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Abstract

We study the interplay between intragenerational and intergenerational equity in an economy with two countries producing and consuming from national capital stocks. We characterize the sustainable development path that a social planner would implement to achieve intertemporal egalitarianism. If intergenerational equity is defined with respect to the global consumption of each generation, regardless of its distribution between countries, consumption in the poor country should be set as low as possible to maximize investment and hasten convergence, resulting in important intragenerational inequalities. When social welfare accounts for intragenerational equity, the larger the intragenerational inequality aversion (IIA), the smaller the sacrifice asked of the poor country, but the lower the sustained level of generational welfare. Along the intertemporal welfare-egalitarian path with IIA, consumption in the poor country increases, while it decreases in the rich country, resulting in a global degrowth.

JEL-Codes: O440, Q560.

Keywords: sustainable development, intergenerational egalitarianism, maximin, intragenerational inequality, differentiated degrowth.

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1 Introduction

The idea of perpetual growth is challenged in the sustainability literature, which puts a strong emphasis on intergenerational equity (Heal, 1998; Martinet, 2012). By depleting natural resources and inducing pollution, growth may jeopardize future generations' ability to enjoy a livable planet earth (Arrow et al., 1995; Rockström et al., 2009), especially when environmental externalities are not accounted for (Arrow et al., 2004), or when departing from the optimal path, even slightly (Bretschger, 2017). Significant environmental policies, such as a carbon tax, are justified to overcome intergenerational environmental issues, but face rejection from developing countries as well as from some categories of population in developed countries (Sterner, 2012). Just as in the climate change example (Heal, 2009), inequalities within generations can get in the way of intergenerational equity.

Part of current inequalities rely on unequal access to resources, broadly speaking, be it at individual, national, or continental levels. Productive assets, including financial and physical capital, knowledge, and environmental assets, are unequally distributed within a generation, inducing inequalities in production and consumption. Growth and capital accumulation could reduce these inequalities gradually. Inclusive economic growth is one of the United Nations' sustainable development goals (SDG8), in particular for the least developed countries, and is presented as a way of reducing poverty (SDG1) and inequalities (Boarini et al., 2018). As such, the call for a stationary state, or even degrowth, for intergenerational equity purposes may seem unfair to the less endowed of a generation. There is a need to account for intragenerational inequalities when defining a sustainable development path. How do sustainability policies affect intragenerational inequalities? How does intragenerational inequality aversion affect sustainable development paths?

In this paper, we investigate the theoretical interplay of intragenerational and intergenerational equity in the design of sustainable development paths. We examine how the optimal development path for a social planner aiming at implementing intergenerational equity is modified when taking into account intragenerational inequality aversion. This allows us to begin to answer the previous questions. First, we show how the pursuing of intergenerational equity affects inequalities within generations in the short and the long run. Second, we examine how greater inequality aversion within generations impacts the prospects for a sustainable development path.

Intragenerational and intergenerational equity issues have mostly been studied formally in different branches of the economics literature.[1] An exception is the literature on climate change,[2] which mainly adopts the discounted utility framework. This framework has been strongly criticized in the sustainability literature as poorly accounting for intergenerational equity concerns, leading to the definition of alternative criteria, such as maximin (Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006; d'Autume and Schubert, 2008a; Cairns and Martinet, 2014; Fleurbaey, 2015a; Cairns et al., 2019), undiscounted utilitarianism (Ramsey, 1928; Dasgupta and Heal, 1979; d'Autume and Schubert, 2008b; d'Autume et al., 2010), the Chichilnisky criterion (Chichilnisky, 1996), the weighting of the worst-off generation (Alvarez-Cuadrado and Long, 2009; Adler and Treich, 2015; Adler et al., 2017), sustainable discounted utilitarianism (Asheim and Mitra, 2010; Dietz and Asheim, 2012), as well as intergenerational egalitarianism (Piacquadio, 2014). Botzen and Bergh (2014) report that applying (some of) these alternative criteria to the climate change issue would result in more stringent climate policies than under discounted utilitarianism. Imposing stronger requirements for intergenerational equity may thus induce greater tensions regarding intragenerational equity.

The literature on intergenerational equity is mostly axiomatic, and uses growth models to explore the consequences of the studied criteria (see Asheim, 2010, for a review). The key models used in this framework are Ramsey's one sector growth model[3] and the Dasgupta-Heal-Solow model (Dasgupta and Heal, 1974; Solow, 1974). In most cases, the analysis is based on a single representative agent,[4] overlooking the intragenerational

[1] Some papers discuss explicitly the interplay of the two dimensions of equity. For example, Baumgärtner et al. (2012) offer an informal discussion of the issue. Baumgärtner and Glotzbach (2012) discuss these links for ecosystem services, without a formal model. Berger and Emmerling (2020) offer a theoretical framework that could lead to interesting discussions combining both equity issues.

[2] Schelling (1992) emphasizes the interplay of the two dimensions of equity in the climate change issue. Anthoff et al. (2009) introduce equity-weights to account for the different impacts of climate change in different countries with different levels of development. Kverndokk et al. (2014) study the effect of inequality aversion on optimal climate policies, using the Fehr and Schmidt (1999) framework for intragenerational equity, and discounted utility as an intertemporal welfare function. Yamaguchi (2019) studies the effect of inequality aversion on the consumption discount rate in a climate economy. Several empirical works based on Integrated-Assessment Models introduce a concern for intragenerational inequalities too. Dennig et al. (2015) develop a nested-inequalities model that accounts for inequalities within regions. Scovronick et al. (2017) examine the effect of population growth on optimal climate mitigation policies, comparing Total Utilitarianism (which is sensitive to the population size) and Average Utilitarianism (which is not). Anthoff and Emmerling (2019) disentangle the effect of intragenerational and intergenerational inequality aversions on the social cost of carbon.

[3] See Asheim and Ekeland (2016) for a discussion of the interest of this model to study sustainability issues, and Asheim and Nesje (2016) for the analysis of the optimal path in this model under various criteria of intergenerational equity. See also Asheim et al. (2020) who discuss time-consistency issues for criteria that do not satisfy stationarity, and illustrate their results in the Ramsey model.

[4] There are a few exceptions, including the axiomatic work on intergenerational equity with varying

equity issue.

We move away from the single representative agent model and consider a dynamic economy with two agents (called countries for simplicity), each endowed with a stock of productive capital. We use intergenerational egalitarianism to represent intergenerational equity.[5] Equality as an ideal of justice has a long tradition in philosophy and economics (Temkin, 1993; Parfit, 1995). It is based on the idea that "unequal distributions have something bad that equal distributions do not have" (Fleurbaey, 2015b, p. 205). Egalitarianism may not be the criterion selected for intergenerational equity and sustainable development, but it is an interesting benchmark for evaluating intertemporal inequalities in different theoretical frameworks (Piacquadio, 2014). Equality may result both from (intrinsic) egalitarianism or prioritarianism (which implies instrumental egalitarianism). When there are no trade-offs between individuals' utility, prioritarianism corresponds to the maximin criterion (Rawls, 1971; Myerson, 1981; Epstein, 1986), which leads to an egalitarian outcome under many circumstances. Asheim (2010, p. 206) emphasizes that "maximin, the principle of maximizing the well-being of the worst-off generation, [...] satisfies the finite anonymity axiom and is thus an alternative way of treating generations equally." According to Fleurbaey (2015b, p. 214), "in welfare economics, [...], the maximin (or leximin) criterion has then been adopted as yielding the most egalitarian among reasonable (that is, Paretian) social rankings." Asheim and Nesje (2016) use the maximin path as a benchmark for intergenerational equity, as the maximin welfare level offers a lower bound for welfare under other, more sophisticated criteria, such as the Calvo criterion, Sustainable Discounted Utility and Rank Discounted Utility, whose solutions may not be easy to compute in particular problems. Also, more sophisticated forms of intergenerational equity imply time-inconsistency, which has to be dealt with using sophisticated game-theoretic equilibrium selection (Asheim et al., 2020). Intertemporal egalitarianism is thus for us a natural first step. It also allows us to avoid the delicate exercise of comparing growing economies (Asheim, 2011; Llavador et al., 2011). We point out that, even in this egalitarian case, there are interactions between the two dimensions of equity that have far reaching consequences.

Regarding intragenerational equity, we consider a welfarist approach with inequality aversion. The welfarist approach allows us to deal with a continuum of cases, especially the two polar ones: utilitarianism (no aversion) and intragenerational maximin (infinite

population, which mainly focuses on population ethics (see, e.g., Asheim and Zuber, 2014).

[5]Other approaches could be considered. For example, Del Campo (2019) studies redistribution effects along the optimal growth path under undiscounted utilitarianism.

aversion).[6] The effective distribution of consumption, both within a generation and across time, is ultimately linked to the chosen degree of inequality aversion. This simple model allows us i) to examine the dynamic effect of intergenerational equity (egalitarianism) on intragenerational inequalities, and ii) to study the effect of inequality aversions on sustainability strategies. We show that, when technologies are symmetric, pursuing intergenerational egalitarianism increases intragenerational inequalities in the short run. In the long-run, however, intragenerational inequalities vanish as the two countries converge. These results are mitigated by inequality aversion. A larger intragenerational inequality aversion reduces inequalities in the short run, but at the cost of a lower intergenerational level of sustained welfare. It also induces a decreasing global consumption, even if welfare is constant over time. This development path corresponds to overall degrowth, but with growth in the poor country. When technologies are heterogeneous, the interplay of the two dimensions of equity is more complex, and depends on the technologies as well as on the initial state of the economy.

We present our model in Section 2, examine the case of a nil inequality aversion in Section 3, and study the effect of inequality aversion in Section 4. We relax some of the simplifying assumptions of the main model in Section 5, and conclude in Section 6. All the mathematical details are in the Appendix.

2 The economic model

Consider an economy composed of two entities, R and P, that we shall call countries for simplicity. The population sizes of the two countries are equal and normalized to unity, for simplicity. In our dynamic, continuous-time framework, we assume that at each time t each country has a single representative agent that lives for that time only. Country R (respectively, P) is endowed with an aggregate stock $X_R(t)$ (respectively, $X_P(t)$) of comprehensive productive assets (including natural resources), capital for short, that can evolve over time. These capital stocks correspond to national wealth. The aggregate capital of a generation, $X_R(t) + X_P(t)$, corresponds to global wealth. Wealth is unequally distributed at initial time 0, with $X_R(0) > X_P(0)$, so that we call R the rich country, and P the poor country.

Capital stocks are productive, according to a production function $F(X_i)$ that is iden-

[6] Note that comparing unequal situations among a finite number of individuals within a generation is much less sensitive than comparing unequal situations in the intertemporal setting of an infinite number of generations (Asheim, 2010).

tical for both countries, meaning that countries differ only in their wealth endowment and related country-specific production.[7] A similar assumption is made in Kverndokk et al. (2014) in a climate change economy with two countries. In Section 5, we relax this assumption and consider country-specific production functions, but for now we consider the symmetric case. Technology can be interpreted as the countries' capacity to derive output from individual endowments. We assume that the technology is strictly increasing and strictly concave, that marginal product $F'(X)$ is bounded from above,[8] and that production is bounded from above, i.e., $\lim_{X\to\infty} F(X) = \bar{F} < \infty$.[9] Under the assumption of similar production functions, a lower capital stock entails lower production, and the poor country produces less initially, i.e., $X_P(0) < X_R(0)$ implies $F(X_P(0)) < F(X_R(0))$.

There may be several reasons for the two countries to have unequal capital stocks at the initial time. The countries may be endowed with different natural resources, or they may be at different stages of development. Another possibility is that they have different time preferences that drove the economy to different stationary states according to the modified golden rule (i.e., $F'(X_i(0)) = \delta_i$), with a larger discount rate in P (i.e., $\delta_P > \delta_R$) so that $F'(X_P(0)) > F'(X_R(0))$. In such a scenario, our analysis amounts to studying the transition from this "individual-based" approach to a global sustainable development taking the form of a "generational-based" equity.

For simplicity, we assume that each country can invest only in its own productive capacities, and that no transfer of capital or consumption occurs. We relax this assumption and discuss the consequences of the possibility of such transfers in Section 5. Output can be either consumed by the producing country or invested in its capital stock, so that

[7] By defining local production as a function of local capital only, we overlook the possible interactions between the two capital stocks. This could be the case if the two countries are involved in trade, or if the model was interpreted as representing two individuals whose productivity in the global economy depends on own endowment as well as on the endowment of the other individual. A way to account for such interactions would be to assume that both production levels depend on both capital stocks, but the solution would depend on the specific type of interactions considered. For example, trade would require to consider at least two different goods in two locations, and thus at least four decision variables. An easier way to include interactions would be to consider two dynasties that own a share of the same capital stock, as in Asheim and Nesje (2016). As we want to keep results as generic as possible, we avoid such specification. Note that when capital stocks correspond to local natural resources, there is no such concern if the two stocks are independent. This is the case, for example, in Quaas et al. (2013).

[8] When this technical assumption is satisfied, the transversality conditions we will use are necessary conditions (see Cairns et al., 2019, footnote 18).

[9] This condition is used in the proof of the existence of an efficient egalitarian path (see Cairns et al., 2019, Proposition 2). Our intuition is that it could be relaxed for maximin problems in which state trajectories are converging to a finite limit.

country-specific capital dynamics is given by

$$\dot{X}_i = F(X_i) - c_i, \quad i = R, P \ .$$

Output can be interpreted as Net National Product, which is shared between national consumption and national investment. These measures are used in national accounts and for international comparisons.

Our objective is to determine the development path that a social planner aiming at intergenerational egalitarianism would implement. To combine sustainability and Pareto-efficiency concerns, we consider the "highest" such intertemporal egalitarian path. We will see that, depending on the way the outcome of a generation is defined – global consumption versus a welfare measure accounting for consumption inequalities –, the results differ.

The sustainability literature emphasizes that, along efficient egalitarian paths, net investment is nil and welfare is constant over time.[10] Applying intergenerational egalitarianism at an individual level would mean that each country consumes its national sustainable income, i.e., its whole production (Cairns and Long, 2006; Cairns and Martinet, 2014). As such, one would get a stationary state in which capital stocks remain constant, with $c_P(t) = F(X_P(0)) < c_R(t) = F(X_R(0))$, the inequalities perpetuating forever. As such, addressing the intergenerational equity issue at an individual level would ensure equity between generations, but not equity within generations.[11] Applied at a global level, however, an intergenerationally egalitarian path may depart from such a stationary state. For example, in an economy with a single representative agent and two reproducible assets, the efficient egalitarian path may not correspond to a stationary state (Burmeister and Hammond, 1977; Asako, 1980; Cairns et al., 2019). We will investigate this possibility in our economy with two agents and two national economies. We start by the case of a social planner aiming at sustaining the highest possible global consumption (Section 3). We then examine how the egalitarian path is modified when aversion to intragenerational inequality is introduced (Section 4).

[10]See the literature on Hartwick's rule and its generalization (Hartwick, 1977; Dixit et al., 1980; Mitra, 2002; Asheim, 2013) and on the characterization of egalitarian paths (Burmeister and Hammond, 1977; Cairns et al., 2019).

[11]See the interesting discussion on the disaggregation of welfare problems in Berger and Emmerling (2020, sect. 4.3.3, pp. 740–741).

3 Sustaining global consumption

We first examine what would be the path of the economy if a social planner aimed at maximizing the level of global consumption to be sustained over time by following an efficient and intergenerationally egalitarian path. This corresponds to a case in which one is concerned only with the total consumption of a generation, without considerations for intragenerational inequalities. This is also equivalent to maximizing the sustained consumption level in a single-agent model.

We base our inequality analysis on consumption, and not on utility, for two reasons. First, it has the advantage of being more concrete than the consideration of individual utility levels. It requires no assumption on how utility is derived from consumption, nor interpersonal utility comparison, and is consistent with the practice of empirical inequality analysis, mostly based on income or consumption data. Second, it allows us to establish a clear link with the single-agent approach, which considers aggregate consumption as a source of utility. Introducing individual utility functions with decreasing marginal utility would induce a motive to smooth consumption within a generation for efficiency purposes, irrespective of the social planner's motive to account for intragenerational inequalities. The results would then be close to that of Section 4, but would not make the analysis of this section possible.

To characterize the global-consumption egalitarian path, we solve a maximin problem.[12] The maximin consumption value $m^0(X_R, X_P)$ is the highest level of global consumption that can be sustained forever from the initial state of the economy:[13]

$$
\begin{aligned}
m^0(X_R, X_P) = \max_{\underline{c}, c_R(\cdot), c_P(\cdot)} \; & \underline{c} \; , && (1) \\
\text{s.t. } & (X_R(0), X_P(0)) = (X_R, X_P) \; ; \\
& \dot{X}_i(t) = F(X_i(t)) - c_i(t), \; i = R, P, \quad \text{and} \\
& c_R + c_P \geq \underline{c} \text{ for all } t \geq 0 \; . && (2)
\end{aligned}
$$

[12]The objective of a maximin problem is to define an equitable development path by maximizing the utility of the worst-off generation, through intergenerational redistribution. It guarantees a procedural equity (finite anonymity) since all generations are equally treated (Lauwers, 1997; Asheim, 2010). Whenever it is possible, i.e., in so-called *regular* maximin problems (see Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006; Cairns and Martinet, 2014), such a redistribution results in a strongly Paretian egalitarian allocation. In such a case, even if this is not formally the objective of a maximin problem, it characterizes an egalitarian and efficient path. This is the case in our model (see Cairns et al., 2019).

[13]The superscript ^{0} refers to the fact that intragenerational inequality aversion is nil in this case.

We follow Cairns and Long (2006) and Cairns et al. (2019) to solve this maximization problem.[14] The details of the resolution are in Appendix A.1. The main results are the following.

Proposition 1 (Sustaining global consumption: stationary states)**.** *The efficient egalitarian path is characterized by* $c_i = F(X_i^*)$ *and* $\dot{X}_i = 0$ *if and only if the capital stocks* (X_P^*, X_R^*) *satisfy* $F'(X_P^*) = F'(X_R^*)$*, i.e.,* $X_P^* = X_R^*$ *in the case of symmetric production functions.*

This result means that, when the objective is to maximize the level of global consumption sustained over time, the optimal path is a stationary path with constant consumption levels at country-specific production levels and constant capital stocks (if and) only if the marginal product of both countries are equal, which corresponds to equal capital stocks under the assumption of symmetric production functions. In that case, consumption levels are equal in the two countries, and there are no inequalities, neither within a generation nor between generations.

If the capital stocks do not satisfy the restrictive condition, i.e., if endowments are unequal, the optimal path is not stationary. Apart from a stationary state, the dynamics is as follows.

Proposition 2 (Sustaining global consumption: Transition path)**.** *For any state such that* $F'(X_P) > F'(X_R)$*, i.e.,* $X_P < X_R$ *in the case of symmetric production functions, the constant consumption path with* $c_R = m^0(X_R, X_P) > F(X_R)$ *and* $c_P = 0 < F(X_P)$ *is an optimal maximin path. Stock* X_R *decreases while stock* X_P *increases.*

This dynamic path corresponds to a corner solution in terms of country-specific consumption levels. This result is due to the fact that both consumption levels are perfect substitutes in global consumption. The country with the highest marginal product of local capital, i.e., P in the case of symmetric production functions, has a higher return on investment. From a global point of view, it is more efficient to reduce that country's consumption as much as possible to invest where capital has the highest return. At the same time, global consumption is sustained through a large consumption of the rich country, where capital stock is larger and its marginal product lower. In the absence of inequality aversion, sustaining global consumption consists in adapting the same strategy

[14]See the discussion in Cairns et al. (2019), and especially their footnotes 9 and 10, for an overview of the different mathematical approaches to the characterization of an efficient and egalitarian development path, their equivalence in most cases, and the differences in their interpretations.

as an annuitant owning two financial accounts with different returns: let the asset with the highest return grow, and withdraw capital to sustain consumption from the other asset.

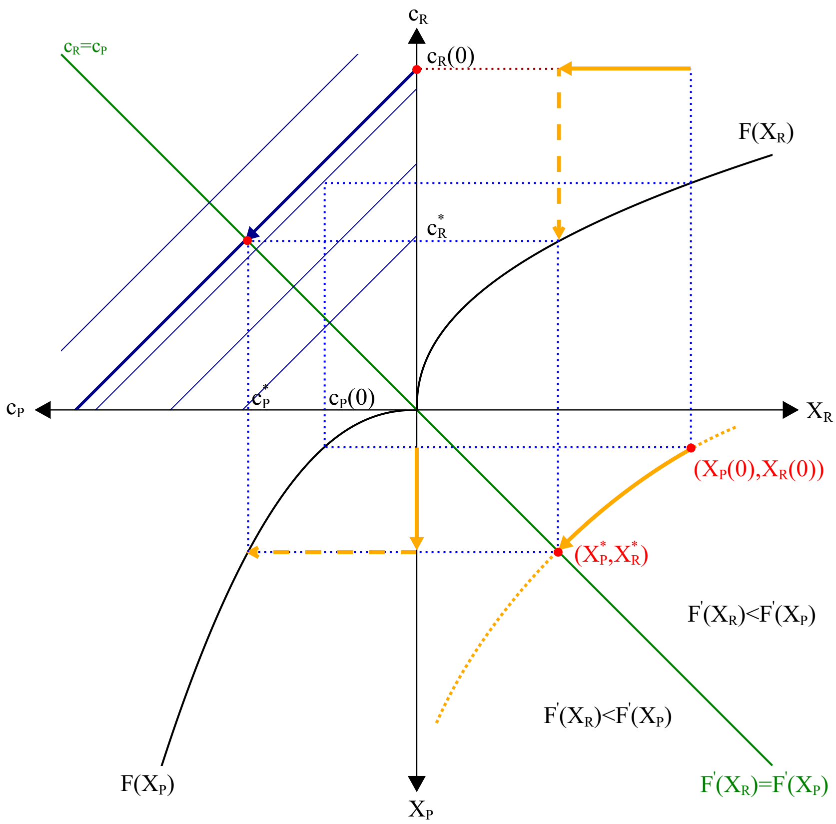

Figure 1: Sustaining global consumption.

These results are illustrated in Fig. 1. It is a four-quadrant graph in which the east axis represents X_R, the south axis X_P, the north axis c_R, and the west axis c_P. The north-east quadrant represents production $F(X_R)$ and the south-west quadrant production $F(X_P)$. The north-west quadrant plots social welfare indifference curves in the consumption map (c_R, c_P), and the south-east quadrant is the state map (X_P, X_R) in which state trajectories can be drawn. Efficient and egalitarian paths converge to stationary states that lie on the "wealth-equality line" $X_R^* = X_P^*$ in the state map. The equilibrium consumption levels $c_R^* = F(X_R^*) = F(X_P^*) = c_P^*$ are characterized by an egalitarian consumption within a generation, and correspond to a sustained global consumption $c_R^* + c_P^*$. During the transition phase, the global consumption is sustained at this level, but with only the rich country consuming. All the production in P is invested to have X_P growing as fast as possible, while X_R is depleted. This situation lasts as long as the marginal products differ in the two countries, i.e., until capital stocks converge.[15]

One could impose a constraint on the minimal consumption of the poor country, e.g., $c^\flat \leq c_P \leq F(X_P)$, without changing qualitatively the result. The optimal solution to maximize sustainable global consumption is to consume as little as possible in the poor country, which has higher marginal product of capital, and invest as much as possible in this high return capital stock.

The consequences of these results are twofold. First, if a policy was put into place to maximize the level of global consumption sustained over time, it would require high savings from the poor country and higher consumption in the rich country, and thus an increase of inequalities in the short-run with respect to a situation in which each country consumes its (unequal) sustainable income. Second, such a policy would lead to capital accumulation in the poor country and capital depletion in the rich county until the equalization of marginal product, and equality of capital and production. The inequalities vanish in the long-run.

The case examined in this section corresponds to a social planner indifferent to intragenerational inequalities. We now turn to the case in which intragenerational inequality matters.

4 Intragenerational inequality aversion

To introduce aversion to intragenerational inequalities, we use a social welfare function (SWF) valuing the allocation of individual consumption. The social planner is assumed to have an *intragenerational inequality aversion* (IIA) measured by a parameter θ (Atkinson, 1970). This parameter restricts the substitutability of the consumption of the two countries in the definition of the welfare of a generation. We use the following welfare function:

$$W^\theta(c_R, c_P) = 2\left(\frac{1}{2}c_R^{1-\theta} + \frac{1}{2}c_P^{1-\theta}\right)^{\frac{1}{1-\theta}} \ , \ \theta \geq 0, \ \theta \neq 1 \ .$$

The chosen welfare function allows us to represent different theories of justice as special cases. For example, Utilitarianism assumes a nil IIA ($\theta = 0$): only the total of consumption matters (Vickrey, 1945; Harsanyi, 1953, 1955, 1977). (Intragenerational) maximin

[15] For a given production function, the trajectory leading to a given stationary state (X_R^*, X_P^*) can be integrated backward from that state, with constant controls $c_P(t) = 0$ and $c_R(t) = F(X_R^*) + F(X_P^*)$.

(Rawls, 1971), on the opposite, depicts an infinite IIA ($\theta = \infty$), and no trade-off between individual consumption is allowed. Finite positive values for θ correspond to intermediate cases. Note that the previous case considering only global consumption corresponds to $\theta = 0$, a situation with no inequality aversion.

With this SWF, in case of equality within a generation (i.e., $c_R = c_P = c$), welfare is equal to global consumption: $W^\theta(c, c) = 2c$.[16] This means that the level of welfare is directly expressed in terms of *equally-distributed-equivalent* (e.d.e.) global consumption.[17] It can thus be compared to the actual global consumption, offering a measure of the effect of intragenerational inequality, and making the comparisons with the previous case easy. As $W^\theta(c_R, c_P) \leq c_R + c_P$, the difference between the actual global consumption of a generation $c_R + c_P$ and its e.d.e. level $W^\theta(c_R, c_P)$ is a measure of the extra-consumption needed to compensate for inequalities within a generation for a given welfare level.

The maximin problem is now

$$m^\theta(X_R, X_P) = \max_{\underline{w}, c_R(\cdot), c_P(\cdot)} \underline{w} \ , \tag{3}$$

$$\begin{aligned} \text{s.t. } & (X_R(0), X_P(0)) = (X_R, X_P) \ ; \\ & \dot{X}_i(t) = F(X_i(t)) - c_i(t), \ i = R, P, \quad \text{and} \\ & W^\theta(c_R, c_P) \geq \underline{w} \text{ for all } t \geq 0 \ . \end{aligned} \tag{4}$$

We conduct the same analysis as before to determine the maximin path. The details of the resolution are in Appendix A.2. The main results are the following.

Proposition 3 (IIA: Stationary states)**.** *The efficient egalitarian path is a constant consumption path with $c_i = F(X_i^*)$ if and only if the capital stocks (X_P^*, X_R^*) satisfy*

[16] Such a construct is discussed in Fleurbaey (2015b, p. 208) and is very convenient to relate welfare to an inequality index, either based on average or total consumption levels.

[17] In his renowned article on inequality measurement, Atkinson (1970) coined the e.d.e. concept. (A similar concept was independently presented by S.G. Kolm in 1968 (see Lambert, 2007).) It represents the level of consumption (or income, utility, etc.) which, if equally distributed, would allow to reach the same level of social welfare as the actual distribution. It leads to the definition of the inequality index $\mathcal{A}_\theta = 1 - \frac{\text{e.d.e.}}{\text{mean level}}$. In our two-agents economy, for given consumption levels (c_R, c_P), the e.d.e. consumption $\bar{c}$ is defined as the level of *per capita* consumption which, if consumed by all agents, would result in the same level of welfare, i.e., $W^\theta(\bar{c}, \bar{c}) = W^\theta(c_R, c_P)$. We shall here often refer to the corresponding *global* consumption $2\bar{c} = W^\theta(\bar{c}, \bar{c})$. This presentation is convenient to compare the results with that of the single-agent case, without modifying the interpretation of the θ parameter, nor the related inequality index, as the ratio of the e.d.e. consumption $\bar{c}$ to the mean consumption $(c_R + c_P)/2$ is equal to the ratio of the e.d.e. global consumption $2\bar{c}$ to the global consumption $c_R + c_P$. Berger and Emmerling (2020) discuss how the e.d.e. concept can be applied jointly to inequality across individuals, time, and states of the world, in a utilitarian framework restricted to nested additively separable welfare functional forms.

$F'(X_P^*) = F'(X_R^*)$, i.e., $X_P^* = X_R^*$ in the case of symmetric production functions.

The potential stationary states are the same as in the case maximizing the level of sustained global consumption. In the absence of initial inequalities within a generation (equal endowments), IIA does not affect the intergenerational egalitarian path.

Apart from a stationary state, the dynamics is as follows.

Proposition 4 (IIA: Transition path)**.** *For any state such that $F'(X_P) > F'(X_R)$, i.e., $X_P < X_R$ in the case of symmetric production functions, the efficient egalitarian path satisfies $0 < c_P < F(X_P)$ and $c_R > F(X_R)$, such that capital X_P increases and capital X_R decreases toward a stationary state characterized by $F'(X_P^*) = F'(X_R^*)$, i.e., $X_R^* = X_P^*$. The consumption levels are characterized by $W^\theta(c_R, c_P) = W^\theta(F(X_R^*), F(X_P^*))$, which is equal to $F(X_R^*) + F(X_P^*)$ when production levels are equal in the symmetric production functions case.*

When the social planner is averse to intragenerational inequalities, the consumption in the poor country is not reduced to a minimum, and investment is lower than in the case with no IIA. As a consequence, intragenerational inequalities are reduced with respect to the situation in which the social planner aims at sustaining global consumption.

The investment pattern is related to the IIA as follows.

Proposition 5 (IIA: Investment pattern)**.** *Along an efficient egalitarian path, at state (X_P, X_R), the convergence pattern (relative investment) is characterized by the relationship -$\frac{\dot{X}_P}{\dot{X}_R} = \frac{W^\theta_{c_R}}{W^\theta_{c_P}}\Big|_{W^\theta=\underline{w}} = \left(\frac{c_P}{c_R}\right)^\theta$.*

The relationship in Prop. 5 relates the relative investment $-\frac{\dot{X}_P}{\dot{X}_R}$ (how many units of capital are invested in P for each unit of capital depleted in R), and thus the investment velocity, to the shape of the social welfare indifference curves. The latter are steeper when the IIA is higher. The investment pattern is also related to the level of inequalities through the term $\left(\frac{c_P}{c_R}\right)^\theta$, which depends on the relative consumption (how much P consumes less than R) and the IIA coefficient θ.

These results are illustrated in Fig. 2. The shape of the capital stocks trajectory is related to the shape of the social welfare indifference curves and thus to the inequality aversion. All the points on the state trajectory correspond to as-good-as-stationary-state loci, and could be interpreted as indifference curves in terms of endowments regarding intergenerational equity.

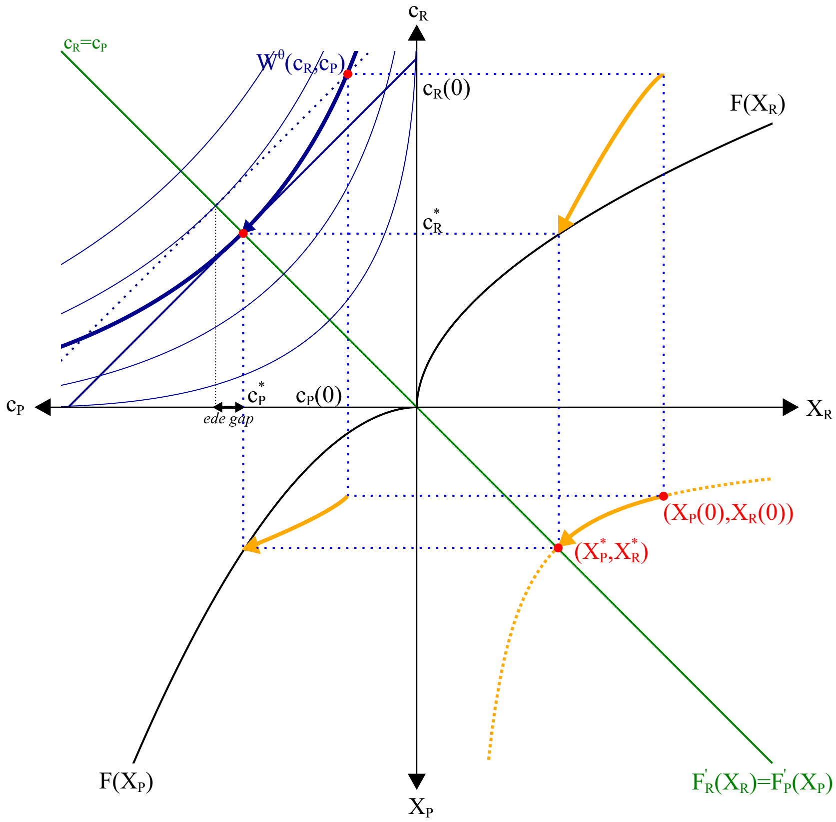

Figure 2: Sustaining IIA social welfare

We can discuss the interplay between IIA and intergenerational equity. The degree of IIA influences the level of sustained welfare in a subtle way. The higher the IIA, the lower the substitutability of consumption levels in the SWF, and the larger the consumption of the rich country has to be to compensate a low consumption by the poor country. This induces a larger global consumption to achieve a given welfare level when there are inequalities. When intragenerational inequalities decrease, the global consumption gets closer to its e.d.e. level. As a consequence, along a given efficient egalitarian path with IIA, the level of global consumption decreases, in spite of a constant e.d.e. level. Accounting for IIA results in unequal global consumption over time. Successive generations have the same welfare but decreasing global consumption.

We can also examine the effect of the degree of IIA on the intergenerational egalitarian path. For this purpose, let us consider the paths that would start from a given state under two scenarios of IIA, i.e., corresponding to two SWFs with different θ. Given our definition of welfare and the interpretation of the e.d.e., we can compare the two trajectories through the corresponding egalitarian consumption at stationary state. The optimal stationary states (satisfying $F'(X_R^*) = F'(X_P^*)$) fall on the line $X_R = X_P$. The corresponding stationary state consumption levels are on the line $c_R = c_P$ in the case of symmetric production functions. Consider two different sets of social welfare indifference curves corresponding to different degrees of IIA. The indifference curves of both sets are tangent along the equal-consumption line. According to Proposition 5, the trajectories converging to a given stationary state are also tangent in the state map along the optimal stationary states line $X_R = X_P$, the ones with the higher IIA having greater curvature and thus lying south-east of the corresponding ones for the lower IIA. This pattern is illustrated in Fig. 3. For any endowment $(X_P(0), X_R(0))$ away from the stationary state, a lower IIA

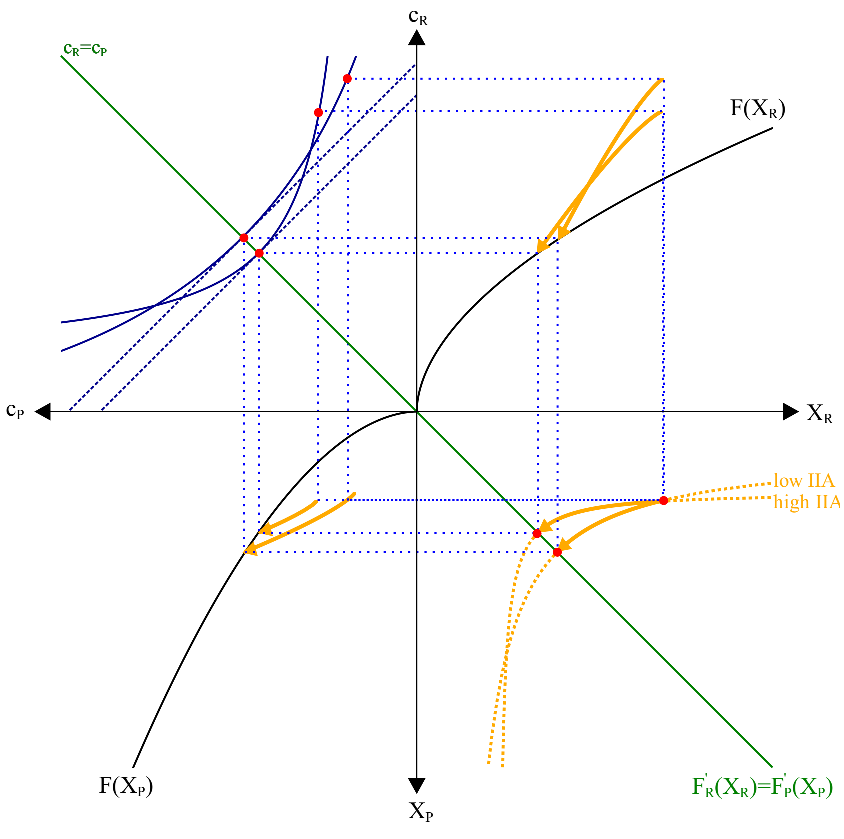

Figure 3: Effect of intragenerational inequality aversion on intergenerational equity

implies higher stationary state levels of the stocks and higher stationary state levels of consumption. The effect of IIA on intergenerational equity is unambiguous. The lower the degree of IIA, the higher the level of welfare sustained over time (apart from the stationary states, where IIA plays no role). The farther from the optimal stationary states curve, the stronger the effect of IIA on intergenerational equity. The degree of inequality aversion strongly influences the optimal stationary state of an economy pursuing an egalitarian efficient path, and thus the level of e.d.e. global consumption.

The fact that the IIA affects the whole development trajectory, including the equilibrium, is a feature of intergenerational egalitarianism. In a maximin problem, the stationary state reached by a trajectory depends on the initial state of the economy (Cairns et al., 2019). In the discounted-utilitarian framework, on the other hand, the stationary state is determined by the (constant) discount rate, and is usually the same whatever the initial state of the economy. For example, in the climate economy with two countries of Kverndokk et al. (2014), higher inequality aversion "generally lifts the consumption path of the poor region, while the rich region must take a greater share of the climate burden" but without modifying the long-run socially optimal consumption levels and capital stocks.[18]

Interestingly, inequality aversion within generations generates an egalitarian generational welfare, but an unequal global consumption over time. Earlier generations consume more than their distant heirs. The larger the IIA, the larger that effect.

Infinite inequality aversion. Let us now briefly consider the case in which the social planner is infinitely averse to inequalities within a generation. As the social planner focuses only on the worst-off individual, the SWF is of the form $W(c_R, c_P) = \min\{c_R, c_P\}$. In this case, total, not marginal, country-specific production levels drive the solution. The level of sustainable welfare depends on the level of production that can be maintained within each country and is given by $m(X_R, X_P) = \min\{F_R(X_R), F_P(X_P)\}$. The social planner would not request the poor country to reduce consumption with respect to the sustainable national income level. No substitution in social welfare is tolerated. No decision in the rich country (especially a high consumption) can justify a sacrifice in the poor country. There is no savings and no growth in P. Inequalities perpetuate forever, just as if an intertemporal egalitarian path were implemented at the country level. The only way to escape from this situation is to consider transfers (of capital or consumption) from the rich to the poor country, an option we shall analyze in next section.

[18] This pattern can also be illustrated by the results in Baumgärtner et al. (2017), who consider a single-agent model with several goods and a CES utility function. Modifying the substitutability between goods in utility (which is, from a technical point of view, close to modifying IIA in a model with several consumers) does not change the optimal stationary state, but only the transition path to it. In the discounted-utility framework, the degree of substitutability between goods affects the social discount rate and is central to the debate between proponents of weak and strong intergenerational equity, as discussed in Traeger (2011) and Drupp (2018), but it does not affect the stationary state of the economy.

This "poverty trap" is a classical outcome of the extreme version of intergenerational equity imposed by intertemporal egalitarianism. It has been the main argument opposing the use of maximin or leximin criteria to represent intergenerational equity (Asheim, 2010). The results in this section offer an interesting sideline to the story, though. This outcome of intertemporal egalitarianism is critical in the single-agent model in which, if the initial capital stock is low, sustaining poverty is not an interesting option for sure. It is usually recognized, however, that the absence of growth would be an acceptable outcome if the level of sustained welfare was high enough (with the underlying argument that this is not currently the case, especially in developing countries). We have shown that, in an economy with several agents, as soon as society is not totally averse to intragenerational inequalities and there is a rich country, targeting an intertemporal egalitarian path for intergenerational equity purposes calls for growth in the poor country, and a way out of the poverty trap.

5 Extensions

In this section, we relax two of the simplifying assumptions of the main model. First, we consider the case of heterogeneous production possibilities. Second, we examine the question of capital and consumption transfers.

5.1 Heterogeneous production capacities

Let us relax the assumption that the two countries have the same technology to generate income from capital, and consider the heterogeneous case $F_R \neq F_P$. In this case, it becomes difficult to say which country is rich, and which is poor, as $X_P < X_R$ does not necessarily entail $F_P(X_P) < F_R(X_R)$ for all capital stocks. We shall, however, consider that this is the case, to ease interpretation, and assume that country P is having initially less capital and a lower production. In this case, the previous results are modified as follows:

- The stationary states are still characterized by the equality of the marginal products, i.e., $F'_P(X^*_P) = F'_R(X^*_R)$, but this no longer implies an equality of wealth at equilibrium. Depending on the technologies *and* on the initial wealth levels, we may either have $X^*_P < X^*_R$ or the opposite. The condition $F'_P(X^*_P) = F'_R(X^*_R)$ defines a curve of potential equilibria in the state map. The location of a particular equilibrium with respect to the "wealth-equality line" $X_P = X_R$ characterizes which country has more capital in the long-run.

- The equilibrium condition on marginal products tells nothing about the consumption inequalities in the long-run, as we may have $F_P(X_P^*) < F_R(X_R^*)$ or the opposite. The curve defined by the condition $F_P(X_P) = F_R(X_R)$ corresponds to situations of equal production. The location of a particular equilibrium with respect to that curve characterizes which country consumes more in the long-run.

- The transition path is still characterized by positive savings (consumption is lower than production) in the country with the larger marginal product, and negative savings (consumption is larger than production) in the country with the lower marginal product. Interestingly, this pattern does not mean that the poor country grows as the rich country declines. The opposite occurs if $F_P'(X_P(0)) < F_R'(X_R(0))$, so that the rich country saves more when its marginal product is larger, in spite of larger capital stock and production. Implementing an intertemporal egalitarian path may even require that the poor country 'optimally' exhausts its capital stock in the case of nil IIA (sustaining global consumption), if $F_P'(0)$ is lower than the marginal product of the rich country.[19]

These results characterize the complex interplay between intragenerational and intergenerational equity concerns when production possibilities are heterogeneous. Fig. 4 illustrates these results. The figure exhibits trajectories starting from different initial states and resulting in optimal intertemporal egalitarian paths with contrasted effects on

[19]The absence of IIA can justify the sacrifice of a country for the sustaining of global consumption. This result is endogenous to the initial stocks. It occurs when the marginal products of the two countries cannot be equalized, i.e., when the poor country's capital stock is still relatively unproductive at the margin when it declines toward zero whereas the marginal product of the rich country does not fall too much as it is built up. This result is in striking contrast to the result of Quaas et al. (2013) in the discounted-utilitarian framework. They study a model with a manufactured good and two renewable natural resources in the discounted-utilitarian framework. They investigate the resilience of this economy to a one-time shock. Solving the post-shock optimum, they show that when a stock is low (and thus has a higher marginal product), building it up is optimal only if the two resources are substitutable enough. The transition requires limiting the consumption of the more productive stock to build it up. If the resources are substitutes, this pattern has only a limited effect on utility. On the other hand, if the resources are complements, building up the stock of the scarce resource has a high utility cost. It may even be optimal to exhaust this stock if the discount rate is high. In their model, exhaustion of the stock with the *higher* marginal product may be optimal when resources are perfect *complements*. In our case, we may exhaust the stock with the lowest marginal product when the consumption levels are perfect *substitutes*. When they are complements, exhaustion is not an option because it would lead to an overall collapse. This difference is due to the unequal treatment of generations under discounting. When the very long run matters, as in our intergenerational egalitarian case, overall collapse is not an option.

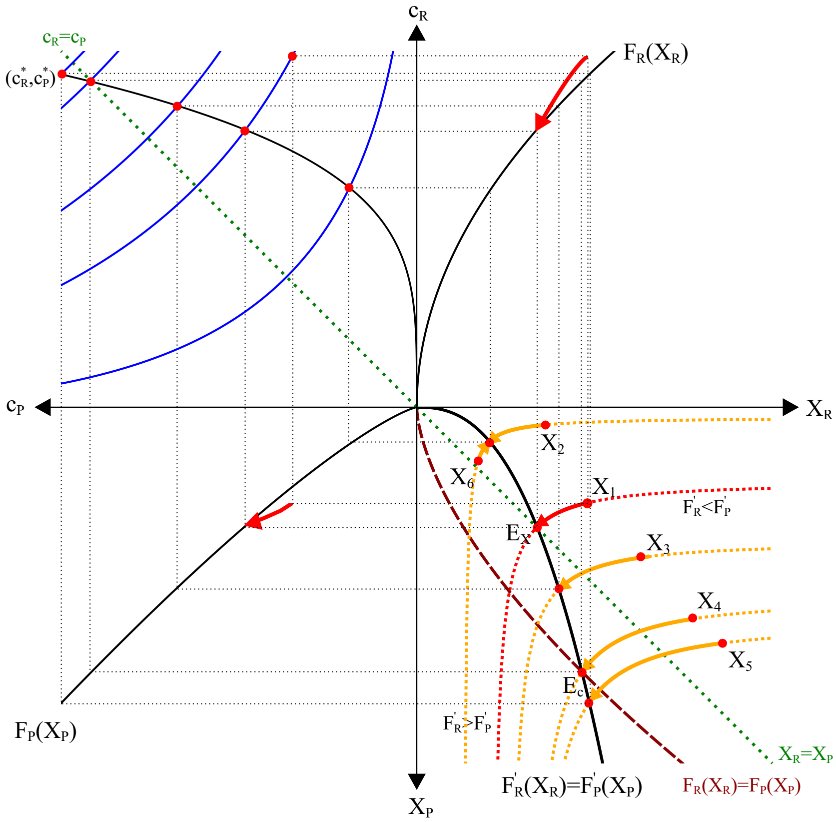

Figure 4: Welfare egalitarian paths with different technologies

intragenerational inequalities. The trajectory starting from the initial state X_1 reaches the wealth-egalitarian state E_X, at the intersection of the equilibrium curve and the wealth-equality line. At this equilibrium, capital stocks are equal but consumption levels are not. Intragenerational inequalities diminish along the intertemporal egalitarian path, but inequalities do not vanish in the long run. The trajectory starting from initial state X_2 follows the same pattern regarding consumption, but leads to a situation in which even capital stocks are not equalized. On the contrary, the trajectory starting from X_3 leads to situations in which wealth inequality are reversed at some point in time, with country P ending up with more capital than country R, but still a lower consumption. The trajectory starting from state X_4 is interesting in that it tends towards the consumption-egalitarian state E_c, at the intersection of the equilibrium curve and the equal-production curve. Consumption levels are equal in the long run, and intragenerational (consumption) inequalities vanish. The consumption inequality could even reverse at some point in time for trajectories that reach an equilibrium on the other side of the equal-production curve, as the trajectory starting from state X_5. The trajectory starting from the initial state X_6, which is characterized by a larger capital stock and a larger production level in country R along with a larger marginal product of capital ($F'_R(X_R(0)) > F'_P(X_P(0))$), illustrates the fact that, with asymmetric production functions, implementing an intertemporal egalitarian path could even lead to an increase of intragenerational inequality of wealth and consumption under particular circumstances.

When technologies are different, the effect of the degree of IIA on the intergenerational egalitarian path is more complex than in the symmetric case.

First of all, let us emphasize that comparing the results for two degrees of IIA in the asymmetric case is less straightforward than in the symmetric case, because the equilibria are no longer on the equal-consumption line. In the symmetric case, the welfare level along a trajectory corresponds to the actual global consumption reached in the long-run, which can serve as a benchmark to assess what is "sustained" objectively, as two social planners with different degrees of IIA would assign the same welfare value to a given equilibrium. This is no longer the case in the asymmetric case, in which the equilibria do not correspond to equal consumption levels and would not be "valued" the same by the two planners. They would, however, agree that the farther from the origin the equilibrium is, the larger the sustained global consumption, and the better for the generations in the very long run.

In the symmetric case, a larger IIA induces a lower sustained global consumption level. In the asymmetric case $F_R \neq F_P$, a higher IIA still induces less inequalities within generations, but the effect on the long-run global consumption depends on the initial state (and on the technologies). This is due to the fact that i) the path can either result in decreasing or increasing inequalities, and ii) the indifference curves, whose shape determines state trajectories (Proposition 5), are not tangent one to another and symmetric at the equilibria, but cross. As such, the effect of the degree of IIA will depend on whether the equilibrium is reached from above or from below, i.e. whether $F'_R(X_R) < F'_P(X_P)$ or the opposite.

Fig. 5 illustrates these two cases. For initial state X_3, satisfying $F'_R(X_R) < F'_P(X_P)$, a larger IIA results in a lower level of sustained global consumption, just as in the symmetric case. This is due to the fact that the convergence is based on sacrifice and savings in the poor country and over-consumption in the rich country, inducing important intragenerational inequalities. With a higher IIA, growth in the poor country is slower and convergence is achieved at a "smaller" stationary state. For initial state X_6 (satisfying

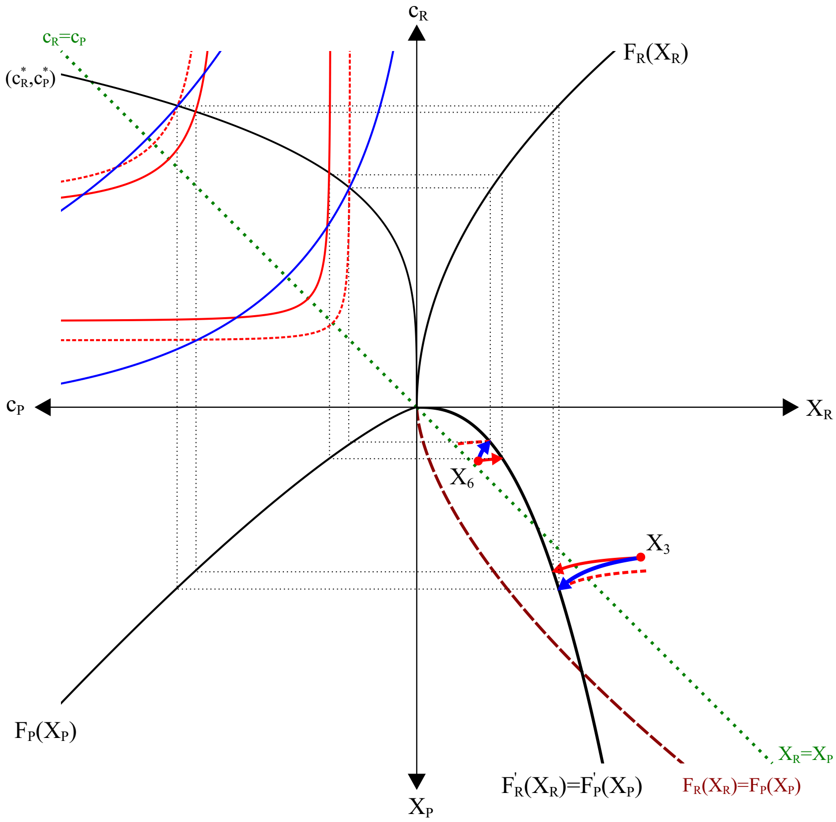

Figure 5: Effect of IIA on intergenerational equity with different technologies

$F'_R(X_R) > F'_P(X_P)$), however, a larger IIA results in a higher level of sustained global consumption, and thus benefits future generations. This is due to the fact that intragenerational inequalities increase along the trajectories starting from this state. A larger IIA induces a lower consumption in the rich country (and thus a larger investment level) combined with a minimal over-consumption in the poor country (and thus a lower capital depletion). Current inequalities are reduced with respect to a trajectory with lower IIA, and the global consumption level reached in the long-run is higher due to the resulting investment pattern. These two cases illustrate that, even in our simple two-agents model, intragenerational equity and intergenerational equity can conflict in some ways.

The effect of IIA on the intertemporal trajectory depends on the degree of IIA in the SWF of course but also, and perhaps less intuitively, on the shape of the production functions. Even though this result may seem surprising, it is not unheard of in the maximin literature. The same type of ambiguous effect of substitutability on intergenerational equity occurs for substitutability in production in the Dasgupta-Heal-Solow model

(Solow, 1974; Dasgupta and Heal, 1979; Martinet and Doyen, 2007). Martinet (2012, pp. 145-146) shows that, for low capital stocks, a higher elasticity between inputs reduces the level of utility that can be sustained from a given state, whereas for larger capital stocks a higher elasticity increases the level of utility that can be sustained.

5.2 The case of capital transfers through foreign investment

If inequalities within a generation are due to unequal endowment of productive assets, a possibility is to reduce the wealth gap through transfers of capital and/or consumption. Of course, some forms of capital do not move easily. This is the case for natural resources, property rights on land, human capital, etc., but there is a possibility to transfer some of it, including knowledge. It is also possible to make direct transfers of consumption.

In the economic model of Section 2, the two capital dynamics are independent. We now relax this assumption by making it possible for R to invest in P. We do so by introducing a new state variable, T, corresponding to the cumulative amount of capital transferred, with an instantaneous transfer rate τ. We assume that this transfer occurs through investment (forgone consumption in R) in the aggregate capital stock in P (e.g., through investment in manufactured capital, infrastructures, education and human capital enhancement, etc.).[20] This investment may generate a return π, which may go from nothing (pure capital transfer without compensation) to a positive return on foreign investment, and even spoliation of income (very high capture of the production in P). The case of consumption transfer from R to P would correspond to $\pi < 0$.

The model reads as follows:

$$\dot{X}_P = F(X_P + T) - c_P - \pi \ , \tag{5}$$

$$\dot{X}_R = F(X_R) - c_R - \tau + \pi \ , \tag{6}$$

$$\dot{T} = \tau \ . \tag{7}$$

We assume that the initial state of the economy is $(X_P(0), X_R(0), 0)$, with $X_P(0) < X_R(0)$ and no foreign capital $T(0) = 0$.

This model could be studied under different settings, including the maximization of countries' intertemporal consumption. For consistency with the previous analysis, we will consider the case of a social planner aiming at sustaining generational welfare, and solve the following maximin problem.

[20]Formally, the model could also encompass the case of capital spoliation, when τ is negative.

$$
\begin{aligned}
m^{\tau}(X_R, X_P, T) \qquad &= \max\nolimits_{\underline{w}, c_R(\cdot), c_P(\cdot), \tau(\cdot), \pi(\cdot)} \ \underline{w} \ , && (8)\\
\text{s.t.} \quad & (X_R(0), X_P(0), T(0)) = (X_R, X_P, 0) \ ; \\
& \text{dynamics } (5-6-7) \ ; \\
& W^{\theta}(c_R, c_P) \geq \underline{w} \text{ for all } t \geq 0 \ . && (9)
\end{aligned}
$$

We derive two results from this model.

Proposition 6 (Maximal capital transfer)**.** *Implementing an efficient egalitarian path would require a maximal transfer of capital from the country with the lowest marginal product to the country with the highest marginal product, until converge.*

In the symmetric technology case, it means that the rich country should transfer capital to the poor country as fast as possible, to accelerate the convergence.

Proposition 7 (Minimal return on foreign investment)**.** *Implementing an efficient egalitarian path would require a minimal return on foreign investment.*

In the symmetric case, it means that the rich country should not get a return on capital transfer, to accelerate the convergence.

Note that these results point in the same direction as the analysis of the role of international transfers in the discounted-utility, two-countries climate economy of Kverndokk et al. (2014). In their setting, the optimal transfer policy is a most rapid approach toward equality too. While such a particular solution is the result of the same mathematical pattern in both approaches (the Lagrangean/Hamiltonian is linear in the transfer decision parameters), one should not conclude that this result is a technical artefact. This pattern emerges from the structure of the model, which is supported by the economic stylized facts it represents. If one is really concerned with inequalities (or with efficiency in a utilitarian approach with decreasing marginal utility of consumption), any cost-free transfer should be implemented right away.

6 Conclusion

In the introduction, we asked two questions on the link between intragenerational inequalities and intergenerational equity. How do sustainability policies affect intragenerational inequalities? How does inequality aversion affect sustainable development paths?

We studied the interplay between intra- and intergenerational equity in the definition of an intertemporal egalitarian path. For this purpose, we developed a dynamic economic model with two countries having heterogeneous endowments. In a situation in which a poor country is endowed with less productive assets, has a lower production but a higher marginal product, the following results hold.

Targeting intergenerational egalitarianism while considering only the global consumption of a generation – which is equivalent to considering a single-agent model – would imply extreme intragenerational inequalities. This result urges to moving away from the single-agent model to consider heterogeneous agents within generations, with inequality aversion, when examining the consequences of criteria aiming at representing intergenerational equity.

When the social planner has an aversion to intragenerational inequalities, implementing an intertemporal egalitarian path sustaining welfare generates inequalities in the short run, but these inequalities are reduced over time and vanish in the long run. As such, targeting intergenerational egalitarianism affects intragenerational inequalities in a dynamic way, answering our first question. More specifically, with symmetric production functions, implementing an egalitarian welfare path induces a convergence toward equal wealth, income, and consumption. The stock of productive assets declines in the rich country while it grows in the poor country. Along that convergence path, consumption in the poor country increases while consumption in the rich country decreases, resulting in a constant welfare over time. Such a pattern is followed as long as the marginal products of capital in the two countries differ. As intragenerational inequalities decrease over time, global consumption can also decrease without reducing the welfare level. The egalitarian path with intragenerational inequality aversion thus corresponds to a differentiated degrowth path, with actual degrowth in the rich country, but growth in the poor country. Such a path may appear consistent with the needed growth in developing countries and the perhaps needed global degrowth for environmental sustainability.

Regarding our second question, we showed that the degree of intragenerational inequality aversion affects both the level of sustainable welfare and the path of global consumption. Along an intertemporal egalitarian path, welfare is constant but the global consumption decreases over time as intragenerational inequality decreases. The larger the intragenerational inequality aversion, the lower the sustainable level of welfare, and the lower the level of global consumption reached in the long-run. The intuition is that one can take a greater advantage of disparities between endowments and capacities of countries to make trade-offs to increase welfare when inequality aversion is lower. In the extreme case of an infinite aversion to inequalities within generations, no sacrifice can be asked of the poor country, which does not grow. Initial inequalities perpetuate. There is thus, here again, a tension between the two types of equity issues. Of course, allowing for transfers of capital or consumption from the rich country to the poor country solves the problem. Any policy that makes such transfers easier would reduce the tension between the two equity requirements.

Just as in the discounted-utility case examined in the two-countries climate economy of Kverndokk et al. (2014), stronger inequality aversion leads to a reduction of the consumption gap, and thus to a larger consumption in the poor country and a lower consumption in the rich country compared to the case with a lower inequality aversion. In the discounted utility case, however, it may be optimal to reduce consumption inequality in the short run by reducing investment in the poor country and increasing it in the rich country, which can lead to higher consumption inequalities in the future. In our intertemporal egalitarian set-up, an higher intragenerational inequality aversion results in a reduction of consumption inequalities at all times.

Of course, all our results rely on a social planner approach. One may wonder who would be legitimate (and able) to implement an intertemporal egalitarian path. But the problem is the same for any global issue. Boarini et al. (2018, pp. 18-19) offer an interesting discussion on the role of national governance on global issues, which we here briefly summarize and quote. Countries could easily pursue their individual interest of growth, disregarding global environmental issues, but global objectives should sometimes get priority over national ones, and even be incorporated in national policy agendas. "These objectives entail a net transfer of resources from high-income countries toward low-income countries. As such, they imply a cost for high-income countries that, strictly speaking, may hinder the achievement of national objectives, at least in the short run. In the long run, however, there is a strong common interest in achieving convergence of living standards over the world in order to construct a peaceful, cooperative and flourishing global society." Intergenerational equity may be such an issue, worth implementing at a global level rather than at a national level (notwithstanding the interest of net transfers we emphasized in Section 5). Applied at a national level, intergenerational equity would result in perpetual, and perhaps unbearable intragenerational inequalities. Our results suggest that combining intragenerational inequality aversion with intergenerational equity concerns could provide arguments for a differentiated degrowth, in addition to what Bretschger (2017) exhibits for reasons for a degrowth of material consumption, with possible compensation if natural conditions improve. Studying the effect of such a degrowth on individual utilities when environmental assets are explicitly accounted for would be of great interest.

Our work could be extended in several ways. First, one could examine how the results are modified when the two countries interact, for example through trade. Second, our results were obtained in the quite extreme version of intergenerational equity implied by intertemporal egalitarianism. Asheim (2010) stresses that such a criterion may have "undesirable consequences," such as no growth at all. It, however, offers a starting point to identify interactions between the intra- and intergenerational dimensions of equity. Analyzing theses interactions for other forms of intergenerational equitable criteria would complete our analysis. Last, studying the interaction of the two equity dimensions in an economic model of interest (e.g., a climate change economy) would be in line with Rawls' reflective equilibrium (Asheim, 2010).

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A Appendix

A.1 Sustaining global consumption: Mathematical details

This section solves the problem of Section 3. To make the analysis more general and cover the results presented in the extensions of Section 5, we distinguish the technologies of the two countries. The symmetric case is retrieved by setting $F_P \equiv F_R \equiv F$.

Problem (3) can be converted into an usual optimal control problem under the constraint (4), the sustained consumption level $\underline{c}$ being a control parameter. From a technical point of view, our problem is close to the problems treated in Burmeister and Hammond (1977) and Cairns et al. (2019).[21] We denote the co-state variables of the stocks by μ_i. The Hamiltonian is

$$\mathcal{H}(X, c, \mu) = \mu_R \left(F_R(X_R) - c_R\right) + \mu_P \left(F_P(X_P) - c_P\right) \ .$$

Denoting the multiplier associated with the constraint (2) by ρ, we get the Lagrangean associated with the maximin problem:

$$\mathcal{L}(X, c, \mu, \underline{c}, \rho) = \mu_R \left(F_R(X_R) - c_R\right) + \mu_P \left(F_P(X_P) - c_P\right) + \rho \left(c_R + c_P - \underline{c}\right) \ .$$

This Lagrangean is linear in the decisions, which implies that corner solutions for the controls are possible.

The necessary conditions are, for $i = R, P$:

$$\frac{\partial \mathcal{L}}{\partial c_i} = -\mu_i + \rho \leq 0 \ , \ c_i \geq 0 \ , \ c_i \frac{\partial \mathcal{L}}{\partial c_i} = 0 \ ; \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial X_i} = -\dot{\mu}_i \ . \tag{11}$$

The complementary slackness conditions are

$$\rho \geq 0 \ , \quad c_R + c_P - \underline{c} \geq 0 \ , \quad \rho \left(c_R + c_P - \underline{c}\right) = 0 \ .$$

By Lemma 1 of Cairns et al. (2019), $c_R = c_P = 0$ cannot be solution of the problem. There are thus three cases to consider: (i) $c_R > 0$ and $c_P > 0$; (ii) $c_R > 0$ and $c_P = 0$; (iii) $c_R = 0$ and $c_P > 0$. We start with the case of positive consumption in both countries

[21] See Cairns et al. (2019, Proposition 2) for a proof of the existence of the optimal solution.

(i). In this case, condition (10) corresponds to $\frac{\partial \mathcal{L}}{\partial c_i} = -\mu_i + \rho = 0$ for both countries. It implies $\mu_R = \mu_P = \rho$. Taking the time derivative of ρ, one gets $\frac{\dot{\rho}}{\rho} = \frac{\dot{\mu}_R}{\mu_R} = \frac{\dot{\mu}_P}{\mu_P}$. Moreover, from conditions (11), we get for $i = R, P$

$$-\frac{\partial \mathcal{L}}{\partial X_i} = -\mu_i F'(X_i) = \dot{\mu}_i, \quad \Leftrightarrow \quad \frac{\dot{\mu}_i}{\mu_i} = -F'(X_i) \; .$$

Combining these conditions, we can state that an internal solution with $c_R > 0$ and $c_P > 0$ is possible only for states (X_R^*, X_P^*) such that $F'(X_R^*) = F'(X_P^*)$. This is dynamically possible only for a stationary state, with $c_R = F_R(X_R^*)$ and $c_P = F_P(X_P^*)$, *proving Proposition 1.* Apart from those stationary states, one must have either $c_P = 0$ or $c_R = 0$ along the maximin path.

We now characterize the transition path described in Proposition 2. Consider a state $(X_P, X_R) >> (0, 0)$ such that $F'(X_P) > F'(X_R)$. We demonstrate that, under these conditions, stock X_R is consumed alone while stock X_P builds up as long as the previous inequality holds by proving that the opposite is not possible. For a regular maximin path, $(\mu_R, \mu_P, \rho) \neq (0, 0, 0)$, $\rho > 0$ and thus $c_R + c_P = m^0(X_R, X_P)$. Assume that $c_P > 0$ and $c_R = 0$. Given the maximin value m^0, along the maximin path, one would have $c_P = m^0$, which is constant. By Lemmata 1 and 2 of Cairns et al. (2019), $m^0(X_P, X_R) > F_P(X_P) + F_R(X_R)$. This implies that $c_P > F(X_P)$. Therefore, $\frac{\mathrm{d}X_P}{\mathrm{d}t} = F_P(X_P) - c_P < 0$. Also, $\frac{\mathrm{d}^2 X_P}{(\mathrm{d}t)^2} = F'(X_P)\frac{\mathrm{d}X_P}{\mathrm{d}t} - \frac{\mathrm{d}c_P}{\mathrm{d}t} = F'(X_P)\frac{\mathrm{d}X_P}{\mathrm{d}t} < 0$. Therefore, stock X_P would be exhausted in a finite time ν. After that time, global consumption would correspond to the sustained production of stock X_R. At time ν, stock X_R would have increased to some level $X_R^\star \equiv X_R(\nu)$ such that $F_R(X_R^\star) = m^0$, allowing consumption c_R to sustain exactly the maximin utility. The stationary state would be $(0, X_R^\star)$. Making a step backward to examine the states through which such a path goes just prior to exhaustion, the dynamics before exhaustion would be

$$\begin{aligned} \dot{X}_P = F_P(X_P) - c_P \quad &\Leftrightarrow \quad \mathrm{d}X_P = \left(F_P(X_P) - m^0\right)\mathrm{d}t \; , \\ \dot{X}_R = F_R(X_R) \quad &\Leftrightarrow \quad \mathrm{d}X_R = F_R(X_R)\mathrm{d}t \; . \end{aligned}$$

Consider an infinitesimal time lapse $\mathrm{d}t$. At time $\nu - \mathrm{d}t$, stock X_P is equal to $\tilde{X}_P = \mathrm{d}X_P = m^0\mathrm{d}t$. Stock X_R is equal to $\tilde{X}_R = X_R^\star - \mathrm{d}X_R = X_R^\star - m^0\mathrm{d}t$. By Lemma 1 of Cairns et al. (2019), we know that the maximin value at time $\nu - \mathrm{d}t$ is greater than or equal to the equilibrium consumption of state $(\tilde{X}_P, \tilde{X}_R)$. Let us denote this consumption level by $\tilde{c} = F_P(m^0\mathrm{d}t) + F_R(X_R^\star - m^0\mathrm{d}t)$. We have $m^0(\nu - \mathrm{d}t) \geq \tilde{c}$. By subtracting $m^0(\nu)$

from both sides of the equation, we obtain the following.

$$\begin{aligned}
m^0(\nu-\mathrm{d}t)-m^0(\nu) &\geq \tilde{c}-m^0(\nu)\ ; \\
&\geq F\left(m^0\mathrm{d}t\right)+F\left(X_R^\star-m^0\mathrm{d}t\right)-F(X_R^\star)\ ; \\
&\geq \left(F\left(0+m^0\mathrm{d}t\right)-F(0)\right)+\left(F\left(X_R^\star-m^0\mathrm{d}t\right)-F(X_R^\star)\right)\ ; \\
&\geq m^0\mathrm{d}t\frac{F\left(0+m^0\mathrm{d}t\right)-F(0)}{m^0\mathrm{d}t}-m^0\mathrm{d}t\frac{F\left(X_R^\star-m^0\mathrm{d}t\right)-F(X_R^\star)}{-m^0\mathrm{d}t}\ .
\end{aligned}$$

Let us note $\epsilon=m^0\mathrm{d}t$ and $\tilde{\nu}=\nu-\mathrm{d}t$ (thus $\nu=\tilde{\nu}+\mathrm{d}t$). We get

$$\begin{aligned}
m^0(\tilde{\nu})-m^0(\tilde{\nu}+\mathrm{d}t) &\geq m^0\mathrm{d}t\frac{F(0+\epsilon)-F(0)}{\epsilon}-m^0\mathrm{d}t\frac{F(X_R^\star-\epsilon)-F(X_R^\star)}{-\epsilon}\ ; \\
\Leftrightarrow\quad \tfrac{1}{m^0}\left(\tfrac{m^0(\tilde{\nu}+\mathrm{d}t)-m^0(\tilde{\nu})}{\mathrm{d}t}\right) &\leq \frac{F(X_R^\star-\epsilon)-F(X_R^\star)}{-\epsilon}-\frac{F(0+\epsilon)-F_i(0)}{\epsilon}\ .
\end{aligned}$$

By taking the limits $\epsilon,\mathrm{d}t\to 0$, we obtain

$$\frac{\dot{m}^0}{m^0}\leq F'(X_R^\star)-F'(0)<0\ .$$

As the maximin value cannot decrease along a maximin path, we have a contradiction. We thus can assert that if $F'(X_P)>F'(X_R)$, $c_P=0$ and $c_R>0$. By regularity, $m^0=c_R+c_P$. Thus, $c_R=m^0(X_R,X_P)$. *This proves Proposition 2.*

A.2 Intragenerational inequality aversion: Mathematical details

The resolution of the maximin problem is similar to that of the previous section, except that both consumption levels have to be positive when $\theta>0$. The Lagrangian is

$$\mathcal{L}(X,c,\mu,\underline{w},\rho)=\mathcal{H}(X,c,\mu)+\rho\left(W^\theta(c_R,c_P)-\underline{w}\right)\ .$$

The necessary conditions include, for $i=R,P$, and for any time t (Cairns and Long, 2006; Cairns et al., 2019):

$$\frac{\partial\mathcal{L}}{\partial c_i}=0\quad\Leftrightarrow\quad\mu_i=\rho W_{c_i}\ ; \tag{12}$$

$$\frac{\partial\mathcal{L}}{\partial X_i}=-\dot{\mu}_i\quad\Leftrightarrow\quad-\frac{\dot{\mu}_i}{\mu_i}=F'(X_i)\ . \tag{13}$$

Deriving an expression for $\frac{\dot{\mu}_i}{\mu_i}$ from eq. (12) and combining the two equations gives

$$-\frac{\dot{\rho}}{\rho} = F'(X_i) + \frac{\dot{W}_{c_i}}{W_{c_i}} \; . \tag{14}$$

Equation (14) holds for both countries, so we can write

$$\frac{\dot{W}_{c_R}}{W_{c_R}} - \frac{\dot{W}_{c_P}}{W_{c_P}} = F'(X_P) - F'(X_R) \; .$$

Since $W_{c_R}/W_{c_P} = (c_P/c_R)^\theta$, the log-differentiation gives

$$\frac{\dot{c}_P}{c_P} - \frac{\dot{c}_R}{c_R} = \frac{F'(X_P) - F'(X_R)}{\theta} \; .$$

The growth gap depends positively on the productivity gap and negatively on the intra-generational inequality aversion.

Stationary states, with $\frac{\dot{c}_i}{c_i} = 0$ and $c_i = F(X_i^\star)$, are characterized by the no-arbitrage condition $F'(X_P^*) = F'(X_R^*)$. *This constitutes the proof of Proposition 3.*

Apart from the stationary state, when $F_P'(X_P) > F_R'(X_R)$, $\frac{\dot{c}_P}{c_P} > \frac{\dot{c}_R}{c_R}$. Along an egalitarian path, both consumptions cannot decrease or increase at the same time. Either (*i*) $\frac{\dot{c}_P}{c_P} > 0, \frac{\dot{c}_R}{c_R} < 0$ or (*ii*) $\frac{\dot{c}_P}{c_P} < 0, \frac{\dot{c}_R}{c_R} > 0$. Obviously, only (*i*) is possible. Then, $c_P < F_P(X_P)$ and $c_R > F_R(X_R)$. The welfare level during the transition equals its value at the stationary state, namely $W(c_R, c_P) = F(X_R^*) + F(X_P^*)$. *This constitutes the proof of Proposition 4.* For a detailed proof of the dynamics, see Cairns et al. (2019, Proof of Prop. 5).

Along the efficient egalitarian path, welfare is constant, i.e., $dW^\theta/dt = 0 = \dot{c}_R W_{c_R}^\theta + \dot{c}_P W_{c_P}^\theta$. This implies $-\frac{\dot{c}_P}{\dot{c}_R} = \left.\frac{W_{c_R}^\theta}{W_{c_P}^\theta}\right|_{W^\theta = \underline{w}}$. Also, we get from the Hartwick rule and condition (12) that $-\frac{\dot{X}_P}{\dot{X}_R} = \left.\frac{W_{c_R}^\theta}{W_{c_P}^\theta}\right|_{W^\theta = \underline{w}}$. Combining these two conditions, and noting that $\left.\frac{W_{c_R}^\theta}{W_{c_P}^\theta}\right|_{W^\theta = \underline{w}} = \left(\frac{c_P}{c_R}\right)^\theta$, *we get the result of Proposition 5.*

A.3 Extensions: capital transfers

With transfers, the Lagrangian becomes

$$\mathcal{L}^\tau = \mu_R \left(F(X_R) - c_R\right) + \mu_P \left(F(X_P + T) - c_P\right) + (\mu_R - \mu_P)\pi + (\mu_\tau - \mu_R)\tau + \rho \left(W^\theta - \underline{w}\right) \; ,$$

which is linear in π and τ. Optimal transfer levels will then respectively depend on the sign of

$$\frac{\partial \mathcal{L}^{\tau}}{\partial \pi} = \mu_R - \mu_P \quad \text{and} \quad \frac{\partial \mathcal{L}^{\tau}}{\partial \tau} = \mu_{\tau} - \mu_R \ .$$

Two new FOCs give $-\frac{\dot{\mu}_P}{\mu_P} = F'\left(X_P + T\right)$ and $-\frac{\dot{\mu}_{\tau}}{\mu_P} = F'\left(X_P + T\right)$, and thus $\dot{\mu}_{\tau} = \dot{\mu}_P$. We have $\mu_P > \mu_R$ before reaching the stationary state and $\mu_P^{\star} = \mu_R^{\star}$ afterward. Either (i) $\mu_{\tau} \geq \mu_P$ or (ii) $\mu_{\tau} < \mu_P$. Case (ii) could lead $\mu_{\tau} - \mu_R$ to pass from positive to negative at a date d before the equilibrium is reached, inducing the optimal transfer to jump from its maximum value to zero at d. Also, the optimal Lagrangian has to be nil during the transition: $\mathcal{L}^{\tau\star} = \mu_R\left(F(X_R) - c_R\right) + \mu_P\left(F(X_P + T) - c_P\right) + (\mu_{\tau} - \mu_R)\tau = 0$. As stocks have no reason to become stationary at d, case (ii) is impossible. We have $\mu_{\tau} \geq \mu_R$, and thus $\frac{\partial \mathcal{L}}{\partial \tau} \geq 0$. Therefore τ^{opt} equals its upper limit (*Proposition 6*).

As $\frac{\mu_P}{\mu_R} = \frac{W^{\theta}_{c_P}}{W^{\theta}_{c_R}} = \left(\frac{c_R}{c_P}\right)^{\theta}$, and $c_R \geq c_P$, $\mu_P \geq \mu_R$, we have $\frac{\partial \mathcal{L}}{\partial \pi} \leq 0$. Therefore $\pi^{*} = 0$ (or a negative value, if consumption transfer is possible) (*Proposition 7*).