

**Prevention and Mitigation of
Epidemics: Biodiversity
Conservation and
Confinement Policies**

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Prevention and Mitigation of Epidemics: Biodiversity Conservation and Confinement Policies

Abstract

This paper presents a first model integrating the relation between biodiversity loss and zoonose pandemic risks in a general equilibrium dynamic economic set-up. The occurrence of pandemics is modeled as Poissonian leaps in economic variables. The planner can intervene in the economic and epidemiological dynamics in two ways: first (prevention), by deciding to preserve a greater quantity of biodiversity, thus decreasing the probability of a pandemic occurring, and second (mitigation), by reducing the death toll through a confinement policy. The class of social welfare functional considered has, as polar cases, a total utilitarian and an average utilitarian specifications. It implicitly considers, at the same time, the effects of policies on mortality and on the productive capacity of the country. Thanks to the Epstein-Zin specification of preferences, we disentangle risk aversion and fluctuation aversion. The model is explicitly solved and the optimal policy completely described. The qualitative dependence of the optimal policy on natural, productivity and preference parameters is discussed. In particular the optimal lockdown is shown to be more severe in societies valuing more human life, and the optimal biodiversity conservation is shown to be more relevant for more “forward looking” societies, with a small discount rate and a high degree of altruism towards individuals of future generations. We also show that societies accepting a large welfare loss to mitigate the pandemics are also societies doing a lot of prevention. After calibrating the model with COVID-19 pandemic data we compare the mitigation efforts predicted by the model with those of the recent literature and we study the optimal prevention-mitigation policy mix.

JEL-Codes: Q560, Q570, Q580, O130, C610.

Keywords: biodiversity, COVID-19, prevention, mitigation, epidemics, Poisson processes, recursive preferences.

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1 Introduction

The hopes of the post-war period that infectious diseases were behind us (Fisher, 1995) have been dashed: the number of emerging diseases has continued to rise since the 1950s (Smith *et al.*, 2014). Despite this increasing frequency of disease, the number of infected people has been reduced thanks to control and treatment improvement (Smith *et al.*, 2014). However, the fear of a pandemic has remained vivid, as shown by the Clade X exercise¹ hosted at Johns Hopkins Center for Health Security in 2018. Indeed, at the same time, the number of outbreaks leading to epidemics and even pandemics has accelerated sharply over the last twenty years.

As 60% of these emerging diseases are linked to zoonoses (Jones *et al.*, 2008), it is natural to make the link between the loss of biodiversity and the emergence of epidemics (Morand *et al.*, 2014, Morand, 2018), especially since the current COVID-19 pandemic is linked to the transmission of a pathogen from humans to animals.

The link between biodiversity and epidemics is complex because both a dilution effect (decreasing relationship between biodiversity and epidemic risk, Civitello *et al.* 2015) and an amplification effect (increasing relationship between biodiversity and epidemic risk, Wood *et al.* 2017) are at work simultaneously (Rohr *et al.*, 2019). The predominance of one effect over the other depends on the spatial scale at which the phenomenon is studied (Johnson *et al.*, 2015). On a national or global scale, the dilution effect dominates (Halliday *et al.*, 2020, Morand *et al.*, 2014) and conserving biodiversity appears as a prevention against emerging diseases.

There are at least two reasons for the dilution effect (Keesing *et al.*, 2010). Firstly, the decline in biodiversity leads to an increase in the prevalence and transmission rates at the local level and to a selection effect of the most harmful pathogenic strains. Second, habitat destruction brings species together and brings them closer to humans (Wolfe *et al.*, 2005)². The promiscuity between several species, in the wild, in captivity or in breeding, increases the risk of transmission and mutation of pathogens, and makes transmission to humans more likely (LoGiudice *et al.*, 2003).

The economic cost of these emerging epidemics is very high (Sands *et al.*, 2016a), leading to a disruption at each emergence (Sands *et al.*, 2016b). The COVID-19 epidemic and the containment measures that led to the containment of more than half of the planet resulted in GDP losses of more than 10% in several countries (see OECD, 2020). The Ebola epidemic led to a 10% loss of GDP in Sierra Leone and Guinea (World Bank, 2014).

¹https://www.centerforhealthsecurity.org/our-work/events/2018_clade_x_exercise/

²The case of bats is emblematic. Bats are reservoirs of pathogens. The reduction of their habitats due to deforestation forces them to move closer to fruit production, livestock farming and other species.

In this work we propose a first stylized model to study the impact of biodiversity conservation on the economic dynamics via the “pandemic frequency” channel. We model the uncertainty of the survenue of a recurrent epidemic outbreak using a Poisson stochastic process, which implies irreversibility due to over-mortality and productivity loss.

In the model a planner can decide about the size of the land devoted to biodiversity conservation and, consequently, the size of the land converted to economic activities. The former impacts the probability (and then the frequency) of having demo-economic pandemic shocks, the latter has an immediate effect on production flow. The planner targets a social welfare functional in a family which includes (varying an altruism degree in the spirit of Palivos and Yip, 1993), as polar cases, total and average utilitarianism.

In addition to preventive intervention determined by the preservation of biodiversity, the planner also has the possibility to decide on lockdown mitigation policies. She can indeed decide that, in the event of a pandemic, a (partial) blockage of economic activity will be put in place in order to reduce population mortality, at the cost of a reduction in productive activities.

Given the simplicity of the model we can solve it explicitly. We characterize the optimal mitigation policy and the optimal allocation of land to biodiversity conservation and study their behaviors in terms of the model’s parameters. We show that the optimal lockdown is more severe in societies valuing more human life, and that the optimal biodiversity conservation is greater in more “forward looking” societies, with a small discount rate and a high degree of altruism towards individuals of future generations. We also show that societies accepting a large welfare loss to mitigate the pandemics are also societies doing a lot of prevention, not to have to incur the loss too often and so all the more since risk aversion or the risk of pandemics absent any biodiversity are higher.

To calibrate the model (Section 4) we use data from Gollier (2020) on the consequences in terms of mortality and economic activity of the *laissez-faire*, a “flatten the curve” mitigation strategy and a “crush the curve”, or suppression, strategy. We exhibit the terms of the trade-off between the loss of lives and the loss of GDP for the whole set of mitigation strategies, from *laissez-faire* to suppression. We compute the optimal mitigation policy as a function of the relative value the planner assigns to life over the economy, and the optimal prevention strategy, depending on the former and on the risk parameters.

The idea of assessing the economic impact of disasters that arise from “environmental” causes is the source of a well-stocked literature, both in the empirical works (see for instance Noy, 2009, and the contained references) and from the theoretical point of view (see for instance Akao and Sakamoto, 2018 or Bakkensen and Barrage, 2016). The general idea of these contributions is to try to understand the impact of environmental disasters on growth and development. Particularly inspiring for our research are the papers by Bretschger and

Vinogradova (2016, 2019) and Douenne (2020). The latter in particular studies in detail the possibility of dealing with disaster of endogenous probability³ and then the idea of disaster prevention. In the present paper the focus is different since, for the first time we link biodiversity conservation, the risk of pandemics, population and economic dynamics and mitigation policies.

The paper is organized as follows. Section 2 introduces model’s assumptions and formulation. In Section 3 we describe the explicit solution of the model. Section 4 contains numerical simulations while Section 5 concludes. All the proofs are reported in the Appendix A.

2 The model

We consider an economy where the planner makes decisions about how to manage biodiversity and how to deal with a pandemic if it happens.

2.1 Biodiversity and the economy

The planner decides at the beginning of the planning horizon how to allocate the land (whose total size is normalized to 1). A part f is devoted to biodiversity conservation while $1 - f$ is used for various economic human activities, such as agriculture, industry, human settlement and infrastructure.

The choice of f influences the economy in two ways. Firstly in a direct way: the land devoted to human activities is used as an input in the production process. Indeed we suppose that the production is described by an aggregate production function of the form

$$Y(t) = F(1 - f(t), A(t)N(t))$$

where $N(t)$ is the size of the population and $A(t)$ is labor productivity at time t . We abstract in particular from the use of capital (but the model can be extended in various ways to a more general production structure).

Consistently with the choice of not modeling factor accumulation and then investment, we suppose that all production is consumed:

$$C(t) = Y(t).$$

³The idea of preserving biodiversity to reduce the probability of future negative outcomes is also used in another bunch of works in the literature, see for instance Baumgärtner (2007), Baumgärtner and Quaas (2010), Baumgärtner and Strunz (2014), Augeraud-Véron *et al.* (2019, 2020) which highlight the *insurance value* of biodiversity.

The second way the choice of f influences the economy is through its effects on zoonoses' outbreaks. We suppose that f influences the probability $h(t)$ that a zoonose appears and becomes a pandemic: the bigger f , the more biodiversity and the smaller the probability of epidemic outbreaks. In this sense, biodiversity conservation is a risk-reducing policy.

2.2 Pandemics and policy response

When a pandemic hits the economy, it produces an instantaneous decrease of the population which size depends on the policy response of the planner, that is the mitigation policy she adopts. Let e represent the intensity of social interactions. In normal times, this intensity is \bar{e} . The planner, by imposing a more or less severe and lengthy lockdown, is able to make the intensity of interactions decrease, eventually down to 0, which corresponds to a total lockdown. The size of the instantaneous decrease of the population at the date τ when the pandemic hits the economy depends on the severity of the lockdown as follows:

$$\tilde{N}(\tau) = k(e)N(\tau)$$

where $k(e) < 1$ is a decreasing function of e : the more severe the lockdown the fewer deaths in the population. We suppose more precisely that $k'(e) < 0$, $k(\bar{e}) = \underline{k} \in (0, 1)$, $k(0) = \bar{k} \in (\underline{k}, 1)$, and $k'' \leq 0$: the marginal effectiveness of the mitigation policy is weakly decreasing (see Figure 1). $1 - \underline{k}$ is the instantaneous deaths rate at the date the pandemic hits, absent any policy response. It is the product of the force of infection, the contagion rate, the number of people each person meets on average. We summarize in this simple statistics the outcome of complex dynamics like the ones described by compartmental models (SIR, SIS, SEIR..., see Hethcote, 1994). In fact, here, in a context where we look at a complete time series of potential pandemic explosions, we work on a longer time scale. This fact also justifies the choice of modeling pandemic-related shocks as punctual.

The drawbacks of the policy response to the pandemics is of course that it reduces labor productivity. “Non-essential” economic activities, that is activities outside the health, food and energy sectors, are stopped when teleworking is not possible. In other sectors the generalization the teleworking for all tasks can affect the productivity of workers, especially if they have a poor working environment at home, or when they must simultaneously take care of small children at home. The size of the instantaneous negative shock on productivity when the pandemics hits is modeled as follows;

$$\tilde{A}(\tau) = \kappa(e)A(\tau)$$

with $\kappa(e) < 1$ an increasing function of e : the more severe the lockdown the larger the productivity loss. We suppose more precisely that $\kappa'(e) > 0$, $\kappa(0) = \underline{\kappa} \in (0, 1)$, $\kappa(\bar{e}) = \bar{\kappa} \in (\underline{\kappa}, 1)$ and $\kappa''(e) \leq 0$ (see Figure 1).

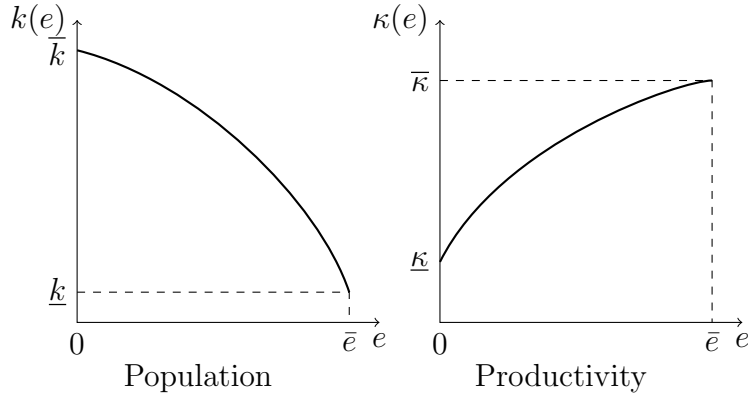


Figure 1: Instantaneous responses to the pandemics as a function of the mitigation policy

2.3 Dynamics

We suppose that a deterministic exponential process of growth of population and productivity takes place when the economy is not hit by a pandemic. Therefore, given the structure of shocks described above, the dynamics of population and labor productivity is described by the following couple of stochastic differential equations:

$$\begin{aligned} \frac{dN(t)}{N(t)} &= n dt - (1 - k(e)) dq(t) \\ \frac{dA(t)}{A(t)} &= g dt - (1 - \kappa(e)) dq(t) \end{aligned}$$

where n and g are the deterministic growth trends of population and productivity, and q is a Poisson process with $\mathbb{E}[dq] = h(f)dt$. The probability of a pandemic outbreak is all the lower since biodiversity is important: $h'(f) < 0$.

2.4 Social welfare

Choosing a social utility functional is delicate since population ethics is a difficult matter (see for instance Arrhenius *et al.*, 2017). We consider here a class of social welfare functions parameterized by $\lambda \in [0, 1]$ by supposing that the planner's instantaneous utility at time t is of the form

$$N(t)^\lambda u\left(\frac{C(t)}{N(t)}\right)$$

with $C(t)$ total consumption and then $\frac{C(t)}{N(t)}$ per-capita consumption.⁴ The value of λ measures the degree of altruism towards individuals of future cohorts (see, *e.g.* Boucekkine *et*

⁴Indeed $\frac{C(t)}{N(t)}$ is actually the per capita *average* consumption but we abstract here from all distributional consideration and we suppose the production to be uniformly divided among population.

al., 2014) and it can be justified, as done by Barro and Becker (1989) in a model of endogenous fertility choice by the “impure altruism” of parents. The two polar cases $\lambda = 0$ and $\lambda = 1$ correspond respectively to standard average utilitarianism and total utilitarianism.

The planner maximizes a non-separable intertemporal utility *à la* Epstein-Zin-Weil, supposing that agents’ preferences are characterized by a constant relative risk aversion $\theta > 0$ (and $\theta \neq 1$), an intertemporal elasticity of substitution $1/\phi > 0$ (ϕ represents aversion to fluctuations) and a discount rate $\rho > 0$.

We can dig a little deeper in the utility functional to understand which choices of parameters are meaningful. If we look at the analogous of the informal representation of preferences given for instance by Svensson (1989) (see also Augeraud-Vron *et al.*, 2019) in our case, we have⁵, for given controls $e(t)$ and $f(t)$,

$$U(t) = \frac{1}{1-\phi} N(t)^\lambda \left(\frac{C(t)}{N(t)} \right)^{1-\phi} + e^{-\rho dt} \left(\mathbb{E}_t \left[U(t+dt)^{\frac{1-\theta}{1-\phi}} \right] \right)^{\frac{1-\phi}{1-\theta}}.$$

Notice that, since in the model the utility of death people is implicitly taken equal to 0, the utility function of living people has to be positive. This requires $\phi \in (0, 1)$, which we impose.

3 The optimal policy

In order to explicitly solve the problem we specify the described setting by considering a linear probability of pandemics:

$$h(f) = \varepsilon(1-f), \quad \varepsilon \in (0, 1],$$

and a linear production function:

$$Y(t) = (1-f(t))A(t)N(t).$$

Thanks to the latter the per-capita consumption specifies to

$$\frac{C(t)}{N(t)} = (1-f(t))A(t).$$

⁵As clarified by Epaulard and Pommeret (2003) (see in particular equation (2.1) and Footnote 2) this expression is equivalent to the one originally proposed by Svensson via a transformation *à la* Duffie and Epstein (1992).

Then the instantaneous utility function becomes:

$$\frac{1}{1-\phi} N(t)^\lambda \left(\frac{C(t)}{N(t)} \right)^{1-\phi} = \frac{1}{1-\phi} N(t)^\lambda (1-f(t))A(t)^{1-\phi}.$$

Observe that, abstracting from dynamics effects, at the level of the instantaneous utility function, the per-capita consumption does not decrease when N increases so no “quality/quantity” trade-off takes place. This is due to the linear specification of the production function.

All in all the HJB equation of the problem reads:

$$\begin{aligned} \rho \frac{1-\theta}{1-\phi} V(A, N) = \max_{f, e} \left[N^\lambda \frac{[(1-f)A]^{1-\phi}}{1-\phi} \frac{1}{((1-\theta)V(A, N))^{\frac{1-\phi}{1-\theta}-1}} \right. \\ \left. + V_A g A + V_N n N + \varepsilon(1-f)(V(\kappa(e)A, k(e)N) - V(A, N)) \right] \quad (1) \end{aligned}$$

Theorem 3.1 *Suppose that the function of the variable $e \in [0, \bar{e}]$*

$$k(e)^{\frac{\lambda}{1-\phi}} \kappa(e) \quad (2)$$

has a unique maximum point of $e^ \in [0, \bar{e}]$. Suppose that the following interior condition*

$$\rho > (1-\phi)g + \lambda n \quad (3)$$

is verified together with the following transversality condition:

$$(1-\theta+\phi)\rho > \frac{1-\theta}{1-\phi} ((1-\phi)g + \lambda n). \quad (4)$$

Then optimal policy $(f^(\cdot), e^*(\cdot))$ is deterministic and constant and it is given by $e^*(\cdot) \equiv e^*$ and $f^*(\cdot) \equiv f^*$ defined by⁶:*

$$f^* := 1 - \min \left[1, \frac{\rho - (1-\phi)g - \lambda n}{\phi \varepsilon} \frac{1-\theta}{1 - \left(k(e^*)^{\frac{\lambda}{1-\phi}} \kappa(e^*) \right)^{1-\theta}} \right]. \quad (5)$$

⁶Observe that, since $\kappa(e), k(e) \in (0, 1)$ then the term $(1-\theta) \left[1 - \left(k(e^*)^{\frac{\lambda}{1-\phi}} \kappa(e^*) \right)^{1-\theta} \right]^{-1}$ is positive for any choice of $\theta > 0$.

The corresponding social welfare is given by

$$V(A, N) = X \frac{\left(N^{\frac{\lambda}{1-\phi}} A\right)^{1-\theta}}{1-\theta}$$

with

$$X = \left(\frac{\phi^{\frac{\phi}{1-\phi}} (1-\theta)}{(\rho - (1-\phi)g - \lambda n)^{\frac{\phi}{1-\phi}} \varepsilon \left[1 - \left(k(e)^{\frac{\lambda}{1-\phi}} \kappa(e)\right)^{1-\theta}\right]} \right)^{1-\theta} \quad (6)$$

Proof. See Appendix A. ■

Theorem 3.1 first allows us to learn about the optimal mitigation policy. Denote by $z(e)$ the function of e defined in equation (2). With our assumptions on the functions k and κ , $z(e)$ is a positive and concave function of e : $z''(e) \leq 0$.

If $z'(e) < 0$ for all $e \in [0, \bar{e}]$, the maximum of z is attained for $e = 0$. The optimal policy is total lockdown: $e^* = 0$. The condition $z'(e) < 0$ is equivalent to $\frac{\lambda}{1-\phi} \frac{k'(0)}{k(0)} + \frac{\kappa'(0)}{\kappa(0)} < 0$, that is $-\frac{\kappa'(0)/\kappa(0)}{k'(0)/k(0)} < \frac{\lambda}{1-\phi}$. Denote $\bar{\tau} := -\frac{\kappa'(0)/\kappa(0)}{k'(0)/k(0)}$. The case under study is then relevant for $\frac{\lambda}{1-\phi} > \bar{\tau}$, that is for large values of the society's relative preference for life. Symmetrically, if $z'(e) > 0$ for all $e \in [0, \bar{e}]$, the maximum of z is attained for $e = \bar{e}$ and the optimal policy is no lockdown: $e^* = \bar{e}$. This case is relevant for $\frac{\lambda}{1-\phi} < \underline{\tau}$, where $\underline{\tau} := -\frac{\kappa'(\bar{e})/\kappa(\bar{e})}{k'(\bar{e})/k(\bar{e})}$, that is for small values of the relative preference for life. Finally, the optimal mitigation policy is interior for values of the relative preference for life between $\underline{\tau}$ and $\bar{\tau}$. e^* is then given by the first order condition for the maximization of (2):

$$-\frac{\kappa'(e)/\kappa(e)}{k'(e)/k(e)} = \frac{\lambda}{1-\phi}. \quad (7)$$

If the pandemic occurs the planner will react instantaneously by no/partial/total lockdown, according to her relative preference for life over the economy $\frac{\lambda}{1-\phi}$, and the shape of the k and κ loss functions. On the left side of equation (7) we can recognize the opposite of the ratio between the elasticities of κ and k with respect to e . This expression can also be rewritten as $-(1-\phi)\kappa'(e)/\kappa(e) = \lambda k'(e)/k(e)$. Unsurprisingly in this expression the contribution of the elasticity of the negative effect on the population is weighted with λ . Indeed, as already mentioned, the per-capita consumption is not affected by the shock and the negative effect of the population loss only comes through the term N^λ appearing in the utility function. This underlines once more the role of λ as a measure of the value of a life in the planner functional. Similarly the shock on productivity is weighted by the exponent $(1-\phi)$ of the productivity in the instantaneous utility. Risk aversion plays no role here. This is not particularly surprising because the choice of e has some impact on the system

only after (and if) the shock occurs. Conversely the value of θ clearly appears in the choice of f which is the “disaster prevention choice” of the planner. Due to the linearity of the model, the growth rates of the productivities (*i.e.* g and n) do not enter in the choice of e .

Turning to the biodiversity conservation policy, we obtain several results by inspection of equation (5).

First, the share of land optimally devoted to biodiversity conservation is a decreasing function of the discount rate: the more the planner is interested in future outcomes, the more she wants to reduce the risk of pandemic outbreaks and the more biodiversity is needed. Conversely the higher the inherent capacity of the system to regenerate after a shock (measured by the deterministic growth rates of the productivity and the population g and n) the smaller the optimal biodiversity conservation.

Then, the role of the parameter ε describing the relationship between biodiversity and the probability of a pandemic is straightforward; a larger ε leads to more biodiversity conservation, which is conform to intuition. Indeed, the larger ε the more powerful is biodiversity conservation in terms of risk reduction.

As for the preference parameters the results are not obvious and are collected in the following proposition.

Proposition 3.2 *Suppose that the hypotheses of Theorem 3.1 are verified and that the optimal value of f^* is interior. Then f^* is an increasing function of θ and λ . Moreover it is an increasing function of ϕ when the discount rate is high enough: $\rho \geq g + \lambda n$.*

Proof. See Appendix A. ■

Even if it is not obvious at a first look the impact of risk aversion is conform to intuition: f^* is an increasing function of risk aversion, meaning that the more risk averse the planner is, the more she wants to reduce the risk of future pandemics.

Similarly Proposition 3.2 tells us that f^* is an increasing function of λ .

We can dig a bit further here. In the limit case of average utilitarianism ($\lambda = 0$), the optimal mitigation policy is no lockdown ($e^* = 0$), since $\frac{\lambda}{1-\phi} = 0 < \underline{\tau}$. But is it necessarily the case that the optimal biodiversity conservation is nil? We have:

$$f^*|_{\lambda=0} = 1 - \min \left[1, \frac{\rho - (1 - \phi)g}{\phi} \frac{1 - \theta}{1 - \bar{k}^{1-\theta}} \right] \quad (8)$$

If the productivity loss absent any social distancing is very small, \bar{k} is close to 1 and $f^* = 0$: there is no biodiversity conservation. Conversely, if the pandemic causes a significant productivity loss even without lockdown (and not only loss of lives), conserving some biodiversity may be optimal to reduce the risk of pandemic outbreaks.

Moreover, we can infer from the previous results the nature of the links between optimal prevention and optimal mitigation. At the beginning of the planning horizon, the planner chooses the optimal mitigation policy she will put in place each times a pandemic hits. This policy is all the more severe since she values life a lot. Therefore, the loss she incurs each time a pandemic hits, $1 - k(e^*)^{\frac{\lambda}{1-\phi}} \kappa(e^*)$ is all the larger (see the proof of Proposition 3.2). To avoid paying this large cost too often, the planner will choose to conserve a lot of biodiversity.

Finally Proposition 3.2 emphasizes that f^* is an ambiguous function of the aversion to fluctuations ϕ but a sufficient condition for f^* being an increasing function of aversion to fluctuations is that the discount rate is high enough: $\rho \geq g + \lambda n$. In the case of average utilitarianism ($\lambda = 0$), this condition reads $\rho > g$ and is necessary and sufficient. Indeed we have, using equation (8) when the solution is interior:

$$\frac{\partial f^*}{\partial \phi} \Big|_{\lambda=0} = \frac{1-\theta}{1-\bar{\kappa}^{1-\theta}} \frac{\rho-g}{\phi^2}.$$

where the term $\frac{1-\theta}{1-\bar{\kappa}^{1-\theta}}$ is positive for any choice of θ since $\kappa \in (0, 1)$.

Corollary 3.3 *Let hypotheses of Theorem 3.1 be satisfied. Then the optimal evolution of $A(t)$ and $N(t)$ are*

$$\begin{aligned} A(t) &= A_0 e^{gt} \kappa(e^*)^{q(t)-q(0)} \\ N(t) &= N_0 e^{nt} k(e^*)^{q(t)-q(0)} \end{aligned}$$

where the increment $q(t) - q(0)$, that is the number of pandemic outbreaks since date 0, is Poisson-distributed with mean $\varepsilon(1 - f^*)$. In particular

$$\mathbb{E}(A(t)) = A_0 e^{(g-\varepsilon(1-f^*)(1-\kappa(e^*)))t}, \quad \text{Var}(A(t)) = (\mathbb{E}(A(t)))^2 (e^{(g-\varepsilon(1-f^*)(1-\kappa(e^*)))t} - 1)$$

$$\mathbb{E}(N(t)) = N_0 e^{(n-\varepsilon(1-f^*)(1-k(e^*)))t}, \quad \text{Var}(N(t)) = (\mathbb{E}(N(t)))^2 (e^{(n-\varepsilon(1-f^*)(1-k(e^*)))t} - 1)$$

The expressions of previous corollary are particularly transparent: the evolution of $A(t)$ and $N(t)$ only depends on their pandemic-free (exponential) dynamics and on the size and the number of shocks (which, in the model, always have the effect of reducing the quantities by the same factor). The growth rates of the expected productivity and population are equal to the pandemic-free growth rates (g and n respectively) adjusted for the effects of possible pandemics given by the size of the loss weighted by the probability of the pandemics. A large weight of life in the social welfare function (large λ) causes a severe optimal mitigation policy (small e^*) and thus a large drop of productivity and a small death toll, together with an important risk-reducing effort of biodiversity conservation (large f^*). Therefore, the growth rate of the expected population is unambiguously an increasing function of λ , whereas the effect of λ on the growth rate of expected productivity is ambiguous.

		Gollier	Greenstone and Nigam	Thurnstrm <i>et al.</i>
Laisser-faire	mortality (% population)	0.54	1.13	0.68
	economic cost (% GDP)	3.73		2
Mitigation	mortality (% population)	0.363	0.52	0.29
	economic cost (% GDP)	5.74		6.2
Suppression	mortality (% population)	0.029		
	economic cost (% GDP)	13.53		

Table 1: Data

4 Numerical illustration

4.1 Specifications

We use the following specifications for the k and κ functions:

$$\begin{aligned}
 k(e) &= \bar{k} - (\bar{k} - \underline{k}) \left(\frac{e}{\bar{e}}\right)^a \\
 \kappa(e) &= \underline{\kappa} + (\bar{\kappa} - \underline{\kappa}) \left(\frac{e}{\bar{e}}\right)^b
 \end{aligned}$$

When $a \geq 1$ and $0 < b \leq 1$ these specifications satisfy the assumptions made above.

4.2 Data

Gollier (2020) calibrates a SIR model on the COVID-19 pandemic data for France and performs cost-benefit analysis exercises. He estimates the death toll and the GDP loss in the no-policy case, and for two strategies in particular: the “suppression”, or “crush the curve” strategy, consisting in confining 90% of the population for 4 months to eradicate the virus, and the “flatten-the-curve” strategy, consisting in confining 30% of the population for 5 months. We use these data for our benchmark calibration.

Needless to say, the parameters used by Gollier to calibrate the SIR model and the results he obtains are highly uncertain. We report in Table 1 Gollier’s results and also the results of two other papers, obtained for different countries and with different methods, to assess whether they give similar or very different information.

	$k(e)$	$\kappa(e)$
<i>laissez-faire</i>	$\underline{k} = 1 - 0.0054$	$\bar{\kappa} = 1 - 0.0373$
mitigation	$\bar{k} - (\bar{k} - \underline{k}) e_m^a = 1 - 0.00363$	$\underline{\kappa} + (\bar{\kappa} - \underline{\kappa}) e_m^b = 1 - 0.0574$
suppression	$\bar{k} = 1 - 0.00029$	$\underline{\kappa} = 1 - 0.1353$

Table 2: Calibration of the loss functions

Greenstone and Nigam (2020) are only interested in the death toll of the pandemic. Using the famous Ferguson *et al.* (2020) simulation model of COVID-19’s spread and mortality impacts in the United States, they estimate the death toll in the *laissez-faire* situation and in the case of a moderate social distancing policy, taking into account, as Gollier does, not only direct deaths but also the deaths due to the overwhelming of hospital intensive care units. Thunstrm *et al.* (2020) also examine the impacts of social distancing in the US, but on both the death and the GDP sides. They use epidemiological and economic forecasting to perform a cost–benefit analysis of controlling the COVID-19 outbreak. We assume here that Gollier’s “flatten the curve” scenario, Greenstone and Nigam “moderate social distancing” scenario and Thunstrm *et al.* “control” scenario are roughly equivalent in terms of severity and length of the lockdown. Table 1 shows that Gollier and Thunstrm *et al.* give similar estimates of the death toll in the *laissez-faire* scenario (no policy) and the mitigation (flatten the curve) scenario whereas Greenstone and Nigam are more pessimistic. For the economic cost of the pandemic in terms of GDP loss, the results from Thunstrm *et al.* we report in the table correspond to immediate losses (the year of the outbreak). The authors also compute the present value of GDP losses on a 30-year horizon, that we do not use for comparability with Gollier’s estimates. Gollier is more pessimistic in the *laissez-faire* scenario, less so in the mitigation scenario. But again these estimates are very uncertain.

4.3 Calibration

We normalize \bar{e} to 1.

We use Gollier’s data, as reported in Table 1, to calibrate the parameters of the k and κ loss functions. The *laissez-faire*, mitigation and suppression scenarios correspond respectively to $e = \bar{e} = 1$, $e = e_m$ unknown and $e = 0$. We have to calibrate parameters \underline{k} , \bar{k} , $\underline{\kappa}$, $\bar{\kappa}$, e_m , a and b . The relationships reported in Table 2 allow us to obtain 6 out of these 7 parameters. We choose to calibrate a/b , set arbitrarily $a = 2$ and deduce b . The results are in Table 3.

\bar{e}	e_m	a	b	\underline{k}	\bar{k}	$\underline{\kappa}$	$\bar{\kappa}$	ρ	g	n	ϕ
1	0.753	2	0.8	0.9946	0.99971	0.8647	0.9627	0.02	0.02	0.005	0.95

Table 3: Results of the calibration of the loss functions, and other parameters

We obtain that the mitigation strategy corresponds to a moderate lockdown: social interactions are reduced by around one quarter ($e_m = 0.753$).

The other parameters of the model, namely the discount rate ρ , the deterministic growth rates of population and productivity n and g , and the aversion to fluctuations ϕ are in the range of the parameters found in the literature (Table 3).

4.4 Results

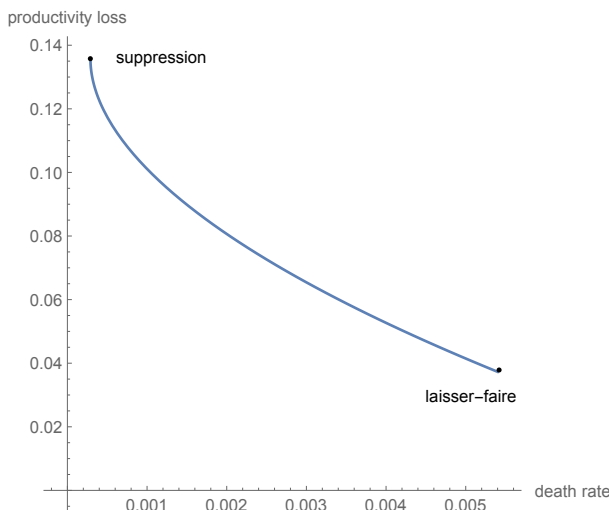


Figure 2: Trade-off lives vs economy

Figure 2 shows the terms of the raw trade-off between the loss of lives and the loss of GDP, for a mitigation strategy between 0 (suppression), on the left end of the curve, and \bar{e} (*laissez-faire*), on the right end. This trade-off is monotonous, in contrast to what Acemoglu *et al.* (2020) obtain in a model with several age classes. In their case, if, from the *laissez-faire* situation, the planner decides to confine the older age classes, most at risk, then she can at the same time save lives and mitigate the GDP loss. For more severe mitigation policies, all age classes are confined and the trade-off becomes similar to ours.

Figure 3 shows the optimal mitigation policies as a function of λ . With our specifications and calibration, the thresholds of $\lambda/(1 - \phi)$ under which there is no mitigation and above which there is suppression are respectively $\underline{\tau} = 10.5672$ and $\bar{\tau} = +\infty$. With $\phi = 0.95$, the lower threshold corresponds to $\lambda = 0.52$. In the case of average utilitarianism ($\lambda = 0$) and for all $\lambda < 0.52$ it is not optimal to engage in mitigation. In the case of total utilitarianism ($\lambda = 1$) we obtain $e^* = 0.44$: optimal mitigation is far more severe than in Gollier (2020)’s mitigation strategy for which $e^* = e_m = 0.753$, that is far more severe than the the “flatten-the-curve” strategy, consisting in confining 30% of the population for 5 months.

The very recent literature on the cost-benefit analysis of COVID-19 mitigation policies commonly uses the Value of a Statistical Life (VSL) to monetize the death toll and compare

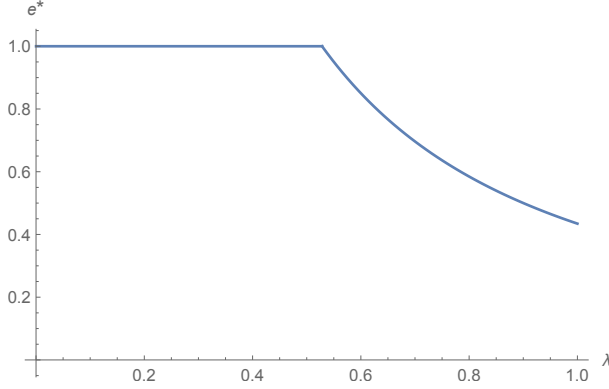


Figure 3: Optimal mitigation policy as a function of λ

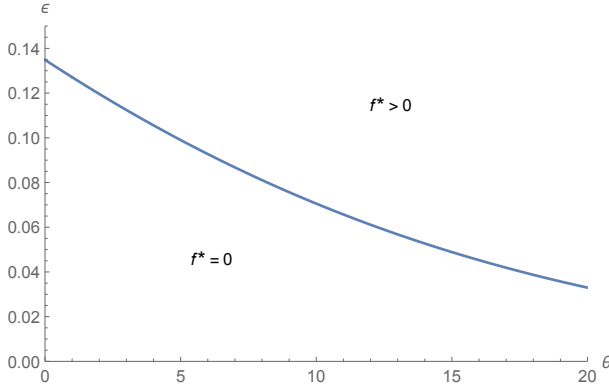


Figure 4: $(\theta - \varepsilon)$ frontier for $\lambda = 0.9$

the benefits of the policy in terms of avoided deaths to its costs in terms of foregone GDP. We do not need to do that here. In our model, the relative value of life is $\frac{V_N N}{V_A A}$, the value of the population over the value of productivity, population and productivity being both valued at the marginal increase of welfare their increase causes. With our specifications the relative value of life is constant and equal to $\frac{\lambda}{1-\phi}$. It only depends on the characteristics of the social welfare function, the degree of partial altruism λ and the aversion to fluctuations ϕ .

The optimal biodiversity conservation f^* , given by equation (5), depends on all the parameters identified above but also on the risk parameters, the risk aversion θ and the probability that the pandemic hits absent biodiversity ε . This last parameter is particularly difficult to calibrate. Instead of engaging in the exercise, we choose to determine the couples (θ, ε) constituting the frontier between no biodiversity conservation and biodiversity conservation or, to put things differently, no prevention and prevention of the pandemics. This frontier is represented on Figure 4 for $\lambda = 0.9$. We see that with our calibration, when risk aversion is around 2 biodiversity conservation becomes optimal for very high values of the probability of pandemics (around 12%), whereas when risk aversion is around 20 it becomes optimal for

values of the probability of pandemics around 4%. Again, these figures are only illustrative.

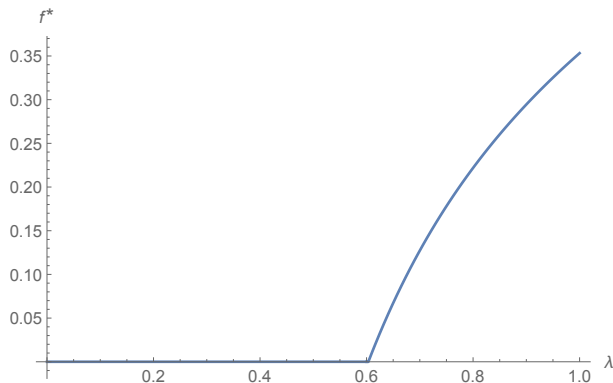


Figure 5: Optimal prevention policy as a function of λ , for $\varepsilon = 0.05$ and $\theta = 8$

To compute the optimal prevention policy as a function of λ we choose $\theta = 10$ and $\varepsilon = 0.1$. Figure 5 shows that there is no biodiversity conservation until $\lambda = 0.6$, and that in the case of total utilitarianism 35% of the land is devoted to biodiversity conservation for the prevention of pandemics.

5 Conclusion

Following the outbreak of the COVID-19 pandemic a long series of contributions in theoretical and applied economics have been written and published.

In this work we dig a little not only into the policies to be implemented to mitigate the effects of the pandemic when it occurs, but also (and above all) into one of the possible vectors to prevent it.

We propose here, for the first time as far as we know, a theoretical dynamic model that looks, in the case of a stylized economy, at the economic importance of reducing the likelihood of the spread of zoonoses through the conservation of biodiversity.

We consider a family of possible social welfare functions in which the elements of production and consumption are combined with the need to combat the excessive mortality caused by the spread of the virus.

Despite a certain technical complexity (due to the presence of jump processes and of an Epstein-Zin type utility) the model is completely solved and a discussion about the impact of various elements at stake (situation of the natural environment, productivity, preferences) is developed.

Emphasizing a role of “forward looking” instrument, we prove that the biodiversity conservation is more important when the social welfare functional is characterized by a lower discount rate and a stronger degree of altruism towards individuals of future cohorts. Not surprisingly, given the risk-reducing (prevention) effect of biodiversity in the model, the biodiversity is also more relevant when the risk aversion or the risk of pandemics is higher.

After calibrating the model using the data from Gollier (2020) we exhibit the terms of the trade-off between the loss of lives and the loss of GDP for the whole set of mitigation strategies, from *laissez-faire* to suppression.

The stylized framework we have developed in this work would enable us to extend our research in several directions. First, we intend to consider a game-theoretical extension of the model to study the cost of non-coordination of policies. Second, we would like to consider uncertainty, disentangling aversion to ambiguity, risk aversion and aversion to fluctuations. Finally in future developments of the model it would be interesting to extend the analysis to production functions which are non-linear in labor. Indeed the variation of the labor supply for health conditions or as a policies’ consequence is one of the relevant points of mitigation policies evaluation for pandemics (see, for instance, Kapicka and Rupert, 2020).

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A Proofs

Proof of Theorem 3.1. As already mentioned in the main text the HJB equation related to our maximization problem is

$$\rho \frac{1-\theta}{1-\phi} V(A, N) = \max_{v, e} \left[N^\lambda \frac{[(1-f)A]^{1-\phi}}{1-\phi} \frac{1}{((1-\theta)V(A, N))^{\frac{1-\phi}{1-\theta}-1}} + V_A g A + V_N n N + \varepsilon(1-f)(V(\kappa(e)A, k(e)N) - V(A, N)) \right] \quad (9)$$

Denote $\tilde{V} = V(\tilde{A}, \tilde{N})$. The optimality condition w.r.t. f yields:

$$1-f = \left(\frac{N^\lambda A^{1-\phi}}{\varepsilon(V - \tilde{V})(1-\theta)V^{\frac{1-\phi}{1-\theta}-1}} \right)^{1/\phi} \quad (10)$$

Replacing $1 - f$ in the HJB equation (1) yields:

$$\begin{aligned}
\rho \frac{1-\theta}{1-\phi} V &= \max_e \frac{N^\lambda A^{1-\phi}}{1-\phi} \left(\frac{N^\lambda A^{1-\phi}}{\varepsilon(V-\tilde{V})((1-\theta)V)^{\frac{1-\phi}{1-\theta}-1}} \right)^{\frac{1-\phi}{\phi}} \frac{1}{((1-\theta)V)^{\frac{1-\phi}{1-\theta}-1}} \\
&\quad + V_A g A + V_N n N + \left(\frac{N^\lambda A^{1-\phi}}{\varepsilon(V-\tilde{V})((1-\theta)V)^{\frac{1-\phi}{1-\theta}-1}} \right)^{1/\phi} \varepsilon(\tilde{V}-V) \\
&= \max_e \frac{\phi}{1-\phi} (N^\lambda A^{1-\phi})^{\frac{1}{\phi}} \left(\varepsilon(V-\tilde{V}) \right)^{-\frac{1-\phi}{\phi}} \left(((1-\theta)V)^{\frac{1-\phi}{1-\theta}-1} \right)^{-\frac{1}{\phi}} + V_A g A + V_N n N \quad (11)
\end{aligned}$$

We try to find a solution of the form:

$$V(A, N) = X \frac{N^\alpha A^{1-\theta}}{1-\theta}.$$

for some real positive parameters α and X . In this case

$$\begin{aligned}
\tilde{V} &= k(e)^\alpha \kappa(e)^{1-\theta} V \\
V_A &= (1-\theta) \frac{V}{A} \\
V_N &= \alpha \frac{V}{N}
\end{aligned}$$

Using these expressions in the HJB equation above we get:

$$\begin{aligned}
\rho \frac{1-\theta}{1-\phi} V &= \max_e \frac{\phi}{1-\phi} (N^\lambda A^{1-\phi})^{\frac{1}{\phi}} \left[\varepsilon(1 - k(e)^\alpha \kappa(e)^{1-\theta}) V \right]^{-\frac{1-\phi}{\phi}} \left(((1-\theta)V)^{\frac{1-\phi}{1-\theta}-1} \right)^{-\frac{1}{\phi}} \\
&\quad + (1-\theta)gV + \alpha nV
\end{aligned}$$

i.e.

$$\begin{aligned}
\rho \frac{1}{1-\phi} - g - \frac{\alpha}{1-\theta} n \\
= \max_e \frac{\phi}{1-\phi} (N^\lambda A^{1-\phi})^{\frac{1}{\phi}} \left[\frac{\varepsilon(1 - k(e)^\alpha \kappa(e)^{1-\theta})}{1-\theta} \right]^{-\frac{1-\phi}{\phi}} (N^\alpha A^{1-\theta})^{-\frac{1}{\phi} \frac{1-\phi}{1-\theta}} X^{-\frac{1}{\phi} \frac{1-\phi}{1-\theta}}
\end{aligned}$$

The maximum point for e is given by e^* which maximizes (2). Choosing

$$\alpha = \lambda \frac{1-\theta}{1-\phi}$$

allows us to obtain:

$$\frac{1}{1-\phi}(\rho - (1-\phi)g - \lambda n) = \frac{\phi}{1-\phi} \left[\frac{\varepsilon(1-k(e^*)^\alpha \kappa(e^*)^{1-\theta})}{1-\theta} \right]^{-\frac{1-\phi}{\phi}} X^{-\frac{1}{\phi} \frac{1-\phi}{1-\theta}}$$

i.e. finally (recall that assumption (3) implies $\rho - (1-\phi)g - \lambda n > 0$):

$$X = \left(\frac{\phi \left[\frac{\varepsilon(1-k(e^*)^\alpha \kappa(e^*)^{1-\theta})}{1-\theta} \right]^{-\frac{1-\phi}{\phi}}}{\rho - (1-\phi)g - \lambda n} \right)^{\phi \frac{1-\theta}{1-\phi}}$$

Then (10) reads:

$$1-f = \left(\frac{1-\theta}{\varepsilon(1-k(e^*)^\alpha \kappa(e^*)^{1-\theta}) X^{\frac{1-\phi}{1-\theta}}} \right)^{1/\phi} = \frac{(1-\theta)(\rho - (1-\phi)g - \lambda n)}{\phi \varepsilon \left[1 - \left(k(e^*)^{\frac{\lambda}{1-\phi}} \kappa(e^*) \right)^{1-\theta} \right]} \quad (12)$$

Since condition (3) is verified this value is in $(0, 1)$.

We finally compute the transversality condition. We need to obtain that

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mathbb{E}V(A(t), N(t)) = 0.$$

The optimal evolution of $A(t)$ and $N(t)$ are

$$\begin{aligned} A(t) &= A_0 e^{gt} \kappa(e^*)^{q(t)-q(0)} \\ N(t) &= N_0 e^{nt} k(e^*)^{q(t)-q(0)} \end{aligned}$$

Then:

$$V(A(t), N(t)) = \frac{X}{1-\theta} N_0^{\lambda \frac{1-\theta}{1-\phi}} A_0^{1-\theta} e^{((1-\theta)g + \lambda \frac{1-\theta}{1-\phi} n)t} \kappa(e^*)^{(1-\theta)(q(t)-q(0))} k(e^*)^{\lambda \frac{1-\theta}{1-\phi} (q(t)-q(0))}$$

and

$$\mathbb{E} \left[\frac{(1-\theta)V(A(t), N(t))}{X N_0^{\lambda \frac{1-\theta}{1-\phi}} A_0^{1-\theta}} \right] = e^{\left((1-\theta)g + \lambda \frac{1-\theta}{1-\phi} n - \left(1 - \kappa(e^*)^{1-\theta} k(e^*)^{\lambda \frac{1-\theta}{1-\phi}} \right) \varepsilon(1-f^*) \right) t}$$

i.e., using the definition of f^* ,

$$\mathbb{E} \left[\frac{(1-\theta)V(A(t), N(t))}{XN_0^{\lambda \frac{1-\theta}{1-\phi}} A_0^{1-\theta}} \right] = e^{((1-\theta)g + \lambda \frac{1-\theta}{1-\phi} n - \frac{1-\theta}{\phi} (\rho - (1-\phi)g - \lambda n))t}$$

So we have $e^{-\rho t} \mathbb{E}V(A(t), N(t)) \rightarrow 0$ as far as

$$-\rho + \left((1-\theta)g + \lambda \frac{1-\theta}{1-\phi} n - \frac{1-\theta}{\phi} (\rho - (1-\phi)g - \lambda n) \right) < 0$$

which reduces to the condition asked in (4). ■

Proof of Proposition 3.2. In the case of an interior solution on f^* , equation (5) reads:

$$f^* = 1 - \frac{\rho - (1-\phi)g - \lambda n}{\varepsilon\phi} \frac{1-\theta}{1 - z(e^*)^{1-\theta}}$$

with

$$z(e^*) = k(e^*)^{\frac{\lambda}{1-\phi}} \kappa(e^*)$$

and e^* does not depend on θ . Then

$$\frac{\partial f^*}{\partial \theta} = \frac{(\rho - (1-\phi)g - \lambda n)[1 - (1 - (1-\theta) \ln z(e^*))z(e^*)^{1-\theta}]}{\varepsilon\phi(1 - z(e^*)^{1-\theta})^2}$$

$$\frac{\partial f^*}{\partial \theta} \geq 0 \iff 1 - (1 - (1-\theta) \ln z(e^*))z(e^*)^{1-\theta} \geq 0 \iff (1 - (1-\theta) \ln z(e^*))z(e^*)^{1-\theta} \leq 1.$$

Let $P(\theta) = (1 - (1-\theta) \ln z)z^{1-\theta}$.

$$P'(\theta) = \ln z z^{1-\theta} + (1 - (1-\theta) \ln z) \frac{\partial e^{(1-\theta) \ln z}}{\partial \theta} = (1-\theta)(\ln z)^2 z^{1-\theta}.$$

$$P'(\theta) \underset{\leq}{\geq} 0 \iff \theta \underset{\leq}{\geq} 1.$$

P is therefore maximum for $\theta = 1$, and as $P(1) = 1$ we can conclude that indeed $P(\theta) \leq 1$. f^* is an increasing function of θ .

Regarding the dependence of f^* on λ , we have:

$$\begin{aligned} \frac{\partial f^*}{\partial \lambda} &= \underbrace{\frac{1}{\varepsilon\phi} \frac{1-\theta}{1 - z(e^*)^{1-\theta}}}_{>0} \left[\underbrace{n - (\rho - (1-\phi)g - \lambda n) \frac{(1-\theta)z(e^*)^{-\theta}}{1 - z(e^*)^{1-\theta}}}_{>0} \frac{\partial z(e^*)}{\partial \lambda} \right] \\ \frac{\partial z(e^*)}{\partial \lambda} &= z(e^*) \left[\frac{1}{1-\phi} \ln k(e^*) + \left(\frac{\lambda}{1-\phi} \frac{k(e^*)'}{k(e^*)} + \frac{\kappa(e^*)'}{\kappa(e^*)} \right) \frac{\partial e^*}{\partial \lambda} \right] \end{aligned}$$

According to the optimality condition defining the mitigation policy (equation (7)) the term in parenthesis on the right-hand side member of this equation is nil. Then

$$\frac{\partial z(e^*)}{\partial \lambda} = z(e^*) \frac{1}{1-\phi} \ln k(e^*) < 0,$$

from which we conclude that $\frac{\partial f^*}{\partial \lambda} > 0$.

Finally, regarding the dependence of f^* on ϕ , we have:

$$\frac{\partial f^*}{\partial \phi} = \frac{1}{\underbrace{\varepsilon \phi}_{>0}} \frac{1-\theta}{1-z(e^*)^{1-\theta}} \left[\frac{\rho-g-\lambda n}{\phi} - \underbrace{(\rho-(1-\phi)g-\lambda n)}_{>0} \frac{(1-\theta)z(e^*)^{-\theta}}{1-z(e^*)^{1-\theta}} \frac{\partial z(e^*)}{\partial \lambda} \right]$$

with

$$\frac{\partial z(e^*)}{\partial \phi} = z(e^*) \frac{1}{(1-\phi)^2} \ln k(e^*) < 0.$$

The sign of $\frac{\partial f^*}{\partial \phi}$ is therefore ambiguous. A sufficient condition for it to be positive is $\rho \geq g + \lambda n$, that is a high enough discount rate. ■

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