

The Rise (and Fall) of Tech Clusters

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The Rise (and Fall) of Tech Clusters

Abstract

Tech clusters play a growing role in knowledge-based economies by accommodating high-tech firms and providing an environment that fosters location-dependent knowledge spillovers and promote R&D investments by firms. Yet, not much is known about the economic conditions under which such entities may form in equilibrium without government interventions. This paper develops a spatial equilibrium model with a competitive final sector and a monopolistically competitive intermediate sector, which allows us to determine necessary and sufficient conditions for a tech cluster to emerge as an equilibrium outcome. We show that strongly localized knowledge spillovers, skilled labor abundance, and low commuting costs are key drivers for a tech cluster to form. Not only is the productivity of the final sector higher when intermediate firms cluster, but a tech cluster hosts more intermediate firms and more R&D and production activities, and yields greater worker welfare, compared to what a dispersed pattern would generate. With continual improvements in infrastructure and communication technology that lowers coordination costs, tech clusters will eventually be fragmented.

JEL-Codes: D510, L220, O330, R130.

Keywords: high-tech city, knowledge spillovers, intermediate firm clustering, land use, commuting, R&D.

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1 Introduction

The concept of *tech clusters* has gained the favor of many analysts and policy-makers (see Kerr and Robert-Nicoud, 2020, for a detailed review and an exhaustive list of references). Even though the idea of industrial district has been around for a long time (Marshall, 1890, ch. X), it was not until the 1990s that the related concept of tech cluster, or science park, has been developed (Castells and Hall, 1994; Saxenian, 1994). Although there is a rich variety of tech clusters (Klepper, 2010; McCarthy *et al.*, 2018), they do share some common features. In particular, a tech cluster (i) accommodates *knowledge-intensive firms*, (ii) encourages *R&D investments* by firms, while researchers and high-skilled workers can be drawn from nearby universities/research institutes, and (iii) provides an environment that fosters location-dependent *knowledge spillovers* (Kerr and Robert-Nicoud, 2020). A tech cluster also offers an opportunity to its workers for earning higher wages by involving in research and production activities in a productive environment. In addition, a city hosting a tech cluster also provides office and lab space, as well as housing, retail, restaurants and other leisure facilities within a given geographic area. A high-tech city thus offers an *urban environment* to its population (Katz and Bradley, 2013; McCarthy *et al.*, 2018). This leads us to consider such a cluster as a special type of city whose spatial structure is determined endogenously by the interaction between multiple stakeholders through the above-mentioned channels.

Despite the importance of tech clusters in the real world, it is fair to say that this concept has attracted little attention in economic theory. This is where we hope to contribute by providing a full-fledged general equilibrium model that allows us to determine under which conditions a high-tech city emerges as a *decentralized equilibrium outcome*. The main tenet of this paper is that *the emergence and efficiency of a tech cluster is intimately related to the spatial structure of the area that hosts it*. To show this, we view a tech cluster as a city formed by firms involved in R&D activities and which interact to determine endogenously knowledge spillovers. We echo Kerr and Robert-Nicoud (2020) by focusing on knowledge spillovers and R&D activities. In line with the empirical literature, we recognize that knowledge spillovers both depend on localization via a distance-decay function and raise the marginal product of R&D and production technologies.¹ These specific features of a high-tech city differentiate our model from typical city formation models where cities are marketplaces, transportation nodes, or factory towns. Additionally, our model contrasts with the conventional ones by allowing the size of the high-tech city to be endogenous. This is an important feature enabling us to explain why tech clusters first grow and, then, fall. Despite several simplifications, it is

¹See Arzaghi and Henderson (2008), Greenstone *et al.* (2010), Lychagin *et al.* (2016), and Buzart *et al.* (2017), as well as critical reviews by Audretsch and Feldman (2004) and Carlino and Kerr (2015).

our belief that our setting will contribute to a better understanding of the formation of high-tech cities and will allow for a more precise quantification of their economic consequences and a better evaluation of relevant policies.

To achieve our goal, we develop a model that captures the following basic features: (i) a composite consumption good is produced by using an endogenous range of specialized inputs provided by intermediate firms whose combination generates coordination problems that require the hiring of production-line designers; (ii) high-skilled and specialized workers are hired to produce intermediate goods or to conduct R&D in the intermediate sector; (iii) the productivity of an intermediate firm depends on its level of R&D investments and inter-firm spillovers, the intensity of which depends on how intermediate firms are distributed across space; and, (iv) both workers and intermediate firms are spatially mobile and use land. The novelty of our approach lies in an agglomeration force that combines firms' R&D investments and the existence of localized knowledge spillovers. On the other hand, the dispersion force, which is generated by both intermediate firms' and workers' demand for land and costly commuting, is common to most models of city formation. As usual, the equilibrium distribution of firms and workers is determined as the balance between these two opposite forces.

Our main results may be summarized as follows. We begin by establishing necessary and sufficient conditions for a high-tech cluster to emerge as a spatial equilibrium outcome. This in turn will allow us to uncover the reasons explaining why high-tech clusters may or may not be formed. More specifically, we identify three key rationales for firms to have incentives for clustering in a high-tech cluster: (i) *highly localized knowledge spillovers*, (ii) *relatively inexpensive commuting costs*, and (iii) *abundance of high-skilled labor*. These are all typical features of new high-tech industries that make a cluster, which accommodates more intermediate firms and fosters research activities, more likely to emerge. This may explain why knowledge-intensive firms form a high-tech cluster such as the Silicon Valley, the Hsinchu Science-Based Industrial Park in Taiwan or the Cambridge Science Park in the U.K. (Saxenian, 1994; Chen, 2008; Helmers, 2019) and why in the absence of localized knowledge spillovers, simple cluster policies are not sufficient for a high-tech city or a local innovative system to develop (see Duranton *et al.*, 2010, for a critical appraisal of such policies). Although our analysis of spatial externalities remains relatively rudimentary, our paper may be viewed as an attempt to open the black box of spatial externalities by means of a setting that take firms' behavior as the driving force.

We also show that abundant availability of high-skilled workers fosters a tech cluster if one emerges, whereas highly localized spillovers leads to a smaller tech cluster but makes it

more likely to arise in equilibrium. By contrast, continual improvements in infrastructure and communication technology that lowers coordination costs may lead to the fragmentation of tech clusters as observed in the case of Silicon Valley (Saxenian, 1994). In addition, tech clusters should not be viewed as the panacea for local development. Since they offer high wages, they attract high-skilled workers. However, too large an inflow of migrants will lead to their geographical fragmentation. Indeed, high land rents and commuting costs become a concern to potential entry of small specialized firms. More specifically, we will see that *tech clusters are more likely to be sustainable in urban environments that do not have too large a population* because commuting may become too costly. This may explain why most successful tech clusters or science parks do not emerge in megacities. For example, Silicon Valley is over 30 miles south of San Francisco, the Cambridge Science Park in the U.K. more than 60 miles north of London, and Hsinchu Science-Based Industrial Park in Taiwan 50 plus miles away from Taipei.

Due to endogenous determination of the size of a high-tech city, our setting has the nature of an “open” city model: despite of a given population of workers, the physical size of the urban space is endogenous. In this context, a full welfare analysis is not obvious. To gain insights, we undertake a “quasi-counterfactual” by considering an urban configuration in which activities are assumed to be evenly dispersed across locations. Comparing a high-tech city with such a flat city enables us to understand better the benefit associated with the emergence of the cluster. For example, we find that *a high-tech city hosts more intermediate firms and more researchers* than if activities were dispersed. What is more, the final sector is more productive while *wages and land rents are higher in a high-tech city*. Last, despite paying higher rent and incurring higher commuting costs, workers are better-off in a high-tech city than in a flat city. All of this shows why high-tech cities have a lot of appeal.

Related literature. A handful of related models have been developed in urban economics. Their common aim is to explain the emergence of business centers within cities. They include O’Hara (1977), Ogawa and Fujita (1980), Fujita and Ogawa (1982), Berliant *et al.* (2002), and Lucas and Rossi-Hansberg (2002). However, these models focus on gravity-like reduced forms in which knowledge spillovers and/or spatial externalities are mechanically presumed. They ignore firms’ R&D policies and the supply of intermediate inputs and knowledge-intensive business services, which affect the overall productivity of firms. Although spatial externalities have attracted a lot of attention in urban economics (see Carlino and Kerr, 2015, for a survey), we are not aware of a microeconomic model developed to investigate how spatial externalities emerge from intentional firms’ R&D and location decisions. Here it is worth mentioning Desmet and Rossi-Hansberg (2014) who study how firms choose to

innovate in a dynamic technology diffusion framework characterized by spatial frictions. However, unlike us, they cannot characterize the equilibrium analytically, so that they have to source to quantitative analysis. Another key difference is that they do not consider workers' commuting costs, which play an important role here in pinning down the pattern of a tech cluster and explain why most successful tech-clusters or science parks do not emerge in megacities. Different from all papers above, our model allows the size of the tech cluster to be endogenously determined, which yields new comparative statics and serves to explain the rise and the fall of tech clusters.

The remainder of the paper is organized as follows. The model is presented in Section 2, while Section 3 describes the optimizing conditions of the different groups of agents. The necessary and sufficient conditions for a high-tech city to emerge are determined and its properties are studied in Section 4. In Section 5, we comparing a high-tech city and a flat city where firms and workers are fully mixed. Section 6 concludes.

2 The model

The economy consists of a featureless one-dimensional space Z and a continuum H of high-skilled workers. Land density and the opportunity cost of land are normalized to one. A unit mass of firms produce the final good under perfect competition by using the basket of intermediate goods. Since our focus is on the intermediate sector, it is convenient to assume that the final producers do not consume land. Assuming that the final sector is located at the city center allows us to focus on symmetric distributions of intermediate firms and workers.² Intermediate firms and workers choose their location within the urban area and consume a fixed amount of land normalized to one. The final and intermediate goods are shipped within the city at no costs, thus implying that the prices of intermediate goods are independent of where intermediate and final producers are located.³

Apart from land, workers consume the final good. Each worker is endowed with one unit of labor that she supplies inelastically. Commuting between the residence $y \in Z$ and workplace $z \in Z$ requires $t|y - z|$ units of the numéraire, where $t > 0$ is the commuting rate.

2.1 The spatial structure of the economy

Since each worker and each firm consumes one unit of land, the total demand for land is equal to $N + H$ where N is the mass of intermediate firms that will be endogenously determined

²Using another location for the final sector does not affect the nature of our main results.

³Dealing with shipping costs is fairly straightforward and does not add much to our results.

in equilibrium. Therefore, the city is given by the interval

$$Z = \left[-\frac{N+H}{2}, \frac{N+H}{2} \right],$$

which implies that the city size varies with the mass of intermediate firms (N) and the population size (H). As will be seen, the size of a high-tech city depends on the location of the intermediate sector because the size of intermediate sector N varies with its spatial structure. Following the literature, we assume that the city is geographically symmetric and centered at $z = 0$.

We focus on a high-tech city where all firms are clustered around city center, which is flanked by two residential areas. In this case, the intermediate producers are uniformly distributed within a tech cluster whose spatial extend is given by,

$$Z^F = [-N/2, N/2],$$

with density $h^F = 1$. Therefore, workers are uniformly distributed over the residential area

$$Z^W = \left[-\frac{N+H}{2}, -\frac{N}{2} \right] \cup \left[\frac{N}{2}, \frac{N+H}{2} \right]$$

with density $h^W = 1$.

2.2 The intermediate sector

The intermediate sector produces a differentiated intermediate good under monopolistic competition using land and labor with endogenous R&D that enhances firm productivity. Each intermediate is provided by a single firm and each firm supplies a single intermediate good. Since a firm located at z makes the same choices in equilibrium regardless of the variety it produces, we index intermediate inputs by their place of production z .

A firm located at $z \in Z^F$ produces the intermediate good $z \in [-N/2, N/2]$ and uses one unit of land, $R(z)$ units of labor for R&D, and $L(z)$ units of labor for production. This firm produces the quantity $x(z)$ according to a Cobb-Douglas technology:

$$x(z) = A(z) \cdot L(z)^\beta l^{1-\beta}, \tag{1}$$

where $\beta \in (0, 1)$ and $l = 1$ is a firm's land requirement. Land is then a fixed factor of production, which implies that firms face decreasing returns to scale.

The total factor productivity (TFP) of an intermediate firm $A(z)$ is given by a Cobb-Douglas aggregator of the firm's R&D employment $R(z)$ and a *spatial externality* that combines a Romer-Lucas external effect weighted by a distance-decay function $S(z)$:

$$A(z) = R(z)^\theta \cdot (\bar{R}S(z))^{1-\theta}, \quad (2)$$

where \bar{R} is the total mass of workers involved in R&D activities, $S(z)$ is a measure of firms' spatial concentration around z , and $\theta \in (0, 1)$. Our setup of the endogenous TFP captures Romer's idea that externalities enter the system through an aggregator of individual firms' decisions, but we recognize that a firm's TFP is also influenced by this firm's R&D policy. Firms' endogenous decisions of R&D interacting with location-dependent knowledge spillovers via $\bar{R}S(z)$ differentiates our paper from all previous studies cited in the introduction.

One of the distinctive features of our setting is that a firm's TFP is endogenized through the following three channels: (i) the number of researchers hired to conduct R&D in each firm, (ii) the total number of researchers working the city, and (iii) the spatial distribution of firms/researchers. This modeling strategy is consistent with the idea that spillovers increase the level and productivity of R&D investments (Aghion and Jaravel, 2015; Helmers, 2019). It is also in line with Chyi *et al.* (2012) who find that knowledge spillovers are especially strong across intermediate firms located in science parks. That said, it is worth stressing that *what distinguishes a high-tech city from a standard city is the presence of the endogenous R&D shifter $R^\theta \bar{R}^{1-\theta}$ in (2)*. In this context, the efficiency of an intermediate firm depends on its level of R&D activities, as well as on the total mass of researchers whose productivity rises with their number (this was highlighted by Romer, 1986, and Lucas, 1988). While the urban economics literature highlights the existence of various types of agglomeration economies, the impact of the spatial structure on the endogenous R&D shifter $R^\theta \bar{R}^{1-\theta}$ is often overlooked although its importance is stressed by empirical evidence (e.g., Siegel *et al.*, 2003).

In (2), the spatial externality at z is generated by both the size of the research pool in the intermediate sector (\bar{R}) and the spatial distribution of intermediate firms ($S(z)$). In line with most of the literature, we assume that the distance-decay effect is given by a negative exponential (Fujita and Ogawa, 1982; Lucas and Rossi-Hansberg, 2002; Desmet and Rossi-Hansberg, 2014; de Palma *et al.*, 2019):

$$S(z) \equiv \int_{-N/2}^{N/2} \exp(-\gamma |z - y|) h^F dy = \frac{1}{\gamma} [2 - e^{-\gamma N/2} (e^{-\gamma z} + e^{\gamma z})] \quad (3)$$

since $h^F = 1$. Admittedly, $S(z)$ has the nature of a reduced-form that aims to capture a rich set of interactions. In this respect, we want to stress that Smith (1978) has provided micro-foundations for (3) that went unnoticed. He shows that, when knowledge flows are drawn

randomly, the distance-decay function that describes the spatial diffusion of knowledge across space is given by a negative exponential if and only if a low-cost knowledge flow between two locations is more likely to be observed than a high-cost flow. Whereas $S(z)$ decreases with the spatial impedance parameter γ that measures the severity of distance-decays of positive spatial externality, it increases at a decreasing rate with the mass of intermediate firms N that gives rise to an agglomeration force encouraging firm clustering. Note also that $S(z)$ is strictly decreasing and concave in z .

Since workers bear commuting costs, the equilibrium wage rate $w(z)$ is location-specific. The immobility of land implies that the land rent $r(z)$ is also location-specific. Taking wages and land rents as given, each firm chooses its location z , output price $p(z)$, and the numbers of workers allocated to its production and R&D activities to maximize profits:

$$\pi(z) = p(z)x(z) - w(z)[R(z) + L(z)] - r(z). \quad (4)$$

The land rent $r(z)$ plays the role of an endogenous and location-specific fixed cost for an intermediate firm located at z . If land were not an input of the intermediate sector, firms would have a zero size under free entry and would not invest in R&D activity. This is reminiscent of Desmet and Rossi-Hansberg (2012).

2.3 The final sector

For any given N , the final sector in the high-tech city produces the numéraire according to the following production function:

$$Y = \int_{Z^F} x(z)^{\frac{\sigma-1}{\sigma}} dz, \quad (5)$$

where $\sigma > 1$. Furthermore, designing the production line gives rise to an expenditure that has the nature of an endogenous fixed cost for the final sector. This cost accounts for the fact that combining specialized and differentiated inputs (or tasks) tends to generate specific coordination costs that grow as the number of inputs increases (Becker and Murphy, 1992). We allow final production to exhibit decreasing returns to scale so that coordination costs can be covered.

Taking both the wage rates and the prices of intermediate goods as given, the final sector located at $z = 0$ maximizes profits given by,

$$\Pi = Y - \int_{Z^F} p(z)x(z)dz - w(0)\phi N, \quad (6)$$

subject to (5). In this expression, $w(0)\phi N$ stands for the coordination costs incurred by the final producers when production involves N differentiated inputs, while $\phi > 0$ is the labor requirement needed to use one additional intermediate input.

We have chosen to work with a homogeneous final good because this assumption vastly simplifies the algebra without affecting our main results. For example, they hold true if the final sector produces varieties of a horizontally differentiated good sold to consumers with CES preferences while firms would produce under monopolistic competition and increasing returns, the fixed costs being endogenous and given by $w(0)\phi N$.

2.4 Workers

A worker chooses her residential site y and workplace z to maximize her consumption of the final good. This amounts to maximizing her net income given by,

$$\max_{y,z} I(y, z) = w(z) - t|y - z| - r(y). \quad (7)$$

At the residential equilibrium, workers reach the same utility level, hence earn the same net income I_0 :

$$I(y, z) = I_0, \quad (8)$$

where I_0 is endogenous. This condition implies that a worker has no incentive to change either her residential or working places.

Workers are used to enhance the efficiency of the intermediate sector by performing the following three tasks: (i) to undertake firm-specific R&D, (ii) to design the production line in the final sector, and (iii) to produce intermediate goods. Labor is allocated to the following three activities: (i) the mass \bar{R} of workers involved in R&D activities, (ii) the mass $M \equiv \phi N$ of workers involved in designing the supply chain of the final sector, and (iii) the mass \bar{L} of workers producing the intermediate goods. Labor market clearing implies the following condition:

$$H = M + \bar{R} + \bar{L}. \quad (9)$$

3 The programs of firms and workers

In this section, we determine the equilibrium conditions for the final and intermediate producers, as well as for workers.

3.1 The final sector

The final sector chooses the mass N of intermediate goods and the quantity x of each variety. Since shipping the intermediate goods to the final sector is costless, the price of an intermediate good i is the same within the city. Therefore, plugging (5) in (6) and differentiating Π

yields the profit-maximization conditions:

$$\begin{aligned}\frac{d\Pi}{dx} &= \frac{\sigma - 1}{\sigma}x(z)^{-\frac{1}{\sigma}} - p(z) = 0, \\ \frac{d\Pi}{dN} &= x(N/2)^{\frac{\sigma-1}{\sigma}} - w(0)\phi - p(N/2)x(N/2) = 0,\end{aligned}\tag{10}$$

because the marginal variety N is produced at the edge of the cluster $N/2$. Thus, the final sector's inverse demand for the variety produced at z is location-specific and given by,

$$p(z) = \frac{\sigma - 1}{\sigma}x(z)^{-\frac{1}{\sigma}}.\tag{11}$$

Plugging (11) into (10), we obtain the output of a firm located at the edge of the cluster:

$$x(N/2)^{\frac{\sigma-1}{\sigma}} = \sigma w(0)\phi.\tag{12}$$

3.2 The intermediate sector

Plugging (11) and (1) into (4) yields the following profit function:

$$\pi(z) = [R(z)^\theta \cdot (\bar{R}S(z))^{1-\theta} \cdot L(z)^\beta]^{\frac{\sigma-1}{\sigma}} - w(z)[R(z) + L(z)] - r(z).\tag{13}$$

Applying the first-order conditions with respect to $R(z)$, $L(z)$, and z leads to the following equations:

$$w(z) = \theta \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{1}{R(z)} x(z)^{\frac{\sigma-1}{\sigma}},\tag{14}$$

$$w(z) = \beta \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{1}{L(z)} x(z)^{\frac{\sigma-1}{\sigma}},\tag{15}$$

$$(1 - \theta) \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{dS(z)}{dz} \frac{1}{S(z)} x(z)^{\frac{\sigma-1}{\sigma}} = \frac{dw(z)}{dz} [R(z) + L(z)] + \frac{dr(z)}{dz}.$$

The first two expressions are standard. The last one means that, by moving away from the center $z = 0$, a firm incurs a decrease in the benefit generated by spillovers, which is exactly compensated by a decrease in wage and land rent. This condition is the counterpart of the Alonso-Muth equation obtained in the monocentric city model of urban economics (Fujita, 1989).

We can combine these three conditions to obtain the following equilibrium conditions:

$$\frac{R(z)}{L(z)} = \frac{\theta}{\beta},\tag{16}$$

$$w(z) = \beta \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{1}{L(z)} x(z)^{\frac{\sigma-1}{\sigma}},\tag{17}$$

$$\frac{dr(z)}{dz} = \frac{L(z)}{\beta} \left[(1 - \theta) \frac{w(z)}{S(z)} \frac{dS(z)}{dz} - (\theta + \beta) \frac{dw(z)}{dz} \right].\tag{18}$$

The first condition means that the research-production labor ratio is equal to the ratio of their output elasticities, while the second states that the wage is proportional to the output per capita of a variety. The third condition implies that the land rent gradient is flatter when the spillover S is stronger because the benefit earned from clustering is higher. We acknowledge that having a constant fraction of workers employed in research vs. production, which does not depend on the city structure, is restrictive. However, this vastly simplifies the analytical analysis of the model.

It remains to show that the first-order conditions are also sufficient. We show in Appendix A that firms' profit functions are strictly concave if and only if the inequality

$$\frac{\sigma}{\sigma - 1} > \theta + \beta \quad (19)$$

holds. Estimations of the elasticity of substitution σ among inputs is 4.7 on average (Peter and Ruane, 2020), while the elasticity of production with respect to R&D θ is about 0.12 (Hall *et al.*, 2010). Since the land share is approximately 0.06 for manufacturing (Caselli and Coleman, 2001) and 0.15 for services (Brinkman *et al.*, 2015), we may safely conclude that the empirical evidence backs up the inequality $\sigma/(\sigma - 1) > \theta + \beta$. In what follows, we assume that (19) holds true.

Using (14)-(15), the profit earned by a firm located at z is given by,

$$\pi(z) = \left(\frac{\sigma}{\sigma - 1} - \theta - \beta \right) \frac{L(z)w(z)}{\beta} - r(z),$$

which is nonnegative due to (19).

In equilibrium, intermediate firms earn the same profits. Therefore, the maximum bid that a firm can offer to set up at z , denoted by $r^F(z)$, is obtained from the zero-profit condition $\pi(z) = 0$:

$$r^F(z) = \left(\frac{\sigma}{\sigma - 1} - \theta - \beta \right) \frac{L(z)w(z)}{\beta}. \quad (20)$$

This defines the bid rent by an intermediate firm at production site z . Thus, the bid rent by intermediate firms is positive if and only if (19) is met.

Differentiating (20) with respect to z and using the envelop theorem yield:

$$\frac{dr^F(z)}{dz} = \left(\frac{\sigma}{\sigma - 1} - \theta - \beta \right) \frac{L(z)}{\beta} \frac{dw(z)}{dz}.$$

Plugging (18) in this expression, we obtain the following differential equation in w :

$$\frac{dw(z)}{dz} = \frac{(1 - \theta)(\sigma - 1)}{\sigma} \frac{w(z)}{S(z)} \frac{dS(z)}{dz}.$$

Solving this differential equation yields:

$$w(z) = C \cdot [S(z)]^{\frac{(1-\theta)(\sigma-1)}{\sigma}}, \quad (21)$$

where $C > 0$ is the constant of integration. Since $S(z)$ is strictly decreasing and strictly concave in z and $(1-\theta)(\sigma-1)/\sigma < 1$, *the equilibrium wage schedule is strictly decreasing and strictly concave in z* . In other words, the wage falls at a decreasing rate as the distance to the center of the cluster increases.

Plugging (1) into (17) leads to:

$$\beta \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{1}{L(z)} [R(z)^\theta \cdot (\bar{R}S(z))^{1-\theta} \cdot L(z)^\beta]^{\frac{\sigma-1}{\sigma}} = w(z).$$

Using (16) and (21), we obtain after simplifications the equilibrium mass of production workers hired by a firm located at z :

$$L(z) = \left[\left(\frac{\beta}{\theta} \right)^{\sigma-1} (C\beta)^\sigma \left(\frac{\sigma-1}{\sigma} \right)^{2\sigma} \cdot N^{-(1-\theta)(\sigma-1)} \right]^{\frac{1}{\beta(\sigma-1)-1}} \equiv L^*(N), \quad (22)$$

which is the same at each location z . It then follows from (16) that the equilibrium mass of R&D workers $R(z) = \theta L^*(N)/\beta$ is also constant for all $z \in Z^F$. Therefore, *intermediate firms hire the same number of production workers and the same number of researchers regardless of their location within the cluster*.

We may rewrite (20) as follows:

$$r^F(z) = \left(\frac{\sigma}{\sigma-1} - \theta - \beta \right) \frac{L^*(N)}{\beta} w(z), \quad (23)$$

which implies that the bid rent of an intermediate firm at z is proportional to the wage that prevails at this location. Furthermore, since $w(z)$ is concave in z , (23) also implies that *firms' bid rent function is strictly decreasing and strictly concave in z* :

$$r^F(z) = \left(\frac{\sigma}{\sigma-1} - \theta - \beta \right) \frac{L^*(N)}{\beta} C \cdot [S(z)]^{\frac{(1-\theta)(\sigma-1)}{\sigma}}.$$

Using (9), it follows from $M = \phi N$, $\bar{L} = NL^*$, and $\bar{R} = NR^* = (\theta/\beta)NL^*$ that

$$L^*(N) = \frac{\beta}{\beta + \theta} \left(\frac{H}{N} - \phi \right), \quad (24)$$

while (16) implies:

$$R^*(N) \equiv \frac{\theta}{\theta + \beta} \left(\frac{H}{N} - \phi \right). \quad (25)$$

Thus, the higher the elasticity of production with respect to R&D (θ), the more researchers are employed by this firms. To put it differently, when the spatial externality matters less ($\theta \uparrow$), each firm invests more in R&D. Note also that N is bounded above by H/ϕ because each firm must earn sufficiently high operating profits to cover the land rent.

Finally, using (1) and (2), it is readily verified that the equilibrium output of a firm located at z is equal to

$$x^*(z) = \frac{\theta}{\beta} \cdot [NS(z)]^{1-\theta} \cdot [L^*(N)]^{1+\beta}.$$

In other words, *firms' size decreases with the distance to the city center*. The equilibrium mass of firms is the outcome of two conflicting effects. It, on the one hand, reduces employment at each firm but, on the other hand, raises the extent of spatial externality. Substituting (24) into this expression, it is readily verified that the employment effect dominates, and hence the equilibrium size of a firm decreases with the mass of firms unambiguously.

3.3 Workers

We first determine the equilibrium mapping J from Z to Z that associates a (potential) job site $J(y) = z$ with a (potential) residential location y . This mapping describes the commuting pattern of workers. More specifically, a worker residing at y works at the location $J(y) = z$ that maximizes her net income:

$$w[J(y)] - t|y - J(y)| = \max_{z \in Z} [w(z) - t|y - z|], \quad y \in Z.$$

Solving workers' program shows that the maximum bid a worker can offer to reside at location y is given by,

$$r^W(y) = \max_z \{w[J(y)] - t|y - J(y)| - I(y, J(y)) \mid I(y, J(y)) = I_0\}. \quad (26)$$

This therefore defines the bid rent by a worker residing at location y .

The following result is intuitively obvious: in equilibrium, cross-commuting does not occur. Indeed, if two groups of workers cross-commute, any worker belonging to any of these groups would strictly increase her net income by choosing a job site in the area where the other group works. In other words, $J(y)$ increases in y . More specifically, we assume that the commuting function $J(y)$ is given by,

$$z = J(y) = \frac{N}{H}(y - N/2), \quad y \in [N/2, (H + N)/2]$$

so that workers living at $y = N/2$ work at $z = 0$, while those at $y = (H + N)/2$ work at $z = N/2$. Hence, a worker living in the residential area $[N/2, (H + N)/2]$ is assigned to a unique location belonging to $[0, N/2]$ with y and $J(y)$ varying in the same direction.

Combining (7) and (8) with (26), we obtain:

$$r^W(y) = w(z) - t|y - J(y)| - I_0, \quad (27)$$

which means that *workers' bid rent functions are linear and downward slopping in distance.*

4 The high-tech city

We determine the conditions for the spatial equilibrium to be a high-tech city in two steps. In the first one, given N , we identify the conditions for a high-tech city to emerge. In the second step, we determine the mass of intermediate firms. To produce clear-cut comparative results, we focus on symmetric firm distributions around the cluster center.

The equilibrium land rent $r^*(z)$ is the upper envelope of the two bid rent functions $r^F(z)$ and $r^W(z)$. In other words, whenever a firm or a worker locates at z , its bid rent must be equal to the equilibrium land rent:

$$\begin{aligned} r^*(z) &= \max \{ r^F(z), r^W(z), 1 \} \\ r^*(z) &= \begin{cases} r^F(z) & \text{if } h^F(z) > 0 \\ r^W(z) & \text{if } h^W(z) > 0 \end{cases} \\ r^*(-(N+H)/2) &= r^*((N+H)/2) = 1, \end{aligned}$$

where the bid rents $r^W(z)$ and $r^F(z)$ are given by (27) and (23), respectively. Since both $r^W(z)$ and $r^F(z)$ decrease with z , the equilibrium land rent also decreases as the distance to the center rises.

A *spatial equilibrium* is defined by the quantity vector $\{L^*, R^*, x^*(z), N^*\}$ and the price vector $\{p^*(z), w^*(z), r^*(z)\}$ for $z \in Z$ such that the following conditions are satisfied:

- (i) profits are zero in the final and intermediate sectors;
- (ii) land and labor markets clear;
- (iii) the location sets of intermediate firms and workers are $Z^F = [-N/2, N/2]$ and $Z^W = [-(H+N)/2, -N/2] \cup [N/2, (H+N)/2]$;
- (iv) market clearing for intermediate goods pins down the mass of intermediate firms;
- (v) population constraint: $\int_{Z^W} h^W(z) dz = H$.

The Walras Law implies that the final good market clears.

We have seen that the workers' bid rent schedule is linear and downward sloping while the firms' bid rent schedule is strictly decreasing and concave in z . In this case, the following two conditions are necessary and sufficient for a tech cluster to be sustained as a spatial equilibrium:

- (a) firms' and workers' bid rent schedules intersect at $z = N^*/2$;
- (b) firms outbid workers at $z = 0$.

Indeed, condition (a) is necessary and sufficient to guarantee that workers outbid firms outside the tech cluster, while condition (b) means that firms outbid workers within the tech cluster. It remains to check what conditions (a) and (b) are for a high-tech city to emerge as an equilibrium outcome (see Figure 1).

Insert Figure 1 about here

4.1 A high-tech city as a spatial equilibrium

We show in Appendix B that the constant of integration C in (21) is uniquely pinned down by the following equation:

$$\left(\frac{\sigma}{\sigma-1} - \theta - \beta \right) \frac{CL^*(N)}{\beta} \left(\frac{1 - e^{-\gamma N}}{\gamma} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}} = 1 + \frac{tH}{2}. \quad (28)$$

We next determine the condition for

$$r^F(0) > r^W(0) = 1 + t(N + H)/2, \quad (29)$$

to hold. That is, intermediate firms outbid workers toward the center of the high-tech city – a necessary condition for intermediate firms to cluster.

Plugging (21) in (23) for $z = 0$ and using (28) to replace $CL^*(N)$ yields the value of the firms' bid rent at $z = 0$:

$$r^F(0) = \left(1 + \frac{tH}{2} \right) \left(2 \frac{1 - e^{-\gamma N/2}}{1 - e^{-\gamma N}} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}.$$

Using this expression, (29) may be rewritten as follows:

$$\left(2 \frac{1 - e^{-\gamma N/2}}{1 - e^{-\gamma N}} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}} > \frac{2 + t(H + N)}{2 + tH}. \quad (30)$$

The left-hand side of (30) is independent of t and larger than 1 for all $N > 0$, while the right-hand side is increasing and equal to 1 at $t = 0$. Therefore, there exists a unique solution in t for (30) to hold in equality, which we denote $T(N)$.

Proposition 1. *For any given mass N of intermediate firms, a high-tech city is a spatial equilibrium if and only if $t < T(N)$.*

This proposition has an important policy implication: commuting must be fairly inexpensive for a high-tech city to emerge.⁴ This highlights the need for modern and efficient transportation and communication infrastructures in a tech cluster. As empirical evidence suggests that knowledge spillovers are very localized, it is worth studying how $T(N)$ varies with γ . Since the left-hand side of (30) shifts upward when γ rises, the value of $T(N)$ increases, which makes it easier for (30) to be satisfied.

Moreover, since the right-hand side of (30) shifts downward when H increases, the value of $T(N)$ increases as well. Therefore, we have:

Proposition 2. *For any given mass N of intermediate firms, very localized spillovers (high γ) or a bigger pool of skilled workers (high H) fosters the making of a tech cluster.*

4.2 The size of a tech cluster

We show in Appendix C that the equilibrium number of intermediate firms that sustains a tech cluster solves the following equation:

$$F(N) \equiv \frac{\sigma\phi}{\beta} \left(\frac{\sigma}{\sigma-1} \right)^2 \left(2 \frac{1 - e^{-\gamma N/2}}{1 - e^{-\gamma N}} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}} = \frac{\beta}{\beta + \theta} \left(\frac{H}{N} - \phi \right) \equiv G(N). \quad (31)$$

The function $F(N)$ is increasing in N with $F(0) > 0$, while $G(N)$ decreases in N with $G(0) \rightarrow \infty$ and $\lim_{N \rightarrow \infty} G(N) < 0$. Therefore, (31) has a unique solution, which yields the equilibrium mass of firms N^* in the intermediate sector.

Summarizing yields the following proposition.

Proposition 3. *If $t < T(N^*)$, then there exists a unique mass of intermediate firms N^* such that a high-tech city is a spatial equilibrium.*

Since $F(N) - G(N)$ is an increasing function of N , which is shifted upward with γ and ϕ but downward with H , we have the following proposition.

⁴Proposition 1 is in accordance with Ogawa and Fujita (1980) who show in a different setting that a monocentric city emerges when commuting costs are sufficiently small.

Proposition 4. *In a high-tech city, a bigger labor pool ($H \uparrow$), a weaker distance-decay effect ($\gamma \downarrow$), or a lower coordination cost ($\phi \downarrow$) leads to a higher mass of intermediate firms and a larger tech cluster.*

Intuitively, abundant availability of skilled workers or lower coordination cost enables high-tech production with a longer production line, thus inducing more intermediate firms to cluster and encouraging more R&D activities. Moreover, a weaker distance-decay effect counters the necessary dispersion of intermediate firms as a result of required land usage and the rising wage cost to compensate longer commuting by workers, thereby permitting a larger tech cluster to form in equilibrium.

We are now equipped to study how firm's R&D policy R^* varies with the main parameters of the cluster. We proceed in two steps. First, (25) is a decreasing function of N^* and does not involve γ . Therefore, since N^* decreases with γ from Proposition 4, R^* increases with γ . Second, substituting (25) into (31) shows that $R^* = (\theta/\beta)F(N^*)$, which is an increasing function of N^* . Since this expression does not include H and ϕ , Proposition 4 implies that N^* increases with H and decreases with ϕ , and thus the same holds for R^* . The following proposition summarizes our findings.

Proposition 5. *In a high-tech city, a bigger labor pool ($H \uparrow$), a stronger distance-decay effect ($\gamma \uparrow$), or a lower coordination cost ($\phi \downarrow$) induces firms to invest more in R&D.*

The intuition behind the impact of γ is worth noting: a stronger distance-decay effect incentivizes firms to invest more in R&D because they benefit less from what each firm does and interact with fewer firms.

How would the likelihood of a high-tech city change in response to these key parameters? Recall that the commuting cutoff $T(N)$ is pinned down by (30) in equality:

$$\left(2 \frac{1 - e^{-\gamma N/2}}{1 - e^{-\gamma N}}\right)^{(1-\theta)\frac{\sigma-1}{\sigma}} = \frac{2 + T(N)(H + N)}{2 + T(N)H},$$

while the equilibrium mass of firms N^* is determined by (31). Define $t^* \equiv T(N^*)$. Plugging (31) into the above expression yields a relationship between N^* and t^* :

$$\left(\frac{\sigma - 1}{\sigma}\right)^2 \frac{\beta^2}{\sigma\phi(\beta + \theta)} \left(\frac{H}{N^*} - \phi\right) = \frac{2 + t^*(H + N^*)}{2 + t^*H}. \quad (32)$$

Totally differentiating this expression and solving for dt^*/dN^* , we obtain:

$$\frac{dt^*}{dN^*} = -\frac{(2 + t^*H)^2}{2N^*} \left[\left(\frac{\sigma - 1}{\sigma}\right)^2 \frac{\beta^2}{\sigma\phi(\beta + \theta)} \frac{H}{(N^*)^2} + \frac{t^*}{2 + t^*H} \right] < 0.$$

In other words, the commuting cutoff t^* decreases with the equilibrium mass of intermediate firms N^* . Note that labor requirement for designers does not affect the commuting cutoff directly. Thus, Proposition 4 implies that a lower coordination cost ϕ would raise the mass of intermediate firms and reduce the likelihood for a high-tech city to form as t^* decreases. More specifically, consider an economy with continual improvements in infrastructure and communication technology that lowers coordination costs ϕ . At an initial equilibrium (N^*, t^*) , the condition for the formation of the high-tech city $t < t^*$ is met. With continual reduction in ϕ , the size of the high-tech city continues to grow until N^{**} at which $t = T(N^{**})$ holds, after which it ceases to grow even if ϕ continues to fall. At this stage a single high-tech city will be decentralized into multiple ones. This fragmentation of tech clusters may serve to explain the evolution of the Silicon Valley, which began with Palo Alto Industrial Park in 1951, expanded to Mountain View, then to Sunnyvale, Santa Clara and, eventually, San Jose and its adjacent municipals (Saxenian 1994, pp. 29-30).

Recall from Proposition 4 that a weaker distance-decay effect ($\gamma \downarrow$) makes high-tech city larger ($N^* \uparrow$). Rearranging (32) yields:

$$\frac{1}{N^*} \left[\left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{\beta^2}{\sigma \phi (\beta + \theta)} \left(\frac{H}{N^*} - \phi \right) - 1 \right] = \frac{t^*}{2 + t^* H},$$

which is not directly affected by γ . Since the left-hand side of this expression is a decreasing function of N^* while the right-hand side increases with t^* , the latter decreases with a weaker distance-decay effect ($\gamma \downarrow$). In other words, *more localized spillovers make high-tech city more likely to form, but Proposition 4 implies that the equilibrium size of the high-tech city is smaller.*

By contrast, the effect of a larger labor pool is a priori ambiguous. Indeed, for a given mass N of intermediate firms, Proposition 2 implies that a higher value of H raises the commuting cutoff t^* . However, Proposition 4 implies that a bigger labor pool also induces a larger equilibrium mass of intermediate firms. This in turn reduces the commuting cutoff. Yet, we show in the Appendix D that the cutoff t^* may decrease with the size of the labor pool when t^* takes intermediate values. Specifically, when the labor pool is sufficiently large, there exist two commuting rates $0 < t_1 < t_2$ such that $dt^*/dH < 0$ holds over (t_1, t_2) . Thus, when $t^* \in (t_1, t_2)$, a further increase in the labor pool leads to a lower cutoff $t^{**} < t^*$. Should the new cutoff t^{**} fall below the current commuting cost rate t , the city would turn out to host too many workers to sustain its tech cluster.

5 The flat city

The aim of this section is to highlight the impact of the spatial distribution of activities on the industrial structure of the city. To this end, we undertake a quasi-counterfactual in which the parameters of the economy are the same as in a high-tech city for the comparison to be meaningful, but where firms' and workers' locations are exogenously given. Specifically, we assume that they are fully dispersed. In such a spatial pattern (denoted with subscript D), workers and intermediate firms cohabit at each location. Since the total demand for land is $N + H$, the worker density is equal to $h_D^W = H/(N + H)$, while the firm density is $h_D^F = N/(N + H)$. Since firms and workers are uniformly distributed across the city, we refer to such a configuration as a *flat city*. We then compare the equilibrium wage schedules, land rents and welfare in a high-tech city and flat city. We acknowledge that the flat city is not a spatial equilibrium for the set of parameters under consideration. This is why we treat all locations parametrically. By contrast, we proceed exactly as in Section 4 to characterize the equilibrium values of the other variables.

In a flat city, the distance-decay effect is as follows:

$$S_D(z) = \frac{1}{\gamma} \frac{N}{N + H} \left[2 - e^{-\gamma \frac{N+H}{2}} (e^{-\gamma z} + e^{\gamma z}) \right] < S(z). \quad (33)$$

In other words, for the same research pool, *the spatial externality is stronger in a high-tech city than in a flat city*.

5.1 The size of a flat city

We show in Appendix E that the equilibrium mass N_D of intermediate firms is a solution to the equation

$$\frac{\sigma \phi}{\beta} \left(\frac{\sigma}{\sigma - 1} \right)^2 \left(2 \frac{1 - e^{-\gamma \frac{N+H}{2}}}{1 - e^{-\gamma(N+H)}} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}} = \frac{\beta}{\beta + \theta} \left(\frac{H}{N} - \phi \right), \quad (34)$$

which has a unique solution. Since the right-hand side of (31) and (34) are the same while the left-hand side of (34) lies above (31), we may conclude as follows:

Proposition 6. *There are more intermediate firms in a high-tech than in a flat city, i.e., $N^* > N_D$.*

The above results show that, in our setting, the efficiency of the intermediate and final sectors is endogenized through the R&D activities undertaken by intermediate firms and the size of the skilled labor pool, which both vary with the spatial configuration adopted by firms and workers.

5.2 Is a tech cluster more efficient than a flat city?

It is plausible that designing a production line is more costly in a flat city than in an agglomerated one because dispersion gives rise to higher coordination costs. We capture this idea by assuming that more coordinating workers are required in a flat city, i.e., $\phi < \phi_D$. How does this change in ϕ affect the structure of employment in the flat city?

Using (24), the total mass of workers employed in producing the intermediate goods is given by,

$$\bar{L} = NL^*(N) = \frac{\beta}{\beta + \theta} (H - \phi N). \quad (35)$$

Clearly, how \bar{L} varies with ϕ depends on whether the elasticity of the equilibrium mass of firms with respect to ϕ is larger or smaller than -1 . Total differentiation of (31) gives the elasticity of N with respect to ϕ :

$$\varepsilon(N) \equiv \frac{\phi}{N} \frac{dN}{d\phi} = - \frac{\frac{H}{N}}{\frac{H}{N} + (1 - \theta)\gamma \frac{\sigma-1}{\sigma} \left(\frac{1}{2} \frac{e^{-\gamma \frac{N}{2}}}{1 - e^{-\gamma \frac{N}{2}}} - \frac{e^{-\gamma N}}{1 - e^{-\gamma N}} \right) \left(\frac{H}{N} - \phi \right)} > -1.$$

From (35), \bar{L} is thus decreasing in ϕ . As a consequence, it follows from (9), (24) and (25) that $\bar{L}^* > \bar{L}_D^*$, $\bar{R}^* > \bar{R}_D^*$ and $M^* < M_D$ when $\phi < \phi_D$. In other words, *a high-tech city hosts more researchers and production workers than a flat city*. By implication, the final sector is more productive when a high-tech city is formed because more intermediate inputs are available while more workers are involved in production. To put it differently, the final sector benefits from a finer division of labor in a high-tech city. This may explain why it is rewarding for a city to host a tech cluster, as suggested by the empirical evidence provided by Delgado *et al.* (2012).

Note that the productivity of the final sector is entirely channelized through employment of researchers and designers, as well as by the total size of the researcher pool in the high-tech city. Moreover, these two effects are reinforced by the magnitude of the spatial spillover effect, which takes on its highest values when firms cluster in a single high-tech city. These results may serve to explain why the extent of the proximity effect may vary by location or by industry (see the survey by Carlino and Kerr, 2015, Section 4.3).

In addition, it follows from (25) that each firm hires fewer researchers in a high-tech city. Hence, even though each firm employs fewer researchers, a high-tech city involves a bigger pool of researchers and undertake more R&D through a larger mass of firms. Since it accommodates more intermediate firms, the size of the city that hosts a high-tech city is also larger than that of a city without a high-tech city.

We summarize the aforementioned results in the following proposition.

Proposition 7. *A high-tech city hosts more researchers, the intermediate sector involves more firms and the final sector is more productive than a flat city.*

5.3 Welfare, wages and land rent

Proposition 1 implies that firms outbid workers over Z^F in a high-tech city, whereas workers' and firms' bid rents are identical and linear in a flat city. Since the land rent at the city limit is equal to 1, *the land rent is higher in a high-tech city than in a flat city over Z^F* while it is the same over Z^W . This is because the higher benefits generated by the spatial externality in a high-tech city ($S(z) > S_D(z)$) are partially capitalized in the land rent.

Likewise, since $L^*(z)$ is smaller in a high-tech city than in a flat city, it follows from (23) that wages are higher in the former than in the latter over Z^F .⁵ Here too, wages capitalize the higher productivity triggered by the spatial externality in a high-tech city. Furthermore, using (23) again, as well as $L^* < L_D^*$ shows that *the wage gap between the high-tech city and the flat city is wider than the difference in land rents over Z^F* . In other words, everything else being equal, the workers benefit more from the clustering of intermediate firms than the land owners. To conclude, in a high-tech city workers earn higher wages but bear positive commuting costs. The difference in welfare is thus a priori ambiguous. To this end, we undertake a welfare comparison of the two configurations.

Using (7) shows that, in a high-tech city, the indirect utility level of a worker who works at $N^*/2$ and lives at $(N^* + H)/2$ is given by,

$$I\left(\frac{N^* + H}{2}\right) = w(N^*/2) - tH/2 - 1.$$

Plugging the equilibrium wage (21) and the constant of integration (28) in this expression and simplifying leads to:

$$I\left(\frac{N^* + H}{2}\right) = \left[\frac{\beta + \theta}{\left(\frac{\sigma}{\sigma-1} - \theta - \beta\right) \left(\frac{H}{N^*} - \phi\right)} - 1 \right] \left(1 + \frac{tH}{2}\right). \quad (36)$$

In a flat city, a worker who works and lives at $(N^* + H)/2$ enjoys an indirect utility level given by,

$$I_D\left(\frac{N_D + H}{2}\right) = \frac{\beta + \theta}{\left(\frac{\sigma}{\sigma-1} - \theta - \beta\right) \left(\frac{H}{N_D} - \phi\right)} - 1. \quad (37)$$

Since $N^* > N_D$, (36) and (37) imply:

$$I\left(\frac{N^* + H}{2}\right) > I_D\left(\frac{N_D + H}{2}\right).$$

⁵We cannot compare wages over $(-(N + H)/2, -N/2) \cup ((N + H)/2, N/2)$ because it is residential area in a high-tech city.

As the indirect utility level is independent of the location z , welfare is higher in a high-tech city, i.e., $I(z) > I_D(z)$ for any z . To put it differently, the finer division of labor in the final sector caused by the geographical concentration of firms leads to a higher utility level in the high-tech city than in the flat city.

To sum up, we have the following proposition.

Proposition 8. *Wages and land rents are higher in a high-tech city than in a flat city. Despite incurring higher commuting costs, workers are better-off in a high-tech city than in a flat city.*

This proposition also implies that, when the high-tech city is allowed to accommodate potential skilled workers from the rest of the world, the high-tech city attracts more workers than the flat city. As a consequence, the former hosts a bigger population of researchers than the latter, which makes the tech cluster even more productive.

6 Concluding remarks

In this paper, we have developed a spatial equilibrium model of a high-tech city in the presence of positive knowledge spillovers between monopolistically competitive intermediate firms. We have shown that a tech cluster hosts an intermediate sector in which firms' TFP is mainly driven by the aggregate R&D expenditure. With this proviso, a high-tech city is more likely to form under strongly localized knowledge spillovers, skilled labor abundance, and low commuting costs. We have also shown that, compared to a counterfactual flat city, a high-tech city grants higher productivity to the production of the final good, hosts more intermediate firms, more researchers and more production workers, and generates greater worker welfare. Finally, we have shown why continual improvements in infrastructure and communication technology that lowers coordination costs may lead to eventual fragmentation of tech clusters.

Although we have confirmed the social desirability of tech clusters from the viewpoints of worker welfare, it is worth checking whether the typical tech cluster promotion policies may work effectively. One of the most common instruments used by governments is to reduce firms' startup costs which can be translated into the reduction in the coordination cost for setting up the production line. Our analysis suggests that implementing such a policy need not achieve its goals. By contrast, subsidizing land may incentivize firms to cluster in a large tech cluster. More specifically, with subsidy Δr , the equilibrium land rent becomes $r_j(z) = \max \{r_j^F(z) + \Delta r, r_j^W(z)\}$. This generates a discontinuity in firms' bid rents at the border between industrial and residential areas, thus enabling firms to outbid workers and to

occupy more land within the inner area of the high-tech city. Having illustrated briefly the working of this land subsidy policy, a full welfare analysis of this and other plausible policy instruments requires, by maintaining neutral government revenues, careful treatments of the open-city issues mentioned above, which is left for future work.

We have considered a closed city model in which the total population is fixed. A promising step for future research is to determine the equilibrium population size of the tech cluster by using an open city setting in which workers are free to migrate in and out while the utility level is exogenously given by the best option available in the rest of the world. In this way, firms' incentive to cluster will interact with workers' incentive to migrate, potentially leading to richer equilibrium outcomes. Another important extension is to dig further into the black box of spillovers by assuming with Davis and Dingel (2019) that face-to-face contacts across workers are costly and freely chosen, and to determine the condition for a high-tech city to emerge in such a social environment. Equally important, like in Behrens *et al.* (2014), our setting should be extended to deal with an urban system that involves heterogeneous workers, as well as cities that host a tech cluster and cities that do not. These are likely to be rewarding avenues for future research.

Appendix

A. Second-Order Condition

To show that firms' profit functions are strictly concave in (R, L) , we compute the second derivatives of (13):

$$\frac{\partial^2 \pi}{\partial R^2(z)} = \theta \left(\theta \frac{\sigma-1}{\sigma} - 1 \right) \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{R(z)^2} < 0,$$

$$\frac{\partial^2 \pi}{\partial L^2(z)} = \beta \left(\beta \frac{\sigma-1}{\sigma} - 1 \right) \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{L(z)^2} < 0,$$

$$\frac{\partial^2 \pi}{\partial R(z) \partial L(z)} = \beta \theta \left(\frac{\sigma-1}{\sigma} \right)^3 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{L(z)R(z)}.$$

Since the Hessian matrix $H(z)$ of $\pi(z)$ is given by,

$$H = \begin{pmatrix} \theta \left(\theta \frac{\sigma-1}{\sigma} - 1 \right) \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{R(z)^2} & \beta \theta \left(\frac{\sigma-1}{\sigma} \right)^3 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{L(z)R(z)} \\ \beta \theta \left(\frac{\sigma-1}{\sigma} \right)^3 \frac{1}{L(i,z)R(i,z)} x_j^{\frac{\sigma-1}{\sigma}}(i,z) & \beta \left(\beta \frac{\sigma-1}{\sigma} - 1 \right) \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{L(z)R(z)} \end{pmatrix}.$$

the second-order condition holds if and only if $|H(z)| > 0$, that is,

$$|H| = \theta \beta \left(\theta \frac{\sigma-1}{\sigma} - 1 \right) \left(\frac{\sigma-1}{\sigma} \right)^4 \frac{x^{2\frac{\sigma-1}{\sigma}}(z)}{L(z)^2 R(z)^2} \left(\beta \frac{\sigma-1}{\sigma} - 1 \right) - \left[\beta \theta \left(\frac{\sigma-1}{\sigma} \right)^3 \frac{x^{\frac{\sigma-1}{\sigma}}(z)}{L(z)R(z)} \right]^2,$$

or, after simplifications,

$$|H| = \beta \theta \left(\frac{\sigma-1}{\sigma} \right)^5 \left(\frac{\sigma}{\sigma-1} - \theta - \beta \right) \frac{x_j^{2\frac{\sigma-1}{\sigma}}(i,z)}{R(i,z)^2 L(i,z)^2} > 0,$$

which is equivalent to (19). Q.E.D.

B. The constant of integration

We begin by determining the value of the constant of integration by equalizing firms' and workers' bit rents at $N/2$:

$$r^F(N/2) = \left(\frac{\sigma}{\sigma-1} - \theta - \beta \right) \frac{L^*(N)}{\beta} w(N/2) = 1 + \frac{tH}{2} = r^W(N/2). \quad (\text{B.1})$$

Plugging (21) evaluated at $z = N/2$ into (B.1), we obtain (28), which pins down the unique value of the constant of integration C . Q.E.D.

C. Proof of Proposition 3

Evaluating (21) at $z = 0$ yields:

$$w(0) = C \left(2 \frac{1 - e^{-\gamma N/2}}{\gamma} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}.$$

Plugging this expression into (12), we obtain:

$$x(N/2)^{\frac{\sigma-1}{\sigma}} = \sigma \phi C \left(2 \frac{1 - e^{-\gamma N/2}}{\gamma} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}},$$

which gives us the constant of integration:

$$C = \frac{x(N/2)^{\frac{\sigma-1}{\sigma}}}{\sigma \phi \left(2 \frac{1 - e^{-\gamma N/2}}{\gamma} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}}.$$

Substituting C into (22) yields:

$$L^*(N)^{\frac{\beta(\sigma-1)-1}{\sigma}} = \beta \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{x(N/2)^{\frac{\sigma-1}{\sigma}}}{\sigma \phi \left(2 \frac{1 - e^{-\gamma N/2}}{\gamma} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}} \frac{1}{\left(\frac{\theta}{\beta} N^{1-\theta} \right)^{\frac{\sigma-1}{\sigma}}}. \quad (\text{C.1})$$

Using (1), we obtain the equilibrium output:

$$x^*(N/2) = \frac{\theta}{\beta} L^*(N)^{1+\beta} N^{1-\theta} S(N/2)^{1-\theta} = \frac{\theta}{\beta} L^*(N)^{1+\beta} N^{1-\theta} \left[\frac{1}{\gamma} (1 - e^{-\gamma N}) \right]^{1-\theta}, \quad (\text{C.2})$$

because the marginal variety N is produced at the edge of the cluster.

Combining (C.1) and (C.2) yields:

$$L^*(N)^{\frac{\beta(\sigma-1)-1}{\sigma}} = \beta \left(\frac{\sigma-1}{\sigma} \right)^2 \cdot \frac{\left\{ \frac{\theta}{\beta} N^{1-\theta} L^*(N)^{1+\beta} \left[\frac{1}{\gamma} (1 - e^{-\gamma N}) \right]^{1-\theta} \right\}^{\frac{\sigma-1}{\sigma}}}{\sigma \phi \left[2 \left(1 - e^{-\gamma N/2} \right) \right]^{\frac{(1-\theta)(\sigma-1)}{\sigma}}} \cdot \frac{\gamma^{\frac{(1-\theta)(\sigma-1)}{\sigma}}}{\left(\frac{\theta}{\beta} N^{1-\theta} \right)^{\frac{\sigma-1}{\sigma}}},$$

or, after simplifications,

$$L^*(N) = \frac{\sigma \phi}{\beta} \left(\frac{\sigma}{\sigma-1} \right)^2 \left[\frac{2 \left(1 - e^{-\gamma N/2} \right)}{1 - e^{-\gamma N}} \right]^{\frac{(1-\theta)(\sigma-1)}{\sigma}}. \quad (\text{C.3})$$

Equalizing (24) and (C.3) leads to (31) that pins down the equilibrium mass of intermediate firms. Q.E.D.

D. The impact of H on $T(N^*)$

Differentiating (31) and (32) with respect to H yields at N^* :

$$F'(N)N' = \frac{N - HN'}{N^2},$$

$$\left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} \frac{N-HN'}{N^2} = \frac{(t^{*'}N+t^*N')(2+t^*H)-t^*N(t^{*'}H+t^*)}{(2+t^*H)^2},$$

or, after simplifications,

$$N' = \frac{1}{NF'(N) + \frac{H}{N}},$$

$$\frac{2t^{*'}N}{(2+t^*H)^2} = \frac{Nt^{*2}}{(2+t^*H)^2} + \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} \frac{1}{N} - \left[\left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} \frac{H}{N^2} + \frac{t^*}{(2+t^*H)} \right] N'.$$

Combining these two equations and simplifying, we obtain:

$$2t^{*'}N \left(NF'(N) + \frac{H}{N} \right) = N^2t^{*2}F'(N) + \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2(2+t^*H)^2}{(\beta+\theta)\sigma\phi} F'(N) - 2t^*,$$

or, equivalently,

$$\begin{aligned} 2(N^2F'(N) + H)t^{*'} &= \Psi(t^*) \\ &\equiv F'(N) \left[N^2 + H^2 \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} \right] t^{*2} \\ &\quad - 2 \left[1 - 2H \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} F'(N) \right] t^* + 4 \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} F'(N). \end{aligned} \quad (\text{D.1})$$

Since $\Psi(t^*)$ is a quadratic function of t^* , it has at most two roots t_1 and t_2 with $t_1 < t_2$.

We now determine conditions for both roots are positive. Note $\Psi(t^*)$ is minimized at

$$\bar{t} = \frac{1}{F'(N)} \frac{1 - 2H \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} F'(N)}{N^2 + H^2 \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi}}.$$

Since $\Psi(0) > 0$ and $\Psi(\cdot)$ is convex, $0 < t_1 < t_2$ holds if $\bar{t} > 0$ and $\Psi(\bar{t}) < 0$. Since $F'(N) > 0$, the former holds if

$$F'(N) < \left(\frac{\sigma}{\sigma-1}\right)^2 \frac{(\beta+\theta)\sigma\phi}{2H\beta^2}. \quad (\text{D.2})$$

Plugging \bar{t} in (D.1), we obtain after simplifications:

$$2 \left[N^2 + H^2 \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} \right] t^{*'}(\bar{t}) = 4 \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{\beta^2}{(\beta+\theta)\sigma\phi} - \frac{1}{F'(N)(N^2F'(N) + H)}. \quad (\text{D.3})$$

Since (D.2) implies that the highest admissible value of $F'(N)$ is given by,

$$F'(N) = \left(\frac{\sigma}{\sigma-1}\right)^2 \frac{(\beta+\theta)\sigma\phi}{2H\beta^2},$$

it then follows from (D.3) that $t^{*'}(\bar{t}) < 0$ when H is sufficiently large. By implication of (A.1), $\Psi(\bar{t}) < 0$ when H is large enough. Q.E.D.

E. Proof of Proposition 7

Without loss of generality, we assume that firms' and workers' bid rents are equal at the city edge $(N + H)/2$:

$$r_D^F((N + H)/2) = \left(\frac{\sigma}{\sigma - 1} - \theta - \beta \right) \frac{L_D^*(N)w_D((N + H)/2)}{\beta} = r_D^H((N + H)/2) = 1.$$

Plugging the corresponding equilibrium wage (21), we obtain:

$$\left(\frac{\sigma}{\sigma - 1} - \theta - \beta \right) C_D L_D^*(N) \left[S_D \left(\frac{N + H}{2} \right) \right]^{\frac{(1-\theta)(\sigma-1)}{\sigma}} = \beta.$$

Second, combining (12) and (21) yields the constant of integration:

$$C_D(N) = \frac{x_D(N/2)^{\frac{\sigma-1}{\sigma}}}{\sigma \phi [S_D(0)]^{(1-\theta)\frac{\sigma-1}{\sigma}}}.$$

Plugging $C_D(N)$ into (22) gives:

$$L_D^*(N)^{\frac{\beta(\sigma-1)-1}{\sigma}} = \beta \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{x_D(N/2)^{\frac{\sigma-1}{\sigma}}}{\sigma \phi S_D(0)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}} \frac{1}{\left(\frac{\theta}{\beta} N^{1-\theta} \right)^{\frac{\sigma-1}{\sigma}}}. \quad (\text{E.1})$$

Using (1), we obtain:

$$x_D(N/2) = \frac{\theta}{\beta} N^{1-\theta} (L_D^*(N))^{1+\beta} [S_D((N + H)/2)]^{1-\theta}. \quad (\text{E.2})$$

Combining (E.1) and (E.2) yields:

$$L_D^*(N)^{\frac{\beta(\sigma-1)-1}{\sigma}} = \beta \left(\frac{\sigma - 1}{\sigma} \right)^2 \frac{\left[\frac{\theta}{\beta} N^{1-\theta} \cdot (L_D^*(N))^{1+\beta} \cdot S_D((N + H)/2)^{1-\theta} \right]^{\frac{\sigma-1}{\sigma}}}{\sigma \phi S_D(0)^{\frac{(1-\theta)(\sigma-1)}{\sigma}}} \frac{1}{\left(\frac{\theta}{\beta} N^{1-\theta} \right)^{\frac{\sigma-1}{\sigma}}}.$$

Evaluating (33) at $z = 0$ and $z = (N + H)/2$ and simplifying leads to:

$$L_D(z) = \frac{\sigma \phi}{\beta} \left(\frac{\sigma}{\sigma - 1} \right)^2 \left(2 \frac{1 - e^{-\gamma \frac{N+H}{2}}}{1 - e^{-\gamma(N+H)}} \right)^{(1-\theta)\frac{\sigma-1}{\sigma}} \equiv L_D^*(N), \quad (\text{E.3})$$

which is independent of z , like in the case of a high-tech city. The same holds for $R_D^*(N)$.

Equalizing (E.3) and (24) yields the desired equation (34). Q.E.D.

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