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Abstract

This paper analyzes the optimal level of public debt when taxes are used not only for funding public expenditures but also for correcting externalities from climate change. Taking into account externalities implies that the optimal policy deviates from tax smoothing. Provided cumulative marginal damages are larger from today's than from tomorrow's emissions, the internalization of externalities decreases [increases] optimal debt if tax rates are on the increasing [decreasing] side of the Laffer curve. The reversed holds if the cumulative marginal damages increase over time. Allowing for endogenous adaptation investments reduces the deviation from tax-smoothing, but nevertheless increases optimal debt.

JEL-Codes: H230, H630, Q540, Q580.

Keywords: environmental externality, public debt, tax smoothing.

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1 Introduction

The substantial social and economic costs of environmental degradation arising from climate change have been thoroughly detailed, amongst others, by Tol (2002a, 2002b) and Stern (2008). For this matter, it should be of little surprise that the problem of global warming is one of the most important topics in the current political debate. In recent years, it became apparent that governments have to take measures to both mitigation, i.e. decelerating climate change, and adaptation, i.e. coping with the consequences of altered, less favorable environmental conditions (see, e.g., the Paris Agreement in United Nations, 2015). Interestingly, the discussion on policy implementation often quickly shifts to questions of financing and, thus, the impact of climate policy on the public budget balance. On the one hand, there is hope for co-benefits, for instance, when revenues from carbon pricing enable the decision maker to cut distortionary taxes on labor or capital (see, e.g., Goulder, 1995, and Proost and Van Regemorter, 1995, for the 'double dividend' theory) or public debt. On the other hand, investments in adaptation technologies or subsidies towards renewable energy production are generally expensive and put additional strains on the public budget. These effects are of particular relevance, as many countries, not only in Europe but worldwide, struggle with the sustainability of public finance in the wake of the Great Recession and the Covid-19 pandemic.

This paper contributes to the discussion on the fiscal implications of climate policy. We investigate the rather unexplored but – as argued above – highly policy relevant relation between emissions taxation and public debt. In doing so, we take a normative point of view and address the following research question: when a tax is implemented not only to satisfy public spending requirements, but also to lower greenhouse gas emissions and internalize the associated environmental externality, will it create incentives to decrease or increase the *optimal* level of public debt? At first glance, one might conjecture that taking into account the internalization of environmental externalities will create additional tax revenues that can be used to lower optimal public debt. However, the central insight of our analysis is that the impact of the environmental externality on optimal public debt may be of either sign, depending on whether the cumulative marginal environmental damages caused by one unit of emissions are decreasing or increasing over time and whether the optimal tax rates are on the increasing or decreasing side of the Laffer curve.

In order to derive this insight, we employ Barro's tax-smoothing approach (Barro, 1979 and 1989) and adapt it to suit our requirements by introducing environmental externalities. We develop a two-period model, where a representative household consumes two goods in each period, one of which pollutes the environment. Emissions are assumed to accumulate over time as a stock and cause environmental damage. The household pays an emissions tax upon consumption of the polluting good. The tax fulfills a dual role by, first, internalizing an environmental externality and, second, providing funds for exogenously given public expenditures. In addition, in the first period, spending requirements can be met through issuing public debt which has to be repaid in the second period. Emissions tax rates and public debt are set such that the household's welfare is maximized.

For a better understanding, the detailed results from analyzing this model are visualized in Table 1. As a benchmark, we first consider the case without an environmental

cumulative marginal damages	optimal tax rates	optimal revenues and debt if τ_1 and τ_2 on ...	
		... increasing side of Laffer curve	... decreasing side of Laffer curve
$\mathrm{MD}_1 = \mathrm{MD}_2 = 0$	$\tau_1 = \tau_2$	$R_1 = R_2$ $\Rightarrow$ tax smoothing by optimal debt b	
$\mathrm{MD}_1 > \mathrm{MD}_2 > 0$	$\tau_1 > \tau_2$	$R_1 > R_2$ $\Rightarrow$ negative effect on optimal b	$R_1 < R_2$ $\Rightarrow$ positive effect on optimal b
$0 < \mathrm{MD}_1 < \mathrm{MD}_2$	$\tau_1 < \tau_2$	$R_1 < R_2$ $\Rightarrow$ positive effect on optimal b	$R_1 > R_2$ $\Rightarrow$ negative effect on optimal b

MD_t=cumulative marginal damages from period t consumption; τ_t = optimal tax rate in period t; R_t = optimal tax revenue in period t; b = optimal public debt in period 1; $t = 1, 2$

Table 1: Main results

externality and, i.e. without marginal damages. In this case, optimal tax rates remain constant over time in order to minimize the present value of the excess burden associated with taxation. Constant tax rates imply constant tax revenues and, thus, optimal public debt is positive [negative] only if the expenditure requirement is larger [smaller] in the first period than in the second period. This represents the traditional tax-smoothing argument of public debt derived by Barro (1979, 1989). Starting from this benchmark, we find that introducing an environmental externality may induce the optimal policy to deviate from tax smoothing. If the cumulative marginal environmental damages from first-period consumption are larger [smaller] than those from second-period consumption, the first-period tax rate will be higher [lower] than the second-period tax rate as the internalization incentive is stronger [weaker] in the first period. In addition, if both tax rates are on the increasing side of the Laffer curve, tax revenues in the first period are larger [smaller] than those in the second period. Compared to the tax-smoothing level, we therefore obtain a negative [positive] effect on the optimal level of public debt. For example, if spending requirements are constant over time optimal public debt becomes negative [positive]. This implication is inverted if both tax rates are on the decreasing side of the Laffer curve. Then, tax revenues are larger [lower] in the second period and we obtain a positive [negative] effect on the optimal level of public debt. We show that these results hold independently of whether tax revenues from the Pigouvian internalization of the environmental externality are already sufficient to finance the spending requirements or whether optimal tax rates need to deviate from the Pigouvian level.

As an extension, we also take endogenous adaptation investments into account. The decision maker can then choose to invest in a technology which requires upfront effort in the first period and adapts the economy to better cope with pollution in the second period. Thus, we further extend the model by adding an endogenous margin to public spending, while the standard tax-smoothing analysis of Barro (1979, 1989) takes spending requirements as exogenously given. Since adaptation investments alleviate the environmental damages experienced from emissions, we move closer to the benchmark without environmental externalities and the optimal tax rates turn out to deviate less from the tax-smoothing principle. Hence, we find that adaptation will shrink the wedge between first- and second-period tax rates previously induced by the environmental externality. Yet, investing in the technology always creates an incentive to issue more debt in order to finance adaptation effort in the first period. That is, if accumulating public debt [savings] was optimal before, adaptation now leads to a higher total level of debt [lower savings].[1]

The plenitude of cases for which we derive results raises the question which combination of tax rate locations on the Laffer curve and cumulative damage curves is most relevant. In the latter respect, we argue that, in the context of climate change, the cu-

[1]As a remark, note that the alleviating effect of adaptation on the deviation from tax-smoothing tax rates intuitively also holds if adaptation reduces already the first-period damages, and not only the second-period damages as assumed in our formal model. The reason is that the model then becomes even closer to the benchmark case without an environmental externality. In contrast, if adaptation requires investments not only in the first period but also in the second period, again in contrast to our formal model, then intuitively the effect of adaptation on optimal debt may be reversed, if optimal adaptation investments are larger in the second period than in the first period.

mulative marginal damages from emissions might be higher in the first period. Many greenhouse gases, especially carbon dioxide, are characterized by exceedingly long atmospheric lifetimes and negligible decay rates (see for instance Archer et al., 2009). Hence, the earlier a unit of the greenhouse gas is emitted, the larger are the cumulative damages caused by this unit. The optimal emissions tax rates should then decrease over time.[2] In contrast, it is less straightforward to determine whether emissions tax rates are more likely to be located on the increasing or decreasing side of the Laffer curve. Frequently cited, Trabandt and Uhlig (2011) provide empirical evidence suggesting that labor and capital taxes are typically located on the increasing side of the Laffer curve. However, it is not clear whether this finding can also be applied to other kinds of taxes. For instance, in a recent study of the corporate income tax in Canada, Dahlby and Ferede (2018) obtain much less consistent results regarding the location of the tax rate on the Laffer curve. Even more important for our purposes, to the best of our knowledge there is no evidence with regard to the Laffer curve of emissions taxes. Intuitively, if the internalization objective of such taxes outweighs revenue-raising incentives, the optimal tax rates may well be found on the decreasing side of the Laffer curve. Yet, providing empirical evidence of this assertion is beyond the scope of our analysis and left for future research.

We contribute to the literature in two ways. First, we add an additional dimension to the discussion on the fiscal implications of climate policy. As already mentioned above, the double dividend is a prominent topic in this strand of literature, see e.g. Bovenberg and De Mooij (1994), Proost and Van Regemorter (1995), Parry (1995) and Goulder (1995). This literature generally addresses the question whether an emissions tax, in addition to its positive effect of increasing environmental quality by reducing emissions, can also improve the efficiency of the tax system by reducing other distortionary taxes. A related topic is discussed in the recent study by Franks et al. (2017). In a dynamic general equilibrium model, these authors investigate whether emissions taxation attains a higher welfare level than taxation of mobile capital, even if environmental externalities are ignored. However, none of these papers examines the link between emissions taxation and optimal public debt, which is the main contribution of our analysis.

Second, our paper introduces the issue of climate change into the literature on public

[2] Note that this conclusion is also consistent with the optimal tax path suggested by the Green Paradox analyzed in Sinn (2012), even though the Green Paradox is not analyzed within a tax-smoothing framework, but in the context of the exploitation of non-renewable natural resources.

debt. The existing literature can basically be divided into positive studies explaining the accumulation of public debt, like the political economy models of, e.g., Persson and Svensson (1989), Tabellini and Alesina (1990) and Woo (2003), and normative studies investigating optimal public debt, like the tax-smoothing theory by Barro (1979, 1989). Our analysis relates to the normative approach and, as already stated above, shows that in the presence of a taxable, polluting consumption good, the optimal public deficit may be non-zero, even if spending requirements are constant over time. To the best of our knowledge, studies that explicitly investigate the relation between public debt and environmental issues are scarce in the debt literature. As an exception, in a recent working paper Boly et al. (2019) employ the concept of a stock of environmental quality to derive a trade-off between public debt and 'environmental debt'. Furthermore, Fodha and Seegmuller (2014) examine the welfare effect of an environmental abatement policy which may either be funded by tax revenues or public debt in a fully dynamic model. They find that pollution abatement should not be conducted at the costs of increased debt when the capital stock is low. Finally, ignoring emissions taxation, Catalano et al. (2020) investigate the impact of fiscal policy on public investments in climate change adaptation. The authors show that early debt-funded adaptation spending has a long-run beneficial effect even though negatively affecting the debt-to-GDP ratio initially. While these papers also investigate the link between public debt and environmental issues, none of them takes into account the dual role of taxation as a means of financing public spending and correcting environmental externalities. Hence, in contrast to our analysis, they cannot investigate the implications of climate policy on the tax-smoothing role of public debt.

Our paper is organized as follows. In Section 2 we introduce the basic framework. In Section 3, we analyze the optimal tax and debt policy. In Section 4, we investigate how our findings are affected when the economy can adapt to pollution by means of investing in an adaptation technology. The final section concludes the paper.

2 Model

2.1 Private Sector

We consider an economy with a representative household that lives for two periods, 1 and 2. In period $t = 1, 2$ the household consumes a composite good Y in quantity y_t and a polluting good X in quantity x_t. The household's utility in period t is given by

$$u_t = y_t + V(x_t), \tag{1}$$

with $V' > 0$ and $V'' < 0$. Without loss of generality, we normalize the household's discount rate to zero, so the present value of the household's utility reads $w = u_1 + u_2$.

In each period, the household receives an exogenous endowment of a numeraire good normalized to one. We assume that goods Y and X can be produced from the endowment by a one-to-one-technology. Hence, the prices of both goods are equal to one. Good Y is untaxed, whereas good X is taxed by a unit tax with tax rate τ_t in period t. The household may receive a lump-sum transfer z_t from the government in period t. For simplicity, we ignore private savings. The private budget constraint in period t is

$$y_t + (1 + \tau_t)x_t = 1 + z_t. \tag{2}$$

Tax rates and lump-sum transfers are taken as given by the household. The household chooses consumption in order to maximize the present value of its utility. Inserting (2) into (1), the maximization problem can be written as

$$\max_{x_1,x_2} w = \sum_{t=1,2} \Big\{ V(x_t) + 1 + z_t - (1 + \tau_t)x_t \Big\}.$$

The first-order condition with respect to x_t reads

$$V'(x_t) = 1 + \tau_t, \qquad t = 1, 2. \tag{3}$$

This condition equates the household's marginal utility to the after-tax price of good X in period t. Hence, the household's optimal consumption of good X in period t is a function of the tax rate in period t. Formally, equation (3) implies $x_t = X(\tau_t)$ with $X'(\tau_t) = 1/V'' < 0$ and $X''(\tau) = -V'''/V''^3 \geq 0$.[3]

2.2 Government

In addition to taxing good X, the government may raise revenues in the first period through issuing public debt b which has to be repaid in the second period. As for the

[3]Here, we implicitly assume $V''' \geq 0$, which is satisfied, for example, if V is quadratic or if V is monotone and has monotone derivatives. In the latter case, $V''' > 0$ is implied by $V' > 0$ and $V'' < 0$.

private discount rate, we normalize the interest rate on public debt to zero. Public policy pursues two goals. First, revenues from taxation and debt are used to finance public spending requirements in both periods. In the basic model, we follow the tax-smoothing literature and assume exogenously given spending requirements $g_1 \geq 0$ and $g_2 \geq 0$ in both periods. Second, the government uses taxation in order to internalize the pollution externality caused by private consumption of good X. In period 1, this externality is reflected by the damage function $D_1(x_1)$ with $D_1' > 0$ and $D_1'' \geq 0$. In period 2, the damage function reads $D_2(x_2 + \gamma x_1)$ with $D_2' > 0$, $D_2'' \geq 0$ and $\gamma \geq 0$. The parameter γ allows distinguishing between flow pollution ($\gamma = 0$) and stock pollution ($\gamma > 0$). Greenhouse gas emissions and climate change provide an example for the latter case.

Formally, the government's welfare maximization problem can be stated as

$$\max_{\{b,\tau_t,z_t\}_{t=1,2}} w = \sum_{t=1,2} \Big\{ V[X(\tau_t)] + 1 + z_t - (1+\tau_t)X(\tau_t) \Big\} \\ - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)], \tag{4}$$

subject to

$$\tau_1 X(\tau_1) + b = g_1 + z_1, \quad \tau_2 X(\tau_2) - b = g_2 + z_2, \tag{5}$$

$$z_1 \geq 0, \qquad z_2 \geq 0. \tag{6}$$

According to (4), the government maximizes the present value of the household's utility net of environmental damages, taking into account the public budget constraints given in (5) and the household's consumption reactions determined by $x_t = X(\tau_t)$. Moreover, due to (6) we restrict the policy space to non-negative lump-sum transfers. The reason is that we follow the tax-smoothing literature referred to in the introduction and focus on the case where the government has to use distortionary taxation in order to meet its spending requirements. If we would allow for negative transfers, the government would have an incentive to use these transfers in order to finance the spending requirements in a non-distortionary way. Note that we nevertheless need the transfers since, in contrast to the previous tax-smoothing literature, in our framework tax revenues may exceed the spending requirements due to the government's second goal of internalizing the pollution externality. Hence, in our framework the transfers only exist in order to redistribute back potential excess revenues from the emissions tax in a non-distortionary way. As shown below, (6) will be binding – and z_1 and z_2 will vanish – in the (most realistic) case where emissions tax revenues are not sufficient to finance the public spending requirements.

The solution to the government's welfare maximization problem (4)–(6) can be characterized with the help of the Lagrangian

$$\begin{aligned} L &= \sum_{t=1,2} \Big\{ V[X(\tau_t)] + 1 + z_t - (1+\tau_t)X(\tau_t) \Big\} - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)] \\ &\quad + \lambda_1[\tau_1 X(\tau_1) + b - g_1 - z_1] + \lambda_2[\tau_2 X(\tau_2) - b - g_2 - z_2], \end{aligned}$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the budget constraint in period 1 and period 2, respectively. The Kuhn-Tucker first-order conditions read

$$L_b = \lambda_1 - \lambda_2 = 0, \tag{7}$$

$$\begin{aligned} L_{\tau_1} &= -X(\tau_1) - \Big\{ D_1'[X(\tau_1)] + \gamma D_2'[X(\tau_2) + \gamma X(\tau_1)] \Big\} X'(\tau_1) \\ &\quad + \lambda_1 \Big[X(\tau_1) + \tau_1 X'(\tau_1) \Big] = 0, \end{aligned} \tag{8}$$

$$L_{\tau_2} = -X(\tau_2) - D_2'[X(\tau_2) + \gamma X(\tau_1)]X'(\tau_2) + \lambda_2 \Big[X(\tau_2) + \tau_2 X'(\tau_2) \Big] = 0, \tag{9}$$

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g_1 - z_1 = 0, \tag{10}$$

$$L_{\lambda_2} = \tau_2 X(\tau_2) - b - g_2 - z_2 = 0, \tag{11}$$

and the slackness conditions are

$$L_{z_1} = 1 - \lambda_1 \leq 0, \quad z_1 \geq 0, \quad z_1 L_{z_1} = 0, \tag{12}$$

$$L_{z_2} = 1 - \lambda_2 \leq 0, \quad z_2 \geq 0, \quad z_2 L_{z_2} = 0, \tag{13}$$

where in (8) and (9) we used (3). For the second-order conditions to be satisfied, the determinant $|H|$ of the bordered Hessian needs to be negative. We determine $|H|$ in Appendix A and will verify that $|H| < 0$ in all relevant cases considered below.

3 Optimal Tax and Debt Policy

To analyze the government's welfare maximum characterized by conditions (7)–(13), we first examine the public budget constraints (10) and (11). Adding both equations gives the government's intertemporal budget constraint

$$\tau_1 X(\tau_1) + \tau_2 X(\tau_2) = g_1 + g_2 + z_1 + z_2, \tag{14}$$

stating that the present value of tax revenues (LHS) has to be equal to the present value of public spending and transfers (RHS). Subtracting (11) from (10) yields

$$b = \frac{g_1 - g_2}{2} + \frac{\tau_2 X(\tau_2) - \tau_1 X(\tau_1)}{2} + \frac{z_1 - z_2}{2}. \tag{15}$$

In the subsequent analysis, we will use (15) in order to compute the optimal level of public debt. Basically, the equation has the same meaning as in previous studies on tax smoothing without pollution. The first term on the RHS shows the central tax-smoothing argument: Public debt is used to equalize variations in exogenous public spending. The reason is that without externalities the optimal tax policy minimizes the excess burden of taxation by charging constant tax rates over time. Consequently, tax revenues also remain constant such that the second term on the RHS vanishes. In contrast, we will show that tax revenues may vary over time in our analysis with environmental externalities. Thus, taxation can affect the optimal debt policy via the second term on the RHS of (15).[4]

Specifically, if tax revenues in the second period, $\tau_2 X(\tau_2)$, are larger than tax revenues in the first period, $\tau_1 X(\tau_1)$, then the second term on the RHS of (15) is positive, providing an additional rational for public debt. To determine tax revenues in period t associated with the tax rate τ_t, we make use of the Laffer curve defined as

$$R(\tau_t) = \tau_t X(\tau_t). \tag{16}$$

We impose the following quite general assumption on the shape of the Laffer curve.

Assumption A1. *The Laffer curve $R(\tau)$ is twice continuously differentiable and satisfies $R'(\tau) = X(\tau) + \tau X'(\tau) \gtreqless 0$ if and only if $\tau \lesseqgtr \bar{\tau}$ with $\bar{\tau} > 0$, $R''(\tau) = 2X'(\tau) + \tau X''(\tau) < 0$, $R(0) = 0$ and $\lim_{\tau \to \infty} R(\tau) < (g_1 + g_2)/2 < R(\bar{\tau})$.*

This assumption states that the Laffer curve is inverted u-shaped with a unique maximum at the positive tax rate $\bar{\tau}$ and vanishing tax revenues at a zero tax rate. The latter properties in Assumption A1 ensure that maximal revenues at $\bar{\tau}$ are more than enough to

[4] As stated above, in the (most realistic) case where emissions tax revenues are not sufficient to fund the public spending requirements, the transfers z_1 and z_2 are zero, so they have no impact on public debt via the third term on the RHS of (15). If taxation revenues exceed the spending requirements, optimal transfers will turn out to be positive, but only the sum $z_1 + z_2$ will be determined by the optimality conditions. Since we introduced the transfers only to redistribute excessive tax revenues, it is natural to assume $z_1 = z_2$ in the case with positive transfers in order to abstract from further effects on public debt.

meet the spending requirements. Together with the inverted u-shape of the Laffer curve, this implies that there are additional tax rates $\tau \gtreqless \bar{\tau}$ on both sides of the Laffer curve which generate sufficient revenues for funding total public spending $g_1 + g_2$.

Next, we rewrite the first-order conditions of welfare maximization in order to identify conditions under which tax rates and revenues differ across the two periods. From (7) we obtain $\lambda_1 = \lambda_2 =: \lambda$. Using this in (8) and (9) yields

$$\lambda = \frac{X(\tau_1) + \Big\{D_1'[X(\tau_1)] + \gamma D_2'[X(\tau_2) + \gamma X(\tau_1)]\Big\}X'(\tau_1)}{X(\tau_1) + \tau_1 X'(\tau_1)}, \tag{17}$$

$$\lambda = \frac{X(\tau_2) + D_2'[X(\tau_2) + \gamma X(\tau_1)]X'(\tau_2)}{X(\tau_2) + \tau_2 X'(\tau_2)}, \tag{18}$$

Since $\lambda \geq 1 > 0$ from the slackness conditions (12) and (13), the respective nominator and denominator on the RHS of (17) and (18) must have the same sign. They may be either both positive or both negative, in each of these equations. The implications, however, depend on whether the slackness conditions are binding or not. As a benchmark, we start with the case where consumption does not cause environmental damages and obtain the following result, which is proven in Appendix B.

Proposition 1. *If $D_1 \equiv D_2 \equiv 0$, then the optimal policy is characterized by $z_1 = z_2 = 0$, $\tau_1 = \tau_2 = \tau$ and $b = (g_1 - g_2)/2$, where τ is implicitly determined by $\tau X(\tau) = (g_1 + g_2)/2$ and lies on the increasing side of the Laffer curve $R(\tau)$.*

Proposition 1 replicates the results from the previous tax-smoothing literature: If good X does not cause externalities, the only purpose of taxation is to meet the spending requirements. Since taxation is distortionary, the government chooses tax rates that minimize the excess burden. The minimum is obtained when the tax rates and, thus, tax revenues are constant over time ($\tau_1 X(\tau_1) = \tau_2 X(\tau_2)$). Due to the excess burden of taxation, the government will not generate more revenues than required for exogenous spending, so transfers are zero in both periods ($z_1 = z_2 = 0$). As a result, equation (15) reduces to $b = (g_1 - g_2)/2$, i.e. public debt or savings will only occur if the exogenous spending requirements are non-constant over time. More precisely, a strictly positive level of debt [savings] is optimal if spending is larger [lower] in period 1 than in period 2.

Having established the classical tax-smoothing benchmark, we can now turn to the case with externalities. Due to (17) and (18), for $D_1, D_2 \neq 0$ there are two important differences to the case without externalities. First, λ may be equal to one such that the slackness conditions are not binding and, second, tax rates may be on the decreasing side of the Laffer curve. To ease exposition, in the subsequent analysis we always assume both tax rates are on the same side of the Laffer curve.[5] Starting with the binding case, we obtain the following proposition that is proven in Appendix C.

Proposition 2. *If $D_1, D_2 \neq 0$ and $\lambda > 1$, then the optimal policy is characterized by*
(i) $z_1 = z_2 = 0$.
(ii) $\tau_1 \gtreqless \tau_2$ *if and only if* $D_1' + \gamma D_2' \gtreqless D_2'$.
(iii) If $D_1' + \gamma D_2' < -x_1/X_1'$ *and* $D_2' < -x_2/X_2'$*, then* τ_1 *and* τ_2 *are both on the increasing side of the Laffer curve and* $\tau_1 > D_1' + \gamma D_2'$ *and* $\tau_2 > D_2'$*. Moreover,*

$$b \gtreqless \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D_1' + \gamma D_2' \lesseqgtr D_2'.$$

(iv) If $D_1' + \gamma D_2' > -x_1/X_1'$ *and* $D_2' > -x_2/X_2'$*, then* τ_1 *and* τ_2 *are both on the decreasing side of the Laffer curve and* $\tau_1 < D_1' + \gamma D_2'$ *and* $\tau_2 < D_2'$*. Moreover,*

$$b \gtreqless \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D_1' + \gamma D_2' \gtreqless D_2'.$$

Let us first take a look at the optimal tax-transfer policy characterized in parts (i) and (ii) of Proposition 2. In order to understand these results, notice that $D_1' + \gamma D_2'$ and D_2' reflect the Pigouvian levels of emissions taxation, i.e. the cumulative marginal environmental damages that one unit of emissions from first-period consumption and second-period consumption, respectively, causes over its whole lifetime in the atmosphere. If the slackness conditions are binding ($\lambda > 1$), then taxing good X according to these Pigouvian levels would not generate enough tax revenues to satisfy the spending requirements. Hence, if the tax rates are on the increasing [decreasing] side of the Laffer curve, the government has to set them above [below] the Pigouvian levels in order to generate more tax revenues and to meet the spending requirements (formally, this property is contained in part (iii) [part (iv)] of Proposition 2). As shown in part (i) of Proposition 2, transfers z_1 and z_2 are not needed in this case, since there are no excess tax revenues from Pigouvian internalization of the environmental externalities. Nevertheless, according to part (ii) of Proposition 2, optimal tax rates are positively correlated with the cumulative marginal damages in the

[5] From the intuition behind these results, which we intensively discuss below Proposition 2, it should become immediately obvious what happens if both tax rates are on different sides of the Laffer curve.

sense that the tax rate is always higher in the period in which consumption of good X is associated with larger cumulative marginal damages, even though tax rates deviate from their Pigouvian level and are thus not equal to the cumulative marginal damages.

The consequences of this emissions tax policy for optimal public debt is characterized in parts (iii) and (iv) of Proposition 2. The basic insight from these results is that the presence of environmental externalities can influence the optimal debt level as b may deviate from $(g_1-g_2)/2$, which is the optimal debt level under tax smoothing in the absence of externalities. To illustrate this, first consider the case where cumulative marginal damages are larger in the first than in the second period ($D_1'+\gamma D_2' > D_2'$), so the optimal tax rate is higher in period 1 than in period 2 ($\tau_1 > \tau_2$) according to part (ii) of Proposition 2. This situation is displayed in Figure 1.

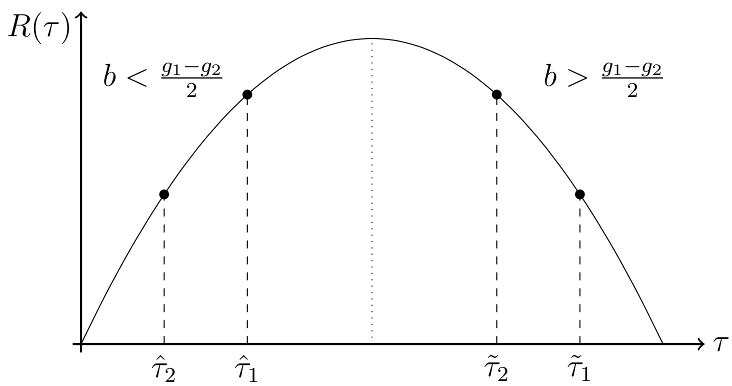

Figure 1: The Laffer curve and optimal public debt

If the cumulative marginal damages are relatively low ($D_1' + \gamma D_2' < -x_1/X_1'$ and $D_2' < -x_2/X_2'$), we obtain part (iii) of Proposition 2 and optimal tax rates like $(\hat{\tau}_1, \hat{\tau}_2)$ on the increasing side of the Laffer curve. Tax revenues are then larger in period 1 than in period 2, implying a negative effect on the optimal level of debt, i.e. b falls short of the tax-smoothing level $(g_1 - g_2)/2$. In contrast, if the cumulative marginal damages are relatively high ($D_1'+\gamma D_2' > -x_1/X_1'$ and $D_2' > -x_2/X_2'$), part (iv) of Proposition 2 holds and the optimal tax rates are represented by $(\tilde{\tau}_1, \tilde{\tau}_2)$ on the decreasing side of the Laffer curve. Tax revenues are then larger in period 2 than in period 1 and we obtain a positive effect on the optimal level of debt, i.e. b is above the tax-smoothing level $(g_1 - g_2)/2$. Not displayed in Figure 1 is the case where the cumulative marginal damages are smaller in the first than in the second period ($D_1' + \gamma D_2' < D_2'$). Accordingly to part (ii) of

Proposition 2, the optimal tax rate is then larger in period 2 than in period 1 and all the results illustrated in Figure 1 are reversed.

To sum up, the impact of environmental externalities on optimal public debt depends on the time path of cumulative marginal damages, on the one hand, and the tax rates' location on the Laffer curve, on the other hand. We obtain four cases with different implications for optimal public debt. In order to illustrate that for each of these four cases there is a non-empty set of parameter constellations satisfying the conditions of the respective case, we present a numerical example with a linear-quadratic specification of our model. The utility function for good X in period t is given by $V(x_t) = (1+\alpha)x_t - \beta x_t^2/2$ with $\alpha, \beta > 0$. The damage function in period 1 and 2 reads $D_1(x_1) = \delta_1 x_1$ and $D_2(x_2 + \gamma x_1) = \delta_2(x_2 + \gamma x_1)$, respectively, with $\delta_1, \delta_2 > 0$. The household's first-order condition (3) then yields the demand function $X(\tau_t) = (\alpha - \tau_t)/\beta$. The Laffer curve in period t is $R(\tau_t) = (\alpha\tau_t - \tau_t^2)/\beta$ with a maximum at $\tau_t = \alpha/2$ and zero tax revenues at $\tau_t = 0$ and $\tau_t = \alpha$. In the following, we present only numerical examples in which the optimal tax rates are between 0 and α. This ensures positive consumption levels and tax revenues in both periods. The second-order conditions for a welfare maximum are always satisfied under the linear-quadratic specification. Details on this and the numerical examples displayed in Table 2 are relegated to Appendix D.

Parameter values							Optimal Policy				Side of Laffer	Marg. damages
α	β	γ	δ_1	δ_2	g_1	g_2	τ_1	τ_2	b	λ	curve: $\tau_t \gtreqless \alpha/2$	$\delta_1 + \gamma\delta_2 \gtreqless \delta_2$
5.0	1.0	1.0	0.1	0.1	1.0	1.0	0.26	0.16	−0.22	1.01	increasing	decreasing
5.0	1.0	0.1	0.1	0.2	1.0	1.0	0.17	0.25	0.18	1.01	increasing	increasing
1.0	1.0	0.1	0.9	0.9	0.2	0.2	0.75	0.70	0.01	1.50	decreasing	decreasing
1.0	1.0	0.1	0.7	0.9	0.2	0.2	0.69	0.76	−0.02	1.28	decreasing	increasing

Table 2: Numerical examples

Note that in all these examples the slackness conditions are binding (since $\lambda > 1$) and the tax-smoothing level of debt would be zero (since $g_1 = g_2$). In the first two examples, optimal tax rates are below $\alpha/2$ and, thus, on the increasing side of the Laffer curve. In the first [second] example, cumulative marginal damages are decreasing [increasing], since $\delta_1 + \gamma\delta_2$ is larger [smaller] than δ_2, implying that it is welfare maximizing to issue a negative [positive] level of debt. The third and fourth example may be interpreted analogously. Hence, the numerical exercise in Table 2 provides examples for each of the four cases identified in Proposition 2.

While this illustrates that each of the four cases is possible from a theoretical perspective, it is worthwhile to establish which of these cases is most likely for policy advise. As already intensively discussed in the introduction, a falling time path of the cumulative marginal damages seems plausible, at least in case of climate change. In terms of our formal model, γ is then close to one and $D_1' + \gamma D_2' = D_1' + D_2' > D_2'$. This reflects the idea that the earlier a unit of greenhouse gas is emitted, the larger are the cumulative damages caused by this unit. Given the mixed empirical results with respect to the Laffer curve of labor, capital and corporate taxes (e.g. Trabandt and Uhlig, 2011 and Dahlby and Ferede, 2018) and the non-availability of Laffer curve estimates for emissions taxes, it is a more intricate puzzle to judge whether emissions tax rates are located on the increasing or decreasing side of the Laffer curve. As the focus of this paper is on the theoretical analysis of the relation between environmental externalities and optimal public debt, we leave the empirical analysis of this relation for future research.

Finally, we briefly turn to the scenario where the slackness conditions are non-binding ($\lambda = 1$) and tax revenues from the Pigouvian internalization of the externality are already sufficient to meet the government's spending requirements. While this scenario seems to be less likely in practice, it is useful from a theoretical point of view since the basic insights from Proposition 2 will turn out to generalize to the non-binding case. Appendix E proves

Proposition 3. *If $D_1, D_2 \neq 0$ and $\lambda = 1$, then the optimal policy is characterized by*

(i) $z_1 = z_2 = \dfrac{(D_1' + \gamma D_2') \cdot X(D_1' + \gamma D_2') + D_2' \cdot X(D_2')}{2} - \dfrac{g_1 + g_2}{2} > 0.$

(ii) $\tau_1 = D_1' + \gamma D_2' \gtreqqless D_2' = \tau_2$ *if and only if* $D_1' + \gamma D_2' \gtreqqless D_2'$.

(iii) If $D_1' + \gamma D_2' < -x_1/X_1'$ and $D_2' < -x_2/X_2'$, then τ_1 and τ_2 are both on the increasing side of the Laffer curve. Moreover

$$b \gtreqqless \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D_1' + \gamma D_2' \lesseqqgtr D_2'.$$

(iv) If $D_1' + \gamma D_2' > -x_1/X_1'$ and $D_2' > -x_2/X_2'$, then τ_1 and τ_2 are both on the decreasing side of the Laffer curve. Moreover

$$b \gtreqqless \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D_1' + \gamma D_2' \gtreqqless D_2'.$$

As becomes obvious from parts (iii) and (iv) of Proposition 3, with respect to the optimal debt level we obtain qualitatively the same results as in Proposition 2: Depending on whether the optimal tax rates are on the increasing or decreasing margin of the Laffer curve and depending on the relation of the cumulative marginal damages in the two periods, introducing externalities into the tax-smoothing analysis may increase or decrease the optimal debt level. The difference to Proposition 2 is that the optimal tax rates are now at their Pigouvian levels (equal to the cumulative marginal damages) and that transfers are positive. The reason is that Pigouvian internalization of the externality requires taxes that are high enough to overfulfill the spending requirements. Hence, positive transfers amount to the difference between tax revenues and the exogenous spending requirement.[6]

4 Adaptation to Climate Change

So far, we assumed that the government's expenditures g_1 and g_2 were exogenously given and unproductive. In this section, we extend our basic model and take into account the option to invest in an adaptation technology today that reduces the future welfare loss from pollution damages. Formally, we suppose that the government has the opportunity to invest a in period 1, funding the adaptation technology $T(a)$ in period 2 with positive but decreasing returns, i.e. $T'(a) > 0$ and $T''(a) < 0$. In order to reduce the number of possible cases, we assume in this section that the slackness conditions are binding and the revenues from Pigouvian internalization are not yet enough to finance public expenditures ($z_1 = z_2 = 0$). Moreover, we ignore the traditional tax-smoothing argument of public debt by assuming that the exogenously given spending requirements are constant over time ($g_1 = g_2 = g$). In (5), the first-period budget constraint therefore changes to $\tau_1 X_1(\tau_1) + b = g + a$, while the second-period budget constraint now reads $\tau_2 X_2(\tau_2) - b = g$. Marginal damages in period 1 are still equal to $D_1(x_1)$, while marginal damages in period 2 are now given by the damage function $D_2[x_2 + \gamma x_1, T(a)]$ with $D_{2,X} := \partial D_2 / \partial(x_2 + \gamma x_1) > 0$, $D_{2,XX} := \partial^2 D_2 / \partial(x_2 + \gamma x_1)^2 \geq 0$, $D_{2,T} := \partial D_2 / \partial T(a) < 0$, $D_{2,TT} := \partial^2 D_2 / \partial T(a)^2 \geq 0$. Hence, adaptation investments in period 1 improve the adaptation technology in period 2

[6] As for the binding case, it is straightforward to identify numerical examples for each of the four cases contained in Proposition 3. In fact, we obtain such examples, if in Table 2 we simply replace $g_1 = g_2 = 1$ by $g_1 = g_2 = 0.1$ in the first two rows and $g_1 = g_2 = 0.2$ by $g_1 = g_2 = 0.02$ in the last two rows. Details on these numerical examples can be obtained upon request.

that, in turn, reduces second-period damages at non-increasing rates. This setup reflects that adaptation investments often have no instantaneous effect, either because the scale of the project requires some time lag or because adaptation will only become effective when global warming has exceeded a critical level. For instance, $T(a)$ can represent construction of sea defense walls protecting lowlands from rising sea levels.

The government again maximizes the household's welfare subject to the modified public budget constraints specified above and subject to the non-negativity constraint $a \geq 0$. In Appendix F, we show that the welfare-maximizing investment level is strictly positive if the adaptation technology satisfies the Inada condition $\lim_{a\to 0} T'(a) = \infty$. In the following, we proceed on the assumption that this condition is satisfied and that the optimal adaptation investment level is $a > 0$. We now examine how a positive investment level affects the deviation of τ_1 and τ_2 from the tax-smoothing principle as well as the effect on optimal public debt b, in comparison to a situation where adaptation is not available. In order to ensure tractability of this analysis, we confine ourselves to the linear-quadratic example already used in the previous section. Thus, consumption demand and the Laffer curve in period t are still given by $X(\tau_t) = (\alpha - \tau_t)/\beta$ and $R(\tau_t) = (\alpha\tau_t - \tau_t^2)/\beta$, respectively. Taking adaptation into account, the damage functions are now specified as $D_1(x_1) = \delta_1 x_1$ and $D_2\big[x_2 + \gamma x_1, T(a)\big] = \delta_2\big[x_2 + \gamma x_1 - \sqrt{a}\big]$, so the adaptation technology is specified as $T(a) = -\sqrt{a}$. Notice that, for this specification, we obtain the optimal policy in the absence of adaptation as a special case, if in the first-order conditions of the welfare maximum we ignore the optimality condition for a and set $a = 0$ in the remaining optimality conditions. Hence, the impact of adaptation a on the optimal policy (τ_1, τ_2, b) can be determined by running a comparative static analysis of the welfare maximum with respect to a and letting a increase from 0 (adaptation not available) to a strictly positive value $a > 0$ (adaptation available). Details on this analysis can be found in Appendix G.

Subtracting the second-period budget $\tau_2 X_2(\tau_2) - b = g$ from the first-period budget $\tau_1 X_1(\tau_1) + b = g + a$ and using $R(\tau_t) = \tau_t X(\tau_t)$, we obtain the central equation

$$b = \frac{a}{2} + \frac{R(\tau_2) - R(\tau_1)}{2}. \tag{19}$$

According to (19), adaptation exerts a direct positive effect on optimal debt as public debt is used to distribute the costs of adaptation over both periods. In addition, adaptation also influences the optimal tax rates τ_1 and τ_2 such that we observe an indirect effect via changes in the tax revenues $R(\tau_1)$ and $R(\tau_2)$. Differentiating (19), the overall effect is

$$\frac{\mathrm{d}b}{\mathrm{d}a} = \frac{1}{2} + \frac{1}{2}\left[R'(\tau_2)\frac{\mathrm{d}\tau_2}{\mathrm{d}a} - R'(\tau_1)\frac{\mathrm{d}\tau_1}{\mathrm{d}a}\right]. \tag{20}$$

Equation (20) shows that the effect of adaptation on optimal debt will typically deviate from 1/2, i.e. from an equal distribution of adaptation costs across both periods. In general, the bracketed term in (20) indicates that optimal debt will additionally be influenced by an unequal change in the tax revenues in each period. For the linear-quadratic specification, we show in Appendix G that the change in period t tax revenues equals

$$\frac{\mathrm{d}R(\tau_t)}{\mathrm{d}a} = \frac{\mathrm{d}\tau_t}{\mathrm{d}a}R'(\tau_t) = \frac{R'(\tau_t)^2}{R'(\tau_1)^2 + R'(\tau_2)^2}. \tag{21}$$

Substituting (21) into (20) yields

$$\frac{\mathrm{d}b}{\mathrm{d}a} = \frac{1}{1 + [R'(\tau_1)/R'(\tau_2)]^2}. \tag{22}$$

From this expression we already see that the overall effect of adaptation on optimal debt will always be positive. However, whether the indirect effect via changes in tax revenues amplifies or mitigates the direct effect, i.e. whether the total effect is larger or smaller than 1/2, depends on the relation between $R'(\tau_1)$ and $R'(\tau_2)$, thus, on the exact location of the optimal tax rates on the Laffer curve. In Appendix G we prove

Proposition 4. *Consider a linear-quadratic specification of the model with adaptation, i.e. assume $V(x_t) = (1+\alpha)x_t - \beta x_t^2/2$, $D_1(x_1) = \delta_1 x_1$ and $D_2\big[x_2 + \gamma x_1, T(a)\big] = \delta_2\big[x_2 + \gamma x_1 - \sqrt{a}\big]$ with $\alpha, \beta, \delta_1, \delta_2 > 0$. Optimal adaptation is then strictly positive ($a > 0$), and $\tau_1 \gtreqless \tau_2$ if and only if $\delta_1 + \gamma\delta_2 \gtreqless \delta_2$, as in the version of the model without adaptation. The impact of adaptation on optimal fiscal policy is described by the following statements:*

(i) $\dfrac{\mathrm{d}R(\tau_t)}{\mathrm{d}a} > 0$ *for* $t = 1, 2$.

(ii) If τ_1 and τ_2 are both on the increasing side of the Laffer curve, then

$$\frac{\mathrm{d}b}{\mathrm{d}a}\begin{cases} \in \left(\frac{1}{2}, 1\right), \\ = \frac{1}{2} \\ \in \left(0, \frac{1}{2}\right), \end{cases} \quad \textit{and} \quad \frac{\mathrm{d}R(\tau_1)}{\mathrm{d}a} - \frac{\mathrm{d}R(\tau_2)}{\mathrm{d}a}\begin{cases} < 0, & \textit{if } \delta_1 + \gamma\delta_2 > \delta_2, \\ = 0, & \textit{if } \delta_1 + \gamma\delta_2 = \delta_2, \\ > 0, & \textit{if } \delta_1 + \gamma\delta_2 < \delta_2. \end{cases}$$

(iii) If τ_1 and τ_2 are both on the decreasing side of the Laffer curve, then

$$\frac{\mathrm{d}b}{\mathrm{d}a}\begin{cases}\in \left(0, \frac{1}{2}\right), \\ = \frac{1}{2} \\ \in \left(\frac{1}{2}, 1\right),\end{cases} \quad \textit{and} \quad \frac{\mathrm{d}R(\tau_1)}{\mathrm{d}a} - \frac{\mathrm{d}R(\tau_2)}{\mathrm{d}a}\begin{cases} > 0, & \textit{if } \delta_1 + \gamma\delta_2 > \delta_2, \\ = 0, & \textit{if } \delta_1 + \gamma\delta_2 = \delta_2, \\ < 0, & \textit{if } \delta_1 + \gamma\delta_2 < \delta_2.\end{cases}$$

Notice first that the insights from part (ii) of Proposition 2 generalize to our model specification with endogenous adaptation. That is, optimal emissions tax rates are higher in the period where consumption emissions causes the higher cumulative marginal damages ($\tau_1 \gtreqless \tau_2$ if and only if $\delta_1 + \gamma\delta_2 \gtreqless \delta_2$). Optimal adaptation investments are strictly positive ($a > 0$), since the adaptation technology $T(a) = -\sqrt{a}$ satisfies the Inada condition.

The most important insight from Proposition 4 regards the impact of adaptation on optimal fiscal policy. In order to finance the additional expenditures for optimal adaptation, the government increases optimal tax revenues in both periods, see $\mathrm{d}R(\tau_t)/\mathrm{d}a > 0$ in part (i) of Proposition 4, as well as optimal debt, see $\mathrm{d}b/\mathrm{d}a > 0$ in all cases of parts (ii) and (iii) of Proposition 4. Moreover, adaptation reduces the deviation of the optimal tax rates from the tax-smoothing principle, i.e. tax rates (or, equivalently, tax revenues) in the two periods move closer together. This can be seen from the sign of the change in the difference between tax revenues, i.e. $\mathrm{d}R(\tau_1)/\mathrm{d}a - \mathrm{d}R(\tau_2)/\mathrm{d}a$ in parts (ii) and (iii) of Proposition 4, which depends on the location of the tax rates on the Laffer curve and the development of the cumulative marginal damages. The sign of this expression also determines whether the direct effect of adaptation on optimal debt is amplified or mitigated by the indirect effect, i.e. whether debt increases by more or less than $1/2$.

To provide an example, we focus on one case from parts (ii) and (iii) of Proposition 4 and leave the discussion of the other cases to the reader.[7] Suppose optimal tax rates are on the decreasing side of the Laffer curve and cumulative marginal damages are decreasing, so we are in part (iii) of Proposition 4 with $\delta_1 + \gamma\delta_2 > \delta_2$. This case is illustrated in Figure 2. Due to $\delta_1 + \gamma\delta_2 > \delta_2$, the government chooses a higher tax rate in the first period than in the second period, $\tau_1 > \tau_2$, and issues a positive level of debt, $b > 0$. The optimal tax rates in the absence of adaptation are simply denoted by τ_1 and τ_2. If adaptation becomes available, the government reduces both tax rates from τ_1 and τ_2 to τ_1^a

[7] We can again provide a numerical example for each of the several cases contained in Proposition 4 in order to show that each case is satisfied by a non-empty set of parameter constellations. Details on this numerical exercise can be obtained from the authors upon request.

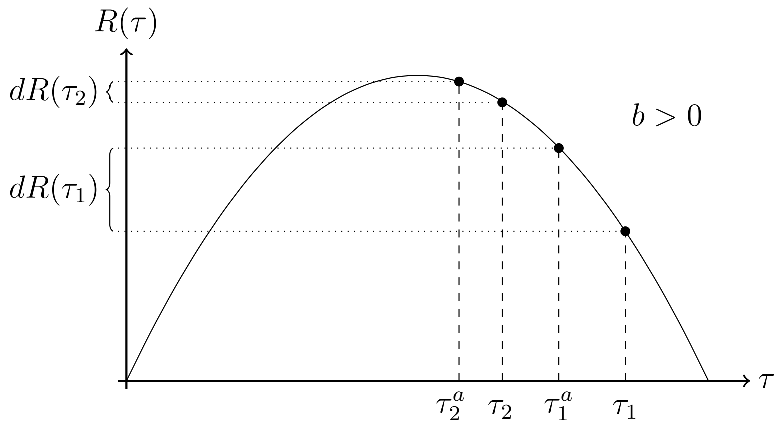

Figure 2: Effect of adaptation on tax rates and public debt

and τ_2^a, respectively, in order to increase tax revenues in both periods. However, investing in adaptation leads to a more significant drop in the first-period tax rate. This implies a larger gain of tax revenues in period 1, see $\mathrm{d}R(\tau_1)/\mathrm{d}a - \mathrm{d}R(\tau_2)/\mathrm{d}a > 0$ in part (iii) of Proposition 4, and the difference in tax revenues, $R(\tau_2) - R(\tau_1)$, falls. The opportunity of adaptation therefore moves optimal taxation closer to the tax-smoothing principle. The intuition is that adaptation reduces second-period environmental damages, so overall the importance of pollution is reduced and we are closer to a world without externalities. Remember that, nevertheless, optimal public debt increases due to the direct financing effect of adaptation investment. Under the conditions of Figure 2, this direct effect is mitigated by the indirect effect, since optimal debt is positive and the difference between first- and second-period tax revenues decreases. Hence, adaptation increases optimal debt by less than 1/2, as shown by $\mathrm{d}b/\mathrm{d}a \in (0, 1/2)$ in part (iii) of Proposition 4.

5 Conclusion

In this paper, we introduce a taxable emissions externality into the standard tax-smoothing framework of public debt. When the government levies an emissions tax not only to raise funds for public expenditures but also in order to restrict private consumption of a polluting good, adhering to a balanced budget rule is no longer optimal even if spending requirements are constant over time. Instead, running a deficit at the end of the first period is welfare maximizing either if the tax rates are on the increasing side of the Laffer curve and cumulative marginal damages from pollution increase over time or if the tax rates are on the decreasing side of the Laffer curve while marginal damages decrease over time. In contrast, for constant spending requirements public savings turn out to be optimal if the tax rates are on the increasing side of the Laffer curve and cumulative marginal damages are decreasing or if the tax rates are on the decreasing side of the Laffer curve and cumulative marginal damages are increasing. In either of these cases, the optimal policy prescribes non-constant tax rates and a non-balanced public budget, deviating from the tax-smoothing principle. In an extension, we introduce adaptation to climate change as an endogenous spending margin. As this technology attenuates environmental damages, we move closer to the tax-smoothing solution with weaker incentives to impose non-constant tax rates. Nevertheless, investments in the adaptation technology always create an additional incentive to increase public debt.

While our paper provides a theoretical analysis of the relation between environmental externalities and optimal public debt, it is an interesting and important task for future research to empirically assess this relation. In terms of our analysis, it is of particular importance to estimate Laffer curves for emissions taxes and to find out on which side of the Laffer curve optimal tax rates are located. In addition, it is likewise important to assess the time path of cumulative marginal environmental damages. Empirical insights on the Laffer curve of emissions taxes and the cumulative marginal damages can then immediately be applied to our analysis, in order to provide guidance for policy makers whether they should optimally increase or decrease public debt as a response to a more intensive use of emissions taxation. Such an empirical analysis is comprehensive and, thus, beyond the scope of the present paper and left for future research.

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Appendix

(A) Determinant of the bordered Hessian of (7)–(11). The bordered Hessian H of the system of equations (7)–(11) can be written as

$$H = \begin{pmatrix} L_{\lambda_1\lambda_1} & L_{\lambda_1\lambda_2} & L_{\lambda_1 b} & L_{\lambda_1\tau_1} & L_{\lambda_1\tau_2} \\ L_{\lambda_2\lambda_1} & L_{\lambda_2\lambda_2} & L_{\lambda_2 b} & L_{\lambda_2\tau_1} & L_{\lambda_2\tau_2} \\ L_{b\lambda_1} & L_{b\lambda_2} & L_{bb} & L_{b\tau_1} & L_{b\tau_2} \\ L_{\tau_1\lambda_1} & L_{\tau_1\lambda_2} & L_{\tau_1 b} & L_{\tau_1\tau_1} & L_{\tau_1\tau_2} \\ L_{\tau_2\lambda_1} & L_{\tau_2\lambda_2} & L_{\tau_2 b} & L_{\tau_2\tau_1} & L_{\tau_2\tau_2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & x_1+\tau_1 X_1' & 0 \\ 0 & 0 & -1 & 0 & x_2+\tau_2 X_2' \\ 1 & -1 & 0 & 0 & 0 \\ x_1+\tau_1 X_1' & 0 & 0 & L_{\tau_1\tau_1} & -\gamma D_2'' X_1' X_2' \\ 0 & x_2+\tau_2 X_2' & 0 & -\gamma D_2'' X_1' X_2' & L_{\tau_2\tau_2} \end{pmatrix}, \tag{23}$$

with

$$L_{\tau_1\tau_1} = -X_1' - (D_1' + \gamma D_2')X_1'' - (D_1'' + \gamma^2 D_2'')X_1'^2 + \lambda_1(2X_1' + \tau_1 X_1''), \tag{24}$$

$$L_{\tau_2\tau_2} = -X_2' - D_2' X_2'' - D_2'' X_2'^2 + \lambda_2(2X_2' + \tau_2 X_2''), \tag{25}$$

and $x_t = X(\tau_t)$,$X_t' := X'(\tau_t)$, $X_t'' := X''(\tau_t)$, $D_t' := D'(x_t)$ and $D_t'' := D''(x_t)$. Calculating the determinant of H with standard methods gives

$$|H| = (x_1+\tau_1 X_1')^2 L_{\tau_2\tau_2} + (x_2+\tau_2 X_1')^2 L_{\tau_1\tau_1} + 2\gamma(x_1+\tau_1 X_1')(x_2+\tau_2 X_2')D_2'' X_1' X_2'. \tag{26}$$

(B) Proof of Proposition 1. From $D_1 \equiv D_2 \equiv 0$ and (17) and (18) we obtain

$$\lambda = X(\tau_1)/[X(\tau_1) + \tau_1 X'(\tau_1)], \quad \lambda = X(\tau_2)/[X(\tau_2) + \tau_2 X'(\tau_2)]. \tag{27}$$

Since $\lambda \geq 1 > 0$ and $X(\tau_t) > 0$, it follows $X(\tau_t) + \tau_t X'(\tau_t) > 0$ for $t = 1, 2$. Hence, in each period the optimal tax rate is on the increasing side of the Laffer curve. As $X'(\cdot) < 0$, we have $X(\tau_t) + \tau_t X'(\tau_t) < X(\tau_t)$ and therefore (27) implies $\lambda > 1$ and $z_1 = z_2 = 0$ by the slackness conditions (12) and (13). Moreover, (27) shows that τ_1 and τ_2 are determined by the same equation implying $\tau_1 = \tau_2 = \tau$. Inserting this into the intertemporal budget constraint (14) gives $\tau X(\tau) = (g_1 + g_2)/2$. Finally, substituting $\tau_1 = \tau_2 = \tau$ and $z_1 = z_2 = 0$ into (15) yields $b = (g_1 - g_2)/2$.

(C) Proof of Proposition 2. Part (i) immediately follows from $\lambda > 1$, (12) and (13). In order to prove part (ii), rewrite (17) and (18) as

$$F(\tau_1) = G_1(\tau_1, \tau_2), \quad F(\tau_2) = G_2(\tau_2, \tau_1), \tag{28}$$

with

$$F(\tau) := \tau - \frac{1-\lambda}{\lambda}\frac{X(\tau)}{X'(\tau)}, \quad F'(\tau) := 1 - \frac{1-\lambda}{\lambda}\frac{[X'(\tau)]^2 - X(\tau)X''(\tau)}{[X'(\tau)]^2} \gtreqless 0, \tag{29}$$

and

$$G_1(\tau_1, \tau_2) := \frac{D_1'[X(\tau_1)] + \gamma D_2'[X(\tau_2) + \gamma X(\tau_1)]}{\lambda}, \quad \frac{\partial G_1(\tau_1, \tau_2)}{\partial \tau_1} = \frac{\left\{D_1''[\cdot] + \gamma^2 D_2''[\cdot]\right\} X'(\tau_1)}{\lambda} \leq 0, \tag{30}$$

$$G_2(\tau_2, \tau_1) := \frac{D_2'[X(\tau_2) + \gamma X(\tau_1)]}{\lambda}, \quad \frac{\partial G_2(\tau_2, \tau_1)}{\partial \tau_2} = \frac{D_2''[\cdot]X'(\tau_2)}{\lambda} \leq 0 \tag{31}$$

where the signs of $\partial G_1(\tau_1, \tau_2)/\partial \tau_1$ and $\partial G_2(\tau_2, \tau_1)/\partial \tau_2$ follow from $D_t''[\cdot] \geq 0$ and $X'(\tau_t) < 0$. Hence, G_1 and G_2 are non-increasing functions in τ_1 and τ_2, respectively, while $F(\tau)$ may be increasing or decreasing in its only argument τ. Consider first the case where $F(\tau)$ is increasing in τ. This case is illustrated in Figure 3. In the left panel of this figure, we consider the case where τ_1 and τ_2 are such that $D_1' + \gamma D_2' > D_2'$

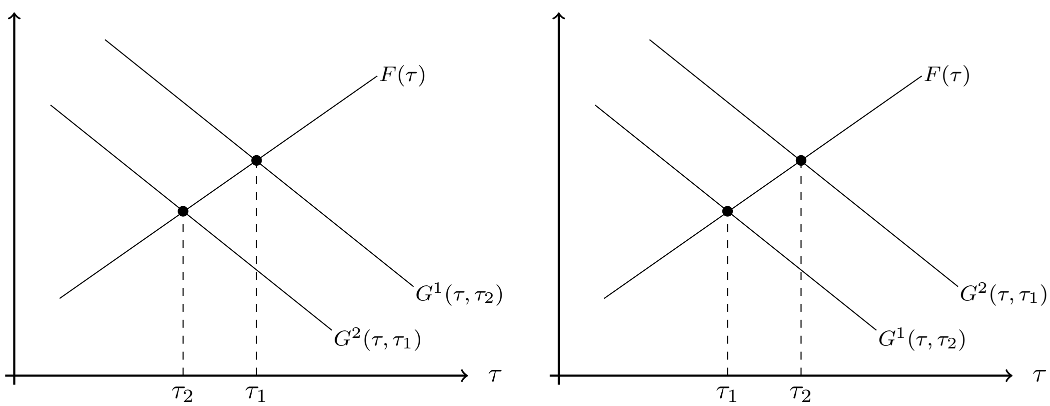

Figure 3: Proof of Proposition 2 part (ii)

and, thus, $G_1(\tau, \tau_2)$ lies above $G_2(\tau, \tau_1)$. It immediately follows that $\tau_1 > \tau_2$. In the right panel, τ_1 and τ_2 are such that $D_1' + \gamma D_2' < D_2'$ and $G_1(\tau, \tau_2)$ lies below $G_2(\tau, \tau_1)$. Hence, we obtain $\tau_1 < \tau_2$. If τ_1 and τ_2 are such that $D_1' + \gamma D_2' = D_2'$, then $G_1(\tau, \tau_2)$ and $G_2(\tau, \tau_1)$ are identical and we obtain $\tau_1 = \tau_2$ (not displayed in Figure 3). The same line of reasoning applies if the function $F(\tau)$ is decreasing but not steeper than $G_1(\tau, \tau_2)$ and $G_2(\tau, \tau_1)$ (also not displayed in Figure 3). This completes the proof of part (ii). Note that it is not possible that $F(\tau)$ is decreasing and steeper than $G_1(\tau, \tau_2)$ and $G_2(\tau, \tau_1)$. In this case, it can be shown that $L_{\tau_1\tau_1} > 0$ and $L_{\tau_2\tau_2} > 0$ and, thus, the bordered Hessian is $|H| > 0$, i.e. the second-order conditions of welfare maximization are violated.[8]

[8] We can rewrite (25) as $L_{\tau_2\tau_2} = (2\lambda_2 - 1)X_2' - D_2''X_2'^2 - (D_2' - \lambda_2\tau_2)X_2''$. From (9) we obtain $D_2' - \lambda_2\tau_2 = (\lambda_2 - 1)X_2/X_2'$. Inserting this expression into the second derivative of the Lagrangian gives

Next turn to part (iii). If $D_1' + \gamma D_2' < -x_1/X_1'$ and $D_2' < -x_2/X_2'$, (17) and (18) imply $x_1 + \tau_1 X'(\tau_1) > 0$ and $x_2 + \tau_2 X'(\tau_2) > 0$, i.e. both tax rates are on the increasing side of the Laffer curve. Moreover, rearranging (17) and (18) in this case gives $\tau_1 > D_1' + \gamma D_2'$ and $\tau_2 > D_2'$. Taking into account part (ii) and that both tax rates are on the increasing side of the Laffer curve, we obtain $\tau_1 X(\tau_1) \lesseqqgtr \tau_2 X(\tau_2)$ if and only if $D_1' + \gamma D_2' \lesseqqgtr D_2'$. Using this together with $z_1 = z_2 = 0$ in (15) proves the result with respect to optimal debt b in part (iii). Finally, the proof of part (iv) is perfectly analogous to that of part (iii).

(D) Details on the numerical examples in Table 2. Under the linear-quadratic specification and binding slackness conditions ($z_1 = z_2 = 0$), the intertemporal budget constraint (14) becomes

$$\frac{\alpha\tau_1 - \tau_1^2}{\beta} + \frac{\alpha\tau_2 - \tau_2^2}{\beta} = g_1 + g_2. \tag{32}$$

The function for optimal debt (15) equals

$$b = \frac{g_1 - g_2}{2} + \frac{\alpha\tau_2 - \tau_2^2 - (\alpha\tau_1 - \tau_1^2)}{2\beta}. \tag{33}$$

The first-order conditions (17) and (18) for welfare maximization can be written as

$$\lambda = \frac{\alpha - \tau_1 - \delta_1 - \gamma\delta_2}{\alpha - 2\tau_1}, \quad \lambda = \frac{\alpha - \tau_2 - \delta_2}{\alpha - 2\tau_2}. \tag{34}$$

For a given parameter constellation $(\alpha, \beta, \gamma, \delta_1, \delta_2, g_1, g_2)$, we solve the system of equations (32)–(34) with respect to the optimal policy $(\tau_1, \tau_2, b, \lambda)$ with the help of the software Mathematica. In order to prove the second-order conditions note that from (24) and (25) we obtain

$$L_{\tau_1\tau_1} = L_{\tau_2\tau_2} = \frac{1 - 2\lambda}{\beta} < 0 \tag{35}$$

since $\lambda > 1$. The determinant of the bordered Hessian (26) turns into

$$|H| = \frac{1 - 2\lambda}{\beta^3}\Big[(\alpha - 2\tau_2)^2 + (\alpha - 2\tau_1)^2\Big] < 0. \tag{36}$$

Hence, under the linear-quadratic specification of our model, the second-order conditions of welfare maximization are always satisfied.

(E) Proof of Proposition 3. For $\lambda = 1$ equation (17) and (18) can be written as

$$\frac{x_1 + (D_1' + \gamma D_2') \cdot X_1'}{x_1 + \tau_1 X_1'} = 1 = \frac{x_2 + D_2' \cdot X_2'}{x_2 + \tau_2 X_2'}, \tag{37}$$

$L_{\tau_2\tau_2} = \big[(2\lambda_2 - 1)(X_2')^2 - D_2'' X_2'^3 + (1 - \lambda_2) X_2 X_2''\big]/X_2'$. If $F(\tau)$ is decreasing and steeper than G_2, it is straightforward to show with the help of (29) and (31) that the bracket term in $L_{\tau_2\tau_2}$ is negative and, thus, $L_{\tau_2\tau_2} > 0$. In the same way we can show $L_{\tau_1\tau_1} > 0$ if $F(\tau)$ is decreasing and steeper than G_1. Using theses signs in (26) and taking into account that we focus on the case where both tax rates are on the same side of the Laffer curve, i.e. $\text{sign}\{x_1 + \tau_1 X_1'\} = \text{sign}\{x_2 + \tau_2 X_2'\}$, we obtain $|H| > 0$.

It follows that the optimal tax rates are $\tau_1 = D_1' + \gamma D_2'$ and $\tau_2 = D_2'$ which proves part (ii) of Proposition 3. For $D_1' + \gamma D_2' < -x_1/X_1'$ and $D_2' < -x_2/X_2'$, equation (37) implies that τ_1 and τ_2 are on the increasing side of the Laffer curve. As we assume $z_1 = z_2$, the optimal debt level in (15) becomes

$$b = \frac{g_1 - g_2}{2} + \frac{D_2' \cdot X(D_2') - (D_1' + \gamma D_2') \cdot X(D_1' + \gamma D_2')}{2}. \tag{38}$$

Since the tax rates are on the increasing side of the Laffer curve, we immediately obtain the result for b in part (iii), which completes the proof of part (iii). The proof of part (iv) is analogous. Finally, inserting (38) into (10) and (11) and solving with respect to z_1 and z_2 shows part (i) of Proposition 3.

(F) Proof of an interior solution with respect to welfare-maximizing adaptation. The Lagrangian for the modified welfare-maximization problem in the presence of adaptation reads

$$\begin{aligned} L = \sum_{t=1,2} \Big\{ V\big[X(\tau_t)\big] + 1 - (1+\tau_t)X(\tau_t) \Big\} - D_1\big[X(\tau_1)\big] - D_2\big[X(\tau_2) + \gamma X(\tau_1), T(a)\big] \\ + \lambda_1\big[\tau_1 X(\tau_1) + b - g - a\big] + \lambda_2\big[\tau_2 X(\tau_2) - b - g\big]. \end{aligned} \tag{39}$$

We obtain the first-order conditions

$$L_b = \lambda_1 - \lambda_2 = 0, \tag{40}$$

$$\begin{aligned} L_{\tau_1} = -X(\tau_1) - \Big\{ D_{1,X}[X(\tau_1)] + \gamma D_{2,X}[X(\tau_2) + \gamma X(\tau_1), T(a)] \Big\} X'(\tau_1) \\ + \lambda_1 \Big[X(\tau_1) + \tau_1 X'(\tau_1) \Big] = 0, \end{aligned} \tag{41}$$

$$L_{\tau_2} = -X(\tau_2) - D_{2,X}[X(\tau_2) + \gamma X(\tau_1), T(a)] X'(\tau_2) + \lambda_2 \Big[X(\tau_2) + \tau_2 X'(\tau_2) \Big] = 0, \tag{42}$$

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g - a = 0, \tag{43}$$

$$L_{\lambda_2} = \tau_2 X(\tau_2) - b - g = 0, \tag{44}$$

as well as the slackness conditions for adaptation investments

$$L_a = -D_{2,T}[X(\tau_2) + \gamma X(\tau_1), T(a)] T'(a) - \lambda \leq 0, \qquad a \geq 0, \qquad a L_a = 0. \tag{45}$$

As long as the adaptation technology satisfies the Inada condition $\lim_{a \to 0} T'(a) = \infty$, the latter condition implies $a > 0$, since for $a \to 0$ we have $L_a \to \infty > 0$ and $L_a \leq 0$ is violated.

(G) Proof or Equation (21) and Proposition 4. In order to derive the marginal effect of adaptation on the tax rates used in (21), note that we can view (40)–(44) as a system of 5 equations that determine the 5 variables $(b, \tau_1, \tau_2, \lambda_1, \lambda_2)$ as a function of a. Due to the linear-quadratic specification of the model, we obtain $D_{1,X} + \gamma D_{2,X} = \delta_1 + \gamma \delta_2$ and $D_{2,X} = \delta_2$. Hence, adaptation a influences $(b, \tau_1, \tau_2, \lambda_1, \lambda_2)$ only via equation (43). We employ Cramer's Rule to obtain

$$\frac{\mathrm{d}\tau_t}{\mathrm{d}a} = \frac{|J_{\tau_t}|}{|J|}, \tag{46}$$

where J represents the Jacobian of (40)–(44) and J_{τ_t} denotes the adjusted Jacobian in which the column containing the derivatives with respect to τ_t is substituted for by the replacement vector of a containing the derivatives with respect to a. The Jacobian J coincides with the bordered Hessian in (23). Hence, in the linear-quadratic example, we obtain

$$|J| = |H| = \begin{vmatrix} 0 & 0 & 1 & R'(\tau_1) & 0 \\ 0 & 0 & -1 & 0 & R'(\tau_2) \\ 1 & -1 & 0 & 0 & 0 \\ R'(\tau_1) & 0 & 0 & (1-2\lambda)/\beta & 0 \\ 0 & R'(\tau_2) & 0 & 0 & (1-2\lambda)/\beta \end{vmatrix} = \frac{1-2\lambda}{\beta}\left[R'(\tau_1)^2 + R'(\tau_2)^2\right] < 0, \quad (47)$$

where we used $\lambda_1 = \lambda_2 = \lambda > 1$ and $R'(\tau_t) = x_t + \tau_t X'(\tau_t)X(\tau)$. In order to obtain J_{τ_1} (J_{τ_2}), we substitute the replacement vector $(-L_{\lambda_1 a}, -L_{\lambda_2 a}, -L_{ba}, -L_{\tau_1 a}, -L_{\tau_2 a})' = (1, 0, 0, 0, 0)'$ for the fourth (fifth) column in the determinant of (47). The adjusted Jacobian can then be computed as

$$|J_{\tau_t}| = \frac{1-2\lambda}{\beta} R'(\tau_t). \quad (48)$$

Dividing (48) by (47) results in

$$\frac{\mathrm{d}\tau_t}{\mathrm{d}a} = \frac{R'(\tau_t)}{R'(\tau_1)^2 + R'(\tau_2)^2}. \quad (49)$$

which completes the proof of equation (21).

In order to proof Proposition 4, note first that we can show $\tau_1 \gtreqless \tau_2$ if and only if $\delta_1 + \gamma\delta_2 \gtreqless \delta_2$ by the same steps as in Proposition 2, since for this proof we only need equations (41) and (42) which do not depend on a under the linear-quadratic model specification. To proof part (i) of Proposition 4 simply verify that (21) is always positive. It remains to show parts (ii) and (iii) of Proposition 4. Consider first part (ii) and focus on the case $\delta_1 + \gamma\delta_2 > \delta_2$, so $\tau_1 > \tau_2$. Since both tax rates are on the increasing side of the Laffer curve in this case, $R''(\tau_t) = -2/\beta < 0$ implies $R'(\tau_1) < R'(\tau_2)$. Hence, the effect in (49), while positive in both periods, is larger on τ_2 than on τ_1. Since τ_2 was initially lower than τ_1 in the absence of adaptation, this implies that the wedge between the tax rates decreases. The same holds true with respect to the changes of tax revenues captured by (21), so we obtain $\mathrm{d}R(\tau_1)/\mathrm{d}a - \mathrm{d}R(\tau_2)/\mathrm{d}a < 0$. The effect of a on public debt in (22) is $\mathrm{d}b/\mathrm{d}a \in (1/2, 1)$ since $R'(\tau_1)/R'(\tau_2) < 1$ due to $R'(\tau_2) > R'(\tau_1) > 0$. In the opposite case, if $\delta_1 + \gamma\delta_2 < \delta_2$, we observe that $\tau_1 < \tau_2$. Then, $R'(\tau_1) > R'(\tau_2) > 0$ implies that $\mathrm{d}\tau_1/\mathrm{d}a > \mathrm{d}\tau_2/\mathrm{d}a$, $\mathrm{d}R(\tau_1)/\mathrm{d}a - \mathrm{d}R(\tau_2)/\mathrm{d}a > 0$ and $\mathrm{d}b/\mathrm{d}a \in (0, 1/2)$. Finally, for constant marginal damages $\delta_1 + \gamma\delta_2 = \delta_2$, we have $\tau_1 = \tau_2$ and, thus, $R'(\tau_1) = R'(\tau_2)$, $\mathrm{d}\tau_1/\mathrm{d}a = \mathrm{d}\tau_2/\mathrm{d}a$, $\mathrm{d}R(\tau_1)/\mathrm{d}a - \mathrm{d}R(\tau_2)/\mathrm{d}a = 0$ and $\mathrm{d}b/\mathrm{d}a = 1/2$, which completes the proof of part (ii) of Proposition 4. The proof of part (iii) can be conducted analogously if we recall that on the decreasing side of the Laffer curve $R'(\tau_t) < 0$, which means that (49) is negative. Therefore, both tax rates decrease in response to a marginal increase in a.