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When the Threat Is Stronger Than the Execution: Trade and Welfare under Oligopoly

Abstract

We compare trade liberalization under Cournot and Bertrand competition in reciprocal markets. In both cases, the critical level of trade costs below which the possibility of trade affects the domestic firm's behavior is the same; trade liberalization increases trade volume monotonically; and welfare is U-shaped under reasonable conditions. However, welfare is typically greater under Bertrand competition; for higher trade costs the volume of trade is greater under Cournot competition, implying a "van-der-Rohe Region" in parameter space; and, for even higher trade costs, there exists a "Nimzowitsch Region", where welfare is higher under Bertrand competition even though no trade takes place.

JEL-Codes: L130, F120.

Keywords: Cournot and Bertrand Competition, Nimzowitsch Region, oligopoly and trade, trade liberalization, van-der-Rohe Region.

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1 Introduction

The sign and magnitude of the gains from trade and from trade liberalization continue to be among the central issues in international trade, all the more so now that globalization is being challenged and trade costs are rising rather than falling in many markets. Recent work has thrown new light on the quantitative extent of these gains under perfect competition and monopolistic competition with heterogeneous firms.[1] However, far less is known about the effects of trade liberalization in oligopolistic markets, despite growing evidence that trade is dominated by large firms.[2] In this paper we compare trade liberalization under Cournot and Bertrand oligopoly in a common framework. Notwithstanding the fact that some results of oligopoly models in trade and industrial organization are sensitive to the mode of competition,[3] we show that many of the predictions about the effect of trade liberalization are qualitatively robust to whether firms compete on quantity or price. At the same time, there are important quantitative differences between the two models, reflecting the fact that firms compete more aggressively in the Bertrand than in the Cournot case.

We focus on a trade interpretation throughout, where a home firm faces foreign competitors that are penalized by a trade cost, and we want to understand the effects of changes in trade restrictiveness. However, our model also throws light on other problems in industrial organization. For example, the model can be reinterpreted as one with a technologically superior firm facing competitors that suffer from a cost disadvantage, where the interest lies in the effects of technological catch-up by the competing firms.

To focus attention on the nature of competition, and to provide intuition for the outcomes we highlight, we first consider a canonical case: a symmetric two-country world with linear

[1]See, for example, Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot and Rodríguez-Clare (2014), Melitz and Redding (2015), and Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2018).

[2]See, for example, Mayer and Ottaviano (2008) and Freund and Pierola (2015).

[3]See, for example: Salant, Switzer, and Reynolds (1983) and Deneckere and Davidson (1985) on merger incentives; Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987) on strategic delegation; and Brander and Spencer (1985) and Eaton and Grossman (1986) on strategic trade policy. As Fudenberg and Tirole (1984) emphasise, the issue in these examples is whether firms' actions are strategic substitutes or complements rather than whether they engage in quantity or price competition. As we shall see in Section 6, most of our results do not hinge on whether actions are strategic substitutes or complements.

demands and differentiated products, in which a single domestic firm faces competition in both home and foreign markets from a single foreign firm. In this context, we show that the qualitative effects of trade liberalization are common irrespective of the mode of competition. In particular, trade liberalization increases trade volume monotonically; welfare as a function of trade costs follows a U-shaped pattern; and there is a critical level of trade costs below which the possibility of trade affects the domestic firm's behavior, which is the same under Cournot and Bertrand competition. On the other hand, there are important quantitative differences between the two cases. For any trade cost below this critical level, welfare is higher under Bertrand than under Cournot competition. This makes intuitive sense, because price competition is more competitive than quantity competition. Much less intuitive are the rankings of trade volumes. When trade costs are sufficiently low, we get the expected result where the volume of trade is always higher under Bertrand competition. However, for higher trade costs, imports are lower under Bertrand competition, though welfare remains higher. We call the region of parameter space in which this outcome holds the "van-der-Rohe Region," after the remark attributed to the architect Mies van der Rohe, "Less is more."[4] More surprisingly still, for even higher trade costs, no trade takes place under Bertrand competition while the volume of trade is positive under Cournot. Nonetheless, the mere threat of trade under price competition is sufficient to discipline the home firm and is more effective in reducing prices and raising welfare than actual trade is under Cournot competition. We call the region of parameter space in which this outcome holds the "Nimzowitsch Region," after the remark attributed to the chess grandmaster Aron Nimzowitsch, "The threat is stronger than the execution."[5]

As noted, we first show that all these results hold in a canonical two-country model with linear demands. Subsequently, we extend the analysis to many countries and to more general demand functions, and show that most of the results are qualitatively robust.

We focus throughout on a comparison between one-stage Cournot and Bertrand games.

[4]See https://en.wikipedia.org/wiki/Ludwig_Mies_van_der_Rohe.

[5]See http://en.wikipedia.org/wiki/Aron_Nimzowitsch.

However, these can be interpreted as alternative outcomes to a two-stage game, as shown by Kreps and Scheinkman (1983) with identical products and by Maggi (1996) with differentiated products.[6] In these models firms first invest in capacity and then set their prices in a Bertrand manner. If the capacity cost is low enough the outcome resembles a one-stage Bertrand game but if it is high enough it resembles a Cournot game. Hence our results can be interpreted as applying to a comparison between otherwise identical industries that differ in the ease with which they can adjust their capacity.

Our results build on and extend a large literature on the welfare effects of trade liberalization under oligopoly. (For overviews, see Leahy and Neary (2011) and Head and Spencer (2017).) The topic was first studied by Brander (1981) and Brander and Krugman (1983) using a model of two-way trade in segmented markets. In this reciprocal-markets setting, they demonstrated that, under Cournot competition, intra-industry trade can occur in equilibrium even when goods are identical. Bernhofen (2001) introduced product differentiation into Cournot and Bertrand oligopoly models of intra-industry trade, focusing on the effects of trade on profits and consumer surplus. The effects of trade liberalization in related models have also been considered by Clarke and Collie (2003), Brander and Spencer (2015) and Collie and Le (2015). All of these papers considered the effects of trade costs, while the effects of quotas were explored by Harris (1985) and Krishna (1989). As we shall see, the effects of quotas serve as a useful contrast with the effects of trade liberalization under Bertrand competition. Our setting also draws on an extensive literature in industrial organization. Our framework of a differentiated-product oligopoly where we compare quantity- and price-setting behavior is similar to that of Singh and Vives (1984) and Vives (1985), extended to allow for asymmetric selling costs and corner solutions. Unlike work on entry deterrence and accommodation that builds on Dixit (1980) and Fudenberg and Tirole (1984), the effects that we identify arise in one-stage games where firms simultaneously choose their outputs or prices.

[6] See also Neary and Tharakan (2012).

The plan of the paper is as follows. Section 2 introduces the setting of a many-country reciprocal-markets model. Section 3 considers the linear duopoly version of the model and illustrates the effects of trade liberalization on outputs, trade volumes, and welfare under Cournot and Bertrand competition when trade costs are sufficiently low that imports occur in equilibrium. Section 4 turns to consider the case where imports do not occur in equilibrium when firms compete on price, but nevertheless the threat of foreign entry serves to discipline the home firm. It shows how this outcome depends on the underlying parameters, and relates the findings to some classic results in game theory. Sections 5 and 6 explore in greater depth the cases with many firms and with general demands, respectively, and show that the main results are robust. Section 7 concludes, while the Appendices give proofs of the main results and also show that they continue to hold when trade is restricted by tariffs rather than by trade costs that yield no revenue, as we assume in the text.

2 The Setting

Throughout the paper, we use a framework in which preferences and technology are independent of the mode of competition. There are $n \geq 2$ countries in the world, in each of which there is a single domestic firm producing a unique good and, except in autarky, competing against imports from foreign firms. Markets are segmented, so each firm's output may sell at different prices in different countries in equilibrium. The results we will highlight do not hinge on exogenous asymmetries between countries, so to avoid unnecessary notation we confine attention to the case where all firms face symmetric demands, the same production cost functions, and the same trade costs. This allows us to focus attention on a single representative country that we refer to as "home", and our main interest is to understand the endogenous asymmetries that arise as a result of trade costs.

On the demand side we assume a home representative consumer with quasi-linear utility:

$$U = z_0 + u(x, \mathbf{y}) \tag{1}$$

where x is home sales of the home firm and $\mathbf{y}$ is the import vector with the sales of a typical foreign firm in the home market represented by y_j. Here z_0 is the consumption of the outside good which we assume is produced under perfect competition. This is a composite commodity defined over all the other goods in the economy, and we treat it as the numéraire good with a price equal to one. The sub-utility function $u(x, \mathbf{y})$ represents the domestic utility from consumption of the oligopolistic goods.

We can write the identity between national expenditure and national income as follows:

$$z_0 + px + \mathbf{p}^* \cdot \mathbf{y} = I + \Pi \tag{2}$$

Here p is the home-market price of the goods produced by the home firm; $\mathbf{p}^*$ is the vector of home-market prices of the foreign firms; I is domestic factor income; and Π is the total profits of the home firm in all markets. As it is written, equation (2) assumes that trade costs do not yield any revenue. If instead they took the form of tariffs, then tariff revenue would appear as an additional term on the right-hand side of (2). We show in Appendix B that our results continue to hold in that case. We can make use of (2) in the utility function (1) to write home welfare as:

$$W = U = \chi + \Pi + I \tag{3}$$

where $\chi \equiv u(x, \mathbf{y}) - px - \mathbf{p}^* \cdot \mathbf{y}$ is a micro-founded expression for home consumer surplus. As is standard, we assume that the non-numéraire sector is small in factor markets and so we treat I as constant.

On the cost side, we assume that marginal costs are constant and we ignore fixed costs. Hence the home and foreign firms' operating profits in the home market are:

$$\pi = (p - c)x \tag{4}$$

and

$$\pi_j^* = (p_j^* - c - t)y_j \tag{5}$$

where c is the marginal production cost of the home and foreign firms, assumed to be constant, and t is the per-unit trade cost.

3 Quantity vs. Price Competition in Linear Duopoly

Turning first to the linear duopoly case, we will first examine the effects of symmetric multilateral trade liberalization under quantity competition and then compare them with the outcome under price competition. Because of symmetry, equilibrium foreign market sales of the home and foreign firms are also equal to the home market sales y and x respectively. As the countries are mirror images of each other we need only consider the effect of a trade cost reduction on equilibrium in the home market. We concentrate on giving intuition in the text, with details of the derivations in Appendix A.[7]

3.1 Quantity Competition

In this section and the next, we assume that the sub-utility function in (1) takes a quadratic form: $u(x, y) = a(x + y) - \frac{1}{2}b(x^2 + 2exy + y^2)$. Maximization of (1) in this case subject to the budget constraint yields linear inverse demand functions:

$$p = a - b(x + ey) \quad \text{and} \quad p^* = a - b(y + ex). \tag{6}$$

The parameter $b > 0$ can be interpreted as an inverse measure of market size. As for e, it is an inverse measure of the degree of product differentiation, which we assume henceforward lies strictly between the cases of perfect substitutes ($e = 1$) and independent demands ($e = 0$).[8]

[7] Some of the results in this section have been shown in different ways by Brander (1981), Clarke and Collie (2003), Leahy and Neary (2011).

[8] We rule out the case of $e = 1$ since it implies that no imports take place under price competition for any strictly positive trade cost. We rule out the case of $e = 0$ since it implies that firms are independent

At free trade ($t = 0$), imports equal the home firm's sales. There is two-way trade in the oligopolistic sector since, from the symmetry of the model, foreign market sales of the home and foreign firms are also equal to their home market sales. As first shown by Brander (1981), this is true even when products are identical ($e = 1$), the case of cross-hauling or two-way trade in identical products. As goods become more differentiated, so e falls below one, the volume of trade rises further: the oligopolistic motive for trade is reinforced by a taste-for-diversity motive. As trade costs increase for a given value of e, two-way trade persists, though at a diminishing level: home sales rise and imports fall, reaching zero at the prohibitive level of trade costs which we denote by $\widehat{t}^C$.

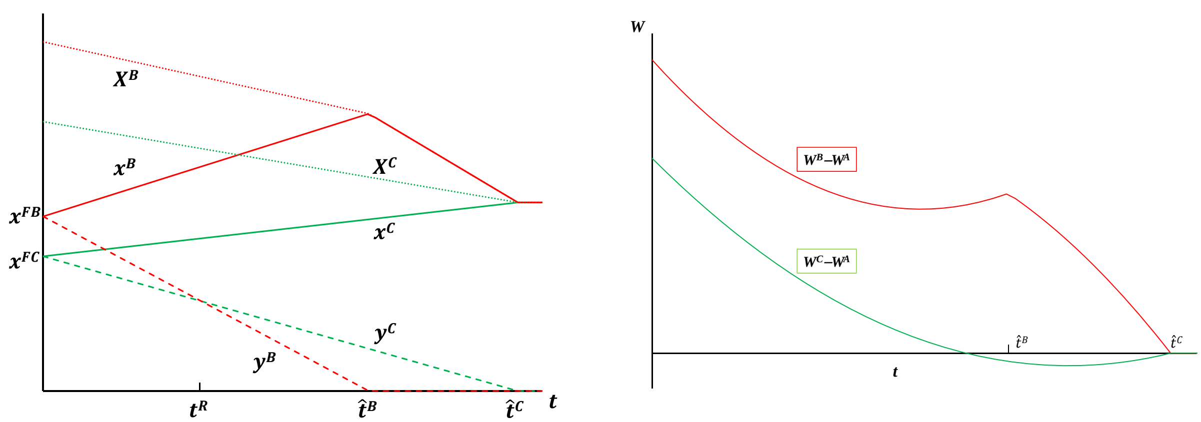
We are mainly concerned with the effect of trade liberalization on welfare, but an important preliminary step, which is also of independent interest, is its implications for profits. Focusing on the home firm, its total profits equal the sum of its profits on home sales and on exports. The first are given by (4) while the second equal the foreign firm's profits in the home market (5): because of the symmetry of the model, home exports x^* equal home imports y, so the home firm's profits on its exports are $\pi^* = (p^* - c - t)x^* = (p^* - c - t)y$.

Profits are decreasing in trade costs at free trade, but increasing in them in the neighborhood of autarky. It follows that profits must be a U-shaped function of trade costs. The intuition for this is straightforward. First, starting from free trade, exports are harmed more by an increase in the firm's own costs than home sales are helped by an equal rise in its rival's costs. Hence total sales and profits fall for a small increase in t at free trade. Second, starting from autarky, exports are initially zero, so a small fall in trade costs has a negligible effect on profits in the export market; by contrast, home sales are initially at the monopoly level, so a small fall in the foreign firm's trade costs has a first-order effect on home-market profits. Hence, overall profits fall for a small reduction in t at autarky.

Combining the results so far on changes in prices and profits, we can consider the full

monopolists with no strategic interaction. We assume that b is independent of e, which is the standard Bowley specification of linear demands. See, for example, Vives (1985). This specification has been criticized on the grounds that the market size increases as products become differentiated. This feature is avoided by the alternative Shubik-Levitan specification, used by Collie and Le (2015), which sets $b = \frac{\beta}{1+e}$ where $\beta > 0$.

effect of changes in trade costs on welfare. Consider in turn the components of welfare in (3). Consumer surplus must rise monotonically as trade costs fall. This is because a reduction in trade costs lowers the prices of both goods to home consumers. To this must be added the U-shaped relationship between profits and trade costs already derived. In the neighborhood of free trade, welfare is clearly falling in trade costs. All that is left is to consider the sum of consumer surplus and profits for a small fall in t starting in autarky (where $t = t^C$). Consumer surplus rises because the price falls, but profits on home sales fall both because the price falls and because sales are reduced. The price effects cancel, so the total fall in profits outweighs the rise in consumer surplus. Thus home welfare (the sum of profits and consumer surplus) is also a U-shaped function of t, reaching its maximum at free trade but its minimum below the prohibitive level of trade costs.



(a) Consumption and Outputs

(b) Welfare

Figure 1: Effects of Trade Liberalization in Cournot and Bertrand Competition

The green loci in Figure 1 summarize the results for the linear duopoly model under quantity competition.[9] In free trade, the home and foreign firm have identical sales, equal to x^{FC}. Higher trade costs reduce imports y^C and raise the home firm's sales x^C though by less, so total sales ($X^C = x^C + y^C$) fall. Finally, welfare is a U-shaped function of the trade cost as we have seen, with autarky welfare below the free-trade level.

[9]The figure is drawn for e equal to 0.8, and with autarky welfare normalized to zero in panel (b). The segments of the curves to the right of $\widehat{t}^B$ will be considered in the next section.

3.2 Price Competition

How do the effects of trade liberalization on trade and welfare differ if firms compete in price rather than quantity? In comparing Bertrand and Cournot competition we make the same assumptions about preferences and technology. However, in the Bertrand case we need to give more attention to corner solutions. We will consider these in detail in Section 4; for the moment we consider only interior equilibria in which both firms export positive quantities.

To solve for the Bertrand equilibrium in this case, it is convenient to use the direct demand functions, which can be obtained by inverting the system in (6) to get:

$$x = \frac{(1-e)a - (p - ep^*)}{b\,(1-e^2)} \quad \text{and} \quad y = \frac{(1-e)a - (p^* - ep)}{b\,(1-e^2)} \tag{7}$$

Details of the solution are given in Appendix A, and are illustrated by the red loci in Figure 1(a). Qualitatively, equilibrium outputs are related to trade costs in the same way as in the Cournot case in Section 3.1: imports equal the home firm's sales at free trade and are decreasing in trade costs, falling to zero when trade costs reach a threshold level denoted by $\widehat{t}^B$ at which imports are eliminated. Quantitatively, the differences reflect the fact that price competition is more competitive than quantity competition, as shown by Vives (1985) for the case of identical costs. Relative to Cournot competition, it is easy to see that, in Bertrand competition, free-trade output is higher, and the prohibitive trade cost is lower, $\widehat{t}^B < \widehat{t}^C$. This implies that, for trade costs higher than a threshold level, denoted by t^R in Figure 1(a), imports are lower in price than in quantity competition.

Profits and welfare also behave quite similarly to quantity competition for trade costs between zero and the prohibitive trade cost level $\widehat{t}^B$. Profits are decreasing in trade costs at free trade, but increasing in them in the neighborhood of the threshold trade cost $\widehat{t}^B$. Hence, it follows that profits must be a U-shaped function of trade costs. Once again, consumer surplus falls monotonically in trade costs and so, as in the Cournot case, welfare is a U-shaped function of trade costs. Figure 1(b) illustrates these outcomes and also shows that welfare is always higher in price competition. (Details are in Appendix A.) This holds even when trade costs are in the interval $(t^R, \widehat{t}^B)$, where as we have seen imports are lower under price competition. Hence we can call this interval a "van-der-Rohe Region" in parameter space, where "less is more": even though imports are lower, welfare is higher in the Bertrand case because competition is more intense.

However, this comparison only holds for $t \leq \widehat{t}^B$. We have not yet considered what happens under price competition for trade costs in the range between $\widehat{t}^B$ and $\widehat{t}^C$. To understand this case we have to give more careful attention to the nature of the game between firms.

4 The Nimzowitsch Region

4.1 Price Competition in the Absence of Imports

Even when trade costs are too high for imports to take place, they may not be too high to prevent the threat of imports from affecting the domestic firm's behavior. As we will show in this section, for all trade costs in the region $t \in (\widehat{t}^B, \widehat{t}^C)$, the home firm chooses a price below the monopoly level such that the foreign firm is just unable to produce. The home firm does not have an incentive to raise its price, since its rival would then make positive sales and this would lower the home firm's domestic profits. Only when trade costs equal $\widehat{t}^C$ or higher can the home firm behave as an unconstrained monopolist.[10] At levels of trade costs above $\widehat{t}^B$ and below $\widehat{t}^C$ the threat of imports implies a higher level of welfare than under Cournot competition, even though no actual trade occurs. Hence, as discussed in the introduction, we call this the Nimzowitsch Region.

To demonstrate these results, we need to consider the firms' best-response functions under price competition in the region $t \in (\widehat{t}^B, \widehat{t}^C)$. As we show in Appendix A, these thresholds

[10]Note the difference between this behavior by the home firm and entry deterrence as in for instance Dixit (1980). In the entry deterrence case, firms move sequentially and the leader commits to an action in advance. Here firms move simultaneously and the home firm's equilibrium price represents a best response.

equal:

$$\widehat{t}^B = \frac{(1-e)(2+e)}{2-e^2}A \quad < \quad \widehat{t}^C = \frac{2-e}{2}A \tag{8}$$

where $A \equiv a - c$ is the difference between the maximum price consumers are willing to pay and the marginal cost of production. Consider first the foreign firm. In an interior Bertrand equilibrium (where imports are strictly positive) of the kind already considered in the last sub-section, the foreign firm's best-response function is:

$$p^* = B^*(p;t) = \underset{p^*}{\text{argmax}}\ \pi^*(p^*, p\,; t) = \tfrac{1}{2}\left((1-e)a + c + t + ep\right) \tag{9}$$

However, if the home firm's price falls to a level where the zero-import constraint binds, then the foreign firm's best response is to charge a price equal to its marginal cost of serving the market, $c + t$.[11] The zero-import constraint, $y(p^*, p) = 0$, defines the maximum foreign price consistent with zero imports as a function of the home price. From (7), this is:

$$p^* = \tilde{p}^*(p) = (1-e)a + ep \tag{10}$$

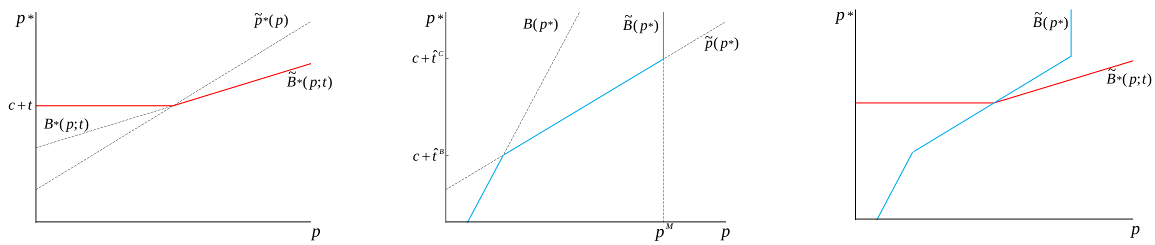
Combining these two regimes, the foreign firm's best-response function is:

$$p^* = \tilde{B}^*(p,t) = \begin{cases} c+t & \text{when } \tilde{p}^*(p) \leq B^*(p;t) \\ B^*(p;t) & \text{when } \tilde{p}^*(p) \geq B^*(p;t) \end{cases} \tag{11}$$

This locus is kinked where it intersects the zero-import locus, and is shown by the bold locus in Figure 2(a).

In the same way, we can examine the best responses of the home firm. In this case there are three distinct regimes. First, for low import prices, the home firm's best response is also

[11] Following most of the literature, we rule out cases where firms, in anticipation of zero sales, set price below marginal cost. For an alternative view and references see Erlei (2002).



(a) Foreign Best-Response Function

(b) Home Best-Response Function

(c) Equilibrium in the Nimzowitsch Region

Figure 2: Strategic Interactions in the Nimzowitsch Region

to charge a low price along its unconstrained best-response function $B(p^*)$, which is:

$$p = B(p^*) = \underset{p}{\operatorname{argmax}}\ \pi(p, p^*) = \tfrac{1}{2}\left((1-e)a + c + ep^*\right) \tag{12}$$

This case corresponds to trade costs below the threshold $\widehat{t}^B$. By contrast, for very high import prices, the home firm is an unconstrained monopolist, and so it charges the monopoly price which we denote by $p^M = \frac{1}{2}(a+c)$. This price is the solution to the zero-import constraint (10) when the foreign firm charges a price just sufficient to choke off import demand: i.e., a price equal to its marginal cost of production plus the prohibitive trade cost $\widehat{t}^C$.[12] Finally, for intermediate import prices, the home firm's best-response is to set its own price at the level such that the zero-import constraint just binds: from its perspective it operates on the inverse of (10), which we denote $\tilde{p}(p^*)$. Combining these three regimes, the home firm's best-response function is:

$$p = \tilde{B}(p^*) = \begin{cases} B(p^*) & \text{when } \tilde{p}(p^*) \leq B(p^*) \\ \tilde{p}(p^*) & \text{when } B(p^*) \leq \tilde{p}(p^*) \leq p^M \\ p^M & \text{when } p^M \leq \tilde{p}(p^*) \end{cases} \tag{13}$$

This function has two kinks, as shown by the bold locus in Figure 2(b).

[12] Combining (10), $\widehat{t}^C$ from (8), and $p^* = c + t$ gives the monopoly price p^M.

Finally, we can bring together the responses of the two firms to show the full equilibrium, as in Figure 2(c). Notice first that the home firm's best-response function (13) is not affected by the actual level of the trade cost t. A change in the trade cost shifts the foreign firm's best-response function only, so the equilibrium moves along the home firm's function. There are therefore three possible regimes, depending on where the intersection point occurs. In the case shown, the trade cost lies between $\widehat{t}^B$ and $\widehat{t}^C$, and so the foreign firm's best-response function intersects the home's along the zero-imports locus. The home firm does not have an incentive to raise its price, since its rival would then make positive sales and this would lower the home firm's domestic profits. This outcome therefore exhibits one of the features of what we will call a Nimzowitsch Region: the home firm's behavior under price competition is affected by the threat of imports, even though no actual imports take place.

4.2 Maximum versus Minimum Import Constraints

It is instructive to compare this outcome with the case studied by Krishna (1989), where the foreign firm faces a quantitative trade barrier, such as a quota or a voluntary export constraint. There the constraint takes the form of a maximum level of imports, whereas in our case the constraint is a minimum one: imports cannot fall below zero. This difference affects the nature of the game in important ways.[13] In particular, it determines whether the home firm's best-response function is continuous or not.

To compare the two cases on a common basis, assume for convenience that the import constraint is strictly positive rather than zero. Whether the constraint is a maximum or minimum, the home firm is choosing between two options, whose implications for its profits can be represented by two concave functions, as shown in Figure 3. One, denoted by π^M, represents the profits it would obtain if it were to act as a monopolist along its residual

[13] A further complication when a maximum import constraint binds is that, since consumers wish to purchase more than the permitted level of imports, some rationing mechanism must be adopted to allocate it. Different rationing mechanisms have different implications for the equilibrium outcome. This problem does not arise in our context, since consumers are always able to purchase the level of imports they desire at the prevailing prices.

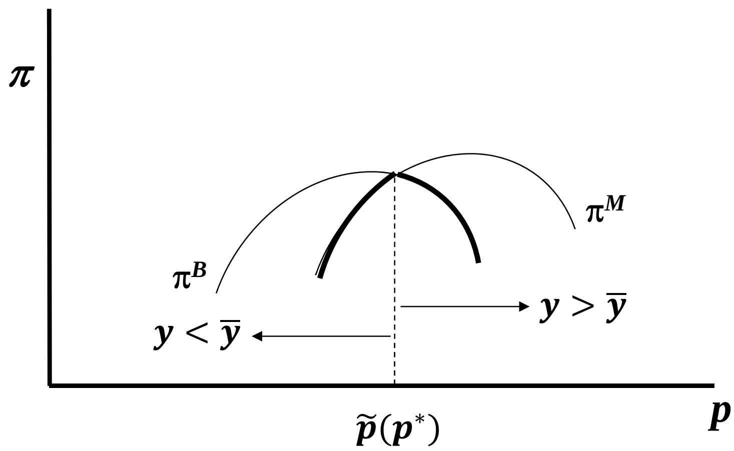


Figure 3: Price Competition in the Nimzowitsch Region:
The Home Firm's Perspective

demand function; this is relevant only when the constraint on the level of imports is binding. The other, denoted by π^B, represents the profits, conditional on a foreign price p^*, that it would obtain in a Bertrand equilibrium where both firms have positive sales; this is relevant only when the constraint on the level of imports is non-binding.

In our case with a minimum import constraint, the home firm's profit function is the lower envelope of these two concave functions, indicated by the solid line in the lower panel of Figure 3. Such a lower envelope is itself concave. Hence, the home firm's best-response function is continuous, and there is a unique equilibrium in pure strategies, just as illustrated in Figure 2(c): the home firm sets a price that is just consistent with the level of imports equal to the permitted level, while the foreign firm sets a price equal to that underlying the π^B locus. As the trade cost changes, the monopoly profit function is unaffected, but the optimal price chosen by the foreign firm changes, and so the function representing home profits in an unconstrained Bertrand equilibrium shifts.

By contrast, in the case of a maximum import constraint considered by Krishna, the home firm's profit function is the upper rather than the lower envelope of the two concave functions. As a result, the profit function of the home firm is non-concave. In addition, for a particular foreign price (which as Krishna shows is profit-maximizing for the foreign firm), the two functions yield the same profit levels for the home firm (unlike the case illustrated in the figure). Hence its best-response function is discontinuous and the game has no equilibrium in pure strategies. As the permitted level of imports changes, the constrained monopoly profit function shifts, but the function representing profits in an unconstrained Bertrand equilibrium is unaffected.

The difference between the two cases can also be related to two classic outcomes in game theory. The case of import quotas considered by Krishna is analogous to Edgeworth's demonstration that an equilibrium in pure strategies does not exist in a Bertrand game with capacity constraints. By contrast, the Nimzowitsch Region is analogous to the contrast between quantity and price competition with identical products, where in price competition the more efficient firm captures the entire market by charging a price equal to the marginal cost of the second most efficient firm. This is exactly what happens in the case of $e = 1$: any non-zero trade cost is prohibitive, with only the home firm selling to consumers at a price equal to the foreign firm's marginal cost of serving the home market: its marginal production cost plus the trade cost. The novel feature of the current model is that a similar outcome arises even when products are differentiated.

4.3 Welfare in the Nimzowitsch Region

Finally, consider the level of welfare in this region of potential though not actual competition from imports. Welfare in the absence of trade is consumer surplus plus home profits. This can be written as $W = Ax - \frac{1}{2}bx^2$. Totally differentiate this to get:

$$dW|_{t\in(\hat{t}^B,\hat{t}^C)} = (A - bx)dx. \tag{14}$$

This is negative since in this region the home firm's output is:[14]

$$x^B\big|_{t\in(\hat{t}^B,\hat{t}^C)} = \frac{A-t}{be} \tag{15}$$

Hence both output and welfare are decreasing in t under price competition in the region $t \in (\hat{t}^B, \hat{t}^C)$, as illustrated in Figure 1. Putting this differently, under price competition, unlike under quantity competition, trade liberalization starting from autarky initially raises welfare: the home firm is disciplined by the threat of trade, without any trade taking place.

The last step is to show that welfare is higher under price than under quantity competition, as illustrated in Figure 1(a). We have already seen in Section 3.2 that welfare in an interior Bertrand equilibrium is higher than in a Cournot equilibrium. It turns out that the same is true for comparisons between welfare in a non-interior Bertrand equilibrium and in a Cournot equilibrium, as we show explicitly in Appendix A. There are two opposing effects. On the one hand, the threat of imports generates a pro-competitive effect that tends to raise welfare under price competition in the interval $t \in (\hat{t}^B, \hat{t}^C)$ relative to the case of quantity competition. On the other hand, imports are excluded in price competition, so quantity competition generates a "love-of-variety" effect. It might be thought that the latter effect might dominate when goods are highly differentiated, i.e., for low values of e. However, the proof in Appendix A shows that this cannot happen: welfare is always higher under price competition even though imports do not take place. This completes the demonstration that the interval $t \in (\hat{t}^B, \hat{t}^C)$ is indeed a Nimzowitsch Region.

Figure 4 summarizes the results to date, showing how the sizes of the different regions vary with the degree of substitutability e and the trade cost t (normalized by A). In particular, we can see that the Nimzowitsch Region is largest when goods are close substitutes and

[14] To see this explicitly, find the level of p that sets $y = 0$ for any given p^*. Inverting (10), this is: $\tilde{p}(p^*) = \frac{p^*-(1-e)a}{e}$. In the Nimzowitsch Region, $t \in (\widehat{t}^B, \widehat{t}^C)$, where the foreign firm is just kept out of the market: $p^* = c + t$. Combine these and make use of $p = a - bx$ to obtain (15). Note that output from (15) reduces to monopoly output, $x^M = \frac{A}{2b}$, when $t = \widehat{t}^C$. Substituting $\widehat{t}^B$ into home output under Bertrand competition, x^B, from equation (52) in Appendix A.2, shows that the home firm's output is $\widehat{x}^B = \frac{A}{b(2-e^2)}$ at this threshold level of trade costs, which is above the unconstrained monopoly output level, $x^M = \frac{A}{2b}$.

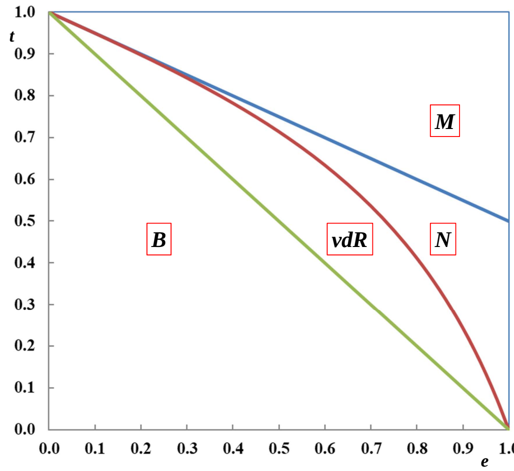


Figure 4: Regions of Trade and Welfare in Duopoly

The three loci show the values of $\widehat{t}^C$, $\widehat{t}^B$, and t^R (in decreasing order), as functions of e.
B: Bertrand yields Higher Trade and Welfare, $y^B > y^C > 0$, $W^B > W^C$.
vdR: van-der-Rohe Region, $y^C > y^B > 0$, $W^B > W^C$.
N: Nimzowitsch Region, $y^C > y^B = 0$, $W^B > W^C$.
M: Unconstrained Monopoly, $y^C = y^B = 0$.

declines as they become more differentiated (so the love-of-variety effect becomes stronger).

5 Many Countries

So far we have restricted attention to the case of only two countries. We now allow for oligopolistic competition with many countries. We continue to assume that there is a single domestic firm in each country and that all firms have the same marginal cost of production and face the same trade cost of selling into the other markets. Our main concern in this section is to show that the results for the duopoly case of Sections 3 and 4 are strengthened as the number of countries increases. We first consider the case where all firms' sales are positive under both quantity and price competition. We then turn to the case where imports are zero in equilibrium yet the best response of the home firm is to charge a lower price than it would in unconstrained autarky. We show that this occurs in the interval between the prohibitive trade cost thresholds, $\widehat{t}^B$ and $\widehat{t}^C$, which is always non-empty and increases in the number of countries. The final step is to show that welfare is higher under price than quantity competition in this interval, so it is indeed a Nimzowitsch region.

A priori, the effect of additional firms could go either way. On the one hand, more firms means that consumers enjoy more variety when imports occur. This tends to increase welfare under Cournot competition relative to price competition. On the other hand, more firms means more competition, including more incipient competition when imports do not occur. This forces the home firm to charge a lower price under price competition, which is clearly welfare-improving. It turns out that the competition effect dominates the variety effect, and by more the greater the number of firms.

There are n countries in the world, in each of which there is a single domestic firm producing a unique good. As before, due to symmetry across countries, we can focus our analysis on a representative market. Define total import volume by: $Y = \sum_{j=1}^{n-1} y_j$. The sub-utility function for the oligopolistic goods from (1) takes the form:

$$u(x, \mathbf{y}) = a(x+Y) - \frac{1}{2}b\left(e(x+Y)^2 + (1-e)\left(x^2 + \sum_{j=1}^{n-1}(y_j)^2\right)\right) \tag{16}$$

The inverse demand faced by the domestic firm and a representative foreign firm can be written as:

$$p = a - b(x+eY) \quad \text{and} \quad p_j^* = a - b(y_j + ex + eY_{-j}) \tag{17}$$

where $Y_{-j} = Y - y_j$. Firm profits are given by (4) and (5) as before. Hence the firms' first-order conditions under Cournot competition are $p - c + x\frac{\partial p}{\partial x} = 0$ and $p_j^* - c - t + y_j\frac{\partial p_j^*}{\partial y_j} = 0$ for all j, where $\frac{\partial p}{\partial x} = \frac{\partial p_j^*}{\partial y_j} = -b$ is the slope of the firms' inverse demand functions. The corresponding first-order conditions under Bertrand can be written as: $p - c + x\left(\frac{\partial x}{\partial p}\right)^{-1} = 0$ for the home firm, and $p_j^* - c - t + y_j\left(\frac{\partial y_j}{\partial p_j^*}\right)^{-1} = 0$ for a typical foreign firm, where $\frac{\partial x}{\partial p} = \frac{\partial y_j}{\partial p_j^*} = -\frac{1}{b}\frac{1+e(n-2)}{1+e(n-2)-e^2(n-1)}$ is the slope of the firms' direct demand functions. It is convenient to rewrite the home and foreign first-order conditions in a way that nests the Cournot and Bertrand special cases:

$$A - b(2+V)x - b(n-1)ey = 0 \tag{18}$$

$$A - b\left(2 + V + (n-2)e\right)y - bex - t = 0 \tag{19}$$

where we have made use of the fact that the outputs of every foreign firm are identical, allowing us to drop the subscript on y. In (18) and (19), we can interpret V as a conjectural variations parameter, though we are only interested in the values it takes in quantity and price competition.[15]

$$V^C = 0 \quad \text{and} \quad V^B = -\frac{(n-1)e^2}{1+(n-2)e} \tag{20}$$

By inspection, $V^B < V^C$ for any $e > 0$ and $V^B > -1$ for $e < 1$. We show in Appendix C.1 that V^B is decreasing in n and in e.

The first-order conditions in (18) and (19) can be solved for the equilibrium quantities. Sales by the home firm, and imports from a typical foreign firm are given by:

$$x = x^F\left(1 + (n-1)\frac{e}{2+V}\frac{t}{\widehat{t}}\right) \qquad y = y^F\left(1 - \frac{t}{\widehat{t}}\right) \tag{21}$$

where:

$$x^F = y^F = \frac{A}{b\Gamma}, \qquad \Gamma \equiv 2 + V + (n-1)e, \qquad \widehat{t} \equiv \left(1 - \frac{e}{2+V}\right)A, \qquad \text{and} \qquad t \leq \widehat{t} \tag{22}$$

Here x^F and y^F denote home sales and imports respectively in free trade; Γ can be interpreted as a measure of the competitive pressure faced by an individual firm for a given value of V;[16] and $\widehat{t}$ denotes the prohibitive trade cost, i.e., the minimum trade cost at which imports become zero. Note that, since V varies with the mode of competition from (20), so also do x^F, y^F, Γ, and $\widehat{t}$; where necessary we use superscripts C and B, e.g., x^{FC}, $\widehat{t}^B$, etc., to denote

[15] Conjectural variations are an unsatisfactory way of modeling strategic interaction in general, but as here they can provide a useful way of comparing Cournot and Bertrand behavior on a common basis. See, for example, Dixit (1986). In particular, the finite comparison between the Cournot and Bertrand equilibria is much more simply done by differentiating with respect to V.

[16] This interpretation follows as in Mrázová (2011), because, for a given value of V, Γ is increasing in the number of firms n and in the degree of substitutability e: increases in either of these make the environment each firm faces more competitive and so tend to reduce individual firm output. In price competition, the endogenous change in V^B works in the opposite direction, but as we show in Appendix C.1, for most parameter values it offsets but cannot reverse the direct effects: Γ continues to be increasing in n and e.

quantity and price competition respectively.

We are interested in comparing outputs and welfare under quantity and price competition for different values of the trade cost t. The response of outputs to trade costs can be deduced immediately from (21):[17]

Lemma 1. *Irrespective of the number of countries:*

(a) In free trade, both home sales and imports are higher in price than in quantity competition: $x^{FB} > x^{FC}$*;*

(b) Higher trade costs raise home sales and reduce imports irrespective of the form of competition: $\frac{\partial x}{\partial t} > 0$ *and* $\frac{\partial y}{\partial t} < 0$*;*

(c) A rise in trade costs raises home sales by more and lowers imports by more in price than in quantity competition: $\frac{\partial x^B}{\partial t} > \frac{\partial x^C}{\partial t} > 0$ *and* $\frac{\partial y^B}{\partial t} < \frac{\partial y^C}{\partial t} < 0$*; and*

(d) The prohibitive trade cost is lower in price than in quantity competition: $\widehat{t}^B < \widehat{t}^C$*.*

Turning to the response of welfare to trade costs, it is clear that consumer surplus falls monotonically as trade costs rise. However, the response of profits is more complex. Focusing on the home firm, its total profits equals the sum of profits on home sales and the profits from exporting to $n-1$ foreign markets. Using the first-order conditions, maximized profits are equal to $b(1+V)x^2$ from home sales and $(n-1)b(1+V)(x^*)^2$ from exports. Total profits for the home firm are then equal to the sum of these, and their behavior as trade costs change can be shown to equal:

$$\frac{\partial \Pi}{\partial t} = 2b(1+V)\left(x\frac{\partial x}{\partial t} + (n-1)x^*\frac{\partial x^*}{\partial t}\right)$$

$$= \frac{2(n-1)}{2+V}\,\frac{p^F - c}{\widehat{t}}\,(ex - (2+V)x^*) \quad \begin{cases} < 0 \text{ when } t = 0 \text{ (so } x = x^*) \\ > 0 \text{ when } t = \widehat{t} \text{ (so } x^* = 0) \end{cases} \tag{23}$$

where $p^F - c = b(1+V)x^F$ is the firms' price-cost margin in free trade. Profits are decreasing in trade costs at free trade, but increasing in them in the neighborhood of the prohibitive

[17] These results and many subsequent ones are stated as strict, which follows from our assumption that e lies strictly between zero and one.

trade cost $\widehat{t}$. Hence, it follows that profits must be a U-shaped function of trade costs, irrespective of the mode of competition.[18] The same must be true of welfare, in line with the intuitive argument given for the duopoly case in Section 3. At free trade, both consumer surplus and profits are falling in trade costs. As for autarky, the fall in consumer surplus is not enough to offset the rise in home profits for an infinitesimal increase in trade costs: $d\chi = -xdp$ (since $y = 0$ at autarky) and $d\Pi = (p-c)dx + xdp$ (since $y = x^* = 0$ and $p^* = c + t$). The sum of these reduces to $dW = (p-c)dx$ and we know from part (b) of Lemma 2 that home sales are increasing in trade costs. Hence it follows that welfare is also increasing in trade costs in the neighborhood of autarky. Summarizing:

Lemma 2. *Assume* $0 < e < 1$. *Then welfare is a U-shaped function of trade costs under both quantity and price competition.*

Having established how welfare depends on trade costs, we next want to compare the levels of welfare in quantity and price competition.

Consider first the range of trade costs consistent with positive imports under both types of competition, which, from part (d) of Lemma 2, is $0 \leq t \leq \widehat{t}^B$. We want to show that, in this range, welfare is always higher under price than under quantity competition for given values of e, n and t. Welfare equals the utility level given by (16), evaluated in equilibrium where all foreign firms have equal sales, minus production and trade costs:

$$W(x, Y) = A(x+Y) - \frac{1}{2}b\left(e(x+Y)^2 + (1-e)\left(x^2 + \frac{1}{n-1}Y^2\right)\right) - tY \qquad (24)$$

where $Y = (n-1)y$ because of symmetry. We can differentiate this with respect to V as a simple way of comparing the Cournot and Bertrand cases, since V^C is unambiguously greater than V^B from (20):

$$\frac{dW(x,Y)}{dV} = W_x \frac{dx}{dV} + W_Y \frac{dY}{dV} = (W_x - W_Y)\frac{dx}{dV} + W_Y \frac{dQ}{dV} \qquad (25)$$

[18] For the special case of quantity competition in duopoly (23) takes a particularly simple form: $\frac{\partial \Pi}{\partial t} = \frac{p^F - c}{\widehat{t}}(ex - 2x^*)$.

where Q denotes total sales in the home market: $Q = x + Y$. It is easy to check that sales of the home firm, and total sales in the home market, are both decreasing in V; hence they are higher in Bertrand than in Cournot competition. (Details are in Appendix C.2.) Similarly, it is immediate that, in the relevant range, welfare increases in both x and Y: intuitively, increases in either of these partially overcome the oligopoly distortion. As for the effect of an equal and opposite change in x and Y, this has a negative effect on welfare by reducing variety, but also a positive effect through a saving in trade costs. It turns out that the latter effect dominates so $W_x > W_Y$.[19] Combining these results, we see that welfare is decreasing in V. This gives our first key result:

Proposition 1. *For trade costs in the range $0 \leq t \leq \widehat{t}^B$, welfare is strictly higher in price competition than in quantity competition: $W^B > W^C$, for $e > 0$.*

Surprisingly, the unambiguous ranking of welfare levels under different modes of competition does not carry over to the levels of imports in the range $0 \leq t \leq \widehat{t}^B$. From part (a) of Lemma 2, we know that imports in free trade are higher under price competition than under quantity competition, while from part (d) imports at the upper point of the range $\widehat{t}^B$ are lower under price competition than under quantity competition. Since imports are linear in t from (21), there must be an intermediate trade cost t^R, $0 \leq t^R \leq \widehat{t}^B$, at which imports are the same under the two modes of competition:

$$t^R : \quad y^B\big|_{t=t^R} - y^C\big|_{t=t^R} = 0 \tag{26}$$

Hence imports must be lower under price competition than under quantity competition for all t such that $t^R < t < \widehat{t}^B$. Substituting for outputs from (21), we can express t^R in terms of free-trade imports under each form of competition, y^{FB} and y^{BC}, and the two prohibitive trade costs:

$$t^R = \frac{\left(y^{FB} - y^{FC}\right)\hat{t}^C}{y^{FB}\hat{t}^C - y^{FC}\hat{t}^B}\,\hat{t}^B \tag{27}$$

[19]From equation (72) in Appendix C.2, the difference $W_x - W_Y$ equals the dumping margin, the difference between the price charged at home, p, and the f.o.b. price received from exporting, $p^* - t$.

By inspection, this is less than $\widehat{t}^B$. It can be shown that t^R is decreasing in both n and e: see Appendix C.3. Combining these results with Proposition 1, we can conclude:

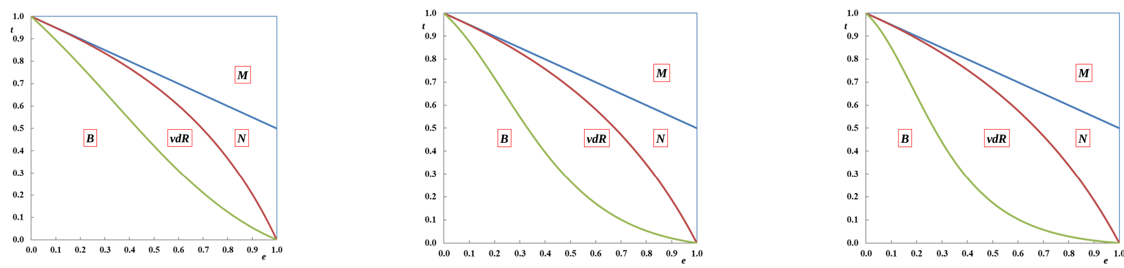
Proposition 2. *The non-empty interval* $(t^R, \widehat{t}^B)$ *is a van-der-Rohe Region in which welfare is strictly higher but imports are strictly lower under price competition than under quantity competition:* $W^B > W^C$ *and* $0 < y^B < y^C$*, for* $0 < e < 1$*. The lower boundary of this interval,* t^R*, is decreasing in the number of firms* n *and the degree of substitutability* e*.*

Finally, we want to consider what happens for trade costs in the interval $(\widehat{t}^B, \widehat{t}^C)$. Consider first the size of the interval. Since $V = 0$ under Cournot, $\widehat{t}^C = \frac{2-e}{2}A$ is independent of the number of countries. As for $\widehat{t}^B$, it is increasing in V^B which is decreasing in n as we have seen, and so overall it is falling in n. It follows that as the number of countries increases the interval between the two prohibitive trade costs gets larger. It can also be shown that this interval is increasing in the degree of substitutability e: see Appendix C.5.

The final part of the proof is to show that welfare is higher under price competition in the interval $(\widehat{t}^B, \widehat{t}^C)$; in other words, that this interval is indeed a Nimzowitsch Region. We show this in three steps, the details of which are given in Appendix C.4. First, we show that $W^N - W^C$ is quadratic and concave in t everywhere. This implies that a sufficient condition for $W^N - W^C$ to be positive throughout the region is that it is non-negative at the end-points of the region. Next, we can show that W^N is equal to W^C at the upper end of the region, $t = \widehat{t}^C$, where no trade occurs and the home firm is an unconstrained monopolist in both price and quantity competition. Finally, we can show that W^N is equal to W^B at the lower end, where $t = \widehat{t}^B$, since welfare depends only on outputs, and at that point outputs under unconstrained price competition, x^B and y^B, are equal to those under constrained price competition, x^N and y^N (with $y^N = y^B = 0$). Since we know from Proposition 1 that W^B is greater than W^C at that point, it follows that W^N is also greater than W^C.

Hence, it follows that welfare is always greater under price than under quantity competition throughout the $(\widehat{t}^B, \widehat{t}^C)$ region, so it is indeed a Nimzowitsch Region. Summarizing our results:

Proposition 3. *The non-empty interval* $(\widehat{t}^B, \widehat{t}^C)$ *is a Nimzowitsch Region in which welfare is higher under price competition than under quantity competition:* $W^N > W^C$*; while imports are positive under quantity competition and zero under price competition. The width of this region is increasing in the number of firms* n *and the degree of substitutability* e*.*



(a) $n = 4$ (b) $n = 12$ (c) $n = 24$

Figure 5: Regions of Trade and Welfare in Oligopoly for Different Values of n

See Figure 4 for legend

Figure 5 illustrates Propositions 1, 2 and 3. The three panels repeat the loci from Figure 4 for values of n equal to 4, 12 and 24, respectively. They show clearly that, as the number of countries increases, the Nimzowitsch Region expands slowly, whereas the B region contracts substantially. As n grows without bound, the lower boundary of the Nimzowitsch Region asymptotes to $2\frac{1-e}{2-e}$, while the lower boundary of the van-der-Rohe Region expands to such an extent that the B region is fully crowded out. (See Appendix C.3.) For very large numbers of firms, there are almost no parameter values for which price competition yields higher imports, yet it always yields higher welfare.

6 General Demands

We now consider welfare under general demands and discuss the extent to which the results obtained in the previous sections are dependent on the assumption of linear demands. We will show that many of the results are qualitatively robust. Figure 6 repeats Figure 1(b), highlighting the properties that we will show are robust when we switch from linear to general demands. We begin with some preliminary discussion of the properties of general demands and their implications for firm behavior, in Sections 6.1 and 6.2, and of the effects of changes in trade costs on outputs and welfare in free trade and in autarky, in Sections 6.3 and 6.4, respectively. We then show in Sections 6.5 and 6.6 that the novel insights from earlier sections continue to hold: both a van-der-Rohe Region and a Nimzowitsch Region always exist. To avoid unnecessary notation, we restrict attention to the canonical case of duopoly.

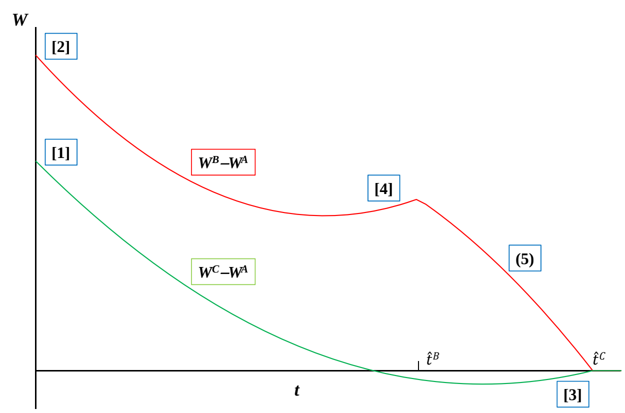


Figure 6: Welfare and Trade Costs with General Demands
The numbers in parentheses correspond to the numbered results given later in the text.

6.1 Properties of General Demands

We write inverse demands as $p(x, y)$ for the home good and $p^*(y, x)$ for the foreign. We impose some mild regularity assumptions on these demands.

First, we assume that the demand functions are twice differentiable and strictly downward-sloping: $p_x < 0$ and $p^*_y < 0$. Next, we assume that the goods are substitutes: $p_y < 0$ and $p^*_x < 0$. Finally, we assume that demand for the imported good has a choke price: $\lim_{y \to 0} p^*(y, x) < \infty$. This assumption guarantees the existence of a prohibitive trade cost that eliminates all imports, which in turn is a necessary condition for the existence of a Nimzowitsch Region, as we show below. The assumption is consistent with linear demands as in previous sections, and also with many of the other demand functions used in applied economics. It rules out the case where consumers have an infinite reservation price for the first unit purchased, as for example with iso-elastic or logistic demand.[20]

We also assume that the demand functions are symmetric, in the sense that their levels and first and second derivatives are equal when evaluated at the same point:

$$\left.\begin{aligned} p(x_0, y_0) &= p^*(y_1, x_1) \\ p_i(x_0, y_0) &= p_j^*(y_1, x_1),\ i, j = x, y,\ i \neq j \\ p_{xi}(x_0, y_0) &= p_{yj}^*(y_1, x_1),\ i, j = x, y,\ i \neq j \end{aligned}\right\} \text{ when: } x_0 = y_1 \text{ and } y_0 = x_1 \tag{28}$$

Finally, we assume that the inverse demand system can be inverted to get the direct demands:

$$x = x(p, p^*) \quad \text{and} \quad y = y(p^*, p) \tag{29}$$

For this inversion to be possible, we require that own-price effects in demand dominate cross-price effects, in the sense that $ee^* < 1$, where $e(x, y) \equiv p_y/p_x$ and $e^*(x, y) \equiv p_x^*/p_y^*$ are inverse measures of product differentiation for the home and foreign demand functions respectively.[21] Our assumption that goods are substitutes ensures that both e and e^* are strictly positive.

6.2 Firm Behavior in Interior Equilibria

In this section we consider the responses of imports and domestic sales when both firms sell positive amounts in either quantity or price competition. The home firm's profit function can be written as a function of either quantities or prices: $\pi(x, y) = [p(x, y) - c]x$ or $\pi(p, p^*) = (p-c)x(p, p^*)$. We assume that both functions are quasi-concave in the firm's own action, and

[20] Requiring that demand must have a choke price for the Nimzowitsch Region to exist is in line with Vives (1985), following Friedman (1977).

[21] Inverting the inverse demand functions to yield well-behaved direct demand functions requires that $p_x p_y^* - p_x^* p_y > 0$, which from the definitions of e and e^* implies that $(1 - ee^*)p_x p_y^* > 0$.

have a unique maximum given the foreign firm's action. We also assume that all equilibria are stable, which requires that own effects dominate cross effects at the margin:

Assumption 1. $\Delta \equiv \pi_{xx}\pi^*_{yy} - \pi_{xy}\pi^*_{yx} > 0$ *and* $\widetilde{\Delta} \equiv \pi_{pp}\pi^*_{p^*p^*} - \pi_{pp^*}\pi^*_{p^*p} > 0$

This in turn implies:[22]

Lemma 3. $\pi_{xx} + \pi_{xy} = 2p_x + p_y + x(p_{xx} + p_{xy}) < 0$ *when outputs are equal* $(x = y)$*; and* $\pi_{pp} + \pi_{pp^*} = 2x_p + x_{p^*} + (p - c)(x_{pp} + x_{pp^*}) < 0$ *when prices are equal* $(p = p^*)$.

This lemma implies that own effects dominate cross effects at the margin in a different sense: home marginal revenue falls following an equal increase in the outputs of both goods, and the marginal responsiveness of home profits to own price falls following an equal increase in the prices of both goods. Finally, for some (though not all) of the results that follow, we need an additional mild restriction on the home firm's profits as a function of outputs:

Assumption 2. $\pi_{xx} = 2p_x + xp_{xx} < 0$ *holds everywhere.*

Assumption 2 is stronger than requiring the second-order condition to hold: it is equivalent to the home firm's marginal revenue being always downward-sloping. This means that the demand function can be convex, but not too much so.

Turning to the production side, we continue to assume identical constant marginal production costs c and per-unit trade costs t. Hence, under monopoly, the home firm has first-order condition:

$$\pi_x = p(x^M, 0) + x^M p_x(x^M, 0) - c = 0 \tag{30}$$

where x^M is the monopoly output, and $p(x^M, 0) = p^M$ is the monopoly price.

Under quantity competition, the home and foreign firm's first-order conditions are:

$$\pi_x = p(x, y) + xp_x(x, y) - c = 0 \quad \text{and} \quad \pi^*_y = p^*(y, x) + yp^*_y(y, x) - c - t = 0 \tag{31}$$

[22]The proof is immediate. In quantity competition when outputs are equal, Δ equals $(\pi_{xx})^2 - (\pi_{xy})^2 = (\pi_{xx} + \pi_{xy})(\pi_{xx} - \pi_{xy}) > 0$. There are two cases to consider. Either outputs are strategic substitutes, $\pi_{xy} < 0$, in which case $\pi_{xx} + \pi_{xy} < 0$ follows immediately, which implies that $\pi_{xx} - \pi_{xy}$ must also be negative. Alternative, outputs are strategic complements, $\pi_{xy} > 0$, in which case $\pi_{xx} - \pi_{xy} < 0$, which implies that $\pi_{xx} + \pi_{xy}$ must also be negative. An analogous proof holds in price competition.

Differentiating these gives the responses of sales to trade costs when both firms sell positive amounts:

$$\frac{dx^C}{dt} = \frac{\pi_{xy}}{\Delta} \lesseqgtr 0 \quad \text{and} \quad \frac{dy^C}{dt} = \frac{\pi_{xx}}{\Delta} < 0 \tag{32}$$

recalling that Δ is positive from Assumption 1. Hence an increase in the trade cost always reduces the volume of imports, while it raises domestic sales if and only if the goods are strategic substitutes ($\pi_{xy} = p_y + xp_{xy} < 0$).

Under price competition, the firms' first-order conditions are:

$$\pi_p = (p-c)\, x_p(p, p^*) + x = 0 \quad \text{and} \quad \pi^*_{p^*} = (p^* - c - t)\, y_{p^*}(p^*, p) + y = 0 \tag{33}$$

For later use, it is very helpful to rewrite these in terms of inverse rather than direct demands and their derivatives; this will facilitate comparison with the corresponding Cournot first-order conditions in (31). To do this, we can use footnote 21 to write $x_p = \frac{p^*_y}{p_x p^*_y - p_y p^*_x} = \frac{1}{p_x}\frac{1}{1-ee^*}$ and $y_{p^*} = \frac{p_x}{p_x p^*_y - p_y p^*_x} = \frac{1}{p^*_y}\frac{1}{1-ee^*}$. Substituting into (33) allows us to write the two firms' first-order conditions under price competition expressed in terms of inverse demands:[23]

$$p(x,y) + xp_x\,(1-ee^*) - c = 0 \quad \text{and} \quad p^*(y,x) + yp^*_y\,(1-ee^*) - c - t = 0 \tag{34}$$

Returning to (33), differentiating shows that the effects of trade costs on prices are: $dp^B/dt = -\left(y_{p^*}/\widetilde{\Delta}\right)\pi_{pp^*}$, which is positive if and only if prices are strategic complements; and $dp^{*B}/dt = \left(y_{p^*}/\widetilde{\Delta}\right)\pi_{pp}$ which is always positive. (Again, recall that $\widetilde{\Delta}$ must be positive from Assumption 1.) These price derivatives can be combined with the derivatives of (29) to obtain the

[23] The same outcome would be obtained if we assumed that the firms have "Bertrand conjectural variations": e.g., the home firm conjectures that, if it increases its output, the foreign firm's output will change so as to exactly offset the home firm's impact on the foreign price. In effect, the home firm conjectures that its own quantity affects its own price as follows: $\frac{dp}{dx}\big|_{\overline{p}^*} = p_x + p_y \frac{dy}{dx}\big|_{\overline{p}^*} = p_x - p_y \frac{p^*_x}{p^*_y}$. Recall the discussion in footnote 15.

effects of trade costs on outputs:

$$\frac{dx^B}{dt} = \frac{y_{p^*}}{\widetilde{\Delta}} \left(x_{p^*}\pi_{pp} - x_p \pi_{pp^*} \right) \quad \text{and} \quad \frac{dy^B}{dt} = \frac{y_{p^*}}{\widetilde{\Delta}} \left(y_{p^*}\pi_{pp} - y_p \pi_{pp^*} \right) \tag{35}$$

The natural case is where these effects are positive and negative respectively: higher trade costs raise home output and restrict imports. Unlike the Cournot case in (32), the latter outcome is not guaranteed. However, it must hold if own effects on demand and marginal profits dominate cross effects ($|y_{p^*}| > |y_p|$ and $|\pi_{pp}| > |\pi_{pp^*}|$). Henceforward we assume that $\frac{dy^B}{dt}$ is negative, so that trade costs reduce imports. The former outcome, $\frac{dx^B}{dt} > 0$, is not guaranteed by dominant own effects. A sufficient condition for it to hold is that prices are not too strongly strategic complements.

Armed with these preliminary results, we can now turn to consider their implications for the robustness of the results derived in previous sections for linear demands.

6.3 Zero Trade Costs

At $t = 0$ the sales of the home and foreign firm are equal. Thus under Cournot, $x^C = y^C$, and under Bertrand, $x^B = y^B$, where C and B denote the Cournot and Bertrand equilibria respectively. We can write the first-order condition for the home and foreign firms under Cournot at $t = 0$ as $\pi_x(q^C) = \pi_y^*(q^C) = 0$, where $q = x = y$ is the symmetric output which is at the level q^C in the zero trade cost Cournot equilibrium. From (34), the first-order condition under Bertrand can be written as $\pi_x(q^B) + \beta(q^B) = 0$, where q^B represents the symmetric output at the zero trade cost Bertrand equilibrium, and where $\beta(q) = -xp_x ee^* = -yp_y^* ee^*$ is positive at any $q > 0$. Since $\beta(q)$ is positive everywhere we have: $\pi_x(q^B) < 0$ but $\pi_x(q^C) = 0$ and since $\pi_x(q)$ falls in q from Lemma 3 it follows that: $q^B > q^C$. Therefore, at zero trade costs, the volume of trade is higher under Bertrand than Cournot competition. It must also be the case that prices at free trade are lower under Bertrand than Cournot. Finally, it is easy to show that welfare is monotonically increasing in output in a symmetric free-trade equilibrium.[24] Hence at zero trade costs welfare is higher in price than in quantity competition. Summarizing:[25]

Lemma 4. *At free trade, price competition leads to lower prices, higher trade volume, and higher welfare than quantity competition.*

Next we want to compare the responsiveness of welfare to trade costs at free trade under the two forms of competition. We first derive a general expression for the change in welfare. Recall from expression (3) above that welfare is the sum of consumer surplus and home profits. The change in consumer surplus is: $d\chi = -(xdp + ydp^*)$ and the change in the profits of the home firm is: $d\Pi = (p-c)dx + xdp + (p^* - c - t)dx^* + x^*dp^* - x^*dt$. Combining these and making use of the symmetry of the model which implies that home exports x^* equal home imports y, we can write the change in welfare as:

$$dW = d\chi + d\Pi = (p-c)dx + (p^* - c - t)dy - ydt \tag{36}$$

Thus consumption of home and imported goods have a positive effect on welfare, while wasteful trade costs have an additional negative effect.

Consider next the welfare effect of an increase in trade costs starting at free trade ($t = 0$). Irrespective of the mode of competition, (36) simplifies in this case to:

$$\left.\frac{dW^h}{dt}\right|_{t=0} = (p-c)\left(\frac{dx^h}{dt} + \frac{dy^h}{dt}\right) - y, \qquad h = C, B \tag{37}$$

It can be shown that total domestic consumption $x + y$ is unambiguously decreasing in trade costs at free trade under both quantity and price competition. Detailed derivations are given

[24]Evaluating welfare from equation (3) in symmetric free trade gives: $W|_{t=0} = u(x,x) - 2cx + I$. Differentiating gives $\frac{dW}{dx}\big|_{t=0} = 2(u_x - c) = 2(p-c)$. This is strictly positive given that there is an oligopoly distortion.

[25]The results on prices and trade volume are implicit in Vives (1985).

in Appendix D.1. Hence we can conclude that

$$[1] \left. \frac{dW^C}{dt} \right|_{t=0,+} < 0 \qquad [2] \left. \frac{dW^B}{dt} \right|_{t=0,+} < 0 \tag{38}$$

which confirms the result found in the linear case. Summarizing:

Lemma 5. *Starting at free trade, an increase in trade costs lowers welfare under both quantity and price competition.*

6.4 Prohibitive Trade Costs

Next we turn to the case of prohibitive trade costs. We first show that they must be lower under price than under quantity competition. We can write the Cournot equilibrium output pair as a function of the trade cost as $\{x^C(t), y^C(t)\}$. The equilibrium level of imports in the Cournot case reaches zero at the threshold prohibitive trade cost, $\widehat{t}^C$. Let $\widehat{x}^C = x^C(\widehat{t}^C)$ and $\widehat{y}^C = y^C(\widehat{t}^C) = 0$ be the Cournot equilibrium outputs at $\widehat{t}^C$. At the prohibitive trade cost the Cournot first-order conditions given in (31) can be written as $p(\widehat{x}^C, 0) + \widehat{x}^C p_x(\widehat{x}^C, 0) - c = 0$ and $p^*(0, \widehat{x}^C) - c - \widehat{t}^C = 0$ for the home and foreign firm respectively. At the prohibitive trade cost the home first-order condition takes exactly the same form as under monopoly, so, from (30), we have $\widehat{x}^C = x^M$. Making use of this allows us to write: $\widehat{t}^C = p^*(0, x^M) - c$. This means that, if the trade cost were at $t = \widehat{t}^C$ and the home firm were to behave as an unconstrained monopolist, the foreign firm's trade-cost-inclusive price-cost margin would be zero and the foreign firm would be unable to export profitably.

Consider next the case of price competition. Let $\widehat{t}^B$ be the threshold prohibitive trade cost under Bertrand. Write the Bertrand equilibrium outputs at $\widehat{t}^B$ as: $\widehat{x}^B = x^B(\widehat{t}^B)$ and $\widehat{y}^B = y^B(\widehat{t}^B) = 0$. At $\widehat{t}^B$, the first-order conditions under Bertrand given in (34) can be written as $p(\widehat{x}^B, 0) + \widehat{x}^B p_x(\widehat{x}^B, 0) - c + \beta(\widehat{x}^B, 0) = 0$ for the home firm, where $\beta = -xp_x ee^*$, and as $p^*(0, \widehat{x}^B) - c - \widehat{t}^B = 0$ for the foreign firm. Since $\beta > 0$ it follows that: $p(\widehat{x}^B, 0) + \widehat{x}^B p_x(\widehat{x}^B, 0) - c < 0$. Making use of this and Assumption 1 we get: $\widehat{x}^B > \widehat{x}^C = x^M$. It then follows that $\widehat{t}^B = p^*(0, \widehat{x}^B) - c < p^*(0, \widehat{x}^C) - c = \widehat{t}^C$. Summarizing:[26]

Lemma 6. *The prohibitive trade cost is lower under price than under quantity competition:* $\widehat{t}^B < \widehat{t}^C$.

Next, consider the effect of a small fall in trade costs on welfare starting at the prohibitive level. At this point we have $p^* - c - t = 0$ and $y = x^* = 0$, which from (36) imply:

$$\left.\frac{dW^h}{dt}\right|_{t=\widehat{t}^h,-} = (p-c)\frac{dx^h}{dt}, \qquad h = C, B \tag{39}$$

The effect of trade costs on outputs in interior equilibria have been derived above: see equations (32) and (35). Substituting these into (39) gives:

$$[3] \quad \left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C,-} = -\frac{p-c}{\Delta}\pi_{xy} \qquad [4] \quad \left.\frac{dW^B}{dt}\right|_{t=\widehat{t}^B,-} = -\frac{p-c}{\widetilde{\Delta}}\left(x_p\pi_{pp^*} - x_{p^*}\pi_{pp}\right)y_{p^*} \tag{40}$$

Inspecting these expressions and recalling equations (32) and (35), we can conclude that:

Lemma 7. *Starting at the relevant prohibitive trade cost, a reduction in trade costs lowers welfare under quantity competition if and only if outputs are strategic substitutes; and it lowers welfare under price competition provided prices are not too strongly strategic complements.*

These results are indicated by the points denoted [3] and [4] in Figure 6.

6.5 The van-der-Rohe Region

Next we want to show that there exists a van-der-Rohe Region, that is, a subset of trade costs at which welfare is higher but imports are lower in price than in quantity competition.

The first step is to show that there must exist at least one trade cost at which imports are equal under the two modes of competition. We know from Lemma 4 that at free trade imports

[26]Both threshold trade costs are unique: $\widehat{t}^C$ from (32), and $\widehat{t}^B$ from (35) given our assumption that dy^B/dt is always negative.

are higher under price competition, and we know from Lemma 6 that at the prohibitive trade cost $\widehat{t}^B$ they are zero under price competition and strictly positive under quantity competition. Hence, since imports are always continuous in trade costs, we can conclude that:

Lemma 8. *There exists at least one trade cost, $t^R \in (0, \widehat{t}^B)$, at which the volume of imports is the same in both quantity and price competition.*

Start with the case where there is a unique trade cost, t^R, at which the volume of imports is the same. Hence, it must be true that an infinitesimal increase in the trade cost must yield higher imports under quantity than under price competition. To see this, let y^R denote the level of imports under both forms of competition when the trade cost equals t^R. We can then show that home sales must be higher under price competition. Consider the response of home profits to a small increase in home sales when imports equal y^R:

$$\pi_x(x, y^R) = p(x, y^R) + xp_x(x, y^R) - c \tag{41}$$

In quantity competition, we know from the first-order condition (31), evaluated at the equilibrium level of home sales which we call x^{CR}, that the expression in (41) is zero: $\pi_x(x^{CR}, y^R) = 0$. By contrast, in price competition, we know from the first-order condition (34) that it must be strictly negative:

$$\pi_x(x^{BR}, y^R) = p(x^{BR}, y^R) + x^{BR}p_x(x^{BR}, y^R) - c = x^{BR}p_x(x^{BR}, y^R)ee^* < 0 \tag{42}$$

where x^{BR} denotes the level of home sales in the Bertrand equilibrium. Recalling from Assumption 2 that the home firm's marginal profit is always decreasing in home sales, this implies that home sales are strictly higher under price than under quantity competition. Since imports are the same, this in turn implies that welfare is strictly higher. It follows that a small increase in t will lead to higher imports but lower welfare under quantity than under price competition.[27] Hence there exists a van-der-Rohe Region in the right-hand-side neighborhood of t^R.

Next, consider the possibility that there are many trade costs at which imports are the same under both forms of competition. Assume that such trade costs arise from multiple but separated intersection points of the two functions giving imports as a function of trade costs. (We rule out the theoretical *curiosum* of a continuum of intersections.) It then follows that the number of such intersections cannot be even, and that in the neighborhood of each odd intersection the argument just given for the case of a unique t^R continues to apply. Hence we can conclude:

Proposition 4. *There must exist a (not necessarily connected) van-der-Rohe Region; that is, a subset of trade costs at which imports are lower but welfare is higher under price than under quantity competition.*

6.6 The Nimzowitsch Region

The final step is to prove the existence of a Nimzowitsch region. With general demands, this requires two distinct conditions. First, "threat" and "execution" must coexist: there must exist a range of trade costs such that, under price competition, imports are zero and the equilibrium price is below the autarky level, while, under quantity competition, imports are strictly positive. We call this "import contestability." By itself it corresponds to a weak version of a Nimzowitsch region, since welfare is higher under price competition than in autarky, even though no imports take place. However, for the strong version of a Nimzowitsch region we require in addition a second condition: the threat must be "superior" to the execution, meaning that welfare must be higher under price competition than under quantity competition. With general demands we cannot expect to prove that this holds throughout the region in which import contestability holds, but we can prove it holds for a small fall in trade

[27] We ignore the contrived case where (42) holds, so welfare is strictly higher in price than in quantity competition at t^R, $W^{CR} < W^{BR}$, but only infinitesimally so, and, for an infinitesimal increase in the trade cost above t^R, the welfare levels in quantity and price competition are equal.

costs in the neighborhood of autarky. We call this "local superiority of price competition." We consider these two conditions in turn.

Formally we can define import contestability as follows:

Definition 1. Import Contestability: Imports are strictly positive under quantity competition, $y^C > 0$, and zero under price competition, $y^B = 0$, with $p^B < p^M$.

We wish to show that import contestability holds if and only if trade costs are in the interval $\left(\widehat{t}^B, \widehat{t}^C\right)$. (Recall that both thresholds are unique and, from Lemma 6, that the interval between them is non-empty: $\widehat{t}^B < \widehat{t}^C$.) To show this, we consider in turn trade costs outside and inside this range.

First, it is immediate that import contestability cannot hold for trade costs outside the interval. For trade costs lower than $\widehat{t}^B$, imports under price competition are strictly positive: $y^B > 0$ for $t \in \left[0, \widehat{t}^B\right)$. While for trade costs equal to or higher than $\widehat{t}^C$, imports under quantity competition are zero: $y^C = 0$ for $t \geq \widehat{t}^C$. In both cases, one or other of the conditions for import contestability is violated.

Second, we turn to explore what happens in the interval $\left(\widehat{t}^B, \widehat{t}^C\right)$. It is immediate that imports are strictly positive under quantity competition, $y^C > 0$, in this range, since we have seen in equation (32) that imports under quantity competition are strictly decreasing in trade costs below autarky, $\frac{dy^C}{dt} < 0$ for $t \in \left[0, \widehat{t}^C\right]$. It remains to prove that, under price competition, the other two conditions for import contestability hold in this range: the equilibrium level of imports is zero, $y^B = 0$, and the equilibrium price is lower than that of an unconstrained monopolist, $p^B < p^M$. To do this, we must consider the properties of the Nash equilibrium in the price game.

Consider first the foreign best-response function. This is unchanged from equation (11), as illustrated in Figure 2(a), except that now the demand functions are general. It is simply $p^* = c + t$, in anticipation of zero sales, and $B^*(p, t)$ when the home price is high enough that the foreign firm anticipates positive sales; with the threshold between the two given by the locus $\widetilde{p}^*(p)$ defined implicitly by $y(p, \widetilde{p}^*) = 0$.

Consider next the home firm's best-response function when $t \in \left(\widehat{t}^B, \widehat{t}^C\right)$. We wish to show that, as in Section 4.1, the home firm's best response is to set its own price at a level $\widetilde{p}$ such that the zero-import constraint, $y(\widetilde{p}, p^*) = 0$, just binds. Using the notation of that section, but keeping in mind that demand is now general, we can write the domestic price at which the zero-import constraint just binds as $\widetilde{p}(p^*)$ for $t \in \left(\widehat{t}^B, \widehat{t}^C\right)$. First, note that $\widetilde{p}(p^*)$ is monotonically increasing (since goods are substitutes, so y_p is positive). Next, note that $\widetilde{p}(p^*)$ equals the monopoly price at the upper bound of the range, $\widetilde{p}(p^*) = p^M$ when $p^* = c + \widehat{t}^C$. This reflects the fact that the prohibitive trade cost in Cournot competition, $\widehat{t}^C$, equals the threshold level of trade costs at or above which the home firm can act as an unconstrained monopolist under price competition. It follows from this and from the monotonicity of $\widetilde{p}$ in p^* that $\widetilde{p}(c+t)$ must be less than p^M for $t < \widehat{t}^C$.

Next, to show that $\widetilde{p}(p^*)$ is the best-response function for $t \in \left(\widehat{t}^B, \widehat{t}^C\right)$, consider price deviations by the home firm. Recall Figures 2(b) and 3, where we illustrated the same points for the linear demand case. First, consider price reductions from $\widetilde{p}(p^*)$. Such reductions in the home firm's price while keeping imports at zero would lower the home firm's profits, since $\widetilde{p}(p^*) \leq p^M$, profits are maximized at p^M, and profits are quasi-concave in p. Next, consider an upward deviation in the price from $\widetilde{p}(p^*)$. The slope dp/dp^* of the home firm's interior Bertrand best-response function, $B(p^*)$, is less than the slope dp/dp^* of the zero-import locus, $\widetilde{p}(p^*)$, at the point where they intersect, where $p^* = c + \widehat{t}^B$.[28] This means that $\widetilde{p}(p^*)$ cannot be on the upward-sloping part of $\pi(p, p^*)$ in Figure 3. That is, $\pi_p(\widetilde{p}(c+t), c+t) \leq 0$ for $t \geq \widehat{t}^B$. Thus an increase in the home firm's price from $\widetilde{p}(p^*)$ would lead to imports ($y > 0$) and result in the home firm's profits falling. Hence $\widetilde{p}(p^*)$ is the best-response function for $t \in \left(\widehat{t}^B, \widehat{t}^C\right)$ and import contestability holds within this region of trade costs: imports are zero, $y^B = 0$, the foreign firm sets the price $p^{*B} = c + t$, and the home firm prices below the

[28]The home firm's interior Bertrand best-response function can only intersect once with $\widetilde{p}(p^*)$. To see this, note first that $y_{p^*}\pi_{pp} - y_p\pi_{pp^*} > 0$, as was assumed earlier: see equation (35) and the subsequent discussion. Second, note that the slope dp/dp^* of the interior Bertrand best-response function is $-\pi_{pp^*}/\pi_{pp}$, while the slope of the zero-import locus $\widetilde{p}(p^*)$ is y_{p^*}/y_p. It is then easy to show that the condition $y_{p^*}\pi_{pp} - y_p\pi_{pp^*} > 0$ implies that $-y_{p^*}/y_p > -\pi_{pp^*}/\pi_{pp}$: relative to the p^* axis, the zero-imports locus is steeper than the home best-response function.

monopoly level, $p^B < p^M$. Summarizing:

Lemma 9. *Import contestability, $y^C > 0$, $y^B = 0$, and $p^B < p^M$, holds throughout the interval $t \in \left(\widehat{t}^B, \widehat{t}^C\right)$.*

Consider next the second property, local superiority of price competition.

Definition 2. Local Superiority of Price Competition: $W^C < W^B$ in the neighborhood of $t = \widehat{t}^C$; i.e.,

$$[6] \quad \left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C,-} > \left.\frac{dW^B}{dt}\right|_{t=\widehat{t}^C,-} \tag{43}$$

To show that this holds, consider first the case of price competition. In this case we can show that welfare is decreasing in trade costs throughout the relevant region and not just at $t = \widehat{t}^C$. This is because, as we have seen, imports are zero, and the foreign firm sets its price equal to $c + t$. Because imports are zero, the change in welfare for a small reduction in trade costs depends only on the change in consumption of the home good:

$$\left.\frac{dW^B}{dt}\right|_{t\in(\hat{t}^B,\hat{t}^C)} = (p-c)\left.\frac{dx^B}{dt}\right|_{t\in(\hat{t}^B,\hat{t}^C)} \tag{44}$$

And, because of the firm's pricing behavior, a fall in trade costs results in an increase in home output sufficient to keep imports equal to zero:

$$p^*(0,x) = c + t \quad \Rightarrow \quad \left.\frac{dx^B}{dt}\right|_{t\in(\hat{t}^B,\hat{t}^C)} = \frac{1}{p^*_x(0,x)} \tag{45}$$

This must be negative, since foreign output is a substitute for home output, which gives the desired result:[29]

$$[5] \quad \left.\frac{dW^B}{dt}\right|_{t\in(\hat{t}^B,\hat{t}^C)} < 0 \tag{46}$$

Consider next the case of quantity competition. Note first that, in the neighborhood of autarky, a small change in imports is valued at $p^* - c - t$ which is zero. Hence, in quantity

[29] This property holds throughout the interval $\left(\hat{t}^B, \hat{t}^C\right)$, but at the end-points it holds only for changes in t that move strictly inside the interval; i.e., $\left.\frac{dW^B}{dt}\right|_{t=\hat{t}^B,+}$ and $\left.\frac{dW^B}{dt}\right|_{t=\hat{t}^C,-}$.

competition, the change in welfare for a small reduction in trade costs starting in autarky depends only on the change in consumption of the home good:

$$\left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C,-} = (p-c)\left.\frac{dx^C}{dt}\right|_{t=\widehat{t}^C,-} \tag{47}$$

From (32), this is positive provided outputs are strategic substitutes; i.e., a small reduction in trade costs starting at $t = \widehat{t}^C$ increases trade but reduces welfare. Since we have already seen in (46) that it increases welfare under price competition, this confirms that a Nimzowitsch Region exists when quantities are strategic substitutes.

We can go further and show that, even when quantities are strategic complements, so that welfare under quantity competition rises for a small reduction in trade costs starting in autarky (i.e., the W^C locus is downward-sloping at $t = \widehat{t}^C$ unlike in Figure 6), welfare is still higher locally under price competition, so equation (43) holds. Intuitively, even if dx^C/dt is negative, it is still the case that dx^B/dt is larger in absolute value in the neighborhood of $\hat{t}^C$. See Appendix D.2 for the proof. Hence we can conclude:

Proposition 5. *A Nimzowitsch Region always exists in the neighborhood of autarky.*

Taken in conjunction with the results in (38), (40) and (46), this confirms that the qualitative features of welfare as a function of trade costs are the same with general demands as in the linear case.

7 Conclusion

In this paper we have compared multilateral trade liberalization under quantity and price competition in a common reciprocal-markets framework. We have shown that the qualitative effects of trade cost reductions are broadly similar in the two cases. In particular, welfare is typically U-shaped in trade costs, first falling (as home firms face more competition) and then rising (as prices fall and consumers gain). Moreover, the critical level of trade costs below which the possibility of trade affects the domestic firm's behavior is the same under both quantity and price competition.

Nonetheless, there are important quantitative differences between the effects of trade liberalization in the two cases. These broadly reflect the fact that price competition is generally more competitive than quantity competition. While this is known from previous work, we have identified two novel and important aspects of the comparison. First is that the pro-competitive effects of trade are stronger under price competition despite the fact that, for sufficiently high trade costs, the volume of trade is greater under quantity competition. We call the region of parameter space in which this outcome obtains a "van-der-Rohe Region" since "less is more": though imports are lower, welfare is higher under price competition. Second is that a potential rival can affect an incumbent's sales and profits under price competition even if no imports take place in equilibrium. Moreover, this has a sufficiently large effect on domestic prices and hence on consumer surplus that it leads to higher welfare. We call the region of parameter space in which this outcome obtains a "Nimzowitsch Region" since "the threat is stronger than the execution": even though imports are zero under price competition, the pro-competitive effect is sufficiently strong that welfare is higher than under quantity competition.

Our analysis clearly has implications for empirical work. For one thing, it shows that the level of imports by itself is a poor measure of the extent to which home firms are exposed to foreign competition. Moreover, a change in foreign access costs may affect the behavior of a domestic firm even though imports themselves do not change much or may not even occur. This chimes with recent empirical work by **?**, who use media reports of potential multinational investments to quantify threats to domestic firms from foreign competition, and show that incumbent firms change their behavior even when foreign entry does not occur. Similar considerations apply in closed-economy applications of our model: a high market share by itself need not imply that an incumbent firm is shielded from competition; it may be disciplined by potential competition from a higher-cost competitor, even though the latter's sales are low or non-existent. Further work is needed to devise empirical measures of these kinds of invisible competition and to quantify their practical relevance.

Appendices

A Outputs, Profits and Welfare in Linear Duopoly

A.1 Outputs and Profits in Quantity Competition

Standard derivations yield the Cournot-Nash equilibrium outputs, x^C and y^C, which we write in terms of free-trade outputs, x^{FC} and y^{FC}, and a measure of trade restrictiveness, equal to the actual trade cost relative to the prohibitive level, $\widehat{t}^C$, as follows:

$$x^C = x^{FC}\left(1 + \frac{e}{2}\frac{t}{\widehat{t}^C}\right) \qquad y^C = y^{FC}\left(1 - \frac{t}{\widehat{t}^C}\right) \tag{48}$$

where:

$$x^{FC} = y^{FC} = \frac{A}{b(2+e)}, \qquad \widehat{t}^C = \frac{2-e}{2}A, \qquad \text{and} \qquad t \leq \widehat{t}^C \tag{49}$$

Here, $A \equiv a - c$ is the difference between the maximum price consumers are willing to pay and the marginal cost of production. Writing the results in this way highlights the fact that both outputs vary linearly with our measure of trade restrictiveness, and that home sales rise by less than foreign sales fall as the trade cost increases. We call the absolute value of the rise in home sales relative to the fall in imports, the "demand-switching effect" of the trade cost. It equals $\frac{e}{2}$, and so is linear in the degree of substitutability in demand, falling from a maximum of a half when goods are identical ($e = 1$), to zero when they are unrelated ($e = 0$).

Turning to the home firm's profits in the home and foreign markets, we can use the first-order conditions, $p - c = bx$ and $p^* - c - t = bx^*$, to show that these are proportional to home and export sales respectively: $\pi = bx^2$ and $\pi^* = b(x^*)^2$. Differentiating these and using the derivatives of the Cournot outputs from (48), the effect of a multilateral change in trade costs on total profits can be shown to equal:

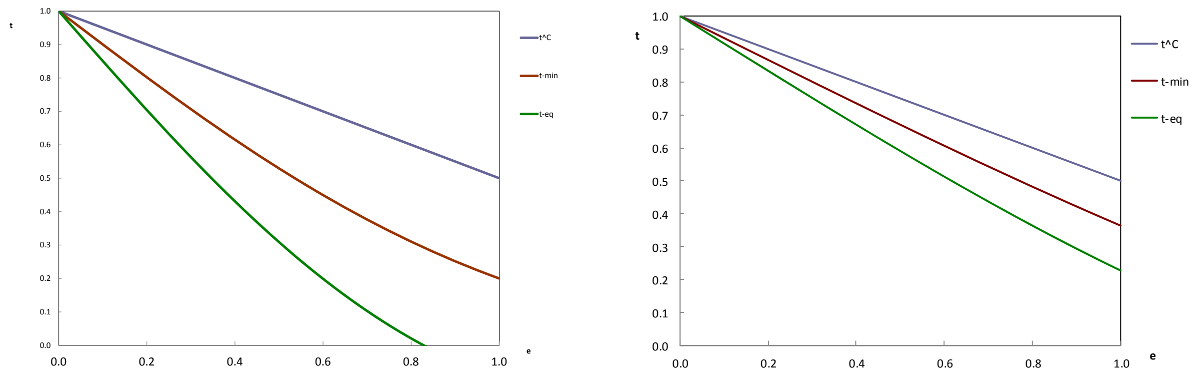
$$\frac{d(\pi+\pi^*)}{dt} = 2b\left(x\frac{dx}{dt} + x^*\frac{dx^*}{dt}\right) = \frac{p^{FC}-c}{\widehat{t^C}}(ex-2x^*) \quad \begin{cases} < 0 \text{ when } t=0 \text{ (so } x=x^*) \\ > 0 \text{ when } t=\widehat{t^C} \text{ (so } x^*=0) \end{cases} \tag{50}$$

where $p^{FC}-c = bx^{FC}$ is the firms' price-cost margin in free trade. This confirms the intuitive argument given in Section 3.1 that profits are a U-shaped function of trade costs.

It is clear that trade is locally bad for profits in the neighborhood of autarky. Must it be globally bad? More precisely, are profits lower in free trade than in autarky? Anderson, Donsimoni, and Gabszewicz (1989) showed that this must be true when goods are perfect substitutes. However, this need not be true for values of the substitution parameter below unity. Equation (50) suggests that the losses from increased foreign competition in the home market (reducing home sales as t falls) are lower as e falls and goods are less substitutable; moreover, the gains from increased export opportunities (increasing x^* as t falls) are higher. The net effect is that total profits can be higher than in autarky for sufficiently low trade costs and sufficiently low values of the substitutability parameter e. To see this, solve for the trade cost at which profits with trade equal profits in autarky:

$$t^{\Pi A} = \frac{4-4e-e^2}{4+e^2}\frac{2-e}{2}A = \frac{4-4e-e^2}{4+e^2}\widehat{t^C} \tag{51}$$

where $\widehat{t^C}$ is the prohibitive trade cost from (49). In the case of $e=1$ considered by Anderson, Donsimoni, and Gabszewicz (1989), $t^{\Pi A}$ is negative, equal to $-0.1A$; i.e., when goods are identical, profits are always lower with trade (whether free or not) than in autarky. However, for any value of e less than $2(\sqrt{2}-1)$ (approximately 0.828), profits are higher in free trade than in autarky. Moreover, as e falls, the range of values of t at which profits are higher than in free trade expands rapidly. This is shown in Figure 7(a), which illustrates in $\{t,e\}$ space the loci corresponding to autarky, $\widehat{t^C}$, the minimum value of profits, $t^{\Pi min}$, and the same level of profits as in autarky, $t^{\Pi A}$.[30]



(a) Profits (b) Welfare

Figure 7: Profits and Welfare in Cournot Competition
Loci of t and e that yield Autarky, Minimum Profits or Welfare,
and the Same Level of Profits or Welfare as in Autarky

A.2 Outputs and Profits in Price Competition

From the demand functions in (7), the first-order conditions for the optimal choice of prices are $p - c = b(1-e^2)x$ for the home firm and $p^* - c - t = b(1-e^2)y$ for the foreign firm. These can be solved for the Bertrand-Nash equilibrium prices which in turn can be combined with the direct demand functions to obtain the equilibrium quantities under Bertrand competition:

$$x^B = x^{FB}\left(1 + \frac{e}{2-e^2}\frac{t}{\widehat{t}^B}\right) \quad \text{and} \quad y^B = y^{FB}\left(1 - \frac{t}{\widehat{t}^B}\right) \tag{52}$$

where:

$$x^{FB} = y^{FB} = \frac{A}{b(1+e)(2-e)}, \qquad \widehat{t}^B = \frac{(1-e)(2+e)}{2-e^2}A, \qquad \text{and} \qquad t \leq \widehat{t}^B \tag{53}$$

Turning to profits, we use the first-order conditions for prices to show that maximized

[30] As we have already seen, Cournot profits are always U-shaped. The level of trade costs at which profits are minimized is: $t^{\Pi min} = \frac{(2-e)^2}{4+e^2}A$, which is always strictly positive. This value is exactly the average of the prohibitive trade cost $\widehat{t}^C$ and the trade cost at which profits return to their autarky level, $t^{\Pi A}$. In this and subsequent figures, we scale trade costs by their autarky level.

profits are equal to $b(1-e^2)x^2$ and $b(1-e^2)y^2$. Total profits for the home firm are then equal to the sum of these, and their behavior as trade costs change is given by:

$$\frac{d\left(\pi+\pi^*\right)}{dt} = 2b(1-e^2)\left(x\frac{dx}{dt}+x^*\frac{dx^*}{dt}\right)$$

$$= \frac{2}{2-e^2}\,\frac{p^{FB}-c}{\widehat{t}^B}\left(ex-(2-e^2)x^*\right) \quad \begin{cases} <0 \text{ when } t=0 \text{ (so } x=x^*) \\ >0 \text{ when } t=\widehat{t}^B \text{ (so } x^*=0) \end{cases} \tag{54}$$

where $p^{FB}-c=b(1-e^2)x^{FB}$ is the firms' price-cost margin in free trade.

A.3 Welfare in Quantity and Price Competition

Finally, we turn to welfare. We first express it as a function of home consumption levels, by combining the welfare function (3) with the quadratic sub-utility function defined in the text, and the expressions for profits (4) and (5):

$$W = A(x+y) - \frac{1}{2}b\left(x^2+y^2+2exy\right) - ty \tag{55}$$

This holds in both quantity and price competition, whether imports are strictly positive or not. (We are only interested in the differences between welfare in different cases, so without loss of generality we set exogenous factor income I from (3) equal to zero.)

In the Cournot case, we substitute for outputs from (48) to obtain:

$$W^C = \frac{1}{b\left(2+e\right)^2}\left((3+e)A(A-t)+\frac{12-e^2}{2(2-e)^2}t^2\right) \tag{56}$$

This holds for trade costs less than the prohibitive level, $t \leq \widehat{t}^C = \frac{2-e}{2}A$. It can be checked that it inherits the U-shape of profits: it is decreasing at free trade, increasing at the prohibitive trade cost, and is always convex in t.

In the Bertrand case with strictly positive imports, we substitute for outputs from (52) to obtain:

$$W^B = \frac{1}{b\left(1+e\right)\left(2-e\right)^2}\left((3-2e)A(A-t) + \frac{12+2e^4-9e^2}{2(1-e)(2+e)^2}t^2\right) \tag{57}$$

This holds for trade costs less than the prohibitive level, $t \leq \widehat{t}^B = \frac{(1-e)(2+e)}{2-e^2}A$. Its qualitative properties are the same as those of W^C.

Subtracting (56) from (57) yields:

$$\left(W^B - W^C\right)\Big|_{0\leq t\leq \widehat{t}^B} = \frac{e^2}{b\left(1+e\right)\left(4-e^2\right)^2}\left(\left(4-2e-e^2\right)A(A-t) + \frac{4+e^2}{2(1-e)}t^2\right) \tag{58}$$

This is clearly positive, which proves the result stated in Section 3.2 that welfare in price competition is always higher than in quantity competition when imports are strictly positive.

Consider next welfare under price competition in the Nimzowitsch Region. This is given by (55) with $y = 0$, which is simply $W^B = Ax - \frac{1}{2}bx^2$. Substituting for home output from (15) yields:

$$W^B = \frac{1}{2be^2}\left((2e-1)A+t\right)\left(A-t\right) \tag{59}$$

where $\widehat{t}^B \leq t \leq \widehat{t}^C$. It is clear that this is decreasing and concave in t.

Subtracting (56) from (59) yields:

$$\left(W^B - W^C\right)\Big|_{\widehat{t}^B\leq t\leq \widehat{t}^C} = \frac{1}{2be^2\left(2+e\right)^2}\left(\left(e^2+4e-4\right)A^2 + 8At - 4\frac{4+e^2}{\left(2-e\right)^2}t^2\right) \tag{60}$$

Inspecting this equation might suggest that it could be negative for low values of t and e. However, a direct argument shows that this cannot happen in the Nimzowitsch Region. Note first that W^B and W^C are equal at the prohibitive trade cost $\widehat{t}^C$, the upper bound of the Region. As for the lower bound, $\widehat{t}^B$, it is clear from (58) that W^B is strictly greater than W^C at that level of trade costs. Finally, recalling that W^B is concave in t from (59) and W^C is convex in t from (56), the difference $W^N - W^C$ must itself be concave. Taken together, these results imply that W^N is always greater than W^C throughout the Nimzowitsch Region.

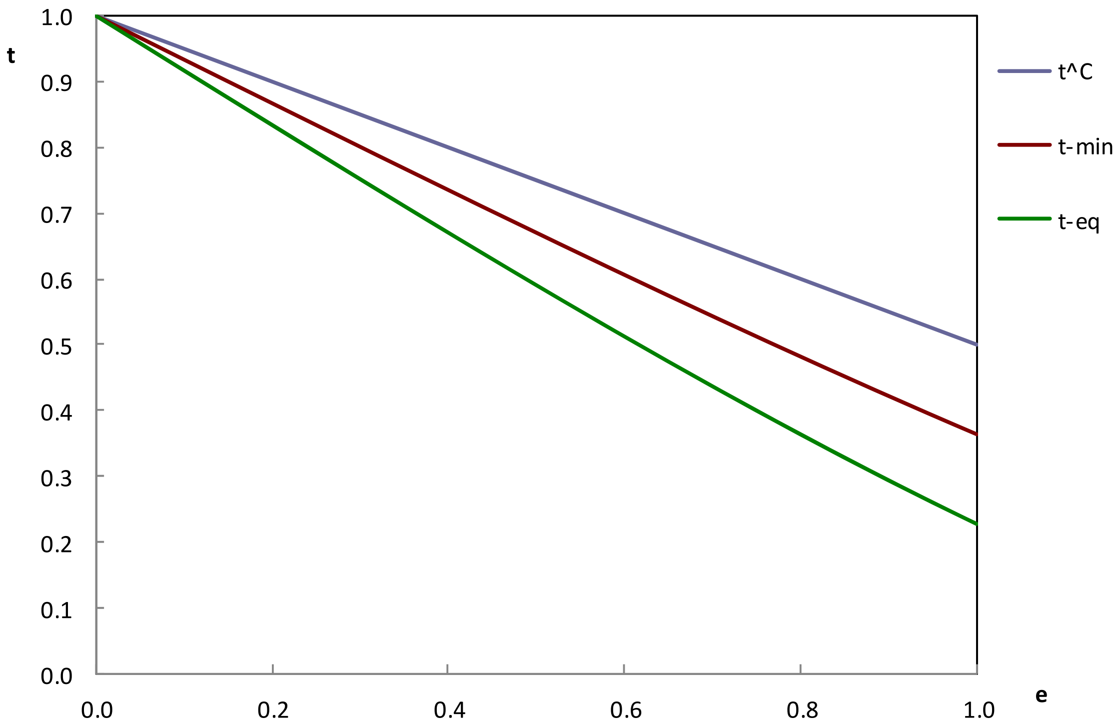


Figure 8: Welfare in Cournot Competition
Loci of t and e that yield Autarky, Minimum Welfare, and Autarky Welfare

The combinations of values of t and e at which welfare is minimized and returns to its autarky level are shown in Figure 7(b).

B Welfare with Tariff Revenue

B.1 Preliminaries

We continue to use W^C and W^B to denote the sum of consumer and producer surplus in the Cournot and Bertrand cases respectively: explicit expressions are given in equations (56), for Cournot, (57), for Bertrand when imports are strictly positive, and (59), for Bertrand when imports are zero. In the presence of tariffs, welfare equals these expressions plus tariff revenue: we denote this by $\widetilde{W} \equiv W + T$. Tariff revenue, $T \equiv ty$ is zero in the Nimzowitsch Region by construction, while in the Cournot and Bertrand cases with strictly positive imports it is given by the following:

$$T^C = \frac{t}{b(2+e)}\left(A - \frac{2}{2-e}t\right) \qquad T^B\Big|_{0\le t\le \widehat{t}^B} = \frac{t}{b(1+e)(2-e)}\left(A - \frac{2-e^2}{(1-e)(2+e)}t\right) \tag{61}$$

Note that both of these are concave in t: a standard property, implying that for a small tariff the welfare loss in the home market is zero. Combining these with (56) and (57) gives tariff-revenue-inclusive welfare:

$$\widetilde{W}^C = \frac{3+e}{b\,(2+e)^2}A^2 - \frac{1}{b\,(2+e)^2}At - \frac{4-3e^2}{2b\,(4-e^2)^2}t^2 \tag{62}$$

$$\widetilde{W}^B\Big|_{0\le t\le \widehat{t}^B} = \frac{3-2e}{b\,(1+e)\,(2-e)^2}A^2 - \frac{1-e}{b\,(1+e)\,(2-e)^2}At - \frac{4-3e^2}{2b\,(1-e^2)\,(4-e^2)^2}t^2 \tag{63}$$

Both of these are monotonically decreasing and globally concave in t, unlike (56) and (57) in the absence of tariff revenue as illustrated in Figure 1(b).

B.2 Difference between $\widetilde{W}^B$ and $\widetilde{W}^C$ when Imports are Positive

Taking the difference between (62) and (63) yields:

$$\left(\widetilde{W}^B - \widetilde{W}^C\right)\Big|_{0\le t\le \widehat{t}^B} = \frac{e^2}{b\,(1+e)\,(4-e^2)^2}\left(\left(4-2e-e^2\right)A^2 + 2eAt - \frac{4-3e^2}{2\,(1-e)}t^2\right) \tag{64}$$

This is a concave quadratic that is positive and increasing in t at $t=0$. Therefore, to show that it is positive throughout the relevant range, it is sufficient to show that it is positive at the maximum value, i.e., at the prohibitive Bertrand tariff $\widehat{t}^B = \frac{(1-e)(2+e)}{2-e^2}A$. By inspection, we have:

$$\left(4-2e-e^2\right)A^2 + 2eA\,\widehat{t}^B - \frac{4-3e^2}{2\,(1-e)}\,(\widehat{t}^B)^2 \ge A^2\left(4-2e-e^2\right) - \frac{4-3e^2}{2\,(1-e)}(\widehat{t}^B)^2 \tag{65}$$

Hence, substituting for $\widehat{t}^B$ into the right-hand side of (65), a sufficient condition for (64) to be positive is:

$$\frac{4-2e-e^2}{4-3e^2} - \frac{1-e}{2}\left(\frac{2+e}{2-e^2}\right)^2 \geq 0 \tag{66}$$

It can be checked that this is indeed positive for all $e \in [0,1]$, which proves that welfare is always higher under Bertrand than under Cournot when imports are strictly positive.

B.3 Difference between $\widetilde{W}^B$ when Imports are Zero and $\widetilde{W}^C$

Next we want to compare welfare under Bertrand and Cournot competition when the tariff is in the $(\widehat{t}^B, \widehat{t}^C)$ interval. This equals $W^B - W^C$ as in (60), less Cournot tariff revenue T^C as in (61). Evaluating this gives:

$$\left(W^B - W^C\right)\Big|_{\widehat{t}^B \leq t \leq \widehat{t}^C} = \frac{1}{be^2}\frac{1}{(2+e)^2}\left(-\frac{(4-e^2-4e)}{2}A^2 + \left(4-2e^2-e^3\right)At - \frac{(4-3e^2+e^4)}{(2-e)^2}2t^2\right) \tag{67}$$

Once again, this is quadratic and concave in t, so to show it is always positive we need only consider its values at the end-points of the relevant range. We know that it must be positive at $t = \widehat{t}^B$: we saw in the last sub-section that $\widetilde{W}^B > \widetilde{W}^C$ at this point. As for the upper bound of the range, i.e., at $t = \widehat{t}^C$, $\widetilde{W}^B$ and $\widetilde{W}^C$ are both equal to monopoly welfare there. Taken with the results in the last sub-section, this confirms that price competition yields higher welfare when tariffs yield revenue.

To summarize, we have shown in this section that the same qualitative results hold whether trade costs yield revenue or not. These can be interpreted as cases of full and zero rent retention respectively. It follows that our results also hold for intermediate degrees of rent retention. Hence, our results apply in the presence of any mix of natural trade costs (such as transport costs), policy-induced trade barriers that yield no revenue (such as red-tape barriers), and tariffs.

C Trade Costs and Welfare with Many Countries

C.1 Comparative Statics of V and Γ

Recalling the definition of $V^B \equiv -\frac{(n-1)e^2}{1+(n-2)e}$ from (20) in Section 5, we have:

$$\frac{\partial V^B}{\partial n} = -\frac{e^2(1-e)}{(1+(n-2)e)^2} < 0 \quad \text{and} \quad \frac{\partial V^B}{\partial e} = -(n-1)\,e\,\frac{2+(n-2)\,e}{(1+(n-2)\,e)^2} < 0 \tag{68}$$

Hence, as a function of n, V^B falls monotonically from its value of $-e^2$ in duopoly, $n=2$, to its limiting value of $-e$ as n grows without bound. As a function of e, it falls monotonically from zero to -1 along the $e \in [0,1]$ interval.

Next we want to see how the competitiveness parameter Γ responds to changes. The only ambiguity arises in the case of price competition, when $\Gamma^B = 2 + V^B + (n-1)\,e$. Differentiating with respect to n and using (68) yields:

$$\frac{d\Gamma^B}{dn} = e + \frac{\partial V^B}{\partial n} = e\left(1 - \frac{e\,(1-e)}{(1+(n-2)\,e)^2}\right) > 0 \tag{69}$$

So the indirect effect of n on the conjectural variations parameter dampens but does not offset the direct effect of n in raising Γ. Similarly, differentiating with respect to e:

$$\frac{d\Gamma^B}{de} = n - 1 + \frac{\partial V^B}{\partial e} = (n-1)\,\frac{1+2\,(n-3)\,e+(n-2)\,(n-3)\,e^2}{(1+(n-2)\,e)^2} \tag{70}$$

For $n \geq 3$ this is definitely positive. The only ambiguity arises at $n=2$, when (70) simplifies to $1-2e$. Hence the indirect effect of e on the conjectural variations parameter dampens the direct effect of e in raising Γ^B; but does not offset it except in the duopoly case when goods are close substitutes ($n=2$ and $e>0.5$).

C.2 Proof of Proposition 1: Welfare Levels in the $[0, \widehat{t^B}]$ Interval

In order to prove that W^B is greater than W^C in the $[0, \widehat{t^B}]$ interval, we wish to evaluate the response of welfare to a change in the conjectural variations parameter V from (25). The change in home sales and in total sales in the home market can be written from (21) as follows:

$$x = \frac{A}{b\Gamma}\left(1 + \frac{(n-1)e}{2+V-e}\frac{t}{A}\right) \qquad Q = \frac{A}{b\Gamma}\left(n - (n-1)\frac{t}{A}\right) \tag{71}$$

These are decreasing in V by inspection. Next, we determine the responsiveness of welfare to changes in x and Y:

$$W_x = A - bx - beY = p - c \qquad W_Y = A - bex - b\frac{1}{n-1}(1+(n-2)e)Y - t = p^* - c - t \tag{72}$$

As for the difference between them, this simplifies to:

$$W_x - W_Y = t - b(1-e)(x-y) = \frac{1+V}{2+V-e}t \tag{73}$$

Both $W_x - W_Y$ and W_Y are strictly positive in the interior of the interval $(0, \widehat{t^B})$. As for the end-points, W_Y is strictly positive at $t = 0$ and $W_x - W_Y$ is strictly positive at $t = \widehat{t^B}$. Substituting into (25) proves Proposition 1.

C.3 Proposition 2: The Lower Boundary of the vdR Region

We wish to show that the lower boundary of the van-der-Rohe Region, denoted by t^R, is decreasing in both n and e. Recall the expression for t^R in (27). Using (22) to eliminate terms, this can be rewritten as follows:

$$t^R = A\frac{\left(2-e+V^B\right)(2-e)}{e\Gamma^C + 2\left(2-e+V^B\right)} \tag{74}$$

where, from (20) and (22), $V^B = -\frac{(n-1)e^2}{1+(n-2)e}$ and $\Gamma^C = 2+(n-1)\,e$. Recalling from Appendix C.1 that V^B must lie in the range $[-e^2, -e]$, it follows that $2 - e + V^B$ is positive.

Consider first how t^R varies with n. Differentiating (74) yields:

$$\frac{dt^R}{dn} = \underbrace{\frac{\partial t^R}{\partial \Gamma^C}}_{-}\underbrace{\frac{\partial \Gamma^C}{\partial n}}_{+} + \underbrace{\frac{\partial t^R}{\partial V^B}}_{+}\underbrace{\frac{\partial V^B}{\partial n}}_{-} \tag{75}$$

To confirm the signs of the individual terms, it is clear by inspection that t^R is decreasing in Γ^C, which itself is increasing in n; hence the first expression on the right-hand-side is negative. As for the second expression, differentiating yields:

$$\frac{\partial t^R}{\partial V^B} = A\frac{e(2-e)\Gamma^C}{\left(e\Gamma^C + 2\left(2-e+V^B\right)\right)^2} > 0 \tag{76}$$

and we have seen in equation (68) in Appendix C.1 that V^B is decreasing in n. This confirms the signs of the last two terms in (75), which completes the proof that t^R is decreasing in n.

We can also establish the limiting values of the t^R locus as n varies. In the duopoly case, where $n = 2$, (74) reduces to a downward-sloping 45-degree line: $t^R = A(1-e)$, as illustrated in Figure 4. As for the other extreme, as $n \to \infty$, $V^B\big|_{n\to\infty} = -e$ and $\Gamma^C\big|_{n\to\infty} = \infty$ (for $e > 0$). Hence it follows that:

$$t^R\big|_{n\to\infty} = A\frac{2\,(1-e)\,(2-e)}{e\Gamma^C + 2\left(2-e+V^B\right)}\Bigg|_{n\to\infty} = 0 \tag{77}$$

This confirms that the van-der-Rohe Region expands to fully crowd out the B region for large n.

Consider next how the boundary varies with e. Differentiating (74) with respect to e yields:

$$\frac{dt^R}{de} = \underbrace{\frac{\partial t^R}{\partial \Gamma^C}}_{-}\underbrace{\frac{\partial \Gamma^C}{\partial e}}_{+} + \underbrace{\frac{\partial t^R}{\partial V^B}}_{+}\underbrace{\frac{\partial V^B}{\partial e}}_{-} + \underbrace{\frac{\partial t^R}{\partial e}}_{-} \tag{78}$$

The signs of the first four terms are straightforward as before, recalling from equation (68) in Appendix C.1 that V^B is decreasing in e. As for the final term, inspecting (74) it can be seen that the numerator is decreasing in e and the denominator is increasing in e (since $\Gamma^C > 2$). It follows that t^R is decreasing in e.

C.4 Proposition 3: Welfare Levels in the $(\widehat{t^B}, \widehat{t^C})$ Interval

We want to show that, in the $(\widehat{t^B}, \widehat{t^C})$ interval when $n \geq 2$, welfare is higher under price competition than under quantity competition.

The first step is to write the level of welfare in general form, generalizing the expression for welfare in duopoly as given in (55):

$$W = u(x, \mathbf{y}) - c(x + Y) - tY \tag{79}$$

Using (16) to eliminate utility, and imposing symmetry across all countries, so $y = \frac{Y}{n-1}$, gives:

$$W = A(x + Y) - \frac{1}{2}b\left((1 - e)\left(x^2 + \frac{1}{n-1}Y^2\right) + e(x + Y)^2\right) - tY \tag{80}$$

To evaluate this in a Cournot equilibrium, we use the expression for imports by firm from (21), specialized to the Cournot case, and the corresponding expression for home output:

$$x = \frac{1}{b\Gamma^C}\left(A + \frac{(n-1)e}{2-e}t\right) \quad \text{and} \quad Y = \frac{n-1}{b\Gamma^C}\left(A - \frac{2}{2-e}t\right) \tag{81}$$

Substituting into (80) yields

$$W^C = \frac{(3 + (n-1)e)(nA - 2(n-1)t)}{2b(\Gamma^C)^2}A + (n-1)\frac{12 + 4(n-2)e - (n-1)e^2}{2b\,(2-e)^2\,(\Gamma^C)^2}t^2 \tag{82}$$

(It can be checked that this collapses to (56) at $n = 2$.) By inspection, this is decreasing at free trade, and is always convex in t.

As for the level of welfare under price competition, W^B, from (80), with $Y = 0$, this equals $W^B = Ax - \frac{1}{2}bx^2$. Output in turn is the same as in the duopoly case, as given in (15): $x\big|_{Y=0} = \frac{A-t}{be}$. By inspection, we see that W^B is concave in t. It follows immediately that $W^B - W^C$ must be positive throughout the relevant interval $(\widehat{t}^B, \widehat{t}^C)$. First, since W^B is concave in t in this interval, and, as already shown, W^C is convex in t, the difference $W^B - W^C$ must itself be concave in t. Next, note that this difference is strictly positive at the lower end-point, $t = \widehat{t}^B$, from Proposition 1, and that it is zero at the upper end-point, $t = \widehat{t}^C$, where the home firm is an unconstrained monopolist. It follows that W^B is equal to or greater than W^C throughout the interval, which confirms that it is indeed a Nimzowitsch Region as stated in Proposition 3.

C.5 Proposition 3: The Width of the $[\widehat{t}^B, \widehat{t}^C]$ Interval

Direct calculation from (21) shows that the width of the $[\widehat{t}^B, \widehat{t}^C]$ interval is:

$$\widehat{t}^C - \widehat{t}^B = -\frac{eV^B}{2(2+V^B)} \geq 0 \tag{83}$$

This is decreasing in V^B, which in conjunction with (68) gives the result in Proposition 3 that the region is increasing in n. To show how it varies with e, totally differentiate:

$$\frac{d(\widehat{t}^C - \widehat{t}^B)}{de} = \frac{\partial(\widehat{t}^C - \widehat{t}^B)}{\partial e} + \frac{\partial(\widehat{t}^C - \widehat{t}^B)}{\partial V^B}\frac{\partial V^B}{\partial e} \tag{84}$$

The first term on the right-hand side is positive from (83); the final two terms are both negative from (83) and (68) respectively. This proves the result in Proposition 3 that the region is increasing in e.

D Trade Costs and Welfare with General Demands

D.1 Interior Equilibria

We begin with the case of quantity competition. Substituting from the expressions for output change in (32) and (35) into the general expression for welfare change in (36) yields an expression that cannot be signed in general. However, we can sign it unambiguously in special cases. First, in free trade, when $t = 0$, what matters for the effect of trade costs on welfare is the effect on total domestic consumption:[31]

$$\left.\frac{dW^C}{dt}\right|_{t=0} = (p-c)\left(\frac{dx^C}{dt} + \frac{dy^C}{dt}\right) - y = \frac{p-c}{\pi_{xx}+\pi_{xy}} - y < 0 \tag{85}$$

So, an increase in trade costs at $t = 0$ reduces welfare. This gives result [1] in equation (38) in the text.

Second, we can sign the effect of trade costs on welfare at the prohibitive trade cost $\widehat{t}^C$. At this point, we have $p^* - c - t = 0$ and $y = x^* = 0$ which when combined with (36) yield:

$$\left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C} = (p-c)\frac{dx^C}{dt} = -\frac{p-c}{\Delta}\pi_{xy} \tag{86}$$

This gives result [3] in equation (40) in the text. We can conclude that, under quantity competition, a small fall in the trade cost from the prohibitive level, $\widehat{t}^C$, reduces welfare if and only if outputs are strategic substitutes.

Next we consider price competition. At $t = 0$ we can make use of the symmetry of the model to obtain:[32]

$$\left.\frac{dW^B}{dt}\right|_{t=0} = (p-c)\left(\frac{dx^B}{dt} + \frac{dy^B}{dt}\right) - y = \frac{p-c}{\pi_{pp}+\pi_{pp^*}}\left(x_p + x_{p^*}\right)y_{p^*} - y < 0. \tag{87}$$

This gives result [2] in equation (38) in the text. Hence a small increase in trade costs

[31]Recall from Lemma 3 that the denominator $\pi_{xx} + \pi_{xy}$ is negative in a symmetric equilibrium.
[32]Recall from Lemma 3 that the denominator $\pi_{pp} + \pi_{pp^*}$ is negative in a symmetric equilibrium.

starting at zero reduces welfare. At $t = \widehat{t}^B$, the volume of trade is zero and the effect of a local change in the trade cost is:

$$\left.\frac{dW^B}{dt}\right|_{t=\widehat{t}^B,-} = (p-c)\frac{dx^B}{dt} = \frac{p-c}{\widetilde{\Delta}}\left(x_{p^*}\pi_{pp} - x_p\pi_{pp^*}\right)y_{p^*} \tag{88}$$

which is positive if $x_{p^*}\pi_{pp} - x_p\pi_{pp^*}$ is negative, the same condition we found above for home sales to be increasing in the trade cost, $dx^B/dt > 0$. Note that this is the left-hand derivative at $t = \widehat{t}^B$. This gives result [4] in equation (40) in the text.

D.2 The Nimzowitsch Region: Proof of Proposition 5

We have shown in the text that, in quantity competition, welfare falls as trade costs fall in the neighborhood of autarky if and only if goods are strategic substitutes, $\left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C} > 0$. This proves that a Nimzowitsch Region exists in this case. Can we go further and prove that it must also exist under strategic complements? In this case, the signs of output and welfare change in quantity competition are reversed: $\left.\frac{dx^C}{dt}\right|_{t=\widehat{t}^C} < 0$ and so $\left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C} < 0$. However, we can show that the same ranking between the changes in quantity and price competition holds as in the case of strategic substitutes. To see this, recall that the foreign firm's first-order condition is $p^*(y,x) - c + yp^*_y(y,x) = t$ throughout the Cournot region. Differentiating this and evaluating it at $y = 0$ gives:

$$2p^*_y(0,x)\left.\frac{dy^C}{dt}\right|_{t=\widehat{t}^C} + p^*_x(0,x)\left.\frac{dx^C}{dt}\right|_{t=\widehat{t}^C} = 1 \tag{89}$$

Hence, recalling from (45) that $\frac{dx^B}{dt} = \frac{1}{p^*_x(0,x)}$, we have:

$$\left.\frac{dx^C}{dt}\right|_{t=\widehat{t}^C} = \frac{1}{p^*_x(0,x)}\left(1 - 2p^*_y(0,x)\left.\frac{dy^C}{dt}\right|_{t=\widehat{t}^C}\right) = \left.\frac{dx^B}{dt}\right|_{t=\widehat{t}^C} - \frac{2p^*_y(0,x)}{p^*_x(0,x)}\left.\frac{dy^C}{dt}\right|_{t=\widehat{t}^C} \tag{90}$$

Hence we can conclude that, at $t = \widehat{t}^C$, home consumption increases faster with the trade cost under quantity than under price competition:

$$\left.\frac{dx^C}{dt}\right|_{t=\widehat{t}^C} - \left.\frac{dx^B}{dt}\right|_{t=\widehat{t}^C} = -\frac{2p_y^*(0,x)}{p_x^*(0,x)} \left.\frac{dy^C}{dt}\right|_{t=\widehat{t}^C} > 0 \tag{91}$$

Invoking (44) and (47) shows the same is true of welfare:

$$\left.\frac{dW^C}{dt}\right|_{t=\widehat{t}^C} - \left.\frac{dW^B}{dt}\right|_{t=\widehat{t}^C} = -\left(p(x,0) - c\right) \frac{2p_y^*(0,x)}{p_x^*(0,x)} \left.\frac{dy^C}{dt}\right|_{t=\widehat{t}^C} > 0 \tag{92}$$

which proves the result that, as trade costs fall in the neighborhood of autarky ($t = \widehat{t}^C$), welfare rises faster under price competition than under quantity competition, when goods are strategic complements.

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