CESifo WORKING PAPERS

8608
2020

October 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp

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CESifo *Working Paper No. 8608*

Perceived Uncertainty Shocks, Excess Optimism-Pessimism, and Learning in the Business Cycle

Abstract

What are the effects of beliefs, sentiment, and uncertainty, over the business cycle? To answer this question, we develop a behavioral New Keynesian macroeconomic model, in which we relax the assumption of rational expectations. Agents are, instead, boundedly rational: they have a finite-planning horizon, and they learn about the economy over time. Moreover, we allow agents to have a potentially asymmetric loss function in forecasting, which creates a direct channel for expected variances to affect the economy. In forming expectations, agents may be subject to shifts in optimism and pessimism (sentiment) and their beliefs may be influenced by their perceptions about future uncertainty. We estimate the behavioral model using Bayesian methods and exploit a large number of subjective expectation series (both point and density forecasts) at different horizons from the Survey of Professional Forecasters. We find that sentiment shocks are the key source of business cycle fluctuations. Shifts in perceived uncertainty can also affect real activity and inflation through a confidence channel, as they play an important role in belief formation. Overall, the results shed light on the importance of behavioral forces over the business cycles, and on the contribution and interaction of first-moment - sentiment - shocks versus second-moment - perceived uncertainty - shocks.

JEL-Codes: C320, E320, E500, E520, E700.

Keywords: uncertainty shocks, sentiment, animal spirits, learning, behavioural New Keynesian model, sources of business cycle fluctuations, observed survey expectations, optimism and pessimism in business cycles, probability density forecasts.

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July 2020

1 Motivation

A growing literature in macroeconomics is making progress in modeling the often neglected impact of psychological factors on the real economy, and it suggests that exogenous variations in sentiment may play an important role toward generating business cycle fluctuations (see Angeletos et al. (2018), Benhabib et al. (2015), Milani (2011, 2017)). In these studies, however, expectations remain purely a function of first moments. On the other hand, another strand of the literature emphasizes the role of uncertainty and singles out shocks to second moments as important sources of business cycle fluctuations (see Bloom (2009), Basu and Bundick (2017), and Bloom et al. (2018)). These studies typically model uncertainty as the time-varying volatility of exogenous disturbances.

In this paper, we aim to investigate the importance of both first and second moment shocks in the context of an estimated microfounded model. Our approach to model sentiment follows Milani (2011, 2017), but here we allow for agents' optimism and pessimism regarding the direction of the economy at longer horizons to matter as well. We provide, instead, an alternative approach compared to the existing literature to study the role of uncertainty in guiding business cycles. We model uncertainty as the subjective expectation of future second moments that can be extracted from economic agents' probability density forecasts. We call this the measure of agents' *perceived uncertainty*. We use micro-level data on probability density forecasts for real GDP growth rate and inflation, to construct our time-varying measures of perceived uncertainty, which captures the uncertainty underlying forecasts made by participants of the Survey of Professional Forecasters. A point of distinction from existing studies, therefore, lies in the interpretation of uncertainty. Our view of uncertainty aligns to the confidence channel guiding expectations and sentiments.

Next, we develop a highly 'behavioral' New Keynesian macroeconomic model, in which we start by relaxing the assumption of rational expectations. Economic agents form expectations from a near-rational model with constant-gain learning. The conventional New Keynesian model is extended to allow for a potential impact of uncertainty shocks on the real economy and for shifts in sentiment, i.e. changes in aggregate optimism or pessimism in the formation of expectations that are unjustified based on observed fundamentals. Excess optimism or pessimism may refer to the direction of the economy in the very short-term (one-quarter-ahead), or for longer terms (one-year-ahead).

Traditionally in behavioral macro models, agents learn about the parameters governing the 'actual laws of motion (ALM)' of the economy by making forecasts, observing data, and updating beliefs. How do agents decide on what guides these expectations? Existing studies typically assume that agents minimize a symmetric loss function such that the expected value at any point of time is simply the conditional mean. Next, the agents forecast the expected value using the equations that describe the 'perceived law of motion (PLM)' of the economy. In this paper we make two changes to the standard framework that governs the learning mechanism. First, instead of assuming that agents minimize a symmetric loss function, we assume that the loss function leading to the formulation of optimal forecasts is potentially asymmetric. The motivation for using this asymmetric loss function is econometric as well as intuitive.

From an econometric perspective, if we assume that variables are conditionally normal, then Christoffersen and Diebold (1997) demonstrate that the optimal forecast consists of two components - one corresponding to the conditional mean and the other corresponding to the conditional variance. Moreover, a priori there is no reason to assume symmetry in the loss function that guides forecasts. By deviating from the benchmark and estimating the parameters that characterize the degree and extent of asymmetry we flexibly evaluate the importance of second-moments in expectation formation and pin down the importance of the 'confidence channel' towards guiding business cycle activity.

While this improved precision is desirable from an econometric perspective, the presence of the conditional variance in the forecast forges a direct channel for forecast-uncertainty to impact the dynamics of the model. Since the PLM is now a function of the conditional variances (and in our case perceived uncertainty), the ALM consequently depends explicitly on agents' forecasts for the second moments along with the usual dependence on the first moments. We are therefore able to study the effects of changes in uncertainty without considering a higher order solution to the equilibrium conditions of the model. By allowing uncertainty to enter through the expectations channel we can work with first order approximations of the equilibrium conditions describing the model economy. We estimate the model using data on expectations and measures of perceived uncertainty and allow for full flexibility on the values of the parameters guiding the degree and the extent of asymmetry.

The second deviation from the benchmark New Keynesian model with learning is to allow agents in the model to have a finite planning horizon. By incorporating a limited planning horizon (in place of the more common infinite planning horizon), we account for the fact that agents now need to consider expectations L periods ahead explicitly in their optimization decision. We implement this form of bounded rationality by using the finite horizon learning approach developed in Branch et al. (2012). Including a finite planning horizon allows us to quantitatively evaluate the relative importance of shorter- and longer-term sentiment (and uncertainty) shocks on macroeconomic variables.

The behavioral model is estimated using a Bayesian approach and exploiting data on realized macroeconomic variables (GDP growth, Inflation, Interest Rate), data on expectations for the same variables at multiple horizons from the Survey of Professional Forecasters, and data on measures of perceived uncertainty regarding output growth and inflation that we construct using forecaster-level probabilistic forecasts.

Results. The findings from the estimation provide evidence in favor of asymmetry in the loss-function, at least for inflation forecasts. This creates a direct channel for changes in perceived uncertainty to impact the laws of motion governing the evolution of macroeconomic variables. Furthermore, the estimated beliefs are also significantly affected by real-time perceptions about uncertainty: output expectations are negatively affected by subjective output uncertainty, while beliefs about the level of inflation positively comove with perceived inflation uncertainty.

On studying the impulse responses of macroeconomic variables to changes in year-ahead perceived uncertainty, we find that this channel can generate a significant impact. An unforeseen change in perceived uncertainty about longer-term output generates a decline in current GDP, inflation, and interest rate. In this way the impact of shocks to changes in the second moment guiding output expectations are comparable to the effects of aggregate uncertainty examined from a stochastic volatility point of view. The channel of transmission, however, relies on both the direct effect of a shock to confidence as well as on how beliefs endogenously respond to this change in confidence. A shock to longer-term perceived uncertainty about inflation, on the other hand, generates recessionary effects on GDP, and it leads to higher inflation and interest rates.

When evaluating the relative importance of shocks towards explaining business cycle fluctuations, we find that first-moment, sentiment, shocks are the dominant driver of fluctuations for both GDP growth and inflation. Short-term optimism and pessimism inject volatility in the economy, but longer-term sentiment, being highly persistent, is responsible for a large share of low frequency output fluctuations.

A novel result that stems from this framework is the interaction between shocks to first moments and shocks to second moments. By modeling beliefs to be a function of endogenous variables as well as structural shocks, our framework can generate endogenous changes in confidence about future outcomes. We demonstrate this by examining the effect of a shock to aggregate demand. A positive shock to aggregate demand endogenously changes the uncertainty underlying expected output in the longer-term. The effects on GDP, inflation, and interest rate, are similar to the effects of changes in upside uncertainty. A positive demand shock endogenously changes confidence about future outcomes as it generates dispersion in beliefs about future realization of GDP growth rate. This change in confidence manifests as an increase in the likelihood of positive tail-events in the model.

Related Literature. The paper is related to three major strands of studies. First, it is connected to the literature examining the effects of shocks to second moments in guiding business cycle fluctuations. Bloom (2009) makes a seminal contribution in studying the effects of uncertainty. Basu and Bundick (2017), and Bloom et al. (2018), among others, quantify the effects of uncertainty in the context of DSGE models, where the sources of uncertainty are derived from the time-varying volatility of exogenous processes. In our paper, we use the micro-level forecaster data to explicitly isolate the expected standard deviation associated with point forecasts, and analyze the effects of changes in this constructed measure of perceived uncertainty within a microfounded behavioral macroeconomic model of learning. Therefore, we offer an alternative perspective and an alternative empirical measure to study the role of uncertainty.

The paper adds to the expanding literature on bounded rationality and learning in macroeconomics (e.g., Evans and Honkapohja (2001), Sargent (1993)). It exploits a wide range of direct data on expectations to inform the estimation of the best-fitting learning process over the sample. Moreover, it shows that, in addition to the role of learning, there are two components of expectations - changes in the first moment/shocks to sentiment and changes in the second moment/shocks to perceived uncertainty, that are key to understand business cycles. In this way, the paper adds to the previous studies that document the empirical importance of learning in macroeconomic models, such as Milani (2007). The paper also contributes to the recent efforts to model 'sentiment' in general equilibrium models, either with learning or rational expectations (e.g., Milani (2017, 2011), Angeletos et al. (2018), Benhabib et al. (2015)).

Finally, from a technical perspective the paper adds to the literature dealing with the estimation of DSGE models using full information Bayesian techniques. Specifically, the paper relates to the studies using survey expectations across different forecast horizons to estimate microfounded models (e.g., Del Negro and Eusepi (2011), Ormeño and Molnár (2015), Milani and Rajbhandari (2012), Cole and Milani (2019)).

2 Calculating Measures of Perceived Uncertainty for Inflation and Output

One of the main motivations for this paper is to examine the impact of perceived uncertainty by agents on learning dynamics and its impact on macroeconomic aggregates of interest. Unlike most of the literature, which models uncertainty as the stochastic volatility of shocks, we extract perceptions of uncertainty from the Subjective Probability Density forecasts about inflation and the GDP growth rate that are available from the Survey of Professional Forecasters (SPF). Therefore, our perceived uncertainty measure is the economic agents' expectations about the future variance of macroeconomic variables.

Each quarter, individual forecasters assign probabilities over bins that summarize possible realizations of the underlying macroeconomic variable for the current, as well as the following, year. Therefore, the model equivalent measure of inflation and output uncertainty can be calculated by aggregating the information across forecasters and estimating the uncertainty associated with forecasts at each horizon.

It is important to note some key features of this forecaster level information. The number of bins that summarize the set of possible realizations for inflation and GDP outlook changes across different sub periods between 1968 and the present. The next-year forecasts are available for GDP growth rate and inflation from 1981 onwards. The surveys between 1968 Q4 and 1981 Q2 records projection for current year estimates only. Finally, the survey records the outlook for nominal GNP between 1968 Q4 and 1981 Q2 and real GNP in subsequent periods.[1] The inflation outlook is calculated by using the implicit deflator for nominal GDP.[2]

[1]We refer to variable PRGDP in the micro dataset provided in https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts

[2]We refer to variable PRPGDP in the micro dataset provided in https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts

There are many ways to carry out this aggregation of forecaster-level information to arrive at a measure of uncertainty in outlook that matches the specification described in the theoretical description. The simplest method is to begin by constructing nonparametric measures of uncertainty as follows:

$$\mu_{i,t+h|t} = \sum_{j=1}^{n} p_{i,,t+h|t}(u_{i,t+h|t} + l_{i,t+h|t})/2$$

where $\mu_{i,t+h|t}$ indicates the average outlook for a variable at time t and for horizon h for forecaster i. The terms $u_{i,t+h|t}$ and $l_{i,t+h|t}$ denote the upper and lower limits of the j^{th} interval, whereas n indicates the number of bins that define the set of expected outcomes. The perceived uncertainty by each forecaster i, for a variable at time t a for horizon h, is denoted by $\sigma_{i,t+h|t}$ and given by

$$\sigma_{i,t+h|t} = \sqrt{\sum_{j=1}^{n} p_{i,t+h|t}\Big[(u_{i,t+h|t} + l_{i,t+h|t})/2 - \mu_{i,t+h|t}\Big]^2 - w_t^2/12}$$

where w_t is the width of the central interval and the term $-w_t^2/12$ represents the Sheppard's correction for the second moment. The individual means across forecasters at each horizon h and each point of time t can be used to construct the aggregate nonparametric measure of average expectation of the relevant variable $\mu_{t+h|t}$ such that:

$$\mu_{t+h|t} = \sum_{i} \mu_{i,t+h|t}/N_t$$

where N_t denotes the total number of forecasters at each point of time t. Likewise, the aggregate measure of uncertainty can be obtained by averaging the standard deviations across forecasters at point of time t

$$\sigma_{t+h|t} = \sum_{i} \sigma_{i,t+h|t}/N_t.$$

While the nonparametric measures are easy to implement, calculating an aggregate measure of uncertainty by considering a simple average across the sample of forecasters at each point of time has drawbacks. This is because some survey respondents tend to round and concentrate their probabilistic responses by utilizing a small fraction of bins relative to the total number of bins, subsequently leading to errors in the aggregated measure. The errors are amplified when the bin widths are larger, and the forecast horizon is short. Given these tendencies, responses that fall in just a few bins introduce errors in the estimation especially when the uncertainty is relatively small. This could be mitigated by adopting a parametric approach towards estimating the aggregate level of uncertainty. We do so by following the approach proposed by D'Amico and Orphanides (2008) and D'Amico and Orphanides (2014). For each individual probabilistic response at time t and horizon h, we fit a normal CDF to the empirical CDFs by choosing the mean and variance such that it minimizes the distance between the empirical CDF and the probabilities implied by the normal CDF. That is,

$$X_{i,t+h|t} \sim N(\mu_{i,t+h|t}, \sigma^2_{i,t+h|t})$$
$$\hat{\mu}_{t+h|t} = \sum_i \mu_{i,t+h|t}/N_t \text{ and } \hat{\sigma}_{t+h|t} = \sum_i \sigma_{i,t+h|t}/N_t.$$

This approach, however, yields an accurate estimate only if the responses for each forecaster are spread across three bins or more. For responses that are concentrated in fewer than three bins, the normal distribution fails to provide an accurate measure of the uncertainty underlying the forecasters beliefs. This is mitigated by adopting the solution described in D'Amico and Orphanides (2008, 2014). Whenever the set of responses for any horizon at any time comprises of a forecaster with probabilistic responses concentrated in two or fewer bins, the relevant measure of uncertainty for that period and horizon is constructed by directly modeling the distribution of the individual uncertainties. This is carried out by treating the uncertainties of forecasters that assign a positive probability only to one or two bins as small, but unobserved. To demonstrate, suppose the individual uncertainty for horizon h at time t originates from a distribution (assumed Gamma) of individual responses $\nu_{i,t+h|t}$:

$$\nu_{i,t+h|t} \sim \Gamma(\nu, \theta_{t+h|t})$$

where $\nu_{i,t+h|t} = \sigma^2_{i,t+h|t}$. Specifically, given a threshold C, individual variances that exceed the threshold $\nu_{i,t+h|t}(c_T)$, for all $c_T > C$, where $c_1, c_2, ...c_{Nc}$ denotes the right end-points of intervals over which the range of uncertainty values have been discretized for each horizon, are considered. Given this characterization, denote $F(\nu_{i,t+h|t}(c_1)), ..F(\nu_{i,t+h|t}(c_T))$ the relevant empirical CDFs defined at these endpoints of the individual nonparametric variances that exceed a certain threshold. Therefore, we incorporate information about the mass of the CDF at the threshold and treat anything below as unobserved. Next, we estimate the parameters of the gamma distribution such that:

$$\hat{\theta}_{t+h|t} = \underset{\theta_{t+h|t}}{argmin} \sum_{\tau=1}^{Nc} [\Gamma((\nu_{i,t+h|t}(c_\tau)), \theta_{t+h|t}) - F(\nu_{i,t+h|t}(c_\tau))]^2$$

with $\theta_{t+h|t} = [\alpha_{t+h|t}, \beta_{t+h|t}]$, and where $\alpha_{t+h|t}$ and $\beta_{t+h|t}$ denote the shape and scale of the gamma distribution. The relevant measure of aggregate uncertainty for horizon h and time t is then calculated as follows:

$$\hat{\sigma}_{t+h|t} = (\hat{\beta}_{t+h|t}^{1/2}) \frac{\Gamma(\hat{\alpha}_{t+h|t} + 1/2)}{\Gamma(\hat{\alpha}_{t+h|t})}.$$

Given that the density forecasts are fixed-event, rather than fixed-horizon, forecasts, we can proceed in two ways. First, we can use all the forecasts at different horizons (a $t+4$ forecast is available in quarter 1, a $t+3$ forecast is available in quarter 2, and so forth) and use them in an estimation with mixed-frequency data and missing observations (where forecasts at each horizon are available only once per year). Second, we can follow D'Amico and Orphanides (2014) and others, in computing an approximate $t+4$ uncertainty series. In this way, the approximate year-ahead uncertainty is equal to the available $t+4$ forecast in Q1, $t+3$ forecast in Q2, $t+6$ forecast in Q3, and $t+5$ forecast in Q4. The slightly different horizons can still introduce seasonality in our series, hence, we seasonally adjust our perceived uncertainty measure before using it in the estimation. We have estimated the model using both the mixed-frequency and the approximate $t+4$ uncertainty series as observable, but we report the results for the latter approach as benchmark.

The different measures of aggregate perceived uncertainty for GDP growth rate and the GDP deflator are shown in Figure 1, along with the other observables that will be used in the Bayesian estimation. We next proceed to outline the micro foundations in which we examine the effects of shocks to sentiment and perceived uncertainty.

3 Behavioral Model

We assume a version of the New Keynesian model extended to include several behavioral features. First, we relax rational expectations and introduce learning with finite-planning horizons following the specification presented by Branch, Evans, and McGough (2012), and also used in Goy et al. (2020).[3] Second, we allow for sentiment shocks at different horizons; sentiment is modeled as in Milani (2011, 2017). Finally, we allow for perceived uncertainty to affect the economy through two channels: a structural channel, by assuming that forecasters adopt an asymmetric loss function, which makes forecasts depend optimally on higher moments, and an expectational channel, through which uncertainty affects the formation of agents' beliefs in real time.

The aggregate dynamics of our behavioral New Keynesian model is given by the following specification:

$$x_t = \tilde{E}_t x_{t+L} - \frac{1}{\sigma}\tilde{E}_t \sum_{j=0}^{L-1}[i_{t+j} - \pi_{t+j+1}] + b_t \tag{1}$$

$$\pi_t = \beta^L \tilde{E}_t \pi_{t+L} + \tilde{E}_t \sum_{j=0}^{L-1} \beta^j \kappa x_{t+j} + u_t \tag{2}$$

$$i_t = \rho i_{t-1} + \chi_\pi \pi_{t-1} + \chi_x x_{t-1} + \epsilon_t^{mp} \tag{3}$$

where x_t denotes the output gap, π_t denotes inflation, i_t denotes the nominal interest rate, and b_t , u_t, ϵ_t^{mp}, denote demand, supply, and monetary policy shocks, respectively.

Eq. (1) is the log-linearized Euler equation that arises from households' consumption decisions; the parameter σ denotes the elasticity of intertemporal substitution of consumption. Eq. (2) is the New Keynesian Phillips curve that arises from firms' profit maximization under Calvo price-setting; the parameter β denotes the household's discount factor and κ denotes the slope of the Phillips curve. The conventional Euler equation and the Phillips curve are iterated L periods ahead such that the model is consistent with the finite horizon learning environment described in Branch et al. (2012). Agents exhibit bounded rationality in the model: expectations up to L periods ahead matter for optimal decisions in the current period, i.e., agents hold a finite planning horizon. Such specification provides us with a microfounded framework that allows us to incorporate

[3]Finite planning horizons have been proposed recently in the literature also as a way to avoid puzzling results in the New Keynesian model, such as the 'forward guidance puzzle'. Woodford (2019) analyzes finite horizons in an alternative framework, which retains the assumption of rational expectations. The models used in Lustenhouwer (2020) and in Lustenhouwer and Mavromatis (2017) extend Woodford's finite-planning horizon approach to study the benefits of fiscal stimulus in a liquidity trap and the effects of fiscal consolidations, respectively. In their frameworks, agents plan ahead only L periods into the future. They make optimal decisions within their finite-horizon plan, while they are boundedly rational in their computation of continuation values for horizons at $L + 1$ and beyond.

both short-term and longer-term expectations in the estimation and to isolate the relative strengths of these two channels on business cycles. In the finite-horizon model, the output gap depends on L-period-ahead expectations of the same variable, and on current and future expected inflation and interest rates. Similarly, inflation depends on the expected inflation rate L periods into the future, as well as on demand conditions, both contemporaneously and in the next $L-1$ quarters. In the model, $\tilde{E}_t$ indicates subjective (possibly non-rational) expectations, the formation of which will be explained in the next section.

Monetary policy is described by Eq. (3), which is a Taylor rule that allows for an interest-rate smoothing term with coefficient ρ, and where χ_π and χ_x represent the feedback coefficients to inflation and the output gap.[4] The policy rule is "operational" in the sense of McCallum (1993), since it responds to lagged variables, which are observable by the policymaker in real time. Shocks to monetary policy are assumed to be *i.i.d.*, $\epsilon_t^{mp} \sim \text{iidN}(0, \sigma_{mp}^2)$. The demand and supply disturbances b_t and u_t evolve, instead, as AR(1) processes:

$$b_t = \rho_b b_{t-1} + \epsilon_t^b, \quad \epsilon_t^b \sim \text{ iidN}(0, \sigma_b^2) \tag{4}$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u, \quad \epsilon_t^u \sim \text{ iidN}(0, \sigma_u^2). \tag{5}$$

4 Expectation Formation and the Asymmetric Loss Function

Asymmetric loss function. Typically, in models where the assumption of rational expectations is relaxed, agents learn about the parameters describing the *Actual Law of Motion* (ALM) in the economy by formulating forecasts and updating their beliefs about parameters. A common assumption in forecasting is that agents use a symmetric, Mean Squared Error, loss function to calculate optimal forecasts.

$$L(x_{t+h} - \hat{x}_{t+h}) = (x_{t+h} - \hat{x}_{t+h})^2.$$

[4]We omit the term $(1-\rho)$ in front of inflation and output gap in the estimation.

A symmetric loss functions weighs positive and negative forecast errors equally and the optimal forecast obtained is simply the conditional mean.

$$\frac{\partial L(.)}{\partial \hat{x}_{t+h}} = 2(x_{t+h} - \hat{x}_{t+h}) = 0 \implies E_t(x_{t+h} - \hat{x}_{t+h}) = 0 \implies \hat{x}_{t+h} = E_t x_{t+h} \quad (6)$$

In this paper, we deviate from this approach by allowing agents to derive the optimal forecast using a possibly asymmetric loss function (the degree of asymmetry will be estimated along with the other parameters). The motivation for using an asymmetric loss function is twofold.

First, there is no reason, a priori, to believe that agents in the economy weigh positive and negative errors in the same way. A recent literature acknowledges the importance of allowing for asymmetric loss functions to evaluate forecasts. Elliott, Komunjer, and Timmermann (2008) suggest a flexible loss function and estimate the corresponding parameters using internal forecasts by the Federal Reserve. They find that the Fed's forecasts can be rationalized as optimal only under an asymmetric loss function. They find that the Fed considers negative forecast errors for output growth as substantially more costly than positive errors: overprediction of output, in fact, may induce the central bank to inaction and, hence, worsens the state of the economy.

Second, from an econometric perspective, Christoffersen and Diebold (1997) show that when the underlying variables are conditionally Gaussian, the optimal forecast will depend not only on the conditional expectation - the first moment, but, also, it will depend on the conditional variance - the second moment, characterizing the distribution underlying forecast errors. To accommodate for these features, we assume a more general loss function. We use a lin-ex (Linear Exponential) loss function to accommodate potential asymmetries in forming forecasts:

$$L(x_{t+h} - \hat{x}_{t+h}) = b\Big[\exp\{a(x_{t+h} - \hat{x}_{t+h})\} - a(x_{t+h} - \hat{x}_{t+h}) - 1\Big].$$

The optimal forecast under such a loss function (as shown below) will imply that the optimal forecast now depends not only on the conditional mean, but on the conditional variance as well:

$$\frac{\partial L(.)}{\partial \hat{x}_{t+h}} = bE_t\Big[\exp\{a(x_{t+h}-\hat{x}_{t+h})\}(-a)-1(-a)\Big] = 0 \implies E_t\Big[\exp\{a(x_{t+h}-\hat{x}_{t+h})\}-1\Big] = 0$$
$$\implies \exp a\hat{x}_{t+h} = E_t \exp a x_{t+h}$$

Using the property that $x_{t+h|t}$ follows a normal distribution, it can be shown that:

$$\hat{x}_{t+h} = \hat{E}_t x_{t+h} + \frac{a}{2}\hat{E}_t \sigma^2_{t+h}. \tag{7}$$

The optimal forecast is thus a linear function of the conditional mean as well as the conditional variance, with the importance of the variance term depending on the degree of asymmetry given by a. The optimal amount of bias in the forecast is a function of the degree of asymmetry, and depends on second-order terms with $a < 0$ implying $(x^{f,t}_{t+h} - x_{t+h}) > 0$ is more costly than $(x^{f,t}_{t+h} - x_{t+h}) < 0$ and $a > 0$ implying $(x^{f,t}_{t+h} - x_{t+h}) < 0$ is more costly than $(x^{f,t}_{t+h} - x_{t+h}) > 0$. For $a \to 0$ the system will revert to the case where the loss function weighs positive and negative forecast errors in a symmetric manner. By estimating the parameter a, which guides the direction and extent of asymmetry in forecasts, we potentially allow for the underlying loss function to be symmetric/asymmetric. The equilibrium conditions of the model (1)-(3), along with the evolution of exogenous shocks (4)-(5), are thus augmented with the expressions for optimal forecasts, at different horizons:

$$\tilde{E}_t x_{t+h} = \hat{E}_t x_{t+h|t} + \frac{a_x}{2}\hat{E}_t \sigma^2_{x_{t+h|t}} \tag{8}$$
$$\tilde{E}_t \pi_{t+h} = \hat{E}_t \pi_{t+h|t} + \frac{a_\pi}{2}\hat{E}_t \sigma^2_{\pi_{t+h|t}} \tag{9}$$

Now, short and longer-term expectations depend on agents' mean forecasts $\hat{E}_t x_{t+h|t}$ and $\hat{E}_t \pi_{t+h|t}$, which will be formed from the agents' near-rational learning model, but also on the expected variance of the same variables, i.e., their *ex-ante* perceived uncertainty.[5]

All that is left to specify is the expectation formation mechanism that guides how agents calculate $\hat{E}_t x_{t+h|t}$, $\hat{E}_t \pi_{t+h|t}$, $\hat{E}_t \sigma^2_{x_{t+h|t}}$ and $\hat{E}_t \sigma^2_{\pi_{t+h|t}}$.[6]

[5] The idea of an asymmetric loss function has also been used in Branch (2014) in the context of 'nowcasting' output and inflation in the Taylor rule.

[6] We don't include an asymmetric loss function related to forecasts about future interest rates, since

Expectations, Learning, and Uncertainty. Agents need to form expectations about future real activity, inflation, and nominal interest rates, at different horizons. Under near-rational expectations, they ignore the reduced-form coefficients in the model solution (for example, by not knowing structural parameters as the degree of price rigidity, or policy coefficients, they cannot successfully recover the rational expectations solution). Therefore, they don't know the magnitudes of key relationships, such as the sensitivity of output to policy rates, the sensitivity of inflation to demand conditions, the persistence of variables, and so forth. They learn them using observed historical data. They form expectations from a perceived model, the Perceived Law of Motion (PLM), which typically includes the same state variables that would appear in the solution of the system under rational expectations (here augmented to allow agents to take their perceived uncertainty into account). The PLM for the macro variables $Y_{1,t} = [x_t, \pi_t, i_t]$ is given by

$$Y_{1,t} = \begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix} = \underset{3\times1}{A_1} + \underset{3\times3}{B_1} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \underset{3\times2}{C_1} \begin{bmatrix} b_{t-1} \\ u_{t-1} \end{bmatrix} + \underset{3\times2}{D_1} \begin{bmatrix} \widehat{E}_t \sigma^2_{x_{t+h}} \\ \widehat{E}_t \sigma^2_{\pi_{t+h}} \end{bmatrix} + \epsilon_{1,t} \tag{10}$$

Therefore, agents learn about the steady-state of endogenous variables (through A_1), the dynamic relationships among variables (in B_1); they are allowed to respond to past disturbances, and they may be influenced by their subjective perceptions about uncertainty.

Moreover, agents need to learn about the dynamics of uncertainty as well, to form their expectations that enter expressions (8)-(9). The PLM for their estimated uncertainty $Y_{2,t} = [\widehat{\sigma}^2_{x_{t+h}}, \widehat{\sigma}^2_{\pi_{t+h}}]$ is given by

$$Y_{2,t} = \begin{bmatrix} \widehat{\sigma}^2_{x_{t+h}} \\ \widehat{\sigma}^2_{\pi_{t+h}} \end{bmatrix} = \underset{2\times1}{A_2} + \underset{2\times3}{B_2} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \underset{2\times2}{C_{2,t}} \begin{bmatrix} b_{t-1} \\ u_{t-1} \end{bmatrix} + \epsilon_{2,t} \tag{11}$$

Therefore, agents learn about uncertainty in real-time based on realized macroeconomic conditions and on recent disturbances. In this way, and unlike a large part of the literature on

probability density forecasts were not available from the SPF for this variable.

uncertainty in macroeconomics, we allow for perceived uncertainty to be partially endogenous, by responding to past economic developments.

The specification for the PLMs described above takes into account the fact that $[\widehat{E}_t\sigma^2_{x_{t+h}}, \widehat{E}_t\sigma^2_{\pi_{t+h}}]$ are computed based on information up to $t-1$. Therefore, agents in the model mimic this feature of real world forecasters, who have access to real-time $t-1$ data when forming their expectations in t. However, changes in measures of perceived uncertainty can have real effects on the current state of the macroeconomy. The structure of the PLM is such that agents are allowed to dynamically use the information across different horizons to update their beliefs. An interesting feature of the PLM is that it creates a direct link between first and second moments in the model. So essentially, changes in sentiments or structural shocks can affect uncertainty, and uncertainty can affect expectations.

Agents update their beliefs, collected in $\hat{\phi}_t = [A_t, B_t, C_t, D_t]$, with $A_t = [A_{1,t}; A_{2,t}]$, $B_t = [B_{1,t}; B_{2,t}]$, $C_t = [C_{1,t}; C_{2,t}]$, $D_t = [D_{1,t}; 0]$, as follows

$$\hat{\phi}_t = \hat{\phi}_{t-1} + \overline{\mathbf{g}} R_t^{-1} X_t (Y_t - \hat{\phi}'_{t-1} X_t)'$$
$$R_t = R_{t-1} + \overline{\mathbf{g}}(X_t X'_t - R_{t-1})$$

where $Y_t = [Y_{1,t}; Y_{2,t}]$, and X_t collects all the regressors in (10) and (11). The parameter $\overline{\mathbf{g}}$ is the constant gain learning parameter that describes the extent to which agents incorporate new information in updating beliefs. In the estimation, we allow $\overline{\mathbf{g}} = [\overline{g}_{mean}, \overline{g}_{unc}]'$, therefore agents may have separate constant gain parameters for updating beliefs about first and second-moments respectively. The second equation represents, instead, the updating of the precision matrix R_t, corresponding to the beliefs $\hat{\phi}_t$. Given the setup of the PLM, L-step ahead expectations evolve as follows:

$$\widehat{E}_t \begin{bmatrix} x_{t+L} \\ \pi_{t+L} \\ i_{t+L} \end{bmatrix} = \hat{A}_{1,t-1} + \hat{B}_{1,t-1} \widehat{E}_t \begin{bmatrix} x_{t+L-1} \\ \pi_{t+L-1} \\ i_{t+L-1} \end{bmatrix} + \hat{C}_{1,t-1} \begin{bmatrix} \rho_b^{L+1} & 0 \\ 0 & \rho_u^{L+1} \end{bmatrix} \begin{bmatrix} b_{t-1} \\ u_{t-1} \end{bmatrix} + \hat{D}_{1,t-1} \begin{bmatrix} \widehat{E}_t \sigma^2_{x_{t+L+h}} \\ \widehat{E}_t \sigma^2_{\pi_{t+L+h}} \end{bmatrix} + \begin{bmatrix} e_t^{x,L} \\ e_t^{\pi,L} \\ e_t^{i,L} \end{bmatrix} \tag{12}$$

and

$$\widehat{E}_t \begin{bmatrix} \sigma^2_{x_{t+L}} \\ \sigma^2_{\pi_{t+L}} \end{bmatrix} = \hat{A}_{2,t-1} + \hat{B}_{2,t-1} Y_{1,t-1} + \hat{C}_{2,t-1} \begin{bmatrix} \rho_b^{L+1} & 0 \\ 0 & \rho_u^{L+1} \end{bmatrix} \begin{bmatrix} b_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_t^{x,L} \\ \mu_t^{\pi,L} \end{bmatrix}, \tag{13}$$

and similarly for different horizons.[7] Data on observed expectations are used to pin down expectations on the left-hand side. Such expectations are assumed to be formed from the near-rational learning model described above. Agents use their updated beliefs and most recent macroeconomic realizations to obtain a point forecast.

There are, however, two deviations from this typical formulation. First, agents recognize that the volatility of fluctuations may be varying over time. This can occur for different reasons: first, the volatilities of the exogenous disturbances are changing over time in a form that is unknown to them. This means that some of the structural coefficients may change - different monetary policies may lead to different levels of volatility in the economy. Finally, learning dynamics may imply endogenous changes in volatility in the economy, which are not obvious to recognize by economic agents that learn using past realized data. Therefore, agents form beliefs about uncertainty and recognize that uncertainty can interact with other variables in the economy. We estimate the model using data on expectations across horizons along with the data on measures of perceived uncertainty that we construct to estimate the parameters guiding the shocks to sentiments - both the first and second moments, to understand the relative contribution of each channel.

As a second deviation from standard learning frameworks that are used in theoretical studies (e.g., Evans and Honkapohja (2001)), agents' expectations may deviate from the point forecasts that arise from the learning model. They do so because of excesses of unjustified optimism or pessimism, as modeled in Milani (2011) and Milani (2017). These waves of optimism and pessimism, or sentiment, shocks are captured by the terms $e_t^{x,h}$, $e_t^{\pi,h}$, $e_t^{i,h}$, and may refer to output, inflation, or interest rate expectations, and to each horizon h, $h = 1, ..., 4$. The same reasoning is used to decompose observed uncertainty into two parts: an endogenous reaction to the state of the economy and an exogenous perceived uncertainty shock, denoted by μ_t^x for

[7] In (12), even if uncertainty at $t+L+h$ enters the expression, we use the expected uncertainty in $t+L$, with $L = 4$, in the estimation, since we only use the approximate year-ahead uncertainty as observable.

output gap uncertainty and μ_t^π for inflation uncertainty.

Combining the ALM and the expectations channel. We substitute the perceived laws of motion that describe expectation formation in equations (1)-(3) and obtain the Actual Laws of Motion for the model economy:

$$\xi_t = A_0(\widehat{\phi}_{t-1}, \theta) + F(\widehat{\phi}_{t-1}, \theta)\xi_{t-1} + G\omega_t \tag{14}$$

where ξ_t is the vector of state variables and θ is the vector of parameters. The vector ω_t summarizes the set of disturbances included in the model. The model includes a diverse menu of disturbances. In this way, we remain agnostic on the main sources of fluctuations in the economy and let the data select the main drivers of business cycles. The set includes fundamental disturbances (shocks to preferences b_t, cost-push shocks u_t, and shocks to monetary policy ϵ_t^{mp}), first-moment expectational disturbances to aggregate optimism/pessimism, and second-moment (perceived) uncertainty disturbances. We include shocks to sentiment and perceived uncertainty across different horizons ranging from the immediate short run: $t+1$ to the relatively longer-term horizon: $t+4$. Shocks to sentiment capture waves of optimism/pessimism, while the confidence channel of uncertainty is captured through the effect of shocks to perceived uncertainty on beliefs.

5 Bayesian Estimation

We estimate the behavioral model using a full-information Bayesian approach. The model is estimated to match the following set of observables. Three realized variables: Real GDP growth rate, Inflation, short-term nominal interest rate (3-month Treasury bill rate).[8] We use real-time second-vintage releases for these variables, to match the information set available by economic agents in real time. We then use twelve observed survey expectation series (mean across forecasters): Expected growth rates of real GDP at horizons from $t+1$ to $t+4$, expected inflation from $t+1$ to $t+4$, and expected nominal interest rates (same definition as the realized

[8] We use the 3-month rate, because SPF forecasts are available for this variable, but not for the Federal Funds rate.

variable, 3-month Treasury bill rate) from $t+1$ to $t+4$. Finally, we add two perceived uncertainty series: approximate $t+4$ uncertainty (expected variance) for output growth and approximate $t+4$ uncertainty for inflation.[9] We use the series computed using D'Amico and Orphanides' approach; the correlation with the alternative non-parametric series are 0.99 and 0.98 and, hence, all results remain identical. Given our available uncertainty data, we use an horizon $L = 4$ in the model. The data on expectations for the first moments are readily available from the Survey of Professional Forecasters. For the data on measures of perceived uncertainty, we use the series that were calculated as described in Section 2, starting from probability density forecasts made available through the SPF. We estimate the model based on data from 1981:I to 2008 IV.[10] All variables are at quarterly frequency. All observable data are shown in Figure 1.

We summarize the information on prior distributions along with the posterior estimate results in Tables 1, 2, and 3. We assume Gamma prior distributions for the parameters denoting the sensitivity of output to interest rates (σ) and of inflation to output (κ). We also use Gamma distributions for the monetary policy response coefficients to output gap and inflation. We center the priors for the asymmetry parameters in the forecasting loss function to zero, and assume a Normal distribution. The gain coefficients follow a Beta prior with mean 0.05 and standard deviation 0.01. All autoregressive parameters for the disturbances follow a Beta(0.5,0.1) prior, while the standard deviation parameters follow an Inverse Gamma distribution with mean equal to 0.5.

We use a block Metropolis-Hastings algorithm to generate draws from the posterior distribution. We divide the parameters in three blocks, to improve the mixing properties of our chain: a block with structural parameters, a block with AR parameters for the disturbances, and a block with the standard deviations of the shocks.[11]

[9] We have also estimated the model using $t+1$ to $t+4$ data for uncertainty and adding them to a state-space model with mixed-frequency observables and missing data. We report results for $t+4$ uncertainty only as our benchmark, since we found uncertainty at shorter horizons to be relatively unimportant.

[10] We stop the sample before the zero-lower-bound period.

[11] We have experimented with different blocks, for example separating sentiment and uncertainty parameters, as well as increasing the number of blocks.

6 Results

Posterior Estimates. Tables 1, 2, and 3 report the posterior estimates. We estimate posterior means equal to 0.118 for σ and to 0.015 κ. The inclusion of various expectations at different horizons in the Euler equation and in the Phillips curve, under finite-horizon planning, likely works to attenuate the elasticities of output and inflation with respect to their driving variables. The monetary policy parameters are estimated in conventional ranges: $\rho = 0.931$, $\chi_\pi = 0.13$ (implying that χ_π in a typical $(1-\rho)\chi_\pi$ expression would equal 1.88), and $\chi_x = 0.03$ (implying $(1-\rho)^{-1}\chi_x = 0.43$).

The constant-gain parameter is estimated at 0.016 for learning about macroeconomic variables; its value is not far from the estimate in Milani (2007). Agents adopt a lower gain when learning about economic uncertainty (with the gain estimated at 0.01).

The evidence related to asymmetry in expectation formation is mixed. The results suggest that asymmetry is not a robust feature for GDP growth, as the posterior mean for a_x is close to 0. There is, instead, evidence for asymmetry in the forecasting loss function for inflation. We estimate a posterior mean equal to -0.082 for a_π; moreover, the 68% and 90% credible sets fall entirely below zero.

The negative value for a_π implies an explicit role for the changes in the uncertainty-confidence channel to impact expectations through the actual laws of motion in the economy. Had we estimated both values of a_x and a_π equal to zero, we would have reverted to the standard finite-horizon learning framework where optimal forecasts arise from minimizing a symmetric loss function.

Shocks to sentiment and uncertainty. The next feature we want to point out is the differences in the persistence of shocks to the first and second moments guiding expectations.

Disturbances accounting for waves of optimism and pessimism are very persistent. In particular, optimism/pessimism about the longer term direction of the economy (at a one-year-ahead horizon, for example), introduce significant low frequency movement into the macroeconomy. Sentiment about one quarter-ahead output is less persistent ($\rho_{x_1} = 0.72$), but more volatile ($\sigma^{x_1} = 0.689$ versus $\sigma^{x_4} = 0.167$ for the $t+4$ shock).

Perceived uncertainty disturbances display lower persistence than sentiment, with autore-

Parameters	Prior Distribution	Posterior Mean	Credible Set (90%)	Credible Set (68%)
σ	$\Gamma(0.1, 0.075)$	0.118	[0.024,0.208]	[0.045,0.184]
$\overline{\gamma}$	$\Gamma(0.63, 0.05)$	0.698	[0.682,0.713]	[0.689,0.708]
κ	$B(0.025, 0.01)$	0.015	[0.011,0.019]	[0.012,0.017]
ρ	$B(0.6, 0.05)$	0.931	[0.842,0.996]	[0.876,0.999]
χ_π	$\Gamma(1, 0.5)$	0.130	[0.052,0.227]	[0.078,0.192]
χ_x	$\Gamma(0.25, 0.1)$	0.030	[0.017,0.045]	[0.022,0.039]
g_{mean}	$B(0.05, 0.01)$	0.016	[0.013,0.019]	[0.014,0.018]
$g_{uncertainty}$	$B(0.05, 0.01)$	0.010	[0.008,0.011]	[0.009,0.011]
a_x	$N(0, 0.1)$	0.008	[-0.153,0.167]	[-0.096,0.102]
a_π	$N(0, 0.1)$	-0.082	[-0.157,-0.012]	[-0.123,-0.041]

Table 1: Reporting the estimates from an estimation using 500,000 draws. A burn-in of 40% has been used.

gressive coefficients ranging from 0.17 to 0.38. They are, however, more volatile than most sentiment shocks, with standard deviations estimated at 0.23 and 0.29.

Parameters	Prior Distribution	Posterior Mean	Posterior Distribution (90%)	Posterior Distribution (68%)
ρ_b	$B(0.5, 0.1)$	0.847	[0.693,0.987]	[0.767,0.926]
ρ_u	$B(0.5, 0.1)$	0.152	[0.012,0.352]	[0.05,0.259]
ρ_{x_1}	$B(0.5, 0.1)$	0.785	[0.675,0.879]	[0.722,0.837]
ρ_{x_2}	$B(0.5, 0.1)$	0.844	[0.736,0.953]	[0.776,0.917]
ρ_{x_3}	$B(0.5, 0.1)$	0.912	[0.761,1]	[0.85,0.998]
ρ_{x_4}	$B(0.5, 0.1)$	0.999	[0.999,0.999]	[0.999,0.999]
ρ_{π_1}	$B(0.5, 0.1)$	0.721	[0.571,0.855]	[0.616,0.834]
ρ_{π_2}	$B(0.5, 0.1)$	0.815	[0.708,0.946]	[0.746,0.873]
ρ_{π_3}	$B(0.5, 0.1)$	0.733	[0.622,0.867]	[0.654,0.804]
ρ_{π_4}	$B(0.5, 0.1)$	0.758	[0.668,0.848]	[0.699,0.806]
ρ_{i_1}	$B(0.5, 0.1)$	0.314	[0.08,0.504]	[0.154,0.445]
ρ_{i_2}	$B(0.5, 0.1)$	0.579	[0.452,0.67]	[0.516,0.641]
ρ_{i_3}	$B(0.5, 0.1)$	0.871	[0.773,0.957]	[0.817,0.924]
$\rho_{\sigma_4^x}$	$B(0.5, 0.1)$	0.171	[0.003,0.351]	[0.07,0.288]
$\rho_{\sigma_4^\pi}$	$B(0.5, 0.1)$	0.380	[0.237,0.505]	[0.286,0.476]

Table 2: Reporting estimated values of the autoregressive coefficients

Short and Longer-Term Sentiment. This paper departs from the previous literature by allowing for sentiment to affect different variables and at different horizons from $t+1$ to $t+4$.

Shocks to optimism/pessimism have a large effect on the economy. Figures 2 and 3 show the

Parameters	Prior Distribution	Posterior Mean	Posterior Distribution (90%)	Posterior Distribution (68%)
σ^b	$\Gamma^{-1}(0.5,1)$	0.490	[0.434,0.546]	[0.454,0.522]
σ^u	$\Gamma^{-1}(0.5,1)$	0.285	[0.256,0.32]	[0.266,0.304]
σ^i	$\Gamma^{-1}(0.5,1)$	0.330	[0.289,0.381]	[0.306,0.357]
σ^{x_1}	$\Gamma^{-1}(0.5,1)$	0.689	[0.615,0.779]	[0.643,0.743]
σ^{x_2}	$\Gamma^{-1}(0.5,1)$	0.273	[0.246,0.303]	[0.255,0.291]
σ^{x_3}	$\Gamma^{-1}(0.5,1)$	0.210	[0.186,0.238]	[0.194,0.223]
σ^{x_4}	$\Gamma^{-1}(0.5,1)$	0.167	[0.148,0.188]	[0.156,0.180]
σ^{π_1}	$\Gamma^{-1}(0.5,1)$	0.146	[0.129,0.164]	[0.135,0.157]
σ^{π_2}	$\Gamma^{-1}(0.5,1)$	0.087	[0.076,0.098]	[0.08,0.093]
σ^{π_3}	$\Gamma^{-1}(0.5,1)$	0.095	[0.085,0.107]	[0.089,0.102]
σ^{π_4}	$\Gamma^{-1}(0.5,1)$	0.084	[0.074,0.093]	[0.078,0.090]
σ^{i_1}	$\Gamma^{-1}(0.5,1)$	0.143	[0.128,0.161]	[0.132,0.153]
σ^{i_2}	$\Gamma^{-1}(0.5,1)$	0.052	[0.046,0.058]	[0.048,0.055]
σ^{i_3}	$\Gamma^{-1}(0.5,1)$	0.030	[0.027,0.034]	[0.028,0.032]
σ^{i_4}	$\Gamma^{-1}(0.5,1)$	0.036	[0.031,0.041]	[0.033,0.038]
$\sigma^{\sigma^{x}_4}$	$\Gamma^{-1}(0.5,1)$	0.287	[0.256,0.321]	[0.267,0.306]
$\sigma^{\sigma^{\pi}_4}$	$\Gamma^{-1}(0.5,1)$	0.230	[0.203,0.255]	[0.215,0.245]

Table 3: Reporting estimated values of the standard deviations of shocks

impulse responses of output gap and inflation to short-term and medium-term sentiment, along with their responses to fundamental demand and supply shocks. The output gap responds in similar ways to a preference and a quarter-ahead optimism shock. These two shocks are the main sources of fluctuations in the very short run. The longer-term optimism shock has lower volatility, but it induces low frequency movement in output, which permeates medium-term business cycles. Structural cost-push shocks dominates quarter-to-quarter fluctuations in inflation; their effects are strongest on impact, but they dissipate rather quickly. Again, fluctuations in the medium term are driven by the state of longer-term (inflationary in this case) sentiment.

Transmission of Shocks to Uncertainty. We present the responses of endogenous variables to changes in perceived uncertainty about longer run GDP growth rate and inflation in Figures 4 and Figure 5 respectively. Shocks to perceived uncertainty about four-quarter-ahead GDP growth rate are recessionary triggering a decline in the output gap. Inflation is not very responsive to changes in confidence about four-quarter-ahead GDP growth rate. Shocks to perceived uncertainty about four-quarter-ahead inflation uncertainty, on the other hand, raise inflation as well as generate a slow down in activity. To understand the role of the different behavioral elements in the model, we analyse the channels of transmission.

An unforeseen change in the confidence (uncertainty) governing the forecasts will have a direct effect on the equilibrium conditions through the parameters governing the extent of asymmetry - a_x and a_π. If we had estimated the loss function independent of beliefs in the model, the effects of perceived uncertainty on forecasts would be captured in entirety through the parameters a_x and a_π respectively. However, the perceived law of motion is specified such that beliefs about future output growth, inflation and the nominal interest rate are contemporaneously affected by the shocks/changes to perceived uncertainty. To isolate the indirect effect operating through beliefs, we revisit the PLM. For the sake of exposition we focus on beliefs about $t+1|t$.

$$\hat{E}_t \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ \sigma^2_{x,t+4} \\ \sigma^2_{\pi,t+4} \end{bmatrix} = \begin{bmatrix} \phi^{1,1}_{t+1|t} \\ \phi^{2,1}_{t+1|t} \\ \phi^{3,1}_{t+1|t} \\ \phi^{4,1}_{t+1|t} \\ \phi^{5,1}_{t+1|t} \end{bmatrix} + \begin{bmatrix} \phi^{1,2}_{t+1|t} & \phi^{1,3}_{t+1|t}\phi^{1,4}_{t+1|t} & \phi^{1,5}_{t+1|t} & \phi^{1,6}_{t+1|t} & \phi^{1,7}_{t+1|t} & \phi^{1,8}_{t+1|t} \\ \phi^{2,2}_{t+1|t} & \phi^{2,3}_{t+1|t}\phi^{2,4}_{t+1|t} & \phi^{2,5}_{t+1|t} & \phi^{2,6}_{t+1|t} & \phi^{2,7}_{t+1|t} & \phi^{2,8}_{t+1|t} \\ \phi^{3,2}_{t+1|t} & \phi^{3,3}_{t+1|t}\phi^{3,4}_{t+1|t} & \phi^{2,5}_{t+1|t} & \phi^{3,6}_{t+1|t} & \phi^{3,7}_{t+1|t} & \phi^{3,8}_{t+1|t} \\ \phi^{4,2}_{t+1|t} & \phi^{4,3}_{t+1|t}\phi^{4,4}_{t+1|t} & \phi^{2,5}_{t+1|t} & \phi^{4,6}_{t+1|t} & 0 & 0 \\ \phi^{5,2}_{t+1|t} & \phi^{5,3}_{t+1|t}\phi^{5,4}_{t+1|t} & \phi^{5,5}_{t+1|t} & \phi^{5,6}_{t+1|t} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ \rho_b b_{t-1} \\ \rho_u u_{t-1} \\ \sigma^2_{x,t+4} \\ \sigma^2_{\pi,t4} \end{bmatrix} + \begin{bmatrix} \epsilon^x_{1,t+1} \\ \epsilon^\pi_{1,t+1} \\ \epsilon^{int}_{1,t+1} \\ \mu^x_{2,t+4} \\ \mu^\pi_{2,t+4} \end{bmatrix}$$

The estimated values of $\phi^{1,7}_{t+1|t}, \phi^{1,8}_{t+1|t}, \phi^{2,7}_{t+1|t}, \phi^{2,8}_{t+1|t}$ capture the responsiveness of beliefs to changes in confidence about GDP growth rate and inflation. The total effect of a shock to perceived uncertainty will transmit both through the parameters guiding the extent of asymmetry as well as through the beliefs. For output gap, the effect of an unforeseen change in $\mu^x_{2,t+h}$ can be decomposed as follows:

$$x_{t+\hat{h}|t} = E_t x_{t+h|t} + \frac{a_x}{2} E_t {\sigma^x_{t+h|t}}^2$$

$$x_{t+\hat{h}|t} = \phi^{1,7}_{t+h|t} E_t {\sigma^x_{t+h|t}}^2 + \frac{a_x}{2} E_t {\sigma^x_{t+h|t}}^2 = [\phi^{1,7}_{t+h|t} + \frac{a_x}{2}] E_t {\sigma^x_{t+h|t}}^2$$

Likewise for inflation the effect of an unforeseen change in $\mu^{\pi}_{2,t+h}$ can be decomposed as follows:

$$\hat{\pi_{t+h|t}} = E_t\pi_{t+h|t} + \frac{a_\pi}{2}E_t{\sigma^{\pi}_{t+h|t}}^2$$

$$\hat{\pi_{t+h|t}} = \phi^{2,8}_{t+h|t}E_t{\sigma^{\pi}_{t+h|t}}^2 + \frac{a_\pi}{2}E_t{\sigma^{\pi}_{t+h|t}}^2 = [\phi^{2,8}_{t+h|t} + \frac{a_\pi}{2}]E_t{\sigma^{\pi}_{t+h|t}}^2.$$

The term $\phi^{1,7}_{t+h|t}$ captures the effects perceived uncertainty about output on expected output gap at horizon h and $\phi^{1,8}_{t+h|t}$ captures the effects perceived uncertainty about inflation on expected output at horizon h. Likewise, $\phi^{2,7}_{t+h|t}$ captures the effects perceived uncertainty about GDP growth rate on expected inflation at horizon h and $\phi^{2,8}_{t+h|t}$ captures the effects perceived uncertainty about inflation on expected inflation at horizon h. We report the estimated beliefs in Table 4.

Table 4: Estimated beliefs: Average - [90% Credible Interval].

$\phi^{1,7}_{t+4\|t}$	-0.1489	[-0.1590 , -0.1373]
$\phi^{2,7}_{t+4\|t}$	0.0039	[0.0027 , 0.0053]
$\phi^{2,8}_{t+4\|t}$	0.1243	[0.1192 , 0.1298]
$\phi^{1,8}_{t+4\|t}$	-0.0289	[-0.0357 , -0.0215]
a_x	0.0077	[-0.1534 , 0.1672]
a_π	-0.0820	[-0.1567 , -0.0119]

The relative sizes of $a_x/2$ ($a_\pi/2$) and $\phi^{1,7}_{t+h|t}$ ($\phi^{2,8}_{t+h|t}$) from Table 4 show that indirect effect dominates in understanding the real effects of changes in perceived uncertainty. What this means is a change in second moment transmits itself endogenously through a change in expectations or the first moment. Thus, forging a direct link between the level confidence and the resulting sentiment in the economy. This result is similar to the literature studying the effects of aggregate uncertainty whereby the effects are magnified when interacted with a change in the first moment (see Bloom (2009)).

This interaction can be seen when we examine the impulse responses of output gap, inflation and the nominal interest rate to a shock to perceived uncertainty about GDP growth and inflation at a horizon of t+4 (Figure 4 and Figure 5).

For output gap, an increase in longer-term perceived uncertainty leads to a downward revision of beliefs (see $\phi^{1,7}_{t+4|t}$ and $\phi^{2,7}_{t+4|t}$ in Table 4). The direct effect of a_x is weakly positive (average effect on impact 0.004); however, after considering the impact on beliefs, the net-effect of changes in perceived uncertainty generates recessionary effects with decline in GDP growth rate, inflation and nominal interest rate. For inflation, the direct and the indirect effects work in the opposite direction (Figure 5). The direct effect is captured by a negative a_π and a positive indirect effect through the beliefs (see $\phi^{2,7}_{t+4|t}$ and $\phi^{2,8}_{t+4|t}$ in Table 4).

To quantify the effect of beliefs in the transmission of shocks to the confidence channel governing expectations, we carry out a counterfactual exercise. We do this by calculating impulse responses of GDP growth rate, inflation and the nominal interest rate, by shutting down the indirect channel operating through beliefs. That is we set $\phi^{1,7}_{t+4|t}, \phi^{1,8}_{t+4|t}, \phi^{2,7}_{t+4|t}, \phi^{2,8}_{t+4|t}, \phi^{3,7}_{t+4|t}, \phi^{3,8}_{t+4|t}$ to zero. As seen from Figure 6, beliefs play a quantitatively important role in transmitting changes in the confidence channel to macro variables of interest.

Interaction between first and second moments. One of the novel features of the framework is that changes in the first moment can endogenously trigger changes the second-moment through the restrictions imposed by the perceived laws of motion in the economy. In Figure 7 we demonstrate the effects of a 1% shock to aggregate demand. A positive shock to demand operates through the PLM and generates a positive change in the perceived standard deviation of output growth at a horizon of 4 quarters.

Rossi and Sekhposyan (2015) differentiate between shocks to upside and downside uncertainty and find that dispersion in the right tail leads to an increase in economic activity. Our results suggest the same, but with a mechanism working through the PLM. Although we do not distinguish between upside (confidence channel reflecting the possibility of higher GDP growth rate in the near future) and downside uncertainty (confidence channel reflecting the possibility of lower GDP growth rate in the near future), our results suggest that positive shocks to demand lead to an increase in upside uncertainty. Similar to Rossi and Sekhposyan (2015) this is accompanied by positive changes in GDP growth rate.

Psychological Sources of Business Cycles. To understand the relative importance of different shocks in explaining business cycle fluctuations, we carry out a forecast error variance decomposition exercise at horizons of 1, 4, and 40 quarters. Shocks to sentiment are the dominant predictor of business cycles across horizons, explaining more than half of fluctuations in GDP growth rate. Within the sentiment component, the $t+1$ optimism/pessimism shock is predominant at short horizons, and the $t+4$ output sentiment becomes more important at longer horizons.

Shocks to uncertainty on their own explain a relatively small part. However, an important point to note is that expectations now endogenously depend on changes in perceived uncertainty. Therefore, the endogenous effects of changes of the second moment may manifest through this channel. For inflation, the contribution of sentiment is even larger at the year horizon, while

Horizon	Fundamentals	Sentiment	Uncertainty
GDP growth rate			
1	40.2	59.7	0.097
4	46.5	53.5	0.001
40	36.8	63.2	0.038
Inflation			
1	90.1	9.6	0.2392
4	21.4	78.6	0.0089
40	55.9	43.8	0.33

Table 5: Here shocks to fundamentals represent the total effect of shocks to aggregate demand (b_t), cost-push shocks (u_t) and shocks to monetary policy (ϵ_t^i). Shocks to sentiment represent the total effect of shocks to $E_t x_{t+1}, , ..., E_t x_{t+4}$, $E_t \pi_{t+1}, , ..., E_t \pi_{t+4}$ and , $E_t i_{t+1}, , ..., E_t i_{t+4}$. Shocks to perceived uncertainty represent the total effect of shocks to ${\sigma^x_{t+4}}^2$ and and $E_t {\sigma^\pi_{t+4}}^2$ respectively.

cost-push shocks explain most of the high-frequency dynamics within one quarter. Sentiment explains between 40% and 70% of inflation variability, for horizons above a year. The direct effects of shocks to perceived uncertainty for inflation are comparable to what we find for GDP growth rate.

7 Conclusion

In this paper we introduce a novel channel that guides how agents form expectations for key macroeconomic variables. We use forecaster-level information to construct measures of aggregate uncertainty as perceived by economic agents in real time. We subsequently estimate a modified behavioral new Keynesian model, which deviates from the assumption of rational expectations. In this environment, agents are boundedly rational and have finite-horizon plans;

they form expectations for the quarters, up to a year, ahead, and their optimal forecasts may be derived using a potentially asymmetric loss-function. We estimate the model using full information Bayesian techniques. We find evidence favoring an asymmetric loss function for inflation, but not for output. Furthermore, the estimates for the perceived law of motion suggests significant effects of changes in uncertainty on the forecasts for GDP growth, inflation and the nominal interest rates.

The empirical results highlight the role of shocks to sentiment, i.e., exogenous shifts in excess optimism and pessimism, both at short and longer horizons, as major drivers of business cycles. Our results also stress the importance of feedbacks between first and second moments for understanding fluctuations.

We recognize that our results are obtained in a specific model with finite-horizon learning modeled as in Branch et al. (2012) and Goy et al. (2020). In this framework, longer-term expectations may play a larger role than one-period-ahead expectations. We leave for future research to investigate the robustness of our findings to alternative settings. For example, the effect of sentiment and uncertainty shocks can be investigated using the finite-planning framework proposed by Woodford (2019), and, particularly, following the approach in Lustenhouwer (2020) and Lustenhouwer and Mavromatis (2017), which maintain the L-period-ahead specification, rather than rewriting it in a recursive formulation where only $(t+1)$-forecasts matter. Agents in the model would actively consider their budget constraint in their planning decisions and learn about their end-of-horizon wealth in a boundedly rational way.

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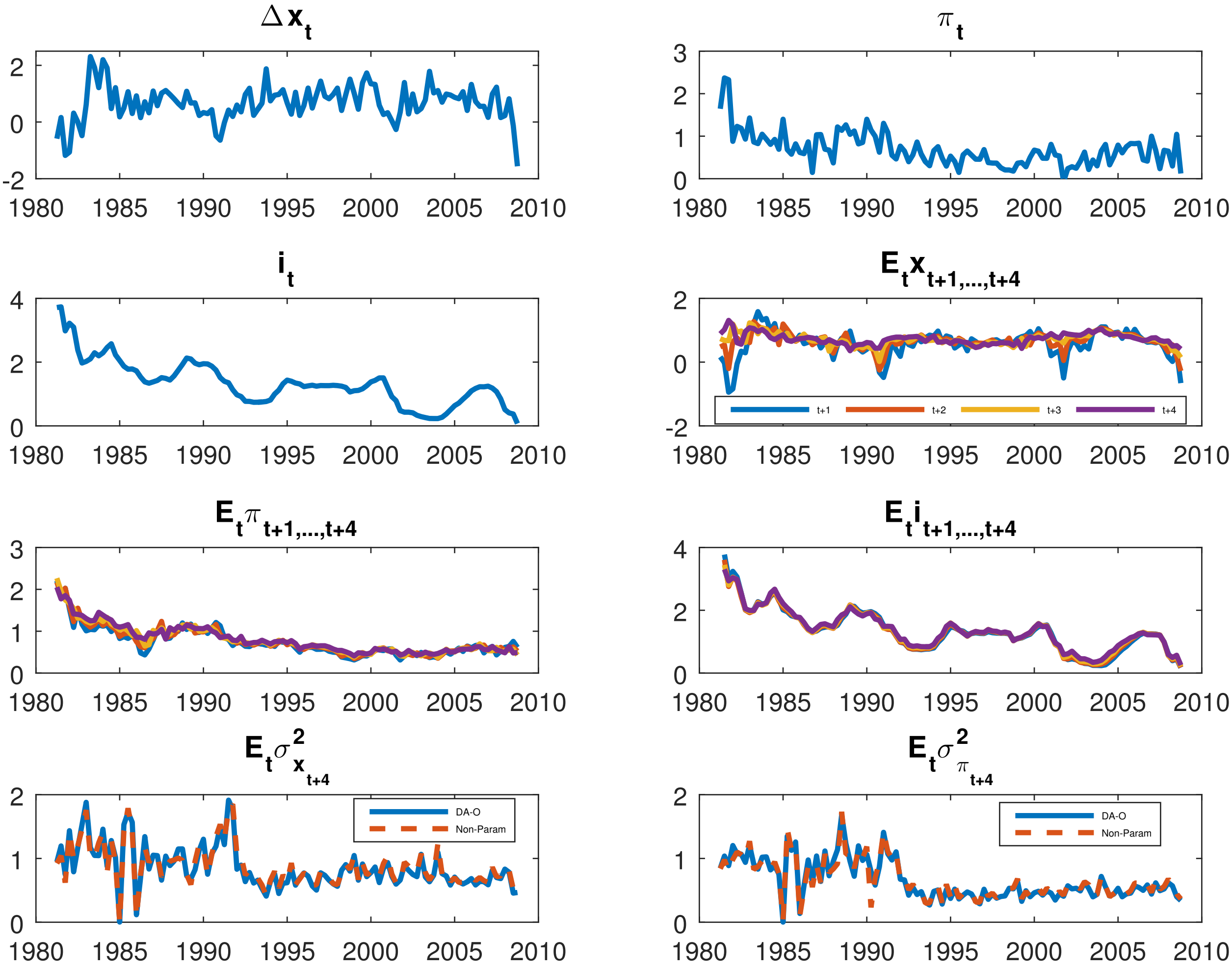


Figure 1: Data Series used as observable in the estimation.

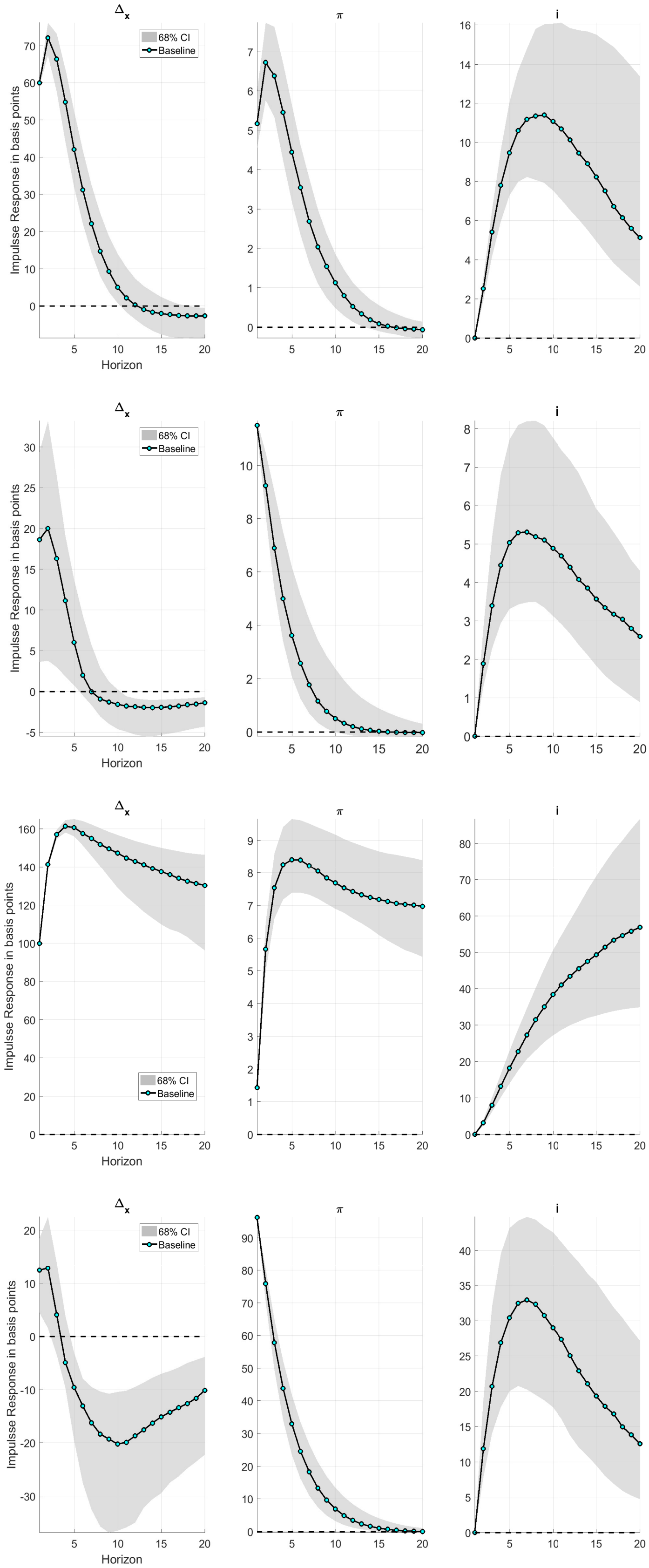


Figure 2: Impulse responses of Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) 1% shock to output sentiment (optimism) at $t+1$ (row 1), 1% shock to inflation sentiment (optimism) at $t+1$ (row 2), 1% shock to output sentiment (optimism) at $t+4$ (row 3) and 1% shock to inflation sentiment (optimism) at $t+4$ (row 4). Shaded areas denote 68% credible interval.

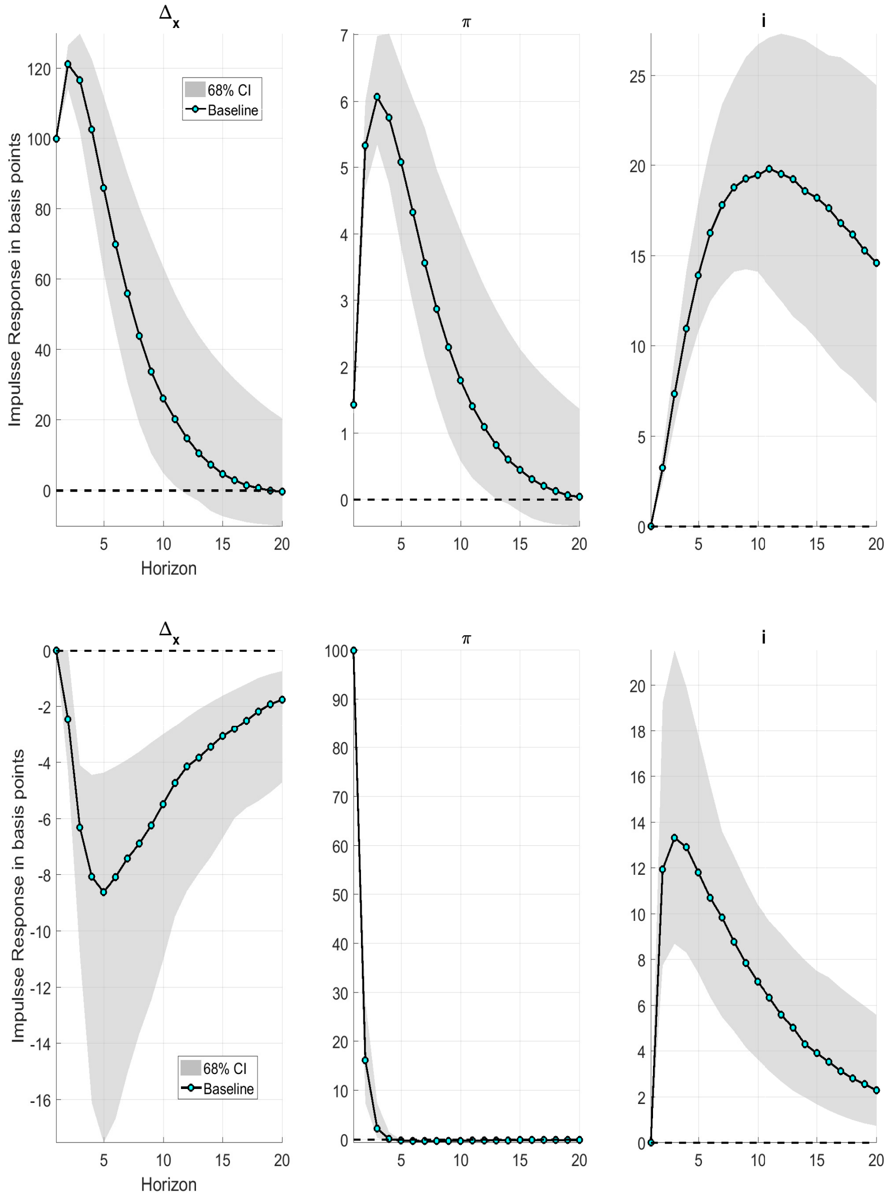


Figure 3: Impulse Responses of Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) to 1% demand (preference) shock (row 1), 1% cost-push shock (row 2). Shaded areas denote 68% credible interval.

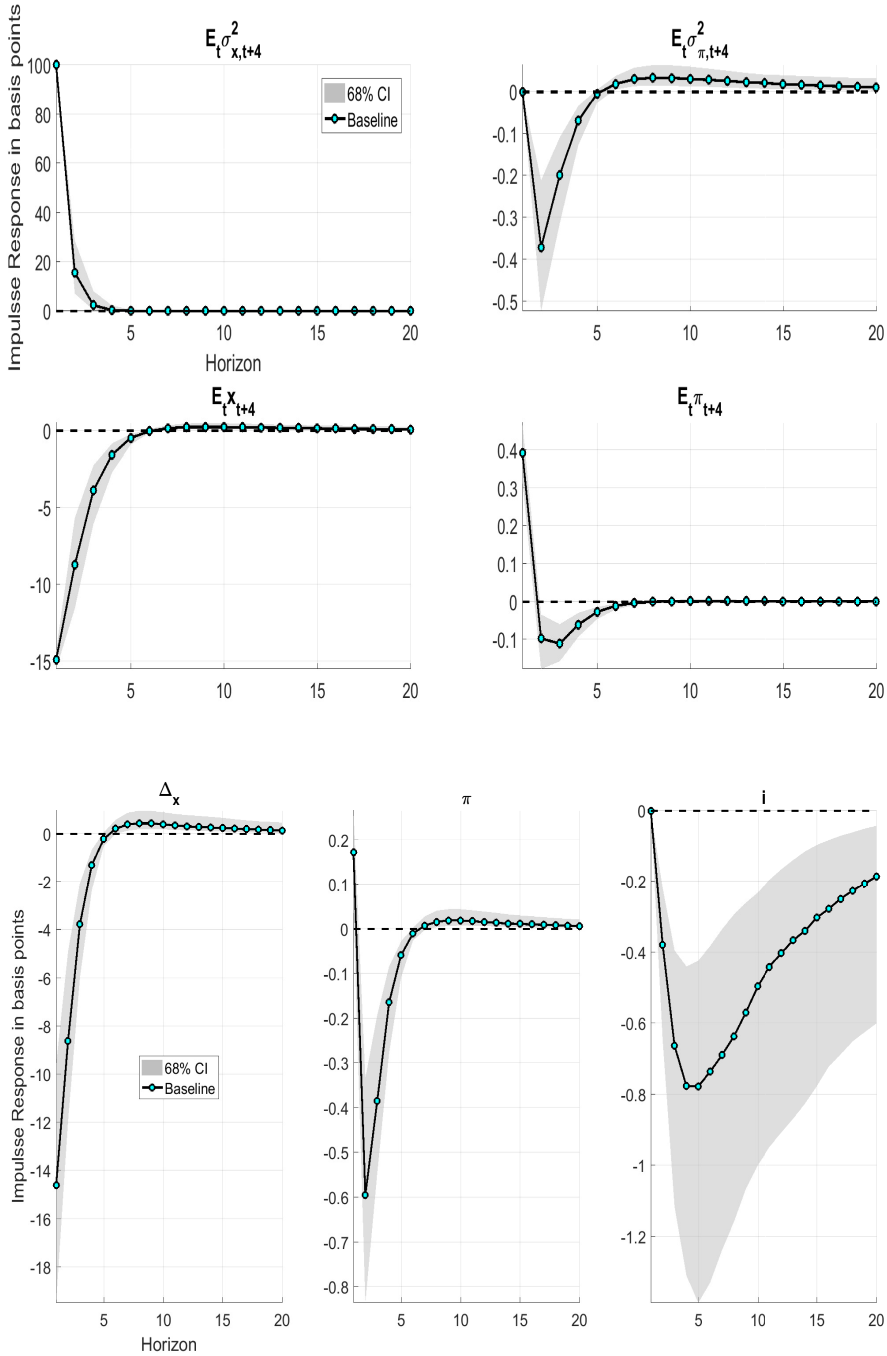
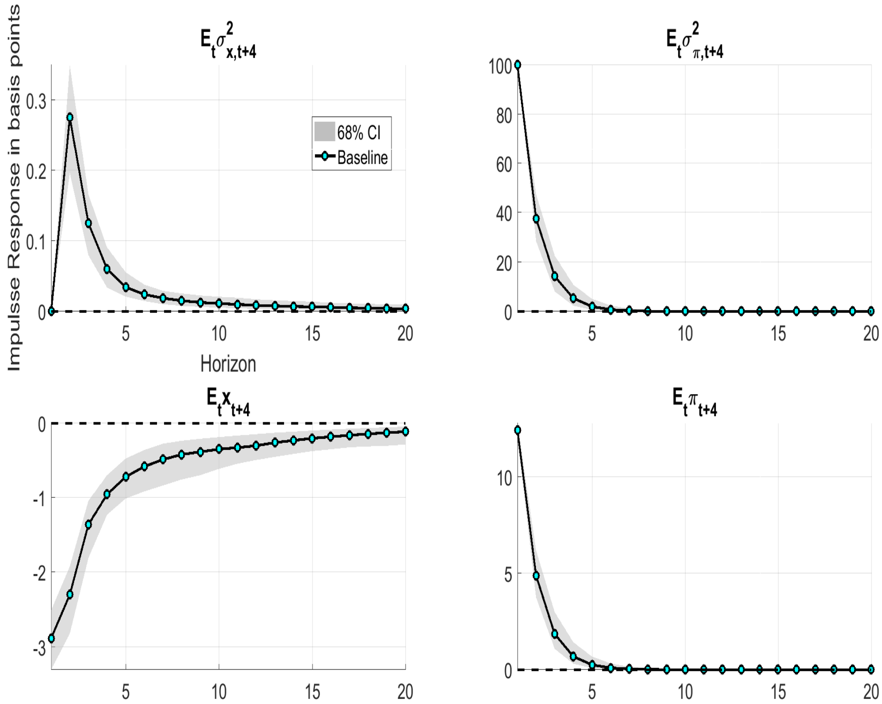


Figure 4: Impulse Responses of Perceived Uncertainty about x_{t+4}, π_{t+4}, Expectations of x_{t+4}, π_{t+4}, Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) for a 1% shock to perceived uncertainty about x_{t+4}. Shaded area denotes 68 % credible interval.



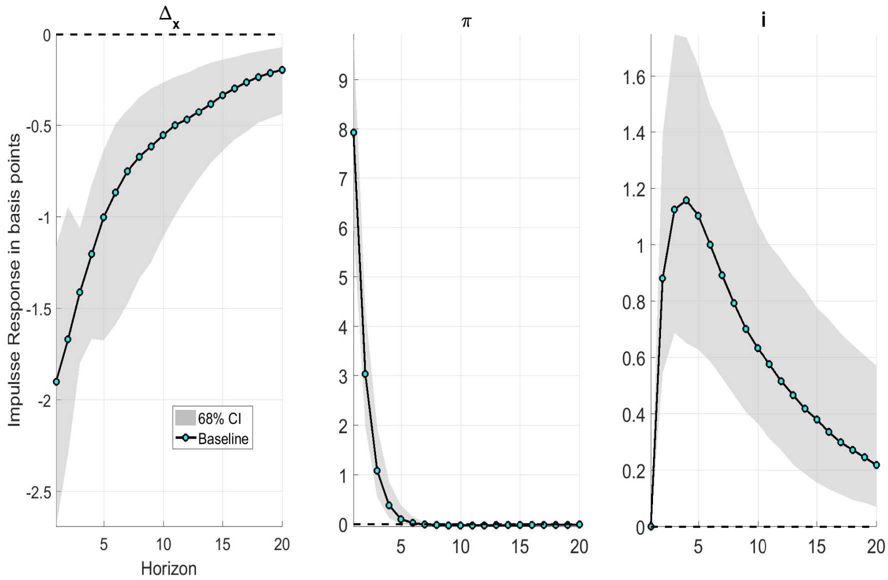


Figure 5: Impulse Responses of Perceived Uncertainty about x_{t+4}, π_{t+4}, Expectations of x_{t+4}, π_{t+4}, Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) for a 1% shock to perceived uncertainty about π_{t+4}. Shaded area denotes 68 % credible interval.

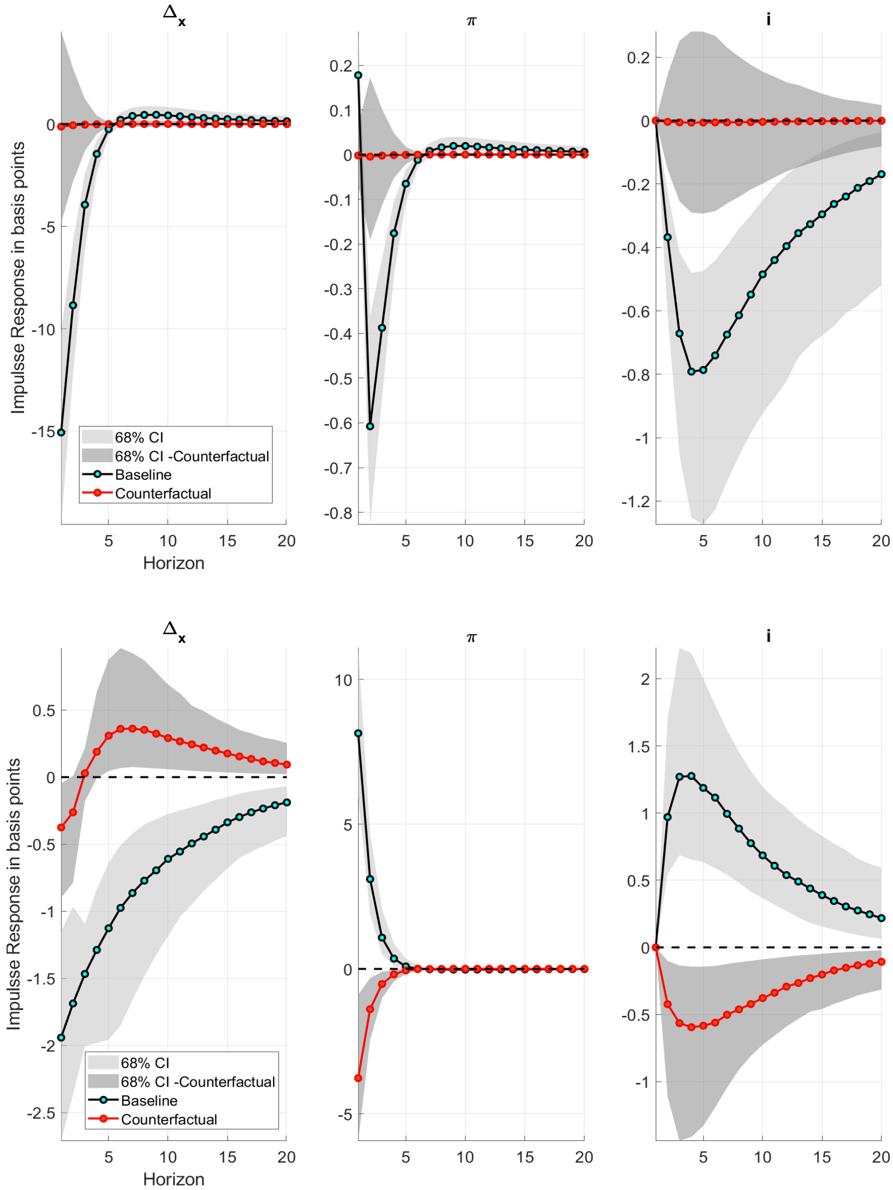


Figure 6: Counterfactual analysis - Row 1 - Black line with circles- Impulse Responses of Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) to perceived uncertainty about x_{t+4}. Row 1 - Crossed Red line - impulse responses calculated by setting beliefs to 0. Shaded area denotes 68 % credible interval.
Row 2 - Black line with circles- Impulse Responses of Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) to perceived uncertainty about π_{t+4}. Row 1 - Crossed Red line - impulse responses calculated by setting beliefs to 0. Shaded area denotes 68 % credible interval.

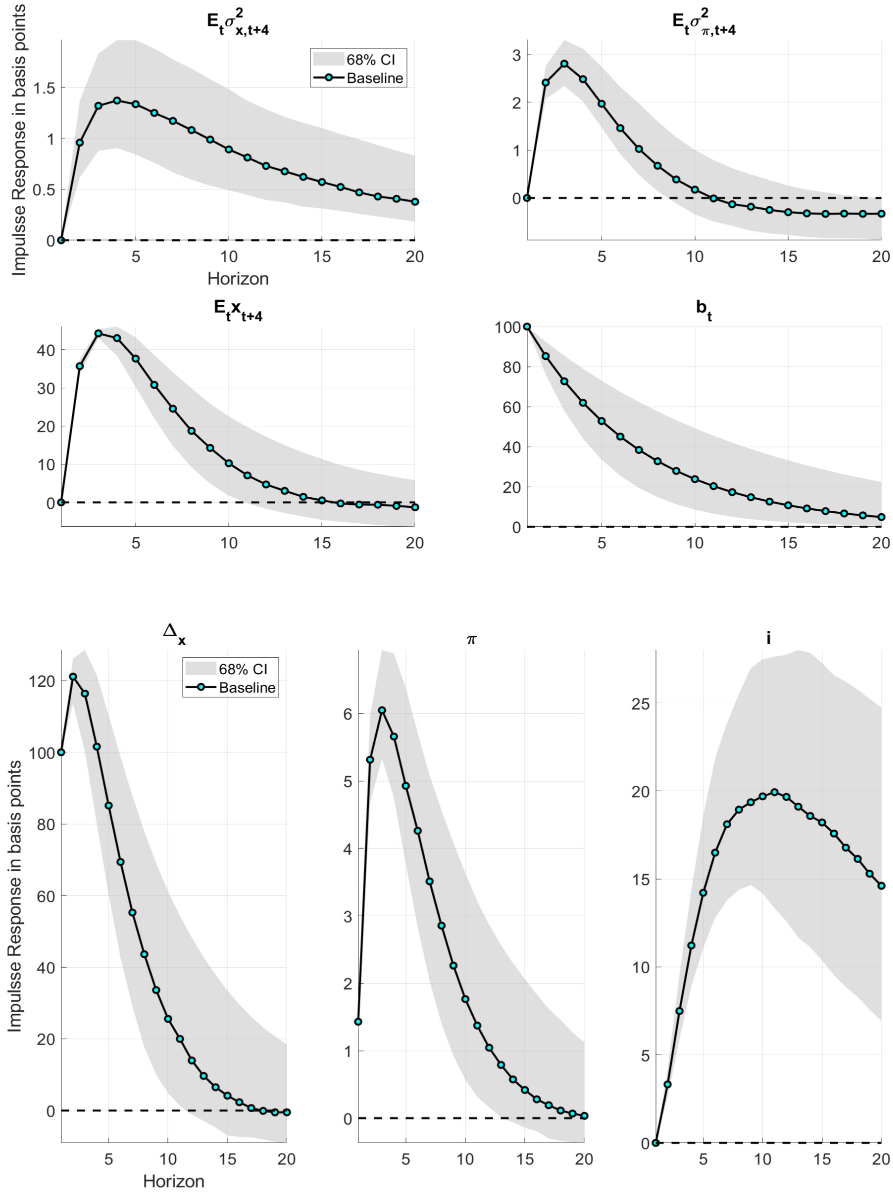


Figure 7: Impulse Responses of Perceived Uncertainty about x_{t+4}, π_{t+4}, Expectations of x_{t+4}, π_{t+4}, Output Gap (Δ_x), Inflation (π) and the nominal interest rate (i) for a 1% shock to demand (preference) b_t. Shaded area denotes 68 % credible interval.