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Inequality and the Value of Public Natural Capital

Abstract

We study how income inequality affects the social value of a dynamic public good, such as natural capital. Our theory shows that both intra- and intertemporal inequality affect the social value of public natural capital. The direction and size of the effects are driven by the degree of substitutability between the public and private consumption goods. While the value of the public good increases (decreases) with intratemporal income inequality in the case of complements (substitutes), it increases (may decrease) with intertemporal income inequality for complements and Cobb-Douglas (substitutes). A problem of major relevance for the accounting of public natural capital as required by international treaties is to transfer values between study and policy contexts, or to up-scale values from study sites to the national scale. Our theory provides closed-form adjustment factors that allow controlling for differences in study and policy contexts.

JEL-Codes: D630, H430, Q510.

Keywords: dynamic public good, inequality, accounting, natural capital, environmental good, non-market valuation, WTP, benefit transfer, biodiversity.

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1 Introduction

The valuation and efficient provision of dynamically evolving public goods, such as open access libraries, public infrastructure, UNESCO World Heritage sites, the Earth's climate system or biodiversity, is a key challenge for economic theory and policy. This is especially true for natural capital that is declining in many parts of the world, as documented by the recent global assessment report of the Intergovernmental Science Policy Platform for Biodiversity and Ecosystem Services (IPBES 2019). Under the United Nations' Convention on Biological Diversity (CBD 1992), countries have committed themselves to integrate natural capital values into national accounts. However, there are so far no methods available to appropriately live up to this commitment in a way that is consistent with economic theory. At the same time, the CBD requires a fair and equitable sharing of benefits derived from the use of biodiversity, especially of genetic resources. Also more generally, concerns about economic inequality have found increasing attention in recent years (e.g. Stiglitz et al. 2010, Alvaredo et al. 2017). Thus, there is a need to develop an economic theory of how to integrate effects pertaining to economic inequality into methods of valuing public-good natural capital.

To advance the theory of public goods valuation, we address the question of how the intra- and intertemporal distribution of income affects the accounting price of a dynamic public good. We conceptualize the social value of a dynamic public good as the aggregate inverse Lindahl-demand for this good. It is given as the sum of individual present-value Lindahl prices that households would pay if they would purchase the dynamic public good in their individual competitive market. We then study how initial inequality across individuals and inequality over time affect the social value of the dynamic public good. For this, we generalize a static model of how intratemporal inequality affects the valuation of a public good (Ebert 2003, Baumgärtner et al. 2017) to a dynamic setting.

Our analysis of the social value of dynamic public goods has three major fields of application: First, the efficient provision of dynamic public goods. Second, cost-benefit analysis (CBA) that involves changes in the stock of public capital. Third, the integration of the value of dynamic public goods into national accounts.

All these applications draw on the practice of *benefit transfer*, to which we contribute methodologically. Benefit transfer means the transfer of estimated values of public goods from a primary 'study context' to a 'policy context', where these transferred values are used for accounting or to inform decision-making. Since primary studies are costly, benefit transfer has become a primary method of non-market valuation (Richardson et al. 2015) and a crucial part of economic analysis for governmental decision-making (OECD 2018). Yet, a simple transfer would only be valid if both contexts—as well as their future developments—were identical. As this is usually not the case, no matter whether transferring values across space or across time, the value of a public good derived in the study context has to be adjusted to the policy context to minimize transfer errors. Most benefit transfer approaches employ meta-regressions, but approaches based purely on statistical fit may not guarantee basic principles for internal consistency (e.g. Newbold et al. 2018). Accordingly, there have been several calls to base benefit transfer more firmly on economic theory (Bateman et al. 2011, Phaneuf and Requate 2017). Here, we derive closed-form analytic benefit transfer factors that allow for transferring the social value of a dynamic public good across space and time while controlling for key differences between study and policy contexts, such as differences in the income distribution or in the (positive or negative) growth rates of income and natural capital.

While our theory can be applied more generally, we frame our exposition in terms of environmental amenities derived from public natural capital. To allow for closed-form solutions that can guide policy appraisal and accounting, we use a number of simplifying assumptions. Our model considers a continuum of infinitely-lived households that have identical preferences over private consumption and the environmental amenities (or: environmental goods) derived from public natural capital, but that differ in their incomes. We assume that initial income is log-normally distributed (cf. Baumgärtner et al. 2017, Emmerling et al. 2017), which implies that relative income inequality remains constant over time. Regarding preferences, we assume that the elasticity of substitution between the private consumption good and the environmental goods from public natural capital is constant at any point in time, and that the intertemporal elasticity of substitution for the consumption good is constant, too. The latter is warranted when the presence of public goods should not distort savings behavior for consumption goods. These conditions are satisfied for utility functions that feature Cobb-Douglas substitutability across goods and constant intertemporal substitution (e.g. Gerlagh and Keyzer 2001), and for utility functions of the intertemporal-constant-elasticity-of-substitution type (Quaas et al. 2020). Furthermore, we consider exogenously given time paths of income and natural capital as resulting from a given 'resource allocation mechanism' (Arrow et al. 2003a). We thus abstract from savings and optimal management, and value natural capital in not-necessarily-optimal economies. In particular, we study the case of constant growth rates, common in the related literature on social discounting that deals with changing relative prices of non-market goods (e.g. Traeger 2011, Zhu et al. 2019). Finally, we use a proportional mapping of natural capital into the environmental goods it provides (cf. Gerlagh and Keyzer 2001). We thereby focus on non-use goods, such as regulating ecosystem services, e.g. local or global climate control or the existence value of biodiversity. It is in particular for these non-use goods that information on their social value is crucial for public policy.

We derive novel results regarding the effect of the intertemporal distribution of income or public natural capital on the social value of natural capital, and generalize effects of the intratemporal income distribution to a dynamic setting. We show that the social value of public natural capital increases with initial mean income in society, and decreases (increases) with initial relative income inequality if and only if the public environmental goods derived from natural capital and manufactured private consumption goods are substitutes (complements). This extends the static analysis of Baumgärtner et al. (2017) to the dynamic setting, which is relevant for many policy contexts including, but not restricted to, natural capital accounting. Regarding the intertemporal distribution of income and capital, we find that the social value of public natural capital increases with the income growth rate for the case of complements or the Cobb-Douglas case, but it is possible that the social value declines with the income growth rate in the case of substitutes. Further, we show that the social value increases (decreases) with the growth rate of natural capital if and only if public environmental goods and consumption goods are substitutes (complements). On this basis, we derive benefit transfer factors that account for differences in the distribution of income, income growth, natural capital growth, interest rates and other characteristics between a study and a policy site.

Drawing on a global case study on the valuation of non-use public ecosystem services derived from biodiversity, we illustrate that ignoring the effects of the intra- and intertemporal distribution can lead to serious errors. For example, using a mean estimate of the degree of substitutability from the literature (cf. Drupp 2018), we show that when the income growth rate is only half as high as the mean forecast for the global level, the social value of biodiversity would be almost 200 percent higher. Likewise, a reduction in the biodiversity loss rate by one half would increase the social value of biodiversity by 12 percent. The illustration also shows that the degree of substitutability not only determines the direction of effects, but is also the key driver of effect sizes.

Our analysis on valuing dynamic public goods contributes to different strands of the literature. First, we contribute to the literature on natural capital accounting. A large part of this research is concerned with determining shadow prices for non-market capital assets (e.g. Hartwick 1990, Dasgupta and Mäler 2000, Arrow et al. 2003a, Dasgupta 2009, Arrow et al. 2012, Fenichel and Abbott 2014). These approaches are based on an intertemporal welfare function and the shadow price of a capital asset is derived through constrained optimization of the intertemporal welfare function, as the increase of welfare from a marginal increase of the capital asset. Here, we develop an alternative approach that is based on Lindahl demand and thus less demanding with respect to normative assumptions, as it is based on (Pareto-)efficiency only. Moreover, while most welfare economic analysis of natural capital values are set in a representative agent framework, we study a population of heterogeneous households described by a continuous distribution of income.

Second, our analysis relates to the literature on social discounting. A number of studies examine intra- and intertemporal distribution in the context of discounting of a single consumption good (e.g. Fleurbaey and Zuber 2015, Gollier 2015, Emmerling et al. 2017, Emmerling 2018). As far as the intertemporal distribution of market-traded and non-market-traded goods is concerned, our paper is related to the literature on dual discounting of non-market goods (e.g. Weikard and Zhu 2005, Hoel and Sterner

2007, Gollier 2010, Traeger 2011, Baumgärtner et al. 2015, Drupp 2018, Yamaguchi 2019, Drupp and Hänsel forthcoming). The difference in good-specific discount rates is determined by the degree of substitutability and the differences in good-specific growth rates. While we do not focus on social discounting *per se*, our analysis combines these two strands by showing how intra- and intertemporal distribution as well as limited substitutability affect the value of dynamic public goods.

Finally, we contribute to the literature on non-market valuation. With few exceptions (see, e.g., Kriström and Riera 1996, Drupp et al. 2018b, Hsiang et al. 2019), this literature does not explicitly consider effects pertaining to the distribution of income. Within this strand, we add primarily to the emerging literature on structural benefit transfer (e.g. Smith et al. 2002, 2006, Baumgärtner et al. 2017, Newbold et al. 2018, Meya et al. 2019, Kling and Phaneuf 2018, Moeltner 2019, Meya 2020). So far, this literature does not specifically deal with the dynamic nature of natural capital. Our contribution is to derive a closed-form benefit transfer function with adjustment (or: transfer) factors that allow controlling for differences in dynamic aspects across contexts, such as the income growth rate and environmental loss rate, or interest rates.

The paper is organised as follows. We present the model in Section 2, our valuation concepts in Section 3, and results in Section 4. We apply our findings empirically for global biodiversity conservation in Section 5, and we report several extensions of the analysis in Section 6. We discuss limitations in Section 7, and conclude in Section 8. All proofs are contained in the Appendix.

2 Model

We consider a society of n households, labelled $i = 1, \dots, n$, who derive utility from the consumption of two goods—a market-traded private consumption good and an environmental public good, which is non-rival and non-excludable in consumption.[1] The flow of the environmental good is proportional to the stock of natural capital. Examples include the existence value of biodiversity or an agreeable climate.

Time is discrete and the horizon is infinite, $t = 0, 1, \dots$. As common in the literature on natural capital valuation, we assume a given 'resource allocation mechanism' (Arrow et al. 2003a) or 'economic program' (Fenichel and Abbott 2014), such that the time paths of both goods are exogenously given.[2] $E_t \geq 0$ denotes the stock of natural capital which equals, with suitable normalization, the flow of the environmental public good, and $C_t^i \geq 0$ denotes private consumption of household i at time t. The market price of the consumption good is $P_t \geq 0$. Household income $Y_t^i \geq 0$ is exogenous and fully expended for consumption at each time, so that $C_t^i = Y_t^i / P_t$. For simplicity, we take consumption as numéraire and set $P_t = 1$. We therefore substitute income Y_t^i for private consumption C_t^i in the following. We refer to the distribution of income over time as 'intertemporal distribution', and to its distribution over households at a given point in time as 'intratemporal distribution'.

Like Zhu et al. (2019), we focus on a balanced growth path where both income and natural capital grow at constant rates, i.e. we study

$$Y_t^i = Y_0^i \left(1 + g_Y\right)^t \text{ and} \tag{1a}$$

$$E_t = E_0 \left(1 + g_E\right)^t , \tag{1b}$$

where Y_0^i is household i's income and thus private consumption at $t = 0$, E_0 is the stock of public natural capital and amount of the environmental public good at $t = 0$,

[1]We study a time-constant population size and thus abstract from how population growth or decline affect natural capital values (see, e.g., Dasgupta 2001, Arrow et al. 2003b and Yamaguchi 2018).

[2]These time paths can be formulated in form of differential equations, which include the resource allocation mechanism as feedback rule, or explicitly as functions of time. We follow the latter approach.

$g_E \in (-1, 0)$ is the growth rate of natural capital, and $g_Y > -1$ is the growth rate of income.[3]

As in Baumgärtner et al. (2017) and Emmerling et al. (2017), income in $t = 0$ is log-normally distributed over households with mean μ_{Y_0} and standard deviation σ_{Y_0}.[4] We assume that the number of households is large enough so that the initial income distribution is adequately described by a continuous density function $f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})$ over non-negative incomes. To measure intratemporal income inequality, we employ the coefficient of variation $\mathrm{CV}_{Y_0} := \sigma_{Y_0}/\mu_{Y_0}$ as a measure of relative inequality. It captures the width of the distribution of income relative to mean income. While there are a number of different notions of income inequality in use, concepts of relative income inequality—often in the form of income shares—feature prominently in academic and policy circles (e.g. Alvaredo et al. 2018). Our model assumes that the income growth rate is the same for all households. This implies that absolute income inequality—as measured by, e.g., the standard deviation—will increase over time, while relative income inequality, as measured by the coefficient of variation, will stay constant, that is $\mathrm{CV}_{Y_t} = \mathrm{CV}_{Y_0}$ for all t. Moreover, income will remain log-normally distributed over households at each future time $t > 0$.

Households are infinitely-lived and have identical preferences over consumption and the environmental good which are described by a standard time-separable, stationary intertemporal utility function U with

$$U(\{Y_t^i\}, \{E_t\}) = \sum_{t=0}^{\infty} \rho^t u(Y_t^i, E_t) \ , \tag{2}$$

where u is an instantaneous utility function and $\rho \in (0, 1)$ is the utility discount factor. Moreover, we assume that households have (i) constant-elasticity-of-substitution (CES) preferences between Y_t^i and E_t for all t, and (ii) constant-intertemporal-elasticity-of-

[3]Here, we consider exogenous growth rates of natural capital and income. One example of a model that endogenizes these growth rates is Riekhof et al. (2019). To focus on the social value of natural capital we avoid the complications that come with setting up such an endogeneous growth model.

[4]There is empirical evidence that the global household income distribution can be approximated by a log-normal distribution (Pinkovskiy and Sala-i-Martin 2009).

substitution (CIES) preferences between Y_t^i and $Y_{t'}^i$ for any t, t'. Except for the case of instantaneous Cobb-Douglas preferences, which we study in Section 6.4, both assumptions (i) and (ii) are fulfilled simultaneously for time-separable, stationary intertemporal utility (Eq. 2) if and only if intertemporal utility is of the intertemporal-constant-elasticity-of-substitution (ICES) form (this is shown in Quaas et al. 2020):

$$U(\{Y_t^i\}, \{E_t\}) = \sum_{t=0}^{\infty} \rho^t \frac{\theta}{\theta - 1} \left(\alpha\, {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha)\, {E_t}^{\frac{\theta-1}{\theta}}\right) , \tag{3}$$

where the parameter $\alpha \in (0, 1)$ measures the preference for private consumption relative to the environmental good. The parameter $\theta > 0$ is the constant elasticity of substitution between Y_t^i and E_t for all t and also the constant intertemporal elasticity of substitution between Y_t^i and $Y_{t'}^i$ for any t, t'. Assumptions (i) and (ii) imply that the intertemporal elasticity of substitution for the environmental good is also constant and identical to θ. The ICES utility function, Eq. (3), comprises the special cases where the consumption good and the environmental good are substitutes ($\theta > 1$), Cobb-Douglas ($\theta = 1$) and complements ($\theta < 1$) at each point in time.[5]

For an infinite time horizon and using the time paths of Y_t^i and E_t given by Eqs. (1a) and (1b), respectively, the sum in Eq. (3) converges if (see Appendix A.1)

$$\rho(1 + g_Y)^{\frac{\theta-1}{\theta}} < 1 \text{ and} \tag{4a}$$

$$\rho(1 + g_E)^{\frac{\theta-1}{\theta}} < 1 , \tag{4b}$$

which we assume to hold in the following. We can then rewrite intertemporal utility U

[5] Assumptions (i) and (ii) imply a single elasticity of substitution for the substitutability between the two goods within any time period as well as for the substitutability of the consumption bundle across time. To relax these assumptions we run numerical simulations in Section 6.3.

(Eq. 3) in reduced form (see Appendix A.1):[6]

$$U(Y_0^i, g_Y, E_0, g_E) = \frac{\theta}{\theta - 1} \left(\frac{\alpha \, {Y_0^i}^{\frac{\theta-1}{\theta}}}{1 - \rho \, (1 + g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha) \, E_0^{\frac{\theta-1}{\theta}}}{1 - \rho \, (1 + g_E)^{\frac{\theta-1}{\theta}}} \right) \, . \tag{5}$$

3 Valuation concepts

We derive the social value of a change in the time path of public natural capital from $\{E_t\}$ to $\{E'_t\}$, through aggregation from individual benefits of such a change to all households. To start with, we focus on a change in the initial stock of natural capital by dE, so that the dynamics of $\{E_t\}$ and $\{E'_t\}$ according to Eq. (1b) differ in the initial level, $E'_0 = E_0 + dE$, but not in the growth rate, $g_{E'} = g_E$. An example of such a change in the stock of public natural capital might be an increase in forest cover through reforestation, or a local re-establishment of an endangered species. We extend our analysis to changes in the growth rate of natural capital in Section 6.1.

3.1 Individual valuation

As the measure of benefit for household i of a change in natural capital, we consider the compensating surplus for a change in the time path of natural capital from $\{E_t\}$ to $\{E'_t\}$.[7] Specifically, we measure compensating surplus as a constant payment fraction, $\omega^i > 0$, at which the household is willing to reduce income in each period on the time path $\{E'_t\}$. It is defined through

$$U(\{(1 - \omega^i)Y_t^i\}, \{E'_t\}) \equiv U(\{Y_t^i\}, \{E_t\}) \, . \tag{6}$$

From here on, we speak of the *willingness to pay* (WTP) for a change in $\{E_t\}$, rather than of the *compensating surplus*, as the term WTP features more prominently in the

[6]To save notation, we employ the same symbol U for both functions Eq. (3) and (5). As we always explicitly specify the arguments of the function, it should be clear what function we mean.

[7]Alternatively, one could take the equivalent surplus. For a marginal change in $\{E_t\}$, as considered in the following, the compensating and the equivalent surplus coincide.

non-market valuation literature. Conceptualizing individual WTP as a payment that is made as a constant fraction of income in each period, as in Eq. (6), is in line with the applied valuation literature which also considers a stream of payments as fraction of income, often in the form of a tax.[8] In Section 6.2 we show that our main results also hold when considering the WTP in form of a one-time payment.

For ICES utility (Eq. 5) household i's individual WTP for an increase in the initial level of natural capital by one marginal unit, expressed as a constant payment fraction as defined by Eq. (6), is given by (see Appendix A.2)

$$\omega^i = \omega(Y_0^i, E_0) = \frac{1-\alpha}{\alpha} \frac{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\, Y_0^{i\,\frac{1-\theta}{\theta}}\, E_0^{-\frac{1}{\theta}} \; . \tag{7}$$

With $\omega(Y_0^i, E_0)$ as the constant fraction of income that household i is willing to pay in each time period, the corresponding absolute amount of income in period t is $\omega(Y_0^i, E_0)Y_t^i$ or, taking into account income dynamics over time (Eq. 1a), $\omega(Y_0^i, E_0)Y_0^i\,(1+g_Y)^t$. The present value, discounted at the constant market interest factor $\delta > 0$,[9] of this stream of what household i is willing to pay over the entire time horizon is

$$\text{WTP}(Y_0^i, E_0) = \sum_{t=0}^{\infty} \delta^t\, \omega(Y_0^i, E_0)\, Y_0^i\, (1+g_Y)^t \; . \tag{8}$$

This intertemporal marginal WTP is the present value of what household i is willing to pay over the entire time horizon for one marginal unit of the initial stock of public natural capital, expressed in units of income. It measures her marginal intertemporal benefit of natural capital. From Eqs. (7) and (8) it is obvious that the household's WTP for one marginal unit of initial natural capital, $\text{WTP}(Y_0^i, E_0)$, decreases with the amount of initially existing natural capital, E_0, and increases with the household's initial income, Y_0^i.

[8]For instance, 91 of the 98 reported WTP values elicited with stated preference methods in the TEEB-Database (Van der Ploeg and de Groot 2010) have been elicited as annual payments.

[9]A constant market interest factor is consistent with the assumption of constant rates of growth or decline for income and natural capital, i.e. a balanced growth path of the overall economy.

3.2 Societal valuation

In a static setting, Pareto-efficiency requires that a public good is supplied to the extent that the sum of households' marginal benefits equals the marginal (opportunity) cost of supplying the public good (Lindahl 1928, Samuelson 1954). For the dynamic setting considered here, we generalise this efficiency condition as follows:

Lemma 1. *Intertemporal Pareto-efficiency implies that at $t = 0$ public natural capital is supplied at the level, denoted by $E_0^\star$, where the marginal costs of initially supplying natural capital equal the sum of households' individual marginal intertemporal benefits of the initial level of public natural capital,*

$$c'(E_0^\star) = \sum_{i=1}^{n} \mathrm{WTP}(Y_0^i, E_0^\star) \ , \tag{9}$$

where $c(E_0)$ are the total cost, including opportunity costs, of initially supplying natural capital at level E_0, and WTP(Y_0^i, E_0) is given by Eq. (8)*.*

Proof. See Appendix A.3. □

Lemma 1 proposes an intertemporal extension of the Lindahl-Samuelson-condition. Formally, Condition (9) is in full analogy with the static Lindahl-Samuelson-condition. The intertemporal extension is in the intertemporally detailed specification of the terms: intertemporal Pareto-efficency requires that the marginal cost of supplying the public natural capital in $t = 0$ equals the aggregate intertemporal marginal WTPs of all households, that is, the sum over all households of the present value, discounted at the market interest factor, of a household's WTP over the entire time horizon for one marginal unit of the initial stock of public natural capital.

Figure 1 illustrates Condition (9) for $n = 2$ households with different initial incomes, $Y_0^1 < Y_0^2$. The blue thin curves show the two individual intertemporal marginal WTPs (on the vertical axis) as functions of the initial stock E_0 of public natural capital (on the horizontal axis), that is, $\mathrm{WTP}(Y_0^i, E_0)$ according to Eq. (8) for $i = 1, 2$. The blue thick curve is the (vertical) sum of the two individual intertemporal marginal WTPs,

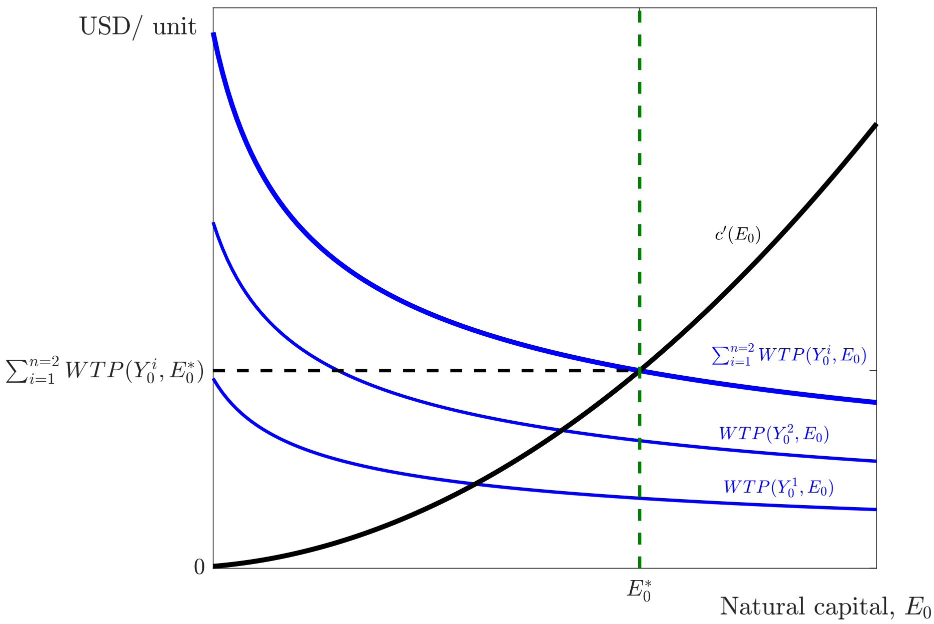

Figure 1: Illustration of the intertemporal Lindahl-Samuelson condition, Condition (9), for $n = 2$ households with different incomes $Y_0^1 < Y_0^2$.

$\text{WTP}^{\Sigma}(Y_0^1, Y_0^2, E_0) = \sum_{i=1}^{2} \text{WTP}(Y_0^i, E_0)$. The efficient initial stock $E_0^\star$ of public natural capital is determined by the intersection of the blue thick curve (aggregate intertemporal marginal WTPs of all households for public natural capital) with the black curve which depicts the marginal costs of providing the initial stock of public natural capital.

Motivated by Lemma 1, we define the social (marginal) value of public natural capital as the term on the right-hand side of Eq. (9), which is the aggregate intertemporal marginal benefit (WTP) of all households for the initial stock of natural capital:

$$\text{WTP}^{\Sigma}(Y_0^1, \ldots, Y_0^n, E_0) := \sum_{i=1}^{n} \text{WTP}(Y_0^i, E_0) \; . \tag{10}$$

The underlying reason for this sum to make the right-hand side of the intertemporal efficiency Condition (9) is that, first, each summand measures a household's individual marginal benefit of public natural capital according to Eq. (8) and, second, it is the simple sum of all those that determines (in conjunction with the marginal costs) the Pareto-efficient level of public natural capital. It therefore makes sense to think of this expression as the (marginal) value of public natural capital to society.

A complementary interpretation of expression (10) as the social (marginal) value of public natural capital is based on Lindahl prices and Lindahl equilibrium in a hypothetical setting where households make individual decisions and each household faces an individual Lindahl price for the public good. In such a setting, the right-hand side of Eq. (10) can be interpreted as the inverse aggregate Lindahl-demand function for the public good, which is the sum of inverse individual Lindahl-demand functions. The latter are given as individual demand for a rationed public good when the household faces an individual virtual price for that public good (Flores and Carson 1997, Ebert 2003, Baumgärtner et al. 2017). As the inverse individual demand for the given amount of the public good equals the household's individual virtual price for the public good, the sum in Eq. (10) is simply the sum of all individual Lindahl prices with a given amount E_0 of the public good for all households. It is plausible to think of this sum of individual Lindahl prices as the social value of public natural capital in a Lindahl setting.

In Figure 1, the blue thick curve depicts this inverse aggregate Lindahl demand for public natural capital. While inverse aggregate Lindahl demand yields a social value for all potential levels E_0 of public natural capital, the level $E_0^\star$ has a special meaning: it is the Lindahl equilibrium amount of public natural capital where aggregate Lindahl demand (blue thick curve) equals supply given by the inverse marginal costs of providing natural capital (black curve). Comparison with the efficiency Condition (9) shows that the Lindahl equilibrium is Pareto-efficient. Hence, while the inverse aggregate Lindahl demand for public natural capital, Eq. (10), may be evaluated at the efficient level $E_0^\star$, it also yields a social value at any other, inefficient, level of public natural capital.

Summing up, we define the social (marginal) value of public natural capital as the inverse aggregate Lindahl demand for public natural capital. This definition of the social value of natural capital is consistent with the approach of accounting for marginal damages of environmental pollution proposed by Muller et al. (2011) in a static setting. It is conceptually more general than the shadow price as obtained from constrained optimization of a given social welfare function, which measures by how much social welfare would increase if one additional marginal unit of natural capital was available initially. We do not presuppose a social welfare function for the aggregation from individuals to society, but we build on the weaker concept of intertemporal Pareto efficiency and on the even weaker concept of a Lindahl economy that may or may not be in equilibrium. One advantage of our more general approach is that the defining Eq. (10) leaves open the level of natural capital, E_0, at which WTP is evaluated. One could do this at the Pareto-efficient level $E_0^\star$ and thus arrive at the *efficiency value* of natural capital. Or one could do this at the actually existing, and presumably inefficient, level of natural capital and thus arrive at an *accounting price* which is consistent with the current (distorted) general market allocation and prices (Muller et al. 2011). While the former aims at justifying policy recommendations for efficiency, the latter is in line with the aim of establishing natural capital accounting to systematically include hitherto neglected environmental goods and assets in the current system of national accounting based on (potentially distorted) market prices. Our general valuation concept allows pursuing either one of these two applications.

Having clarified the concept of social (marginal) value of public natural capital, we can now calculate it in our model framework. To this end, we switch to the mean (or: per-capita) marginal value,

$$\overline{\mathrm{WTP}} := \frac{1}{n}\mathrm{WTP}^{\Sigma}(Y_0^1, \ldots, Y_0^n, E_0) \ , \tag{11}$$

which is—for given number of households—fully equivalent to the social marginal value defined by Eq. (10) and more in line with applied studies who typically report the mean WTP found in a study population. Taking into account that income in $t = 0$ is log-normally distributed over households and therefore employing the continuous income density function $f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})$ for aggregation over households with different incomes, the mean marginal value of public natural capital (Eq. 11) can be expressed as a function of the moments of the initial income distribution, the initial stock of natural capital, and the growth rates of income and natural capital (see Appendix A.4):

$$\begin{aligned}
\overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) &= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \mathrm{WTP}(Y_0, E_0)\, dY_0 && (12) \\
&= \kappa \mu_{Y_0}^{1/\theta} \left(1 + \mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} && (13) \\
&\text{with} \quad \kappa = \frac{1-\alpha}{\alpha} \frac{1 - \rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1 - \rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t \, .
\end{aligned}$$

Eq. (13) shows how the mean social value $\overline{\mathrm{WTP}}$, and hence the total social value WTP^{Σ}, of public natural capital depends on intra- and intertemporal inequalities, mediated by the degree of substitutability between the consumption good and natural capital, and substitutability over time.[10] In the following, we study in detail the effects of intra- and intertemporal distribution on this value.

4 Results

In this section, we address five questions. The first two questions address how social WTP is affected by a change in the *intra*temporal distribution, and in particular we ask about the effects of changes in (i) mean income, μ_{Y_0} and (ii) income inequality, CV_{Y_0}. The second two questions address how social WTP is affected by a change in the *inter*temporal distribution, and in particular we ask about the effects of changes in (iii) the growth rate of income, g_Y, and (iv) the environmental loss rate, g_E. Finally, we ask (v) by which factors the social WTP has to be adjusted for benefit transfer from a study to a policy site.

Our first result answers the question, how does society's current mean income affect the social WTP? The answer is that the social WTP for public natural capital increases with mean income:

Proposition 1. *Social WTP for the public natural capital, $\overline{WTP}$ (Eq. 13), increases with current mean income, μ_{Y_0}:*

[10] This details earlier findings that the shadow price of natural capital is determined by, i.a., the substitutability of assets across each other as well as across time (Arrow et al. 2012, Dasgupta 2009).

$$\frac{\partial\, \overline{WTP}(\mu_{Y_0}, CV_{Y_0}, g_Y, E_0, g_E)}{\partial\, \mu_{Y_0}} > 0. \tag{14}$$

Proof. See Appendix A.5. □

Our second result answers the question, how does the relative income inequality affect social WTP? Here, the effect is ambiguous and the following proposition derives conditions under which the social WTP for public natural capital increases or decreases with relative inequality of the intratemporal income distribution:

Proposition 2. *Social WTP for public natural capital, $\overline{WTP}$ (Eq. 13), decreases (increases) with intratemporal income inequality, CV_{Y_0}, if and only if the public natural capital and the private consumption good are substitutes (complements):*

$$\frac{\partial\, \overline{WTP}(\mu_{Y_0}, CV_{Y_0}, g_Y, E_0, g_E)}{\partial\, CV_{Y_0}} \lesseqgtr 0 \quad \textit{if and only if} \quad \theta \gtreqless 1. \tag{15}$$

Proof. See Appendix A.6.

□

Prop. 2 states that the sign of the effect of the intratemporal income inequality on the social WTP depends on whether the environmental public goods derived from natural capital are a substitute or a complement to private consumption. For the case of substitutes, social WTP for natural capital decreases with income inequality, whereas it increases in the case of complements. Prop. 2 generalizes the finding in Baumgärtner et al. (2017) to an intertemporal setting: The degree of substitutability mediates how intratemporal income inequality affects social WTP. In our setting where relative income inequality is constant over time, this finding holds for relative income inequality at each point in time. It is also in line with intuition. In the case of complements, a rich household's individual WTP for natural capital is more than proportionally higher, and a poor household's individual WTP is less than proportionally smaller, than the average household's individual WTP. Thus, increasing inequality increases mean WTP. In the case of substitutes, this is just the other way around.

Our third result addresses the question, how does income growth affect social WTP?

Proposition 3. *Social WTP for public natural capital, $\overline{WTP}$ (Eq. 13), for a time-constant market interest factor, $\delta < \frac{1}{1+g_Y}$, increases with the growth rate of income, g_Y, if the public natural capital and the private consumption good are complements or Cobb-Douglas:*

$$\frac{\partial\, \overline{WTP}(\mu_{Y_0}, CV_{Y_0}, g_Y, E_0, g_E)}{\partial\, g_Y} > 0 \quad \textit{if} \quad \theta \leq 1. \tag{16}$$

Proof. See Appendix A.7. □

How the intertemporal distribution of income affects social WTP depends on the level of the growth rate of income, the relative sizes of the utility discount and market interest factor as well as on the degree of substitutability. The condition that the market interest rate exceeds the growth rate, $\delta < \frac{1}{1+g_Y}$, holds typically for industrialized economies (Piketty 2014). For the case of complements, an increase in intertemporal inequality in terms of consumption goods increases social WTP. As a larger income growth rate leads to more private goods consumption relative to the complementary environmental goods from natural capital, the household is willing to sacrifice more of the private good to have a more balanced consumption of both goods. We find the same effect for the special case of Cobb-Douglas substitutability.

However, if natural capital is a substitute to manufactured consumption goods there are cases where an increase in intertemporal inequality in terms of consumption goods leads to a decrease of social WTP. This depends on the relative magnitudes of the elasticity of substitution, the growth rate of income as well as utility discount factor and the market interest factor.[11] The intuition is as follows: For sufficiently substitutable goods, the household cares more about the absolute amount of goods consumed and less about their composition. A higher income growth rate makes a constant payment fraction more 'expensive' in absolute terms regarding the trade-off between the private consumption and the environmental goods from natural capital. Thus, for the sign of the effect, the discount and market interest factors play a role as the trade-off is sensitive

[11] We illustrate the range of conditions for which social WTP in case of substitutes may fall with the growth rate of income in Fig. 8 and Fig. 9 in Appendix A.8.

to how strongly those periods in the long-term future with more consumption due to higher growth matter. Our analysis of the boundary conditions in Appendix A.8 shows that lower individual (market) discounting shifts the switch from a negative growth impact on social WTP to a positive growth impact towards a lower (higher) degree of substitutability.

Our fourth result answers the question, how does the growth rate of natural capital affect the social WTP for natural capital? Again, we derive a condition that determines the sign of the effect:

Proposition 4. *Social WTP for public natural capital, $\overline{WTP}$ (Eq. 13), increases (decreases) with the growth rate of natural capital, g_E, if and only if the public natural capital and the private consumption good are substitutes (complements):*

$$\frac{\partial\, \overline{WTP}(\mu_{Y_0}, CV_{Y_0}, g_Y, E_0, g_E)}{\partial\, g_E} \gtreqless 0 \quad \textit{if and only if} \quad \theta \gtreqless 1. \tag{17}$$

Proof. See Appendix A.9. □

The intertemporal distribution of natural capital, captured by the growth rate of natural capital, has no effect on social WTP for the special case of Cobb-Douglas substitutability, and only for this case. For substitutes, an increase in intertemporal inequality in terms of natural capital increases social WTP, while it is the reverse case when natural capital is a complement to manufactured consumption goods. Thus, if there is a complementarity relationship, a ceteris paribus higher growth rate and thus consumption of environmental goods from natural capital tends to make consumption goods relatively scarcer and thus decreases WTP for an extra unit of natural capital.

Fifth, and finally, we address the question how should one adjust per-capita social WTP when transferring estimates from a study to a policy site? Specifically, we derive transfer factors to adjust for differences in the present income distribution in a dynamic context and for differences in growth rates and market interest rates. These generalizations and extensions make the benefit function approach more suitable for natural capital valuation and accounting. Social WTP for a policy site, $\overline{\mathrm{WTP}}^{\mathrm{policy}}$, is the product of a transfer function $\mathcal{T}$ and the mean WTP elicited at a study site, $\overline{\mathrm{WTP}}^{\mathrm{study}}$.

Proposition 5. *Assume that households' preferences (θ, α, ρ) are identical at the study and the policy sites. Per-capita social WTP for public natural capital at the policy site is given by* $\overline{WTP}^{policy} = \mathcal{T}(\ldots) \cdot \overline{WTP}^{study}$ *with the following transfer function*

$$\begin{aligned} \mathcal{T}(\ldots) =& \mathcal{T}_E(E_0^{policy}, E_0^{study}; \theta) \cdot \mathcal{T}_{g_E}(g_E^{policy}, g_E^{study}; \theta, \rho) \cdot \mathcal{T}_\mu(\mu_{Y_0}^{policy}, \mu_{Y_0}^{study}; \theta) \\ & \cdot \mathcal{T}_{CV}(CV_{Y_0}^{policy}, CV_{Y_0}^{study}; \theta) \cdot \mathcal{T}_{g_Y,\delta}(g_Y^{policy}, \delta^{policy}, g_Y^{study}, \delta^{study}; \theta, \rho), \end{aligned} \tag{18}$$

where the corresponding disentangled transfer factors are:

$$\mathcal{T}_E(E_0^{policy}, E_0^{study}; \theta) = \left(\frac{E_0^{policy}}{E_0^{study}}\right)^{-1/\theta}, \tag{19}$$

$$\mathcal{T}_{g_E}(g_E^{policy}, g_E^{study}; \theta, \rho) = \frac{1 - \rho(1 + g_E^{study})^{\frac{\theta-1}{\theta}}}{1 - \rho(1 + g_E^{policy})^{\frac{\theta-1}{\theta}}}, \tag{20}$$

$$\mathcal{T}_\mu(\mu_{Y_0}^{policy}, \mu_{Y_0}^{study}; \theta) = \left(\frac{\mu_{Y_0}^{policy}}{\mu_{Y_0}^{study}}\right)^{1/\theta}, \tag{21}$$

$$\mathcal{T}_{CV}(CV_{Y_0}^{policy}, CV_{Y_0}^{study}; \theta) = \left(\frac{1 + CV_{Y_0}^{policy\,2}}{1 + CV_{Y_0}^{study\,2}}\right)^{\frac{1-\theta}{2\theta^2}}, \tag{22}$$

$$\begin{aligned} &\mathcal{T}_{g_Y,\delta}(g_Y^{policy}, \delta^{policy}, g_Y^{study}, \delta^{study}; \theta, \rho) \\ &\qquad = \frac{1 - \rho\,(1 + g_Y^{policy})^{\frac{\theta-1}{\theta}}}{1 - \rho\,(1 + g_Y^{study})^{\frac{\theta-1}{\theta}}} \cdot \frac{\sum_{t=0}^{\infty} \left(\delta^{policy}\right)^t (1 + g_Y^{policy})^t}{\sum_{t=0}^{\infty} \left(\delta^{study}\right)^t (1 + g_Y^{study})^t}. \end{aligned} \tag{23}$$

Proof. See Appendix A.10. □

Prop. 5 shows how to adjust per-capita social WTP for dynamic public goods such as natural capital for site specific differences in benefit transfers. Adjustment for differences in the income distribution can be done by exploiting information on the currently prevailing intratemporal income distribution using $\mathcal{T}_{\mathrm{CV}}$ and $\mathcal{T}_\mu$. In addition, Prop. 5 shows how to adjust per-capita social WTP for differences in income growth and interest rates by using $\mathcal{T}_{g_Y,\delta}$ and for differences in the environmental loss rate by employing $\mathcal{T}_{g_E}$.

5 Application: Global biodiversity conservation

To illustrate our results and to demonstrate the applicability of the theory, we now present an empirical case study of how inequalities affect the value of global ecosystem services related to biodiversity. For this, we consider individuals instead of households, as using per-capita values from national statics is typical for benefit transfer applications.

5.1 Data

Table 1 gives an overview of the data used. For the initial global income distribution we draw on Pinkovskiy and Sala-i-Martin (2009) to use an estimate of global per-capita income distribution for 2006, with a mean of $\mu_{Y_0} = 9,550$ [2006-PPP-USD] and a standard deviation of $\sigma_{Y_0} = 15,400$ [2006-PPP-USD]. This corresponds to a coefficient of variation of $\text{CV}_{Y_0} = 1.61$. For the forecasted growth rate of income, g_Y, we draw on an expert survey by Drupp et al. (2018a). Almost two hundred experts on long-term societal decision-making were asked to provide their best guess of the global average, long-term annual growth rate of real per-capita consumption. They find a mean consumption growth rate of $g_Y^{\text{mean}} = 1.7$ percent. The lower bound (abbreviated as 'lb') is -2 percent and the upper bound ('ub') is $g_E^{\text{ub}} = 5$ percent. As only three experts stated negative growth rates, we take $g_Y^{\text{lb}} = 0.1$ percent as lower bound value in our application.

We interpret E_0 as an index of global biodiversity, with the reference level at $t = 0$ set to $E_0 = 100$. An example for such a global biodiversity index is the global 'Living Planet Index' (WWF 2020). Hence, the marginal value of biodiversity studied here is the value of an increase of the initial index by one point, from 100 to 101. The scale of E_0 has no effect on the adjustments of social WTP for differences in the intra- and intertemporal distribution, on which we focus in the following (see Prop. 5), but it scales the absolute WTP estimates.

For the growth rate of natural capital, g_E, we focus on non-use ecosystem services, as this provides the best fit for our proportional mapping from natural capital to ecosystem service provisioning. We take the global mean annual growth rate of cultural ecosystem services estimated by Baumgärtner et al. (2015), based on the best available time-series data for different ecosystem services and countries. These include as ecosystem service measures landscape connectedness, forest area, living planet index, red-list-index and national biodiversity indicators. Baumgärtner et al. (2015) estimate the global growth

Table 1: Variable and parameter values used in the application

Parameter	Value(s)	Description	Source
CV_{Y_0}	1.61	Coefficient of variation of global per-capita income	Pinkovskiy and Sala-i-Martin (2009)
μ_{Y_0}	9,550	Mean global per-capita income in 2006-PPP-USD	Pinkovskiy and Sala-i-Martin (2009)
g_Y	0.017 [0.001; 0.050]	Annual real per-capita (consumption) growth rate	Drupp et al. (2018a)
E_0	100	Global biodiversity index; set to 100 for $t = 0$	–
g_E	-0.0052 [-0.0128; -0.0008]	Growth rate of global non-use environmental goods	Baumgärtner et al. (2015)
α	0.85 [0.7; 1]	Utility share parameter for consumption goods	Kopp et al. (2012)
θ	2.31 [0.86; 7.14]	Elasticity of substitution	Drupp (2018)
ρ	0.989 [0.926; 1]	Pure time discounting factor	Drupp et al. (2018a)
δ	0.977 [0.94; 1]	Risk-free market interest factor	Drupp et al. (2018a)

Note: Numbers in brackets correspond to lower and upper bound values. We approximate the upper bound discount and interest factors with 0.999 instead of 1.

rate as $g_E^{\text{mean}} = -0.52$, with a lower (upper) bound of $g_E^{\text{lb}} = -1.28$ ($g_E^{\text{ub}} = -0.08$) percent.

We take preference parameters from the literature. For the utility share parameter of consumptions goods relative to environmental goods, α, we draw on the parameter range considered by Kopp et al. (2012), ranging from 0 to 0.3, with a mean of 0.15, for the environmental good. Thus, the parameter value for the consumption good ranges from 0.7 to 1, with a mean of 0.85.[12] For the elasticity of substitution between the environmental and the market-traded consumption good, we use data from Drupp (2018), who gathers indirect evidence from 18 non-market valuation studies. This yields a mean elasticity of substitution of $\theta^{\text{mean}} = 2.31$, implying that environmental goods are considered as substitutes to market-traded goods on average, with a lower and upper bound of $\theta^{\text{lb}} = 0.86$ and $\theta^{\text{ub}} = 7.14$ respectively.[13] For the pure time discount factor, the elas-

[12]This encompasses parameter values chosen by Sterner and Persson (2008), who assume $1 - \alpha = 0.1$, and Gollier (2010), who assumes $1 - \alpha = 0.29$.

[13]These values are similar to those found for environmental goods in a more recent meta-study by Drupp and Hänsel (forthcoming), but values implying stronger complementarity have been used in the

ticity of marginal utility of consumption and the market interest factor, we again draw on survey data from Drupp et al. (2018a), who elicited expert recommendations and long-run forecasts. They obtain a mean rate of pure time preference of 1.1 percent, with a lower and upper bound of 0 and 8 percent. This translates into an initial mean pure time discount factor of $\rho^{\text{mean}} = 0.989$, with a lower and upper bound of 0.926 and 1.[14] The mean forecasted real risk-free market interest rate is 2.38 percent, with a lower and upper bound of 0 and 6 percent. This translates into an initial mean market interest factor $\delta^{\text{mean}} = 0.977$, with a lower and upper bound of $\delta^{\text{lb}} = 0.943$ and $\delta^{\text{ub}} = 1$.[15] We quantify our results for a time horizon of hundred years, $T = 100$.

We provide a detailed discussion of the growth path conditions (Eqs. 4a and 4b) for the existence of the closed-form ICES utility function (Eq. 3) in Appendix A.11. When we compare the infima and suprema of the growth rates, g_E and g_Y, for which a closed-form intertemporal utility function exists, with empirical data on growth rates, we see that these conditions appear generally uncritical for the loss rate of ecosystem services g_E.[16] This is not the case for the income growth rate g_Y. However, for the mean pure time discount factor and the best guess estimate for the degree of substitutability, $\theta^{\text{mean}} = 2.31$, the income growth rate supremum is with $g^{\max}_{\theta=2.31} = 0.0197$ higher than the mean expected long-term growth rate of $g_Y^{\text{mean}} = 0.017$, so that the growth path condition is fulfilled. Thus, while there are cases for which our growth path conditions are not satisfied,[17] which we discuss further below, the growth path condition is fulfilled for the main specification of our model.

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applied modelling literature. For example, Sterner and Persson (2008) used a central value of 0.5.

[14]We use a value of 0.999 instead of 1 to ensure that our intertemporal welfare function is bounded.

[15]We use a value of 0.999 to ensure that the social WTP as a constant payment fraction is bounded.

[16]For the mean estimate on the pure time discount factor, $\rho^{\text{mean}} = 0.989$, and complements with the strongest complementarity we consider here, $\theta^{\text{lb}} = 0.86$, the growth rate infimum $g^{\min}_{\theta=0.86} = -0.0657$ is well below the lower bound rate of global loss of ecosystem services $g_E^{\text{lb}} = -0.0128$ estimated by Baumgärtner et al. (2015), so that the growth path condition is satisfied.

[17]For instance, the upper bound of the expected annual global income growth rate of $g_Y^{\text{ub}} = 0.05$ does not meet the growth path condition for the mean substitutability parameter $g^{\max}_{\theta=2.31}$. Moreover, for the upper bound of the substitutability parameter, $\theta = 7.14$, the mean growth rate, g_Y^{mean}, is already higher than the supremum $g^{\max}_{\theta=7.14}$ and thus the growth path condition is not fulfilled.

5.2 Quantification of main results

We now estimate how intra- and intertemporal distribution affects per-capita social WTP, $\overline{\text{WTP}}$, for public-good ecosystem services derived from the public natural capital stock of biodiversity. Moreover, we compute transfer factors that allow controlling for the intertemporal aspects of natural capital valuation.

Fig. 2 depicts how mean income (left) and income inequality (right) affect global per-capita social WTP. Examining the left panels of Fig. 2, we see that social WTP for public biodiversity is increasing with mean income (see Prop. 1). For substitutes social WTP is a strictly increasing concave function of mean income (Fig. 2 top left subplot), while it is a convex function for complements (bottom left subplot). We find that for a hypothetical doubling of global mean income, social WTP would be 34.99 percent higher for the mean substitutability estimate. In this case, WTP-estimates would need to be adjusted with a factor of $\mathcal{T}_\mu(2\mu_{Y_0}^{\text{GLO}}, \mu_{Y_0}^{\text{GLO}}; \theta^{\text{mean}}) = 1.35$. For the lower bound range of complements, $\theta^{\text{lb}} = 0.86$, it would be even 123.89 percent higher as initially, corresponding to a transfer factor of $\mathcal{T}_\mu(2\mu_{Y_0}^{\text{GLO}}, \mu_{Y_0}^{\text{GLO}}; \theta^{\text{lb}}) = 2.24$. Foremost, however, the two left subfigures show that the degree of substitutability, θ, is crucial for determining social WTP: Social WTP is multiple factors higher for the lower bound complementarity value ($\overline{\text{WTP}}$=3.97×10^3 [2006-PPP-USD]) as compared to social WTP for the mean substitutability estimate ($\overline{\text{WTP}}$=8.61 [2006-PPP-USD]).

The subplots on the right of Fig. 2 illustrate how social WTP for public biodiversity conservation changes for a change in relative intratemporal income inequality as measured by the coefficient of variation of per-capita income, CV_{Y_0}. While social WTP decreases with income inequality for the mean case of substitutes (top right subplot), it increases for complements (bottom right subplot) (see Prop. 2). A hypothetical reduction of global income inequality, $\text{CV}_{Y_0}^{\text{GLO}} = 1.61$, to zero would increase social WTP by 17.00 percent, corresponding to a transfer factor of $\mathcal{T}_{\text{CV}}(0, \text{CV}_{Y_0}^{\text{GLO}}; \theta^{\text{mean}}) = 1.17$ given the mean empirical estimate for the elasticity of substitution, $\theta^{\text{mean}} = 2.31$. In contrast, the lower bound elasticities of substitution, $\theta^{\text{lb}} = 0.86$, produces a decrease by 11.40 percent, corresponding to a transfer factor of $\mathcal{T}_{\text{CV}}(0, \text{CV}_{Y_0}^{\text{GLO}}; \theta^{\text{lb}}) = 0.89$.

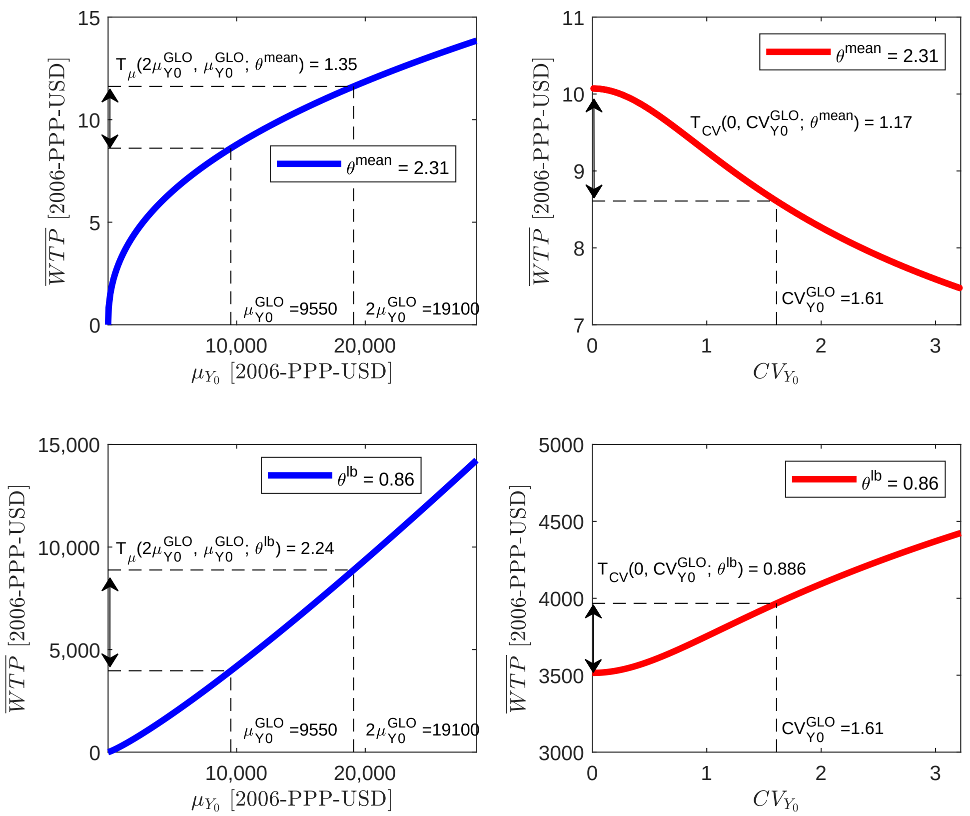

Figure 2: Effect of mean income, μ_{Y_0}, (left side) and relative intratemporal income inequality, CV_{Y_0}, (right side) on the per-capita social WTP, $\overline{\mathrm{WTP}}$, for a one percent increase in public biodiversity for different degrees of substitutability, θ, between the consumption good and ecosystem services derived from public biodiversity.

The left side of Fig. 3 illustrates how social WTP for global biodiversity conservation depends on the intertemporal distribution of income, i.e. how it changes with the income growth rate. Social WTP increases with income growth for complements and Cobb-Douglas substitutability (Prop. 3). Yet, for the case of substitutes, social WTP can also decreases with income growth.[18] We find that if the expected global income growth rate would be only half as large, social WTP would be 192.83 percent higher for a substitutability of $\theta^{\mathrm{mean}} = 2.31$ and decrease by 36.54 percent for a substitutability of $\theta^{\mathrm{lb}} = 0.86$. Concerning the effect of the growth rate of biodiversity and related ecosystem services, the right side of Fig, 3 shows that social WTP increases with the growth rate of biodiversity for substitutes, but decreases for complements (Prop. 4). If the loss rate

[18]In our central calibration and for an infinite time horizon, social WTP starts to decrease with income growth at g_Y^{mean} already for a degree of substitutability of $\theta^* = 1.93$ (see Fig. 9 and Eq. A.68).

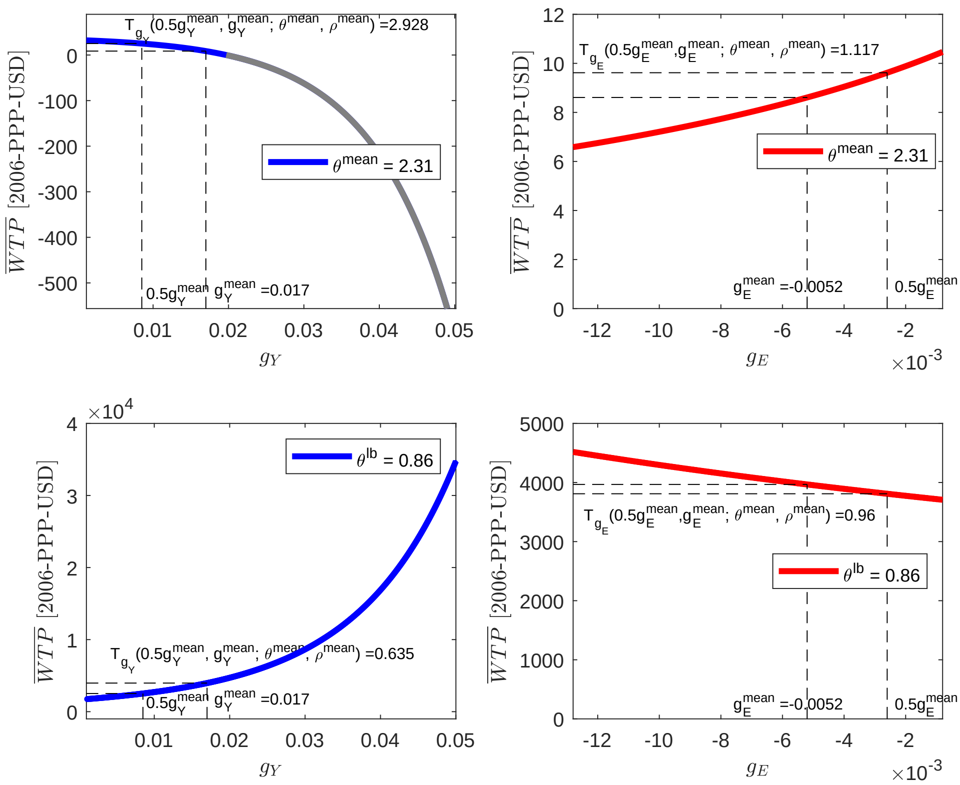

Figure 3: Effect of the global per-capita income growth rate, g_Y, (left side) and the growth rate of biodiversity and related ecosystem services, g_E, (right side) on the per-capita social WTP, $\overline{\text{WTP}}$, for a one percent increase in biodiversity for different degrees of substitutability, θ, between the consumption good and ecosystem services derived from biodiversity. Grey coloured lines indicate parameter combinations that do not meet the growth path condition for the closed-form intertemporal utility function (Eq. 4a).

of of biodiversity would be reduced by half, social WTP would increase by 11.72 percent for our mean substitutes case, $\theta^{\text{mean}} = 2.31$, and decrease social WTP by 3.97 percent for our lower bound complements, $\theta^{\text{lb}} = 0.86$. In this case, the effect of income growth on the social WTP is relatively stronger than the effect of the growth rate of biodiversity.

Next, we study whether the benefit transfer factors for differences in growth rates and market interest rates (Prop. 5) lead to notable WTP adjustments. Specifically, we perform hypothetical transfers of per-capita social WTP elicited at the mean of empirical estimates to sites characterized by the lower or upper bound parameters within the empirically plausible parameter a value ranges (cf. Table 2).

First, we turn to the transfer factor for differences in the growth rate of biodiversity $\mathcal{T}_{g_E}$ (Eq. 20). Fig. 4 displays the required adjustment when transferring social WTP from

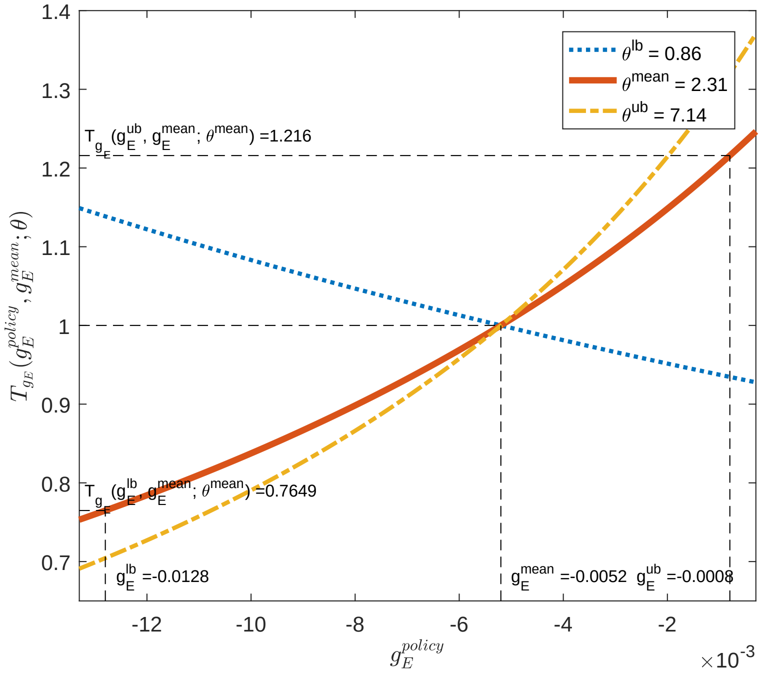

Figure 4: Transfer factor to adjust per-capita social WTP, $\overline{\text{WTP}}$, for a one percent increase in biodiversity from a study site with a growth rate of $g_E^{\text{study}} = -0.0052$ to the growth rate g_E^{policy} at a policy site. Colours depict different degrees of substitutability, θ, between the consumption good and ecosystem services derived from biodiversity.

a study site with the global average growth rate of non-use ecosystem services, $g_E^{\text{mean}} = -0.0052$, to a policy site with a growth rate within the range of global growth rates for different non-use ecosystem services estimated by Baumgärtner et al. (2015). Applying environmental values elicited at a study site with $g_E^{\text{study}} := g_E^{\text{mean}}$ at a policy site with a higher growth rate of the environmental good, that is where the loss of biodiversity is at a lower rate, equal to $g_E^{\text{ub}} = -0.0008$, would require an upward adjustment of social WTP by 21.58 percent, corresponding to a transfer factor of $\mathcal{T}_{g_E}(g_E^{\text{ub}}, g_E^{\text{mean}}; \theta^{\text{mean}}) = 1.22$. To the contrary, social WTP-estimates for a transfer to a policy context with a higher rate of biodiversity loss of $g_E^{\text{lb}} = -0.0128$ would need to be lowered by 23.51 percent, that is they require adjustment by the factor $\mathcal{T}_{g_E}(g_E^{\text{lb}}, g_E^{\text{mean}}; \theta^{\text{mean}}) = 0.77$. Again, the transfer factor crucially depends on the substitutability between the two goods. A higher degree of substitutability would reinforce these required adjustments, $\mathcal{T}_{g_E}(g_E^{\text{ub}}, g_E^{\text{mean}}; \theta^{\text{ub}}) = 1.32$ and $\mathcal{T}_{g_E}(g_E^{\text{lb}}, g_E^{\text{mean}}; \theta^{\text{ub}}) = 0.70$, but complementarity would reverse the direction of the required adjustments, with $\mathcal{T}_{g_E}(g_E^{\text{ub}}, g_E^{\text{mean}}; \theta^{\text{lb}}) = 0.94$ and $\mathcal{T}_{g_E}(g_E^{\text{lb}}, g_E^{\text{mean}}; \theta^{\text{lb}}) = 1.14$.

Second, adjusting social WTP for differences in income growth rates, $\mathcal{T}_{g_Y,\delta}$ (Eq. 23), can be substantial (Fig. 5). To isolate the effect of the income growth rate, we assume for now that the market interest factor is identical at the policy and the study site and constant over time, $\delta^{\text{policy}} = \delta^{\text{study}}$, so that the market interest factors cancel out in $\mathcal{T}_{g_Y,\delta}$ (Eq. 23). Note that depending on the degree of substitutability we end up in parameter constellations in which the growth path condition on g_Y is not fulfilled and the closed-form transfer factor $\mathcal{T}_{g_Y,\delta}$ cannot be applied any more. In Fig. 5 the estimates of the transfer factor for income growth rates are coloured grey at growth rates where the growth path condition does not hold (Eq. 4a). Applying WTP-estimates elicited for an income growth rate at the expected global mean, $g_Y^{\text{study}} := g_Y^{\text{mean}} = 0.017$, in a policy context where the income growth rate is $g_Y^{\text{lb}} = 0.001$ would imply a transfer factor of $\mathcal{T}_{g_Y,\delta}(g_Y^{\text{lb}}, g_Y^{\text{mean}}; \theta^{\text{mean}}) = 3.75$. That is, the social WTP-estimate would have to be adjusted upwards by 274.65 percent. The direction of adjustment is reversed for complements requiring a downward adjustment, with $\mathcal{T}_{g_Y,\delta}(g_Y^{\text{lb}}, g_Y^{\text{mean}}; \theta^{\text{lb}}) = 0.44$. For upper bound substitutability, $\theta^{\text{ub}} = 7.14$, the growth path condition is not meet at g_Y^{mean}. Hence, we cannot apply the transfer factor. The required adjustments $\mathcal{T}_{g_Y,\delta}$ are even more pronounced, when applying WTP-estimates in contexts with higher income growth equal to the maximal expected rate, $g_Y^{\text{ub}} = 0.05$. Yet, the growth path condition is also not met for this parameter constellation for the mean substitutability estimate, θ^{mean}, at $g_Y^{\text{ub}} = 0.05$.[19] For the case of complements, $\theta^{\text{lb}} = 0.86$, applying WTP-estimates from a site characterised by the mean expected income growth rate in a context characterised by the maximal expected rate implies a transfer factor of $\mathcal{T}_{g_Y,\delta}(g_Y^{\text{ub}}, g_Y^{\text{mean}}; \theta^{\text{lb}}) = 8.76$ and thus an upward adjustment of social WTP of 776.30 percent.

Third, Fig. 6 depicts again the transfer factor $\mathcal{T}_{g_Y,\delta}$, but this time for differences in the market interest factor between a study and a policy site. For illustration, the income growth rate at policy and study site are identical and equal to the global average, $g_Y^{\text{study}} = g_Y^{\text{policy}} = g_Y^{\text{mean}}$. For identical growth rates, the first factor in $\mathcal{T}_{g_Y,\delta}$ reduces to one and hence the entire transfer factor does not depend on the elasticity of substitution, θ,

[19] The maximum value for applying $\mathcal{T}_{g_Y,\delta}$ is for $\theta^{\text{mean}} = 2.31$ at $g_{\theta=2.31}^{\text{max}} = 0.0197$, where the transfer factor approaches zero, while it is generally uncritical for complements.

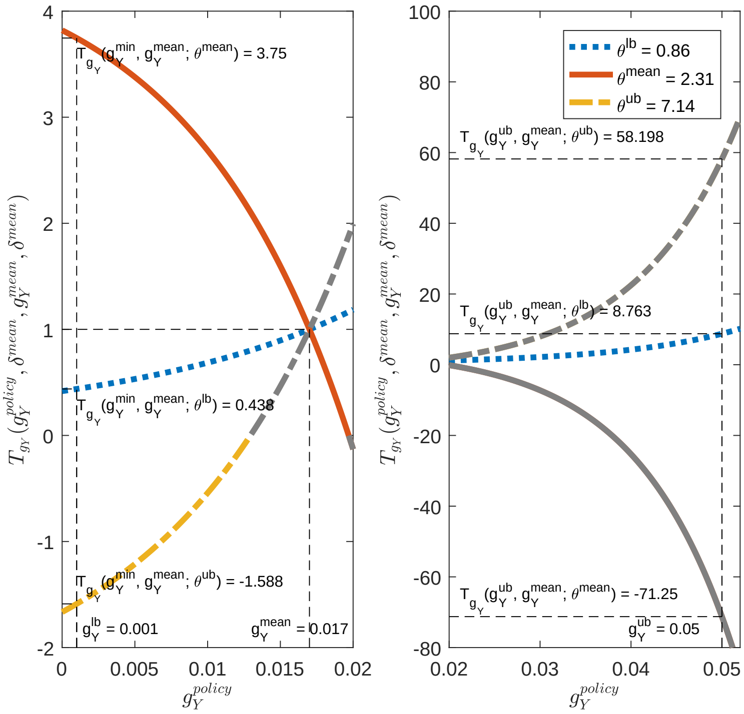

Figure 5: Transfer factor to adjust per-capita social WTP, $\overline{\text{WTP}}$, for a one percent increase in public biodiversity from a study site with an income growth rate of $g_Y^{\text{study}} := g_Y^{\text{mean}} = 0.017$ to an income growth rate g_Y^{policy} at a policy site, such as the lower bound growth rate $g_Y^{\text{lb}} = 0.001$. Coloured lines depict different degrees of substitutability, θ, between the consumption good and ecosystem services derived from biodiversity. Grey coloured lines indicate parameter combinations that do not meet the growth path condition for the closed-form intertemporal utility function (Eq. 4a).

any more. It shows that differences in market interest rates within the range expected by international experts lead to substantial WTP adjustments: For a hypothetical transfer of per-capita social WTP elicited at $\delta^{\text{mean}} = 0.977$ to a policy site with $\delta^{\text{ub}} = 0.999$ the required adjustment would be $\mathcal{T}_{g_Y,\delta}(g_Y^{\text{mean}}, \delta^{\text{ub}}, g_Y^{\text{mean}}, \delta^{\text{mean}}) = 3.35$ and thus increase social WTP by 235.39 percent. In contrast, a hypothetical transfer to the lower bound forecasted market interest factor, $\delta^{\text{lb}} = 0.94$, i.e. a situation with a high market interest rate, would imply a transfer factor of $\mathcal{T}_{g_Y,\delta}(g_Y^{\text{mean}}, \delta^{\text{lb}}, g_Y^{\text{mean}}, \delta^{\text{mean}}) = 0.29$. Moreover, we see that the required adjustment of social WTP in absolute terms, that is $|\mathcal{T}_{g_Y,\delta} - 1|$, is larger for higher levels of the common income growth rate at study and policy site.

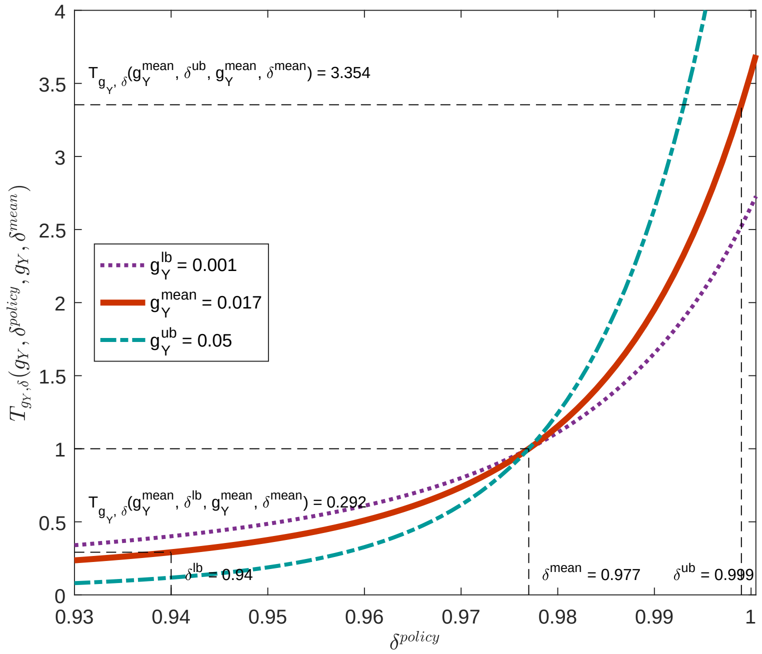

Figure 6: Transfer factor to adjust mean WTP, $\overline{\text{WTP}}$, for a one percent increase in biodiversity from a study site with a market interest rate of $\delta^{\text{study}} = 0.977$ to the market interest rate δ^{policy} at the policy site when the income growth rate at both the study and policy site is identical. Colours indicate different levels of the income growth rate common to the study and policy site.

6 Extensions

This section provides a number of extensions to our main modelling framework. We show how our results generalise to a different policy (Section 6.1) or payment vehicle (Section 6.2) as well as to other utility functions (Section 6.3, 6.4) or more general preference structures and time paths (Section 6.5).

6.1 Change in the growth rate of natural capital

First, an environmental policy could also increase the growth rate of natural capital. That is, the dynamics of $\{E_t\}$ and $\{E'_t\}$ according to Eq. (1b) would differ in the growth rate, $g_{E'} = g_E + dg_E$, but not in the initial stock, $E'_0 = E_0$. For instance, an increase in the growth rate of biodiversity (which is, at the currently negative growth rates, a slowing down of the loss of biodiversity) might result from improving habitat conditions for insects and birds, thus increasing their net reproduction rates.

Following the same steps as for the derivation of Eq. (13), yields the social WTP for a marginal increase of the growth rate by one unit (see Appendix A.12.1)

$$\overline{\mathrm{WTP}}_{dg_E}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) = \kappa' \mu_{Y_0}^{1/\theta} \left(1 + \mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}}, \tag{24}$$

$$\text{with} \quad \kappa' = \frac{1-\alpha}{\alpha} \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} E_0^{\frac{\theta-1}{\theta}} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t.$$

Props. 1, 2 and 3 carry over to the growth rate as a different object of valuation (see Appendices A.12.2, A.12.3, A.12.4): the social WTP for a marginal increase of the grothe rate of public natural capital, $\overline{\mathrm{WTP}}_{dg_E}$, always increases with mean income, μ_{Y_0}, and decreases (increases) with income inequality, CV_{Y_0}, if and only if the public natural capital and the private consumption good are substitutes (complements). $\overline{\mathrm{WTP}}_{dg_E}$ increases with g_Y for complementarity or Cobb-Douglas, but it may decrease with g_Y in the case of substitutes. When valuing a change in the growth rate of natural capital, the transfer function is slightly different and given by (see Appendix A.12.5):

$$\begin{aligned}\mathcal{T}_{dg_E}(\ldots) =& \mathcal{T}_E'(E_0^{\text{policy}}, E_0^{\text{study}}; \theta) \cdot \mathcal{T}_{g_E}'(g_E^{\text{policy}}, g_E^{\text{study}}; \theta, \rho) \cdot \mathcal{T}_\mu(\mu_{Y_0}^{\text{policy}}, \mu_{Y_0}^{\text{study}}; \theta) \\ &\cdot \mathcal{T}_{\mathrm{CV}}(\mathrm{CV}_{Y_0}^{\text{policy}}, \mathrm{CV}_{Y_0}^{\text{study}}; \theta) \cdot \mathcal{T}_{g_Y,\delta}(g_Y^{\text{policy}}, \delta^{\text{policy}}, g_Y^{\text{study}}, \delta^{\text{study}}; \theta, \rho),\end{aligned} \tag{25}$$

with $\mathcal{T}_{\mathrm{CV}}$, $\mathcal{T}_\mu$ and $\mathcal{T}_{g_Y,\delta}$ as in Prop. 5. Thus, adjusting for differences in the intra- and intertemporal income distribution works exactly the same for both objects of valuation, i.e. is independent of whether the valuation is done for a marginal change in the initial stock or in the growth rate of natural capital. In contrast, one has to apply $\mathcal{T}_E'$ and $\mathcal{T}_{g_E}'$, instead of $\mathcal{T}_E$ and $\mathcal{T}_{g_E}$ given in Prop. 5, when valuing a change in the growth rate of natural capital. These transfer factors read $\mathcal{T}_E' = \left(\frac{E_0^{\text{policy}}}{E_0^{\text{study}}}\right)^{\frac{\theta-1}{\theta}}$ and $\mathcal{T}_{g_E}' = \frac{\rho(1+g_E^{\text{policy}})^{-1/\theta}\left(1-\rho(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}\right)^2}{\rho(1+g_E^{\text{study}})^{-1/\theta}\left(1-\rho(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}\right)^2}$.

6.2 Single payment

For measuring WTP, we have considered so far a payment made as a constant fraction of income in each time period. While this is the most-often used payment vehicle in non-market valuation studies using stated preference techniques, sometimes WTP is considered as a single payment (hereafter: SP) – usually made in the initial period. For standard time preferences (Eq. 2) and if the single payment is made in $t = 0$, the compensating surplus $\omega^i_{\mathrm{SP}} > 0$ is

$$u(Y^i_0 - \omega^i_{\mathrm{SP}}, E'_0) + \sum_{t=1}^{\infty} \rho^t\, u(Y^i_t, E'_t) \;=\; \sum_{t=0}^{\infty} \rho^t\, u(Y^i_t, E_t). \tag{26}$$

Using ICES utility in Eq. (26), considering a marginal improvement in the initial stock of public natural capital, $E'_0 = E_0 + dE$, and conducting a Taylor series expansion at $\omega^i_{\mathrm{SP}} = 0$ and $dE = 0$ gives (see Appendix A.13.1)

$$\omega_{\mathrm{SP}}\left(Y^i_0, E_0\right) = \frac{1-\alpha}{\alpha}\, \frac{{Y^i_0}^{1/\theta}\, E_0^{-1/\theta}}{1 - \rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}, \tag{27}$$

where $\omega_{\mathrm{SP}}\,(Y^i_0, E_0)$ is household i's individual WTP for a marginal increase in the initial stock of natural capital by one unit. Compared to a constant payment, the intertemporal Lindahl-Samuelson condition reads slightly simpler as $c'(E^\star_0) = \sum_{i=1}^{n} \omega_{\mathrm{SP}}(Y^i_0, E^\star_0)$ (see Appendix A.13.2). Taking the expected value of Eq. (27), the corresponding mean WTP in terms of a single payment at $t = 0$ is

$$\begin{aligned} \overline{\mathrm{WTP}}_{\mathrm{SP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, E_0, g_E) \;&=\; \kappa''\, \mu_{Y_0}^{1/\theta}\, \left(1 + \mathrm{CV}^2_{Y_0}\right)^{\frac{1-\theta}{2\theta^2}} \\ \text{with} \qquad \kappa'' \;&=\; \frac{1-\alpha}{\alpha}\, \frac{E_0^{-1/\theta}\, dE}{1 - \rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}, \end{aligned} \tag{28}$$

which does not depend on, and is thus not affected by, the growth rate of income g_Y (in contrast to a constant payment fraction, see Prop. 3). Props. 1, 2 and 4 carry over to a single payment (see Appendices A.13.4, A.13.5, A.13.6). Moreover, the benefit transfer function is identical to Prop. 5, except that one does not need to control for differences in income growth and market interest rates. It reads (see Appendix A.13.7)

$$\begin{aligned} \mathcal{T}_{\mathrm{SP}}(\ldots) =& \mathcal{T}_E(E_0^{\mathrm{policy}}, E_0^{\mathrm{study}}; \theta) \cdot \mathcal{T}_{g_E}(g_E^{\mathrm{policy}}, g_E^{\mathrm{study}}; \theta, \rho) \cdot \mathcal{T}_\mu(\mu_{Y_0}^{\mathrm{policy}}, \mu_{Y_0}^{\mathrm{study}}; \theta) \\ & \cdot \mathcal{T}_{\mathrm{CV}}(\mathrm{CV}_{Y_0}^{\mathrm{policy}}, \mathrm{CV}_{Y_0}^{\mathrm{study}}; \theta), \end{aligned} \tag{29}$$

where the transfer factors are those stated in Prop. 5.

6.3 CES-CIES preferences

To study how sensitive the effect sizes of our main analysis are to the assumed ICES utility function (Eq. 3), we consider the case of constant-elasticity-of-substitution (CES) and constant-intertemporal-elasticity-of-substitution (CIES) utility function that is prominently applied in the dual-discounting literature (e.g. Sterner and Persson 2008, Gollier 2010, Traeger 2011, Zhu et al. 2019). In contrast to ICES utility, the CES-CIES specification implies non-constant intratemporal elasticities of substitution for each good when a bundle of goods is considered. Household i's instantaneous CES utility reads

$$u(Y_t^i, E_t) = \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha)\, {E_t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \ , \tag{30}$$

where θ, with $0 < \theta < +\infty$, is the CES between the two goods (Y_t^i, E_t), and $0 < \alpha < 1$ determines the initial weight of the consumption good in utility. Intertemporal utility is the aggregated discounted instantaneous utility represented by the CIES form

$$U(\{Y_t^i\}, \{E_t\}) = \sum_{t=0}^{\infty} \rho^t \, \frac{1}{1-\eta} \, u(Y_t^i, E_t)^{1-\eta} \ , \tag{31}$$

where $0 < \rho < 1$ is the pure time discount factor and $0 \leq \eta < \infty$ is the inverse of the constant intertemporal elasticity of substitution with respect to the within-period aggregate consumption bundle, composed of Y_t^i and E_t. For instantaneous CES (Eq. 30) and intertemporal CIES utility (Eq. 31) a household's intertemporal utility reads

$$U(\left\{Y_t^i\right\}, \{E_t\}) = \sum_{t=0}^{\infty} \rho^t \, \frac{1}{1-\eta} \, \left(\alpha \, {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha)\, {E_t}^{\frac{\theta-1}{\theta}}\right)^{\frac{(1-\eta)\theta}{\theta-1}} \ . \tag{32}$$

Note, that in a CES-CIES framework, a constant single elasticity can be recovered by assuming that the inverse of the CIES with respect to the aggregate consumption bundle, η, equals the inverse of the elasticity of substitution between market consumption goods and environmental goods, $1/\theta$. For $\eta = 1/\theta$, CES-CIES utility, Eq. (32), reduces to

ICES utility, Eq. (3), which we studied in our main analysis.[20]

For general CES-CIES utility the social WTP—discounted at market interest factor δ—expressed as a constant fraction of income is (see Appendix A.14):

$$\begin{aligned}
&\overline{\mathrm{WTP}}(\mu_{Y_0}, \sigma_{Y_0}, g_Y, E_0, g_E) \\
&= \kappa''' \int_0^\infty f_{\ln}(.) \, \frac{\sum_{t=0}^\infty u(Y_0^i\,(1+g_Y)^t, E_0\,(1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^\infty u(Y_0^i\,(1+g_Y)^t, E_0\,(1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t} \, {Y_0^i}^{1/\theta} \, dY_0 \qquad (33) \\
&\quad \text{with} \qquad \kappa''' = \frac{1-\alpha}{\alpha}\, {E_0}^{-1/\theta} \sum_{t=0}^\infty \delta^t (1+g_Y)^t,
\end{aligned}$$

where $u(Y_t^i, E_t)$ is instantaneous CES utility (Eq. 30).

We estimate Eq. (33) for the parameter values of our central calibration (see Table 1), $\theta^{\mathrm{mean}} = 2.31$ and a range of values for η.[21] To specify η, we draw on expert responses surveyed by Drupp et al. (2018a). These range from $\eta^{\mathrm{lb}} = 0$ to $\eta^{\mathrm{ub}} = 5$, with a mean of $\eta^{\mathrm{mean}} = 1.35$.[22] We compare these with the case of $\eta^{\mathrm{ICES}} = 1/\theta$.

Fig. 7 depicts the effects of the intra- and intergenerational distribution on $\overline{\mathrm{WTP}}$ for a broad range of estimates on the intertemporal elasticity of substitution. We observe that the assumption of $\eta^{\mathrm{ICES}} = 1/\theta$ produces a too conservative estimate compared to $\overline{\mathrm{WTP}}$ for the mean expert value of η^{mean} in all cases. Furthermore, the choice of η within the lower and upper bound estimates matters increasingly as initial mean income, μ_{Y_0}, or the income growth rate, g_Y, become larger. In contrast, the variation induced by the range of η's considered remains relatively constant for variation in initial income

[20] There is considerable scope for $\eta = 1/\theta$ to hold, as typical values for η range from 0 to 5 (e.g. Drupp et al. 2018a, Groom and Maddison 2018), and those for $1/\theta$ range from 0.14 to 2 (e.g. Sterner and Persson 2008, Drupp 2018).

[21] To numerically estimate Eq. (33) we take 10.000 draws from a log-normal distribution of Y_0. We slightly recalibrate κ''' by a linear factor to resemble the best-guess social WTP estimate in the central calibration (Section 5).

[22] This range encompasses estimates of the elasticity of marginal utility in the literature. A meta-analysis by Havranek et al. (2015) suggest that the mean estimate for the elasticity of intertemporal substitution is 0.5, implying an η of 2, and that estimates for η below 1.3 are not in line with empirical evidence. Groom and Maddison (2018) use a number of methods to estimate the elasticity of marginal utility in the UK, including the equal-sacrifice approach using income tax schedules, the Euler-equation approach, the Frisch Formula, risk aversion measures from insurance data and estimates derived from subjective well-being. They find that η ranges from around 1.3 to 3.6, with a mean value of around 1.5.

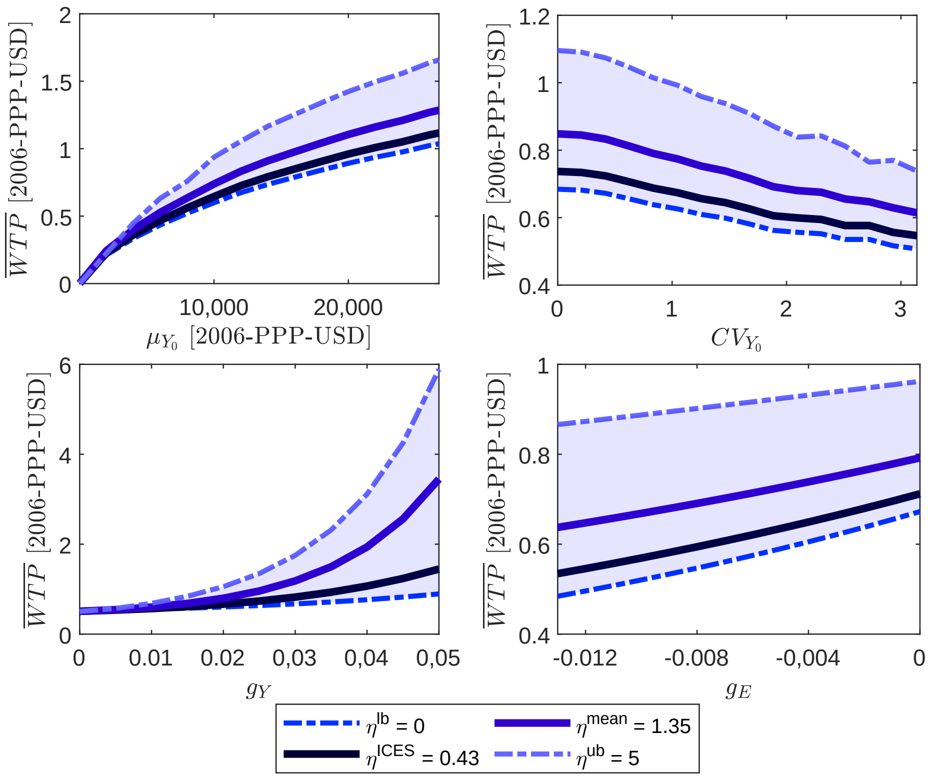

Figure 7: How different intertemporal elasticities of substitution change the effect of mean income, μ_{Y_0} (top left), income inequality, CV_{Y_0} (top right), income growth, g_Y (bottom left) and environmental loss rate, g_E (bottom right) on $\overline{\text{WTP}}$. We study the mean, η^{mean}, lower η^{lb} and upper bound η^{ub} estimates as surveyed among leading international experts and contrast it with ICES utility implying $\eta^{\text{ICES}} = 1/\theta^{\text{mean}} = 0.43$.

inequality, CV_{Y_0} or the environmental loss rate, g_E. While the choice of η changes mean WTP estimates quantitatively, the qualitative predictions remain unaffected.

6.4 Cobb-Douglas preferences

The only other case beyond ICES utility for which the intertemporal utility function satisfies both a constant elasticity of substitution between goods at each point in time and a constant intertemporal elasticity of substitution for the consumption good is if instantaneous utility is of the Cobb-Douglas form (Quaas et al. 2020). In this case, and in contrast to ICES utility (Eq. 3), the elasticity of substitution between goods might differ from each good's intertemporal elasticity of substitution. We thus study Cobb-Douglas-CIES utility (as used by Gerlagh and Keyzer 2001) as an another extension.

For $\theta \to 1$ the instantaneous CES utility function (Eq. 30) converges to $u(Y_t^i, E_t) = {Y_t^i}^{\alpha} {E_t}^{1-\alpha}$. Using this in the CIES utility function (Eq. 31), and assuming $\rho(1 +$

$g_Y)^{\alpha(1-\eta)}\,(1+g_E)^{(1-\alpha)(1-\eta)} < 1$ to ensure that intertemporal utility converges, gives the intertemporal utility function for Cobb-Douglas preferences (see Appendix A.15.1)

$$U(Y_0^i, g_Y, E_0, g_E) = \frac{1}{1-\eta}\,\frac{\left(Y_0^{i\,\alpha}\,E_0^{1-\alpha}\right)^{1-\eta}}{1-\rho\,\left[(1+g_Y)^{\alpha}\,(1+g_E)^{(1-\alpha)}\right]^{(1-\eta)}}\,. \tag{34}$$

Following the same steps as for Eq. (13), the social WTP reads (see Appendix A.15.2)

$$\overline{\mathrm{WTP}}(\mu_{Y_0}, g_Y, E_0) = \frac{(1-\alpha)}{\alpha}\,E_0^{\;-1}\left[\sum_{t=0}^{\infty}\delta^t\,(1+g_Y)^t\right]\mu_{Y_0}, \tag{35}$$

which is strictly increasing with mean income (as in Prop. 1) and does not depend on income inequality (as expected from Prop. 2; compare to case $\theta = 1$). As expected from Props. 3 and 4 (compare each to the case $\theta = 1$), $\overline{\mathrm{WTP}}$ increases in g_Y, but does not depend on g_E. As for ICES utility (Eq. 5) $\overline{\mathrm{WTP}}$ is not determined by η.

6.5 General intertemporal utility and time paths

Finally, we derive social WTP for general utility specification to facilitate a broader understanding on what composes WTP. Consider ordinal, continuous intertemporal utility, $U : \mathbb{R}_+^{2T} \to \mathbb{R}$. Households have identical preferences over two goods, Y and E, and are purely self-regarding. $U(\{Y_t^i\}, \{E_t\})$ is once differentiable in both arguments. Income in period t is a differentiable function of its initial level, growth rate and time, $Y_t^i(Y_0^i, g_Y, t)$, and so is the natural capital, $E_t(E_0, g_E, t)$.

At the Pareto-efficient level $E_0^\star$ of public natural capital and for compensated income, social WTP is (see Appendix A.16)

$$\overline{\mathrm{WTP}} := \frac{1}{n}\sum_{i=1}^{n}\mathrm{WTP}\left(\{Y_t^i\}, \{E_t\}\right), \tag{36}$$

$$\text{where}\quad \mathrm{WTP}\left(\{Y_t^i\}, \{E_t\}\right) = \frac{\sum_{t=0}^{\infty}\frac{\partial U}{\partial E_t}\left(\{Y_t^i\}, E_t\right)\frac{\partial E_t}{\partial E_0}}{\sum_{t=0}^{\infty}\frac{\partial U}{\partial Y_t^i}\left(Y_t^i, \{E_t\}\right)\frac{\partial Y_t^i}{\partial Y_0^i}}\sum_{t=0}^{\infty}\delta^t\frac{\partial Y_t^i}{\partial Y_0^i}, \tag{37}$$

is the household's WTP for a marginal increase in the initial level of natural capital.

Eq. (37) can be rewritten as

$$\mathrm{WTP}\left(\{Y_t^i\},\{E_t\}\right) = \mathrm{MRS}^i_{Y_0^i,E_0} \frac{\sum_{t=0}^{\infty} \mathrm{MRS}^i_{E_t,E_0} \frac{\partial E_t}{\partial E_0}}{\sum_{t=0}^{\infty} \mathrm{MRS}^i_{Y_t^i,Y_0^i} \frac{\partial Y_t^i}{\partial Y_0^i}} \sum_{t=0}^{\infty} \delta^t \frac{\partial Y_t^i}{\partial Y_0^i}, \tag{38}$$

where $\mathrm{MRS}^i_{Y_0^i,E_0} := -\frac{\mathrm{d}E_0}{\mathrm{d}Y_0^i} = \frac{\partial U}{\partial Y_0^i} / \frac{\partial U}{\partial E_0}$ is the marginal rate of substitution (MRS) between income at time $t = 0$ and the natural capital at time $t = 0$, $\mathrm{MRS}^i_{E_t,E_0}$ is the MRS between the natural capital at t and at $t = 0$, and $\mathrm{MRS}^i_{Y_t^i,Y_0^i}$ is the MRS between income at t and at $t = 0$. Eq. (38) shows that individual WTP depends on the initial MRS between both goods and on both good-specific MRSs over time.

7 Discussion

A number of assumptions limit the generality of our analysis. These include (i) the proportional mapping between natural capital and environmental services, (ii) the constant (de)growth rates of income and of environmental services, (iii) the purely self-regarding character of infinitely long-lived households, (iv) identical preferences, (v) the pure public-good character of natural capital, and (vi) the deterministic development.

First, we considered a simple proportional mapping between natural capital and environmental goods. Yet, the mapping of different forms of natural capital into the services it provides are multifaceted. Our analysis thus applies only to those cases of that may be reasonably described by this simplification. In particular, our work focusses on non-use services derived from public natural capital for which WTP information is key for public policy. Beyond existence values, our analyses is relevant for regulating services and provisioning services of sustainably managed stocks of natural capital. For an analysis of provisioning services derived from natural capital, such as the fishery, featuring more complex dynamics see, e.g. Fenichel et al. (2016).

Second, as exemplary time paths for income and natural capital we have considered exponential growth or decline. We show in Appendix A.17 how, for the case of income, such a time-constant growth rate, g_Y, can be derived as the balanced growth path of a general equilibrium endogenous growth model. While these time paths are prominent case in the long-term analysis of environmental-economic problems (e.g. Hoel and Sterner 2007, Baumgärtner et al. 2015), there may be different relevant time paths. As non-market valuation studies often do not specify the exact time path of the evolution of natural capital or environmental goods and services, we leave an analysis of other types of growth dynamics for future research.

Third, our analysis assumes purely self-regarding and infinitely long-lived households. Yet, there may also be behavioral responses to income inequality within and across generations. These may include relative consumption concerns (e.g. Johansson-Stenman and Sterner 2015) or variants of inequality aversion (e.g. Cairns et al. 2020).

Fourth, we have assumed that all households are identical except that they differ in initial income. Yet, households may also face different income or consumption growth rates. For example, recent empirical evidence from the World Inequality Report (Alvaredo et al. 2018) shows that the growth rate of income differs over income groups. Different growth rates of income give rise to convergence or divergence of income and hence changing relative income inequality over time. Future work should try to relax the assumption of equal growth rates for all households. Furthermore, households may have heterogenous preferences. An emerging body of research on heterogeneous time preferences (Gollier and Zeckhauser 2005, Millner 2020) tends to suggest that the social pure time discount rate falls over time in the presence of heterogeneities. In our setting, this may imply that over time the WTPs of the more patient households will dominate. Again, we leave an extension to heterogenous preferences to future work.

Fifth, we have restricted our analysis to the case of pure public goods. While this is a reasonable representation for several important goods and services humans derive from natural capital, such as the existence value of biodiversity studied in our application, there are certainly several environmental goods that vary spatially. The provision of these heterogeneously distributed public goods will frequently be correlated with income. For instance, Lee and Lin (2018) show for US metropolitan areas that neighbourhoods close to environmental amenities, such as hills or coastlines, have persisted a high level of income since 1880, and that a heterogeneous endowment with environmental amenities shapes the spatial distribution of incomes. Meya (2020) shows that the effect of income inequality on social WTP for heterogenously distributed public goods generally also depends on how their provision is correlated with income as well as on environmental inequality, but that the main result of Baumgärtner et al. (2017) on how income inequality affects social WTP generalizes to heterogeneously distributed public goods if these are distributed independently of income. We leave an extension of our analysis to a heterogeneous endowment with environmental goods from natural capital and how this distribution evolves over time for future research.

Finally, we have considered a deterministic setting throughout. However, when it comes to issues of intertemporal distribution the world is, of course, full of uncertainties. Besides parameter uncertainty, this applies in particular to uncertainty about the growth rates of income and of natural capital. There is a large body of literature on discounting in the presence of uncertainty about baseline consumption growth (e.g. Gollier 2002, 2008). Gollier (2010) considers uncertainty about the growth rate of environmental goods. More recently, Gollier (2019) also analyses how uncertainty about the elasticity of substitution interacts with other forms of uncertainty about growth rates.

8 Conclusion

We have studied how intra- and intergenerational inequalities affect the value of a dynamic public good. This is particularly relevant for the valuation and management of non-use environmental goods derived from natural capital, but holds also for all other dynamic public goods, such as culture, knowledge, open access journals or national security. To focus our analysis on valuation, we have developed an analytic dynamic model in which income is distributed unevenly among otherwise identical households, who have constant-elasticity-of-substitution preferences across private and public goods both at one point in time as well as over time.

Intertemporal Pareto-efficiency requires a stock of public natural capital such that the marginal (opportunity) cost of supplying the stock equals the aggregate inverse Lindahl-demand for the environmental goods from this natural capital, which is given as sum of individual intertemporal marginal willingness to pay (WTP). We find that this social value of natural capital (i) increases with society's mean income; (ii) decreases (increases) with society's intra-temporal income inequality if and only if natural capital is a substitute (complement) to private consumption goods; (iii) increases with income growth for the case of Cobb-Douglas and complements, but it might decrease for case of substitutes; (iv) increases (decreases) with the growth rate of natural capital if and only if natural capital is a substitute (complement). Our empirical application for a global case study on biodiversity conservation shows that the intra- and intertemporal distribution, mediated by the substitutability of natural capital, is a quantitatively strong determinant of natural capital values.

One important contribution of our theory is to provide structural, dynamic benefit transfer formulas to adjust the values from a primary study context for applications in diverse policy contexts. Approaches to account for natural capital in monetary units usually draw on a set of non-market values and scale these up by means of benefit transfer. Consequently, governmental bodies emphasise that "[g]enerally, it will be necessary to apply benefit transfer methods" (United Nations 2017, p.102) and call for improving such benefit transfer methods (United Nations et al. 2014). The structural transfer factors that follow from our analysis allow adjusting for differences in the initial income distribution in a dynamic context and for differences in growth rates and market interest rates, among others. These benefit transfer formulas are relevant for two key purposes of natural capital valuation: First, for monitoring the performance of the economy, such as in natural capital accounts; Second, for evaluating the efficiency of policies or projects, such as by CBA. Both purposes have a high political mandate as the vast majority of countries declared to integrate biodiversity in national accounts and to mainstream the value of biodiversity in decision making (CBD, Aichi Target 2).[23]

In closing, we briefly highlight how the developed dynamic natural capital valuation methods are relevant for both purposes. First, to consistently integrate natural capital in standard national accounts, the UN *System of Environmental-Economic Accounting*

[23] These targets are reinforced in several national and international political documents. For instance, EU member states agreed to "promote the integration of these [economic values of ecosystems and their services] into accounting and reporting systems at EU and national level by 2020" (European Commission 2011, p.15).

– *Experimental Ecosystem Accounting* (SEEA-EEA) proposes that accounting prices for goods and services from natural capital are hypothetical exchanges values. Accordingly, the *"concept [..] of exchange values entails obtaining valuations of ecosystem services and assets that are consistent with values that would have been obtained if a market for the ecosystem services or assets had existed"* (United Nations et al. 2014, p. 106). While shadow prices obtained in the wealth accounting literature are inconsistent with this principle (Obst et al. 2016), the Lindahl demand concept that we employ here precisely matches the SEEA-EEA understanding of accounting prices as hypothetical exchange values: The (inverse) Lindahl demand is the price households would have been willing to pay if the prevailing level of services consumed from the stock of natural capital was chosen voluntarily on a market. A key question of natural capital accounting—"how the concept of exchange value can be applied when non-monetary transactions are involved"(Obst et al. 2016, p. 7)—thus seems naturally answered by Lindahl prices, which do not require that the economy is on an optimised path.[24] This also provides a theoretical framework to draw on stated preference studies for accounting, which stands in contrast to concerns that these were in general unsuitable (e.g. by Obst et al. 2016).

Second, our dynamic transfer factors can be used to estimate WTPs from secondary data when evaluating the economic efficiency of projects, such as by the means of CBA. This is in particular relevant for evaluating the environmental effects of long-term public investments, for instance in the transportation sector where CBA is routinely carried out in many countries (OECD 2018). Finally, and relatedly, aking to using ex-post equity weights for cost-benefit analysis (e.g. Dasgupta et al. 1972, Johansson-Stenman 2000, 2005, Nyborg 2014), our transfer factors for intra- and intergenerational inequality can be used to adjust WTP-estimates (Drupp et al. 2018b). In the context of natural capital management, equity might require redistributing resources within and across generations. The proposed transfer factors can be used to derive the corresponding equity-adjusted social WTP-estimates for cost-benefit analysis, analogue to distributional weights when the underlying social welfare function is utilitarian.

[24]Dasgupta (2009) argues that the economy being in optimum is one of the key assumptions that renders a large share of the wealth accounting literature unsuitable for practical applications.

Appendix

A.1 Derivation of the intertemporal utility function (Eq. 5)

Rearranging Eq. (3) gives

$$
\begin{aligned}
U(\{Y_t^i\},\{E_t\}) &= \sum_{t=0}^{\infty} \rho^t \, \frac{\theta}{\theta-1} \left(\alpha \, {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha)\, {E_t}^{\frac{\theta-1}{\theta}} \right) \\
&\overset{(1a),\,(1b)}{=} \sum_{t=0}^{\infty} \rho^t \, \frac{\theta}{\theta-1} \left(\alpha \, \left(Y_0^i \,(1+g_Y)^t\right)^{\frac{\theta-1}{\theta}} + (1-\alpha) \left(E_0\,(1+g_E)^t\right)^{\frac{\theta-1}{\theta}} \right) \\
&= \frac{\theta}{\theta-1} \left(\sum_{t=0}^{\infty} \rho^t \alpha \, \left(Y_0^i\,(1+g_Y)^t\right)^{\frac{\theta-1}{\theta}} + \sum_{t=0}^{\infty} \rho^t\,(1-\alpha)\left(E_0\,(1+g_E)^t\right)^{\frac{\theta-1}{\theta}} \right) \\
&= \frac{\theta}{\theta-1} \left(\alpha \sum_{t=0}^{\infty} \left(\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t {Y_0^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) \sum_{t=0}^{\infty} \left(\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^t {E_0}^{\frac{\theta-1}{\theta}} \right).
\end{aligned}
$$

As $|\rho(1+g_Y)^{\frac{\theta-1}{\theta}}| \overset{g_Y \geq -1}{=} \rho(1+g_Y)^{\frac{\theta-1}{\theta}} < 1$ and $|\rho(1+g_E)^{\frac{\theta-1}{\theta}}| \overset{g_E \geq -1}{=} \rho(1+g_E)^{\frac{\theta-1}{\theta}} < 1$ hold by assumption (Eqs. 4a and 4b), the geometric series can be simplified so that one obtains the following intertemporal utility function

$$
U(Y_0^i, g_Y, E_0, g_E) = \frac{\theta}{\theta-1} \left(\alpha \, \frac{{Y_0^i}^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}} + (1-\alpha)\, \frac{{E_0}^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}} \right). \quad \text{(A.39)}
$$

A.2 Derivation of ω (Eq. 7)

ω^i as defined by Eq. (6), for the ICES intertemporal utility function (Eq. 5), is given by

$$
U\left((1-\omega^i)Y_0^i, g_Y, E_0', g_E'\right) = U\left(Y_0^i, g_Y, E_0, g_E\right) \quad \text{(A.40)}
$$

$$
\frac{\alpha\,(1-\omega^i)^{\frac{\theta-1}{\theta}}\,{Y_0^i}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,{E_0'}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E')^{\frac{\theta-1}{\theta}}} = \frac{\alpha\,{Y_0^i}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,{E_0}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\,. \quad \text{(A.41)}
$$

We derive ω^i as the individual WTP for a change dE in the initial stock of natural capital, by setting $g_E = g'_E$ and $E'_0 = E_0 + dE$ in Eq. (A.41):

$$\begin{aligned}
&\frac{\alpha\,(1-\omega^i)^{\frac{\theta-1}{\theta}}\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,(E_0+dE)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} = \frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \\
&\iff 0 = \frac{\alpha(1-\omega^i)^{\frac{\theta-1}{\theta}}\,Y_0^{i\,\frac{\theta-1}{\theta}} - \alpha Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)(E_0+dE)^{\frac{\theta-1}{\theta}} - (1-\alpha)E_0^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}}.
\end{aligned} \tag{A.42}$$

Applying Taylor series expansion of degree one at $\omega^i = 0$ and $dE = 0$, respectively, yields the following approximations:

$$(1-\omega^i)^{\frac{\theta-1}{\theta}} \approx 1 + \frac{1-\theta}{\theta}\,\omega^i \text{ and} \tag{A.43}$$

$$(E_0+dE)^{\frac{\theta-1}{\theta}} \approx E_0^{\frac{\theta-1}{\theta}} + \frac{\theta-1}{\theta} E_0^{-1/\theta} dE \ , \tag{A.44}$$

where the approximation error is the smaller, the smaller dE. Therefore, for marginal dE the approximations hold almost exactly. Using these approximations in Eq. (A.42) yields

$$0 = \frac{\alpha\,\frac{1-\theta}{\theta}\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}\,\omega^i + \frac{(1-\alpha)\,\frac{\theta-1}{\theta}\,E_0^{-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\,dE \tag{A.45}$$

$$\iff \omega^i = \underbrace{\frac{1-\alpha}{\alpha}\,\frac{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\,Y_0^{i\,\frac{1-\theta}{\theta}}\,E_0^{-1/\theta}}_{=:\,\omega(Y_0^i,E_0)}\,dE. \tag{A.46}$$

Considering a change in the initial stock of natural capital by one marginal unit, that is, setting $dE = 1$ in Eq. (A.46), reveals that $\omega(Y_0^i, E_0)$ is household i's WTP – expressed as a constant fraction of income – for one marginal unit of the initial stock E_0 of public natural capital.

A.3 Proof of Lemma 1

Consider a hypothetical situation in which the initial stock of natural capital can be freely, but uniformly across households, chosen at any level $E_0 > 0$. The total costs, including opportunity costs, of supplying the natural capital at this level are $c(E_0)$ with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$ for all E_0, the full amount of which accrues at once at $t = 0$. These costs are financed, at a constant market interest factor δ, by the households who contribute to the financing of public natural capital through an individual constant fraction $\omega^i \geq 0$ of income in each period. Hence, household i's actual consumption in period t is a fraction $(1 - \omega^i) \leq 1$ of her income in this period.

This hypothetical setting is similar to the setting studied in the main text where ω^i measures consumer surplus, but differs in two aspects: (i) the amount of natural capital E_0 is not given but can be chosen; (ii) households do not simply state a willingness to pay for a marginal increase dE of the initial stock E_0 of natural capital, but make actual payments for the full initial amount E_0.

In regards of (ii), in order to make allocations in the two settings comparable one needs to make exogenously given income in the two settings comparable. Because households in the main model do not actually pay for natural capital, the exogenously given incomes Y_t^i in the main model allow for private consumption at levels that cannot be reached if a fraction of income is contributed to the financing of natural capital. For households to reach these levels of private consumption and, consequently, intertemporal utility also in the hypothetical setting in which they have to make actual payments for the initial stock of natural capital, they would need to have higher incomes. Denote by $\tilde{Y}_0^i$ the exogenously given initial level of income of household i in period t in the hypothetical setting, which is such that it allows the same level of consumption and intertemporal utility as in the main setting with exogenously given initial income Y_0^i:

$$\tilde{Y}_0^i := Y_0^i/(1 - \omega^i) \ . \tag{A.47}$$

In the hypothetical setting with exogenously given initial income distribution $(\tilde{Y}_0^1, \ldots, \tilde{Y}_0^n)$, an intertemporal allocation $(\{C_t^1\}, \ldots, \{C_t^n\}, \{E_t\})$ with $C_t^i = (1 - \omega^i)\,\tilde{Y}_t^i$ is called *Pareto-efficient* if and only if no household i $(i = 1, \ldots, n)$ can be made better of, in terms of her intertemporal utility $U(\{C_t^i\}, \{E_t\})$, without making any other household j $(j = 1, \ldots, n$ and $j \neq i)$ worse off, in terms of her intertemporal utility $U(\{C_t^j\}, \{E_t\})$.

We denote the time path for income starting in period 1 by ${}_1\tilde{\mathbf{Y}}^{\mathbf{i}} = (\tilde{Y}_1^i, \tilde{Y}_2^i, ...)$ and that for natural capital by ${}_1\mathbf{E} = (E_1, E_2, ...)$. The Pareto-efficient initial amount $E_0^\star$ of public natural capital is the solution to the following maximization problem:

$$\max_{E_0,\omega^i} U\left(\left\{(1-\omega^1)\,\tilde{Y}_t^1\right\}, \{E_0, {}_1\mathbf{E}\}\right) \tag{A.48}$$

$$\text{s.t.}\;\; U\left(\left\{(1-\omega^i)\,\tilde{Y}_t^i\right\}, \{E_0, {}_1\mathbf{E}\}\right) = \bar{U}^i \text{ for all } i > 1 \;, \tag{A.49}$$

$$\sum_{i=1}^{n}\sum_{t=0}^{\infty} \delta^t\,\omega^i\,\tilde{Y}_t^i = c(E_0)\;, \tag{A.50}$$

$$\text{Eqs. (1a), (1b)}\;.$$

Here, without loss of generality household 1's utility is maximised (Eq. A.48), while no other household falls below its reference utility level $\bar{U}^i$ (Eq. A.49). In addition, an aggregate intertemporal budget constraint must hold, Eq. (A.50), according to which the sum of all households' net present value of contributions to finance (at the constant market interest factor δ) the provision of public natural capital over the entire time horizon equals the total costs of providing the initial stock of public natural capital. Thus, the initial amount E_0 of public good provision – which implies a certain subsequent time path ${}_1\mathbf{E}$ due to Eq. (1b) – is balanced against the reduction in private good consumption due to the households' contribution to finance this, in such a manner as to maximize the utility of one household while not making any other household worse off.

For ICES utility, Eq (5), the Lagrangian for the maximization problem reads

$$\begin{aligned}\mathcal{L} = {} & \frac{\theta}{\theta-1}\left(\frac{\alpha\,(1-\omega^1)^{\frac{\theta-1}{\theta}}\,(\tilde{Y}_0^1)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,E_0{}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\right) \\ & + \sum_{i=2}^{n}\varphi^i\left(\frac{\theta}{\theta-1}\left(\frac{\alpha\,(1-\omega^i)^{\frac{\theta-1}{\theta}}\,(\tilde{Y}_0^i)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,E_0{}^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\right) - \bar{U}^i\right) \\ & + \lambda\left(-c(E_0) + \sum_{i=1}^{n}\sum_{t=0}^{\infty}\delta^t\,\omega^i\,(1+g_y)^t\,\tilde{Y}_0^i\right)\;,\end{aligned} \tag{A.51}$$

where φ^i (for $i = 2, \ldots, n$) and λ are Lagrange multipliers.

Taking the first derivatives of $\mathcal{L}$ (Eq. A.51) with respect to ω^i for all $i = 1, \ldots, n$

and to E_0, and setting each to zero gives the first-order conditions for a Pareto-efficient allocation, $E_0^\star$ and $\omega^{i\star}$:

$$\frac{\alpha(1-\omega^{1\star})^{-1/\theta}\,(\tilde{Y}_0^1)^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}} = \lambda \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t\, \tilde{Y}_0^1 \;, \tag{A.52}$$

$$\varphi^i\, \frac{\alpha(1-\omega^{i\star})^{-1/\theta}\,(\tilde{Y}_0^i)^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}} = \lambda \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t\, \tilde{Y}_0^i \quad \text{for all } i = 2,\ldots,n \;, \tag{A.53}$$

$$\lambda\, c'(E_0^\star) = \frac{(1-\alpha)\, E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} + \sum_{i=2}^{n} \varphi^i\, \frac{(1-\alpha)\, E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \;. \tag{A.54}$$

Defining $\varphi^1 = 0$ and rearranging, these can be re-written as follows:

$$\varphi^i = \lambda\, \frac{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}}{\alpha(1-\omega^{i\star})^{-1/\theta}(\tilde{Y}_0^i)^{-1/\theta}} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t \quad \text{for all } i = 1,\ldots,n \;, \tag{A.55}$$

$$\lambda\, c'(E_0^\star) = \sum_{i=1}^{n} \varphi^i\, \frac{(1-\alpha)\, E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \;. \tag{A.56}$$

Inserting Eqs. (A.55) into Eq. (A.56) yields the first-order condition for the Pareto-efficient initial level $E_0^\star$ of public natural capital:

$$\begin{aligned}
c'(E_0^\star) &= \sum_{i=1}^{n} \frac{1-\alpha}{\alpha}\, \frac{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \left((1-\omega^{i\star})\, \tilde{Y}_0^i\right)^{1/\theta} E_0^{\star\,-1/\theta} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t \\
&\overset{(A.47)}{=} \sum_{i=1}^{n} \frac{1-\alpha}{\alpha}\, \frac{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\, Y_0^{i\,1/\theta}\, E_0^{\star\,-1/\theta} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t \\
&\overset{(7)}{=} \sum_{i=1}^{n} \sum_{t=0}^{\infty} \delta^t\, \omega(Y_0^i, E_0^\star)\, Y_0^i\, (1+g_Y)^t \\
&\overset{(8)}{=} \sum_{i=1}^{n} \mathrm{WTP}(Y_0^i, E_0^\star) \;.
\end{aligned} \tag{A.57}$$

A.4 Derivation of $\overline{\mathrm{WTP}}$ (Eq. 13)

The density function of the log-normal distribution of initial income Y_0 with mean μ_{Y_0} and standard deviation σ_{Y_0} is given by

$$
\begin{aligned}
f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0}) &= \frac{1}{Y_0\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) && \text{(A.58)} \\
\text{with} \quad m &= \ln \mu_{Y_0} - \frac{1}{2} \ln\left(1 + \sigma_{Y_0}^2/\mu_{Y_0}^2\right) \ , && \text{(A.59)} \\
s^2 &= \ln\left(1 + \sigma_{Y_0}^2/\mu_{Y_0}^2\right) \ . && \text{(A.60)}
\end{aligned}
$$

$\overline{\mathrm{WTP}}$, as defined by Eq. (12), can be rewritten as

$$
\begin{aligned}
\overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) &= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \mathrm{WTP}(Y_0, E_0)\, dY_0 \\
&\overset{(8)}{=} \int_0^\infty \sum_{t=0}^\infty \delta^t\, f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \omega(Y_0, E_0)\, Y_0\, (1 + g_Y)^t\, dY_0 \\
&= \sum_{t=0}^\infty \delta^t\, \overline{\mathrm{WTP}}_t(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \ , && \text{(A.61)}
\end{aligned}
$$

where $\overline{\mathrm{WTP}}_t(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)$ is the current social WTP at time t:

$$
\begin{aligned}
&\overline{\mathrm{WTP}}_t(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \\
&= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \omega(Y_0, E_0)\, Y_0 (1+g_Y)^t \, dY_0 \\
&\overset{(A.58),(7)}{=} \int_0^\infty \frac{1}{Y_0\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) \frac{1-\alpha}{\alpha} \frac{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} Y_0^{\frac{1-\theta}{\theta}}\, Y_0 (1+g_Y)^t dY_0 \\
&= \underbrace{\frac{1-\alpha}{\alpha} \frac{\left(1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)(1+g_Y)^t}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta}}_{=:\tilde{\kappa}} \int_0^\infty \frac{Y_0^{\frac{1-\theta}{\theta}}}{\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) dY_0 \\
&= \tilde{\kappa}\, \exp\left[\frac{1}{2\theta^2} s^2 + \frac{1}{\theta} m\right] \\
&\overset{(A.59),(A.60)}{=} \tilde{\kappa}\, \exp\left[\frac{1}{2\theta^2} \ln\left(1+\frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right) + \frac{1}{\theta}\ln \mu_{Y_0} - \frac{1}{2\theta}\ln\left(1+\frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right)\right] \\
&= \tilde{\kappa}\, \exp\left[\frac{1-\theta}{2\theta^2} \ln\left(1+\frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right) + \frac{1}{\theta}\ln(\mu_{Y_0})\right] \\
&= \tilde{\kappa}\, \exp\left[\ln\left(\left(1+\frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right)^{\frac{1-\theta}{2\theta^2}}\right)\right] \exp\left[\ln(\mu_{Y_0}^{\frac{1}{\theta}})\right] \\
&= \tilde{\kappa}\, \mu_{Y_0}^{1/\theta} \left(1+\frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right)^{\frac{1-\theta}{2\theta^2}}, \qquad \text{(A.62)}
\end{aligned}
$$

and, for relative inequality in initial income, $\mathrm{CV}_{Y_0} = \sigma_{Y_0}/\mu_{Y_0}$,

$$
\overline{\mathrm{WTP}}_t(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) = \tilde{\kappa}\, \mu_{Y_0}^{1/\theta} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}}. \qquad \text{(A.63)}
$$

The corresponding present value – discounted at constant market interest factor δ – is

$$
\begin{aligned}
&\overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \\
&= \sum_{t=0}^{\infty} \delta^t \, \overline{\mathrm{WTP}}_t(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \\
&\overset{(A.63)}{=} \sum_{t=0}^{\infty} \delta^t \frac{1-\alpha}{\alpha} \frac{\left(1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)(1+g_Y)^t}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} \, \mu_{Y_0}^{1/\theta} \, \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \\
&= \underbrace{\frac{1-\alpha}{\alpha} \frac{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} \left[\sum_{t=0}^{\infty} \delta^t (1+g_Y)^t\right]}_{=:\kappa} \mu_{Y_0}^{1/\theta} \, \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \\
&= \kappa \, \mu_{Y_0}^{1/\theta} \, \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} .
\end{aligned}
\tag{A.64}
$$

A.5 Proof of Prop. 1

Differentiating $\overline{\mathrm{WTP}}$ (Eq. 13) with respect to initial mean income, μ_{Y_0}, yields

$$
\frac{\partial\,\overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial\,\mu_{Y_0}} = \kappa \frac{1}{\theta} \mu_{Y_0}^{\frac{1-\theta}{\theta}} \, \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \tag{A.65}
$$

$$
\text{with } \kappa = \frac{1-\alpha}{\alpha} \frac{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} \left[\sum_{t=0}^{\infty} \delta^t (1+g_Y)^t\right],
$$

which is strictly greater zero as $E_0, \mu_{Y_0}, \mathrm{CV}_{Y_0}, \theta > 0$, $\alpha \in (0,1)$ and by assumption $\rho(1+g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4b), $\rho(1+g_Y)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4a).

A.6 Proof of Prop. 2

Differentiating $\overline{\mathrm{WTP}}$ (Eq. 13) with respect to relative income inequality, CV_{Y_0}, yields

$$
\frac{\partial\,\overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial\,\mathrm{CV}_{Y_0}} = \kappa \frac{1-\theta}{\theta^2} \mu_{Y_0}^{1/\theta} \mathrm{CV}_{Y_0} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta-2\theta^2}{2\theta^2}} . \tag{A.66}
$$

The sign of the derivative is determined by the sign of the factor $(1-\theta)$, as $\mu_{Y_0}, \mathrm{CV}_{Y_0}, \kappa > 0$. It holds that $1-\theta \lesseqgtr 0$ if and only if $\theta \gtreqless 1$.

A.7 Proof of Prop. 3

Differentiating $\overline{\mathrm{WTP}}$ (Eq. 13) with respect to the growth rate of income, g_Y, yields

$$
\begin{aligned}
&\frac{\partial \overline{\mathrm{WTP}}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial g_Y} \\
&= K\left(\left[1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right]\sum_{t=0}^{\infty} t\delta^t(1+g_Y)^{t-1} - \frac{\theta-1}{\theta}\rho(1+g_Y)^{-1/\theta}\sum_{t=0}^{\infty}\delta^t(1+g_Y)^t\right) \\
&= K\left(\frac{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}}{\delta}\sum_{t=1}^{\infty} t[\delta(1+g_Y)]^{t-1} - \frac{\theta-1}{\theta}\rho(1+g_Y)^{-1/\theta}\sum_{t=0}^{\infty}[\delta(1+g_Y)]^t\right) \\
&\text{with } K := \frac{1-\alpha}{\alpha}\frac{E_0^{-1/\theta}\,\mu_{Y_0}^{1/\theta}(1+\mathrm{CV}_{Y_0}^2)^{\frac{1-\theta}{2\theta^2}}}{1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}} > 0.
\end{aligned}
$$

For

$$
\delta(1+g_Y) < 1, \tag{A.67}
$$

the geometric series converge as $t \to \infty$ and this becomes

$$
\begin{aligned}
\frac{\partial \overline{\mathrm{WTP}}}{\partial g_Y} &= K\left[\frac{(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}})}{\delta\,(1-\delta(1+g_Y))^2} - \frac{\frac{\theta-1}{\theta}\,\rho(1+g_Y)^{-\frac{1}{\theta}}}{1-\delta(1+g_Y)}\right] \\
&= K\,\frac{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}} - \delta\,(1-\delta\,(1+g_Y))\,\rho\,\frac{\theta-1}{\theta}\,(1+g_Y)^{-\frac{1}{\theta}}}{\delta(1-\delta(1+g_Y))^2} \\
&= \frac{K}{\delta(1-\delta(1+g_Y))^2}\left[1-\rho(1+g_Y)^{-\frac{1}{\theta}}\left((1+g_Y)+\frac{\theta-1}{\theta}\,\delta\,(1-\delta\,(1+g_Y))\right)\right] \\
&= \frac{K}{\delta(1-\delta(1+g_Y))^2}\left[1+\rho(1+g_Y)^{-\frac{1}{\theta}}\left(-(1+g_Y)+\frac{1-\theta}{\theta}\,\delta\,(1-\delta\,(1+g_Y))\right)\right] \\
&= \frac{K\,(1+g_Y)^{-\frac{1}{\theta}}}{\delta(1-\delta(1+g_Y))^2}\left[(1+g_Y)^{\frac{1}{\theta}}+\rho\left(-(1+g_Y)+\frac{1-\theta}{\theta}\,\delta\,(1-\delta\,(1+g_Y))\right)\right] \\
&= \frac{K\,(1+g_Y)^{-\frac{1}{\theta}}}{\delta(1-\delta(1+g_Y))^2}\left[(1+g_Y)^{\frac{1}{\theta}}-\rho(1+g_Y)+\frac{1-\theta}{\theta}\,\rho\,\delta\,(1-\delta\,(1+g_Y))\right] \\
&= \frac{K\,(1+g_Y)^{-\frac{1}{\theta}}}{\delta(1-\delta(1+g_Y))^2}\left[(1+g_Y)\underbrace{\left((1+g_Y)^{\frac{1-\theta}{\theta}}-\rho\right)}_{>0,\,\text{cf. Eq. 4a}}+\frac{1-\theta}{\theta}\,\rho\,\delta\,\underbrace{(1-\delta\,(1+g_Y))}_{>0,\,\text{cf. Eq. A.67}}\right].
\end{aligned} \tag{A.68}
$$

Recall the growth path condition $\rho(1+g_Y)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4a) which is equivalent to $(1+$

$g_Y)^{\frac{1-\theta}{\theta}} > \rho$, and thus $(1+g_Y)\left((1+g_Y)^{\frac{1-\theta}{\theta}} - \rho\right) > 0$. As $E_0, \mu_{Y_0}, \mathrm{CV}_{Y_0}, \theta, \rho, g_Y, \delta > 0$, $\alpha \in (0,1)$, $\rho(1{+}g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4b) the only term that can turn negative is $\frac{1-\theta}{\theta}\rho\,(1-\delta\,(1+g_Y))$. The sign of this term is fully determined by the factor $1{-}\theta$, as by assumption $\delta(1{+}g_Y) < 1$ (Eq. A.67). It thus holds that

$$\frac{\partial\overline{\mathrm{WTP}}}{\partial g_Y} > 0 \qquad \text{if} \qquad \theta \leq 1.$$

The reader might stumble when comparing the term in square brackets of Eq. (A.68), with Fig. 3 or Fig. 8. The difference in parameter values zeroing the derivative $\frac{\partial\overline{\mathrm{WTP}}}{\partial g_Y}$ results from different time horizons. While we here consider $t \to \infty$, we assume $t \in (0,T)$ with $T = 100$ for the application (Sec. 5) as this is a common time horizon in cost-benefit analysis. For longer time horizons the parameter values setting the derivative to zeros in the Fig. 3 and Fig. 8 converge to the ones implicit given by Eq. (A.68).

A.8 Prop. 3: Parameter combinations for which WTP declines with income growth

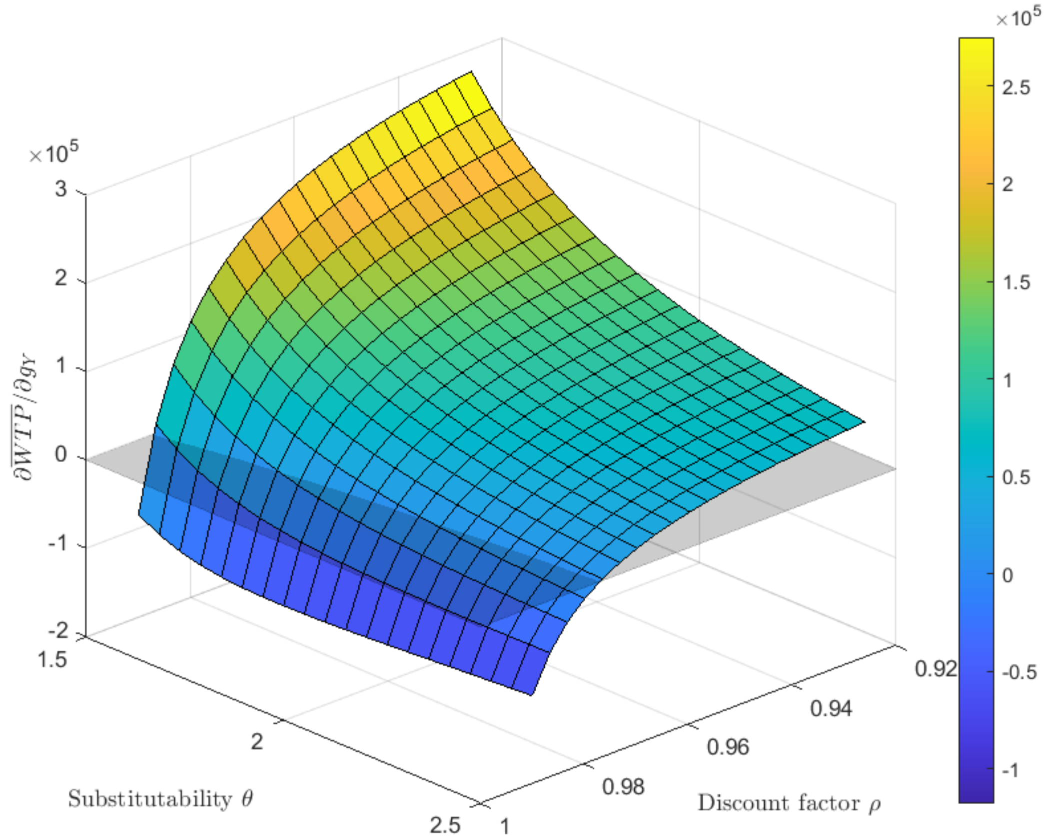

Figure 8: The derivative of mean WTP with respect to the growth rate of income and how its sign and magnitude depend on the elasticity of substitution, θ, and the pure time discount factor, ρ. The remaining parameter values are those of the central calibration in the global biodiversity conservation case study (Table 2), in particular the time horizon goes to $T = 100$.

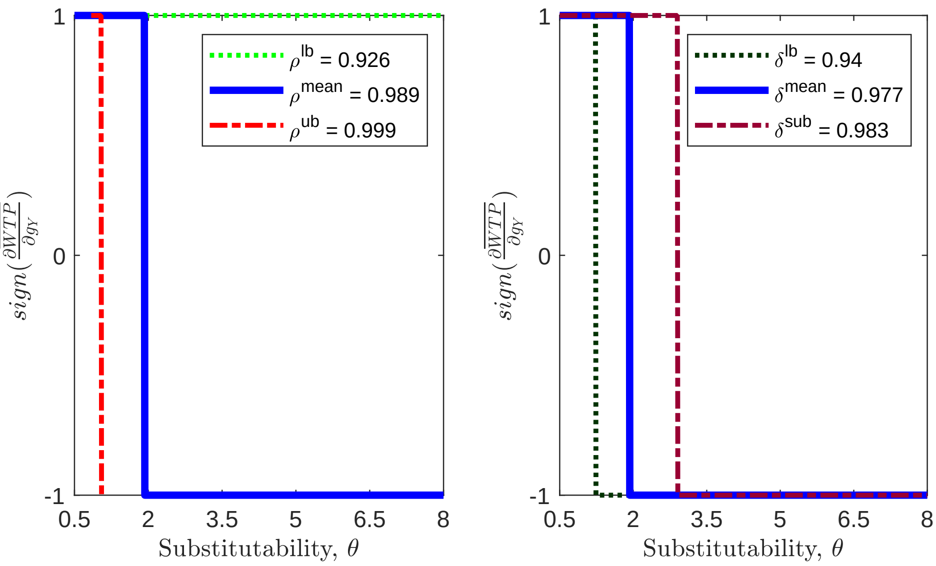

Figure 9: The derivative of mean WTP with respect to the growth rate of income and how its sign depends on the elasticity of substitution, θ, for different pure time discount factors, ρ, (left subplot) or market interest rates, δ (right subplot). In the left subplot the sign of the derivative is only depicted where the growth path conditions are fulfilled (Eqs. 4a and 4b). In the right subplot the convergence condition given by Eq. A.67 is not fulfilled for the upper bound interest factor, δ^{ub}, and therefore the supremum $\delta^{\text{sup}} = (1 + g_Y^{\text{mean}})^{-1}$ is depicted. Results are given for $t \to \infty$ by populating Eq. A.68 with the parameter values of the central calibration in the global biodiversity conservation case study (Table 2).

A.9 Proof of Prop. 4

Differentiating $\overline{\text{WTP}}$ (Eq. 13) with respect to the growth rate of natural capital, g_E, yields

$$\frac{\partial \overline{\text{WTP}}(\mu_{Y_0}, \text{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial g_E} = K' \frac{\theta - 1}{\theta} \, \rho \, \frac{(1 + g_E)^{-1/\theta}}{(1 - \rho(1 + g_E)^{\frac{\theta-1}{\theta}})^2} \tag{A.69}$$

$$\text{with } K' := \frac{1-\alpha}{\alpha}(1 - \rho(1+g_Y)^{\frac{\theta-1}{\theta}}) \, E_0^{-1/\theta} \left[\sum_{t=0}^{\infty} \delta^t \, (1+g_Y)^t \right] \mu_{Y_0}^{1/\theta} (1 + \text{CV}_{Y_0}^2)^{\frac{1-\theta}{2\theta^2}}.$$

As $E_0, \mu_{Y_0}, \text{CV}_{Y_0}, \theta, \rho, g_Y, \delta > 0$, $\alpha \in (0,1)$, $\rho(1+g_Y)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4a), $\rho(1+g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4b) the sign of $\frac{\partial \overline{\text{WTP}}}{\partial g_E}$ is determined by the sign of $\theta - 1$.

A.10 Proof of Prop. 5

The transfer function is defined by the quotient of the mean WTPs at the policy site and study site. For $\overline{\text{WTP}}$ (Eq. 13) it reads:

$$
\begin{aligned}
\mathcal{T}(\ldots) &= \frac{\overline{\text{WTP}}^{\text{policy}}}{\overline{\text{WTP}}^{\text{study}}} \\
&\overset{(13)}{=} \frac{\frac{1-\alpha}{\alpha}\,\frac{1-\rho\,(1+g_Y^{\text{policy}})^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}}\,E_0^{\text{policy}\,-1/\theta}\,\mu_{Y_0}^{\text{policy}\,1/\theta}\left(1+\text{CV}_{Y_0}^{\text{policy}\,2}\right)^{\frac{1-\theta}{2\theta^2}}}{\frac{1-\alpha}{\alpha}\,\frac{1-\rho\,(1+g_Y^{\text{study}})^{\theta}}{1-\rho\,(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}}\,E_0^{\text{study}\,-1/\theta}\,\mu_{Y_0}^{\text{study}\,1/\theta}\left(1+\text{CV}_{Y_0}^{\text{study}\,2}\right)^{\frac{1-\theta}{2\theta^2}}} \\
&\quad\cdot\frac{\sum_{t=0}^{\infty}\left(\delta^{\text{policy}}\right)^t(1+g_Y^{\text{policy}})^t}{\sum_{t=0}^{\infty}\left(\delta^{\text{study}}\right)^t(1+g_Y^{\text{study}})^t} \\
&= \left(\frac{E_0^{\text{policy}}}{E_0^{\text{study}}}\right)^{-1/\theta}\cdot\frac{1-\rho(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}}\cdot\left(\frac{\mu_{Y_0}^{\text{policy}}}{\mu_{Y_0}^{\text{study}}}\right)^{1/\theta}\cdot\left(\frac{1+\text{CV}_{Y_0}{}^{\text{policy}\,2}}{1+\text{CV}_{Y_0}^{\text{study}\,2}}\right)^{\frac{1-\theta}{2\theta^2}} \\
&\quad\cdot\frac{1-\rho\,(1+g_Y^{\text{policy}})^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y^{\text{study}})^{\frac{\theta-1}{\theta}}}\cdot\frac{\sum_{t=0}^{\infty}\left(\delta^{\text{policy}}\right)^t(1+g_Y^{\text{policy}})^t}{\sum_{t=0}^{\infty}\left(\delta^{\text{study}}\right)^t(1+g_Y^{\text{study}})^t}\,. \qquad \text{(A.70)}
\end{aligned}
$$

A.11 Growth path conditions for existence of the closed-form utility function

The set of growth rates that meet the existence condition (Eqs. 4a and 4b) for the closed-form intertemporal utility function for a given elasticity of substitution, θ, and discount factor, ρ, is given as[25]

$$
\rho(1+g)^{\frac{\theta-1}{\theta}} < 1 \iff \begin{cases} g < \rho^{\frac{-\theta}{\theta-1}} - 1 =: g^{\max} & \text{for} \quad \theta > 1 \\ g > \rho^{\frac{-\theta}{\theta-1}} - 1 =: g^{\min} & \text{for} \quad \theta < 1 \end{cases}\,. \qquad \text{(A.71)}
$$

Thus, the growth path condition for substitutes implies a supremum defined by $g^{\max}$, which is always positive and thus bites only for the income growth rate g_Y, but not for the growth rate of natural capital which is by definition always negative, $g_E < 0$. In contrast, the growth rate condition for complements implies an infimum for the growth

[25] As the condition is identical for g_E and g_Y we suppress the subscript on the growth rate in the following formula and only write g.

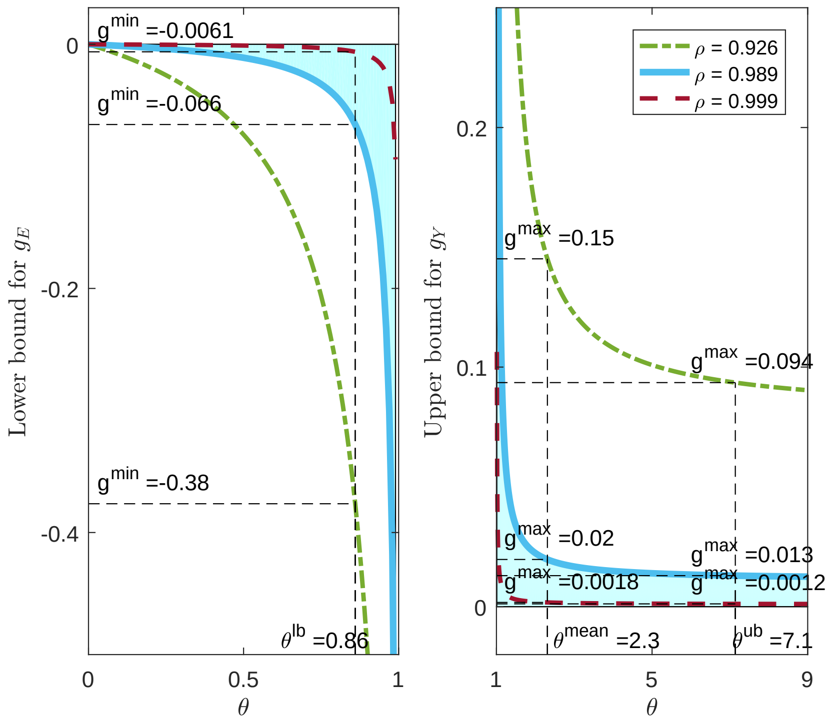

Figure 10: Minimal value for g_E in case of complements, $\theta < 1$, (left side) and maximal value for g_Y in case of substitutes, $\theta > 1$, (right side) to ensure the existence of a closed-form intertemporal utility function (Eq. 31) for different values of the discount factor, ρ, and the elasticity of substitution, θ. The shaded area depicts the set of growth rates g_E (left side) or g_Y (right side) that meets the growth path condition (Eq. 4b or 4a) for the transfer factor for the mean pure time discount factor, $\rho = 0.989$.

rates $g^{\min}$, below which the closed-form intertemporal utility function does not exist. As $g^{\min}$ is always negative, this condition is generally fulfilled for the income growth rate, $g_Y > 0$, but applies for the growth rate of natural capital, $g_E < 0$.

Fig. 10 displays this frontier for the growth rates of income and natural capital for a range of empirical elasticities of substitution, θ, and pure time discount factors, ρ, depicted in Table 2. The supremum for the income growth rate in the mean case of substitutes, with $\theta^{\text{mean}} = 2.31$, depends on $\rho = 0.989\,[0.926; 0.999]$ and is given by $g^{\max}_{\theta=2.31} = 0.0197\,[0.1452, 0.0018]$. For the upper bound substitutability, with $\theta^{\text{ub}} = 7.14$, it is given by $g^{\max}_{\theta=7.14} = 0.0129\,[0.0935; 0.0012]$. For the lower bound complementarity case, with $\theta^{\text{lb}} = 0.86$, the infimum for the growth rate of natural capital is $g^{\min}_{\theta=0.86} = -0.0657\,[-0.3764; -0.0061]$. We observe that the closer the discount factor ρ is to unity, the smaller is the set of g_E in case of complements and of g_Y in case of substitutes that fulfils the growth path condition. Moreover, the higher the degree of substitutability, the smaller is the set of g_Y that still meets the condition, and the stronger the complementarity, the larger is the set of g_E that meets the condition.

A.12 Proofs for Section 6.1

Here, we proof that Prop. 1 to Prop. 3 also hold for an environmental policy that is concerned with a change in the growth rate of natural capital and derive a transfer function for this cases (analogue to Prop. 5).

A.12.1 Derivation of $\overline{\mathbf{WTP}}_{dg_E}$ (Eq. 24)

We derive $\omega^i_{dg_E}$ as the individual WTP for a change dg_E in the growth rate of natural capital, by setting $E_0 = E_0'$ and $g_E' = g_E + dg_E$ in Eq. (A.41):

$$\frac{\alpha\,(1-\omega^i_{dg_E})^{\frac{\theta-1}{\theta}}\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E+dg_E)^{\frac{\theta-1}{\theta}}}=\frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}$$

$$\begin{aligned}\Longleftrightarrow 0 = &\frac{\alpha Y_0^{i\,\frac{\theta-1}{\theta}}-\alpha(1-\omega^i_{dg_E})^{\frac{\theta-1}{\theta}}\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}}\\ &+(1-\alpha)E_0^{\frac{\theta-1}{\theta}}\left(\frac{1}{1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}}-\frac{1}{1-\rho(1+g_E+dg_E)^{\frac{\theta-1}{\theta}}}\right).\end{aligned} \tag{A.72}$$

Applying Taylor series expansion of degree one at $\omega^i_{dg_E}=0$ and $dg_E=0$, respectively, yields the approximations:

$$(1-\omega^i_{dg_E})^{\frac{\theta-1}{\theta}}\approx 1-\frac{\theta-1}{\theta}\omega^i_{dg_E}\text{ and}$$
$$\frac{1}{1-\rho\,(1+g_E+dg_E)^{\frac{\theta-1}{\theta}}}\approx\frac{1}{1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}}+\frac{\frac{\theta-1}{\theta}\rho(1+g_E)^{-1/\theta}}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2}\,dg_E,$$

where the approximation error is the smaller, the smaller dg_E. Using these approximations in Eq. (A.72) yields

$$0 = \frac{\alpha {Y_0^i}^{\frac{\theta-1}{\theta}} - \alpha(1 - \frac{\theta-1}{\theta}\omega^i_{dg_E})\, {Y_0^i}^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}} - (1-\alpha) E_0^{\frac{\theta-1}{\theta}} \frac{\rho\frac{\theta-1}{\theta}(1+g_E)^{-1/\theta}}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} dg_E$$

$$\Longleftrightarrow \; \omega^i_{dg_E} = \underbrace{\frac{1-\alpha}{\alpha} \, \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} \, {Y_0^i}^{\frac{1-\theta}{\theta}} \, E_0^{\frac{\theta-1}{\theta}} \, dg_E}_{=:\omega_{dg_E}(Y_0^i, E_0)}. \tag{A.73}$$

Considering a change in the growth rate of natural capital by one marginal unit, that is, setting $dg_E = 1$ in Eq. (A.73), reveals that $\omega_{dg_E}(Y_0^i, E_0)$ is household i's WTP – expressed as a constant fraction of income – for one marginal unit of the growth rate g_E of public natural capital.

The current social WTP at time t is

$$\begin{aligned}
&\overline{\mathrm{WTP}}_{dg_E;t}(\mu_{Y_0}, \sigma_{Y_0}, g_Y, E_0, g_E) \\
&= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \omega_{dg_E}(Y_0, g_Y, E_0, g_E)\, (1+g_Y)^t Y_0 \, dY_0 \\
&\overset{(A.58),\ (A.73)}{=} \int_0^\infty \frac{1}{Y_0\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) \frac{1-\alpha}{\alpha} \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} \\
&\qquad Y_0^{\frac{1-\theta}{\theta}} E_0^{\frac{\theta-1}{\theta}} (1+g_Y)^t Y_0 \, dY_0 \\
&= \underbrace{\frac{1-\alpha}{\alpha} \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} (1+g_Y)^t E_0^{\frac{\theta-1}{\theta}}}_{=:\kappa'''''} \int_0^\infty \frac{Y_0^{\frac{1-\theta}{\theta}}}{\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) dY_0 \\
&\overset{Sec.\ A.4}{=} \kappa''''' \, \mu_{Y_0}^{1/\theta} \left(1 + \frac{\sigma^2_{Y_0}}{\mu^2_{Y_0}}\right)^{\frac{1-\theta}{2\theta^2}},
\end{aligned} \tag{A.74}$$

and, for relative inequality in initial income, $\mathrm{CV}_{Y_0} = \sigma_{Y_0}/\mu_{Y_0}$,

$$\overline{\mathrm{WTP}}_{dg_E;t}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) = \kappa''''' \, \mu_{Y_0}^{1/\theta} \left(1 + \mathrm{CV}^2_{Y_0}\right)^{\frac{1-\theta}{2\theta^2}}. \tag{A.75}$$

The corresponding present value - discounted at market interest factor δ - is

$$\begin{aligned}
&\overline{\mathrm{WTP}}_{dg_E}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \\
&= \sum_{t=0}^{\infty} \delta^t \, \overline{\mathrm{WTP}}_{dg_E;t}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E) \\
&\overset{A.75}{=} \sum_{t=0}^{\infty} \delta^t \, \frac{1-\alpha}{\alpha} \, \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} (1+g_Y)^t E_0^{\frac{\theta-1}{\theta}} \mu_{Y_0}^{1/\theta} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \\
&= \underbrace{\frac{1-\alpha}{\alpha} \, \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} E_0^{\frac{\theta-1}{\theta}} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t}_{=:\kappa'} \mu_{Y_0}^{1/\theta} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \\
&= \kappa' \mu_{Y_0}^{1/\theta} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} . \qquad \text{(A.76)}
\end{aligned}$$

A.12.2 Proof of Prop. 1 for growth rate as object of valuation

Differentiating $\overline{\mathrm{WTP}}_{dg_E}$ (Eq. 24) with respect to initial mean income, μ_{Y_0}, yields

$$\frac{\partial \, \overline{\mathrm{WTP}}_{dg_E}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial \, \mu_{Y_0}} = \kappa' \frac{1}{\theta} \mu_{Y_0}^{\frac{1-\theta}{\theta}} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \qquad \text{(A.77)}$$

$$\text{with } \kappa' = \frac{1-\alpha}{\alpha} \, \frac{\rho(1+g_E)^{-1/\theta}\left(1-\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{\left(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}}\right)^2} E_0^{\frac{\theta-1}{\theta}} \sum_{t=0}^{\infty} \delta^t \, (1+g_Y)^t,$$

which is strictly greater zero, as $\alpha \in (0,1)$, $g_E > -1$, $\rho(1+g_Y)^{\frac{\theta-1}{\theta}} < 1$ and $g_Y, E_0, \mathrm{CV}_{Y_0}, \mu_{Y_0}, \theta, \rho > 0$.

A.12.3 Proof of Prop. 2 for growth rate as object of valuation

Differentiating $\overline{\mathrm{WTP}}_{dg_E}$ (Eq. 24) with respect to relative income inequality, CV_{Y_0}, yields

$$\frac{\partial \, \overline{\mathrm{WTP}}_{dg_E}(\mu_{Y_0}, \mathrm{CV}_{Y_0}, g_Y, E_0, g_E)}{\partial \, \mathrm{CV}_{Y_0}} = \kappa' \frac{1-\theta}{\theta^2} \mu_{Y_0}^{1/\theta} \, \mathrm{CV}_{Y_0} \left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta-2\theta^2}{2\theta^2}} . \qquad \text{(A.78)}$$

The sign of which is determined by the factor $(1-\theta)$, as $\mu_{Y_0}, \mathrm{CV}_{Y_0}, k' > 0$. It again holds that $1-\theta \lesseqqgtr 0$ if and only if $\theta \gtreqqless 1$.

A.12.4 Proofs of Prop. 3 for growth rate as objective of valuation

As the components relating to the growth rate of income in Eq. (24) are the same as those in Eq. (13), the proof for $\frac{\partial \overline{\text{WTP}}_{dg_E}}{\partial g_Y} > 0 \quad \text{if} \quad \theta \leq 1$ proceeds analogously (see Appendix A.7). Likewise, the conditions for which the sign of the effect changes from positive to negative are the same. Figures 8 and 9 in Appendix A.8 also applies to $\overline{\text{WTP}}_{dg_E}$.

A.12.5 Derivation of the transfer function for growth rate as object of valuation, Eq. (25).

For $\overline{\text{WTP}}_{dg_E}$ (Eq. 24) the transfer function is

$$
\begin{aligned}
\mathcal{T}_{dg_E}(\ldots) &= \frac{\overline{\text{WTP}}^{\text{policy}}_{dg_E}()}{\overline{\text{WTP}}^{\text{study}}_{dg_E}()} \\
&\overset{(24)}{=} \frac{\frac{1-\alpha}{\alpha} \frac{\rho(1+g_E^{\text{policy}})^{-1/\theta}}{\left(1-\rho(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}\right)^2} \left(1-\rho(1+g_Y^{\text{policy}})^{\frac{\theta-1}{\theta}}\right) E_0^{\text{policy}\frac{\theta-1}{\theta}} \mu_{Y_0}^{\text{policy}1/\theta} \left(1+\text{CV}_{Y_0}^{\text{policy}2}\right)^{\frac{1-\theta}{2\theta^2}}}{\frac{1-\alpha}{\alpha} \frac{\rho(1+g_E^{\text{study}})^{-1/\theta}}{\left(1-\rho(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}\right)^2} \left(1-\rho(1+g_Y^{\text{study}})^{\frac{\theta-1}{\theta}}\right) E_0^{\text{study}\frac{\theta-1}{\theta}} \mu_{Y_0}^{\text{study}1/\theta} \left(1+\text{CV}^{\text{study}2}_{Y_0}\right)^{\frac{1-\theta}{2\theta^2}}} \\
&\quad \cdot \frac{\sum_{t=0}^{\infty} \delta^{\text{policy}t}(1+g_Y^{\text{policy}})^t}{\sum_{t=0}^{\infty} \delta^{\text{study}t}(1+g_Y^{\text{study}})^t} \\
&= \left(\frac{E_0^{\text{policy}}}{E_0^{\text{study}}}\right)^{\frac{\theta-1}{\theta}} \cdot \frac{\rho(1+g_E^{\text{policy}})^{-1/\theta}\left(1-\rho(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}\right)^2}{\rho(1+g_E^{\text{study}})^{-1/\theta}\left(1-\rho(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}\right)^2} \cdot \left(\frac{\mu_{Y_0}^{\text{policy}}}{\mu_{Y_0}^{\text{study}}}\right)^{1/\theta} \\
&\quad \cdot \left(\frac{1+\text{CV}_{Y_0}{}^{\text{policy}2}}{1+\text{CV}_{Y_0}^{\text{study}2}}\right)^{\frac{1-\theta}{2\theta^2}} \cdot \frac{1-\rho\left(1+g_Y^{\text{policy}}\right)^{\frac{\theta-1}{\theta}}}{1-\rho\left(1+g_Y^{\text{study}}\right)^{\frac{\theta-1}{\theta}}} \cdot \frac{\sum_{t=0}^{\infty} \delta^{\text{policy}t}(1+g_Y^{\text{policy}})^t}{\sum_{t=0}^{\infty} \delta^{\text{study}t}(1+g_Y^{\text{study}})^t} \quad .
\end{aligned}
\tag{A.79}
$$

A.13 Proofs for Section 6.2

A.13.1 Derivation of $\omega_{\mathbf{SP}}$ (Eq. 27)

ω^i_{SP}, as defined by Eq. (26), for the ICES intertemporal utility function (Eq. 3), is:

$$\alpha\,(Y_0^i-\omega_{\text{SP}}^i)^{\frac{\theta-1}{\theta}}+(1-\alpha)\,E_0'^{\frac{\theta-1}{\theta}}+\sum_{t=1}^{\infty}\rho^t\left(\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}+(1-\alpha)\,E_t'^{\frac{\theta-1}{\theta}}\right)=\sum_{t=0}^{\infty}\rho^t\left(\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}+(1-\alpha)\,E_t^{\frac{\theta-1}{\theta}}\right)$$

$$\Leftrightarrow\ \alpha\,(Y_0^i-\omega_{\text{SP}}^i)^{\frac{\theta-1}{\theta}}+\sum_{t=1}^{\infty}\rho^t\,\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}+\sum_{t=0}^{\infty}\rho^t(1-\alpha)\,E_t'^{\frac{\theta-1}{\theta}}=\sum_{t=0}^{\infty}\rho^t\left(\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}+(1-\alpha)\,E_t^{\frac{\theta-1}{\theta}}\right)$$

$$\overset{Appendix A.1}{\Leftrightarrow}\ \alpha\,(Y_0^i-\omega_{\text{SP}}^i)^{\frac{\theta-1}{\theta}}+\sum_{t=1}^{\infty}\rho^t\,\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}+\frac{(1-\alpha)\,E_0'^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E')^{\frac{\theta-1}{\theta}}}=\frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}. \tag{A.80}$$

Where

$$\begin{aligned}
\sum_{t=1}^{\infty}\rho^t\,\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}} &= \sum_{t=1}^{\infty}(\rho(1+g_Y)^{\frac{\theta-1}{\theta}})^t\,\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}\\
&= \sum_{t=0}^{\infty}(\rho(1+g_Y)^{\frac{\theta-1}{\theta}})^t\,\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}-\left(\rho(1+g_Y)^{\frac{\theta-1}{\theta}}\right)^0\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}\\
&= \frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}-\alpha\,Y_t^{i\,\frac{\theta-1}{\theta}}\\
&= \frac{\left(1-1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)\,\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}\\
&= \frac{\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\,\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}.
\end{aligned}$$

Using this in Eq. (A.80) gives

$$\alpha\,(Y_0^i-\omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}}+\frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}\,\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0'^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E')^{\frac{\theta-1}{\theta}}}=\frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}. \tag{A.81}$$

$$\begin{aligned}\iff \alpha\,(Y_0^i-\omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}}+\frac{(1-\alpha)\,E_0'^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E')^{\frac{\theta-1}{\theta}}}&=\frac{\alpha\,Y_0^{i\,\frac{\theta-1}{\theta}}\left(1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}+\frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\\ \iff (Y_0^i-\omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}}&=Y_0^{i\,\frac{\theta-1}{\theta}}+\frac{\frac{1-\alpha}{\alpha}\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}-\frac{\frac{1-\alpha}{\alpha}\,E_0'^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E')^{\frac{\theta-1}{\theta}}}.\end{aligned} \tag{A.82}$$

We derive ω_{SP}^i as the individual WTP at $t=0$ for a change dE in the initial stock of natural capital, by setting $g_E=g_E'$ and $E_0'=E_0+dE$ in Eq. (A.82):

$$\begin{aligned}(Y_0^i-\omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}}&=Y_0^{i\,\frac{\theta-1}{\theta}}+\frac{\frac{1-\alpha}{\alpha}\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}-\frac{\frac{1-\alpha}{\alpha}\,(E_0+dE)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\\&=Y_0^{i\,\frac{\theta-1}{\theta}}+\frac{1-\alpha}{\alpha}\,\frac{E_0^{\frac{\theta-1}{\theta}}-(E_0+dE)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}.\end{aligned} \tag{A.83}$$

Applying Taylor series expansion of degree on at $\omega_{dE}^i=0$ we approximate $(Y_0^i-\omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}}\approx Y_0^{i\,\frac{\theta-1}{\theta}}-\frac{\theta-1}{\theta}Y_0^{i\,-1/\theta}\,\omega_{\mathrm{SP}}^i$ and applying Taylor expansion of degree one at $dE=0$ we approximate $(E_0+dE)^{\frac{\theta-1}{\theta}}\approx E_0^{\frac{\theta-1}{\theta}}+\frac{\theta-1}{\theta}E_0^{-1/\theta}dE$. Using these approximations in Eq. (A.83) gives

$$\begin{aligned}Y_0^{i\,\frac{\theta-1}{\theta}}-\frac{\theta-1}{\theta}\,Y_0^{i\,-1/\theta}\,\omega_{\mathrm{SP}}^i=Y_0^{i\,\frac{\theta-1}{\theta}}+\frac{1-\alpha}{\alpha}\,\frac{E_0^{\frac{\theta-1}{\theta}}-E_0^{\frac{\theta-1}{\theta}}-\frac{\theta-1}{\theta}\,E_0^{-1/\theta}\,dE}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\\ \iff \omega_{\mathrm{SP}}^i=\underbrace{\frac{1-\alpha}{\alpha}\,\frac{Y_0^{i\,1/\theta}\,E_0^{-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}}_{=:\omega_{\mathrm{SP}}(Y_0^i,E_0)}\,dE.\end{aligned} \tag{A.84}$$

Considering a change in the initial stock of natural capital by one marginal unit, that is, setting $dE=1$ in Eq. (A.84), reveals that $\omega(Y_0^i,E_0)$ is household i's WTP – expressed as a single payment made at $t=0$ – for one marginal unit of the initial stock

E_0 of public natural capital.

A.13.2 Proof of Lemma 1 for a single payment

Consider a hypothetical setting in which the initial stock of natural capital can be chosen freely, but uniformly across households, at any level $E_0 > 0$. The costs of supplying natural capital E_0 are financed by the households who contribute by a single payment in the first period $t = 0$.

In contrast to the setting in the main text, in this setting households do not simply state a willingness to pay for a marginal increase dE of the initial stock E_0 of natural capital, but make actual payments to finance it. To make both settings comparable, one needs to make the exogenously given income comparable. We therefore consider compensated income $\tilde{Y}_0^i$, which is such that it allows the same level of consumption as in the main setting

$$\tilde{Y}_0^i := Y_0^i + {\omega_{\mathrm{SP}}^i}^* . \tag{A.85}$$

For a single payment, the Pareto-efficient initial amount E_0^* of public natural capital is the solution to the following maximization problem:

$$\max_{E_0, \omega_{\mathrm{SP}}^i} U\left(\left\{\tilde{Y}_0^1 - \omega_{\mathrm{SP}}^1, {}_1\mathbf{Y^1}\right\}, \{E_0, {}_1\mathbf{E}\}\right), \tag{A.86}$$

$$\text{s.t.} \quad U\left(\left\{\tilde{Y}_0^i - \omega_{\mathrm{SP}}^i, {}_1\mathbf{Y^i}\right\}, \{E_0, {}_1\mathbf{E}\}\right) = \bar{U}^i \qquad \text{for all } i > 1, \tag{A.87}$$

$$\sum_{i=1}^{n} \omega_{\mathrm{SP}}^i = c(E_0), \tag{A.88}$$

$$\text{Eqs. (1a), (1b)} .$$

Here, without loss of generality, household 1's utility is maximised, while all other households obtain at least the reference utility level $\bar{U}^i$. Note, the households decides upon the total amount of the natural public good it will consume and balances it against the reduction in consumption from its individual contribution to finance this ω_{SP}^1. For a single payment this reduces consumption in the first period to $\tilde{Y}_0^1 - \omega_{\mathrm{SP}}^1$, while income path ${}_1\mathbf{Y^1}$ remains unchanged.

For ICES utility, Eq. (5), the Langrangian reads (compare LHS of Eq. A.81)

$$\mathcal{L} = \frac{\theta}{\theta-1}\left(\alpha\,(\tilde{Y}_0^1 - \omega_{\mathrm{SP}}^1)^{\frac{\theta-1}{\theta}} + \frac{\alpha\,(\tilde{Y}_0^1)^{\frac{\theta-1}{\theta}}\,\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\right) + \sum_{i=2}^{n}\varphi^i\,\frac{\theta}{\theta-1}$$
$$\left(\alpha\,(\tilde{Y}_0^i - \omega_{\mathrm{SP}}^i)^{\frac{\theta-1}{\theta}} + \frac{\alpha\,(\tilde{Y}_0^i)^{\frac{\theta-1}{\theta}}\,\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1-\alpha)\,E_0^{\frac{\theta-1}{\theta}}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}\right) + \lambda\left(\sum_{i=1}^{n}\omega_{\mathrm{SP}}^i - c(E_0)\right).$$

Defining $\varphi^1 = 0$ the first-order conditions for the Pareto-efficient initial level $E_0^\star$ can be written as

$$\lambda = \varphi^i\,\alpha\,\left(\tilde{Y}_0^i - \omega_{\mathrm{SP}}^{i\;*}\right)^{-1/\theta} \quad \text{for all} \quad i = 1,...,n, \tag{A.89}$$
$$\lambda\,c'(E_0^\star) = \sum_{i=1}^{n}\varphi^i\,\frac{(1-\alpha)\,E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}. \tag{A.90}$$

Rearranging Eq. (A.89) for φ^i, inserting this in Eq. (A.90), dividing by λ and slightly rearranging yields the first-order condition for the Pareto-efficient initial level E_0^* of natural capital:

$$\begin{aligned} c'(E_0^\star) &= \sum_{i=1}^{n}\frac{(1-\alpha)}{\alpha}\,\frac{\left(\tilde{Y}_0^i - \omega_{\mathrm{SP}}^{i\;*}\right)^{1/\theta}\,E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \\ &\overset{(A.85)}{=} \sum_{i=1}^{n}\frac{1-\alpha}{\alpha}\,\frac{Y_0^{i\,1/\theta}\,E_0^{\star\,-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} \\ &\overset{(27)}{=} \sum_{i=1}^{n}\omega_{\mathrm{SP}}(Y_0^i, E_0^\star). \end{aligned} \tag{A.91}$$

A.13.3 Derivation of $\overline{\text{WTP}}_{\text{SP}}$ (Eq. 28)

Mean WTP expressed as single payment at $t = 0$ is

$$\begin{aligned}
\overline{\text{WTP}}_{\text{SP}}(\mu_{Y_0}, \sigma_{Y_0}, E_0, g_E) &= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \omega_{\text{SP}}(Y_0, E_0)\, dY_0 \\
&\overset{(27),\ (A.58)}{=} \int_0^\infty \frac{1}{Y_0\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) \frac{1-\alpha}{\alpha} \frac{Y_0^{1/\theta}\, E_0^{-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}} dY_0 \\
&= \underbrace{\frac{1-\alpha}{\alpha} \frac{E_0^{-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}}}_{=:\kappa''} \int_0^\infty \frac{Y_0^{1/\theta}}{Y_0\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) dY_0 \\
&= \kappa'' \int_0^\infty \frac{Y_0^{\frac{1-\theta}{\theta}}}{\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) dY_0 \\
&\overset{Sec\ A.4}{=} \kappa''\, \mu_{Y_0}^{1/\theta} \left(1 + \frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2}\right)^{\frac{1-\theta}{2\theta^2}}, \qquad \text{(A.92)}
\end{aligned}$$

and, relative inequality in initial income, $\text{CV}_{Y_0} = \frac{\sigma_{Y_0}}{\mu_{Y_0}}$,

$$\overline{\text{WTP}}_{\text{SP}}(\mu_{Y_0}, \text{CV}_{Y_0}, E_0, g_E) = \kappa''\, \mu_{Y_0}^{1/\theta} \left(1 + \text{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}}. \qquad \text{(A.93)}$$

A.13.4 Proof of Prop. 1 for a single payment

Differentiating $\overline{\text{WTP}}_{\text{SP}}$ (Eq. 28) with respect to initial mean income, μ_{Y_0}, yields

$$\begin{aligned}
\frac{\partial\, \overline{\text{WTP}}_{\text{SP}}(\mu_{Y_0}, \text{CV}_{Y_0}, E_0, g_E)}{\partial\, \mu_{Y_0}} &= \kappa'' \frac{1}{\theta} \mu_{Y_0}^{\frac{1-\theta}{\theta}} \left(1 + \text{CV}_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} \qquad \text{(A.94)} \\
\text{with} \qquad \kappa'' &= \frac{1-\alpha}{\alpha} \frac{E_0^{-1/\theta}}{1-\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}},
\end{aligned}$$

which is strictly greater zero, as $E_0, \mu_{Y_0}, \text{CV}_{Y_0}, \theta > 0$, $\alpha \in (0, 1)$ and by assumption $\rho(1 + g_E)^{\frac{\theta-1}{\theta}} < 1$.

A.13.5 Proof of Prop. 2 for a single payment

Differentiating $\overline{\text{WTP}}_{\text{SP}}$ (Eq. 28) with respect to relative income inequality, CV_{Y_0}, yields

$$\frac{\partial\,\overline{\mathrm{WTP}}_{\mathrm{SP}}(\mu_{Y_0},\mathrm{CV}_{Y_0},E_0,g_E)}{\partial\,\mathrm{CV}_{Y_0}} = \kappa''\,\frac{1-\theta}{\theta^2}\mu_{Y_0}^{1/\theta}\mathrm{CV}_{Y_0}\left(1+\mathrm{CV}_{Y_0}^2\right)^{\frac{1-\theta-2\theta^2}{2\theta^2}}\,. \tag{A.95}$$

The sign of the derivative is determined by the sign of the factor $(1-\theta)$, as $\mu_{Y_0}, \mathrm{CV}_{Y_0}, \kappa'' > 0$. It holds that $1-\theta \lesseqgtr 0$ if and only if $\theta \gtreqless 1$.

A.13.6 Proof of Prop. 4 for a single payment

Differentiating $\overline{\mathrm{WTP}}_{\mathrm{SP}}$ (Eq. 28) with respect to the growth rate of natural capital, g_E, yields

$$\frac{\partial\overline{\mathrm{WTP}}_{\mathrm{SP}}(\mu_{Y_0},\mathrm{CV}_{Y_0},g_Y,E_0,g_E)}{\partial g_E} = K''\frac{\theta-1}{\theta}\,\rho\,\frac{(1+g_E)^{-1/\theta}}{(1-\rho(1+g_E)^{\frac{\theta-1}{\theta}})^2} \tag{A.96}$$

$$\text{with } K'' := \frac{1-\alpha}{\alpha}\,E_0^{-1/\theta}\,\mu_{Y_0}^{1/\theta}(1+\mathrm{CV}_{Y_0}^2)^{\frac{1-\theta}{2\theta^2}}.$$

As $E_0, \mu_{Y_0}, \mathrm{CV}_{Y_0}, \theta, \rho$, $\alpha \in (0,1)$, $g_E > -1$, $\rho(1+g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 4b) the sign of $\frac{\partial\overline{\mathrm{WTP}}_{\mathrm{SP}}}{\partial g_E}$ is determined by the sign of $\theta-1$ and it follows directly that

$$\frac{\partial\,\overline{\mathrm{WTP}}_{\mathrm{SP}}(\mu_{Y_0},\mathrm{CV}_{Y_0},g_Y,E_0,g_E)}{\partial\,g_E} \gtreqless 0 \quad \text{if and only if} \quad \theta \gtreqless 1.$$

A.13.7 Derivation of the transfer function for a single payment, Eq. (29)

For WTP elicited at study site and policy site as a single payment for a change in the initial stock of natural capital, $\overline{\mathrm{WTP}}_{\mathrm{SP}}$ (Eq. 28), the transfer function is given as

$$\begin{aligned}
\mathcal{T}_{\mathrm{SP}}(\ldots) &= \frac{\overline{\mathrm{WTP}}^{\text{policy}}_{\mathrm{SP}}()}{\overline{\mathrm{WTP}}^{\text{study}}_{\mathrm{SP}}()} \\
&\overset{(28)}{=} \frac{\frac{1-\alpha}{\alpha}\,\frac{E_0^{\text{policy}\,-1/\theta}}{1-\rho\,(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}}\,\mu_{Y_0}^{\text{policy}\,1/\theta}\left(1+\mathrm{CV}_{Y_0}{}^{\text{policy}\,2}\right)^{\frac{1-\theta}{2\theta^2}}}{\frac{1-\alpha}{\alpha}\,\frac{E_0^{\text{study}\,-1/\theta}}{1-\rho\,(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}}\,\mu_{Y_0}^{\text{study}\,1/\theta}\left(1+\mathrm{CV}_{Y_0}{}^{\text{study}\,2}\right)^{\frac{1-\theta}{2\theta^2}}} \\
&= \left(\frac{E_0^{\text{policy}}}{E_0^{\text{study}}}\right)^{-1/\theta} \cdot \frac{1-\rho(1+g_E^{\text{study}})^{\frac{\theta-1}{\theta}}}{1-\rho(1+g_E^{\text{policy}})^{\frac{\theta-1}{\theta}}} \cdot \left(\frac{\mu_{Y_0}^{\text{policy}}}{\mu_{Y_0}^{\text{study}}}\right)^{1/\theta} \cdot \left(\frac{1+\mathrm{CV}_{Y_0}^{\text{policy}\,2}}{1+\mathrm{CV}_{Y_0}^{\text{study}\,2}}\right)^{\frac{1-\theta}{2\theta^2}} .
\end{aligned} \tag{A.97}$$

A.14 Mean WTP for CES-CIES preferences, Eq. (33)

Here, we first derive WTP for CIES preferences (Eq. 31) and a general instantaneous utility function and than study CIES-CES preferences.

ω^i, as defined by Eq. (6), for general instantaneous preferences and intertemporal CIES utility (Eq. (31)), is given by using Eq. (31) in (Eq. A.40):

$$\begin{aligned}
\sum_{t=0}^{\infty}\rho^t\,\frac{1}{1-\eta}\,u\left((1-\omega^i)Y_0^i(1+g_Y)^t, E_0'(1+g_E')^t\right)^{1-\eta} &= \sum_{t=0}^{\infty}\rho^t\,\frac{1}{1-\eta}\,u\left(Y_0^i(1+g_Y)^t, E_0(1+g_E)^t\right)^{1-\eta} \\
\Longleftrightarrow \sum_{t=0}^{\infty}\rho^t\,u\left((1-\omega^i)\,Y_0^i(1+g_Y)^t, E_0'(1+g_E')^t\right)^{1-\eta} &= \sum_{t=0}^{\infty}\rho^t\,u\left(Y_0^i(1+g_Y)^t, E_0(1+g_E)^t\right)^{1-\eta} .
\end{aligned} \tag{A.98}$$

We derive ω^i as the individual WTP for a change dE in the initial stock of natural capital, by setting $g_E = g_E'$ and $E_0' = E_0 + dE$ in Eq. (A.98):

$$\sum_{t=0}^{\infty}\rho^t\,u\left((1-\omega^i)Y_0^i(1+g_Y)^t, (E_0+dE)(1+g_E)^t\right)^{1-\eta} = \sum_{t=0}^{\infty}\rho^t\,u\left(Y_0^i(1+g_Y)^t, E_0(1+g_E)^t\right)^{1-\eta} . \tag{A.99}$$

Approximating the left hand side of Eq. (A.99) by a Taylor series expansion of degree one at $\omega^i = 0$ and $dE = 0$ yields

$$\sum_{t=0}^{\infty} \rho^t \, u \left((1-\omega^i) Y_0^i (1+g_Y)^t, (E_0 + dE)(1+g_E)^t\right)^{1-\eta}$$
$$\approx \sum_{t=0}^{\infty} \rho^t \, u \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right)^{1-\eta}$$
$$-\, \omega^i \, Y_0^i \, (1-\eta) \sum_{t=0}^{\infty} \rho^t \, u \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right)^{-\eta} \frac{\partial u}{\partial Y_t^i} \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right) (1+g_Y)^t$$
$$+\, dE \, (1-\eta) \sum_{t=0}^{\infty} \rho^t \, u \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right)^{-\eta} \frac{\partial u}{\partial E_t} \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right) \, (1+g_E)^t. \tag{A.100}$$

Using Eq. (A.100) in Eq. (A.99) and rearranging gives the individual WTP for a general instantaneous utility function:

$$\omega^i = \frac{\sum_{t=0}^{\infty} \rho^t u \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right)^{-\eta} \frac{\partial u}{\partial E_t} ((Y_0^i (1+g_Y)^t, E_0(1+g_E)^t) \, (1+g_E)^t}{\sum_{t=0}^{\infty} \rho^t u \left(Y_0^i (1+g_Y)^t, E_0(1+g_E)^t\right)^{-\eta} \frac{\partial u}{\partial Y_t^i} ((Y_0^i (1+g_Y)^t, E_0(1+g_E)^t)(1+g_Y)^t} dE \, {Y_0^i}^{-1}$$
$$\overset{(30)}{=} \frac{\sum_{t=0}^{\infty} \rho^t u \left(Y_t^i, E_t\right)^{-\eta} \frac{\partial u}{\partial E_t} (Y_t^i, E_t) \, (1+g_E)^t}{\sum_{t=0}^{\infty} \rho^t u \left(Y_t^i, E_t\right)^{-\eta} \frac{\partial u}{\partial Y_t^i} (Y_t^i, E_t) \, (1+g_Y)^t} dE \, {Y_0^i}^{-1}. \tag{A.101}$$

For instantaneous CES utility (Eq. 30) and its partial derivatives

$$\begin{aligned} \frac{\partial u(Y_t^i, E_t)}{\partial Y_t^i} &= \alpha \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} \, {Y_t^i}^{-1/\theta}, \\ \frac{\partial u(Y_t^i, E_t)}{\partial E_t} &= (1-\alpha) \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} \, {E_t}^{-1/\theta}, \end{aligned}$$

individual WTP, Eq. (A.101), is

$$\omega^i = \frac{1-\alpha}{\alpha} \frac{\sum_{t=0}^{\infty} \rho^t \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{-\eta\theta}{\theta-1}} \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} {E_t}^{-1/\theta} (1+g_E)^t}{\sum_{t=0}^{\infty} \rho^t \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{-\eta\theta}{\theta-1}} \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} {Y_t^i}^{-1/\theta} (1+g_Y)^t} dE \, {Y_0^i}^{-1}$$

$$= \frac{1-\alpha}{\alpha} \frac{\sum_{t=0}^{\infty} \rho^t \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\eta\theta}{\theta-1}} {E_t}^{-1/\theta} (1+g_E)^t}{\sum_{t=0}^{\infty} \rho^t \left(\alpha {Y_t^i}^{\frac{\theta-1}{\theta}} + (1-\alpha) E_t^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\eta\theta}{\theta-1}} {Y_t^i}^{-1/\theta} (1+g_Y)^t} dE \, {Y_0^i}^{-1}$$

$$\overset{(1a),(1b)}{=} \frac{1-\alpha}{\alpha} \frac{dE}{{E_0}^{1/\theta}} {Y_0^i}^{\frac{1-\theta}{\theta}} \frac{\sum_{t=0}^{\infty} \rho^t \left(\alpha \big(Y_0^i (1+g_Y)^t\big)^{\frac{\theta-1}{\theta}} + (1-\alpha) \big(E_0 (1+g_E)^t\big)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\eta\theta}{\theta-1}} \left((1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^{\infty} \rho^t \left(\alpha \big(Y_0^i (1+g_Y)^t\big)^{\frac{\theta-1}{\theta}} + (1-\alpha) (E_0 (1+g_E)^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\eta\theta}{\theta-1}} \left((1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t}$$

$$\overset{(30)}{=} \underbrace{\frac{1-\alpha}{\alpha} {E_0}^{-1/\theta} {Y_0^i}^{\frac{1-\theta}{\theta}} \frac{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t}}_{=:\omega(Y_0^i, g_Y, E_0, g_E)} dE. \tag{A.102}$$

Considering a change in the initial stock of natural capital by one marginal unit, that is, setting $dE = 1$ in Eq. (A.102), gives $\omega(Y_0^i, g_Y, E_0, g_E)$ as the household i's WTP – expressed as a constant fraction of income – for one marginal unit of E_0 in case of general instantaneous utility.

The corresponding current social WTP at time t reads

$$\overline{\mathrm{WTP}}_t(\mu_{Y_0}, \sigma_{Y_0}, g_Y, E_0, g_E) = \int_0^{\infty} f_{\ln}(Y_0^i; \mu_{Y_0}, \sigma_{Y_0}) \, \omega(Y_0^i, g_Y, E_0, g_E) (1+g_Y)^t Y_0^i \, dY_0$$

$$\overset{A.102}{=} \int_0^{\infty} f_{\ln}(.) \, (1+g_Y)^t Y_0^i \frac{1-\alpha}{\alpha} {E_0}^{-1/\theta} {Y_0^i}^{\frac{1-\theta}{\theta}} \frac{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t} dY_0$$

$$= \frac{1-\alpha}{\alpha} (1+g_Y)^t {E_0}^{-1/\theta} \int_0^{\infty} f_{\ln}(.) \frac{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^{\infty} u(Y_0^i (1+g_Y)^t, E_0 (1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \left(\rho (1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t} {Y_0^i}^{1/\theta} dY_0 \, , \tag{A.103}$$

and the associated present value - discounted at market interest factor δ - is

$$
\begin{aligned}
&\overline{\mathrm{WTP}}(\mu_{Y_0}, \sigma_{Y_0}, g_Y, E_0, g_E) = \sum_{t=0}^{\infty} \delta^t \, \overline{\mathrm{WTP}}_t(\mu_{Y_0}, \sigma_{Y_0}, g_Y, E_0, g_E) \\
&\overset{A.103}{=} \underbrace{\frac{1-\alpha}{\alpha} \, {E_0}^{-1/\theta} \sum_{t=0}^{\infty} \delta^t (1+g_Y)^t}_{=:\kappa'''} \int_0^{\infty} f_{\ln}(.) \, \frac{\sum_{t=0}^{\infty} u(Y_0^i\,(1+g_Y)^t, E_0\,(1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \, \left(\rho\,(1+g_E)^{\frac{\theta-1}{\theta}}\right)^t}{\sum_{t=0}^{\infty} u(Y_0^i\,(1+g_Y)^t, E_0\,(1+g_E)^t)^{\frac{1-\eta\theta}{\theta}} \, \left(\rho\,(1+g_Y)^{\frac{\theta-1}{\theta}}\right)^t} \, {Y_0^i}^{1/\theta} \, dY_0
\end{aligned}
\tag{A.104}
$$

A.15 Proofs for Section 6.4 (Cobb-Douglas preferences)

A.15.1 Intertemporal utility for Cobb-Douglas-CIES preferences, Eq. (34)

Using household i's instantaneous Cobb-Douglas utility function

$$
u(Y^i, E) = {Y^i}^{\alpha} \, E^{1-\alpha} \tag{A.105}
$$

in the intertemporal CIES utilty function, Eq. (31), gives

$$
\begin{aligned}
U(\{Y_t^i\}, \{E_t\}) &= \sum_{t=0}^{\infty} \rho^t \, \frac{1}{1-\eta} \, u(Y_t^i, E_t)^{1-\eta} \\
&\overset{\text{(A.105)}}{=} \sum_{t=0}^{\infty} \rho^t \, \frac{1}{1-\eta} \left({Y_t^i}^{\alpha} \, E_t^{1-\alpha}\right)^{1-\eta} \\
&\overset{\text{(1a),(1b)}}{=} \frac{1}{1-\eta} \sum_{t=0}^{\infty} \rho^t \left(\left[Y_0^i(1+g_Y)^t\right]^{\alpha} \left[E_0(1+g_E)^t\right]^{1-\alpha}\right)^{1-\eta} \\
&= \frac{1}{1-\eta} \sum_{t=0}^{\infty} \rho^t \left({Y_0^i}^{\alpha} [(1+g_Y)^{\alpha}]^t \, E_0^{1-\alpha} \left[(1+g_E)^{1-\alpha}\right]^t\right)^{1-\eta} \\
&= \frac{1}{1-\eta} \sum_{t=0}^{\infty} \left({Y_0^i}^{\alpha} \, E_0^{1-\alpha}\right)^{1-\eta} \left[\rho\,(1+g_Y)^{\alpha(1-\eta)} \, (1+g_E)^{(1-\alpha)(1-\eta)}\right]^t .
\end{aligned}
$$

If

$$
\rho(1+g_Y)^{\alpha(1-\eta)} \, (1+g_E)^{(1-\alpha)(1-\eta)} < 1 \tag{A.106}
$$

than the geometric series converges for $t \to \infty$ and one obtains the following intertemporal utility function, representing Cobb-Douglas-CIES preferences,

$$U(Y_0^i, g_Y, E_0, g_E) = \frac{1}{1-\eta} \frac{\left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta}}{1-\rho \, \left[(1+g_Y)^\alpha \, (1+g_E)^{(1-\alpha)}\right]^{(1-\eta)}} \; . \tag{A.107}$$

A.15.2 Derivation of $\overline{\textbf{WTP}}$, Eq. (35)

ω^i as defined by Eq. (6), for the Cobb-Douglas-CIES intertemporal utility function (Eq. A.107), is given by

$$\begin{aligned} U((1-\omega^i)Y_0^i, g_Y, E_0', g_E') &= U(Y_0^i, g_Y, E_0, g_E) \\ \frac{\left([(1-\omega^i)\, Y_0^i]^\alpha \, E_0'^{1-\alpha}\right)^{1-\eta}}{1-\rho \, \left[(1+g_Y)^\alpha \, (1+g_E')^{(1-\alpha)}\right]^{(1-\eta)}} &= \frac{\left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta}}{1-\rho \, \left[(1+g_Y)^\alpha \, (1+g_E)^{(1-\alpha)}\right]^{(1-\eta)}}. \end{aligned} \tag{A.108}$$

We derive ω^i as the individual WTP for a change dE in the initial stock of natural capital, by setting $g_E' = g_E$ and $E_0' = E_0 + dE$ in Eq. (A.108):

$$\begin{aligned} \frac{\left([(1-\omega^i)\, Y_0^i]^\alpha \, (E_0+dE)^{1-\alpha}\right)^{1-\eta}}{1-\rho \, \left[(1+g_Y)^\alpha \, (1+g_E)^{(1-\alpha)}\right]^{(1-\eta)}} &= \frac{\left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta}}{1-\rho \, \left[(1+g_Y)^\alpha \, (1+g_E)^{(1-\alpha)}\right]^{(1-\eta)}} \\ \Longleftrightarrow \; \left(\left[(1-\omega^i)\, Y_0^i\right]^\alpha (E_0+dE)^{1-\alpha}\right)^{1-\eta} &= \left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta} \end{aligned} \tag{A.109}$$

Applying Taylor series expansion of degree one at $\omega^i = 0$ and $dE = 0$, yields the following approximation:

$$\begin{aligned} &\left(\left[(1-\omega^i)\, Y_0^i\right]^\alpha (E_0+dE)^{1-\alpha}\right)^{1-\eta} \\ &\approx \left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta} + \left((1-\alpha)\, dE \, E_0^{-1} - \alpha\, \omega^i\right) \, (1-\eta) \left(Y_0^{i\alpha} \, E_0^{1-\alpha}\right)^{1-\eta}. \end{aligned} \tag{A.110}$$

Using Eq. (A.110) in Eq. (A.109) and slightly rearranging yields

$$\omega^i \;\; = \;\; \underbrace{\frac{(1-\alpha)}{\alpha} \, E_0{}^{-1}}_{=:\omega(E_0)} \, dE, \tag{A.111}$$

so that $\omega(E_0)$ is the individual WTP for one marginal unit of the initial stock E_0 of natural capital. Since ω only depends on E_0 (and the preference parameter α), which are identical for all households, this reveals that for Cobb-Douglas-CIES preferences all households have the same WTP.

The current social WTP at time t is

$$
\begin{aligned}
&\overline{\mathrm{WTP}}_t(\mu_{Y_0}, g_Y, E_0) \\
&= \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \omega(E_0)\, Y_t(Y_0)\, dY_0 \\
&\overset{(1a),(A.111)}{=} \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, \frac{(1-\alpha)}{\alpha}\, E_0{}^{-1}\, (1+g_Y)^t Y_0\, dY_0 \\
&= \underbrace{\frac{(1-\alpha)}{\alpha}\, E_0{}^{-1}\, (1+g_Y)^t}_{=:\kappa''''''} \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0})\, Y_0\, dY_0 \\
&\overset{(A.58)}{=} \kappa'''''' \int_0^\infty \frac{1}{\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln Y_0 - m)^2}{2s^2}\right) dY_0 \\
&= \kappa'''''' \exp\left(m + \frac{s^2}{2}\right) \\
&\overset{(A.59),(A.60)}{=} \kappa'''''' \exp\left(\ln \mu_{Y_0} - \frac{1}{2} \ln\left(1 + \sigma_{Y_0}^2/\mu_{Y_0}^2\right) + \frac{1}{2} \ln\left(1 + \sigma_{Y_0}^2/\mu_{Y_0}^2\right)\right) \\
&= \kappa''''''\, \mu_{Y_0}. \qquad \text{(A.112)}
\end{aligned}
$$

The corresponding present value - discounted at market interest factor δ - is

$$
\begin{aligned}
\overline{\mathrm{WTP}}(\mu_{Y_0}, g_Y, E_0) &= \sum_{t=0}^{\infty} \delta^t\, \overline{\mathrm{WTP}}_t(\mu_{Y_0}, g_Y, E_0) \\
&\overset{(A.112)}{=} \frac{(1-\alpha)}{\alpha}\, E_0{}^{-1} \left[\sum_{t=0}^{\infty} \delta^t\, (1+g_Y)^t\right] \mu_{Y_0}. \qquad \text{(A.113)}
\end{aligned}
$$

A.16 Marginal WTP for a general intertemporal utility

Here, we derive for a general intertemporal utility function the individual WTP, expressed as constant fraction of income. The maximisation problem defining the Pareto-efficient allocation is given by Eqs. (A.48), (A.49), (A.50). The corresponding Lagrangian reads

$$\mathcal{L} = U\left(\{(1-\omega^1)\,\tilde{Y}_t^1\}, \{E_0, {}_1\mathbf{E}\}\right) + \sum_{i=2}^{n} \varphi^i \left(U\left(\{(1-\omega^i)\,\tilde{Y}_t^i\}, \{E_0, {}_1\mathbf{E}\}\right) - \bar{U}^i\right)$$
$$+ \lambda \left(-c(E_0) + \sum_{i=1}^{n}\sum_{t=0}^{\infty} \delta^t\, \omega^i\, \tilde{Y}_t^i\right),$$

with the first-order conditions for a Pareto-efficient allocation, $E_0^\star$ and ω^{i^*}, (defining $\varphi^1 := 0$):

$$\lambda \sum_{t=0}^{\infty} \delta^t \tilde{Y}_t^i = \varphi^i \sum_{t=0}^{\infty} \tilde{Y}_t^i \, \frac{\partial U}{\partial Y_t^i} \left((1-x^{i^*})\,\tilde{Y}_t^i, \{E_t\}\right), \qquad \text{for all} \quad i = 1, ..., n, \tag{A.114}$$

$$\lambda\, c'(E_0^\star) = \sum_{i=1}^{n} \varphi^i \sum_{t=0}^{\infty} \frac{\partial U}{\partial E_t} \left(\{(1-\omega^{i^*})\,\tilde{Y}_t^i\}, E_t\right) \frac{\partial E_t}{\partial E_0}. \tag{A.115}$$

Rearranging Eqs. (A.114) for φ^i, inserting this in Eq. (A.115), dividing by λ and slightly rearranging yields the first-order condition for the Pareto-efficient initial level $E_0^\star$ of public natural capital

$$\begin{aligned}
c'(E_0^\star) &= \sum_{i=1}^{n} \frac{\sum_{t=0}^{\infty} \delta^t Y_t^i \, \sum_{t=0}^{\infty} \frac{\partial U}{\partial E_t} \left(\{(1-\omega^{i^*})\,\tilde{Y}_t^i\}, E_t\right) \frac{\partial E_t}{\partial E_0}}{\sum_{t=0}^{\infty} Y_t^i \, \frac{\partial U}{\partial Y_t^i} \left((1-\omega^{i^*})\,\tilde{Y}_t^i, \{E_t\}\right)} \\
&\overset{\text{(A.47)}}{=} \sum_{i=1}^{n} \frac{\sum_{t=0}^{\infty} \delta^t Y_t^i \, \sum_{t=0}^{\infty} \frac{\partial U}{\partial E_t} \left(\{Y_t^i\}, E_t\right) \frac{\partial E_t}{\partial E_0}}{\sum_{t=0}^{\infty} Y_t^i \, \frac{\partial U}{\partial Y_t^i} \left(Y_t^i, \{E_t\}\right)} \\
&\overset{Y_t = \frac{\partial Y_t}{\partial Y_0} Y_0}{=} \sum_{i=1}^{n} \underbrace{\frac{\sum_{t=0}^{\infty} \frac{\partial U}{\partial E_t} \left(\{Y_t^i\}, E_t\right) \frac{\partial E_t}{\partial E_0}}{\sum_{t=0}^{\infty} \frac{\partial U}{\partial Y_t^i} \left(Y_t^i, \{E_t\}\right) \frac{\partial Y_t^i}{\partial Y_0^i}} \sum_{t=0}^{\infty} \delta^t \frac{\partial Y_t^i}{\partial Y_0^i}}_{=:\, \mathrm{WTP}\left(\{Y_t^i\}, \{E_t\}\right)} .
\end{aligned} \tag{A.116}$$

A.17 Endogenizing the growth and interest rates in a general equilibrium endogenous growth model

The constant growth rate of income, g_Y, considered in this paper can be derived as the balanced growth path outcome of an endogenous growth model. To demonstrate this, consider a model with product innovation. Output Y_t is produced by means of labor L

and a mass M_t of different types i of machines with input quantities $q_t(i)$, according to the constant-returns-to-scale production function

$$Y_t = \frac{1}{\varphi}\, L^{1-\varphi} \int_0^{M_t} q_t(i)^{\varphi}\, di \ , \tag{A.117}$$

where φ is the output elasticity of machinery input.

We normalize labor input to one, $L = 1$, based on the assumption that each of the n households inelastically supplies $1/n$ units of labor. We choose the final output good as the numéraire. Machines fully depreciate after one period of use.

Using $p_t(i)$ to denote the price of a machine of type i, input demand by competitive firms in the final goods sector is given by the condition that the value of the marginal product of this machine is equal to its price, i.e.

$$q_t(i)^{\varphi-1} = p_t(i). \tag{A.118}$$

Blueprints for new types of machines are generated by research and development, which uses the output as the final good as input. Using Z_t to denote the input into R&D at time t, the expected mass of new varietis developed is

$$M_{t+1} = M_t + \frac{1}{\Phi}\, Z_t, \tag{A.119}$$

with a constant $\Phi > 0$. A firm being successful in R&D becomes the monopolistic supplier for this type of machine. Machines are produced using the final good, such that one unit of the final good is required to build one unit of a machine. The profit-maximizing price $p_t(i)$ of a machine of type i is obtained by maximizing $p_t(i)\, q_t(i) - q_t(i)$ subject to A.118, which yields $p_t(i) = 1/\varphi$. Using this in A.118, market equilibrium input of machine type i is $q_t(i) = \varphi^{1/(1-\varphi)}$, and total output is

$$Y_t = M_t\, \varphi^{\frac{1}{1-\varphi}}. \tag{A.120}$$

We consider a balanced growth path, such that the interest factor δ_t is constant, $\delta_t = \delta$.

The present value of monopoly profits for a firm successful in R&D is $(p-1)\,q/\delta = (1-\varphi)/(\delta\,\varphi)$. Under free entry, the expected present value of profits from one dollar spent on R&D must just be equal to this one dollar, i.e.

$$\Phi\,\frac{\delta}{1-\delta}\,\frac{1-\varphi}{\varphi} = 1, \tag{A.121}$$

from which we obtain the interest factor

$$\delta = \left(1 + \frac{1-\varphi}{\Phi\,\varphi}\right)^{-1}. \tag{A.122}$$

Market equilibrium for final goods implies

$$Y_t = n\,C_t + Z_t + \int_0^{M_t} q_t(i)\,di = n\,C_t + \Phi\,\left(M_{t+1} - M_t\right) + M_t\,\varphi^{\frac{\varphi}{1-\varphi}}. \tag{A.123}$$

In a balanced growth path, Y_t, C_t,and M_t must thus all grow at the same rate g_Y.

Households choose the intertemporal distribution of consumption to maximize

$$\sum_{t=0}^{\infty} \rho^t\,\frac{\theta}{\theta-1}\,\left(\alpha\,C_t^{\frac{\theta-1}{\theta}} + (1-\alpha)\,E_t^{\frac{\theta-1}{\theta}}\right), \tag{A.124}$$

where ρ is the pure time discount factor of the household. As households are facing a constant interest factor δ, the optimal intertemporal distribution of consumption is determined by the discrete-time Keynes-Ramsey rule

$$\begin{aligned} & (1+g_Y)^{\frac{1}{\theta}} = \frac{\rho}{\delta} \\ \Leftrightarrow \qquad & g_Y = (\rho/\delta)^{\theta} - 1 = \left(\rho\,\left(1 + \frac{1-\varphi}{\Phi\,\varphi}\right)\right)^{\theta} - 1. \end{aligned} \tag{A.125}$$

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