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Call It a Day: History Dependent Stopping Behavior

Abstract

In this paper, we provide evidence of history-dependent stopping behavior. Using data from an online chess platform, we estimate a dynamic discrete choice model in which an agent may have time non-separable preferences over the stochastic outcomes of their actions. We show that the agent's decisions cannot be reconciled in a model with time separable preferences and that there is substantial heterogeneity in preferences across players. In particular, there are two types of people: those who get discouraged by a loss and stop, and others, who get encouraged by failure and keep playing until a win. We show how to leverage the information about an agent's type in market design to achieve various welfare goals. A counterfactual analysis demonstrates that a matching algorithm that incorporates stopping behavior can significantly increase the length of play.

JEL-Codes: D900, C500, C130, D400.

Keywords: time non-separable preferences, history dependence, stopping behaviour, chess.com.

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1 Introduction

What determines our decision of when to stop a given endeavor? Does our past success motivate the stopping decision, or is failure the primary determining factor? In this paper, we use online chess platform data to show history dependence in stopping decision. Furthermore, we provide evidence that different players react differently to wins and losses when deciding whether to play another game. We argue that existing economic theories do not explain such history dependence, and we propose an alternative theory with time non-separable preferences that rationalizes our data. Our theory suggests a practical and straightforward way of identifying types of history dependent behavior. We show how a market designer can leverage this information to achieve specific welfare improving objectives.

We collect the data from the most prominent online chess platform, chess.com, which has over 30 million users and hosts, on average, 3 million chess games every day. We scraped the entire history of play for a random sample of about 20,000 users in 2017 and 2018. Based on the data for 2017, we find that about 60% of players are *loss-stoppers* (players who are substantially more likely to stop playing after a loss), and 25% are *win-stoppers* (players who are considerably more likely to stop playing after a win). The remaining players are *neutral* types. We use the 2018 data to reclassify the same players and find that individuals are, in their vast majority, stable over time in terms of how they react to wins and losses.

We develop a structural model that allows for time non-separable preferences, where *future* game utility can depend on the history of play. Our estimates from the structural model are consistent with the above mentioned reduced form evidence. For some people, loss in a given game decreases the utility from playing another game. For others, it increases utility from playing another game.[1]

We then use our findings to propose a matching algorithm that increases the number of games played and discuss its implications for platform profits and players' welfare. Currently, the platform only uses players' ratings to match similarly rated players with higher probability, ignoring additional information such as a player's "type."[2] The matching mechanism we put forward leverages the following simple observation: loss-stoppers play

[1] Our classification may best be understood not as a reference to a fundamental, underlying bias, but as a reference to a reduced-form phenomenon. It is worth noting that several different underlying psychological forces could generate this behavior. See the discussion in Appendix A.

[2] The rating system reflects how well a person plays chess.

more when they win, while win-stoppers play more when they lose. Thus, loss-stoppers can be matched with relatively lower rated individuals, so that they have a higher likelihood of winning and a higher likelihood of continuing to play. On the other hand, win-stoppers can be matched with players with higher ratings, so that they are more likely to lose and start one more game. We use our structural model to conduct counterfactual analyses. We show that incorporating user's type can lead to a 5.44% increase in the number of games played while only minimally changing the user experience.[3] In general, we can apply our theory to any environment in which practice can improve overall outcomes. For example, loss-stopper types may benefit from smaller challenges that they can succeed in to avoid discouragement. On the other hand, win-stopper types may benefit from bigger challenges that could lead to their longer engagement.

Fundamentally, this paper presents and estimates a dynamic discrete choice model in which the agent may have non-separable preferences over the stochastic outcomes of their actions. In that sense, the application is analogous to the optimal stopping problems faced, for example, by taxi drivers, whose decisions to end their shifts may be influenced by their recent fares.[4] Recent empirical research on this topic is complicated by spatial search frictions and limited by the imperfect observability of the decision-makers' identities and histories of outcome. In contrast, we perfectly observe actions, payoff relevant outcomes, and independent realizations of each agent's decision problem. We take advantage of these rich data to demonstrate that the agent's decisions cannot be reconciled in a model without non-separable preferences and that there is substantial heterogeneity in preferences across players. Although our quantitative findings do not speak to the design of taxi markets, our counterfactual analysis provides an illustrative example of a policy that leverages such preference heterogeneity to further the market designer's goals.[5] More broadly, our paper is contributing to a growing body of literature on structural behavioral economics by testing a standard model with time separable preferences.[6]

Our paper is also related to the literature studying psychological biases. For example, reference dependence, gambler's fallacy, and hot hand fallacy could explain the behavior

[3] User behavior may change if we make changes to the matching algorithm, especially if the changes are drastic and inferred by the users. In this paper, we abstract from this concern, but we still focus on alternative matching algorithms that we believe would minimally affect the user experience.

[4] see Camerer et al. [1997], Thakral and Tô [2017], Crawford and Meng [2011], Frechette et al. [2019], Farber [2005, 2008, 2015], Abeler et al. [2011], Morgul and Ozbay [2015], Cerulli-Harms et al. [2019]

[5] In principle ride-sharing platforms such as Uber and Lyft can use a similar approach to increase drivers labor supply, for example, by using the information on user's tipping behavior to match riders with drivers.

[6] See DellaVigna [2018] for a review of recent studies that use structural models to test behavioral theories.

found in the data.[7] In our data, we do not find evidence of reference dependence. Gambler's fallacy and hot hand fallacy can account for only one type but are silent on the other. The entire population may be a mixture of people who are guided by a different theories or principles. However, in this paper, we do not take a stand about underlying psychological forces that motivate such behavior. Instead, we propose a theory that explains the actions found in the data and allows for a unified framework that can capture observed heterogeneity in players' behavior.

Data from chess games have been used to study risk, time, and other behavioral preferences for different age and gender groups.[8] Closest to our work is the paper by Anderson and Green [2018]. The authors show that players are more likely to stop playing when they set a new personal best rating. Behaviorally these patterns are interesting but rare. Anderson and Green [2018] show that, on average, a person reaches their personal best twice every 15 years. In contrast, we study stopping behavior that affects the agent's decision after every game.

2 Data

In this section, we first provide details on the data collection and information about the platform. We then provide descriptive statistics that illustrate consistent behavioral patterns.

2.1 Collection

We scraped the data from an online chess platform, chess.com, which is the most frequently visited chess website.[9] The website has over 30 million users, and it hosts around 3 million chess games every day. Users range from amateur players to the world's best chess players, including Magnus Carlsen, the World Chess Champion.[10] This platform is free to use, and anyone can register to play against other people or a computerized opponent. The website also provides some lessons and chess puzzles.

[7] See Tversky and Kahneman [1991], Miller and Sanjurjo [2017, 2018], Miller et al. [2018], Aharoni and Sarig [2012], Arkes [2010, 2013], Avugos et al. [2013], Cervone et al. [2014], Clotfelter and Cook [1993], Brown and Sauer [1993], Camerer [1989], Croson and Sundali [2005], Suetens et al. [2016], Gilovich et al. [1985], Green and Zwiebel [2017], Koehler and Conley [2003], Rabin and Vayanos [2010], Sundali and Croson [2006], Tversky and Gilovich [1989].

[8] See recent papers using chess data to study behavior Gerdes and Gränsmark [2010], Gobet and Charness [2018], Dreber et al. [2013b], Gränsmark [2012], Bertoni et al. [2015], Dreber et al. [2013a], Linnemer and Visser [2016].

[9] Based on Alexa internet rating, www.alexa.com.

[10] Magnus Carlsen, a Norwegian chess grandmaster, and chess prodigy is the highest-rated player in the world, and the highest-rated player in the history of chess.

We use the public Application Programming Interface (API) to collect data. Each observation includes information about the players and the game: usernames, their self-identified country of association, platform ratings, the time at which the game was played, which player had white pieces, the length of the game, and the final results. We focus on a subsample of the data that contains "blitz"[11] games and games played against human players (see Table 1 for a summary).

Number of games	35,517,526
Number of sessions	6,063,366
Number of players	13,027
Average Session Length	4.14
Average Number of Session	348
Average Rating	1311
Rating Range	[100, 2816]
Pr(Win | White Pieces)	0.51
Pr(Win | Black Pieces)	0.47
Pr(Draws)	3.4

Table 1: Data description

2.2 Definitions

A *game* g is a single game with a human opponent. A collection of games ordered by time stamp, $(g_1, g_2, ..., g_n)$, is called a *session* if there is no game played T minutes before g_1 or after g_n, and for any $i \in \{1, ..., n-1\}$, the time between g_i and g_{i+1} is less than T.[12] We call sessions that contain only one game ($n = 1$) *only game* (O-game). For sessions with $n \geq 2$, g_1 is the first game, g_n is the last game and any game in between the first and the last is referred to as *a middle game*.

Based on the terms above, we categorize sessions into three mutually exclusive groups: (1) sessions that consist of a single game – the only game $S = (O)$; (2) session with exactly two games $S = (F, L)$; (3) sessions with more than two games $S = (F, M_1, ..., M_m, L)$ where $m \in \{1, ..., n-2\}$.

Let $f_W(\cdot)$ be a function that calculates the fraction of wins in a particular type of game, for example, $f_W(L)$ is a player's fraction of wins in the last games.[13] In some cases, when

[11] Blitz is a type of chess game where each player has somewhere between 3 to 15 minutes for the game.

[12] For the main section of the results we set $T = 30$ minutes, we vary T to check the robustness of our results and we find no substantial differences.

[13] For the most part of the paper, we look at a user's playing history in year 2017. For each user we calculate

the context is clear, instead of writing $f_W(F)$, $f_W(M)$, $f_W(L)$, $f_W(O)$, we write F, M, L, and O, and we mean the fraction of wins in first, middle, last, and only games, respectively.

2.3 Descriptive Results

In this section, we first establish that session-stopping behavior is history-dependent. We find that a sizable fraction of players fall into two categories: some people consistently leave their session after they win a game, while others exit after a loss; we refer to these types as *win-* and *loss-stoppers*, respectively. In section 3, we present a theoretical model that can account for the behavior observed in the data.

2.3.1 History Dependence

We first take all sessions in our data that lasted at least three games and calculate the average winning frequency in the first, middle, and last games for *each player*, as defined in Section 2.2. If a player makes a decision to stop the game randomly and stopping behavior is history independent, then the fraction of wins in the last game should be similar to the fraction of wins in any other game. Our null hypothesis is:

H_0*: Players stopping behavior is independent of the outcome of the last game.*

H_0 implies that the correlation between the fraction of wins in the last and middle games to be close to 1.[14] Figure 1a presents the relationship between the fraction of wins in the last and middle scores (the solid line represents the linear regression line, where the dependent variable is the fraction of wins in the middle games and the independent variable is the fraction of wins in the last game). The correlation between the fraction of wins in the last and middle games is $-.41$ and statistically different from 1 with $p < 0.001$. Thus, at the aggregate level, the decision to stop is not random and H_0 is rejected.

2.3.2 Behavioral Types

We rejected the null hypothesis of history independence; however, the alternative hypothesis does not identify the nature of the relationship between the outcome of the game just played and the decision to play another game. Are players more likely to end the session after a win or after a loss? We state the following two alternative hypotheses:

the fraction of wins in different types of games.

[14] We do not include first games at this stage because the data on first games is used later to check the robustness of the findings.

(a) Fraction of wins (b) Density of fraction of wins

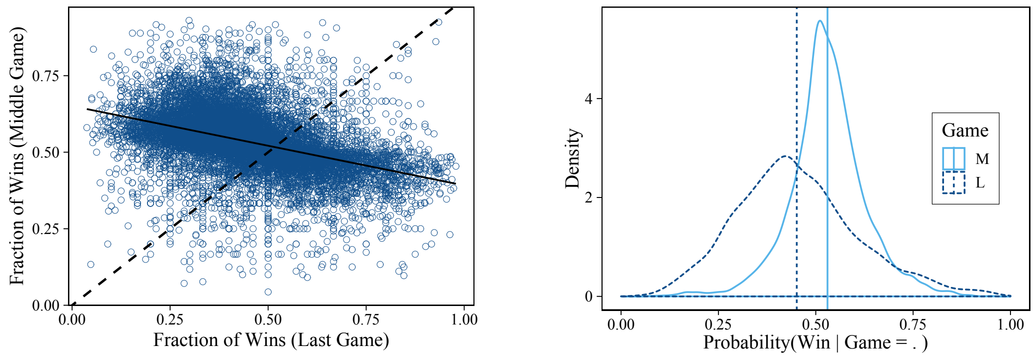


Figure 1: Fraction of wins in the Last and Middle games.

H_A^1: *players are more likely to end a session after a win.*

H_A^2: *players are more likely to end a session after a loss.*

If only one of the two alternative hypotheses is correct, we should see a skew or a shift of the distribution of the fraction of wins in the last game compared to the fraction of wins in the middle games (Figure 1b presents both distributions). The test of medians for the two distributions shows that the median fraction of wins in the last game is statistically lower than the median fraction of wins in the middle game. If we only look at this aggregate result, it supports the hypothesis that players are more likely to stop playing after a loss. However, if we take a closer look at the distribution, we see an interesting pattern. The standard deviation of the fraction of wins in the last game is twice that of the middle game. Both plots in Figure 1 inform us that there are some people with a much higher fraction of wins in the last game compared to the middle game and that there are others with a much lower fraction of wins in the last game compared to the middle game. We define players that are more likely to end the session on a loss and others who are more likely to end the session on a win in the following way:

Definition 1 *A player is behavioral type at the tolerance level of* τ *and he is*

- *a win-stopper if* $f_W(L) > f_W(M) + \tau$,
- *a loss-stopper if* $f_W(L) < f_W(M) - \tau$,

neutral types have $f_W(L) \in [f_W(M) - \tau, f_W(M) + \tau]$.

Using data from sessions that last 3 or more games, we classify players according to Definition 1 (see Figure 8 in the appendix for population decomposition by types using different tolerance levels of τ). Intuitively, as we increase the tolerance level, fewer players are classified as win- or loss-stopper types. Interestingly, when we change τ the ratio of win-stoppers to loss stoppers stays stable at around 40%. At tolerance level τ of 5%, 84% of the players are classified as behavioral types, with about 30% of them being win-stoppers and a larger fraction, 70%, loss-stoppers.[15]

To further examine that Definition 1 captures the patterns in the data, let us examine only-games—sessions that contain only one game. Note that we have not used the sessions with only one game for the classification; so far, we used sessions of length three and above. For ease of exposition, let us take Definition 1 to the extreme, where we assume that a win-stopper type **always** stops after a win and a loss-stopper type **always** stops after a loss. This extreme definition of types implies the following two observations (see Figure 2). First, the fraction of wins for win-stoppers in the only and last games must be 1. This is because if a player wins this game, he ends a session, and the game is classified as the only game. If a player loses the first game, he will start another game, making this session at least two games long, in which case whenever a player wins, we classify that game as the last game. Therefore the fraction of wins in the last game is also 1. Second, the fraction of wins for loss-stopper types in the only and last games must be 0 (the argument is similar to the win-stopper case). Combining these two observations leads to the following prediction.

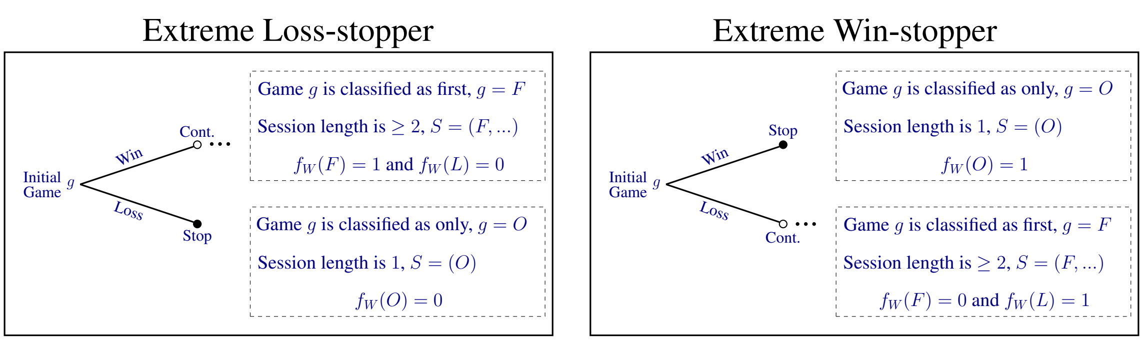


Figure 2: Extreme Behavioral Types' Actions

Prediction 1 *The correlation between the fraction of wins in the last and only game is positive.*

We find that win-stoppers' fraction of wins in the only-game is two times higher than that of loss-stoppers. Figure 3 presents a scatter plot with the fraction of wins in the last-

[15] Unless specified otherwise, we use 5% tolerance level.

game and the fraction of wins in the only-game. A strong and significant positive relationship implies that the types who are more likely to stop playing on a win similarly have a higher fraction of wins in the only game and vice versa, as stated in Prediction 1.

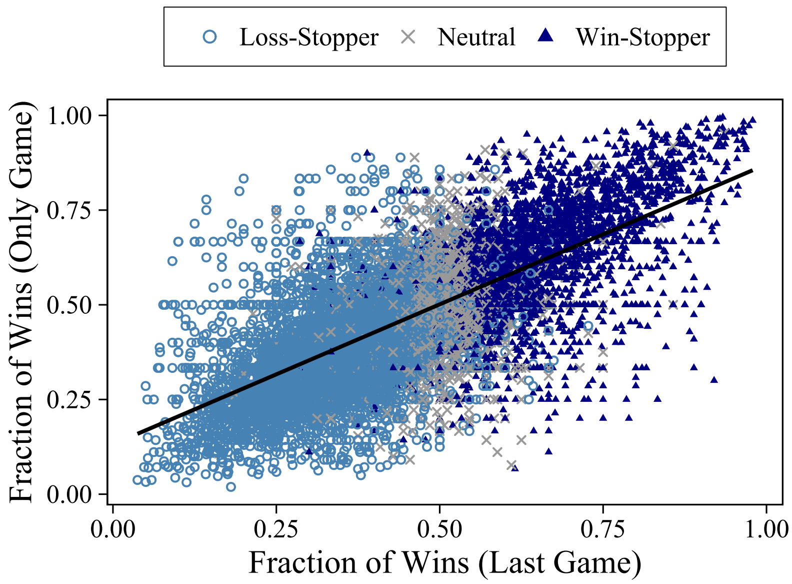


Figure 3: Fraction of wins in the Last and Only Game

So far, we used a fraction of wins in the middle, last, and only games. We have not used a fraction of wins in the first games. The diagram in Figure 2 describes what each type would do if they won or lost the initial game. It gives a relationship between the fraction of wins in the first and only games. If the loss-stopper wins the initial game he plays another one, which makes that game classified as the first game, while if loss-stopper loses the initial game, he stops playing, which makes that game classified as an only game. Therefore loss-stopper's fraction of wins on the first game must be one, and in the only games must be 0. Similarly, we argue that for win-stoppers, the fraction of wins in the first game must be 0, while the fraction of wins in the only game must be 1. Note that while the prediction for last- and only-game relationship is positive, the prediction for the first- and only-game relationship is negative.

Prediction 2 *The correlation between the fraction of wins in the first and only game is negative.*

Figure 4 presents the correlation matrix with p-values in parentheses. A negative relationship between the fraction of wins in the first and only games implies that the types who are more likely to stop playing on a win have a higher fraction of wins in the only-games since they would have kept playing if they had lost the initial game. Similarly, the types who are more likely to stop playing on a loss have a lower fraction of wins in the only-games.

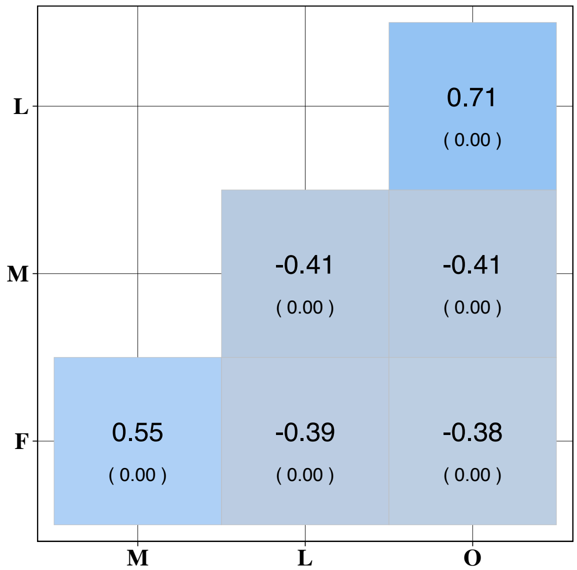


Figure 4: Correlation Matrix

Similar to predictions 1 and 2, we can use the diagram in Figure 2 to write four more predictions about relationships between the fraction of wins in the first, middle, last, and only games. All those predictions are accurately matched to the data (see Figure 4).

3 The Model

We do not take a stand on whether people end a session after a win or loss. Instead, we model different types of behavior and let the data inform us about the types. We capture the different types of people by allowing the utility from playing a new game to depend on the last game outcome and the person's type. We call a person *loss-stopper* type, if the utility from playing one more game for that person decreases after a loss compared to that after a win. Similarly, we call a person *win-stopper* type, if the utility from playing another game increases after a loss compared to that after a win. We say that a person is *neutral* type if the person's utility is not affected by the outcome of the last game.

3.1 Description

This section lays out a chess player's dynamic choice problem. A chess player is characterized by his type that consists of an element observable to the player, player's opponents and econometrician, and an element privately known to the player.

Let y be a vector of characteristics that may change over time, and are observable to the player, opponents, and econometrician (e.g., player's rating). We assume that y lives in finite space Y. A player can be one of the following 3 types: win-stopper (θ_W), loss-stopper (θ_L), or neutral (θ_N). Let $\Theta = \{\theta_W, \theta_L, \theta_N\}$ be the set of all types and let θ be an element of this set. The player's type is fixed over time. A player's type profile at time t, (y_t, θ), consists of player's time-variable characteristics, y_t, and fixed unobservable type, θ. We use variables without time subscripts to denote current states and 'prime' superscripts to denote the next period's state.

Each period, a player is facing the following decision: given the previous history of the play, the player needs to decide whether to play an additional game or to go offline and take an outside option. Before making this decision, the player's utility from another game is

$$U(y, \theta, \chi) = u(y) + (1 - \chi)l_\theta \tag{1}$$

where y is the player's current rating, θ is the player's type, and χ is the outcome of the last game. If a player won the last game ($\chi = 1$), the utility from playing another game is $u(y)$. We can think of this term as how much the player likes playing chess independently of his type. If the player lost the last game, then his utility from playing another game depends on his type.

Definition 2 *A player is*

i) Loss-Stopper, if $l_\theta < 0$*;*

ii) Win-Stopper, if $l_\theta > 0$*;*

iii) Neutral, if $l_\theta = 0$.

There is an outside option, c, that every period is independently drawn from a distribution with density $f(c)$. If a player ends a session, he takes an outside option c. Otherwise, the player's utility is $U(y, \theta, \chi)$ from playing a new game and he moves to the next period, at which point the player faces the same decision based on the new history of the last game (χ'). In each period (after a game is over) a player is facing the following problem:

$$V(y,\theta,\chi,c) = \max\left\{c,\ u(y) + (1-\chi)l_\theta + \delta \sum_{\substack{\chi',y'\in\\ \{0,1\}\times Y}} p(y'|y,\chi')p(\chi'|y)V(y',\theta,\chi')\right\} \tag{2}$$

where δ is the discount factor; $p(y'|y,\chi')$ is the probability of receiving a rating y' given that the player's current rating is y and the outcome of the next game is χ'; $p(\chi'|y)$ is the probability of the subsequent game outcome given the current period rating y. Note that we can directly recover the law of motion of y from the data (player's rating updating rule). The player matching mechanism, which is based on the player's rating, affects the probability of winning the next game. This is the way through which changing the matching mechanism can influence a player's decision to start a new game. In particular, $p(\chi'|y) = \sum(p(\chi'|y,y_{-i})p(y,y_{-i}))$, where $p(y,y_{-i})$ is a probability that a player with rating y will be matched to a player with rating y_{-i}. The set $\{p(y,y_{-i})\}$ is a control for a designer through which it can influence a player's decision to start a new game.

It is important to note that we focus on a reduced-form phenomenon: win-stoppers are more likely to stop playing after a win compared to a loss, and loss-stoppers are more likely to end the session after a loss compared to a win. Some papers evaluated what psychological biases may lead to behavior similar to loss-stopper or wins-stopper separately but not the heterogeneity we observe in this paper (see section A in the appendix for a theoretical discussion and more details).[16]

3.2 Identification

In this section, we prove the identification of players' types, l_θ, δ, probabilities of winning, outside option distribution parameter, and matching probabilities. We start with the identification of behavioral types.[17]

[16] See Miller and Sanjurjo [2018], Miller et al. [2018], Miller and Sanjurjo [2017], Aharoni and Sarig [2012], Arkes [2010, 2013], Avugos et al. [2013], Cervone et al. [2014], Brown and Sauer [1993], Camerer [1989], Croson and Sundali [2005], Suetens et al. [2016], Gilovich et al. [1985], Green and Zwiebel [2017], Koehler and Conley [2003], Rabin and Vayanos [2010], Rao [2009b,c,a], Rinott and Bar-Hillel [2015], Sinkey and Logan [2014], Stone [2012], Stone and Arkes [2018], Sundali and Croson [2006], Tversky and Gilovich [1989], Wardrop [1999], Xu and Harvey [2014], Yaari and Eisenmann [2011].

[17] The identification and estimation of the theoretical model are in the tradition of Hotz and Miller [1993]. We show how we can forgo numerical dynamic programming to compute the value functions for every parameter vector, and propose an estimation procedure that is simple to implement and computationally efficient.

Claim 1 *Optimal stopping rule is a threshold rule in c.*

Proof. Note that in equation (2), continuation values do not depend on the **current** realization of c. Hence, fixing the continuation values and current period utility from playing another game, the second term under the $\max$ operator is lower than outside option c, for sufficiently high c. So, we have a threshold, $\bar{c}(y, \theta, \chi)$, such that for realizations of c above this threshold, the player stops playing and takes the outside option. ■

Therefore $\bar{c}(y, \theta, \chi)$ is a threshold such that a player with type profile (y, θ) having an outcome χ in the last game ends a session if and only if the realized c is at least as large as $\bar{c}(y, \theta, \chi)$.

From (2), we have,

$$\bar{c}(y, \theta, \chi) = u(y) + (1 - \chi)l_\theta + \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi'|y)V(\theta, y', \chi') \tag{3}$$

The following proposition leads to the identification of behavioral types.

Proposition 1
i) $\bar{c}(\theta_W, y, 0) > \bar{c}(\theta_W, y, 1)$*;*
ii) $\bar{c}(\theta_L, y, 0) < \bar{c}(\theta_L, y, 1)$*;*
iii) $\bar{c}(\theta_N, y, 0) = \bar{c}(\theta_N, y, 1)$.

Proof. The proof follows from equation (3). ■

Proposition 1 implies that the win-stopper types' probability of playing one more game is higher if they lose the previous game compared to when they win, and vice versa for the loss-stopper types. For the neutral types, that probability is the same no matter the history of outcomes. By Proposition 1 we can identify a behavioral type of a player from the data by looking at his stopping probabilities after losses and wins.

3.2.1 Model Parameters

The following parametric assumption is made on the distribution of outside option, $F(c)$,

Assumption 1 *$F(c)$ is an exponential distribution with parameter λ.*

We now argue that under the assumption 1 and by normalizing one parameter of our choice in the model, we can identify δ, λ, l_θ and $u(\cdot)$. In the estimation, we normalize $\lambda = 1$. Let,

$$H(y,\theta) = u(y) + \delta \sum_{\chi',y' \in \{0,1\}\times Y} p(y'|y,\chi')p(\chi'|y)V(\theta,y',\chi') \tag{4}$$

Under the assumption 1 and from equation 3, we get that the probability of stopping and taking outside option, $h(y,\theta,\chi)$, can be written as

$$h(y,\theta,\chi) = e^{-\lambda(H(y,\theta)+(1-\chi)l_\theta)} \tag{5}$$

Claim 2 *$\lambda H(y,\theta)$ and λl_θ are identified for all (y,θ).*

Proof. Let us look at equation (5) for $\chi = 1$. LHS, $h(\theta,y,1)$ is coming from the data as probability of stopping after a win. On the RHS, the second term in the power, $(1-\chi)l_\theta = 0$, therefore we can say that $\lambda H(\theta,y)$ is identified from (5).

Now substitute $\chi = 0$ in equation (5). Again the LHS is coming from the data as probability of stopping after a loss $h(\theta,y,0)$. On the RHS, first term in the power $H(\theta,y)$ is coming from the first part of this prove. Thus, λl_θ is identified from (5) as well. ■

Claim 3 *$\lambda V(y,\theta,\chi)$ are identified for all (y,θ,χ).*

Proof. Let us denote $\bar{c}(\theta,y,\chi) \equiv \bar{c}$. We can rewrite (2) as:

$$V(\theta,y,c,\chi) = \mathbf{1}(c > \bar{c}) * c + \mathbf{1}(c \leq \bar{c})\Big(H(\theta,y) + (1-\chi)l_\theta\Big) \tag{6}$$

Where $\mathbf{1}(\cdot)$ is an indicator function. Taking expectations of both hand sides of (6) with respect to c, gives

$$\begin{aligned} V(y,\theta,\chi) &= E(c|c > \bar{c}) + Pr(c \leq \bar{c})\Big(H(\theta,y) + (1-\chi)l_\theta\Big) \\ &= Pr(c > \bar{c})\Big(E(c) + \bar{c}\Big) + Pr(c \leq \bar{c})\Big(H(\theta,y) + (1-\chi)l_\theta\Big) \\ &= Pr(c > \bar{c})\Big(\frac{1}{\lambda} + \bar{c}\Big) + Pr(c \leq \bar{c})\Big(H(\theta,y) + (1-\chi)l_\theta\Big) \\ &= Pr(c > \bar{c})\Big(\frac{1}{\lambda} + \bar{c} - H(\theta,y) - (1-\chi)l_\theta\Big) + H(\theta,y) + (1-\chi)l_\theta \end{aligned} \tag{7}$$

multiplying both hand sides by λ and substituting $\bar{c}(y,\theta,\chi)$ from (3), we get,[18]

$$\lambda V(y,\theta,\chi) = e^{-\lambda[H(y,\theta)+(1-\chi)l_\theta]} + \lambda H(y,\theta) + (1-\chi)\lambda l_\theta \tag{8}$$

[18] Notice that $Pr(c \leq \bar{c}) = 1 - Pr(c > \bar{c}) = 1 - e^{-\lambda[H(y,\theta)+(1-\chi)l_\theta]}$

Claim 2 and expression (8) imply that $\lambda V(y, \theta, \chi)$ are identified for all (y, θ, χ). ■

Claim 4 *δ and $\lambda u(y)$ are identified.*

Proof.

We can consider the difference $\lambda(H(y,\theta) - H(\theta', y))$ for some $\theta \neq \theta'$. This gives us,

$$\delta = \frac{\lambda(H(y,\theta) - H(\theta', y))}{\lambda(\sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi'|y)(V(\theta, y', \chi') - V(\theta', y', \chi')))}$$

By claims 2 and 3, numerator and denominator are identified in the above equation.

We identify $\lambda u(y)$ from,

$$\lambda u(y) = \lambda H(y, \theta) - \lambda \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi', y)V(\theta, y', \chi')$$

■

Finally, we can normalize all the parameters and value functions by λ. This completes the identification of the parameters of the model.

4 Estimation and Counterfactual Analysis

In this section, we first introduce the structural estimation results. Then we present the counterfactual analysis results that highlight how a market designer can change how much time a player spends on the platform.

To better understand the counterfactual analysis, we begin by describing the rating system on the platform. Once a player signs up for chess.com, he gets an initial rating (1200 points). Rating changes after every game based on the outcome of the game. Intuitively, the rating goes up after a win and goes down after a loss. Thus, a player's current rating reflects his current expertise in chess: higher the rating, better the player. We recover the rating updating rules from the data. To estimate parameters structurally, we divide the rating range into the grids of 20 since the rating has a wide range ($[100, 2798]$).[19] Since we have few observations where the rating is below 800 or above 2200, we place all the players with a rating below 800 in the first rating grid and those above 2200 in the last rating grid (grid 71). The rest of the rating range is divided into 20 point intervals.

[19] For the counterfactual analysis, we change the grids, more details are provided in the counterfactual section. Changing the grid size or the number of grids does not qualitatively affect any of our results.

4.1 Structural Estimates

Our estimation strategy parallels the identification proof. First, we estimate $H(y, \theta)$ and l_θ using an empirical counterpart of equation 5. Second, we recover $V(y, \theta, \chi)$ using equation 8. Finally, we use empirical counterparts of equations in Claim 4 to estimate delta and $u(y)$. The main parameters that we focus on are l_θ for $\theta \in \{\theta_W,\ \theta_L,\ \theta_N\}$. The estimates are presented in Table 2. We sub-sample the data from the entire sample 300 times with replacement and find that our estimates are stable.

Parameter	Mean	SD	[min, max]
l_{θ_W}	0.810	0.009	$[0.776, 0.838]$
l_{θ_N}	-0.067	0.004	$[-0.077, 0.057]$
l_{θ_L}	-0.698	0.004	$[-0.706, -0.686]$

Table 2: Bootstrapped values for l_θ

Table 2 shows that for a win-stopper type, utility from playing another game increases by 0.810 after a loss compared to a win. The effect is opposite for loss-stopper types, loss in the last game decreases utility from playing another game by 0.696 compared to a win. Intuitively for neutral types, last game result has only a minor effect on utility.

4.2 Counterfactual Analysis

Can the market designer leverage information on behavioral types to increase a player's expected time spent on the platform? To answer this question, we need to know what can be controlled by the market designer. In our setting, the player-to-player matching algorithm is controlled by the market designer. Therefore we need to know what is the current matching algorithm, which can be recovered directly from the data. The platform has a simple matching rule. Two players with closer ratings are matched into pairs with higher probability. In other words, the platform only uses the players' rating to decide who plays with whom.[20] In our counterfactual exercise, we allow the platform to choose from matching mechanisms that can be contingent on players' behavioral type in addition to the rating.

Before presenting counterfactual results, we need to explain the data that we use for counterfactual analysis. In the last subsection, we presented estimates for the entire data, but for counterfactual analyses, we focus on a sub-sample. The reason is that we want

[20] Obviously we assume that both players who are matched are on the platform and requested a particular type of chess game, for example, 3 minute blitz game.

players to be homogeneous in their rating. In other words, we want to consider players who have similar rating to avoid matching players, one with a high rating and other with low rating. Therefore, we consider only players who have an average yearly rating from 900 to 1300. Then we divide rating space [800, 1400] into 102 grids, each grid size is 6 points.

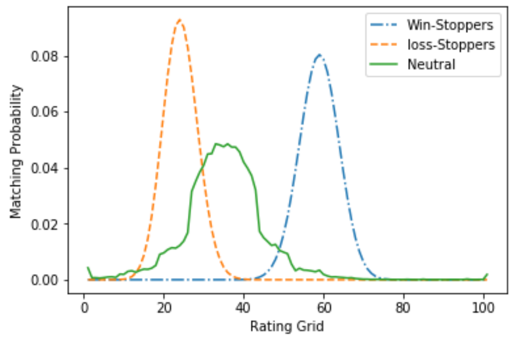
We re-estimated the model and find that even in the restricted data, the parameter estimates of l_θ are similar to what we had for the entire data (Table 3). To check the stability, we bootstrap the data 300 times.

Parameter	Mean	SD	[min, max]
l_{θ_W}	0.718	0.005	[0.711, 0.726]
l_{θ_N}	-0.047	0.002	[-0.050, 0.045]
l_{θ_L}	-0.613	0.002	[-0.615, -0.609]

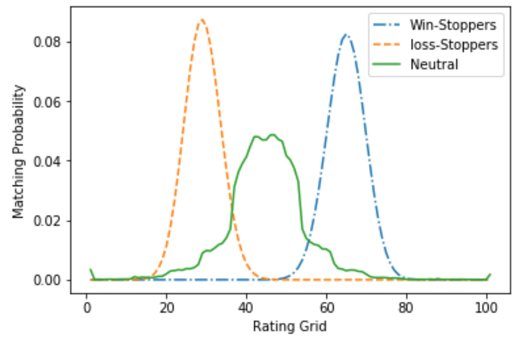
Table 3: Bootstrapped values for l_θ with restricted data

The goal of this section is to provide a matching algorithm that is better than the current one. Ideally, we would find an optimal matching algorithm among the class of algorithms that accounts for rating as well as players' types. However, due to the high dimensionality of the problem (we have $3n(n-1)$ variables to optimize, where n is the number of rating grids), our current computational capabilities do not allow us to find the best such algorithm. Instead, we answer the following question: at least, by how much can the platform increase the expected time spent on the platform? To answer this question, we consider optimization over a smaller class of matching algorithms and find that the platform can increase the time spent by the user theoretically by at least 62%.

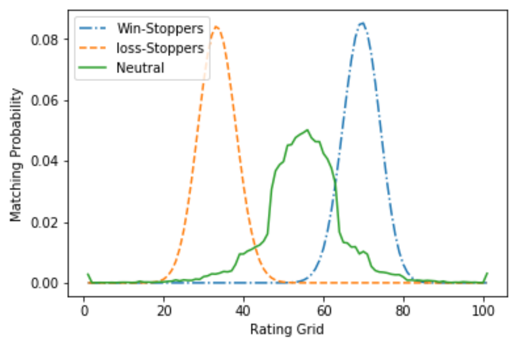
There is a concern with this algorithm. The algorithm matches players with very low ratings to players with a very high rating. Such drastic change is matching algorithm could lead players to learn about the matching or become uninterested in a platform that frequently matches them with either very low or very high rating players. Therefore we consider the matching algorithm, which does not allow for such radical matching pairs. Intuitively, such an algorithm results in less improvement – a 5.44% increase. To put this number in perspective, let us look at simple statistics. An average player plays 348 sessions a year, and an average session lasts about 4.14 games. Since we consider only blitz games and average blitz games in our sample lasts for 8 minutes and 12 seconds, 5.44% improvement results in 87 more games per year (on average 11 hours and 54 minutes more time spend on the platform per year per player).



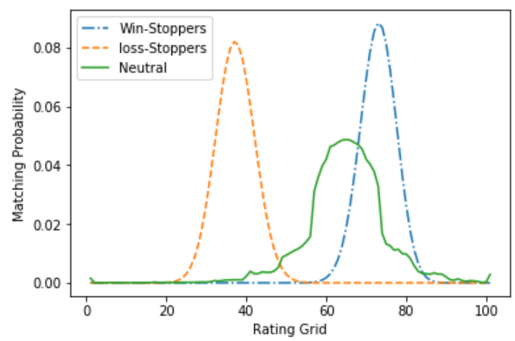
(a) Rating grid 35 (rating 1004-1010)



(b) Rating grid 45 (rating 1064-1070)



(c) Rating grid 55 (rating 1124-1130)



(d) Rating grid 65 (rating 1184-1190)

Figure 5: Matching probabilities for four different rating grids

Figure 5 presents the resulting matching probabilities for four different rating grids. Green (solid) distribution represents current matching probabilities, while orange (dashed) and blue (dotted-dashed) distributions are new (improved) matching probability distributions respectively for loss-stopper and win-stopper types.

We choose middle grids because, for them, we have both higher as well as lower-rated possible opponents. If we match win-stoppers with relatively higher rated players, we decrease the probability that the current player wins leading them to play another game with higher probability. For loss-stoppers argument is reversed. As figure 5 shows, loss-stoppers should be matched with lower-rated players so that they have a higher chance of winning and continue playing with higher probability. Since the neutral types are not affected by the outcome of the last game result, we use these players to clear the market. The matching algorithm explained above and presented in figure 5 improves average session length by 5.44%.

5 Robustness of the Model and Time Stability

5.1 Time Stability of Behavioral Types

We use data for 2017 for the estimation. However, we also have data for the same people for 2018, which we keep as a holdout sample to check the time stability of behavioral types. Figure 6 shows the relationship between these two years. First, we created individual variable, break probability difference, as the difference between the fraction of times a player took a break after a win and after a loss. Figure 6a shows that there is a positive correlation between those two years. People who have a higher fraction of breaks after a win than after a loss in 2017, also have very similar break probability difference in 2018.

(a) Time Invariance of Types (b) Transition From 2017 to 2018

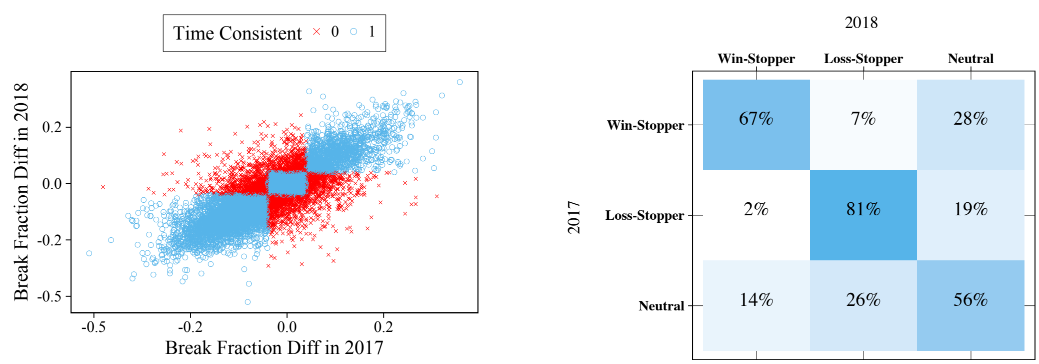


Figure 6: Time Invariance of Types (2017 to 2018)

As Figure 6b shows, the major differences in classification occur for neutral types. This result is not surprising since the definition of types is based on the threshold level. Among neutral types, players who change the type are threshold cases (see Figure 6a).

5.2 Consistency of Behavioral Types

In this paper we have two distinct definitions of behavioral types (Definitions 1 and 2). The two definitions are intuitively related, but these two methods of identifying behavioral types do not have to overlap at all. That is, given some data, a player could be identified as win-stopper according to the model but be classified as loss-stopper according to the last-game and middle-game definition. For example, suppose a player's complete playing history contains the following set of 3 sessions:

$$\{WWWWW, WLW, WLL\},$$

Let us first calculate the stopping probability after a loss, $\Pr(Stop|Loss) = 1/3$. Then, given all the wins and we calculate the stopping probability after a win, $\Pr(Stop|Win) = 2/7$. Given that the probability to stop playing is higher after a loss than after a win, $\Pr(Stop|Loss) > \Pr(Stop|Win)$, our model would qualify the player as loss-stopper. However, according to our definition through the last game of the session, the player's behavior corresponds to win-stopper, because the last game is won more often than middle game.

This example demonstrates that two definitions are intuitively related; however, one does not imply the other. This fact strengthens any relationship we find between the two classifications, highlighting the consistency of our intuition with the proposed theory. Let us look at how the two classifications pair. We take all the players, and in Figure 7a we put a plus sign if the model identifies a win-stopper player as win-stopper, loss-stopper as loss-stopper and neutral as neutral. For 81% of the players in the data, the two classifications match. This result provides strong evidence that the model captures the players' behavior and that the game outcome affects the utility of the next game.

(a) Consistency (b) Model VS Data Types

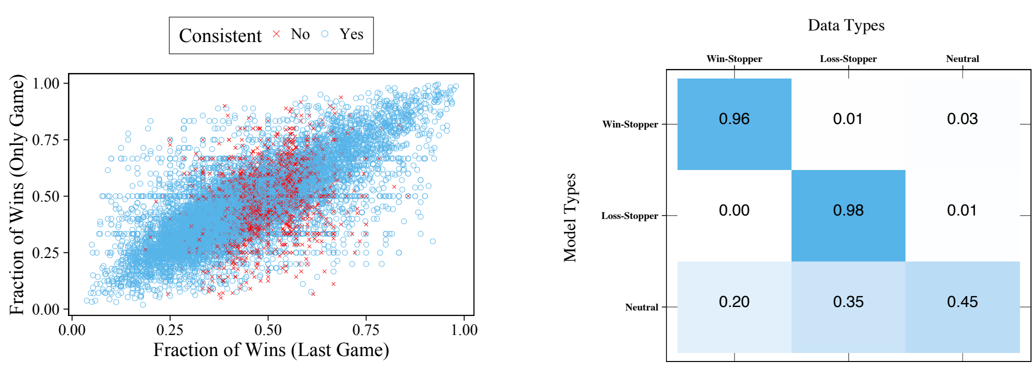


Figure 7: Model Identified Types and Consistency

Figure 7b presents a transition matrix between model types to behavioral types. We observe a large mass on the diagonal, meaning that the two classifications are consistent. For example, 96% of win-stopper types identified by the model were identified as win-stopper types by Definition 1. However, there are mismatches; for example, some neutral types are classified as behavioral types by the model and vice versa. Notice that the cases in which a win-stopper (loss-stopper) is identified as loss-stopper (win-stopper) by the model account happen only 1% of the time.

5.3 Validity of the Modeling Choices

To show that the relevant factor that affect players' stopping decision is the last game result, we estimate the Cox Proportional Hazard (CPH) model. The purpose is to evaluate the effect of a number of factors on survival.[21] Survival analysis allows us to examine how specified factors influence the session stopping rate. Such a rate is referred to as hazard rate, and the examined factors are called covariates.

We use Cox's proportional hazard model with time-dependent covariates. We proceed by first estimating the model using aggregate data and then, later on, we estimate the model for each player separately.[22] The general description of the model is as follows:

$$h_j(t, x_j(t)) = h_0(t) \exp\{x_j(t)'\beta\} \tag{9}$$

LHS of the equation 9 represents risk that game j with characteristics $x_j(t)$ is the last game of the session (session terminates after that games). RHS is comprised of two components: baseline risk and relative risk. The baseline risk, $h_0(t)$, represents the risk that a game will be the last game in a session when all the covariates equal to zero, $x_j(t) = \mathbf{0}$. The relative risk, $\exp\{x_j(t)'\beta\}$, is a proportionate increase or reduction in risk associated with the set of characteristics $x_j(t)$.

Let us look at the CPH results when we pool the data and examine the effect of the last game on the decision to stop a session. The estimation results are presented in Table 4.

Covariate	Coef	exp(Coef)	p-value
Score	-0.23	0.80	< 0.005

Table 4: CPH without type heterogeneity

The variable *Score* takes value 1 if a player wins the last game and 0 otherwise. Interpreting the result of the analysis is easier using the third column (exp(Coef)), where $\exp(\text{Coef}) = 1$ implies that whether the last game is a win or a lose has no effect on the decision to stop the session. The value of exp(Coef) is $.8$, and it shows that a player is 20% less likely to stop playing after a win than after a loss. Recall that this behavior is

[21] Survival in our setting means "not ending" a session

[22] Appendix C shows the results of individual estimates for the CPH model. We also explain how the type identification using the CPH model coincides with our structural model type identification.

observed among loss-stopper types, but not among the win-stopper types who are more likely to continue playing after a loss. This result is hiding an important heterogeneity that we are aware of based on the analysis in the previous sections of the paper. The estimation does not take into account any differences between players and assumes there is a universal effect of the last game outcome for all players. The negative relationship that we observe is due to the fact that more player are loss-stoppers than win-stoppers.

We introduce behavioral type heterogeneity in players according to our model estimation in Section 3. For the ease of exposition of the results, we assume that there are no neutral types, and we have only two types of players: loss-stoppers and win-stoppers. We include the player's type and interaction of type and the last game result to the covariates and re-estimated the CPH model. The results are presented in Table 5.

Covariate	Coef	$\exp$(Coef)	p value
Score	-0.54	0.58	< 0.005
Type	-0.55	0.58	< 0.005
Type $\times$ Score	1.01	2.76	< 0.005

Table 5: CPH with type heterogeneity

The variable *Type* is 1 for win-stopper types and it is 0 for loss-stoppers. Therefore, the baseline in the estimation is loss-stopper losing in the last game. Table 5 shows that for a win-stopper type (Type$= 1$), the hazard rate is higher after a win than after a loss.[23] In other words, a chance that a win-stopper type ends a session after a loss is lower than after a win. That is what we expected. For loss-stopper types, the relationship is reversed. A win in the last game decreases the hazard rate compared to a loss (baseline) by 42%.

5.3.1 History dependence of length 2

Up until now we have focused on the effect of win/loss history in one game. Now, we examine whether there is an effect of more than one lag on the decision to stop a session (last two games). We estimated the CPH model as before, but now we add Lag 1 score and interaction of this variable and player's type. Table 6 presents the results of the estimation.

Adding the outcome of the game previous to the last game has little effect on the estimates. The last game outcomes have effects similar to what we observed in Table 5. The lagged values have statistically significant effects, but the magnitude of the effect is much

[23] Hazard rate is lower by -0.54-0.55+1.01=-0.08 for a win-stopper type after a win compared to baseline and lower by -0.55 after a loss. Therefore loss decreases the hazard rate much more than a win.

Covariate	Coef	exp(Coef)	p-value
Score	-0.62	0.54	$<$ 0.005
Type	-0.65	0.52	$<$ 0.005
Type $\times$ Score	1.13	3.08	$<$ 0.005
Lag 1 score	0.04	1.04	$<$ 0.005
Type $\times$ Lag 1 score	-0.04	0.96	$<$ 0.005

Table 6: CPH with type heterogeneity and Lag 1 game

smaller than the effect of the last game outcome. Note that the sign of the effect of the lagged variable is predicted using the arguments presented in Section 2.3.2.

5.3.2 Starting rating as a reference point

One possible factor to effect player's stopping decision could be the session's starting rating. If a player's stopping rule is to end a session once he reaches a higher rating compared to what he started with, then the rating difference between the first and last game of a session should be significant. To test this hypothesis, we included rating change since the start of the session in the CPH and estimated the model.

Covariate	Coef	exp(Coef)	p-value
Score	-0.61	0.54	$<$ 0.005
Type	-0.67	0.51	$<$ 0.005
Type $\times$ Score	1.10	3.01	$<$ 0.005
Rating Change	0.00	1.00	$<$ 0.005
Type $\times$ Rating Change	-0.00	1.00	$<$ 0.005

Table 7: CPH with type heterogeneity and rating change since the start of the session

Table 7 shows that the change of the rating since the session started is not an important variable to explain stopping behavior, neither is interaction term between the type of the player and rating change. It is important to note that change in rating after every game is in the interval of [-12, 12], the coefficient for rating change is 0.002, and for rating change and interaction term is -0.001; therefore, the effect of rating change seem not essential for the stopping decision.

6 Conclusion

In this paper, we investigate stopping behavior in an environment free of monetary incentives and identify factors that determine how people make a stopping decision. We use rich data that provides information about an individual over the years in many occurrences. We identify two behavioral types: win-stoppers and loss-stoppers. Win-stoppers are more likely to stop playing after a win, while Loss-stoppers are more likely to stop playing after a loss. With conservative parameter values, we classify 84% of players as behavioral types, one-third of which are win-stoppers, and the rest loss-stoppers.

We develop a dynamic discrete choice model in which the agent may have time non-separable preferences. The model allows for the future game utility to depend on the outcomes of the current game and can capture the heterogeneity in stopping behavior. We use the model to estimate the model parameters and then test alternative market designs using counterfactual analysis. The results show that using type identification in the matching algorithm can increase session length by 5.44%.

Although the industry for online games is sizable, little is known about the determinants of length of play. In this paper, we document a robust behavioral force that drives players' stopping decision. The results in the paper might not be limited to a chess game, and the results can be applied to online games more broadly. Our approach can also be applied to other settings. For example, ride-sharing platforms such as Uber and Lyft can use a similar approach of looking heterogeneity to increase drivers' labor supply by using the information on the user's tipping behavior to match riders with drivers.

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Appendices

A Theoretical Discussion

What would explain the patterns of behavior that we find in the data? Can reference dependence, fatigue, gambler's fallacy, or hot hand fallacy explain the observed patterns? Let us start with reference dependence. One's personal best rating could act as a reference point for a player: a player ends a session whenever he sets his new personal best rating, and he plays longer otherwise. Reference dependence could only predict one type of behavior – stopping the session on a win. However, loss-stoppers can not be explained by reference-dependence unless we assume players have goals to lose a certain number of games every time they play. While theoretically possible, we have neither intuitive nor statistical support for such targets.

Another theory that we discuss is fatigue. Suppose that as a player keeps playing games, he gets fatigued over time; hence, fatigue would lead to worsened play over time. While lower last game scores are observed among loss-stoppers, it is opposite for win-stoppers, who have a much higher score in the last game. Moreover, even for loss-stoppers, we do not find any support of the fact that players' performance worsens as they play more games during a session.[24]

The last two theories that we consider are the gambler's fallacy and the hot hand fallacy. We call them belief-based explanations because both of them are based on players' beliefs about their future performance. The first one – the gambler's fallacy – implies the maturity of chances, if something happens more frequently than normal during a given period, it will happen less frequently in the future. The gambler's fallacy suggests that if a player wins a few games in a row, then a player may think that it is less likely that he wins again, and therefore, stops on a win. Similar logic applies if a player has lost a few games in a row, then he may think that he is more likely to win, therefore assuming person likes winning more than losing, he should stay and not end a session on a loss. While gambler's fallacy could explain some patterns found among win-stoppers, it goes against the behavior of loss-stoppers.[25]

[24] We ordered games for every session and find that fraction of wins is not affected by the order number of the game. We also conducted logit estimation to see if winning probability is affected by the order number of the game. We find that order number is not a statistically significant variable.

[25] In addition, if a player believes in the gambler's fallacy, that player's stopping decision should depend not only on the last game but on the game previous to last (lag 1 game). We show in Section 5.3 that the

The second belief based theory is the hot hand fallacy. Some athletes (as well as their fans) believe that if they succeed several times in a row, they have a "hot hand", i.e. higher chances of success in the next attempt too. It implies that if a player believes in the hot hand, that player should keep playing in case of win and stop after lose since it indicates that he does not have a hot hand anymore. Such behavior can predict the last game result of the loss-stoppers, however it goes against the behavior of win-stoppers.

To sum up, all the existing theories can explain one part of the story but is silent about the other. Win-stoppers might be the players who believe in gamblers fallacy and loss-stoppers be the one's who believe in hot hand fallacy. It can be that the entire population is a mixture of people who believe in different theories or principles. In this paper, we do not take a stand about underline psychological forces that motivate such behavior. Instead, we propose a theory that explains the actions found in the data and accounts for heterogeneity. Therefore, our model can accommodate win-stoppers, loss-stoppers, and neutral types together in the same model.

B Robustness

B.1 Robustness of Tolerance Threshold

Figure 8 presents the players' population decomposition by types as we vary τ from 0 to .2. We see that behavioral types are robust to changing the allowed tolerance.

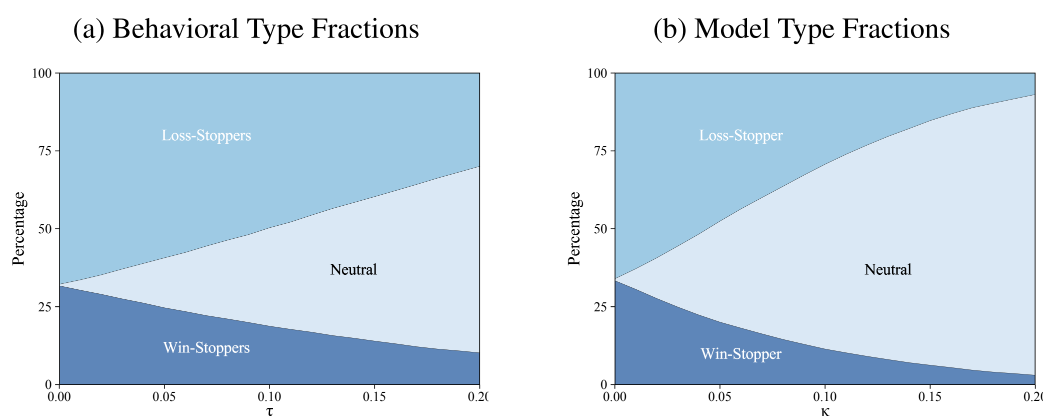


Figure 8: Type Decomposition

Similarly, changes in κ, also have consistent effect on model type decomposition, Fig-

effect of lag 1 game result is weak compared to the last game result.

ure 8b.

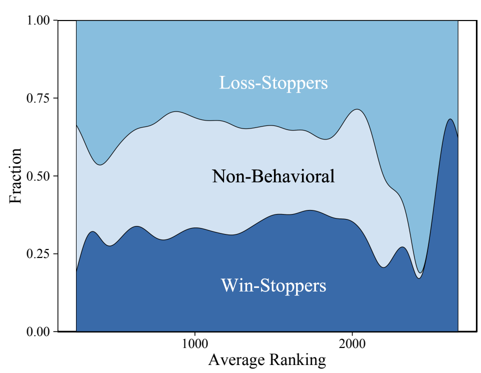
B.2 Rating and Behavioral Type

In this section, we examine whether a person's type changes over time and whether the rating in the chess game is correlated with the type classification. To study these questions we take two approaches: (i) we use existing data, and (ii) we collect additional data.

Rating Figure 9a presents the break down of types over different rating categories. All three classification types are represented at every rating level. Moreover, the ratio of types for non-extreme rating levels are similar. Figure 9b presents the the distribution of the rating by type classification and we find that the distributions are practically the same. Kolmogorov-Smirnov test between win and loss-stoppers finds no difference with $p = 0.4$. Neutral types have on average lower rating than behavioral types with $p < 0.01$.

To further examine and look for any possible differences in rating and type classification we collect data for a special subset of chess players that have rating in top 2000. We do not find any qualitative difference in the distribution of behavioral types or in the ratio of win and loss-stoppers. These result suggest that the rating in the chess game is not related to the player's type.

(a) Number of Session



(b) Session Length

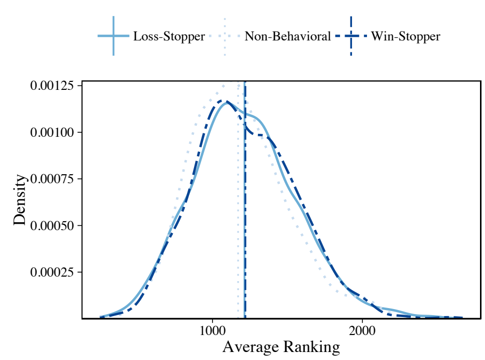


Figure 9: Ability and Behavioral Types

B.3 Crowding Out

Our counterfactual shows that taking into account players behavioral type for matching algorithm can increase average number of games played during the session. One might think of several crowding out effects that increase of a session length might have. Even though we can not fully address such concerns without RCT, in this subsection we provide evidence that such effects is not likely.

B.3.1 More games does not guarantee more time

The goal of counterfactual exercise was to increase number of games during the session, but the goal of market designer (platform) is to increase time spend on the platform. To address this issue we calculated correlation for every individual between minutes spend on the platform during a session and number of games played in the same session. Figure 10 shows that correlation between these two variables is high. Median correlation between minutes and games during the session is 0.98 across players.

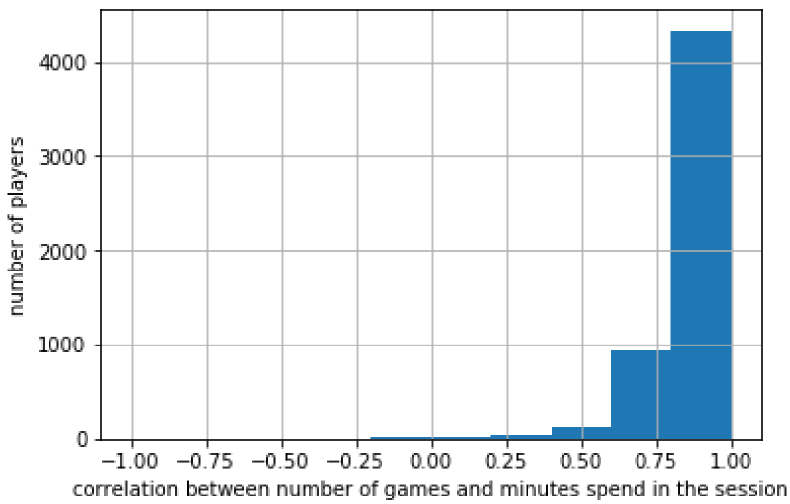


Figure 10: Correlation between minutes spend for a session and number of games

B.3.2 Asymmetric matching can decrease playing time

Second issue that one might worry about is that asymmetric matching can cause fast games, in the sense that strong players can win with a weak players very fast. To show that that will not be an issue we calculated correlation between rating difference and minutes spend on a game. Rating difference gives as measure of how much better one player is compared to another. Figure 11 shows correlation between how much better an opponent is and how much time the game lasts is close to zero for most of the players.

B.3.3 One long session can cause next session to be short

One might worry that if a first session time increases during the day, it can decrease next session length (if players set out a certain amount of time to spend on the app every day).

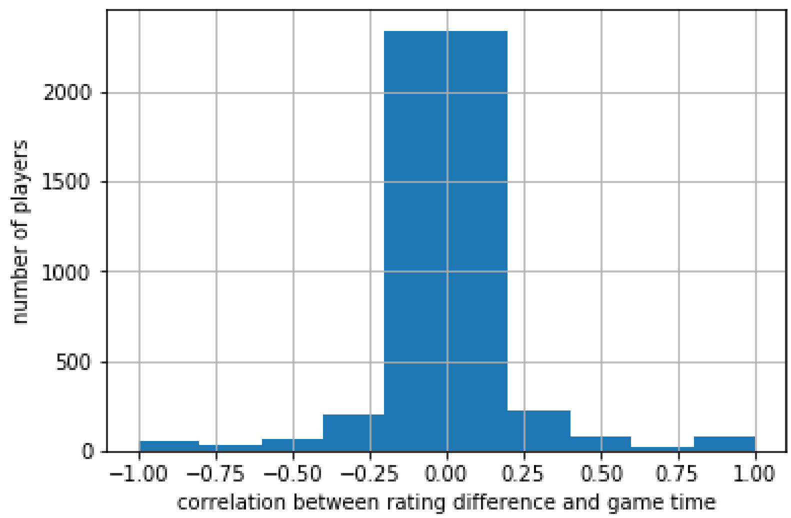


Figure 11: Correlation between minutes spend for a game and rating difference

We find that correlation between number of sessions played during a day and average length of a session is 0.0002. We also find that one session length does not have any explanatory power on the length of the next session.

B.3.4 Players adjust game type based on the time they have played

The last issue that we address here is about changing the type of the game. It might be that person who started a session with 5-minute blitz game can play shorter last game (for example 3-minute game) because he has only certain time allocated to play on the platform. If that is the case we should see that people change game types during the session. We find that 96% of sessions are homogeneous in the sense of the game type. This includes not only change if game type in the last game but during any other time. Which makes our argument even stronger that players do not choose last game type based on the remaining time that they have allocated for plying chess.

C Estimating CPH Model for Each Individual Separately

In our data set, an average player plays 348 session a year and an average session lasts 4 games. Therefore, we have sufficient data to estimate CPH model for each individual player. By examining the individual estimates we can identify win-stopper, loss-stopper and neutral types by looking at individual coefficients and the corresponding p-values. Since we already identified types using two different methods (one comparing winning fractions in the last and non-last games and other from the structural model), this is another robustness check of consistency of types over different methods.

Figure 12 presents the distribution of individual coefficients for the last game outcome variable. The figure highlights heterogeneity among the players.

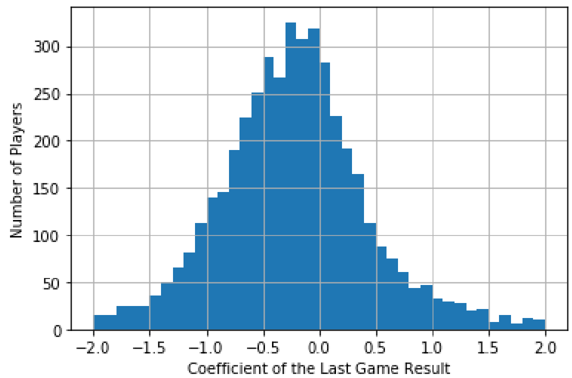
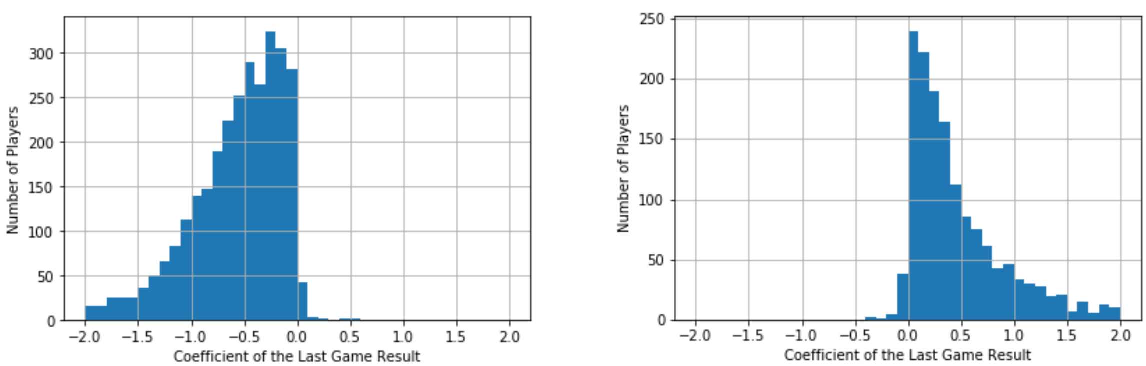


Figure 12: Individual coefficient distribution

Let us look at the differences in the hazard rate coefficient distributing of CPH model by behavioral types using the structural model estimation. Figure 13 shows the coefficient distributions for loss-discourage and win-stopper type separately. As we see win-stopper types have positive coefficients (Figure 13b), which means that win in the last game increases the chance that the player will end a session. For loss-stopper types (Figure 13a), the effect is opposite, win in the last game decreases the hazard of ending a session.



(a) loss-stopper

(b) win-stopper

Figure 13: Individual coefficient distributions for each behavioral type