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Fiscal Sustainability and Low Interest Rates: A Note

Abstract

In this paper, I demonstrate that an indicator which is commonly used to assess the long-term fiscal sustainability of public finances in EU member states (“*S*2”) is also defined if government borrowing rates are assumed to be permanently lower than the growth rate of GDP. I illustrate this finding based on simulations prepared for the Fifth Sustainability Report published by the German Federal Ministry of Finance. In addition, I discuss the interpretation of the indicator in a low-interest environment and the assumption that relevant interest rates continue to be low if there are substantial challenges for fiscal sustainability, *e.g.*, through demographic ageing.

JEL-Codes: H600, J110, E430.

Keywords: public budget, public debt, fiscal sustainability, interest rates.

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1 Introduction

A standard indicator for long-term fiscal sustainability that the European Commission (most recently, see European Commission, 2019) and some of the member states are using to monitor fiscal policies in the EU under the Stability and Growth Pact cannot be directly applied if interest rates are very low or, more specifically, if the government borrowing rate is assumed to be below the growth rate of GDP not just temporarily, but permanently. In the present note, I elaborate on this observation and try to fill this gap.

The paper starts by introducing the relevant indicator ("$S2$") and explaining why it cannot be derived in the usual fashion if the current low-interest situation lasts too long (section 2). Section 3 illustrates the problem using simulations prepared for the Fifth Sustainability Report published by the German Federal Ministry of Finance (BMF, 2020) and discusses whether this non-result is really harmful. In section 4, I suggest an alternative approach to deriving the $S2$-indicator in a continued low-interest environment and demonstrate how it can be applied to the German example. Section 5 concludes, discussing the implications of the findings for analyses of fiscal sustainability and for long-term fiscal planning.

2 The sustainability indicator $S2$

In the regular reports monitoring fiscal sustainability in all member states at an EU-level, several indicators have been used to address short-term fiscal risks and fiscal sustainability in a medium-term perspective. At the same time, long-term fiscal sustainability has been consistently measured using an indicator called "$S2$" since the first EU-level report on sustainability of public finances was published in 2006 (European Commission, 2006). It was first adopted in preliminary work carried out in the years 1999 to 2003 (EU Economic Policy Committee, 2001, 2003) and can be traced back to suggestions developed by Blanchard (1990).

The $S2$-indicator is based on the intertemporal government budget constraint which requires that – over a virtually infinite time horizon – all future public revenues must be sufficiently high to cover all future public expenditure, plus public debt which has been accumulated up to the present. In order to state this requirement formally, I use the following notation. Let D_t denote public debt and P_t the primary surplus (revenues minus expenditure, disregarding interest payments) in a given year $t \in \{1, 2, 3...\}$. Since nominal (and even real) amounts for both these variables are difficult to compare over time, I will mainly look at the corresponding GDP-ratios, $d_t = D_t/Y_t$ and $p_t = P_t/Y_t$. Annual GDP, Y_t, is assumed to grow at a (nominal) rate of g_t against the preceding year. Interest payments derive from the (nominal) interest rate r_t applying to government bonds. To simplify the notation, I define the discount factor $q_t = (1+g_t)/(1+r_t)$ which converts period-t aggregates into period-t–1 present values and assume that q_t is constant from period 1 onwards, so that $q_t = q$.[1]

Given that, the intertemporal budget constraint (IBC) the government is faced with can be written as

$$d_0 - \sum_{t=1}^{\infty} q^t p_t = 0, \tag{1}$$

where period 0 is a baseline year and $t \geq 1$ are the years for which fiscal policy is to be monitored.

With real-world figures for d_0 and simulations regarding how p_t may develop under the current legal framework for revenues and expenditure, equation (1) often does not hold. Therefore, an improvement of current and future primary balances may be required to meet the IBC. Assuming that this improvement is constant in terms of its GDP-ratio and invariably applies from $t = 1$ onwards leads to

$$d_0 - \sum_{t=1}^{\infty} q^t (p_t + \sigma) = 0, \tag{2}$$

as a new version of the IBC. Here, σ is called the "sustainability gap". It measures consolidation needs involved in the combination of d_0 and simulated time-series for p_t in a rather stylized way, *viz.* as a single figure reflecting a constant, permanent correction that would shift the entire time path of the GDP-ratios of annual primary balances by some fraction of GDP.

For practical applications, a difficulty arises from the fact that simulations regarding the future development of p_t necessarily span only a finite time horizon, until some year T. Lacking better information, it is therefore assumed that p_t (and, if they are assumed to vary for $t < T$, also g_t and r_t, hence q_t) remain constant from T onwards. Given that, equation (2) can be re-written as follows.

$$d_0 - \sum_{t=1}^{T} q^t p_t - \sum_{t=T+1}^{\infty} q^t p_T - \sum_{t=1}^{\infty} q^t \sigma = 0 \tag{3}$$

Provided that $q < 1$ (because $r > g$), the sums with infinite numbers of elements included in (3) can be simplified, based on a general rule for geometric series by which $\sum_{t=1}^{\infty} q^t = q/(1-q)$ if $q < 1$. In this case, the single terms included in each series converge towards zero as t approaches infinity, and the series of infinite length

[1] In the illustrative simulations considered in sections 3 and 4, both g_t and r_t may vary over the entire simulation period.

assume finite values. Applying this to equation (3), the IBC can be re-written once more, *viz.* as

$$d_0 - \sum_{t=1}^{T} q^t p_t - \frac{q^{T+1}}{1-q} p_T - \frac{q}{1-q}\sigma = 0, \tag{4}$$

which can easily be solved for the sustainability gap σ:

$$\sigma = \frac{1-q}{q}\left(d_0 - \sum_{t=1}^{T} q^t p_t\right) - q^T p_T \tag{5}$$

This is how the indicator $S2$ can be derived from the IBC under the standard assumption that $q < 1$ (at least in the long run, *i.e.*, for q_t with $t \geq T$).[2] Thus far, this assumption has been considered to hold true in all scenarios that were looked at in sustainability analyses conducted at an EU-level and also in the vast majority of scenarios covered in the German sustainability reports.

However, in a number of countries including Germany, current government borrowing rates ("r_0") are below the growth rate of GDP ("g_0"). If this situation is assumed to continue (until year T and even beyond), the discount factor q which is relevant for the above calculations exceeds unity and, hence, equation (3) can no longer be transformed to (4) and solved for σ.

The standard indicator for long-term fiscal sustainability, $S2$, is then no longer defined – at least, at first sight –, and a simple, but telling measure of consolidation needs that may arise to keep annual budget balances and the debt ratio d_t on a sustainable time path is lacking. On the other hand, interest rates that are low and even fall short of GDP-growth could in themselves contribute to improving on the sustainability of public finances – again, at least at first sight. In sections 3 and 4, I will show that both of these assertions are premature.

3 Illustrative results for Germany

The simulations prepared for the latest sustainability report of the German Federal Ministry of Finance include a number of scenarios which are explicitly meant to test the sensitivity of the results with respect to differing assumptions on future trends in the interest rate applying to German government bonds (Werding et al., 2020, pp. 123–133). Here, I will use these scenarios to illustrate the implications for long-term developments of the debt ratio and for the sustainability indicator $S2$ (section 3.1) and then discuss the observations (section 3.2).

[2] The formal derivation of $S2$ provided in European Commission (2019, annex A2.4) looks much more complex but is mathematically equivalent if one accepts a few simplifications made here.

3.1 Debt projections and the $S2$-indicator

Most of the work devoted to preparing simulations for the German sustainability reports is spent on projecting future time paths for public expenditure that is expected to be influenced by the on-going process of demographic ageing.[3] Age-related expenditure in Germany[4] amounts to almost 26% of GDP or close to 60% of total primary expenditure today. Under the current legal framework, these shares must be expected to go up in the future, since Germany is faced with an ageing process that is rather pronounced by international standards and will enter an acute phase rather soon.

While the EU Fiscal Sustainability Report rests on simulations for one "baseline" scenario (combined with numerous alternative scenarios), the German sustainability reports regularly provide two diverging baseline scenarios (again combined with numerous alternative scenarios) which both rest on current policies, but on differing assumptions affecting the age composition of the population, labour-force participation, employment and productivity growth. The underlying assumptions are either basically optimistic (scenario "$T+$") or basically pessimistic (scenario "$T-$"), but in no case extreme.[5] Taken together, the two scenarios are meant to indicate a range of possible future developments which, as of today, can be considered plausible. Therefore, important intermediate results of the simulations relate to changes in the GDP-ratio of total age-related expenditure expected for both these scenarios. Projected increases amount to +2.7 percentage points (pp) until 2040 and +3.6 pp until 2060 in the optimistic scenario $T+$. Over the same time periods, projected increases are +4.8 and +7.2 pp, respectively, in the pessimistic scenario $T-$.

Based on these results, analyses regarding the long-term sustainability of German public finances follow the same logic as in parallel work done at the EU-level. "Other" (*i.e.*, non-age-related) public expenditure is assumed to stay constant as a percentage of GDP over the entire simulation period and the same applies to public revenues. The first of these assumptions is a simplification, while the second one is a convention, or an identifying assumption, that is meant to indicate the full dimension of any problems with fiscal sustainability which may be involved in

[3] This work parallels the preparation of the EU Ageing Reports (most recently, see European Commission and EU Economic Policy Committee, 2018) which regularly precede the EU Fiscal Sustainability Reports.

[4] This includes public expenditure on old-age provision (Statutory Pension Scheme, civil servants' pensions), health and long-term care (Statutory Health Insurance, Social Long-term Care Insurance, related benefits for civil servants and their family members), education (pre-primary through tertiary) and core measures of family policies, plus expenditure on unemployment and measures of active labour-market policies – due to the important role of employment for financing all other items on a pay-as-you-go basis.

[5] Assumptions and many intermediate results are documented in Werding et al. (2020, ch. 2). An English summary covering some of these materials can be provided upon request.

current policies before entering discussions on how to bring about consolidations (either through reductions in expenditure or through higher revenues) that may be required. Under these two assumptions, projected changes in age-related expenditure turn into changes in GDP-ratios of total primary expenditure as well as the primary balance on a one-for-one basis. In other words, in the absence of any fiscal reactions, the primary balance must be expected to deteriorate by 3.6 to 7.2 pp of GDP until 2060 in the two baseline scenarios considered here.

Consequences for the total fiscal balance (including interest payments) depend on government borrowing rates and on the projected time path of government debt. Especially in cases with high interest rates, an unfavourable interaction can arise between annual budget deficits and accumulated public debt, by which both figures start to increase at accelerating speed from some point in time onwards. If the underlying increase in the primary deficit is rather strong, the same can happen even if relevant interest rates are low. This is demonstrated in figure 1 which exhibits simulated developments of the German debt-to-GDP ratio for the two baseline scenarios under differing assumptions for future trends in government borrowing rates.

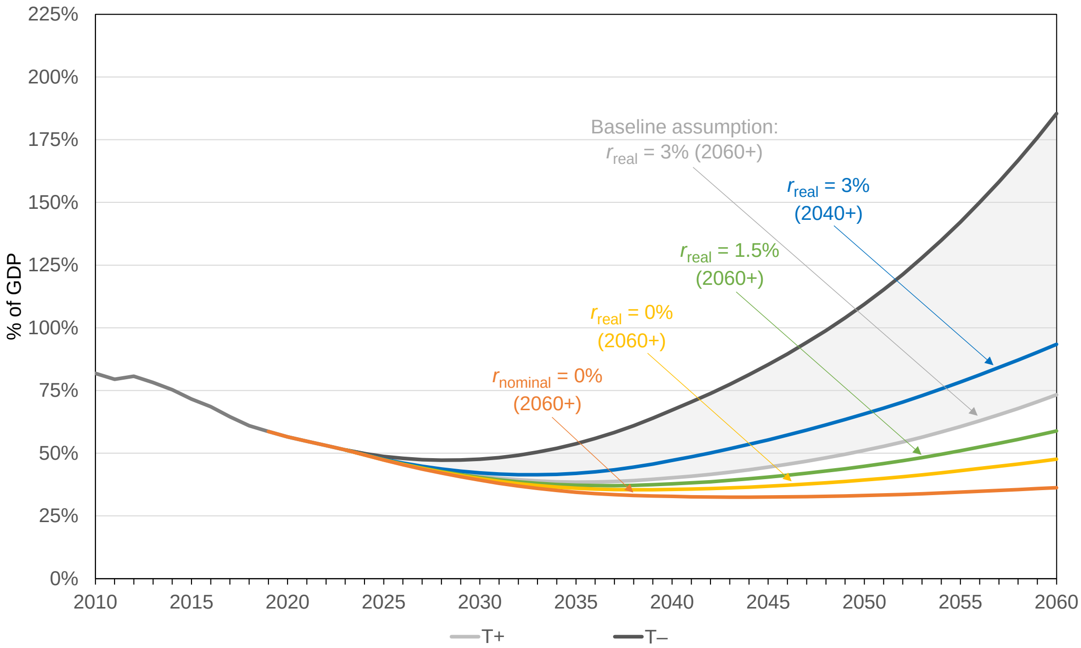
For these simulations, GDP-growth is derived from an aggregate production function, with diverging time paths resulting for the two baseline scenarios. The baseline assumption regarding the interest rate for German government bonds which is uniformly applied to $T+$ and $T-$ is as follows. Starting from the current situation with a very low interest rate ($r < g$), a normalization is expected to take place over time, even though this may take quite a while. Here, "normalization" is taken to mean that the real interest rate returns to its long-term average figure from the period before the Great Recession. For simplicity, this idea is implemented through a linear transition from the most recent figure (2018: real interest rate -0.4%, nominal interest rate 1.5%; average values for outstanding government bonds of all maturities) to the target level (3% on real terms, about 5% on nominal terms, assuming a constant inflation rate of 2% from 2025 onwards) that lasts until 2060, *i.e.*, the end of the simulation period.[6]

The sensitivity tests included in figure 1 leave all other assumptions for scenarios $T+$ and $T-$ unchanged, but are based on alternative assumptions for the interest rate. Besides a scenario with a faster recovery (lasting only until 2040, with a constant real interest rate of 3% starting from then), three scenarios with lower interest rates are considered (with linear increases of real interest rates until 2060

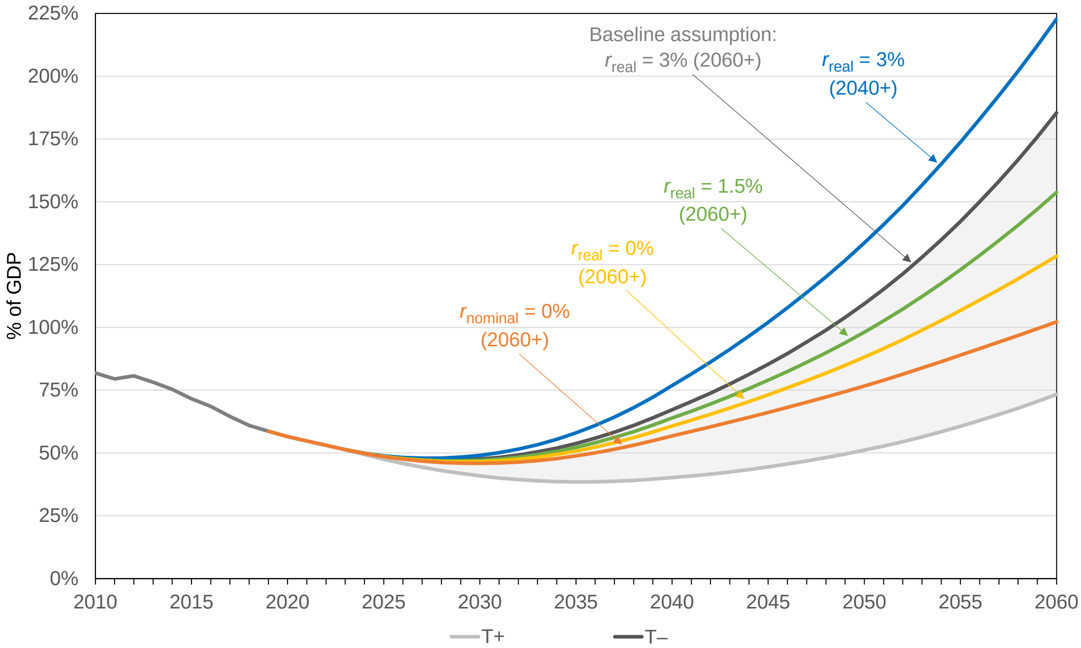
[6]Instead of a linear transition, a further decline with subsequent reversal could be more realistic, even in the case of a long-term normalization, since the current market rate for fresh debt is well below the average figure indicated above. Also, one might question the return to pre-2008 figures and choose a lower target level. For the considerations made here, all this is of secondary importance.

Figure 1: Projected debt ratios

a) Scenario T+ under different interest-rate assumptions



b) Scenario T− under different interest-rate assumptions



Sources: Deutsche Bundesbank, SIM.17 (Werding et al. 2020).

to 1.5% or 0%, respectively, corresponding to nominal interest rates of about 3.5% or 2%, or a linear decline to a nominal interest rate of 0%, implying a real interest rate of -2%). Here, it is important to note that in the latter two cases, interest rates remain below the growth rate of GDP even until 2060, so that the standard assumption of $r > g$ is violated.[7]

Resulting trends in the debt ratio (see figure 1) are clearly diverse. In any of the cases considered, debt ratios start to increase again at some point in time and continue to do so until 2060 – which is a first indication of a lack in long-term sustainability. However, the levels of debt are rather low until 2060 for the scenarios combining all other assumptions for $T+$ with low interest rates, while the debt ratio eventually increases to no less than 102% or even 128%, if assumptions for $T-$ are combined with interest rates permanently ranging at $r < g$.[8]

The sustainability indicator $S2$, as introduced in section 2, is meant to measure consolidations (as a constant correction of the entire time series of primary balances simulated for the scenarios $T+$ and $T-$ starting from 2020 until 2060) which are required to prevent the debt ratio from increasing beyond any limit in the long run, that is, also after 2060. In addition, one should keep in mind that, when calculating $S2$, primary balances, government borrowing rates and GDP-growth rates are assumed to stay constant at their year-2060 levels until infinity. Under these assumptions, the debt ratio must therefore also reach a corrected level in 2060 and stay constant afterwards to be sustainable in a long-term perspective.

For the optimistic scenario $T+$ and baseline assumptions regarding the interest rate, $S2$ turns out to be 1.5 (pp of GDP), remaining basically unchanged in the two sensitivity tests where $r > g$.[9] For the pessimistic scenario $T-$, $S2$ becomes 4.1 under baseline interest-rate assumptions and ranges between 4.0 (if real interest rates increase to 3% until 2040) and 4.6 (if real interest rates increase to 1.5% until 2060) for the sensitivity tests with $r > g$. For all the scenarios in which $r < g$, the indicator $S2$ was said to be “not defined” in the study preparing simulations for the latest German sustainability report (Werding et al., 2020, pp. 130f.) – for the reasons explained in section 2.

[7] According to the macroeconomic background scenarios for the simulations relating to $T+$ and $T-$, real GDP-growth in 2060 is 1.1% and 0.4%, respectively, or 3.1% and 2.4% on nominal terms.

[8] Note that the simulations presented here were essentially prepared in the second half of 2019. Therefore, they do not include any effects of the current covid-19 pandemic on GDP and public budgets. It is now expected that the German debt ratio will jump up to about 72% in 2020 (German Council of Economic Advisors, 2020, pp. 60f.). Even in the case of a quick recovery of the German economy, but in the absence of specific consolidation measures, this would shift all curves shown in figure 1 upward and speed up their increase towards 2060.

[9] Rounded figures are 1.5 again, with some variation at the second digit of more precise results. For a full overview over these and further results see table 1 in section 4.2 below.

3.2 Is there a problem?

To assess whether this non-result is really harmful, one should keep in mind that the indicator $S2$ rests on calculations that are highly stylized.[10] Specifically, the interest rate r (or the time path of r_t) is taken to be exogenously given and is not influenced by any endogenous mechanisms in a general-equilibrium model, by stochastic shocks, by the behaviour of political actors or by expectations of potential creditors of government debt – among other things, regarding the long-term sustainability of the accumulated debt level. Therefore, considering a host of possible complications that are ignored in the stylized calculations, scenarios combining strongly increasing debt ratios with permanently low interest rates may simply appear to be unrealistic.

In this sense, Andersen (2020) argues in a recent contribution that analyses of fiscal sustainability which are based on currently observed low interest rates would be misleading. His main point is that, in a situation with systematic budget deficits and, hence, increasing debt, creditors would start to ask for a credit risk premium if debt levels pass critical thresholds, so that government borrowing rates cannot remain constant. At least, uncertainty regarding such changes should be taken into account when assessing potential sustainability problems. As a consequence, sustainability analyses should always be based on interest rates that exceed the current growth rate (starting from some point in time during the simulation period and certainly in the final year), as otherwise the problems that are to be addressed are defined away. In other words, following Andersen (2020) results for $S2$ under interest rates with $r < g$ are simply not needed.

Nevertheless, the non-existence of indicator values for a situation with permanently low interest rates creates difficulties in current debates about fiscal policy. After all, a number of prominent experts have stated recently that low government borrowing rates are a reason to re-consider ancient wisdom regarding the fiscal costs of debt and the strictness of fiscal rules (see, *e.g.*, Blanchard, 2019). Some experts even argue that there are fundamental reasons why the "natural" interest rate has declined and must be expected to be zero or even negative in the long run (von Weizsäcker, 2017).[11] Many politicians tend to see this as an invitation to expand public deficits without further thinking about the long-term consequences. In this context, it is unfortunate if a lack of results for the $S2$ indicator seemingly suggests that, in a situation with $r < g$, long-term fiscal sustainability is no longer an issue.

[10] That is probably one of the reasons why measures of this kind and sustainability analyses using them are largely disregarded in the research literature on fiscal sustainability, where fiscal reaction functions, fiscal space or distributions of fiscal limits play a core role (see, *e.g.*, the strands of literature sparked off by the contributions of Bohn, 1998; Heller, 2005; Bi and Leeper, 2013)

[11] An English translation of the book by von Weizsäcker and Krämer (2019) which fully spells out this idea is in preparation (under the title "Saving and Investment in the 21st Century").

4 Assessing $S2$ for low interest rates

In section 3, I have demonstrated that debt ratios can reach rather high levels even if government borrowing rates continue to be very low. Probably, this is not so much surprising. In fact, it is what experts openly accept who recommend to deliberately expand fiscal deficits under current interest rates, as they do not think this could be harmful. What could be surprising is that this may happen – at least in Germany – not only following a discretionary expansion of public expenditure on investment projects that might be worth consideration, but also if policy simply adheres to the current legal framework for age-related public expenditure, *i.e.*, on items which mainly serve consumptive purposes. In this section, I will show that – against the impression which may have come across in section 2 – the $S2$ indicator can also be determined for a situation with permanently low interest rates.

4.1 What happens if $r < g$ for $t > T$?

The nice thing about a situation with $r < g$ is that, if this lasts long enough, an economy can grow out of any level of debt, simply by reducing future deficits to zero at some point in time. According to the simulations used in section 3, however, this will never happen, even not at $t = T$ or afterwards. Therefore, the debt ratio can increase to rather high levels (at least under the assumptions for scenario $T-$), even if $r < g$ (*e.g.*, if nominal or real interest rates are zero; see, again, figure 1). As a consequence, creditors of fresh debt could start to ask for risk premiums leading to higher interest rates at some stage, which would accelerate further increases in the debt ratio. If $r > g$ were re-established through these risk premiums, this would openly render the situation of public finances unsustainable, as the sustainability indicator $S2$ would then confirm.

To avoid such a scenario, some orientation about fiscal consolidations that are appropriately sized and are not postponed for too long would be helpful even under the stylized assumption that $r < g$ until $t = T$ and beyond. A few candidate measures for this orientation are not fully convincing. For instance, an improvement of p_t that would stabilize d_t from year T onwards might be too less ambitious – and might also come too late. Improvements in p_t from $t = 1$ onwards which would limit d_t to 60% until $t = T$ and keep it constant at this level afterwards are based on requirements which are rather *ad-hoc*.

I therefore suggest to assess consolidation needs which, under the assumption of a permanent low-interest situation, lead to a time path of the debt ratio d_t that has the same properties as under a consolidation by $S2$ – in cases where this indicator can be derived from the intertemporal budget constraint (see section 2). I am thus looking for improvements in the GDP-ratio of annual primary surpluses which become effective from $t = 1$ onwards, are constant over time, do not aim at a pre-defined level of the debt ratio at $t = T$ (or any other point in time), but will perfectly stabilize d_t from year T onwards, as all other determinants of the debt ratio – *viz.* p_t and (r_t, g_t, hence q_t or) q – are assumed to be constant as well at $t > T$.

Using the same notation as in section 2, the period-0 present value of the debt ratio d_T is given by

$$q^T d_T = d_0 - \sum_{t=1}^{T} q^t(p_t + \sigma) = d_0 - \sum_{t=1}^{T} q^t p_t - \sum_{t=1}^{T} q^t \sigma. \tag{6}$$

More generally, the period-0 present value of d_{T+s} for subsequent years, with $s \in \{1, 2, 3...\}$, is given by

$$q^{T+s} d_{T+s} = d_0 - \sum_{t=1}^{T} q^t p_t - \sum_{t=T+1}^{T+s} q^t p_T - \sum_{t=1}^{T+s} q^t \sigma, \tag{7}$$

assuming once more that p_T remains unchanged for $t > T$. Here, the expressions $\sum_{t=T+1}^{T+s} q^t p_T$ and $\sum_{t=1}^{T+s} q^t \sigma$ can be simplified using the rules applying to geometric series, as long as $q \neq 0$.[12]

In addition, I now impose the condition that d_t remains constant from period T onwards, so that $d_{T+s} = d_T$. This yields

$$q^{T+s} d_T = d_0 - \sum_{t=1}^{T} q^t p_t - q^{T+1} \frac{1 - q^s}{1 - q} p_T - q \frac{1 - q^{T+s}}{1 - q} \sigma. \tag{8}$$

Multiplying equation (8) by $(1 - q)/q$ and re-arranging terms leads to

$$(1 - q^{T+s})\sigma + (1 - q) q^{T+s-1} d_T = \frac{1 - q}{q} \left(d_0 - \sum_{t=1}^{T} q^t p_t \right) - (q^T - q^{T+s}) p_T, \tag{9}$$

which can be further re-arranged to form

$$\sigma + q^{T+s} \left[\frac{1 - q}{q} d_T - p_T - \sigma \right] = \frac{1 - q}{q} \left(d_0 - \sum_{t=1}^{T} q^t p_t \right) - q^T p_T. \tag{10}$$

Now, if the number of years after T included in the calculation goes to infinity, $s \to \infty$, equation (10) behaves as follows.

1) If $q < 1$ (*i.e.*, $r > g$), q^{T+s} converges to zero, and σ becomes equal to the right-hand side of (10). In other words, σ can be determined through equation (5) which was already derived in section 2 – as expected.

[12] Note that for finite series, these rules are applicable also if $q > 1$.

2) However, if $q > 1$ (with $r < g$), the left-hand side of (10) could diverge, because $\lim_{s\to\infty} q^{T+s} = \infty$. A necessary condition for the left-hand side of (10) to converge is that the term in square brackets is equal to zero. For $q > 1$, σ must therefore satisfy

$$\sigma = \frac{1-q}{q} d_T - p_T. \tag{11}$$

If this holds true, the expression including the term in square brackets disappears as s goes to infinity, and σ can again be assessed based on the right-hand side of equation (10) or based on equation (5). Note that equation (11) by no means contradicts equation (5) or over-determines σ. It simply states that σ should keep the debt ratio d_t constant from year T onwards,[13] which is a natural property of σ implied in (5).

3) In the case that $q = 1$ (because $r = g$), some of the transformations made here are not applicable. In this case, one has to go back to equation (7), also using equation (6) to obtain

$$q^{T+s} d_{T+s} = q^T d_T - \sum_{t=T+1}^{T+s} q^t (p_T + \sigma). \tag{7'}$$

Taking into account that $q = 1$ and imposing the condition that d_t remains constant for $t > T$, so that $d_{T+s} = d_T$, this leads to

$$d_T = d_T - s(p_T + \sigma). \tag{12}$$

Equation (12) implies that $\sigma = -p_T$, regardless how large s, which is already included as a special case in equations (11) and (5). In other words, for $q = 1$ the effective primary balance needs to become zero from year T onwards.[14]

In any case, the main result of this exercise is that equation (5) is also applicable – or that the sustainability indicator $S2$ is also defined – in a situation with $q > 1$ (or $r < g$).

[13] This can easily be seen, if equation (11) is re-written, replacing q with $(1 + g)/(1 + r)$. It follows that $-(p_T + \sigma) = (g - r)d_T/(1 + g)$. Here, $-(p_T + \sigma)$ is the effective primary deficit, after consolidation, accruing in any year $t > T$; $d_T/(1+g)$ is the debt ratio of the preceding year adjusted to annual GDP-growth; current interest payments on existing debt amount to $rd_T/(1 + g)$ on a per-GDP basis; to keep the debt ratio constant, a total deficit ratio corresponding to $gd_T/(1 + g)$ can be incurred in the new year, making sure that public debt grows exactly as fast as GDP; a correction of projected figures for p_T through σ may therefore be needed to meet this condition.

[14] In this case, interest payments per GDP of $rd_T/(1 + g)$ create a fiscal deficit in each year $t > T$ that makes public debt increase exactly in line with GDP-growth, *viz.* at rate g.

4.2 Illustrative application

With this new result in mind, the simulations presented in section 3.1 can be taken up once again, to fill the gap in the results regarding the sustainability indicator $S2$ for scenarios with $r < g$ in 2060, *i.e.*, in the final year of the simulation period. Table 1 shows the indicator values for all of the scenarios considered above.

Table 1: The $S2$-indicator: results for all scenarios

	Underlying scenarios	
Interest-rate assumptions	$T+$	$T-$
$r_{\text{real}} = 3\%$ (2040+)	1.54	4.01
$r_{\text{real}} = 3\%$ (2060+): baseline	1.49	4.10
$r_{\text{real}} = 1.5\%$ (2060+)	1.48	4.55
$r_{\text{real}} = 0\%$ (2060+)*	1.52	5.39
$r_{\text{nominal}} = 0\%$ (2060+)*	1.82	8.63

Annotations: All figures are measured as a percentage of GDP, indicating permanent improvements in annual primary balances of the general-government budget which are required starting from 2020 to meet the intertemporal government budget constraint over an infinite time horizon.

* Scenarios with $r < g$.

Sources: SIM.17 (Werding et al. 2020); own calculations.

Two points are remarkable about the amended set of results for $S2$ displayed in the table. First, since assumptions on interest rates are arranged in descending order – meaning that interest rates tend to become lower in each new row – it is now visible that the sustainability gap measured by $S2$ gets smaller as r decreases only if primary deficits simulated for year T are relative low (as they are under the assumptions for the optimistic baseline scenario $T+$) and if interest rates are normal (with $r > g$). In any other case, $S2$ increases with declining interest rates.

With respect to differing results for the size of the year-T primary deficit, this property of the $S2$-indicator is well-known. It is due to the fact that closing the sustainability gap through corrections of the primary balance from year-1 onwards is effectively based on a strategy of pre-funding for future deficits. With lower interest rates, this strategy becomes more difficult. Another way of describing the same effects can be based on the fact that, when assessing $S2$, interest rates also influence the present-value weights of future deficits. With lower interest rates, high primary and total deficits accruing in the more remote future become more important, as they are discounted less heavily. This is nicely demonstrated in the recent contribution by (Andersen, 2020, pp. 32f.).

Second, the results summarized in table 1 also reveal that the indicator $S2$, which exhibits a relatively low sensitivity with respect to r as long as $r > g$,[15] becomes

[15]Under this assumptions, changes in $S2$ are negligible for the variants on scenario $T+$, and even

highly sensitive to government borrowing rates if the latter are assumed to be very low (with $r < g$). In fact, the $S2$ indicator tends to overshoot in the cases with very low interest rates, considering the fact that primary deficits simulated for the year 2060 (which are assumed to remain constant afterwards) are 1.5% of GDP in the scenario $T+$ and 5.1% of GDP in scenario $T-$.

In the optimistic scenario, long-term fiscal sustainability essentially requires to bring the primary balance close to zero from 2060 onwards, as long as nominal interest rates are positive. But if the interest rate is assumed to fall to 0% on nominal terms until 2060, $S2$ seems to suggest a stronger consolidation. In the pessimistic scenario, consolidations needed to render public finances sustainable in the long run even allow for a (corrected) primary deficit from 2060 onwards, if $r > g$. In these cases, early consolidations by $S2$ make sure that, until 2060, the state holds a sufficient amount of public wealth (not debt) to cover the remaining primary deficit by earned interest.[16] With very low interest rates, however, this is possible only under extreme forms of accumulation of public wealth which do not appear to be sensible.

Figure 2 shows simulated time paths of the German debt-to-GDP ratio for the scenarios with differing trends in interest rates that would result from adjustments of primary balances by the $S2$-values indicated in table 1. These time paths illustrate what has just been explained. Under extreme assumptions regarding r, *i.e.*, with nominal interest rates of 0% from 2060 onwards, the resulting accumulation of public wealth has next to no impact on the room for manoeuvre of future fiscal policy. At the same time, fiscal costs arising from high levels of public debt arising in the absence of any consolidation are relevant only if government borrowing rates return to normal levels – which could happen especially under the pessimistic assumptions for scenario $T-$. In this case, however, smaller steps to consolidation would be sufficient, as is indicated by the results for $S2$ under assumptions with $r > g$.

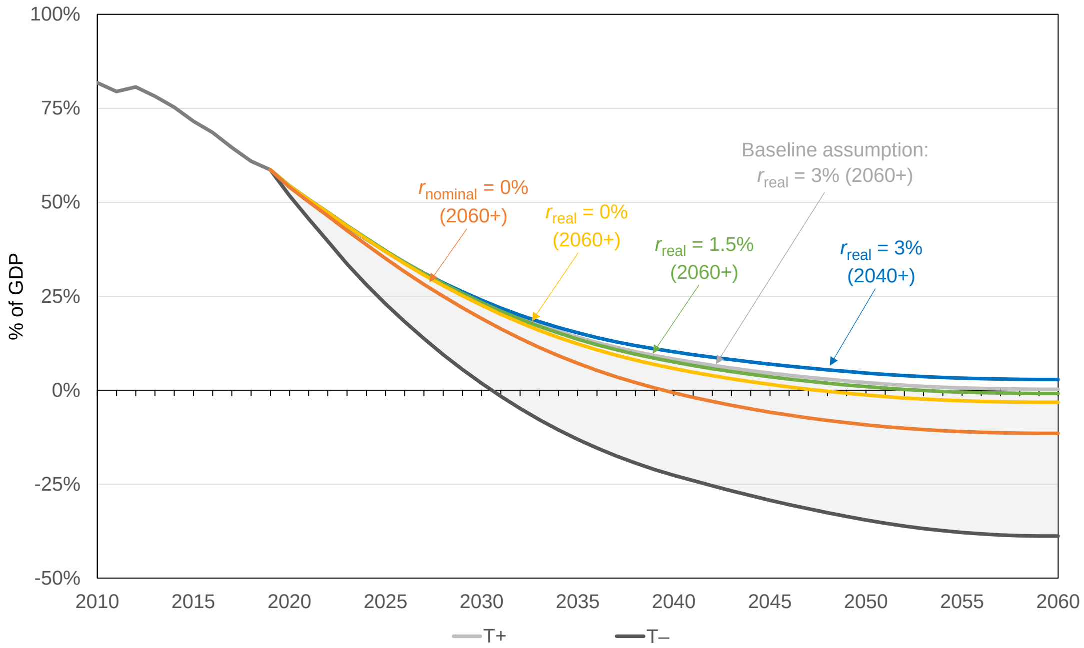
The main lesson to take away from this illustration is therefore the following. While it is indeed possible to determine the $S2$-indicator for long-term fiscal sustainability for scenarios with $r < g$ (as shown in section 4.1), the results can be interpreted only in the context of the underlying simulations for primary balances and debt ratios without any fiscal consolidations (see figure 1). If primary deficits expected towards the end of the simulation period are low, because projected in-

the range of results deriving from the pessimistic scenario $T-$ appears small compared to changes in projected increases in future debt ratios that are associated with the differing interest-rate assumptions considered here (see figure 1 above).

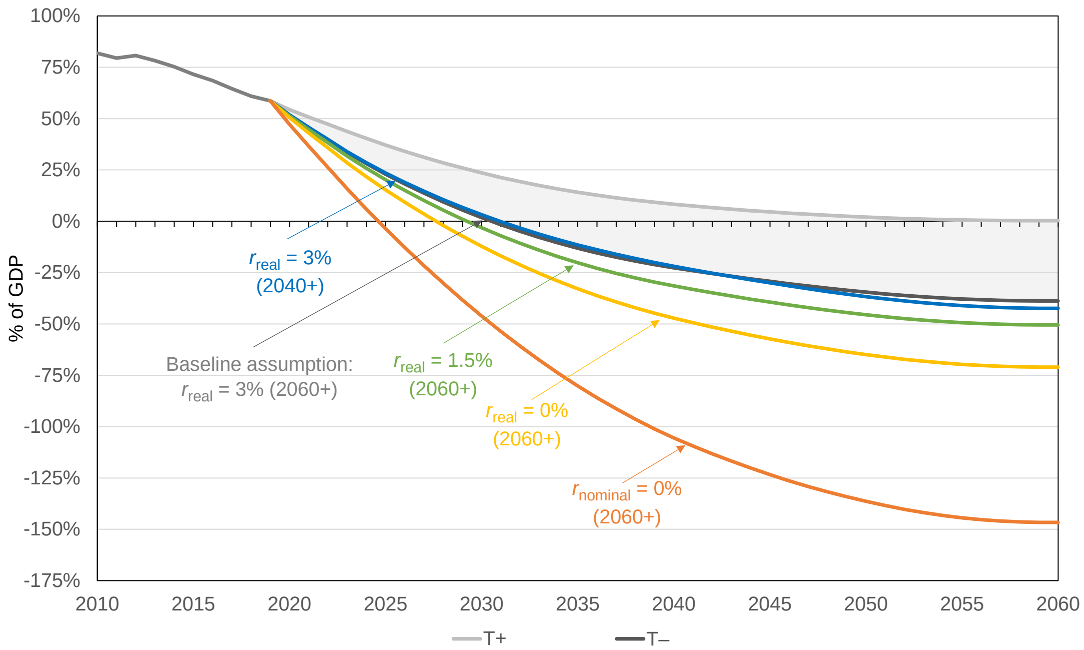
[16] To keep the (negative) debt ratio constant, part of the interest earnings must also be used to adjust D_{T+x} in line with GDP-growth. Total annual surpluses – deriving from primary deficits plus interest earnings – must therefore equal increases in public debt (or wealth) by the growth rate of GDP, $-gD_{T+x-1}$ (see footnote 13).

Figure 2: Projected debt ratios after consolidation by $S2$

a) Scenario T+ under different interest-rate assumptions



b) Scenario T− under different interest-rate assumptions



Sources: Deutsche Bundesbank, SIM.17 (Werding et al. 2020).

creases in age-related public expenditure are relatively small, and if the debt ratio stays well below its current level over the entire time horizon, due to very low government borrowing rates, sustainability problems involved in current legal rules are not very pressing. In such a case – that is, under the optimistic assumptions for scenario $T+$ – consolidation could possibly be postponed until interest rates show signs of a recovery.

If, on the other hand, the primary deficit projected for year T is substantial and if the debt ratio clearly exceeds the current level towards the end of the simulation period even under $r < g$ (and would continue to increase indefinitely afterwards), the situation is different. In this case – *e.g.*, under the assumptions for scenario $T-$ – fiscal consolidation is definitely needed. However, the precise dimension of consolidations needed to render public finances sustainable would be overstated by figures for $S2$ relating to a permanent low-interest-rate environment. Instead, results for the $S2$-indicator under assumptions with $r > g$ offer a reliable orientation for the size of fiscal reactions which are required. Assessing the sustainability gap also for situations with $r < g$ is therefore useful, as it may help to avoid possible misperceptions. At the same time, Andersen (2020) is right in saying that precise results for $S2$ under $r < g$ are not needed because, in themselves, they are not very telling.

5 Conclusions

In the end, all observations made in sections 3 and 4 point in the same direction. When it comes to assessing the long-term sustainability of public finances, assumptions regarding future trends of government borrowing rates are of secondary importance. The current situation with low interest rates, and even the expectation that this may last for quite some time, does not provide good news in this respect, if primary balances must be expected to deteriorate substantially over the next decades as the ageing process unfolds. Conversely, if interest rates start to increase again at some point in time in the future, the situation of public finances does not get worse than it already is. What needs to be done to make public finances sustainable is fully captured by results for the $S2$-indicator under assumptions with $r > g$. Also, in such a "normal" constellation, sensitivity of $S2$ with respect to the precise level of r is not very strong.

Higher results for $S2$ which are obtained under assumptions with $r < g$ are therefore a kind of warning sign. They ought to remind those in charge of fiscal policy that, when they exist, problems regarding the long-term sustainability of public finances are not removed through low government borrowing rates. There is still a need for correcting primary deficits that drive up the debt ratio. Otherwise, borrowing rates may normalize through risk premiums that creditors would ask for even if fundamental reasons for "safe" interest rates to be permanently low apply. In turn, being considered a "safe haven" is not guaranteed if – for instance, in the case of Germany – a country is faced with an ageing process that is more pronounced than elsewhere and if age-related public expenditure must be expected to increase considerably under the existing legal framework.

For practical fiscal planning, implications of the results obtained here are limited, but not entirely negligible. Closing the long-term sustainability gap $S2$ has never been considered a hard fiscal rule, due to uncertainties involved in the underlying simulations and the highly stylized nature of the calculations it rests on. For instance, the "medium-term objectives" set in the year-2012 European Fiscal Compact are not influenced by the results which are regularly published in the EU Fiscal Sustainability Reports. If, however, current fiscal rules are to replaced by less well-defined "fiscal standards" based on stochastic debt sustainability analyses (as is suggested by Blanchard et al., 2020), taking into account that future trends in age-related expenditure have systematic, not purely stochastic, components may well be required.[17] Where these trends must be expected to be strong and unfavourable, they ought to have an impact on planning and monitoring fiscal policy already in the short to medium run.

In addition, the considerations made here also have implications for the research agenda of those who are interested in budget planning and fiscal sustainability. Specifically, further research may be needed regarding the nature of trend reversals in government borrowing rates. Thus far, it has been demonstrated that reversals of this kind occur with much regularity and that, currently, a reversal indeed appears to be delayed.[18] More and in-depth empirical work may be needed regarding the causes of interest-rate reversals as well as the determinants of risk premiums related to government bonds, capturing not only the role of debt-to-GDP ratios, but also the structure of debt (*e.g.*, by types of creditors or by currencies in which debt is denominated), the existence and ideally also the strictness of fiscal rules, and other aspects of the governance of fiscal policy and debt management.

[17] How this can be fitted together may deserve further thinking. Stochastic population projections have long become a standard in demographic research (see, *e.g.*, Lee, 1992). Fully capturing the uncertainties relating to other determinants of future age-related expenditure – such as labour-force participation, trend unemployment, productivity growth, retirement behaviour, morbidity, *etc.*, plus future changes in relevant legal rules – in a stochastic framework may turn out to be difficult, though.

[18] See, for instance, Mehrotra (2017), building on the database compiled by Jordà et al. (2020). Further work that is of interest here has been carried out by Sergeyev and Mehrotra (2019) or Lian et al. (2020), based on calibrated simulations and on regressions of the $r-g$-differential on the debt ratio and a few macroeconomic variables, respectively.

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