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Abstract

I first summarize and critically discuss the different normative views that are operational in the academic literature on education. Afterwards, I analyze how prioritarians would allocate resources in a dynamic model of skill formation and compare it to utilitarians and Rawlsians. I finish with a brief discussion of several plausible extensions of the resource allocation problem.

Keywords: prioritarianism, education, opportunities versus outcomes, intrinsic versus instrumental value, resource allocation, dynamic skill formation.

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1. Introduction

Equity in education is a big topic in various disciplines (economics, philosophy, psychology, and sociology). Yet, *operational* normative views—i.e., views that provide a clear mathematical formulation of how to compare the goodness of different 'social states'—turn out to be rare.[1] The first aim of this chapter is to summarize the normative views that are operational in the academic literature on education. This summary reveals that prioritarianism is compatible with the existing normative views.[2] Not the idea of priority as such, but "priority of what?" turns out to be the dividing line between the different normative views.

The answer to the question "priority of what?" leads to four normative views in the education literature, depending on (*i*) whether the relevant metric is educational (e.g., test scores and attitudes) or economic (e.g., future income and leisure) in nature and (*ii*) whether one wants to provide outcomes or opportunities in these metrics. The second aim of this chapter is to critically discuss these four normative views as applied to sufficiently young pupils. I do so by directly challenging the philosophical foundations underlying the different views. I complement this direct approach by assessing whether counterintuitive policy recommendations result from adopting these views in the context of a simple human capital investment problem: whose education should we invest in if we have an extra euro to spend? Both critiques highlight that outcomes, rather than opportunities, should be prioritized.

While the previous critical discussion was based on a simple static setting, it is interesting to look at optimal human capital investment in a dynamic setting. The third aim of this chapter is to examine the policy recommendations derived in a dynamic model of skill formation from an outcome-prioritarian—rather than an outcome-utilitarian or an outcome-Rawlsian—point of view. The results show that some utilitarian or Rawlsian claims are not true for prioritarians. For example, the utilitarian recommendation to reinforce (rather than to compensate) the impact of background in later stages of education does not necessarily hold for prioritarians. And, as another example, also the Rawlsian recommendation to equally compensate for ability and background (over all stages together) does not hold for prioritarians.

[1] In the case of economics, Waltenberg (2010:107) observes: "While the handbook of health economics […] contains three chapters explicitly concerned with normative health economics, […] the analogous handbook of economics of education [...] does not contain any chapter or part devoted exclusively to equity and education."

[2] Prioritarianism is explained at length by Adler (chapter 2, this volume). For now it suffices to know that prioritarianism provides some middle ground between utilitarianism and Rawlsianism. We come back to it later.

In the aforementioned static and dynamic setting, I consider a *school team* in *basic education* that optimally *designs* its *resource* allocation over pupils based on a prioritarian objective defined over an *educational index*. The fourth and final aim of this chapter is to check whether the derived policy recommendations also hold if I change the italicized elements in the benchmark story. In particular, I will focus on a different decision-maker (*minister* versus *school team*), a different age of pupils (*higher* versus *basic education*), a different normative exercise (*evaluation* versus *design*), different policy tools (beyond simple *resources*), and multidimensional outcomes (*multiple skills* versus an *educational index*).[3]

This chapter is organized as follows. Section 2 summarizes the different normative views that are operational in the academic education literature. Section 3 critically discusses these normative views as applied to sufficiently young pupils and argues that educational outcomes should be prioritized. Section 4 analyses outcome-prioritarian human capital investment in a dynamic model of skill formation. Section 5 briefly discusses several extensions. A final Section 6 concludes. Technical details are relegated to the Appendix.

2. Normative views in education

The aim of this section is to summarize the normative views adopted in the different disciplines that study education from an academic point of view. As this literature is huge, I use the following stringent selection principles. First, I focus on operational views, i.e., views for which there exists a clear mathematical formulation of how to compare the goodness of different social states in education.[4] Second, I focus on the broad methodological principles underlying the different research strands, not on their research findings. Third, references to publications will be limited, for obvious reasons, to a selection of key publications (with, admittedly, a bias towards research in economics).

In the remainder of this section, I first provide a brief overview of the literature. While this overview is quick and dirty, my hope is that the resulting classification encompasses the bulk

[3] One extension that I do not discuss is how to deal with (historical) discrimination. In other words, I will assume that educational differences that are linked to differences in background are based only on differences in resources (such as time, skills, and money) that families can and want to invest in their children, and not on discrimination. See, e.g., Schouten (2012) for a discussion.

[4] This principle excludes most of the philosophical literature. However, we will come back to that literature in the next section to critically discuss the operational normative views adopted in other disciplines.

of the literature.[5] Afterwards, I discuss in somewhat more detail whether prioritarianism is compatible with these approaches. In my view, the answer is clearly yes: not the idea of prioritarianism itself, but what should be prioritized, turns out to be the dividing line in the normative literature on education.

2.1. A brief overview

Table 1 classifies the academic education literature on the basis of its research aim: evaluation or design.

A first strand wants to *evaluate* either policy interventions (e.g., based on experimental or quasi-experimental data) or educational systems (e.g., based on large-scale internationally comparable data). This strand encompasses all disciplines with a quantitative interest in education (economics, psychology, and sociology).[6] The research in this literature is data-driven and focuses mostly on (pre-)primary and secondary education, probably because data is more readily available for these education levels in many countries.

Table 1: A classification of the education literature.

Research aim	Evaluation	Design
Disciplines	economics, psychology, sociology	mainly economics
Research type	data-driven (empirical)	model-driven (theoretical)
Education level	often (pre-)primary and secondary	often not specified
Cardinalization	educational opportunities economic opportunities	educational outcomes economic outcomes
Aggregation	dashboard, i.e., no aggregation	welfarist, including prioritarian

A second strand wants to optimally *design* educational policies (such as educational resources, accountability schemes, or vouchers). Especially economists are very active in this strand. The research in this literature is model-driven and the exact education level is often not specified, in which case it is meant to capture all levels in an abstract way.

I now turn to the normative views adopted in both strands, summarized in the second part of Table 1. I distinguish two steps: a cardinalization step (what is the appropriate metric?) and an

[5] I will briefly mention some exceptions that do not fit in the provided straitjacket.

[6] Still, there are some small differences between the different disciplines. Economists focus more on policy evaluations, sociologists prefer system evaluations, and psychologists (including educational scientists) do both.

aggregation step (how should these metrics be aggregated into an overall judgment of the goodness of social states?).

First, I discuss the cardinalization step.

In the *evaluation* literature, the (distributional) purpose of education is usually thought to be the provision of equal opportunities, defined as the same opportunities for all pupils independent of background characteristics (such as gender, race, ethnicity, family income, and parental education). Depending on whether equality of opportunity must hold for educational outcomes (e.g., skills and attitudes) or economic outcomes (e.g., future earnings and leisure), two sub-strands result: the equality of educational opportunity literature (initiated by Coleman, 1966) and the equality of economic opportunity literature (initiated by Bowles, 1973).[7]

In the *design* literature, the purpose of education is usually thought to provide outcomes, rather than opportunities.[8] Depending again on the outcome of interest, two sub-strands emerge. One strand focuses on optimal investment in education (initiated by Arrow, 1971), while the optimal redistribution literature (initiated by Ulph, 1977) designs educational subsidies, alongside earnings taxes, to generate and redistribute earnings.[9] The optimal redistribution literature is the only sub-strand that introduces preferences over net earnings and effort (to study and/or work). In these cases, the metric is usually utility, rather than earnings.

Second, I discuss the aggregation step in both literatures.

The *evaluation* literature usually follows a dashboard approach. In such a dashboard approach, researchers report several indices in isolation and do not specify how to trade off the different reported indices to arrive at a global evaluation.[10] The dashboard typically includes the average outcome, the dispersion in outcomes, and the correlation (or, more generally, the association) between outcomes and background characteristics. The correlation is important in the

[7] I refer to Bowles (1973) as an example of seminal work because he focuses specifically on the role of education in explaining unequal economic opportunities. See Brunori, Ferreira, and Peragine (chapter 11, this volume) for additional references on the equality of opportunity literature.

[8] There exists a small literature that focus on *designing* education policies from an equality-of-opportunity point of view; see, e.g., Betts and Roemer (2007) and Keane and Roemer (2009).

[9] Interested readers may also consult (*i*) Bruno (1976), de Fraja (2002), and Cunha and Heckman (2007) on optimal investment in education and (*ii*) Hare and Ulph (1979), Tuomala (1986), Fleurbaey, Gary-Bobo, and Maguain (2002), Brett and Weymark (2003), and Bovenberg and Jacobs (2005) on the joint design of education and taxation.

[10] The dashboard approach does not specify how to commensurate the different dashboard indicators and therefore, somewhat contrary to our definition of an operational normative view, it does not provide a complete mathematical formulation of how to judge the goodness of social states.

evaluation literature as it measures to what extent equality of opportunity is realized, where a correlation closer to zero is considered as better.

The *design* literature is deeply rooted in economics and therefore welfarist in nature. In other words, the global evaluation of goodness is an increasing function of the individual metric only. This welfarist approach often uses a prioritarian aggregation (to be defined in a more precise way later on), usually for pragmatic, rather than deep normative reasons.

2.2. Compatibility with prioritarianism

In the previous subsection I made a rough classification of different normative views in the academic education literature. While I initially started with two strands depending on research aim (evaluation or design), two sub-strands emerged within each strand, depending on the outcome of interest (educational or economic).

While prioritarianism as an aggregation device is often used in the 'welfarist' design literature, and is thus clearly compatible with this strand, the remaining question is therefore whether prioritarianism is also compatible with the evaluation literature. Nevertheless, as it is interesting to see the implications of employing a prioritarian approach in each case, I will discuss all four sub-strands in the sequel.

I introduce a simple example that will be used, with some slight modifications, throughout the rest of this chapter. Educational achievement e is an increasing function of innate ability a, background b, and (class) resources c; I write $e = E(a,b,c)$.[11] Economic achievement y is an increasing function of ability, background, and education; I write $y = Y(a,b,e)$ and thus I also have $y = Y(a,b,E(a,b,c))$.[12]

Some quick remarks. First, educational and economic achievement are assumed to be cardinally measurable. Second, for simplicity, all variables are assumed to be unidimensional (for now).[13]

[11] Ability, background, and resources are (assumed to be) good for educational achievement.

[12] Ability, background, and educational achievement are (assumed to be) good for economic achievement. Note that ability and background have a direct and indirect effect on economic achievement. Controlling for educational achievement, the direct effect of background on economic achievement captures, e.g., parental network effects that give access to more/better job offers or parental gifts that affect income.

[13] So, educational achievement is an index based on, e.g., acquired skills and attitudes; economic achievement is an index capturing, e.g., earned and unearned income; innate ability captures the influence of inborn genetic endowments; background is an index based on, e.g., gender, race, ethnicity, and socio-economic characteristics of the family (including the resources they invest); and class resources capture all resources at class level, e.g., class size, teacher quality/time, and peers.

Third, there is no effort at this stage, an issue to which I return when I discuss higher, rather than basic education.[14]

A common practice in the equality of educational opportunity literature is to measure opportunities by the expected (educational or economic) outcome, conditional on background.[15] These conditional expectations are denoted $\mathbb{E}[e \mid b]$ and $\mathbb{E}[y \mid b]$. According to opportunity-egalitarians, equality of this conditional expectation is, ceteris paribus, 'ideal' because it means that all pupils can expect the same outcome irrespective of their background. The metrics used in the four different sub-strands are summarized in Table 2.

Table 2. Four metrics underlying the different sub-strands.

	Opportunity	Outcome
Educational	$\mathbb{E}[e \mid b] = \mathbb{E}[E(a,b,c) \mid b]$	$e = E(a,b,c)$
Economic	$\mathbb{E}[y \mid b] = \mathbb{E}[Y(a,b,E(a,b,c)) \mid b]$	$y = Y(a,b,E(a,b,c))$

To aggregate these metrics over pupils, I define a prioritarian evaluation function. Let w be one of the four metrics defined in Table 2. The global prioritarian evaluation is essentially based on the average transformed metric, i.e., on

$$g^{-1}\left(\int g(w)f(w)dw\right), \tag{1}$$

where g is a strictly increasing and strictly concave transformation function and f is the density of the metric under consideration.[16] In particular, note that increasingness of the transformation function g guarantees that higher (expected) outcomes are better (no levelling down), while concavity ensures that increases in (expected) outcomes matter more for the worse off, being those with lower (expected) outcomes. How much more priority should be given to

[14] I do not claim that effort, as such, does not exist in basic education, but I will argue later on, that, contrary to higher education, effort does not require a different normative treatment compared to ability or background.

[15] This approach to equality of opportunity is called the ex ante (Van de gaer) approach, which differs from the ex post (Roemer) approach; see Brunori, Ferreira, and Peragine (chapter 11, this volume) for a discussion of both approaches.

[16] See Adler (chapter 2, this volume) for a detailed discussion of prioritarianism. Adding the inverse g^{-1} in Equation (1) does not change the ranking of social states (if there is no uncertainty), but allows for simple and clean decomposition results later on (in Table 3).

a worse off pupil is left open. Prioritarians lie in between utilitarians (all pupils have equal priority) and Rawlsians (the worst off pupil has absolute priority over all other pupils).[17]

I add some auxiliary assumptions to simplify the analysis.[18] To be clear, these additional assumptions are introduced only in this subsection and for ease of exposition; results in later sections will hold more generally. Under these auxiliary assumptions, the evaluation in each of the four sub-strands reduces to a simple formula based on some key statistics—the mean (μ), the variance (σ^2), and the squared correlation (ρ^2)—and the degree of priority π, which lies in between utilitarians (with $\pi \to 0$) and Rawlsians (with $\pi \to \infty$).

Table 3 summarizes.

Table 3. Four evaluations decomposed in key statistics and the degree of priority.

	Opportunity	Outcome
Educational	$\mu_e - \frac{\pi}{2} \cdot \rho_{be}^2 \cdot \sigma_e^2$	$\mu_e - \frac{\pi}{2} \cdot \sigma_e^2$
Economic	$\mu_y - \frac{\pi}{2} \cdot \rho_{by}^2 \cdot \sigma_y^2$	$\mu_y - \frac{\pi}{2} \cdot \sigma_y^2$

In both the opportunity-prioritarian and outcome-prioritarian approach, the mean is essentially corrected downwardly for inequalities. The correction depends on the degree of priority: the higher the degree of priority assigned to the worse off, the higher the downward correction will be, ceteris paribus.

The measure of inequality is different, however, in both approaches. While outcome-prioritarians only care about the variance, being a measure of dispersion, opportunity-prioritarians also care about the squared correlation between background and outcome. In particular, being squared, a zero correlation between background and outcome is, ceteris paribus, optimal according to opportunity-prioritarians. Alternatively, opportunity-prioritarians

[17] Utilitarians and Rawlsians are defined in this chapter as limiting cases of prioritarians, with either an infinitely small or an infinitely large degree of priority. As the latter limit is not well-defined, a Rawlsian strictly prefers one state over another if the worst off is strictly better off in the former state. This is compatible with both maximin and leximin objectives.

[18] First, the transformation function g is of the Kolm-Pollak type. Second, there is a linear relation between either educational or economic achievement and background and this relation is estimated using standard regression techniques. Third, background and regression errors are independent and follow normal distributions. These assumptions, as well as the results, can be found in Appendix A.1. I also show that if one starts from an Atkinson transformation function and adds log-linear relations and lognormal distributions, one ends up with a very similar but multiplicative decomposition that does not change the main story line.

only care about that part of the total variance that is morally objectionable in their view, i.e., the part that is correlated with background.[19]

2.3. A brief summary

While prioritarianism, as an aggregation device, is often used in the 'welfarist' design literature, and thus clearly compatible with it, I investigated whether prioritarianism is also compatible with the evaluation literature.

If I apply prioritarianism to an opportunity metric, the resulting opportunity-prioritarian view turns out to evaluate social states based on those key statistics that are typically reported in the dashboard approach that underlies most of the evaluation literature. So, while prioritarianism turns out to be compatible with the evaluation literature, prioritarians go one step further in fact, as they trade off these different statistics in a specific way.

To conclude, not so much priority as such, but the metric over which prioritarianism should be defined—i.e., the question "priority of what?"—is the bone of contention in my view. The next section will therefore critically discuss the four metrics of Table 2, while taking the prioritarian aggregation of Equation (1) for granted.

3. A critical discussion of the four metrics

In this section, I use a prioritarian aggregation procedure, i.e., Equation (1), and critically discuss the four metrics of Table 2 as applied to sufficiently young pupils (an assumption that will be made precise and relaxed later on in Section 5.2).

I will use two types of arguments to uncover whether a metric is reasonable or not. First, I use arguments from the philosophical literature on education that directly question the normative position underlying some of these metrics. Second, I use the Gospel of Matthew (7:16), "Ye shall know them by their fruits," and look at the policy consequences of adopting either approach in a simple human capital investment setting.[20] If counterintuitive results obtain in

[19] Note that this correlation does not only capture the direct effect of background on the outcome under consideration, but also the indirect effect of background via its correlation with ability (caused, e.g., by genetic inheritance from the parents) and with available class resources (caused, e.g., by the school choice of parents).

[20] This citation of Matthew is borrowed from Samuelson (1977) and has been applied to education by, e.g., Jencks (1988). I will adapt Jencks' example of 'teacher Higgins' later on.

simple settings, then the underlying normative goal seems questionable. This strategy is what philosophers refer to as a reflective equilibrium.[21]

The analysis below is structured along two questions that directly relate to the four metrics discussed before. What is the value of education: intrinsic or instrumental (or both)? And what should education provide: outcomes or opportunities? I start with the latter question.

3.1. Outcomes or opportunities?

Suppose that the aim of education is the provision of opportunities, as adopted in the bulk of the evaluation literature. From a distributional point of view the desire is that (educational and/or economic) outcomes do not depend on background characteristics. Two critical remarks can be made.

First, let us look at the philosophical literature on equality of opportunity. One normative reason underlying equality of opportunity is the idea that pupils are not (or cannot be held) responsible for their background. This seems a logical normative starting point in my view. Still, there exist many other traits—think, e.g., of variations in innate ability and perseverance for a given background—that also influence outcomes and that sufficiently young pupils cannot be held responsible for too.

The so-called luck-egalitarians have criticized the Rawlsian distinction between social contingencies (such as background) and natural contingencies (such as innate ability).[22] But if one endorses the position that these natural contingencies are equally arbitrary from a moral point of view, then the next question is: what are the normative consequences of this stance?

Recall that if pupils are not responsible for their background, then a common operationalization is to focus on the expected outcome conditional on background. However, if sufficiently young pupils cannot be held responsible for anything, then one should condition over all traits. But this implies that opportunities, understood as expectations conditional on all factors for which

[21] See, e.g., Adler (2019:22-23) for a more detailed elaboration.

[22] Admittedly, Rawls recognizes this problem: "[t]here is no more reason to permit the distribution of income and wealth to be settled by the distribution of natural assets than by historical and social fortune [...] since social contingencies and natural abilities are 'equally arbitrary' from a moral point of view (Rawls, 1999:64-65)." However, Rawls's view is that disadvantages caused by natural contingencies (e.g., innate ability) and disadvantages caused by social contingencies (e.g., social background) must be treated differently: the natural disadvantages in the final distribution of resources (regulated by the 'difference principle') and the social disadvantages in education itself (regulated by the principle of 'fair equality of opportunity').

a pupil cannot be held responsible, reduce to outcomes. In other words, promoting opportunities implies promoting outcomes.[23]

Second, let us investigate some of the policy consequences of equalizing opportunities. I use a teacher called Higgins, first introduced in Jencks (1988), to set up a simple story. Teacher Higgins is a third-grade teacher in a small town. During the school year he notices that two pupils start lagging behind, so he decides to devote some extra help to them right after each lunch break. According to teacher Higgins, both pupils have currently about the same educational achievement. Yet, both pupils are different in terms of innate ability and family background. The first pupil has limited abilities, but these are somewhat moderated by an advantageous family background. The second pupil has favorable abilities, but has a disadvantaged family background. Teacher Higgins knows that the same amount of extra time will lead to the same progress in educational achievement for both pupils: in the language of economists, there are no efficiency considerations to worry about.[24]

Teacher Higgins wonders how he should divide his extra time over both pupils. For now, I assume that (only) education matters.[25] Suppose that teacher Higgins would adopt an opportunity-prioritarian view, i.e., Equation (1) defined over educational opportunities, i.e., $\mathbb{E}[e \mid b]$, as the appropriate metric. As a consequence, he will devote more time and effort to the pupil with the lowest opportunities, being the second pupil with a disadvantaged background. The reason for teacher Higgins—and for all those adopting an opportunity-prioritarian view—to devote more resources to the second pupil is not that this pupil needs it more than the first pupil, or that this pupil will progress more from the same help. Rather, the reason is group-based: devoting more resources to the second pupil helps the group of pupils who are, like the second pupil, disadvantaged in terms of background and who are expected to lag behind. Teacher Higgins must tell the parents of the first pupil that, while their child also needs extra time and would equally benefit from it, he still has decided to give less extra help because their child does not belong to a group that is considered to be disadvantaged. This seems a violation of the first pupil's right to equal treatment in an otherwise equal educational situation.

[23] Recall that the focus here is on sufficiently young pupils in basic education. We come back to older pupils, say at college level, in the extensions.

[24] We focus here on educational opportunities, rather than economic opportunities. A similar story, with the same conclusions, can be told for economic opportunities.

[25] In the next subsection, I introduce well-being, a function of both education and economic achievement, as a broader currency of justice.

This violation would not have occurred if teacher Higgins used educational outcome *e* as the appropriate metric. In that case he would devote the same amount of extra resources to both pupils as they currently reach the same education level and as they would equally benefit from extra resources. The adoption of an outcome-prioritarian view implies therefore that teacher Higgins will give extra help to all pupils who lag behind, including pupils, like the first one, with learning difficulties, and pupils, like the second one, with a disadvantaged background.

I would like to stress several things.

First, to be clear, outcome-prioritarians, similar to opportunity-prioritarians, will on average assign more resources to pupils with a disadvantaged background. But, contrary to opportunity-prioritarians, outcome-prioritarians do not provide extra help to every pupil with a disadvantaged background. They provide it only to those pupils who need it, with needs defined on the basis of educational achievement rather than background. In other words, outcome-prioritarians also equalize opportunities to some extent. Somewhat related is the often heated discussion on (preferential) affirmative action. Opportunity-prioritarians would indeed assign more resources to the pupil with a disadvantaged background even if educational backlog and prospects for progress are identical. In other words, if education is the currency of justice, then they would provide an educational advantage to pupils with a disadvantaged background that is, in my view, preferential and thus hard to justify.[26]

Second, one could also wonder whether the choice of teacher Higgins, allocating his resources over pupils, is different from similar choices at higher levels, e.g., minister Higgins allocating her resources over schools. Jencks (1988:519) writes "I believe, but will not try to prove, that all the principled claims about how […] Higgins ought to allocate […] time among […] pupils recur in essentially the same form when we argue about how school principals, boards of education, or legislatures ought to allocate scarce resources." Indeed, if teachers and ministers agree that children cannot be held responsible for background and ability, then teachers and ministers should agree to focus on outcomes, rather than opportunities. But this does not imply that teachers and ministers should allocate resources over pupils in the same way.

[26] Two remarks. First, while (preferential) affirmative action is not desirable for young pupils, this can be different at later stages of education. In the context of college admissions, affirmative action can be justified if ability is thought to be (one of) the eligibility criteria. For example, mediocre high school results, caused by low abilities and an advantaged family background, could be a reason to refuse admission, while this is not necessarily true if these mediocre results are caused by high abilities and a disadvantaged background. Second, affirmative action can be justified if it is not preferential. Later on, we will discuss such a possibility when the currency of justice is more broadly defined (in terms of well-being based on both educational and economic achievement).

I briefly mention two cases here that will be discussed in more detail later on. One: even if teachers and ministers have the same normative goal, they may have different information and instruments at their disposal such that they could make different choices in the end.[27] For example, it is plausible that minister Higgins, in contrast to teacher Higgins, can only observe background and must therefore allocate resources over schools based only on the background of the pupils at the different schools. Two: teachers and ministers may agree that outcomes, not opportunities, are the appropriate metric, by they may focus on different outcomes. For example, it could be argued that teachers should focus on educational outcomes, while ministers should adopt a broader definition of well-being, based, e.g., on both educational and economic outcomes.

To recapitulate, my first argument shows that the responsibility-logic underlying equality of opportunity, consistently applied to sufficiently young pupils who cannot be held responsible for ability, background, and other traits, leads to the promotion of outcomes. My second argument shows that policies that promote opportunities deny some pupils the right to receive equal help, even if they equally need help and would equally benefit from receiving help. The reason is that, based on their ability deficit, they do not belong to a group that is recognized as being disadvantaged. This problem does not occur if one focuses on outcomes, because it implies that all educational backlog, whether caused by ability or background, is considered to be a reason for help (at least in the absence of efficiency considerations). I conclude that not opportunities, but outcomes are the appropriate metric for sufficiently young pupils. I have indicated that outcome-prioritarians also equalize opportunities, albeit in a different way. I have also stressed that teachers and ministers should both focus on outcomes, but this does not imply that they allocate resources over pupils in the same way, an issue to which I return in the next section.

3.2. Intrinsic or instrumental?

In the previous subsection, I have argued that it is appropriate to promote outcomes, not opportunities in case of sufficiently young pupils. In this subsection, I adopt this focus on outcomes, but investigate what outcome one should promote: education as an intrinsic goal or education as an instrument to improve, e.g., future economic achievement.

[27] In fact, Jencks (1988:519) seems to allude to this possibility: "I recognize, however, that the practical arguments for various possible distributions of […] Higgins's time are often quite different from those that come into play when a board of education or a legislature is allocating resources."

The normative position that education has both intrinsic and instrumental value seems to be common ground in the philosophical literature on education.[28] Such a position immediately discards using only economic achievement as the appropriate metric. Moreover, the position that only economic achievement should be the goal of education can also be criticized if one looks at its consequences. For example, in the (first-best) redistribution literature, it implies that equity and efficiency considerations can be separated. As a consequence, one should organize educational systems efficiently and redistribute optimally afterwards. Such an efficient organization of education would imply that those who are not efficient in attaining education (e.g., because of a limited ability or a disadvantaged background) will get a far more limited education that will be compensated for by monetary transfers in later life.[29] Such a policy seems objectionable and can be avoided by assigning intrinsic value to education too.

While the use of *only* economic achievement is thus not reasonable, the question whether *only* educational achievement would be reasonable is less evident. Suppose that I would define well-being as a strictly increasing function of education e and economic achievement y, say, $w(e, y)$. Using the definition $y = Y(a,b,e)$ introduced before, well-being can be rewritten as

$$w(e, Y(a,b,e)), \tag{2}$$

which is a function of ability, background, and education. Contrary to the previous section, well-being is now the currency of justice, which is broader than education only. Education has a double impact on well-being: directly (intrinsically) and indirectly (instrumentally, via its impact on economic achievement).

Ability and background play a role in Equation (2) and I must investigate whether this is reasonable or not. To judge this, I look again at teacher Higgins and his two pupils. Recall that both pupils had the same educational achievement and would equally benefit from receiving extra help. The first pupil had limited abilities, but these were somewhat moderated by an advantageous family background. The second pupil had normal abilities, but had a disadvantaged family background.

[28] See, e.g., Rawls (1999:87): "The value of education should not be assessed solely in terms of economic efficiency and social welfare. Equally if not more important is the role of education in enabling a person to enjoy the culture of his society and to take part in its affairs, and in this way to provide for each individual a secure sense of his own worth." While this view is common, note that the distinction between intrinsic and instrumental is not always very clear; see, e.g., Kristjánsson (2017).

[29] The same trade-off also holds in second-best settings, but to a lesser extent; see, e.g., Fleurbaey, Gary-Bobo, and Maguain (2002).

Important additional information, for the current story, is that teacher Higgins correctly anticipates that the same amount of progress in *educational achievement* will also lead to the same amount of progress in the *well-being* of both pupils. So, to sum up, the same extra resources would not only lead to the same educational progress of both pupils (as before), but the same progress in education would, in turn, also lead to the same progress in the well-being of both pupils. This additional assumption ensures, again, that there are no efficiency considerations in the story and, as a consequence, the allocation problem of teacher Higgins is a matter of equity only.

Teacher Higgins thinks it is best to adopt a prioritarian view, i.e., Equation (1), but now defined over well-being, i.e., Equation (2). Who should get priority? One way to examine priority, is to compare the marginal social welfare weights for both pupils, obtained by differentiating (1) with respect to class resources. As both pupils currently have the same educational achievement (say, $e_1 = e_2 \equiv e$), the marginal social welfare weight is, up to a common efficiency factor, equal to[30]

$$\phi'(w(e, Y(a_i, b_i, e))), \tag{3}$$

for pupil $i = 1, 2$. As a consequence, the pupil with a higher (expected) economic achievement, will get a lower priority (as ϕ' is strictly decreasing in w and w is strictly increasing in y).

Without loss of generality, I assume that, *given the same educational achievement*, background is (always) more valuable than ability for economic achievement, e.g., because parental network effects (to get a job) and parental gifts (to get started in adult life) are relatively more important for economic success. If true, then $Y(a_2, b_2, e) < Y(a_1, b_1, e)$ holds for the two pupils (for all education levels) and the marginal social welfare weight of the second pupil will be larger. This implies that the second pupil receives priority over the first pupil. As a consequence, teacher Higgins will devote more (extra) resources to the second pupil, but the reason is now that this second pupil can be expected to lag behind in future economic achievement. In other words,

[30] The efficiency factor captures the (direct and indirect) effect of resources on well-being and is, at the margin, equal to $[w_e'(e, y_i) + w_y'(e, y_i) Y_e'(a_i, b_i, e)] E_c'(a_i, b_i, c)$ for pupil $i = 1, 2$. This factor is, by assumption, equal for both pupils and therefore not relevant to assess their relative priority.

the second pupil receives priority in education, and will, in the end, be better educated than the first pupil, but only to compensate for a lower future economic success.[31]

Is this a reasonable outcome? Can teacher Higgins tell the parents of the less able pupil that he devotes less extra time to their child because it has better opportunities to succeed later in life anyway? In my view, he cannot justify this unequal treatment. Compensating pupils in education for having worse economic prospects in the future does not seem attractive, at least for a teacher.

While this conclusion is probably intuitive for teacher Higgins, it is, in the current setting, less obvious for minister Higgins. In the previous subsection, education was the only currency of justice. Assigning more educational resources to the second pupil would have given that second pupil an advantage (a higher education level) that could not be justified. In the current setting, however, the currency of justice is well-being. So, assigning more educational resources to the second pupil will, of course, also imply a higher education level, but only to compensate for the fact that this pupil would, even with the same education level, still lag behind in future economic achievement and thus also in well-being.[32] While I think that such a compensation should not be pursued by teacher Higgins, it is a plausible policy for minister Higgins. Let us discuss the consequences for teacher and minister Higgins separately.

For teacher Higgins I must adjust the metric in Equation (2) to ensure that it does no longer depend on ability and background. One possibility is to integrate out ability and background to arrive at an average well-being metric that depends only on education, i.e.,

$$\overline{w}(e) = \iint w(e, Y(a,b,e)) f(a,b) da db \,, \tag{4}$$

where f denotes the joint density function of ability and background.[33] The new metric $\overline{w}$ only depends on education, but still captures, on average, the intrinsic and instrumental impact of education on well-being. The resulting average well-being metric will be country-specific (or even region-specific) if, e.g., the instrumental importance of education for future economic

[31] Vice-versa, if background is (always) less valuable compared to ability, then the first pupil receives priority over the second pupil. It could also be that neither background nor ability are always less valuable; we do not discuss this possibility further.

[32] While there is (preferential) treatment in education, this treatment is meant to compensate and therefore does not provide a final advantage in terms of well-being. So, if well-being is the currency of justice, I would not necessarily classify this treatment as (preferential) affirmative action.

[33] I use f through this chapter to denote a density function. While somewhat loose, confusion is not possible as it is always clear from the context which density is meant.

achievement is different between countries.[34] This may matter because, intuitively, the higher the instrumental value of education in a country, the more one should care about the educational backlog of pupils in that country.

For minister Higgins, there are two possible normative positions. Either, the objective of minister Higgins coincides with the one of teacher Higgins: a prioritarian view, i.e., Equation (1), with well-being defined as in Equation (4). Minister Higgins will therefore not preferentially treat schools serving disadvantaged pupils to compensate for lower future economic success. This will be the benchmark case in the next section. Or, minister Higgins adopts a prioritarian view, i.e., Equation (1), but now with well-being defined as in Equation (2). In such a case she allows for educational policies that preferentially treat schools serving disadvantaged pupils to compensate for lower economic success. I will discuss this case in Section 5.1.2.

3.3. A brief summary

I have argued that both teachers and ministers should focus on outcomes, rather than opportunities in case of sufficiently young pupils.

For teachers, the relevant outcome is educational, but, still, it can accommodate for both the intrinsic and instrumental value of education, albeit only in some average way as in Equation (4).

For ministers, there are two possibilities. Either their outcome coincides with the one of teachers (Equation (4); the benchmark case) or their outcome is given by Equation (2) and thus deviates from teachers. In the latter case, ministers may preferentially treat pupils in education to compensate for lower future economic success. Whether pupils with a lower ability or pupils with a disadvantaged background should be preferentially treated depends on whether background or ability has a higher impact on economic success for the same education level. This is, in the end, an empirical question.

In the simple story of teacher and minister Higgins, I have up to now neglected at least two important considerations. First, to be sure that the discussion was about equity, I neglected efficiency considerations, assuming that the two pupils under consideration would equally benefit from an extra amount of help. Second, the model was static, while education is, in

[34] It is therefore not clear how one must compare educational systems across countries (as is often the purpose in the evaluation literature). We come back to it later on in Section 9.5.3.

reality, a dynamic cumulative process with different stages. The next section will look at the allocation of resources while taking these considerations into account.

4. Resource allocation and skill formation

The allocation of resources in education consists of at least two questions. How should one divide resources between the different stages of education (the vertical allocation question)? And how should one allocate resources over pupils with different characteristics at each stage (the horizontal allocation question)?

To answer both questions, I extend the previous static model to allow for different stages. In a first subsection I will therefore introduce a model of dynamic skill formation. In a second subsection I formulate the problem of an outcome-prioritarian school team that wants to optimally allocate resources over pupils in each stage. As mentioned before, ministers may allocate resources differently, because of different information or a different view on the relevant outcome; I discuss this extension, and other ones, later on.

4.1. A simple dynamic model of skill formation

The simplest possible way to get insight in the dynamics of education is to allow for two stages only. I will refer to stage 1 as representing the early stages of education and stage 2 as the later stages of education. Using obvious notation, the production of education at the different stages is governed by a simple additive process:[35]

$$\begin{aligned} e_1 &= E_1(a,b,c_1), \\ e_2 &= e_1 + E_2(a,b,c_2). \end{aligned} \tag{5}$$

As a consequence, I can define the final education level as

$$e \equiv E(a,b,c_1,c_2) \equiv E_1(a,b,c_1) + E_2(a,b,c_2). \tag{6}$$

I assume, as before, that the functions E_1 and E_2, and thus also the function E, are strictly increasing in all arguments. In addition, the law of diminishing returns implies that the functions E_1 and E_2, and thus also the function E, are strictly concave in class resources. Moreover, the

[35] It is best to think of this model as a growth model, i.e., E_1 and E_2 measure the change in skills (respectively from e_0=0 and e_1) in period 1 and 2. Without loss of generality, one could make the starting point e_0 non-zero (e.g., a function of ability and background only).

empirical literature tells us that some additional assumptions are plausible. I discuss these assumptions here in an informal way and refer to Appendix A.2 for the formal details.

First, *everything else equal*, investment is more effective at earlier stages, a result that Cunha et al. (2006:710) call "a major finding of an entire literature." I refer to this assumption as Assumption 1.[36]

Second, background and resources are substitutes in early stages, and complements in later stages, which is Assumption 2. It simply means that extra class resources have a higher impact on the education of pupils with a more disadvantaged background at early stages and vice-versa at later stages. This is 'in line' with the empirical literature. Cunha, Heckman, and Schennach (2010) observe that, especially for cognitive skills, the elasticity of substitution between background and resources decreases over time from substitutes to complements.[37] This makes intuitive sense, indeed. Suppose that background and class resources are proxies for the time and effort that, respectively, parents and teachers invest. Then it seems indeed plausible that background and class resources must be relatively good substitutes at early stages of education, while at later stages, requiring more specialized knowledge, this substitutability decreases and even turns into complementarity.

Third, the empirical literature focuses on background and not so much on (innate) ability. So, the next assumption is more of a reasonable a priori, rather than a falsified statement about the real world. I split the assumption in two parts. First, I assume that, at each stage, class resources are better substitutes for background than for ability (Assumption 3a). This can again be related to the previous story, because class resources and background (as proxies for the invested time and effort of, respectively, teachers and parents) are quite close in nature and therefore probably better substitutes than class resources and innate ability. Second, while class resources remain a reasonable substitute for background over the different stages, this might be less the case for ability: class resources can compensate for ability at the early stages, but much less, or even not at all, at later stages (Assumption 3b).

[36] While it is called "a major finding," the technology of skill formation in this literature, based on nested CES production functions, does not satisfy assumption 1 if the elasticities of substitution are different at different stages. The latter seems to be true for cognitive skills in the United States; see, e.g., Cunha, Heckman, and Schennach (2010).

[37] As this empirical literature focuses on background only, we do not impose a similar assumption for ability. We briefly come back to this issue later on.

While this dynamic model extends our previous simple static view on educational production, it is also worth looking at some limitations. Dynamic complementarity— the possibility that investment and educational skills reinforce each other—is not present in our additive model.[38] Yet, all known empirical results from the dynamic skill formation literature can be obtained in an additive model, which suggests that, while a limitation, dynamic complementarity is not important for the (empirical) story that we know up to now.[39] More important, skills are unidimensional in my model and must therefore capture both non-cognitive and cognitive skills. This is a clear limitation, because the dynamics, and thus also the resulting policy implications, can be quite different for both types of skills. I will come back to it in the extensions. Finally, because background is also a proxy for parental investment, and because parental investment is endogenous, parental investment may change as class resources change. This feedback effect is not captured in the main results, but could be important. For example, the assumption that class resources are better substitutes for background than for ability may no longer be true if the use of class resources to compensate for background is counteracted by higher parental investment of resource-rich families.

4.2. The problem of an outcome-prioritarian school team

An outcome-prioritarian school team must decide how to allocate resources over their pupils at the different stages of the educational process. Let $C_1(a,b)$ and $C_2(a,b)$ denote the resources invested in a pupil with characteristics (a,b) at each of the two stages, $\overline{C}^S$ is the available amount of resources (expressed per pupil) that the school team received from the minister of education, and f^S is the density function over types at school *S*. So, the outcome-prioritarian problem of the school team is to choose the functions C_1 and C_2 to maximize an outcome-prioritarian evaluation function based on Equations (1) and (4), i.e.,

$$g^{-1}\left(\iint_{a,b} g(\overline{w}(E(a,b,C_1(a,b),C_2(a,b))))f^S(a,b)dadb\right), \tag{7}$$

[38] Additivity implies $\partial^2 E / \partial e_1 \partial c_2 = 0$.

[39] The opposite might even be true. Figure 1 in Cunha, Heckman, and Schennach (2010:889) shows that, in a simplified version of the model, the ratio of early to late optimal investment is larger than one if and only if the skill multiplier is larger than one half. Actually, the larger the degree of complementarity, the lower this ratio becomes for a given skill multiplier, which suggests that complementarity reduces the case in favor of early over late investment. In any case, in Appendix A.2 I also present the more general formula that allows for dynamic complementarity.

subject to the budget constraint of the school team (i.e., allocated resources cannot exceed available resources), which, neglecting discounting, is given by

$$\iint_{a,b} (C_1(a,b) + C_2(a,b)) f^S(a,b) da db \leq \overline{C}^S . \quad (8)$$

The current exercise is similar to Arrow (1971), but is now applied to a model of dynamic skill formation as popularized by Heckman (2006). The Arrow-Heckman model presented here extends both literatures, because the analysis in Arrow is essentially prioritarian, but static only, and the analysis in Heckman is dynamic, but utilitarian only.[40] Moreover, I introduce heterogeneity in both (innate) ability and background.[41] This additional extension allows to investigate, e.g., whether one should compensate more for background than for ability.

I discuss all results in an informal way and refer to Appendix A.2 for formal details. Table 4 summarizes the answers of a utilitarian, prioritarian, and Rawlsian school team to the questions listed below the table.[42]

Table 4: A comparison of different normative positions.

Question	key assumption	Utilitarian	Prioritarian	Rawlsian
Q1	A1	yes	yes	yes
Q2a	A2	yes	yes	yes
Q2b	A2	no	no/?	?
Q2c	A2	?	?/yes	yes
Q3a	A3	yes	yes/no	no
Q3b	A3	yes	yes	yes
Q3c	A3	yes	yes	=

Q1: Should a school team invest more resources in early education?

Q2a: Should a school team invest more in pupils from a more disadvantaged background in early education?

Q2b: Should a school team invest more in pupils from a more disadvantaged background in later education?

Q2c: Should a school team invest more in pupils from a more disadvantaged background *in total*?

[40] Admittedly, the title of Arrow's (1971) article is somewhat misleading in this respect because Arrow's approach is, in the notation of this chapter, essentially prioritarian. The work of Heckman and colleagues often focuses explicitly on maximizing average test scores, which is, in the notation of this chapter, utilitarian.

[41] Arrow (1971:410) focuses on ability, which he very broadly defines as the "capacity to benefit from the [educational] expenditures in question." This may include, e.g., parental education. Cunha, Heckman, and Schennach (2010:886) mainly focus on the parental environment, but do recognize that "genetic factors may influence […] initial conditions."

[42] Note that the second question is not posed with respect to ability because little is empirically known about the interaction between resources and ability (as innate ability is difficult to observe). We will briefly come back to it later on.

Q3a: Should a school team compensate more for background than for ability in early education?

Q3b: Should a school team compensate more for background than for ability in later education?

Q3c: Should a school team compensate more for background than for ability *in total*?

Possible answers are 'yes', 'no', '?' (meaning that the answer is ambiguous without further assumptions), '=' (meaning that equality must hold), and 'x'/'y' (meaning that for low degrees of priority the answer is 'x' and for high degrees of priority the answer is 'y'). I also highlight in Table 4 the prioritarian answers that are different from the utilitarian or the Rawlsian answer.

4.2.1. Vertical allocation

The first question deals with the vertical allocation problem, i.e., how to allocate resources over the different stages of education. The answer is that a school team should invest more in early stages than in later stages for each type of pupil (i.e., $C_1^S(a,b) > C_2^S(a,b)$ for all a,b).[43] This answer is well-known in the academic literature and follows directly from the empirical observation that investment matters more at earlier stages (assumption 1). It holds for utilitarians, prioritarians, and Rawlsians alike as it is essentially an efficiency result.

4.2.2. Horizontal allocation: background

The second question deals with the horizontal allocation problem, i.e., how to allocate resources over different types of pupils. It asks whether a school team should invest more in pupils from a more disadvantaged background (i.e., *compensate* for background) or invest less in these pupils (i.e., *reinforce* background) at the different stages of education (questions 2a and 2b) as well as in total over both stages (question 2c).[44] Assumption 2 plays a crucial role here.[45]

In the early stages of education, utilitarians, prioritarians, and Rawlsians agree that background should be compensated (i.e., $\partial C_1^S(a,b)/\partial b < 0$ for all a,b). Again, this answer is well-known in the academic literature and follows from the fact that there seems to be no conflict between equity and efficiency in the early stages of education. To be more precise, it is not only equitable

[43] C_1^S and C_2^S denote the optimal solution of school S in both stages and $C^S = C_1^S + C_2^S$ is their sum.

[44] I use compensation and reinforcement rather than Arrow's (1971) input-progressivity and input-regressivity.

[45] Recall that, based on the existing empirical literature, we only made assumption 2 with respect to background. If we would make the same assumption for ability, then all results for background would hold for ability too.

to compensate for background, but it is also more efficient to do so in stage 1 as resources have a larger impact on pupils from disadvantaged backgrounds.

In later stages, utilitarians, prioritarians, and Rawlsians no longer agree. Only the answer of utilitarians and prioritarians with a sufficiently low degree of priority is clear: background should be reinforced at later stages (i.e., $\partial C_2^S(a,b)/\partial b > 0$ for all a,b). The reason is simply because it is efficient to do so in stage 2 as extra resources now have a larger impact on advantaged pupils. The answer of prioritarians with a sufficiently high degree of priority and Rawlsians is ambiguous as they trade off such efficiency gains against the equity losses.

Seen over both stages together, utilitarians, prioritarians, and Rawlsians also disagree. As utilitarians want to compensate for background in early stages, but reinforce it in later stages, their answer to the question whether to compensate for background in total is ambiguous. Rawlsians want to compensate for background over both stages together (i.e., $\partial C^S(a,b)/\partial b < 0$ for all a,b). This stands to reason as Rawlsians actually want to fully compensate for background (as well as for ability).[46] Full compensation for background means that the opportunity-egalitarian ideal will be reached, i.e., that educational achievement will be independent of background. Yet, as we have seen before, Rawlsians will not necessarily fully compensate in each stage separately as they want to reach full total compensation in the most efficient way, which may require to compensate differently in the different stages.[47] Prioritarians with a sufficiently high degree of priority will also compensate for background in total.[48]

[46] Arrow (1971) calls full compensation output-neutrality.

[47] While Rawlsians always fully compensate for background, an interesting question is under what additional assumptions also utilitarians and prioritarians would go for the opportunity-egalitarian ideal. It turns out that they will do so if, e.g., background and class resources are perfect substitutes in stage 1 and background plays no role in stage 2. So, opportunity-egalitarians and prioritarians might still agree if these stringent conditions on educational production would be true (which is not in line with, e.g., the evidence for the United States).

[48] This is in line with Arrow's (1971) tentative conclusion that, in case of education, more resources should go to the less able (broadly defined) if the degree of priority is strong enough.

4.2.3. Horizontal allocation: ability versus background

The third question also deals with the horizontal allocation problem, but focuses on the degree of compensation for ability relative to the degree of compensation for background. The answers now depend on Assumption 3 or subparts of it.[49] For the interpretation, note two things.

First, if I write that "X *compensates more* for background than for ability," with X being a utilitarian, prioritarian or Rawlsian, I mean that the educational gap caused by a more disadvantaged background is counteracted more by X than the gap caused by a lower ability. Somewhat loosely, it is probably best to think of it in a proportional way: "X *compensates more* for background than for ability," means that, e.g., 50% of the gap caused by a more disadvantaged background is eliminated via extra resources, while, e.g., only 40% of the gap caused by a lower ability is counteracted.[50] I stress therefore that, compensating more for background than for ability does not necessarily mean that more resources are allocated to pupils with a more disadvantaged background compared to pupils of lower ability. The size of the gap, for example, plays a role as well.

Second, although I often write that "X wants to *compensate more* for background than for ability," note that this is only relative and could thus also mean that "X wants to *reinforce* background *less* than ability." In other words, and again somewhat loosely, "X wants to *compensate more* for background than for ability," could also mean that the gap caused by a more disadvantaged background is enlarged to, e.g., 120% by providing less resources to these pupils, while the gap caused by a lower ability is enlarged to, e.g., 140%.

In the early stages of education, utilitarians will compensate more for background than for ability. The reason is that class resources are better substitutes for background than for ability (assumption 3a), so it is simply more efficient to compensate for background than ability. A Rawlsian proposes the exact opposite. A Rawlsian, recall, wants to fully compensate for both ability and background in total (over both stages together) and chooses to do so in the most efficient way. Because the possibility to compensate for lower abilities compared to compensating for a disadvantaged background declines over time (assumption 3b), they will

[49] Recall that assumption 3, especially 3b, is not based on empirical evidence. Alternatively, if one were to assume that the elasticity of background and resources decreases faster over time, then the answers to questions 3a and 3b must be switched, which has implications for prioritarians and Rawlsians, but not for utilitarians.

[50] The comparison is based on a normalized degree of compensation, which is, in fact, a marginal concept. A normalized degree of compensation (for ability or background) equal to zero corresponds with no compensation and a normalized degree of compensation equal to one corresponds with full compensation; see Appendix A.2 for more details.

compensate more for ability differences in early stages and more for background differences in later stages. For prioritarians it all depends on the degree of priority: if sufficiently small, they follow the utilitarian recommendation, if sufficiently large, they follow the Rawlsian recommendation.

In later stages of education, utilitarians still compensate more for background as class resources are also better substitutes for background than for ability in stage 2 (assumption 3a). Rawlsians, being concerned with full compensation, will also compensate more for background in stage 2 for the reason that I mentioned before. And prioritarians are again intermediate between both positions, but, as utilitarians and Rawlsians now agree, prioritarians will also compensate more for background in later stages.

In total over both stages, utilitarians and prioritarians will compensate more for background than for ability. Rawlsians will compensate fully for both ability and background, so the degree of compensation will be exactly equal. The finding for prioritarians may seem in contrast with the story of teacher Higgins. Only if there would be no efficiency considerations (as in the story of teacher Higgins), then the (total) degree of compensation for ability and background should be the same for prioritarians. But, as resources are here assumed to compensate more effectively for background than for ability in both stages (assumption 3a), the degree of compensation must be higher for background compared to ability according to a prioritarian. In other words, prioritarians might in the end agree with opportunity-egalitarians that, in total over both stages, more should be done for pupils having a disadvantaged background, but for efficiency reasons.

4.3. A brief summary

Prioritarian school teams may differ from utilitarian and Rawlsian ones in terms of policy recommendations. Contrary to Rawlsians, and in line with utilitarians, they are inconclusive whether background should, in total, be compensated or reinforced (question 2c) and they agree that background should, in total, be compensated more than ability (question 3c). Contrary to utilitarians, and in line with Rawlsians, they are inconclusive whether background should be compensated or reinforced in later stages of education (question 2b). Finally, whether background should be compensated more than ability in early stages, depends for outcome-prioritarians on the degree of priority: they are in line with utilitarians—who want to compensate more for background in early stages—if the degree of priority is sufficiently low and they are in line with Rawlsians—who want to compensate more for ability in early stages—if the degree of priority is sufficiently high.

5. Extensions

In the previous section, I analyzed a *school team* in *basic education* that optimally *designs* its *resource* allocation over pupils based on a prioritarian objective defined over an *educational index*. Question is whether the derived results also hold if I change some elements in the story.

In this section, I focus on a different decision-maker (*minister* versus *school team*), pupils at different ages (*higher* versus *basic education*), a different policy exercise (*evaluation* versus *design*), different policy tools (beyond *resources*), and different outcomes (*multiple skills* versus *educational index*).

The first extension is closely related to the previous section from an analytical viewpoint and therefore discussed in more detail. The other extensions are less related such that prioritarian recommendations are harder to derive without a detailed analysis. These other extensions should therefore best be read as caveats of the previous sections, useful after-thoughts, additional literature, or suggestions for future research.

5.1. Minister versus school team

In this section I look at a higher decision level, i.e., a minister of education who allocates resources over schools.

First, I will focus on a minister who successfully convinced all school teams to pursue the same outcome-prioritarian goal based on Equation (4). However, the difference is informational: the school team can observe both the ability and background of each pupil, while the minister observes only the background of the pupils at the different schools. I call this case the “same goal, other information” case.

Second, I will look at the opposite case, called “other goal, same information.” So, the minister has the same information about pupils as the school teams, but has a different goal. In particular, the minister and the school teams are both outcome-prioritarian, but the school teams focus on education only, i.e., Equation (4), while the minister focuses on a broader well-being concept, based on both education and economic success, i.e., Equation (2). Recall that in this case the minister, contrary to the school teams, allows for policies that treat disadvantaged pupils ‘preferentially’ in education to compensate for lower future economic success.

5.1.1. Same goal, other information

If the minister has the same goal, but only observes the background of the pupils at different schools, a paradox may occur. I discuss this paradox here informally and refer to Appendix A.3 for the technical details.

Suppose that both the minister and the school teams have the same outcome-prioritarian goal based on education, i.e., Equation (4). Suppose also that school teams would find it optimal to compensate for background over both stages together and to reinforce ability over both stages. This position is consistent with Table 4 if the degree of priority is sufficiently large.

This position would also hold for the minister if she would be able to observe ability and if she could directly allocate resources to pupils. Yet, while the minister would like to compensate for background if she did not face informational constraints, she will not necessarily allocate more resources (per pupil) to schools serving pupils with a more disadvantaged background if she cannot observe ability. So, the decision level, and in particular, the information available at the decision level, may matter.

To understand why, suppose first that there is no association (or, that there is a negative association) between ability and background. In this case the previous paradox cannot arise. In other words, the result at the pupil level ("give, as a school team, more resources to pupils with a more disadvantaged background") extends towards the school level ("give, as a minister, more resources per pupil to schools serving pupils with a more disadvantaged background").

However, if there is a positive association between ability and background, then this result does not necessarily hold anymore. On the one hand, the minister would like to compensate for a disadvantaged background. On the other hand, ability is not observable: so, given a positive association, a more advantaged background signals a higher ability, which is what the minister wants to reinforce. Both forces counteract each other. The final allocation is therefore not clear and depends, among other things, on the strength of the association between ability and background. Indeed, even if the minister agrees that school teams should compensate pupils for a disadvantaged background, she might somewhat paradoxically decide not to give more resources per pupil to schools serving pupils with a more disadvantaged background.

5.1.2. Other goal, same information

I have argued before that it is not very compelling for teacher Higgins to preferentially treat some children in education to compensate for lower future economic success. However,

minister Higgins could maybe adopt a more holistic view towards well-being, based on education and economic success, as defined in Equation (2). The problem becomes far more complex now as one obtains a so-called principal-agent problem, where the principal (the minister) has a different goal than the agents (the school teams). I therefore do not analyze this case in detail here, but discuss some intriguing questions.

First, is it inconsistent that the minister and the school teams have different objectives? Let us suppose that the holistic view of the minister towards well-being is the true goal in society. The fact that teachers are not allowed to preferentially treat some pupils then acts as a deontological constraint on their actions.[51] But if the holistic view is the true goal, then it is better, from a consequentialist point of view, that everyone works towards that same goal. The point is not that constraints are not allowed, but only that everyone should work towards the same goal, either with or without constraints. So, the penultimate question is, in my view, what position society should endorse towards the education of children; the positions of ministers, school teams, and teachers should then be aligned with this societal goal.

Second, I realize that the previous view is contentious or the alignment is not feasible. So, what happens if the minister adopts a holistic view, but teachers are not allowed to preferentially treat pupils in education? Recall the story of the two pupils (with the same educational achievement, but pupil one has limited abilities and pupil two has a disadvantaged background) and teacher Higgins who must choose how to allocate resources over both pupils from an equity perspective, i.e., ignoring efficiency considerations. I look at two cases.

In the first case, pupil one and two go to school A led by teacher Higgins and two exact copies of both pupils go to school B led by an exact copy of teacher Higgins. For ease of exposition, no other pupil is involved. Because schools A and B are exact copies, the minister cannot, but allocate the same resources to both schools. The teachers in both schools are not allowed to preferentially treat one pupil over the other, so they will both allocate the same resources to both pupils. In other words, pupil one and two are treated the same in both schools.

In the second case, pupils are re-sorted over schools such that pupil one and his copy go to school A and pupil two and her copy go to school B, ceteris paribus. In this case, teachers cannot, but allocate the same resources to both pupils in each school. But the minister can allocate different resources to the schools. If future economic success depends more on ability

[51] Note that a constraint on a shared goal (as discussed here) is in fact different from integrating out ability and background in the shared goal (as we did in Equation (4)). We do not discuss this difference further.

than background (for the same educational achievement), then the minister will allocate more resources to school A serving the pupils of limited ability. And if future economic success depends more on background than ability (for the same educational achievement), then the minister will allocate more resources to school B serving the disadvantaged pupils.[52]

Both cases also show that different allocations may arise depending on the sorting of pupils over schools. More diverse schools (case 1) imply that the goal of school teams will receive more weight in the final allocation and less diverse schools (case 2) imply that the goal of the minister receives more weight. But, as sorting can be linked to parental decisions, for which sufficiently young pupils are not held responsible, this dependency is puzzling.[53] In any case, such a dependency would not have occurred if ministers and teachers adopted the same normative position towards the relevant outcome.

5.2. Higher versus basic education

In this section I look at a higher education level, i.e., tertiary rather than basic (pre-primary and primary) education. Up to now, I indeed focused on sufficiently young pupils, say basic education. This focus avoided the discussion of responsibility such that, in the end, priorities were defined over outcomes rather than opportunities.

While I do not know the exact age at which responsibility kicks in, it seems reasonable to allow outcomes to be ambition-sensitive in tertiary education. Indeed, individual ambitions may lead to different decisions about continuing studying after high school, the study field, the time and effort to devote to your studies, and so on. And ambitions do not only impact educational outcomes, but also future economic success, depending, e.g., on the type of job one chooses in later life and how hard one is willing to work. I again discuss some intriguing questions.

Suppose I indeed allow outcomes to be ambition-sensitive. For example, if two individuals are equally well-prepared for tertiary education, but one of them wants to study philosophy and the other prefers to become a surgeon, differences in future economic success could be considered legitimate to some extent. Or, as another example, if two individuals end up with the same

[52] It could also be undefined, in fact, if, e.g., ability is, at some education levels, more important than background, but vice-versa for other education levels. While plausible, we do not discuss it further for ease of exposition.

[53] Sorting can also be influenced by school teams and ministers. In any case, as sufficiently young pupils cannot be held responsible for anything, the same puzzling result obtains.

tertiary degree, but one genuinely decides to work half-time and the other prefers to work full-time, then, again, the resulting differences in earnings could be justified.

A complication arises, however. These ambitions are also formed during previous education. Gintis (1972:593), for example, writes "The fallacy, of course, is the basing of all recommendations concerning education on the postulate of fixed preferences, in a situation where the educational system is a central instrument in the formation of preferences." The question then is how to allow ambitions to play a role during tertiary education and in the labor market if these ambitions are formed during basic education. Is it plausible to be ambition-sensitive in later life if one pupil has learnt to work hard and the other not?

It is, in my view, not necessarily wrong to assign responsibility over ambitions at later ages even if these ambitions are formed (or even partly imposed) on pupils at earlier ages. Such ambitions are not under control, but one could still be held responsible as long as one identifies with one's own ambitions.[54] The key question then is how the allocation of resources would change.

Suppose that I introduce effort, besides ability and background, as a factor that determines (higher) education outcomes as well as economic success. Suppose also that I keep individuals responsible for their preferences that govern the choice of effort. The presence of effort probably implies that the extent to which one wants to compensate for ability and background in higher education will be smaller compared to basic education. Whether one wants to reinforce ability relatively more than background in higher education will now also depend on their complementarities with effort. For example, if effort and ability are more complementary than effort and background (i.e., extra effort has a stronger effect on educational outcomes at higher ability levels and this is not (or to a lesser extent) true for background), then this will favor reinforcing ability more in higher education.

5.3. Evaluation versus design

In this section I look at a different normative exercise, i.e., the evaluation of educational systems rather than its design. Up to now, I indeed focused on the optimal allocation of resources over pupils and schools within a specific country. Recall that the goal was outcome-prioritarian and the outcome was country-specific, irrespective of whether Equation (2) or (4) was used.

[54] The responsibility cut can indeed be based on what individuals 'control' or on 'preferences' with which individuals identify.

As long as one sticks to policy design, country differences do not raise problems, at the contrary. Educational systems are designed within a country and must therefore take the specific context of a country into account. For example, in countries where educational differences matter less for future economic success (e.g., because of high levels of earnings redistribution), educational inequalities are also less problematic. Educational differences may therefore be mitigated less, resulting in higher levels of educational inequality in such countries.

However, if I switch from design to evaluation, an interesting question arises: how can one compare the educational systems of different countries? Because the evaluation is country-specific, it is possible that system A is best for country A and system B is best for country B. This implies that, from the viewpoint of country A, system A is better than B, and from the viewpoint of country B, system B is better than system A. While this is, in my view, not problematic, it implies that the comparison of educational systems is a delicate exercise, far more delicate than simply comparing key educational statistics, which, as discussed before, is the common practice in the evaluation literature.

5.4. Beyond resources

In this section I look beyond the allocation of resources. Up to now, resources was used as a catch-all word whose meaning depends, e.g., on the decision level. At the level of the minister one typically thinks of money to be allocated to schools. At the level of the school team, resources could refer to the money, time, and effort to run the school and, in particular, to educate the pupils. But, beyond resources, there exist many other ways to influence educational outcomes.[55]

One way to improve outcomes is to introduce school competition by, e.g., extending school choice (e.g., using school vouchers or charter schools) or introducing school accountability. Such school competition policies promise to be "a tide that lifts all boats" and are therefore, in theory, efficiency-improving policies that ensure that resources are used in their most productive way. If true, prioritarians would not oppose it.

In practice, however, such overall improvements are rather unlikely.[56] School accountability, for example, may lead to incentives for pupil selection. Such unproductive uses of resources by

[55] I will restrict attention to institutional features (at the decision level of ministers) rather than pedagogical features (at the decision level of school teams and teachers).

[56] See, e.g., Epple, Romano, and Urquiola (2017) for a review of school vouchers, Epple, Romano, and Zimmer (2016) for a review of charter schools, and Figlio and Loeb (2011) for a review of school accountability schemes.

teachers and school teams may counteract potential efficiency gains of competition. But if overall improvements are not possible, equity comes in again. Depending on who wins and loses, prioritarians may or may not be in favor of school accountability. If school accountability leads to student retainment and the removal of low-achieving students, as observed in some empirical studies, then it is very likely to be discarded by prioritarians, especially if their degree of priority is high.

Another way to improve educational outcomes is to group pupils in different ways. Mixing, tracking, and streaming are examples of such policies.

Mixing refers to obtaining a sufficiently diverse mix of pupils at schools in terms of background. While the intrinsic argument is that schools should simply mirror society, there may also be instrumental reasons to mix pupils depending on the direction of so-called peer effects (i.e., cognitive and non-cognitive spillover effects between pupils). Such mixing may be enforced directly (e.g., via busing policies) or can be introduced in a more subtle way in school assignment policies.

Tracking and streaming refer to obtaining a less diverse mix of pupils in terms of ability.[57] Tracking is more popular in Europe and typically puts pupils with different abilities in different schools (i.e., between-school grouping); streaming is more popular in Anglo-Saxon countries (e.g., the United States) and puts pupils with different abilities either in different classes within schools (within-school grouping) or in different groups within classes (within-class grouping) for a selection of courses.

Little can be said about the outcome-prioritarian design of such alternative policies without explicit modelling. Moreover, as peer effects are highly non-linear and the underlying mechanisms not well-understood, such models will be fairly complex and clear policy recommendations cannot be guaranteed at all.[58] In any case, the core questions underlying Table 4 will definitely be influenced by diversity policies, which are therefore an intriguing topic for further research.

[57] While mixing on the one hand and tracking and streaming on the other hand seem opposite, mixing and (especially) streaming are compatible as one can strive for a more diverse mix of pupils at schools in terms of background and still group these pupils, for one or more courses, in ability groups.

[58] While peer effects do exist in specific contexts, the mechanisms and policy consequences are not fully grasped yet; see Sacerdote (2014) for a review.

5.5. Multiple skills versus educational index

In this final section I look at different outcomes, i.e., non-cognitive and cognitive skills as separate outcomes. Up to now, education was indeed measured by an index of different skills, including cognitive and non-cognitive skills, that is produced as a whole. However, cognitive and non-cognitive skills are produced in different ways; see, e.g., Kautz et al. (2014) for an overview. For example, non-cognitive skills remain relatively more plastic at later ages.[59] Moreover, there are also indications that non-cognitive skills improve cognitive skills, but not vice-versa.

Splitting up our educational index in cognitive and non-cognitive skills is likely to change the prioritarian allocation of resources over pupils. Probably the best starting point for such an ambitious exercise is Cunha, Heckman, and Schennach (2010). They estimate the educational production of cognitive and non-cognitive skills and simulate optimal policies based on a utilitarian objective defined over either the number of schooling years or crime, which are both a function of cognitive and non-cognitive skills. I leave a prioritarian evaluation, eventually based on a less instrumental view of education, for future research.

6. Conclusion

The first aim of this chapter was to summarize the operational normative views that have been adopted in the academic literature on education. This summary reveals that while prioritarianism as an aggregation device is often used in the 'welfarist' design literature, and thus clearly compatible with it, prioritarianism is also compatible with the dashboard approach used in the evaluation literature. Not prioritarianism as such, but the metric over which prioritarianism should be defined—the question "priority of what?"—turns out to be the real bone of contention.

The answer to the question "priority of what?" leads to four normative views in the education literature, depending on whether the metric is educational or economic and whether one should provide outcomes or opportunities in these metrics. The second aim of this chapter was to critically analyze these four existing normative views. I did so by directly engaging its philosophical foundations and by looking at its policy recommendations in a simplified setting.

[59] In the technical jargon we used before, the elasticity of substitution between resources and either background or ability decreases less rapidly over time for non-cognitive skills.

My analysis strongly suggests that outcomes—and not opportunities—should be the appropriate metric for sufficiently young pupils. In addition, the appropriate outcome should capture both the intrinsic and instrumental value of education, but the precise formulation may be different for school teams and ministers.

The third aim of this chapter was to introduce a dynamic model of skill formation and to analyze the policy recommendations from an outcome-prioritarian—rather than outcome-utilitarian or outcome-Rawlsian—point of view. It turns out that, whenever equity is at stake, prioritarian school teams may recommend different policies compared to utilitarians or Rawlsians.

The fourth and final aim was to discuss several extensions to see whether and how the derived policy recommendations would change if I change the benchmark story. Some of these extensions were analyzed in detail and revealed paradoxical results. Some extensions were only dealt with briefly and can therefore best be seen as an invitation to further study the consequences of prioritarianism for educational policies.

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A.1. Decomposing the four metrics

I start with formally defining the assumptions.

First, the transformation function g is of the Kolm-Pollak type, i.e., $g: w \mapsto \exp(-\pi w)$ with π the degree of priority in between utilitarian ($\pi \to 0$) and Rawlsian ($\pi \to \infty$).

Second, 'reduced-form' linear regression techniques are used to regress either educational or economic achievement on background. So, these regression equations, including errors, are $e = \alpha_e + \beta_e b + \varepsilon_e$ and $y = \alpha_y + \beta_y b + \varepsilon_y$. I call it a reduced form, because the error terms capture, among other things, remaining differences associated with innate ability (after having corrected for background).

Third, background and regression errors are independent and follow normal distributions, so, I have $b \perp \varepsilon_e \perp \varepsilon_y$ and $b \sim N(\mu_b, \sigma_b^2)$, $\varepsilon_e \sim N(0, \sigma_{\varepsilon_e}^2)$, and $\varepsilon_y \sim N(0, \sigma_{\varepsilon_y}^2)$.

The decomposition is based on two observations.

First, if a metric, say *w*, is normally distributed, say, $w \sim N(\mu_w, \sigma_w^2)$ and if the transformation function g is of the Kolm-Pollak type (as defined before), then the evaluation in Equation (1) reduces to $-\frac{1}{\pi}\ln(\exp(-\pi\mu_w + \frac{1}{2}\pi^2\sigma_w^2)) = \mu_w - \frac{\pi}{2}\sigma_w^2$. So, given that also e and y follow normal distributions, I directly get $\mu_e - \frac{\pi}{2}\sigma_e^2$ (if $w = e$) and $\mu_y - \frac{\pi}{2}\sigma_y^2$ (if $w = y$) for the outcome-prioritarians (to the right of Table 2).

Second, the conditional expectations $\mathbb{E}[e|b] = \alpha_e + \beta_e b$ and $\mathbb{E}[y|b] = \alpha_y + \beta_y b$ also follow normal distributions. Replacing β_e and β_y by their best linear unbiased estimators, i.e., $\hat{\beta}_e = \rho_{be}\sigma_e / \sigma_b$ and $\hat{\beta}_y = \rho_{by}\sigma_y / \sigma_b$ respectively, I get for the opportunity-prioritarians (to the left of Table 2)

$$\mu_{\alpha_e + \hat{\beta}_e b} - \frac{\pi}{2} \cdot \sigma^2_{\alpha_e + \hat{\beta}_e b} = \mu_e - \frac{\pi}{2} \cdot \hat{\beta}_e^2 \cdot \sigma_b^2 = \mu_e - \frac{\pi}{2} \cdot \rho_{be}^2 \cdot \sigma_e^2,$$

and

$$\mu_{\alpha_y + \hat{\beta}_y b} - \frac{\pi}{2} \cdot \sigma^2_{\alpha_y + \hat{\beta}_y b} = \mu_y - \frac{\pi}{2} \cdot \hat{\beta}_y^2 \cdot \sigma_b^2 = \mu_y - \frac{\pi}{2} \cdot \rho_{by}^2 \cdot \sigma_y^2.$$

As a side note, one can also combine an Atkinson transformation function, i.e., $g: w \to w^{1-\pi}$, with log-linear relations (i.e., $\ln e = \ln \alpha_e + \beta_e \ln b + \ln \varepsilon_e$ and $\ln y = \ln \alpha_y + \beta_y \ln b + \ln \varepsilon_y$) and independent and lognormal distributions (i.e., $\ln b \perp \ln \varepsilon_e \perp \ln \varepsilon_y$ and $\ln b \sim N(\mu_{\ln b}, \sigma^2_{\ln b})$, $\ln \varepsilon_e \sim N(0, \sigma^2_{\ln \varepsilon_e})$, and $\ln \varepsilon_y \sim N(0, \sigma^2_{\ln \varepsilon_y})$). I show that it leads to a very similar (multiplicative) decomposition that does not change the main story line.

To see this, note first that if a metric w is lognormally distributed, say, $\ln w \sim N(\mu_{\ln w}, \sigma^2_{\ln w})$ and the transformation function g is of the Atkinson type (as defined before), then the evaluation in Equation (1) reduces to $\mu_w \exp(-\frac{\pi}{2} \cdot \sigma^2_{\ln w})$.[60] So, given that now e and y follow lognormal distributions, I directly get $\mu_e \exp(-\frac{\pi}{2} \cdot \sigma^2_{\ln e})$ and $\mu_y \exp(-\frac{\pi}{2} \cdot \sigma^2_{\ln y})$ for the outcome-prioritarians.

Second, given loglinear relations, the conditional expectations, being, $\mathbb{E}[e \mid b] = \alpha_e b^{\beta_e} \mathbb{E}[\varepsilon_e]$ and $\mathbb{E}[y \mid b] = \alpha_y b^{\beta_y} \mathbb{E}[\varepsilon_y]$, also follow lognormal distributions and β_e and β_y can again be replaced by their best linear unbiased estimators, i.e., $\hat{\beta}_e = \rho_{\ln b \ln e} \sigma_{\ln e} / \sigma_{\ln b}$ and $\hat{\beta}_y = \rho_{\ln b \ln y} \sigma_{\ln y} / \sigma_{\ln b}$ respectively. So, for the opportunity-prioritarians I get

$$\mu_{\mathbb{E}[e|b]} \exp(-\frac{\pi}{2} \cdot \sigma^2_{\ln \mathbb{E}[e|b]}) = \mu_e \exp(-\frac{\pi}{2} \cdot \hat{\beta}_e{}^2 \sigma^2_{\ln b}) = \mu_e \exp(-\frac{\pi}{2} \cdot \rho^2_{\ln b \ln e} \cdot \sigma^2_{\ln e}),$$

and

$$\mu_{\mathbb{E}[y|b]} \exp(-\frac{\pi}{2} \cdot \sigma^2_{\ln \mathbb{E}[y|b]}) = \mu_y \exp(-\frac{\pi}{2} \cdot \hat{\beta}_y{}^2 \sigma^2_{\ln b}) = \mu_y \exp(-\frac{\pi}{2} \cdot \rho^2_{\ln b \ln y} \cdot \sigma^2_{\ln y}).$$

[60] Note that we use $\mu_w = \exp(\mu_{\ln w} + \frac{1}{2} \cdot \sigma^2_{\ln w})$ here.

A.2. The allocation of a school team

In this appendix, I first discuss the assumptions of the dynamic model of skill formation in a formal way. Afterwards, I derive the optimal outcome-prioritarian allocation of resources of the school team over pupils.

A.2.1. Assumptions

I assume that (final) education is strictly increasing in all arguments and strictly concave in class resources, i.e.,

$$E_a^{'} > 0, E_b^{'} > 0, E_1^{'} > 0, E_2^{'} > 0, E_1^{''} < 0, E_2^{''} < 0,$$

where subscripts 1 and 2 refer to the partial derivatives with respect to resources c_1 and c_2, respectively.

On top of these standard assumptions, I impose three additional assumptions.

First, *everything else equal*, investment is more effective at earlier stages. With two stages, I impose

Assumption 1. $E_1^{'}(a,b,c,c) > E_2^{'}(a,b,c,c)$ for all (a,b,c).

Note that, given additivity, assumption 1 implies $E_{1,1}^{'}(a,b,c) > E_{2,2}^{'}(a,b,c)$ for all (a,b,c).

Second, it is more effective to compensate for background at earlier stages, but this result turns around at later stages. With two stages, I get

Assumption 2. $E_{b1}^{''}(a,b,c_1,c_2) < 0 < E_{b2}^{''}(a,b,c_1,c_2)$ for all (a,b,c_1,c_2).

Define the 'Hicksian' measure of complementarity as

$$\varsigma_{ij}(a,b,c_1,c_2) = \frac{E(a,b,c_1,c_2)E_{ij}^{''}(a,b,c_1,c_2)}{E_i^{'}(a,b,c_1,c_2)E_j^{'}(a,b,c_1,c_2)},$$

where $i, j \in \{a,b,1,2\}$. For later use, note that assumption 2 implies $\varsigma_{b1} < 0 < \varsigma_{b2}$.

The third assumption imposes that ability is more complementary to resources than background in all stages (assumption 3a) and this difference in complementarity increases over time (assumption 3b). As mentioned before, this assumption is a priori plausible, especially assumption 3a, but there is no empirical evidence that supports it. With two stages, I impose:

Assumption 3. $\varsigma_{a1}-\varsigma_{b1}>0$ and $\varsigma_{a2}-\varsigma_{b2}>0$ (A3a) and $\varsigma_{a2}-\varsigma_{b2}>\varsigma_{a1}-\varsigma_{b1}$ (A3b).

A.2.2. The problem of the school team

Start from Equation (7), leave out the inverse g^{-1} (as strictly increasing transformations do not matter to derive the optimal solution), and define the composite function $\hat{g}=g\circ\overline{w}$. The system of first-order conditions with respect to resources in each stage is

$$\begin{aligned}\hat{g}'(E(a,b,C_1^S(a,b),C_2^S(a,b)))\cdot E_1'(a,b,C_1^S(a,b),C_2^S(a,b))=\lambda^S,\\ \hat{g}'(E(a,b,C_1^S(a,b),C_2^S(a,b)))\cdot E_2'(a,b,C_1^S(a,b),C_2^S(a,b))=\lambda^S,\end{aligned} \tag{9}$$

for some λ^S, being the Lagrange multiplier of the budget constraint of school *S*. Dividing the first-order conditions and using additivity (i.e., $E_t'=E_{t,t}'$ holds at each stage *t*), one must have equal marginal productivities, i.e.,

$$E_{1,1}'(a,b,C_1^S(a,b))=E_{2,2}'(a,b,C_2^S(a,b)), \tag{10}$$

at the optimum. Assigning equal class resources cannot be optimal because assumption 1 implies $E_{1,1}'(a,b,c)>E_{2,2}'(a,b,c)$ everywhere. As both production functions are concave in resources, I must have $C_1^S(a,b)>C_2^S(a,b)$, which is the answer to question 1.

The system of first-order conditions (9) can be 'abbreviated' as

$$\begin{aligned}\hat{g}'\cdot E_1'=\lambda^S,\\ \hat{g}'\cdot E_2'=\lambda^S.\end{aligned} \tag{11}$$

Differentiating (11) with respect to either ability or background (denoted $z=a,b$), I get

$$\begin{aligned}\hat{g}''(E_z'+E_1'C_{1,z}^{S'}+E_2'C_{2,z}^{S'})E_1'+\hat{g}'(E_{z1}''+E_{11}''C_{1,z}^{S'}+E_{12}''C_{2,z}^{S'})=0,\\ \hat{g}''(E_z'+E_1'C_{1,z}^{S'}+E_2'C_{2,z}^{S'})E_2'+\hat{g}'(E_{z2}''+E_{12}''C_{1,z}^{S'}+E_{22}''C_{2,z}^{S'})=0,\end{aligned} \tag{12}$$

for $z=a,b$, which can be written in matrix form as

$$\begin{bmatrix}\hat{g}'E_{11}''+\hat{g}''(E_1')^2 & \hat{g}''E_1'E_2'+\hat{g}'E_{12}''\\ \hat{g}''E_1'E_2'+\hat{g}'E_{12}'' & \hat{g}'E_{22}''+\hat{g}''(E_2')^2\end{bmatrix}\begin{bmatrix}C_{1,z}^{S'}\\ C_{2,z}^{S'}\end{bmatrix}=-\begin{bmatrix}\hat{g}'E_{z1}''+\hat{g}''E_1'E_z'\\ \hat{g}'E_{z2}''+\hat{g}''E_2'E_z'\end{bmatrix} \tag{13}$$

Using Cramer's rule, the solution is

$$C_{1,z}^{S'}=-\frac{E_z'}{E_1'}\frac{(\pi-\varsigma_{z1})(\pi-\varsigma_{22})-(\pi-\varsigma_{z2})(\pi-\varsigma_{12})}{(\pi-\varsigma_{11})(\pi-\varsigma_{22})-(\pi-\varsigma_{12})^2}, \tag{14}$$

and

$$C_{2,z}^{S'} = -\frac{E_z^{'}}{E_2^{'}} \frac{(\pi - \varsigma_{z2})(\pi - \varsigma_{11}) - (\pi - \varsigma_{z1})(\pi - \varsigma_{12})}{(\pi - \varsigma_{11})(\pi - \varsigma_{22}) - (\pi - \varsigma_{12})^2}, \tag{15}$$

with $\pi = -\frac{\hat{g}''(E)}{\hat{g}'(E)} E > 0$ the degree of priority and $\varsigma_{xy} = \frac{EE_{xy}^{''}}{E_x^{'} E_y^{'}}$ the Hicksian complementarity measure that we defined before. I assume that the denominator is strictly positive, i.e., we have $(\pi - \varsigma_{11})(\pi - \varsigma_{22}) - (\pi - \varsigma_{12})^2 > 0$ (to satisfy the second-order conditions).

Background

Using additivity ($\varsigma_{12} = 0$) and focusing first on background ($z = b$), the comparative statics reduce to

$$C_{1,b}^{S'} = -\frac{E_b^{'}}{E_1^{'}} \frac{\varsigma_{b1}\varsigma_{22} + \pi(\varsigma_{b2} - \varsigma_{b1} - \varsigma_{22})}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})}, \tag{16}$$

and

$$C_{2,b}^{S'} = -\frac{E_b^{'}}{E_2^{'}} \frac{\varsigma_{11}\varsigma_{b2} + \pi(\varsigma_{b1} - \varsigma_{b2} - \varsigma_{11})}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})} \tag{17}$$

Several results can be deduced.

First, as assumption 2 implies $\varsigma_{b1} < 0 < \varsigma_{b2}$ and as the standard assumptions imply $\varsigma_{11}, \varsigma_{22} < 0$, I must have $C_{1,b}^{S'} < 0$ because the numerator (and denominator) of the second fraction in (16) is always positive. This means that, irrespective of the degree of priority, the impact of background must be compensated in stage 1. This is the answer to question 2a.

Second, the sign of $C_{2,b}^{S'}$ is not clear in general. For utilitarians ($\pi \to 0$) the sign is positive, so they will reinforce the impact of background in stage 2. For prioritarians with a sufficiently low degree of priority, the sign must be positive too (using continuity). For Rawlsians ($\pi \to \infty$) the sign is not clear without further assumptions. This is the answer to question 2b.

As a side note (which is also mentioned in the main text), a special case arises if background and resources are perfect substitutes in stage 1 (i.e., $E_1(a,b,c_1) = f(a, b + \gamma c_1)$ everywhere for some function f and scalar γ) and if background has no impact on outcomes in period 2 (i.e.,

$E_2(a,b,c_2)=f(a,c_2)$ everywhere for some function *f*).[61] Both conditions lead to $\varsigma_{b1}=\varsigma_{11}$ and $\varsigma_{b2}=0$, implying $C_{1,b}^{S'}=-\frac{E_b^{'}}{E_1^{'}}=-\frac{E_{1,b}^{'}}{E_{1,1}^{'}}$ and $C_{2,b}^{S'}=0$. Because $-\frac{E_b^{'}}{E_1^{'}}=-\frac{E_{1,b}^{'}}{E_{1,1}^{'}}$ is the marginal rate of technical substitution between resources and background (in stage 1, using additivity), the interpretation is simple. In stage 1, the impact of background on outcomes must be fully compensated and thus equality of opportunity applies (as the outcome does not depend on background). In stage 2 there is no effect of background on education and resources therefore do not change with background. So, after the policy intervention, equality of opportunity applies also in the second stage.

Ability versus background

Using additivity ($\varsigma_{12}=0$) and focusing now on ability ($z=a$), the comparative statics are

$$C_{1,a}^{S'}=-\frac{E_a^{'}}{E_1^{'}}\frac{\varsigma_{a1}\varsigma_{22}+\pi(\varsigma_{a2}-\varsigma_{a1}-\varsigma_{22})}{\varsigma_{11}\varsigma_{22}-\pi(\varsigma_{11}+\varsigma_{22})}, \tag{18}$$

and

$$C_{2,a}^{S'}=-\frac{E_a^{'}}{E_2^{'}}\frac{\varsigma_{11}\varsigma_{a2}+\pi(\varsigma_{a1}-\varsigma_{a2}-\varsigma_{11})}{\varsigma_{11}\varsigma_{22}-\pi(\varsigma_{11}+\varsigma_{22})}. \tag{19}$$

First, the product $-\frac{E_t^{'}}{E_z^{'}}\cdot C_{t,z}^{S'}$ (for $z=a,b$) measures the normalized degree of compensation for characteristic *z* in stage *t*. To see this, note that the lower the second factor $C_{t,z}^{S'}$, the less reinforcement or the more compensation there is for ability or background and, as the first factor is negative, the higher the degree of compensation. Furthermore, the first factor is the (inverse of the) marginal rate of technical substitution. The marginal rate of technical substitution tells us how much extra resources in period *t* are needed to compensate for a small change in *z*. The first factor therefore is a normalization that guarantees that full compensation corresponds with a degree of compensation equal to one. As this normalization holds irrespective of whether I

[61] Actually, this result holds more generally. Consider a (not necessarily additive) educational production function in stage 2, say, $E_2(e,a,b,c_2)$. This implies that final educational achievement would be given by $E(a,b,c_1,c_2)=E_2(E_1(a,b,c_1),a,b,c_2)$. If this function can be written as $E(a,b,c_1,c_2)=f(a,b+\gamma c_1,c_2)$ everywhere, then $\varsigma_{b1}=\varsigma_{11}$ and $\varsigma_{b2}=\varsigma_{12}$ hold, which, plugged in in Equations (16) and(17), lead to the same result (full compensation for background in period 1 and no compensation for it in period 2). An open question is what other specifications, besides the additive one that I use, are made possible by this generalization.

look at ability or background, it allows to compare the degree of compensation for ability and background.

So, whether ability should be compensated more or less compared to background in each stage depends on the sign of $\Delta_t = -\frac{E_t^{'}}{E_a^{'}} C_{t,a}^{S'} + \frac{E_t^{'}}{E_b^{'}} C_{t,b}^{S'}$. If negative, for example, then ability is, compared to background, compensated less (or reinforced more) in stage *t*. I get:

$$\Delta_1 = \frac{\varsigma_{22}(\varsigma_{a1} - \varsigma_{b1}) + \pi((\varsigma_{a2} - \varsigma_{b2}) - (\varsigma_{a1} - \varsigma_{b1}))}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})}, \tag{20}$$

and

$$\Delta_2 = \frac{\varsigma_{11}(\varsigma_{a2} - \varsigma_{b2}) + \pi((\varsigma_{a1} - \varsigma_{b1}) - (\varsigma_{a2} - \varsigma_{b2}))}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})}. \tag{21}$$

I can now use assumption 3, being

$$\varsigma_{a2} - \varsigma_{b2} > \varsigma_{a1} - \varsigma_{b1} > 0, \tag{22}$$

to derive the answers to questions 3a and 3b.

For a utilitarian ($\pi \to 0$) both Δ_1 and Δ_2 turn out to be negative, i.e., always more compensation for background than for ability, indeed.

For a Rawlsian, I take the limit, leading to

$$\lim_{\pi \to \infty} \Delta_1 = \frac{(\varsigma_{a2} - \varsigma_{b2}) - (\varsigma_{a1} - \varsigma_{b1})}{-(\varsigma_{11} + \varsigma_{22})} > 0,$$

and

$$\lim_{\pi \to \infty} \Delta_2 = \frac{(\varsigma_{a1} - \varsigma_{b1}) - (\varsigma_{a2} - \varsigma_{b2})}{-(\varsigma_{11} + \varsigma_{22})} < 0,$$

as required.

For a prioritarian, Δ_2 is negative for sure, but the result for Δ_1 depends on the degree of priority. If sufficiently small, then the prioritarian agrees with the utilitarian to compensate more for background, and if sufficiently large, then the prioritarian agrees with the Rawlsian to compensate more for ability in stage 1.

Total investment over both stages

Finally, I analyze total resources, i.e., $C^S(a,b) = C_1^S(a,b) + C_2^S(a,b)$. The comparative statics with respect to $z=a,b$ are

$$\begin{aligned} C_z^{S'}(a,b) &= C_{1,z}^{S'}(a,b) + C_{2,z}^{S'}(a,b), \\ &= -\frac{E_z^{'}}{E_{1/2}^{'}} \frac{\varsigma_{z1}\varsigma_{22} + \varsigma_{11}\varsigma_{z2} - \pi(\varsigma_{11} + \varsigma_{22})}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})}, \end{aligned} \tag{23}$$

where I use (10) to define $E_{1/2}^{'} = E_1^{'} = E_2^{'}$. This expression highlights four things.

First, for utilitarians with $\pi \to 0$, the sign depends on the sign of $\varsigma_{b1}\varsigma_{22} + \varsigma_{11}\varsigma_{b2}$, which is ambiguous. Taking the limit $\pi \to \infty$, the second fraction in Equation (23) is equal to one. So, an outcome-Rawlsian school team will fully compensate for both ability and background over both stages together. For prioritarians with a sufficiently high degree of priority the sign of the second fraction must be positive too (invoking continuity). These results together provide an answer to question 2c.

Second, the total degree of compensation, appropriately normalized, for characteristic z is $-\frac{E_z^{'}}{E_{1/2}^{'}} C_z^{S'}(a,b)$. The difference between the degree of compensation for ability and background is therefore

$$\begin{aligned} \Delta &= -\frac{E_{1/2}^{'}}{E_a^{'}} C_a^{S'} + \frac{E_{1/2}^{'}}{E_b^{'}} C_b^{S'}, \\ &= \frac{(\varsigma_{a1} - \varsigma_{b1})\varsigma_{22} + \varsigma_{11}(\varsigma_{a2} - \varsigma_{b2})}{\varsigma_{11}\varsigma_{22} - \pi(\varsigma_{11} + \varsigma_{22})}. \end{aligned} \tag{24}$$

Using Equation (22) this difference is always negative; in other words, irrespective of the degree of priority, background will always be compensated more than ability *in total*. But, as this difference becomes zero in the limit $\pi \to \infty$, the degree of compensation for ability and background is equal for Rawlsians. This is the answer to question 3c.

A.3. The allocation of a minister

I look at the optimal outcome-prioritarian allocation of resources by the minister over schools, where the minister can only observe the background of the pupils at each school.

First, it is useful to consider how the minister would allocate resources if he could (hypothetically) allocate directly over pupils (rather than over schools) and if both ability and background were observable (rather than only background). In this first hypothetical case, the minister would solve the same problem as a school team, given by Equations (7) and (8), but now replacing the school resources by the total resources (per pupil) in society, say $\overline{C}$, and replacing the school density by the overall density, say f. In other words, the problem of the minister would be to choose resource functions C_1 and C_2 to maximize

$$g^{-1}\left(\iint_{a,b} g(\overline{w}(E(a,b,C_1(a,b),C_2(a,b))))f(a,b)dadb\right), \tag{25}$$

subject to the budget constraint, being

$$\iint_{a,b}(C_1(a,b)+C_2(a,b))f(a,b)dadb \leq \overline{C}\,. \tag{26}$$

Define $\hat{g} = g \circ \overline{w}$. The optimal solution of the minister, denoted C_1^M and C_2^M, must satisfy the first-order conditions:

$$\begin{aligned} \hat{g}' \cdot E_1' &= \lambda, \\ \hat{g}' \cdot E_2' &= \lambda, \end{aligned} \tag{27}$$

for some λ, being the Lagrange multiplier of the minister. For later use, define $C^M = C_1^M + C_2^M$ as the total resources over both stages.

The problem is that the minister cannot directly allocate resources over pupils. She must allocate resources over schools, which are then (optimally) reallocated afterwards over pupils by each school team according to the functions C_1^S and C_2^S that I derived before.

Second, it is useful to consider now how the minister would allocate resources over schools, but still assuming that ability and background are observable to the minister. Let p^S be the fraction of pupils at school *S*. The problem of the minister is now to allocate resources $\overline{C}^S$ to the different schools to maximize[62]

[62] Be aware that the superscript *S* in $C_1^S(a,b)$ and $C_2^S(a,b)$ indicates that in fact the school team solution also depends on the amount of resources allocated to school *S* ($\overline{C}^S$) and on the density of types at school *S* (f^S).

$$g^{-1}\left(\sum_S p^S \iint_{a,b} g(\overline{w}(E(a,b,C_1^S(a,b),C_2^S(a,b)))) f^S(a,b)dadb\right), \tag{28}$$

subject to the budget constraint

$$\sum_S p^S \overline{C}^S \leq \overline{C}\,. \tag{29}$$

The solution to this problem of the minister is relatively easy because the goal of the minister and school teams are fully aligned and the prioritarian evaluation is separable. In particular, the optimal solution of the minister can be decentralized by giving each school S an amount of resources per pupil equal to

$$\overline{C}^S = \iint_{a,b} C^M(a,b) f^S(a,b)dadb\,. \tag{30}$$

This allocation is feasible at societal level as it satisfies the budget constraint (29) of the minister (note that, by definition, $\sum_S p^S f^S = f$ must hold). Moreover, because also the first-order conditions of the minister and the schools coincide (i.e., Equations (27) and (11), choosing $\lambda = \lambda^S$), each school will implement the optimal allocation of the minister (as if the latter could directly allocate resources over pupils).

Third, I now also drop the assumption that the minister can observe ability. As $f^S(a,b) = f^S_{a|b}(a\,|\,b) f^S_b(b)$ holds by definition, the allocation in Equation (30) can be rewritten as

$$\overline{C}^S = \iint_{a,b} C^M(a,b) f^S_{a|b}(a\,|\,b) f^S_b(b)dadb\,. \tag{31}$$

If the minister assumes that the distribution of ability conditional on background is common over all schools (i.e., $f^S_{a|b} = f_{a|b}$ for each school S), then the per-pupil resources allocated to school S become[63]

$$\overline{C}^S = \int_b \tilde{C}^M(b) f^S_b(b)db\,, \tag{32}$$

[63] This assumption is not entirely realistic. Implicitly, it assumes that schools do not select pupils on the basis of ability (for a given background), e.g., for status or for money depending on the school funding rules, and that parents (with a given background) do not sort over schools on the basis of ability, e.g., to enjoy peer effects. Even if not entirely realistic, it is plausible that, without information on ability, the minister will assume it.

with

$$\tilde{C}^M(b) = \int_a C^M(a,b) f_{a|b}(a \mid b) da \,, \tag{33}$$

the resources invested in a pupil with background *b*. The allocation of resources over schools in Equation (32) only depends on background, as required.

Suppose that f_b^S, the pupil background distribution at school *S*, changes, i.e., school *S* attracts more pupils with a disadvantaged background.[64] This may arise because there are new pupils in the system or because pupils move from one school to another. The question whether school *S* will receive more resources depends on the derivative of Equation (33) with respect to *b*. If the sign of this derivative is negative, then, using standard (first-order) stochastic dominance techniques, the school will for sure get extra resources; see, e.g., Lambert (2001).

The derivative $\tilde{C}^{M'}(b)$ is equal to

$$\tilde{C}^{M'}(b) = \int_a C_b^{M'}(a,b) f_{a|b}(a \mid b) da + \int_a C^M(a,b) \frac{\partial f_{a|b}(a \mid b)}{\partial b} da \,. \tag{34}$$

The second term can be rewritten as

$$\int_a C^M(a,b) d\left(\frac{\partial F_{a|b}(a \mid b)}{\partial b} \right), \tag{35}$$

with $F_{a|b}(a \mid b) = \int_{\underline{a}}^{a} f(t,b) dt / f_b(b)$ the distribution of ability conditional on background. To verify Equation (35), note that

$$\frac{\partial F_{a|b}(a \mid b)}{\partial b} = \frac{f_b(b) \int_{\underline{a}}^{a} \frac{\partial f(t,b)}{\partial b} dt - \frac{\partial f_b(b)}{\partial b} \int_{\underline{a}}^{a} f(t,b) dt}{f_b(b)^2} \,, \tag{36}$$

which indeed implies that

[64] In technical terms, the old distribution (first-order) dominates the new one.

$$d\left(\frac{\partial F_{a|b}(a\,|\,b)}{\partial b}\right)=\frac{f_b(b)\dfrac{\partial f(a,b)}{\partial b}-\dfrac{\partial f_b(b)}{\partial b}f(a,b)}{f_b(b)^2}da,$$
$$=\frac{\partial}{\partial b}\left(\frac{f(a,b)}{f_b(b)}\right)da, \tag{37}$$
$$=\frac{\partial f_{a|b}(a\,|\,b)}{\partial b}da,$$

as required. Now, using integration by parts, Equation (35) is equal to

$$C^M(a,b)\frac{\partial F_{a|b}(a\,|\,b)}{\partial b}\Bigg|_{\underline{a}}^{\overline{a}}-\int_a C_a^{M'}(a,b)\frac{\partial F_{a|b}(a\,|\,b)}{\partial b}da\,, \tag{38}$$

where the first term is zero (as $\dfrac{\partial F_{a|b}(\underline{a}\,|\,b)}{\partial b}=\dfrac{\partial F_{a|b}(\overline{a}\,|\,b)}{\partial b}=0$ and $C^M(a,b)$ finite at the boundaries). To sum up, Equation (34) can be written as

$$\tilde{C}^{M'}(b)=\int_a C_b^{M'}(a,b)f_{a|b}(a\,|\,b)da-\int_a C_a^{M'}(a,b)\frac{\partial F_{a|b}(a\,|\,b)}{\partial b}da\,. \tag{39}$$

What does this equation tells us? Consider an outcome-prioritarian minister with $C_a^{M'}(a,b)>0$ and $C_b^{M'}(a,b)<0$, a possible case, given Table 4.

If there is no association between background and ability, then $\dfrac{\partial F_{a|b}(a\,|\,b)}{\partial b}=0$ holds everywhere, and the second term drops out. In that peculiar case, the sign of the numerator depends on the sign of $C_b^{M'}(a,b)$, which was negative for the prioritarian minister under consideration. In this case, the result at the individual level extends towards the school level, i.e., with no association between background and ability, the minister will always provide more resources per pupil to schools serving pupils with a more disadvantaged background.[65]

If there is a positive association between background and ability, i.e., $\dfrac{\partial F_{a|b}(a\,|\,b)}{\partial b}<0$ holds everywhere, then the second term no longer drops out and is negative (because $C_a^{M'}(a,b)>0$). The sign of the derivative is no longer defined. So, while the minister would like to give more resources per pupil to schools serving pupils with a more disadvantaged background (the first term), being more disadvantaged also signals a lower ability, on average, which the minister

[65] If the association were negative, then the same result obtains.

does not want to reinforce (the second term). This may indeed lead to a paradox if the second term dominates: the minister would like to provide more resources to schools serving more disadvantaged pupils, but, not being able to observe ability and given a positive dependence between ability and background, she decides to give less resources to schools serving more disadvantaged pupils.