CESifo WORKING PAPERS

8502
2020

Original Version: August 2020
This Version: Februar 2023

Mechanism Design with Narratives

Matthias Lang

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 8502*

Mechanism Design with Narratives

Abstract

Individuals use narratives as rationales or justifications to make their claims more convincing. I provide a general framework for partial verifiability based on narratives. Narratives give many reasons and arguments. The receiver derives the message's meaning by aggregating these reasons; her private information tells her whether some potential reasons support the sender's claims. Therefore, the receiver might detect some misreports. Narratives flexibly allow for different degrees of partial verifiability and allow using the revelation principle. Considering mechanism design as an example, I prove that narratives are sufficiently powerful to implement efficient trade in the canonical bilateral-trade setting.

JEL-Codes: D810, D820, D860.

Keywords: narrative, communication, partial verifiability, mechanism design, bilateral trade.

Matthias Lang
University of Munich (LMU)
Geschwister-Scholl-Platz 1
Germany – 80539 Munich
matthias.lang@econ.lmu.de

February 2023
I thank Andreas Asseyer, Yves Breitmoser, Matthew Ellman, Leonardo Felli, Joachim Henkel, Alida Johannsen, Sebastian Kodritsch, Nenad Kos, Stefan Lauermann, Yves Le Laouanq, Johannes Maier, Moritz Meyer-ter-Vehn, Marco Ottaviani, Martin Pollrich, Larry Samuelson, Roland Strausz, Gabriel Vollert, Jonas von Wangenheim, Leeat Yariv and the audiences at the CMID, ESEM, EEA, SAET, NASMES, EARIE, VfS, the Committee for Organizational Economics and the Japanese-German Workshop on Contracts and Incentives as well as the participants of seminars at Bocconi, HU Berlin, Lancaster, Konstanz, Duisburg-Essen, and TU Munich for helpful comments. Financial support by the German Research Foundation (DFG) through the CRC/TRR190 (Project number 280092119) is gratefully acknowledged.

1 Introduction

A lot of communication involves not just direct statements but also provides rationales or justifications. Start-ups present product ideas to venture capitalists or crowdfunders. Their products and services are still under development but adding project details and milestones makes projects more convincing. Companies supply earning figures to attract funding and to determine tax liabilities. Auditors and the SEC ask companies for many details, breakdowns and segmentations of revenues and costs to make earning figures reliable. Companies talk to regulators about their costs, market power or consumer data to avoid regulation. Regulators require many details of market interactions. Most organizations interview job candidates to learn about their skills. Candidates tell how they acquired skills instead of just listing their skills. All these senders provide many reasons, arguments, points, details, and breakdowns to make and support their claims.

I capture this kind of communication by offering a general and flexible framework for partial or probabilistic verifiability; a goal the literature has pursued at least since Green and Laffont (1986).[1] I call this communication narratives. This interpretation of "narratives as rationales or justifications" follows previous literature, like Bénabou et al. (2020, p. 1).[2] I then apply this framework to mechanism design. Demonstrating their possibilities, narratives implement efficient trade in the canonical bilateral-trade setting in contrast to famous impossibility results. Moreover, the framework satisfies the revelation principle making analyses very tractable. In particular, classical tools and methods of mechanism design are applicable to determine optimal contracts and study implementability.

It is helpful to contrast narratives with traditional models of asymmetric information. Traditionally, communication is brief and concise. Building on ideas in Holmström (1979) and Kim (1995), communication usually focuses on sufficient statistics of payoff-relevant information, like valuations or costs. Narratives in my framework are longer and more elaborate messages that spread out the payoff-relevant information in many pieces. Instead of making direct statements, narratives provide many reasons, arguments and points to make and support the sender's claim. For example, firms do not tell regulators that 'costs are high' but provide reasons 'why costs are high.' Thus, the narrative provides rationales or justifications for the sender's claim following above interpretation of narratives by Bénabou et al. (2020). Indeed, the reasons in the narrative establish the sender's statement. Spiegler

[1]See Glazer and Rubinstein (2006) and Carroll and Egorov (2019) for more recent contributions and the related literature section for detailed discussions.

[2]Shiller (2017) considers a different interpretation of narratives as 'a gem for conversation.'

(2020) and Eliaz and Spiegler (2020) suggest a similar interpretation of narratives in which the receiver combines smaller details to establish the overall meaning of the message. While I share the same interpretation of narratives with this literature, technically my approach is entirely new. I assume that the receiver knows whether some potential reasons are suitable and support the sender's statement. For example, the receiver could learn this information from some big data collection revealing certain details about the sender's valuation, technology or productivity. If the sender misreports, she might coincidentally use in her narrative some of those reasons which the receiver privately learned as unsuitable. If the receiver truthfully reveals his information about unsuitable reasons, a mechanism can detect such a deviation. Senders trade off gains from misreporting and probabilities of detection. Mechanisms might, thus, be able to deter some misreporting. Hence, narratives make the sender's claims more convincing. Senders can send any messages as they face no communication costs and no restrictions on messages spaces; third parties who lack the agents' private information cannot observe whether narratives are truthfully and only the equilibrium establishes a message's meaning. Narratives contain a lot of 'cheap talk' for all the reasons not contained in the receiver's signal. Nonetheless, messages are still informative along this dimension. This can explain why we see so much exchange of unverifiable arguments in real life - which is difficult to reconcile with standard cheap talk.

Consider as a leading example the canonical setting of bilateral trade by Myerson and Satterthwaite (1983) addressing a major problem of allocative efficiency: A seller and a buyer bargain over the terms of trade. The buyer's valuation for the good and the seller's costs are private information. The seller's costs are the sum of the costs of different inputs. The seller knows all inputs required for production. She could directly tell her costs as in traditional models. Classical results tell us that doing so, however, implies that valuable opportunities for trade are lost. In a narrative, instead, the seller tells all necessary inputs to justify her costs and causally explain why her costs are high or low. The narrative breaks down the seller's costs into smaller pieces of information about the required inputs. By providing this additional information, she makes herself vulnerable to scrutiny. The buyer reads some product reviews or inspects the product. This way, he learns an exogenous signal telling him for a few inputs whether those inputs are necessary. If the buyer truthfully reveals his signal, a mechanism can use this information to crosscheck the seller's narrative. Narratives, thus, enable ex-post efficient trade in contrast to previous impossibility results that the literature discusses extensively and that are part of the core curriculum in economics. Thus, narratives mitigate widely documented frictions caused by asymmetric information.

For illustration, consider a car manufacturer developing a new car. The car manufacturer bargains with a supplier to procure, e.g., a high-end sound system, some novel sensors or a head-up display. Due to its technology or its brand name, only this supplier can produce the specific gadget and the car manufacturer has to revert to its outside option of the car without that gadget or of using a generic equivalent if negotiations fail. In her narrative, the supplier will provide many specifications, inputs and sometimes even some blueprints of the sound system to justify its price offer. Indeed, narratives disclosing information about inputs and their respective costs are so common in some industries that the accounting literature coined the terms open-book negotiation or open-book accounting: "open books concern the exchange of cost information between suppliers and customers" (Romano and Formentini, 2012, p. 69).[3] The car manufacturer might ask for a prototype of the gadget for physical inspection or reverse engineering to learn about some required inputs. Alternatively, manufacturers derive such data from "from the internal production of similar parts, as well as by re-engineering the part." (Mueller et al., 2016, p. 9)

To show the possibilities of narratives, I then turn to a special case which limits the buyer's information as far as possible. The buyer observes only one input out of a continuum. Indeed, the buyer's sample is completely uninformative about the seller's costs: the seller's costs and the buyer's information are stochastically independent in this limit case. Even then, the buyer can check the seller's narrative to some extent. The seller's narrative containing the required inputs ensures that a mechanism can detect any relevant misreports with positive probability if the buyer truthfully reports his sample. Moreover, the more the sender wants to increase her reported costs as implied by her narrative, the more inputs she must report as necessary. Thus, the probability of detection is proportional to the gains of misreporting ensuring that mechanisms can deter small *and* large deviations with common contractual clauses and bounded payments. Thus, efficient trade occurs.

To demonstrate the flexibility of my framework, I introduce the strength of narratives. The narratives' strength – a measure of the support of the distribution of the buyer's sample – varies the efficiency of second-best contracts. This strength could relate to common cultural backgrounds, length and depth of contractual relationships between sender and receiver, the familiarity of the buyer with the product category or the familiarity of

[3] Mueller et al. (2016, p. 9) survey negotiations in the car industry: manufacturers "elicit cost information from the supplier." Carr and Ng (1995) first describe this practice in the car industry. See also Alenius et al. (2015) and Axelsson et al. (2002) for additional evidence on open-book negotiations.

the seller with buyers' tastes, e.g., based on big data analysis. Stronger narratives allow the receiver to detect more deviations making incentive compatibility easier to satisfy and second-best contracts more efficient. When narratives are sufficiently strong, optimal contracts approximate first-best welfare arbitrarily closely. These different degrees of partial verifiability make my framework very flexible.

Narratives apply to most economic environments, including mechanism design, taxation, market regulation, finance, auditing and many other settings whenever contracting parties interact and communicate. Narratives do *not* assume verification, testing or auditing technologies, type-dependent message spaces, or messages that are verifiable by third parties in contrast to other approaches for partial verifiability, which I discuss next. Therefore, the framework of narratives differs substantially from other approaches to partial verifiability.

After reviewing the related literature, this paper is organized as follows. In Section 2, I consider the canonical model of bilateral trade. Section 3 conveys the basic intuition of narratives in a simplified setting. Section 4 provides the details of my framework and proves the power of narratives – ensuring efficiency in bilateral trade. In Section 5, I introduce the strength of narratives and calculate second-best contracts. Section 6 discusses the connection of the model to the applications. Section 7 concludes and discusses a more familiar indirect implementation. Most formal proofs are relegated to the Appendix A. Appendix B considers a symmetric extension with both sides telling narratives.

Related Literature

The literature offers different approaches concerning partial verifiability. First, agents disclose hard information that does not fully reveal senders' types. For example, in Shin (1994), senders present hard information. Messages indicate whether the type is above or below certain privately-known thresholds. Dziuda (2011) considers more general disclosure but only one sender. Again full revelation is impossible. Alternatively, Fishman and Hagerty (1990) assume that senders learn finitely many signals about types. The sender can disclose only one of these signals. They determine the optimal discretion for senders' choices. Schweighofer-Kodritsch and Strausz (2022) discuss the game-theoretic problems created by disclosure and why the revelation principle is usually violated in these settings – I will return to this issue throughout this paper.

Second, agents' message spaces depend on their types (Milgrom, 1981, Green and Laffont, 1986). This approach is more general than the first one. Green and Laffont (1986) focus on direct mechanisms allowing for any dependency of message spaces on types. Deneckere and Severinov (2008) show that sequential and password mechanisms are superior to direct mechanisms – violating the revelation principle. Okuno-Fujiwara et al. (1990) provide conditions on a game's payoffs for full revelation. Lipman and Seppi (1995) provide conditions on how message spaces depend on types for full revelation. Glazer and Rubinstein (2006) and Sher (2014) characterize optimal reporting strategies. Hart et al. (2017) show that randomization and commitment are irrelevant in certain classes of evidence games. Returning to cheap-talk settings of Crawford and Sobel (1982), Seidmann and Winter (1997) and Mathis (2008) provide conditions on how message spaces depend on types for existence of fully separating equilibria. Correspondingly, Giovannoni and Seidmann (2007) provide conditions on preferences for existence of full revelation. Lang (2019) considers moral hazard with partial verifiability. Hagenbach et al. (2014) study pre-play communication. Finally, Bull and Watson (2004), Ben-Porath and Lipman (2012), and Kartik and Tercieux (2012) characterize implementable social choice functions for complete information.[4]

Third, Caragiannis et al. (2012) and Ball and Kattwinkel (2022) consider probabilistic verification. Mechanisms choose optimal tests, i.e., exogenous verification technologies, to audit messages. Tests specify the probability for each type of the agent to pass or fail the test. Narratives can be seen as a microfoundation of probabilistic verification – not relying on exogenous verification technologies but based on private and soft information. In probabilistic verification, verification is a choice of the designer. In my framework, the agents' strategic reports create partial verifiability.

Fourth, senders' types are multidimensional and receivers can verify one dimension of senders' types. Usually in that literature, the designer chooses the verification rule but cannot choose payoffs, which are exogenously given. With binary actions and a two-dimensional state space, Glazer and Rubinstein (2004) show that finding optimal mechanisms is equivalent to solving linear programming problems. Carroll and Egorov (2019) consider finite multidimensional type spaces and monotone payoffs. They prove that full revelation is possible for submodular payoffs but impossible for (strictly) supermodular payoffs.

Fifth, agents cannot write or understand all contracts. In Al-Najjar et al. (2006) and

[4] Kartik and Tercieux (2012) use an even more general approach of partial verifiability with the costs of sending certain messages depending on players' types.

Anderlini and Felli (1994), the contracting language is restricted and some events are ex-ante impossible to describe. These restrictions make incomplete contracts optimal. In Jakobsen (2020), the principal writes complex contracts that agents do not fully understand.

Finally, I relate to Myerson and Satterthwaite (1983) who study the bilateral trade setting to which I apply narratives. They show that efficient trade is impossible without narratives. Segal and Whinston (2016) confirm this impossibility for general property right allocations. Recently, a couple of papers re-examine this impossibility for different preferences. Ambiguity aversion (De Castro and Yannelis, 2018, Wolitzky, 2016) and concave utilities (Garratt and Pycia, 2023) can sometimes enable efficient trade. Benkert (2022) proves impossibility for loss-averse agents.

2 Trade Model

Begin with the canonical model of bilateral trade addressing a major problem of allocative efficiency. A seller (she) has an object to sell to a buyer (he). The seller and the buyer have to agree on a price and whether to trade at all. Seller and buyer are asymmetrically informed. In particular, the buyer's valuation, v, is distributed according to a distribution F on $[0, \nu)$ with $\nu > 0$. The buyer's valuation, v, is his private information. The seller's costs, c, are distributed independently according to a distribution G on $[0, \gamma)$ with $\gamma > 0$. The seller's costs, c, are her private information. If trade occurs at a price p, the seller's utility is $p - c$ and the buyer's utility is $v - p$; reservation utilities are zero. Both the seller and the buyer maximize expected payoffs.[5] The following properties are common objectives in mechanism design: First, the mechanism should be efficient.

Definition (**Ex-post efficiency**). A mechanism is ex-post efficient if there is trade if and only if $v \geq c$.

Second, both seller and buyer should be willing to enter the mechanism as participation is voluntary.

Definition (**Individual Rationality**). A mechanism is individually rational if both seller and buyer participate voluntarily given their costs and valuations.

[5]I assume, in line with mechanism design traditions, that the designer can choose an equilibrium of the ensuing game.

Finally, the mechanism should be budget-balanced, so that it requires no outside subsidies.[6] I return to the issue of budget balance in the conclusion.

Definition (**Budget Balance**). A mechanism is budget-balanced if it requires no outside subsidies.

Unfortunately, for bilateral trade, it is impossible to find a mechanism satisfying these three properties.

Proposition 1. *[Myerson and Satterthwaite (1983)] Assume that the distributions $F(v)$ and $G(c)$ have continuous and positive densities for all $v, c \in [0, \min\{\gamma, \nu\}]$. Then no mechanism is ex-post efficient, individually rational, and budget-balanced.*

Proof: See Myerson and Satterthwaite (1983, Corollary 1) or Segal and Whinston (2016, Proposition 1). □

Before introducing the general model of narratives, I discuss the basic intuition of narratives in a simplified setting with discrete valuations and correlation.

3 Narratives: Basic Intuition

Remember that narratives are longer and more elaborate messages. Hence, I first specify the source of the additional information. In the context of bilateral trade, I frame this information in terms of inputs required for production. Obviously, the framing does not matter much. In the examples provided in the introduction, like reporting earnings or presenting product ideas, other breakdowns and details will provide the additional information. I discuss these alternative frames in greater detail at the end of this section. For the moment, I focus on bilateral trade and inputs required for production.

Assume that production requires up to $I \geq 2$ potential inputs. Denote $c_i = 1$ if input i is necessary and $c_i = 0$ if input i is unnecessary. Each input has the same costs. The seller knows all necessary inputs and her costs are $c = \sum_{i=1}^{I} c_i$. The buyer only knows the distribution of inputs: $\text{Prob}(c_i = 1) \in (0, 1)$ and $\text{Corr}(c_i, c_j) \in (-1, 1)$ for $i \neq j$. By inspection of the good, reverse engineering or reading some reviews, the buyer learns for one input i^* whether that input is necessary. That input i^* is drawn with positive probabilities $h(i)$. Denote $h_{min} = \min_{i \in \{1,2,\ldots,I\}} h(i)$. The buyer's valuation v is drawn

[6] Outside subsidies can implement efficient trade as demonstrated by Vickrey (1961), Clarke (1971), and Groves (1973).

- In period 1, buyer and seller learn their private information.
- In period 2, agents can accept or reject the mechanism.
- In period 3, agents report their narrative/information.
- In period 4, the mechanism implements trade and transfers.

Figure 1: Timing of the Model

from a distribution $F(v)$ with finite or continuous support on $\mathbb{R}_0^+$. To summarize, the seller's private information is the vector $(c_1, \dots, c_I)$ while the buyer's private information is (v, i^*, c_{i^*}). Payoffs are the same as in the trade model of Section 2. For ease of exposition, I study an asymmetric setting with only sellers using narratives about their costs. The cases of buyers using narratives about their valuations or both sides using narratives are analogous, however. If buyers use narratives, their narratives contain, for example, all the features they appreciate instead of their overall valuation. Appendix B considers such a symmetric setting.

After learning their information, seller and buyer decide whether to participate in the mechanism. Then they report their narrative/information. Finally, the mechanism implements trade and payments by specifying probabilities of trade t and payments conditional on the buyer's message $\mathcal{M}_b$ and the seller's message $\mathcal{M}_s$. In addition to the buyer paying a price p to the seller, the seller pays π to a third party. I return to these payments in the conclusion and discuss common interpretations. Figure 1 summarizes the timing.

Proposition 2. *The following mechanism is ex-post efficient, individually rational, and budget-balanced: The seller's narrative describes the necessary inputs*

$$M_s = \{i \in \{1, 2, \dots, I\} | c_i = 1\}.$$

The buyer reports his valuation $M_b = v$ and his sample of the seller's inputs by stating the name of the observed input if that input is unnecessary: $m_b = i^*$ if $c_{i^*} = 0$ and $m_b = 0$ otherwise. Payments and probabilities of trade are

$$\begin{cases} t = 0, \quad p = 0, \quad \pi = 0 & \textit{if } M_b < |M_s| \\ t = 1, \quad p = |M_s|, \quad \pi = 0 & \textit{if } M_b \geq |M_s| \text{ and } m_b \notin M_s \\ t = 1, \quad p = |M_s|, \quad \pi = 1/h_{min} & \text{otherwise.[7]} \end{cases}$$

In equilibrium, the buyer can easily infer the seller's costs, $c = |M_s|$, from her narrative. The mechanism implements no trade if the seller's costs (inferred from her narrative) are above the buyer's (reported) valuation. Otherwise, trade takes places and the buyer pays a price equal to seller's costs.

Intuitively, truthful reports of the buyer's sample makes the seller's narrative partially verifiable because they allow the mechanism to detect some (off-equilibrium) misreports by the seller. If the buyer learns about an unnecessary input and the seller includes this input in her narrative, the seller has to make an additional payment $1/h_{min}$ as a punishment. The last case does not occur in equilibrium because the seller reports only necessary inputs in equilibrium. Hence, trade occurs for $v \geq c$ and the mechanism is ex-post efficient in equilibrium. Obviously, the mechanism is budget-balanced because it requires no outside subsidies. The proof in the Appendix A shows that equilibrium strategies are incentive compatible and satisfy individual rationality.

To attain efficiency, all the surplus has to go to the buyer in this toy model. In general, narratives do not require this. In symmetric models, any split of the surplus between seller and buyer is possible. See Appendix B for an example. As the buyer's information is correlated with the seller's costs, this correlation might prima facie remind of ideas by McAfee and Reny (1992), Crémer and McLean (1988), and Myerson (1981) to exploit correlation in agents' beliefs.[8] It is obvious from the mechanism that narratives work differently. To emphasize these differences and formalize them, the main model has no such correlation and I will prove that the spanning conditions are violated. Indeed, the buyer's information and the seller's costs will be stochastically independent emphasizing the power of narratives.

Within bilateral trade, the framing of narratives as required inputs or appreciated features seems plausible and natural. Alternatively, consider the procurement of complex goods as in Herweg and Schmidt (2020). Then the narratives could also entail design flaws or design improvements. The seller's costs then consists of commonly known costs c and costs c_i for fixing the design flaw i or improving design aspect i. The buyer might have learned about one design flaw or design aspect i^* when setting up the initial design or employing third-party experts or consultants for setting up the procurement process. The use of narratives in the procurement process then improve outcomes compared to fixing flaws ex post. Narratives are general and apply to other settings, too. In these other

[7] Let $|M|$ denote the cardinality of the set M.

[8] Also Gresik (1991) uses correlation to achieve efficient trade but for binary valuations.

settings, however, the detailed information contained in the narrative might come from different sources. Returning to the examples in the introduction, I briefly discuss these breakdowns here. When start-ups present product ideas to acquire funding, their narratives might consist of project details about past R&D, planned development processes, future milestones, features of the product, valuation of those features by consumers, revenue forecasts (including temporal and geographical segmentations), tools and methods employed, skills and abilities of the entrepreneurs and much more. Venture capitalists usually cannot inspect products but gather a lot of data themselves equivalent to the buyer's sample in the bilateral-trade example. When companies supply earing figures to attract funding and/or to determine tax liabilities, they not only report headline numbers but provide many details, breakdowns and segmentation of revenues and costs in temporal and geographical dimensions as well as along product lines. Tax authorities might have data on some transactions in their sample while investors might know the development or the size of some particular markets or the level or changes in some costs. The more elaborate communication in these examples seems to involve narratives that make the communication more convincing. I now turn to the general model to formalize this intuition.

4 First-Best by Narratives

For the general model of narratives, I return to the continuous valuations of Section 2. Thus, the buyer's valuation $v \in [0, \nu)$ and the seller's costs $c \in [0, \gamma)$ are drawn from distributions $F(v)$ and $G(c)$ with continuous and positive densities. Again I use the framing of inputs required for production for consistency reasons but refer to the alternatives previously discussed. Instead of I inputs, production requires potential inputs $i \in [0, \gamma)$. Each input i is necessary with probability c/γ. The inputs are essentially pairwise independent as defined by Sun (2006, Definition 2.7).[9] For most readers, it will be convenient to neglect these technical details and think about identically and independently distributed inputs.

[9]See also the discussion about independence on a continuum in Ali and Bénabou (2020, Footnote 18).

Lemma 1. *There is a probability space that satisfies these assumptions and guarantees a law of large numbers:*

$$\int c_i \mathrm{d}\lambda(i) = c \quad \textit{almost surely}$$

with an extension λ of the Lebesgue measure on $[0, \gamma)$ as introduced by Sun and Zhang (2009, Theorem 1).

By inspection of the good or reverse engineering, the buyer learns one input i^* that it is *un*necessary. That input i^* is drawn uniformly. To summarize, the seller's private information is c and $(c_i)_{i\in[0,\gamma)}$, while the buyer's private information is (v, i^*). The buyer cannot infer anything about the seller's costs c from his sample i^* because the buyer's information is stochastically independent and uncorrelated with the seller's costs c. Nevertheless, narratives make efficient trade feasible.

Theorem 1. *The following mechanism is ex-post efficient, budget-balanced, and individually rational: The seller's narrative describes the necessary inputs*

$$M_s = \{i \in [0, \gamma) | c_i = 1\}.$$

The buyer reports his valuation $M_b = v$ and the name of the observed input $m_b = i^*$. Payments and probabilities of trade are[10]

$$\begin{cases} t = 0, & p = 0, \qquad\quad \pi = 0 \quad \textit{if } M_b < \lambda(M_s) \\ t = 1, & p = \lambda(M_s), \quad \pi = 0 \quad \textit{if } M_b \geq \lambda(M_s) \text{ and } m_b \notin M_s \\ t = 1, & p = \lambda(M_s), \quad \pi = \gamma \quad \text{otherwise.} \end{cases} \tag{1}$$

Trade and payments depend on the buyer's reported valuation M_b, the seller's costs as inferred from her narrative $\lambda(M_s)$, and whether the seller's narrative is consistent with the buyer's report. The seller's narrative is *in*consistent with the buyer's report if the buyer learns about an unnecessary input and the seller includes this input in her narrative. Intuitively, the mechanism must deter the seller from overstating her costs, i.e., reporting more inputs than she requires for production. In this case, the seller's narrative contains

[10] Remember that t is the probability of trade, the buyer pays the price p to the seller, and the seller pays π to a third party.

unnecessary inputs. If the buyer reports any unnecessary inputs he observed, the mechanism can detect such deviations by the seller with positive probability. As the benefits of such a deviation for the seller are proportional to probabilities of detection, the mechanism can deter such deviations with bounded payments π.

Equilibrium strategies ensure that the seller's narrative is consistent with the buyer's report. Thus, trade occurs iff $M_b \geq \lambda(M_s)$ – making the mechanism ex-post efficient. The mechanism is also budget-balanced as I only consider payments made by the seller and the buyer. In the next steps, I investigate individual rationality and incentive compatibility.

To study the seller's incentives, suppose the buyer follows his strategy. In equilibrium, the seller expects the price to equal her costs and, hence, payoffs of zero making the seller willing to participate in the mechanism. Consider deviations by the seller. A deviation with $\lambda(M_s) \leq c$ makes the seller worse off by reducing the price. A deviation with $\lambda(M_s) > c$ has three effects: the price increases, probabilities of trade decrease, and additional payments π could be triggered. The first effect benefits the seller while the second and third effect hurt her. In particular, payoffs of such a deviation are below

$$\begin{aligned}
&\Big(1 - F(\lambda(M_s))\Big) \Big[\underbrace{\lambda(M_s) - c}_{\text{First Effect}} - \underbrace{\text{Prob}(m_b \in M_s)\gamma}_{\text{Third Effect}}\Big] \\
\leq &\Big(1 - F(\lambda(M_s))\Big) \Big[\lambda(M_s) - c - \frac{1}{\gamma}(\lambda(M_s) - c)\gamma\Big] = 0.
\end{aligned}$$

As the seller's narrative must contain at least a mass of $\lambda(M_s) - c$ unnecessary inputs, the probability that the buyer observed one of these inputs is at least $(\lambda(M_s) - c)/\gamma$. Hence, $\text{Prob}(m_b \in M_s) \geq (\lambda(M_s) - c)/\gamma$ and such a deviation is unprofitable. Consequently, her strategy is optimal for the seller. As the buyer gets the entire surplus, the mechanism is also incentive compatible for him. I postpone the remainder of the proof to the Appendix A.

As a theoretical limit, this section limits the information of the counterpart to a minimum showing the possibilities of narratives. Indeed, the buyer's information has mass zero and is stochastically independent from the seller's costs. Narratives nevertheless allow the buyer to cross-check the seller's reported costs learning with some probability if the reported costs are incorrect. This kind of partial verifiability allows the mechanism designer to attain first best in many settings. Narratives differ from ideas of McAfee and Reny (1992), Crémer and McLean (1988), and Myerson (1981). Crémer and McLean (1988) consider only finite type spaces. Hence, McAfee and Reny (1992) is the relevant comparison.

As the buyer's valuation and the seller's costs are independently distributed the difference seems suggestive. In addition, I implement efficient trade with bounded payments while McAfee and Reny (1992) can only approximate first best and require arbitrarily large payments the closer they approximate first best. Finally, I prove that my setting violates their spanning condition.

Lemma 2. *The setting violates the spanning condition of McAfee and Reny (1992, Theorem 2).*

Lemma 2 emphasizes the differences between the narratives proposed here and earlier ideas in the literature to use correlated information also shedding new light on the results of McAfee and Reny (1992) and Crémer and McLean (1988). If narratives allow the mechanism designer to attain first best, finding an optimal mechanism is relatively straightforward. In general, finding optimal mechanisms can be tricky. For this purpose, the revelation principle is very useful. Previous models of partial verifiability often made it difficult to find optimal mechanisms as the revelation principle fails. For instance, Green and Laffont (1986) provide this example: An agent has one of three types, $\theta \in \{\theta_1, \theta_2, \theta_3\}$. Type θ_1 can report θ_1 or θ_2. Type θ_2 can report θ_2 or θ_3. Type θ_3 can report θ_3. The principal takes a binary action, e.g., whether to procure a good from the agent. The agent prefers procurement independently of her type. The principal wants to procure the good from types θ_2 or θ_3 but not from type θ_1. This outcome is implementable by the allocation rule: Procure the good if and only if the agent reports θ_3. Then types θ_2 and θ_3 report θ_3. It does not matter whether type θ_1 reports θ_1 or θ_2 because she will never get the procurement contract. This outcome function cannot be implemented truthfully, though. Whenever type θ_2 receives the contract upon reporting θ_2, type θ_1 can ensure that she also gets the procurement contract by reporting θ_2. Hence, the revelation principle fails in the framework of Green and Laffont (1986). Using narratives, messages are partially verifiable *and* the revelation principle is valid.

Proposition 3. *The revelation principle is valid: Any implementable outcome can be implemented truthfully.*

Therefore, I can focus on direct mechanisms to find a second-best mechanism as I do in the next section.

5 The Strength of Narratives

Sometimes, we are interested in partial verifiability that does not immediately yield first-best. Narratives allow doing so by restricting (the support of) the receiver's sample. The narratives' strength measures the familiarity of the buyer with the product or the depth of the relationship between buyer and seller. I denote the narratives' strength by an even $\delta \in \mathbb{N}$. If a big car producer procures some well-known components, a high δ shows her ability to comprehend and crosscheck the supplier's narratives. If a little experienced consumer buys some antique furniture or painting, a low δ shows her limited ability to crosscheck a seller's narratives. Vice versa, if the buyer tells the narratives, the higher δ, the better the seller understands the value-generating aspects for the buyer. Thus, longer relationships between banks and borrowers or the availability of big data increase δ – making narratives more powerful and strengthening the receiver's hand even if the additional information or knowledge does not affect the receiver's prior about the sender's valuation. Therefore, I call δ the narratives' strength. The idea is to restrict the support of the buyer's sample to a finite set with δ elements. Thus, the seller knows that the mechanism cannot detect many misreports limiting the partial verifiability of her narrative.

For ease of exposition, I describe one of many possible specifications instead of aiming for maximum generality. Think about correlated inputs so that you can only use some inputs together but not individually. For example, a car with a petrol engine needs oil filters and a gearbox, while an electric car needs a battery and a hybrid car needs all these inputs. The necessary inputs are correlated in the following way: Consider a value x drawn from a distribution $\bar{G}(\cdot)$ on $[0, 1/2]$ and a starting point ρ drawn from a distribution with full support on a grid $\mathcal{L} = \{0, 1/\delta, 2/\delta, \ldots, (\delta - 1)/\delta\}$. Beginning from ρ, the next inputs with mass x are necessary: $c_s^i = 1$ if and only if $\rho \leq i \leq \rho + x$ or $i \leq \rho + x - 1$. A simple illustration depicted in Figure 2 is to think about inputs $[0, 1)$ as a circle with circumference one. Assume λ to be the Lebesgue measure so that the seller's costs c equal x. Both x and ρ are private information of the seller. The buyer only knows their distribution. Assume that the distributions $F(v)$ and $\bar{G}(x)$ have continuous and positive densities for all $v, x \in [0, 1/2]$. The buyer privately learns for exactly one input i^* whether it is necessary. I assume that the buyer's signal is $(i^*, c_s^{i^*})$ with the input i^* drawn uniformly from all unnecessary inputs on the grid $\mathcal{L}$. The seller only knows the distribution of this signal and, hence, its support.

The seller's knowledge about the grid $\mathcal{L}$ allows her to overstate her costs to some extent

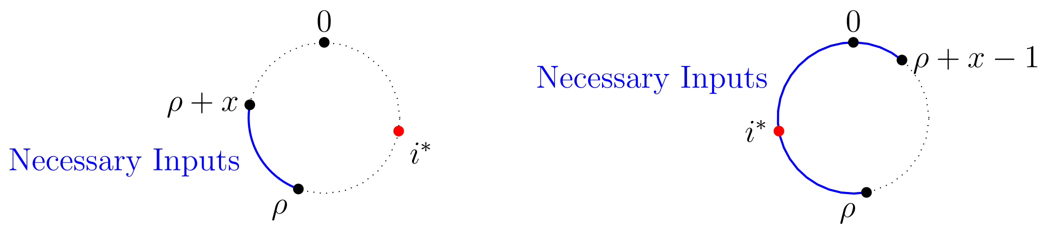

Figure 2: Two Examples of Necessary Inputs $[\rho, \rho + x]$ and Buyer's Sample i^*

without the buyer or the mechanism being able to detect such deviations. Therefore, it is impossible to achieve efficiency. Narratives still make even simple mechanisms surprisingly efficient. For this purpose, define a consistency function

$$A(m_b, \tilde{\rho}, \tilde{c}) = \begin{cases} 0 & \text{if } \tilde{\rho} \leq m_b \leq \tilde{\rho} + \tilde{c} \text{ or } m_b \leq \tilde{\rho} + \tilde{c} - 1 \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

Correspondingly, a narrative $(\tilde{\rho}, \tilde{c})$ is *in*consistent, $A = 0$, with the buyer's report m_b if the buyer reports to have observed an unnecessary input and the seller includes this input in her narrative. Proposition 4 gives an example of such a simple mechanism.

Proposition 4. *The following mechanism requires no subsidies, is individually rational, and incentivizes buyer and seller to report truthfully: The seller's narrative describes the necessary inputs, $m_s = \rho$ and $M_s = c$. The buyer reports his valuation $M_b = v$ and his sample $m_b = i^*$. Payments and probabilities of trade are*

$$\begin{cases} t = 0, \quad p = 0, & \pi = 0 \quad \textit{if } M_b < p^*(M_s) \\ t = 1, \quad p = p^*(M_s), & \pi = 0 \quad \textit{if } M_b \geq p^*(M_s) \text{ and } A(m_b, m_s, M_s) = 1 \\ t = 1, \quad p = p^*(M_s), & \pi = 1 \quad \text{otherwise} \end{cases} \tag{3}$$

with a price equal to the closest grid point above M_s, i.e.,

$$p^*(c) = \frac{k}{\delta}$$

if $(k-1)/\delta \leq c < k/\delta$ for a $k \in \{1, 2, \ldots, \delta\}$.

If the seller's narrative is inconsistent with the buyer's reported sample, the seller has to pay a penalty $\pi = 1$. Otherwise, the mechanism offers the good to the buyer at a price slightly above the seller's reported costs. Indeed, the price can be at most $1/\delta$ above the seller's costs with $1/\delta$ being the inverted strength of the narrative. This ensures that the mechanism incentivizes the seller to report her costs truthfully. The mechanism does not achieve efficiency. If narratives become stronger, however, welfare increases and converges to first best.

Lemma 3. *Welfare generated by the mechanism in Proposition 4 converges to first best for sufficiently strong narratives, $\delta \to \infty$. In particular, for sufficiently strong narratives, this mechanism attains higher efficiency than any mechanism that does not use narratives.*

A natural follow-up question regards the characterization of a second-best mechanism in this setting. Due to the revelation principle in Proposition 3, I can focus on direct mechanisms to find a second-best mechanism. It turns out that, as in the classical setting without narratives, additional assumptions on the distributions are necessary. Assume that virtual values are piecewise monotone:

$$c + \frac{\bar{G}(c) - \bar{G}((k-1)/\delta)}{\bar{g}(c)} \text{ increases in } c \in \left[\frac{k-1}{\delta}, \frac{k}{\delta}\right) \text{ for all } k \in \mathbb{N}^{\delta} := \{1, 2, \dots, \delta/2\}, \quad (4)$$

$$\text{and } v - \frac{1 - F(v)}{f(v)} \text{ increases in } v. \quad (5)$$

For $\delta = 2$, these assumptions about increasing virtual values are common in mechanism design.[11] See, e.g., Myerson (1981). To ease notation, define $Z(c) = p^*(c) - 1/\delta$ indicating the closest grid point below c. Furthermore, define a trading rule

$$Y(\tilde{c}, \tilde{v}, \tilde{\alpha}) = \begin{cases} 1 & \text{if } \tilde{c} + \tilde{\alpha}\frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})} \leq \tilde{v} - \tilde{\alpha}\frac{1-F(\tilde{v})}{f(\tilde{v})} \\ 0 & \text{otherwise} \end{cases}$$

depending on a parameter $\tilde{\alpha}$ such that trade occurs if virtual valuations are above virtual costs. For higher values of $\tilde{\alpha}$ trade occurs less often. Define the value $\alpha \in (0, 1]$ so that

$$\int_0^1 \int_0^1 \left(\tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})} - \tilde{c} - \frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})} \right) Y(\tilde{c}, \tilde{v}, \alpha) \mathrm{d}G(\tilde{c}) \mathrm{d}F(\tilde{v}) = 0.$$

The value α governs the efficiency of the second-best mechanism. Lower α indicates that the second-best mechanism is more efficient. Finally, define the expected probabilities of

[11] Remember that δ is even and $\delta \in \mathbb{N}$. Thus, $\delta = 2$ is the smallest possible value.

trade conditional on the trading rule Y

$$\mathcal{T}_s(c) = \int_0^1 Y(c, \tilde{v}, \alpha)\mathrm{d}F(\tilde{v}) \text{ and } \mathcal{T}_b(v) = \int_0^1 Y(\tilde{c}, v, \alpha)\mathrm{d}G(\tilde{c})$$

for all $c, v \in [0, 1)$. The value $\mathcal{T}_b(M_b)$ equals the expected probability of trade from the buyer's point of view if he reports M_b. The value $\mathcal{T}_s(M_s)$ equals the expected probability of trade from the seller's point of view if she reports M_s.

Theorem 2. *If distributions $F(v)$ and $\bar{G}(x)$ satisfy conditions* (4) *and* (5)*, the following mechanism is optimal, i.e., surplus maximizing: Equilibrium strategies are the same as in Proposition 4. Trade occurs if and only if $Y(M_s, M_b, \alpha) = 1$. The seller's transfers to the principal equal $\pi = 2(1 - A(m_b, m_s, M_s))$ and the buyer's expected transfers to the seller equal*

$$p = p^*(M_s, M_b) = \mathcal{T}_s(M_s)M_s + \int_{M_s}^{Z(M_s)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} + \mathcal{T}_b(M_b)M_b - \int_0^{M_b} \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v}$$
$$- \int_0^{1/2} \left(\mathcal{T}_s(c')c' + \int_{c'}^{Z(c')+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \right) \mathrm{d}\bar{G}(c')$$

with the consistency function $A(m_b, m_s, M_s)$ defined in (2)*.*

Narratives limit the seller's scope for deviations. Large deviation might make narratives inconsistent and trigger payments $\pi = 2$. Small deviations, however, are undetectable. Therefore, classical tools of mechanism design are useful in determining the solution in contrast to previous partial-verifiability settings like Green and Laffont (1986). It is impossible to attain efficiency in this setting — reinforcing the earlier result that the setting violates the spanning conditions of McAfee and Reny (1992) and Crémer and McLean (1988). Accordingly, there is no trade if the buyer's valuation is only marginally above the seller's costs. Narratives decreases this inefficiency and increase welfare. Thus, narratives should be used from a welfare perspective and communication should not be limited to summary statistics, like costs c and valuations v in the example of bilateral trade. Stronger narratives increase welfare. In particular, similarly to Lemma 3, welfare can approach efficiency arbitrarily close for sufficiently strong narratives.

6 Discussion

This sections aims to identify the essential assumptions of the model and connect these to possible applications. Narratives break down an aggregate information into smaller pieces. In the framing of bilateral trade, the total costs of the seller are the sum of the costs of different inputs. Suppose a car manufacturer develops a new car. For this purpose, the car manufacturer bargains with a supplier to procure, e.g., a high-end sound system, some novel sensors or a head-up display. Due to its technology or its brand name, only this supplier can produce the specific gadget and the car manufacturer has to revert to his outside option of building the car without that gadget or of using a generic equivalent if negotiations fail.

In her narrative, the supplier will provide many specifications, inputs and sometimes even some blueprints of the sound system to justify its price offer.[12] Indeed, narratives disclosing information about inputs and their respective costs are so common in some industries that the accounting literature coined the terms open-book negotiation or open-book accounting: "open books concern the exchange of cost information between suppliers and customers" (Romano and Formentini, 2012, p. 69). Carr and Ng (1995) first describe this practice in the car industry. See also Alenius et al. (2015) and Axelsson et al. (2002) for additional evidence on open-book negotiations.

The main model of Section 4, makes a couple of assumptions on the costs structure. The required inputs are identically and essentially independently distributed from the point of the car manufacturer. The (discrete) example in Section 3 shows that both assumptions are made mainly for notational convenience. Narratives also work for inputs whose distributions differ and which might be correlated with other inputs as long as there is no perfect correlation.[13] In addition, I assume that each input has the same costs. Again, this assumption is made for ease of notation. If the costs differ between inputs, narratives still make messages partially verifiable and the intuition remains valid and unchanged. Optimal mechanisms change, though. Mechanisms might need larger off-equilibrium payments to satisfy incentive compatibility, in particular, if costs differ a lot between inputs.

In the model, I assume that the buyer learns for few inputs whether these inputs are required for production. Accordingly, the car manufacturer might ask for a prototype of the sound system for physical inspection or reverse engineering to learn about some

[12] Mueller et al. (2016, p. 9) survey negotiations in the car industry: manufacturers "elicit cost information from the supplier."

[13] This robustness with respect to correlated inputs is confirmed by my results in Section 5.

inputs. Indeed, manufacturers derive such data from “from the internal production of similar parts, as well as by re-engineering the part.” (Mueller et al., 2016, p. 9) I analyze the case of a sample of one input out of a continuum of inputs to show how powerful narratives are. Arguably, in reality buyers might frequently observe several inputs in their sample. Narratives still work in these cases and the intuition remains unchanged. I also assume that the buyer's sample is uniformly distributed saving a lot of notation. Again, other distributions for the sample do not change the intuition as long as there is full support and densities are bounded away from zero. Section 5 provides some insights what happens when these conditions are violated.

Finally, the narratives in main model of Section 4 are very rich and complicated messages that might be difficult to convey in actual negotiations but Section 5 demonstrates that it is possible to reduce the complexity of the narratives employed in negotiations. Hence, Section 4 shows what is possible in the limit but already much simpler narratives such as those in Section 5 can attain high level of efficiency. Alternatively, mediators might help conveying the narratives explaining some of the benefits of using mediators in bilateral trade.[14]

So far I consider direct mechanisms which are very tractable for analysis, but sometimes too abstract for direct applications. Indirect implementation makes contracts more familiar. Consider, for example, the setting of Section 4: Buyer and seller participate in a contract with dispute procedures. First, the seller makes an offer M_s that contains her narrative and all necessary inputs reflecting the complexity of the object. Observing this offer, the buyer decides whether to buy the good at a price, $p = \lambda(M_s)$. The buyer has the possibility to dispute the seller's claim. If he disputes the seller's claim or claims loss of trust, the seller has to sell the good to the buyer for a price, $p = \lambda(M_s)$. In addition, the seller has to pay the costs of arbitration $\pi = \gamma$.[15] It is easy to see that this contract is pay-off equivalent to the mechanism in Theorem 1. Consequently, this contract with dispute procedures is individually rational, ex-post efficient and budget-balanced.

Such dispute procedures can be implemented internally, by a mediator or by the legal system. These dispute procedures are quite common. See Fenn et al. (1997) for examples in the construction industry and the chemical process industry. Notice that this indirect implementation is also feasible if disputing a claim is costly by adjusting the contract to

[14] See also Copic and Ponsati (2008) and Larsen et al. (2022) for the benefits of using mediators in in bilateral trade.

[15] The buyer disputes the seller's claim if he wants to trade, i.e., $v \geq \lambda(M_s)$ and $i^* \in M_s$.

reimburse these costs. For this purpose, some of the payments by the seller are paid to the buyer to reimburse his costs for disputing the seller's claim. If the costs for disputing the seller's claim are not prohibitively high, this reimbursement is possible by diverting the seller's payments.

7 Conclusions

I consider narratives as rationales or justifications following previous literature, for example, Bénabou et al. (2020). I develop a general framework for partial verifiability based on narratives. Narratives enrich messages to make messages partially verifiable. Narratives provide a number of reasons, arguments and points to make the sender's claim instead of just stating the claim. The receiver derives the sender's claim from these reasons. Therefore, misreporting requires changing several arguments. Indeed, the larger the intended change, the more arguments the sender has to adjust. The receiver knows whether some potential reasons are appropriate and support the sender's claim. Hence, if the sender deviates from equilibrium the receiver notices such a deviation with positive probability. Narratives work even if the receiver knows only one reason out of a continuum and cannot infer anything about the sender's valuation from the reasons she knows. I apply narratives to mechanism design. As an example, I consider the canonical setting of bilateral trade. I show that narratives allow for ex-post efficient trade and ensure individual rationality as well as budget balance. Consequently, narratives are so powerful to overturn the famous impossibility result by Myerson and Satterthwaite (1983). In addition, I demonstrate how to adjust the strength of narratives. Thus, narratives offer a powerful, general, and flexible framework to work with partial verifiability. With narratives, the classical revelation principle is still valid so that classical tools can be used to determine optimal contracts and study feasibility as well as implementability.

The mechanisms considered so far do not require outside subsidies and have no payments to third parties on the equilibrium path. The mechanisms require payments to third parties off the equilibrium path, however. Many results in the literature, e.g., Wolitzky (2016), are in line with this property. For example, Kojima and Yamashita (2017, p. 1399) "do not regard [strong] budget balance to be indispensable as long as the mechanism runs no budget deficit." In the case of discrete type spaces and symmetric settings, it is possible to find conditions on the distributions of costs and valuations that ensure strong budget balance, i.e., no payments to third parties.

In line with the canonical bilateral-trade setting, I assume simultaneous reporting of all messages. The mechanisms of Theorems 1 and 2 as well as Propositions 2 and 4, however, are robust to different timings. In particular, sequential communication is possible as long as narratives are told first or other messages are not made public before the sender tells her narratives. Thus, in bilateral trade, the seller must not observe the buyer's message m_b about his sample before telling the narrative about her costs (and vice versa for the buyer in symmetric settings).

A Appendix (Proofs)

Proof of Proposition 2: The proposition states a budget-balanced mechanism and corresponding equilibrium strategies. For sake of completeness, let me write down the messages spaces here: The buyer reports $\mathcal{M}_b = (m_b, M_b) \in \{0, 1, \ldots, I\} \times$ support of F. The seller reports her narrative[16]

$$M_s \in 2^{\{1,2,\ldots,I\}}.$$

The equilibrium strategies guarantee that $M_b = v$ and $|M_s| = c$ such that there is trade if and only if $v \geq c$. Therefore, the mechanism is ex-post efficient.

To study the buyer's incentives, suppose the seller follows her strategy. In equilibrium, the buyer expects nonnegative payoffs making the mechanism individually rational for the buyer. The buyer receives the entire surplus in equilibrium so that he cannot gain by deviating: A deviation with $M_b > v$ lowers his utilities by making him buy the object at a price above his valuation. A deviation with $v > M_b$ lowers the buyer's utilities by not being able to buy the object at some prices below his valuation. Any other deviation by the buyer does not change his payoffs. Therefore, his strategy is optimal for the buyer.

Turning to the seller, suppose the buyer follows his strategy. In equilibrium, the seller expects payoffs

$$\sum_{i=c}^{I} [c - c] f(i) = 0$$

making the mechanism individually rational for the seller. A deviation with $|M_s| \leq c$ at least weakly lowers the price and, hence, cannot yield positive profits. A deviation with $|M_s| > c$ has three effects: the price increases, probabilities of trade decrease, and additional payments $\pi = 1/h_{min}$ could be triggered. The first effect benefits the seller while the second and third effect hurt her. In particular, expected payoffs of such a deviation

[16] For any finite set M, let 2^M denote the power set (the set of all subsets of M).

are below

$$\sum_{i=|M_s|}^{I}\left[|M_s|-c-\text{Prob}(m_b\in M_s)\frac{1}{h_{min}}\right]f(i)\leq(|M_s|-c)[1-1]\sum_{i=|M_s|}^{I}f(i)=0.$$

These deviation payoffs are nonpositive, because $\text{Prob}(m_b \in M_s) \geq h_{min}(|M_s| - c)$ as the probability that a given input is observed by the buyer is at least h_{min} and the seller has to include at least $|M_s|-c$ unnecessary inputs in her report for such a deviation. Consequently, any deviation in M_s is unprofitable and her strategy is optimal for the seller. To sum up, the mechanism is ex-post efficient, individually rational, and budget-balanced. □

Proof of Lemma 1: A probability space for the random variable c is constructed in the usual way using the Borel σ-algebra. For given c and probability c/γ, I require a suitable probability space for the inputs c_i. Following the approach by Sun (2006), I consider a Fubini extension instead of the usual continuum product based on the Kolmogorov construction. Sun and Zhang (2009, Theorem 1 and Corollary 2) prove that there exist a set Ω, a probability space on Ω, an extension λ of the Lebesgue measure on $[0,\gamma]$, a Fubini extension on $[0,\gamma]\times\Omega$ and a measurable process $z\colon [0,\gamma]\times\Omega\mapsto\mathbb{R}$, such that the random variables $z(i,\cdot)$ are essentially pairwise independent with

$$\text{Prob}(\{\omega\in\Omega|z(i,\omega)=1\})=c/\gamma$$

for all $i\in[0,\gamma)$ almost surely. By definition of a Fubini extension, the integral $\int_{[0,\gamma)} z(i,\omega)\mathrm{d}\lambda(i)$ is well defined for all $\omega\in\Omega$. Finally, Sun (2006, Theorem 2.8) shows that the integral equals $\gamma c/\gamma = c$ almost surely. □

Proof of Theorem 1: The theorem states a budget-balanced mechanism and corresponding equilibrium strategies. For sake of completeness, let me write down the messages spaces here: The buyer reports $\mathcal{M}_b = (m_b, M_b) \in [0,\gamma)\times[0,\nu)$. The seller reports her narrative

$$M_s\in\{X\subseteq[0,\gamma)|X\text{ is }\lambda\text{-measurable}\}.$$

The equlibrium strategies guarantee that $M_b = v$ and $\lambda(M_s) = c$ such that there is trade if and only if $v \geq c$. Therefore, the mechanism is ex-post efficient. In main text, I show that the strategy is optimal and the mechanism is individually rational for the seller. The seller's strategy is also feasible because the suggested messages are measurable as realizations of a measurable process according to Lemma 1.

Turning to the buyer, denote the conditional expectation of the seller's costs by $c^*(v) = \mathbb{E}(c|v\geq c)$ and suppose the seller follows her strategy. Then the buyer expects payoffs

$$\text{Prob}(\text{trade})(v-\mathbb{E}(p|\text{trade}))=G(v)(v-c^*(v))=\int_0^v v-\tilde{c}\mathrm{d}G(\tilde{c}).$$

in equilibrium. These equilibrium payoffs are nonnegative so that the mechanism is individually rational for the buyer. Turning to incentive compatibility, consider deviations by the buyer. A deviation with $M_b \geq v$ makes the buyer worse off by potentially triggering trade at a price above his valuation. Deviations with $M_b < v$ decrease probabilities of trade. In particular, such deviations imply utilities below

$$G(M_b)\left[v - c^*(M_b)\right] = \int_0^{M_b} v - \tilde{c}\mathrm{d}G(\tilde{c})$$

Hence, such a deviation is unprofitable if

$$\int_0^v v - \tilde{c}\mathrm{d}G(\tilde{c}) \geq \int_0^{M_b} v - \tilde{c}\mathrm{d}G(\tilde{c}) \Leftrightarrow \int_{M_b}^v v - \tilde{c}\mathrm{d}G(\tilde{c}) \geq 0$$

The last inequality is valid because $v - \tilde{c} > 0$ for all $\tilde{c} \in [M_b, v)$. Therefore, any deviation in M_b is unprofitable. A deviation in m_b does not affect the buyer. Therefore, any deviation in m_b is unprofitable and, in addition, any joint deviation in M_b and m_b is unprofitable. Consequently, his strategy is optimal for the buyer.

To sum up, there is a mechanism that is individually rational, ex-post efficient and budget-balanced. □

Proof of Lemma 2: Denote the seller's private information and, hence, her type by τ_1 with the type space T_1 and the buyer's private information by τ_2 with the type space T_2. The seller's type is a profile of necessary inputs, $\tau_1 = (c_i)_{i\in[0,\gamma)}$. The buyer's type is his valuation and his sample of the seller's inputs, $\tau_2 = (v, i^*)$. I denote by h the conditional densities of a type conditional on the other player's type. According to McAfee and Reny (1992, Theorem 2) their mechanisms can attain efficient trade if and only if for every type $\tau_j^0 \in T_j$ and every probability measure μ on the type space T_j, $\mu(\{\tau_j^0\}) \neq 1$ implies $h(\cdot|\tau_j^0) \neq \int_{T_j} h(\cdot|\tau_j)\mathrm{d}\mu(\tau_j)$.

Begin with the buyer and $j = 2$. The conditional density of τ_1 conditional on τ_2 is either 0 or equal to the unconditional density of $(c_i)_{i\in[0,\gamma)\setminus\{i^*\}}$ and, hence, determined by the distribution $G(c)$ and the essential independence. The first case occurs if $c_{i^*} = 1$. The second case occurs if $c_{i^*} = 0$. Pick any $\tau_2^0 = (v_0, i_0^*) \in T_2$ and denote by $\cdot$ the dot product.[17] Consider the measure μ_2 on T_2 determined by

$$\mu_2(\hat{T}_2) = 0 \text{ with } \hat{T}_2 = \{\tau_2 = (v, i^*) \in T_2 | i^* \neq i_0^*\} \text{ and}$$
$$\mu_2(\tilde{T}_2) = \frac{1}{\nu}\lambda(\{(1,0)\cdot\tau_2 | \tau_2 \in \tilde{T}_2\}) \text{ with the Lebesgue measure } \lambda$$

[17]Notice that $(1,0)\cdot\tau_2 = v \in (0,\nu]$ for any $\tau_2 = (v, i^*)$.

for all $\tilde{T}_2 \subseteq T_2 \setminus \hat{T}_2 = \{\tau_2 = (v, i^*) \in T_2 | i^* = i_0^*\}$. The definition of μ_2 guarantees countable additivity. Thus, it is enough to verify that $\mu_2(T_2) = \mu_2(T_2 \setminus \hat{T}_2) = \nu/\nu = 1$ and, hence, $0 \leq \mu_2(T) \leq 1$ for all $T \subseteq T_2$ to ensure that μ_2 is a probability measure.

It is obvious that $\mu_2(\{\tau_2^0\}) = 0 < 1$ and $h(\tau_1|\tau_2^0) = h(\tau_1|\tau_2)$ for all $\tau_2 \in T_2 \setminus \hat{T}_2$. Hence,

$$h(\tau_1|\tau_2^0) = \int_{T_2} h(\tau_1|\tau_2)\mathrm{d}\mu_2(\tau_2) \qquad \forall \tau_1 \in T_1$$

violating the spanning condition of McAfee and Reny (1992, Theorem 2).

Continue with the seller and $j = 1$. The conditional density of τ_2 conditional on τ_1 is either 0 or determined by the distribution $F(v)$ and the uniform distribution of i^* on $\{i \in [0, \gamma) | c_i = 0\}$. The first case occurs if $c_{i^*} = 1$, i.e., the $(0, 1) \cdot \tau_2 = i^*$ component of τ_1 equals one. The second case occurs if $c_{i^*} = 0$. Pick any $\tau_1^0 = (c_i^0)_{i \in [0,\gamma)} \in T_1$. Consider the measure μ_1 on T_1 determined by

$$\mu_1(T_1 \setminus \hat{T}_1) = 0 \text{ with } \hat{T}_1 = \{\tau_1 \in T_1 | \exists \hat{i}\colon c_i = c_i^0 \text{ for all } i \in ([0,\gamma) \setminus \{\hat{i}\})\} \text{ and}$$
$$\mu_1(\tilde{T}_1) = \frac{1}{\gamma}\lambda(\{\hat{i} \in (0,\gamma] | \exists \tau_1 \in \tilde{T}_1\colon c_i = c_i^0 \text{ for all } i \in ([0,\gamma) \setminus \{\hat{i}\})\}) \text{ for all } \tilde{T}_1 \subseteq \hat{T}_1$$

with the Lebesgue measure λ. The definition of μ_1 guarantees countable additivity. Thus, it is enough to verify that $\mu_1(T_1) = \mu_1(\hat{T}_1) = \gamma/\gamma = 1$ and, hence, $0 \leq \mu_1(T) \leq 1$ for all $T \subseteq T_1$ to ensure that μ_1 is a probability measure.

It is obvious that $\mu_1(\{\tau_1^0\}) = 0 \neq 1$ and

$$h(\tau_2|\tau_1^0) = \int_{T_1} h(\tau_2|\tau_1)\mathrm{d}\mu_1(\tau_1) \qquad \forall \tau_2 \in T_2$$

violating the spanning condition of McAfee and Reny (1992, Theorem 2). Therefore, the mechanisms of McAfee and Reny (1992) and Crémer and McLean (1988) to achieve efficiency do not work here. $\square$

Proof of Proposition 3: The framework consists of well-defined type spaces, typical quasi-linear preferences and common priors. In addition, any communication is possible. Therefore, any implementable outcome function can be implemented truthfully according to Mas-Colell et al. (1995, Proposition 23.D.1). This is valid in all settings considered in this paper. $\square$

Proof of Proposition 4: The proposition states a budget-balanced mechanism and corresponding equilibrium strategies. For sake of completeness, let me write down the messages spaces: The buyer reports $\mathcal{M}_b = (m_b, M_b) \in \{0, 1/\delta, 2/\delta, \ldots, (\delta - 1)/\delta, 2\} \times [0, 1)$ and the seller reports her narrative $\mathcal{M}_s = (m_s, M_s) \in \mathcal{L} \times [0, 1)$. The equilibrium strategies ensure that narratives are consistent, $A(m_b, m_s, M_s) = 1$.

To study the seller's incentives, suppose the buyer follows her strategy. In equilibrium, the seller expects payoffs

$$\text{Prob}(\text{trade})(\mathbb{E}(p|\text{trade}) - c) = (1 - F(p^*(c)))(p^*(c) - c) > 0$$

making the mechanism individually rational for the seller. A deviation with $M_s \leq c$ could make the seller worse off by trading at prices below costs and/or by triggering additional payments π. A deviation with $M_s > c$, $p^*(M_s) = p^*(c)$ and $m_s = \rho$ does not change the seller's payoffs. A deviation with $M_s > c$ and ($p^*(M_s) \neq p^*(c)$ or $m_s \neq \rho$) increases the price and could trigger additional payments π. By deviating and claiming more necessary inputs, the price $p^*(M_s)$ increases by $1/\delta$ for each grid point that the seller additionally includes in her report $[m_s, m_s + M_s]$. At the same time, she has to make an additional payment of 1 with probability $1/\delta$ for each grid point that she additionally includes in her report. The decrease in the probability of trade disadvantages the seller. Hence, such a deviation is unprofitable for the seller. Therefore, any deviation in M_s and m_s is unprofitable. Consequently, her strategy is optimal for the seller.

Now turn to the buyer. If there is no trade, his payoffs are zero. In addition, trade takes place only if his reported valuation is above the price p^* set in the mechanism. These expected payoffs are clearly nonnegative in equilibrium making the mechanism (3) individually rational for the buyer. A deviation with $M_b \geq v$ makes the buyer worse off by potentially triggering trade at a price above his valuation. Deviations with $M_b < v$ decrease probabilities of trade making him worse off. Therefore, any deviation in M_b is unprofitable. A deviation in m_b has no effect for the buyer. Therefore, any (joint) deviation in M_b and m_b is unprofitable. Consequently, his strategy is optimal for the buyer. $\square$

Proof of Lemma 3: For $\delta \to \infty$, $\lim p^*(c) = c$ and $p^*(c) \geq c$ for all $c \in [0, 1)$. Therefore, trade occurs in the mechanism (3) with probability 0 if $v < c$ and for $\delta \to \infty$ with probability 1 if $v \geq c$. Hence, the mechanism (3) is ex-post efficient for $\delta \to \infty$. Therefore, for sufficiently large δ, the mechanism generates a higher surplus than any fixed-price mechanism or any other classical mechanism that does not use narratives because these mechanisms cannot attain efficiency according to Myerson and Satterthwaite (1983, Corollary 1). $\square$

Proof of Theorem 2: Although messages spaces are the same as in Proposition 4, let me write them down for sake of completeness: The buyer reports $\mathcal{M}_b = (m_b, M_b) \in \{0, 1/\delta, 2/\delta, \ldots, (\delta - 1)/\delta, 2\} \times [0, 1)$ and the seller reports $\mathcal{M}_s = (m_s, M_s) \in \mathcal{L} \times [0, 1)$.

Equilibrium strategies ensure that narratives are consistent, $A(m_b, m_s, M_s) = 1$, in equilibrium. The mechanism in Theorem 2 is budget-balanced. I begin by proving incentive compatibility. Then I turn to individual rationality before considering optimality. Finally, I show existence of α.

Incentive Compatibility

To study the buyer's payoffs, suppose the seller follows her equilibrium strategy. Then the buyer's expected payoffs are

$$\begin{aligned} v\mathcal{T}_b(M_b) - \mathbb{E}(p^*(c, M_b)) &= v\mathcal{T}_b(M_b) - M_b\mathcal{T}_b(M_b) + \int_0^{M_b} \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v} \qquad (6) \\ &- \int_0^1 \left(\mathcal{T}_s(c')c' + \int_{c'}^{Z(c')+\frac{1}{\delta}} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \right) \mathrm{d}G(c') + \int_0^1 \left(\mathcal{T}_s(c')c' + \int_{c'}^{Z(c')+\frac{1}{\delta}} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \right) \mathrm{d}G(c') \\ &= (v - M_b)\mathcal{T}_b(M_b) + \int_0^{M_b} \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v} \end{aligned}$$

If the buyer reports $M_b = v$, his expected payoffs $U_b(v)$ are $\int_0^v \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v}$. Turning to incentive compatibility, consider deviations by the buyer. If the buyer deviates in M_b, his expected payoffs change by

$$\frac{\partial v\mathcal{T}_b(M_b) - \mathbb{E}(p^*(c, M_b))}{\partial M_b} = (v - M_b)\frac{\partial \mathcal{T}_b(M_b)}{\partial M_b} - \mathcal{T}_b(M_b) + \mathcal{T}_b(M_b) = \frac{\partial \mathcal{T}_b(M_b)}{\partial M_b}(v - M_b).$$

For all $\tilde{\alpha} \in [0, 1]$, assumptions (4) and (5) ensure that $c + \tilde{\alpha}(G(c) - G(Z(c)))/g(c)$ strictly increases in $c \in [(k-1)/\delta, k/\delta)$ for all $k \in \mathbb{N}^\delta$ and $v - \tilde{\alpha}(1 - F(v))/f(v)$ strictly increases in v. Hence, for all $\tilde{\alpha} \in [0, 1]$, $Y(\tilde{c}, \tilde{v}, \tilde{\alpha})$ decreases in $\tilde{c} \in [(k-1)/\delta, k/\delta)$ for all $k \in \mathbb{N}^\delta$ and increases in $\tilde{v}$. Therefore, $\mathcal{T}_s(c)$ decreases in c in each interval $[(k-1)/\delta, k/\delta)$ and $\mathcal{T}_b(v)$ increases in v. Hence, it is optimal to report $M_b = v$ because the positive derivative of $\mathcal{T}_b(v)$ ensures that the buyer's expected payoffs are maximised at $M_b = v$. Any deviation in m_b does not change the buyer's expected payoffs. Consequently, his strategy is optimal for the buyer.

Now turn to the seller. To study the seller's payoffs, suppose the buyer follows his equilibrium strategy. If the seller reports any messages (m_s, M_s) that guarantee a consistent narrative, i.e., $A(m_b, m_s, M_s) = 1$ for all m_b consistent with the buyer's strategy and the seller's information, her expected payoffs are

$$
\begin{aligned}
\mathbb{E}(p^*(M_s, v)) - c\mathcal{T}_s(M_s) = (M_s - c)\mathcal{T}_s(M_s) + \int_{M_s}^{Z(M_s)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \qquad (7)\\
+ \int_0^1 \left(\mathcal{T}_b(v')v' - \int_0^{v'} \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v} \right) \mathrm{d}F(v') - \int_0^1 \left(\mathcal{T}_s(c')c' + \int_{c'}^{Z(c')+\frac{1}{\delta}} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \right) \mathrm{d}G(c')
\end{aligned}
$$

The last two terms are constant in the seller's message. If the seller deviates in (m_s, M_s) still ensuring a consistent narrative, her expected payoffs change by

$$
\begin{aligned}
\mathbb{E}(p^*(M_s, v)) - &c\mathcal{T}_s(M_s) - \mathbb{E}(p^*(c, v)) + c\mathcal{T}_s(c) \\
&= (M_s - c)\mathcal{T}_s(M_s) + \int_{M_s}^{Z(M_s)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} - \int_{c}^{Z(c)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c}. \qquad (8)
\end{aligned}
$$

If $Z(c) = Z(M_s)$, it is optimal to report $M_s = c$ because the derivative of the seller's expected payoffs equals $(M_s - c)(\partial\mathcal{T}_s(M_s)/\partial M_s)$ and $\mathcal{T}_s(\cdot)$ is decreasing. If $Z(c) > Z(M_s)$, $c \geq Z(M_s) + 1/\delta > M_s$ and, hence, $(M_s - c)\mathcal{T}_s(M_s) < 0$. In addition, the last term in Eq. (8) is negative and the sum of the first two terms is also nonpositive as

$$
\int_{M_s}^{Z(M_s)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \leq (Z(M_s) + 1/\delta - M_s)\mathcal{T}_s(M_s) \leq (c - M_s)\mathcal{T}_s(M_s) = -(M_s - c)\mathcal{T}_s(M_s).
$$

Therefore, expected payoffs decrease and deviations with $Z(c) > Z(M_s)$ are unprofitable. If $Z(c) < Z(M_s)$, any such deviations cannot ensure a consistent narrative. In particular, the seller expects to pay π with probability of at least $Z(M_s) - Z(c)$ because there are at least $\delta Z(M_s)$ grid points contained in the set of inputs $[m_s, m_s + M_s]$ reported by the seller and the buyer observes each grid point with probability $1/\delta$. The mechanism charges the penalty only if trade occurs. Therefore, the gains of such a deviation are at most

$$
\begin{aligned}
&(M_s - c)\mathcal{T}_s(M_s) + \int_{M_s}^{Z(M_s)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} - \int_{c}^{Z(c)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} - \pi(Z(M_s) - Z(c)) \\
&\leq (Z(M_s) + \frac{1}{\delta} - c - \pi(Z(M_s) - Z(c)))\mathcal{T}_s(M_s) - \int_{c}^{Z(c)+1/\delta} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \\
&\leq (\frac{1}{\delta} - (\pi - 1)(Z(M_s) - Z(c)))\mathcal{T}_s(M_s) \leq (\frac{1}{\delta} - (\pi - 1)\frac{1}{\delta})\mathcal{T}_s(M_s) = 0.
\end{aligned}
$$

The first inequality results from the fact that the expected probability of trade $\mathcal{T}_s(\cdot)$ is decreasing in $[M_s, Z(M_s)+1/\delta)$. The second inequality results from $c \geq Z(c)$ and a positive integral in the second line. The last equality results from $\pi = 2$ and $Z(M_s) - Z(c) \geq 1/\delta$.[18]

[18] Notice that $\pi = 1$ is also incentive compatible if $\lim_{c' \nearrow k/\delta} \mathcal{T}_s(c') \geq \mathcal{T}_s((k' - 1)/\delta)$ for all $k, k' \in \mathbb{N}^\delta$ with $k < k'$. The reason is that this condition guarantees $\lim_{c' \nearrow Z(c)+1/\delta} \mathcal{T}_s(c') \geq \mathcal{T}_s(M_s)$ and the second line of above inequality is at most $(1/\delta - c + Z(c))\mathcal{T}_s(M_s) - (Z(c) + 1/\delta - c) \lim_{c' \nearrow Z(c)+1/\delta} \mathcal{T}_s(c')$ for $\pi = 1$.

Hence, it is optimal to report $M_s = c$. Any deviation in m_s weakly decreases the seller's expected payoffs. Therefore, any deviation in M_s and m_s is unprofitable. Consequently, her strategy is optimal for the seller.

Individually Rationality

Notice that the buyer's expected payoffs are zero for $v = 0$ according to Eq. (6) and they increase in his valuation v with derivative $\mathcal{T}_b(v) \geq 0$ in equilibrium. Hence, the mechanism in Theorem 2 is individually rational for the buyer. The seller's expected payoffs $U_s(c)$ decrease in her costs $c \in [(k-1)/\delta, k/\delta)$ for all $k \in \mathbb{N}^\delta$ with derivative $-\mathcal{T}_s(c) \leq 0$. Denote $U_s^-(c') = \lim_{\tilde{c} \nearrow c'} U_s(\tilde{c})$. It remains to show that $U_s^-(k/\delta) \geq 0$ for all $k \in \mathbb{N}^\delta$ to ensure individually rationality. For this purpose, consider a more general mechanism with prices $p(c, v)$ and probabilities of trade $t(c, v)$ such that the buyer's expected utilities $U_b(v)$ increase in v and equal

$$U_b(v) = \int_0^1 vt(\tilde{c}, v) - p(\tilde{c}, v)\mathrm{d}G(\tilde{c}) = U_b(0) + \int_0^v \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}G(\tilde{c})\mathrm{d}\tilde{v} \tag{9}$$

and the seller's expected utilities $U_s(c)$ decrease in $c \in [(k-1)/\delta, k/\delta)$ for all $k \in \mathbb{N}^\delta$ and equal

$$U_s(c) = \int_0^1 p(c, \tilde{v}) - ct(c, \tilde{v})\mathrm{d}F(\tilde{v}) = U_s^-(\frac{k}{\delta}) + \int_c^{k/\delta} \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} \tag{10}$$

for all $c \in [(k-1)/\delta, k/\delta)$ and $k \in \mathbb{N}^\delta$. Notice that $\mathcal{T}_s(c) = \int_0^1 t(c, \tilde{v})\mathrm{d}F(\tilde{v})$ and $\mathcal{T}_b(v) = \int_0^1 t(\tilde{c}, v)\mathrm{d}G(\tilde{c})$ for the special case of the mechanism in Theorem 2, that satisfies these conditions. Denote $\Xi = \sum_{k=1}^{\delta} U_s^-\left(\frac{k}{\delta}\right)\left(G\left(\frac{k}{\delta}\right) - G\left(\frac{k-1}{\delta}\right)\right)$. Hence, the seller's ex-ante expected utilities are

$$\begin{aligned}
\int_0^1 U_s(c')\mathrm{d}G(c') &= \sum_{k=1}^{\delta} \int_{(k-1)/\delta}^{k/\delta} U_s(c')\mathrm{d}G(c') \qquad (11)\\
&\overset{\text{Eq. (10)}}{=} \sum_{k=1}^{\delta} \int_{(k-1)/\delta}^{k/\delta} \left(U_s^-\left(\frac{k}{\delta}\right) + \int_{c'}^{k/\delta} \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} \right) \mathrm{d}G(c') \\
&= \sum_{k=1}^{\delta} \left[\int_{(k-1)/\delta}^{k/\delta} \int_{c'}^{k/\delta} \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c}\mathrm{d}G(c') + U_s^-\left(\frac{k}{\delta}\right)\left(G\left(\frac{k}{\delta}\right) - G\left(\frac{k-1}{\delta}\right)\right) \right] \\
&\overset{\text{integration by parts}}{=} \sum_{k=1}^{\delta} \left[\int_{(k-1)/\delta}^{k/\delta} G(\tilde{c}) \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} - G\left(\frac{k-1}{\delta}\right) \int_{(k-1)/\delta}^{k/\delta} \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} \right] + \Xi \\
&= \int_0^1 \Big(G(\tilde{c}) - G\left(Z(\tilde{c})\right) \Big) \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} + \Xi
\end{aligned}$$

Next, plug Eq. (11) into the total expected surplus.

$$\int_0^1 \int_0^1 (v' - c')\, t(c', v')\mathrm{d}G(c')\mathrm{d}F(v') = \int_0^1 U_b(v')\mathrm{d}F(v') + \int_0^1 U_s(c')\mathrm{d}G(c') \tag{12}$$

$$= U_b(0) + \int_0^1 \int_0^{v'} \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}G(\tilde{c})\mathrm{d}\tilde{v}\mathrm{d}F(v') + \int_0^1 U_s(c')\mathrm{d}G(c')$$

$$\overset{\text{int. by parts \& Eq.(11)}}{=} U_b(0) + \int_0^1 (1 - F(\tilde{v})) \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}G(\tilde{c})\mathrm{d}\tilde{v} + \int_0^1 \Big(G(\tilde{c}) - G\left(Z(\tilde{c})\right)\Big) \int_0^1 t(\tilde{c}, \tilde{v})\mathrm{d}F(\tilde{v})\mathrm{d}\tilde{c} + \Xi$$

$$= U_b(0) + \int_0^1 \int_0^1 t(\tilde{c}, \tilde{v}) \left(\Big(G(\tilde{c}) - G(Z(\tilde{c}))\Big) f(\tilde{v}) + (1 - F(\tilde{v}))g(\tilde{c}) \right) \mathrm{d}\tilde{c}\mathrm{d}\tilde{v} + \Xi$$

Combining the first and last line of Eq. (12) yields

$$\int_0^1 \int_0^1 \left(\tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})} - \tilde{c} - \frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})} \right) t(\tilde{c}, \tilde{v})\mathrm{d}G(\tilde{c})\mathrm{d}F(\tilde{v}) = U_b(0) + \Xi \tag{13}$$

The definition of α ensures that the left-hand side of this equation equals zero for the mechanism in Theorem 2. Therefore,

$$\Xi = \sum_{k=1}^{\delta} U_s^- \left(\frac{k}{\delta}\right) \left(G\left(\frac{k}{\delta}\right) - G\left(\frac{k-1}{\delta}\right) \right) = 0$$

in this mechanism because $U_b(0) = 0$. According to Eq. (7),

$$U_s^- \left(\frac{k}{\delta}\right) = \int_0^1 \left(\mathcal{T}_b(v')v' - \int_0^{v'} \mathcal{T}_b(\tilde{v})\mathrm{d}\tilde{v} \right) \mathrm{d}F(v') - \int_0^1 \left(\mathcal{T}_s(c')c' + \int_{c'}^{Z(c')+\frac{1}{\delta}} \mathcal{T}_s(\tilde{c})\mathrm{d}\tilde{c} \right) \mathrm{d}G(c')$$

is independent of k for all $k \in \mathbb{N}^\delta$. Hence, $U_s^-(k/\delta) = U_s^-(k'/\delta)$ for all $k, k' \in \mathbb{N}^\delta$ and $U_s^-(k/\delta) = 0$ for all $k \in \mathbb{N}^\delta$. Consequently, the mechanism in Theorem 2 is individually rational for the seller.

Optimality

The seller's costs c are her private information. For the moment, assume that the designer learns $Z(c)$. In this case, the designer cannot infer anything about the seller's costs c and the necessary inputs even if the designer were to know the buyer's sample. Therefore, the narratives do not matter and this setting is equivalent to the classical setting considered by Myerson and Satterthwaite (1983). Consider a mechanism with probabilities of trade $t(c, v)$ and price $p(c, v)$. Standard arguments show that any incentive compatible mechanism has to satisfy conditions (9) and (10). Therefore, Eq. (13) is valid in such a mechanism.

Individual Rationality is satisfied if $U_b(0) \geq 0$ and $U_s^-\left(\frac{k}{\delta}\right) \geq 0$ for all $k \in \mathbb{N}^\delta$. Hence, any incentive compatible and individual rational mechanism has to ensure that the left-hand side of Eq. (13) is nonnegative. Therefore, optimal mechanisms are equivalent to probabilities of trade $t(c, v)$ that maximize

$$\int_0^1 \int_0^1 (v' - c')t(c', v')\mathrm{d}G(c')\mathrm{d}F(v')$$

subject to this constraint. Using κ as the Lagrange multiplier, the Lagrangian equals

$$(1+\kappa)\int_0^1 \int_0^1 \left(\tilde{v} - \frac{\kappa}{1+\kappa}\frac{1-F(\tilde{v})}{f(\tilde{v})} - \tilde{c} - \frac{\kappa}{1+\kappa}\frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})}\right) t(\tilde{c}, \tilde{v})\mathrm{d}G(\tilde{c})\mathrm{d}F(\tilde{v}).$$

This Lagrangian is maximized by $t(\tilde{c}, \tilde{v}) = 1$ if $\tilde{v} - \frac{\kappa}{1+\kappa}\frac{1-F(\tilde{v})}{f(\tilde{v})} \geq \tilde{c} + \frac{\kappa}{1+\kappa}\frac{G(\tilde{c})-G(Z(\tilde{c}))}{g(\tilde{c})}$ and $t = 0$ otherwise. The mechanism in Theorem 2 implements such probabilities of trade and is incentive compatible as well as individually rational as shown above. Furthermore, the definition of α ensures that the left-hand side of Eq. (13) is zero and, hence, the constraint is satisfied with equality. Finally, the mechanism guarantees that $U_b(0) = 0$ and $U_s^-(k/\delta) = 0$ for all $k \in \mathbb{N}^\delta$. Consequently, the mechanism in Theorem 2 is optimal.

Existence of $\boldsymbol{\alpha}$

Define

$$I(\tilde{\alpha}, k) = \int_0^1 \int_{(k-1)/\delta}^{k/\delta} \left(\tilde{v} - \frac{1-F(\tilde{v})}{f(\tilde{v})} - \tilde{c} - \frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})}\right) Y(\tilde{c}, \tilde{v}, \tilde{\alpha})\mathrm{d}G(\tilde{c})\mathrm{d}F(\tilde{v})$$

for all $k \in \mathbb{N}^\delta$. Obviously, $I(1, k) \geq 0$ for all k because $Y(\tilde{c}, \tilde{v}, 1) = 0$ if the term in brackets is negative. Thus, $\sum_{k=1}^{\delta} I(1, k) \geq 0$. Next, $\sum_{k=1}^{\delta} I(0, k) < 0$ because by the steps above $\sum_{k=1}^{\delta} I(0, k) \geq 0$ would imply the existence of an ex-post efficient mechanism contradicting Myerson and Satterthwaite (1983, Corollary 1). Finally, it remains to prove continuity of $I(\tilde{\alpha}, k)$. Notice that $c + \tilde{\alpha}\frac{G(c)-G(Z(c))}{g(c)}$ strictly increases in $c \in [(k-1)/\delta, k/\delta)$ for all $\tilde{\alpha} \in [0, 1)$. Hence, $c + \tilde{\alpha}\frac{G(c)-G(Z(c))}{g(c)} = v - \tilde{\alpha}\frac{1-F(v)}{f(v)}$ has at most one solution in c for given v and $\tilde{\alpha} \in [0, 1)$. This solution is continuous in v and $\tilde{\alpha}$. Therefore, there is a continuous function $\zeta^k(v, \tilde{\alpha})$: $[0, 1) \times [0, 1) \mapsto [(k-1)/\delta, k/\delta]$ so that

$$I(\tilde{\alpha}, k) = \int_0^1 \int_{(k-1)/\delta}^{\zeta^k(\tilde{v}, \tilde{\alpha})} \tilde{v} - \frac{1-F(\tilde{v})}{f(\tilde{v})} - \tilde{c} - \frac{G(\tilde{c}) - G(Z(\tilde{c}))}{g(\tilde{c})}\mathrm{d}G(\tilde{c})\mathrm{d}F(\tilde{v}).$$

Thus, $\sum_{k=1}^{\delta} I(\tilde{\alpha}, k)$ is continuous in $\tilde{\alpha}$. Consequently, Bolzano's theorem ensures that there is an $\alpha \in (0, 1]$ such that $\sum_{k=1}^{\delta} I(\alpha, k) = 0$. □

B Appendix (Symmetric Model)

Return to the setting of Section 4. To achieve symmetry, assume that the buyer appreciates some features $f \in [0,\nu)$ of the product. Denote $v_f = 1$ if feature f is appreciated and $v_f = 0$ if feature f is not appreciated. Each feature $f \in [0,\nu)$ is appreciated with probability v/ν and yields the same utility. The features are essentially pairwise independent. The buyer knows his preferences and, hence, which features of the product he appreciates. His valuation v equals $\int_0^\nu v_f \mathrm{d}\lambda(f)$ almost surely with an extension λ of the Lebesgue measure on $[0,\max\{\gamma,\nu\}]$. The seller only knows the distribution of preferences. By analyzing social media data or cookies in the buyer's browser, the seller learns for one feature f^* whether it is appreciated by the buyer. That feature f^* is drawn uniformly. In contrast to Section 4, I also assume that the buyer's sample i^* is drawn uniformly from all possible inputs $[0,\gamma)$.

Equivalently to Theorem 1, Theorem 3 provides an ex-post efficient mechanisms with both seller and buyer telling narratives. The mechanisms can implement any distribution of the surplus between buyer and seller in contrast to the asymmetric settings above where all the surplus goes to one side. For this purpose, define $p^*(M_b, M_s) = \chi\lambda(M_b) + (1-\chi)\lambda(M_s)$ with the seller's bargaining power $\chi \in [0,1]$.

Theorem 3. *The following mechanism implements any distribution of the surplus between buyer and seller. The mechanism is individually rational, ex-post efficient and budget-balanced:*
The buyer's narrative describes the features he appreciates $M_b = \{f \in [0,\nu) | v_f = 1\}$. He also reports his sample i^* of the seller's inputs: $m_b = i^*$ if $c_{i^*} = 0$ and $m_b = \gamma$ otherwise. Correspondingly, $\mathcal{M}_b = (m_b, M_b) \in [0,\gamma] \times \{X \subseteq [0,\nu) |\ X \text{ is } \lambda\text{-measurable}\}$. The seller's narrative describes the necessary inputs $M_s = \{i \in [0,\gamma) | c_i = 1\}$. She also reports her sample f^* of the buyer's preferences: $m_s = f^*$ if $v_{f^*} = 1$ and $m_s = \nu$ otherwise. Correspondingly, $\mathcal{M}_s = (m_s, M_s) \in [0,\nu] \times \{X \subseteq [0,\gamma) |\ X \text{ is } \lambda\text{-measurable}\}$. Payments and probabilities of trade are

$$\begin{cases} t = 0, & p = 0, \quad \pi = 0 \quad \text{if } \lambda(M_b) < \lambda(M_s) \\ t = 1, & p = p^*(M_b, M_s), \quad \pi = 0 \quad \text{if } \lambda(M_b) \geq \lambda(M_s), m_b \notin M_s \text{ and } m_s \in (M_b \cup \{\nu\}) \\ t = 1, & p = p^*(M_b, M_s) + \nu\chi, \quad \pi = \nu\chi \quad \text{if } \lambda(M_b) \geq \lambda(M_s), m_b \notin M_s \text{ and } m_s \notin (M_b \cup \{\nu\}) \\ t = 1, & p = p^*(M_b, M_s), \quad \pi = \gamma(1-\chi) \quad \text{if } \lambda(M_b) \geq \lambda(M_s), m_b \in M_s \text{ and } m_s \in (M_b \cup \{\nu\}) \\ t = 0, & p = \nu\chi, \quad \pi = \nu\chi + \gamma(1-\chi) \quad \text{otherwise.} \end{cases}$$

The equilibrium strategies ensure that $m_b \notin M_s$ and $m_s \in (M_b \cup \{0\})$. Hence, trade occurs iff $\lambda(M_b) \geq \lambda(M_s)$ and the mechanism is ex-post efficient in equilibrium. It is easy to see that the mechanism is budget-balanced.

To study the seller's incentives, suppose the buyer follows his strategy. Denote the conditional expectation of the buyer's valuation by $v^*(c) = \mathbb{E}(v|v \geq c)$. In equilibrium, the seller expects payoffs

$$\text{Prob(trade)}(\mathbb{E}(p|\text{trade}) - c) = (1 - F(c))(v^*(c) - c)\chi = \chi \int_c^\nu \tilde{v} - c \mathrm{d}F(\tilde{v})$$

which are nonnegative making the mechanism individually rational for the seller. Turning to incentive compatibility, consider deviations by the seller. A deviation with $\lambda(M_s) \leq c$ makes the seller worse off by reducing the price. Her expected payoffs are below

$$\begin{aligned}&(1 - F(\lambda(M_s)))(\chi v^*(\lambda(M_s)) + (1 - \chi)\lambda(M_s) - c) < (1 - F(\lambda(M_s)))(v^*(\lambda(M_s)) - c)\chi \\ &= \chi \int_{\lambda(M_s)}^\nu \tilde{v} - c \mathrm{d}F(\tilde{v})\end{aligned}$$

which is below her equilibrium payoffs because $\tilde{v} - c < 0$ for all $\tilde{v} \in [\lambda(M_s), c)$. A deviation with $\lambda(M_s) > c$ has three effects: the price increases, probabilities of trade decrease, and additional payments $\pi = \gamma(1 - \chi)$ could be triggered. The first effect benefits the seller while the second and third effect hurt her. In particular, payoffs of such a deviation are below

$$\begin{aligned}&(1 - F(\lambda(M_s))) \left[\chi v^*(\lambda(M_s)) + (1 - \chi)\lambda(M_s) - c - \frac{\lambda(M_s) - c}{\gamma}\gamma(1 - \chi)\right] \\ &= (1 - F(\lambda(M_s)))(v^*(\lambda(M_s)) - c)\chi = \chi \int_{\lambda(M_s)}^\nu \tilde{v} - c \mathrm{d}F(\tilde{v})\end{aligned}$$

As the seller's narrative must contain at least a mass of $\lambda(M_s) - c$ unnecessary inputs, the probability that the buyer observed one of these inputs is at least $(\lambda(M_s) - c)/\gamma$. Hence, $\text{Prob}(m_b \in M_s) \geq (\lambda(M_s) - c)/\gamma$ and such a deviation is unprofitable if

$$\chi \int_c^\nu \tilde{v} - c \mathrm{d}F(\tilde{v}) \geq \chi \int_{\lambda(M_s)}^\nu \tilde{v} - c \mathrm{d}F(\tilde{v}) \Leftarrow \int_c^{\lambda(M_s)} \tilde{v} - c \mathrm{d}F(\tilde{v}) > 0$$

The last inequality is valid because $\tilde{v} - c > 0$ for all $\tilde{v} \in (c, \lambda(M_s)]$. A deviation in m_s has no effect for the seller. Therefore, any (joint) deviation in M_s and m_s is unprofitable. Consequently, her strategy is optimal for the seller.

Turning to the buyer, denote the conditional expectation of the seller's costs by $c^*(v) = \mathbb{E}(c|v \geq c)$ and suppose the seller follows her strategy. In equilibrium, the buyer expects payoffs

$$\text{Prob(trade)}(v - \mathbb{E}(p|\text{trade})) = G(v)(v - c^*(v))(1-\chi) = (1-\chi)\int_0^v v - \tilde{c}\mathrm{d}G(\tilde{c})$$

which are nonnegative making the mechanism individually rational for the buyer. Turning to incentive compatibility, consider deviations by the buyer. A deviation with $\lambda(M_b) \geq v$ makes the buyer worse off by increasing the price. His expected payoffs are below

$$\begin{aligned} &G(\lambda(M_b))\left[v - \big(\chi\lambda(M_b) + (1-\chi)c^*(\lambda(M_b))\big)\right] < G(\lambda(M_b))\big(v - c^*(\lambda(M_b))\big)(1-\chi) \\ &= (1-\chi)\int_0^{\lambda(M_b)} v - \tilde{c}\mathrm{d}G(\tilde{c}) \end{aligned}$$

which is below his equilibrium payoffs because $v - \tilde{c} < 0$ for all $\tilde{c} \in (v, \lambda(M_s)]$. A deviation with $\lambda(M_b) < v$ has three effects: the price decreases, the probability of trade decreases, and a price surcharge of $\nu\chi$ could be triggered. The first effect benefits the buyer while the second and third effect hurt him. In particular, payoffs of such a deviation are below

$$\begin{aligned} &G(\lambda(M_b))\left[v - \big(\chi\lambda(M_b) + (1-\chi)c^*(\lambda(M_b))\big) - \frac{v - \lambda(M_b)}{\nu}\nu\chi\right] \\ &= G(\lambda(M_b))\big(v - c^*(\lambda(M_b))\big)(1-\chi) = (1-\chi)\int_0^{\lambda(M_b)} v - \tilde{c}\mathrm{d}G(\tilde{c}). \end{aligned}$$

As the buyer's narrative must miss at least a mass of $v - \lambda(M_b)$ appreciated features, the probability that the seller observed one of these features is at least $(v - \lambda(M_b))/\nu$. Hence, $\text{Prob}(m_s \notin (M_b \cup \{\nu\})) \geq (v - \lambda(M_b))/\nu$ and such a deviation is unprofitable because $v - \tilde{c} > 0$ for all $\tilde{c} \in [\lambda(M_s), v)$. Therefore, any deviation in M_b is unprofitable. A deviation in m_b has no effect for the buyer. Therefore, any (joint) deviation in M_b and m_b is unprofitable. Consequently, his strategy is optimal for the buyer.

To sum up, the mechanism is ex-post efficient, budget-balanced, and individually rational. Choosing χ, the seller's bargaining power, appropriately, the mechanism can implement any distribution of the surplus. □

References

Al-Najjar, N. I., Anderlini, L., and Felli, L. (2006). Undescribable Events. *The Review of Economic Studies*, 73(4):849–868.

Alenius, E., Lind, J., and Strömsten, T. (2015). The role of open book accounting in a supplier network: Creating and managing interdependencies across company boundaries. *Industrial Marketing Management*, 45:195–206.

Ali, N. and Bénabou, R. (2020). Image Versus Information: Changing Societal Norms and Optimal Privacy. *American Economic Journal: Microeconomics*, 12(3):116–164.

Anderlini, L. and Felli, L. (1994). Incomplete Written Contracts: Undescribable States of Nature. *The Quarterly Journal of Economics*, 109(4):1085–1124.

Axelsson, B., Laage-Hellman, J., and Nilsson, U. (2002). Modern management accounting for modern purchasing. *European Journal of Purchasing & Supply Management*, 8(1):53–62.

Ball, I. and Kattwinkel, D. (2022). Probabilistic Verification in Mechanism Design. *Working Paper*.

Ben-Porath, E. and Lipman, B. L. (2012). Implementation with partial provability. *Journal of Economic Theory*, 147(5):1689–1724.

Bénabou, R., Falk, A., and Tirole, J. (2020). Narratives, Imperatives and Moral Reasoning. *NBER Working Paper*, 24798.

Benkert, J.-M. (2022). Bilateral Trade with Loss-Averse Agents. *University of Zurich Working Paper*.

Bull, J. and Watson, J. (2004). Evidence disclosure and verifiability. *Journal of Economic Theory*, 118(1):1–31.

Caragiannis, I., Elkind, E., Szegedy, M., and Yu, L. (2012). Mechanism Design: from Partial to Probabilistic Verification. In *Proceedings of the 13th ACM Conference on Electronic Commerce*, pages 266–283.

Carr, C. and Ng, J. (1995). Total cost control: Nissan and its UK supplier partnerships. *Management Accounting Research*, 6:347–366.

Carroll, G. and Egorov, G. (2019). Strategic Communication With Minimal Verification. *Econometrica*, 87(6):1867–1892.

Clarke, E. H. (1971). Multipart pricing of public goods. *Public Choice*, 11(1):17–33.

Copic, J. and Ponsati, C. (2008). Robust bilateral trade and mediated bargaining. *Journal of the European Economic Association*, 6(2-3):570–580.

Crawford, V. P. and Sobel, J. (1982). Strategic Information Transmission. *Econometrica*, 50(6):1431–1451.

Crémer, J. and McLean, R. P. (1988). Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions. *Econometrica*, 56(6):1247–1257.

De Castro, L. and Yannelis, N. C. (2018). Uncertainty, efficiency and incentive compatibility: Ambiguity solves the conflict between efficiency and incentive compatibility. *Journal of Economic Theory*, 177:678–707.

Deneckere, R. and Severinov, S. (2008). Mechanism design with partial state verifiability. *Games and Economic Behavior*, 64(2):487–513. Special Issue in Honor of Michael B. Maschler.

Dziuda, W. (2011). Strategic Argumentation. *Journal of Economic Theory*, 146(4):1362–1397.

Eliaz, K. and Spiegler, R. (2020). A Model of Competing Narratives. *American Economic Review*, 110(12):3786–3816.

Fenn, P., Lowe, D., and Speck, C. (1997). Conflict and dispute in construction. *Construction Management and Economics*, 15(6):513–518.

Fishman, M. J. and Hagerty, K. M. (1990). The Optimal Amount of Discretion to Allow in Disclosure. *The Quarterly Journal of Economics*, 105(2):427–444.

Garratt, R. and Pycia, M. (2023). Efficient bilateral trade. *Working Paper*.

Giovannoni, F. and Seidmann, D. J. (2007). Secrecy, two-sided bias and the value of evidence. *Games and Economic Behavior*, 59(2):296–315.

Glazer, J. and Rubinstein, A. (2004). On Optimal Rules of Persuasion. *Econometrica*, 72(6):1715–1736.

Glazer, J. and Rubinstein, A. (2006). A Study in the Pragmatics of Persuasion: a game theoretical Approach. *Theoretical Economics*, 1(4):395–410.

Green, J. R. and Laffont, J.-J. (1986). Partially Verifiable Information and Mechanism Design. *The Review of Economic Studies*, 53(3):447–456.

Gresik, T. A. (1991). Efficient bilateral trade with statistically dependent beliefs. *Journal of Economic Theory*, 53(1):199–205.

Groves, T. (1973). Incentives in teams. *Econometrica*, 41(4):617–631.

Hagenbach, J., Koessler, F., and Perez-Richet, E. (2014). Certifiable pre-play commnication: Full disclosure. *Econometrica*, 82(3):1093–1131.

Hart, S., Kremer, I., and Perry, M. (2017). Evidence games: Truth and commitment. *American Economic Review*, 107(3):690–713.

Herweg, F. and Schmidt, K. M. (2020). Procurement with Unforeseen Contingencies. *Management Science*, 66(5):2194–2212.

Holmström, B. (1979). Moral Hazard and Observability. *Bell Journal of Economics*, 10(1):74–91.

Jakobsen, A. M. (2020). A Model of Complex Contracts. *American Economic Review*, 110(5):1243–1273.

Kartik, N. and Tercieux, O. (2012). Implementation with evidence. *Theoretical Economics*, 7(2):323–355.

Kim, S. K. (1995). Efficiency of an Information System in an Agency Model. *Econometrica*, 63(1):89–102.

Kojima, F. and Yamashita, T. (2017). Double auction with interdependent values: Incentives and efficiency. *Theoretical Economics*, 12(3):1393–1438.

Lang, M. (2019). Communicating Subjective Evaluations. *Journal of Economic Theory*, 179:163–199.

Larsen, B. J., Lu, C. H., and Zhang, A. L. (2022). Intermediaries in bargaining: Evidence from business-to-business used-car inventory negotiations. *Working Paper*.

Lipman, B. L. and Seppi, D. J. (1995). Robust Inference in Communication Games with Partial Provability. *Journal of Economic Theory*, 66(2):370–405.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press.

Mathis, J. (2008). Full revelation of information in Sender-Receiver games of persuasion. *Journal of Economic Theory*, 143(1):571–584.

McAfee, R. P. and Reny, P. J. (1992). Correlated Information and Mechanism Design. *Econometrica*, 60(2):395–421.

Milgrom, P. R. (1981). Good News and Bad News: Representation Theorems and Applications. *Bell Journal of Economics*, 12(2):380–391.

Mueller, F., Stahl, K. O., and Wachtler, F. (2016). Upstream Relationships in the Automotive Industry: A Contractual Perspective. *Working Paper*, Available at www.vwl.uni-mannheim.de/media/Lehrstuehle/vwl/Stahl/MueStaWa_2016_03_19.pdf.

Myerson, R. B. (1981). Optimal Auction Design. *Mathematics of Operations Research*, 6:58–73.

Myerson, R. B. and Satterthwaite, M. A. (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29(2):265–281.

Okuno-Fujiwara, M., Postlewaite, A., and Suzumura, K. (1990). Strategic Information Revelation. *The Review of Economic Studies*, 57(1):25–47.

Romano, P. and Formentini, M. (2012). Designing and implementing open book accounting in buyer-supplier dyads: A framework for supplier selection and motivation. *International Journal of Production Economics*, 137(1):68–83.

Schweighofer-Kodritsch, S. and Strausz, R. (2022). Principled mechanism design with evidence. *Working Paper*.

Segal, I. and Whinston, M. D. (2016). Property Rights and the Efficiency of Bargaining. *Journal of the European Economic Association*, 14(6):1287–1328.

Seidmann, D. J. and Winter, E. (1997). Strategic Information Transmission with Verifiable Messages. *Econometrica*, 65(1):163–169.

Sher, I. (2014). Persuasion and dynamic communication. *Theoretical Economics*, 9(1):99–136.

Shiller, R. J. (2017). Narrative Economics. *American Economic Review*, 107(4):967–1004.

Shin, H. S. (1994). The Burden of Proof in a Game of Persuasion. *Journal of Economic Theory*, 64(1):253–264.

Spiegler, R. (2020). Can Agents with Causal Misperceptions be Systematically Fooled? *Journal of the European Economic Association*, 18(2):583–617.

Sun, Y. (2006). The exact law of large numbers via Fubini extension and characterization of insurable risks. *Journal of Economic Theory*, 126(1):31–69.

Sun, Y. and Zhang, Y. (2009). Individual risk and Lebesgue extension without aggregate uncertainty. *Journal of Economic Theory*, 144(1):432–443.

Vickrey, W. (1961). Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of Finance*, 16(1):8–37.

Wolitzky, A. (2016). Mechanism design with maxmin agents: Theory and an application to bilateral trade. *Theoretical Economics*, 11(3):971–1004.