CESifo WORKING PAPERS

8751
2020
December 2020

Postponing Retirement and Social Security in a Two Sector Model

Partha Sen

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 8751*

Postponing Retirement and Social Security in a Two Sector Model

Abstract

Pay-as-you-go (PAYG) social security schemes in the OECD countries are facing solvency problems, as people are living longer and birth rates have declined. Postponing the full retirement age (FRA), when retirees are entitled to full pension, has been proposed as a solution. This effectively lowers the payroll tax rate since pension is paid only in the post-FRA period. In a two-period two-sector overlapping generations model, I show that this shift lowers savings (because a part of the expected old age income is consumed in the first period), as employment increases. In the transition to the new steady state, capital is decumulated and the wage rate falls. Contrast this with a reduction of the payroll tax rate where the initial old suffer reduced consumption, but the young have higher post-tax income and this spurs capital accumulation.

JEL-Codes: H550.

Keywords: overlapping generations, social security reform, postponing retirement.

Partha Sen
Centre for Development Economics
Delhi School of Economics
India – Delhi 110007
partha@econdse.org

An earlier version of this paper was presented at the Singapore Economic Review Conference, August 2019 and TU Chemnitz. I thank Yuki Kitamura, Charles Horioka, Thomas Kuhn and Jochen Hartwig for comments. A very special “thank you” to Thomas Breuer, with whom I had long discussions on retirement in Europe.

1. INTRODUCTION

Social security systems became pervasive in the advanced capitalist countries after the Second World War. Although there were other schemes that formed a part of these (e.g. disability insurance), it was the old-age pensions that was the main focus of any debate on the social security systems. The pension schemes were mainly of the unfunded or pay-as-you-go (PAYG) type, where a payroll tax on the working population was used to pay the pensions of the retired.

The PAYG schemes were instituted at a time of high wage and population growth. Both of these factors have disappeared over time in the OECD countries, and with longer lives of the citizens, social security systems are facing a solvency crunch.[1] Hence some modification, or a "reform", of the PAYG system seems to be warranted.

In the literature, three features of a PAYG scheme have been highlighted. The first, that contributes to their popularity and poses problems for change, is the provision of "insurance" that private markets are unwilling to offer (due to e.g. moral hazard) (see Diamond (1977), (2004) and Aaron (2011)). Second, because the PAYG schemes are financed through payroll taxes, labor supply is distorted. Finally, it reduces the need to save for old age, and, hence, lowers capital accumulation.[2]

A specific reform lowers the payroll tax rate, and (given the wage rate) lowers the transfers from the working population (the "young") to the retired (the "old") in that period. In its extreme form there would be a "privatization" of social security i.e. moving from the PAYG system to a fully-funded system. This lowering of transfers is, in effect, a "reneging" on an

---

[1] Between 1970 and 2020 in the OECD countries the average life expectancy at 65, increased by six years, and retirement was brought forward by three years. (OECD (2019)).

[2] See e.g. Casamatta, Cremer and Pestieau (2000), Krueger and Kubler (2006), Nishiyama and Smetters (2007).

implicit promise to the retired. After all, that generation had paid into the PAYG scheme when young, and are now being told that there is a lower pension for them because of a solvency problem.[3]

A cut in the payroll tax rate is also expected to increase saving and capital accumulation. In effect this ameliorates the reduced saving (when the economy is dynamically efficient) due to the PAYG system.[4]

Given that the retirees in the period of the introduction of the reform are hit, there are a number of papers that try to see if there is a way to compensate this generation and at the same time, scale down the PAYG system.

One way of removing the burden on the retired generation(s) in the period of reform, is to allow them to work for a fraction of their retirement period(s) by deferring the age of retirement when full pension benefits (called Full Retirement Age (FRA)) are received. This would allow the income levels of the retirees to be protected, while reducing the burden on the social security systems.

In this paper, we consider the implications of rolling back the FRA and keep the payroll tax rate unchanged (while allowing the old to work). Social security payment is made only for the fraction of the old-age spent in retirement (i.e. not working). This effectively reduces the payroll tax on the young, since now the given tax rate is multiplied by the fraction of the second period spent in retirement. We contrast this with a reform that allows full retirement but lowers the pay-roll tax.

We address the general equilibrium implications of the deferring of FRA in a PAYG social security framework. We assume there is no uncertainty, so insurance is not an issue; and

[3] This is certainly the view of those opposing President Macron's reform in France.

[4] The evidence, though, of such a policy change on saving across countries is ambiguous. Samwick (2000) concludes, that in his sample only Chile saw a rise in saving with a move towards a system based on defined contributions.

that labor is supplied inelastically (both in youth and old age), and so the distortion introduced in the labor-leisure choice is absent. A two-sector two-period overlapping structure is used. Solow (2005) had lamented the lack of two (or more) sector dynamic macro models.[5] A two-sector set-up adds to our understanding of social security issues because this allows for both a demand channel (with delayed FRA, the young tend to save more out of a higher take-home wage, but less in anticipation of income in old age), and a goods-supply channel because the postponed retirement increases labor supply in every period. Resources then move towards the production of the labor-intensive good (in our model the consumption good).[6]

With a delay in FRA, in initial period(s), the supply of consumption goods has to increase since the old spend all their income on consumption. This results in lower saving (due to life cycle reasons) and a fall in wages (as the pool of workers grows). The lower saving reduces capital accumulation. The future generations are hurt by this lower capital accumulation. In this paper while there are no bequests or inheritance (i.e. there is an absence of intergenerational caring) but with capital as a store of value, there are intergenerational spillovers of postponing the FRA. If a generation saves less, then in the subsequent periods there would be a lower capital stock and wages.

Empirically, the FRA is an important reference point because, while it is possible that depending on incentives, agents may prefer to retire before reaching that age, there is ample evidence that people do not do so (see Behagel and Blau (2012) for the USA and Seibold (2019) for Germany).[7]

There are a few papers analyzing a change in the mandatory retirement age. Zhang and Zhang (2005) address the effect of this on the accumulation of human capital (in their paper

[5] See Sen (2016) and Fedotenkov et al (2019), for a sample of papers of two sector models that discuss social security.

[6] A reform involving the cutting of the payroll tax has the opposite effect—saving increases and so does the demand for the investment good.

[7] The Eurobarometer (2001) surveys also suggest that workers are unwilling to accept an increase in their retirement age. Most reform proposals that would increase in the retirement age allow for long transition periods.

human capital is accumulated via having more children); but they do not deal with social security issues. Miyazaki (2014) looks at postponing FRA in a one sector overlapping generations model.

A distinct, but analytically related, literature deals with social security with aging and endogenous retirement; see e.g. Cremer and Pestieau (2003), Galasso (2008), Cipriani (2018) and Hirono and Mino (2019). In these papers, retirement is decided by the leisure-labor choice in old age, and longevity by a parameter that tells us what fraction of the second period an individual is going to live. Aging, typically, increases the steady state level of capital and increases the retirement age. Some papers have a representative member of the older generation, whereas in others heterogeneity of workers is allowed for. In Hirono and Mino (2019), also the old are heterogeneous and have different productivity levels. Those who have a productivity level above a cutoff (determined endogenously), work, and those below it retire.

2. THE MODEL

The closed economy consists of overlapping generations of individuals (or households). No individual is altruistically linked to any future generations i.e., there are no bequests or inheritances. Every individual lives for two periods. In the first period of its life (youth) the individual supplies one unit of labor, pays the social security contribution via a payroll tax, and saves for the second period (old age). In old-age, the individual consumes the saving from the first period plus the return on these savings and the receipts from the social security. We start off by focussing on the usual scenario where in old age everyone is retired. We then allow the old to work for a part of the second period ("postponing retirement"). The population is assumed, without loss of generality, to be constant. We shall study the properties of the model by linearizing it around the initial steady state.

The Households

The representative household born in time period t maximizes the following utility function:

$$U_t \equiv \mathrm{U}(D_t^1, D_{t+1}^2) \qquad \text{t=0,1,2….} \qquad (1)$$

where $D_t^1 (D_{t+1}^2)$ is the consumption when young (old) of a household born in period t.

The utility function U(.) is increasing and strictly concave in its arguments and satisfies the Inada conditions. Both period consumptions are assumed to be normal.

Its lifetime budget constraint is

$$W_t(1-\tau(1-\alpha)) + \left(\frac{(\tau(1-\alpha)+\alpha)W_{t+1}}{\chi_{t+1}}\right) = D_t^1 + \left(\frac{1}{\chi_{t+1}}\right) D_{t+1}^2 \qquad (2)$$

Where W_t is the wage rate in time period t (in terms of the consumption good, which is the numeraire), population is constant over time and its size is normalized to 2—each generation in every period is of size 1. χ_{t+1} is the own interest factor on one period consumption loans between t and t+1, and τ is the payroll tax. We assume that the system is "dynamically efficient" so $\chi_{t+1} \geq 1$ (for all t). Initially all the agents in the second period of their lives are fully retired. Then we look at a binding postponement of the FRA by α fraction of old age. Once we allow the old to work, the labor force will increase from 1 to 1+α.

In equation (2), we have used the fact that a PAYG scheme pays the proceeds of the payroll tax to the old in that period. Hence the young in period t expect to receive (1-α) τW_{t+1} per worker in their old age. This expected future transfer is discounted to date t by using the discount factor χ_{t+1}.

The maximization yields the Euler equation:

$$\frac{\partial \mathrm{U}(.)}{\partial D_t^1} = \chi_{t+1} \frac{\partial U(.)}{\partial D_{t+1}^2} \tag{3}$$

Using equations (2) and (3) we derive the demand functions

$$D_t^1 = D^1(W_t(1-\tau(1-\alpha)) + \left(\frac{(\tau(1-\alpha)+\alpha)W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4a}$$

and

$$D_{t+1}^2 = D^2(W_t(1-\tau(1-\alpha)) + \left(\frac{(\tau(1-\alpha)+\alpha)W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4b}$$

The saving function is given by:

$$S_t \equiv W_t\big(1-\tau(1-\alpha)\big) - D^1(W_t(1-\tau(1-\alpha)) + \left(\frac{(\tau(1-\alpha)+\alpha)W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4c}$$

Savings are assumed to be an non-decreasing function of the real rate of interest.[8] An increase in the interest rate works through three channels—(i) the substitution effect causing a

---

[8] See e.g. Azariadis (1993). But this is not always assumed in the literature; see e.g. Casamatta et al. (2000) where they assume the contrary.

postponement of consumption; (ii) an income effect that would increase consumption in both periods; and (iii) by reducing the present value of future social security receipts it would reduce consumption in both periods.

The Firms

The two goods—a pure consumption good (C) and a pure investment good (I)--are produced under conditions of constant returns to scale using the two inputs, capital and labor. $K^C(K^I)$ is the capital employed in the consumption goods (investment goods) sector. Similarly, for L^C (L^I) is the labor employed in the consumption goods (investment goods) sector. All inputs are mobile between sectors instantaneously. The production functions are given by

$$C_t = \mathrm{F}(K_t^C, L_t^C) \tag{5a}$$

$$I_t = \mathrm{G}(K_t^I, L_t^I) \tag{5b}$$

The functions F(.) and G(.) have positive but diminishing marginal productivities and are homogeneous of degree one. They are also assumed to satisfy the Inada conditions.

The consumption good is assumed to be labor-intensive at all relative factor prices. Two justifications are given for assuming this: first, Galor (1992) has shown that in the other case (i.e. when the consumption good is capital-intensive) there is indeterminacy (multiple perfect foresight paths). And, second, in a two-sector model the old spend all their incomes on consumption goods--a large fraction of these are services and are possibly relatively labor-intensive.

Firms maximize profits with perfect competition in all markets. In equilibrium, the firms set the minimized unit cost equal to the market price of the product (this specification is borrowed from Atkinson and Stiglitz (1980)):

$$a_{LC}W_t + a_{KC}R_t = 1 \quad (6a)$$

$$a_{LI}W_t + a_{KI}R_t = p_t \quad (6b)$$

where a_{ij} is the requirement of the i^{th} input (i = K, L) in the production of the j^{th} good (j = C, I). Note that the a_{ij}'s are functions of the relative factor-prices. The relative price of the investment good in terms of (the numeraire) good C is given by p and R is the (gross) return on capital. We assume capital depreciates completely in the process of production. We have in equilibrium for the consumption interest factor $\chi_{t+1} = R_{t+1}/p_t$.

The Government

Definition: A pay-as-you-go system (PAYG) with defined contribution rate is simply a sequence (τ, E_t)), where both τ and all E_t are strictly positive and where $E_t = \tau W_t$ for all t.

The government collects the payroll taxes from the young in each period and transfers it to the old. Given τ, below when we allow for the old to work for part of their old age, we will talk of an "effective tax rate". This effective tax is the actual payroll tax τ adjusted for the proportion of the old age spent not working (i.e. retired)—given by (1-α), where α is the proportion of old age spent working. Pension is paid only for the latter sub-period. With full retirement (α=0), the effective and actual tax rates coincide; and with no retirement (α=1), the effective tax rate is zero.

$$E_t = W_t\tilde{\tau} = W_t(1-\alpha)\tau \quad (7a)$$

We do not have the old paying pay-roll tax, since in a balanced budget set-up, it would be transferred to them within the period.[9] In a set-up where each old individual supplies labor based on a net of payroll tax wage, τ distorts the labor-leisure choice and the (old) agents do not internalize the effect of their part of the payroll tax collection being "rebated" via a pension. Here, with labor supplied inelastically by the old (as well as the young) --we "pierce the veil" and have the old paying no payroll tax.

A Digression

In a simple two period model with full retirement, the PAYG budget constraint is

$$\tau W_t = E_t \tag{7b}$$

The left hand side is the proceeds from the payroll tax per (young) worker. The right-hand side is the receipt per old person from retirement. (Remember that both generations are of size one.)

Miyazaki's (2014) specification is different. For the time being, assume the old pay no taxes on their labor—more on this anon. He keeps the left hand side unchanged, while introducing delayed retirement by a proportion α of old age. So now:

$$\tau W_t = E_t' \tag{7c}$$

Again, the left hand side is the tax revenue and the right hand side is the total receipt of pension by the older generation per capita. But each older person is eligible for pension in (1-α) of the

[9] Hirono and Mino (2019) also do not have the old paying payroll taxes, although in their model the old are heterogeneous.

period. So pension per capita for retirement is Q_t . And we have $Q_t(1-\alpha) = E_t'$. Thus for the period of retirement, the pension actually goes up! Logically, there is no inconsistency there, except as pointed out in the Introduction above, the raison d'etre for introducing delayed FRA is that otherwise there might be a cut in pension benefits. Comparing the right hand sides of (7b) and (7c), even if FRA is pushed back, and people do not work extra at FRA age, their receipts will equal what they were getting with a postponement of FRA i.e., their lifetime budget is not impinged by the roll-back of FRA.

I divided both sides of equation (7a) by $(1-\alpha)$, so that retirement period being $(1-\alpha)$ effectively makes the payroll tax $\tau(1-\alpha)$. Thus pension for the retirement period remains unchanged. But retirement is $(1-\alpha)$ of old age.

It does matter in the dynamics whether we use (7a) or Miyazaki's (7c), because the young will save more (because the effective payroll tax is lower). In both cases the old-age income causes saving to fall (due to life-cycle reasons). In my case (7a) we will assume (as seems reasonable) that the dissaving due to the old-age earnings outweigh the increased saving due to the reduced effective tax rate.

To see that ignoring the effect of taxing the wages of the old has no effect on the analysis, note that the left hand side becomes $\{(1-\alpha)\tau + \alpha\tau\}\, W_t = E_t''$ (E_t'' is the total pension for the period of retirement). But the second term on the left hand side is transferred within the period to the old—an intra-generational transfer. Since labor is supplied inelastically by both the young and the old (for the latter the FRA is binding), we get back to (Q_t' is the pension per capita):

$$(1-\alpha)Q_t' \quad = (1-\alpha)\tau W_t \qquad \text{(7d)}$$

It is obvious from (7d) (which is the same as (7a)), that $Q_t' = \tau W_t$ that is the pension rate is the same as the full retirement rate; but the payment and the receipts are scaled down by the lower fraction of old age (1-α) for which the pension is payable.

Market-Clearing

In any period, there are two goods markets and two factor markets. By Walras' Law, if three of these are in equilibrium in any period, then so is the fourth one. We thus have

$$a_{LC}C_t + a_{LI}I_t = 1 + \alpha \tag{8a}$$

$$a_{KC}C_t + a_{KI}I_t = K_t \tag{8b}$$

$$W_t\big(1-\tau(1-\alpha)\big) - D^1\left(\left(W_t(1-\tau(1-\alpha)) + \frac{(\tau(1-\alpha)+\alpha)W_{t+1})}{\chi_{t+1}}\right), \chi_{t+1}\right) = p_t\mathrm{I}(p_t, K_t; \alpha) \tag{8c}$$

Equations (8a), (8b) and (8c) are the market-clearing conditions for the labor, capital and investment goods markets respectively. In equations (8c) (and in (9a) and (9b) below) we have incorporated the assumption of one hundred per cent depreciation. The variable C_t is the product per worker of the consumption good, I_t is the output per worker of the investment good and K_t is the capital stock (all in time period t).

Dynamics

The dynamics of the capital stock comes from the fact that the investment good this period will be the next period's capital stock.

$$K_{t+1} = I(p_t, K_t;\ \alpha) \tag{9a}$$

Equation (8c) using (4c)) gives us the other dynamic equation (9b):

$$W(p_t)(1-\tau(1-\alpha)) - D^1\left(\left(W(p_t)(1-\tau(1-\alpha)) + \frac{(\tau(1-\alpha)+\alpha)W(p_{t+1})p_t}{R(p_{t+1})}\right), \frac{R(p_{t+1})}{p_t}\right) = p_t\mathrm{I}(p_t, K_t; \alpha) \tag{9b}$$

3. GENERAL EQUILIBRIUM

Competitive Equilibrium

Definition: A competitive equilibrium is, given τ and the initial stock of capital K(0), a sequence of prices and capital stocks $(p_t, K_t)_{t=0}^{\infty}$,of wages and the rental rates $(W_t, R_t)_{t=0}^{\infty}$, and the consumption pairs (D_t^1, D_{t+1}^2) such that:

(i) households maximize utility (equations (2) and (3)),

(ii) firms maximize profits (equations (6a) and (6b)),

(iii) markets clear (equations (8a), (8b) and (8c)),

(iv) and the capital stock dynamics is given by (9a).[10]

Existence of equilibrium for the above system is shown in Galor (1992) and Azariadis (1993).

[10] For the existence of a steady state with a positive capital stock, τ cannot be very "large".

The Dynamical System

Equation (9a) and (8b) can be log-linearized and we obtain a system of two difference equations expressing $\widehat{K}_{t+1}$ and $\hat{p}_{t+1}$ in terms of $\widehat{K}_t$ and $\hat{p}_t$, and (dτ and dα) (a ^ over a variable is the percentage deviation from the steady state):

$$\begin{bmatrix} \widehat{K}_{t+1} \\ \hat{p}_{t+1} \end{bmatrix} = A \begin{bmatrix} \widehat{K}_t \\ \hat{p}_t \end{bmatrix} + Hd\tau + Gd\alpha \qquad (10)$$

The elements of matrix A and H are given in Appendix 2. Matrix A has, under some reasonable assumptions, two positive roots, lying on either side of unity (see Appendix 2).

We can draw a phase diagram (Figure 1) for a given τ, and α=0. Both $\hat{K}_{t+1} - \hat{K}_t = 0$ (the KK curve) and $\hat{p}_{t+1} - \hat{p}_t = 0$ (the IS curve) are downward sloping, with the latter curve being the steeper of the two. The horizontal arrows point away from the KK curve and the vertical arrows point away from the IS curve. The steady state is a saddle point and the stable arm is flatter than the IS line. The adjustment path is monotonic, given the signs of the roots.

Steady State

The steady state of this economy is obtained by setting $K_{t+1} = K_t = K$ and $p_{t+1} = p_t = p$ (a steady state value is denoted without a time subscript) and solving for the other (now time-

$$K = I(p, K; \alpha) \qquad (11a)$$

$$W(p)(1-\tau(1-\alpha)) - D^1\left(\mathrm{W(p)}\big(1-\tau(1-\alpha)\big) + \left(\tau(1-\alpha)+\alpha\right)W(p)/(\tfrac{R(p)}{p})\right), \tfrac{R(p)}{p}\Big) =$$

$$pI(p,k;\alpha) \qquad (11b)$$

Or, in matrix form:

$$(I-A)\begin{bmatrix}\hat{K}\\ \hat{p}\end{bmatrix} = Hd\tau + Gd\alpha$$

The analysis of steady states is given in Appendix 3.

4. LOWERING PAYROLL TAXES WITH FULL RETIREMENT

Let us discuss very briefly a reform of the social security system without postponing retirement but by lowering the payroll tax (i.e. α=0, dτ<0)—this is brief because it has been dealt with at length in Sen (2016)[11]. This raises the disposable income of the young, who consume a part of this and save a part. Saving rises both because of higher disposable income this period and lower pension receipts in old age. Capital accumulation rises. In the new steady state, the capital stock is higher, and so are wages. The starting point of this is by reducing the pension of the initial older generation—they were promised higher pensions than they actually get.

Thus we see that the effects of a cut in payroll tax spurs capital accumulation and makes future implantation of the payroll tax cut. We will contrast this with the delayed FRA regime where capital accumulation falls, dragging down wages with it.

We focus on equations (9a) and (9b) above (with α=0), reproduced as (12a) and (12b) below:

[11] A full "privatization" of the PAYG pension system would set τ =0.

$$K_{t+1} = I(p_t, K_t) \quad (12a)$$

$$W(p_t)(1-\tau)) - D^1\left(\left(W(p_t)(1-\tau) + \frac{\tau W(p_{t+1})p_t}{R(p_{t+1})}\right), \frac{R(p_{t+1})}{p_t}\right) = p_t \mathrm{I}(p_t, K_t) \quad (12b)$$

In the phase in figure 1, IS curves shift out (KK does not depend on τ)

The steady state is given equations (11a) and (11b), reproduced as (13a) and (13b) here:

$$K = I(p, K) \quad (13a)$$

$$W(p)(1-\tau) - D^1\left(\mathrm{W(p)}(1-\tau) + \left(\tau W(p)/(\frac{R(p)}{p})\right), \frac{R(p)}{p}\right) = pI(p,k) \quad (13b)$$

With a change in the payroll tax, τ, we have.

$$\widehat{K}\ /d\tau = \ -W(1-D_W^1)\eta_{Ip}/(\Gamma\Delta) < 0 \quad (14a)$$

$$\hat{p}\ /d\tau = W(\eta_{Ik}-1)(1-D_W^1)/(\Gamma\Delta) > 0 \quad (14b)$$

where

$$\Delta \equiv= (\eta_{IK}-1)\Gamma^{-1}\left\{(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right) - \Gamma - \mathrm{pI}\right\} + \Gamma^{-1}\, pI(1+\eta_{Ip})$$

is the determinant of (A-I) and η_{ij} is an elasticity of the ith variable with respect to the jth variable (e.g. η_{Wp} is the elasticity of the wage with respect to the price of investment good)—see the Appendix 1 and 2 for details.

Thus, across steady states, the social security reform (dτ<0) crowds "in" capital and raises the wage rate—exactly what the proponents of reform say it would do.

In the phase diagram in figure 1, IS curves shift out (KK does not depend on τ). The economy jumps up to the new stable arm. On impact, when the policy change is implemented, the wage rate falls. The reason is straightforward—the old consumed all their income and with a reduction in social security transfers, the demand for the labor-intensive consumption good falls. And so does the return to labor.

5. POSTPONING RETIREMENT WITH UNCHANGED PAYROLL TAXES

Suppose, starting off from a regime where the older generation were fully retired, the government mandates that everybody will be eligible for pension after working for α fraction of the second period (i.e. FRA is at 1+ α).

First, we look at the steady state effects of this:

$$\hat{p}/d\alpha = (\Delta\Gamma)^{-1}[(1\text{-}\eta_{IK})\left\{(1 - D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi} - pI\right\} \tag{15a}$$

and

$$\frac{\hat{K}}{\mathrm{d}\alpha} = (\Delta\,\Gamma)^{-1}\left\{-\eta_{IL}\left\{(1 - D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi\left(1-\eta_{Rp}\right) - D_W^1\left(1-\eta_{Rp}+\eta_{Wp}\right)\left(\frac{\tau W}{\chi}\right) - pI\right\} + \eta_{Ip}\left((1 - D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi}\right)\right\} \tag{15b}$$

If the payroll tax rate is not high and the interest sensitivity of consumption is also not high, we shall have across steady states: $\frac{\hat{p}}{\mathrm{d}\alpha} > 0$ and $\frac{\hat{K}}{\mathrm{d}\alpha}<0$

To get a better grip on the mechanism at work, let us focus on the demand and supply of the investment good. There are three channels at work—two of them work through the demand channel for the investment good (equivalently the consumption good).

To see the demand channels, I reproduce the left hand side (the lifetime income of a representative young person) from equation (2): $W_t\big(1-\tau(1-\alpha)\big)+\left(\frac{(\tau(1-\alpha)+\alpha)W_{t+1}}{\chi_{t+1}}\right)$. First, an increase in α reduces the effective payroll tax rate $(1-\alpha)\tau$, and for a given wage rate, W_t (a smaller fraction of the old age is entitled to social security). This reduces the size of the pension transferred to the old in that period and increases the take-home pay of the young and increases savings, ceteris paribus. Also saving tends to go up because this young individual's social security receipts will be less in the next period (for the fraction of old age spent in employment). These effects increase the demand for the investment good. Second, increased income from work by postponing retirement next period makes saving go down in this period--it is the propensity to save out of the discounted value of future income. It would be reasonable to assume that this reduction in saving outweighs the increased saving (via first effect mentioned above) as long as the initial payroll tax rate not very high.

The third channel is the fall in the supply of the investment good—since increased labor supply increases the supply of the labor-intensive (consumption) good, ceteris paribus. The investment good is capital-intensive, and its supply would fall.

If we bring these effects together--lower demand and lower supply of the investment good--the effect on the relative price of the investment good is ambiguous. It is reasonable to assume that the saving effect of a future rise in wage income is outweighed by the production effect. This causes the wage rate to fall (and the price of the investment good rises), as capital accumulation falls. Thus, as in a one-sector model, an increase in labor supply causes the wage rate to fall. The increased supply of the consumption good is met with increased demand by the older generation that is now earning a wage for α part of the period.

One has to be a little careful here. There are two effects--the first concerns the old in this period. They earn a wage income for the postponed retirement part (and receive a lower pension), which they use for consumption. But they also supply labor, which given our factor intensity assumptions, increases the supply of the consumption good.

The second effect of the postponing of retirement is on saving and investment. The young save less because they expect higher income when they are old--but the saving behavior depends on the discounted value of future income.

Thus if we look at the market clearing equation for the investment good, the demand (the saving by the young) depends on, inter alia, on wage they will receive in the next period (when they are old) but the supply depends on, inter alia, the increased supply of labor by the old today.

The overall effect of a lower pension implies that the future generations will have lower wages because of a lower capital stock. Although I do not do welfare analysis here, I note in passing that this form of social security reform i.e. postponing retirement via a lower capital stock, makes future generations worse off, although they may have a higher wage income (over their lifetimes via the postponed retirement channel). The social security payments (given α and τ) will also be lower, as the wage rate falls along the adjustment path. The labor input increases but capital stock falls in the new steady state. The effect on GDP is ambiguous in our model.

This is shown diagrammatically in Figure 2, the KK curve shifts to the right and the IS also does---the former rightward shift is more (the shifts are not shown to avoid cluttering).[12] The new steady state equilibrium is to the northwest of the original equilibrium. On impact, with the

[12] $\left(\partial\widehat{K}/\partial\alpha\right)_{KK} = -\frac{\eta_{IL}}{\eta_{IK}-1} > \left(\partial\widehat{K}/\partial\alpha\right)_{IS} = \frac{(1-D_W^1)W\tau - D_W^1 W(1-\tau)/\chi - pI\eta_{IL}}{pI\eta_{IK}} > 0$

capital stock given at K_0, the impact effect is for the economy to jump up to a point such as E_{01}. From that point onwards, as capital is decumulated (with the life cycle effect dominating). We move to a new steady state at E_1.

A lesson for empirical work emerges here: it has been shown that in an open economy setting, the country with a PAYG pension will run a current account deficit; or in a two-commodity setting, it will have a comparative advantage in the labor-intensive good.[13] While these statements are unexceptionable, the analysis section shows that these tendencies are exacerbated when an attempt is made to restore solvency to pension system by postponing retirement.

6. CONCLUSIONS

There is an urgency in the advanced capitalist countries to do something to prevent a collapse of the PAYG social security systems. The impending crisis results from longer lives of the population and the fall in fertility rates. The solutions on offer vary from abolishing the system altogether ("privatize social security"), to its moderate version involving a lowering of the payroll tax (with a lower transfer to the retired). The problem with these schemes is the lack of fairness to the first generation of pensioners, who had paid a higher payroll tax when they were young. To get around this, a postponing of retirement has been proposed. Doing so would enable the older people to augment their incomes.

I looked at the two solutions proposed in a two sector two period overlapping generations model. When the payroll tax is reduced, saving increases and the future generations benefit from higher wages. But the cost of this is borne by pensioners in the period of reform.

In the delayed retirement case, life cycle saving considerations lower saving, and hence the future capital stock. Thus a part of the cost of reforming the system by postponing the age of retirement is borne by the future generations.

---

[13] See e.g. Eugeni (2015)

APPENDIX 1

TWO-SECTOR PRODUCTION

Equations (6a) and (6b) yield by logarithmic differentiation

$$\theta_{LC}\widehat{W}_t + \theta_{KC}\hat{R}_t = 0 \qquad \text{(A1.1a)}$$

$$\theta_{LI}\widehat{W}_t + \theta_{KI}\hat{R}_t = \hat{p}_t \qquad \text{(A1.1b)}$$

From (A1.1a) and (A1.1b), we can solve for $\hat{W}_t$ and $\hat{R}_t$ in terms of $\hat{p}_t$. We thus have

$$\eta_{Wp} \equiv \hat{W}_t / \hat{p}_t = -\theta_{KC} / \Delta \qquad \text{(A1.2a)}$$

$$\eta_{Rp} \equiv \hat{R}_t / \hat{p}_t = \theta_{LC} / \Delta \qquad \text{(A1.2b)}$$

where $\Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC}$ and η_{ij} is the (partial) elasticity of variable i with respect to j. From equations (A1.2a) and (A1.2b) we see that η_{Wp} and η_{Rp} depend on pital intensities. Given our assumption that the consumption good is labor-intensive, $\Delta>0$. And, hence, by the Stolper-Samuelson Theorem, $\eta_{Wp} < 0, \eta_{Rp} > 1$.

Similarly, by logarithmically differentiating (6a), (6b) and (6c) we have

$$\lambda_{LC} \cdot \hat{C}_t + \lambda_{LI} \cdot \hat{I}_t = \hat{L}_t + [\hat{W}_t - \hat{R}_t][\lambda_{LC} . \theta_{KC} . \varepsilon_C + \lambda_{LI} . \theta_{KI} . \varepsilon_I] \qquad \text{(A1.3a)}$$

$$\lambda_{KC} \cdot \hat{C}_t + \lambda_{KI} \cdot \hat{I}_t = \hat{K}_t - [\hat{W}_t - \hat{R}_t][\lambda_{KC} . \theta_{LC} . \varepsilon_C + \lambda_{KI} . \theta_{LI} . \varepsilon_I] \qquad \text{(A1.3b)}$$

where λ_{ij} is the share of sector j in the total employment of input i and ε_j is the elasticity of substitution between inputs in the j^{th} industry.

From equations (A1.3a) and (A1.3b), we have the Rybczinski effects (which depend on assumed capital intensities)

$$\eta_{IK} \equiv \hat{I}_t / \hat{K}_t = \lambda_{LC} / \Omega > 0 \qquad \text{(A1.4a)}$$

$$\eta_{CK} \equiv \hat{C}_t / \hat{K}_t = -\lambda_{LI} / \Omega < 0 \qquad \text{(A1.4b)}$$

$$\eta_{IL} \equiv \hat{I}_t / \hat{L}_t = -\lambda_{KC} / \Omega < 0$$

$$\eta_{CL} = \frac{\widehat{C_t}}{\hat{L}_t} = \frac{\lambda_{KI}}{\Omega} > 0$$

where $\Omega \equiv \lambda_{LC} - \lambda_{KC} > 0$. (by assumption).

From (A1.3a) and (A1.3b), we have the supply elasticities (which are independent of capital intensities but depend on the elasticities of factor substitution). In particular, we have:

$$\eta_{Ip} \equiv \hat{I}_t / \hat{p}_t = \{\lambda_{LC}\lambda_{KC}\varepsilon_I + (\lambda_{LC}\lambda_{KI}\theta_{LI} + \lambda_{KC}\lambda_{LI}\theta_{KI})\varepsilon_C\}/(\Delta\Omega) \geq 0 \qquad \text{(A1.4c)}$$

APPENDIX 2

DYNAMICS

Matrices A and the vector H in equation (9) are given by:

$$A \equiv \begin{bmatrix} \eta_{IK} & \eta_{Ip} \\ -pI\eta_{IK}/\Gamma & [(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1(\frac{\tau W}{\chi})\} - pI(1+\eta_{Ip})]/\Gamma \end{bmatrix},$$

$$G \equiv \begin{bmatrix} \eta_{IL} \\ \{(1-D_W^1)W\tau - \dfrac{D_W^1 W(1-\tau)}{\chi} - p\eta_{IL}\}/\Gamma \end{bmatrix}$$

$$H \equiv \begin{bmatrix} 0 \\ -W(1-D_W^1)/\Gamma \end{bmatrix}$$

$$\Gamma \equiv \eta_{Rp} D_\chi^1 \chi - D_W^1(\frac{\tau W}{\chi})\}(\eta_{Rp} - \eta_{Wp}) < 0 \qquad \text{(A2.1)}$$

Where $D_j^1 \equiv \partial D_t^1 / \partial j_t$ is the derivative with respect to variable j (j=W,χ).

All the elements of matrix A are positive. The two roots are ξ_S and ξ_U.

$$TrA = \xi_S + \xi_U = a_{11} + a_{22} > 0$$

$$DetA = \eta_{IK}[(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1(\frac{(\tau W)}{\chi})\} - pI]/\Gamma > 0$$

Since $Det.A \equiv \xi_S\xi_U$ and all the elements of A are positive, both the roots are positive.

Now

$$(TrA)^2 - 4DetA > 0$$

(Proof: All the elements of A are positive. Hence $(a_{11}+a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12}) =$ $(a_{11}-a_{22})^2 + a_{21}a_{12} > 0$.

So the (two positive) roots are real.

Next the requirement

$$1 - TrA + DetA < 0$$

$1 - TrA + DetA$ndition that two the roots of the matrix A-I (i.e. $\xi_S - 1$and $\xi_U - 1$) are of opposite signs; or the roots of A lie on the opposite side of unity)

$$1 - TrA + DetA =$$

$$= 1 - (a_{11}+a_{22}) + a_{11}a_{22} - a_{21}a_{12}$$

$$= (a_{11}-1)(a_{22}-1) - a_{21}a_{12}$$

$$= Det\ (A - I)$$

$$\Delta \equiv \text{Det}\begin{bmatrix} \eta_{IK}-1 & \eta_{Ip} \\ -pI\eta_{IK}/\Gamma & \left[(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right)\right\} - pI\left(1+\eta_{Ip}\right) - \Gamma]/\Gamma \end{bmatrix},$$

$$= (\eta_{IK}-1)\Gamma^{-1}\left\{\left(1-D_W^1\right)(1-\tau)W\eta_{W_p} + \{D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right) - \Gamma - \text{pI}\right\} + \Gamma^{-1}\,pI(1+\eta_{Ip})$$

A “high enough” value of η_{Ip} is a sufficient condition to deliver this (requiring “high” elasticities of substitution in production). Also a “small” η_{IK} i.e. close to 1 is also sufficient.

We want to sign the shifts in the two curves KK and IS in Figure 2 with a change in α. The horizontal shifts are given by:

$$\left(\partial\widehat{K}/\partial\alpha\right)_{KK} = -\frac{\eta_{IL}}{\eta_{IK}-1} = \frac{\lambda_{LI}}{\lambda_{KC}} > 0 \qquad \text{(A2.2)}$$

$$\left(\partial\widehat{K}/\partial\alpha\right)_{IS} = \frac{(1-D_W^1)W\tau - D_W^1 W(1-\tau)/\chi c - pI\eta_{IL}}{pI\eta_{IK}} = ? \qquad \text{(A2.3)}$$

Given the assumption made above i.e. $-pI\eta_{IL} > (1-D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi c} > 0$. Thus, while both the IS curve and the KK curve shift to the right, and KK shifts more, as shown in Figure 2.

APPENDIX 3

STEADY STATE

$$A - I \equiv \begin{bmatrix} \eta_{IK}-1 & \eta_{Ip} \\ -pI\eta_{IK}/\Gamma & \left[(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right)\} - pI\left(1+\eta_{Ip}\right) - \Gamma\right]/\Gamma \end{bmatrix},$$

$$\mathrm{H} \equiv \begin{bmatrix} \eta_{IL} \\ \{(1-D_W^1)W\tau - \dfrac{D_W^1 W(1-\tau)}{\chi} - pI\eta_{IL}\}/\Gamma \end{bmatrix}$$

$$\frac{\hat{p}}{d\alpha} = \begin{bmatrix} \eta_{IK}-1 & -\eta_{IL} \\ -pI\eta_{IK}/\Gamma & -\{(1-D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi} - pI\eta_{IL}\}/\Gamma \end{bmatrix}$$

$$\hat{p}/d\alpha = (\Delta\Gamma)^{-1}[(1\text{-}\eta_{IK})\left\{(1-D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi} - pI\right\}$$

Where Δ≡Det.(A-I)<0 (the condition of saddle-point stability)

Note $(\eta_{IK}-1) = -\eta_{IL}$ and $\Gamma < 0$ (see (A2.1) above).

$\widehat{K}/d\alpha =$

$$(\Delta\Gamma)^{-1}\begin{bmatrix} -\eta_{IL} & \eta_{Ip} \\ -(1-D_W^1)W\tau + \dfrac{D_W^1 W(1-\tau)}{\chi} & \left\{(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi(1-\eta_{Rp}) - D_W^1(1-\eta_{Rp}+\eta_{Wp})(\tau W/\chi)\right\} - pI \end{bmatrix}$$

$$= (\Delta\, \Gamma)^{-1}\left\{-\eta_{IL}\left\{(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi\big(1-\eta_{Rp}\big) - D_W^1\big(1-\eta_{Rp}+\eta_{Wp}\big)\left(\frac{\tau W}{\chi}\right) - pI\right\} + \eta_{Ip}\left((1-D_W^1)W\tau - \frac{D_W^1 W(1-\tau)}{\chi}\right)\right\}$$

This is likely to be negative for small values of τ and D_χ^1 s we have assumed.

REFERENCES

Aaron, Henry J. (2011) "Social Security Reconsidered" *National Tax Journal* **64**, 385–414.

Atkinson, Anthony and Joseph Stiglitz (1980) *Lectures on Public Economics*, Princeton University Press, Princeton.

Azariadis, Costas (1993) *Intertemporal Macroeconomic*, Basil Blackwell, Oxford UK.

Behaghel, L. and Blau, D. M. (2012) "Framing Social Security Reform: Behavioral Responses to Changes in the Full Retirement Age" *American Economic Journal: Economic Policy*, 4, 41–67.

Casamatta, Georges., Helmuth Cremer, and Pierre Pestieau (2002). "The Political Economy of Social Security". *Scandinavian Journal of Economics* **102**, 503-522.

Cipriani, Giam Petro (2018) "Aging, Retirement and Pay-as-you-go Pensions", *Macroeconomic Dynamics* **22**, 1173-1183.

Cremer, Helmuth, and Pierre Pestieau (2003) "The Double Dividend of Postponing Retirement", *International Tax and Public Finance* **10**, 419-434.

Diamond, Peter. A. (1977). "A Framework for Social Security Analysis," *Journal of Public Economics* **8**, 275-298.

Diamond, Peter A. (2004) "Social Security" *American Economic Review* **94**, 1-24.

Eugeni, Sara (2015) "An OLG Model of Global Imbalances," *Journal of International Economics* **95**, 83-97.

Fedotenkov, Igor, Bas van Groezen and Lex Meijdam (2019) "International Trade with Pensions and Demographic Shocks" *Journal of Pension Economics and Finance* **18,** 140-164.

Galasso, Vincenzo (2008) "Postponing Retirement: The Political Effect of Aging" *Journal of Public Economics*, **92**, 2157-2169.

Galor, Oded (1992) "A Two-sector Overlapping Generations Model: A Global Characterization of the Dynamical System", *Econometrica*, **60**, 1351-1386.

Hirono, Makoto, and Kazuo Mino (2019) "Pension, Retirement, and Growth in the Presenc Heterogeneous Elderly" Doshisha University, Kyoto (Mimeo).

Krueger, Dirk and Felix Kubler, (2006) “Pareto-improving Social Security Reform when Financial Markets are Incomplete” *American Economic Review* **96**, 737-755.

Miyazaki, Koichi (2014) "The Effects of Raising the Official-Pension-Age-Policy in an Overlapping Generations Economy," *Economics Letters* **123**, 329-332.

Nishiyama, Shinichi and Kent Smetters (2007). “Does social security privatization produce efficiency gains?” *Quarterly Journal of Economics* **122**, 1677-1719.

OECD (2019) *Pensions at a Glance: OECD and G20 Indicators*, (OECD, Paris).

Samwick, Andrew A (2000) “Is Pension Reform Conducive to Higher Saving?” Review of Economics and Statistics 82, 264-272.

Seibold, A. (2019). Reference Points for Retirement Behavior: Evidence from German Pension Discontinuities. CESifo Working Papers No. 7799.

Sen, Partha (2016) “Anyone for Social Security Reform?”. CESifo Working Paper No. 6035.

Solow, Robert M., (2005). “Reflections on Growth Theory” in *The Handbook of Economic Growth*, Volume 1, Philippe Aghion and Steven Durlauf (ed.), pages 3-10 Elsevier, Amsterdam.

Zhang, Jie, and Junsen Zhang (2009) "Longevity, Retirement, and Capital Accumulation in a Recursive Model with an application to Mandatory Retirement," *Macroeconomic Dynamics* **13**, 327-348.

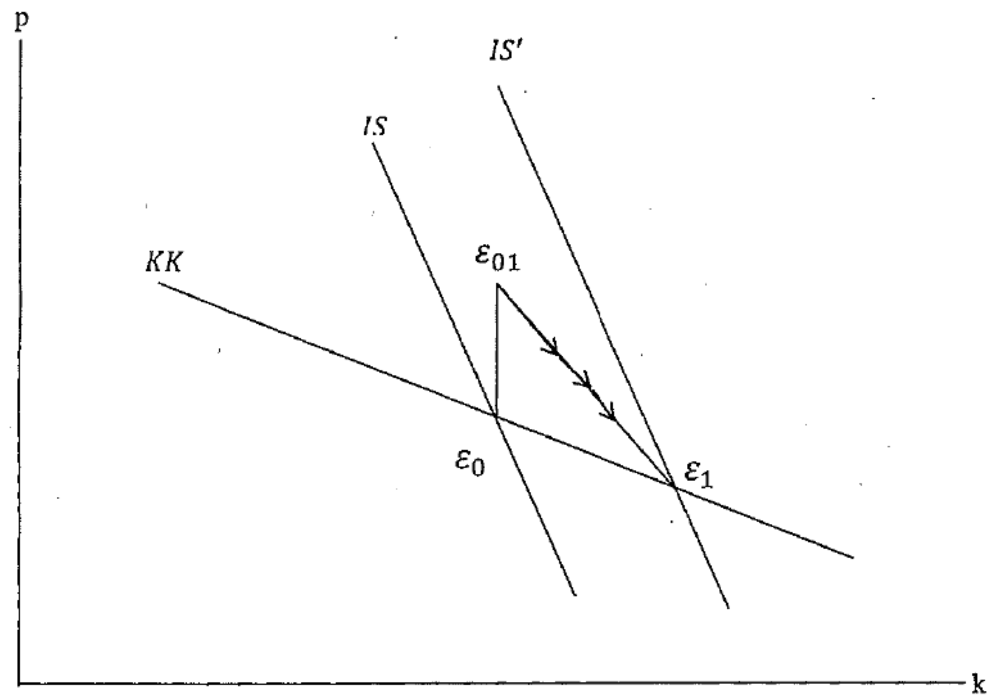

Figure 1

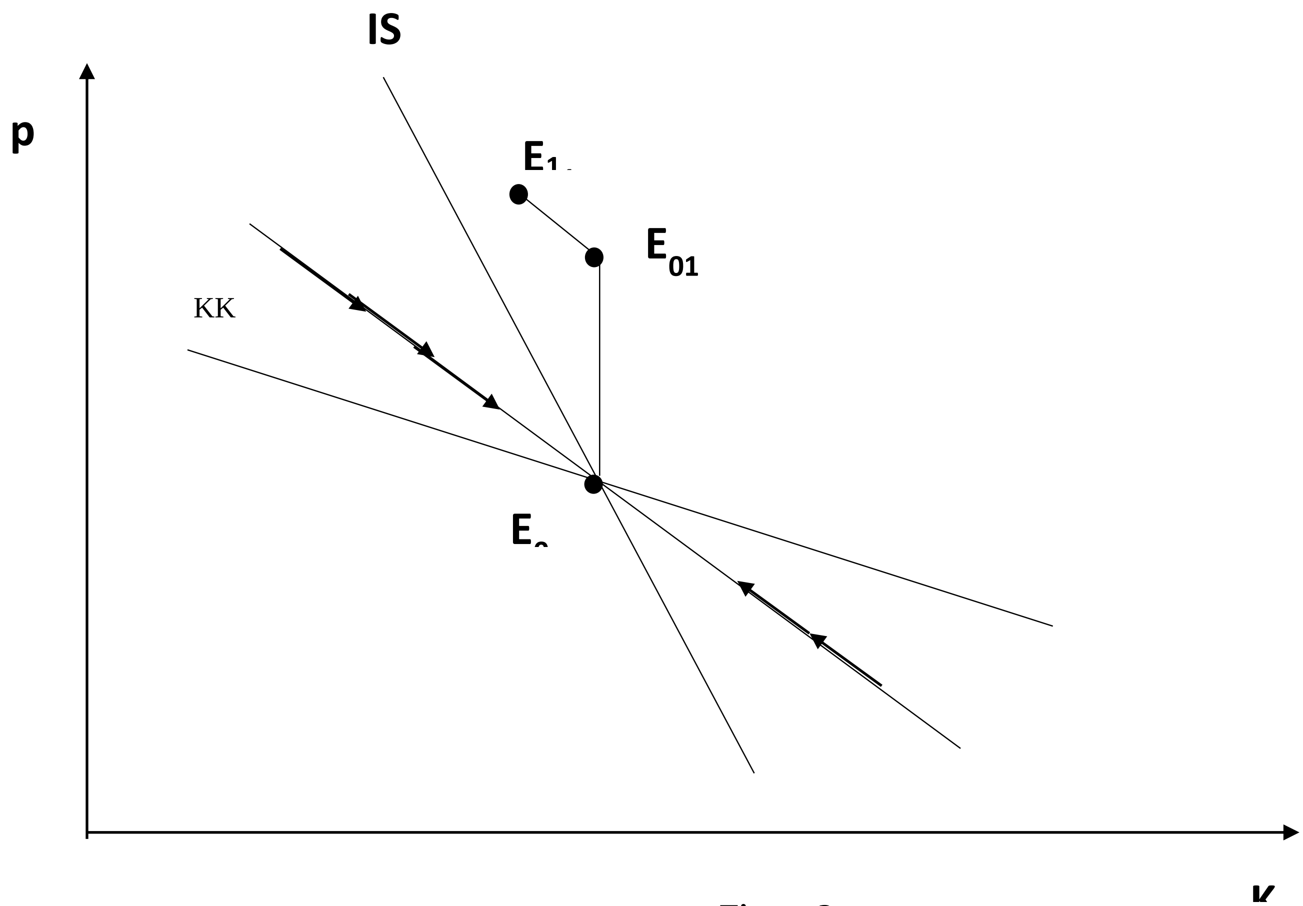

Figure 2