CESifo WORKING PAPERS

8758
2020

December 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 8758*

A Pandemic Business Interruption Insurance

Abstract

We analyze how pandemic business interruption coverage can be put in place by building on capitalization mechanisms. The pandemic risk cannot be mutualized since it affects simultaneously a large number of businesses, and furthermore, it has a systemic nature because it goes along with a severe decline in the real economy. However, as shown by COVID-19, pandemics affect economic sectors in a differentiated way: some of them are very severely affected because their activity is strongly impacted by travel bans and constraints on work organisation, while others are more resistant. This opens the door to risk coverage mechanisms based on a portfolio of financial securities, including long-short positions and options in stock markets. We show that such financial investment allow insurers to offer business interruption coverage in pandemic states, while simultaneously hedging the risks associated with the alternation of bullish and bearish non-pandemic states. These conclusions are derived from a theoretical model of corporate risk management, and they are illustrated by numerical simulations, using data from the French stock exchange.

JEL-Codes: G110, G220, G320.

Keywords: pandemic, business interruption, insurance, corporate risk management.

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December 1, 2020
Simulations presented in the paper have been realized in cooperation with Lou Saison (student at Ecole Polytechnique). We are very grateful to her for an outstanding contribution to the realization of these simulations.

1 Introduction

Although the full extent of COVID-19 economic consequences is not yet perfectly known to date, there is no doubt that, in many countries, it has been at the origin of dramatic losses due to business interruption. Because of the pandemic, workers, customers and entrepreneurs were not in a position to pursue their activity normally, and therefore firms were prevented from conducting usual business operations.[1] In this context, many firms turned to their insurance policies, in the hope that business interruption claims could be filed to recover losses resulting from the ongoing sanitary crisis. Business interruption coverage is typically included as part of a company's commercial property insurance policy, and is most commonly triggered when there is direct damage to insured property, particularly in case of fire or during natural disasters, such as floods, hurricanes or earthquakes. Contingent business interruption coverage can also apply when a government limits access to a specific geographic area, thereby impairing access to the policyholder's premises. However, for claims related with COVID-19, policy wording appears to be critical, and many insurers have denied coverage, by contending that claims do not meet the "direct physical loss" requirement contained within standard business interruption policies.[2]

Court decisions reflect the diversity of situations and policy wordings, and we do not intend here to express a view on the validity of the arguments made by different parties.[3] There ought, however, to be agreement about the fact that insurers were not prepared to face such a pandemic risk. In practice, business interruption was merely viewed as an indirect loss induced by property damage, with specific loss evaluation principles, that should be covered through similar mutualization mechanisms. In the recent period, the emergence of cyber risk was a first reason for considering that the mutualization of business interruption risk could be undermined by common factors affecting the whole economy. COVID-19 is another step in this direction, but with a much greater magnitude.

Put in simple terms, a worldwide pandemic is an insurance risk that cannot be covered by usual mutualization mechanisms, because it is characterized by a very large degree of correlation between policyholders. In other words, and at the risk of stating the obvious, characterizing an optimal pandemic insurance scheme requires that we start from the fact that a pandemic affects a large

[1]Estimates from the US Census Bureau (Buffington et al. (2020)) about the first phase of the pandemic show that 89.9% of small businesses have experienced a negative effect on operations due to the COVID-19, including 51.4% seeing a large negative effect and 38.5% a moderate negative effect. The large negative effect was especially pronounced in the Accommodation and Food Services industry where 83.5% of businesses experienced a large negative effect. In a survey of more than 5,800 small businesses conducted between March 28 and April 4, 2020, Bartik et al. (2020) find that 43% of the small businesses in their sample had temporarily closed and that businesses have – on average – reduced their employee counts by 40 percent relative to January. They document that mass layoffs and closures had already occurred, just a few weeks into the crisis, and they also find that many small businesses are financially fragile, with less than one month of cash on hand.

[2]See DBRS-Morningstar (2020).

[3]For a discussion of these issues, see French (2020).

number of individuals or businesses simultaneously. In other words, it would not make sense to look for a pandemic insurance scheme in which, as with most other property lines, the misfortunes suffered by a few policyholders would be compensated by the contributions of all the other ones. In this regard, while insurance pooling arrangements are part of the usual arsenal against catastrophic risks,[4] they are not of great help here, since when a pandemic occurs, it is feared that it affects all the insurers in the pool. In other words, pandemic risks are correlated not only within an insurer's portfolio, but also between insurers.

A further question relates to the role of governments in guaranteeing the sustainability of a pandemic corporate insurance. Many think that national governments should ultimately backstop the insurance coverage, as is the case in the Terrorism Risk Insurance Act created in the U.S. in the aftermath of 9/11, but also in natural disaster insurance regimes existing in European countries, especially France and Spain. However, even with such a backstop, pandemic risk is considered by many insurers as too unwieldy and too potentially widespread for underwriting to take place. In other words, the insurability of the pandemic risk itself is in question.

In the limited framework of the present study, we will leave aside this important question of the role of governments, and we will focus attention on the issue of how the pandemic risk could be covered through insurance mechanisms. It is well-known that, in the usual activity of the insurance industry, the two basic insurance mechanisms, namely mutualization and capitalization, are relevant in areas that are clearly separated from one another: P&C lines and health insurance are based on mutualization, while life insurance works through capitalization. This dichotomy has to be abandoned when it comes to corporate pandemic insurance, since the coverage of business interruption is a key line of business of P&C insurers, but it cannot be mutualized in the case of pandemics. The objective of the present paper is to explore this avenue, by analyzing how capitalization mechanisms may yield business interruption coverage in the case of a pandemic.

At first sight, going through the capitalization channel conflicts with the systemic nature of the pandemic risk.[5] To put it in simple terms, the per-

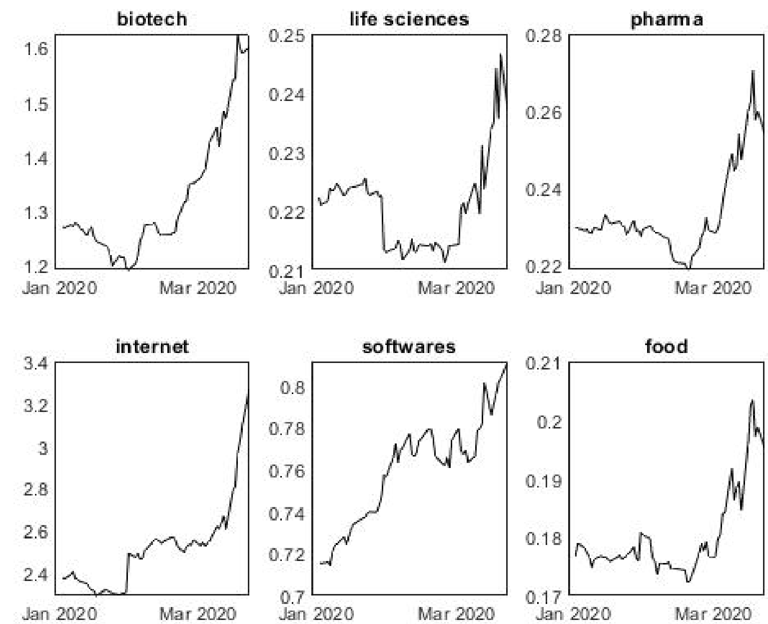
[4]This includes terrorism risk (e.g., Pool Re in UK and GAREAT in France), flood risk (e.g., Flood Re in UK) and nuclear liability risk in the US and for signatory countries of Paris and Brussels international conventions.

[5]According to the definition of Cummins & Weiss (2013), a systemic risk is "the risk that an event will trigger a loss of economic value or confidence in a substantial segment of the financial system that is serious enough to have significant adverse effects on the real economy with a high probability". In this definition, the transmission chain starts from an economic event, whatever it may be, that destabilizes the financial sector, and thereby induces a severe decline of the activity in the real sector. The collapse of the U.S. housing bubble that peaked in 2006, and was at the origin of a global credit crunch in 2007-2008, causing huge losses in stock markets, and that ultimately created a worldwide downturn in economic activity, is a typical example of such a sequence that goes from financial markets to the real economy. In the case of COVID-19, the causality chain is reversed, in the sense that the trigger affects the real sphere first (i.e., the pandemic has prevented many firms from carrying on their business in a normal manner), with effects that are widespread enough to affect the global economy, and ultimately the financial markets. Whatever the direction of causality, in both

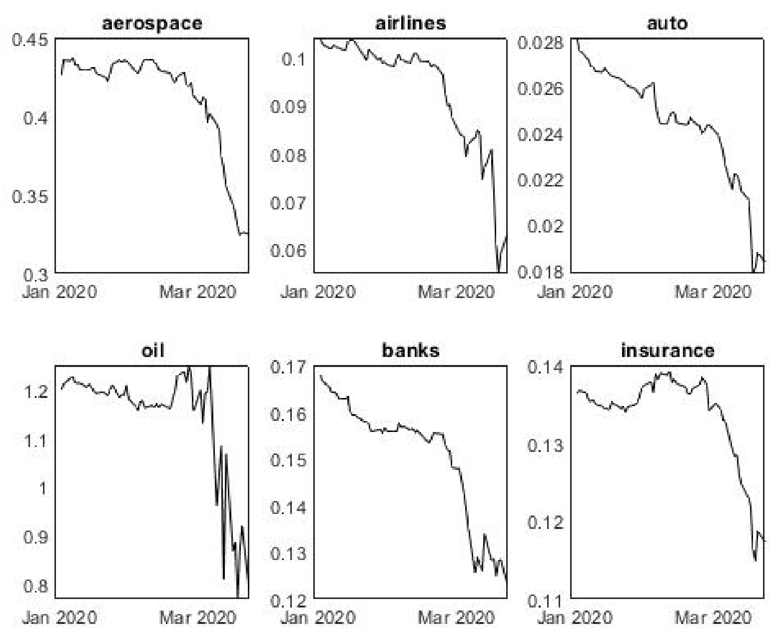
spective of suffering from a major macroeconomic downturn in the case of a pandemic does not make it easy to create risk sharing mechanisms to the benefit of firms suffering from business interruption. However, this concomitance between a well-defined event (mainly, the limited ability to move and to exert a normal business activity) causing severe corporate losses, and a macroeconomic crisis is too narrow a view that overlooks the uneven impacts of the pandemic throughout the economy. Pandemics affect sectors of the economy more or less, according to the effect of travel and work restrictions on their activity, while some of them may even take advantage of the situation. Tourism and restaurants, transportation and distribution, manufacturing and craft, entertainment industries, retail and luxury industries, and all industries based on international supply chain have been most severely penalized by COVID-19, while pharmaceutical and biotech industries, online BtoB and BtoC platforms and high-tech industries have benefited either from the increase in demand for health care, or from changes in consumption patterns, or from the propensity of firms to reorganize their activity through a more intense use of digital tools.

These uneven effects of the current pandemic across sectors is reflected in stock market performances. A decomposition of the S&P500 index at the industry level allows us to see these differential effects clearly. Panels (a) and (b) in Figure 1 display the evolution of several industry specific indices relative to the global S&P500 index, during the first quarter of 2020. While the aerospace and defense, airlines, automobiles, oil, gas and consumable fuels, banks and insurance indices have experienced a drop more important than the global index, industries such as biotechnologies, life sciences and tools, pharmaceutical, internet and direct marketing, software and food products have fared relatively better. In France, the CAC40 index is a capitalization-weighted measure of the 40 most significant stocks on the Euronext Paris. In 2020, while the CAC40 has experienced a severe fall of 30 % between January 1st and April 1st, the individual stock responses have presented a large degree of heterogeneity. Table A lists twelve French stocks that have relatively well resisted to the COVID-19 shock. The luxury brand Hermes even won two percent on its quotation and its associated calls increased by 10.40%. At the bottom of the table, LVMH, another company from the luxury sector, lost "only" 15% while its associated call options lost 14%. Table B in contrast, lists twelve stocks that have significantly under-performed the CAC40. With a drop of 64.1% of its quotation, the retail real estate trust UNIBAIL-RODAM-WESTFIELD suffered the largest loss, while its associated put options skyrocketed to provide a 807.92% return to their holders. The performance of all stocks listed in Table B is below that of the media group PUBLICIS, that lost 35% of its value in four months.

cases the risk is said to be systemic because it affects the real and financial spheres of the global economy, and not only a limited number of victims. Natural disasters (at least, those we have experienced so far) are not systemic, although they may be at the origin of dramatic losses for the population concerned.



(a) Over-performing sub-indices



(b) Under-performing sub-indices

Figure 1: Historical values of the S&P500 sub-indices, at the industry level relative to global S&P500 from January 1st, 2020 to April 1st, 2020. *Source : The Financial Times.*

STOCK	SUB-INDUSTRY	YTD	CALL YTD
HERMES	Clothing	2.0%	10.40%
SANOFI	Pharmaceutical	-0.1%	44.90%
STMICRO	Semi-conductors	-2.5%	2.80%
L'OREAL	Cosmetics	-4.0%	-28%
DASSAULT SYSTEMES	Software	-4.2%	37.90%
AIR LIQUIDE	Chemicals	-5.0%	-7.50%
ATOS	Computer services	-11.0%	44.20%
CARREFOUR	Food retailer	-11.1%	-41.90%
SCHNEIDER ELECTRIC	Electrical components	-12.1%	0.30%
DANONE	Food products	-13.3%	-46.40%
PERNOD RICARD	Distiller and vintner	-14.2%	-27.50%
LVMH	Clothing	-15.0%	-14.00%

Table A: Over-performing stocks with their activity sectors, their yields measured between January 1st, and April 1st 2020 and their associated call yields measured over the same time period. The call yields reported are averages of all warrant calls traded during the first quarter of 2020.

STOCK	SUB-INDUSTRY	YTD	PUT YTD
UNIB-RODAM-WES	Real Estate	-64.1%	807.92%
RENAULT	Automobiles	-58.4%	252.25%
SOCIETE GENERALE	Bank	-57.2%	152.61%
AIRBUS	Aerospace	-57.0%	164.47%
BNP PARIBAS	Bank	-45.5%	116.70%
ACCOR	Hotels	-45.0%	220.33%
CREDIT AGRICOLE	Bank	-43.8%	118.29%
SAFRAN	Aerospace	-41.1%	170.40%
SODEXO	Restaurants	-38.0%	538.70%
PEUGEOT	Automobiles	-37.0%	122.60%
AXA	Insurance	-35.4%	157.20%
PUBLICIS GROUPE	Media Agency	-35.0%	192.40%

Table B: Under-performing stocks with their activity sectors, their yields measured between January 1st, and April 1st 2020 and their associated put yields measured over the same time period. The put yields reported are averages of all warrant puts traded during the first quarter of 2020.

The remainder of the paper exploits this heterogeneity to build a corporate insurance scheme against pandemic risks. It is organized as follows. Section 2 develops a conceptual framework that shows how corporate pandemic insurance can be based on a self-funding mechanism, by building on the heterogeneity of sectoral reactions to a pandemic event. We start with a one-period model of an economy, where two stocks are traded, differing according to how their returns react to the occurrence of a pandemic. Type 1 stocks perform relatively well, and may even benefit from the pandemic event. In contrast, type 2 stocks are struck with full force, with a strong decline in their return, should a pandemic occur. A risk-averse firm seeks protection against the consequences of a pandemic on its cashflows. This is done by contributing to an insurance fund, managed either directly by the firm itself, or, more realistically, by a mutual or stock insurer acting on behalf of the firm. This insurance fund portfolio, formally analogous to a unit-linked fund, includes a riskless asset (or the issuance of riskless debt), and long or short positions in stocks 1 and 2. An optimal portfolio maximizes the expected utility of the firm's cashflow. Since the firm is risk-averse, this means covering the pandemic risk faced by the firm, while hedging the non-pandemic risks that affect the stock returns. As we will see, the optimal financial investment strategy consists in going long on stock 1 and short on stock 2, so as to obtain benefits from their opposite reaction to pandemic events, while hedging the bearish and bullish non-pandemic episodes affecting the stock market. This results in total or partial coverage of the pandemic and non-pandemic risks, according to whether or not stock returns include a risk premium. In other words, the risk premium required by representative investors in the stock market play a role similar to loading in usual insurance models: they make the cost of transferring risks greater, which affects the optimal level of coverage. Furthermore, in a classical way, the holding of riskless asset (akin to liquid reserves) or the issuance of riskless debt, should equalize the marginal productivity of the firm's capital and the risk-free rate of interest.

Long-short is a widespread strategy among hedge funds: by going long in stocks which have the potential to appreciate, and simultaneously going short in stocks expected to decline in value, hedge fund managers aim at making a profit, while being immunized against the market risk, and without incurring stock delivery costs. This strategy should be adapted, but its underlying logic remains valid, when it is a question of designing an insurance coverage against pandemics, and no more of taking advantage of market mispricing: going short or long in stocks, according to whether or not they are expected to suffer strongly from a pandemic event provides the required coverage, while hedging the non-pandemic risks. Although the long-short strategy is very usual in the hedge fund industry, it is not without disadvantages in terms of transaction costs induced by margin calls, and risk exposure when long and short position do not exactly match. This may justify using put options rather than short positions, and call options rather than long positions. We show that the state-contingent payoff of the insurance fund can be replicated through a portfolio of call options on stock 1 and put options on stock 2, without incurring the disadvantages of the long-short strategy.

We extend our analysis in several directions. First, a multi-period model is analysed, in which the firm can transfer funds from period to period, with qualitatively unchanged conclusions. Second, we consider the case where the future pandemic's characteristics are imperfectly known, in which case several forms of market incompleteness may arise. In this setting, we show that when the pandemic creates uncertain losses for the insured firm, prudent firms use the long-short strategy to obtain a higher level of coverage than in the absence of uncertainty. We draw a parallel with the precautionary motive for insurance highlighted by Schlesinger (2013). Finally, the case where future stock prices are imperfectly known and markets are incomplete is discussed. It is shown that the firm may be lead to choose a portfolio that delivers higher expected cashflows in the pandemic state than in the non-pandemic states.

Section 3 is devoted to numerical simulations of a capitalization-based pandemic insurance, using data from the French stock exchange. We first conduct a backtest, in which we assess the insurance strategy described above, over a period of twelve years. Starting from January 2009, an insured firm is assumed to contribute €20,000 each year to purchase put and call options on the stocks listed in Tables A and B, respectively. At the beginning of each year, the firm purchases put and call options with a maturity of two year. At the end of each year, these options are sold and new ones are purchased. This strategy allows the firm to build a fund that can be liquidated if a pandemic disrupts its activities. We find that the liquidation of the two-fund portfolio on April 1st, in the midst of the COVID-19 crisis, would have delivered a €868,690 pay-out, hence alleviating the cost of business disruptions. In a forward-looking perspective, we then simulate future price paths for the stocks listed in Tables A and B, and we analyze the performance of the option-based two-fund portfolio. This prospective exercise results in a distribution of potential payoffs should a crisis, similar to the one of 2020, occur in the future. We find that a firm contributing during twelve years, starting from April 1st, 2020 would benefit from an expected payoff of €1.0690 million, should such a crisis occur in 2031. We also characterize the probability distribution of the two-fund portfolio liquidation value.

We then appraise the robustness of the insurance strategy to uncertainty concerning the effect of a pandemic on stock markets. We show that an increase in the underlying returns' variances makes the strategy more profitable due to the positive response of option values to an increase in underlying return volatility. In contrast, selecting the wrong underlying assets may undermine the strategy's performance when a small number of assets is selected.

Finally, Section 4 concludes and Section 5 is an Appendix containing the proofs of the theoretical sections.

2 The model

2.1 One-period setting[6]

We analyze the financial choices of a firm facing a pandemic risk, in a one-period model, starting at time $t = 0$ and ending at $t = 1$. The firm owns initial assets, including productive assets and financial reserves, with total value A. A part I of the financial reserves available at $t = 0$ is kept as financial assets from $t = 0$ to $t = 1$, and the remainder is invested as additional productive assets. Hence,

[6]Section 5.5 in the Appendix presents an extended version of this model, with arbitrary numbers of assets and states

the firm has productive assets $K = A - I$ during the current period. In the absence of a pandemic, these productive assets provide cashflows $f(K)$, with $f' > 0, f'' \leq 0$, available at $t = 1$.

The firm's environment is characterized by two types of uncertainty. Firstly, a pandemic occurs with probability $\pi \in (0, 1)$. Secondly, in the absence of pandemic, stock markets are bullish or bearish, with probability α_u and α_d, respectively, with $\alpha_u + \alpha_d = 1$. Thus, there are three states $s \in \{u, d, p\}$, where u (up) and d (down) are the two non-pandemic states (with bullish and bearish market, respectively) and p is the pandemic state, with probability $\alpha_u(1 - \pi), \alpha_d(1 - \pi)$ and π, respectively.

Three securities, indexed by $i \in \{0, 1, 2\}$, are traded in the financial market, and may be held as reserves by the firm: security $i = 0$ is a risk-free asset, with interest rate r_f, and $i = 1$ and 2 are two types of stocks. These two stocks are similarly affected by the ups and downs of the business cycle in the non-pandemic states. In the absence of pandemic, the expected return of stock i is denoted R_i, with returns $R_i + h$ and $R_i - h'$ in states u and d, respectively, with $\alpha_u h = \alpha_d h'$. Stocks $i = 1$ and $i = 2$ differ in their reaction to the occurence of a pandemic: stock 1 is a defensive asset, while stock 2 is severely affected should a pandemic occur: their returns in state p are $R_1 + H$ and $R_2 - H'$, respectively. We assume $H' > h'$ and $H + H' > 0$. Condition $H' > h'$ reflects the fact that stock 2 is severely affected by the pandemic. As regards stock 1, H may be positive or negative: the assumption $H > -H'$ simply means that stock 1 reacts better than stock 2 to the occurence of a pandemic. We have $R_2 > R_1$ because the higher expected return of stock 2 in the no-pandemic states compensates its stronger downward reaction should a pandemic occur. Overall, the returns of the three assets are summarized in Table C.

$s\backslash i$	0	1	2
u	r_f	$R_1 + h$	$R_2 + h$
d	r_f	$R_1 - h'$	$R_2 - h'$
p	r_f	$R_1 + H$	$R_2 - H'$

Table C: Return of securities

One easily checks that the matrix of security returns is of rank 3, and thus financial markets are complete. In particular, for each state s, an Arrow-Debreu security (i.e., a security that pays one unit of numeraire in state s, and zero otherwise) can be obtained through a portfolio of available assets.

2.2 Covering pandemic losses

We view the firm as a small or medium-sized enterprise, whose business is not significantly correlated with the ups and downs of the stock market, except when a pandemic is at the origin of a business interruption, causing losses in an amount equal to L. Thus, the final cashflow derived from productive assets is

$f(K)$ in states u and d, and $f(K)-L$ in state p. The firm is supposed to be risk-averse with respect to its final net cashflow including the payoff of its financial holdings (in short, its cashflow), either because of the investment crowding-out mechanism analyzed by Froot-Scharfstein and Stein (1993), or because its owner has a non-diversified wealth, or because a decrease in cashflow exacerbates the risk of bankruptcy in the future.

The firm makes its financial choices so as to maximize the expected utility of its cashflows

$$\sum_{s\in\{u,d,p\}} \pi_s u(w_s),$$

where w_s is the firm's cashflow in state s (more precisely defined below), π_s is the probability of state s (i.e. $\pi_u = \alpha_u(1-\pi), \pi_d = \alpha_d(1-\pi)$ and $\pi_p = \pi$), and $u(.)$ is a von Neumann-Morgenstern utility function that represents the firm's risk aversion, with $u' > 0$ and $u'' < 0$.

The firm allocates a part x_i of its reserves to security i with

$$\sum_{i\in\{0,1,2\}} x_i = I.$$

The firm's cashflow is the sum of the cashflow $f(K) = f(A-I)$ derived from its productive assets, cut by the loss L in the case of a pandemic, and of the payoff of its financial holdings, which gives

$$w_s = f\left(A - \sum_{i\in\{0,1,2]} x_i\right) + \sum_{i\in\{0,1,2]} (1+r_{si})x_i \text{ if } s \in \{u,d\}, \tag{1}$$

$$w_p = f\left(A - \sum_{i\in\{0,1,2]} x_i\right) + \sum_{i\in\{0,1,2]} (1+r_{pi})x_i - L, \tag{2}$$

where r_{si} is the return of security i in state s, as described in Table 1. The firm's optimal financial policy is obtained by maximizing the expected utility of cashflows with respect to x_0,x_1 and x_2. Furthermore, and more realistically, the financial positions of the firm may be intermediated by a financial institution, such as an insurance company, offering self-funded hedging mechanisms.

Since financial markets are complete, the price of Arrow-Debreu securities (or state prices) q_u, q_d and q_p can be recovered from the matrix of asset returns. When there are risk-neutral investors in the financial markets, the expected return of stocks are equal to the risk-free interest rate, i.e.

$$r_f = R_1 + \pi H = R_2 - \pi H', \tag{3}$$

and in that case, the vector of state prices is proportional to the state probability vector. More realistically, we may assume that the return on stocks include a risk premium because of investors' risk aversion, a case in which we have

$$r_f < R_1 + \pi H \text{ and } r_f < R_2 - \pi H'. \tag{4}$$

States u, d and p correspond to various degrees of macroeconomic prosperity, state p corresponding to a most severe economic downturn. Lemma 1 provides a sufficient condition on the security returns for this to be reflected in a simple hierarchy of probability-weighted state prices.

Lemma 1 *When (3) holds, i.e. there are risk-neutral investors, the Arrow-Debreu security prices are such that*

$$\frac{1}{1+r_f} = \frac{q_u}{\alpha_u(1-\pi)} = \frac{q_d}{\alpha_d(1-\pi)} = \frac{q_p}{\pi}. \tag{5}$$

When (4) holds, i.e. all investors are risk-averse, then we have

$$\frac{q_u}{\alpha_u(1-\pi)} < \frac{1}{1+r_f} < \frac{q_d}{\alpha_d(1-\pi)}. \tag{6}$$

If in addition we have

$$\frac{R_2 - \pi H' - r_f}{R_1 + \pi H - r_f} > \frac{h + \pi H'}{h - \pi H}, \tag{7}$$

then

$$\frac{q_d}{\alpha_d(1-\pi)} < \frac{q_p}{\pi}. \tag{8}$$

Condition (7) means that the risk premium is substantially larger for stock 2 than for stock 1. This reflects the catastrophic nature of state p in which stock 2 has a very low return $R_2 - H'$ by comparison with its no-pandemic expected return R_2, while the relative performance is better for stock 1. In what follows, we refer to (5), and (6)-(8) as the investors' risk neutrality and risk aversion cases, respectively, in relationship with the attitude toward risk of a representative investor who may require risk premiums to hold the securities in its portfolio. As assumed, the firm under consideration is risk averse.

The firm chooses its portfolio (x_0, x_1, x_2) of assets $0, 1$ and 2 (or, equivalently, it chooses a portfolio of Arrow-Debreu Securities) in order to maximize its expected utility. Let y_s denote the quantity of type s Arrow-Debreu security purchased by the firm, with $s \in \{u, d, p\}$, with

$$\begin{aligned} w_s &= f(A - q_u y_u - q_d y_d - q_p y_p) + y_s \text{ if } s \in \{u, d\}, && (9) \\ w_p &= f(A - q_u y_u - q_d y_d - q_p y_p) + y_p - L. && (10) \end{aligned}$$

The firm chooses y_u, y_d and y_p in order to maximize its expected utility $\sum_{s\in\{u,d,p\}} \pi_s u(w_s)$, and its portfolio (x_0, x_1, x_2) can then be deduced from the data on asset returns.

Proposition 1 *If investors are risk neutral, then*

$$w_u = w_d = w_p, \tag{11}$$

and the firm's portfolio is such that

$$x_0 \;=\; A - K, \tag{12}$$

$$x_1 \;=\; -x_2 = \frac{L}{H + H'}, \tag{13}$$

If investors are risk averse, then

$$w_u > w_d > w_p, \tag{14}$$

and the firm's portfolio is such that

$$x_0 < A - K, \tag{15}$$

$$x_1 + x_2 = \frac{w_u - w_d}{h + h'} > 0, \tag{16}$$

$$-x_2 < \frac{L}{H + H'}. \tag{17}$$

In both cases, we have $K = K^*$ *given by*

$$f'(K^*) = 1 + r_f. \tag{18}$$

Hence, when investors are risk-neutral, an optimal financial strategy of the firm consists of going long on stock 1 and short on stock 2, for exactly the same amount, i.e. $x_1 + x_2 = 0$. In more concrete terms, at $t = 0$ the firm sells stock 2 after borrowing it on the spot market and uses the proceeds of this sale to purchase stock 1. Hence, no net disbursement is required for these stock market operations. At $t = 1$, the firm purchases stock 2 and cancels its short position, Because of the zero aggregate net position of the firm's stock portfolio from $t = 0$ to $t = 1$, in state u the high return from the long position on stock 1 exactly compensates the low return from the short position on stock 2, and vice versa in state d. Hence, these opposite positions allow the firm to perfectly hedge its market exposure in the non-pandemic states u and d, with an aggregate return equal to the risk-less interest rate r_f. The size of these long and short positions is chosen in order to perfectly cover the firm's loss L in the case of a pandemic, which will be the case when (13) holds. The firm holds reserves or borrows money at the risk-less interest rate r_f, according to whether A is larger or lower than K^*, respectively. To put it simply, when there are risk-neutral investors, the firm can use stock market operations to fully cover the losses caused by a pandemic, while perfectly hedging its exposures to non-pandemic market fluctuations, and ultimately its cash-flows are independent from the state that may occur.

When investors are risk-averse, the return on stocks includes risk premiums and the higher these returns, the higher the probability-weighted state prices. More explicitly, from (6) and (8), substituting wealth from state u to state d can be done at rate q_d/q_u which is larger than the odds ratio α_d/α_u, and similarly q_p/q_d is larger than $\pi/\alpha_d(1-\pi)$. These distortions between state-price ratios and odds ratios make the hedging of pandemic and non-pandemic risks more costly, hence the partial coverage of cashflows reflected in inequalities (14): the firm is better off in state u than in state d, the pandemic state p being the worse. In the case of risk-neutral investors, it was optimal to hedge the non-pandemic risk (i.e., to substitute wealth in state d to wealth in state u) by going long on stock 1 and short on stock 2, with equal positions in absolute value. When the aggregate firm's position in the stock market is long, i.e., when $x_1 + x_2 > 0$, the firm's financial performances are higher in state u than in state d, which corresponds to the partial hedging of non-pandemic financial risk. As expressed by (16), the aggregate position x_1+x_2 is proportional to $w_u - w_d$, which depends on the firm's degree of risk aversion. Conditions (13) and (17) show that partial coverage of the pandemic risk goes through a short position on stock 2 which is smaller than in the risk neutral case.

Finally, in both cases, for an optimal level of productive capital K^*, the discounted marginal productivity of capital $f'(K^*)/(1 + r_f)$ should equal 1, which corresponds to a standard corporate value maximization rule.

Remark 1 *Sofar we have assumed that financial assets are held by the firm itself. It is more realistic, particularly for a small or medium-sized firm, to restrict its financial operations to the holding of a remunerated bank account if* $x_0 > 0$ *and to the issuance of debt if* $x_0 < 0$*, the riskless interest rate* r_f *applying in both cases, and to relate the return from stocks to contractual links with a financial institution. In this interpretation, the return on stocks* 1 *and* 2 *correspond to the payout of a self-funded pandemic insurance scheme managed by an insurance company or a bank. At* $t = 0$*, the firm (or its owner, in the case of a single owner) contributes an amount* $x_1 + x_2$ *to the insurance scheme, and has the right to decide how this financial investment gets allocated between stocks* 1 *and* 2*. The firm receives the proceeds of the insurance scheme at* $t = 1$*. The objective of this scheme is to provide coverage against the risks that affect the firm's cashflow, in a setting where no risk mutualization is feasible. This insurance dimension is particularly obvious in the case of risk-neutral investors: the scheme allows the firm's owner to perfectly hedge its non-pandemic risks (i.e., the ups and downs of financial markets), and also to fully cover the firm's loss, should a pandemic occur. This insurance feature remains true when investors are risk-averse, but the scheme only provides partial hedging of non-pandemic risks and partial coverage of the losses that may result from a pandemic.*

Remark 2 *We have assumed that the returns of stocks* 1 *and* 2 *have the same standard deviation* $(\alpha_u h^2 + \alpha_d h'^2)^{1/2}$ *in the non-pandemic states* u *and* d*. This assumption was made to simplify calculations, but our qualitative conclusions would remain unchanged in a more general setting, were the variability of returns may differ between between stocks* 1 *and* 2*. Table D corresponds to such a more general setting, in which stocks 1 and 2 may react more or less to the bullish or bearish state of the market.*

$s\backslash i$	0	1	2
u	r_f	$R_1 + h_1$	$R_2 + h_2$
d	r_f	$R_1 - h_1'$	$R_2 - h_2'$
p	r_f	$R_1 + H$	$R_2 - H'$

Table D: Case where the non-pandemic volatility of stock returns differ between stocks

We assume $\alpha_u h_i = \alpha_d h_i'$ for $i = 1, 2$, so that R_i still denotes the expected return of stock i in the non-pandemic states, and stock 1 is more volatile than stock 2 if $h_1 > h_2$, and vice versa. When investors are risk neutral, it is still possible to hedge the market risk in the non-pandemic states while covering the loss L in the case of a pandemic, by choosing x_1 and x_2 such that

$$\frac{x_1}{h_1} = \frac{x_2}{h_2} = \frac{L}{h_1 H + h_2 H'},$$

which is an extension of condition (13) to this broader setting. Our conclusions when investors are risk-averse could be adapted in a similar way.

The coverage mechanism at work is to have a portfolio of long and short positions in the stock market in order to hedge the non-pandemic risk and simultaneously to compensate the firm for the loss incurred in the case of a pandemic. To do so, the firm (or the insurance company managing the self-funded insurance scheme) goes long on stock 1 and short on stock 2. Alternatively, an adequate portfolio of call and put options purchased at $t = 0$ with maturity date $t = 1$ may allow the firm to hedge its risks, in the same way as if it were going long and short on the stocks themselves. For illustration purposes, consider call options on stocks 1 and put options on stock 2, with strike price $1 + R_1$ and $1 + R_2$, respectively, the value of each stock being equal to 1 at $t = 0$. Hence, for each option the strike price is equal to the expected payoff of the stock in the non-pandemic states. We consider the case where $H > 0$ so that the call option on stock 1 is in the money in states u and p (with payoffs h and H, respectively), while the put option on stock 2 is in the money in states d and p (with payoffs h' and H', respectively). Let q_{c1} and q_{p2} be the price of the call and put options, on stocks 1 and 2, respectively. For simplicity, consider the case where investors are risk-neutral, and thus the price of securities is equal to the discounted value of their expected payoff, which gives

$$\begin{aligned} q_{c1} &= \frac{\alpha_u h(1-\pi) + \pi H}{1 + r_f}, \\ q_{p2} &= \frac{\alpha_d h'(1-\pi) + \pi H'}{1 + r_f}. \end{aligned}$$

Let z_{c1} and z_{p2} be the value of calls and puts (on stocks 1 and 2, respectively) purchased at $t = 0$, and z_{c1}/q_{c1} and z_{p2}/q_{p2} the corresponding numbers of options.[7] When investors are risk-neutral, the optimal insurance scheme provides

[7] z_{c1} and z_{p2} are positive or negative, according to whether the firm goes long or short in each option market.

perfect hedging of non-pandemic risks and full coverage of the loss in the case of a pandemic, which gives

$$y = y_u = y_d = y_p - L. \tag{19}$$

These state-dependent financial cashflows are associated with a portfolio (x_0, z_{c1}, z_{c2}) if the following conditions are satisfied:

$$x_0(1 + r_f) + z_{c1}\frac{h}{q_{c1}} = y, \tag{20}$$

$$x_0(1 + r_f) + z_{p2}\frac{h'}{q_{p2}} = y, \tag{21}$$

$$x_0(1 + r_f) + z_{c1}\frac{H}{q_{c1}} + z_{p2}\frac{H'}{q_{p2}} = y + L. \tag{22}$$

Solving (20)-(22) for x_0, z_{c1} and z_{p2} yields

$$\begin{aligned} x_0 &= \frac{1}{1 + r_f}\left[y - \frac{hh'L}{Hh' + h(H' - h')}\right] , \\ \frac{z_{c1}}{q_{c1}} &= \frac{h'L}{Hh' + h(H' - h')} > 0, \\ \frac{z_{p2}}{q_{p2}} &= \frac{hL}{Hh' + h(H' - h')} > 0. \end{aligned}$$

Hence following a long-short portfolio management strategy and holding a portfolio of call and put options are two ways to reach the same goal, i.e., covering simultaneously the non-pandemic market risks and the loss that may result from a pandemic. Most interestingly, it turns out that these two financial strategies, usually restricted to pure portfolio management with performance benchmarking, are relevant when it comes to designing the coverage strategy of a firm facing a risk with a systemic dimension, i.e., whose occurrence coincides with a non-diversifiable shock on the stock market. Portfolio management and corporate insurance are converging here, in the search for an optimal pandemic insurance scheme.

Nevertheless, behind this methodological alignment, there are important practical differences between the two strategies. Short stock positions are typically only given to accredited investors (which is in accordance with our interpretation of intermediation through an insurance company or a bank), who are usually required to place a margin deposit or collateral with the broker in exchange for the loaned shares. In practice, this creates non-negligible transaction costs and thereby reduces the attractiveness of such self-funded insurance schemes for small or medium-sized firms, with limited financial resources. The option-based financial strategy would exonerate these firms from such transaction costs, by shifting the margin calls to a much larger set of financial investors who act as counter-parts in option markets. Furthermore, short positions on stock 2 expose investors to potentially unlimited risk of loss if, for any reason, this stock performs better than expected, while, with the put options, the maximum loss is restricted to the price paid for the puts. Finally, borrowing stock 2 to short it - or, equivalently, going through a Contract For Difference broker - entails transaction costs that have been ignored, as well as interest payable on the margin account, while put options only require an up-front cost to purchase the puts, but no other ongoing expenses. For all these reasons, although long-short and options are two ways to reach the same goal, options may be considered as a cheaper and less risky strategy.

2.3 Multi-period setting

Let us extend our results to an infinite horizon model, where time periods are indexed by $t = 0, 1...$, and the firm can transfer financial resources across time. The notations are adapted from the one-period setting as follows. At each period t, the total value of the firm's initial assets is equal to A_t, allocated between productive assets K_t (including investment made at the beginning of period t) and financial reserves $A_t - K_t$. Productive assets provide cashflows $f(K_t)$, with $f' > 0, f'' \leq 0$, available at the end of period t. We still assume that three states $s \in \{u, d, p\}$ may occur at each period t, with probability π_s equal to $\alpha_u(1-\pi), \alpha_d(1-\pi)$ and π, respectively, and that the firm incurs loss L in the pandemic state p. The probability distributions of states are assumed to be independent between periods. Financial reserves are allocated between securities $i = 0, 1$ and 2 with return r_{si} in state s as specified in Table 1. We denote x_{ti} the value of security i held by the firm at period t. Since the firm's assets are allocated between productive and financial assets at the beginning of each period, we have

$$A_t = K_t + \sum_{i \in \{0,1,2\}} x_{ti}.$$

The total cashflows come from productive and financial assets, and they are allocated either to the owner's consumption (through the distribution of dividend) or to corporate investment, i.e. to the increase in the value of the firm's assets. This allocation of cashflows depends on the state prevailing during the period. Let c_{st} denote the consumption level in state s at period t, and let $A_{t+1,s}$ be the value of the firm's assets at the beginning of period $t+1$ when state s prevails at period t, hence with total investment $A_{t+1,s} - A_t$. The cashflows coming from productive and financial assets, possibly reduced by loss L, are equal to the sum of consumption and investment, and thus we have

$$c_{ts} + A_{t+1,s} - A_t = f(K_t) + \sum_{i \in \{0,1,2\}} (1 + r_{si}) x_{ti} \text{ if } s \in \{u, d\}, \quad (23)$$

$$c_{tp} + A_{t+1,p} - A_t = f(K_t) + \sum_{i \in \{0,1,2\}} (1 + r_{pi}) x_{ti} - L \ . \quad (24)$$

The firm chooses its portfolio of financial assets (x_{t0}, x_{t1}, x_{t2}) and its state-contingent investment level $A_{t+1,s} - A_t$, in order to maximize the discounted sum of consumption expected utility

$$\sum_{t=0}^{\infty}\delta^t \sum_{s_t\in\{u,d,p\}} \pi_{s_t} u(c_{ts_t}),$$

where δ is the discount factor, such that $\delta < 1$, and s_t is the state at period t.

The intertemporal strategy of the firm is characterized by functions $A_t(h_t)$ and $x_{ti}(h_t)$ for all $t \geq 0$ and all $i \in \{0,1,2\}$, where $h_t = (s_0, s_1, ..., s_{t-1}) \in H_t \equiv \{u,d,p\}^t$ denotes the sequence of states from period 0 to period $t-1$ if $t \geq 1$ and $h_0 \in \varnothing$. Let $\Pi_t(h_t)$ be the probability of sequence h_t when $t \geq 1$, with $\sum_{h_t \in H_t} \Pi_t(h_t) = 1$. For notational consistency, we denote $\Pi_0(h_0) = 1$. In words, $A_t(h_t)$ is the value of the firm's assets at the beginning of period t, with the constraint $A_0(h_0) \equiv A_0$ imposed by the value of initial assets, and thus $A_{t+1}(h_{t+1}) - A_t(h_t)$ is its investment at period t, while $(x_{t0}(h_t), x_{t1}(h_t), x_{t2}(h_t))$ is its portfolio of financial assets held during period t. The firm maximizes its discounted expected utility

$$\sum_{t=0}^{\infty}\delta^t \Bigg[\sum_{h_t\in H_t}\Pi_t(h_t) \sum_{s_t\in\{u,d,p\}} \pi_{s_t} u\Bigg(f\big(A_t(h_t) - \sum_{i\in\{0,1,2\}} x_{ti}(h_t)\big) \\ + \sum_{i\in\{0,1,2\}} (1+r_{s_t i})x_{ti}(h_t) + A_t(h_t) - A_{t+1}(h_t,s_t) - \mathbb{1}(s_t = p)L\Bigg)\Bigg],$$

with respect to $A_t(.) : H_t \to R_+$ for all $t \geq 1$ and $x_{ti}(.) : H_t \to R$ for $i \in \{0,1,2\}$ and all $t \geq 0$. Let $v(A_0)$ be the optimal discounted expected utility level as a function of initial assets A_0, with $v'(A_0) > 0$ and $v''(A_0) < 0$.[8] At period $t = 0$, the firm chooses its portfolio (x_{00}, x_{01}, x_{02}) and its state-dependent investment level $A_1(s_0) - A_0$ in order to maximize

$$\sum_{s_0\in\{u,d,p\}} \pi_{s_0}\Bigg[u\Bigg(f(A_0 - \sum_{i\in\{0,1,2\}} x_{0i}) + \sum_{i\in\{0,1,2\}}(1+r_{s_0 i})x_{0i} \\ + A_0 - A_1(s_0) - \mathbb{1}(s_t = p)L\Bigg) + \delta v(A_1(s_0))\Bigg],$$

where the effect of current decisions on the discounted expected utility in next periods goes through the last term $\delta v(A_1(s_0))$. Conditionally on state s_0, the future assets $A_1(s_0)$ are chosen so as to maximize the discounted expected utility

$$u\Bigg(f(A_0 - \sum_{i\in\{0,1,2\}} x_{0i}) + \sum_{i\in\{0,1,2\}}(1+r_{s_0 i})x_{0i} \\ + A_0 - A_1(s_0) - \mathbb{1}(s_t = p)L\Bigg) + \delta v(A_1(s_0)).$$

[8]Note that the expected utility is a concave function of the parameter A_0 and of the unknowns $A_t(h_t)$ and $x_{ti}(h_t)$, which implies the concavity of the value function $v(A_0)$.

Hence, the portfolio (x_{00}, x_{01}, x_{02}) maximizes

$$\sum_{s_0 \in \{u,d,p\}} \pi_{s_0} \overline{u} \Bigg(f(A_0 - \sum_{i \in \{0,1,2\}} x_{0i}) + \sum_{i \in \{0,1,2\}} (1 + r_{s_0 i}) x_{0i} - \mathbb{1}(s_t = p) L \Bigg) ,$$

where indirect utility function $\overline{u}(.)$ is defined by

$$\overline{u}(w) \equiv \max_{A_1} \{ u(w + A_0 - A_1) + \delta v(A_1) \},$$

with $\overline{u}' > 0$ and $\overline{u}'' < 0$.[9] Hence, in this multi-period setting, the optimal portfolio is the solution to an optimization problem deduced from the one-period problem by replacing utility function u by the indirect utility function $\overline{u}$. The concavity of function $\overline{u}$ allows us to conclude that the results obtained in the one-period setting are also valid in this multi-period setting, with unchanged qualitative conclusions.

2.4 Various types of pandemic

So far we have restricted ourselves to a unique type of pandemic, with well-defined effects on the firm's cashflows and on stock returns. We may extend our analysis to a more general setting, where pandemics take several forms. As we will see, this may lead to two forms of market incompleteness. For the sake of simplicity, we explore these issues in the one-period setting.

2.4.1 Uncertain losses of the insured firm

Let us first consider the case where pandemics are at the origin of random losses $\widetilde{L}$ for the firm, depending on their specific caracteristics, with expected value $\overline{L} = \mathbb{E}\widetilde{L}$. The matrix of security returns is unchanged and still defined by Table C. In particular, the set of stocks available in the market (limited to stocks 1 and 2) does not reflect the diversity of pandemics, with more or less major losses for the firm, which corresponds to a first form of market incompleteness. In this setting, the firm's cashflow in the pandemic state is random and written as

$$\widetilde{w}_p = f \left(A - \sum_{i \in \{0,1,2]} x_i \right) + \sum_{i \in \{0,1,2]} (1 + r_{pi}) x_i - \widetilde{L}, \tag{25}$$

with $\overline{w}_p = \mathbb{E}\widetilde{w}_p$. The firm chooses x_0, x_1 and x_2 in order to maximize its expected utility

$$\pi_u u(w_u) + \pi_d u(w_d) + \pi_p \mathbb{E} u(\widetilde{w}_p), \tag{26}$$

where $\mathbb{E}u(\widetilde{w}_p)$ is the expected utility, conditionally on the occurrence of a pandemic event. Proposition 2 characterizes the firm's portfolio choices in this setting.

[9] The envelope theorem gives $\overline{u}'(w) = u'(w + A_0 - A_1) > 0$. Differentiating the first-order optimality conditions yields $dA_1/dw = u''/(\delta v'' + u'')$, which implies $\overline{u}''(w) = \delta u'' v''/(\delta v'' + u'') < 0$.

Proposition 2 *Assume that the firm is downward risk-averse (i.e. $u''' > 0$). When facing a risk of random loss $\widetilde{L}$, the firm's optimal portfolio choices are the same as in the case of a deterministic loss L larger than expected loss $\overline{L}$. In particular, when investors are risk-neutral, we have $x_1 = -x_2 > \overline{L}/(H + H')$ and $w_u = w_d < \overline{w}_p$.*

Hence, when pandemics create losses of uncertain amount, the prudent (i.e. downward risk averse) firm behaves as if its losses in case of pandemic were known in advance, but larger than their expected value. In other words, the uncertainty regarding losses incentivizes the firm to overinsure its exposure to the pandemic risk, by comparison with the case where losses would be certain and equal to their expected value. In the particular case where investors are risk-neutral, the return of the firm's portoflio should provide resources that exceed expected losses $\overline{L}$.

It is easy to see where this conclusion comes from. Writing $\widetilde{L} = \overline{L} + \varepsilon$, with $\mathbb{E}\varepsilon = 0$, shows that assuming uncertain pandemic losses is equivalent to adding a zero-mean background risk ε to a deterministic pandemic loss $L = \overline{L}$, which makes insurance even more attractive to prudent policyholders. In fact, Proposition 3 is similar to the precautionary motive of the prudent agent highlighted by Schlesinger (2013), whose intuition can be found in Eeckhoudt & Schlesinger (2006), and which states that uncertainty about losses exacerbates insurance demand. As far as we are concerned, we can conclude that the uncertainty about the pandemic cost is likely to incentivize firms to extend their insurance coverage beyond the level that they would choose if pandemics affect their cashflows in a perfectly predictable way.

2.4.2 Uncertain stock returns in the pandemic state

Let us now turn toward the case where stock returns react differently, according to the type of pandemic that may occur. As shown in Appendix 5.5, our analysis would be qualitatively unchanged if financial markets were complete, with a large number of stocks reacting differently according to the type of pandemic. More explicitly, if the set of state-contingent stock returns span the full space of state-contingent claims (each type of pandemic corresponding to a specific state), then asset portfolios would allow us to replicate Arrow-Debreu securities for all pandemic and non-pandemic states. If there are risk-neutral investors, Arrow-Debreu security prices are proportional to state probabilities, and it is optimal for the risk-averse firm to fully cover all its pandemic and non-pandemic risks, through a portfolio of long-short positions. If investors are risk-averse, then partial coverage is optimal. In other words, in this complete market setting, the diversity of pandemic types would not affect our conclusions.

Things are different when there are various types of pandemics, but the set of available stocks is not large enough to allow investors to reach a target payoff whatever the pandemic. To get a hint of how this second form of market incompleteness affects the firm's risk management strategy, let us extend our base model to a case where the returns of stocks 1 and 2 in the pandemic state $s = p$ are affected by independently-distributed random variables ε_1 and ε_2, respectively, with $\mathbb{E}\varepsilon_1 = \mathbb{E}\varepsilon_2 = 0$. The diversity of possible pandemics under incomplete market is reflected in the distribution of ε_1 and ε_2, while only two stocks are traded. Table E describes the state-contingent asset returns.

$s\backslash i$	0	1	2
u	r_f	$R_1 + h$	$R_2 + h$
d	r_f	$R_1 - h'$	$R_2 - h'$
p	r_f	$R_1 + H + \varepsilon_1$	$R_2 - H' + \varepsilon_2$

Table E: Return of securities under multiple pandemic states

Let r_{si} be the expected state-contingent return of asset i as defined in Table 1. The firm's cashflow w_s in state s is still defined by (1) for $s = u$ and p, while the state-p random payoff $\widetilde{w}_p$ with deterministic loss L is written as

$$\widetilde{w}_p = f\left(A - \sum_{i\in\{0,1,2]} x_i\right) + \sum_{i\in\{0,1,2]} (1 + r_{pi})x_i + \varepsilon_1 x_1 + \varepsilon_2 x_2 - L, \qquad (27)$$

with $\overline{w}_p = \mathbb{E}\widetilde{w}_p$. Comparing (27) and (25) shows that portfolio choices now affect the probability distribution of the firm's cashflow in the pandemic state, and not only its expected value. Thus, analyzing how multiple pandemic types affect portfolio choices cannot be reduced simply to adding a background risk to a deterministic loss level. Proposition 3 characterizes the optimal solution to this problem when investors are risk neutral, under technical assumptions that simplify our analysis.

Proposition 3 *Assume $h' = h, \alpha_u = \alpha_d = 1/2, H = 0$. When investors are risk-neutral, the firm's optimal portfolio choices are such that $w_u < w_d$, and $x_1 + x_2 < 0$. Furthermore, if $\pi H' < h$ and $u(.)$ is quadratic, then we have $\overline{w}_p > w_u$.*

We know from Proposition 1 that under complete financial markets and risk-neutral security pricing, it is optimal for the firm to perfectly hedge its non-pandemic risks and also to fully cover the loss that may result from a pandemic event. Proposition 3 shows that this is no more the case under incomplete markets. When $h' = h$ and $\alpha_u = \alpha_d = 1/2$, the bullish and bearish non-pandemic states are treated symmetrically, which makes the analysis more straightforward. Assumption $H = 0$ is a sufficient condition, but not a necessary one, to derive the first part of the Proposition. It means that stock 1 plays a neutral role in the case of a pandemic (its return being equal to its expected value R_1), while the effect of the pandemic shock is fully concentrated on stock 2, with a downward effect $-H'$.[10]

In our setting, non-pandemic risks can be perfectly hedged by long-short positions that balance out each other, with $x_1 + x_2 = 0$. Should a pandemic occur, then a long position on stock 1 (when H was assumed to be positive) and a short position on stock 2 provide expected financial resources to the

[10] Needless to say, this assumption is made to highlight the mechanism at work, much more than for reasons of realism.

firm that compensate its loss L. However, perturbations ε_1 and ε_2 make this compensation random, which is detrimental to the risk-averse firm. When $H = 0$, the balance of these effects tips in favor of obtaining compensation more through the short position on stock 2, than through the long position on stock 1, which gives $-x_2 > x_1$, i.e. $x_1+x_2 < 0$.[11] Consequently, we have $w_u < w_d$, the firm being better off when the stock market is bearish than when it is bullish. Since the short position on stock 2 is larger than the long position on stock 1, we may even have the paradoxical result $\overline{w}_p > w_u$, meaning that the expected cashflow is larger in the pandemic state than when the market is bullish without pandemic. As stated in the second part of Proposition 3, this is the case when $\pi H' < h$ and $u(.)$ is quadratic.

These results are obtained under the assumption of investors' risk neutrality. As shown in the second part of Proposition 1, the investors' risk aversion pulls in the opposite direction, the long position on stock 1 being larger than the short position on stock 2 when there is only one type of pandemic. In practice, the optimal structure of the firm's portfolio results from the complex interaction between these two effects: the pricing of securities under investors' risk aversion incentivizes the firm to have more long positions on stock 1 than short positions on stock 2, while the uncertainty of stock returns in the pandemic state may exert an opposite effect.[12]

3 Numerical simulations

In this section, we conduct simulations that illustrate the functioning of the insurance scheme discussed in the previous sections. This exercise will allow us to assess the coverage made possible by pandemic self-funded insurance, by using data from the French stock exchange. Our perspective will be first retrospective, and then prospective. From a retrospective standpoint, we will ask the question of what amount of coverage would have been available at the outbreak of the COVID-19 crisis, if the insured firm had invested during the previous decade in two option-based funds, with underlyings similar to stocks 1 and 2 of our theoretical model. In a forward-looking perspective, we will simulate the coverage from such a financial investment strategy when the returns of the underlying stocks are random and the next pandemic occurs a decade after the start of the financial investment phase.

[11] In the proof of Proposition 3, it is assumed that the support of ε_1 and ε_2 is not too large, so that we still have $x_1 > 0$ and $x_2 < 0$ at an optimal solution of the firm's maximization problem. If ε_1 and ε_2 correspond to large perturbations, then covering the firm's risk through financial assets could be sub-optimal, meaning that the firm would choose $x_1 = 0$ and/or $x_2 = 0$.

[12] The proof of Proposition 3 shows that we still have $K = I - A = K^*$, i.e. $f'(K) = 1 + r_f$ when stock returns in the pandemic state are random.

3.1 Backtest

We first perform a backtest of the following strategy: each year, from January 1st 2009 to January 1st 2020, €20,000 are invested in the insurance scheme composed of two funds. Fund 1 is composed of call options, whose underlyings are the twelve stocks listed in Table A, that are expected to resist well to a pandemic shock. Fund 2 is composed of put options on the twelve stocks of Table B, that are expected to under-perform the CAC40 in the pandemic state. Each year, the strategy re-balances the two funds values. The symmetry between the funds aims at neutralizing the effect of a potential trend in stock prices since a general price increase benefits the calls and harms the puts, while a general price drop benefits the puts at the expense of the calls.

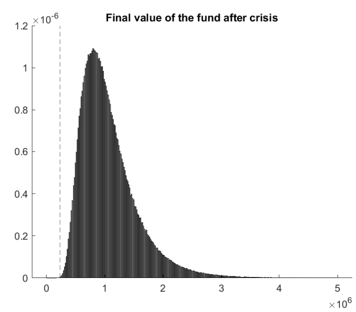
Between 2009 and 2019, the French stock market experienced a 85.8% increase in the CAC40 index. This marked evolution favours capitalization in fund 1. In contrast, fund 2 incurs a loss each year. A negative trend over the period would have had the opposite effect. Since it is hard to predict the direction of stock markets, a symmetrical strategy of investment in the two funds may be viewed as a prudent approach.[13]

The yields of the two funds where computed by reconstituting each year the value of options with the following characteristics : options purchased at the beginning of each year have a maturity of two years and are sold at the end of the year, with a remaining maturity of one year. The strike price is equal to the current price of the underlying asset at the moment of purchase. Option prices were calculated retrospectively using the Black-Scholes formula on the basis of the historical underlying prices and option characteristics.[14]

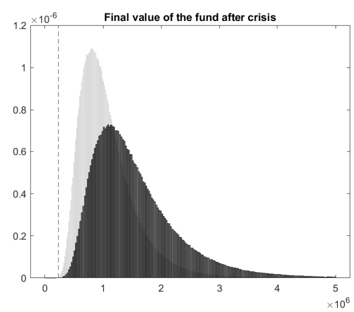
At the beginning of each year, the additional €20,000 contribution is added to the total fund value which is split in two equal parts to finance the purchase of new put and call options. This results in a year-by-year evolution of the two funds' values. Results are presented in Table E. The total contribution is equal to €240,000 on January 1st 2020. With the notable exceptions of 2011 and 2018, the fund invested in put options, i.e., fund 2, yields a negative payoff due to the positive trend of the stock market over the period. In contrast, the fund invested in call options, i.e. fund 1, provides positive payoffs that more than compensates the losses of fund 2 most years and helps to increase the overall fund value. At the beginning of year 2020, the fund accumulated a total of €395,005. At the beginning of each year, financial investments are evenly spread between and within each fund.

[13] In portfolio management, a straddle consisting in simultaneous purchases of call and put options with the same underlying stock, is a way to hedge the uncertainty about the return of this stock. The same logic applies when it comes to hedge the market risk that affects the underlyings of the two funds.

[14] Hence, the procedure we followed consisted in estimating the price of options whose characteristics may be considered as representative of calls and puts traded in the Paris stock exchange. Another approach would have consisted in deriving option prices from the list of actual options listed in the warrant market. We have opted for the first approach for practical reasons, related to data availability, but also because, for each stock listed in the CAC40, there are many put and call options, with various maturities and strike prices, which would have made our analysis intractable.



(a) Strike prices equal to underlying prices. Mean value: €1.0690 million, median: €967,030, 5th percentile: €497,660, 1st percentile: €385,260.



(b) Strike prices of the put lower than underlying prices. Mean value: €1.5478 million, median: €1.3793 million, 5th percentile: €677,120, 1st percentile: €515,130.

Figure 2: Liquidation value of the two-fund portfolio, with change in option strike prices.

The portfolio of puts, with underlying assets particularly vulnerable to the pandemic shock, generates a yield that offsets the limited losses incurred by the calls, whose underlying assets have been chosen to resist the crisis. This results in a total of €868,690 available to the firm when the crisis hits, the options being sold on April 1st, 2020. The back-test therefore results in a €628,690 excess payoff for the firm. If contributions to the fund were considered as tax-deductible, then the net excess payoff would even be larger, and would reach €664,690 and €700,690 when corporate tax rate is 15% and 30%, respectively.

3.2 Forward-looking analysis

We now turn to an exercise of forward-looking simulations. Instead of using the historical values of the underlying asset prices, we analyse the performance of the two-fund investment strategy, by simulating a large number of future stock price trajectories. We use the log-normal assumption, on which the Black-Scholes model is built. The means and variances of the return processes are calibrated to their historical values, measured before the 2020 pandemics, between January 1st 2007 and February 1st 2020. The outcome of the insurance strategy (i.e., of financial investments made by the firm at the beginning of each year) is evaluated on each simulated price path. This allows us to construct a histogram of potential payoffs to the insured firm, should another pandemic occur in the future.

The data-set is composed of daily price fluctuations of the twenty-four stocks listed in Tables A and B between January 1st 2007 and April 1st 2020. In our baseline scenario, the firm starts contributing to the funds on April 1st, 2020. The firm makes twelve yearly payments of €20,000 and a pandemic crisis,

Dates	Contribution	Yield F.1	Yield F.2	A. Yield	Capital	Net gain CT = 0%	Net gain CT = 15%	Net gain CT = 30%
01/01/2009	20 000							
01/01/2010	40 000	1.6380	0.3696	1.0038	20 076.17	76.17	3 076.17	6 076.17
01/01/2011	60 000	1.5776	0.3970	0.9873	39 567.20	-432.80	5 567.20	11 567.20
01/01/2012	80 000	0.7141	1.3108	1.0124	60 305.83	305.83	9 305.83	18 305.83
01/01/2013	100 000	1.7326	0.4011	1.0668	85 670.27 5	670.27	17 670.27	29 670.27
01/01/2014	120 000	1.1022	0.1927	0.6474	68 410.93	-31 589.07	-16 589.07	- 1589.07
01/01/2015	140 000	0.8040	0.7288	0.7664	67 758.13	-52 241.87	-34 241.87	-16 241.87
01/01/2016	160 000	1.5619	0.7034	1.1326	99 394.86	-40 605.14	-19 605.14	1394.86
01/01/2017	180 000	1.4615	0.7350	1.0982	131 119.44	-28 880.56	-4 880.56	19 119.44
01/01/2018	200 000	1.4220	0.3645	0.8932	134 979.88	-45 020.12	-18 020.12	8 979.88
01/01/2019	220 000	0.7599	1.5496	1.1547	178 955.27	-21 044.73	8 955.27	38 955.27
01/01/2020	240 000	3.4556	0.5152	1.9854	395 005.79	175 005.79	208 005.79	241 005.79
01/04/2020	240 000	0.6222	3.5642	2.0932	868 690.12	628 690.12	664 690 .12	700 690.12

Table E: Back-test summary. The column Contribution gives the total contribution up to each date. The column Yield F.1 (F.2) gives the annual yield on fund 1 (fund 2). The column A. Yield provides the average yield. Capital indicates the value of the two funds and the columns Net gains CT = $x\%$ report the difference between total contributions, net from corporate tax at rate $x\%$, and total funds value.

affecting assets in the same proportions as the 2020 crisis, occurs in 2031. More specifically, we assume that the expected value of each daily stock return during a future pandemic is equal to the average return observed during the 2020 pandemic. We consider the case of a pandemic crisis starting in August 1st, 2031, the funds being liquidated in October 1st 2031 to provide a compensation to the firm. It is assumed that the risk-free interest rate remains constant and equal to its last value in the data-set (April 1st 2020).[15] This allows us to calculate the price of call and put options with strike prices being equal to the current underlying prices.

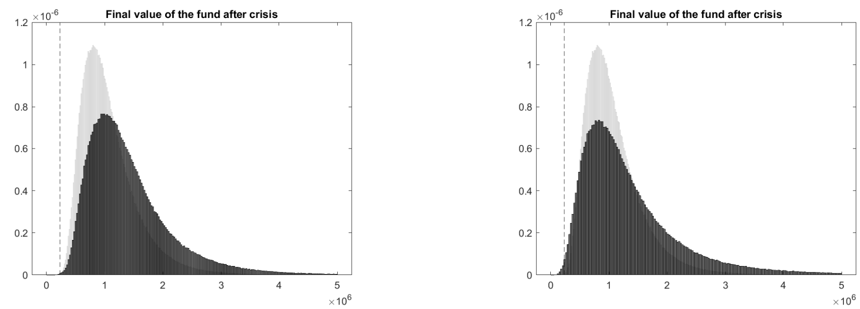
Figure 2a displays the histogram of this baseline scenario with 1 million random draws. The average value of the insurance strategy across draws is equal to €1.069 million while the median is €967,030. 95% of the simulated paths deliver a final value above €497,660 and 99% of them provide a value higher than €385,260. The cumulative investment cost of this strategy is €240,000, reduced to €204,000 and €168,000 if the firm's contributions are tax-deductible, with corporate tax rate of 15% and 30%, respectively. In comparison, if the firm adopts the conservative strategy of investing at the risk-free rate, it is only able to constitute a €235,730 buffer to face the 2031 pandemic crisis, a level higher than the long-short strategy with a probability of only 0.0077 in our baseline scenario.

It is worth considering how put and call option parameters affect the liquidation value of the two-fund portfolio. In particular, in the previous simulations, the option strike prices were taken equal to the spot underlying prices, but other assumptions are of course possible. For illustrative purposes, Figure 2b represents the distribution of the liquidation value, under the same assumptions as in Figure 2a, except for put options, that have strike prices equal to 80% of the underlying's current prices. Since a lower strike price increases leverage, the resulting distribution is more spread-out with a higher mean and median, at €1.5478 million and €1.3793 million respectively. In the case considered here, the first and fifth percentiles are also higher at €677,120 and €515,130, respectively.

The simulations presented so far rely on the assumption that the future pandemic is similar to the one experienced in 2020. In particular, the return processes are assumed to have identical expected values and volatilities in the future pandemic and in the current one. A future pandemic however, is likely to have its own specificities and to differ from what we have experienced during the 2020 COVID-19 crisis. We therefore allow the expected values of the 2031 pandemic stock returns to be imperfectly known, and thus modelled as random variables.[16] Figure 3a displays the case where the 2031 pandemic expected

[15]Time-varying interest rates and volatilities could be acknowledged easily through Ornstein-Uhlenbeck and GARCH processes for example.

[16]In the baseline scenario, it is assumed that stock returns are distributed during the two pandemic episodes (2020 and 2031) according to normal laws with expected values μ_i and standard errors σ_i for each stock $i = 1, \ldots 24$. Investors know the values of μ_i and σ_i and these values correspond to what has been observed during the 2020 crisis. We now assume that investors have imperfect information about the parameters μ_i for the 2031 crisis. Available



(a) No correlation between future expected returns. Mean value: €1.3894 million, median: €1.2187 million, 5th percentile: €571,530, 1st percentile: €430,510.

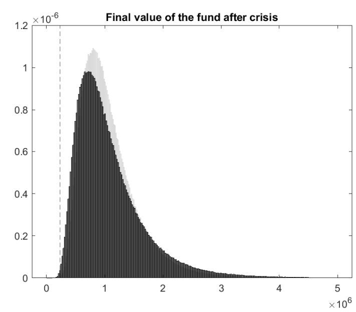
(b) Positive correlation between future expected returns. Mean value: €1.3971 million, median: €1.1300 million, 5th percentile: €442,990, 1st percentile: €298,400.

Figure 3: Liquidation value of the two-fund portfolio with random shocks on stock return expected values during the pandemic event.

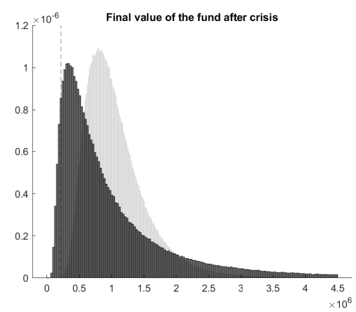
returns are independently distributed random variables. Compared with the baseline distribution, represented by the light grey histogram, the new dark grey distribution is more spread-out and displays higher mean, median, 5th and 1st percentiles. The higher level of volatility is due to the added layer of uncertainty associated with random expected returns during the future pandemic event. The fact that the new distribution dominates the baseline scenario is a by-product of the Black-Scholes model, that provides higher option valuations for more volatile stocks. In contrast, Figure 3b presents the case where the expected values of future returns are correlated random variables. The new dark grey distribution also features more volatility, but with more density on both the high and low outcomes, which means that correlation reduces the gain from the volatility of expected returns. While mean values are almost identical (€1.3971 million for the correlated case and €1.3894 million for the uncorrelated case), the fifth and first percentiles are lower in the correlated case (€442,990 and €298,400 against €571,530 and 430,510 for the uncorrelated case), due to lower diversification across stock returns.

In one respect, a perfectly informed investor would be able to anticipate how each stock reacts to a pandemic event. Thus, he would concentrate short and long positions in a small number of stocks that benefit or suffer most from

information about μ_i's is captured by distributions of the random variables $\tilde{\mu}_i$. For simplicity, we assume that each $\tilde{\mu}_i$ is normally distributed with expected value μ_i (which was observed during the 2020 pandemic). The variance of $\tilde{\mu}_i$ is arbitrarily set at 10% of $1/24\sum_i \sigma_i^2$, the average variance during the 2020 pandemic. Furthermore random variables $\tilde{\mu}_i$ are pairwise correlated with parameter $\rho \in [0, 1]$. In this setting, option prices are evaluated through the Black-Scholes formula applied to each draw of the expected value. This crude way of considering parametric uncertainty could be improved by a proper bayesian analysis. We nevertheless expect the results of such an analysis to deliver conclusions qualitatively similar to the ones reported here.



(a) Random selection of 9 out of the 12 assets. Mean value: €1.0823 million, median: €934,940, 5th percentile: €425,980, 1st percentile: €316,350.



(b) Random selection of 3 out of the 12 assets. Mean value: €1.1718 million, median: €720,660, 5th percentile: €201,990, 1st percentile: €133,530.

Figure 4: Liquidation value of the two-fund portfolio with random choice of underlying stocks

the pandemic. In this regard, our approach so far was that of an imperfectly-informed investor, only able to select two groups of underlyings, with one group including pandemic-resistant stocks, and with stocks affected adversely by the pandemic in the other. Option purchases were evenly spread between the twelve underlyings of each fund, which may be viewed as the choice of a conservative-minded investor who would minimise the risk of choosing the wrong stocks.

The robustness of our results may be appraised by considering the case of a less conservative investor who randomly selects a sub-sample of underlyings.[17] Figures 4a and 4b show the cases where nine and three stocks are randomly drawn out of the twelve stocks of our baseline scenario. These random draws are repeated to produce a sample of 1 million outcomes for the liquidation value. Choosing a lower number of stocks produces more volatility and more skewness since shocks across stocks are less likely to compensate each other. In particular, the effect on skewness is similar to the effect produced by an increase in the correlation between expected returns, presented in Figure 3. Indeed, a large number of stocks allows a reduction in the probability of extreme low and high performances thanks to compensation across individual stocks. Figure 4b shows that with only three stocks, the liquidation value may fall below the risk-free strategy payoff with a non-negligible probability, since the fifth and first percentiles are below the €235,730 threshold at €201,990 and €133,530, respectively. This is not the case when nine stocks are selected.

Last but not least, it is worth considering the uncertainty about the timing of a future pandemic. Our baseline scenario assumes a new pandemic crisis in 2031. Figure 5 represents the evolution of the two-fund portfolio's liquidation value,

[17] This is conceptually equivalent to assuming uncertain future returns. However, choosing underlying stocks randomly allows for an additional robustness check and yields different quantitative responses.

as a function of the year of the pandemic event, under the assumptions of our baseline scenario. The full line represents the expected payoff while the dotted lines represent the ninety-fifth and fifth percentiles of the simulated distributions. The crossed circles mark the risk-free strategy payoff. The liquidation value increases non-linearly with time. Indeed, the larger the size of the fund accumulated before the pandemic, the higher the strategy payoff. Delivering a significant payoff therefore requires some time for the fund to build potential value, but it is comforting to observe that the fifth percentile of the distribution is above the risk-free strategy payoff at all horizons.

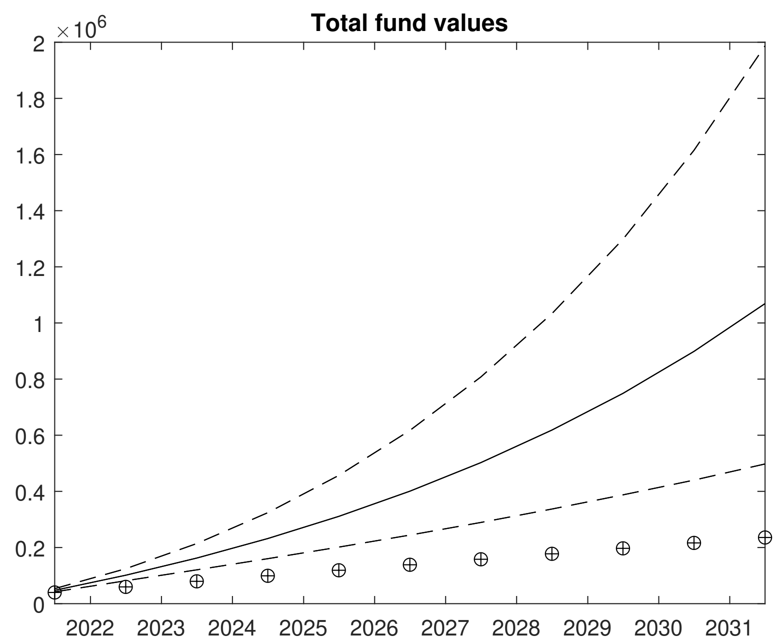


Figure 5: Liquidation value of the two-fund portfolio at different pandemic horizons

4 Conclusion

In many countries, including the U.S. and in Europe, the COVID-19 crisis has highlighted inadequate preparation for pandemics, and the current state of business interruption insurance is illustrative of this deficiency. Pandemic risk displays features that are deeply different from those of other insurance risks: it affects simultaneously a large fraction of businesses, which makes risk mutualization unfeasible, and in addition, it is systemic in nature, since it goes along with a worldwide severe economic downturn. As result, in responding to the demand for corporate pandemic insurance, innovation is required. We cannot merely pour new wine into old bottles!

Having recognized that pandemics do not affect all sectors of the real economy in the same way, we have analyzed how corporate insurance could be built through a capitalization strategy. This consists either in following a long-short strategy in the stock market, or in investing in stock options, or of course in a mixture of the two. The logic of this strategy is to take advantage of the downturn of the stocks that are most exposed to pandemics, and, if possible, of the stimulus given to specific sectors, in order to generate substantial gains in the case of a pandemic, while hedging the risks associated with the bearish or bullish nature of the stock market in non-pandemic states. Simulations using data from the French stock market, both retrospectively or prospectively, have illustrated the potential and robustness of this approach for putting a corporate pandemic insurance in place.

We did not get beyond the basic principles of such a self-funded insurance scheme, and a deeper exploration of this approach would require further studies in various directions. One of them is about the drivers of the insured firm's risk aversion and the design of the insurance mechanism. We have limited ourselves to the case where the insurance coverage level corresponds to the post-pandemic value of the financial assets held by the insurer on behalf of the insured firm. If the firm's risk aversion results from an investment crowding-out mechanism under increasing marginal cost of external capital, as in the Froot et al. (1993) approach, or if it reflects bankruptcy costs, then a committed capital facility, under the form of contingent debt or equity, could meet the firm's needs more effectively, than merely covering the cash-flow losses by selling its financial assets. This would consist in adding another stage in the insurance mechanism, where the value of the assets would be traded against such an option on paid-in capital. It would be worth exploring such mechanisms where a committed capital facility is bundled with asset management. Another issue is related to complementary guarantees that the insurer could provide to the insured firm, in order to reduce the uncertainty on the post-pandemic value of assets. This may go through the securitization of the residual risk, that corresponds to the difference between a guaranteed rate of return and the post-pandemic return of assets. Pandemic catbonds, issued by insurers, may be the right instrument to make such a guarantee feasible, by transferring the residual risk to dedicated investors. This suggests that the self-funded insurance mechanism that we have examined and the more traditional risk transfer through catbonds or other instruments, may have complementary roles in the coverage of the pandemic risk.

5 Appendix

5.1 Proof of Lemma 1

A portfolio $\{x_{0u,}x_{1u}, x_{2u}\}$ pays one unit of numeraire in state u and zero otherwise if

$$x_{0u}(1 + r_f) + x_{1u}(1 + R_1 + h) + x_{2u}(1 + R_2 + h) = 1,$$
$$x_{0u}(1 + r_f) + x_{1u}(1 + R_1 - h') + x_{2u}(1 + R_2 - h') = 0,$$
$$x_{0u}(1 + r_f) + x_{1u}(1 + R_1 + H) + x_{2u}(1 + R_2 - H') = 0.$$

Solving this system of three equations with three unknowns yields

$$x_{0u} = \frac{(h' - 1)(H + H') + (h' - H')R_1 - (H + h')R_2}{(1 + r_f)(h + h')(H + H')}, \tag{28}$$

$$x_{1u} = \frac{H' - h'}{(h + h')(H + H')}, \tag{29}$$

$$x_{2u} = \frac{H + h'}{(h + h')(H + H')}, \tag{30}$$

which gives

$$\begin{aligned} q_u &= x_{0u} + x_{1u} + x_{2u} \\ &= \frac{(H + H')(r_f + h') - (H' - h')R_1 - (H + h')R_2}{(1 + r_f)(h + h')(H + H')}. \end{aligned} \tag{31}$$

When there are risk-neutral investors, using $\alpha_u = h'/(h + h')$ yields

$$q_u = \frac{\alpha_u(1 - \pi)}{1 + r_f}. \tag{32}$$

Straightforward calculations with similar notations for states d and p yield

$$x_{0d} = \frac{(1 + h)(H + H') + (H' + h)R_1 + (H - h)R_2}{(1 + r_f)(h + h')(H + H')}, \tag{33}$$

$$x_{1d} = \frac{-(H' + h)}{(h + h')(H + H')}, \tag{34}$$

$$x_{2d} = \frac{h - H}{(h + h')(H + H')}, \tag{35}$$

and

$$x_{0p} = \frac{R_2 - R_1}{(1 + r_f)(H + H')}, \tag{36}$$

$$x_{1p} = -x_{2p} = \frac{1}{H + H'}, \tag{37}$$

which gives

$$\begin{aligned} q_d &= x_{0d} + x_{1d} + x_{2d} \\ &= \frac{(H+H')(h-r_f) + (H'+h)R_1 + (H-h)R_2}{(1+r_f)(h+h')(H+H')}, \qquad (38) \\ q_p &= x_{0p} + x_{1p} + x_{2p} \\ &= \frac{R_2 - R_1}{(1+r_f)(H+H')}. \qquad (39) \end{aligned}$$

In particular, when there are risk-neutral investors, we have

$$q_d = \frac{\alpha_d(1-\pi)}{1+r_f}, \qquad (40)$$

$$q_p = \frac{\pi}{1+r_f}, \qquad (41)$$

when there are risk-neutral investors.

If (3) holds, then (32),(40) and (41) give (5). If (4) holds, then (31) and (38) give (6). Furthermore, using (7),(38), (39) and $\alpha_d = h/(h+h')$ yield (8).

5.2 Proof of Proposition 1

The first-order optimality conditions for the maximization of

$$\begin{aligned} \mathbb{E}u \equiv (1-\pi)\alpha_u u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_u) \\ + (1-\pi)\alpha_d u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_d) \\ + \pi u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_p - L), \end{aligned}$$

with respect to y_u, y_d and y_p are written as

$$(1-\pi)\alpha_u u'(w_u) = q_u \mathbb{E}u', \qquad (42)$$

$$(1-\pi)\alpha_d u'(w_d) = q_d \mathbb{E}u', \qquad (43)$$

$$\pi u'(w_p) = q_p \mathbb{E}u', \qquad (44)$$

where we denote

$$\mathbb{E}u' = (1-\pi)[\alpha_u u'(w_u) + \alpha_d u'(w_d)] + \pi u'(w_p).$$

Assume first that there are risk-neutral investors, i.e. (3) holds. In that case, (5) and (42)-(44) give (11), and (9),(10) show that there exists y such that

$$y = y_u = y_d = y_p - L. \qquad (45)$$

We have

$$x_i = (x_{iu} + x_{id})y + x_{ip}(y+L) \text{ for } i = 0,1,2,$$

which gives (12),(13) and

$$x_0 = \frac{y(H+H') + (R_2 - R_1)L}{(1+r_f)(H+H')} = \frac{y + \pi L}{1+r_f}. \qquad (46)$$

We have $K = A - I = A - x_0 - x_1 - x_2$. Maximizing the (state-independent) final cashflow of the firm

$$f(A - x_0 - x_1 - x_2) + y = f\left(A - \frac{y + \pi L}{1 + r_f}\right) + y,$$

w.r.t. y gives (18).

Assume now that investors are risk-averse, i.e., (6) and (8) hold. In that case, (42)-(44) give (14) and

$$y_p - L < y_d < y_u. \tag{47}$$

Thus, we may write

$$\begin{aligned} y_d &= y_u - \Delta_d, \\ y_p &= y_u + L - \Delta_p, \end{aligned}$$

with $0 < \Delta_d < \Delta_p$. Using $y_u - y_d = \Delta_d > 0$ gives (16), and thus (15). Using $y_p - L < y_d$ and (16) yields

$$x_1 H - x_2 H' - L < -h'(x_1 + x_2) < 0,$$

and thus

$$L > x_1 H - x_2 H' > -x_2(H + H'),$$

which gives (17). Furthermore, maximizing the firm's expected utility

$$\begin{aligned} \mathbb{E}u \equiv (1-\pi)\alpha_u u(f(A - \frac{y_u}{1+r_f} + q_d\Delta_d + q_p(\Delta_p - L)) + y_u) \\ + (1-\pi)\alpha_d u(f(A - \frac{y_u}{1+r_f} + q_d\Delta_d + q_p(\Delta_p - L)) + y_u - \Delta_d) \\ + \pi u(f(A - \frac{y_u}{1+r_f} + q_d\Delta_d + q_p(\Delta_p - L)) + y_u + L - \Delta_p), \end{aligned}$$

w.r.t. y_u, for Δ_d and Δ_p given, yields (18), with

$$K = A - \frac{y_u}{1+r_f} + q_d\Delta_d + q_p(\Delta_p - L) = K^*.$$

5.3 Proof of Proposition 2

The first-order optimality conditions for the maximization of

$$\begin{aligned} \mathbb{E}u \equiv (1-\pi)\alpha_u u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_u) \\ + (1-\pi)\alpha_d u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_d) \\ + \pi\mathbb{E}[u(f(A - q_u y_u - q_d y_d - q_p y_p) + y_p - \widetilde{L}), \end{aligned}$$

with respect to y_u, y_d and y_p are written as

$$
\begin{aligned}
(1-\pi)\alpha_u u'(w_u) &= q_u \mathbb{E}u', & (48)\\
(1-\pi)\alpha_d u'(w_d) &= q_d \mathbb{E}u', & (49)\\
\pi \mathbb{E}u'(\widetilde{w}_p) &= q_p \mathbb{E}u', & (50)
\end{aligned}
$$

where we denote

$$\mathbb{E}u' = (1-\pi)[\alpha_u u'(w_u) + \alpha_d u'(w_d)] + \pi \mathbb{E}u'(\widetilde{w}_p).$$

Using $u''' > 0$ gives

$$\mathbb{E}u'(\widetilde{w}_p) > u'(\overline{w}_p),$$

and thus, using $u'' < 0$, we deduce that there exists $\gamma > 0$ such that

$$\mathbb{E}u'(\widetilde{w}_p) = u'(\overline{w}_p - \gamma).$$

Comparing with the optimality conditions of Proposition 1 shows that the optimal portfolio choices correspond to the case of deterministic losses $L = \overline{L} + \gamma$. In particular, in the case of risk-neutral investors, the long-short positions allow the firm to cover losses L larger than expected losses $\overline{L}$.

5.4 Proof of Proposition 3

We still denote y_u and y_d the firm's portfolio payoff in states u and d, respectively. Let $\overline{y}_p$ denote the expected portfolio payoff in state p. The state-contingent expected payoffs y_u, y_d and $\overline{y}_p$ can be obtained through portfolio choices

$$x_i = x_{iu} y_u + x_{id} y_d + x_{ip} \overline{y}_p, \tag{51}$$

with $i \in \{0, 1, 2\}$, where x_{is} is defined as in the proof of Proposition 1 for all i, s. Choosing x_0, x_1 and x_2 is equivalent to choosing y_u, y_d and $\overline{y}_p$, with financial investment cost

$$I = q_u y_u + q_d y_d + q_p \overline{y}_p, \tag{52}$$

where q_u, q_d and q_p correspond to the Arrow-Debreu security prices of the complete-market model (i.e. when $\varepsilon_1 \equiv \varepsilon_2 \equiv 0$), and are given by (31), (38) and (39) . The firm chooses y_u, y_d and $\overline{y}_p$ in order to maximize its expected utility

$$\mathbb{E}u = \sum\nolimits_{s=u,p} \pi_s u(w_s) + \pi_p \mathbb{E}u(\widetilde{w}_p),$$

where

$$
\begin{aligned}
w_s &= f(A-I) + y_s \text{ for } s = u \text{ and } d,\\
\widetilde{w}_p &= f(A-I) + x_1\varepsilon_1 + x_2\varepsilon_2 + \overline{y}_p - L,
\end{aligned}
$$

with (x_1, x_2) and I given by (51) and (52), respectively. First-order optimality conditions are written as

$$
\begin{aligned}
\frac{\partial \mathbb{E}u}{\partial y_s} &= \pi_s u'(w_s) - q_s f' \mathbb{E}u' + \pi_p \mathbb{E}\{(x_{1s}\varepsilon_1 + x_{2s}\varepsilon_2)u'(\widetilde{w}_p)\} = 0 \\
&\quad \text{for } s = u, d, \qquad (53) \\
\frac{\partial \mathbb{E}u}{\partial \overline{y}_p} &= \pi_p \mathbb{E}u'(\widetilde{w}_p) - q_p f' \mathbb{E}u' + \pi_p \mathbb{E}\{(x_{1p}\varepsilon_1 + x_{2p}\varepsilon_2)u'(\widetilde{w}_p)\} = 0, \qquad (54)
\end{aligned}
$$

where

$$\mathbb{E}u' = \sum\nolimits_{s=u,d} \pi_s u'(w_s) + \pi_p \mathbb{E}u'(\widetilde{w}_p).$$

Summing (53) and (54), and using

$$x_{1u} + x_{1d} + x_{1p} = x_{2u} + x_{2d} + x_{2p} = 0,$$

and

$$q_u + q_d + q_p = \frac{1}{1 + r_f},$$

yields

$$f'(A - I) = 1 + r_f, \tag{55}$$

i.e.,

$$K = A - I = K^*.$$

When investors are risk-neutral, equations (53) and (54) give

$$
\begin{aligned}
u'(w_u) &= \mathbb{E}u' - \frac{\pi}{\alpha_u(1-\pi)}\mathbb{E}\left\{\left[\frac{(H'-h')\varepsilon_1 + (H+h')\varepsilon_2}{(H+H')(h+h')}\right]u'(\widetilde{w}_p)\right\}, \qquad (56) \\
u'(w_d) &= \mathbb{E}u' - \frac{\pi}{\alpha_d(1-\pi)}\mathbb{E}\left\{\left[\frac{-(H'+h)\varepsilon_1 + (h-H)\varepsilon_2}{(H+H')(h+h')}\right]u'(\widetilde{w}_p)\right\}, \qquad (57) \\
\mathbb{E}u'(\widetilde{w}_p) &= \mathbb{E}u' - \mathbb{E}\left\{\left[\frac{\varepsilon_1 - \varepsilon_2}{H+H'}\right]u'(\widetilde{w}_p)\right\}. \qquad (58)
\end{aligned}
$$

When the supports of ε_1 and ε_2 are not too large, we have $x_1 > 0$ and $x_2 < 0$ at an optimal solution, because $x_1 = -x_2 = L/(H + H') > 0$ when $\varepsilon_1 \equiv \varepsilon_2 \equiv 0$. Hence $\widetilde{w}_p$ is increasing w.r.t. ε_1 and decreasing w.r.t. ε_2. Using $u'' < 0$ then gives

$$
\begin{aligned}
\mathbb{E}[\varepsilon_1 u'(\widetilde{w}_p)] &= cov[\varepsilon_1, u'(\widetilde{w}_p)] < 0, \qquad (59) \\
\mathbb{E}[\varepsilon_2 u'(\widetilde{w}_p)] &= cov[\varepsilon_2, u'(\widetilde{w}_p)] > 0. \qquad (60)
\end{aligned}
$$

Assume $\alpha_u = \alpha_d = 1/2, h' = h$ and $H = 0$. In that case, (56) and (57) are written as

$$
\begin{aligned}
u'(w_u) &= \mathbb{E}u' - \frac{\pi}{1-\pi}\mathbb{E}\left\{\left[\frac{(H'-h)\varepsilon_1 + h\varepsilon_2}{hH'}\right]u'(\widetilde{w}_p)\right\}, \qquad (61) \\
u'(w_d) &= \mathbb{E}u' - \frac{\pi}{1-\pi}\mathbb{E}\left\{\left[\frac{-(H'+h)\varepsilon_1 + h\varepsilon_2}{hH'}\right]u'(\widetilde{w}_p)\right\}. \qquad (62)
\end{aligned}
$$

Using (59),(61) and (62) gives

$$u'(w_u) - u'(w_d) = -\frac{2\pi}{h(1-\pi)}\mathbb{E}[\varepsilon_1 u'(\widetilde{w}_p)] > 0,$$

which implies $w_u < w_d$, and thus $x_1 + x_2 < 0$.

Let us further assume that $u(.)$ is quadratic, and written as $u(w) = -aw^2/2 + bw$, with $a, b > 0$ and $w < b/a$ for all relevant values. Since ε_1 and ε_2 are independently distributed, (61) and (58) give

$$\begin{aligned} w_u &= \overline{w} + \frac{\pi}{hH'(1-\pi)}[(H'-h)x_1\sigma_1^2 + hx_2\sigma_2^2], \\ \overline{w}_p &= \overline{w} + \frac{1}{H'}[x_1\sigma_1^2 - x_2\sigma_2^2], \end{aligned}$$

where $\overline{w}_p = \mathbb{E}\widetilde{w}_p$ and $\overline{w} = (1-\pi)(w_u + w_p)/2 + \pi\overline{w}_p$. This yields

$$\overline{w}_p - w_u = \frac{x_1\sigma_1^2(h - \pi H')\sigma_1^2 - x_2 h\sigma_2^2}{hH'(1-\pi)}.$$

Hence, $\pi H' < h$ is a sufficient condition for $\overline{w}_p > w_u$.

5.5 One-period model with an arbitrary number of pandemic states

This appendix presents an extended version of the model presented in Section 2. It is based on similar assumptions about risk aversion and technology, but with S states and $n+1$ securities. There are m pandemic states and $S-m$ non-pandemic states, with $m < S$, and we assume $n+1 \geq S$. States and assets correspond to index s and i, respectively, with $s = 1, ..., S-m$ the non-pandemic states and $s = S-m+1, ..., S$ the pandemic states. Index $i = 0$ corresponds to the risk-free assets and $i = 1, ..., n$ are the other assets. The return of asset i in state s is denoted r_{si}, with $r_{s0} = r_f$ for all s, and $f(K)$ still denotes the cashflow from productive assets. The model of Section 2 corresponds to the case with one pandemic state, two non-pandemic states and three assets, i.e., $m = 1, S = 3$ and $n = 2$. . We denote L_s the firm's loss in state s, with $L_s = 0$ if $s = 1, ..., S-m$, and $L_s > 0$ if $s = S-m+1, ..., S$. Hence, the firm's cashflow in state s is

$$w_s = f\left(A - \sum\nolimits_{i=0}^{n} x_i\right) + \sum\nolimits_{i=0}^{n}(1 + r_{si})x_i - L_s.$$

Let π_s be the probability of state s, with $\sum_{s=1}^{S}\pi_s = 1$.The firm maximizes its expected utility

$$\mathbb{E}u = \sum\nolimits_{s=1}^{S}\pi_s u(w_s),$$

w.r.t. $x_0, ..., x_n$. Let $e = (1, ..., 1)$ be the $(n+1)-$dimension row vector with all components being 1, and $r_s = (r_{s0}, ..., r_{sn})$. Let

$$E = \begin{pmatrix} e + r_1 \\ ... \\ e + r_s \\ ... \\ e + r_S \end{pmatrix},$$

be the $S \times (n+1)$ matrix, with $1+r_{si}$ the term of row s and column i. We assume that E has full rank, and thus for all $y = (y_1, ..., y_S)'$ there exists $x = (x_{0,} ..., x_n)'$ such that $Ex = y$. Let

$$C(y) = \min\{e \cdot x \mid Ex = y\},$$

be the lowest financial investment $\sum_{i=0}^{n} x_i$ that provides payoff vector $y = (y_1, ..., y_S)'$. Function $C(y)$ is linear, and may be written as

$$C(y) = \sum_{s=1}^{S} q_s y_s,$$

where q_s is the price of the type$-s$ Arrow-Debreu security, with $q_s = \pi_s/(1+r_f)$ for all s when the representative investor is risk-neutral. The firm's expected utility is rewritten as

$$\mathbb{E}u = \sum_{s=1}^{S} \pi_s u(f(A - C(y)) + y_s - L_s).$$

It is maximized w.r.t. $y = (y_1, ..., y_S)'$, and an optimal portfolio of assets $x' = (x_0, ..., x_n)$ is such that $e \cdot x = C(y)$.

The first-order optimality conditions for this problem are written as

$$\frac{\partial \mathbb{E}u}{\partial y_s} = \pi_s u'(f(A - C(y)) + y_s - L_s) - q_s \mathbb{E}u' = 0,$$

for all $s = 1, ..., S$, where

$$\mathbb{E}u' = \sum_{s=1}^{S} \pi_s u'(f(A - C(y)) + y_s - L_s).$$

When the representative investor is risk-neutral, we have

$$\frac{q_s}{\pi_s} = \frac{1}{1 + r_f} \text{ for all } s = 1, ..., S,$$

which gives

$$u'(f(A - C(y)) + y_s - L_s) = \frac{\mathbb{E}u'}{1 + r_f} \text{ for all } s = 1, ..., S,$$

and using $u'' < 0$ yields

$$y_1 - L_1 = y_2 - L_2 = ... = y_S - L_S.$$

In other words, the financial choices of the firm provides full coverage of the pandemic loss and perfect hedging of financial risks.

When the representative investor is risk-averse, we have

$$y_s - L_s < y_{s'} - L_{s'} \Leftrightarrow \frac{q_s}{q_{s'}} > \frac{\pi_s}{\pi_s} \text{ if } s \neq s',$$

and final cashflows are affected by residual uncertainty. In other words, there is partial coverage of pandemic losses and imperfect hedging of financial risks. In particular, comparing states $s \in \{1, ..., s-m\}$ and $s' \in \{s-m+1, ..., S\}$) gives

$$y'_s - y_s < L_{s'} \Leftrightarrow \frac{q_{s'}}{q_s} > \frac{\pi_{s'}}{\pi_s}.$$

Thus, when the firm moves from the non-pandemic state s to the pandemic state s', the additional payoff of its portfolio is lower than the loss caused by the type$-s'$ pandemic, when the cost of transferring wealth from s to s' is larger than the corresponding ratio of state probabilities.

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