friedman, and Rockoff (2014) demonstrate that a better school with high-quality teachers substantially increases students' educational attainment.

Third, even in developed countries where all children attend primary and middle schools under a free public school system, parents are required to pay for their children's pre-school education. Rich children have access to better early-age educational resources in a high-quality kindergarten where their teachers often receive professional training. It is well established in the literature that early childhood intervention lays the foundation for subsequent academic performance (Shonkoff and Phillips 2000) and plays a powerful role in shaping human capital formation (Heckman, Pinto, and Savelyev 2013).

Fourth, richer parents tend to spend more on their children's after-school training. The global private tutoring market is booming, with East Asia leading the way. For example, Korea's household expenditure on private tutoring was estimated at 2.8% of GDP in 2006, equivalent to 80% of government spending on primary and secondary school education (Kim and Lee 2010). In Hong

Kong, the average tutoring spending for secondary school students in 2010 was 8.7% of household income (Bray et al. 2014). According to the 2009 survey of the Programme for International Student Assessment, about half of students in many western countries used after-school tutoring. In the UK, children from some ultra-rich families take lessons at home with a rotation of tutors who charge £60–100 per hour.[8] Poor families, however, do not have the money to hire private tutors.

3 Model Setup

Consider an overlapping generations economy where a mass of individuals live for two periods – childhood and adulthood. Each child receives education to pursue a college degree but does not make decision. Each adult bears one child and works for a salary. Adults who successfully obtain a degree become high-paid skilled workers (s), and the others are low-paid unskilled workers (u).

3.1 Preferences

There is a population L_t of adults in period t, who are labelled as members of "generation t." A representative member spends her wage, w_t, on private consumption, c_t, and her child's education, e_t, without leaving any bequest. Her budget constraint amounts to

$$w_t = c_t + e_t. \tag{1}$$

The representative member's utility, v_t, increases with both consumption, c_t, and the quality of her child, q_t, as given by

$$v_t = \ln c_t + \alpha q_t, \tag{2}$$

where the parameter $\alpha > 0$ represents the preference over the quality of the child.

Denote θ_t as the proportion of skilled workers in generation t. In the beginning of period t, a member expects to obtain v_t^s with a probability θ_t and v_t^u with a probability $1 - \theta_t$. Social welfare in period t, V_t, is defined as the expected individual utility:

$$V_t = \theta_t v_t^s + (1 - \theta_t) v_t^u. \tag{3}$$

[8] See "School's out forever: Why super-rich parents are opting to educate their children using private tutors?" by Joshi Herrmann, *Evening Standard*, 13 February 2015.

3.2 Skill and Ability

The quality of a child is measured by the probability that she receives a degree and becomes a skilled worker in her adulthood. A simply formulation of $q_t^i \in [0,1]$, where $i \in \{s,u\}$ denotes the parent type, is

$$q_t^i = q(a_{t+1}, e_t^i) = \ln(1 + a_{t+1}) + \beta^i \ln e_t^i, \tag{4}$$

which increases with the child's innate ability, a_{t+1}, and the received educational spending, $e_t^i \geqslant 1$, and exhibits decreasing returns to both arguments. The parameter β^i measures the relative weight of educational spending in determining the quality of the child and also reflects to what extent the prevailing labor market and education system are meritocratic. Assume that parents with a college degree perform no worse at using educational resources than those who have no degree, namely $\beta^s \geqslant \beta^u > 0$.[9]

Suppose that an individual's innate ability is unrelated to the family she was born into, and it is not revealed in childhood and remains publicly unobservable in adulthood (see Bernasconi and Profeta 2012). For simplicity, a_t is a binary random variable satisfying

$$a_t = \begin{cases} a^H > 0 & \text{with a probability of } \lambda \\ a^L = 0 & \text{with a probability of } 1-\lambda \end{cases}, \tag{5}$$

where $\lambda \in (0,1)$ is exogenous. Equation (5) shows that an individual may be endowed with high ability, a^H, or low ability, a^L, which is normalized to zero. By law of large numbers, generation t is composed of a fraction λ of high ability members and a fraction $1-\lambda$ of low ability members. Rewrite (4) by using (5):

$$q_t^i = \lambda\Big[\ln(1+a^H) + \beta^i \ln e_t^i\Big] + (1-\lambda)\Big[\ln(1+a^L) + \beta^i \ln e_t^i\Big] = \lambda \ln\left(1+a^H\right) + \beta^i \ln e_t^i, \tag{6}$$

where the first term captures the role of "nature" in shaping child outcome, while the second term reflects the effect of "nurture" (including family environment and school input).

Following Iyigun (1999), we measure the intergenerational social mobility by the odds ratio

[9] For example, educated parents tend to be better at encouraging and motivating their children, have a more positive attitude toward learning, own more books and read regularly, and provide healthier diets (e.g., Papageorge and Thom 2020).

– the probability that children of unskilled workers become skilled relative to the probability that children of skilled workers become skilled. Accordingly, we propose the definition as below:

Definition 1 *Intergenerational social mobility in period t is measured by*

$$M_t := \frac{q_t^u}{q_t^s} = \frac{\lambda \ln(1 + a^H) + \beta^u \ln e_t^u}{\lambda \ln(1 + a^H) + \beta^s \ln e_t^s}. \tag{7}$$

Because $\beta^u \leqslant \beta^s$ by assumption and also $e^u \leqslant e^s$ in equilibrium, we have $M_t \in (0, 1)$. When M gets close to one, children from different families have an equal opportunity of becoming skilled in future. If parents differ more significantly in their educational spending and the efficiency of using educational resources, then M will be lowered, indicating that the prospect for convergence tends to be constrained. As M approaches zero, almost all skilled workers have a skilled parent.

3.3 Production and Wage

There are two sectors in the economy – the unskilled and skilled labor sectors – which produce the unique final good competitively using their own type of labor with a constant-returns-to-scale technology. Let Y_t^u and Y_t^s be the output in the unskilled and skilled labor sectors in period t and L_t^u and L_t^s be their labor inputs. Their production functions are specified as follows:

$$Y_t^u = \phi L_t^u \qquad\qquad Y_t^s = f(\bar{a}_t) L_t^s, \tag{8}$$

where $\phi > 0$ represents the productivity of unskilled workers, which is independent of their ability and remains constant over time. The productivity of skilled workers, $f(\bar{a}_t)$, is a function of their average innate ability $\bar{a}_t$, which in turn can be written as

$$\bar{a}_t = \mu_t a^H, \tag{9}$$

where μ_t is the share of the high ability in skilled workers of generation t. Equation (9) essentially means that only high ability workers contribute to the sectoral productivity.[10]

[10] This result reflects the empirical observations that talented people, as opposed to the "trivial many", are making a big difference and are the main driver of a company's success (Chamorro-Premuzic 2016).

With the price of final goods in every period being normalized to unity, it is straightforward to determine that the equilibrium wage of an unskilled worker is $w_t^u = \phi$ under perfect competition. An unskilled worker earns a wage of $w_t^s = f(\bar{a}_t)$ in equilibrium.[11] We assume that a skilled worker is more productive and is paid a higher wage rate than an unskilled worker:[12]

$$f(\bar{a}_t) = \delta \bar{a}_t > \phi > 0, \tag{10}$$

where the parameter $\delta > 0$ measures the marginal return to skilled workers' average innate ability.

3.4 Population Composition

We proceed to discuss some demographic characteristics from the macroeconomic perspective. The population of generation t, L_t, can be decomposed with respect to labor skill into the following two cohorts:

$$L_t^s = \theta_t L_t, \qquad\qquad L_t^u = (1 - \theta_t) L_t. \tag{11}$$

Skilled and unskilled workers expect to have a number of $q_t^s L_t^s$ and $q_t^u L_t^u$, respectively, of children who will become skilled workers in future. In generation t, the proportion of skilled workers is $\theta_{t+1} = (q_t^s L_t^s + q_t^u L_t^u)/L_{t+1}$. Given that the fertility rate is one (i.e., $L_{t+1} = L_t$ for all t), we can rewrite this proportion by using (6) and (11):

$$\theta_{t+1} = \lambda \ln(1 + a^H) + \beta^s \theta_t \ln e_t^s + \beta^u (1 - \theta_t) \ln e_t^u, \tag{12}$$

which governs the evolutionary path of θ_t.

By (4), (5), and (11), skilled workers of generation $t+1$ can be further categorized into four groups whose population sizes are as follows

[11] In our stylized model, individual talent is not rewarded in the labor market. An alternative assumption would be that a skilled worker's wage depends on both her own talent and the average talent, but this would complicate analysis without a qualitative change in our results under transitional dynamics. The steady states under such an assumption would be hard to derive, since it would imply that the individual heterogeneity would increase over time.

[12] Skilled workers earn a higher wage than unskilled workers regardless of their intelligence. As Borghans, Golsteyn, Heckman, and Humphries (2016) find, financial success and high innate talent have little correlation.

High ability from a rich family:	$\lambda q(a^H, e_t^s)L_t^s$;
Low ability from a rich family:	$(1-\lambda)q(a^L, e_t^s)L_t^s$;
High ability from a poor family:	$\lambda q(a^H, e_t^u)L_t^u$;
Low ability from a poor family:	$(1-\lambda)q(a^L, e_t^u)L_t^u$.

The proportion of the high ability in skilled workers, μ_{t+1}, can be expressed by

$$\begin{aligned}\mu_{t+1} &= \frac{\lambda q(a^H, e_t^s)L_t^s + \lambda q(a^H, e_t^u)L_t^u}{\theta_{t+1}L_{t+1}} \\ &= \frac{\lambda\left[\ln(1+a^H) + \beta^s \ln e_t^s\right]\theta_t + \lambda\left[\ln(1+a^H) + \beta^u \ln e_t^u\right](1-\theta_t)}{\theta_{t+1}} \\ &= \lambda\left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta_{t+1}}\right] > \lambda, \end{aligned} \tag{13}$$

which implies that the majority of high ability individuals work in the skilled labor sector. Combining (9) and (13) derives the average innate ability of skilled workers of generation $t+1$, which is higher than the average innate ability of the whole population:

$$\bar{a}_{t+1} = \lambda a^H\left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta_{t+1}}\right] > \lambda a^H. \tag{14}$$

4 Equilibrium under Different Education Systems

In this section, we characterize the educational spending per child (e_t^i), the proportion of skilled workers (θ_t), the average ability of skilled workers ($\bar{a}_t$), and intergenerational mobility (M_t) under private and public education systems. We first consider that parents send their children to private schools and optimally choose their educational investments. Next, we discuss the case in which the government funds public schools by imposing an income tax rate on all workers, who then decide on private after-school education for their children.

4.1 Private Education (*Laissez-faire*)

We start with the case where people make decentralized decisions on their educational spending without any government intervention. Taking into account the budget constraint (1) and the quality of child (6), every i-type member of generation t obtains a utility at the level of

$$v_t^i = \alpha\lambda \ln(1 + a^H) + \ln(w_t^i - e_t^i) + \alpha\beta^i \ln e_t^i, \tag{15}$$

where $i \in \{s, u\}$. She chooses educational spending to maximize her utility according to $\frac{\partial v_t^i}{\partial e_t^i} = 0$, which can be solved as

$$e_t^i = \gamma^i w_t^i \qquad \text{where} \quad \gamma^i \equiv \frac{\alpha\beta^i}{\alpha\beta^i + 1} \in (0, 1).$$

Clearly, an i-type worker allocates a fraction γ^i of her wage to her child's education in equilibrium, where $\gamma^s \geqslant \gamma^u$ under the assumption of $\beta^s \geqslant \beta^u$. The optimal choices of a skilled worker and an unskilled worker hence satisfy

$$e_t^s = \gamma^s \delta \bar{a}_t \qquad\qquad e_t^u = \gamma^u \phi. \tag{16}$$

It follows that $e_t^s > e_t^u$: a skilled worker pays a premium for her child's education not only because she receives a higher income ($\delta\bar{a} > \phi$) but also because she is more efficient in using educational resources ($\gamma^s \geqslant \gamma^u$).

The next lemma follows directly from (6) and (16):

Lemma 1 *Under the private school system, a skilled parent is more likely to have a skilled child than an unskilled parent.*

Lemma 1 suggests that a child is more likely to become skilled if her parent is a skilled worker. As equation (6) demonstrates, a rich child's advantage comes from two sources. First, she receives more educational resources from her parent ($e^s > e^u$). Second, her parent performs at least as well as a poor parent in using these resources ($\beta^s > \beta^u$). The intergenerational transmission of human capital and labor earnings is advocated in earlier theories (e.g., Becker and Tomes 1979, 1986) and is supported by empirical examinations (e.g., Behrman and Rosenzweig 2002, Schneider, Hastings, and LaBriola 2018).

We now relate intergenerational mobility with income inequality. Inserting (16) into (7) rewrites intergenerational mobility in period t as

$$M_t = \frac{\lambda \ln(1 + a^H) + \beta^u \ln(\gamma^u \phi)}{\lambda \ln(1 + a^H) + \beta^s \ln(\gamma^s \delta \bar{a}_t)}. \tag{17}$$

At the aggregate level, the skilled and unskilled of generation t receive a payroll of $\delta \bar{a}_t L_t^s$ and ϕL_t^u, respectively. To measure income inequality in period t, we rely on the Gini coefficient, G_t, which can be computed as the share of high-paid workers' income in total income minus their proportion in the working-age population:

$$G_t = \frac{\delta \bar{a}_t L_t^s}{\delta \bar{a}_t L_t^s + \phi L_t^u} - \theta_t = \theta_t \left[\frac{1}{\theta_t + (1 - \theta_t)\phi/(\delta \bar{a}_t)} - 1 \right], \tag{18}$$

which is a function of (ϕ, δ, $\bar{a}_t$, θ_t).

We can see from (17) and (18) that the common factors influencing M_t and G_t are (ϕ, δ, $\bar{a}_t$). A rise in unskilled workers' wage rate (higher ϕ) promotes social mobility and remedies income inequality (higher M_t and lower G_t), and a fall in skilled workers' wage rate (lower δ or $\bar{a}_t$) achieves the same result. In sum, M_t and G_t are inversely correlated. We develop the following proposition to summarize the result:

Proposition 1 *In period t, a lower income inequality (smaller G_t) is associated with a greater intergenerational social mobility (larger M_t) under the private school system.*

Proposition 1 formalizes the important conceptual link between economic inequality and social mobility in a certain period, namely that more equal societies tend to be more mobile. All else being equal, narrowing the wage gap (i.e., small $\delta \bar{a}_t$ and/or large ϕ) helps to mitigate *intra*-generational earnings inequality. The income effect suggests that the rich will then tend to spend less on education, with the result that their children are less likely to become rich in the future, but the poor will do the opposite, resulting in more opportunities for their children.

Our prediction is consistent with the empirically observed Great Gatsby curve. Recent research based on cross-sectional data finds that higher inequality in childhood is related with lower mobility in adulthood in Sweden (Brandén 2019) and Latin America (Neidhöfer 2019). It is also consistent with the empirical finding that educational attainment mediates the link between social origin and destination (Jerrim and Macmillan 2015).

We proceed to characterize the evolution of $(\theta_t, \bar{a}_t)$. Plugging (16) into (12) and (14) obtains

$$\theta_{t+1}(\bar{a}_t, \theta_t) = \lambda \ln(1 + a^H) + \beta^s \theta_t \ln(\gamma^s \delta \bar{a}_t) + \beta^u (1 - \theta_t) \ln(\gamma^u \phi) \tag{19}$$

$$\bar{a}_{t+1}(\bar{a}_t, \theta_t) = \left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\lambda \ln(1+a^H) + \beta^s \theta_t \ln(\gamma^s \delta \bar{a}_t) + \beta^u (1-\theta_t) \ln(\gamma^u \phi)}\right] \lambda a^H. \tag{20}$$

These two equations show that θ_t and $\bar{a}_t$ are both path-dependent, namely θ_{t+1} is determined by θ_t, and $\bar{a}_{t+1}$ is determined by $\bar{a}_t$. Define the steady state of the economy as that both θ_t and $\bar{a}_t$ achieve their time-invariant levels, θ and $\bar{a}$.

Definition 2 *The economy reaches the steady state when $\theta_t = \theta$ and $\bar{a}_t = \bar{a}$.*

We characterize the steady-state solutions in the following proposition:

Proposition 2 *Under the private school system, the steady state of the economy is determined by the following two equations*

$$\left[1 + \frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)}\right] \frac{\bar{a}}{a^H} + \beta^s (1-\lambda) \ln(\gamma^s \delta \bar{a}) = 1 + \left[1 - \lambda + \frac{1}{\ln(1+a^H)}\right] \beta^u \ln(\gamma^u \phi), \tag{21}$$

$$\theta = \frac{\lambda(1-\lambda) a^H \ln(1+a^H)}{\bar{a} - \lambda a^H}. \tag{22}$$

Proof. See Appendix. ■

Equation (21) demonstrates that the average innate ability of skilled workers in the steady state, $\bar{a}$, is governed by six parameters, namely $\bar{a}(a^H, \lambda, \alpha, \beta^s, \beta^u, \delta, \phi)$. Inserting $\bar{a}$ solved in equation (21) into equation (22) derives the solution to θ. As equation (22) shows, holding (a^H, λ) constant, θ and $\bar{a}$ are inversely correlated in the steady state. Since a skilled worker is more likely to be high ability than an unskilled worker on average (equation (13)), the average talent of skilled workers tends to decline when workers are increasingly moved from the unskilled to the skilled labor sector.

We proceed to examine the comparative statics of $\bar{a}$ in the following proposition:

Proposition 3 *Under the private school system, the steady-state average innate ability of skilled workers $\bar{a}$: (i) increases with a^H and λ, (ii) decreases with α, (iii) decreases with β^s and β^u, and (iv) decreases with δ and ϕ.*

Proof. See Appendix. ■

Proposition 3(i) is straightforward: all else being equal, if talented people have a higher level of talent and account for a larger population size, then the average talent of skilled workers will be higher in the steady state. The other parts of Proposition 3 can be interpreted from the perspective of parents' educational investments. A parent tends to invest more in her child's schooling if she is more concerned about the educational attainment of her child (larger α), more efficient in using educational resources (larger β^s or β^u), or earns a higher wage (larger δ or ϕ). By (4), educational expenditure and innate ability are substitutes in governing an individual's quality and skill. As a rise in educational spending mitigates the relative weight of natural ability in occupational choice, more mediocre people born into rich families are employed in the skilled labor sector, which undermines the productivity of skilled occupation.

In our model, since natural talent is usually exogenously given, children from poor and rich families tend to have the same probability of having high innate ability. Therefore, an economy becomes more efficient if intergenerational mobility is higher, which will lead to a higher level of innate ability among skilled workers. The next question that arises is how to enhance the efficiency and equity of society by reallocating educational resources across families. This goal can be achieved by taxing the rich and subsidizing the poor. The income effect means that after income redistribution, the rich will spend less on their children's education whereas the poor will spend more. Another solution would be the provision of public education, which aims to provide children with equal educational spending. In the next subsections, we investigate the equilibrium outcome when the government finances public education by levying an income tax.

4.2 Public Education

Suppose that the government imposes a proportional income tax on all workers and spends tax revenues on public education such that each child attains an equal amount of educational expenditure. Public education then generates two benefits. First, rich families give pecuniary supports to poor families through this in-kind redistribution. Second, public education helps to eliminate the gap in the received educational resources between rich and poor children, which in turn promotes intergenerational mobility.

It is also well noted that parents may provide their own children, who receive formal education at public schools, with supplementary remedial classes. After-school private tutoring, which is often referred to as "shadow education," is an example of programs existing alongside the formal education system with a growing size. But tutoring is usually found to be less effective than mainstream schooling. For example, after-school tutoring often takes place off campus; to participate in private tutoring therefore incurs a fixed adjustment cost, such as the time and expense of commuting between school and the tutoring center (Fashola 2001). Besides, the curriculum emphasized by tutoring institutions may not match well with that in school (e.g., Bray 2003). In our model, we consider the discount in private tutoring effectiveness (relative to public education).

In period t, the interaction between the government and workers proceeds in a two-stage game. The government moves first to fund public education by imposing the welfare-maximizing income tax. Then, workers choose to make the optimal private educational investments for their children.

Consider that the government levies an income tax at the rate of $\tau_t \in (0, 1)$ and provides each child with public educational spending e_t^P in period t. The balanced government budget implies:

$$e_t^P = \frac{\tau_t(w_t^s L_t^s) + \tau_t(w_t^u L_t^u)}{L_{t+1}} = \tau_t[\theta_t \delta \bar{a}_t + (1 - \theta_t)\phi], \tag{23}$$

which strictly increases with τ_t. A skilled worker has an incentive to send her child to after-school programs, where a tuition fee of e_t^A is charged, so that the child receives an education investment at the aggregate level of

$$e_t^s = e_t^P + \kappa e_t^A, \tag{24}$$

where e_t^P is expressed in (23) and the parameter $\kappa \in (0, 1)$ measures the efficiency loss of tutoring.

A skilled worker's utility can be written as

$$v_t^s = \alpha\lambda \ln(1 + a^H) + \ln\left[(1 - \tau_t)\delta\bar{a}_t - e_t^A\right] + \alpha\beta^s \ln\left(e_t^P + \kappa e_t^A\right).$$

Given the tax rate τ_t and the public educational expenditure per child e_t^P, a skilled worker optimally chooses private educational expenditure to maximize her utility. Taking the first order condition of the above equation with respect to e_t^A and rearranging obtains

$$e_t^A = \gamma^s(1 - \tau_t)\delta\bar{a}_t - \frac{e_t^P}{(1 + \alpha\beta^s)\kappa}. \tag{25}$$

The negative relationship between e_t^A and e_t^P in equation (25) implies a substitution effect between public schooling and private tutoring. By (24) and (25), the educational investment received by a rich child is

$$e_t^s = e_t^P + \kappa \left[\gamma^s (1-\tau_t)\delta \bar{a}_t - \frac{e_t^P}{(1+\alpha\beta^s)\kappa} \right] = \gamma^s \left[\kappa (1-\tau_t)\delta \bar{a}_t - e_t^P \right]. \tag{26}$$

In light of discount of private tutoring (i.e., $\kappa < 1$), the government will choose sufficiently large public educational expenditure in equilibrium so that a poor parent needs not to spend on tutoring.[13] In other words, a poor child in period t receives public education only (i.e., $e_t^u = e_t^P$).

We develop the following proposition to compare the private and public school systems:

Proposition 4 *Compared with the private school system, the public school system leads rich children to receive less education and poor children more. Hence, intergenerational mobility improves.*

Proof. See Appendix. ■

If the private education system is replaced by a public one, educational resources tend to gravitate from rich to poor children. This result follows directly from the income effect: the public school system involves a redistributive process that effectively transfers wealth from rich to poor families. In equilibrium, more talented poor children attaining a college degree, crowding out some mediocre wealthy children. Consequently, a poor child faces a higher probability of becoming rich in the future while a rich child has a lower probability, which mitigates the transmission of earnings inequality across generations. The public school system creates a more equal society in which an individual's career success hinges more on innate ability and less on family background.[14]

Proposition 4 not only provides a rationale for the wide adoption of public education around the world but also finds some empirical support. For example, Neidhöfer (2019) shows that public education is significantly and positively associated with intergenerational mobility in Latin American countries.

[13] This outcome has a clear empirical counterpart: it is found that family socioeconomic status is positively related with student participation in private tutoring in South Korea (Kim and Lee 2010) and Hong Kong (Bray et al. 2014).

[14] Hassler and Rodríguez Mora (2000) present a similar result in a different model setup. They suggest that when the world changes slowly, children of skilled workers have an informational advantage for skilled occupations over other children, but if the world changes a great deal between generations, then parents' information becomes less valuable and innate ability becomes more important in social selection. Rather than discuss the environment of economic growth, our paper addresses the role of the education system.

In period t, the government chooses the tax rate to maximize social welfare as expressed by

$$
\begin{aligned}
V_t &= \theta_t \left\{ \ln\left[(1-\tau_t)w_t^s - e_t^A\right] + \alpha\lambda\ln(1+a^H) + \alpha\beta^s \ln\left[\gamma^s\kappa(1-\tau_t)\delta\bar{a}_t - \gamma^s e_t^P\right]\right\} \\
&\quad + (1-\theta_t)\left\{\ln[(1-\tau_t)w_t^u] + \alpha\left[\lambda\ln(1+a^H) + \beta^u \ln e_t^P\right]\right\} \\
&= \alpha\lambda\ln(1+a^H) + \theta_t \ln\left\{(1-\tau_t)(1-\gamma^s)\delta\bar{a}_t + \frac{\tau_t[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi]}{(1+\alpha\beta^s)\kappa}\right\} \\
&\quad + \theta_t\alpha\beta^s\ln\gamma^s + \theta_t\alpha\beta^s\ln\left\{\kappa(1-\tau_t)\delta\bar{a}_t - \tau_t\left[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi\right]\right\} + (1-\theta_t)\ln\phi \\
&\quad + (1-\theta_t)\ln(1-\tau_t) + (1-\theta_t)\alpha\beta^u\ln[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi] + (1-\theta_t)\alpha\beta^u\ln\tau_t. \qquad (27)
\end{aligned}
$$

Since V_t is continuous function of τ_t, which belongs to a compact set $[0,1]$, the optimal solution to τ_t must exist. We denote τ_t^* as the optimal tax rate and propose the next lemma:

Proposition 5 *Under the public school system, the rich will provide their children with extra private education in period t if and only if*

$$\alpha\beta^s\kappa\left(\frac{1}{\tau_t^*} - 1\right) > \theta_t + (1-\theta_t)\frac{\phi}{\delta\bar{a}_t}. \qquad (28)$$

Proof. See Appendix. ■

Proposition 5 presents a sufficient and necessary condition for tutoring classes to operate (i.e., $e^A > 0$). Condition (28) is more likely to hold if the rich concern greatly about the quality of their children (large α) and are efficient in using educational resources (large β^s) and if private tutoring is efficient (large κ). Moreover, this condition is more likely to hold in period t if the government provides a small educational fund (small e_t^P and thus small τ_t^*), the income gap is wide (small $\frac{\phi}{\delta\bar{a}_t}$), and skilled workers account for a small fraction in the working-age population (small θ_t).

Analogous to Proposition 2, the next proposition examines the steady state of the economy:

Proposition 6 *Under the public school system, if condition (28) is violated, then the steady state is determined by*

$$\theta - [\beta^s\theta + \beta^u(1-\theta)]\ln\left\{\frac{\lambda\delta a^H[(1-\lambda)\ln(1+a^H)+\theta] + (1-\theta)\phi}{\{\alpha[\beta^s\theta + \beta^u(1-\theta)]\}^{-1} + 1}\right\} = \lambda\ln(1+a^H), \qquad (29)$$

$$\bar{a} = \lambda a^H\left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta}\right]. \qquad (30)$$

Proof. See Appendix. ■

Equation (29) demonstrates that under the public school system, the steady-state proportion of skilled workers in the economy, θ, is governed by $(a^H, \lambda, \alpha, \beta^s, \beta^u, \delta, \phi)$. Consequently, it can be inferred from equation (30) that the average innate ability of skilled workers in the steady state is also influenced by the seven parameters. Furthermore, τ and e^P are a constant in the steady state when $(\theta, \bar{a})$ remain constant.

Finally, we analyze the comparative statics of $\bar{a}$. To simplify the algebra, we focus on a special case of perfect meritocracy (i.e., $\beta^s = \beta^u$).

Proposition 7 *Suppose that $\beta^i = \beta$ and condition (28) is violated. Under the public school system, the steady-state average ability of skilled workers, $\bar{a}$, decreases with $(\alpha,\ \delta,\ \phi)$ if and only if*

$$\beta < \frac{(\delta\bar{a} - \phi)\theta + \phi}{\lambda a^H - \phi}. \tag{31}$$

Proof. See Appendix. ■

Proposition 7 presents a result similar to that in Proposition 3, with the only difference being that condition (31) is imposed. This condition emphasizes that parents' efficiency in using educational expenditure should be sufficiently low (small β). Parents who care little about the quality of their children (small α) or earn a low income (small δ and ϕ) tend to spend a small amount of money on their children's education. Limited educational resources, coupled with low efficiency in using educational resources, will make innate ability a predominant determinant of one's placement. Intelligent children are thus very likely to become skilled workers, which in turn increases the average ability of the skilled labor sector.

5 A Quantitative Analysis

To compare the equilibrium outcome of the three education systems, we rely on a simple quantitative example in this section. We conduct three sets of simulations – (i) a dynamic Great Gatsby curve under the private school system, (ii) the steady-state socioeconomic features under different systems, and (iii) social welfare in transitional dynamics and steady state under different systems. Our simulations are performed based on the parameter values given in Table 1.

Table 1. Benchmark Values for Key Parameters

Parameter	Description	Value
a^H	Level of high ability	3
λ	Probability that an individual is endowed with high ability	0.15
α	Preference over the quality of the child	5
β^s	Educational efficiency of rich families	0.2
β^u	Educational efficiency of poor families	0.12
δ	Impact of average ability on skilled labor productivity	5
ϕ	Unskilled labor productivity	5
θ_0	The initial fraction of skilled workers	0.25
κ	Discount of after-school private education	0.8

Figure 1 charts income inequality versus social mobility over time under *laissez-faire*. In the initial period (period 0), the Gini coefficient is $G_0 \approx 0.211$ while mobility is $M_0 \approx 0.489$. In the next period, the income distribution becomes more even as the Gini coefficient falls to $G_1 \approx 0.161$; meanwhile, intergenerational mobility increases to $M_1 \approx 0.542$. An intertemporal comparison indicates "more equality, more mobility." The arrows depict the direction of motion: the economy sees continuous and simultaneous improvements in equality and mobility until it reaches the steady state ($G \approx 0.155$, $M \approx 0.547$). In short, Figure 1 shows a Great Gatsby curve along the *dynamic* path, which complements Proposition 1 that implies a *static* inverse relationship between economic inequality and social mobility under *laissez-faire*.

Figure 1: The Great Gatsby Curve along the Dynamic Path under the Private School System

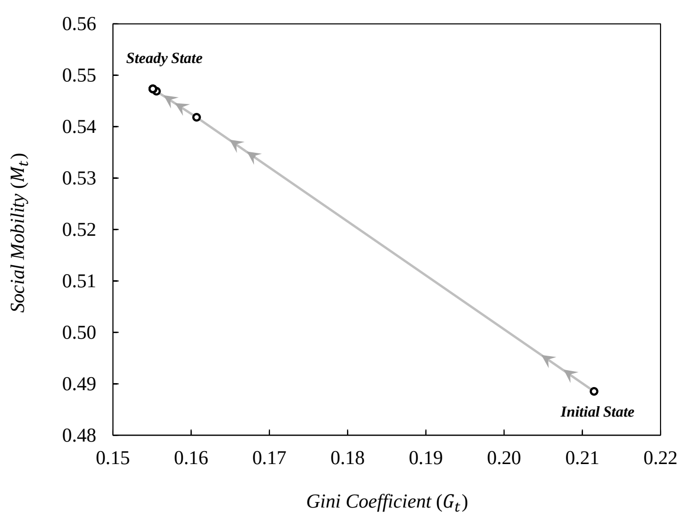


Table 2 reports the simulated outcomes of (θ, $\bar{a}$, M, G) in cases where the formal education is private and where it is public based on the parameter values as in Table 1. For the private school system, we consider [1] the *laissez-faire* (section 4.1) and [2] the government levies an income tax at the rate of τ_t^* as in section 4.2 and returns the revenue equally to all workers. Comparing these two scenarios captures the effect of government interference via "transfer in cash". For the public school system, the parameter configuration suggests that parents provide their children with extra private education (i.e., condition (28) holds). Comparing the last two rows shows different impacts of "transfer in cash" and "transfer in kind".

We see from the first column of Table 2 that under the private school system, income redistribution enables a greater fraction of population to work in the skilled labor sector (larger θ). Transfer in cash is a useful instrument to help more poor children to obtain a college degree and then get hired in skilled occupations, although it inevitably undermines the quality of rich children through the income effect. Given this parameter configuration, the poor's benefits outweigh the rich's costs, thereby leading to the overall improvement in children's quality. However, when public education is provided, the steady-state proportion of skilled workers is smallest ($\theta = 36.74\%$). Note that in equation (22) there is a tradeoff between $\bar{a}$ and θ in the steady state. Public schooling seems the most effective way of placing smart people into the skilled labor sector. Holding other things fixed, income redistribution tends to decrease the average ability of skilled workers.

Let us proceed to examine the values of (M, G). Clearly, both transfer in cash and transfer in kind help to make future generations more mobile. Intergenerational mobility is at its highest under the public school system, which echoes Proposition 4 that public schooling offers a relatively equal opportunity of achieving skilled professions. In the last column, it is intuitive that a direct transfer of wealth from rich to poor mitigates earnings inequality. Yet it is a surprising result that

Table 2. The Steady-state Properties Under Different Systems

	θ	$\bar{a}$	M	G
Private School System				
Laissez-faire	0.3701	1.8827	0.5473	0.1551
Transfer-in-cash with Tax Rate τ_t^*	0.3703	1.8820	0.5983	0.1001
Public School System	0.3674	1.8933	0.6673	0.1563

the public school system gives rise to a greater Gini coefficient. We explain this result as follows. The public school system facilitates high-ability people from disadvantaged families to crowd out silver-spooned low-ability people when competing for jobs in skilled occupations. As the increased average talent of skilled workers raises their wage rate, the between-sector pay gap widens given that the wage of unskilled workers ϕ remains constant. A downsizing of high-paid skilled workers (small θ) also helps to fuel the increase in the Gini coefficient.

We are now in a position to investigate social welfare under different policies. Our simulation analysis on social welfare have two goals. First, we aim to show the advantage of public education if some people are myopic (i.e., who care little about their children). Second, by assigning different values to β^u, we aim to illustrate how social welfare changes in response to the varying degree of meritocracy.

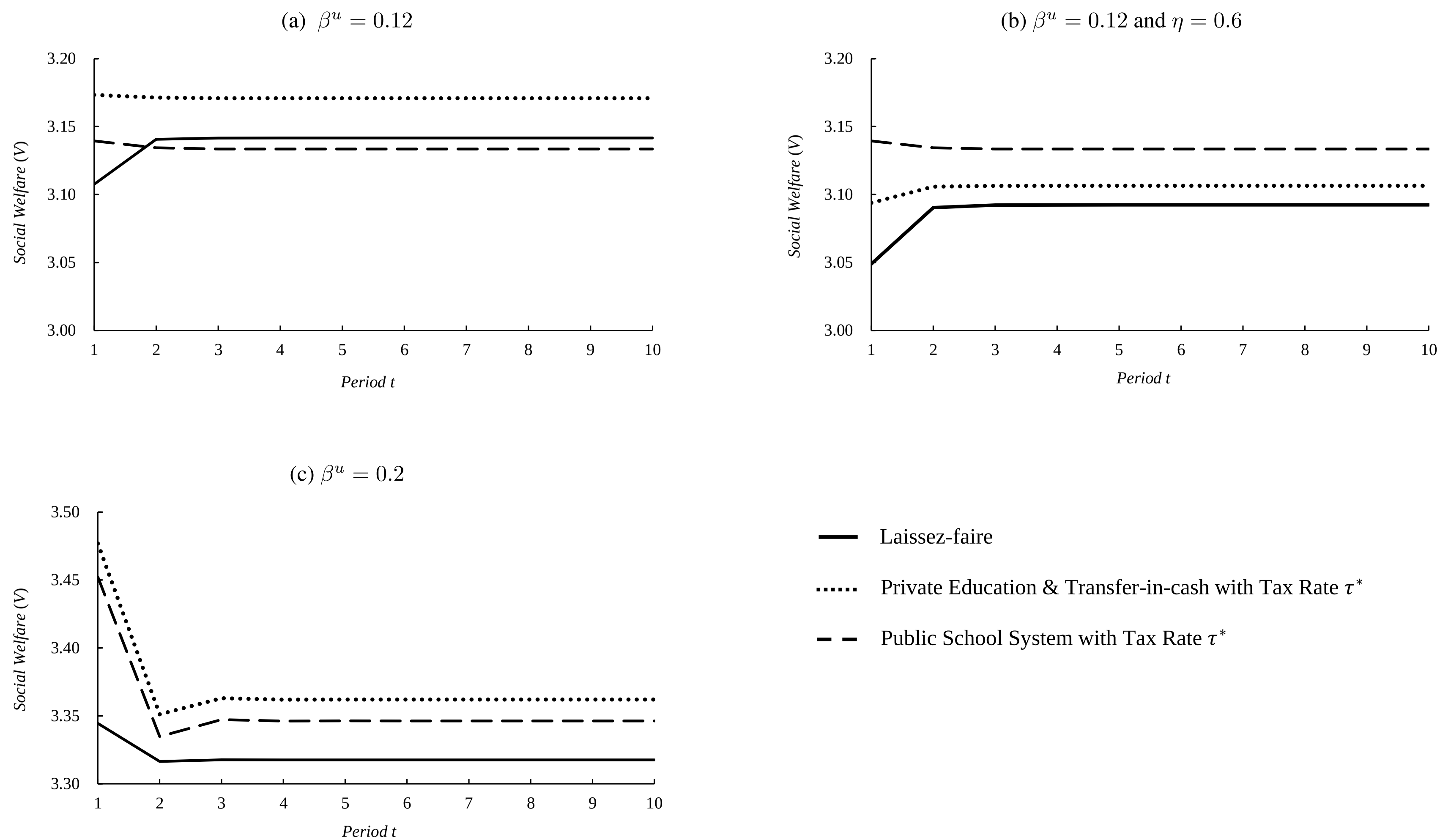
Figure 2(a) plots the dynamic welfare consequence based on parameter values given in Table 1 (the benchmark case). It illustrates that the private school system with income redistribution leads to a higher social welfare than other systems. Through the income effect, educational investments increase in poor families and fall in rich families, which enhances social welfare due to the law of diminishing marginal utility (e.g., Boadway and Keen 2000, Boadway and Sato 2015). This result sheds light on the practice in many countries of focusing government interference on redistributing income from the rich to the poor. Another salient feature is that the public school system results in the lowest welfare level (even lower than the *laissez-faire* outcome). This is largely because poor parents are very inefficient in using the educational resources although their children receive a greater amount of resources after the implementation of public education.

Compared with Figure 2(a), Figure 2(b) examines how the introduction of myopic people alters the simulation result. Assume that a fraction η of unskilled workers and none of skilled workers are myopic. When parents decide to make their educational investments (e.g., section 4.1), myopics will optimally choose the minimum level ($e = 1$). In that case, intergenerational social mobility amounts to

$$M_t = \frac{\lambda \ln(1 + a^H) + (1 - \eta)\beta^u \ln(\gamma\phi)}{\lambda \ln(1 + a^H) + \beta^s \ln(\gamma\delta\bar{a}_t)}. \tag{32}$$

Comparing (32) with (17) indicates that the economy becomes less mobile when myopia matters. With public schooling, the benevolent government ignores the myopic acts so that the tax and

Figure 2: Welfare Consequences of Different Systems



educational policies (and thus the equilibrium outcomes) are maintained as in section 4.2.

Figures 2(a) and 2(b) displays different solid and dotted curves (for private education), although they share the same dashed curve (for public education). In Figure 2(b), we set the proportion of myopic people in poor parents to 60% ($\eta = 0.6$). The presence of myopia makes public education more appealing: the public school system now yields a higher social welfare than any case under the private school system.

Figure 2(c) depicts the case in which unskilled workers perform as efficient as skilled workers in using the educational resources ($\beta^u = 0.2$) all else being equal. Comparing Figures 2(a) and 2(c) tells us which policy is socially preferable as the economy becomes increasingly meritocratic. In Figure 2(c), the dashed curve lies above the solid curve, which implicates that public education is socially preferable to the laissez-faire outcome. Moreover, the dotted curve lies above the dashed curve, implying that transfer in kind is more desirable than transfer in cash when poor parents have no disadvantage in using educational resources. Since social welfare is lowest under *laissez-faire*, government intervention can always make people better off.

6 Conclusion

One reason behind the populist vote in traditional European welfare states is that policies have aimed at alleviating economic inequality while neglecting the issue of social mobility. What is observed in these states is a quite stable income distribution, in great part due to proactive redistributive policies, but little concern for what has been called a broken social elevator. To illustrate, 60 years ago over 90% of parents expected their children to do better than them; today this fraction has fallen to one half.

This paper develops an overlapping-generations model analyzing the influences of educational policies on the intergenerational mobility of individuals with different innate abilities and family backgrounds. We consider that an economy's productivity and income depend on both the fraction of skilled individuals and their average talent. Our model suggests that enhancing intergenerational mobility may enhance economic efficiency and equity simultaneously, which in turn increases social welfare.

In the *laissez-faire* economy where parents invest in their own children's education, the income effect suggests that the rich tend to spend more on education, leading to rich children being more likely to be rich in the future, and the poor do the opposite, giving their children fewer opportunities. Our comparative static analysis shows that, under some reasonable conditions, the proportion of skilled workers in the steady state increases with parental concern about children's educational attainment and parental income, while the average ability of skilled workers decreases with these factors. We also illustrate a Great Gatsby curve along the *dynamic* path, which implies that more equal societies tend to be more mobile over time.

We then suggest that the implementation of public schooling may improve social welfare. Our model derives the condition in which parents will top up their children's education will be operative in equilibrium. While the public school system leads to a greater social mobility than the private education system does, it may also exacerbate income inequality under some configurations. When some poor parents are myopic, the introduction of public school system may be welfare-improving. Besides, as an economy becomes more meritocratic, it will be socially desirable for the government to intervene.

In future research, we may extend our analysis to a setting in which an economy is concerned with not only the wellbeing of the current generation but also the equity of future generations. In that case, people tend to find the provision of public education ("transfer in kind") preferable to income redistribution ("transfer in cash"). It is also conceivable that a greater concern about social mobility leads to a lower level of public educational expenditure per child. This is because skilled workers are often more efficient in utilizing educational resources that their children receive from public schools, and an overly generous educational spending may put poor children at a disadvantage of becoming rich. This consideration would imply a potential tradeoff between efficiency and *dynamic* equity.

Appendix

Proof of Proposition 2

In the steady state, equations (19) and (20) can be rewritten as

$$\theta = \frac{\lambda \ln(1+a^H) + \beta^u \ln(\gamma^u \phi)}{1 - \beta^s \ln(\gamma^s \delta \bar{a}) + \beta^u \ln(\gamma^u \phi)}, \tag{A.1}$$

$$\bar{a} = \lambda a^H \left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta} \right]. \tag{A.2}$$

Plugging (A.1) into (A.2) yields

$$\begin{aligned}
&\frac{\bar{a}}{\lambda a^H} - 1 = \frac{(1-\lambda)\ln(1+a^H)[1 - \beta^s \ln(\gamma^s \delta \bar{a}) + \beta^u \ln(\gamma^u \phi)]}{\lambda \ln(1+a^H) + \beta^u \ln(\gamma^u \phi)} \\
\Leftrightarrow \quad & \frac{\bar{a}}{a^H} - \lambda + \left(\frac{\bar{a}}{\lambda a^H} - 1 \right) \frac{\beta^u \ln(\gamma^u \phi)}{\ln(1+a^H)} = 1 - \lambda - (1-\lambda)[\beta^s \ln(\gamma^s \delta \bar{a}) - \beta^u \ln(\gamma^u \phi)] \\
\Leftrightarrow \quad & \frac{\bar{a}}{a^H} + \frac{\bar{a}}{\lambda a^H} \frac{\beta^u \ln(\gamma^u \phi)}{\ln(1+a^H)} + (1-\lambda)[\beta^s \ln(\gamma^s \delta \bar{a})] = 1 + (1-\lambda)\beta^u \ln(\gamma^u \phi) + \frac{\beta^u \ln(\gamma^u \phi)}{\ln(1+a^H)},
\end{aligned}$$

which can be rewritten as (21). Rearranging (A.2) obtains (22).

Proof of Proposition 3

(i) Totally differentiating (21) with respect to $\bar{a}$ and a^H and then using (14) obtains

$$\begin{aligned}
&\left[1 + \frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)} \right] \frac{d\bar{a}}{a^H} + \frac{\beta^s(1-\lambda) d\bar{a}}{\bar{a}} - \left\{ \frac{\bar{a}}{a^{H2}} + \frac{\beta^u \bar{a} \ln(\gamma^u \phi) \left[\frac{a^H}{1+a^H} + \ln(1+a^H) \right]}{\lambda [\ln(1+a^H) a^H]^2} \right\} da^H \\
&= -\frac{\beta^u \ln(\gamma^u \phi)}{[\ln(1+a^H)]^2} \frac{da^H}{1+a^H} \\
\Leftrightarrow \quad & a^H \Omega d\bar{a} = \left\{ \bar{a} + \frac{\beta^u \ln(\gamma^u \phi)[\bar{a} a^H + \bar{a}(1+a^H)\ln(1+a^H) - \lambda a^{H2}]}{\lambda [\ln(1+a^H)]^2 (1+a^H)} \right\} da^H \\
\Leftrightarrow \quad & \frac{d\bar{a}}{da^H} = \frac{1}{a^H \Omega} \left\{ \bar{a} + \frac{[\lambda(1-\lambda)a^{H2}/\theta + \bar{a}(1+a^H)]\beta^u \ln(\gamma^u \phi)}{\lambda(1+a^H)\ln(1+a^H)} \right\} > 0.
\end{aligned} \tag{A.3}$$

where, for notational simplification, we denote $\Omega := 1 + \frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)} + \frac{\beta^s(1-\lambda)a^H}{\bar{a}} > 0$. Totally differentiating (21) with respect to $\bar{a}$ and λ and using (16) obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}-\frac{\beta^u\ln(\gamma^u\phi)\bar{a}d\lambda}{\lambda^2\ln(1+a^H)a^H}=[\beta^s\ln(\gamma^s\delta\bar{a})-\beta^u\ln(\gamma^u\phi)]d\lambda$$
$$\Leftrightarrow\quad\frac{d\bar{a}}{d\lambda}=\frac{1}{\Omega}\left\{\frac{\beta^u\bar{a}\ln(\gamma^u\phi)}{\lambda^2\ln(1+a^H)}+a^H\Big[\beta^s\ln(\gamma^s\delta\bar{a})-\beta^u\ln(\gamma^u\phi)\Big]\right\}>0. \tag{A.4}$$

(ii) Totally differentiating (21) with respect to $\bar{a}$ and α and then using (16) obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{a^H}\frac{\beta^u}{\lambda\ln(1+a^H)}\frac{\beta^u d\alpha}{\gamma^u(\alpha\beta^u+1)^2}$$
$$+\frac{\beta^s(1-\lambda)}{\gamma^s}\frac{\beta^s d\alpha}{(\alpha\beta^s+1)^2}=\beta^u\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\beta^u d\alpha}{\gamma^u(\alpha\beta^u+1)^2}$$
$$\Leftrightarrow\quad\frac{\Omega d\bar{a}}{a^H}=\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\gamma^u d\alpha}{\alpha^2}-\frac{\bar{a}}{a^H}\frac{\gamma^u d\alpha}{\alpha^2\lambda\ln(1+a^H)}-\frac{(1-\lambda)\gamma^s d\alpha}{\alpha^2}$$
$$\Leftrightarrow\quad\frac{d\bar{a}}{d\alpha}=-\frac{a^H}{\Omega\alpha^2}\left[(1-\lambda)(\gamma^s-\gamma^u)+\frac{\gamma^u(\bar{a}-\lambda a^H)}{\lambda a^H\ln(1+a^H)}\right]<0. \tag{A.5}$$

(iii) Totally differentiating (21) with respect to $\bar{a}$ and β^s obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+(1-\lambda)\ln(\gamma^s\delta\bar{a})d\beta^s+\frac{\beta^s(1-\lambda)\alpha d\beta^s}{\gamma^s(\alpha\beta^s+1)^2}=0$$
$$\Leftrightarrow\quad\frac{\Omega d\bar{a}}{a^H}=-(1-\lambda)\left[\ln(\gamma^s\delta\bar{a})+\frac{1}{\alpha\beta^s+1}\right]d\beta^s$$
$$\Leftrightarrow\quad\frac{d\bar{a}}{d\beta^s}=-\frac{(1-\lambda)a^H\left[\ln(\gamma^s\delta\bar{a})+1-\gamma^s\right]}{\Omega}<0. \tag{A.6}$$

Totally differentiating (21) with respect to $\bar{a}$ and β^u and using (14) obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{\lambda a^H\ln(1+a^H)}\left[\ln(\gamma^u\phi)+\frac{\alpha\beta^u}{\gamma^u(\alpha\beta^u+1)^2}\right]d\beta^u$$
$$=\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\left[\ln(\gamma^u\phi)+\frac{\alpha\beta^u}{\gamma^u(\alpha\beta^u+1)^2}\right]d\beta^u$$
$$\Leftrightarrow\quad\frac{\Omega d\bar{a}}{a^H}=\left[\ln(\gamma^u\phi)+\frac{1}{\alpha\beta^u+1}\right]\left[1-\lambda+\frac{\lambda a^H-\bar{a}}{\lambda a^H\ln(1+a^H)}\right]d\beta^u$$
$$\Leftrightarrow\quad\frac{d\bar{a}}{d\beta^u}=-\frac{a^H(1-\lambda)(1-\theta)[\ln(\gamma^u\phi)+1-\gamma^u]}{\theta\Omega}<0. \tag{A.7}$$

(iv) Totally differentiating (21) with respect to $\bar{a}$ and δ and rearranging obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\beta^s(1-\lambda)\left(\frac{d\bar{a}}{\bar{a}}+\frac{d\delta}{\delta}\right)=0\quad\Leftrightarrow\quad\frac{d\bar{a}}{d\delta}=-\frac{\beta^s(1-\lambda)a^H}{\delta\Omega}<0. \tag{A.8}$$

Totally differentiating (21) with respect to $\bar{a}$ and ϕ and using (14) obtains

$$\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{a^H}\frac{\beta^u d\phi}{\lambda\ln(1+a^H)\phi}=\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\beta^u d\phi}{\phi}$$
$$\Leftrightarrow\ \ \Omega d\bar{a}=\frac{\beta^u}{\phi}\left[(1-\lambda)a^H+\frac{\lambda a^H-\bar{a}}{\lambda\ln(1+a^H)}\right]d\phi$$
$$\Leftrightarrow\ \ \frac{d\bar{a}}{d\phi}=-\frac{(1-\theta)(1-\lambda)a^H\beta^u}{\theta\phi\Omega}<0. \tag{A.9}$$

Proof of Proposition 4

Given $\kappa,\tau_t\in(0,1)$, it is easy to see that $e_t^s=\gamma^s[\kappa(1-\tau_t)\delta\bar{a}_t-e_t^P]$ in (26) is smaller than $e_t^s=\gamma^s\delta\bar{a}_t$ in (16). Under the top-up education system, the proportional income tax leads to an effective rise in an unskilled worker's disposable income, which means that $e_t^u>\gamma^u\phi$. It follows from equations (7) and (17) that $M_t^{private}<M_t^{public}$.

Proof of Proposition 5

By (23) and (25), rich children will receive private tutoring in equilibrium if and only if

$$e_t^A=\gamma^s(1-\tau^*)\delta\bar{a}_t-\frac{\tau_t^*[\theta_t\delta\bar{a}_t+(1-\theta_t)\phi]}{(1+\alpha\beta^s)\kappa}>0$$
$$\Leftrightarrow\ \ \frac{\alpha\beta^s}{1+\alpha\beta^s}\cdot\frac{1-\tau^*}{\tau^*}\delta\bar{a}_t>\frac{\theta_t\delta\bar{a}_t+(1-\theta_t)\phi}{1+\alpha\beta^s}\frac{1}{\kappa}. \tag{A.10}$$

which can be rearranged as (28).

Proof of Proposition 6

If condition (28) violates, we have $e^A=0$ and then $e^u=e^s=e^P$. It follows that social welfare in (3) can be rewritten as

$$\begin{aligned}V_t &= \theta_t\Big\{\ln[(1-\tau_t)w_t^s]+\alpha\left[\lambda\ln(1+a^H)+\beta^s\ln e_t^P\right]\Big\}\\ &\quad+(1-\theta_t)\Big\{\ln[(1-\tau_t)w_t^u]+\alpha\left[\lambda\ln(1+a^H)+\beta^u\ln e_t^P\right]\Big\}\\ &= \ln(1-\tau_t)+\alpha[\beta^s\theta_t+\beta^u(1-\theta_t)]\big\{\ln\tau_t+\ln[\theta_t\delta\bar{a}_t+(1-\theta_t)\phi]\big\}\\ &\quad+\theta_t\ln(\delta\bar{a}_t)+(1-\theta_t)\ln\phi+\alpha\lambda\ln(1+a^H).\end{aligned} \tag{A.11}$$

Given $(\theta_t, \bar{a}_t)$, differentiating V_t in (A.11) with respect to τ_t and setting it to zero yields

$$\frac{dV_t}{d\tau_t} = \frac{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)]}{\tau_t} - \frac{1}{1-\tau_t} = 0.$$
$$\Leftrightarrow \quad \tau_t = 1 - \frac{1}{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)] + 1}. \tag{A.12}$$

Substituting (A.12) into (23) and then using (14) yields e_t^P as a function of θ_t:

$$e_t^P = \frac{\lambda\delta a^H[(1-\lambda)\ln(1+a^H) + \theta_t] + (1-\theta_t)\phi}{\{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)]\}^{-1} + 1}. \tag{A.13}$$

Inserting (A.13) into (12) and then focusing on the steady state obtains

$$\theta = \lambda\ln(1+a^H) + [\beta^s\theta + \beta^u(1-\theta)]\ln\left\{\frac{\lambda\delta a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi}{\{\alpha[\beta^s\theta + \beta^u(1-\theta)]\}^{-1} + 1}\right\}, \tag{A.14}$$

which can be rearranged as (29). Inserting (29) into (14) and rearranging yields the steady-state average ability of skilled labor as in (30).

Proof of Proposition 7

Given $\beta^s = \beta^u = \beta$, we simplify (A.12) as $\tau = \frac{\alpha\beta}{\alpha\beta+1}$ and thus rewrite (29) as

$$K = \theta - \lambda\ln(1+a^H) - \beta\ln\frac{\alpha\beta}{\alpha\beta+1} - \beta\ln\left\{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi\right\} = 0,$$

where $K = K(\theta; \alpha, \beta, \delta, \phi, \lambda, a^H)$. We have the following four partial derivatives:

$$\frac{\partial K}{\partial\theta} = 1 - \frac{\beta(\delta\lambda a^H - \phi)}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} = 1 - \beta\frac{\delta\lambda a^H - \phi}{(\delta\bar{a} - \phi)\theta + \phi}, \tag{A.15}$$
$$\frac{\partial K}{\partial\alpha} = -\beta\cdot\frac{\alpha\beta+1}{\alpha\beta}\cdot\frac{\beta(\alpha\beta+1) - \alpha\beta^2}{(\alpha\beta+1)^2} = -\frac{\beta}{\alpha(\alpha\beta+1)} < 0, \tag{A.16}$$
$$\frac{\partial K}{\partial\delta} = -\frac{\beta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta]}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} < 0, \tag{A.17}$$
$$\frac{\partial K}{\partial\phi} = -\frac{\beta(1-\theta)}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} < 0, \tag{A.18}$$

where $\frac{\partial K}{\partial \theta} > 0$ if and only if (31) is satisfied. In that case, total differentiation of K implies

$$\frac{\partial K}{\partial \theta} d\theta + \frac{\partial K}{\partial \alpha} d\alpha = 0 \qquad \Leftrightarrow \qquad \frac{d\theta}{d\alpha} = -\frac{\partial K}{\partial \alpha} \Big/ \frac{\partial K}{\partial \theta} > 0. \tag{A.19}$$

$$\frac{\partial K}{\partial \theta} d\theta + \frac{\partial K}{\partial \delta} d\delta = 0 \qquad \Leftrightarrow \qquad \frac{d\theta}{d\delta} = -\frac{\partial K}{\partial \delta} \Big/ \frac{\partial K}{\partial \theta} > 0. \tag{A.20}$$

$$\frac{\partial K}{\partial \theta} d\theta + \frac{\partial K}{\partial \phi} d\phi = 0 \qquad \Leftrightarrow \qquad \frac{d\theta}{d\phi} = -\frac{\partial K}{\partial \phi} \Big/ \frac{\partial K}{\partial \theta} > 0. \tag{A.21}$$

Because $\bar{a}$ and θ are inversely related by equation (22), we can infer that $\bar{a}$ decreases with (α, δ, ϕ) if and only if condition (31) holds.

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