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Abstract

Frequent non-completion in optional education can be e¢ cient if dropouts optimally exercise an option rationally foreseen by previous enrollment choices. This paper shows that in educational opportunities and groups of students where enrollment resolves more pronounced individual uncertainty both enrollment and dropout are higher, with ambiguous degree completion e¤ects, and educational outcomes are unambiguously better in expectation and on average.

JEL Codes: I220, I280.

Keywords: option value, higher education.

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1 Motivation

Policy reports and empirical research abundantly document that dropout is frequent in higher-education degree programs, and varies across degree programs and countries in ways related to characteristics of the student population and to labor market conditions.[1] Public opinion and policymakers view high dropout rates, such as those observed in American and Italian public universities, as an indicator of poor educational performance. Because higher education is not mandatory, however, students can drop out only if they previously chose to enrol, so understanding why they did is at least as important as understanding why they drop out. And while dropout is hardly ever planned at enrollment, it can be an optimal choice in light of new information that arrives after enrollment: information is valuable, and so are dropout options.

Degree non-completion need not be a source of concern if it results from worthwhile experimentation, as pointed out by Manski (1989) and earlier contributions.[2] A number of more recent papers analyze dropout and continuation options in structural models of educational choices, like those of Altonji (1993) and Keane and Wolpin (1997), where only completion offers a lumpy and discrete wage premium. In Stange (2012), students update their own aptitude and taste assessments while pursuing completion, so dropout is sensible and valuable in light of uncertainty about such factors. Ozdagli and Trachter (2014) show in a more stylized calibrated model that the option to drop out contributes a large portion of the *ex ante* value of college education for students who are uncertain about their own ability. Stinebrickner and Stinebrickner (2012, 2014) model the theoretical and empirical role of heterogeneous progressively updated degrees of confidence in own ability among enrolled students. Athreya and Eberly (2013; forthcoming) let dropout be triggered by changing individual circumstances as well as by academic performance, Chatterjee and Ionescu (2012) assess the role of uninsured drop-out risk in a similar framework, and Hendricks and Oksana (2018) model dropout choices in a setting where degree completion has heterogenous value and is achieved at individual-specific speeds. These papers do not

[1] See e.g. Vossensteyn et al. (2015) for European countries, Bound et al. (2010) for the US, Ghignoni (2016) and Contini, Cugnata, and Scagni (2018) for Italy.

[2] Besides references in Manski (1989), see e.g. Comay, Melnik, and Pollatschek (1976), Alfred Lord Tennyson ("It's better to have tried and failed than to live life wondering what would've happened if I had tried") and, somewhat more cryptically, Bob Dylan ("She knows there's no success like failure, and that failure's no success at all").

particularly focus on how different uncertainty across educational opportunities and groups of students may determine enrollment and dropout. Some of their relevant theoretical insights and empirical findings will be further discussed below in the context of the present paper's modeling approach, which is focused on arrival of new information about the value of dropout and completion, and is simple enough to afford rigorous characterization of that issue.

Treating individual outcomes as continuous random variables and deploying convenient distributional assumptions to illustrate more general results, Section 2 shows that wider dispersion of possible educational outcomes makes dropout more likely *ex post* if dropout is expected to be worse than completion, but also increases the value of the option to escape bad news by dropping out, and makes enrollment more appealing *ex ante*. Aggregating individual choices, Section 3 shows that average expected completion outcomes move enrollment and dropout in opposite directions but other features of an educational opportunity and of its potential student population do not, with ambiguous implications for overall completion. Degree programs and potential student populations with better expected outcomes conditional on completion do feature higher enrollment and lower dropout rates. An incomplete education can be valuable, however, and a better expected outcome conditional on dropout increases both enrollment and dropout. More interestingly, both enrollment and dropout also increase, and expected and average educational outcomes unambiguously improve, when there is more uncertainty around enrollment-time expectations that are worse for dropout than for completion. Section 4 discusses how real-life problems and empirical findings relate to the model's stylized exogenous outcome distributions, and concludes outlining the results' implications for educational assessments and corrective policies.

2 Individual choices

In what follows "payoff" refers to the discounted expectation (accounting for any predictable trends) of benefits and costs along a future educational and labor market trajectory. At enrollment time, denote an individual's payoff from enrolling and completing an educational program with y_c, that from enrolling and dropping out with y_d. The choice of whether to enroll depends on the payoff of non-enrollment, denoted y_o, which may account for current and future choices to enroll and drop out of other education and work opportunities.

The sign of uncertainty's implications for dropout is shown below to depend on whether dropout is expected to be worse than completion,

$$y_d < y_c. \tag{1}$$

As long as better labor market prospects for graduates more than compensate the effort and the direct and opportunity costs entailed by degree completion, this inequality is plausible, and underlies the notion that dropout is wasteful and should be avoided.[3]

At the time dropout may occur, the continuation and dropout payoffs are updated by innovations, denoted ϵ_c and ϵ_c, that depend on relevant events occurring between enrollment and possible dropout. Both ϵ_c and ϵ_c have zero expectation at enrollment time. Their realizations can trigger dropout, which is the optimal choice if $y_c + \epsilon_c < y_d + \epsilon_d$ and has probability $P_d \equiv$prob$(\epsilon_c - \epsilon_d < y_d - y_c)$ as of enrollment time.

2.1 Uncertain completion outcomes

It is easier to follow derivations if before the dropout choice new information arrives about the completion payoff only. With $\epsilon_d \equiv 0$, the expected value of enrollment is

$$V \equiv (y_c + E\left[\epsilon_c | \epsilon_c > y_d - y_c\right]) (1 - P_d) + y_d P_d. \tag{2}$$

Under certainty, ϵ_c is zero, and condition (1) makes it optimal to enroll and complete when $y_c > y_o$.[4] If completion outcomes are random, an enrolled individual drops out and obtains y_d with probability P_d, completes and obtains $y_c + E\left[\epsilon_c | \epsilon_c > y_d - y_c\right] > y_d$ with probability $(1 - P_d)$. Hence, $y_o < y_d$ is a sufficient condition for enrollment to be optimal. If y_c and y_d are both below y_o enrollment would entail an expected loss if it were irreversible, but the option to observe ϵ_c and possibly drop out adds to enrollment a value that is non-negative and, if $y_c < y_o$, must be positive for the enrolled. The extent and character of uncertainty about mean-zero news determines whether possible positive news make $E\left[\epsilon_c | \epsilon_c > y_d - y_c\right]$ sufficiently large,

[3] As discussed in some of the following footnotes the model is applicable to situations where this condition is violated, as it may be for individuals who receive a generous scholarship while pursuing a worthless degree, or at enrollment expect that "dropout" after obtaining an intermediate degree will be better than continuing to an advanced degree.

[4] If condition (1) does not hold, under certainty it is optimal to enrol, and subsequently drop out, when $\max\{y_c, y_d\} > 0$.

and possible negative news make P_d =prob$(\epsilon_c < y_d - y_c)$ sufficiently small, to make enrollment optimal.

To parameterize uncertainty, consider a uniform mean-preserving spread. Let prob$(\epsilon_c < y)$ =prob$(\epsilon_c/\sigma_c < y/\sigma_c) \equiv F(y/\sigma_c)$. Parameter σ_c is the standard deviation of ϵ_c if $F(z)$ is the probability distribution function of a random variable z with unitary variance. It is larger when more information arrives between the enrollment and possible dropout times.

Supposing that $F(\cdot)$ is differentiable (at least) at the dropout boundaries, the dropout probability $P_d = F(y_d - y_c)/\sigma_c$ depends on expectation and spreads according to

$$\frac{\partial P_d}{\partial y_d} = F'\left(\frac{y_d - y_c}{\sigma_c}\right)\frac{1}{\sigma_c}, \ \frac{\partial P_d}{\partial y_c} = -F'\left(\frac{y_d - y_c}{\sigma_c}\right)\frac{1}{\sigma_c},$$

$$\frac{\partial P_d}{\partial \sigma_c} = F'\left(\frac{y_d - y_c}{\sigma_c}\right)\frac{y_c - y_d}{\sigma_c^2}. \tag{3}$$

If the density $F'((y_d - y_c)/\sigma_c)$ is positive and $y_c - y_d > 0$ by (1),[5] this establishes

Result 1 *If the probability distribution of the relevant innovations is continuous and differentiable, dropout is more likely if at enrollment its payoff is larger, less likely if the completion payoff is larger, more likely if completion is expected to be better than dropout and more uncertainty resolves after enrollment.*

Consider next the enrollment choice. Rearranging (2), its value

$$V(y_d, y_c, \sigma_c) \equiv y_c + E[\epsilon_c | \epsilon_c > y_d - y_c](1 - P_d) + (y_d - y_c) F\left(\frac{y_d - y_c}{\sigma_c}\right). \tag{4}$$

includes $E[\epsilon_c|\epsilon_c > y_d - y_c](1 - P_d) = \int_{y_d - y_c}^{\infty} \epsilon_c dF\left(\frac{\epsilon_c}{\sigma_c}\right)$, the excess over y_c of the expected outcome conditional on not dropping out. Changing variables to $z = \epsilon_c/\sigma_c$, $\epsilon_c = \sigma_c z$

$$V(y_d, y_c, \sigma_c) = y_c + \sigma_c \int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} z F'(z) dz + (y_d - y_c) F\left(\frac{y_d - y_c}{\sigma_c}\right) \tag{5}$$

[5] If (1) does not hold dropout is 100% at $\sigma_c = 0$ and declines in σ_c: students who do not expect completion (or an advanced degree) to be better for them than an incomplete (or basic) degree are more likely to continue when enrollment provides more information.

and differentiating[6]

$$\frac{\partial V(y_d, y_c, \sigma_c)}{\partial \sigma_c} = \int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} zF'(z)dz \geq 0. \tag{6}$$

The inequality follows from the fact that the rational expectation of innovations is zero,

$$\int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} zF'(z)dz = \int_{y_d - y_c}^{\infty} \epsilon_c F'(\epsilon_c/\sigma_c)d\epsilon_c \geq \int_{-\infty}^{\infty} \epsilon_c F'(\epsilon_c/\sigma_c)d\epsilon_c = 0,$$

and is strict if $\int_{-\infty}^{y_d - y_c} \epsilon_c F'(\epsilon_c/\sigma_c)d\epsilon_c > 0$. Mathematically, uncertainty makes dropout more likely but this has no first-order value effects when the dropout choice is optimal: the value of enrollment varies by the expected truncated completion outcome, formalizing the intuitive notion that the option to drop out is more valuable when more important news may arrive, and establishing

Result 2 *When dropout occurs with positive probability, a proportional spread of innovations increases the value of enrollment by the truncated mean of innovations.*

Differentiation of (5) also makes it easy to characterize the more obvious enrollment value effects of the dropout and completion payoffs:

$$\begin{aligned} \frac{\partial V(\cdot)}{\partial y_d} &= -y_d F'(y_d - y_c) + F(y_d - y_c) + y_d F'(y_d - y_c) = F(y_d - y_c) = P_d, \\ \frac{\partial V(\cdot)}{\partial y_c} &= 1 + y_c F'(y_d) - F(y_d - y_c) - y_c F'(y_d - y_c) = 1 - F(y_d - y_c) = 1 - P_d. \end{aligned} \tag{7}$$

Optimality of the dropout choice again implies that different cutoffs have no first-order effects for the optimized enrollment value, as the variation of the truncated expected completion outcome offsets the changing probability of obtaining only the dropout payoff. The derivatives are both positive as long as $0 < P_d < 1$, and establish

Result 3 *When dropout is possible, the value of enrollment increases in the dropout payoff with slope equal to the dropout probability, and in the completion payoff with slope equal to the completion probability.*

[6] The arguments of $V(\cdot)$ appear in the integration limit and in the argument of the dropout probability function: $\frac{\partial V(\cdot)}{\partial \sigma_c} = \int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} zF'(z)dz - \sigma_c \frac{y_d - y_c}{\sigma_c} F'\left(\frac{y_d - y_c}{\sigma_c}\right)\left(-\frac{y_d - y_c}{\sigma_c^2}\right) + (y_d - y_c) F'\left(\frac{y_d - y_c}{\sigma_c}\right)\left(-\frac{y_d - y_c}{\sigma_c^2}\right).$

2.2 Normal innovations

Parameter σ determines all higher moments of the mean-zero news if

$$F'(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) \equiv \varphi(z),$$

the standard Gaussian density whose properties, as usual, afford tractability and neatness to derivations and results. The density of innovations is symmetric around zero, so condition (1) ensures that the probability of dropout is less than 50%, and always positive, so completion would be possible for the not enrolled and dropout is possible for all the enrolled.

Because the dropout probability is bounded away from zero and unity, the expectations y_d and y_c strictly increase the value of enrollment by (7). And so does σ_c, because the expectation of the truncated distribution in (6), using $\int_y^\infty z \exp(-z^2/2)dz = \exp\left(-y^2/2\right)$, coincides with the normal density evaluated at the lower limit of integration: the derivative

$$\frac{\partial V(\cdot)}{\partial \sigma_c} = \int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} z\varphi(z)dz = \varphi\left(\frac{y_d - y_c}{\sigma_c}\right) \tag{8}$$

is strictly positive, and tends to $\varphi(0) = 1/2\pi$ as $\sigma_c \to \infty$. Intuitively, $V(\cdot)$ increases without bound as σ_c grows, because the value of the option to escape bad news and exploit good news becomes arbitrarily large as extremely favorable or unfavorable information may arrive with increasingly large probability. When dropout and completion are both expected to pay less than non-enrollment, enrollment is not optimal at $\sigma_c = 0$, but a sufficiently large yet finite σ_c can bring (2) to exceed any y_o.

2.3 Random dropout outcomes

In reality, at enrollment time there can be uncertainty also around the dropout payoff. The implications of ϵ_d realizations are symmetric to those of ϵ_c realizations in determining the realization of $\epsilon_c - \epsilon_d$, a random variable with mean zero and variance $\sigma^2 = \sigma_c^2 + \sigma_d^2 - 2\sigma_{SD}$ for σ_{SD} a covariance that may be subjectively positive or negative at enrollment time, but cannot be identified in data where only one of the two shocks is observed for each individual. Setting $\sigma_{SD} = 0$ for notational simplicity, the spread of payoff difference innovations $\sigma \equiv \sqrt{\sigma_c^2 + \sigma_d^2}$ increases in both σ_c and σ_d and has the same implications as those of σ_c above for the probability of dropout,

$$P_d = \text{prob}\left(\epsilon_c - \epsilon_d < y_d - y_c\right) = F\left(\frac{y_d - y_c}{\sigma}\right). \tag{9}$$

The value of enrollment,

$$V(\cdot) = \left(y_c + E\left[\epsilon_c | \epsilon_d - \epsilon_c < y_c - y_d\right]\right)(1 - P_d) + \left(y_d + E\left[\epsilon_d | \epsilon_c - \epsilon_d < y_d - y_c\right]\right) P_d, \tag{10}$$

includes truncated (by optimal dropout) innovations that can be shown to be larger in expectation when more information is revealed between enrollment and dropout. To see this, denote the density of ϵ_c with $f\left(\epsilon_c/\sigma_c\right)$ and that of ϵ_d with $g\left(\epsilon_d/\sigma_d\right)$, and write out the relevant integrals of convolutions:

$$E\left[\epsilon_c | \epsilon_d - \epsilon_c < y_c - y_d\right](1 - P_d) = \int_{x=-\infty}^{\infty} \left(\int_{z=x+y_d-y_c}^{\infty} z\, f\left(\frac{z}{\sigma_c}\right) dz\right) g\left(\frac{x}{\sigma_d}\right) dx, \tag{11}$$

$$E\left[\epsilon_d | \epsilon_c - \epsilon_d < y_d - y_c\right] P_d = \int_{z=-\infty}^{\infty} \left(\int_{x=z+y_d-y_c}^{\infty} x\, g\left(\frac{x}{\sigma_d}\right) dx\right) f\left(\frac{z}{\sigma_c}\right) dz. \tag{12}$$

In each expression, as in the derivation of Result 2, the inner integral increases in one innovation's spread given the other innovation's realization, and so does its expectation taken across all realizations of the other innovation.

A larger spread of either innovation increases the value of enrollment (10), which is larger when either the completion or dropout outcome are more uncertain. To illustrate this generally valid result it is convenient to suppose that innovations are normally distributed, and so are their convolutions. The Appendix shows that in this case

$$V(y_d, y_c, \sigma) = y_c + \sigma\varphi\left(\frac{y_d - y_c}{\sigma}\right) + (y_d - y_c)\,\Phi\left(\frac{y_d - y_c}{\sigma}\right) \tag{13}$$

for $\Phi(\cdot)$ the standard Gaussian probability function. Hence, the standard deviation σ of the payoff difference has the same role as σ_c above in determining the value of enrollment.

enrollment is the optimal choice when $V(y_d, y_c, \sigma) > y_o$. It can be helpful to inspect numerical solutions for the sets of y_d, y_c, σ parameters that equate (13) to a given y_o outside option. In the $\sigma = 0$ certainty case, enrollment is optimal when $y_d > y_o$ for all y_c, and also when $y_c > y_o$ for all y_d. When neither y_c nor y_d exceed y_o, Figure 1 shows that an increasingly positive σ adds option value to the enrollment choice and let a smaller y_d suffice to make enrollment optimal for each y_c, along lines with an increasingly negative slope. The slope becomes vertical as y_c

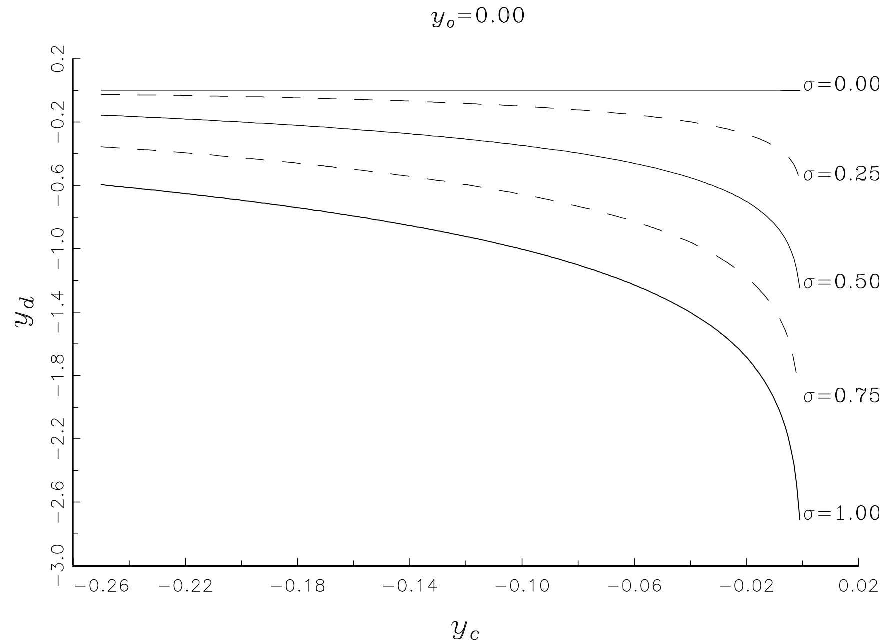

Figure 1: Individual enrollment is optimal for payoff pairs y_c, y_d above and to the right of lines drawn for various degrees of uncertainty σ.

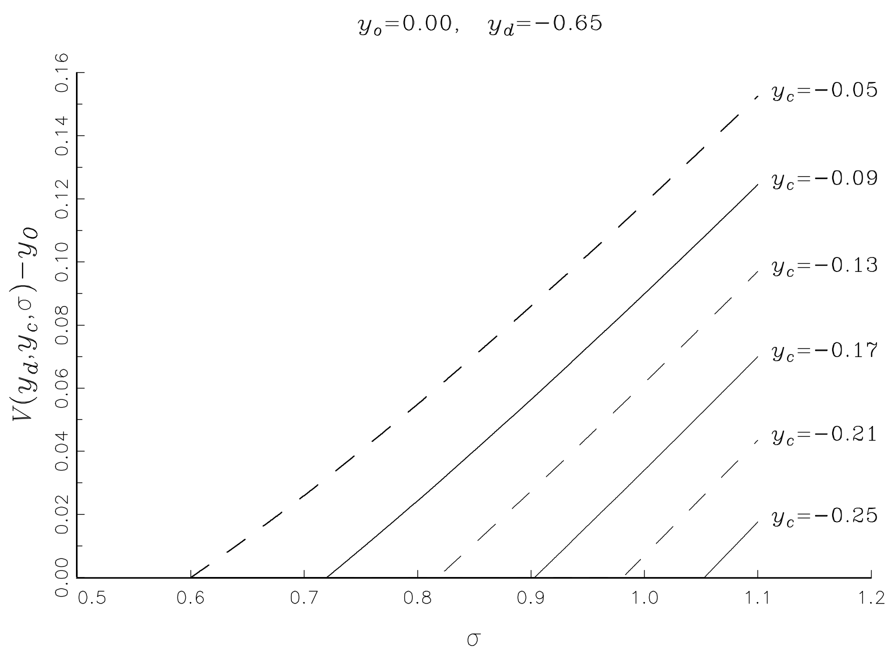

Figure 2: Value added of enrollment opportunity.

approaches y_o, because when $y_c > y_o$ enrollment is optimal for all y_d and σ: its value does not exceed y_o with probability one, as discussed after (2) in the non-random dropout outcome case, but does in expectation, because symmetrically distributed positive and negative news about both completion and dropout payoffs cancel out on average.

It can also be helpful to visualize the implications of uncertainty for the value of enrollment in the normal case. Figure 2 shows that, as a fatter positive tail of innovations increases the mean of the truncated innovation distribution, the value of enrollment increases in σ, and does so almost linearly because its slope $\varphi\left(\left(y_d - y_c\right)/\sigma\right)$ is nearly constant in the parameter range considered. Numerical values of the stylized model's parameters are not meant to be realistic, but it is worth mentioning that for the normal distribution the probability of dropout is 50% at $y_d = y_c$, and about 20-30% for $\left(y_d - y_c\right)/\sigma_c \approx -0.8$.[7] The slope can of course be different in other parameter ranges and for other distributions, but is positive for all the innovations distributions that have positive density at the dropout cutoff and assign probability to both dropout and completion. In what follows, this straightforward option-value effect of individual uncertainty plays an important role in determining aggregate outcomes.

3 Aggregate enrollment, dropout, and achievement

The above characterization of individual choices has straightforward implications for enrollment and dropout of heterogeneous individuals at specific educational programs. When innovations are normally distributed, each individual expects the excess value of enrollment over non-enrollment to be

$$\begin{aligned} V(y_d, y_c, \sigma) - y_o &= \left(\tfrac{y_c - y_o}{\sigma} + \varphi\left(\tfrac{y_d - y_c}{\sigma}\right) + \tfrac{y_d - y_c}{\sigma}\Phi\left(\tfrac{y_d - y_c}{\sigma}\right)\right)\sigma \\ &= \left(\tilde{y}_o + \varphi\left(\tilde{y}_d\right) + \tilde{y}_d \Phi\left(\tilde{y}_d\right)\right)\sigma \qquad (14) \\ &\text{for } \tilde{y}_d \equiv \frac{y_d - y_c}{\sigma},\ \tilde{y}_o \equiv \frac{y_o - y_c}{\sigma}. \end{aligned}$$

[7] Overall non-completion is in the order of 35-50% in the US and in Italy (Bound et al, 2010; Ghignoni, 2016). It is heterogeneous across degrees and groups of students, and well below 50% in some cases. This would require condition (1) to be violated and/or the distribution of innovations to be asymmetric, with large unlikely positive payoffs, rather than normally distributed.

This expression depends on payoff differences scaled by the spread σ, all of which generally vary across individuals. An individual enrols when the expectation and uncertainty parameters introduced in the previous section make it positive. The sign of $\tilde{y}_o$ depends on whether degree completion would be expected to be more or less valuable than not enrolling in the absence of the dropout option. The expression on the right-hand side of (14) also depends on the scaled payoff difference $\tilde{y}_d$, which is negative by (1) but less negative when a smaller y_c or larger y_d make dropout less damaging and more likely. At given expectations, when σ is larger both $\tilde{y}_d$ and $\tilde{y}_o$ are smaller in absolute value. This condition, like the dropout probability $\Phi(\tilde{y}_d)$, depends on scaled payoff differences: the value added of enrollment (14) increases in $\tilde{y}_o$ as well as in $\tilde{y}_d$, because $d[\varphi(\tilde{y}_d) + \tilde{y}_d\Phi(\tilde{y}_d)]/d\tilde{y}_d = \Phi(\tilde{y}_d) > 0$.[8]

Aggregate enrollment and dropout are readily characterized using the Results above, which are valid more generally than in the tractable case of normally distributed innovations. To pin down enrollment rates at specific educational programs by specific populations either or both of $\tilde{y}_d$ and $\tilde{y}_o$ must be heterogeneous across individuals, who enroll if (14) is positive. The individuals who enroll are those for whom $\tilde{y}_d$ and $\tilde{y}_o$ lie below the $\tilde{y}_o = \varphi(\tilde{y}_d) + \tilde{y}_d\Phi(\tilde{y}_d)$ line in the top panel of Figure 3, so the population enrollment fraction is the integral over the $\tilde{y}_d$ marginal density of the $\tilde{y}_o|\tilde{y}_d$ conditional cumulative distribution.

That calculation is feasible for any continuous cross-sectional distribution of the $\tilde{y}_d$ and $\tilde{y}_o$ scaled differences, but is again more tractable when that distribution is normal. Suppose, for illustration purposes, that the payoffs y_o, y_c, y_d in the numerator are normally distributed in the population and independent of each other, and the spread σ of innovations is constant across individuals.[9] The means $\tilde{\mu}_d$ and $\tilde{\mu}_o$, standard deviations $\tilde{\sigma}_d$ and $\tilde{\sigma}_o$, and correlation ρ of $\tilde{y}_d$ and $\tilde{y}_o$ are related to the means and variances of the cross-sectional distribution of individual payoffs

[8]To see this, note that $\varphi'(z) = -z\exp(-z^2/2)/\sqrt{2\pi} = -z\varphi(z)$ cancel out in the derivative, for the same optimal-dropout reasons that deliver the simple form of (7) above.

[9]The distribution of scaled differences cannot be Gaussian if the denominator is random. Numerical experimentation with σ draws from a χ_1^2 distribution (and a Student's t distribution for the ratio) finds that the implications of population parameter variation are qualitatively similar to those illustrated in this section.

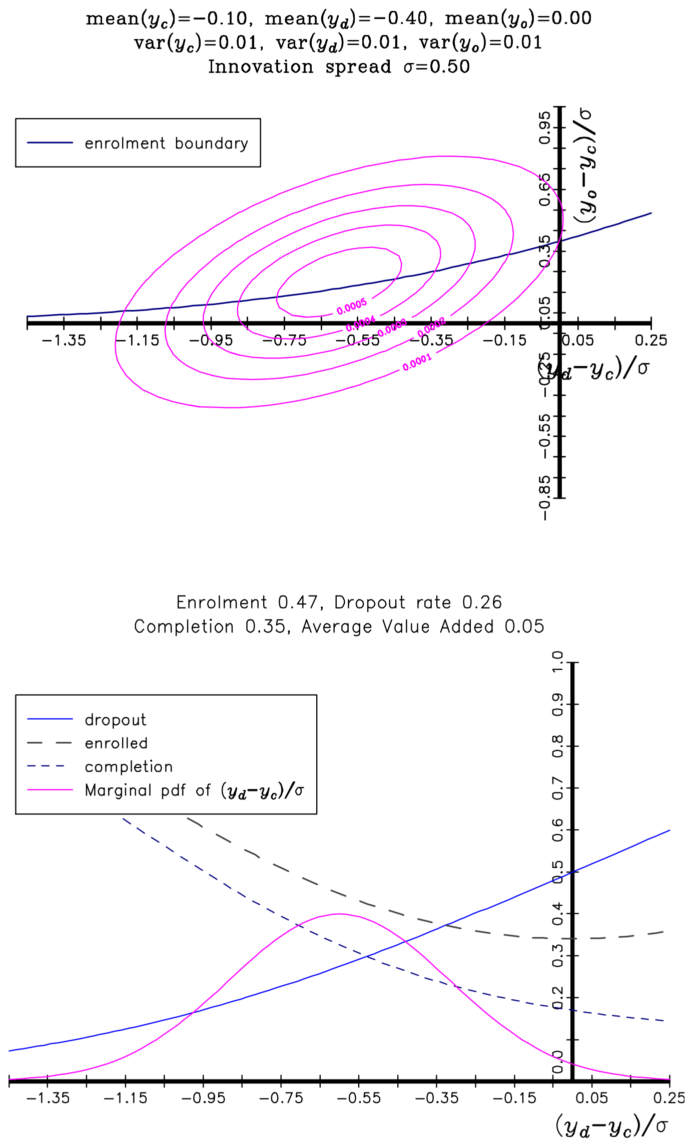

Figure 3: Distribution of bivariate normal scaled expected outcome differences, and conditional and unconditional enrollment and dropout rates.

by

$$\begin{aligned}\tilde{\mu}_d &= \text{mean}(y_d) - \text{mean}(y_c),\ \tilde{\mu}_o = \text{mean}(y_o) - \text{mean}(y_c), \\ \tilde{\sigma}_d &= \sqrt{\text{var}(y_c)+\text{var}(y_d)},\ \tilde{\sigma}_o = \sqrt{\text{var}(y_c)+\text{var}(y_o)},\ \rho = \frac{\text{var}(y_c)}{\tilde{\sigma}_d \tilde{\sigma}_o}.\end{aligned} \tag{15}$$

The correlation ρ is generally positive, as y_c appears in both scaled differences with the same sign, and close to unity if most cross-sectional variation is driven by y_c.

In the top panel the figure plots density contour lines. How many individuals enroll at each $\tilde{y}_d$ depends on the distribution of $\tilde{y}_o$ conditional on $\tilde{y}_d$, which is normal with mean $\tilde{\mu}_o + \rho(\tilde{y}_d - \tilde{\mu}_d)\tilde{\sigma}_o/\tilde{\sigma}_d$ and variance $(1-\rho^2)\tilde{\sigma}_o^2$. Hence,

$$\text{prob}\left(\tilde{y}_o < \varphi(\tilde{y}_d) + \tilde{y}_d \Phi(\tilde{y}_d) \,|\tilde{y}_d\right) = \Phi\left(\frac{\varphi(\tilde{y}_d) + \tilde{y}_d \Phi(\tilde{y}_d) - (\tilde{\mu}_o + \rho(\tilde{y}_d - \tilde{\mu}_d)\tilde{\sigma}_o/\tilde{\sigma}_d)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right).$$

Whether it is increasing or decreasing in $\tilde{y}_d$ depends on whether the conditional density of $\tilde{y}_o$ grows faster or slower than the enrollment trigger, plotted in the bottom panel of Figure 3 along with the dropout rate $\Phi(\tilde{y}_d)$ and the resulting completion rate.

Integration over the marginal distribution of $\tilde{y}_d$ yields the population enrollment rate

$$\int_{-\infty}^{\infty} \Phi\left(\frac{\varphi(\tilde{y}_d) + \tilde{y}_d \Phi(\tilde{y}_d) - (\tilde{\mu}_o + \rho(\tilde{y}_d - \tilde{\mu}_d)\tilde{\sigma}_o/\tilde{\sigma}_d)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right) \varphi\left(\frac{\tilde{y}_d - \tilde{\mu}_d}{\tilde{\sigma}_d}\right) d\tilde{y}_d$$

and the aggregate dropout rate of the enrolled

$$\int_{-\infty}^{\infty} \Phi(\tilde{y}_d)\, \Phi\left(\frac{\varphi(\tilde{y}_d) + \tilde{y}_d \Phi(\tilde{y}_d) - (\tilde{\mu}_o + \rho(\tilde{y}_d - \tilde{\mu}_d)\tilde{\sigma}_o/\tilde{\sigma}_d)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right) \varphi\left(\frac{\tilde{y}_d - \tilde{\mu}_d}{\tilde{\sigma}_d}\right) d\tilde{y}_d.$$

The figure also reports the average educational value added, computed by numerical integration of the positive values of the function (14) of $\tilde{y}_d$ and $\tilde{y}_o$, weighted by the bivariate normal density parametrized as in (15). It includes average realized outcomes of completers as well as of dropouts among the enrolled.

Figure 4 illustrates the effects of the average completion payoff with two pictures like the previous one. The mean of $y_c$is more positive on the right than on the left. In in the top panel the distribution of scaled payoff differences shifts diagonally down and to the left, so a larger fraction of the source population enrols. In the bottom panel, the dropout probability is increasing, so the total dropout rate declines. The effects of y_d, shown in Figure 5, are similarly intuitive. A more positive dropout payoff shifts the bivariate distribution horizontally to the right. Because

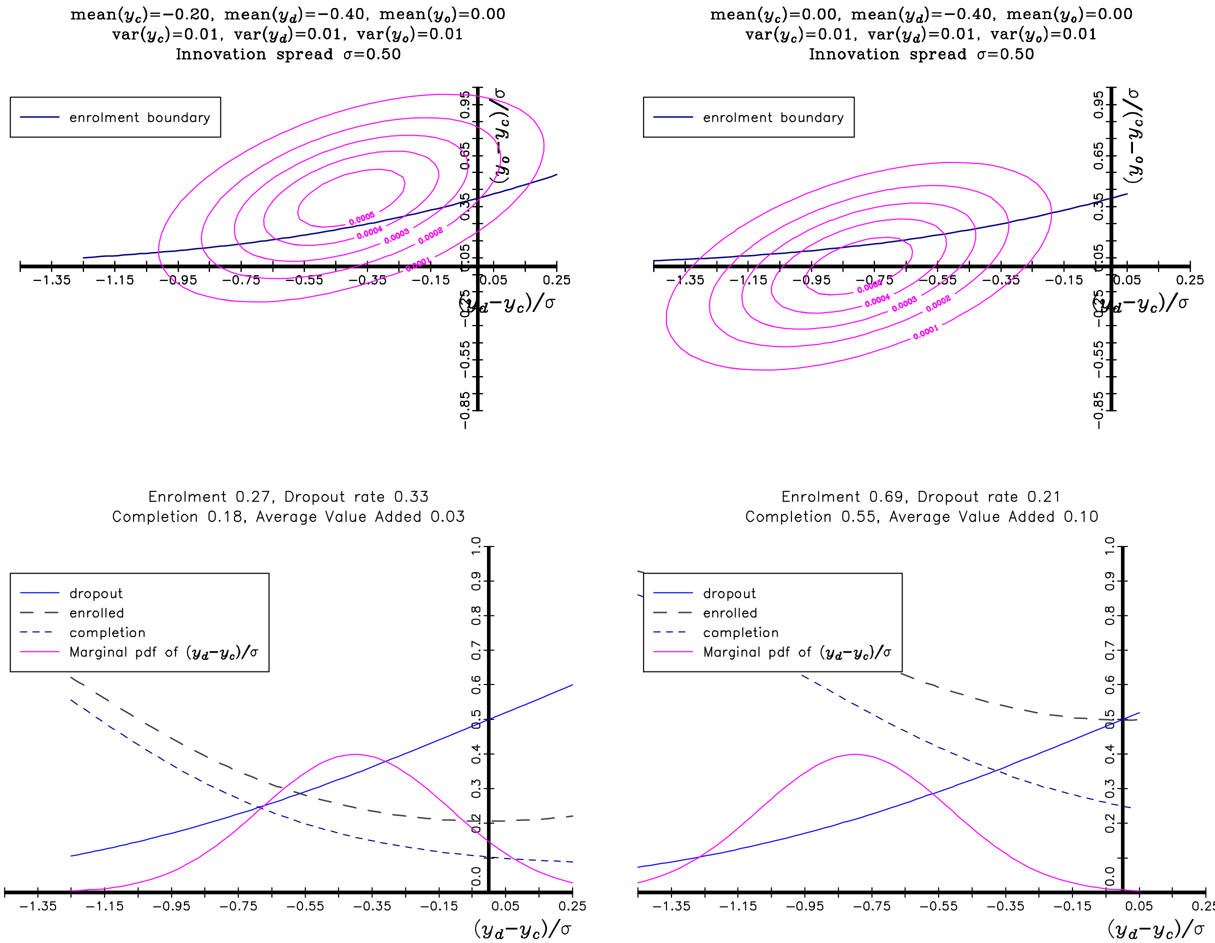

Figure 4: Implications of average completion payoff for enrollment, dropout, completion rates and average value added.

the enrollment cutoff is increasing, a larger fraction of the source population enrols and, because in the bottom panel the dropout probability is also increasing, the total dropout rate increases. In both figures, the expected and average value added (14) is larger when the distribution of its determinants has more positive means.

The same reasoning is valid for more general cross-section distributions than the bivariate normal used to illustrate these effects, which follow from the equally intuitive Result 3, and establishes

Result 4 *Across student populations and educational opportunities, a larger average completion payoff reduces dropout, a larger average dropout payoff increases dropout, and both increase enrollment and the average value added of education.*

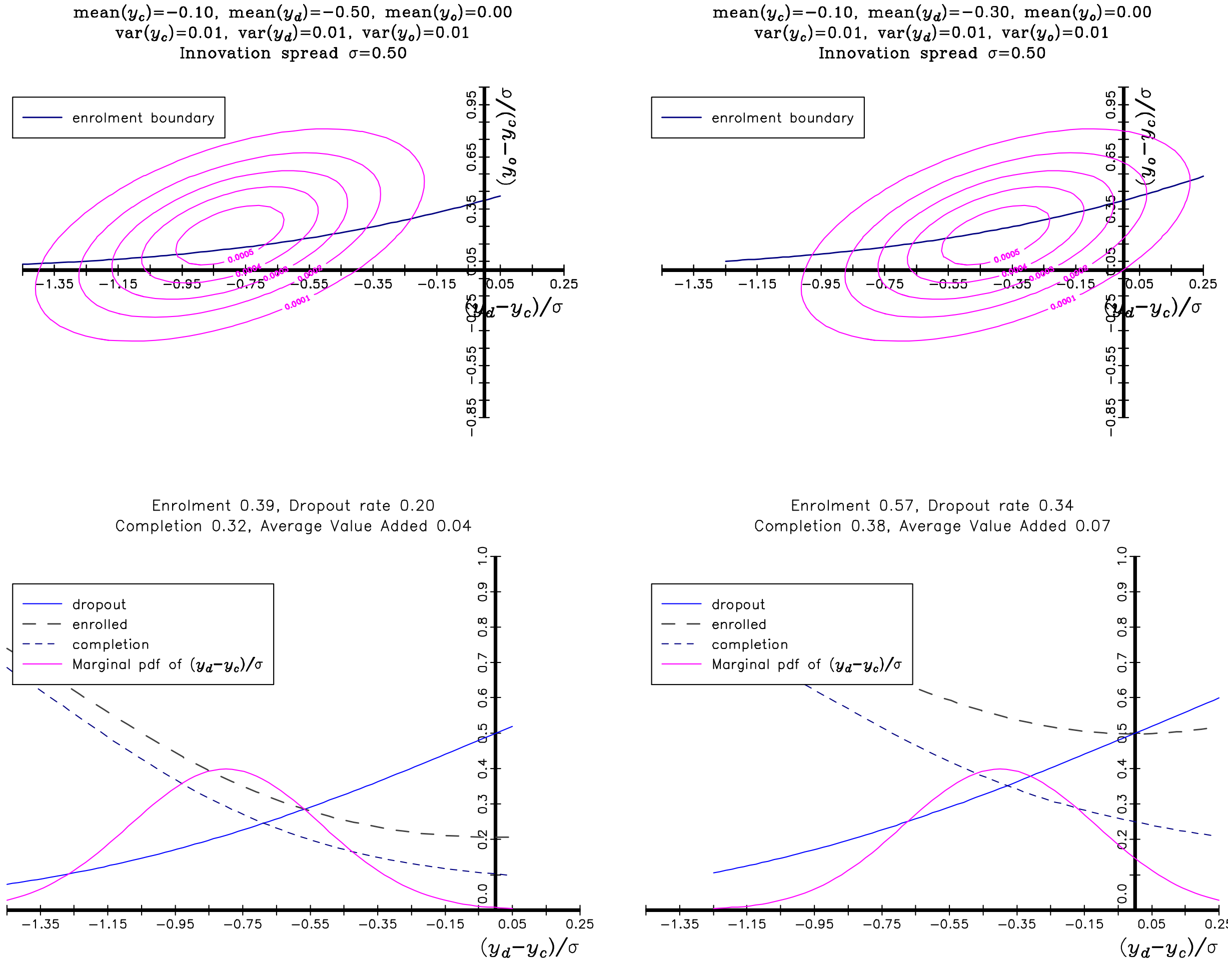

Figure 5: Implications of average dropout payoff for enrollment, dropout, completion rates and average value added.

The non-enrollment value y_o is set to zero in the figures, but its implications are a simple composition of those of $y_d$and y_c: a larger $y_o$is equivalent to the same decline of both y_d and y_c, and reduces $\tilde{y}_o$ leaving $\tilde{y}_d$ unchanged. The bivariate distribution shifts vertically upwards, and enrollment and average value added both decline. The marginal distribution of $\tilde{y}_d$ shown in the bottom panel for the population remains unchanged but dropout declines through a composition effect, because $\tilde{y}_d$is more negative among the enrolled.

The effects of uncertainty around payoffs are somewhat less obvious and more interesting than those just discussed. Figure 6 shows that, moving from the left to the right set of picture panels, a larger σshrinks the distribution of scaled payoff differences along both dimensions and, as (1) holds for most of the population, moves it to the right.[10] A larger fraction of the source

[10]Condition (1) does not hold in the region to the right of the vertical axis in the figure, where the

population enrols, because the enrollment cutoff is increasing in $\tilde{y}_d$. Because in the bottom panel the dropout probability is also increasing in $\tilde{y}_d$, the total dropout rate increases.[11] More uncertainty encourages enrollment by Result 2, as the thicker tail of the truncated distribution increases the option value of dropout, and improves average completed payoffs even as, by Result 1, it increases dropout among enrolled students.

A larger σincreases completion for the parameters used in plotting Figure 6 but need not do so in general because, when (1) holds, it increases dropout as well as enrollment. Inspection of (8) indicates that the dropout effect of $\sigma$is strongest at $\tilde{y}_d \approx 0$, where the normal density is largest. The strength of its enrollment effect depends on the cross-sectional distribution of payoffs: it declines as enrollment increases in the normal and other plausible cases, and can be dominated by the dropout effect. The numerical average of the value-added expression (14) is also larger when $\sigma$is larger, and it is not difficult to see that this holds in general. Value added is zero for the non-enrolled, positive for those who enrol: a larger σmakes it positive for individuals who would not have enrolled and also, as in Figure 2, increases it for all enrolled individuals, including the inframarginal ones who find it optimal to enroll at the initial σ. To the extent that idiosyncratic *ex post* uncertainty averages out across the population considered, this reasoning establishes

Result 5 *When dropout is possible, the aggregate value added of an educational opportunity is higher if enrollment resolves more pronounced uncertainty about individual educational outcomes.*

Higher uncertainty increases enrollment to the point where, for the marginal individual who rationally chooses to enroll and possibly drop out, the larger expected completion outcome implied by the option to take advantage of good news and escape bad news exactly offsets expected welfare losses in the event of dropout. As the expected and average realized outcomes also increase across all inframarginally enrolled individuals, they are better in the aggregate when stronger uncertainty generates more valuable dropout options and, if (1) holds, higher dropout rates.

normal density of payoffs is positive in the normal case: some individuals do expect dropout to be better than completion.

[11] Where condition (1) fails a larger σ moves the distribution of $\tilde{y}_d$ the the left, and reduces dropout.

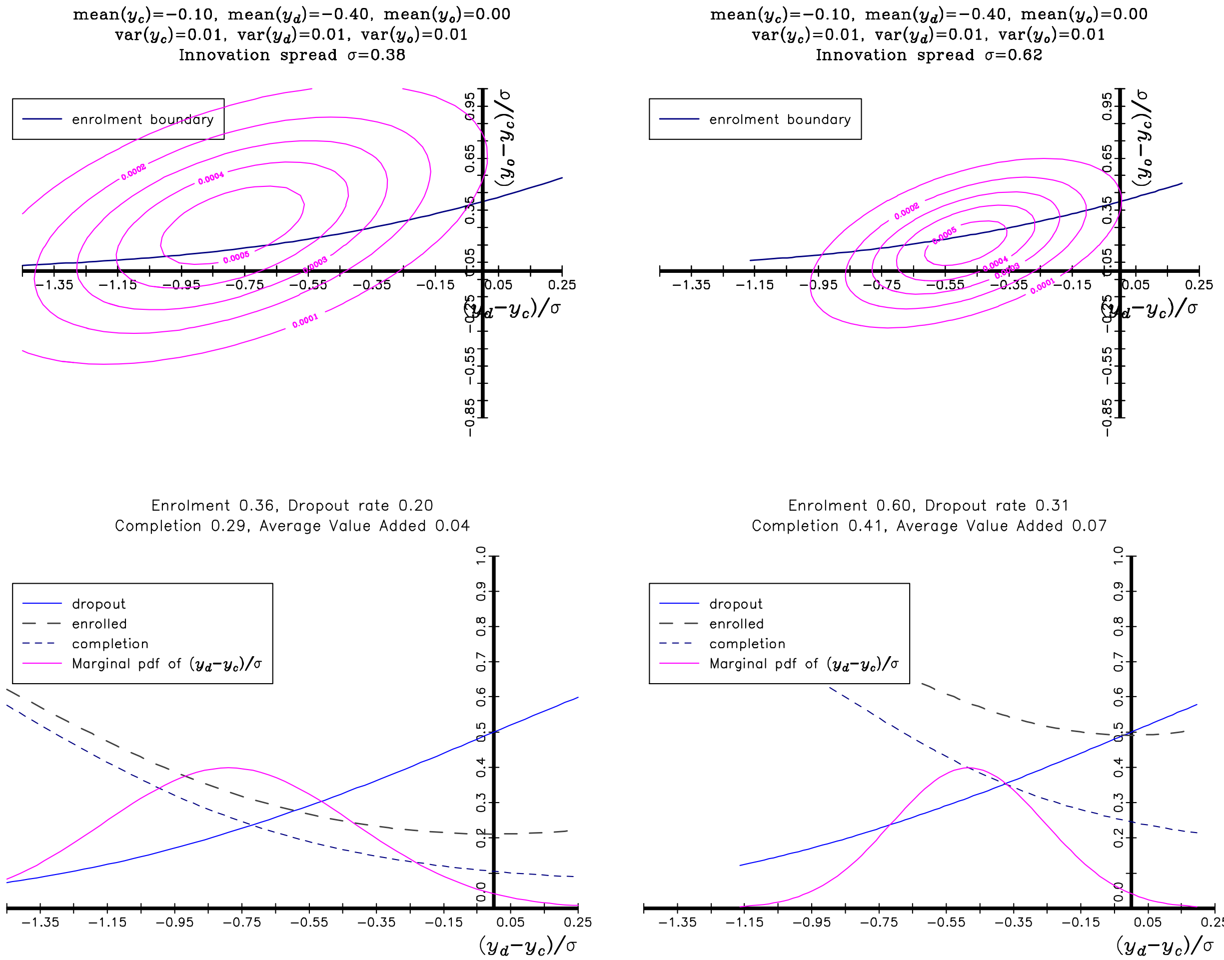

Figure 6: Implications of uncertainty around payoffs for enrollment, dropout, completion, and average value added.

4 Discussion

Stronger uncertainty around expected *ex post* educational outcomes attracts more enrollment and, despite higher dropout, implies a larger aggregate value added of educational opportunities. This may sound puzzling, even to readers of academic journals that treat high rejection rates as a badge of honor, so it is important to understand what the results mean and qualify them in light of the model's assumptions and structure.

The results should not be misconstrued to mean that riskiness is an attractive feature of educational opportunities. Like stock options, dropout options should be valued on a risk-adjusted basis. Stronger uncertainty about educational outcomes increases not only the average but also the dispersion of outcome realizations that in a fully specified economic model, such as those of papers referenced in Section 1 and below, are concave functions of educational outcomes. If potential students are risk averse, then payoffs are smaller when there is more risk.

It would also be wrong to think that the results imply that imprecise and nasty exams are beneficial. To be relevant, the information that arrives after enrollment should reveal structural features of the individual's future outlook. Adding noise to grades or randomly failing some enrolled students would increase dropout, but reduce value added and enrollment. Exams that gauge individual ability produce valuable information that should also not be obscured by lenient grading and generous pass criteria in order to prevent dropout.

Because attempting is a necessary and not sufficient condition for succeeding, dropout is a key peg of success mechanics. The present paper reboots this narrative, previously voiced by Manski (1989) and others, in a setting that models randomness not as the probability of a single completion payoff, but as the standard deviation of continuous realizations that for some students may be exceptionally favorable. As usual, some of a sleek model's features are less than fully general. What follows discusses their tractability and realism advantages over other approaches, and outlines their more or less distinctive empirical and policy implications.

4.1 Modeling strategy

Result 1 hinges on differentiability of the payoff innovations probability distribution function, which rules out discrete distributions. This is not just a technical assumption. While derivatives can somewhat inconveniently be replaced by discrete differences, discreteness of the educational outcome distribution has substantive implications if it makes that distribution irrelevant to dropout. For example, if $\epsilon_c = \pm\sigma$ with probability $p = 0.5$ and $\sigma > y_c - y_d$ the value $(y_c + \sigma + y_d)/2$ of enrolling and possibly dropping out is increasing in y_c, y_d, and σ, hence Results 2 and 3 are valid (and so are the portions of Results 4 and 5 that rely on them). But Result 1 is not valid when the probability of dropout is fixed (at 50% in this example) and does not depend on contingent payoffs and uncertainty.

In Manski's (1989) formal derivations optimal educational choices can result in dropout when degree completion offers a discrete premium and completion probabilities are heterogeneous (and possibly depend on endogenous effort) among potential students. Recent research similarly supposes that exam failures may force dropout and make it impossible to obtain a lumpy degree-completion premium. Most dropout events are indeed exogenous in that sense, or "involuntary," in Athreya and Eberly (2013, forthcoming) and Chatterjee and Ionescu (2012). In such settings, the probability p of dropout determines the expected value of enrollment as well as the variance of possible outcomes (which increases with p only if $p < 0.5$). Given the contingent outcomes, and for a given student, the value of enrollment is lower when p is larger. From this point of view, high dropout rates may appear to be a defect of educational technologies. But if p is heterogeneous across students, then degrees that offer better completion payoffs can feature high dropout rates because they attract enrollment by students who are relatively likely to dropout. Similarly, scholarships or lenient grading or remedial education that reduce *ex post* dropout rates also make enrollment *ex ante* more attractive for students who are likely to drop out.

The relationship between uncertainty and dropout established in Result 1 also hinges on condition (1) which, like the rest of the derivations, focuses on the educational payoffs y_c and y_d of completion and dropout at two points in time. Recalling that y_c and y_d are expected (as of enrollment) present values of risk-adjusted net benefits, (1) requires the predictable balance of costs and benefits to be better in the later than in the earlier portions of the relevant future. In reality, the the bulk of predictable educational costs indeed accrues before innovations may trigger dropout, while most predictable enrollment benefits accrue later and throughout life.[12] The results derived here and in Manski (1989) for a two-period model readily apply in multiple-period extensions. Should potential students be choosing between educational and work opportunities that all allow choices at future times, then y_o would be expressed in the form of (13). In multiple-alternatives and multiple-period extensions of the model, the preferred choice is that which offers better expectations and/or option values than feasible alternatives. A more complicated structure would obscure the role of uncertainty in shaping enrollment and dropout choices, but it would be conceptually straightforward to frame the analysis in terms of a structural model of dynamic dropout choices at various points in time, such as the rich and empirically useful one in Stinebrickner and Stinebrickner (2014).

[12] The extent to which this is the case depends on features of educational programs. Those that initially focus on generally useful education offer more favorable dropout payoffs than those that immediately teach and test highly specialized knowledge and skills. And enrollment is more attractive when it is easier to switch major fields within a degree program, for the same reasons that make submission more appealing when journals offer to forward rejected papers to lesser outlets.

4.2 Assumptions and extensions

Three other features of the model deserve a brief discussion. First, higher uncertainty is parameterized by an increase of the model's proportional spread σ. Outside the convenient and familiar set of normal distributions, not all mean preserving spreads of random variables with differentiable probability distributions take that form, and those that do not need not have the same implications: for example it is possible, if not very interesting, for uncertainty to increase (in the second-order stochastic dominance sense) without changing the dropout probability. The enrollment and dropout effects of $\sigma$are generally valid, however, and normality is not unreasonable if realized welfare is well approximated by a logarithmic function of multiplicative shocks accruing over many periods and along numerous effort and wage dimensions.

Second, the timing of dropout is fixed at a single point in time. It could be the solution of an optimal stopping problem that, like other financial and real options, can be characterized more tractably in a continuous-time approximation than for multiple discrete periods of time. Options to act in the future are more valuable in the presence of stronger uncertainty, which makes it optimal to wait for more information in this and other applications of option values to economic choices. In the model enrollment is also treated as a one-time opportunity. Delaying enrollment is pointless if only enrollment provides new information. Stange (2012) makes a similar assumption and provides a useful discussion of its realism and possible relaxation in studies of educational choices.

Third, individual outcomes and payoffs are statistically and functionally independent of each other, and aggregation simply sums them. In reality, educational technologies are not linear: if returns are decreasing at a given degree program, then stronger enrollment worsens the marginal educational payoffs y_c, and increases dropout. Rational choices remain efficient as long as congestion is a pecuniary externality. Non-pecuniary externalities, such as those generated by peer effects, can be addressed by selective admissions. In a more selective program, $y_c - y_d$ is more positive on average and dropout is less likely. Screening is unavoidably imprecise, however, and high-quality institutions competing for high-quality students may find it less costly and more effective to screen after rather than before enrollment.[13] As long as dropout is an unforced

[13] Some spectacularly successful completing students do emerge from high-quality degree programs that, like the University of Chicago's among top Economics Ph.D. in the 1980s, admit students with relatively

student choice made in light of information that arrives after accepting admission and enrolling, the model's perspective and insights remain qualitatively relevant for even the most selective degree programs.[14]

4.3 Empirical counterparts of theory

It is reasonable for empirical research to focus on "the" completion premium, which is more easily measured than individual-specific educational payoffs. But as in the model above, so in reality dropout probabilities depend on the means and spreads of payoff distributions. Within non-enrolled, completers, and dropout groups of individuals the observed distribution of wages is selected not only at enrollment by permanent heterogeneity, but also at dropout times by new information about continuously-distributed future wages (Stinebrickner and Stinebrickner 2014, their references, and Hendricks and Leukhina, 2018).

The results illustrated in the figures above can help interpret empirical associations of enrollment and dropout across segments of student populations in the same or different degree programs, or over time. A negative association of enrollment and dropout suggests that they are mainly driven by different completion payoffs. A positive association indicates they are driven by different dropout payoffs or, as long as (1), by different *ex ante* uncertainty. Should only the probability and not the payoff of degree completion be heterogeneous among potential students, then a positive association of enrollment and dropout could also be explained by composition effects. Treating educational payoffs as continuously distributed random variables makes it possible to distinguish expectations from the uncertainty surrounding them, and a distinctive empirical implication of this modeling approach is that higher enrollment and dropout should be associated with larger dispersion of outcomes (such as grades, or first-job wages) among enrolled students.

In empirical analysis of individual students' data, maximum likelihood estimation (Stange, 2012) can allow uncertainty to vary with observable characteristics and/or unobservable types, or

little selection and obtain relatively low completion rates.

[14] About 2 per cent of Harvard University undergraduates fail to complete in 6 years. The relative appeal of dropout had very positive realizations for Bill Gates, Mark Zuckerberg, Matt Damon, Bonnie Raitt, and many less successful individuals. Comprehensive data on the fate of dropouts are rare, but Paul's (2015) analysis of a German training program finds that dropout is an opportunity rather than a problem for many individuals.

assess it in survey data (Stinebrickner and Stinebrickner, 2014). To identify its effects the data-generating process needs to be appropriately restricted, for example assuming normality and supposing that the dynamics of dropout probabilities are driven only by learning, rather than innovations to uncertainty. Because only scaled differences matter for individual choices and in probit estimation, less structural regression analysis of enrollment and dropout observations would also need identifying restrictions in order to detect separate effects of expectations and uncertainty. The rich empirical exercise of Lee, Shinz and Lee (2015), who mention in a footnote that estimates are not sensitive to allowing variances to differ across individuals, does not specify such restrictions.

4.4 Dropout: bug or feature?

From this paper's modeling perspective dropping out is an unforced individual choice and a physiological feature of schooling choices and careers, rather than a pathological signal of poor educational performance. Observed dropout is ambiguously related to an educational program's value added. There is less dropout when degree completion is expected to be more valuable, but not when more valuable education is obtained before the dropout decision is made, or when a degree program's enrollment and dropout resolve stronger uncertainty and elicit more valuable information.

The dropout-relevant information generated by enrollment may to some extent be gathered by outside observers of students' exam-taking speed and grades (Hendricks and Leukhina, 2018) or by surveying their expectations (Stinebrickner and Stinebrickner, 2012). It is certainly better known to individuals when they choose whether to drop out. Having met very many students, and remembering how one's younger self chose to enroll in higher education, it is fair to doubt that young people and their families are capable of assessing and comparing the values of educational opportunities. Because those with low ability find it difficult to realize that it is low, self-assessments are empirically inflated at the low end of the ability distribution (Dunning and Kruger, 1999). But if it is true that this personal trait is revealed by behavior and results rather than by any observable characteristics (Cipolla, 2011), society cannot prevent them from trying and failing (repeatedly, if they lack the ability to learn from mistakes).

While educational choices may not be as rational as economic models make them, they may well be as rational as possible. Society can paternalistically mandate schooling for small children, and for parents who need not choose in their children's best interest. To the extent that nobody need know more than young adults themselves about what is good for them, however, they are entitled to experiment and learn about their own ability. High dropout rates are not always due to uncertainty, but when they are, there is nothing wrong with them.

4.5 Dropout as a symptom

Concern about dropout, particularly among disadvantaged students, is justified if dropout rates are inefficiently high and a symptom of underlying imperfections. While student groups with higher dropout rates may genuinely face more uncertainty about their own ability as well as future financial shocks, the information on which enrollment and dropout choices are based can be imprecise or biased. If the subjective probability distribution differs from the one that generates the innovations, the same positive association of enrollment and dropout that uncertainty implies for rational choices can be due to misinformation about a degree program's difficulty or labor-market value.

Individual choices are socially inefficient also when market imperfections make them excessively risk-averse or myopic. Asymmetric information about effort and ability prevents insurance against idiosyncratic educational risk, and reduces individual propensities to choose riskier educational paths. The relative riskiness of educational and labor market choices is not generally obvious, but if higher education is a risky bet with lumpy costs, and risk aversion is decreasing, then enrollment and continuation are less attractive for poorer individuals, who would suffer more from failing to complete after incurring a large cost (unless they debt-finance it and default, or are so poor as to be covered at dropout by a social safety net that makes higher education an essentially one-way bet). Risk does not strongly influence choices at the margin, however, because it determines consumption volatility and *ex ante* welfare on both sides of the relevant indifference conditions. In the carefully calibrated models of Athreya and Eberly (2015; forthcoming) and Chatterjee and Ionescu (2012), inframarginal welfare effects are much larger than marginal enrollment and completion effects, which are somewhat larger when student dropout is not forced by exogenous exam failures but, as in the model above, is a choice triggered by new outside opportunities.

The higher individual discount rate implied by credit rationing discourages enrollment, because the balance of education's expected costs and benefits is initially negative and becomes positive in the more or less distant future, and also makes dropout more likely if throughout a degree program costs are incurred sooner than benefits.[15] Liquidity constraints do not appear to be quantitatively important in empirical studies ranging from Cameron and Heckman (2001), who document that they have a small effect on higher education choices of disadvantaged American youth, to Vergolini et al (2014), who find that an Italian regional experiment offering subsidies to poor capable students had essentially no effect on their enrollment while less capable students, in a counterfactual extrapolation of interaction effects, would have been more likely to enroll (and presumably drop out) if subsidized. Evidence on dropout effects is similar: Hendricks and Leukhina (2018) use college transcripts to estimate ability and find its role to be much more important that financial constraints in determining enrollment and completion, and Stinebrickner and Stinebrickner's (2008) survey evidence documents that self-declared liquidity constraints have no effect on dropout.

4.6 Remedies

Correcting the underling imperfections is desirable if inadequately informed, uninsured, or liquidity constrained individuals inefficiently refrain from enrolling and drop out too frequently, but information is problematic for public policy as well as for markets. Individual preparedness and characteristics of degree programs can be assessed in other ways, which need not be less costly or more accurate than allowing individuals to experiment and drop out: brief aptitude tests are less expensive, but cannot be more precise, than long sequences of assessments. There is an obvious role for policy in gathering and publishing accurate information about dropout (possibly conditional on observable characteristics) in non-selective public degree programs, because choices to enroll are inefficient if such data are noisy or not public, and in accrediting and regulating private degree programs, because asymmetric and better information on the supply side of the enrollment market gives ample opportunities for false advertising.

[15] In the model's notation liquidity constraints discourage enrollment by decreasing both y_c and y_d relative to y_o, and make dropout more likely by decreasing $y_c + \epsilon_c$ relative to $y_d + \epsilon_d$. If more stringently constrained individuals also face more uncertainty about the costs and benefits of higher education, their larger σ offsets the enrollment effect and reinforces the dropout effect.

Not only material resources and information, but also ability to exploit educational opportunities are unfortunately scarce among students from financially and culturally disadvantaged families. Treating a symptom does not cure a disease, and can make it worse. If possible dropout is rationally taken into account by optional enrollment choices, preventing enrollment of disadvantaged students reduces dropout at the cost of excluding those among them who would achieve the outstanding results represented by large positive realizations of the present paper's continuously-distributed innovations. And offering financial help and pedagogical assistance to students with observably weak family background can certainly reduce dropout rates among the enrolled but, attracting enrollment of individuals who are relatively likely to drop out, may well increase the number of dropouts.

Just like there can be too little dropout when misinformed students persist in pursuing unsuitable degrees that contribute little to their welfare, dropout can be too low from the social point of view if policy reduces it for poor performers. Remedial education is appropriate in mandatory educational programs that provide citizens and workers with essential social and cultural skills and generate positive externalities. But disadvantaged individuals can be helped more efficiently than by preventing their dropout from optional higher education programs meant to identify and exploit comparative advantage.

Appendix: Derivation of (13).

With $f(z) = g(z) = \varphi(z) = \exp(-z^2/2)/\sqrt{2\pi}$ and using $\int_{z=x+y_d-y_c}^{\infty} z\exp(-(z/\sigma_c)^2/2)dz = (\sigma_c)^2 \exp\left(-\frac{1}{2}\left(\frac{z+y_d-y_c}{\sigma_c}\right)^2\right)$ to evaluate the inner integral in (11), that truncated convolution expectation reads

$$\int_{-\infty}^{\infty} \sigma_c \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z+y_d-y_c}{\sigma_c}\right)^2}\right) \frac{1}{\sigma_d\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{\sigma_d}\right)^2} dz = \frac{\sigma_c}{\sigma_d}\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{\epsilon_d+y_d-y_c}{\sigma_c}\right)^2} e^{-\frac{1}{2}\left(\frac{\epsilon_d}{\sigma_d}\right)^2} dz$$

or, changing variables to $x = z/\sigma_d$,

$$\begin{aligned} &\frac{\sigma_c}{\sigma_d}\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\left(x\frac{\sigma_d}{\sigma_c}+\frac{y_d-y_c}{\sigma_c}\right)^2+(x)^2\right)}\sigma_d dx = \\ = &\left|\frac{\sigma_c}{\sqrt{2\pi}}\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}} e^{-\frac{1}{2}\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right)^2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\frac{x\sigma_c^2+\frac{y_d-y_c}{\sigma_c}\sigma_c\sigma_d+x\sigma_d^2}{\sqrt{\sigma_c^2\left(\sigma_c^2+\sigma_d^2\right)}}\right)\right|_{x=-\infty}^{\infty} \\ = &\frac{\sigma_c}{\sqrt{2\pi}}\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}} e^{-\frac{1}{2}\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right)^2} \end{aligned}$$

where $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_o^z e^{-t^2}dt$ is the error function, and the last step uses $\operatorname{erf}(\infty) - \operatorname{erf}(-\infty) = 2$. Thus, in the Gaussian case (11) can be written

$$E\left[\epsilon_c|\epsilon_d-\epsilon_c<y_c-y_d\right](1-P_d) = \sigma_c\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}\varphi\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right).$$

Symmetric derivations for (12) yield

$$E\left[\epsilon_d|\epsilon_c-\epsilon_d<y_d-y_c\right]P_d = \sigma_d\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}\varphi\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right).$$

Inserting these and (9) in (10), and using

$$\sigma_c\sqrt{\frac{\sigma_c^2}{\sigma_c^2+\sigma_d^2}}+\sigma_d\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}} = \sqrt{\sigma_c^2+\sigma_d^2} \equiv \sigma$$

to simplify the resulting expression, yields (13).

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