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On the Provision of Excludable Public Goods General Taxes or User Prices?

(An extended version)

Abstract

This paper compares some of the most common and debated ways of financing the provision of impure public goods/services in a unified dynamic general equilibrium framework. We study and rank a wide variety of ways ranging from provision without any user charges, to provision with full user charges; we also study mixed systems. In our quest for the best system, we address both efficiency and distribution issues. The focus is on the effects of user prices on individual incentives and choices. We identify circumstances under which a market system of user prices not only enhances aggregate efficiency but is also Pareto improving meaning that makes all types of income groups better off. But we also provide examples of circumstances under which user prices on impure public goods do not make sense.

JEL-Codes: H400, H200, D600.

Keywords: user prices, taxes, efficiency, equity.

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1 Introduction

Economic crises bring into the spotlight the need to reform the public sector. In addition to the obvious anxiety, which is public debt sustainability, there is the classic issue of how to improve the provision of public goods/services without increasing the social burden for this provision and, if possible, without worsening income inequality. Here, we compare some of the most common and debated ways of financing the provision of impure public goods/services in a unified dynamic general equilibrium framework.

We focus on impure public goods and services simply because their provision is more debateable politically. In modern democracies, national defense and police are usually treated as pure public goods; as such, they are typically produced by the state and are offered uniformly and free of charge (free of charge means their cost is paid by the general taxpayer). However, most publicly provided goods and services are actually impure or quasi-public or even private, which means that they are excludable and/or congestable.[1] Tertiary education and several medical services are the most common examples, but the list can also include child care, elderly care and other family services, as well as motorways, public libraries, museums and sporting facilities, policing of football matches, as well as certain aspects of urban infrastructure.[2] Most of these goods bear the characteristics of private goods, but, nevertheless, they are usually provided free of charge or at low prices, usually on distribution grounds. Interestingly, although there has always been a debate about the advantages and disadvantages of the various ways of their financing in policy circles (free of charge, full user charges, or a mixed system?), there has not been - as far as we know - a ranking of those ways in a unified micro-founded dynamic general equilibrium setup (for a review of the literature, see at the end of this Introduction).

In this paper, we evaluate the aggregate and distributional implications of introducing user prices for excludable publicly provided goods into a dynamic general equilibrium model consisting of heterogeneous social groups. Our focus will be on the role of individual incentives. We will address both aggregate and distributional issues. By aggregate, we mean per capita output and welfare, whereas, by distribution, we mean differences in income and welfare between private sector agents and public sector employees. Distributional implications, and a potential conflict of interests, are at the heart of the debate on the reform of the public sector. Our aim is to give answers to questions like "What is the best way of financing impure public goods?", "Is it good to have user charges and, if yes, up to what degree?", "Who wins and who loses from the introduction of user charges?", "When do user prices make no sense?"

Our vehicle of analysis will be a neoclassical growth model augmented by three distinct types of households (labeled as capitalists, private sector workers and public sector employees),[3] two distinct types of firms (private firms produc-

[1] We will use the terms "impure public goods", "quasi-public goods" or "publicly provided private goods" interchangeably.

[2] According to the ECB, pure public goods include the COFOG categories "defense" and "public order safety" only, which amount to less than 5% of GDP in most countries (see ECB, Monthly Bulletin, 2009, April). The so-called merit goods, namely goods that are underestimated in value by individuals, and club goods, namely goods whose consumption is excludable, are also examples of impure public goods (see e.g. Cullis and Jones, 1998, chapter 3).

[3] The distinction between capitalists and workers has been a popular theme of agent het-

ing a private good and state firms producing an impure public good/service) and a relatively rich menu of fiscal policy instruments (consisting of taxes, government bonds and user charges on the revenue side, and various categories of public spending on the expenditure side including the cost of producing the impure public good/service). We will assume that this publicly provided good/service plays a utility-enhancing role (but we will also report results for the case in which these goods/services play a productivity-enhancing role, where the latter means that they can augment the productivity of inputs used by private firms and/or function as complements to households' work time).

Before we report our results, we need to clarify how the impure public good/service is financed and how state firms make their production decisions in our model. Regarding financing, we start with two polar systems: we compare the case in which the impure public good is provided uniformly and free of user charges to all agents (i.e. it is financed by general taxes) to the case in which individuals are free to choose the amount they wish by paying a price (typically called a user price) where this price is determined by a market-based mechanism. In other words, in the latter case, private agents' optimization problem gives, among other things, individuals' demand for this good as a negative function of the user price and, then, this user price equates demand (i.e. the sum of voluntary individual demands) to supply (i.e. the quantity produced by state firms). But, we will also report results for mixed public financing systems, which combine policy-based and market-based mechanisms, as well results for other pricing systems like average-cost pricing according to which state firms relate the user price to their average cost, or marginal-cost pricing according to which optimizing state firms equate their marginal cost to the user price. Regarding production, we will assume that the inputs used by state firms or, strictly speaking, their spending on these inputs (like goods purchased from the private sector, public investment and the public wage bill, all as shares of GDP) are exogenously set as in the eurozone data. But, we will also report results for more sophisticated cases in which state firms act optimally by choosing their inputs so as to maximize their discounted sum of net cash flows or to minimize their costs given an output target. In all experiments, the benchmark will be the one in which state firms do not act optimally and do not charge user prices either (this is what we shall label the status quo).

Our first result is about aggregate efficiency. The introduction of user prices for the excludable publicly provided good, other things equal, leads to a more efficient macro economy: both per capita net income and welfare rise. The intuition is as follows. The introduction of user charges for this good, and the creation of a new market for it, forces individuals to realize that, in order to afford the provision of publicly provided goods, they need higher income and hence they need to work harder and/or save more. In other words, with user prices and a new market, the cost of publicly provided goods/services can be internalized at personal, as opposed to social level, and this strengthens the individual incentives to work and/or save.[4] It should be stressed that this is the opposite from an increase in income taxes, which typically leads to less

erogeneity in the literature on fiscal, and in particulat optimal tax, policy. To this, we add the group of public employees, who play a key role in public sector reforms and, perhaps more importantly, produce the publicly provided goods.

[4] This is consistent with Newbery (1990) who argues for "the possibility of addressing the problem of market incompleteness directly ... by creating new markets".

hours of work and/or lower savings. But there can also be a second-round (or double-dividend) effect that reinforces the first one. The switch to a more efficient aggregate economy, with higher output and larger tax bases, allows the government to reduce distorting taxes. In out experiments, if the government takes advantage of the fiscal space created by the introduction of user charges to cut, say, labor tax rates, this further improves individual incentives leading to further gains in aggregate efficiency. Note that these results hold both when we compare steady state solutions without, and with, user prices, as well as along the transition from an initial solution without to a solution with user charges over time.

Second, regarding individual outcomes and hence distribution, all types of agents (capitalists, workers and public employees) can benefit over time from the introduction of user charges other things equal. That is, user prices are Pareto efficient intertemporally. Specifically, when we compare steady state solutions, all agents gain from such an introduction, especially in terms of net incomes. On the other hand, when we study transition results, some agents may suffer a temporary reduction in their net income on impact, although, as the economy travels towards its new steady state with user prices, the lifetime gains more than outweigh the initial losses, making eventually everybody better off intertemporally.

Third, the above results remain robust to an extensive sensitivity analysis including, among others, changes in the values of parameters and policy instruments, different functional forms for the utility and production functions, as well as additional roles for the publicly provided good such as productivity-enhancing services as discussed above. The above results also hold when we assume average-cost or marginal-cost pricing on the part of state firms again as discussed above. Naturally, when state firms act optimally (which is the case under marginal-cost pricing), the aggregate gains become bigger although this comes at the cost of public employees.

Fourth, we also study some cases in which a system of user prices is not beneficial or simply collapses. These cases include, for instance, the case in which the good in question exhibits strong social externalities, the case in which the government adopts a mixed system that combines user prices with subsidization of user costs and finally the case in which there is a minimum amount of the good that everybody needs. In the first case, strong enough social externalities lead to standard free-riding problems and the market mechanism does not make sense similarly to the polar case of pure public goods. In the second case, when individuals realize that the government will cover a large enough part of their user charges, they demand excessive quantities of it. In the third case, after a critical size of the minimum amount required, there are affordability issues that violate the optimality conditions of individuals starting with the relatively poor ones. By the way, we believe that this third case can mimic a situation in which the good is a necessity featuring indivisibilities which reminds us of the need for special publicly provided medical services in the ongoing pandemic crisis.

The rest of the paper is organized as follows. After a note on the related literature, section 2 presents the baseline model. Section 3 adds user prices to the baseline model of section 2. Section 4 compares the economy without user prices to the same economy with user prices and presents robustness checks. Section 5 focus on cases that provide counter-arguments for the use of user prices. Section 6 closes the paper. Algebraic details are in an Appendix.

How this paper differs As is well recognized, the word "provision" of public goods/services needs clarification. Publicly provided goods/services can be classified according to their degree of publicness, their way of financing and the identity of their producer (see e.g. the textbooks of Atkinson and Stiglitz (1980, chapters 15 and 16) and Cullis and Jones (1998, chapters 3 and 5), as well as the review paper by Sørensen (2016)). In this paper, we focus on the degree of publicness and the way of financing; we thus leave production issues like privatization and public sector outsourcing aside. As said in the opening paragraphs above, regarding the degree of publicness they possess, public goods/services are distinguished between pure and impure,[5] while, regarding their way of financing, impure public goods/services can be provided free of charge, or with user charges, or with a mix of general taxes and user charges.[6]

There has been a rich literature on user charges in general. Papers include Fraser (1996) who focuses on the provision of public goods under different public financing schemes including user fees; Ott and Turnovsky (2006) who use a representative agent general equilibrium model with endogenous tax bases and focus on the implementation of the first-best; Swope and Janeba (2006) who analyze how populations with different preferences choose different public financing schemes; Fuest and Kolmar (2007) who focus on the use of user fees under cross-border externalities; Blomquist et al. (2010) who study optimal tax-transfer schemes with, and without, user charges; Ellingsen and Paltseva (2012) who focus on the possible inefficiencies of Lindahl prices; etc.

Within this literature, special attention has been given to tuition fees. Papers include Hanushek et al. (2003) who focus on the role of schooling and transfers in an economy with subsidized tuition fees; Hanushek and Yilmaz (2007) who investigate the interaction between household location and the financing and quality of schools; Hanushek et al. (2014) who use a general equilibrium model, in which individuals differ in family wealth and opportunities of completing college, to provide a comparison among tuition subsidies, need-based student aid, merit-based student aid and income-contingent loans; Economides et al. (2017) who provide an example in which the introduction of tuition fees for public education services can be both efficient and equitable; Yilmaz (2018) who analyzes, among other things, the impact of basic education subsidies and college subsidies on welfare, inequality and intergenerational mobility; etc.

To the best of our knowledge, our paper differs from the literature in that it evaluates the aggregate, as well as the distributional, implications of introducing user prices for excludable publicly provided goods/services within a unified dynamic general equilibrium model with heterogeneous income groups one of which is public employees used for the production of publicly provided goods/services. Using this vehicle, we identify circumstances under which a market system of user prices works or does not work. The focus of our analysis is on how user prices shape individual incentives and, in turn, how these incentives affect both the macroeconomy and income distribution.

[5] See e.g. Cullis and Jones (1998, chapter 12), Hillman (2009, chapter 3), Blomquist et al. (2010) and Picot et al. (2015)).

[6] See e.g. Cullis and Jones (1998, Table 5.2) and Picot et al. (2015, Figures 1.1 and 1.2)).

2 Free of charge uniform provision

In this section, we focus on the case in which an excludable publicly provided good is made available uniformly and free of charge to all agents, which means that its production is in the hands of the state and the associated cost is paid by the general tax payer.

We start with an informal description of the model used.

2.1 Informal description of the model

We build on the neoclassical growth model. The model will be an enriched version of the framework used by most of the related macro literature on public employment and public production (see e.g. Ardagna, 2007, Linnemann, 2009, Forni et al., 2009, and Economides et al., 2014, 2016, 2017). There are three types of households, two types of firms/goods, as well as a government.

Regarding households, there are three distinct types labeled capitalists, private workers and public employees. Capitalists participate in asset markets, receive labor income for their managerial services and also, since they own the private firms, they receive their profits. Private workers work in private firms and public employees work in state firms.[7] The difference between public employees and private workers is that they earn different wages.[8]

Regarding firms, we distinguish between private and state firms. Private firms produce a private good by choosing capital and labor inputs where the latter are supplied by capitalists and private workers. State firms produce an excludable publicly provided good by using the labor services of public employees, public capital and a part of the GDP purchased from the private sector. This good provides utility-enhancing services to all households (but see below for the case in which the same good also provides productivity-enhancing services). We assume that the inputs used for the production of the excludable publicly provided good are exogenously set as implied by the data (but see below for the case in which the state firms' production decisions are made optimally).

Regarding fiscal policy, in order to finance its various categories of public spending, including the cost of the public good/service produced, the government levies distorting taxes and issues bonds (but see below for the case in which state firms can also charge user charges for the use of the excludable publicly provided good).

The population size at time t is N. Among N, there is a pool of identical capitalists indexed by $k = 1, 2, ..., N^k$, a pool of identical private workers indexed by $w = 1, 2, ..., N^w$, and a pool of identical public employees indexed by $b = 1, 2, ..., N^b$, where $N^k + N^w + N^b = N$ at each t. Equivalently, the population shares are denoted as $\nu^k = N^k/N$, $\nu^b = N^b/N$ and $\nu^w = N^w/N = 1 - \nu^k - \nu^b$. There are also $f = 1, 2, ..., N^f$ identical private firms producing a single private good, where, for simplicity, the number of private firms equals the number of private agents, namely, $N^f = N^k + N^w$. Similarly, we assume that there are $g = 1, 2, ..., N^g$ identical state firms producing a single public good and that the number of state firms equals the number of public employees, namely, $N^g = N^b$.

[7] The assumption that only capitalists participate in asset markets is made for simplicity.

[8] Public and private employees can differ in many other dimensions, like job security and non-monetary privileges (see e.g. Economides et al., 2016). Here, we focus on differences in wages only.

It should be said that these population assumptions are made in order to avoid scale effects in equilibrium and are not important to our results. Throughout the paper, the fractions of the three agents in total population are exogenously set and are assumed to remain constant over time. We thus rule out occupational choice and social mobility across groups.[9]

We now model the above story.

2.2 Households

We start by modeling the behavior of the three distinct types of households.

2.2.1 Households as capitalists

There are $k = 1, 2, ..., N^k$ identical capitalists. Each capitalist derives utility from private consumption, c_t^k, leisure, $(1 - l_t^k)$, and the per capita quantity of a publicly provided good, $\overline{y}_t^g$ Without user charges and related individual choices, the amount $\overline{y}_t^g$ is uniform across agents (this is relaxed in the next subsection). The discounted lifetime utility of each k is:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^k, l_t^k, \overline{y}_t^g) \tag{1a}$$

where $0 < \beta < 1$ is the private discount rate.

For simplicity, we use a log-linear utility function (our results are robust to the functional form used):

$$u(c_t^k, l_t^k, \overline{y}_t^g) = \mu_1 \log(c_t^k) + \mu_2 \log(1 - l_t^k) + \mu_3 \log(\overline{y}_t^g) \tag{1b}$$

The budget constraint of each k is:

$$(1+\tau_t^c)c_t^k + k_{t+1}^k - (1-\delta)k_t^k + b_{t+1}^k - b_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} \tag{2a}$$

where k_{t+1}^k and b_{t+1}^k are each capitalist's capital and bond holdings respectively at the end of period t, $0 < \delta < 1$ is the depreciation rate of private capital, π_t^k is profit distributed by private firms to each capitalist, l_t^k is each capitalist's work hours, r_t and r_t^b are the returns to private capital and public debt respectively between $t-1$ and t, w_t^k is the wage rate earned by capitalists, τ_t^c, τ_t^k, τ_t^l are tax rates on consumption, capital income, and labor income respectively, and g_t^{tr} is a lump-sum transfer made by the government to each household.[10]

Each k chooses the paths $\{c_t^k, l_t^k, k_{t+1}^k, b_{t+1}^k\}_{t=0}^{\infty}$ to maximize (1a)-(1b) subject to (2a). The first-order conditions include (2a) and:

$$\frac{\mu_2}{1 - l_t^k} = \frac{\mu_1(1 - \tau_t^l)w_t^k}{(1 + \tau_t^c)c_t^k} \tag{2b}$$

$$\frac{(1 + \tau_{t+1}^c)c_{t+1}^k}{(1 + \tau_t^c)c_t^k} = \beta[1 - \delta + (1 - \tau_{t+1}^k)r_{t+1}] \tag{2c}$$

[9] We could allow the number of public employees (or, strictly speaking, their fraction in population) to be endogenous by adding an equation which links the wage rate in the public sector to the wage rate in the private sector (for instance, they become equal); see e.g. Economides et al. (2016). We report that this is not important to our main results.

[10] We have experimented with household-specific transfers; the main results do not change. For redistributive transfers in general equilibrium, see e.g. Park and Philippopoulos (2003).

$$\frac{(1+\tau_{t+1}^{c})c_{t+1}^{k}}{(1+\tau_{t}^{c})c_{t}^{k}}=\beta(1+r_{t+1}^{b}) \tag{2d}$$

where (2b) is the first-order condition for work effort, while (2c) and (2d) are the Euler conditions for capital and bonds respectively.

2.2.2 Households as workers in the private sector

There are $w = 1, 2, ..., N^w$ identical private workers. As said, for simplicity, workers do not save, and thereby their problem is static. Thus, each w maximizes:

$$u(c_t^w, l_t^w, \overline{y}_t^g) = \mu_1 \log(c_t^w) + \mu_2 \log(1 - l_t^w) + \mu_3 \log(\overline{y}_t^g) \tag{3}$$

subject to the budget constraint:

$$(1+\tau_t^c)c_t^w = (1-\tau_t^l)w_t^w l_t^w + g_t^{tr} \tag{4a}$$

where w_t^w is the wage rate earned by private workers.

Each worker chooses c_t^w and l_t^w in each period. The first-order conditions include the constraint (4a) and the optimality condition for work effort:

$$\frac{\mu_2}{1-l_t^w} = \frac{\mu_1(1-\tau_t^l)w_t^w}{(1+\tau_t^c)c_t^w} \tag{4b}$$

which is similar to (2b).

2.2.3 Households as public employees

There are $b = 1, 2, ..., N^b$ identical public sector employees. Public employees, like workers, do not save for simplicity. Therefore, each b faces a problem similar to that of private workers and so maximizes:

$$u(c_t^b, l_t^b, \overline{y}_t^g) = \mu_1 \log(c_t^b) + \mu_2 \log(1 - l_t^b) + \mu_3 \log(\overline{y}_t^g) \tag{5}$$

subject to the budget constraint:

$$(1+\tau_t^c)c_t^b = (1-\tau_t^l)w_t^g l_t^b + g_t^{tr} \tag{6a}$$

where w_t^g is the wage rate earned by public employees.

Each public employee chooses c_t^b and l_t^b in each period. The first-order conditions include the constraint (6a) and the optimality condition for work effort:

$$\frac{\mu_2}{1-l_t^b} = \frac{\mu_1(1-\tau_t^l)w_t^g}{(1+\tau_t^c)c_t^b} \tag{6b}$$

which is similar to (2b) and (4b).

2.3 Private firms and production of the private good

We now model private firms and the production of the private good. There are $f = 1, 2, ..., N^f$ identical private firms. Each firm uses capital (supplied by capitalists) and labor services (supplied by capitalists and private workers) to produce a single private good in a perfectly competitive market.

In each period, each f produces y_t^f and its profit is:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^k l_t^{f,k} - w_t^w l_t^{f,w} \quad (7)$$

where k_t^f is the firm's capital input, $l_t^{f,k}$ is capitalists' labour services used by the firm and $l_t^{f,w}$ is workers' labour services used by the firm.

The production function is assumed to be of the form:[11]

$$y_t^f = A^f (k_t^f)^\alpha (A^k l_t^{f,k} + A^w l_t^{f,w})^{1-\alpha} \quad (8)$$

where $0 < \alpha < 1$, A^f, A^k, $A^w > 0$ are technology parameters (as discussed below, we can also allow the publicly provided good, $\overline{y}_t^g$, to provide productivity-enhancing services to firms).

Each firm acts competitively maximizing (7) subject to (8) in each period. The first-order conditions for the three inputs are simply:

$$w_t^w = \frac{(1-\alpha) A^w y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \quad (9a)$$

$$w_t^k = \frac{(1-\alpha) A^k y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \quad (9b)$$

$$r_t = \frac{\alpha y_t^f}{k_t^f} \quad (9c)$$

which imply $\pi_t^f = 0$.

2.4 Public sector

We will first model the way in which state enterprises produce the publicly provided good and then present the budget constraint of the consolidated public sector.

2.4.1 Production of the publicly provided good and state firms

There are $g = 1, 2, ..., N^g$ identical state firms producing the single publicly provided good. The cost of producing this good for each state firm g is:

$$w_t^g l_t^g + g_t^g + g_t^i \quad (10)$$

where l_t^g is the labor input used by each state firm, g_t^g is goods purchased from the private sector and used for the production of the public good by each state firm and g_t^i is investment spending by each state firm. As said above, w_t^g is the wage rate paid in the public sector.

The production function of each state firm is assumed to be:

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2} \quad (11)$$

where k_t^g denotes the stock of capital used by the government at the beginning of the current period and $0 < \theta_1, \theta_2 < 1$, $A^g > 0$ are technology parameters.

[11] See also e.g. Hornstein et al. (2005) who distinguish between different types of labor services in a similar manner.

The stock of each state firm's capital evolves over time as:

$$k_{t+1}^{g}=(1-\delta^{g})k_{t}^{g}+g_{t}^{i} \tag{12}$$

where $0<\delta^{g}<1$ is the depreciation rate of public capital.

To specify the level of output, y_{t}^{g}, produced by each g, and in turn the total amount of this good provided to the society, we obviously have to specify the amounts of the three inputs, g_{t}^{i}, l_{t}^{g} and g_{t}^{g}, in (10)-(12). We will start with the relatively simple case in which these inputs are set at values implied by the data, meaning that the number of public employees as a fraction of population, as well as government spending on public investment, public wages and goods purchased from the private sector, all three as shares of GDP, will be set as in actual data. Specifically, as shown in Appendix A, for the three inputs in (10)-(12), we have $l_{t}^{g}=l_{t}^{b}$ (from the market-clearing conditions), $g_{t}^{i}=\frac{s_{t}^{i}\nu^{k}y_{t}^{f}}{\nu^{b}}$ and $g_{t}^{g}=\frac{s_{t}^{g}\nu^{k}y_{t}^{f}}{\nu^{b}}$ (from the definition of public spending shares), while, for the public wage rate, we have $w_{t}^{g}=\frac{s_{t}^{w}\nu^{k}y_{t}^{f}}{\nu^{b}l_{t}^{b}}$, where ν^{b} denotes the fraction of public employees in population, and s_{t}^{i}, s_{t}^{g} and s_{t}^{w} denote respectively the GDP shares of government expenditure on public investment, goods purchased from the private sector and public wages. The values of ν^{b}, s_{t}^{i} , s_{t}^{g} and s_{t}^{w} will be set as in the data (see below) and then the equilibrium solution will give the values of l_{t}^{g}, g_{t}^{i}, g_{t}^{g} (and in turn y_{t}^{g}) and w_{t}^{g}. Nevertheless, we will also report results for richer cases in which these inputs, and hence the level of the excludable public good produced, are determined optimally by state firms (see below in subsection 4.4).

2.4.2 Government budget constraint

The within-period budget constraint of the consolidated public sector is (written in aggregate terms):

$$Ng_{t}^{tr}+(1+r_{t}^{b})N^{k}b_{t}^{k}+N^{g}(w_{t}^{g}l_{t}^{g}+g_{t}^{g}+g_{t}^{i})=N^{k}b_{t+1}^{k}+$$

$$\tau_{t}^{c}(N^{k}c_{t}^{k}+N^{w}c_{t}^{w}+N^{b}c_{t}^{b})+\tau_{t}^{k}N^{k}(r_{t}k_{t}^{k}+\pi_{t}^{k})+\tau_{t}^{l}(N^{k}w_{t}^{k}l_{t}^{k}+N^{w}w_{t}^{w}l_{t}^{w}+N^{b}w_{t}^{g}l_{t}^{b}) \tag{13}$$

where one of the fiscal variables needs to follow residually to close the budget (see below).

2.5 Decentralized equilibrium

The equilibrium system consists of 25 equations in 25 endogenous variables, which are $\{c_{t}^{k}, l_{t}^{k}, k_{t+1}^{k}\}_{t=0}^{\infty}$, $\{c_{t}^{w}, l_{t}^{w}\}_{t=0}^{\infty}$, $\{c_{t}^{b}, l_{t}^{b}\}_{t=0}^{\infty}$, $\{r_{t}, r_{t}^{b}, w_{t}^{k}, w_{t}^{w}, \pi_{t}^{k}, w_{t}^{g}\}_{t=0}^{\infty}$, $\{y_{t}^{f}, k_{t+1}^{f}, l_{t}^{f,w}, l_{t}^{f,k}, \pi_{t}^{f}\}_{t=0}^{\infty}$, $\{y_{t}^{g}, l_{t}^{g}, g_{t}^{g}\ g_{t}^{i}, k_{t+1}^{g}, g_{t}^{tr}\}_{t=0}^{\infty}$ and one of the public financing policy variables, $\{b_{t+1}^{k}, \tau_{t}^{c}, \tau_{t}^{k}, \tau_{t}^{l}\}_{t=0}^{\infty}$, which follows residually to close the government budget constraint in each period. See Appendix A for the system and algebraic details. In the numerical solutions below, when we solve for the steady state, the residually determined instrument will be the labor tax rate, τ_{t}^{l}, with the public debt to GDP ratio being set at its average value in the data; on the other hand, along the transition, as is usually the practice, the residually determined instrument will be the end-of-period public debt, b_{t+1}^{k}, which means that now τ_t^l will be set at its average data value along with the rest of the exogenously set fiscal variables. This system is for given values of the exogenously set policy instruments, the fraction of capitalists or self-employed in population (ν^k), the fraction of public sector employees in population (ν^b) and initial values for the state variables. The system will be solved numerically in section 4.

3 Adding user charges to the above model

In this section, to the above model, we add user prices for the excludable publicly provided good. That is, now, there is a new market and an associated new (user) price for this good. Given this price, individual demand functions are derived. In other words, private agents' optimization problem gives, among other things, the individual demand for this good as a negative function of the user price. Then, this demand side, together with the supply side coming from state firms, will determine the market-clearing user price for the publicly provided impure public good.

To account for social externalities, we use the modeling of e.g. Alesina et al. (2005) by assuming that spending on the "public" good by one agent creates positive spillovers for all other agents and this is measured by the parameter $\gamma \geq 0$. In particular, the amounts of the good enjoyed by the three types of agents (k, w, b) are respectively defined as:

$$\widetilde{g}_t^k \equiv \gamma^k g_t^k + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b) \tag{14a}$$

$$\widetilde{g}_t^w \equiv \gamma^w g_t^w + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b) \tag{14b}$$

$$\widetilde{g}_t^b \equiv \gamma^b g_t^b + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b) \tag{14c}$$

where g_t^k, g_t^w and g_t^b denote the amount purchased by each k, w and b respectively, while $\gamma^k > 0$, $\gamma^w > 0$, $\gamma^b > 0$ are parameters.[12] This specification can nest various cases. For example, when $\gamma^k = \gamma^w = \gamma^b = 0$ and $\gamma = 1$, we have the case in section 2. On the other hand, when $\gamma^k = \gamma^w = \gamma^b = 1$ and $\gamma = 0$, the good is private.

We next sketch what changes relative to the model in the previous section. A detailed analysis is in Appendix B.

3.1 Households

Since households are free to choose the amount of the publicly provided good they wish to have by paying the user price, p_t^g, the within-period utility function and the budget constraint of each capitalist change to:

$$u(c_t^k, l_t^k, \widetilde{g}_t^k) = \mu_1 \log(c_t^k) + \mu_2 \log(1 - l_t^k) + \mu_3 \log(\widetilde{g}_t^k) \tag{15a}$$

$$(1 + \tau_t^c)c_t^k + k_{t+1}^k - (1 - \delta)k_t^k + b_{t+1}^k - b_t^k + p_t^g g_t^k =$$

[12] We have also exprimented with alternative specifications like

$$\widetilde{g}_t^k = \gamma^k g_t^k + \gamma \left(\frac{N^k g_t^k + N^w g_t^w + N^b g_t^b}{N} - g_t^k \right)$$

and similarly for $\widetilde{g}_t^w$ and $\widetilde{g}_t^b$. We prefer to work with the specification in (14a-c) simply because we find it to be more intuitive. We report that our results do not depend on this.

$$(1-\tau_t^k)(r_t k_t^k+\pi_t^k)+(1-\tau_t^l)w_t^k l_t^k+r_t^b b_t^k+g_t^{tr} \tag{15b}$$

Similarly, for each private worker:

$$u(c_t^w,l_t^w,\widetilde{g}_t^w)=\mu_1\log(c_t^w)+\mu_2\log(1-l_t^w)+\mu_3\log(\widetilde{g}_t^w) \tag{16a}$$

$$(1+\tau_t^c)c_t^w+p_t^g g_t^w=(1-\tau_t^l)w_t^w l_t^w+g_t^{tr} \tag{16b}$$

and for each public employee:

$$u(c_t^b,l_t^b,\widetilde{g}_t^b)=\mu_1\log(c_t^b)+\mu_2\log(1-l_t^b)+\mu_3\log(\widetilde{g}_t^b) \tag{17a}$$

$$(1+\tau_t^c)c_t^b+p_t^g g_t^b=(1-\tau_t^l)w_t^g l_t^b+g_t^{tr} \tag{17b}$$

We thus have three new first-order conditions (one for each household type):

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^k}=\frac{\mu_3\gamma^k}{\gamma^k g_t^k+\gamma(\nu^k g_t^k+\nu^w g_t^w+\nu^b g_t^b)} \tag{15c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^w}=\frac{\mu_3\gamma^w}{\gamma^w g_t^w+\gamma(\nu^k g_t^k+\nu^w g_t^w+\nu^b g_t^b)} \tag{16c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^b}=\frac{\mu_3\gamma^b}{\gamma^b g_t^b+\gamma(\nu^k g_t^k+\nu^w g_t^w+\nu^b g_t^b)} \tag{17c}$$

where (15c), (16c) and (17c) are the demand functions for the impure public good by each capitalist, each worker and each public employee respectively. Thus, now the optimal behavior of the three types of households is summarized by equations (2b), (2c), (2d), (15b) and (15c) for each capitalist, (4b), (16b) and (16c) for each worker, and (6b), (17b) and (17c) for each public employee.

3.2 Public sector

We now present what changes in the public sector.

3.2.1 State firms charging user prices

The inputs used (l_t^g, g_t^g and g_t^i), and hence the output produced (y_t^g), are determined as explained in the previous section. The only difference is that now each state firm g can also charge a price p_t^g so that it can make a profit or loss equal to $p_t^g y_t^g-w_t^g l_t^g-g_t^g-g_t^i$ at each t (this is the firm's current net cash flow) which ends up in the government budget constraint. Recall, as said above, that in subsection 4.4 we will also report results for the case in which state firms act optimally.

3.2.2 Government budget constraint

The within-period government budget constraint changes from (13) to:

$$Ng_t^{tr}+(1+r_t^b)N^k b_t^k+N^g(w_t^g l_t^g+g_t^g+g_t^i)=N^g p_t^g y_t^g+N^k b_{t+1}^k+$$

$$+\tau_t^c(N^k c_t^k+N^w c_t^w+N^b c_t^b)+\tau_t^k N^k(r_t k_t^k+\pi_t^k)+\tau_t^l(N^k w_t^k l_t^k+N^w w_t^w l_t^w+N^b w_t^g l_t^b) \tag{18}$$

where the new term $N^g p_t^g y_t^g$ is the total revenue obtained from the sales of the excludable publicly provided good.

3.2.3 Market-clearing condition for the excludable publicly provided good

Since we have a new market, namely, the market for the impure public good, its market-clearing condition is:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b \tag{19}$$

where the left hand-side of equation (19) is the supply of the impure public good coming from state firms and the right hand-side is the total demand for it coming from private agents. This equilibrium condition will determine the associated price, p_t^g.

3.3 Decentralized equilibrium

The new equilibrium system consists of 30 equations in 30 endogenous variables which are $\{c_t^k, l_t^k, k_{t+1}^k, g_t^k\}_{t=0}^{\infty}$, $\{c_t^w, l_t^w, g_t^w\}_{t=0}^{\infty}$, $\{c_t^b, l_t^b, g_t^b\}_{t=0}^{\infty}$, $\{r_t, r_t^b, w_t^k, w_t^w, \pi_t^k, w_t^g, p_t^g\}_{t=0}^{\infty}$, $\{y_t^f, k_{t+1}^f, l_t^{f,k}, l_t^{f,w}, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g, l_t^g, g_t^g, g_t^i, \pi_t^g, k_{t+1}^g, g_t^{tr}\}_{t=0}^{\infty}$, and one of the public financing policy variables, $\{b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l\}_{t=0}^{\infty}$, that follows residually to satisfy the government budget constraint in each period. On the other hand, (s_t^{tr}, s_t^w, s_t^i, s_t^g, and ν^g), the rest of the public financing policy variables, as well as the population fractions of capitalists (or self-employed) and public employees (ν^k and ν^b), are exogenously set. The system and algebraic details are in Appendix B. This system will be solved numerically jointly with the equilibrium system of the previous section.

4 Parameterization and solutions

In this section, we compare the numerical solution without user prices (see subsection 2.5 and Appendix A) to the numerical solution with user prices (see subsection 3.3 and Appendix B). In subsection 4.1 we present the chosen parameterization. In subsection 4.2, we present and compare steady state solutions. In subsection 4.3, we study the dynamic path of the economy when it starts without user prices and switches to user prices. Finally, subsection 4.4 reports robustness checks.

4.1 Parameterization

The model (wihout, or with, user prices) contains two types of "parameters". Structural parameters related to preferences/technology and fiscal policy variables.[13] We will employ commonly used values for preferences/technology parameters and will use data averages from the eurozone for policy and population variables (the data source is Eurostat). These values are listed in Table 1. The time unit is meant to be a year. Before we proceed, we wish to report that we have conducted a rather rich sensitivity analysis and our qualitative results are robust to changes in these values except otherwise stated (details are in subsection 4.4 below).

[13] Our goal in this paper is to provide numerical solutions that remain robust under a wide range of commonly used values for structural parameters and policy instruments. It is not to solve a realistically calibrated economy.

Table 1 here: Baseline parameterization

Consider first structural parameters. Starting with technology, in the private sector production function, the Cobb–Douglas exponents of labor and capital are set respectively at 0.67 and 0.33 (= 1 − 0.67). The TFP parameters in the production functions of private and state firms, A^f and A^g respectively, are both normalized at 1 (public sector efficiency, and why it may differ from private sector efficiency, is not an issue in this paper). In the private production function, we set $A^w = 1$ for workers' labor producticity and calibrate A^k, namely capitalists' labour productivity, so as to get positive work hours. The private and public capital depreciation rates, δ and δ^g, are both set at 0.05. In the public sector production function, the Cobb–Douglas exponents of public employment and public capital, θ_2 and θ_1, are set respectively at 0.569 and 0.1078; these values correspond to payments to public wages and payments to public investment, expressed as shares of total public payments to all inputs used in the production of public goods, as they are in the data; in other words, $0.569 = s^w/(s^w + s^g + s^i)$ and $0.1078 = s^i/(s^w + s^g + s^i)$, where, as said above, s^w is the GDP share of government spending on public wages in the data, s^g is the GDP share of public spending on goods and services purchased from the private sector in the data, and s^i is the GDP share of government spending on public investment in the data (see below for these values); for similar calibration practice, see the real business cycle literature and, in related models, see e.g. Linnemann (2009) and Economides et al. (2014 and 2017). In turn, the Cobb-Douglas exponent of goods purchased from the private sector in the public sector production function follows residually so that it is $0.3232 = 1 - 0.569 - 0.1078$.

Consider next preference parameters. The time discount rate, β, is set at 0.9. The weight given to public goods/services in the private utility function is set at 0.05, which is within the range used in the related literature.[14] The other two preference parameters related to private consumption and leisure, μ_1 and μ_2, are set at 0.35 and 0.6 respectively; these values give hours of work within usual ranges. Regarding γ^k, γ^w and γ^b, we simply set all of them at 1, while we give a rather moderate value to the parameter measuring social externalities, $\gamma = 0.15$ (our results remain qualitatively the same as long as $\gamma \leq 2$; see section 5 below for this parameter value).

Finally, consider policy and population variables. The share of public employees in total population (ν^b) is set at 0.215, which is the average value in the data. The share of capitalists (ν^k), defined as those who are self-employed, is set at 0.148 as in the data. Since $N^f = N^k + N^w$ and $N^g = N^b$ as discussed in subsection 2.1, $\nu^f = \nu^k + \nu^w$ and $\nu^g = \nu^b$. The data values of s^w, s^g and s^i (as defined above), as well as the data value of s^{tr} (which denotes total transfers as share of GDP), are respectively 0.132, 0.075, 0.025 and 0.2170. The effective tax rates on consumption, capital income and labor income, (τ^c, τ^k and τ^l) are set at 0.1938, 0.2903 and 0.378 as in the euro area data. Finally, in the steady state, the public debt to GDP ratio is set at 0.9, which is the data average value, while, we treat τ^l as the endogenously determined fiscal variable in all steady state solutions.

[14] Chari et al. (1995) use a zero value. In contrast, Guo and Lansing (1999) use a high value, around 0.36, in a similar utility function.

4.2 Steady state results

We start with steady state solutions. They are reported in Table 2. The first column in Table 2 reports the steady state solution of the economy without user charges in section 2 (or what we call the status quo, SQ), while the third, last column reports the steady state solution of the same economy with user charges; in all solutions, we use the parameter values and the exogenous policy variables in Table 1.

Srarting with the SQ in the first column, this solution is meaningful. Regarding the aggregate economy, the solution does relatively well at mimicking the GDP ratios of the key macroeconomic aggregates, like consumption and capital, as shares of GDP, in the euro data. Notice that the value of the endogenously determined labor tax rate is also close to its actual value in the data. Regarding distribution, the solution implies that capitalists (or "the rich") enjoy the highest net income, the highest consumption level and the highest utility level. Private workers consume more, and also work harder, than public employees; in terms of utility, the benefit from higher consumption more than outweighs the pain of harder work, so that private workers are better off than public employees. These comparative results are in accordance with public perceptions and hard data (see e.g. Economides et al., 2016).

Table 2 here: Steady state solutions

The third, last column in Table 2 reports the steady state solution with user charges other things equal. Regarding aggregate outcomes or efficiency, the comparison of the SQ solution to the solution in the third, last column reveals that a switch to user prices can generate substantial efficiency gains. Private, public and total output (y^f and y respectively)[15] are both higher in the last column than in first, SQ column. Specifically, with the baseline parameterization, private output, y^f, rises by around 10.8% from column 1 to colunn 3, and, similarly, public output, y^g, increases by 16.4%. Per capita utility, u, is also higher in column 3 than in column 1. Notice that the switch to a more efficient macro-economy with larger tax bases allows for a cut in the labor tax rate (τ^l), which serves as the residually determined fiscal instrument in our steady states solutions, so that the allowed cut in a particularly distorting tax rate, like the labor tax rate, triggers a further improvement of individual incentives to work and save. In particular, work hours and capital accumumation are higher in an economy with user prices (compare l^k, l^w, l^b and k^k in column 3 relative to column 1).

Regarding individual outcomes and hence distribution, the net income of all agents (y^k, y^w, y^b)[16] is higher in column 3 than in the status quo in column 1. The same applies to individual utilities (compare u^k, u^b and u^w in column 3 relative to column 1). In other words, a switch to user prices is Pareto efficient. These developments shape in turn relative net incomes like y^w/y^k and y^b/y^k. In particular, a switch from column 1 to column 3 makes public employees better off in terms of relative income, whereas the relative position of private

[15] Total per capita output is defined as $y_t \equiv v^f y_t^f + v^g p_t^g y_t^g$.

[16] From the budget constraints above, $y_t^k \equiv (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l) w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} - \tau_t^c c_t^k - p_t^g g_t^k$ is the net income of each capitalist; $y_t^w \equiv (1-\tau_t^l) w_t^w l_t^w + g_t^{tr} - \tau_t^c c_t^w - p_t^g g_t^w$ is the net income of each worker; and $y_t^b \equiv (1-\tau_t^l) w_t^g l_t^b + g_t^{tr} - \tau_t^c c_t^b - p_t^g g_t^b$ is the net income of each public employee.

workers vis-a-vis capitalists remains almost unchanged (recall however that s^w (i.e. the GDP share of public wages) is kept constant as in the data in this set of experiments).

4.2.1 Decomposition of the full effect and discussion

Summing up, in section 3, relative to the status quo economy in section 2, we allowed for user prices and a market mechanism for the excludable publicly provided good. As we saw, the introduction of user prices and the creation of a new market for this good not only created significant aggregate gains but it was also Pareto efficient benefiting all types of households.

Let us call this extra market effect, whose solution was reported in the last column of Table 2, the full effect. Inspection of our model implies that this full effect is shaped by two closely interrelated, albeit different, channels; one direct and one indirect. The former refers to the direct effect on individual incentives to work and save from the introduction of user prices, whereas, the indirect channel refers to a second-round effect on incentives caused by the cut in the tax burden which is made possible by the switch to a more efficient aggregate economy. It is the joint interaction of these two channels that shapes the full effect. The rest of this subsection quantifies this decomposition so as to understand the importance of each channel. To capture the direct channel, we simply assume that the introduction of user prices is not accompanied by a cut in the tax burden but, instead, the latter remains as in the SQ economy.

The second column in Table 2 quantifies the effect of the new market mechanism for the excludable publicly provided good in the case in which the labour tax rate remains at its value in the SQ economy. As can be seen, even in this case, private and total output (y^f and y respectively) are higher than in the SQ in the first column. Specifically, with the baseline parameterization, private output rises by around 6.6% from the first column to the second colunn. Thus, from the total increase, which is around 10.8%, 6.6% is due to the direct channel and 4.2% to the lower tax burden. Also, all labour efforts (l^k, l^w, l^b) and capital accumulation (k^k) are higher in the second column relative to the SQ in the first column 1 but less in the second column relative to the third column.

The intuition is clear. The introduction of user charges for the excludable publicly provided good, and the creation of a new market for it, forces individuals to realize that, in order to afford the provision of this goods they need higher income and hence they need to work harder and/or save more. And this occurs irrespectively of whether the tax burden falls or not. However, to the extent that the fiscal space (made available by the increase in output and hence in the tax bases generated by the direct channel) is used to cut a distorting tax, this triggers a second round of beneficial effects.

Finally, regarding the impact of the direct channel on individual outcomes, the net incomes of private sector agents (y^k and y^w) fall slightly, whereas the net income of public sector employees (y^b) increases substantially. By contrast, as said above, the full effect, which incorporates the impact of both channels, is positive for all individual incomes. This implies that, for the reform to be Pareto improving, it is necessary the introduction of a market mechanism for the excludable publicly provided good to be accompanied by a cut in taxes made possible by the switch to a more efficient economy.

4.3 Transition results

We now study transition results as we depart from the steady state without user prices (see the first column in Table 2) and travel towards the steady state with user prices (see the last column in Table 2). In other words, transition dynamics will be driven by a change in the policy regime.[17]

Before we present results, we typically enrich the policy rules along the transition path. As said above, along the transition to a steady state, the residually determined public financing instrument is the end-of-period public debt, while, once we reach a steady state, the residually determined instrument is the labor tax rate with the public debt-to-output ratio being set as in the data. To capture this, we assume that the labor tax rate, τ_t^l, adjusts gradually to its new long-run value by following the determintistic AR(1) rule:

$$\tau_t^l = (\tau^l)^{1-\varphi}(\tau_{t-1}^l)^{\varphi} \tag{20}$$

where the persistence parameter φ will be set at say 0.85, while the values of the tax rate in the new reformed steady state, τ^l, as well as the initial value, τ_{-1}^l, depend on the policy scenarios assumed.

In addition, and as is usually the case, dynamic stability requires at least one of the exogenously set fiscal policy instruments to react to the gap between the public debt to GDP ratio and its steady state value. Here, we will assume that this role is played by the GDP ratio of transfer payments. Namely, along the transition, instead of assuming that s_t^{tr} equals its steady state (data average) value, s^{tr}, all the time, we impose the feedback policy rule:

$$s_t^{tr} = s^{tr} - \gamma^{tr}\left(\frac{v^k b_t^k}{y_t} - \frac{v^k b^k}{y}\right) \tag{21}$$

where we will set the feedback policy coefficient, γ^{tr}, at say 0.3 (which is within usual ranges in the related literature; see e.g. Philippopoulos et al., 2017).

Given the above, simulations are shown in Figures 1 and 2. The lines show the paths of private output and total output (y_t^f and y_t), as well as individual net incomes (y_t^k, y_t^w and y_t^b), when we depart from the SQ without user prices and switch to user prices other things equal. Variables are measured as percentage deviations from their value in the initial, departing steady state solution. In these experiments, we use a Newton-type non-linear method implemented in Dynare. We solve the model under perfect foresight in the sense that the structural changes (policy reforms), with which we feed the model, are fully known to the agents of the economy.

Figure 1 shows that both private and total output, shown by the dashed and solid lines respectively, rise when we add user prices. The same applies to public output which is the difference between total and private output. Figure 2 shows that, intertemporally, this is Pareto efficient. The net incomes of all households rise over time, although private agents (capitalists and workers) suffer a temporary reduction in net income on impact. This happens because,

[17] Specifically, the economy is at the steady state of the status quo regime without user prices in the first period, $t = 0$, and it then switches immediately and permanently to a new regime with user prices at $t \geqq 1$. We have also computed the model when the transition to the new policy regime takes place gradually over time. Since the key results do not change, here we just report immediate regime switches. Gradual results are available upon request.

on impact, although we switch to a more efficient economy, the labour tax rate has not yet decreased enough, which means that the gains in net income from the lower tax burden have not managed yet to offset the extra burden on individual agents due to spending on the impure public good. Notice, on the other hand, the rise in the net income of public employees, y_t^b. Recall however that state firms have been assumed to set their inputs exogenously as in the data rather than to choose them optimally; hence, as we switch to a more efficient economy and since the public wage bill as share of GDP remains constant, the level of public wages simply rises.

Figures 1 and 2 here: From SQ to user prices

Summing up, transition results are in accordance with the steady state results analysed in the previous subsection.

4.4 Robustness

In this subsection, we report a number of robustness checks which seem to confirm the generality of the above results. To save on space, here we will just report the types of sensitivity tests we have conducted (details for each case one by one are provided in what follows).

First, as already said throughout the paper, our results are robust to changes in the values of parameters and policy instruments listed in Table 1 (exceptions are explicitly stated in the next section). Moreover, they are robust to different functional forms for the utility and production functions, like CES forms. These results are available upon request.

Second, our results remain qualitatively the same when we add pure public goods (of course without user charges). These results are available upon request.

Third, our results remain qualitatively the same when we allow the publicly provided good to provide productivity enhancing services. The latter can take the form of an extra productive factor in the private firms' production function (as in e.g. Barro (1990)) or of a work-complement service that augments households' labor productivity (see e.g. Blomquist et al., 2010, and the references therein). For details on these cases, see Appendices D and E.

Fourth, our key results do not change even when we assume that state firms act optimally (by e.g. by maximizing profits or minimizing costs). In our experiments, in the case of profit maximization, each state firm is assumed to maximize the present value of the discounted sum of its net cash flows,[18] whereas, in the case of cost minimization, its output target can be, for instance, the level of output that a profit maximizing state firm would have chosen or the level that a social planner would have chosen.[19] The only noticable difference, when state firms act optimally, is that their optimality conditions lead to a cut in the public wage rate which makes public employees worse off. In other words, a switch to user prices combined with optimizing behavior on the part of state firms, hurts public employees and so is not Pareto improving vis-a-vis the status

[18] When state firms maximize their profit, this produces a conventional supply function where the quantity supplied increases with the user price. This modelling enables us to treat the state firms in a similar manner to private firms. Also in this case, any profits or losses made by state firms go to the government budget.

[19] See e.g. Atkinson and Halvorsen (1986) and De Borger (1995) for the behaviour of state enterprises offering quasi-public goods.

quo. It is worth adding here that the case in which the state firm maximizes its profits acting competitively, so that the user price equals the marginal cost, is also known as the marginal-cost pricing practice in the literature of public enterprises (see e.g. Webb (1976), Atkinson and Stiglitz (1980, chapter 15) and Bos (1985) for early studies); this is also what happens here when state firms act as profit maximizers. For details, see Appendix C.

Finally, we have also experimented with other pricing rules for the excludable publicly provided good. In particular, we have assumed that the user price, p_t^g, is set equal to the average cost of producing this good, where the latter is determined by dividing the total cost by the total planned output, that is $p_t^g = \frac{w_t^g l_t^g + g_t^g + g_t^i}{y_t^g}$. This case is known as the average-cost pricing practice in the literature of public enterprises (again see e.g. Webb (1976) and Bos (1985) for early studies). In this case, since p_t^g is set at its average cost, and to the extent that demand is equal to supply in this market, we need to allow one of s^w, s^g and s^i to be determined endogenously (it takes the place of p_t^g). We report that only in the case in which it is the public wage bill as share of GDP, s^w, that is determined endogenously, we get a well-defined solution, the key properties of which are similar to our baseline scenario with the exception that such a pricing rule hurts the public sector employees (in other words, this is like the above case in which state firms acted optimally). When s^g or s^i are determined endogenously, their values become negative which makes our solutions ill-defined (although this is a numerical problem only because the starting values of s^g or s^i are small as in the data and they naturally turn to negative once they get smaller).

5 Counter-arguments for the use of user prices

In this section, we focus on three cases in which user prices do not work. The first case has to do with the degree of social externalities generated by the publicly provided good. The second one studies the case in which there is a mix of user prices and public financing. The third one focuses on the case in which there is some minimum amount of the good that each individual needs. This list is not exhaustive but we believe these three cases deserve special attention.

The first case is straightforward. As might be anticipated, when the degree of social externalities, as measured by the parameter γ in equations (14a-c), is sufficiently large (in our baseline solutions, this happens for $\gamma > 2$), the market mechanism collapses in the sense that the optimally chosen individual demands for the excludable publicly provided good become negative. The intuition is obvious: with strong social external effects, free-riding behavior is inconsistent with the charge of user prices (the polar subcase is a pure public good). The second and the third cases require minor changes in the baseline model of section 3; the second case is presented in subsection 5.1 and the third one in subsection 5.2.

5.1 Mix of user prices and public finance

In this subsection, we allow for partial public finance of the impure public good. In particular, we assume that each agent chooses his/her demand for the impure public good knowing that he/she will pay only a fraction $0 \leq \mu \leq 1$ of user charges, while the rest, $0 \leq 1-\mu \leq 1$, will be covered by the government meaning the general tax payer. In what follows, we explain what changes relative to the model in section 3.

5.1.1 Households

Since now households are aware that they will get back a fraction of their total spending on the impure public good, their budget constraints change.

For each capitalist, the budget constraint is:

$$(1+\tau_t^c)c_t^k + k_{t+1}^k - (1-\delta)k_t^k + b_{t+1}^k - b_t^k + \mu p_t^g g_t^k =$$
$$= (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} \tag{22a}$$

For each worker, the budget constraint is:

$$(1+\tau_t^c)c_t^w + \mu p_t^g g_t^w = (1-\tau_t^l)w_t^w l_t^w + g_t^{tr} \tag{23a}$$

For each public employee, the budget constraint is:

$$(1+\tau_t^c)c_t^b + \mu p_t^g g_t^b = (1-\tau_t^l)w_t^g l_t^b + g_t^{tr} \tag{24a}$$

Then, the first-order conditions with respect to the quantity of the publicly provided good demanded (one for each type of households) change to:

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3 \gamma^k}{\gamma^k g_t^k + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{22b}$$

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3 \gamma^w}{\gamma^w g_t^w + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{23b}$$

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3 \gamma^b}{\gamma^b g_t^b + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{24b}$$

5.1.2 Public sector

Since now the government finances a fraction, $0 \leq 1-\mu \leq 1$, of agents' spending on the impure public good, the within-period government budget constraint changes to:

$$N g_t^{tr} + (1+r_t^b)N^k b_t^k + N^g(w_t^g l_t^g + g_t^g + g_t^i) +$$
$$+(1-\mu)p_t^g(N^k g_t^k + N^w g_t^w + N^b g_t^b) =$$
$$= N^g p_t^g y_t^g + N^k b_{t+1}^k + \tau_t^c(N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k(r_t k_t^k + \pi_t^k) +$$
$$+\tau_t^l(N^k w_t^k l_t^k + N^w w_t^w l_t^w + N^b w_t^g l_t^b) \tag{25}$$

5.1.3 Decentralized equilibrium

The new equilibrium system consists of the same number of equations and the same endogenous variables as in section 3 and Appendix B with the changes shown right above. The new system is solved numerically using the same parameter values and policy variables as in Table 1.

5.1.4 Steady state results

Steady state solutions for three different degrees of $0 \leq \mu \leq 1$, namely, different degrees of user charges vis-á-vis public finance, are presented in Table 3a. In this table, for convenience and comparison, we repeat the status quo solution (see the first column of Table 3) and the solution with full user prices (this is the case with $\mu = 1$ in Table 3 which coincides with the solution in the last column of Table 2). The columns in Table 3, between the status quo case and the case with $\mu = 1$ present solutions with three interior values of μ, for instance, $\mu = 0.15$, $\mu = 0.50$ and $\mu = 0.75$.

Table 3a here: Mix of user prices and public finance

Several interesting results emerge from Table 3a. We first report that for very low values of μ (under our parameterization, for $\mu < 0.1$), we cannot obtain well-defined solutions. This is because a very low value of μ means a heavy subsidization which pushes individual agents to demand a very high quantity of the impure public good. For relatively low values of μ, say $\mu = 0.15$, we do get solutions but these solutions are inferior - both in terms of the private output and individuals' income/utility - and this is not only relative to the polar case of $\mu = 1$ but also relative to the status quo. This is for the same reason: a heavy subsidization gives the wrong incentives; capitalists' work hours and capital holdings, as well as workers' work hours, fall relative to the status quo. Only when μ gets high enough, say 0.5 or above in our calculations, so that private incentives to work and invest are restored, the mixed system gets better than the status quo. Therefore, the main message is clear: if the market mechanism is distorted, it is better not to have a market mechanism at all and, instead, we should rely on centralized policy mechanisms.

Our computations also show that, if the criterion is the production of the private good (y^f), the value of μ that maximizes this type of output is 1, while, if the criterion is total output (y), the maximizing value of μ is around 0.5. This again makes sense: if the criterion is private good production only, we should choose a public finance scheme that delivers the best incentives for work and saving and this is $\mu = 1$. If, on the other hand, the criterion is total output, which includes the production of the impure public good (y^g), we should choose a public financing mix that is relatively biased towards the public good; hence $\mu < 1$.

With respect to individual outcomes and distribution, the net incomes of all types of households are higher under relatively high values of μ; namely, y^k, y^w and y^b are all higher when μ is higher than 0.5 relative to the status quo case. That is, reliance on the market mechanism is Pareto efficient. On the other hand, looking at relative incomes, as μ rises, the big winners as μ rises are private workers and public employees ($y^w/\ y^k$ and $y^b/\ y^k$ increase as μ rises, although the latter non-monotonically). Therefore, a mix of user prices and public finance (with the lion's share going to the former), although less efficient at aggregate level and less desirable for capitalists and workers than a regime of full user prices, is Pareto efficient for all agents' net incomes vis-a-vis the status quo.

5.2 Minimum quantity is needed by everybody

In this subsection we study the case in which there is some minimum quantity of the excludable publicly provided good/service that each individual needs, while he/she is also free to top up if he/she wishes to do so. Our aim is to capture the case in which the publicly provided good is a necessity or features indivisibilities (as said above, examples can include certain types of education and medical care).

Our thought experiment is as follows. Assume that individuals benefit from the total amount provided, that is the minimum amount, which is exogenous to them, plus possible top ups, which are optimally chosen by them as in section 3. In other words, agents do not choose optimally the full amount of the good they wish to consume but instead a part of it is set exogenously. We assume for generality that individuals have to pay user charges for the total amount (this is simply because we want to understand the implications of user charges).

Then, as we shall see below, our solutions imply that, to the extent that the exogenously set minimum amount is relative small, our main results remain the same as in section 3, meaning that user prices help individuals to internalize the social cost of publicly provided goods and to reduce other more distorting taxes. However, as the exogenously set minimum amount gets larger, some individuals (the relatively poor agents are being hit first) cannot afford the user charges and their optimization problem ceases to be well defined. Only the relatively rich individuals can continue to afford the user charges and, from their own point of view, user prices are still a good thing for the reasons discussed above. Nevertheless, after a point, at which the exogenously set minimum amount becomes large enough, nobody can afford the user charges (even the rich agents) and then the economy has either to resort to public financing by general taxes or to stop public good provision.

In what follows, we explain what changes in the model of section 3 and Appendix B.

5.2.1 Households

Since everybody has to consume a minimum uniform level of the impure public good, denoted as $\overline{g}_t$, the within-period utility functions of the three household types become:

$$u(c_t^k, l_t^k, g_t^k) = \mu_1 \log(c_t^k) + \mu_2 \log(1 - l_t^k) + \mu_3 \log(\gamma^k \overline{g}_t + \widetilde{g}_t^k) \tag{26a}$$

$$u(c_t^w, l_t^w, g_t^w) = \mu_1 \log(c_t^w) + \mu_2 \log(1 - l_t^w) + \mu_3 \log(\gamma^w \overline{g}_t + \widetilde{g}_t^w) \tag{27a}$$

$$u(c_t^b, l_t^b, g_t^b) = \mu_1 \log(c_t^b) + \mu_2 \log(1 - l_t^b) + \mu_3 \log(\gamma^b \overline{g}_t + \widetilde{g}_t^b) \tag{28a}$$

For each capitalist, the budget constraint becomes:

$$(1 + \tau_t^c)c_t^k + k_{t+1}^k - (1 - \delta)k_t^k + b_{t+1}^k - b_t^k + p_t^g(\overline{g}_t + g_t^k) =$$

$$= (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l) w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} \tag{26b}$$

for each worker, the budget constraint becomes:

$$(1+\tau_t^c) c_t^w + p_t^g (\overline{g}_t + g_t^w) = (1-\tau_t^l) w_t^w l_t^w + g_t^{tr} \tag{27b}$$

for each public employee, the budget constraint becomes:

$$(1+\tau_t^c) c_t^b + p_t^g (\overline{g}_t + g_t^b) = (1-\tau_t^l) w_t^g l_t^b + g_t^{tr} \tag{28b}$$

Each agent maximizes her/his welfare (in (26a)-(27a)-(28a) respectively) subject to her/his budget constraint (in (26b)-(27b)-(28b) respectively). The first-order conditions for the quantity demanded (one for each type of household) are now:

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c) c_t^k} = \frac{\mu_3 \gamma^k}{\gamma^k (\overline{g}_t + g_t^k) + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{26c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c) c_t^w} = \frac{\mu_3 \gamma^w}{\gamma^w (\overline{g}_t + g_t^w) + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{27c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c) c_t^b} = \frac{\mu_3 \gamma^b}{\gamma^b (\overline{g}_t + g_t^b) + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{28c}$$

5.2.2 Market-clearing condition for the excludable public good

The market-clearing condition for the impure public good is now:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b + N \overline{g}_t \tag{29}$$

where, on the right hand-side, we also have the exogenously determined minimum quantity of the publicly provided good.

5.2.3 Decentralized equilibrium

The new equilibrium system, consists of the same equations as in the baseline model with user prices of section 3 and Appendix B with the changes shown above, plus an equation that specifies the minimum quantity of the public good. For computational reasons only, the latter is defined to be an exogenous fraction of total output:

$$\overline{g}_t = \overline{s}_t (v^k y_t^f + v^g p_t^g y_t^g) \tag{30}$$

where $\overline{s}_t \geqslant 0$ is a parameter.

The new system is solved numerically using the same parameter values and policy variables as in Table 1. Regarding $\overline{s}_t$, and in order to capture the impact of an increasing $\overline{g}_t$ on our solutions, we allow it to vary between 0.03 and 0.15.[20]

[20] The value $\overline{s}_t = 0.03$ is close to the share of government spending on "defence" and "public order and safety" in most EU countries. These two categories of public spending are closer to "pure" public goods according to the European Central Bank (see e.g. ECB, Monthly Bulletin, April 2009, p. 96).

5.2.4 Steady state results

Since almost all of the endogenous variables maintain the quantitative and qualititative features of the baseline solution under user prices in section 3, we choose to present steady state solutions only for g_t^k, g_t^w, g_t^k and $\overline{g}_t$ the values of which are directly affected by the value of $\overline{s}_t$ and thus change quantitatively as $\overline{s}_t$ increases. These solutions are presented in Table 3b. Again, for convenience and comparison, we repeat the status quo solution without user prices (see the first column of Table 3b). Then, in the next five columns, we report solutions with user prices for the cases in which $\overline{s}_t = 0.03$, $\overline{s}_t = 0.05$, $\overline{s}_t = 0.08$, $\overline{s}_t = 0.10$ and $\overline{s}_t = 0.15$.

Table 3b here: Minimum quantity is needed by everybody

When $\overline{s}_t > 0.08$, the solution (with a minimum uniform level of the impure public good provided with user charges) implies that public employees find it optimal not to top up by paying user prices; in other words, g^b goes to zero or becomes even negative as $\overline{s}_t$ further rises. The same applies to private workers for a critical threshold above $\overline{s}_t > 0.10$.

The mechanism and the intuition behind these results are as follows. The first-order conditions in (26c)-(27c)-(28c) reveal that, as the compulsory user payments, $p_t^g\overline{g_t}$, rise, individuals, in order to afford these higher payments, have to reduce their voluntary tops; otherwise, their consumption will turn to negative. That is, there is now an issue of affordabilty. This is confirmed by our numerical solutions in Table 3b which imply that after a critical amount, which is individual-specific depending on the sources of personal income, individual agents' tops ups turn to negative meaning that they have to become subsidies by the government so as consumption to stay positive. But a negative top up, or equivalently a subsidy to personal incomes, is essentially a denial of the user charge system in the first place (this becomes obvious if the top ups are constrained to be non-negative in which case, after of point of $\overline{g}_t$, the personal income is not enough to finance the user charges plus a positive consumption). In sum, after a critical amount of $\overline{g}_t$ and the associated user charges, $p_t^g\overline{g}_t$, the market mechanism of user prices does not work.

6 Closing the paper

In this paper, which belongs to the broad literature on the "dilemma" between efficiency and equity, we studied the aggregate and distributional implications of introducing user prices for publicly provided excludable goods/services. Working within a rather standard general equilibrium setup, we showed that the introduction of user prices can crowd in private incentives and hence improve aggregate efficiency. We also showed that the introduction of user prices can be Pareto efficient. But we also identified conditions under which a system of user prices does not work.

Since the main results have already been listed in the Introduction, we close with some possible extensions. It would be interesting to add incomplete information so that some agents may try not to reveal their valuation of publicly provided goods. It would also be interesting to study what happens in a multi-country model with free riding on publicly provided goods with cross-border spillovers.

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Appendices

Appendix A: Free of charge uniform provision

This is an appendix to section 2 in the main text.

Market-clearing conditions

The market-clearing conditions in the dividend, capital and labor markets are:

$$v^f \pi_t^f = v^k \pi_t^k \tag{A.1}$$

$$v^f k_t^f = v^k k_t^k \tag{A.2}$$

$$v^f l_t^{f,w} = v^w l_t^w \tag{A.3}$$

$$v^f l_t^{f,k} = v^k l_t^k \tag{A.4}$$

$$v^g l_t^g = v^b l_t^b \tag{A.5}$$

Equilibrium system

Households The behaviour of each capitalist is summarised by:

$$(1+\tau_t^c)c_t^k + k_{t+1}^k - (1-\delta)k_t^k + b_{t+1}^k - b_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} \tag{A.6}$$

$$\frac{\mu_2}{1-l_t^k} = \frac{\mu_1(1-\tau_t^l)w_t^k}{(1+\tau_t^c)c_t^k} \tag{A.7}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta[1-\delta+(1-\tau_{t+1}^k)r_{t+1}] \tag{A.8}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta(1+r_{t+1}^b) \tag{A.9}$$

The behaviour of each worker is summarised by:

$$(1+\tau_t^c)c_t^w = (1-\tau_t^l)w_t^w l_t^w + g_t^{tr} \tag{A.10}$$

$$\frac{\mu_2}{1-l_t^w} = \frac{\mu_1(1-\tau_t^l)w_t^w}{(1+\tau_t^c)c_t^w} \tag{A.11}$$

The behaviour of each public employee is summarised by:

$$(1+\tau_t^c)c_t^b = (1-\tau_t^l)w_t^g l_t^b + g_t^{tr} \tag{A.12}$$

$$\frac{\mu_2}{1-l_t^b} = \frac{\mu_1(1-\tau_t^l)w_t^g}{(1+\tau_t^c)c_t^b} \tag{A.13}$$

Private firm Each private firm maximizes profits:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^k l_t^{f,k} - w_t^w l_t^{f,w} \tag{A.14}$$

subject to the production function:

$$y_t^f = A^f (k_t^f)^{\alpha_1} (A^k l_t^{f,k} + A^w l_t^{f,w})^{\alpha_2} \tag{A.15}$$

The optimality conditions for $l_t^{f,w}$, $l_t^{f,k}$ and k_t^f are:

$$w_t^w = \frac{\alpha_2 A^w y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{A.16}$$

$$w_t^k = \frac{\alpha_2 A^k y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{A.17}$$

$$r_t = \frac{\alpha y_t^f}{k_t^f} \tag{A.18}$$

State enterprises The state-owned firm produces public output according to the production function:

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2} \tag{A.19}$$

where the law of motion of public capital is:

$$k_{t+1}^g = (1 - \delta^g) k_t^g + g_t^i \tag{A.20}$$

Government budget constraint

$$g_t^{tr} + (1 + r_t^b) v^k b_t^k + v^g (w_t^g l_t^g + g_t^g + g_t^i) = v^k b_{t+1}^k +$$

$$\tau_t^c (v^k c_t^k + v^w c_t^w + v^b c_t^b) + \tau_t^k v^k (r_t k_t^k + \pi_t^k) + \tau_t^l (v^k w_t^k l_t^k + v^w w_t^w l_t^w + v^b w_t^g l_t^b) \tag{A.21}$$

Resource constraint

$$v^k c_t^k + v^w c_t^w + v^b c_t^b + v^k (k_{t+1}^k - (1 - \delta) k_t^k) + v^g (g_t^g + g_t^i) = v^f y_t^f \tag{A.22}$$

Public spending policy rules The public inputs are determined by the exogenously set policy rules:

$$w_t^g = \frac{s_t^w v^f y_t^f}{v^b l_t^b} \tag{A.23}$$

$$g_t^g = \frac{s_t^g v^f y_t^f}{v^g} \tag{A.24}$$

$$g_t^i = \frac{s_t^i v^f y_t^f}{v^g} \tag{A.25}$$

where s_t^w, s_t^g and s_t^i, namely, spending on public wages, goods purchased from the private sector and public investment respectively, all as shares of GDP, are fiscal policy instruments.

For transfers, we use a similar policy rule:

$$g_t^{tr} = s_t^{tr} v^f y_t^f \tag{A.26}$$

where s_t^{tr} is a policy instrument.

Decentralized equilibrium without user prices

The equilibrium system consists of 25 equations, (A.1)-(A.21) and (A.23)-(A.26),[21] in 25 endogenous variables, which are $\{c_t^k, l_t^k, k_{t+1}^k\}_{t=0}^{\infty}$, $\{c_t^w, l_t^w\}_{t=0}^{\infty}$, $\{c_t^b, l_t^b\}_{t=0}^{\infty}$, $\{r_t, r_t^b, w_t^k, w_t^w, \pi_t^k, w_t^g\}_{t=0}^{\infty}$, $\{y_t^f, k_{t+1}^f, l_t^{f,w}, l_t^{f,k}, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g, l_t^g, g_t^g, g_t^i, k_{t+1}^g, g_t^{tr}\}_{t=0}^{\infty}$ and one of the public financing policy variables, $\{b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l\}_{t=0}^{\infty}$, which follows residually to close the government budget constraint in each time period. This is for given values of the exogenously set policy instruments as defined in the main text, the fraction of capitalists or self-employed in population (ν^k), the fraction of public employees in population (ν^b) and initial values for the state variables.

Appendix B: Adding user charges to the above model

This is an appendix to section 3 in the main text.

Market-clearing conditions

The market-clearing conditions are again:

$$v^f \pi_t^f = v^k \pi_t^k \tag{B.1}$$

$$v^f k_t^f = v^k k_t^k \tag{B.2}$$

$$v^f l_t^{f,w} = v^w l_t^w \tag{B.3}$$

$$v^f l_t^{f,k} = v^k l_t^k \tag{B.4}$$

$$v^g l_t^g = v^b l_t^b \tag{B.5}$$

Moreover, since we have a new market (i.e. the market for the impure publicly provided good), its market-clearing condition is:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b$$

or written in terms of population fractions:

$$v^g y_t^g = v^k g_t^k + v^w g_t^w + v^b g_t^b \tag{B.6}$$

[21] If we include the budget constraints of all agents, namely (A.6), (A.10), (A.12) and (A.21), the economy's resource constraint in (A.22) is linearly dependent and hence redundant. Equivalently, we can work with (A.22) and drop one of the other budget constraints.

Equilibrium system

Households The behaviour of each capitalist is summarised by:

$$(1+\tau_t^c)c_t^k + k_{t+1}^k - (1-\delta)k_t^k + b_{t+1}^k - b_t^k + p_t^g g_t^k = \\ (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k l_t^k + r_t^b b_t^k + g_t^{tr} \tag{B.7}$$

$$\frac{\mu_2}{1-l_t^k} = \frac{\mu_1(1-\tau_t^l)w_t^k}{(1+\tau_t^c)c_t^k} \tag{B.8}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta[1-\delta+(1-\tau_{t+1}^k)r_{t+1}] \tag{B.9}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta(1+r_{t+1}^b) \tag{B.10}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3\gamma^k}{\gamma^k g_t^k + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{B.11}$$

The behaviour of each worker is summarised by:

$$(1+\tau_t^c)c_t^w + p_t^g g_t^w = (1-\tau_t^l)w_t^w l_t^w + g_t^{tr} \tag{B.12}$$

$$\frac{\mu_2}{1-l_t^w} = \frac{\mu_1(1-\tau_t^l)w_t^w}{(1+\tau_t^c)c_t^w} \tag{B.13}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3\gamma^w}{\gamma^w g_t^w + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{B.14}$$

The behaviour of each public employee is summarised by:

$$(1+\tau_t^c)c_t^b + p_t^g g_t^b = (1-\tau_t^l)w_t^g l_t^b + g_t^{tr} \tag{B.15}$$

$$\frac{\mu_2}{1-l_t^b} = \frac{\mu_1(1-\tau_t^l)w_t^g}{(1+\tau_t^c)c_t^b} \tag{B.16}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3\gamma^b}{\gamma^b g_t^b + \gamma(\nu^k g_t^k + \nu^w g_t^w + \nu^b g_t^b)} \tag{B.17}$$

Private firm The problem of the private firm is as in Appendix A above. They are repeated here for convenience:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^k l_t^{f,k} - w_t^w l_t^{f,w} \tag{B.18}$$

$$y_t^f = A^f (k_t^f)^{\alpha_1}(A^k l_t^{f,k} + A^w l_t^{f,w})^{\alpha_2} \tag{B.19}$$

$$w_t^w = \frac{\alpha_2 A^w y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{B.20}$$

$$w_t^k = \frac{\alpha_2 A^k y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{B.21}$$

$$r_t = \frac{\alpha y_t^f}{k_t^f} \tag{B.22}$$

State enterprise Since the state firm does not act optimally, its problem remain the same as in Appendix A above, which for the convenience of the reader is repeated here.

The state-owned firm produces public output according to the production function:

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2} \tag{B.23}$$

where the law of motion of public capital is:

$$k_{t+1}^g = (1-\delta^g) k_t^g + g_t^i \tag{B.24}$$

Government budget constraint

$$g_t^{tr} + (1+r_t^b) v^k b_t^k + v^g (w_t^g l_t^g + g_t^g + g_t^i) = v^g p_t^g y_t^g + v^k b_{t+1}^k +$$

$$+\tau_t^c (v^k c_t^k + v^w c_t^w + v^b c_t^b) + \tau_t^k v^k (r_t k_t^k + \pi_t^k) + \tau_t^l (v^k w_t^k l_t^k + v^w w_t^w l_t^w + v^b w_t^g l_t^b) \tag{B.25}$$

Resource constraint

$$v^k c_t^k + v^w c_t^w + v^b c_t^b + v^k (k_{t+1}^k - (1-\delta) k_t^k) + v^g (g_t^g + g_t^i) = v^f y_t^f \tag{B.26}$$

Public spending policy rules The spending shares s_t^w, s_t^g and s_t^i, namely, spending on public wages,on goods purchased from the private sector and on public investment respectively, all as shares of GDP, are obtained from the following equations:

$$s_t^w = \frac{w_t^g v^b l_t^b}{v^f y_t^f + v^g p_t^g y_t^g} \tag{B.27}$$

$$s_t^g = \frac{v^g g_t^g}{v^f y_t^f + v^g p_t^g y_t^g} \tag{B.28}$$

$$s_t^i = \frac{v^g g_t^i}{v^f y_t^f + v^g p_t^g y_t^g} \tag{B.29}$$

For transfers, we have the following policy rule:

$$g_t^{tr} = s_t^{tr} (v^f y_t^f + v^g p_t^g y_t^g) \tag{B.30}$$

where s_t^{tr} is a policy instrument.

Decentralized equilibrium with user prices

The new equilibrium system consists of 30 equations, (B.1)-(B.30),[22] in 30 endogenous variables which are $\{c_t^k, l_t^k, k_{t+1}^k, g_t^k\}_{t=0}^{\infty}$, $\{c_t^w, l_t^w, g_t^w\}_{t=0}^{\infty}$, $\{c_t^b, l_t^b, g_t^b\}_{t=0}^{\infty}$, $\{r_t, r_t^b, w_t^k, w_t^w, \pi_t^k, w_t^g, p_t^g\}_{t=0}^{\infty}$, $\{y_t^f, k_{t+1}^f, l_t^{f,k}, l_t^{f,w}, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g, l_t^g, g_t^g,$

[22] As above, if we include the budget constraints of all agents, the economy's resource constraint in (B.26) is linearly dependent and hence redundant.

g_t^i, π_t^g, k_{t+1}^g, $g_t^{tr}\}_{t=0}^{\infty}$, and one of the public financing policy variables, $\{b_{t+1}^k$, τ_t^c, τ_t^k, $\tau_t^l\}_{t=0}^{\infty}$, that follows residually to satisfy the government budget constraint in each period. This is for given values of the exogenously set policy instruments as defined in the main text, the fraction of capitalists or self-employed in population (ν^k), the fraction of public employees in populations (ν^b) and initial values for the state variables.

Appendix C: Optimizing state firms

In this appendix, we study the case in which state firms act optimally. In particular, we start by assuming that each state firm maximizes its present value which is the discounted sum of its net cash flows. This will produce a conventional supply function where the quantity supplied increases with the user price. Thus, we treat the state firms in a similar manner to private firms. As in the main text, any profits or losses made by state firms go to the government budget.

Formally, each g chooses its inputs to maximize the discounted sum of its present value which is the discounted sum of its net cash flows:

$$\sum_{t=0}^{\infty} \beta^t (p_t^g y_t^g - w_t^g l_t^g - g_t^g - g_t^i) \tag{C.1}$$

subject to the technology constraints in section 2.[23]

Taking p_t^g as given, the first-order conditions for the three inputs, k_{t+1}^g, l_t^g and g_t^g, are respectively:

$$1 = \beta(1 - \delta^g + \frac{\theta_1 y_{t+1}^g}{k_{t+1}^g} p_{t+1}^g) \tag{C.2a}$$

$$w_t^g = \frac{\theta_2 y_t^g}{l_t^g} p_t^g \tag{C.2b}$$

$$g_t^g = (1 - \theta_1 - \theta_2) y_t^g p_t^g \tag{C.2c}$$

where (C.2a-c) equate the marginal product of each input to its cost. Note that when inputs are chosen optimally, y_t^g is determined by the state firm's optimality conditions, (C.2a-c). The new equilibrium system is as the one presented in Appendix B above with the addition of (C.2a-c). The last column in Table 4 (for reasons of comparison we also repeat the results from Table 2) reports the steady state solution with user charges plus optimizing state firms. As in all cases studied, at the steady state, the residually determined fiscal policy instrument is the labor tax rate. As can seen, the main results remain unchanged as reported in the robustness subsection 4.4 in the main text.

Table 4 here: Steady state solutions with optimizing state firms

[23] We use the same time discount factor for private agents and state firms. Alternatively we could assume that the state firm's discount factor is the intertemporal marginal rate of substitution of a "representative household". These assumptions affect the quantitative effects along the transition path but do not affect our main results regarding policy regimes. We thus use a simple modeling here.

In the case in which the state firms act optimally as cost minimizers rather than as profit maximizers their output target can be, for instance, the level of output that profit maximizing state firms would have chosen or the level that a social planner would have chosen. We report that even in this case the main results, which are available upon request, continue to hold.

Appendix D: Adding productivity enhancing services

In this appendix, we allow the publicly provided good to enter the private firm's production function as an external factor a la Barro (1990). For notational simplicity, we will present the model by setting $\gamma = 0$ (however, we report that our numerical results do not depend on this).

What changes relative to Appendix A

In the SQ economy, and since the publicly provided good is freely available to everybody, the only change concerns the private production function which now becomes:

$$y_t^f = A(k_t^f)^{\alpha_1}(A^k l_t^{f,k} + A^w l_t^{f,w})^{\alpha_2}(\frac{\overline{y}_t^g}{v^f})^{1-\alpha_1-\alpha_2} \tag{D.1}$$

where the per capita amount of the publicly provided productive service, $\overline{y}_t^g$, is taken as given by private firms. Therefore, in this case, the equilibrium system of the SQ economy is similar to the one presented in Appendix A apart from the new production function (D.1) which replaces equation (A.15). In our numerical solutions, the parameter α_2 is set equal to 0.64, so as the exponent of the publicly provided service in private firms' production function, $1 - \alpha_1 - \alpha_2$, is equal to 0.03 which is a rather common value in the relevant business-cycle literature.

What changes relative to Appendix B

By contrast, in the market-based regime, private firms (like households in the case of utility-enhancing goods/services) are free to choose the amount of the publicly provided service they wish to use by paying the user price, p_t^g. In particular, in each period, each private firm, f, produces y_t^f and its profit is:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^k l_t^{f,k} - w_t^w l_t^{f,w} - p_t^g g_t^f \tag{D.2}$$

where g_t^f is the amount of the publicly provided productive service used and purchased by the firm.

The production function is:

$$y_t^f = A^f(k_t^f)^{\alpha_1}(A^k l_t^{f,k} + A^w l_t^{f,w})^{\alpha_2}(g_t^f)^{1-\alpha_1-\alpha_2} \tag{D.3}$$

Each firm acts competitively maximizing (D.2) subject to (D.3) in each period. The first-order conditions for capital and labour inputs remain the same as in Appendices A and B. However, now, we have an extra first-order condition for the amount of the publicly provided productive service deamanded. In particular,

$$p_t^g = \frac{(1 - \alpha_1 - \alpha_2)y_t^f}{g_t^f} \tag{D.4}$$

The market-clearing condition in the market for the impure publicly provided productive service now changes to:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b + N^f g_t^f \tag{D.5a}$$

or in terms of population fractions:

$$v^g y_t^g = v^k g_t^k + v^w g_t^w + v^b g_t^b + v^f g_t^f \tag{D.5b}$$

As above, we can focus on two cases: the case in which state firms do not act optimally and the more general case in which they do. In the latter more general case, the equilibrium system consists of 34 equations, (B.1)-(B.5), (B.7)-(B.17), (B.20)-(B.30), (C.2a-c) (presented in Appendices B and C above), (D.2), (D.3), (D.4) and (D.5b), in 34 endogenous variables which are $\{c_t^k,\, l_t^k,\, k_{t+1}^k,\, g_t^k\}_{t=0}^{\infty}$, $\{c_t^w,\, l_t^w,\, g_t^w\}_{t=0}^{\infty}$, $\{c_t^b,\, l_t^b,\, g_t^b\}_{t=0}^{\infty}$, $\{r_t,\, r_t^b,\, w_t^k,\, w_t^w,\, \pi_t^k,\, w_t^g,\, p_t^g\}_{t=0}^{\infty}$, $\{y_t^f,\, k_{t+1}^f,\, l_t^{f,k},\, l_t^{f,w},\, g_t^f,\, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g,\, l_t^g,\, g_t^g,\, g_t^i,\, \pi_t^g,\, k_{t+1}^g,\, s_t^w,\, s_t^g,\, s_t^i,\, g_t^{tr}\}_{t=0}^{\infty}$, and one of the public financing policy variables, $\{b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l\}_{t=0}^{\infty}$, that follows residually to satisfy the government budget constraint in each period. That is, in the latter case, the public spending shares $\{s_t^w,\, s_t^g,\, s_t^i\}_{t=0}^{\infty}$ become endogenous variables determined by the state firm's optimality conditions, (C.2a-c). In the former case, we simply ignore (C.2a-c). Numerical solutions of these equilibrium systems and their comparison to SQ are in Table 5. The main results do not change as reported in the robustness subsection 4.4 in the main text.

Table 5 here: Adding productivity enhancing services

Appendix E: Adding work-complement services

In this appendix, we assume that the publicly provided good is a work-complement in the sense that it enhances the productivity of households' work time. For notational simplicity, we will again set $\gamma = 0$, although our results do not depend on this.

Market-clearing conditions

The market-clearing conditions are again:

$$v^f \pi_t^f = v^k \pi_t^k \tag{E.1}$$

$$v^f k_t^f = v^k k_t^k \tag{E.2}$$

$$v^f l_t^{f,w} = v^w l_t^w (g_t^w)^\eta \tag{E.3}$$

$$v^f l_t^{f,k} = v^k l_t^k (g_t^k)^\eta \tag{E.4}$$

$$v^g l_t^g = v^b l_t^b (g_t^b)^\eta \tag{E.5}$$

Moreover, since we have a new market (i.e. the market for the impure publicly provided good), its market-clearing condition is:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b$$

or written in terms of population fractions:

$$v^g y_t^g = v^k g_t^k + v^w g_t^w + v^b g_t^b \tag{E.6}$$

Equilibrium system

Households The budget constraint of each capitalist is now:

$$(1+\tau_t^c)c_t^k + k_{t+1}^k - (1-\delta)k_t^k + b_{t+1}^k - b_t^k + p_t^g g_t^k =$$

$$(1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k l_t^k (g_t^k)^\eta + r_t^b b_t^k + g_t^{tr} \tag{E.7}$$

where $(g_t^k)^\eta$ formalizes the work-complement nature of the publicly provided good/service with $0 \leq \eta \leq 1$.

Then, the capitalist's optimality conditions are:

$$\frac{\mu_2}{1-l_t^k} = \frac{\mu_1(1-\tau_t^l)w_t^k(g_t^k)^\eta}{(1+\tau_t^c)c_t^k} \tag{E.8}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta[1-\delta+(1-\tau_{t+1}^k)r_{t+1}] \tag{E.9}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta(1+r_{t+1}^b) \tag{E.10}$$

$$g_t^k = \left(\frac{p_t^g}{\eta(1-\tau_t^l)w_t^k l_t^k}\right)^{\frac{1}{\eta-1}} \tag{E.11}$$

Similarly, we have for each worker:

$$(1+\tau_t^c)c_t^w + p_t^g g_t^w = (1-\tau_t^l)w_t^w l_t^w (g_t^w)^\eta + g_t^{tr} \tag{E.12}$$

$$\frac{\mu_2}{1-l_t^w} = \frac{\mu_1(1-\tau_t^l)w_t^w(g_t^w)^\eta}{(1+\tau_t^c)c_t^w} \tag{E.13}$$

$$g_t^w = \left(\frac{p_t^g}{\eta(1-\tau_t^l)w_t^w l_t^w}\right)^{\frac{1}{\eta-1}} \tag{E.14}$$

and for each public employee:

$$(1+\tau_t^c)c_t^b + p_t^g g_t^b = (1-\tau_t^l)w_t^g l_t^b (g_t^b)^\eta + g_t^{tr} \tag{E.15}$$

$$\frac{\mu_2}{1-l_t^b} = \frac{\mu_1(1-\tau_t^l)w_t^g(g_t^b)^\eta}{(1+\tau_t^c)c_t^b} \tag{E.16}$$

$$g_t^b = \left(\frac{p_t^g}{\eta(1-\tau_t^l)w_t^g l_t^b}\right)^{\frac{1}{\eta-1}} \tag{E.17}$$

Private firm The problem of the private firm is as in Appendices A and B above. They are repeated here for convenience:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^k l_t^{f,k} - w_t^w l_t^{f,w} \tag{E.18}$$

$$y_t^f = A^f (k_t^f)^{\alpha_1} (A^k l_t^{f,k} + A^w l_t^{f,w})^{\alpha_2} \tag{E.19}$$

$$w_t^w = \frac{\alpha_2 A^w y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{E.20}$$

$$w_t^k = \frac{\alpha_2 A^k y_t^f}{(A^k l_t^{f,k} + A^w l_t^{f,w})} \tag{E.21}$$

$$r_t = \frac{\alpha y_t^f}{k_t^f} \tag{E.22}$$

State enterprise When acting optimally, the state firm maximizes the discounted sum of its intertemporal profits:

$$\sum_{t=0}^{\infty} (\beta)^t (p_t^g y_t^g - w_t^g l_t^g - g_t^g - g_t^i) \tag{E.23}$$

subject to:

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2} \tag{E.24}$$

$$k_{t+1}^g = (1 - \delta^g) k_t^g + g_t^i \tag{E.25}$$

Taking p_t^g as given, the first-order conditions for the three inputs, k_{t+1}^g, l_t^g and g_t^g are respectively:

$$1 = \beta^g (1 - \delta^g + \frac{\theta_1 y_{t+1}^g}{k_{t+1}^g} p_{t+1}^g) \tag{E.26}$$

$$w_t^g = \frac{\theta_2 y_t^g}{l_t^g} p_t^g \tag{E.27}$$

$$g_t^g = (1 - \theta_1 - \theta_2) y_t^g p_t^g \tag{E.28}$$

Government budget constraint

$$g_t^{tr} + (1 + r_t^b) v^k b_t^k + v^g (w_t^g l_t^g + g_t^g + g_t^i) = v^g p_t^g y_t^g + v^k b_{t+1}^k +$$

$$+ \tau_t^c (v^k c_t^k + v^w c_t^w + v^b c_t^b) + \tau_t^k v^k (r_t k_t^k + \pi_t^k) + \tau_t^l (v^k w_t^k l_t^k (g_t^k)^\eta$$

$$+ v^w w_t^w l_t^w (g_t^w)^\eta + v^b w_t^g l_t^b (g_t^b)^\eta) \tag{E.29}$$

Resource constraint

$$v^k c_t^k + v^w c_t^w + v^b c_t^b + v^k(k_{t+1}^k - (1-\delta)k_t^k) + v^g(g_t^g + g_t^i) = v^f y_t^f \qquad \text{(E.30)}$$

Public spending policy rules Given the optimal choices of $\{k_{t+1}^g, l_t^g, g_t^g\}_{t=0}^{\infty}$ in (D.26)-(D.28), the spending shares s_t^w, s_t^g and s_t^i follow from the following equations:

$$s_t^w = \frac{w_t^g v^b l_t^b (g_t^b)^\eta}{v^f y_t^f + v^g p_t^g y_t^g} \qquad \text{(E.31)}$$

$$s_t^g = \frac{v^g g_t^g}{v^f y_t^f + v^g p_t^g y_t^g} \qquad \text{(E.32)}$$

$$s_t^i = \frac{v^g g_t^i}{v^f y_t^f + v^g p_t^g y_t^g} \qquad \text{(E.33)}$$

For transfers, we have the following policy rule:

$$g_t^{tr} = s_t^{tr}(v^f y_t^f + v^g p_t^g y_t^g) \qquad \text{(E.34)}$$

Decentralized equilibrium

We can again study two cases: the case in which state firms do not act optimally and the more general case in which they do. In the former case, we just add a new market and a new price (the user price) which means that we ignore the state firm's first-order conditions (E.26), (E.27) and (E.28). In the latter more general case, the equilibrium system consists of 33 equations, (E.1)-(E.29), and (E.31)-(E.34), in 33 endogenous variables which are $\{c_t^k, l_t^k, k_{t+1}^k, g_t^k\}_{t=0}^{\infty}$, $\{c_t^w, l_t^w, g_t^w\}_{t=0}^{\infty}$, $\{c_t^b, l_t^b, g_t^b\}_{t=0}^{\infty}$, $\{r_t, r_t^b, w_t^k, w_t^w, \pi_t^k, w_t^g, p_t^g\}_{t=0}^{\infty}$, $\{y_t^f, k_{t+1}^f, l_t^{f,k}, l_t^{f,w}, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g, l_t^g, g_t^g, g_t^i, \pi_t^g, k_{t+1}^g, s_t^w, s_t^g, s_t^i, g_t^{tr}\}_{t=0}^{\infty}$, and one of the public financing policy variables, $\{b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l\}_{t=0}^{\infty}$, that follows residually to satisfy the government budget constraint in each period. That is, in the latter case, the public spending shares $\{s_t^w, s_t^g, s_t^i\}_{t=0}^{\infty}$, become endogenous variables being determined by the state firm's optimality conditions, (E.26), (E.27) and (E.28). Numerical solutions and their comparison to SQ can be found in Table 5. Again the main results do not change as reported in the robustness subsection 4.4 in the main text.

Table 6: Adding work-complement services

Tables

Table 1: Baseline parameterization

Parameters and policy instruments	Description	Value
α_1	Capital share in private production	0.33
α_2	Labor share in private production	0.67
θ_1	Capital share in public production	0.1078
θ_2	Labor share in public production	0.5690
δ	Private capital depreciation rate	0.05
δ^g	Public capital depreciation rate	0.05
β	Time discount rate	0.9
μ_1	Preference on private consumption	0.35
μ_2	Preference on leisure	0.6
s^w	Public wage bill (% GDP)	0.132
s^g	Public purchases of private goods (% GDP)	0.075
s^i	Public investment (% GDP)	0.025
s^{tr}	Government transfers (% GDP)	0.217
τ^c	Tax rate on consumption	0.1938
τ^k	Tax rate on capital income	0.2903
τ^l	Tax rate on labor income	0.378
B/Y	Public debt (% GDP)	0.9
ν^k	Capitalists (% of population)	0.148
ν^b	Public employees (% of population)	0.215
ν^w	Private workers (% of population)	$1-\nu^k-\nu^w$
ν^f	Private firms (% of population)	$\nu^k+\nu^w$
ν^g	Public firms (% of population)	ν^b
γ	Degree of social externalities	0.15
A^f	TFP in private firms' production function	1
A^g	TFP in state firms' production function	1
A^k	Labor productivity of capitalists	3
A^w	Labor productivity of private workers	1

Table 2: Steady state solutions

Variable	Status Quo	User Prices (Direct channel)	User Prices (Full effect)
c^k	0.6521	0.6415	0.7001
l^k	0.1301	0.1442	0.1599
k^k	2.8656	3.0545	3.1765
g^k	0.0470	0.1551	0.1591
y^k	0.7954	0.7942	0.8589
c^w	0.1774	0.1735	0.1915
l^w	0.2902	0.3055	0.3108
g^w	0.0470	0.0361	0.0375
y^w	0.1774	0.1735	0.1915
c^b	0.1483	0.1580	0.1771
l^b	0.2726	0.2946	0.3024
g^b	0.0470	0.0322	0.0341
y^b	0.1483	0.1580	0.1771
y^w/y^k	0.2230	0.2185	0.2229
y^b/y^k	0.1864	0.1989	0.2062
y	0.2917	0.3465	0.3625
y^f	0.3717	0.3961	0.4120
y^g	0.2187	0.2462	0.2547
p^g	$-$	0.6708	0.7138
c/y	0.8273	0.6912	0.7273
τ^l	0.3652	0.3652	0.2943
u^k	-0.3861	-0.3394	-0.3188
u^w	-0.9639	-0.9877	-0.9561
u^b	-1.0118	-1.0159	-0.9801
u	-0.8887	-0.8979	-0.8669

Table 3a: Mix of user prices and public finance

Variable	Status Quo	$\mu = 0.15$	$\mu = 0.50$	$\mu = 0.75$	$\mu = 1$
c^k	0.6521	0.3801	0.6412	0.6803	0.7001
l^k	0.1301	0.0209	0.1368	0.1523	0.1599
k^k	2.8656	1.8499	2.9576	3.1043	3.1765
g^k	0.0470	0.1346	0.1582	0.1589	0.1591
y^k	0.7954	0.4725	0.7890	0.8355	0.8589
c^w	0.1774	0.0994	0.1738	0.1855	0.1915
l^w	0.2902	0.2313	0.2978	0.3065	0.3108
g^w	0.0470	0.0301	0.0369	0.0373	0.0375
y^w	0.1774	0.0994	0.1738	0.1855	0.1915
c^b	0.1483	0.1031	0.1688	0.1746	0.1771
l^b	0.2726	0.2390	0.2942	0.2997	0.3024
g^b	0.0470	0.0315	0.0356	0.0347	0.0341
y^b	0.1483	0.1031	0.1688	0.1746	0.1771
y^w/y^k	0.2230	0.2105	0.2203	0.2220	0.2229
y^b/y^k	0.1864	0.1031	0.2139	0.2090	0.2062
y	0.2917	0.3285	0.3728	0.3667	0.3625
y^f	0.3717	0.2399	0.3835	0.4026	0.4120
y^g	0.2187	0.2136	0.2539	0.2548	0.2547
p^g	$-$	3.0523	1.3144	0.9254	0.7138
c/y	0.8273	0.4316	0.6489	0.6992	0.7273
τ^l	0.3652	0.6712	0.37094	0.3203	0.2943
u^k	-0.3861	-0.4489	-0.3334	-0.3234	-0.3188
u^w	-0.9639	-1.1304	-0.9793	-0.9636	-0.9561
u^b	-1.0118	-1.1218	-0.9881	-0.9818	-0.9801
u	-0.8887	-1.0277	-0.8856	-0.8804	-0.8669
s^w	0.132	0.132	0.132	0.132	0.132

Table 3b: Minimum quantity is needed by everybody

Variable	Status Quo	$\overline{s}_t = 0.03$	$\overline{s}_t = 0.05$	$\overline{s}_t = 0.08$	$\overline{s}_t = 0.10$	$\overline{s}_t = 0.15$
g^k	0.0470	0.1481	0.1409	0.1300	0.1228	0.1046
g^w	0.0470	0.0266	0.0194	0.0085	0.0012	-0.0168
g^b	0.0470	0.0232	0.0159	0.0050	-0.0021	-0.0202
$\overline{g}$	0	0.0108	0.0181	0.0290	0.0362	0.0543

Table 4: Steady state solutions with optimizing state firms

Variable	Status Quo	User Prices (Direct channel)	User Prices (Full effect)	Optimizing state firms
c^k	0.6521	0.6415	0.7001	0.8477
l^k	0.1301	0.1442	0.1599	0.1929
k^k	2.8656	3.0545	3.1765	3.4369
g^k	0.0470	0.1551	0.1591	0.1034
y^k	0.7954	0.7942	0.8589	1.0196
c^w	0.1774	0.1735	0.1915	0.2372
l^w	0.2902	0.3055	0.3108	0.3223
g^w	0.0470	0.0361	0.0375	0.0252
y^w	0.1774	0.1735	0.1915	0.2372
c^b	0.1483	0.1580	0.1771	0.1468
l^b	0.2726	0.2946	0.3024	0.2636
g^b	0.0470	0.0322	0.0341	0.01366
y^b	0.1483	0.1580	0.1771	0.1468
y^w/y^k	0.2230	0.2185	0.2229	0.2326
y^b/y^k	0.1864	0.1989	0.2062	0.1440
y	0.2917	0.3465	0.3625	0.3956
y^f	0.3717	0.3961	0.4120	0.4457
y^g	0.2187	0.2462	0.2547	0.1597
p^g	−	0.6708	0.7138	1.3307
c/y	0.8273	0.6912	0.7273	0.7788
τ^l	0.3652	0.3652	0.2943	0.1104
u^k	−0.3861	−0.3394	−0.3188	−0.2974
u^w	−0.9639	−0.9877	−0.9561	−0.9116
u^b	−1.0118	−1.0159	−0.9801	−1.0538
u	−0.8887	−0.8979	−0.8669	−0.8513
s^w	0.132	0.132	0.132	0066

Table 5: Adding productivity-enhancing services

Variable	Status Quo	User Prices	Optimizing State Firms
c^k	0.5891	0.5940	0.6782
l^k	0.1101	0.1536	0.1806
k^k	2.5579	2.7148	2.8402
g^k	0.0447	0.1150	0.0861
y^k	0.7170	0.7297	0.8202
c^w	0.1568	0.1609	0.1872
l^w	0.2894	0.3122	0.3215
g^w	0.0447	0.0312	0.0238
y^w	0.1568	0.1609	0.1872
c^b	0.1320	0.1564	0.1406
l^b	0.2722	0.3093	0.2908
g^b	0.0447	0.0303	0.0178
y^b	0.1320	0.1564	0.1406
y^w/y^k	0.2187	0.2205	0.2282
y^b/y^k	0.1841	0.2143	0.1715
y	0.2604	0.3229	0.3405
y^f	0.3318	0.3521	0.3684
y^g	0.2081	0.2455	0.1775
p^g	$-$	0.8809	1.3441
c/y	0.8273	0.6938	0.7339
τ^l	0.3681	0.2773	0.1306
u^k	-0.4105	-0.3905	-0.3780
u^w	-1.0088	-1.0374	-1.0062
u^b	-1.0548	-1.0463	-1.0940
u	-0.9301	-0.9436	-0.9321
s^w	0.132	0.132	0.0857
s^g	0.075	0.075	0.0487
s^i	0.025	0.025	0.0050

Table 6: **Adding work-complement services**

Variable	Status Quo	User Prices	Optimizing State Firms
c^k	0.2794	0.3370	0.3793
l^k	0.1196	0.1377	0.1794
k^k	1.2346	1.4446	1.4284
g^k	0.0203	0.0347	0.0247
y^k	0.3412	0.4092	0.4507
c^w	0.0768	0.0840	0.0924
l^w	0.2744	0.3028	0.3128
g^w	0.0203	0.0235	0.0125
y^w	0.0768	0.0840	0.0924
c^b	0.0639	0.0772	0.0603
l^b	0.2570	0.2924	0.2532
g^b	0.0203	0.0205	0.0061
y^b	0.0639	0.0772	0.0603
y^w/y^k	0.2250	0.2054	0.2051
y^b/y^k	0.1873	0.1886	0.1338
y	0.1257	0.1640	0.1647
y^f	0.1601	0.1874	0.1853
y^g	0.0944	0.1141	0.0602
p^g	−	0.6880	1.4889
c/y	0.8273	0.7319	0.7771
τ^l	0.3652	0.2978	0.1111
u^k	−0.5291	−0.4770	−0.4678
u^w	−1.1069	−1.1013	−1.0772
u^b	−1.1558	−1.1214	−1.1726
u	−1.0319	−1.0132	−1.0075
s^w	0.132	0.132	0.0666
s^g	0.075	0.075	0.0378
s^i	0.025	0.025	0.0039

Figures

Figure 1
From SQ to user prices -Aggregate outcomes
(% deviation from initial steady state solution)

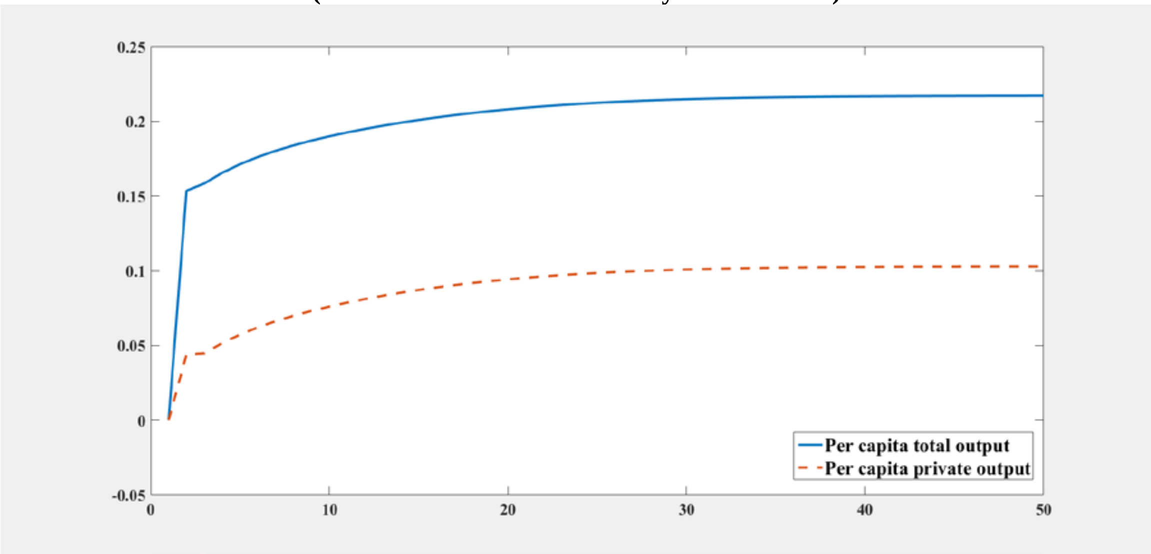

Figure 2
From SQ to user prices - Net incomes
(% deviation from initial steady state solution)