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Abstract

We consider an international cartel whose members interact repeatedly in their own as well as in third-country segmented markets. Cartel discipline-an inverse measure of the degree of competition between firms-is endogenously determined by the cartel's incentive compatibility constraint (ICC), which links strategically markets that are seemingly unrelated. Owing to this linkage, trade cost reductions induce cartel members to adjust their sales, not only due to direct effects, but also due to spillover effects related to cartel discipline. We apply these ideas to preferential trade agreements (PTAs) and show that the indirect effects can give rise to trade diversion. We also characterize the welfare effects of preferential tariff cuts for all countries under various circumstances regarding the determination of external PTA trade policy. A persistent finding is that, in the absence of appropriate regulation, preferential trade liberalization can be welfare-reducing even when external policy is jointly optimal.

JEL-Codes: D430, F100, F120, F130, F150, L120, L130.

Keywords: multimarket contact, repeated interactions, constrained collusion, intra-industry trade, welfare, optimal trade policies.

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1 Introduction

Conventional wisdom holds that globalization intensifies competition, undermines the cohesion of international cartels, and improves efficiency. Yet, evidence on prosecuted cartels operating in multiple international markets suggests that they can thrive when trade costs fall due to cost-saving innovations or trade liberalization.[1] Are trade cost reductions conducive to collusion? If they are, how do they affect trade flows, prices, and overall welfare? In particular, do tariff cuts in preferential trade agreements (PTAs) dilute the market power of cartels and benefit PTA members? And what do they imply for the rest of the world (ROW)? This paper aims to address questions of this type.

To shed light on the possible connection between regional economic integration and collusive behavior, we develop a symmetric, segmented markets duopoly model (Brander, 1981; Krugman and Brander, 1983) in which the two firms located in distinct national markets collude via repeated interactions (Bernheim and Whinston, 1990).[2] Importantly, though, the operations of these firms are not confined within their home borders. They may engage in two-way trade and also export to third-country markets. Our approach is distinguished by its emphasis and treatment of the cartel's incentive compatibility constraint (ICC).[3] In contrast to static analyses of segmented markets—which, understandably, treat international sales as independent—the ICC links markets via a hitherto under-explored channel which serves as the cornerstone to our analysis: cartel discipline. With its help we are able to systematically explore the emergent spillover/feedback effects of economic

[1] Since the inception of the World Trade Organization (WTO) in 1995, over 400 agreements in goods and services have been notified to it. (https://www.wto.org/english/tratop_e/region_e/regfac_e.htm). In 2012 the European Commission fined seven international groups of companies from Taiwan, France, Netherlands, Japan and South Korea for collusive practices in the cathode ray tubes sector. (See *http://europa.eu/rapid/press-release_IP-12-1317_en.htm.)* The European Commission and the United States Department of Justice also charged four firms from Japan and South Korea for price-fixing, customer allocation, and the exchange of confidential information in the nucleotides (food flavor enhancers) sector. (For information on the decision of the European Commission see: *http://europa.eu/rapid/press-release_IP-02-1907_en.htm?locale=en.* The U.S. Department of Justice's decisions are available at: *http://www.justice.gov/atr/cases/f9200/9297.htm*, *http://www.justice.gov/opa/pr/2001/August/435at.htm*, and *http://www.justice.gov/atr/cases/f9300/9301.htm.*) A common feature of the activities of these firms is that their actions also affected third-county markets.

[2] Head and Spencer (2017) lamented the relative decline of interest in oligopoly models among trade economists and applauded the renewed interest in them with a number of supportive arguments. In their words, "...oligopoly is a robust characteristic of a broad set of industries in the US and around the world. Concentration of sales among the four largest firms is trending up across a wide variety of industries... While it is certainly possible to explain rising concentration and profit shares in a monopolistic competition model, the assumption of 'massless' firms does not accord with the data. Many industries are dominated worldwide by a few *massive* firms. Furthermore, the free-entry assumption made in most monopolistic competition models is hard to reconcile with the observation of large and rising profits" (p. 1423). These authors also rebutted the claim that policy recommendations based on oligopolistic models cannot be applied in practice. Also see Neary (2003, 2010) for powerful arguments in favor of using oligopoly models in international trade.

[3] See Harrington (2006) and the contributions to the related literature cited therein.

integration that static analyses fail to capture.

In our setting, firms interact in quantities and use trigger strategies to sustain collusion. Specifically, they promise to reward cooperative conduct with continued adherence to a collusive agreement while threatening to punish deviations from it with reversion to the Cournot-Nash equilibrium. Because these ideas shape the cartel's ICC, we begin our formal analysis with a description of the properties of global profits under collusive agreements (including pure monopoly), unilateral deviations, and Cournot-Nash competition.

We first detail how the minimum discount factor that sustains maximal collusion (i.e., pure monopoly) depends on *internal trade costs* (i.e., the costs that separate cartel members' home markets) and *external trade costs* (i.e., the costs that separate home and third-country markets).[4] This analysis is salient for two reasons: because it studies the stability of maximal collusion—which is of interest in its own right; and, perhaps more importantly for our purposes, because it serves as a benchmark to our subsequent exploration of cartel discipline, welfare and optimal policies. In this context, we find that the minimum discount factor is non-monotonic in trade costs and that the initial level of internal trade costs is paramount. For example, reductions in internal trade costs undermine the stability of maximal collusion when these costs are sufficiently low but bolster it when these costs are sufficiently high. In contrast, when internal trade costs are low (high), reductions in external trade costs facilitate (erode) collusive stability.

But while the emphasis on maximal collusion is warranted, the cartel may remain profitable even if maximal collusion is unsustainable by modifying its engagement in international markets. We address this issue by considering a cartel that maximizes the representative member's global profit subject to its ICC. This enables us to characterize the dependence of local output, trade flows and global profits on trade costs and *cartel discipline*. Still, cartel discipline is endogenous. Herein lies our first substantive contribution to the literature: our analysis of the ICC enables us to characterize the dependence of cartel discipline on trade costs, the sizes of the various markets, and time preferences.

When internal trade costs are sufficiently low (perhaps due to past successes in liberalizing trade), cartel members find it appealing to engage in cross hauling (i.e., maintain a

[4] A related strand of the existing literature has explored the effects of trade policy (in the form of tariffs or quotas) on collusion among domestic and/or foreign firms, operating in a single market (Davidson, 1984; Rotemberg and Saloner, 1989; Fung, 1992; Syropoulos, 1992). Pinto (1986) was the first to consider multimarket interactions in the context of the "reciprocal dumping" model of Brander and Krugman (1983). He showed that the monopoly outcome, which requires firms to separate markets geographically, is sustainable for certain discount factor values. Lommerud and Sørgard (2001) proved that choosing prices may make multimarket collusion easier to sustain, while the opposite holds true for quantity-setting collusion. Akinbosoye et al. (2012) demonstrated that trade liberalization enhances cartel stability when goods are close substitutes and initial trade cost levels are sufficiently high. Another related (though more distant) branch of the literature explored the effectiveness of leniency programs and the benefits of international antitrust cooperation in deterring multimarket collusion. Choi and Gerlach (2012) is a notable contribution.

presence in each other's market through exports). This is so because this activity, though costly, tempers deviation incentives and improves enforcement. Reciprocal reductions in internal trade costs also boost cartel discipline. But cartel discipline also depends on external trade costs. We find that, when cross hauling is present, external trade cost reductions also improve cartel discipline. Thus, the relationship between cartel discipline and external trade costs, too, hinges on the level of internal trade costs.

Of course, trade costs exert a direct effect on equilibrium quantities, prices and welfare. However, trade costs also affect these variables indirectly via cartel discipline. Herein lies our second contribution. After showing how cartel discipline affects production, prices and trade (and, through them, consumer surplus and overall welfare), we substantiate the emergence of cross market effects via cartel discipline. Our third contribution to the literature is related to PTAs. Specifically, we use these findings to characterize the combined (i.e., direct and feedback) effects of trade liberalization on the welfare of members and non-members as well as the jointly optimal trade policies within a PTA.[5]

Normally, *internal* trade cost reductions promote intra-industry trade and expand aggregate output. But when the ICC is active, these reductions may also strengthen cartel discipline thereby undermining the provision of aggregate output, increasing domestic prices, and harming consumers. Furthermore, the improved cartel discipline tends to reduce external trade flows, causing trade deflection that could harm ROW. By the same token, *external* trade cost reductions promote external trade and tend to benefit ROW. However, these reductions, too, may bolster cartel discipline, thereby generating a secondary (and possibly adverse) effect on welfare in ROW and the cartel hosts.

The trade costs we refer to above may take the form of import tariffs and/or export taxes (which are revenue-generating instruments) or the form of shipping costs (which absorb real resources). To understand the significance of the forces noted above in the context of PTAs, we focus on revenue-generating costs and analyze their welfare effects on PTA and ROW. We examine PTAs and the spillover effects of deepening integration in them under three scenarios regarding a PTA's common external policy: (i) it consists of fixed taxes on trade with ROW;[6] (ii) it is set in a way that keeps external trade flows fixed;[7] and (iii)

[5] A significant portion of the literature on PTAs considers two types of unions: customs unions (CUs) and free trade areas (FTAs). While both forms of regional integration normally aim to reduce internal barriers to trade there is an important difference. In CUs members determine the union's common external tariff structure jointly, whereas in FTAs members determine it independently.

[6] This type of intervention is consistent with Aricle XXIV of the GATT (now the WTO) which constrains PTA members not to raise their external tariffs beyond pre-integration levels. See Syropoulos (1999) and Mrázová et al. (2013), among numerous others.

[7] This intervention is related to Kemp and Wan (1976) who established, in the context of perfect competition, that there exists a common external tariff structure in CUs and appropriate compensating intra-union transfers that (i) leave welfare of non-members unaffected and (ii) improve welfare of members. Syropoulos (1999) and Bond et al. (2004) used Kemp-Wan adjustments as benchmarks to identify the welfare

it is jointly optimal.[8] We show that internal tariff cuts may affect adversely welfare of PTA members—a finding that normally does not arise in static segmented-markets based analysis. Strikingly, welfare of PTA members may fall under all circumstances regarding the determination of external policy we consider. The driving force behind this finding is the possible improvement in cartel discipline due to trade liberalization that supports domestic price hikes that reduce consumer surplus.

Our analysis also demonstrates that, when external taxes are fixed or optimally determined, trade may be diverted away from ROW and cause its welfare to fall. Thus, the deepening of regional integration may reduce world efficiency in the Pareto sense. We view this aspect of our work as providing theoretical support to the need for appropriate regulation within PTAs and the world economy as a whole.[9] A corollary to these ideas is that PTAs that aim to dismantle "virtually all" internal trade barriers—which is sanctioned by Article XXIV of the WTO and which is typical in FTAs and CUs—may be Pareto dominated by a policy that maintains some barriers to internal trade.

Our work is related to the contributions of Auqier and Caves (1979) and Brander and Spencer (1984) which also study the operation of cartels in export markets and emphasize their favorable terms-of-trade externalities for host countries. While these works emphasize the tension between terms-of-trade improvements and the exercise of domestic market power, there is a fundamental difference between them and ours: we treat cartel discipline as endogenous thereby generating new insights regarding welfare and the design of optimal policies.[10] Syropoulos (1992) and Bond and Syropoulos (2008) also treated firm conduct as endogenous. However, the former study abstracted from multimarket interactions and the latter could not address issues related to PTAs because it was based on a two-country model. By placing firm behavior at center stage, this paper captures the cross-market effects of regional integration and generates new insights that may be more relevant empirically.[11]

Having already commented on how our work contributes to the literature that is con-

implications of actual (as opposed to potential) tariff adjustments in CUs and FTAs.

[8] Syropoulos (1999) demonstrated that CU members have an incentive to dismantle all barriers to internal trade when their common external tariff is set optimally. Bond et al. (2004) argued that FTA members may find the complete elimination of internal tariffs unappealing due to the presence of uninternalized external terms-of-trade externalities.

[9] In our analysis of a PTA's optimal common external policy, we find that external taxes may be complements or substitutes to internal tariffs (e.g., Bagwell and Staiger, 1997; Syropoulos, 1999) depending on their initial levels. We also find that there exist conditions under which the optimal external tax is negative.

[10] As already emphasized, the endogeneity of cartel discipline links markets even though marginal costs are constant. Auqier and Caves (1979) are aware of this separation but do not explore it.

[11] The welfare portion of our analysis is related to Deltas et al. (2012) who find that monopoly may enhance welfare by reducing costly cross-hauling. These authors focus on horizontal differentiation à la Hotelling in segmented markets and find that collusion may be "consumer-surplus-enhancing" if trade costs are considerably high—so that the cartel expands the share of the efficiently produced variety by reducing its price to cover the entire market. Our work differs in model specifics and orientation.

cerned with the stability of collusion, it may be of value to reiterate how we depart from it. First, while most trade-related contributions focus exclusively on the sustainability of the monopoly outcome, we solve the cartel's constrained optimization problem and derive a measure of cartel discipline as part of the solution. Moreover, we characterize discipline and shed light on its implications for welfare. Second, to our knowledge, this is the first study to incorporate third-country markets into an analysis that explores the cross-country effects of preferential trade liberalization in the presence of collusion. Third, our focus on revenue-generating costs enables us to shed light on optimal external and internal PTA trade policies. It also helps shed light on the implications of resource-using trade costs.

The rest of the paper is organized as follows. Section 2 presents the nmodel and prepares the ground for our analysis of multimarket collusion. Section 3 describes the ICC and the stability of maximal collusion as a function of trade costs and other parameters of interest. This section also solves the cartel's optimization problem and characterizes the dependence of cartel discipline on trade costs, time preferences and the relative size of ROW. Section 4 examines the effects of trade costs on domestic output and international shipments. Section 5 focuses on welfare, paying special attention to the effects of revenue generating trade costs and the consequences of preferential trade liberalization. Section 6 concludes. All technical proofs have been placed in Appendices.

2 The Basic Model

We consider a duopoly model in which firm i is located in country $i \in I \equiv \{1, 2\}$ and produces a homogeneous good for delivery to three countries/markets indexed by $j \in J \equiv \{1, 2, ROW\}$. The two firms are the prospective cartel members. And, since these firms reside in countries 1 and 2, we label these countries "cartel hosts." For simplicity, we assume each firm's marginal cost of production is zero and ROW's internal demand for the cartel product is satisfied solely through imports.[12]

Let q_{ij} and $Q_j \equiv \sum_{i \in I} q_{ij}$ respectively capture the quantity supplied by firm $i \in I$ to market $j \in J$ and the total output delivered to market j. Consumer preferences in country j take the quasi-linear form $U_j = u(Q_j) + q_{0j}$, where q_{0j} captures the consumption of a numeraire (produced in positive quantities by perfectly competitive firms) and $u(Q_j) = AQ_j - \frac{1}{2}\beta_j Q_j^2$. Optimization in consumption gives $p_j \equiv p(Q_j) = \max(0, A - \beta_j Q_j)$, where A and β_j respectively capture the choke-off price and slope of the inverse demand function the non-numeraire good in market j. Since $1/\beta_j$ is the measure of identical consumers with

[12] The model can be extended to consider $n > 2$ cartel hosts with $m \geq 1$ firms in each and to identify ROW with a set of $s > 1$ independent countries, with each country containing a fringe of price-taking firms. Neither of these extensions changes substantively our analysis and our findings on the relationship between cartel discipline and trade costs.

identical preferences residing in country j, we may interpret it as country j's market size.

The profit function of firm i in market j is defined as $\pi_{ij} \equiv \pi_{ij}(q_{ij}, Q_j, t_{ij}) = [p(Q_j) - t_{ij}]\, q_{ij}$, where t_{ij} is a per unit trade cost of shipping firm i's product from its own market to market j. We assume $t_{ii} = 0$. As discussed in detail in Section 5, one can interpret t_{ij} as a geographic trade barrier that generates no revenues or as a trade tax.[13] Therefore, firm i's global profit is defined as

$$\Pi_i = \textstyle\sum_{j \in J} \pi_{ij}(q_{ij}, Q_j, t_{ij}) \quad \text{for} \quad i \in I. \tag{1}$$

We focus on firm interactions in quantities and use superscripts "N", "C" and "D" to identify functions and variables associated with the "Cournot-**N**ash" equilibrium, a "**C**ollusive" agreement, and an optimal "**D**eviation" from that agreement, respectively. To keep the notation simple and the analysis compact, we impose the following symmetry conditions and normalization:

$$\beta = \beta_{ROW} \text{ and } \beta_1 = \beta_2 = 1 \tag{C1}$$

$$t = t_{12} = t_{21} \geq 0 \tag{C2}$$

$$\tau = t_{1ROW} = t_{2ROW} \geq 0. \tag{C3}$$

Condition ($C1$) requires the markets of cartel hosts to be equally sized and normalizes the measure of their (common) size to unity. Thus, $1/\beta$ is a measure of ROW's *relative size*.[14] Conditions ($C2$) and ($C3$) impose symmetry on the *internal* costs (t) of trading between cartel hosts and the *external* costs (τ) of exporting to ROW, respectively. As we will see, in addition to bypassing the difficult problem of determining the allocation of market shares between asymmetric cartel members, ($C1$) – ($C3$) enable us to treat multimarket collusion as a constrained optimization problem. In particular, these conditions ensure the two firms face a symmetric environment that has the following implications: (i) Cournot-Nash quantities satisfy $q_{11}^N = q_{22}^N$, $q_{12}^N = q_{21}^N$ and $q_{1ROW}^N = q_{2ROW}^N$; (ii) the best-response functions of firms follow a similar pattern of symmetry; and (iii) we may focus on cartel agreements that involve identical actions by "mirror-image" firms. As we explain next, these conditions of symmetry help simplify notation and the related analysis.

Denote with $q \equiv (x, y, z)$ the triple of output levels the representative cartel member may supply to its own market (x), the market of the other host (y), and ROW (z).[15]

[13] The key difference between the technology-based and policy-related interpretations of trade costs is that the latter may generate revenues. (The case of trade subsidies can be explored by considering $t_{ij} < 0$.) Due to the partial-equilibrium nature of the model, this distinction between trade costs is inconsequential for the behavior of firms. However, it is important for our analysis of welfare.

[14] For brevity, hereafter we refer to β as "ROW size."

[15] We omit superscipt "C" from these variables to avoid clutter.

Moreover, let $q^N \equiv (x^N, y^N, z^N)$ be the triple associated with Cournot-Nash quantities and $q^D \equiv (x^D, y^D, z^D)$ the triple when a cartel member deviates optimally from q.

Starting with Cournot-Nash competition, one can show that $x^N = \min\left(\frac{A+t}{3}, \frac{A}{2}\right)$, $y^N = \max\left(\frac{A-2t}{3}, 0\right)$ and $z^N(\tau, \beta) = \max\left(\frac{A-\tau}{3\beta}, 0\right)$. Clearly, $\bar{t} \equiv A/2$ and $\bar{\tau} \equiv A$ are the lowest prohibitive trade cost levels to internal and external trade, respectively. One can now see that, provided $t < \bar{t}$, reciprocal reductions in internal trade costs ($t \downarrow$) spur internal trade ($y^N \uparrow$), partially displace local supplies ($x^N \downarrow$), and expand aggregate domestic output ($Q^N \uparrow$). However, because marginal costs are constant, the quantities delivered to the three markets are independent (i.e., markets are segmented). Thus, in the Cournot-Nash equilibrium, changes in t *do not affect* shipments to ROW (z^N). Similarly, while reductions in external trade costs ($\tau \downarrow$) expand z^N, they do not affect either x^N or y^N.[16]

The representative firm's global profit under Cournot-Nash competition is

$$\Pi^N = \frac{1}{9}(A+t)^2 + \frac{1}{9}(A-2t)^2 + \frac{1}{9\beta}(A-\tau)^2, \tag{2}$$

for $t \leq \bar{t}$ and $\tau \leq \bar{\tau}$, and is strictly convex in these costs. Reciprocal reductions in t enhance the appeal of exporting to the other firm's host but also reduce a firm's profitability in its own market. This renders Π^N non-monotonic in t with $\arg\min_t \Pi^N = A/5$. Moreover, $\Pi^N(0, \tau, \beta) < \Pi^N(\bar{t}, \tau, \beta)$ because industry profits are lower under a symmetric duopoly than under a pure monopoly with a similar cost structure. Turning to the role of τ and β, we find that $\Pi^N_\tau \equiv \partial\Pi^N/\partial\tau < 0$ and $\Pi^N_\beta < 0$, with $\Pi^N_{\tau\tau} > 0$ and $\Pi^N_{\beta\beta} > 0$ for $\tau < \bar{\tau}$. In words, larger external trade costs ($\tau \uparrow$) and/or lower ROW size ($\beta \uparrow$) deliver lower global profit in the Nash equilibrium.

The representative cartel member's global profit under collusion is

$$\Pi^C = (A - x - y)(x + y) - ty + (A - \tau - 2\beta z) z \quad \text{for} \quad x, y, z \geq 0, \tag{3}$$

which is concave in $q \equiv (x, y, z)$. The first two terms in (3) capture a firm's profit obtained in the two hosts. The third term captures its profit in ROW (π_{ROW}). A special case of a collusive agreement is when cartel members solve $\max_q \Pi^C$ in the absence of antitrust regulation and/or incentive compatibility issues. The solution to this (unconstrained optimization) problem involves geographic separation of markets and is identified with "pure monopoly" or "maximal collusion", which we capture with superscript "M". If internal trade costs are absent ($t = 0$), any combination of x and y that satisfies $x + y = \frac{A}{2}$ $(= Q^M)$ satisfies this solution. However, if $t > 0$ the cartel can avoid trade costs by foreclosing on

[16] The just noted independence of output decisions across national markets is standard in this context. As we will see, this independence may disappear when firms collude.

internal trade ($y^M = 0$) and supplying the monopoly output locally ($x^M = Q^M$). Moreover, for $\tau \leq \bar{\tau}$, exports to ROW are $z^M = \frac{A-\tau}{4\beta} = \arg\max_z \pi_{ROW}$. One can verify that the per firm global profit under pure monopoly is

$$\Pi^M = \frac{1}{4}A^2 + \frac{1}{8\beta}(A-\tau)^2, \text{ for } \tau \leq \bar{\tau}. \tag{4}$$

Clearly, Π^M is convex and decreasing in τ.

To prepare the ground for our upcoming analysis of the ICC, one must also examine a cartel member's incentive to deviate from q. A firm's best-response to q is to deviate optimally in all markets and supply: $x^D(y) = \max\left(\frac{A-y}{2}, 0\right)$, $y^D(x;t) = \max\left(\frac{A-t-x}{2}, 0\right)$, and $z^D(z;\tau,\beta) = \max\left(\frac{A-\tau-\beta z}{2\beta}, 0\right)$. Substituting these quantities in (1) and simplifying a bit delivers the following global profit under an optimal deviation from q:

$$\Pi^D = \left(x^D\right)^2 + \left(y^D\right)^2 + \beta\left(z^D\right)^2. \tag{5}$$

Provided trade costs are below their prohibitive levels, Π^D is strictly convex and increasing in q. Moreover, for any feasible q, reductions in internal and/or external trade costs enhance the deviating firm's profit by enabling it to expand its volume of exports to the relevant market. Additionally, Π^D is decreasing and strictly convex in t, τ and β.

3 Multimarket Collusion

The theory of repeated games suggests that recurrent contact enables firms to sustain collusion through strategies that reward "cooperation" with adherence to cartel agreements and punish defections with "retaliation." Multimarket contact enables firms to sustain more collusion by pooling their incentive constraints across markets (Bernheim and Whinston, 1990). Focusing on cartel agreements that allocate a triple $q = (x, y, z)$ to each cartel member, we assume that firms "enforce" collusion with a grim trigger strategy that prescribes adherence to the provision of q if all firms supply it and permanent reversion to the Cournot-Nash equilibrium if a firm defects.[17] Our objective in this section is to characterize the most profitable, incentive-compatible cartel agreement. We pursue this objective in three steps. *First*, we describe a cartel member's ICC. *Second*, we study the stability of maximal collusion and its dependence on internal (t) and external (τ) trade costs. (This analysis also deepens our understanding of the circumstances under which the ICC is active

[17] We chose reversion to the static Nash equilibrium as the punishment mechanism for two reasons. Because this allows us to compare how the economies would perform under collusion relative to "competition" (when "competition" is identified with the provision of q^N); and to render our analysis comparable to numerous other contributions in the trade the literature that adopted a similar approach. Later on we briefly discuss how the analysis may change if firms choose more severe punishments.

or inactive.) *Third*, we develop our theory of endogenous cartel discipline and study its dependence on parameters, including trade costs.

3.1 The Cartel Problem

Let δ be the common discount factor to all firms. A cartel member finds supplying q appealing if

$$\Phi(q,t,\tau,\beta,\delta) \equiv \Pi^{C}(q,t,\tau,\beta) - (1-\delta)\Pi^{D}(q,t,\tau,\beta) - \delta\Pi^{N}(t,\tau,\beta) \geq 0, \tag{6}$$

where Π^C, Π^D and Π^N respectively denote the global profit under a collusive agreement q, an optimal deviation from it, and the Nash punishment. (ICC) requires q to generate a collusive global profit that exceeds the weighted sum of the global profit under an optimal deviation and reversion to the Cournot-Nash equilibrium. By virtue of the facts that Π^C is concave in q and Π^D strictly convex in q, $\Phi(\cdot)$ is strictly concave in q. The set of incentive-compatible agreements $F(q,t,\tau,\beta,\delta) \equiv \{q \mid \Phi(\cdot) \geq 0 \text{ and } q \geq 0\}$ is convex in q.

We may now describe the cartel's optimization problem as $\max_q \Pi^C(q,t,\tau,\beta)$, s.t. $q \in F(q,t,\tau,\beta,\delta)$. The solution to this problem is captured by the saddle point problem of the Lagrangian function

$$\max_{q} \min_{\lambda,\mu} \mathcal{L}(q,\lambda,t,\tau,\beta,\delta) = \Pi^{C}(q,t,\tau,\beta) + \lambda\Phi(q,t,\tau,\beta,\delta) + \mu y, \tag{7}$$

where $\lambda \geq 0$ and $\mu \geq 0$ are the Lagrange multipliers associated with (ICC) and the non-negativity of internal trade $(y \geq 0)$, respectively.[18] The first-order conditions (FOCs) for an interior solution to (7) are

$$\begin{aligned} &\Phi(q^*) \geq 0,\ \ \lambda^* \geq 0,\ \ \lambda^*\Phi(q^*) = 0,\ \ y^* \geq 0,\ \ \mu^* \geq 0,\ \ \mu^* y^* = 0, \\ &\nabla\mathcal{L}(q^*,\lambda^*,\mu^*) = \nabla\Pi^{C}(q^*) + \lambda^*\nabla\Phi(q^*) + \mu^* = 0, \end{aligned} \tag{8}$$

where a star "$*$" identifies the solution.

3.2 Stability of the Monopoly Outcome

Denote with $\delta^M(t,\tau,\beta)$ the minimum discount factor that ensures $q^* = q^M$. This is the value of δ, labeled δ^M, that solves $\Phi\left(q^M(\tau,\beta), t,\tau,\beta,\delta\right) = 0$ in (ICC) (and, consequently,

[18] The non-negativity constraint $y \geq 0$ must be included in the Langrangian function because, as we will see, the volume of intra-industy trade y vanishes at some $t < \bar{t}$.

implies $\lambda^* = 0$) and is given by

$$\delta^M \equiv \delta^M(t, \tau, \beta) = \frac{\Pi^D(q^M(\tau, \beta), t, \beta) - \Pi^M(\tau, \beta)}{\Pi^D(q^M(\tau, \beta), t, \beta) - \Pi^N(t, \tau, \beta)}. \tag{9}$$

We will now examine the dependence of $\delta^M(\cdot)$ on parameters. A noteworthy benefit of this exercise is that it helps study the stability of maximal collusion and sheds light on the circumstances under which the ICC is inactive ($\lambda^* = 0$) as well as the circumstances under which the ICC is active ($\lambda^* > 0$). The latter point is especially important because, as noted earlier, it serves as a stepping stone to our characterization of collusive optima.

We have already seen how trade costs and market size affect Π^M and Π^N. To complete our analysis of $\delta^M(\cdot)$ we must also characterize $\Pi^D\left(q^M(\tau, \beta), t, \beta\right)$. Suppose $t = 0$ initially, which implies that the monopoly outcome in the cartel's home markets can be implemented with combinations of x and y that satisfy $x + y = Q^M$. This observation combined with the strict convexity of Π^D in (x, y) though implies this: It may be in the interest of cartel members to maintain a presence in each other's market (i.e., engage in cross hauling) because doing so reduces Π^D.[19]

Now consider a small $t > 0$. To avoid trade costs the cartel could foreclose on internal trade ($y^M = 0$). If at the same time the cartel supplied the monopoly output ($x^M = Q^M$) it could potentially still obtain Π^M. However, this strategy also raises the incentive to defect (i.e., $\Pi^D \uparrow$). In short, δ^M is discontinuous in t at $t = 0$. This discontinuity has important implications for the dependence of cartel discipline on internal trade costs t (see below).

Turning to external trade costs, there are two possibilities: either $\tau \geq \bar{\tau}$ or $\tau < \bar{\tau}$. We unveil the dependence of $\delta^M(\cdot)$ on trade costs in the former case in Proposition $A1$ of Appendix A. This case, which effectively assumes the absence ROW, has been treated in Bond and Syropoulos (2008). We consider it because it serves as a useful benchmark in the latter case (where $\tau < \bar{\tau}$) that highlights the importance of external trade. In addition to the discontinuity of δ^M at $t = 0$ noted above, a valuable result due to Bond and Syropoulos (2008) is that δ^M is monotonically decreasing in t with $\lim_{t \to \bar{t}} \delta^M \to 0$, as shown by the downward sloping dashed-line schedule in Fig. 1.

Henceforth, we focus on $\beta < \infty$ and $\tau < \bar{\tau}$. Letting $\underline{\delta} \equiv \frac{9}{17}$ and $\widehat{\delta} \equiv \lim_{t \to 0} \delta^M(t, \tau, \beta)$, we may describe the stability of maximal collusion as follows:

Proposition 1 *(Stability of maximal collusion) Provided $\tau < \bar{\tau}$, the minimum discount factor that sustains maximal collusion, $\delta^M(t, \tau, \beta)$, has the following properties:*

a) Internal trade costs (t)

i) $\delta^M(0, \tau, \beta) = \delta^M(\bar{t}, \tau, \beta) = \underline{\delta} < \widehat{\delta}$;

[19] Π^D is lowest when $x = y = Q^M/2$ and largest when $x = Q^M$ while $y = 0$.

ii) $t \lesseqqgtr t_1 \Rightarrow \delta^M \gtreqqless \underline{\delta}$, *where* $t_1 = \left[\delta^M\right]^{-1}(\underline{\delta}, \cdot) = \frac{A}{14}$;

iii) $t \lesseqqgtr t_2 \Rightarrow d\delta^M/dt \lesseqqgtr 0$, *where* $t_2 \equiv \arg\min_t \delta^M$.

b) *External trade costs* (τ)

$t \lesseqqgtr t_1 \Rightarrow d\delta^M/d\tau \gtreqqless 0$.

c) *Market size* (β)

i) $t \lesseqqgtr t_1 \Rightarrow d\delta^M/d\beta \gtreqqless 0$;

ii) $\lim_{\beta\to 0}\delta^M = \underline{\delta}$ *and* $\lim_{\beta\to\infty}\delta^M = \frac{18(\bar{t}-t)}{13A+22t}$.

Proposition 1, which can be visualized with the help of the thick, solid-line, blue curve in Fig. 1, highlights the behavior of δ^M when export opportunities to ROW are present. Part (a) reveals that, while the discontinuity of δ^M at $t = 0$ is still present (compare $\underline{\delta}$ and $\widehat{\delta}$), δ^M now is a U-shaped function of internal trade costs t.[20] In particular, as t rises above 0, δ^M falls below $\underline{\delta}$ to reach a minimum at $\underline{\underline{\delta}} \equiv \min_t \delta^M(t, \tau, \beta)$ and then rises back to $\underline{\delta}$ and remains there for $t \geq \bar{t}$. Thus, in contrast to the case of no-trade with ROW ($\tau \geq \bar{\tau}$) studied in Bond and Syropoulos (2008), the cartel's ability to access third-country markets via exports alters substantively the link between collusive stability and trade costs t.

The intuition behind the U-shaped relationship noted above is simple. Because Π^M is independent of internal trade costs t the effect of t on δ^M is solely due to changes in the deviation and punishment payoffs Π^D and Π^N, respectively. For $t < \bar{t}$, an increase in t reduces Π^D because it raises the cost of cross hauling. This reduction in Π^D causes δ^M to fall and thus facilitates collusion.[21] But, as noted earlier, Π^N is U-shaped in t with $\arg\min_t \Pi^N = A/5$. Thus, when t is sufficiently low, δ^M falls because defection becomes less appealing and punishment more severe. However, as t approaches the prohibitive level $\bar{t}$, δ^M must rise back to $\underline{\delta}$ because only the effects on global profits in ROW matter.[22] The continuity of δ^M implies that it is minimized at some internal trade cost level t_2.[23]

The trade cost level t_1 noted in part ($a.ii$) is a "pivot" point that clarifies how the initial level of t conditions the dependence of δ^M on external trade costs τ and market size β. Parts (b) and (c) elaborate on this dependence.[24] These parts also clarify how the initial

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[20] As explained in part ($a.i$), $\underline{\delta}$ is the minimum discount factor that supports the monopoly outcome in ROW (i.e., $\underline{\delta} = \frac{\pi^D_{ROW} - \pi^M_{ROW}}{\pi^D_{ROW} - \pi^N_{ROW}}$). Moreover, when $t = 0$, $\underline{\delta}$ is also the the minimum discount that sustains the monopoly outcome in the home markets of the cartel. This explains why $\delta^M(0, \tau, \beta) = \underline{\delta}$ in all markets.

[21] This effect vanishes $t \to \bar{t}$ and $\partial\Pi^D/\partial t < 0$ for $t < \bar{t}$, $\lim_{t\to\bar{t}} \partial\Pi^D/\partial t = 0$.

[22] In other words, $\lim_{t\to\bar{t}}\left(\Pi^D - \Pi^M\right) = \pi^D_{ROW} - \pi^M_{ROW}$ and $\lim_{t\to\bar{t}}\left(\Pi^D - \Pi^N\right) = \pi^D_{ROW} - \pi^N_{ROW}$ which imply $\lim_{t\to\bar{t}}\delta^M = \underline{\delta}$. When there is no trade with ROW ($\tau \geq \bar{\tau}$), the negative effect of t on δ^M through Π^D always dominates the opposing effect through Π^N as $t \to \bar{t}$.

[23] In Appendix A we establish the following properties of t_2: $\partial t_2/\partial\tau > 0$, $\partial t_2/\partial\beta > 0$ and $\lim_{\tau\to\bar{\tau}} t_2 = \lim_{\beta\to\infty} t_2 = \bar{t}$.

[24] Fig. 1 also clarifies the dependence of δ^M on τ by highlighting its shape for two extreme values: $\tau = 0$ and $\tau = \bar{\tau}$. At low (high) t values, increases in τ away from 0 cause δ^M to rotate clockwise around pivot

levels of τ and β shape the relationship between δ^M and t.[25]

Proposition 1 implies that the ICC is *inactive* at $t = 0$ for $\delta > \underline{\delta}$ and at $t > 0$ for $\delta \geq \delta^M$. The same logic also points out that the ICC is *active* at $t = 0$ for $\delta < \underline{\delta}$ and at $t \in (0, \bar{t})$ for $\delta < \delta^M$. The non-monotonicity of δ^M in t suggests that the ICC is also *inactive* at intermediate levels of t for $\delta \in [\underline{\underline{\delta}}, \underline{\delta})$. Once again, the U-shaped dependence of δ^M on t is due to the presence of export opportunities to ROW.

3.3 Cartel Discipline

Having just shown the sense in which the presence of third-country markets affects the stability of maximal collusion, we now turn to the determination of cartel discipline when the ICC in active. To do this we relate the shadow price of the ICC constraint (i.e., λ which is endogenous) to cartel discipline.

Accordingly, define $\theta \equiv \frac{\lambda(1-\delta)}{1+\lambda}$. Henceforth, we view θ as an *inverse* measure of cartel discipline. (This is so because θ is increasing in λ.)[26] Also note that $\theta \in [0, 1)$ (because $\lambda \geq 0$ and $\delta \in [0, 1)$). To deepen one's understanding of the importance of θ, it helps to temporarily treat it as a parameter. Consider, for example, the extreme cases of $\theta = 0$ and $\theta \to 1$. The former case arises when the ICC is inactive and is identified with pure monopoly (i.e., $\theta = 0$ implies $q = q^M$). The latter case is associated with Cournot-Nash competition (so $\theta \to 1$ implies $q \to q^N$). Thus, $\theta \in [0, 1)$ spans the entire spectrum of collusive outcomes (or, alternatively, captures various degrees of "competition"). Importantly, according to our interpretation, the lower the value of θ the more robust "cartel discipline."

First note that if t is sufficiently low and $\delta < \widehat{\delta}$, cross hauling is profitable ($y > 0$) and sustainable. On the other hand, if internal trade costs t are sufficiently large (not necessarily larger than $\bar{t}$), cross hauling becomes unprofitable ($y = 0$). This suggests that, for sufficiently low discount factor values, there will exist a range of internal trade costs under which both the ICC and the non-negativity constraint on y will be binding. To address this issue we separate the analysis into two distinct cases, identified with superscripts "1" and "2", respectively, and study them sequentially. Case 1 below focuses on $y > 0$; case 2 examines $y = 0$.[27]

Denote with $\Phi^1 \equiv \Phi^1(\theta, \delta, t, \tau, \beta) = 0$ and $\Phi^2 \equiv \Phi^2(\theta, \delta, t, \tau, \beta) = 0$ the ICC in the two cases noted above, respectively. Henceforth we focus on discount factor values that

point F. This substantiates the idea that increases in external trade costs τ are anti-collusive for $t < t_1$ but pro-collusive for $t > t_1$, as noted in part (*b*).

[25] One can also show that an increase in the number of cartel members would shift δ^M upwards (which would imply that the difficulty of sustaining collusion would rise).

[26] Since $\theta_\lambda \equiv \partial\theta/\partial\lambda > 0$, we may use θ and λ interchangeably.

[27] A third possibility is that $y = 0$ even when the ICC is inactive. We address this possibility later by writing the ICC as a weak inequality. See Lemma 3 for a description of the values of t that imply $y > 0$ and $y = 0$, respectively.

satisfy $\delta < \widehat{\delta}$ because q^M is sustainable for all $\delta \geq \widehat{\delta}$. (The solution (x^*, y^*) at $t = 0$ and $\delta \in [\underline{\delta}, \widehat{\delta})$ is a correspondence because the ICC is inactive.) To reduce the dimensionality of the problem and develop intuition we, first, use the FOCs in (8) to express the cartel's constrained optimal output levels as functions of θ, our inverse measure of cartel discipline. We then substitute these values back into $\Phi^1 = 0$ and $\Phi^2 = 0$ to obtain θ^* (for which there is no explicit solution).

When Cross Hauling is Present ($y > 0$ and $\mu = 0$)**.** Focusing on $y > 0$, we may rewrite the FOCs in (8) as

$$\mathcal{L}_j^1 = \Pi_j^C + \lambda \Phi_j^1 = 0 \quad \Rightarrow \quad \Pi_j^C = \theta \Pi_j^D < 0, \;\; j \in \{x, y, z\}, \tag{10}$$

where subscript j now denotes a partial derivative (e.g., $\mathcal{L}_j^1 = \partial \mathcal{L}^1 / \partial j$). The equality in the right-hand side (RHS) of (10) is obtained by utilizing the fact that $\Phi_j^1 = \Pi_j^C - (1-\delta)\,\Pi_j^D$ and the definition of θ ($\equiv \frac{\lambda(1-\delta)}{1+\lambda}$), and implies $\Phi_j^1 = (1-\delta-\theta)\,(-\Pi_j^D) > 0$.

Keeping in mind that $Q^1 = x^1 + y^1$, we can write the solution to (10) as

$$Q^1(\theta, t) = \frac{(2A - t)(2+\theta)}{8+\theta}, \tag{11a}$$

$$x^1(\theta, t) = \frac{1}{2}\left(Q^1 + \frac{2-\theta}{\theta} t\right), \tag{11b}$$

$$y^1(\theta, t) = \frac{1}{2}\left(Q^1 - \frac{2-\theta}{\theta} t\right), \tag{11c}$$

$$z^1(\theta, \tau, \beta) = \frac{(A - \tau)(2+\theta)}{\beta(8+\theta)}. \tag{11d}$$

The equations in (11) unveil how the cartel's local deliveries and export supplies depend on trade costs, market size and, of course, discipline. One can verify that Q^1, y^1 and z^1 are increasing and concave in θ.[28] Thus, the more disciplined the cartel ($\theta \downarrow$), the lower the volumes of aggregate output (Q^1) and internal (y^1) and external (z^1) shipments. Moreover, Q^1 and y^1 (z^1) are linear and decreasing in internal (external) trade costs t (τ). Notably, x^1 and y^1 (and thus Q^1) do not depend directly on external trade costs (τ) and market size (β). Similarly, z^1 does not depend directly on internal trade costs (t).

Focusing on y^1, one can verify from (11c) that there exists a positively-sloped schedule $t_o(\theta) \equiv \frac{A\theta(2+\theta)}{2(4-\theta)}$, such that $y^1(\theta, t_o(\theta)) = 0$ for any $\theta \in [0,1]$.[29] Henceforth, it is convenient to work with the inverse $\theta_o(t) \equiv t_o^{-1}(t)$ for $t \in [0, \bar{t}]$. This function, which is increasing and concave in t (see Fig. 2), divides the space $[0,\bar{t}] \times [0,1]$ of (t,θ) as follows: $y^1 \gtreqqless 0$

[28] See the proof of Lemma 1 below for a detailed description of the properties of these functions. Inspection of (11a) confirms our earlier claim that $\theta \to 1$ implies $Q^1 \to Q^N$ and $\theta = 0$ implies $Q^1 = Q^M$.

[29] This schedule has the following properties: $t_o(0) = 0$ and $t_o(1) = \bar{t}$; $t_o'(\theta) > 0$ and $t_o''(\theta) > 0$.

if $\theta \gtreqless \theta_o(t)$. Our current requirement that $y^1 > 0$ implies that (11) holds true only for $\theta > \theta_o(t)$.[30] In words, cross hauling arises for any non-prohibitive internal trade cost level if cartel discipline is sufficiently lax.

Turning to the behavior of local production x^1, differentiation of (11b) gives $x_t^1 > 0$, which conforms to economic intuition: Local cartel deliveries serve as a substitute for more expensive imports from a cartel host. One can also verify that x^1 is strictly quasi-convex in θ and $x_\theta^1 \gtreqless 0$ for $t \lesseqgtr t_d$, where $t_d \equiv \frac{3A\theta^2}{2(16+4\theta+\theta^2)} < t_o(\theta)$. Thus, x^1 falls with improvements in cartel discipline ($\theta \downarrow$) when t is sufficiently small but rises when t is large enough.

We now take a closer look at the determination of cartel discipline θ^{1*}. Substituting q^1 from (11) into $\Phi^1 = 0$ defines $\theta^{1*} \equiv \theta^{1*}(\delta, t, \tau, \beta)$ implicitly. Naturally, $\theta^{1*} = 1$ for $t \in [0, \bar{t}]$ is a generic solution associated with $q^1 = q^N$. But there also exists another (more "collusive") solution $\theta^{1*} \in [0, 1)$.

Lemma 1 *(Cartel discipline with cross hauling) Suppose $\delta < \widehat{\delta}$. Then $\Phi^1(\theta, \cdot) = 0$ has a unique interior solution $\theta^{1*} \equiv \theta^{1*}(t, \cdot)$ which has the following properties. If $t = 0$, then $\theta^{1*} = \max(\theta_g, 0)$, where $\theta_g \equiv \frac{17(\underline{\delta}-\delta)}{9+\delta}$. However, if $t \in (0, \bar{t})$, then $\theta^{1*} \in (0, 1-\delta)$. Moreover,*

- *a) $d\theta^{1*}/d\delta \leq 0$ (with equality if $\delta \geq \delta^M$), $\lim_{\delta \to 0} \theta^{1*} = 1$ and $\lim_{\delta \to \delta^M} \theta^{1*} = 0$;*
- *b) $d\theta^{1*}/dt > 0$ for $t > 0$;*
  - *i) if $\delta \in [0, \underline{\delta}]$, then $\lim_{t \to 0} \left(d\theta^{1*}/dt\right) = 0$;*
  - *ii) if $\delta \in (\underline{\delta}, \widehat{\delta})$, then $\lim_{t \to 0} \left(d\theta^{1*}/dt\right) = \left[\frac{17}{32}\left(\frac{\delta-\underline{\delta}}{1-\delta}\right)\Pi^N|_{t=0}\right]^{-1/2}$;*
- *c) $sign\left(d\theta^{1*}/d\tau\right) = sign\left(d\theta^{1*}/d\beta\right) \geq 0$, with equality if $t = 0$;*
- *d) $sign\left(d^2\theta^{1*}/dtd\tau\right) = sign\left(d^2\theta^{1*}/dtd\beta\right) > 0$.*

The key ideas behind Lemma 1 are contained in Fig. 2 which depicts several families of θ^{1*} associated with three discount factor values: high, intermediate and low. The curves within each family are associated with different external trade cost levels τ. Each of these curves describes how cartel discipline responds to changes in internal trade costs t in case 1.[31,32]

[30] We recognize and explicitly take into account this restriction after we complete our analysis of the individual cases 1 and 2.

[31] Note that θ_g (which is defined only when $\delta \leq \underline{\delta}$) is independent of external trade costs τ and market size β. Also note that $\theta^{1*} = 0$ for $t = 0$ and $\delta > \underline{\delta}$ because full collusion is sustainable in this case. As we will see, the dashed-line portions of the contours in Fig. 2 are irrelevant because they violate the non-negativity constraint on y. (See Proposition 1 below for details.) Also, as discussed in Lemma 3 below, these contours intersect curve $\theta_o(t)$ at a unique (t, θ) pair that implies $y = 0$.

[32] In Appendix A we establish the existence of θ^{1*} by showing that $\lim_{\theta \to 0} \Phi^1 < 0$ and $\lim_{\theta \to 1-\delta} \Phi^1 > 0$, and by utilizing the fact that Φ^1 is continuous in $\theta \in (0, 1-\delta)$. We then prove that $\Phi_\theta^1 \equiv d\Phi^1/d\theta|_{\Phi^1=0} > 0$, which confirms uniqueness. From (ICC) changes in θ affect Φ^1 solely through their impact on q^1; that is, $\Phi_\theta^1 = \Phi_x^1 x_\theta^1 + \Phi_y^1 y_\theta^1 + \Phi_z^1 z_\theta^1$. But, as explained above, $\Phi_j^1 = (1-\delta-\theta)(-\Pi_j^D) > 0$ for $j = x, y, z$ which, by (5), is proportional to a cartel member's best-reply in the relevant market (e.g., $-\Pi_x^D = y^D$). These effects

The intuition behind part (a) is simple. Since $\Pi^D - \Pi^N > 0$, placing more value on future profits ($\delta \uparrow$) improves cartel discipline by loosening the ICC. This is shown in Fig. 2 by a downward shift of a family of curves.

Part (b) points out that internal trade cost reductions ($t \downarrow$) strengthen cartel discipline ($\theta^{1*} \downarrow$), as indicated by the upward sloping curves within each family in Fig. 2. By the implicit function theorem, $d\theta^{1*}/dt = -\Phi^1_t/\Phi^1_\theta$. Since $\Phi^1_\theta > 0$, cartel discipline improves with reductions in t only if $\Phi^1_t < 0$. Decreases in t reduce the marginal cost of cross hauling, expanding global profits under a collusive agreement (Π^C) and under an optimal deviation (Π^D). The effect of t on Φ^1 through Π^C relaxes the ICC, whereas its effect on Φ^1 through Π^D tightens the ICC. But the direction of change in Nash profits also depends on the initial level of t (recall Π^N is U-shaped in t). Thus, the effect of a fall in t on Φ^1 through its impact on Π^N tightens (relaxes) the ICC at low (high) t values. Lastly, a reduction in t also affects Φ^1 through (x^1, y^1). In the proof, we show that the expansionary effect of internal trade cost reductions on volume of intra-industry trade (y), is a dominant force that loosens the ICC (i.e., $\Phi^1_t < 0$) that explains the strengthening of cartel discipline.[33]

Part (c) shows that, in the presence of cross hauling, external trade cost reductions ($\tau \downarrow$) also improve cartel discipline because $\Phi^1_\tau < 0$. In words, reductions in τ create slack in the ICC which, to restore incentive compatibility, requires cartel discipline to improve ($\theta^{1*} \downarrow$). This effect is depicted in Fig. 2 by the downward shift of a curve within each family of curves.[34] The effect of expanding ROW size ($\beta \downarrow$) is similar since $\Phi^1_\beta < 0$.

Part (d) reveals that improvements in export opportunities to ROW ($\beta \downarrow$ or $\tau \downarrow$) reduce the sensitivity of cartel discipline to internal trade cost changes and conversely. This point is captured by the fact that the curves within each family become flatter as β and/or τ fall.

At higher discount factor values, the curves in Fig. 2 become steeper, so cartel discipline becomes more sensitive to internal trade cost changes. At the same time, the curves move further apart from each other, so the impact of external trade cost changes on cartel discipline becomes more pronounced.

When Cross Hauling is Absent ($y = 0$ and $\mu > 0$)**.** We now consider the case of $\Phi^2 \geq 0$, which requires the ICC to rule out (arbitrarily for now) cross hauling ($y = 0$).

together with the fact that increases in θ induce cartel members to expand their exports—both absolutely and in comparison to local output (i.e., $y^1_\theta > 0$ and $y^1_\theta > x^1_\theta$)—establishes that $\Phi^1_\theta > 0$. This is key to understanding the determination of cartel discipline for $y^1 > 0$.

[33] The fact that $\Phi^1_t < 0$ also explains why $\partial\delta^M/\partial t < 0$ in Proposition 1. Parts ($b.i$) and ($b.ii$) describe how cartel discipline responds to changes in internal trade costs in the neighborhood of internally free trade for alternative discount factor values. As explained later, these parts play key roles in the determination of cartel shipments and welfare.

[34] The lowest contour within a family arises when external trade is free ($\tau = 0$) and the highest contour arises when there external trade is eliminated ($\tau = \bar{\tau}$). The curve in the middle arises for some $\tau \in (0, \bar{\tau})$.

Focusing on $(t, \theta) \in [0, \bar{t}] \times [0, 1]$, the relevant FOCs in this case are

$$\mathcal{L}_j^2 = \Pi_j^C + \lambda \Phi_j^2 = 0 \quad \Rightarrow \; \Pi_j^C = \theta \Pi_j^D < 0, \; j \in \{x, z\}. \tag{12}$$

The solution to these equations is captures by:

$$Q^2(\theta, t) = x^2(\theta, t) = \frac{A(2+\theta) - \theta t}{4+\theta}, \tag{13a}$$

$$z^2(\theta, \tau, \beta) = \frac{(A-\tau)(2+\theta)}{\beta(8+\theta)}. \tag{13b}$$

Inspection of (13) reveals that, in contrast to $y > 0$ considered earlier, decreases in t expand x^2, even though there is no competition with imports from a cartel host. The effect of cartel discipline θ on local output level x^2 $(= Q^2)$ also differs. Since $x_\theta^2 = \frac{4(\bar{t}-t)}{(4+\theta)^2} > 0$ for $t < \bar{t}$, improvements in cartel discipline ($\theta \downarrow$) now induce cartel members to reduce local supplies.[35] As before, $z_\tau^2 < 0$ and $z_\theta^2 = \frac{6(\bar{\tau}-\tau)}{\beta(8+\theta)^2} \geq 0$.

To determine θ^{2*} we proceed as follows. First, we substitute (x^2, z^2) from (13) into Φ^2 $(\theta, t, \cdot)$ and study the shape of the resulting surface for all $(t, \theta) \in [0, \bar{t}] \times [0, 1]$. Second, we identify the (t, θ) pairs that ensure the ICC is binding (i.e., $\Phi^2 = 0$).

As will be detailed shortly in Lemma 3, let t_y be the "prohibitive" internal trade cost level (i.e., the lowest internal trade cost that ensures $y = 0$ in the presence of endogenous discipline). (See Fig. 2.) With the help of several lemmas detailed in Appendix A,[36] we may characterize cartel discipline in the absence of cross hauling as follows:

Lemma 2 *(Cartel discipline without cross hauling) Suppose cross hauling is absent ($y = 0$) and $\delta < \widehat{\delta}$. Moreover, define $\underline{\underline{\delta}} \equiv \min_t \delta^M(t, \cdot)$, $\underline{t}_m \equiv \min\{[\delta^M]^{-1}(\delta)\}$, and $\bar{t}_m \equiv \{\max [\delta^M]^{-1}(\delta)\}$. Then $\theta^{2*} \equiv \max(\theta_s, 0)$, where θ_s solves $\Phi^2(\theta, t, \cdot) = 0$. In this case, increases cartel discipline in non-decreasing is the discount factor (i.e., $d\theta^{2*}/d\delta \leq 0$). Furthermore, cartel discipline depends on trade costs and market size as follows:*

a) ***Internal Trade Costs (t)***

i) If $\delta \leq \underline{\underline{\delta}}$, then $\theta^{2*} = \theta_s$ and $d\theta^{2*}/dt \lesseqgtr 0$ for $t \lesseqgtr t_{\min} \equiv \arg\min_t \theta_s(t, \cdot)$.

ii) If $\delta \in (\underline{\underline{\delta}}, \underline{\delta}]$, there is a subset $[\underline{t}_m, \bar{t}_m] \subset (t_y, \bar{t})$ such that

- $\theta^{2*} = \theta_s$ *and* $d\theta^{2*}/dt < 0$ *for* $t \in (t_y, \underline{t}_m)$;
- $\theta^{2*} = 0$ *for* $t \in [\underline{t}_m, \bar{t}_m]$;
- $\theta^{2*} = \theta_s$ *and* $d\theta^{2*}/dt > 0$ *for* $t \in (\bar{t}_m, \bar{t}]$.

[35] As discussed earlier, x^1 was non-monotonic in θ in case 1. Still, the dependence of Q^2 on θ is similar to the dependence of Q^1 on θ. In particular, Q^2 is increasing and concave in θ. Moreover, $Q^2 = Q^M$ for $\theta = 0$.

[36] Lemma 2 is obtained after a series of lemmas (specifically, Lemmas $A1$-$A3$) in Appendix A. Fig. 3 depicts graphically the definitions of several threshold values of t (e.g., $\underline{t}_m$, $\bar{t}_m$ and t_g) that appear in Lemma 2 (as well as in Propositions 2-4 below).

iii) If $\delta \in (\underline{\delta}, \widehat{\delta}]$*, then* $\bar{t}_m = \bar{t}$ *and*

- $\theta^{2*} = \theta_s$ *and* $d\theta^{2*}/dt < 0$ *for* $t \in (t_y, \underline{t}_m)$*;*
- $\theta^{2*} = 0$ *for* $t \in [\underline{t}_m, \bar{t}]$*.*

b) ***External Trade Costs*** $(\boldsymbol{\tau})$ ***and Market Size*** $(\boldsymbol{\beta})$

i) $d\theta^{2*}/d\xi > 0$ *for* $t \in [t_y, t_g)$ *if* $\delta \in (0, \underline{\delta})$ *and for* $t \in [t_y, \underline{t}_m)$ *if* $\delta \in (\underline{\delta}, \widehat{\delta}]$*, where* t_g *is a pivot point and* $\xi \in \{\tau, \beta\}$*;*

ii) $d\theta^{2*}/d\xi = 0$ *for* $t \in \{t_g, \bar{t}\} \cup [\underline{t}_m, \bar{t}_m]$*, where* $\xi \in \{\tau, \beta\}$*;*

iii) $d\theta^{2*}/d\xi < 0$ *for* $t \in (t_g, \underline{t}_m) \cup (\bar{t}_m, \bar{t})$*, where* $\xi \in \{\tau, \beta\}$*.*

iv) Increases in τ *or* β *expand the range of internal trade costs that imply* $\theta^{2*} = 0$*.*

Several points related to Lemma 2 deserve emphasis. *First*, depending on the values of the discount factor, external trade costs and market size, cartel discipline θ^{2*} may be non-monotonic or insensitive to changes in internal trade costs t. Clearly, this finding differs from the result in in Lemma 1 that $d\theta^{1*}/dt > 0$. The reason why θ^{2*} varies with t, even though $y = 0$, is that internal trade costs affect deviation and punishments payoffs. *Second*, for $t > t_y$, there always exists a range of t values adjacent to t_y such that $d\theta^{2*}/dt < 0$. Going to the other extreme, there may exist a range of t values adjacent to $\bar{t}$ that imply $d\theta^{2*}/dt > 0$. The possibility that $d\theta^{2*}/dt = 0$ arises for intermediate values of t between these extremes where $t \in [\underline{t}_m, \bar{t}_m]$.[37] *Third*, there exist several "pivot" points (i.e., (t, θ) pairs) that ensure θ^{2*} is insensitive to changes in external trade costs and market size. One such point is (t_g, θ_g) which arises for $\delta \leq \underline{\delta}$ (see Lemma $A2$ in Appendix A for details). Part $(b.i)$ implies that increases in τ and/or β weaken cartel discipline θ^{2*} for $t < t_g$ but improve it (or maintain it at 0, the level that sustains maximal collusion) for $t > t_g$.

Having described the determination of cartel discipline both in the presence and absence of intra-industry trade, for completeness, we now detail the key features of the prohibitive trade cost level t_y noted above.

Lemma 3 *If, for given* $\delta < \widehat{\delta}$*, cartel discipline is endogenously determined, there will exist a unique internal trade cost level* $t_y \equiv t_y(\delta, \tau, \beta) \in (0, \bar{t})$*, such that* $y > 0$ *for* $t < t_y$ *while* $y = 0$ *for* $t \geq t_y$*. Moreover,*

a) $dt_y/d\delta < 0$ *with* $\lim_{\delta \to \widehat{\delta}} t_y = 0$ *and* $\lim_{\delta \to 0} t_y = \bar{t}$*;*

b) $sign(dt_y/d\tau) = sign(dt_y/d\beta) > 0$ *with* $\lim_{\beta \to 0} t_y = 0$*.*

If firms value the future highly (e.g., if $\delta \to \widehat{\delta}$) or if the relative size of ROW is very large ($\beta \to 0$), the prohibitive internal trade cost is small ($t_y \to 0$).

[37] Fig. 3a depicts the $\underline{t}_m$ and $\bar{t}_m$ threshold levels for $\delta \in (\underline{\underline{\delta}}, \underline{\delta})$. (See also Fig 4$a$.) It is worth keeping in mind that if $\delta = \underline{\underline{\delta}}$, then $\underline{t}_m = \bar{t}_m = t_{\min}$. Moreover, if $\delta \in [\underline{\delta}, \widehat{\delta})$, then $\underline{t}_m \leq t_g$ and $\bar{t}_m = \bar{t}$. (See Figs 3a, 3b, 4a and 4b for illustrations of t_g.)

Equilibrium Cartel Discipline. With the help of Lemmas 1-3, we now describe the salient features of cartel equilibrium discipline as follows:

Proposition 2 *(Equilibrium cartel discipline) For* $\delta < \widehat{\delta}$ *and* $t \in [0, \bar{t}]$, *equilibrium cartel discipline is defined as*

$$\theta^* \equiv \begin{cases} \theta^{1*} & \text{if} \quad t \in [0, t_y) \\ \theta^{2*} & \text{if} \quad t \in [t_y, \bar{t}] \end{cases} .$$

a) This discipline is

i) weakest at the prohibitive internal trade cost level t_y*;*

ii) strongest at

- $t_{\min} \in (t_y, \bar{t})$ *if* $\delta < \underline{\underline{\delta}}$*;*
- $t \in [\underline{t}_m, \bar{t}_m]$ *if* $\delta \in (\underline{\underline{\delta}}, \underline{\delta}]$*;*
- $t = 0$ *and* $t \geq \underline{t}_m$ *if* $\delta \in (\underline{\delta}, \widehat{\delta}]$.

b) Cartel discipline improves with increases in the discount factor (i.e., $\partial\theta^*/\partial\delta < 0$*).*

c) Internal trade cost reductions ($t \downarrow$*)*

i) strengthen cartel discipline when cross hauling is present ($t < t_y$*) and possibly when cross hauling is absent (provided* t *is large enough);*

ii) weaken cartel discipline if cross hauling is absent and t *is close to* t_y.

d) Expansion of export opportunities to ROW ($\tau \downarrow$ *or* $\beta \downarrow$*)*

i) strengthen cartel discipline if $t \in (0, t_g)$ *or if* $t \in (0, \underline{t}_m)$ *depending on whether* $\delta \in (0, \underline{\delta})$ *or* $\delta \in (\underline{\delta}, \widehat{\delta}]$*, respectively;*

ii) do not affect cartel discipline for $t \in \{0, t_g, \bar{t}\} \cup [\underline{t}_m, \bar{t}_m]$*;*

iii) weaken cartel discipline for $t \in (t_g, \underline{t}_m) \cup (\bar{t}_m, \bar{t})$.

Fig. 4 illustrates Proposition 2. The blue, solid-line curve in panel (a) unveils the dependence of equilibrium cartel discipline on internal trade costs, under the assumption that $\delta < \underline{\underline{\delta}}$ initially. As noted in part (a), the peak of this curve is attained at $t = t_y$, affirming the point that cartel discipline is weakest at t_y. Cartel discipline is strongest at the global minimum which in the context of this curve is attained at $t_{\min}$. These findings, as well as the other ones in part (a), are direct consequences of Lemmas 1-3. The dependence of cartel discipline on the discount factor detailed in part (b)—which would be captured by downwards shifts of the curves in Fig. 4a (not shown)—is due to the fact that increases in it create slack in the ICC which boosts discipline.

Fig. 4 also captures the response of cartel discipline to internal trade cost reductions ($t \downarrow$) considered in part (c). For t sufficiently close to $\bar{t}$, such reductions do not affect Π^C

because cross hauling is absent. Thus, decreases in t improve cartel discipline because they intensify the severity of punishments ($\Pi^N \downarrow$) more than they raise deviation profits ($\Pi^D \uparrow$) in the ICC.[38] However, when internal trade costs fall below $t_{\min}$, additional reductions in t weaken cartel discipline because they bring about a reversal in the intensity of their effects on Π^N and Π^D. Once t falls below t_y (to support cross hauling), cartel discipline is strengthened because the positive effect on Π^C in the ICC prevails.

A valuable insight of part (d) is that the presence of third-country markets and the possible expansion and/or promotion of trade opportunities there ($\tau \downarrow$ or $\beta \downarrow$) *strengthen cartel discipline when internal trade costs are sufficiently low and may weaken it when these costs are large enough.* Fig. 4a helps visualize these ideas. The dotted-line curve there is associated with some $\tau > 0$; the thick solid-line curve is associated with $\tau = 0$. Together these curves confirm the effects of τ on cartel discipline described above—note, however, that cartel discipline remains intact at the "pivot" points A, G and E.

Parts (c) and (d) contain an alarming finding: If regional trade liberalization has advanced significantly (i.e., if t is low), then all types of trade cost reductions (regional and/or multilateral) promote cartel discipline. In light of the fact that many countries have implemented preferential trade agreements over the last three decades, this finding raises uncomfortable questions about the welfare implications of subsequent trade cost reductions. Do such reductions promote efficiency? Is there a role for activist competition policy?

In summary, Proposition 2 places at center stage a novel channel through which international shocks affect equilibrium outcomes: cartel discipline. By paying careful attention to the ICC, the proposition unveils the determination of cartel discipline in the context of multimarket interactions and explains how trade costs, trade policies and market characteristics affect firm conduct. We now use these insights to examine their implications for the international allocation of cartel output.

4 Equilibrium Production and Trade

Prior to studying cartel discipline, we showed that Q^i is linearly decreasing in internal trade costs t but increasing and concave in θ. Thus, $\arg\max_{(t,\theta)} Q^i = (0, 1)$ while $\arg\min_\theta Q^i = 0$. In the former case, the maximized output coincides with the Cournot-Nash level Q^N under

[38] The two panels of Fig. 4 can also help link our earlier analysis of the minimum discount factor to our current analysis of cartel discipline.

Would this analyses change if firms adopt more severe punishments? As emphasized in the literature on repeated games in relation to the Folk Theorem, one possibility is that cartel members use min-max strategies for a number of periods. It can be shown that the most collusive outcome in this case is sustainable for a larger range of discount factors. Relatedly, one can show that all parts of Proposition 2 remain intact except this: θ^* does not rise with increases in internal trade costs t as $t \to \bar{t}$.

How would the analysis change if the number of cartel members considered resided in $n > 2$ distinct countries? Consistent with one's intuition, one can show that this tends to undermine cartel discipline, however, without altering the key insights.

$t = 0$; in the latter case, the minimized output coincides with Q^M under maximal collusion. But when cartel discipline is endogenous, Q^* requires a deeper investigation.

In Proposition 2 we argued that θ^* is increasing in internal trade costs t when these costs permit cross hauling ($t < t_y$) and possibly when t is sufficiently close to $\bar{t}$. This suggests that the initial level of these costs shapes their impact on Q^*. Proposition 3 below clarifies this relationship and discusses the role of time preferences and export opportunities to ROW. It unveils several noteworthy traits of internal and external trade volumes y^* and z^*, respectively, which shape welfare.

Proposition 3 (Quantities) *For any given $t < \bar{t}$ and $\delta < \widehat{\delta}$, aggregate output Q^* and shipments y^* and z^* have the following characteristics:*

a) ***Output (Q^*)***

i) If $\delta \gtreqless \underline{\delta}$, then $\lim_{t\to 0} dQ^*/dt \gtreqless 0$ while $\lim_{t \searrow t_y} dQ^*/dt < 0$.

ii) If $\delta \in [\underline{\delta}, \widehat{\delta})$, then Q^*

- *attains a unique maximum $Q^*_{\max} > Q^M$ at $t_Q \in (0, t_y]$, where $\lim_{\delta \nearrow \widehat{\delta}} t_Q = t_y$;*
- *equals Q^M at $t \in \{0\} \cup [\underline{t}_m, \bar{t}]$.*

iii) If $\delta \in (0, \underline{\delta})$, then Q^*

- *may have multiple peaks, including one at $t = 0$, another in $(0, t_y]$, and possibly a third at some t close to $\bar{t}$; $t_Q = 0$ if export opportunities to ROW abound;*
- *equals Q^M for $t \in [\underline{t}_m, \bar{t}_m]$.*

iv) Time preferences and export opportunities to ROW affect Q^* solely through cartel discipline; thus sign $(dQ^*/d\xi) =$ sign $(\partial\theta^*/\partial\xi)$ for $\xi \in \{\delta, \beta, \tau\}$.

b) ***Internal Trade (y^*)****: If $t \in (0, t_y)$, then y^* rises with*

i) reductions in internal trade costs and/or the discount factor;

ii) improvements in export opportunities to ROW if these improvements weaken cartel discipline.

c) ***External Trade (z^*)****: The volume z^* of trade with ROW rises with*

i) reductions in the discount factor;

ii) improvements in export opportunities to ROW;

iii) reductions in internal trade costs if these reductions weaken cartel discipline.

Part (*a.i*) describes the dependence of Q^* on internal trade costs t. Provided the initial level of t is sufficiently low, Q^* falls with reductions in t for $\delta \in [\underline{\delta}, \widehat{\delta})$ but rises with such reductions for $\delta \in (0, \underline{\delta})$. In the former case, this is so because the contractionary effect of the incipient strengthening in cartel discipline ($\theta^* \downarrow$) dominates the direct and expansionary effect of t on Q^*. Exactly the opposite is true in the latter case. Part (*a.i*) also points out that, if internal trade costs are high enough to eliminate cross hauling, then $dQ^*/dt < 0$ for t sufficiently close to the prohibitive level t_y. The reason for this rests in Proposition 2c, which explained that the disciplinary and direct effects of t on Q^* move in the same direction.

Parts ($a.ii$) and ($a.iii$) elaborate further on the dependence of Q^* on t. Suppose $\delta \in [\underline{\delta}, \widehat{\delta})$ and $t \in (0, t_y)$, so that cross hauling is present. Part ($a.ii$) shows that, if δ is close to $\widehat{\delta}$ (which materializes if δ and/or export opportunities to ROW are sufficiently large), then $t_Q = t_y$. Thus, Q^* is decreasing in t for $t < t_y$, as indicated in Fig. 5. In contrast, part ($a.iii$) suggests that, if the discount factor is sufficiently low, then $dQ^*/dt < 0$ for $t < t_y$. Therefore, $t_Q = 0$, as shown in Fig. 5c. Fig. 5b reveals that Q^* may have multiple peaks in t when the discount factor is moderately large.

Part ($a.iv$) argues that parameters δ, β and τ influence Q^* solely through their effect on cartel discipline θ^*, an effect that is simply absent in static analyses. To see this, consider the consequences of reducing external trade costs ($\tau \downarrow$) for sufficiently low t. Because such reductions strengthen cartel discipline ($\theta^* \downarrow$ by Proposition 2d), local output Q^* necessarily falls. Similarly, $dQ^*/d\delta \leq 0$ because increases in δ do not weaken cartel discipline.

Part (b) sheds light on the behavior of internal trade y^*. Reductions in internal trade costs t promote cross hauling because their direct (and favorable) effect outweighs their indirect (and negative) effect due to improvements in cartel discipline. The dependence of y^* on all other parameters hinges solely on their effect on cartel discipline.[39]

Part (c) considers the possible effects on the volume of external trade z^*. The logic behind this part is similar to that in part (b), so we only discuss the significance of internal trade cost reductions ($t \downarrow$). Once again, this effect is transmitted exclusively through cartel discipline and is at work regardless of whether cross hauling is present or not. Interestingly, if cross hauling is present, then reductions in t divert external trade because they bring about an improvement in cartel discipline. In contrast, if cross hauling is absent and t is close to t_y, reductions in t promote external trade. These insights, and the ones in parts (a) and (b), emphasize the strategic linkages among markets via cartel discipline through the ICC. We now turn to welfare. As we will see, the behavior of Q^* is salient.

5 Preferential Trade Liberalization and Welfare

In this section, we suppose cartel hosts have agreed to join each other in a PTA and study the welfare effects of internal trade cost cuts due to preferential trade liberalization. Accordingly, we interpret t and τ as tariffs and taxes/subsidies on intra-PTA (internal) and

[39] For example, because $\partial y^*/\partial\theta > 0$ and $d\theta^*/d\delta < 0$, larger discount factor values reduce y^*. In contrast, external trade cost reductions ($\tau \downarrow$) and/or increases in ROW's market ($\beta \downarrow$) cause y^* to fall because they strengthen cartel discipline (for $t < t_y$).

extra-PTA (external) trade flows, respectively.[40] Although we also consider situations in which internal trade is absent, our primary focus is on the presence of cross hauling ($y > 0$). We start with a general decomposition of welfare and a discussion of unconstrained optimal policies in a PTA. Then, abstracting from the possible presence of domestic competition policies, for contrast we analyze policies under two situations: when cartel discipline is exogenous and when it is endogenous.

Denote with W and W_{ROW} the welfare levels of the representative PTA member and ROW, respectively, and with CS and CS_{ROW} their corresponding consumer surpluses. Since $CS = u - pQ$, $\Pi = pQ - ty + p_{ROW}z - \tau z$ and $W \equiv CS + \Pi + ty + \tau z$, we have $W = u + p_{ROW}z$. Moreover, $W_{ROW} = CS_{ROW} = u_{ROW} - 2p_{ROW}z$ $(= 2\beta z^2)$.

Denote with $MR_{ROW} \equiv d\left(p_{ROW}z\right)/dz$ the cartel's marginal revenue in ROW. Totally differentiating W and W_{ROW} and utilizing the fact that optimization in consumption implies $u' = p$ and $u'_{ROW} = p_{ROW}$ gives

$$dW = pdQ + MR_{ROW}dz, \tag{14a}$$

$$dW_{ROW} = \left(-4p'_{ROW}z\right)dz. \tag{14b}$$

Clearly, PTA welfare is increasing in Q and z if $p > 0$ and $MR_{ROW} > 0$, respectively. Moreover, since $p'_{ROW} < 0$, ROW welfare is increasing in z.

As a benchmark, suppose domestic competition and common external trade policies are available and consider the PTA's first-best policies. Internal optimality requires firms to price according to marginal cost, so $p = 0$. Similarly, external optimality requires firms to exploit their collective power in ROW and extract the largest possible rents; so $MR_{ROW} = 0$. The PTA's first-best optimal domestic policy can be implemented with a price ceiling at $p = 0$. As explained in some detail below, the PTA's optimal external policy requires an intervention that induces the cartel to behave as a pure monopolist in ROW.[41]

5.1 Exogenous Discipline

Assume now (for future purposes) that cartel discipline θ is exogenous. Also assume that domestic competition policy in a PTA is unavailable so that, depending on the level of t, output Q is determined by (11a) or (13a) while z is determined by (11d) or (13b).

[40] Our identification of internal trade costs with tariffs makes sense because trade liberalization is normally associated with reductions in revenue-generating trade barriers. But our focus on tariffs is also justified on the grounds that our welfare findings stand up to scrutiny even when reductions in resource-using (e.g., transportation) costs are considered. Due to the prevalence of the latter costs (Anderson and van Wincoop, 2004) and for comparability with related work we will discuss the implications of these costs later.

[41] The $p = 0$ equation requires firms to supply $Q = x + y = A$ for $t = 0$, and $x = A$ and $y = 0$ for $t > 0$. The $MR = 0$ condition requires each firm to supply ROW with $z = z^M\big|_{\tau=0}$ units.

Utilizing the latter observation, one can show that $MR_{ROW} = -\frac{4(2+\theta)}{8+\theta}\left(\tau^C - \tau\right)$, where $\tau^C \equiv \tau^C(\theta) = \frac{3A\theta}{4(2+\theta)} \geq 0$. This paves the way to:

Lemma 4 *Suppose cartel discipline θ is exogenous and domestic competition policy in a PTA is unavailable. Then, regardless of whether the PTA's common external trade policy is fixed or optimal, preferential trade liberalization*

a) improves PTA welfare and maximizes it at $t = 0$;

b) does not affect welfare in ROW.

The PTA's optimal external policy is a tax that coincides with $\tau^C(\theta)$, which falls with improvements in cartel discipline (i.e., $d\tau^C/d\theta > 0$).

Part (*a*) of Lemma 4 holds true because $p > 0$ and $dQ/dt = Q_t < 0$ in (14a), for a given θ, while $dz/dt = 0$ because z (as well as τ) are independent of t. The latter observation also establishes part (*b*).[42]

The last statement in the lemma follows from the fact that $dQ/d\tau = 0$ which requires the PTA's optimal external policy to satisfy $MR_{ROW} = 0$ in (14a), for given θ. But, as can be ascertained from the definition of MR_{ROW}, this policy coincides with $\tau^C(\theta)$ and is a tax for $\theta > 0$.[43]

5.2 Endogenous Discipline

To highlight the importance of endogenous cartel discipline, we consider three situations regarding external policy and its response to internal tariff cuts. In the first, we suppose the common external tax is held fixed at some predetermined level; in the second, we allow this tax to adjust in a neutral way; that is, in a way that leaves ROW welfare unchanged; in the third, we allow the tax to be optimally determined. These exercises aim to assess the new channel (i.e., cartel discipline) through which the effects of regional economic integration affect welfare and policy. Furthermore, the exercises also aim to shed light on the effects of preferential tariff cuts, including the issue of whether free intra-PTA trade is desirable in the first place. Henceforth, we retain the assumption that domestic policies are unavailable.

[42] By virtue of the facts that neither θ nor τ respond to changes in t in this setting (and, as a consequence, z and W_{ROW} remain constant while W rises), external taxes adjust to internal tariff cuts in a Kemp-Wan fashion trivially (i.e., by not responding changes in t).

[43] The reason τ^C falls with improvements in cartel discipline is because $d\tau^C/d\theta = \frac{3A\theta}{2(2+\theta)^2} > 0$. The intuition is simple: the more disciplined the cartel, the higher its ability to extract rents from ROW and thus the lower the PTA's need to intervene with a high export tax. For additional insight, consider the following two extremes: (i) $\theta = 0$, which holds true under maximal collusion ($z^M = \frac{A-\tau}{4\beta}$); and (ii) $\theta = 1$, which arises under Cournot-Nash competition ($z^N = \frac{A-\tau}{3\beta}$). In case (i), the optimal external policy is $\tau^C = 0$ and implies $z^M\big|_{\tau=0} = \frac{A}{4\beta}$. In case (ii), $\tau^C = A/4$ which, again, implies $z^N\big|_{\tau=A/4} = \frac{A}{4\beta}$. Thus, in both cases, the optimal external policy is given by $\tau^C = \frac{3A\theta}{4(2+\theta)}$ and has the property that it induces the cartel to exploit its market power in ROW fully.

Fixed External Taxes. With cartel discipline being determined endogenously, we fix external trade taxes at some level τ and consider the representative PTA member's change in welfare due to a change in internal tariffs t. Utilizing (14a), this change is given by

$$dW^*/dt = p^*(\overset{(?)}{dQ^*/dt}) + MR^*_{ROW}(\overset{(+)}{z^*_\theta})\theta^*_t, \tag{15}$$

where $MR^*_{ROW} = -\frac{4(2+\theta^*)}{8+\theta^*}\left(\tau^C - \tau\right)$, as noted earlier. To prepare the ground for our analysis to follow and develop intuition, temporarily assume the absence of trade with ROW ($\tau \geq \bar{\tau}$), so that only the effect on local consumers (captured by the first term on the RHS of (15)) matters. Recall that $dQ^*/dt = Q^*_t + Q^*_\theta \theta^*_t$ and focus on t values that sustain cross hauling ($y^* > 0$). With the help of Proposition 3, we make two observations. First, in sharp contrast to Lemma 4*a*, the global maximizer of W^* need not coincide with internally free trade. Second, W^* may have multiple peaks. The logic behind the first observation hinges on the direct and indirect effects of t on Q^* studied in Proposition 3. The direct effect of reducing t on W^* is positive because $Q^*_t < 0$. But, as we have seen, the *reduction* in t *strengthens* cartel discipline ($\theta^*_t > 0$) for $y^* > 0$, which causes output Q^* to *fall* ($Q^*_\theta > 0$). As emphasized in Proposition 3, this pro-collusive effect may dominate the direct effect so that $dQ^*/dt > 0$. More generally, though, the net effect of reducing t on W^* depends on parameter values (e.g., the values δ and β). The second point on the possible existence of multiple peaks in Q^* also follows from Proposition 3.

Now suppose $\tau < \bar{\tau}$, so that $z^* > 0$. Because preferential tariff cuts ($t \downarrow$) affect cartel discipline—and, actually, boost it in the presence of cross hauling ($\theta^* \downarrow$)—the impact on cartel rents in ROW (i.e., the second term) in (15) matters. But, as already emphasized, the sign of this term depends on the value of τ relative to $\tau^C(\theta^*)$. The nature of the PTA's common external policy determines whether exporting to ROW amplifies or diminishes the appeal of regional integration to PTA members. If $\tau < \tau^C(\theta^*)$ (which includes the case of export subsidies, $\tau < 0$, and implies $MR^*_{ROW} < 0$), the presence of ROW enhances the appeal of preferential tariff reductions to PTA members more than one would predict on the basis of conventional static analyses.[44]

Is it possible for the pro-collusive (and harmful to PTA members) effect of preferential tariff cuts to dominate the just described (and possibly positive) effect due to the presence of ROW? The short answer is: YES. However, the size of ROW's market matters because it shapes the cartel's earning potential there. Interestingly, this possibility always arises if $\delta \in (\underline{\delta}, \widehat{\delta})$, the initial level of t is sufficiently low (perhaps due to past successful liberalization

[44] A special case one could consider is the possibility of free trade with ROW ($\tau = 0$). In addition to serving as a valuable benchmark, interest in this case may be justified on two grounds: a number of countries prohibit the use of export taxes; WTO commitments discourage the use of export subsidies. The important point is that, in this case, $W^* \uparrow$ as $t \downarrow$ due to the presence of ROW.

efforts), and export taxes are nonnegative. Why? Because the welfare effect of reducing t through the external channel is negligible in this case, thus causing the impact on W^* to be determined by the change in Q^*.[45] For clarity, we illustrate this possibility in the first two panels of Fig. 6 that are associated with three external trade cost levels: $\tau \in \{0, \frac{1}{2}\overline{\tau}\}$. In addition to highlighting the importance of considering the presence of ROW, these panels also illustrate the dependence of W^* on t for several discount factor values. First, the value of δ plays a key role in determining the desirability of additional internal tariff cuts; second, depending on the initial values of t and τ, W^* may depend on δ non-monotonically; and, third, when cross hauling is absent ($y^* = 0$), the dependence of W^* on t is complex.

How does preferential trade liberalization ($t \downarrow$) affect ROW welfare ("spillover" effect)? From (14b) one can see that this relationship is determined solely by the dependence of z^* on t. But, as explained in Proposition 3*c*, the direction of change in z^* coincides with the direction of change in cartel discipline θ^*. Unfortunately for ROW, in the presence of cross hauling ($y^* > 0$), the deepening of regional integration ($t \downarrow$) categorically diverts external trade (because $\theta^* \downarrow$) and reduces its welfare.

The following proposition elaborates on the above ideas.

Proposition 4 *(Fixed external taxes) Suppose external taxes are fixed at some exogenous level $\tau \in [0, \overline{\tau})$ and cartel discipline is endogenous.*

a) In the presence of cross hauling, preferential trade liberalization ($t \downarrow$) ***always*** *brings about external trade diversion ($z^* \downarrow$) and* ***harms*** *ROW.*

b) If preferential trade liberalization has advanced substantially, then the deepening of regional integration

i) ***benefits*** *PTA members if $\delta \in (0, \underline{\delta})$, and*

ii) ***hurts*** *PTA members if $\delta \in (\underline{\delta}, \widehat{\delta})$.*

c) There exists a threshold level $\widetilde{\delta} \equiv \widetilde{\delta}(\tau, \beta) < \underline{\delta}$ such that internally free trade maximizes PTA welfare for $\delta < \widetilde{\delta}$, but not necessarily otherwise.

d) PTA welfare may be ***non-monotonic*** *in minimum discount factor δ.*

We already discussed part (*a*). The novel insight here—which contrasts part (*b*) of Lemma 4—is that the deepening of integration in an existing PTA (or the formation of a new one) diverts external trade and generates negative spillover effects on ROW welfare, when τ is fixed. Part (*b*) summarizes the welfare effects of internal tariff cuts for PTA members that were discussed.

[45] This is so because $\lim_{t \to 0} \tau^C = \lim_{t \to 0} \theta^* = 0$, which for $\tau \in [0, \overline{\tau})$ implies that the welfare effect of internal tariff cuts through the external channel is non-positive while the welfare effect through the internal channel is finitely negative (Proposition 3*a*).

But while the deepening of integration may benefit PTA members when internal tariffs are low to start with, this does not necessarily imply that totally free internal trade is welfare superior to regimes associated with restrictive (perhaps even prohibitive) internal tariffs. Part (c) posits that free internal trade may be optimal here.[46] However, unlike part (a) of Lemma 4, a key requirement for this eventuality is that the discount factor be smaller than a certain threshold level $\widetilde{\delta}$.[47] Turned on its head, part (c) suggests that the PTA may prefer to restrict internal trade if $\delta \in (\widetilde{\delta}, \widehat{\delta})$.[48] What's more, ROW may also find restrictions to internal trade for $t \in (0, t_y)$ appealing because such restrictions limit trade diversion by weakening cartel discipline.

The possible non-monotonicity of W^* in δ featured in part (d) also deserves to be mentioned here.[49] The intuition behind it is simple. For $t < t_y$, increases in δ strengthen cartel discipline (Proposition 2a), causing both Q^* and z^* to fall and pushing W^* in correspondingly opposite directions. If ROW is infinitesimal ($\beta \to \infty$), external rents are insignificant and the adverse pro-collusive effect of δ through Q^* prevails. In contrast, if the size of ROW is large enough, the favorable effect on rents through z^* may overwhelm the adverse effect through Q^*. This suggests that there exist intermediate values of ROW size that render the dependence of W^* on δ non-monotonic.

We already explored the circumstances under which the deepening of regional integration reduces welfare in ROW. Can this integration also reduce world efficiency? The following corollary provides the answer (with the help of parts (a) and (b) of Proposition 4).

Corollary 1 *If external trade taxes are fixed and $\delta \in (\underline{\delta}, \widehat{\delta})$, then the deepening of integration in a PTA that has already reduced internal tariffs substantially is welfare-reducing in the Pareto sense.*

The finding that regional trade liberalization may inflict welfare losses to all trade partners is noteworthy. It is also consistent with the theory of the Second Best and is due to

[46] In the context of a perfectly competitive model, Bond et al. (2004) showed that a key requirement for internally free trade to be optimal is that PTA members adopt a common external tariff structure that internalizes terms-of-trade externalities on trade with ROW. As a consequence, in that model, internally free trade is optimal for CUs aiming to maximize joint welfare, but not for FTAs in which members determine their external policies indepedently. We will address the question of whether the adoption of optimal policies on trade with ROW resurrects the optimality of internally free trade shortly.

[47] Note that $\delta < \widetilde{\delta}$ ($< \underline{\delta}$) implies that the most collusive outcome is not sustainable for $t = 0$. Moreover, one can show that $\widetilde{\delta}_\tau < 0$ and $\widetilde{\delta}_\beta < 0$ which imply that the more extensive the cartel's export opportunities to ROW, the larger the set of discount factor values under which internally free trade is preferred by the PTA over all other outcomes.

[48] In fact, if δ is sufficiently close to $\widehat{\delta}$, it is possible to have $\arg\max_t W^* = t_y$ which requires the elmination of internal trade.

[49] This is interesting because it seems to defy the perceived wisdom that increases in δ may facilitate collusion and thus welfare. As shown in Fig. 6, the validity of this point remains intact only if τ is sufficiently high. In other words, the dependence of W^* on δ hinges on the external tax level.

the presence of market power by firms and the absence of appropriate regulation within the PTA to address it. Corollary 1 and Proposition 4 add value to the theory of trade policy in PTAs for the following reasons. First, they identify concrete circumstances under which preferential trade liberalization is globally inefficient. Second, they shed new light on the importance of WTO's Article XXIV—which directs PTA members to refrain from imposing higher restrictions on trade with ROW and urges them to dismantle barriers to internal trade. We have shown that strict enforcement of this principle may be unappealing, not just to PTA members, but also to ROW. Third, they underline the importance of antitrust policies (which are prominently missing here) and flag the need to supplement WTO-based rules with international coordinating of antitrust policies.

A couple of words on the implications of resource-using trade (e.g., transportation) costs are in order here. Due to the partial equilibrium nature of the model, the effects of such costs on cartel discipline, prices and quantities are identical to the ones associated with the revenue-generating taxes discussed above. Consequently, the impact of internal trade costs on welfare in ROW is qualitatively similar to the one studied above. The key difference is that the representative PTA member's welfare will be defined as $V = W - ty - \tau z$ because policy-related revenues are no longer present. Therefore, $V \leq W$ for $\tau \in [0, \overline{\tau})$ and $t \in [0, \overline{t})$.[50] I follows that the welfare levels associated with the preceding analysis in this section serve as upper bounds to the welfare levels we would observe in the presence of resource-using trade costs. In particular, the negative effects of regional integration we have identified for internal trade costs that support cross hauling will be amplified. Moreover, PTA welfare for $\delta \in (\underline{\delta}, \widehat{\delta})$ may fall for a larger set of initial trade cost levels.

Modified Kemp-Wan Policies. A key feature of Kemp-Wan tariff structures is that they adjust to internal tariff cuts in a way that leaves trade with ROW intact while providing efficiency gains to PTA members (in the presence of appropriate intra-union income transfers and/or commodity taxes/subsidies). As noted earlier (see footnote #38), this insight holds true *trivially* in one-shot games with firms interacting in spatially separated markets under constant marginal costs and exogenous discipline.[51] Our primary goal in this section is to examine the validity of this insight to the presence of endogenous cartel discipline.

Denote with $\tau_K = \tau_K(t)$ the external tax that keeps the volume of trade z^* with ROW fixed, for any internal tariffs t. This tax is implicitly defined as the solution to $z(\theta^*(t, \tau), \tau) = fixed$ and, for specificity, we refer to it as a "modified Kemp-Wan" or "neutral" external policy. Focusing on trade cost values that sustain cross hauling ($y^* > 0$),

[50] In the special cases where $\tau \in \{0\} \cup [\overline{\tau}, \infty)$ and $t \in \{0\} \cup [t_y, \infty)$ (which imply either that trade costs are absent or that their levels preclude the emergence of any type of trade), we have $V = W$ implying $V < W$ in all other cases.

[51] We say "trivially" because the Kemp-Wan criterion requires taxes to remain constant in this case.

an adjustment in τ_K to cuts in t requires τ to fall because $d\tau_K/dt = -\frac{z_\theta^* \theta_t^*}{z_\tau^* + z_\theta^* \theta_\tau^*} > 0$.[52] To assess the implications of τ_K on PTA welfare, we utilize the fact that $dz^*|_{\tau=\tau_K}/dt = 0$ in (14a) and simplify expressions there to obtain

$$dW^*/dt|_{\tau=\tau_K} = p^* \overset{(?)}{[dQ^*/dt]} = p^* \left\{ \overset{(-)}{Q_t^*} + \overset{(+)}{Q_\theta^*} \left[\overset{(+)}{\theta_t^*} + \overset{(+)}{\theta_\tau^*} \overset{(+)}{(d\tau_K/dt)} \right] \right\}. \tag{16}$$

Clearly, the policy adjustments described in (16) affect PTA welfare solely through their impact on aggregate output Q^*. As before, the net effect of reducing t on Q^* consists of a positive direct effect ($t \downarrow \Rightarrow Q^* \uparrow$) and a negative indirect effect due to the improvement in cartel discipline ($t \downarrow \Rightarrow \theta^* \downarrow \Rightarrow Q^* \uparrow$). However, the latter effect of reducing t now is augmented by the second term inside the brackets of the second equation. This additional term is related to the reduction in τ_K due to the fall in t that compounds the strengthening of cartel discipline. On the basis of these observations we arrive at

Proposition 5 *(Modified Kemp-Wan adjustments) If preferential trade liberalization has advanced significantly (so that t is sufficiently low), then the deepening of economic integration, accompanied by external policy adjustments that keep external trade flows constant,* ***reduces*** *PTA welfare for $\delta \in [\underline{\delta}, \widehat{\delta})$.*

The intuition behind Proposition 5 follows from Proposition $3a.i$ and is related to the fact that the indirect effect of preferential trade liberalization on output Q^* through cartel discipline θ^* (which dominates under the noted circumstances) is strengthened by the accompanying reduction in external taxes. The novel insight here is that preferential trade liberalization, accompanied by modified Kemp-Wan policy adjustments, makes it more likely that PTA welfare will fall in the case considered here. Thus, once again, in the absence of domestic regulation, real Kemp-Wan adjustments may fail to exist. We hasten to add that the validity of this finding is driven, not by imperfect competition per se, but by the presence of endogenous discipline. It is easy for one to verify that, when δ is sufficiently low, "modified Kemp-Wan" adjustments *become* "Kemp-Wan" adjustments.

Optimal External Policy. Suppose now external policy $\tau = \tau_{opt}$ maximizes joint PTA welfare. How does the presence of endogenous discipline shape the welfare effects of preferential trade liberalization for PTA members and ROW? Is totally free internal trade optimal for the PTA?

[52] We abstract from specifying the initial level of the external volume of trade here. The positive sign of this expression follows from Proposition 3, where we showed that $z_\theta^* > 0$, $\theta_t^* > 0$, $z_\tau^* < 0$, $z_\theta^* > 0$, and $z_\tau^* + z_\theta^* \theta_\tau^* < 0$. We label this adjustment "modified" because it is silent on the impact of t on PTA welfare.

Suppose $\tau = \tau^C(\theta) = \frac{3A\theta}{4(2+\theta)}$. Substituting $\tau^C(\theta)$ in the cartel's ICC delivers a solution for cartel discipline which, with a slight abuse of notation, we denote $\theta^c \equiv \theta^c(t, \beta, \delta)$. Now evaluate $\tau^C(\theta)$ at $\theta = \theta^c$ to obtain $\tau^c \equiv \tau^c(t, \beta, \delta)$. A salient trait of τ^c is that it implies $MR^c_{ROW} = 0$ which delivers the monopoly export volume $z^c = z^M = \frac{A}{4\beta}$ and ROW welfare $W^c_{ROW} = W^M_{ROW} = \frac{A^2}{8\beta}$.[53] For these reasons, τ^c is a modified Kemp-Wan policy. We use τ^c as a benchmark to assess the PTA's optimal common external policy τ_{opt}.

The positive dependence of τ^C on θ unveiled in Lemma 4 implies that the relationship between θ^c and t is qualitatively similar to the one between θ^* and t studied in Proposition 2. For clarity, we illustrate this relationship with the blue, solid-line curve in Fig. 7*a* for $\delta < \underline{\underline{\delta}}$. (Ignore the other curve for now.) Similarly, the blue, solid-line curves in Figs 7*b* and 7*c* illustrate the dependence of z^c and W^c_{ROW}, respectively, on internal tariffs t for $\delta < \underline{\underline{\delta}}$.

Let us now consider the determination of τ_{opt}. Being aware of the dependence of cartel discipline θ^* on internal and external policies, the PTA policymaker can utilize the welfare decomposition in (14a) to obtain

$$dW^*/d\tau = p^*(dQ^*/d\tau) + MR^*_{ROW}p(dz^*/d\tau) = p^* \overset{(+)}{Q^*_\theta} \overset{(?)}{\theta^*_\tau} + MR^*_{ROW}(z^*_\tau + \overset{(-)}{z^*_\theta\theta^*_\tau}), \quad (17)$$

where the expressions for $dQ^*/d\tau$ and $dz^*/d\tau$ in (17) follow from Proposition 3. The first (second) term in the RHS of (17) captures the welfare effect of τ through the internal (external) channel Q^* (z^*). The sign of θ^*_τ depends on the level of t (Proposition 2).[54] With the help of (17) we can establish

Proposition 6 *(Optimal external policy) Suppose $\tau = \tau^c$. Then, the PTA's optimal common external policy (τ_{opt}) can be characterized as follows (consult Figs.* 4 *and* 7*b):*

a) For given δ and t, τ_{opt} has the following traits:

i) If $\delta \in [\underline{\delta}, \widehat{\delta})$, then $\tau_{opt} \begin{cases} = \tau^c \text{ for } t \in \{0\} \cup [\underline{t}_m, \bar{t}] \\ > \tau^c \text{ for } t \in (0, \underline{t}_m). \end{cases}$

ii) If $\delta \in (\underline{\underline{\delta}}, \underline{\delta})$, then $\tau_{opt} \begin{cases} = \tau^c \text{ for } t \in \{0, t_g, \bar{t}\} \cup [\underline{t}_m, \bar{t}_m] \\ > \tau^c \text{ for } t \in (0, t_g) \\ < \tau^c \text{ for } t \in (t_g, \underline{t}_m) \cup (\bar{t}_m, \bar{t}). \end{cases}$

iii) If $\delta \in (0, \underline{\underline{\delta}}]$, then $\tau_{opt} \begin{cases} = \tau^c \text{ for } t \in \{0, t_g, \bar{t}\} \\ > \tau^c \text{ for } t \in (0, t_g) \\ < \tau^c \text{ for } t \in (t_g, \bar{t}). \end{cases}$

b) If there is no cross hauling ($y^* = 0$), then, depending on time preferences and ROW

[53] Note that τ^c is not identical with $\tau^M = 0$.

[54] The ambiguity in the sign of θ^*_τ here is due to the fact that we are not limiting our attention to the presence of cross hauling.

size, the optimal external policy may be an ***export subsidy*** *(i.e.,* $\tau_{opt} < 0$*).*

Part (a) displays the traits of τ_{opt} for three discount factor values: (i) high, (ii) moderate, and (iii) low.[55] One feature of τ_{opt} observed in this part is that $\tau_{opt} = \tau^c$ under two distinct circumstances: when t equals the tariff level associated with any of the three pivot points 0, t_g and $\bar{t}$ discussed earlier; and when t supports maximal collusion (i.e., when $t \in [\underline{t}_m, \min(\bar{t}_m, \bar{t})]$).[56] The reason $\tau_{opt} = \tau^c$ in these cases is that cartel discipline is insensitive to external policy (Proposition 2c).

But while $\tau_{opt} = \tau^c$ under the circumstances noted above, $\tau_{opt} \neq \tau^c$ for all other internal tariff levels. Specifically, $\tau_{opt} > \tau^c$ in all cases in part (a) when t is sufficiently small. To see this, suppose, contrary to the claim, $\tau = \tau^c$ so that $MR_{ROW} = 0$ which causes the second term in the RHS of (17) to vanish. But $\theta^*_\tau > 0$ at these values of t and τ (Proposition 2c); therefore, $p^* Q^*_\theta \theta^*_\tau > 0$ and so $dW^*/d\tau|_{\tau=\tau^c} > 0$, which requires $\tau_{opt} > \tau^c$. In words, an increase in the external tax beyond τ^c can improve PTA welfare by undermining cartel discipline and raising consumer surplus. Fig. 7b illustrates the values of τ_{opt} and τ^c for $\delta \in (0, \underline{\delta}]$ and $t \in [0, t_g]$.[57]

Parts (ii) and (iii) highlight a third possibility, not present in part (i). When δ is sufficiently small and t is large enough to eliminate cross hauling, we obtain $\tau_{opt} < \tau^c$. The logic behind this finding is the reverse of the one associated with $\tau_{opt} > \tau^c$ described above, as evaluation of (17) at $\tau = \tau^c$ now implies $dW^*/d\tau|_{\tau=\tau^c} < 0$ for the internal tariff values considered. Strikingly, as underlined in part (b), the optimal external policy may turn out to be an export subsidy (i.e., $\tau_{opt} < 0$).[58] A necessary condition for this possibility (in addition to requiring $dW^*/d\tau|_{\tau=\tau^c} < 0$) is that τ^c be sufficiently close to 0 (which, as we

[55] See Fig. 4 for a description of the discount factor intervals and (some) of the internal tariff levels considered in Proposition 5. Keep in mind that, while $\tau = 0$ in Fig. 4, now $\tau = \tau^c$ in the background. Also recall that $\underline{t}_m$ and $\bar{t}_m$ (if $\bar{t}_m \neq \bar{t}$) are identified with the lower and upper bounds, respectively, (if present) of the internal tariff intervals that imply $\theta^c = 0$.

[56] The following additional ideas should also be kept in mind for $\tau_{opt} = \tau^c$. In parts (i) and (ii), we have $\tau_{opt} = \tau^c = 0$ for $t \in \{0\} \cup [\underline{t}_m, \bar{t}]$ and $t \in [\underline{t}_m, \bar{t}_m]$, respectively. This is so for these internal tariff levels because they sustain the monopoly outcome and thus preclude the necessity to shift rents via external policy. In parts (ii) and (iii), however, $\tau_{opt} = \tau^c > 0$ for $t \in \{0, t_g, \bar{t}\}$, which are the tariff levels associated with the pivot points. The difference is that now there is room for an export tax to shift rents.

[57] The determination of τ_{opt} for $t \in \left(t_y|_{\tau=0}, t_y|_{\tau=\bar{\tau}}\right)$ is more involved than may appear on first sight. For any given t in this interval, gradually raise τ from 0 all the way to $\bar{\tau}$. Since θ^* is continuously increasing in τ and, moreover, $\theta^* = \theta^{2*}$ at $\tau = 0$ while $\theta^* = \theta^{1*}$ at $\tau = \bar{\tau}$, there is a value of τ between these extremes, call it τ_y, that implies $\theta^* = \theta^{1*} = \theta^{2*}$. Moreover, $W^* = W^{2*}$ (since $y^* = 0$) for $\tau \leq \tau_y$ whereas $W^* = W^{1*}$ (since $y^* > 0$) for $\tau > \tau_y$. Hence, W^* has a kink at τ_y that may affect the determination of τ_{opt}. Specifically, we have $\arg\max_\tau W^* = \arg\max_\tau W^{1*}$ for t close to $t_y|_{\tau=0}$ whereas $\arg\max_\tau W^* = \arg\max_\tau W^{2*}$ for t close to $t_y|_{\tau=\bar{\tau}}$. It follows that there exists a proper subset of $\left(t_y|_{\tau=0}, t_y|_{\tau=\bar{\tau}}\right)$ such that $\tau_y = \arg\max_\tau W^*$ for all t in this subset.

[58] Notwithstanding the prohibition of export subsidies by the WTO, our argument differs from the well-known argument in the strategic trade policy literature (Brander and Spencer, 1985; Maggi, 1986); in our setting the policy is jointly optimal to PTA members.

have already seen, happens when internal tariff levels are moderately large). Fig. 7*b* depicts this possibility for $\delta \in (0, \underline{\underline{\delta}})$.

Fig. 7*b* also sheds light on how τ_{opt} may respond to internal tariff changes t. Two interesting possibilities stand out. First, a reduction in t can bring about a fall in external policy τ_{opt}. This possibility, which arises when t is sufficiently small or sufficiently large, is reminiscent of corresponding findings in the literature (e.g., Bagwell and Staiger, 1997; Syropoulos, 1999; and Bond et al. 2004) which find, in the context of very different models, that external trade policy is a complement to internal trade policy. Second, if internal tariffs exceed the prohibitive level t_y, but are sufficiently close to it so that cross hauling is absent, then further reduction in these tariffs cause τ_{opt} to rise; thus, in this case, τ_{opt} is a substitute for t. What separates these findings from the ones in earlier contributions should be clear by now: ours hinge on the endogeneity of cartel discipline.

Below are a couple of additional observations on the welfare effects of preferential tariff reductions in this context:

Proposition 7 *(Optimal external policy adjustments and welfare) Suppose the PTA's external policy maximizes joint PTA welfare (τ_{opt}).*

- *a) If internal tariffs are sufficiently low, then the deepening of regional integration ($t \downarrow$) will spur external trade ($z^* \uparrow$) and* ***benefit*** *ROW.*
- *b) The deepening of regional integration may reduce welfare of PTA members.*
- *c) PTA welfare is decreasing in the discount factor.*

In contrast to Proposition 5*a*, part (*a*) suggests that, if regional integration has already progressed substantially, then the deepening of this integration will be appealing to ROW! What's more, ROW will favor the complete elimination of internal tariffs (i.e., $t = 0$). The intuition behind this seemingly paradoxical finding can be explained by noting that changes in t affect z^* (and thus W^*_{ROW}) through two channels: (i) a strengthening in cartel discipline ($\theta \downarrow$), which reduces z^*; and (ii) a reduction in the external tax ($\tau_{opt} \downarrow$), which spurs z^*.[59] The latter effect dominates the former for t sufficiently close to 0.

It may be tempting for one to conclude that ROW finds the PTA's optimal tax appealing. That would be incorrect. The analysis examines how welfare in ROW responds to a change in t (that, in turn, affects τ_{opt}) and does not compare the welfare levels associated with $\tau = \tau_{opt}$ and $\tau = 0$. In fact, ROW would have preferred the cartel to behave as a pure monopoly or the PTA to impose τ^c instead of τ_{opt}. This is so because $\tau_{opt} > \tau^c$ for all $t < t_g$. Fig. 7*c* (viewed together with Figs 7*a* and 7*b*) illustrates these ideas.

[59] The improvement in cartel discipline due to a fall in t is moderated by the incipient reduction in τ_{opt} where, naturally, the two effects are jointly determined.

Turning to PTA welfare, part (b) reveals that the deepening of integration may harm PTA members, even if external policy is optimal. Reductions in t promote efficiency within the PTA for low discount factor values but not for $\delta \in (\underline{\delta}, \widehat{\delta})$. This possibility arises when regional integration has progressed significantly and is illustrated in Fig. 6c.[60] Once again, this finding is due to the pro-collusive effect of preferential trade liberalization, which can reign supreme even when the PTA's common external trade policy is optimal. This novel and thought-provoking finding contrasts Lemma 4 and challenges the existing wisdom in the literature on PTAs which has shown that internally free trade is optimal in CUs whose external intervention is optimal. The reason for this divergence in findings is that, in the current setting, internal trade liberalization does not serve as a substitute for regulation and, of course, cartel discipline is endogenous.

Finally, it is interesting to note this: a policy of free trade within the PTA coupled with an external tax τ_{opt} may enhance welfare in ROW and in the PTA. Nonetheless, as already discussed above, this is not always the case.[61]

6 Concluding Remarks

We have considered a duopoly model in which firms collude, not only in their own, but also in third-country markets. A notable objective of our work has been to identify, understand and characterize the importance of interdependence in cartel shipments to multiple markets through the cartel's ICC. First, we identified the conditions on trade costs, the relative size of markets, and the salience of the future that ensure the sustainability of maximal collusion. Then, with the help of a simple index (related to the shadow price of the cartel's ICC) whose inverse captures cartel discipline, we studied the dependence of that discipline on fundamentals. This enabled us to pursue a second objective: to characterize, not just the direct effects of trade costs on internal and external trade flows, but also their indirect effects through cartel discipline. Among other things, our analysis revealed that preferential trade liberalization boosts cartel discipline when cross hauling between cartel hosts is present. Expectably, this type of trade liberalization also affects cartel discipline even when there is no cross hauling.

Our third objective is related to the welfare implications of regional economic integration for PTA members and ROW. In the spirit of much of the existing literature on PTAs, we pursued this issue by considering three distinct circumstances regarding the PTA's common

[60] We did not include in Fig. 6 a panel that describes the response of PTA welfare in the presence of modified Kemp-Wan adjustments because their pattern is similar to the one depicted in Fig. 6c.

[61] This possibility arises when the discount factor is sufficiently low. One can show that a sufficient condition for this eventuality is $\delta < \tilde{\tilde{\delta}}$, where $\tilde{\tilde{\delta}}$ is an appropriately defined threshold level. On the other hand, if $\delta > \tilde{\tilde{\delta}}$, then the combined optimal policy package for the PTA can turn out to be an external tax τ_{opt} ($> \tau^c$) accompanied with a positive (and possibly prohibitive) tariff on internal trade.

external policy: fixed taxes, modified Kemp-Wan, and optimal external policies. Perhaps our most notable finding in this context is that the deepening of regional integration can reduce welfare of PTA members. This possibility arises because the adverse effect of internal tariff reductions on consumer surplus (through the strengthening in cartel discipline) may outweigh the accompanying direct and beneficial effect. Moreover, the possible emergence of trade diversion could also prove detrimental to ROW. Strikingly, the effect on PTA welfare persists under all processes regarding the determination of external tariffs we considered. Since in all cases domestic regulation is absent, we view this as indicative of the importance of such regulation: in its absence, preferential trade liberalization may be unappealing on welfare grounds to PTA members, to ROW, and to the world as a whole; in short, regional integration may be welfare-reducing in a Pareto sense. Since these possibilities do not emerge in standard analyses of segmented markets, it is imperative to note that they hinge on the cartel's ICC and are driven by the endogeneity of cartel discipline.

Last, but not least, our analysis shed light on the implications of cartel discipline for the nature of optimal external and internal policies in PTAs. Utilizing the modified Kemp-Wan policy as a benchmark, we characterized the PTA's optimal external policy and showed, among other things, that: (i) it may serve as a complement (substitute) to preferential trade liberalization; (ii) it may turn out to be a subsidy, and (iii) that, in its presence, internal tariff cuts may improve welfare in ROW. We also found that, in the absence of appropriate regulation, restrictions to internal trade may enhance welfare of PTA members.[62]

The analysis can be extended in numerous directions. First, one could study more severe punishments than the permanent reversion to the Nash equilibrium. We expect the gist of our conclusions to remain intact especially if one considered strategies that stipulate zero profits upon defection from the cartel agreement. Second, it is of interest to extend the analysis to study the presence of asymmetric trade costs. This is a challenging (if not intractable) problem that requires careful modeling of the disposition of profits among cartel members.[63] Third, the analysis could be extended to consider an oligopoly that serves multiple third-country markets. Though we do not expect this extension to change the key insights of our analysis, it is nonetheless more descriptive of real-world cartels and therefore desirable. Fourth, antitrust policy should be incorprated in the analysis to study its implications for cartel discipline and the design of trade policies. Finally, several aspects of our theory could be tested empirically. We plan to address variants of this issue using an extensive dataset we have constructed that contains information on discovered and prosecuted international cartels operating in 34 OECD countries.

[62] It is important to note that such restrictions may be called for, not to reduce the waste of resources used in transportation (as, for example, in Brander (1981) and Brander and Krugman (1983)), but to moderate the effect on cartel discipline.

[63] See Vasconcelos (2005) and Bos and Harrington (2010) for important contributions in this direction.

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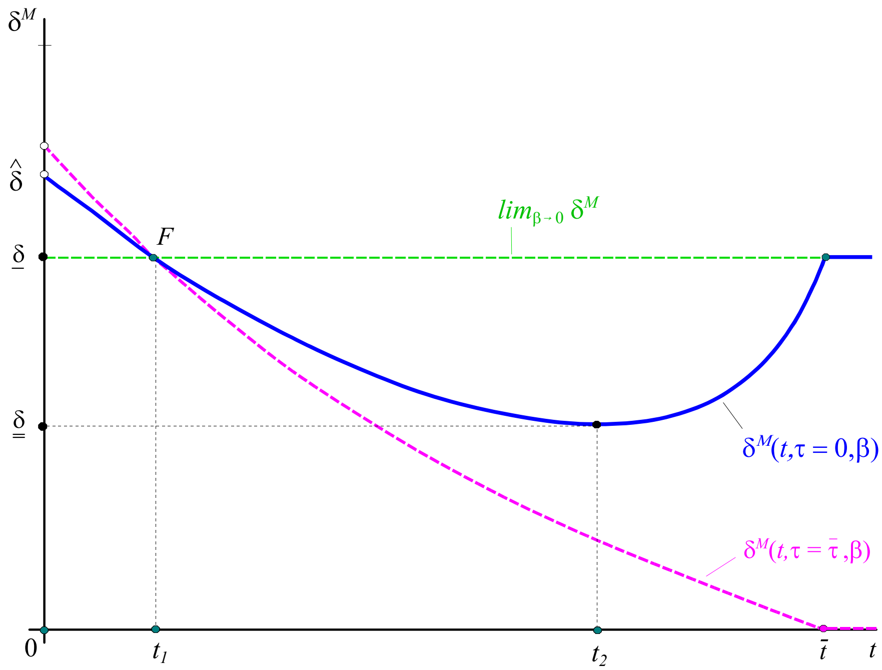

Figure 1: Minimum Discount Factors and Trade Costs

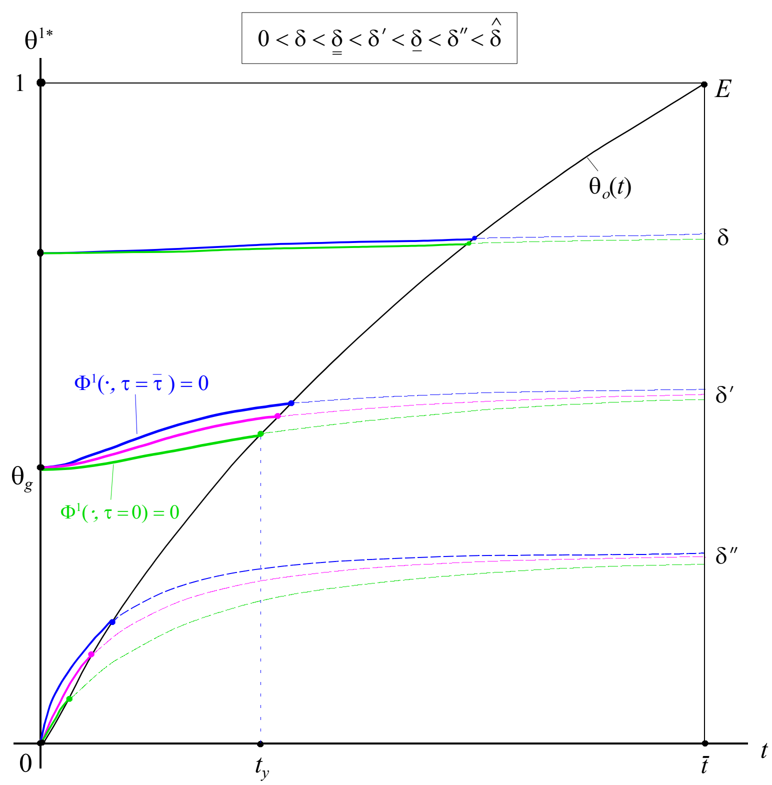

Figure 2: Cartel Discipline in the Presence of Cross Hauling ($\Phi^1 = 0$) under Various Trade Costs and Discount Factors

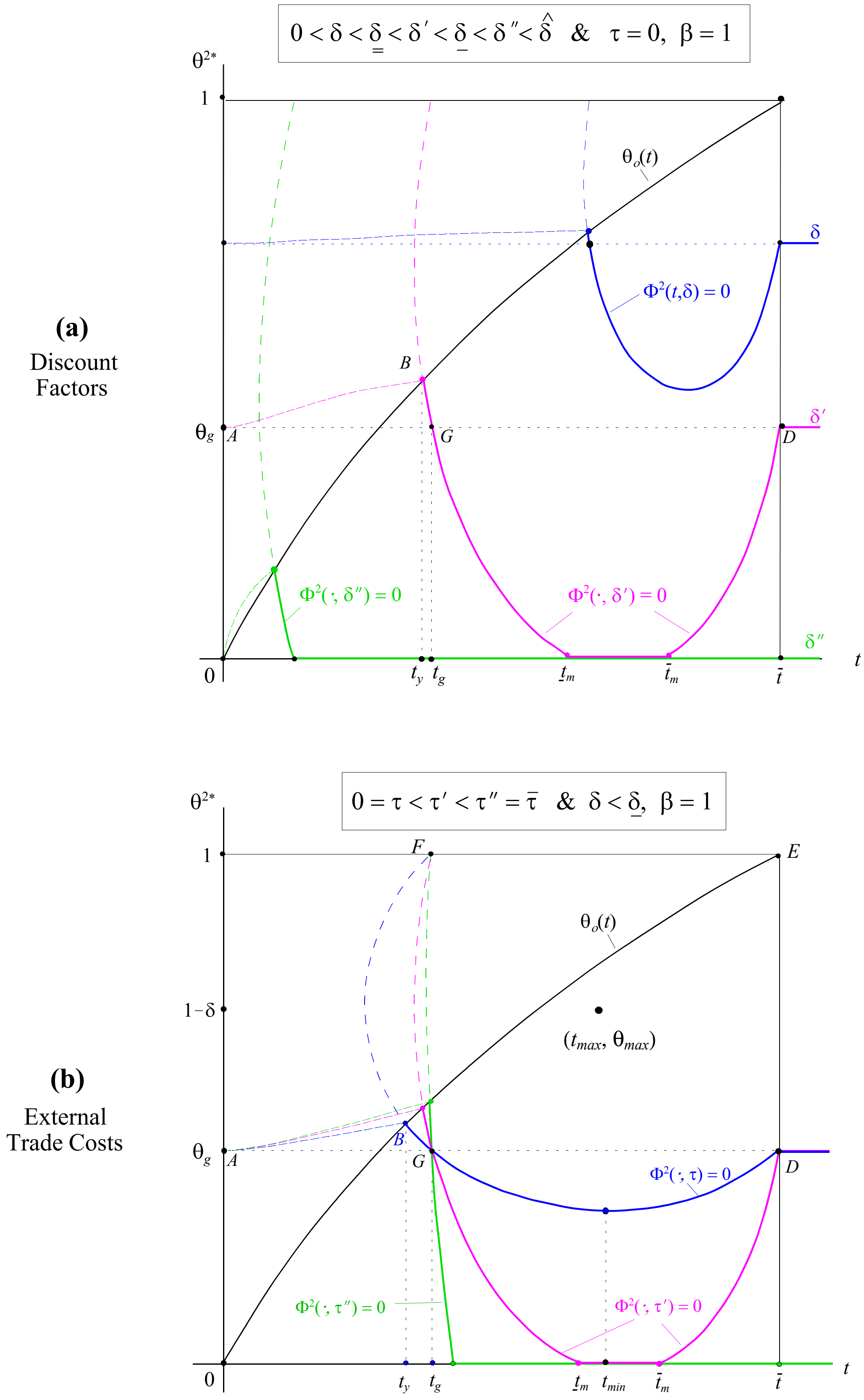

Figure 3: Cartel Discipline in the Absence of Cross Hauling ($\Phi^2 = 0$)

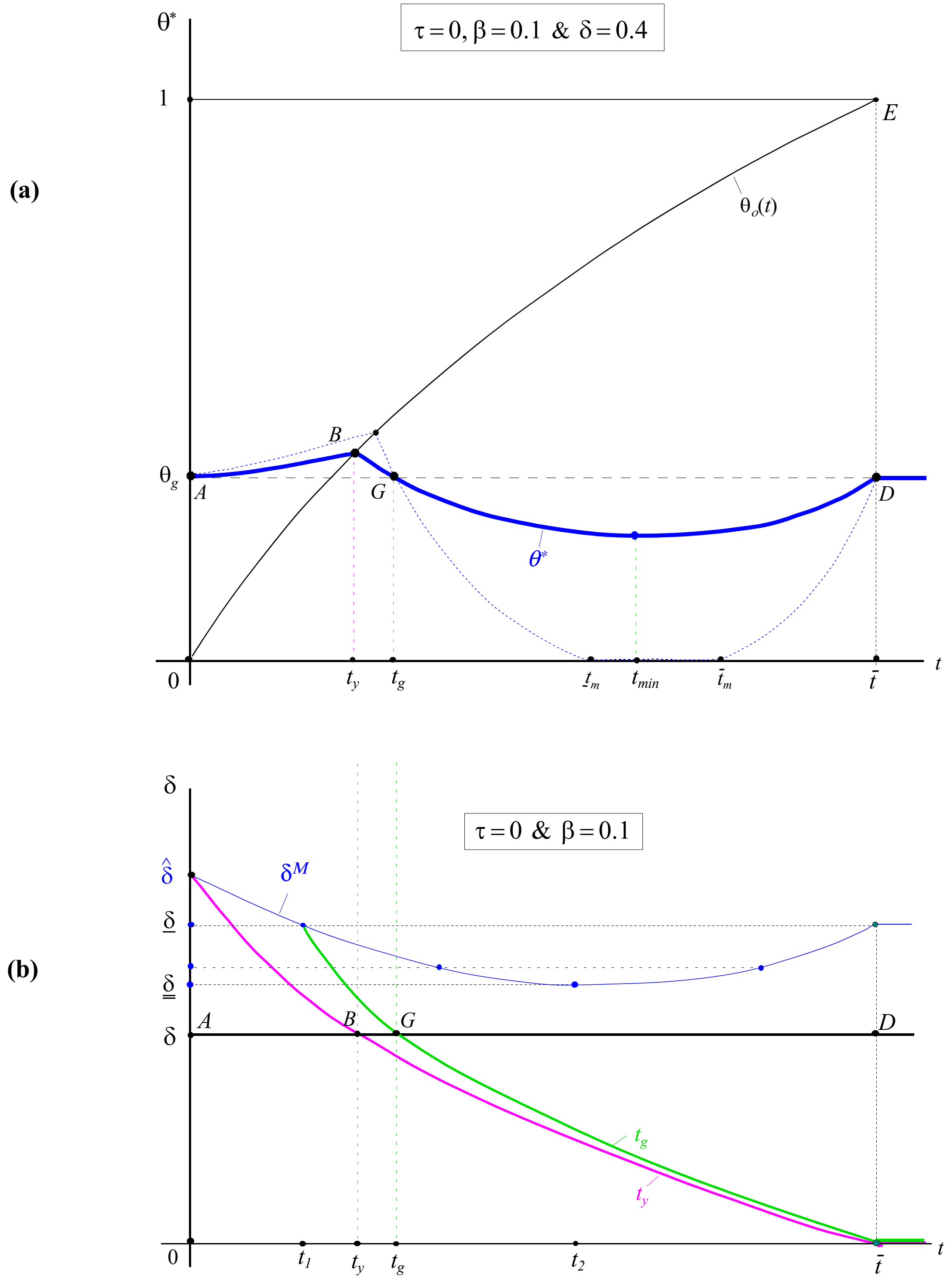

Figure 4: Equilibrium Cartel Discipline (θ^*) and Its Dependence on Internal Trade Costs (t) and the Discount Factor (δ)

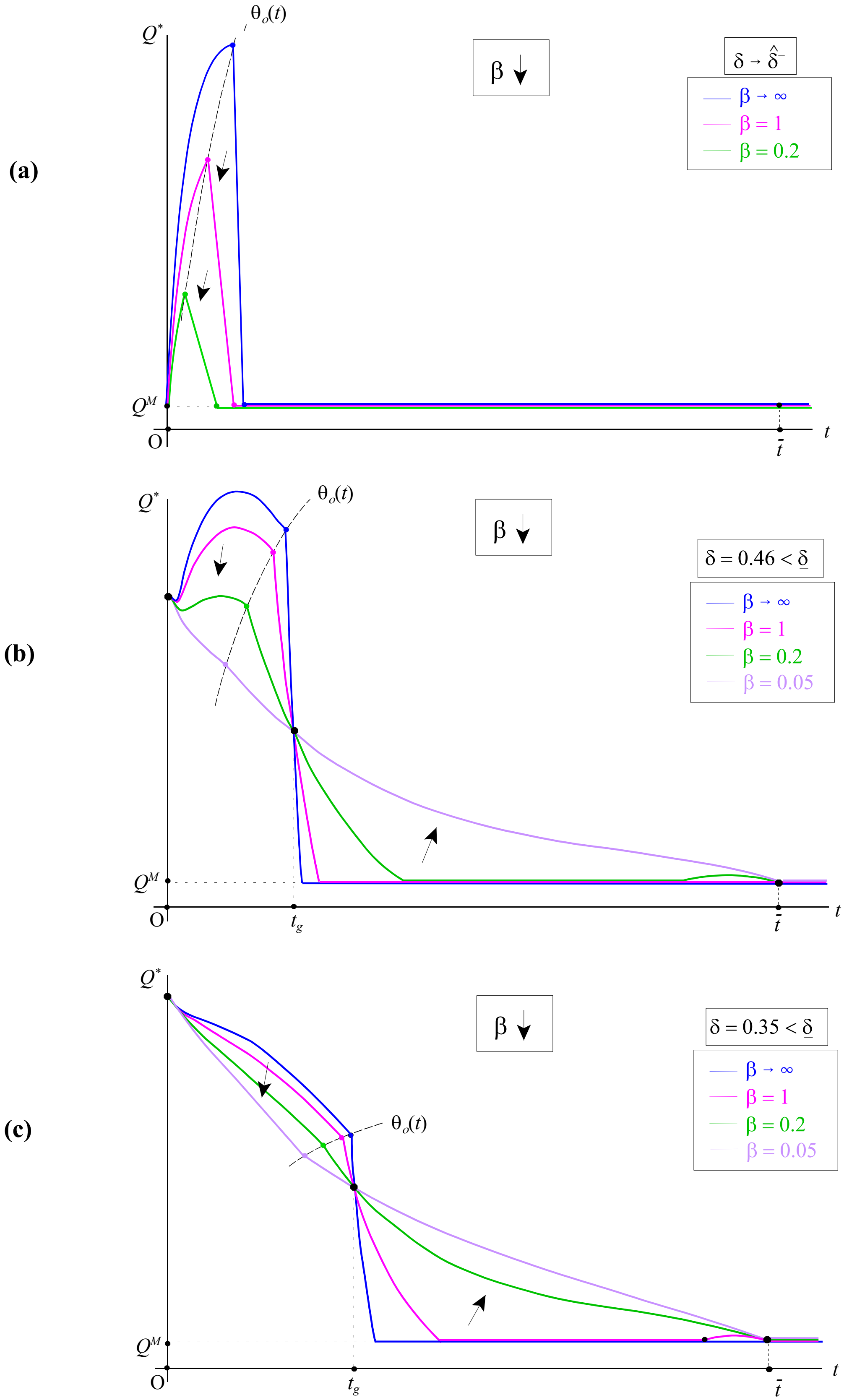

Figure 5: Output in Cartel Hosts and Its Dependence on the Discount Factor, Internal Trade Costs and ROW Size

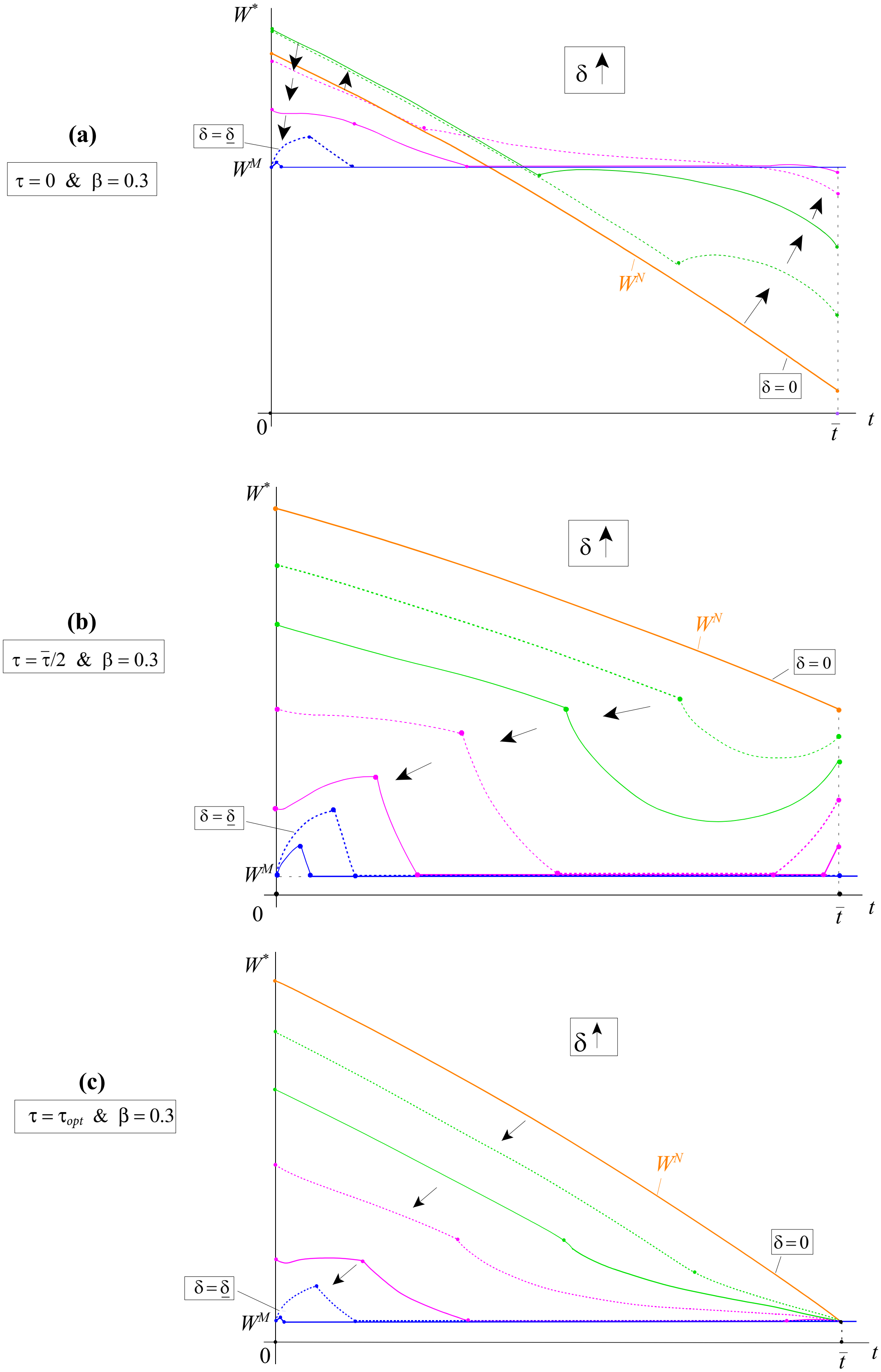

Figure 6: The Dependence of PTA Welfare on Internal Tariffs, the Discount Factor and External Policy

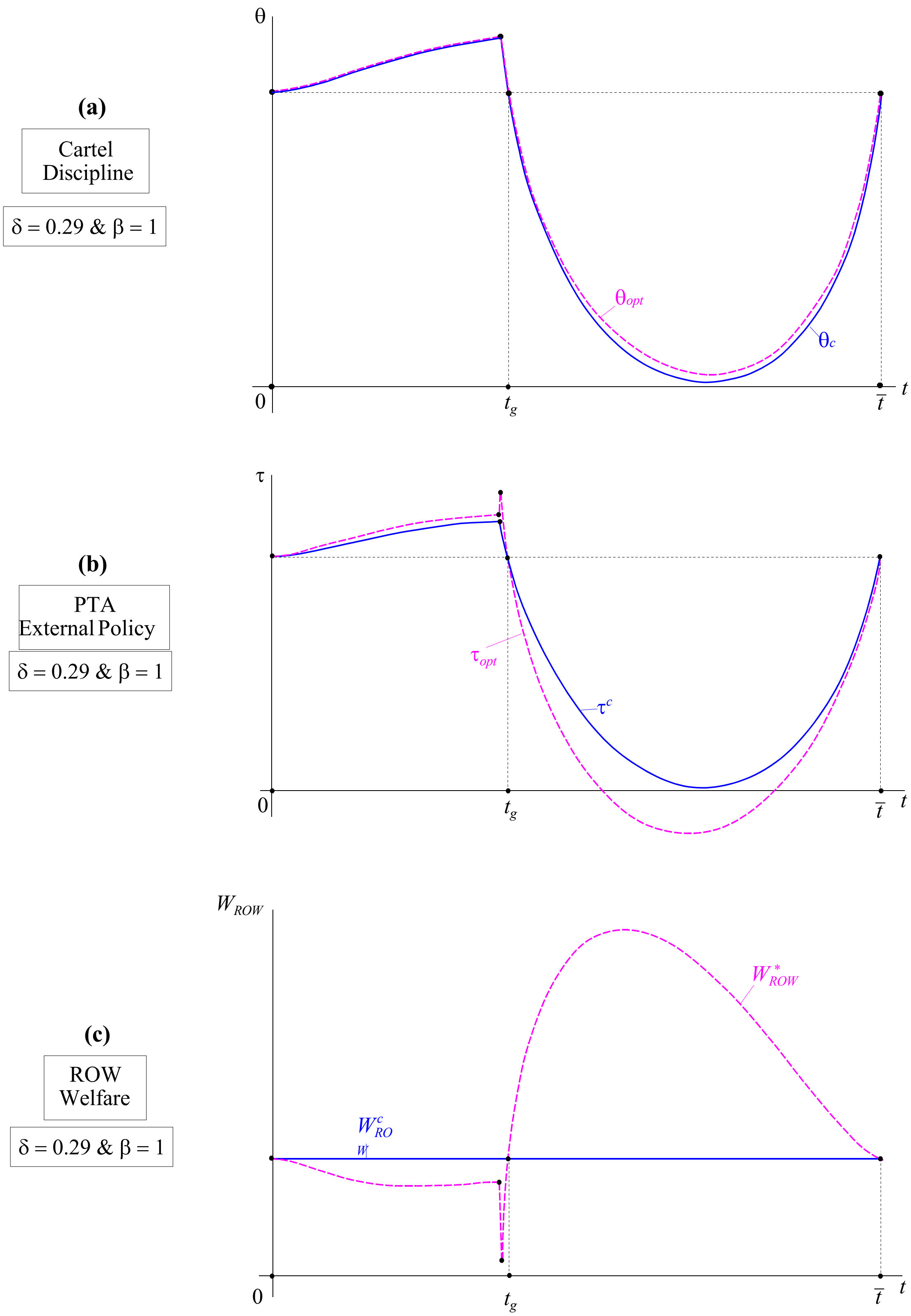

Figure 7: Cartel Discipline, External Trade Policies and Welfare in ROW

Appendix A (Online)

Proposition A1: *(Bond and Syropoulos, 2008) Suppose $\tau \geq \bar{\tau}$ so that $z = 0$. Then the minimum discount factor associated with maximal collusion, $\delta^M(t, \bar{\tau}, \beta)$, has the following properties:*

a) $\delta^M(0, \bar{\tau}, \beta) = \underline{\delta} = \frac{9}{17}$ *whereas* $\delta^M(t, \bar{\tau}, \beta) = \frac{18(\bar{t}-t)}{13A+22t}$ *for* $t \in (0, \bar{t}]$*;*

b) $\lim_{t\to 0} \delta^M(t, \bar{\tau}, \beta) = \widehat{\delta} = \frac{9}{13}$, $\lim_{t\to\bar{t}} \delta^M = 0$ *and* $d\delta^M/dt < 0$ *for* $t \in (0, \bar{t}]$*.*

Proof of Propositions A1 and 1. To describe the dependence of δ^M on trade costs and market size, we need to specify Π^D, Π^C and Π^N in the definition of Φ in (ICC) for $q = q^M$ (or, equivalently, in (8)). Recall from our discussion in the main text that the value of Π^D varies depending on whether $t = 0$ or $t > 0$. If $t = 0$ then $q^M = (x, y, \frac{A-\tau}{4\beta})$ for any $x + y = Q^M$ and $\Pi^D = \frac{(A-y)^2}{4} + \frac{(A-x)^2}{4} + \frac{9(A-\tau)^2}{64\beta}$. In this case Π^D is lowest when $x = y = Q^M/2$; hence, $\Pi^D = \frac{9A^2}{32} + \frac{9(A-\tau)^2}{64\beta}$ for $t = 0$. On the other hand, if $t > 0$ then $q^M = (Q^M, 0, \frac{A-\tau}{4\beta})$ and thus $\Pi^D = \frac{A^2}{4} + \frac{(A/2-t)^2}{4} + \frac{9(A-\tau)^2}{64\beta}$. Next, note that $\Pi^C = \Pi^M$, with Π^M satisfying (4) regardless of the value of t. Lastly, note that Π^N conforms to (2).

Define the local variable $\Psi = \Psi(t, \tau, \beta) \equiv 72\Phi(q^M(\tau, \beta), t, \tau, \beta, \delta)$. Applying the above ideas onto Φ in (ICC), simplifying the resulting expression and searching for the lowest discount factor that ensures $\Psi = 0$ implies the following: *First*, for $t = 0$ and $\tau \geq \bar{\tau}$, we find $\Psi = (\delta^M - \underline{\delta})\left[\frac{17}{4}A^2\right] = 0$ where $\underline{\delta} = \frac{9}{17}$; therefore, $\delta^M(0, \tau, \beta) = \underline{\delta}$ as stated in part (a) of Proposition 1. Similarly, for $t = 0$ and $\tau < \bar{\tau}$, we have $\Psi = (\delta^M - \underline{\delta})\left[\frac{17}{4}A^2 + \frac{17(\bar{\tau}-\tau)^2}{8\beta}\right] = 0$ which, once again, implies $\delta^M(0, \tau, \beta) = \underline{\delta}$, as stated in part $(a.i)$ of Proposition 1. *Second*, if $t \geq \bar{t}$ $(= A/2)$ and $\tau < \bar{\tau}$, then $\Psi = (\delta^M - \underline{\delta})\left[\frac{17(\bar{\tau}-\tau)^2}{8\beta}\right] = 0$ which demands $\delta^M = \underline{\delta}$, as required in part $(a.i)$ of Proposition 1.

Let us now focus on $t \in (0, \bar{t})$. If $\tau \geq \bar{\tau}$, then

$$\Psi = \left[\delta^M - \frac{18(\bar{t}-t)}{13A+22t}\right](\bar{t}-t)(13A+22t) = 0.$$

This helps prove the remainder of Proposition $A1$. On the other hand, if $\tau < \bar{\tau}$, as required in Proposition 1, we find (after some algebra)

$$\Psi = \left[\delta^M - \frac{18(\bar{t}-t)}{13A+22t}\right](\bar{t}-t)(13A+22t) + \left(\delta^M - \underline{\delta}\right)\left[\frac{17(\bar{\tau}-\tau)^2}{8\beta}\right] = 0.$$

The above expression can be rewritten as

$$\Psi = \frac{504}{17}(t-t_1)(\bar{t}-t) + \left(\delta^M - \underline{\delta}\right)\left[(\bar{t}-t)(13A+22t) + \frac{17(\bar{\tau}-\tau)^2}{8\beta}\right] = 0, \qquad \text{(A.1)}$$

where $t_1 \equiv A/14$. Solving $(A.1)$ for δ^M and simplifying the resulting expression gives

$$\delta^M = \frac{18\left(\bar{t}-t\right)^2 + \frac{9(\bar{\tau}-\tau)^2}{8\beta}}{\left(\bar{t}-t\right)\left(13A+22t\right) + \frac{17(\bar{\tau}-\tau)^2}{8\beta}} = \kappa\left[\frac{18\left(\bar{t}-t\right)}{13A+22t}\right] + (1-\kappa)\left[\frac{9}{17}\right], \tag{A.2}$$

where $\kappa \equiv \frac{(\bar{t}-t)(13A+22t)}{(\bar{t}-t)(13A+22t)+\frac{17(\bar{\tau}-\tau)^2}{8}} < 1$. The last expression in $(A.2)$ is the weighted sum of the minimum discount factors that would arise in the hosts of the cartel and in ROW if firms did not pool their incentives constraints in these regions. One can verify that $\delta^M < 1$.

Remainder of parts (a), (b) and (c) of Proposition 1. Since the focus is on $t < \bar{t}$, part $(a.ii)$ follows readily from $(A.1)$. We will demonstrate part $(a.iii)$ shortly. First note though that $d\delta^M/d\gamma = -\Psi_\xi/\Psi_\delta$, for $\xi \in \{t,\tau,\beta\}$. Differentiating $(A.1)$ and simplifying terms gives

$$\begin{aligned}
\Psi_\delta &= \left(\bar{t}-t\right)\left(13A+22t\right) + \frac{17\left(\bar{\tau}-\tau\right)^2}{8\beta} > 0, \\
\Psi_t &= -2\left(A+22t\right)\left[\delta^M - \frac{18(\bar{t}-t)}{A+22t}\right] = 4\left[\frac{9}{4}A\left(3-\delta^M\right) + \left(9+11\delta^M\right)\left(t_1-t\right)\right], \\
\Psi_\tau &= -\left(\delta^M - \underline{\delta}\right)\left[\frac{17\left(\bar{\tau}-\tau\right)}{4\beta}\right] \gtreqless 0 \text{ if } \delta^M \lesseqgtr \underline{\delta}, \\
\Psi_\beta &= -\left(\delta^M - \underline{\delta}\right)\left[\frac{17\left(\bar{\tau}-\tau\right)^2}{8\beta^2}\right] \gtreqless 0 \text{ if } \delta^M \lesseqgtr \underline{\delta}.
\end{aligned}$$

Parts (b) and (c) follow from inspection of the above expressions and from studying the limits of δ^M in $(A.2)$ as $\beta \to 0$ and $\beta \to \infty$, respectively.

To prove part $(a.iii)$, note the following. First, $\Psi_t(t,\tau,\beta) > 0$ for any $t \in (0,t_1]$; therefore, $\delta^M_t < 0$ for $t \in (0,t_1]$. Second, from $(A.2)$ we have $\lim_{t\to\bar{t}} \delta^M \to \underline{\delta}$ as $t \to \bar{t}$ from below. Moreover, $\lim_{t\to\bar{t}} \delta^M_t > 0$ since $\lim_{t\to\bar{t}} \Psi_t(t,\tau,\beta) < 0$. By the continuity of δ^M in $t \in (0,\bar{t})$, there will exist a trade cost $t_2 \equiv \arg\min_t \delta^M\left(t,\tau,\beta\right) \in (t_1,\bar{t})$ for any $\tau < \bar{\tau}$. Setting $\Psi_t = 0$, utilizing the definition of δ^M in the resulting expression, and solving for t gives

$$t_2\left(k\right) \equiv \frac{1}{6A}\left[3A^2 + 7k - \sqrt{k\left(18A^2+49k\right)}\right] \quad \text{where} \quad k = k\left(\tau,\beta\right) \equiv \frac{17\left(A-\tau\right)^2}{8\beta}.$$

Differentiating $t_2\left(\cdot\right)$ appropriately gives $\partial t_2/\partial\xi = t_2'\left(k\right)k_\xi$ for $\xi \in \{\tau,\beta\}$ where

$$t_2'\left(k\right) = -\left[\frac{27A^3}{2k\left(18A^2+49k\right)}\right]\left[7 + \frac{9A^2+49k}{\sqrt{k\left(18A^2+49k\right)}}\right]^{-1} < 0 \quad \text{and} \quad k_\xi < 0.$$

Thus $\partial t_2/\partial\xi > 0$ for $\xi \in \{\tau,\beta\}$. $\lim_{\tau\to\bar{\tau}} t_2 = \lim_{\beta\to\infty} t_2 = \bar{t}$ because $\lim_{\tau\to\bar{\tau}} k = \lim_{\beta\to\infty} k =$

0. ||

Proof of Lemma 1: Henceforth, we simplify notation by defining $\tilde{x} \equiv x^D$, $\tilde{y} \equiv y^D$, $\tilde{z} \equiv z^D$ and to avoid cluttering we drop superscript "*". The active ICC requires $\Phi^1(\theta, \delta, t, \tau, \beta) = 0$ and, as noted in the text, $1 - \delta - \theta > 0$. Recalling from (9) that $\Phi_j^1 = \Pi_j^C - (1-\delta)\Pi_j^D$ for $j = x, y, z$, taking into account (10) and defining $\Phi_\theta^1 \equiv d\Phi^1/d\theta$ yields

$$\begin{aligned}\Phi_\theta^1 &= (1-\delta-\theta)\left[\left(-\Pi_x^D\right)x_\theta^1 + \left(-\Pi_y^D\right)y_\theta^1 + \left(-\Pi_z^D\right)z_\theta^1\right] \quad \text{from (7)} \\ &= (1-\delta-\theta)\left[\tilde{y}x_\theta^1 + \tilde{x}y_\theta^1 + \beta\tilde{z}z_\theta^1\right] \quad \text{from (5).}\end{aligned} \tag{A.3}$$

We will argue that $\Phi_\theta^1 > 0$ which is tantamount to showing that $\Pi^D\left(q^1(\theta,\cdot),\cdot\right)$ is decreasing in θ. Differentiation of the expressions in (10a) with respect to θ yields

$$Q_\theta^1 = \frac{3(2A-t)}{(8+\theta)^2} > 0, \tag{A.4a}$$

$$x_\theta^1 = \frac{1}{2}\left(Q_\theta^1 - \frac{t}{\theta^2}\right), \tag{A.4b}$$

$$y_\theta^1 = \frac{1}{2}\left(Q_\theta^1 + \frac{t}{\theta^2}\right) > 0, \tag{A.4c}$$

$$z_\theta^1 = \frac{6(A-\tau)}{\beta(8+\theta)^2} > 0. \tag{A.4d}$$

The above equations enable us to transform the expression inside the square brackets of $(A.3)$ into

$$\begin{aligned}\tilde{y}x_\theta^1 + \tilde{x}y_\theta^1 + \beta\tilde{z}z_\theta^1 &= \tilde{y}\left(Q_\theta^1 - \frac{t}{\theta^2}\right) + \tilde{x}\left(Q_\theta^1 + \frac{t}{\theta^2}\right) + \tilde{z}\frac{6(A-\tau)}{(8+\theta)^2} \\ &= (\tilde{x}+\tilde{y})Q_\theta^1 + (\tilde{x}-\tilde{y})\frac{t}{\theta^2} + \tilde{z}\frac{6(A-\tau)}{(8+\theta)^2} \\ &= \frac{9(2A-t)^2}{(8+\theta)^3} + \frac{t^2}{\theta^3} + \frac{18(A-\tau)^2}{\beta(8+\theta)^3} > 0,\end{aligned}$$

where the last term was obtained from $(A.4)$ and the facts that

$$\tilde{x} = \frac{1}{2}\left[\frac{3(2A-t)}{8+\theta} + \frac{t}{\theta}\right], \tag{A.5a}$$

$$\tilde{y} = \frac{1}{2}\left[\frac{3(2A-t)}{8+\theta} - \frac{t}{\theta}\right], \tag{A.5b}$$

$$\tilde{z} = \frac{3(A-\tau)}{\beta(8+\theta)}, \tag{A.5c}$$

which imply $\tilde{x}+\tilde{y}=\frac{3(2A-t)}{8+\theta}$ and $\tilde{x}-\tilde{y}=t/\theta$.[1] For clarity, we rewrite $(A.3)$ as

$$\Phi^1_\theta=(1-\delta-\theta)\left[\frac{9(2A-t)^2}{(8+\theta)^3}+\frac{t^2}{\theta^3}+\frac{18\,(A-\tau)^2}{\beta\,(8+\theta)^3}\right]>0. \tag{A.3$'$}$$

The positive sign of $(A.3')$ establishes that weaker cartel discipline ($\theta\uparrow$) relaxes the ICC.

To take a closer look at the solution θ^{1*} to $\Phi^1=0$, substitute (10) into the ICC and simplify the resulting expression to obtain

$$\Phi^1=\frac{(1-\theta)\,(17+\theta)}{(8+\theta)^2}\Omega^1=0, \tag{A.6a}$$

where

$$\Omega^1=-\frac{9\,(1-\theta)}{17+\theta}\left[\frac{4t^2\,(4-\theta)\,(2+\theta)}{9\theta^2}+\Pi^N\right]+\delta\left[\frac{8t^2\,(4+5\theta)}{\theta^2(17+\theta)}+\Pi^N\right] \tag{A.6b}$$

for $t>0$ and $\delta<\delta^M$. One can see from $(A.6a)$ that, indeed, $\theta^{1*}=1$ is a generic solution, as noted in the text. Moreover, the ICC is active as $\delta\to 0$ only if $\lim_{\delta\to 0}\Omega^1=0$ for $t\leq\bar{t}$, which is possible only if $\lim_{\delta\to 0}\theta^{1*}=1$.

Now suppose $t=0$. If $\delta\in[\underline{\delta},\widehat{\delta})$, maximal collusion will be sustainable, so $\theta^{1*}=0$. On the other hand, if $\delta\in[0,\underline{\delta})$ the ICC will bind, so there will exist a θ that ensures $\Omega^1=0$. Utilizing $(A.6b)$ one can show that $\theta^{1*}=\theta_g$, where $\theta_g\equiv 17(\underline{\delta}-\delta)/(9+\delta)>0$ and $\underline{\delta}=9/17$. In short, $\theta^{1*}=\max(0,\theta_g)$ for $t=0$ and $\delta\in[0,\widehat{\delta})$.

Let us now focus on $t\in(0,\bar{t})$. From $(A.6b)$ one can verify that $\Omega^1<0$ as θ becomes arbitrarily small whereas $\lim_{\theta\to 1-\delta}\Omega^1>0$. Since Ω^1 is continuous and increasing in θ on $(0,1-\delta)$ there exists a unique solution $\theta^{1*}\in(0,1-\delta)$ to $\Omega^1=0$ (and thus to $\Phi^1=0$) as claimed in Lemma 1. Inspection of $(A.6b)$ also reveals that we can rearrange $\Omega^1=0$ to obtain

$$(t/\theta)^2=\frac{\left[\delta-\frac{9(1-\theta)}{17+\theta}\right]\Pi^N}{\frac{4(1-\theta)(4-\theta)(2+\theta)}{17+\theta}-\delta\frac{8(4+5\theta)}{17+\theta}}.$$

Since $\lim_{t\to 0}\theta^{1*}=\theta_g$ for $\delta\in[0,\underline{\delta})$ the above equation readily implies $\lim_{t\to 0}(t/\theta^{1*})^2=0$ in this case. In contrast, because $\lim_{t\to 0}\theta^{1*}=0$ for $\delta\in[\underline{\delta},\widehat{\delta})$, we will have

$$\lim_{t\to 0}(t/\theta^{1*})^2=\frac{17}{32}\left[\frac{\delta-\underline{\delta}}{1-\delta}\right]\Pi^N_0,\ \text{ where }\Pi^N_0=\frac{2A^2}{9}+\frac{(A-\tau)^2}{9\beta}. \tag{A.6c}$$

[1] Direct comparison of (10c) and $(A.5b)$ reveal that $\tilde{y}>0$ for all values of t that ensure $y^1>0$; therefore, keeping track of the non-negativity constraint on y^1 also takes care of the non-negativity constraint on $\tilde{y}$.

The above observations will prove helpful in some of the proofs that will follow.

Part (a). Since $d\theta^{1*}/d\delta = -\Phi^1_\delta/\Phi^1_\theta$ by the implicit function theorem and we know $\Phi^1_\theta > 0$, to prove this part it suffices to prove that $\Phi^1_\delta > 0$. But this is trivially true because $\Phi^1_\delta = \Pi^D - \Pi^N > 0$.

Part (b). Since $d\theta^{1*}/dt = -\Phi^1_t/\Phi^1_\theta$, to prove part (b) we must show that $\Phi^1_t < 0$ for $t > 0$. Differentiating Φ^1 with respect to t and utilizing (10a) together with the fact that $\Pi^C_j - (1-\delta)\,\Pi^D_j = (1-\delta-\theta)\,(-\Pi^D_j) > 0$ for $j = x, y$ gives

$$\begin{aligned}\Phi^1_t &= \Pi^C_t - (1-\delta)\,\Pi^D_t - \delta\Pi^N_t + (1-\delta-\theta)\left[-\Pi^D_x x^1_t - \Pi^D_y y^1_t\right]\\ &= -y^1 + (1-\delta)\,\tilde{y} - \delta\Pi^N_t + (1-\delta-\theta)\left[\tilde{y}x^1_t + \tilde{x}y^1_t\right].\end{aligned} \tag{A.7}$$

For clarity and future reference, note that

$$Q^1_t = -\frac{2+\theta}{8+\theta} < 0, \tag{A.8a}$$

$$x^1_t = \frac{1}{2}\left(Q^1_t + \frac{2-\theta}{\theta}\right) = \frac{8-4\theta-\theta^2}{\theta(8+\theta)} > 0, \tag{A.8b}$$

$$y^1_t = \frac{1}{2}\left(Q^1_t - \frac{2-\theta}{\theta}\right) = -\frac{4-\theta}{\theta(8+\theta)} < 0. \tag{A.8c}$$

Equation $(A.7)$ reveals that, for a given level of cartel discipline, changes in t affect Φ^1 through four channels. The first three channels involve the direct effects of t on Π^C, Π^D and Π^N, respectively. The fourth channel is indirect and is associated with the effect of t on x^1 and y^1. The direct effects of t on Φ^1 through Π^C and Π^D are clear. Since $\Pi^C_t = -y^1$ and $\Pi^D_t = -\tilde{y}$, the former effect on Φ^1 is negative whereas the latter effect on Φ^1 is positive (see $(A.7)$). The direct effect of t on Φ^1 through Π^N depends on the level of t. Since $\arg\min_t \Pi^N = A/5$, as discussed earlier, the punishment effect on the ICC is positive for low t levels and negative at high t levels.[2] One can verify (by utilizing $(A.5)$ and $(A.8)$ in $(A.7)$) that $\tilde{y}x^1_t + \tilde{x}y^1_t = \tilde{y}Q^1_t + (\tilde{x}-\tilde{y})y^1_t < 0$, which implies that the fourth (and indirect) effect of t on Φ^1 is negative. It is worth pointing a feature of Φ^1_t from $(A.7)$ and $(A.8)$ that helps prove part (d) below: Φ^1_t is invariant to changes in β and τ.

Interestingly, despite the apparent ambiguity in the sign of Φ^1_t noted above, it is possible to determine its sign by using the fact that the ICC is binding (i.e., $\Phi^1 = 0$). Substituting

[2] Natuarlly, if the punishment payoff is invariant to changes in trade costs, this effect vanishes.

the value of δ implied by this constraint in $(A.7)$ and simplifying expressions yields:

$$\Phi_t^1 = -\frac{8(1-\theta)^3 t \left[\Pi^N - t\Pi_t^N/2\right]}{\left[8t^2(4+5\theta)+\theta^2(17+\theta)\Pi^N\right]}$$
$$= -\frac{8(1-\theta)^3 t \left[A(2A-t)+(A-\tau)^2/\beta\right]}{9\left[8t^2(4+5\theta)+\theta^2(17+\theta)\Pi^N\right]} < 0. \qquad (\text{A.7}')$$

This affirms the idea that the direct effect of t on Π^C, together with its indirect effect on Π^D through (x^1, y^1), dominate the positive direct effects on Π^D and Π^N, thereby tightening the ICC. Thus, $d\theta^{1*}/dt = -\Phi_t^1/\Phi_\theta^1 > 0$ for $t > 0$.

Differentiation of $(A.6b)$ gives $d\theta^{1*}/dt = -\frac{\theta\Omega_t^1}{\theta\Omega_\theta^1}$, where

$$-\theta\Omega_t^1 = \theta\Pi_t^N\left[\frac{9(1-\theta)}{17+\theta}-\delta\right] + (t/\theta)\frac{8}{17+\theta}\left[(1-\theta)(4-\theta)(2+\theta)-2\delta(4+5\theta)\right]$$

and

$$\theta\Omega_\theta^1 = \theta\frac{162}{(17+\theta)^2}\left[\Pi^N + \frac{4}{9}(t/\theta)^2(4-\theta)(2+\theta)\right] + (t/\theta)^2\frac{8(8+\theta)}{17+\theta}\left[1-\theta-\delta\frac{17+10\theta}{17+\theta}\right].$$

Because $\lim_{t\to 0}\theta^{1*} = \theta_g > 0$ for $\delta \in (0, \underline{\delta}]$, we have $\lim_{t\to 0}\left(-\theta\Omega_t^1\right) = 0$ and

$$\lim_{t\to 0}\left(\theta\Omega_\theta^1\right) = \frac{162\Pi_0^N(9+\delta)^2}{269(9+\underline{\delta})^2} > 0,$$

where Π_0^N was defined in $(A.6c)$; therefore, $\lim_{t\to 0}\left(d\theta^{1*}/dt\right) = 0$ in this case. Turning to $\delta \in (\underline{\delta}, \widehat{\delta})$, recall that $\lim_{t\to 0}\theta^{1*} = 0$ and

$$\lim_{t\to 0}(t/\theta^{1*})^2 = \frac{17}{32}\left[\frac{\delta-\underline{\delta}}{1-\delta}\right]\Pi_0^N > 0,$$

(from $(A.6c)$), imply

$$\lim_{t\to 0}\left(-\theta\Omega_t^1\right) = \left[\lim_{t\to 0}(t/\theta)\right]\frac{64}{17}[1-\delta] \text{ and } \lim_{t\to 0}\left(\theta\Omega_\theta^1\right) = \left[\lim_{t\to 0}(t/\theta)^2\right]\frac{64}{17}[1-\delta];$$

therefore,

$$\lim_{t\to 0}\left(d\theta^{1*}/dt\right) = \lim_{t\to 0}\left(\theta^{1*}/t\right) = \left[\frac{17}{32}\left(\frac{\delta-\underline{\delta}}{1-\delta}\right)\Pi_0^N\right]^{-1/2} > 0.$$

Part (c). Since $\theta^{1*} = \max(\theta_g, 0)$ for $t = 0$ and $\delta \in [0, \widehat{\delta})$, θ^{1*} is invariant to changes in external trade costs (τ) and market size (β) in this case. For $t \in (0, \bar{t})$ and $\delta \in [0, \delta^M)$, however, θ^{1*} depends on variables related to ROW's market through their impact on Π^C, Π^D and Π^N. Differentiation of Φ^1 with respect to τ gives

$$\begin{aligned}\Phi^1_\tau &= \Pi^C_\tau - (1-\delta)\,\Pi^D_\tau - \delta\Pi^N_\tau + (1-\delta-\theta)\left[-\Pi^D_z z^1_\tau\right] \\ &= -z^1 + (1-\delta)\,\tilde{z} + \delta\frac{2\,(A-\tau)}{9\beta} + (1-\delta-\theta)\left[\beta\tilde{z}z^1_\tau\right],\end{aligned} \tag{A.9}$$

where $\tilde{z}$ is defined in $(A.5c)$ and thus $z^1_\tau = -\frac{2+\theta}{\beta(8+\theta)} < 0$. Inspection of the terms in $(A.9)$ reveals that the channels of transmission of changes in τ are similar to the ones associated with internal trade cost t changes. One difference is that now the impact of τ on Φ^1 through the punishment payoff Π^N is unambiguously positive. Still, the sign of Φ^1_τ seems ambiguous. However, this ambiguity disappears when we substitute the values of $\tilde{z}$, z^1 and z^1_τ into Φ^1_τ. Doing so gives

$$\Phi^1_\tau = \frac{2(9+\delta)\,(A-\tau)}{9\beta\,(8+\theta)^2}\,(1-\theta)\,(\theta_g-\theta)\,, \tag{A.9$'$}$$

where, again, $\theta_g \equiv 17(\underline{\delta}-\delta)/(9+\delta) \gtreqless 0$ as $\delta \lesseqgtr \underline{\delta}$. Since part (b) implies $\theta^{1*} > \theta_g$ for $t \in (0,\bar{t})$, we will have $\Phi^1_\tau < 0$ for changes along the ICC. Thus, an increase in external trade costs tightens the ICC and relaxes cartel discipline. The analysis of the effect of market size β on θ^{1*} is qualitatively similar and thus omitted.

Part (d). As noted earlier, $\theta^{1*}_t = -\Phi^1_t/\Phi^1_\theta$, where $\Phi^1_t < 0$ and $\Phi^1_\theta > 0$. In the proof of part (b) we noted that $\partial\Phi^1_t/\partial\beta = \partial\Phi^1_t/\partial\tau = 0$. On the other hand, one can see from $(A.3')$ that $sign\left(\Phi^1_{\theta\beta}\right) = sign\left(\Phi^1_{\theta\tau}\right) < 0$; therefore, $\theta^{1*}_{t\xi} > 0$ for $\xi \in \{\beta,\tau\}$, as claimed in this part. ||

Lemma A1: $\Phi^2(\theta,t,\cdot)$ *is strictly concave in* $(t,\theta) \in [0,\bar{t}] \times [0,1]$ *and is maximized at* $(t_{\max},\theta_{\max}) = \left(\frac{A}{2}\left[\frac{9+\delta-2\delta^2}{9+16\delta-5\delta^2}\right], 1-\delta\right)$. *Moreover,*

$$\Phi^2_{\max} \equiv \Phi^2(\theta_{\max}, t_{\max}, \cdot) = \delta^2\left[\frac{A^2\,(5-\delta)}{4\,(9+16\delta-5\delta^2)} + \frac{(\bar{\tau}-\tau)^2}{9\beta\,(9-\delta)}\right] > 0 \textit{ for } \delta > 0.$$

Proof: Utilizing a procedure similar to the one in the proof of Lemma 1, we may differentiate Φ^2 with respect to θ and use (11) to obtain

$$\Phi^2_\theta = (1-\delta-\theta)\left[\tilde{y}x^2_\theta + \beta\tilde{z}z^2_\theta\right].$$

To sign this expression—and several others to follow—note that

$$\tilde{y} = \frac{2(A/2-t)}{4+\theta} \quad \text{and} \quad \tilde{z} = \frac{3\,(A-\tau)}{\beta\,(8+\theta)}, \tag{A.10a}$$

$$x^2_\theta = \frac{4\,(A/2-t)}{(4+\theta)^2} > 0 \quad \text{and} \quad z^2_\theta = \frac{6\,(A-\tau)}{\beta\,(8+\theta)^2} > 0, \tag{A.10b}$$

$$x^2_t = -\frac{\theta}{4+\theta} < 0, \quad z^2_\tau = -\frac{2+\theta}{\beta\,(8+\theta)} < 0 \quad \text{and} \quad z^2_\beta = -\frac{z^2}{\beta} < 0. \tag{A.10c}$$

The partial derivatives of x^2 and z^2 in $(A.10b)$ and $(A10c)$ were obtained by differentiating (12). With the help of the above expressions we find

$$\Phi^2_\theta = (1-\delta-\theta)\left[\frac{8\,(A/2-t)^2}{(4+\theta)^3} + \frac{18\,(A-\tau)^2}{\beta\,(8+\theta)^3}\right] \;\Rightarrow\; \Phi^2_\theta \gtreqless 0 \;\text{ if }\; \theta \lesseqgtr 1-\delta. \tag{A.11a}$$

In due course we will recognize that $1-\delta-\theta>0$ (for the reasons outlined in our study of Φ^1 which require $\theta < \theta_o\,(t)$). However, to obtain a complete view of the properties of Φ^2 we initially abstract from this possibility.

Differentiating Φ^2 with respect to t gives

$$\begin{aligned}\Phi^2_t &= \Pi^C_t - (1-\delta)\,\Pi^D_t - \delta\Pi^N_t + (1-\delta-\theta)\left(-\Pi^D_x x^2_t\right)\\ &= 0 + (1-\delta)\,\tilde{y} + \delta\frac{10}{9}\,(A/5-t) + (1-\delta-\theta)\left(\tilde{y}x^2_t\right)\\ &= \delta\frac{10}{9}\left(\frac{A}{5}-t\right) + \frac{2\left[4\,(1-\delta)+\theta^2\right]}{(4+\theta)^2}\left(\frac{A}{2}-t\right).\end{aligned}$$

To obtain the last expression for Φ^2_t we used $(A.10)$. It now follows that

$$\Phi^2_t \lesseqgtr 0 \;\text{ if }\; t \lesseqgtr \frac{A}{2}\left[\frac{\frac{2\delta}{9} + \frac{4(1-\delta)+\theta^2}{(4+\theta)^2}}{\frac{5\delta}{9} + \frac{4(1-\delta)+\theta^2}{(4+\theta)^2}}\right] < \frac{A}{2} = \bar{t}. \tag{A.11b}$$

Differentiating Φ^2_θ and Φ^2_t appropriately gives

$$\Phi^2_{\theta\theta} = -\left[\frac{8\,(A/2-t)^2\,(7-3\delta-2\theta)}{(4+\theta)^4} + \frac{18\,(A-\tau)^2\,(11-3\delta-2\theta)}{\beta\,(8+\theta)^4}\right] < 0,$$

$$\Phi^2_{tt} = -\left[\frac{10}{9}\delta + \frac{2\left[4(1-\delta)+\theta^2\right]}{(4+\theta)^2}\right] < 0,$$

$$\Phi^2_{\theta t} = -\frac{16\,(1-\delta-\theta)\,(A-t)}{(4+\theta)^3}.$$

It is tedious but straightforward for one to show that $\Phi^2_{\theta\theta}\Phi^2_{tt} - (\Phi^2_{\theta t})^2 > 0$. The strict concavity of Φ^2 in (t,θ) follows from this inequality and $\Phi^2_{\theta\theta} < 0$ and $\Phi^2_{tt} < 0$.[3] The solution to $\Phi^2_{\theta} = 0$ and $\Phi^2_{t} = 0$, which is given by $(t_{\max}, \theta_{\max}) = \left(\frac{A}{2}\left[\frac{9+\delta-2\delta^2}{9+16\delta-5\delta^2}\right], 1-\delta\right)$, is the unique maximizer of Φ^2. Substituting this solution back into Φ^2 and simplifying the resulting expression gives $\Phi^2_{\max} > 0$. $||$

Fig. 3 depicts several contours associated with $\Phi^2 = 0$ under the assumption of equally sized countries ($\beta = 1$). Panel (a) illustrates the role of time preferences (δ) for cartel discipline under the assumption of free external trade ($\tau = 0$). Panel (b) highlights the importance of external trade costs (τ) for cartel discipline. In both panels only the thick, solid-line parts of the curves are relevant.[4]

The following traits of the figures in these panels stand out. Naturally, $\theta^{2*} = \theta^{1*}$ at $t = t_y$. Further, even though there is no cross hauling for $t \geq t_y$, internal trade costs matter for cartel discipline in this case because they affect deviation and punishment payoffs. However, the resulting relationship differs markedly from the one studied in Lemma 1 where cross hauling was present. Now increases in internal trade costs t improve cartel discipline ($\theta^{2*} \downarrow$) when these costs are close to t_y. At higher t levels, though, the dependence of θ^{2*} on t hinges on the value of the discount factor δ, the level of external trade costs τ, and market size β. Panel (a) depicts the behavior of θ^{2*} when $\tau = 0$, for discount factor values in three distinct ranges: (i) $(0, \underline{\delta})$, (ii) $[\underline{\delta}, \underline{\underline{\delta}})$, and (iii) $[\underline{\underline{\delta}}, \widehat{\delta})$. In case (i), cartel discipline θ^{2*} is U-shaped in t because maximal collusion is unsustainable. In case (ii), the dependence of θ^{2*} on t differs in that $\theta^{2*} = 0$ for an intermediate range of t values as they sustain the most collusive outcome. In case (iii), increases in t beyond t_y strengthen cartel discipline up to $\underline{t}'_m$ with $\theta^{2*} = 0$ for $t > \underline{t}'_m$.

Higher δ values reduce the range of internal trade costs under which cartel discipline varies with internal trade costs. On average, increases in δ strengthen cartel discipline. Next, we investigate how external trade costs (and ROW market size)—captured in panel (b)—matter.

Lemma A2: *For given $\delta < \widehat{\delta}$, $\tau \leq \bar{\tau}$ and $\beta < \infty$, $\Phi^2(\theta, t, \cdot) = 0$ implicitly defines a contour over the (t, θ) plane that goes through four (pivot) points that are independent of τ and β.*

[3] In contrast, Φ^1 studied earlier need not be concave in (t,θ).

[4] Points on the $\Phi^2 = 0$ contour that are above $\theta_o(t)$ must be discarded because they violate the $y = 0$ constraint. Points below the horizontal axis must also be ignored because they are associated with t values that sustain the fully collusive outcome (so $\theta^{2*} = 0$ in this case). The effect of an increase in ROW size ($\beta \downarrow$) is similar to the effect of external trade cost cuts ($\tau \downarrow$).

These points are given by: $D = (\bar{t}, \theta_g)$, $E = (\bar{t}, 1)$, $F = (t_f, 1)$ *and* $G = (t_g, \theta_g)$, *where*

$$\theta_g = \frac{17\left(\underline{\delta} - \delta\right)}{9 + \delta}, \quad t_g = \frac{A}{2}\left[\frac{(2 + \theta_g)\left(4 + 10\theta_g + \theta_g^2\right)}{56 - 9\theta_g^2 - 2\theta_g^3}\right], \quad t_f = \frac{A}{2}\left[\frac{45 - 61\delta}{45 + 89\delta}\right].$$

Proof: The impact of τ on Φ^2 is given by

$$\Phi^2_\tau = -\frac{2\left(A - \tau\right)\left(9 + \delta\right)}{9\beta\left(8 + \theta\right)^2}\left(\theta - \theta_g\right)\left(1 - \theta\right). \tag{A.12a}$$

Changes in τ do not affect Φ^2 if: (i) $\theta = \theta_g$, or (ii) $\theta = 1$. The impact of β on Φ^2 is similar since

$$\Phi^2_\beta = -\frac{\left(A - \tau\right)^2\left(9 + \delta\right)}{9\beta^2\left(8 + \theta\right)^2}\left(\theta - \theta_g\right)\left(1 - \theta\right). \tag{A.12b}$$

To find the values of t that are associated with the pivot points noted in the lemma, we sequentially consider cases (i) and (ii). Starting with case (i), invert $\theta = \theta_g\left(\delta\right)$ to obtain $\delta_g = 9(1 - \theta)/(17 + \theta)$. Substituting δ_g into Φ^2 yields, after some algebra,

$$\Phi^2\left(\cdot\right) = -\frac{2\left(56 - 9\theta^2 - 2\theta^3\right)}{\left(4 + \theta\right)^2\left(17 + \theta\right)}\left(t_g - t\right)\left(\bar{t} - t\right) = 0.$$

The solutions to the above equation deliver the values of internal trade costs t_g and $\bar{t}$ associated with $\theta = \theta_g$ noted in the lemma. Since $dt_g/d\delta = \left(dt_g/d\theta_g\right)\left(d\theta_g/d\delta\right)$ and $dt_g/d\theta_g > 0$ while $d\theta_g/d\delta < 0$ we will have $dt_g/d\delta < 0$.

Turning to case (ii), we set $\theta = 1$ in $\Phi^2 = 0$ and simplify the resulting expression to obtain

$$\Phi^2\left(\cdot\right) = -\frac{\left(45 + 89\delta\right)}{225}\left(t_f - t\right)\left(\bar{t} - t\right) = 0,$$

where $dt_f/d\delta < 0$. ||

Lemma $A2$ establishes that, for $\delta < \underline{\delta}$, the $\Phi^2 = 0$ contours go through four stationary points (captured by D, E, F, and G) that are independent of the levels of external trade costs τ and market size β. Panel (b) of Fig. 3 depicts three contours under the assumptions that $\beta = 1$ and $\delta < \underline{\underline{\delta}}$ for $\tau = 0$ initially. These contours are associated with free external trade ($\tau = 0$), costly external trade ($\tau' \in (0, \bar{\tau})$), and the absence of external trade ($\tau'' = \bar{\tau}$), respectively. Importantly, $\partial\theta^{2*}/\partial t < 0$ at $t = t_y$ (point B) and at $t = t_g$ (point G). Moreover, point G is a pivot with θ^{2*} rotating clockwise around it when external trade costs τ rise.[5] Additionally, higher external trade cost levels τ (and, using similar logic, higher values in β) reduce the range of internal trade cost levels that affect cartel discipline. The impact of τ and β on cartel discipline differs from the corresponding impact of δ in that higher τ and/or β values weaken cartel discipline for $t < t_g$ but not for $t \in [t_g, \bar{t}]$.

In Lemma $A3$ below, we describe the dependence of cartel discipline on internal trade costs t along the ICC defined by $\Phi^2(t, \theta, \cdot) = 0$.

Lemma A3: *Suppose the conditions of Lemma* $A2$ *are satisfied. Then the contour defined by* $\Phi^2(t, \theta, \cdot) = 0$ *for* $(t, \theta) \in (0, \bar{t}) \times (-\infty, 1-\delta)$, labeled $\theta_s(t, \cdot)$, *is a convex function of* t *such that* $\partial\theta_s/\partial t \lesseqgtr 0$ *as* $t \lesseqgtr t_{\min}$, *where* $t_{\min} = \arg\min_t \theta_s(t, \cdot) \in (t_g, \bar{t})$.

Proof: The proof follows from the properties of $\Phi^2(t, \theta, \cdot)$ described in Lemma $A2$ and can be visualized with the help of Fig. 3. It's important to note that, as defined, θ_s can take negative values—which, in due course, will be discarded (see Lemma 2 below) because they imply the monopoly solution is sustainable (i.e., $\theta = 0$). These portions of θ_s are not drawn in Fig. 3 to avoid cluttering. Also note that, depending on market size β and external trade costs τ, $\theta_s(t, \cdot)$ may intersect the vertical axis below $1 - \delta$. But this is inconsequential since all points of $\theta_s(t, \cdot)$ above $\theta_o(t)$ are irrelevant in this case (because they imply $y > 0$, which is a contradiction). ||

Proof of Lemma 2: The proof relies on Lemmas $A1 - A3$ which establish the key properties of the ICC in the absence of cross hauling ($y = 0$). Since $t \in [t_y, \bar{t}]$ in this case, only the portion of $\theta_s(t, \cdot)$ that lies below $\theta_o(t)$ is potentially admissible. Moreover, whenever $\theta_s < 0$, maximal collusion will be sustainable, so $\theta^{2*} = 0$ in this case because the ICC is inactive.

Part (a). Panel (a) of Fig. 3 illustrates the various possibilities for alternative values of δ. The condition $\delta \leq \underline{\underline{\delta}}$ in $(a.i)$ ensures that maximal collusion is unsustainable for all $t \in [t_y, \bar{t}]$, so $\theta^{2*} = \theta_s$, as indicated by the blue, solid-line curve. The proof to this part then follows from Lemma $A3$.

If $\delta \in (\underline{\underline{\delta}}, \underline{\delta}]$, there exists a range of internal trade costs $[\underline{t}_m, \bar{t}_m]$ that imply $\theta^{2*} = 0$ for all t in this range, and $\theta^{2*} = \theta_s$ for all other values of t. The pink, solid-line curve in panel (a) illustrates θ^{2*} in this case. Part $(a.iii)$ is illustrated by the green, solid-line curve in the same figure.

[5] Note that $\theta = \theta_g$ at points A, G and D. When only external trade is present (i.e., $t \geq \bar{t}$ and $\tau < \bar{\tau}$), we have $\theta^* = \theta_g$. When both internal and external trade are absent, all points in $[\bar{t}, \infty) \times [0, 1]$ are acceptable as they imply $\Phi^2 \geq 0$.

Part (b). This part follows from Lemmas $A2$ and $A3$. The key point here is that, for given δ, increases in external trade costs ($\tau \uparrow$) or decreases in market size ($\beta \downarrow$) weaken cartel discipline if internal trade costs are sufficiently close to t_y and may strengthen it if these costs are sufficiently high. Panel (b) of Fig. 3 sheds further light on this case. ||

Proof of Lemma 3: The conditions of this lemma, the properties of $\theta^{1*}(t,\cdot)$ described in Lemma 1 and the properties of $\theta_s(t,\cdot)$ described in Lemma $A3$ (including the facts that $\partial\theta^{1*}/\partial t > 0$ and $\partial\theta_s/\partial t < 0$) imply that θ^{1*} and θ_s intersect at a unique point t_y along $\theta_o(t)$, so $\theta^{1*}(t_y) = \theta_s(t_y) = \theta_o(t_y)$. The proofs to parts ($a$) and ($b$) then follow from Lemma 1 (and/or Lemma 2) and can be visualized with the help of the two panels in Fig. 4. ||

Proof of Proposition 2: The proofs to all parts follow readily from Lemmas 1 and 2. Two additional properties of θ^* for $t \in [0, t_y]$ ought to be emphasized here. First, if $\delta \in [0, \underline{\delta}]$, then $\theta^*_{tt} > 0$ for t close to 0 and $\theta^*_{tt} < 0$ for t close to t_y. Second, if $\delta \in (\underline{\delta}, \widehat{\delta})$, then $\theta^*_{tt} < 0$ for $t \in [0, t_y]$.[6] ||

Proof of Proposition 3: We first establish several properties of $Q^i(\theta,t)$ for $i = 1,2$ which facilitate the proof of part (a). From (11a) and (13a) one can see that $Q^i(\theta,t)$ is twice differentiable in θ and t. Ignoring for the moment the nature of the domains of these functions, define the bordered Hessians of order 1 and 2 of Q^i as

$$H^i_1 \equiv \begin{pmatrix} 0 & Q^i_\theta \\ Q^i_\theta & Q^i_{\theta\theta} \end{pmatrix} \text{ and } H^i_2 \equiv \begin{pmatrix} 0 & Q^i_\theta & Q^i_t \\ Q^i_\theta & Q^i_{\theta\theta} & Q^i_{\theta t} \\ Q^i_t & Q^i_{t\theta} & Q^i_{tt} \end{pmatrix},$$

respectively, and let D^i_1 and D^i_2 ($i = 1,2$) be their corresponding determinants. One can see from ($A.4a$), ($A.8a$) and the help ($A.10b$) and ($A.10c$) that $Q^i_\theta > 0$ and $Q^i_t < 0$ ($i = 1,2$) for $t < \bar{t}$. It follows that $D^i_1 < 0$. Additionally, one can show that Q^i is linear in t and concave in θ (because $Q^i_{tt} = 0$, $Q^i_{\theta\theta} < 0$ and $Q^i_{t\theta} < 0$). Furthermore, the determinants of the bordered Hessians of order 2 are:

$$D^1_2 = \frac{12\,(2A-t)\,(2+\theta)}{(8+\theta)^4} > 0 \text{ and } D^2_2 = \frac{4\,(A-2t)\,\theta}{(4+\theta)^4} > 0 \text{ for } t < \bar{t}.$$

Because $D^i_1 < 0$ and $D^i_2 > 0$ for all $(t,\theta) \in (0,\bar{t}) \times (0,1)$ one might infer that $Q^i(\theta,t)$ is strictly quasi-concave in (θ,t) or, equivalently, that every upper level set of Q^i ($i = 1,2$) is strictly convex. It turns out that this is true for $Q^2(\theta,t)$ but not for $Q^1(\theta,t)$. The reason

[6] These properties of θ^* can be established with the help of tedious algebra and numerical analysis. They can be visualized with the help of Fig. 2.

for this is that the domain of Q^2 is a strictly convex set whereas the domain of Q^1 is not.[7] However, because we are primarily interested in the behavior of Q^* (which coincides with the values of Q^i along $\theta^*(t,\cdot)$), the just described issue is inconsequential.

Part (a). First note that

$$sign\left(dQ^{1*}/dt\right) = sign\left(\frac{\theta_t^{1*}}{-Q_t^1/Q_\theta^1} - 1\right).$$

But, from Lemma 1 (b), we have $\lim_{t\to 0}\theta_t^{1*} = 0$ for $\delta \in [0, \underline{\delta})$ and

$$\lim_{t\to 0}\theta_t^{1*} = \lim_{t\to 0}\left(\theta^{1*}/t\right) = \left[\frac{32}{17}\left(\frac{1-\delta}{\delta-\underline{\delta}}\right)/\Pi_0^N\right]^{1/2} \quad \text{for } \delta \in [\underline{\delta}, \widehat{\delta}),$$

where $\Pi_0^N \equiv \Pi^N|_{t=0}$ (see $(A.6c)$). Moreover, from $(A.4a)$ and $(A.8a)$ we have

$$\lim_{t\to 0}(d\theta/dt)\,|_{dQ^1=0} = -Q_t^1/Q_\theta^1 = \frac{(2+\theta)(8+\theta)}{12A} > 0.$$

It follows that $\lim_{t\to 0}\left(dQ^{1*}/dt\right) < 0$ for $\delta \in [0, \underline{\delta})$, so local output Q^{1*} rises as t falls for t close to 0.

In contrast, for $\delta \in [\underline{\delta}, \widehat{\delta})$ we have

$$R \equiv \frac{\lim_{t\to 0}\theta_t^{1*}}{\lim_{t\to 0}(d\theta/dt)\,|_{dQ^1=0}} = \left[\frac{18A^2}{17\Pi_0^N}\left(\frac{1-\delta}{\delta-\underline{\delta}}\right)\right]^{1/2} > \lim_{\delta\to\widehat{\delta}} R = 3.$$

Thus, $\lim_{t\to 0}\left(dQ^{1*}/dt\right) > 0$ in this case, so local output Q^{1*} is decreasing in t.

As explained in the text, the second portion of part $(a.i)$ follows from the facts that $(d\theta/dt)\,|_{dQ^2=0} = -Q_t^2/Q_\theta^2 > 0$ and $\theta_t^{2*} < 0$ for t larger than but close to t_y. Since $sign\left(dQ^{2*}/dt\right) = sign\left(\frac{\theta_t^{2*}}{-Q_t^2/Q_\theta^2} - 1\right)$, internal trade cost reductions expand output Q^{2*} in the absence of cross hauling for t close to t_y.

The easiest way to establish parts $(a.ii)$ and $(a.iii)$ is with the use of numerical methods. But the intuition behind these is relatively easy to understand. For example, the uniqueness of t_Q in part $(a.ii)$ is due to the fact that θ^* is concave in $t \in [0, t_y]$ for $\delta \in [\underline{\delta}, \widehat{\delta})$ (as noted in the proof of Proposition 2) and the contours of Q^1 are strictly convex in $t \in [0, t_y]$. Additionally, the various components of part $(a.iii)$ (which studies the case of $\delta \in (0, \underline{\delta})$) can be understood by noting that the strictly convex contours of Q^1 may be tangent to θ^* at several points as θ^* is convex in t for t close to 0 but concave in t for t close to t_y.

[7] This follows from the facts that $\theta_o(t)$ is concave in t and $Q^2(\theta, t)$ is defined for $t < \bar{t}$ and $\theta \in (0, \theta_o(t))$ whereas $Q^1(\theta, t)$ is defined for $t < \bar{t}$ and $\theta \in (\theta_o(t), 1)$. Thus, we could describe $Q^1(\theta, t)$ as a piecewise quasi-concave function.

Part $(a.iv)$ follows from the facts that $dQ^*/d\xi = Q^i_\theta \theta^*_\xi$ for $\xi \in \{\delta, \beta, \tau\}$ and $Q^i_\theta > 0$ $(i = 1, 2)$.

Part (b). We establish the first portion of part $(b.i)$ via exhaustive numerical analysis. The second portion of part $(b.i)$ and part $(b.ii)$ follow readily from the fact that $dy^*/d\xi = y^*_\theta \theta^*_\xi$ for $\xi \in \{\delta, \beta, \tau\}$ and the observation that $y^*_\theta > 0$.

Part (c). Parts (i) and (iii) can be established by noting that $dz^*/d\xi = z^*_\theta \theta^*_\xi$ for $\xi \in \{\delta, t\}$, recalling that $z^*_\theta > 0$ and invoking Proposition 2. Part (ii), which is easiest to establizh with the help of numerical methods, asserts that the direct effects of β and τ on z^* dominate their possibly opposing effects on z^* through cartel discipline. $||$

In Lemma $A4$ below we maintain the view that t stands for "tariffs" and characterize "optimal" cartel discipline θ^i_W and its dependence on parameters. This serves as a valuable benchmark that can be used to assess the welfare consequences of cartels with endogenous discipline.

Lemma A4: *There exist unique optimal cartel-discipline functions $\theta^i_W(\cdot)$ for $i = 1, 2$ (i.e., functions that maximize welfare W^i). These functions have the following traits:*

a) $\theta^1_W \in (\theta_o(t), 1)$ *for* $\beta \in (\underline{\beta}^1, \overline{\beta}^1)$ *and* $\theta^1_W = 1$ *for* $\beta \geq \overline{\beta}^1$ *where*

$$\overline{\beta}^1 \equiv \frac{(A-\tau)^2}{(2A-t)(A+t)} \text{ and}$$

$$\underline{\beta}^1 \equiv \frac{3(A-\tau)^2 \theta_o}{(2A-t)[A(4-\theta_o)+t(2+\theta_o)]} \quad \text{for} \quad t < \bar{t}.$$

b) $\theta^2_W \in (0, \theta_o(t))$ *for* $\beta \in (0, \overline{\beta}^2)$ *where*

$$\overline{\beta}^2 \equiv \frac{9(A-\tau)^2 \theta_o (4+\theta_o)^3}{(2A-t)(8+\theta)^3(2A+t\theta)} \quad \text{and } \underline{\beta}^2 = 0 \quad \text{for} \quad t < \bar{t}.$$

c) *Suppose* $\beta \in (\underline{\beta}^i, \overline{\beta}^i)$ *for* $i = 1, 2$. *Then*

i) $\partial\theta^1_W/\partial t > 0$ *and* $\partial\theta^2_W/\partial t < 0$.

ii) $\text{sign}\left(\partial\theta^i_W/\partial\beta\right) = \text{sign}\left(\partial\theta^i_W/\partial\tau\right) > 0$.

Proof: Note that superscripts "1" and "2" in variables and functions below refer to situations where $y > 0$ and $y = 0$, as analyzed in Section 3.3. Utilizing the welfare functions in Section 5 and taking into account the decomposition in $(14a)$ and the values of Q^i and

z^i $(i = 1, 2)$ studied in Section 3.3, gives

$$W_\theta^1 = \frac{6\,(2A - t)\,[A\,(4-\theta) + t(2+\theta)]}{(8+\theta)^3} - \frac{18\,(A-\tau)^2\,\theta}{\beta\,(8+\theta)^3}, \tag{B.13a}$$

$$W_t^1 = -\frac{(2+\theta)\,[A\,(4-\theta) + t(2+\theta)]}{(8+\theta)^2} < 0, \tag{B.13b}$$

and

$$W_\theta^2 = \frac{2\,(2A - t)\,[2A + t\theta]}{(4+\theta)^3} - \frac{18\,(A-\tau)^2\,\theta}{\beta\,(8+\theta)^3}, \tag{B.14a}$$

$$W_t^2 = -\frac{\theta\,(2A + t\theta)}{(4+\theta)^2} < 0. \tag{B.14b}$$

Moreover, differentiation of the above expressions gives

$$W_{\theta\theta}^1 = -\frac{36\,(A-\tau)^2\,(4-\theta)}{\beta\,(8+\theta)^3} - \frac{12\,(2A-t)\,[9A + (A-t)\,(1-\theta)]}{(8+\theta)^3} < 0, \tag{B.15a}$$

$$W_{tt}^1 = -\frac{(2+\theta)^2}{(8+\theta)^2} < 0, \tag{B.15b}$$

$$W_{\theta t}^1 = \frac{6\,[3A\theta - 2t(2+\theta)]}{(8+\theta)^3} > 0 \text{ (since } t < t_y) \tag{B.15c}$$

and

$$W_{\theta\theta}^2 = -\frac{36\,(A-\tau)^2\,(4-\theta)}{\beta\,(8+\theta)^3} - \frac{4\,(A-2t)\,(3A - 2t + t\theta)}{(8+\theta)^3} < 0, \tag{B.16a}$$

$$W_{tt}^2 = -\frac{\theta^2}{(4+\theta)^2} < 0, \tag{B.16b}$$

$$W_{\theta t}^2 = -\frac{[2A\,(4-\theta) + 8t\theta]}{(4+\theta)^3} < 0. \tag{B.16c}$$

Equations $(A.13b)$ and $(A.14b)$ imply that W^i is decreasing in t and equations $(A.15b)$ and $(A.16b)$ imply that W^i is strictly concave in t. It follows that W^i attains a maximum at $t = 0$. Now observe from $(A.15a)$ and $(A.16a)$ that W^i is also concave in θ and keep in mind that $\theta_o\,(t, \cdot)$ is the lower (upper) bound of possible θ values for W^1 (W^2) and $t \in [0, \bar{t}]$.[8] For clarity, we illustrate the welfare contours (the pink curves) in Fig. A.1 under the assumptions that $\beta = 0.45$ in panel (a) and $\beta = 0.15$ in panel (b). Turning to the welfare-maximizing values of θ, the concavity of W^i in θ ensures that θ_W^i $(i = 1, 2)$ are

[8] It is not difficult to show that W^1 is piecewise concave in its arguments—we say "piecewise" because its domain is not a convex set. In contrast, W^2 is strictly concave in (θ, t) only if *ROW*'s market size is sufficiently large.

unique.

Parts (a) and (b). Temporarily suppose that $(A.13a)$ and $(A.14a)$ hold with equality. The values of β that ensure $W_\theta^1 = 0$ and $W_\theta^2 = 0$ are

$$\beta^1 = \frac{3(A-\tau)^2\theta}{(2A-t)[A(4-\theta)+t(2+\theta)]}, \tag{B.17a}$$

$$\beta^2 = \frac{9(A-\tau)^2\theta(4+\theta)^3}{(2A-t)(8+\theta)^3(2A+t\theta)}. \tag{B.17b}$$

Differentiation of β^i readily gives $\partial\beta^i/\partial\theta > 0$ for $i = 1, 2$. Thus, the upper and lower bounds for β^1 identified in part (a) can be obtained by setting θ equal to 1 and θ_o, respectively, in $(A.17a)$. Similarly, $\overline{\beta}^2$ and $\underline{\beta}^2$ in part (b) are obtained by setting $\theta = \theta_o$ and $\theta = 0$ in $(A.17b)$.

Part (c). The effect of $\xi \in \{t, \beta, \tau\}$ on θ_W^i can be found by differentiating $W_\theta^i = 0$ $(i = 1, 2)$ and using the implicit function theorem to obtain: $\partial\theta_W^i/\partial\xi = -W_{\theta\xi}^i/W_{\theta\theta}^i$. Since $W_{\theta\theta}^i < 0$, we will have $sign\left(\partial\theta_W^i/\partial\xi\right) = sign\left(W_{\theta\xi}^i\right)$. But $(A.15c)$ and $(A.16c)$ reveal that $W_{\theta t}^1 > 0$ while $W_{\theta t}^2 < 0$. This substantiates part (i). Part (ii) follows from the fact that $W_{\theta\xi}^i > 0$ $(i = 1, 2)$ for $\xi \in \{\beta, \tau\}$. ||

For additional insight, suppose $t = 0$ (which implies $\theta_o = 0$) and $\tau = 0$. Then, $\overline{\beta}^1 = 0.5$ and $\underline{\beta}^1 = \overline{\beta}^2 = \underline{\beta}^2 = 0$ in this case. Thus, if $\beta < 0.5$, then $\theta_W^1(0) \in (0, 1)$, as shown in both panels of Fig. $A.1$. Consistent with part (c), reductions in β move $\theta_W^1(0)$ in the direction of the origin. Fig. $A.1$ also depicts the dependence of the welfare-maximizing functions θ_W^i (captured by the green dashed-line curves) on t. Depending on the values of β and t, these functions may coincide with $\theta_o(t)$ or equal 1. Lastly, ROW size determines the comparison among the welfare levels associated with the Cournot-Nash equilibrium (i.e., W^N at $\theta = 1$), monopoly (i.e., W^M at $\theta = 0$) and at θ_W^1.

Fig. $A.1$ sets the stage for our analysis of welfare W^* in the presence of endogenous cartel discipline, the focus of Proposition 4. To facilitate this pursuit, we superimpose $\theta^*(t)$ on this figure for two discount factor values: $\delta = \underline{\delta}$ and $\delta = 0.3 < \underline{\delta}$. It's useful to keep in mind that functions $\theta_W^i(\cdot)$ do not depend on δ. Moreover, while $\theta^*(t)$ responds to changes in the relative size of ROW (as detailed in Proposition 2), point θ_g remains stationary.

Proof of Proposition 4: *Part (a).* From $(14b)$, $dW_{ROW}^*/dt = (\partial W_{ROW}^*/\partial z)\, z_\theta^*\theta_t^*$, where $\partial W_{ROW}^*/\partial z > 0$ and $z_\theta^* > 0$ (from $(A.4d)$ and/or $(A.10b)$). Therefore, internal trade cost reductions ($t \downarrow$) impact ROW welfare solely through cartel discipline, as described in Proposition 2.

Part (b). This part explains how the value of the discount factor conditions the effect of internal trade liberalization on a representative PTA member's welfare. Since the focus is on low initial values of t, we prove this part by studying the behavior of W^* in the neighborhood of $t = 0$. As noted in (15), W^* depends on t through Q^* and z^*. Further, as noted in Proposition 3, the dependence of Q^* hinges on whether $\delta \in (\underline{\delta}, \widehat{\delta})$ or $\delta \in (0, \underline{\delta})$.

From $(14a)$ we have

$$\lim_{t\to 0} \frac{dW^*}{dt} = \lim_{t\to 0} p\,(dQ^*/dt) - \lim_{t\to 0} \left[\frac{3\,(A-\tau)\,\theta^*}{8+\theta^*} \quad \overset{(+)}{\partial z^*/\partial\theta} \Bigg)\,(d\theta^*/dt) \right]. \tag{A.13}$$

First, focus on $\delta \in (\underline{\delta}, \widehat{\delta})$. From Proposition $3a$, we know that $\lim_{t\to 0}(dQ^*/dt) > 0$, so the first term in $(A.13)$ is positive. Further, from $(A.4d)$ we know that $\partial z^*/\partial\theta$ is finitely positive and, from part $(b.ii)$ of Lemma 1, we know that $\lim_{t\to 0}(d\theta^*/dt)$, too, is finitely positive. But Lemma 1 posits that $\lim_{t\to 0}\theta^* = 0$; therefore, the second term in the RHS of $(A.13)$ vanishes. It follows that internal trade liberalization is necessarily welfare-reducing in this case provided t is sufficiently low to start with. What about the implications of $\delta \in (0, \underline{\delta})$? Since, by Proposition $3a$, $\lim_{t\to 0}(dQ^*/dt) < 0$ in this case, welfare W^* rises with reductions in t in this case.

Parts (a) and (b) deal with the behavior of W^* in the neighborhood of $t = 0$. To develop a view on the dependence of W^* at higher tariff levels, we can use the help of Fig. $A.1$, especially the shapes of the welfare contours in this figure in relation to endogenous cartel discipline $\theta^*(t)$ for various values in ROW size (β) and the discount factor (δ). For example, one can see why $W^*(t)$ may have multiple peaks when β is large (i.e., the size of ROW is small). One can also see why this is no longer an issue (and why $W^*(t)$ is monotonic in t) when β is small.

Parts (c) and (d). These parts can be understood with the help of Fig. $A.1$. For example, to understand and explain part (c), focus on $t = 0$ and suppose $\beta = 0.15$ (as shown in panel (b)) which is consistent with the idea that ROW's size is relatively large and, consequently, export opportunities to it are extensive. Starting at $\delta = \underline{\delta}$ (so that $W^* = W^M$) let δ fall. By Proposition $2a$, cartel discipline will weaken thereby causing W^* to rise until $\theta_W^1(0)$ is reached and W^* is maximized. The non-monotonicity of W^* in δ becomes apparent when $\theta_W^1(0)$ is crossed and W^* begins to fall. Part (d) also follows upon inspection of Fig. $A.1$. Additional details can be provided from the authors by request. ||

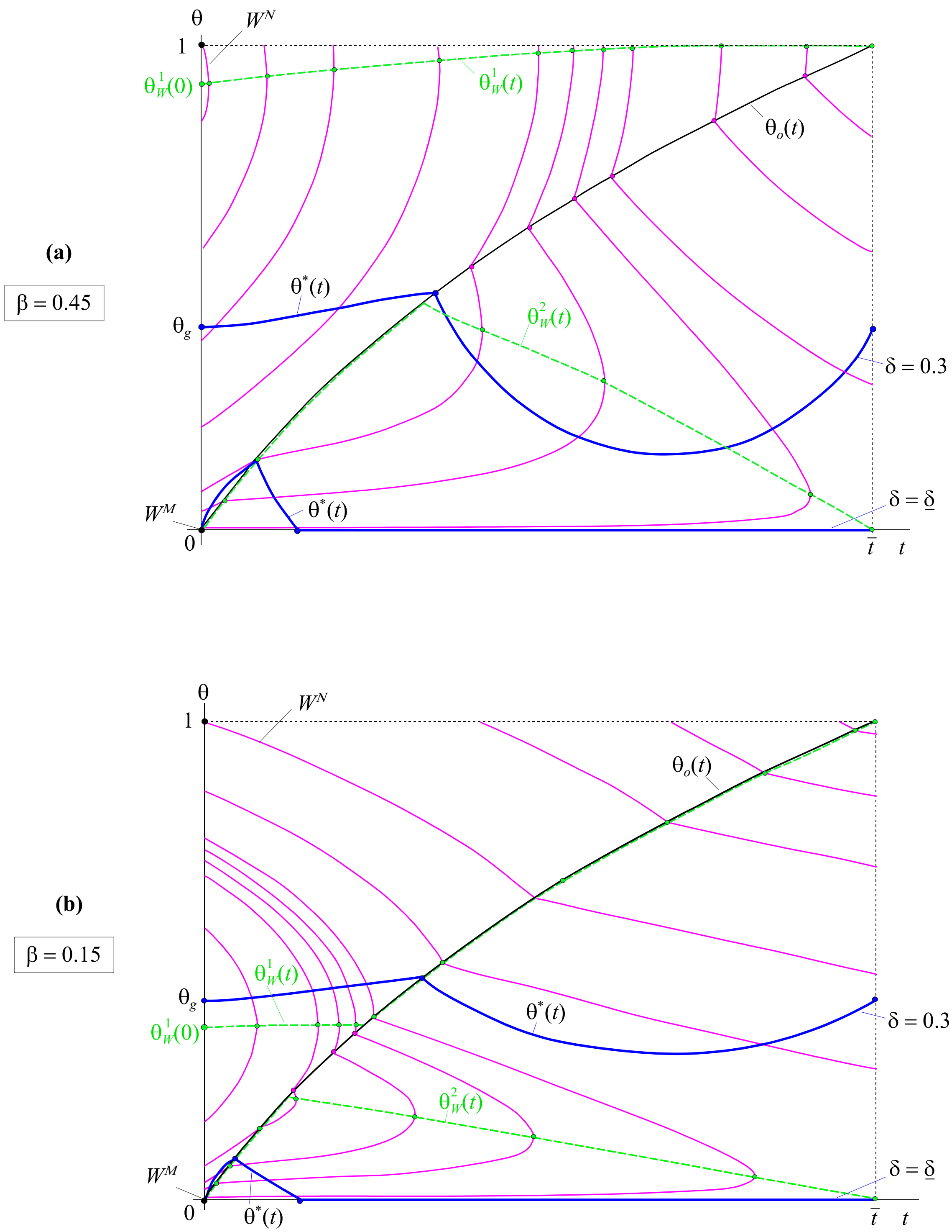

Figure A.1: Welfare Contours in (t,θ) Space and Optimal Cartel Discipline

Appendix B (Online)

The following lemmas clarify the nature of welfare in the static (i.e., one-shot) version of the model where incentive constraints are not considered. As such they could be used for contrast to our findings in Section 5. Lemma $B1$ explains the dependence of welfare under Cournot-Nash competition on the nature of trade costs—i.e., whether they are revenue generating (W) or resource using (V)—the levels of these costs, and the size of ROW's market. Lemma $B2$ compares the welfare levels associated with Cournot-Nash competition and pure monopoly.

Lemma B1: *(Welfare under Cournot-Nash competition) In the Cournot-Nash equilibrium, ROW welfare $W^N_{ROW} = V^N_{ROW}$ is invariant to changes in t and decreasing in τ (and β). In contrast, in the presence of tariffs/taxes, welfare of cartel hosts W^N is decreasing in t for $t < \bar{t}$ and attains a maximum at $\tau^N_{opt} = A/4$. In the presence of resource-using trade costs, welfare of cartel hosts V^N is U-shaped in t and decreasing in τ for $\tau < \bar{\tau}$). Moreover,*

a) $\arg\min_t V^N(t, \tau, \beta) = \frac{4A}{11} \in (0, \bar{t})$.

b) $V^N(t, \tau, \beta) \gtreqless V^N(\bar{t}, \tau, \beta)$ *if* $t \lesseqgtr t_V$, *where* $t_V \equiv \frac{5A}{22} \in (0, \bar{t})$.

c) $W^N(t, \tau, \beta) > V^N(t, \tau, \beta)$ *for all* $t \in (0, \bar{t})$ *and* $\tau \in (0, \bar{\tau})$.

Proof: Utilizing the definition of ROW's welfare in Section 5, we have $W^N_{ROW} = V^N_{ROW} = 2\beta \left(z^N\right)^2$, where $z^N = \frac{A-\tau}{3\beta}$ from our analysis in Section 2. Clearly, W^N_{ROW} is invariant to changes in t and decreasing in τ (and β).

The definitions of welfare for cartel hosts in Section 5 imply $W^N = u\left(Q^N\right) + p_{ROW}\left(z^N\right) z^N$ and $V^N = W^N - ty^N - \tau x^N$. Utilizing the values of Q^N and z^N from Section 2 and simplifying the resulting expressions for W^N and V^N gives

$$W^N = \frac{3}{8}A^2 + \frac{1}{36}\left(A/2 - t\right)\left(5A + 2t\right) + \frac{1}{9\beta}\left(A - \tau\right)\left(A + 2\tau\right), \tag{B.1a}$$

$$V^N = \frac{3}{8}A^2 + \frac{1}{36}\left(A/2 - t\right)\left(5A - 22t\right) + \frac{\left(A - \tau\right)^2}{9\beta}, \tag{B.1b}$$

for $t \leq \bar{t}$ $(= A/2)$ and $\tau \leq \bar{\tau}$ $(= A)$ and

$$W^N = V^N = \frac{3A^2}{8} + \frac{\left(A - \tau\right)^2}{9\beta}, \tag{B.2}$$

for $t > \bar{t}$ $(= A/2)$. Inspection of $(B.1a)$ reveals that, in the case of tariffs/taxes, $sign\left(\partial W^N/\partial t\right) = sign\left(\partial W^N/\partial \beta\right) < 0$. Thus, reciprocal reductions in internal trade costs ($t \downarrow$) and/or increases in ROW's market size ($\beta \downarrow$) unambiguously benefit cartel hosts.

Part (a). Differentiating $(B.1b)$ partially gives $\partial V^N/\partial t = (-4A+11t)/9$ and $\partial^2 V^N/\partial t^2 > 0$, which readily implies $\arg\min_t V^N = 4A/11 \in (0, \bar{t})$.

Part (b). This part follows by utilizing $(B.1b)$ to form the difference $V^N(t,\tau,\beta) - V^N(\bar{t},\tau,\beta) = \frac{11}{18}(\bar{t}-t)(\frac{5A}{22}-t)$ for $t < \bar{t} \equiv \frac{A}{2}$.

Part (c). Obvious. ||

Parts (a) and (b) of Lemma $B1$ confirm the ideas that $V^N(t,\tau,\beta)$ is U-shaped in resource-using trade costs t under Cournot-Nash competition and that internal trade cost reductions harm cartel hosts if t is sufficiently large (but below $\bar{t}$). This possibility arises for the same reason as it does in the 2-country, reciprocal dumping model of Brander and Krugman (1983): for t close to $\bar{t}$, the welfare cost of cross hauling (due to the waste of resources—e.g., transportation costs) outweighs the benefit due to increased competition.

Lemma B2: *(Cournot-Nash competition vs pure monopoly) Suppose $\tau < \bar{\tau}$ and interpret internal trade costs t as "tariffs". Then*

a) $W^M > W^N$ *for* $t \geq \bar{t}$.
b) If $t \in [0, \bar{t})$, *then* $W^M \gtreqless W^N$ *for* $\beta \lesseqgtr \breve{\beta}$, *where* $\breve{\beta} \equiv \frac{(A-\tau)^2}{2(A/2-t)(5A+2t)}$.
c) $W^M_{ROW} < W^N_{ROW}$ *for all* $t \geq 0$.

Proof: The proof to parts (a) and (b) follows readily by noting that

$$W^M = V^M = \frac{3A^2}{8} + \frac{(A-\tau)^2}{8\beta} \tag{B.3}$$

for any t and by comparing W^N in $(B.1b)$ to W^M in $(B.3)$ for the different values of t considered.[9] Part (c) follows from the fact that $z^M < z^N$ and the discussion in the text which pointed out that welfare in ROW is increasing in z. ||

Part (a) of Lemma $B2$ may appear counter-intuitive. $W^M > W^N$ in this case, because (i) the presence of prohibitive internal tariffs implies that each firm is de facto a monopolist in its own market, and (ii) profits in ROW under monopoly exceed profits under competition. Part (b) points out that the $W^M > W^N$ ranking remains intact in the presence of internal trade if ROW's relative market size is sufficiently large (specifically, if $\beta \leq \breve{\beta}$) and gets reversed when the size of ROW's market is relatively small. In the former case, the magnitude of monopoly rents in ROW outweighs the losses in consumer surplus (due to the exercise of monopoly power) in the hosts. Exactly the opposite is true in the latter case.

[9] See also Lemma $B3$ and Fig. $A.2$ for a discussion and illustration of the ranking between W^N and W^M.

One can also verify (by differentiating $\breve{\beta}$ appropriately) that $\breve{\beta}_t > 0$ and $\breve{\beta}_\tau < 0$. Thus, the larger the volume of internal (external) trade the stronger (weaker) the size requirement on β for pure monopoly to dominate Cournot-Nash competition. Part (c) underscores the point that ROW always prefers competition over monopoly.

For additional insight, Fig. $B.1$ below illustrates how the welfare levels associated with Cournot-Nash competition and pure monopoly compare to the levels of welfare associated with endogenous cartel discipline.

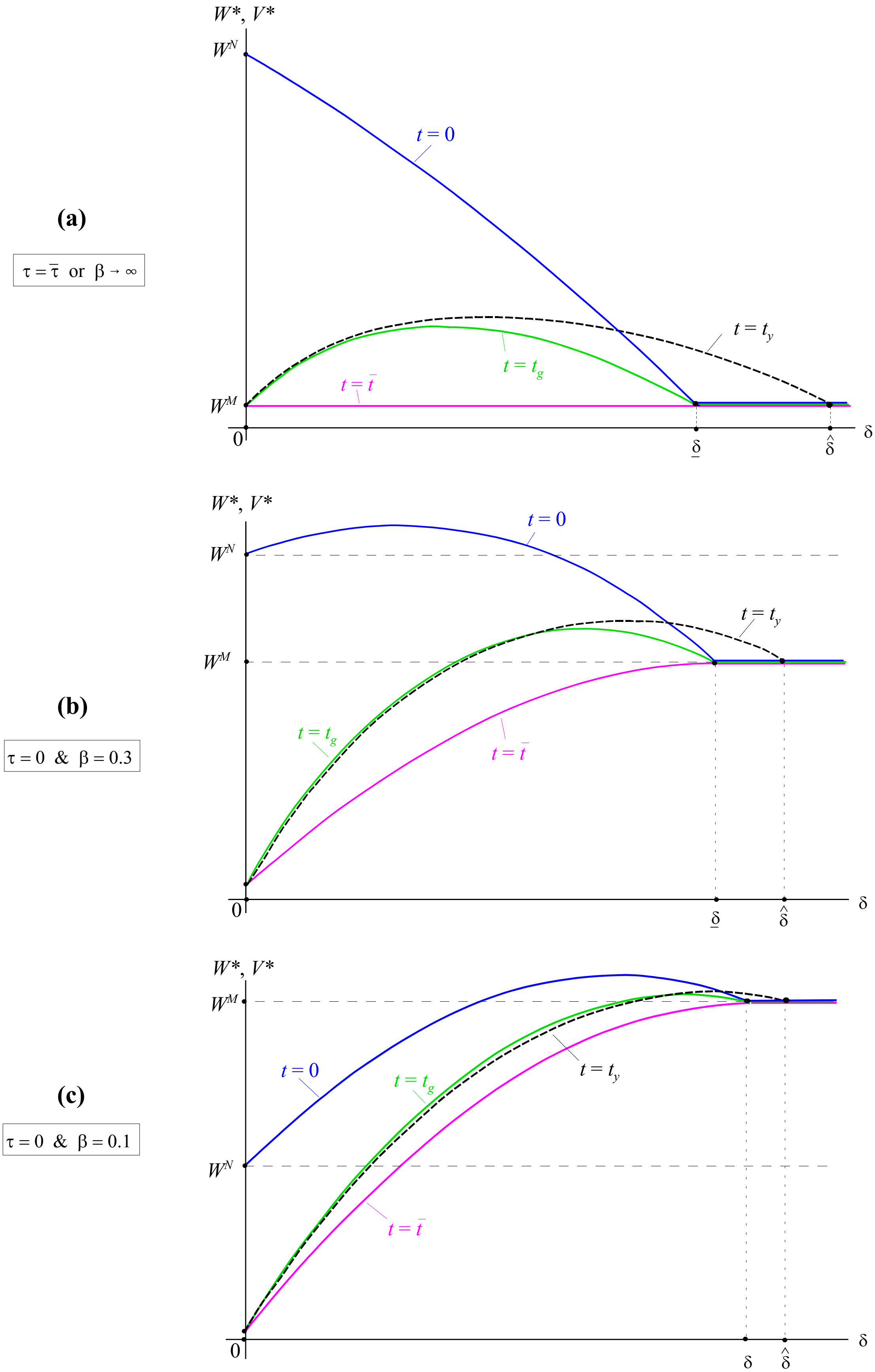

Figure B.1: The Dependence of Welfare on the Discount Factor (δ) under Alternative Internal Trade Cost Levels (t) and Market Size (β)