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When the Decisive Voter Stays at Home

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Abstract

We develop a model of political competition with endogenous turnout and endogenous platforms. Parties trade off incentivizing their supporters to vote and discouraging the supporters of the competing party from voting. We show that the latter objective is particularly pronounced for a party with an edge in the political race. Thus, an increase in political support for a party may lead to the adoption of policies favoring its opponents so as to asymmetrically demobilize them. We study the implications for the political economy of redistributive taxation. Equilibrium tax policy is typically aligned with the interest of voters who are demobilized.

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Introduction

This paper has two main contributions. First, it develops a model of political competition in which the parties' platform choices and voters' participation in elections are jointly determined in equilibrium. Second, it uses this framework for a political economy analysis of redistributive taxation. The previous literature has focused on exogenous turnout, see Meltzer and Richard (1981) and Bénabou (2000) for seminal references. With exogenous turnout, changes in the distribution of incomes among those who actually vote shift redistributive policies in the same direction: if voters get poorer, tax policies get more redistributive; if voters get richer, tax polices get less redistributive. We revisit this relationship.

Political competition. Most of the previous political economy literature has focused either on platform choices or on endogenous turnout. By combining the two we obtain a framework where parties face a tradeoff between, on the one hand, appealing to as many voters as possible, and on the other hand, ensuring that these potential voters turn out to vote. A potential voter of, say, party 1 is weakly better off if party 1 wins and implements its platform. Being among those who prefer party 1 over party 2 is, however, only a necessary condition for voting in favor of party 1. Potential voters are turned into actual voters only if the stakes are sufficiently high, i.e., they must be incentivized to fight for a victory of their party. A voter who is close to being indifferent between the two parties lacks such incentives, since in this case the gain in utility from having her preferred party elected does not justify incurring the voting cost. Thus, parties face a tradeoff between adopting polices that increase the size of their base and policies that foster mobilization.

We draw on the probabilistic voting model – see Coughlin and Nitzan (1981) and Lindbeck and Weibull (1987) – to determine how voters sort into the two parties' bases. Specifically, voters have both policy preferences and idiosyncratic party preferences. A voter can therefore be attracted to the base of party 1 because she likes the platform of party 1 better, or because she likes party 1 for exogenous reasons. With well-behaved distributions of these preferences, a party's base responds in a continuous way to changes in the party's platform, and there are pure strategy equilibria even with multi-dimensional policy spaces. The probabilistic voting model is one of the workhorses in the formal analysis of party competition. However, this literature typically assumes that voter turnout is exogenous.

We draw on models of ethical voting – originally proposed by Harsanyi (1980) and more recently analyzed by Coate and Conlin (2004) and Feddersen and Sandroni (2006) – to endogenize turnout. These models have been proposed as a way of adressing the paradox of voting.[1] It is assumed that voting is costly and that voting behavior is driven by a desire to fulfill a civic duty – formalized as a group rule-utilitarian criterion for turnout. Individuals choose a turnout rule that is optimal on the assumption that everyone with the same party preferences behaves according to the same rule.[2] Such group behavior is able to affect the outcome of the election, thus leading to non-trivial equilibrium turnout rates. These depend on how much voters have at stake: when their aggregate benefit from winning the election is higher, more individuals of a given group turn out to vote. This literature delivers predictions that are consistent with empirical facts on turnout, but it generally considers exogenous policy platforms.

Our formal analysis merges these two models so that both policies and turnout are endogenous outcomes. We focus on the implications of the trade-off between the number of potential voters and mobilization. We establish conditions for equilibrium existence, fully characterize the equilibrium analytically, and provide a comparative statics analysis.

With endogenous turnout, either party has an incentive to propose a platform that is very attractive for its own followers so that they have a good reason to vote. In addition, there is also a demobilization objective for the followers of the competing party. This generates a countervailing incentive to propose a platform that is, from their perspective, as good as the platform proposed by their own party, so that they may as well stay at home on election day. A main finding of our comparative statics analysis is that this demobilization objective gets more weight for a party with strong support from its potential voters. By contrast, a party that only has lukewarm support from its followers, and is thus unlikely to win the election, should put more weight on the mobilization of its own base.

Campaign strategies where a front-runner avoids controversial positions or even

[1] The paradox is that observed turnout in elections is positive even though rational agents have no incentive to participate since the probability of being pivotal in large elections is negligible.

[2] Ethical voter models differ in some aspects: for instance, Feddersen and Sandroni (2006) model the electorate as being split between ethical and non-ethical voters. Coate and Conlin (2004) only have ethical voters in their framework. Our analysis is closer to Feddersen and Sandroni (2006), but we could as well have adopted the modelling choices of Coate and Conlin (2004). We provide a more detailed comparison of these approaches in the Online-Appendix, where we also show that these modelling choices are inconsequential for our main results.

adopts positions of the rival are also referred to as strategies of *asymmetric demobilization.* Direct empirical evidence for our mechanism is given by Chen (2013), who shows in the context of hurricane disaster aid in Florida in 2002 that "an incumbent who delivers distributive benefits to the opposing party's voters partially mitigates these voters' ideological opposition to the incumbent, hence weakening their motivation to turn out and oust the incumbent." Another prominent example are the campaigns of the Christian democrats (CDU) in Germany in the era of Angela Merkel.[3] Our analysis sheds light on the strategic considerations that rationalize such a strategy. Moreover, in part E of the Online-Appendix we present a detailed case study of the federal elections in Germany between 2005 and 2017 and argue that outcomes in various dimensions – margin of victory, overall turnout, relative turnout for the incumbent and the challenger, economic policy orientation – are aligned with the comparative statics predictions of our model.

Redistributive taxation. A classic hypothesis in the political economy of taxation is that increased inequality leads to more redistributive taxation. This hypothesis is usually derived from a model with a *decisive voter* who has below-average income. Meltzer and Richard (1981) is a seminal reference.[4] Meltzer and Richard argue moreover that, historically, extensions of the franchise added voters with below-average income and thus reduced the income of the decisive voter. Bénabou (2000), also a seminal reference, documents that *"every reported form of political activity rises with income and eduction."* Assuming that turnout is, for exogenous reasons, larger among "the rich", Bénabou presents an analysis of redistributive taxation that is based on the assumption that there is a decisive voter with an above-median income.

Both Meltzer and Richard (1981) and Bénabou (2000) exemplify a common and natural perspective on the political economy of redistributive taxation. First, there is a decisive or pivotal voter, defined as the voter whose preferred policy coincides with the policy implemented in a political equilibrium. Second, changes in who participates in elections and changes in the preferences of the decisive voter go together.

[3]It seems that the term *asymmetric demobilization* had its first appearance in an analysis of a regional election in Catalonia, see Lago et al. (2007). In the German context, this strategy is associated with Matthias Jung, an advisor of Angela Merkel and the head of the *Forschungsgruppe Wahlen* research institute that studies elections in Germany. He published on the rationale for this deliberate strategy and its impact on voting behavior and election outcomes: see Jung et al. (2010, 2015); Jung (2019). For an account in English language, see Schmidt (2014).

[4]In their framework, under universal suffrage and majority rule, the decisive voter is the voter with median income.

If the electorate becomes poorer due the extension of the franchise, then the decisive voter becomes also poorer and demands for redistributive taxation go up. If incomes among those who actually vote or otherwise participate in the political process are higher than incomes of those with the right to vote, then the decisive voter is richer, and demands for redistribution are more limited – as compared to a situation with universal or even turnout.

Our analysis of redistributive taxation with endogenous turnout gives rise to a different logic. The *decisive voter* and *turnout of "the rich" relative to "the poor"* may change in different directions when the political environment changes. In particular, the decisive voter may become poorer when turnout gets larger among "the rich" and smaller among "the poor". This is an implication of the asymmetric demobilization logic. To illustrate this result, suppose that there is a race between a pro-market party and a more left-leaning party. The pro-market party gets more support from richer voters who are also more opposed to redistribution. The left-leaning party gets more support from poorer voters who benefit if redistributive taxes go up. Let there be an initial situation that is balanced, i.e., where both parties are equally likely to win the election and neither party has a turnout advantage. Now suppose that, for exogenous reasons, the supporters of the pro-market become more willing to fight for a victory of their party – in the model, a shock that raises the intensity of their party preferences – then, in the resulting new equilibrium, the pro-market party has a turnout advantage, with the implication that turnout gets larger among "the rich" than among "the poor." The pro-market party is now more likely to win the election and the demobilization objective gains in importance. It therefore adopts a more redistributive platform and equilibrium taxes go up: the decisive voter gets poorer.[5]

This finding is shown to be robust in a variety of dimensions. For instance, it does not depend on which model of redistributive taxation is used. It holds for a model with affine income taxes, tax schedules with a constant rate of progressivity or Mirrleesian non-linear income tax schedules. It also holds for a broad class of comparative statics experiments which all imply that a pro-market party gains strength over a more left-leaning competitor. As a response, the pro-market party increasingly seeks to demobilize the supporters of the more left-leaning party by adopting a more redistributive platform.

[5]Again, the case study of federal elections in Germany between 2005 and 2017 in the Online-Appendix gives an empirical example for this mechanism.

Related Literature. Our analysis relates to the literature that seeks a response to the paradox of voting. We draw on one strand of this literature, models of ethical voting, due to Harsanyi (1980), Coate and Conlin (2004) and Feddersen and Sandroni (2006), see also Feddersen (2004) for a survey, and Callander and Wilson (2007), Degan and Merlo (2011) or Aldashev (2015) for more recent contributions. Rational voting, see e.g. Ledyard (1984), is a prominent alternative to ethical voting. Coate et al. (2008) argue that the ethical voter model provides a better fit for data on turnout than the pivotal voter model.

We contribute to a rich literature on the political economy of redistributive taxation. Different models of redistributive taxation are employed by this literature. For instance, Roberts (1977) and Meltzer and Richard (1981) use a model of linear income taxation. Bénabou (2000) considers tax schedules with a constant rate of progressivity, see Heathcote et al. (2017) for a detailed analysis of such tax schedules in dynamic settings. Our analysis of redistributive taxation applies both to linear income taxes and to tax schedules with a constant rate of progressivity. It also applies to fully non-linear income taxes. Political economy treatments of non-linear taxation have been provided by, e.g., Fahri and Werning (2008), Acemoglu et al. (2008, 2010), Brett and Weymark (2017) or Bierbrauer and Boyer (2016).

Different turnout rates among "the rich" and "the poor" are frequently discussed as a potential explanation for limited redistribution, see e.g. Bénabou (2000); Larcinese (2007); Sabet (2016) and the references therein. This literature treats turnout as an exogenous variable; i.e., the possibility that turnout may depend on the parties' policy proposals has not been taken into account. Our analysis of the election campaigns in the era of Angela Merkel in Germany shows that this feedback channel can be important.

There is a rich literature in political science that investigates to what extent parties cater towards their core voters or to swing voters; Cox (2009) provides a survey. It has been shown empirically that parties may also have an incentive to target their promises to the core voters of the competing party to mitigate their turnout, see Chen (2013). We contribute to this literature by developing a tractable theoretical framework that rationalizes these competing effects, and by finding that the incentive for demobilization is stronger for a party that is the likely winner of an election. Bernhardt et al. (2018) derive a similar result, albeit from a model with exogenous turnout.

Outline. The remainder of the paper is organized as follows. Section 1 introduces a general setup for an analysis of political competition that connects probabilistic voting with endogenous turnout. In Section 2 we clarify what this framework implies for the political economy of redistributive taxation. We provide conditions for equilibrium existence in Section 3. Proofs of propositons and of all other formal statements in the paper are in the Online-Appendix.

1 Party competition with endogenous turnout

Two political parties $j \in \{1, 2\}$ compete by choosing policies from a set of feasible policies $\mathcal{P}$. Party j's proposal is denoted by $p^j \in \mathcal{P}$. The policy space $\mathcal{P}$ can be one-dimensional or multi-dimensional.

Preferences. There is a continuum of citizens of mass one. Citizens differ in their preferences over policies. For any $\omega \in \Omega$, we denote by $u(p, \omega)$ the utility that a type-ω citizen realizes under policy $p \in \mathcal{P}$. In the income tax application, ω will determine an individual's position in the income distribution and will thus shape preferences over redistributive taxation.[6] The cross-sectional distribution of types $\omega \in \Omega$ is common knowledge and represented by a cumulative distribution function F_ω with density f_ω.

Individuals also have party preferences. These preferences may be shaped by cultural and ethnic identities, party histories, or fixed party positions in certain policy domains. Formally, the random variable $\varepsilon \in \mathbb{R}$ denotes an agent's idiosyncratic preference for party 2. Conditional on ω, party preferences ε of different voters are independent and identically distributed. Thus, an individual with type ω and party preference ε supports party 1 if

$$u(p^1, \omega) - u(p^2, \omega) \geq \varepsilon.$$

We denote by $B(\cdot \mid \omega)$ the cumulative distribution function of party preferences ε among individuals of type ω, and by $b(\cdot \mid \omega)$ the corresponding density function. Therefore, the fraction of type-ω individuals supporting party 1 is $B\left(u(p^1, \omega) - u(p^2, \omega) \mid \omega\right)$.

Ethical voting. The mass of type-ω supporters of each party j is split into two groups: a fraction $1-\tilde{q}^j(\omega)$ of these agents always abstains from voting, and a fraction

[6] The Online Appendix contains an application where ω represents public goods preferences.

$\tilde{q}^j(\omega)$ decides whether to vote based on a rule-utilitarian calculation.[7,8] The literature often refers to this last group as *ethical voters*.

We seek a framework where the election outcome is uncertain both from the voters' and the parties' perspectives. A convenient approach, adopted by Feddersen and Sandroni (2006), is to assume that $\tilde{q}^j(\omega)$ is a random variable and that its realization is unknown both when parties choose platforms and when potential voters decide whether or not to turn out. More specifically, we assume that $\tilde{q}^1(\omega)$ and $\tilde{q}^2(\omega)$ have the same expected value $\bar{q}(\omega) \in (0,1)$. That is, a type-$\omega$ supporter of party 1 is, on average, as likely to be of the ethical type as a type-ω supporter of party 2. The following assumption puts structure on how realizations of $\tilde{q}^1(\omega)$ and $\tilde{q}^2(\omega)$ relate to the mean.

Assumption 1. *For each party j, there is a non-negative random variable η^j with mean 1 such that $\tilde{q}^j(\omega) = \eta^j \cdot \bar{q}(\omega)$, for all $\omega \in \Omega$.*

The possibility that party 1 is affected by a positive shock $\eta^1 > 1$ and party 2 is affected by a negative shock $\eta^2 < 1$, or vice versa, generates uncertainty in election outcomes. The random variable η^j can be interpreted as capturing the success of an election campaign that is revealed only on election day.[9] Assumption 1 is imposed in the sequel without further mention.

Bases. The ethical supporters are a party's potential voters. For ease of exposition, we also refer to the expected mass of these agents as a party's *base*. That is, given two policies p^1 and p^2, the base of party 1 is given by

$$\mathbf{B}^1(p^1, p^2) \;\; = \;\; \mathbb{E}\left[\bar{q}\,(\omega)\, B\left(u(p^1, \omega) - u(p^2, \omega) \mid \omega\right)\right], \tag{1}$$

where the expectation operator $\mathbb{E}$ indicates the computation of a population average with respect to different types ω. We define the base of party 2, $\mathbf{B}^2(p^1, p^2)$, analogously.

[7] We follow Coate and Conlin (2004) and Feddersen and Sandroni (2006) and assume that there are no "always-voters", i.e., individuals who come to the ballot regardless of how high their voting costs are. In the Online Appendix, we present a version of our model that includes such voters and gives rise to an equilibrium analysis that is equivalent to the one developed in the body of the text.

[8] Below we assume that ethical voters only care about the payoffs of those on their side; thus, they are acting as *group*-rule-utilitarians.

[9] Note that the shocks to the two parties' bases may be correlated. We do not impose an assumption of independence.

Stakes. The *stakes* for the potential voters of party 1 are defined as the expected (utilitarian) welfare gain that is realized if a victory by party 2 is avoided. Formally,

$$W^1(p^1, p^2) \;=\; \mathbb{E}\left[\int_{\mathbb{R}} \max\left\{u(p^1, \omega) - u(p^2, \omega) - \varepsilon\;,\;0\right\} b\,(\varepsilon \mid \omega)\,\mathrm{d}\varepsilon\right]. \qquad (2)$$

The integrand in equation (2) is the difference in utilities realized under the policies p^1 and p^2, including the gains or losses due to party preferences. The *max* indicates that the summation over ε takes into account only the agents for whom this utility difference is positive, i.e., the supporters of party 1. We define $W^2(p^1, p^2)$ analogously.

Voting costs. We denote by σ^j the fraction of ethical supporters of party j who actually turn out to vote. We define the aggregate voting cost of the ethical supporters of party j by $\kappa(\sigma^j)\mathbf{B}^j(p^1, p^2)$, where, for some scalars $\chi > 0$ and $\lambda \in (0, 1]$,

$$\kappa(\sigma^j) \;=\; \chi\,(\sigma^j)^{1/\lambda}. \qquad (3)$$

This isoelastic functional form unifies several cases. First, suppose that all the ethical voters have a common per capita voting cost χ and choose an individual probability of voting σ^j.[10] We then obtain a linear voting cost function $\kappa(\sigma^j) = \chi\sigma^j$, corresponding to $\lambda = 1$ in (3). Second, as we argue below, an inelastic cost function with $\lambda \to 0$ turns our setup into a standard probabilistic voting model with exogenous turnout. Third, our framework nests the case of quadratic voting costs as in Coate and Conlin (2004) and Feddersen and Sandroni (2006), if $\lambda = 1/2$.[11]

Endogenous turnout. The ethical supporters of each party j adhere to a rule σ^j for participation in the election that maximizes their aggregate expected utility, taking the costs of voting into account. As a consequence, turnout depends on the parties' policy proposals. Specifically, the problem of the ethical supporters of party j admits the following representation. Taking as given the policies (p^1, p^2), and the

[10] With an appeal to a law of large numbers, such a probability can also be interpreted as the percentage share of ethical voters who actually turn out to vote.

[11] These papers derive the quadratic cost function from a setup in which individual voting costs are i.i.d. draws from a uniform distribution, and a cutoff rule σ^j so that all individuals with voting costs below that threshold turn out to vote.

other party's turnout rule σ^{-j}, choose $\sigma^j \in [0,1]$ to maximize

$$\pi^j(p^1, p^2, \sigma^1, \sigma^2)\, W^j(p^1, p^2) - \kappa(\sigma^j)\, \mathbf{B}^j(p^1, p^2), \tag{4}$$

where π^j is the probability that party j wins the election. This problem involves a trade-off between the probability of winning and the costs of voting, as both π^j and $\kappa(\sigma^j)\, \mathbf{B}^j$ are increasing in σ^j.

Given p^1 and p^2, an equilibrium of the *turnout game* is a pair of turnout rates $(\sigma^{1*}(p^1, p^2), \sigma^{2*}(p^1, p^2))$ that are mutually best responses. We are interested in equilibria that are interior, i.e., such that turnout responds at the margin to changes in proposed policies. Corner solutions where all or none of the ethical voters participate are conceivable. In this case, turnout is locally irresponsive to the policies that the parties propose, with the implication that the parties' strategic considerations are as in a model with exogenous turnout. In what follows, we assume interior turnout rates.[12]

1.1 Base vs. turnout

Parties face a trade-off between adopting polices that enlarge the size of their base and adopting policies that increases the turnout of their own supporters relative to the supporters of the competing party. In this section, we elaborate on this trade-off. Let

$$\bar{\pi}^1(p^1, p^2) := \pi^1(p^1, p^2, \sigma^{1*}(p^1, p^2), \sigma^{2*}(p^1, p^2))$$

denote party 1's probability of winning, taking into account that policy choices affect the equilibrium of the turnout game. A pair of *equilibrium policies* (p^1, p^2) satisfies $\bar{\pi}^1(p^1, p^2) \geq \bar{\pi}^1(\hat{p}^1, p^2)$, for all $\hat{p}^1 \in \mathcal{P}$ and $\bar{\pi}^1(p^1, p^2) \leq \bar{\pi}^1(p^1, \hat{p}^2)$, for all $\hat{p}^2 \in \mathcal{P}$.

Maximizing the probability of winning for party 1 is equivalent to maximizing

$$\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} \times \frac{\sigma^{1*}(p^1, p^2)}{\sigma^{2*}(p^1, p^2)}, \tag{5}$$

whereas party 2's objective is to minimize this expression. The first term in (5) is

[12]Inada conditions ensure that the equilibrium of the turnout game is interior: the c.d.f. F_η of the random variable η^2/η^1 is concave with a bounded and strictly positive density f_η, and the cost function k is convex with $\lim_{\sigma \to 0} k'(\sigma) = 0$ and $\lim_{\sigma \to 1} k'(\sigma) = \infty$. Alterative conditions are conceivable, e.g., we could also impose Inada-like conditions on the function F_η to ensure an interior equilibrium when the cost function is not as convex.

a measure of the party's relative *base advantage*. The second is a measure of its relative *turnout advantage*. If turnout was exogenous, party 1 would simply focus on maximizing its base advantage, or equivalently[13] its own base $\mathbf{B}^1(p^1, p^2)$. If instead the base was exogenously given, party 1 would maximize its turnout advantage. With both an endogenous base and endogenous turnout, party 1 faces a trade-off between maximizing the number of its supporters and maximizing their relative propensity to actually vote. Solving explicitly for the relative turnout advantage using the ethical voters' optimization problems and substituting the resulting expression into equation (5) yields the following characterization.

Proposition 1 (Base versus Stakes)**.** *Party 1 maximizes and party 2 minimizes*

$$\Pi^1(p^1, p^2) \quad := \quad (1-\lambda)\ln\left(\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right) + \lambda \ln\left(\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right). \tag{6}$$

According to Proposition 1, with endogenous turnout, maximizing the probability of winning an election is equivalent to maximizing a weighted average of two terms, a first term that measures relative support in the population at large, and a second term that measures the party's advantage or disadvantage in the stakes that its supporters have in the election. Exogenous turnout is nested as a special case: for $\lambda \to 0$, party 1 focuses on maximizing its base $\mathbf{B}^1$.

The payoff relevance of the stake advantage when turnout is endogenous ($\lambda > 0$) implies that parties face a tradeoff between mobilizing their own supporters and demobilizing the supporters of the competing party.[14] On the one hand, party 1 would like to propose a policy p^1 that makes W^1 as large as possible, i.e., that makes its own supporters as well off as possible compared to the welfare they would obtain under the opposition's platform p^2. Doing so encourages its own supporters to turn out by increasing how much they have at stake in the election. On the other hand, party 1 would also like to propose a policy p^1 that makes W^2 as small as possible, i.e., that does not hurt party 2's supporters too much compared to the welfare they could obtain under *their* preferred policy p^2. Doing so discourages the opposition

[13] It is straightforward to show that the two parties' bases add up to the constant: $\mathbf{B}^2(p^1, p^2) = \mathbb{E}[\bar{q}(\omega)] - \mathbf{B}^1(p^1, p^2)$. Hence, a change in policies that increases, say, the base of party 1, translates one-for-one into a decrease of party 2's base.

[14] The polar case $\lambda = 1$ corresponds to linear voting costs. In this case, an increase of the base translates one-for-one into additional voting costs, so that the probability of winning only depends on what an average voter has at stake, and not on how numerous potential voters are. Thus, party 1's objective is to maximize its stake advantage, regardless of the implied size of its base.

from turning out by lowering how much they have at stake. Below, when discussing comparative statics, we argue that, in equilibrium, the weight on the demobilization objective is larger for a party that has a larger probability of winning the election.

1.2 Equilibrium policies

The next Proposition provides a characterization of equilibrium policies. Assume that the following regularity condition holds. *For every $p^2 \in \mathcal{P}$ there exists a unique solution to* $\max_{p^1} \Pi^1(p^1, p^2)$*, and for every $p^1 \in \mathcal{P}$ there exists a unique solution to* $\min_{p^2} \Pi^1(p^1, p^2)$*. The relationship between best responses and solutions to first-order conditions is one-to-one.* Under this condition, the analysis of equilibrium policies can focus on first-order conditions.[15]

Henceforth, we use shorthand expressions for the main variables of our model when both parties propose the same policy $p^1 = p^2$. Specifically, we denote the mass of type-ω citizens supporting party 1 by $B^s(\omega) := B(0 \mid \omega)$, and the fraction of agents who are on the verge of indifference between the two parties by $b^s(\omega) := b(0 \mid \omega)$. Furthermore, for $j \in \{1, 2\}$ we denote party j's aggregate base $\mathbf{B}^j(p^1, p^2)$ and stakes $W^j(p^1, p^2)$ when $p^1 = p^2$ by $\mathbf{B}^{js}$ and W^{js}, respectively. The sorting of ethical voters into the parties' bases is then entirely driven by idiosyncratic party preferences $\varepsilon \lesseqgtr 0$. Therefore, the variables $\mathbf{B}^{js}, W^{js}$ are exogenous primitives of the model akin to moments of the distributions $B(\cdot \mid \omega)$, $\omega \in \Omega$. In particular, we have

$$W^{1s} = \mathbb{E}\left[\int_{-\infty}^{0} |\varepsilon| \, \mathrm{d}B(\varepsilon \mid \omega)\right], \quad \text{and} \quad W^{2s} = \mathbb{E}\left[\int_{0}^{\infty} \varepsilon \, \mathrm{d}B(\varepsilon \mid \omega)\right]. \tag{7}$$

The variables W^{1s}, W^{2s} are thus respectively equal to the average values of the political biases of supporters of party 1 (for whom $\varepsilon < 0$) and party 2 (for whom $\varepsilon > 0$). These two *preference intensity* parameters play an important role in the sequel.

Proposition 2 (Political equilibrium)**.** *There is a unique pure strategy equilibrium.*

[15] It holds, for instance, if for every p^2 the function $p^1 \mapsto \Pi^1(p^1, p^2)$ is globally concave, and for every p^1 the function $p^2 \mapsto \Pi^1(p^1, p^2)$ is globally convex. In this case, there is a pure strategy equilibrium, the saddle point of the function Π^1. In Section 3, we give an example of conditions on the primitives of the model – the functions $B(\cdot \mid \omega)$, $\omega \in \Omega$ that govern the joint distribution of party and policy preferences – which imply that this regularity condition is satisfied. We also provide sufficient conditions for the existence of pure and mixed strategy equilibria beyond this special case.

This equilibrium is symmetric. The equilibrium policy solves

$$\max_{p \in \mathcal{P}} \mathbb{E}\left[\gamma^*(\omega)\, u(p,\omega)\right], \qquad \textit{with} \qquad \gamma^*(\omega) = (1-\lambda)\,\gamma_B^*(\omega) + \lambda \gamma_S^*(\omega),$$

where $\gamma_B^*(\cdot)$ *and* $\gamma_S^*(\cdot)$ *are given by*

$$\gamma_B^*(\omega) = \frac{\mathbb{E}\left[\bar{q}(\omega)\right]}{\mathbf{B}^{1s}\mathbf{B}^{2s}}\, \bar{q}(\omega)\, b^s(\omega) \quad \textit{and} \quad \gamma_S^*(\omega) = \frac{1}{W^{1s}} B^s(\omega) + \frac{1}{W^{2s}}\left(1 - B^s(\omega)\right). \quad (8)$$

According to Proposition 2, both parties propose the policy that maximizes a weighted utilitarian welfare function. The weights reflect that parties choose platforms so as to strike a balance between two considerations: the benefit of enlarging their set of potential supporters and the benefit of having a better turnout margin. More formally, the political equilibrium weight of type ω, $\gamma^*(\omega)$, is an average of two weights. The first one, $\gamma_B^*(\omega)$, reflects a party's gain from enlarging its base. The ratio

$$\frac{\gamma_B^*(\omega)}{\gamma_B^*(\omega')} = \frac{\bar{q}(\omega)\, b^s(\omega)}{\bar{q}(\omega')\, b^s(\omega')}$$

can be interpreted as a marginal rate of substitution that measures a party's willingness to trade-off favors to voter types ω and ω' when the size of the base is all that matters. This expression highlights the benefits of catering to swing voters that are familiar from probabilistic voting models.

The second weight, $\gamma_S^*(\omega)$, captures the contribution of the stakes margin to the probability of winning. The corresponding trade-off between the interests of different voters can be understood by considering the ratio

$$\frac{\gamma_S^*(\omega)}{\gamma_S^*(\omega')} = \frac{B^s(\omega) + \frac{W^{1s}}{W^{2s}}\left(1 - B^s(\omega)\right)}{B^s(\omega') + \frac{W^{1s}}{W^{2s}}\left(1 - B^s(\omega')\right)}. \quad (9)$$

From the perspective of party 1, the terms $B^s(\omega)$ and $B^s(\omega')$ stem from the incentives to increase the stakes of voters who belong to uts own base, whereas $1 - B^s(\omega)$ and $1 - B^s(\omega')$ reflect the benefits of reducing the stakes for voters who belong to the base of party 2. These terms reflect party attachments, rather than propensities to swing vote. Moreover, $\frac{W^{1s}}{W^{2s}}$ is the ratio of party preference intensities among the supporters of parties 1 and party 2, conditional on $p^1 = p^2$.

Asymmetric demobilization. The weights $\gamma_S^*(\omega)$ point to the returns from mobilizing the party's own base and those from demobilizing the opponent's base. Their respective contributions to the overall objective of winning the election depend, moreover, on the ratio $\frac{W^{1s}}{W^{2s}}$. From the perspective of party 1, the larger is this ratio, the more important is the demobilization objective relative to the mobilization objective. This observation gives rise to a relationship between a party's likelihood of winning the election and its incentive to demobilize the supporters of its rival – by adopting a platform closer to their preferred policy. When W^{1s} goes up and/or W^{2s} goes down, party 1's equilibrium probability of winning goes up (see equation (6)), and so does the party's incentive to cater to ω-types where party 2 has a large mass of core voters (see equation (9)). Proposition 3 below contains a formal statement of this insight and, moreover, clarifies its implications for redistributive taxation.

The campaigns of the Christian democrats (CDU) in Germany in the era of Angela Merkel are a prominent empirical example of an *asymmetric demobilization* strategy. Asymmetric demobilization was adopted in the 2009, 2013 and 2017 elections in response to the 2005 experience, in which Merkel ran on a pro-market platform and almost lost despite a significant lead in the polls over the main competitor, the Social Democrats (SPD). In subsequent elections the CDU adopted many positions previously held only by the SPD.[16] This strategy paid off: the CDU's margin of victory over the SPD increased from 1 percent in 2005 to more than 10 percent in 2009 and the subsequent elections. Overall turnout went down to all-time lows after WWII, but was lower among potential SPD voters than among potential CDU voters, see Jung et al. (2010); Forschungsgruppe Wahlen (2013b,a, 2015, 2018).

2 Taxes

Models of redistributive taxation have in common that policy preferences are derived from a framework with the following properties. Individuals value consumption, or after-tax income, c. The generation of earnings, or pre-tax income, y, requires costly

[16] Josef Joffe, a well-known German journalist, summarized the CDU's strategy in colorful language: "*Ms Merkel's plan is to lull the other side; don't rile them and win by keeping them at home. How did she do it after the near-disaster of 2005? By shifting to the left. An apostle of free markets and low taxes ten years ago, Merkel simply outflanked the left on the left ... She is the best Social Democrat the SPD could have asked for*" (Financial times, 08-05-2013). In part E of the Online-Appendix, we document the shift in the CDU's positions more systematically using quantitative analyses of party positions by political scientists.

effort. The parameter ω captures individual heterogeneity in effort costs: workers with low (resp., high) effort costs choose high levels of earnings and therefore end up being "rich" (resp., "poor"). For reasons of tractability, it is common to assume that preferences are additively separable between consumption utility $v(c)$ and effort costs $k(y,\omega)$. The effort cost function k is decreasing in y, and has a non-negative cross-derivative k_{12}, so that the marginal effort costs of higher types are lower.[17] Frequent assumptions are that the consumption utility v is linear or logarithmic, and that the cost function is iso-elastic, $k(y,\omega) = \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}$ with $e > 0$.

The policy instruments under consideration are classes of tax functions $T : y \mapsto T(y)$ that specify tax payments as a function of earnings. Thus, consumption is given by $c = y - T(y)$. A redistributive tax system typically has negative tax payments for low values of y. Tax systems have to satisfy a government budget constraint so that the transfers received by "the poor" are financed by the positive taxes paid by "the rich." The policy preferences of a type-ω individual are therefore captured by

$$u(T,\omega) \quad = \quad \max_{y} \{v(y - T(y)) - k(y,\omega)\}.$$

Models of redistributive taxation differ in the classes of tax functions that they consider. In the following, we discuss three classes that are prominent in the literature: affine tax schedules, non-linear tax schedules with a constant rate of progressivity, and the full class of all non-linear tax schedules. We first provide a characterization of political equilibrium taxes for all these models, presuming that the regularity conditions described in Section 1.2 are satisfied. Section 2.2 then contains comparative statics results on what changes in economic or political inequality imply for taxes and turnout. While the models of redistributive taxation differ in many aspects, we can provide a unified analysis: we identify forces that lead to more or less redistributive taxes and which apply in all these models.[18]

[17] It is common to interpret ω as an hourly wage, or, more generally, as a measure of individual productivity. More productive individuals need less time to generate a given earnings level and thus have lower effort costs.

[18] For ease of exposition, we do not consider idiosyncratic income risks. In models with incomplete markets, a redistributive income tax system may be desirable as an insurance device against such risks, see Bénabou (2000). The characterization of political equilibrium weights in Proposition 2 could also be used to characterize political equilibrium taxes in such an extended framework.

2.1 Models of redistributive taxation

2.1.1 Linear income taxation

The affine income tax, introduced by Sheshinski (1972), is frequently employed for political economy analyses under the assumption of exogenous turnout.[19] In this model, the tax function takes the form $T(y) = \tau y - r$, where τ is the constant tax rate and r is a uniform lump-sum transfer. Consequently, marginal tax rates are the same for all levels of income, whereas average tax rates increase with income. Assuming quasi-linear in consumption preferences, $v(c) = c$, leads to utility-maximizing earnings that depend on τ and ω but not on r, that is, $y^* = y^*(\tau, \omega)$. Via the government budget constraint, transfers are a function of τ, $r(\tau) = \tau \mathbb{E}[y^*(\tau, \omega)]$. Therefore, policy preferences can be represented by

$$u(\tau, \omega) = r(\tau) + (1 - \tau)\, y^*(\tau, \omega) - k(y^*(\tau, \omega), \omega). \tag{10}$$

For later reference, we note that policy preferences in this model satisfy a single-crossing property according to which higher types or, equivalently, richer taxpayers benefit less from an increase in the tax rate.[20] This implies that the ideal tax rate of richer agents is lower than the ideal tax rate of poorer ones.[21]

Applying the characterization of equilibrium policies in Proposition 2 to the linear income tax model leads to a characterization of the equilibrium tax rate τ^*. Specifically, with-isoelastic effort costs and quasi-linearity in consumption, τ^* satisfies

$$\frac{\tau^*}{1 - \tau^*} = -\frac{1}{e} Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \frac{\omega^{1+e}}{\mathbb{E}[\omega^{1+e}]}\right), \tag{11}$$

where the weights $\gamma^*(\cdot)$ are given by (8). The left-hand side of this equation is an increasing function of τ^*. Thus, the equilibrium tax rate depends on the covariance between political weights and productive abilities. The equilibrium tax policy is closer

[19] Well-known references include Roberts (1977), Meltzer and Richard (1981) or Alesina and Angeletos (2005).

[20] More formally, note that, by the envelope theorem, $u_1(\tau, \omega) = r'(\tau) - y^*(\tau, \omega)$. Using that y^* is a non-decreasing function of ω, we have $u_{12}(\tau, \omega) = -y_2^*(\tau, \omega) \leq 0$.

[21] It also implies that linear income taxation is covered by the equilibrium existence result in Proposition 5 below which provides conditions for the existence of a pure strategy equilibrium. For some of our results, we assume, moreover, that policy preferences are concave: $u_{11}(\tau, \omega) < 0$, for all τ and ω. In the linear income tax model, concavity holds, for instance, with an isoelastic effort cost function for $e \leq \frac{1}{2}$.

to the ideal of those with high political weights: the more the weights are concentrated on "the poor", the higher is the equilibrium tax rate; conversely, the more they are concentrated on "the rich", the lower is the tax rate.

2.1.2 Constant rate of progressivity

Tax schedules with a constant rate of progressivity, *CRP* schedules for short, allow significant tractability in a variety of redistributive taxation problems. Gans and Smart (1996) and Bénabou (2000) study voting equilibria with such taxes, assuming exogenous turnout. In this setup, the tax function is given by $T(y) = y - ry^{1-\tau}$, so that both the average and the marginal tax rate are increasing functions of income. With $c(y) := y - T(y) = ry^{1-\tau}$, the elasticity of after-tax income with respect to pre-tax income is constant and equal to $1 - \tau$. Hence, τ is measure of the progressivity of the tax code. Given τ, the parameter r governs how redistributive the tax system is: the larger r is, the more people receive transfers. Assuming logarithmic consumption utility, we can proceed along the same lines as for linear income taxation to show that y^* is a function of τ and ω, and that r is determined as a function of τ through the government budget constraint by $r(\tau) = \frac{\mathbb{E}[y^*(\tau,\omega)]}{\mathbb{E}[y^*(\tau,\omega)^{1-\tau}]}$. Policy preferences are now captured by

$$u(\tau, \omega) = \ln r(\tau) + (1 - \tau) \ln y^*(\tau, \omega) - k(y^*(\tau, \omega), \omega). \tag{12}$$

These preferences also satisfy the single-crossing property;[22] thus, a poorer agent would opt for a higher value of τ.

Again, Proposition 2 can be used to obtain a characterization of the political equilibrium tax system. Assuming that $\ln \omega$ is normally distributed with mean μ_ω and variance σ_ω^2, we obtain

$$\frac{\tau^*}{1 - \tau^*} = \left(1 + \frac{1}{e}\right)\left((1 - \tau^*)\,\sigma_\omega^2 - Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \ln \omega\right)\right), \tag{13}$$

where the weights $\gamma^*(\cdot)$ are again given by (8). The left-hand side of this equation is increasing in τ^*, the right-hand side is decreasing in τ^*. The point of intersection is the uniquely-determined equilibrium value of τ. As in the model of linear income

[22] By the envelope theorem, $u_1(\tau, \omega) = \frac{r'(\tau)}{r(\tau)} - \ln y^*(\tau, \omega)$ and this implies $u_{12}(\tau, \omega) = -\frac{y_2^*(\tau,\omega)}{y^*(\tau,\omega)} \leq 0$, where the last inequality follows from $y_2^*(\tau, \omega) \geq 0$. Additional assumptions can be imposed to ensure concave policy preferences. Concavity holds, for instance, with isoelastic effort costs.

taxation, high political weights on "the poor" yield high values of τ^*, and high political weights on "the rich" yield low values of τ^*.

2.1.3 Non-linear income taxation

The Mirrleesian approach to optimal income taxation does not impose any *a priori* restriction on the set of tax schedules. Here, this means that parties are not constrained by predetermined functional forms, but free to propose any tax schedule that satisfies the government budget constraint. They might, for instance, choose high marginal tax rates on "the rich" and earnings subsidies, i.e. negative marginal tax rates, for "the poor". Linear income taxes or *CRP* schedules are less flexible. Preferences are often assumed quasi-linear in consumption with iso-elastic effort costs, see for instance Diamond (1998). Under these assumptions, part B.3 of the Online-Appendix contains a derivation of policy preferences over non-linear taxes. Using arguments from mechanism design, we show that a non-linear income tax schedule can equivalently be represented by a bounded and monotonic earnings function $\mathbf{y} : \omega \mapsto \mathbf{y}(\omega)$. By the Taxation Principle (see Hammond (1979) or Guesnerie (1995)) any tax schedule implements such an earnings function, and conversely, for any such earnings function one can find a tax schedule that implements it. Policy preferences can thus be represented by preferences over earnings functions, $u(\mathbf{y}, \omega)$.[23]

The political equilibrium tax function $T^* : y \mapsto T^*(y)$ satisfies

$$\frac{T^{*\prime}(\mathbf{y}^*(\omega))}{1 - T^{*\prime}(\mathbf{y}^*(\omega))} = \left(1 + \frac{1}{e}\right) \frac{1 - F_\omega(\omega)}{\omega f_\omega(\omega)} \left(1 - \Gamma^*(\omega)\right), \tag{14}$$

where $\mathbf{y}^*(\omega)$ is the equilibrium earnings of type ω and $T^{*\prime}(\mathbf{y}^*(\omega)$ is the corresponding marginal income tax rate. Moreover,

$$\Gamma^*(\omega) = \mathbb{E}\left[\frac{\gamma^*(z)}{\mathbb{E}[\gamma^*(\omega)]} \mid z \geq \omega\right],$$

where $\gamma^*(\cdot)$ are again given by (8), is the average political weight among people who are richer than type ω.

Equation (14) determines marginal tax rates in a political equilibrium with endogenous turnout.[24] The left hand side of this equation is an increasing function of

[23] In Section 3 we provide conditions that ensure the existence of pure and mixed strategy equilibria for this policy space.

[24] This formula is akin to the *ABC* formula for optimal, welfare-maximizing taxes due to Diamond

the marginal income tax rate faced by agents with productive ability ω. Hence, the larger the right-hand side, the larger the marginal tax rate for these types in equilibrium. The right-hand side, in turn, is inversely related to the behavioral responses to taxation as measured by the elasticity parameter e. It is also inversely related to the hazard rate of the type distribution $\frac{\omega f_\omega(\omega)}{1-F_\omega(\omega)}$. The logic is that a local increase in marginal taxes to be paid by types close to ω generates additional revenue from all individuals with a type above ω, i.e., from a mass of taxpayers equal to $1 - F_\omega(\omega)$. It provokes a behavioral response from all individuals with a type close to ω whose incentives to generate income are reduced. The size of the corresponding revenue loss is measured by $\omega f_\omega(\omega)$. Thus, the lower the hazard rate, the more revenue potential there is. Finally, the political weights determine the electoral return from exhausting this revenue potential. These returns are large if the average weight $\Gamma^*(\omega)$ among those who would have to pay this bill is small. By contrast, if people with types above ω have, on average, a lot of political weight, the marginal tax rate for type ω is low.

2.1.4 How redistributive is the tax system?

The political equilibrium weights characterized in Proposition 2 can be used to measure how redistributive a tax system is in a political equilibrium. Consider two specifications of the model's primitives giving rise to two different weighting functions that are respectively denoted by $\gamma_0^* : \omega \mapsto \gamma_0^*(\omega)$ and $\gamma_1^* : \omega \mapsto \gamma_1^*(\omega)$. Moreover, suppose that there is a decreasing function $\delta : \omega \mapsto \delta(\omega)$ with $\mathbb{E}[\delta(\omega)] = 0$ so that

$$\frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]} = \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]} + \delta(\omega). \tag{15}$$

Thus, the weighting function γ_1^* assigns more weight to low income types and less weight to high income types. This implies that the equilibrium tax rate is higher with the weighting function γ_1^* in the model of affine income taxation. For the class of tax functions with a constant rate of progressivity, the equilibrium degree of progressivity is higher with weighting function γ_1^*. Finally, for fully flexible non-linear taxes, marginal tax rates for all levels of income are higher under γ_1^*. Consequently, when condition (15) holds, we can order tax systems according to how redistributive

(1998), except that the welfare weights in Diamond's formula are replaced by the political equilibrium weights derived in Proposition 2.

they are. This order is robust in the sense that it does not depend on the fine details that distinguish different models of redistributive taxation.

2.2 Comparative statics of taxes and turnout

How does the level of redistributive taxation respond to changes in economic inequality? Why is turnout typically lower among "the poor" than among "the rich"? The first is a classic question in the political economy of taxation. A well-known hypothesis, due to Meltzer and Richard (1981), is that increases in inequality should lead to higher taxes. Moreover, an explanation for why taxes are not as high as predicted by such a median voter model is that the "decisive voter" has an above-median income as "the poor" turn out in lower proportions than "the rich", see Bénabou (2000). In this section, we discuss the implications of endogenous turnout for these questions.

As discussed in Kasara and Suryanarayan (2015), empirically the relationship between economic inequality, participation in the political process, and redistributive taxation is multi-faceted. Our analysis cannot do justice to all these aspects; it can, however, illuminate one important channel. Consider an increase in polarization. In particular, suppose that the opponents of redistributive taxation become more zealous about a victory of their pro-market party. Then, our analysis implies that turnout becomes larger among "the rich" and that the equilibrium tax system becomes more redistributive. Thus, while political outcomes get tilted towards "the rich", tax policy gets tilted towards "the poor".

Symmetric benchmark. We start by presenting a benchmark where turnout, although endogenous, does not vary with income, and determine the implications for taxes in equilibrium. Suppose that $\mathbf{B}^{1s} = \mathbf{B}^{2s}$ and $W^{1s} = W^{2s} \equiv W^s$, meaning that the parties have equal numbers of potential voters, and moreover, that the intensity of party preferences among the potential voters of party 1 is, on average, equal to the intensity of party preferences among the potential voters of party 2. We show in the Online-Appendix that, in equilibrium,

$$\frac{\sigma^{1*}}{\sigma^{2*}} = \left(\frac{W^{1s} \,/\, \mathbf{B}^{1s}}{W^{2s} \,/\, \mathbf{B}^{2s}} \right)^{\lambda} = 1. \tag{16}$$

These assumptions also determine the equilibrium value of the political weights. We have

$$\gamma_S^*(\omega) = \frac{1}{W^{1s}} B^{1s}(\omega) + \frac{1}{W^{2s}} B^{2s}(\omega) = \frac{1}{W^s},$$

so that $\gamma_S^*(\omega)$ is now constant across types. Thus, for λ close to 1, the equilibrium tax policy maximizes an unweighted utilitarian welfare function.[25] Importantly, this finding does not depend on the within-base income distributions. E.g., utiliarianism prevails either if "the rich" overwhelmingly support party 1 and "the poor" overwhelmingly support party 2, or if the distribution across the parties' bases is even – all that matters is that the bases are of equal size.[26]

Taxes and asymmetric demobilization. In the following, we describe how shifts in the distribution of party preferences lead to departures from this symmetric benchmark for taxes and turnout. For ease of exposition, we focus on the case $\lambda = 1$ or, equivalently, on the case of linear voting costs. This enables us to highlight the implications of endogenous turnout for equilibrium taxes in the starkest possible way. We also impose the assumption that, all else equal, party 1 gets weakly more support from high-income voters and party 2 gets more support from low-income voters. We refer to party 1 as right-leaning and to party 2 as left-leaning. Formally, we assume that $B^s : \omega \mapsto B^s(\omega)$ is a non-decreasing function. Under these assumptions, higher turnout among "the rich" is equivalent to the turnout ratio σ^{1*}/σ^{2*} taking a value above 1. We also assume that $W^{1s} \geq W^{2s}$, so that, in equilibrium, the stakes of the supporters of the right-leaning party are at least as large as the stakes of the supporters of the left-leaning party.

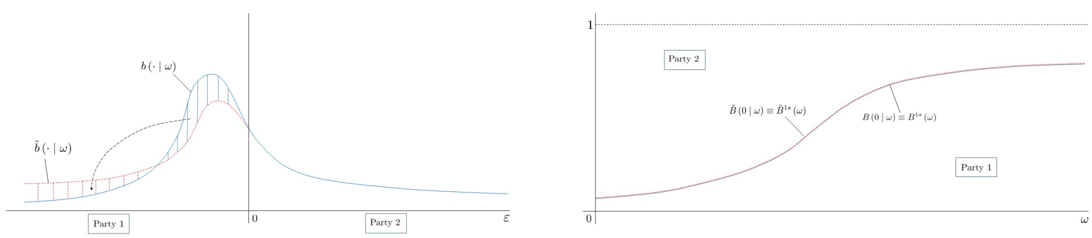
Consider a shift in idiosyncratic party preferences such that the ratio W^{1s}/W^{2s} increases. Recall that, as discussed in Section 1.2, W^{1s}/W^{2s} measures the intensity of political preferences of the supporters of party 1, relative to those of party 2. Thus, on average, the supporters of party 1 now feel more strongly about their party than do

[25] This observation extends to all possible values of λ if $\bar{q}(\omega)b^s(\omega)$ is also constant across types, implying that the inclination to swing from being a potential voter of party 1 to being a potential voter of party 2 does not depend on income.

[26] Using a framework with CRP schedules, Heathcote et al. (2020) argue that US tax policy has been close to maximizing an unweighted utilitarin welfare function between the early 1980s and the early 2010s. For this period, Bierbrauer et al. (2021) document six reforms of the US federals income tax, three of which involved higher taxes on "the rich" and where enacted by Democratic governments, the other three were enacted by Republican governments and involved lower taxes on "the rich". This observation lends some plausibility to the balancedness condition that is needed for utilitarianism to prevail.

Figure 1: Comparative statics

(a) Shift in the density $b(\varepsilon \mid \omega)$ for a fixed ω (b) Shift in the cdf $B(0 \mid \omega)$ as a function of ω



the supporters of party 2. Also assume that this change in preferences does not affect the size of the parties' bases $\mathbf{B}^{1s}$ and $\mathbf{B}^{2s}$, nor the within-base income distributions captured by $B^{1s} : \omega \mapsto B^{1s}(\omega)$. For instance, such a shift takes place if mass is shifted from values of ε that are negative but close to zero to values that are much smaller; see Figure 1.

Proposition 3 (Taxes and asymmetric demobilization)**.** *Suppose that* $\frac{W^{1s}}{W^{2s}} \geq 1$ *increases, keeping* $\mathbf{B}^{1s}$ *and* $B^{1s} : \omega \mapsto B^{1s}(\omega)$ *fixed. Then the tax system becomes more redistributive: the new equilibrium weighting function* γ_1^* *and the old weighting function* γ_0^* *satisfy (15). Moreover, the relative turnout ratio* σ^{1*}/σ^{2*} *and party 1's winning probability go up.*

The results of Proposition 3 follow from party 1's tradeoff between the returns from mobilizing its own base and the returns from demobilizing its competitor's base. These depend on the party's position in the electoral race: if its own supporters have, on average, stronger party biases than the opposition ($W^{1s} \geq W^{2s}$), and therefore have more at stake in the election, then by Proposition 1 party 1 has an edge in the race. By Proposition 2, this goes together with increased returns from demobilization. Since party 1 is the pro-market party and party 2 is the more interventionist party, this leads to a *more* redistributive tax policy in equilibrium. Thus, somewhat paradoxically, the supporters of the pro-market party are not rewarded for their enthusiasm by a more market-oriented policy. They are rewarded by a victory of the party that has the label "pro-market", and the price to be paid for this victory is a less market-oriented policy. The driving force of this argument is the logic of *asymmetric demobilization*: by advocating more redistributive policies, party 1 depresses the turnout of its opponents, whose attachment to their party is, in comparison, weak, disproportionately more than it reduces the turnout of its own supporters.[27] As a result, the *decisive voter* becomes poorer at the same time that the turnout of "the rich" rises relative to that of "the poor".

The previous thought experiment consisted of changing the ratio W^{1s}/W^{2s} while keeping the size of the parties' bases and the within-base income distributions fixed. The Online-Appendix contains further comparative statics results that reinforce the asymmetric demobilization logic. One of these results focuses on an increase in the size of the pro-market party's base, while keeping the within-base income distributions fixed (Section B.4.3). Another keeps the size of the parties bases fixed, but makes the pro-market party even more pro-market, i.e., its within-base income distribution shifts so that its share of rich supporters increases further relative to its share of poor supporters (Section B.4.4). In response to all of these changes, the equilibrium tax policy becomes more redistributive.[28]

2.3 Taxes, turnout, and inequality: a parametric example

We now explore the implications of our framework for the classic question of whether increases in economic inequality lead, via the political process, to more redistributive taxation.[29] Throughout, we assume that productivity types ω are lognormally distributed with parameters $(\mu_\omega, \sigma_\omega^2)$, and we use the variance σ_ω^2 as the measure of economic inequality.

To be specific, we employ the model of affine income taxation. We apply a monotone and concave transformation $U : u(\tau, \omega) \mapsto U(u(\tau, \omega))$ to the policy preferences described in equation (11): thus, a type ω-individual votes for party 1 if $U(u(\tau^1, \omega)) - U(u(\tau^1, \omega)) \geq \varepsilon$. Doing so ensures that the symmetric benchmark tax

[27]These observations are complemented by Proposition 6 in the Online-Appendix, which sheds light on what happens off-equilibrium if the leading, pro-market, party does *not* take recourse to an asymmetric demobilization strategy.

[28]These comparative statics results can also be used to think through the implications of systematic changes in party allegiances. For the US, Enke et al. (2021) document the following pattern: "*rich-but-socially-liberal voters (the educational elite) switched from Republicans to Democrats, while poor-but-socially-conservative voters (e.g., manufacturing workers) switched from Democrats to Republicans*". In our model, this can be represented by a fraction of the rich supporters of party 1 switching to the base of party 2, and a fraction of the poor supporters of party 2 switching to the base of party 1. Our comparative statics results in the Online-Appendix suggest that such a preference shift leads parties to propose less redistributive tax policies in equilibrium.

[29]The hypotheis that more inequality yields higher taxes is due to Meltzer and Richard (1981). Bénabou (2000), by contrast, derives a U-shaped relation between econmic inequality and the level of taxation.

rate is strictly positive, the empirically plausible case. The weighting function $\gamma^*(\omega)$ characterized in Proposition 2 is then replaced by $\gamma_U^*(\omega) = U'(u(\tau^*,\omega))\gamma^*(\omega)$, and the formula (11) characterizing the equilibrium tax rate is otherwise unchanged. For specificity, we let $U(u) = \ln u$.

As our analysis of Section 2.2 has shown, with endogenous turnout, what economic inequality implies for redistribution depends on whether the political competition is balanced – in which case the equilibrium tax policy is utilitarian – or tilted in favor of one party. The following assumptions enable us to distinguish between these different political scenarios in a tractable way. First, we suppose that idiosyncratic party preferences are, for each type ω, uniformly distributed:

$$B\left(u(p^1,\omega) - u(p^2,\omega) \mid \omega\right) = \alpha\left(\omega\right) + \beta\left(\omega\right)\left[u(p^1,\omega) - u(p^2,\omega)\right],$$

so that $B^s(\omega) = \alpha(\omega)$. Second, we assume that $\alpha\left(\omega\right) \in (0,1)$ is increasing and given by the c.d.f. of a lognormal distribution with parameters $(\mu_\alpha, \sigma_\alpha^2)$,

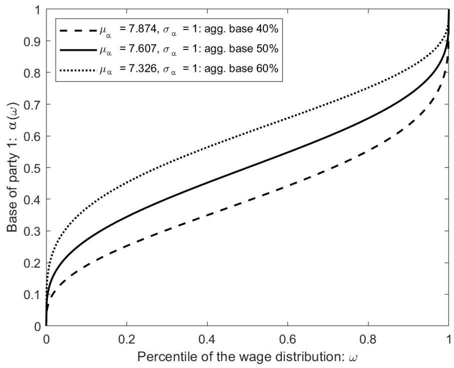
$$\alpha\left(\omega\right) = \frac{1}{\sigma_\alpha\sqrt{2\pi}}\int_0^\omega \frac{1}{w}\exp\left(-\frac{\left(\log w - \mu_\alpha\right)^2}{2\sigma_\alpha^2}\right)dw.$$

This functional form allows us to disentangle two different sources of political inequality: differences in the size of the parties' bases, on the one hand, and differences in the within-base income distributions, on the other. To see this, Figure 2 depicts the function $\alpha : \omega \mapsto \alpha(\omega)$ as the mean parameter μ_α varies (panel (a)), and as the variance parameter σ_α^2 varies (panel (b)). Lower values of μ_α shift up the support for party 1 at every income level, and hence raise its aggregate base B^{1s}, without affecting the variance of this support. On the other hand, σ_α^2 determines how polarized is the electorate for fixed aggregate bases: a low value of σ_α^2 implies that "the rich" overwhelmingly support party 1 while "the poor" overwhelmingly support party 2; a high value σ_α^2 implies that all income groups are split between the two groups of supporters in the same proportion as the whole population.

Panel (a) of Figure 3 shows the relationship between economic inequality and equilibrium tax rates for different sizes of party 1's base, keeping polarization σ_α constant. The asymmetric demobilization logic is visible here: the stronger the pro-market party, the larger is the equilibrium tax rate. An increase in economic inequality σ_ω^2 makes the covariance between weights $\gamma_U^*(\omega)$ and incomes more negative, and this

Figure 2: Base of party 1 along the income distribution

(a) Varying μ_α (aggregate base)



(b) Varying σ_α (polarization)

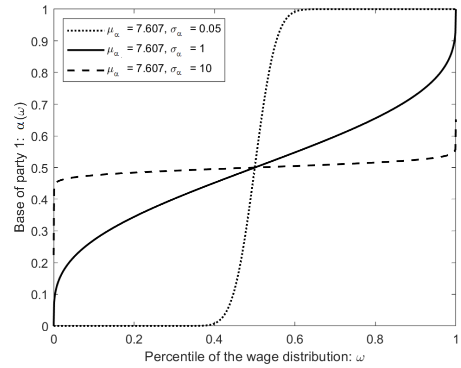
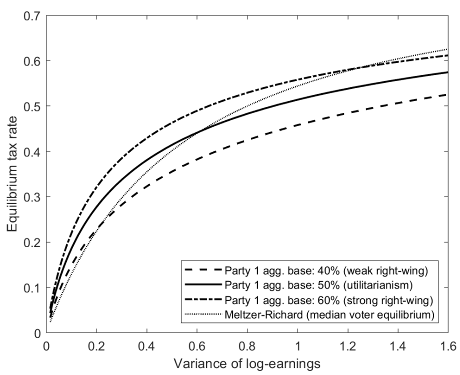
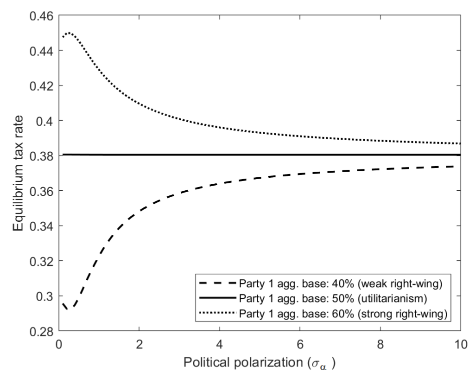


Figure 3: Comparative Statics

(a) Political and Economic Inequality



(b) Polarization



pushes the equilibrium tax rate up. This holds irrespective of which party is dominant. Panel (b) of Figure 3 keeps economic inequality ($\sigma_\omega^2 = 0.5$) constant, and shows the relationship between polarization σ_α^2 and equilibrium tax rates for different sizes of party 1's base. Polarization leads to more extreme equilibrium taxes – either above or below utilitarianism depending on whether the pro-market party is strong or weak.

The interaction between economic and political inequality can dampen, and potentially reverse, the increase in equilibrium taxes that results from an increase in economic inequality. Suppose that some middle-class people get poorer and some get richer, while everyone keeps their party preferences. Thus, economic inequality goes up while political inequality goes down. Given an initial situation in which

$B^s : \omega \mapsto B^s(\omega)$ is increasing, among the new rich are more people who support the left-leaning party, and among the new poor are more people who support the pro-market party. If the initial situation is one with a strong pro-market party, the previous analysis implies that this decrease in polarization pushes equilibrium taxes down, and hence counteracts the increase in taxes due to the higher level of economic inequality.

3 Existence of equilibrium

An example. We begin by imposing specific assumptions on the voting cost function and the distribution of idiosyncratic party preferences. Under these conditions the existence of a pure strategy equilibrium is ensured with linear income taxes, *CRP* schedules and, most notably, non-linear income taxes. Subsequently, we provide conditions for equilibrium existence that transcend this special case.

Assumption 2. *Suppose that voting costs are linear so that $\lambda = 1$. Also suppose that idiosyncratic party biases are, for each type ω, uniformly distributed:*

$$B\left(u(p^1,\omega) - u(p^2,\omega) \mid \omega\right) = \alpha\left(\omega\right) + \beta\left(\omega\right)\left[u(p^1,\omega) - u(p^2,\omega)\right].$$

Moreover, the distributions have a wide support and are close to symmetric. Formally,

i) *There exists $\bar{\beta}$ close to zero so that $0 < \beta(\omega) \leq \bar{\beta}$, for all ω.*

ii) *There exists $\bar{\alpha}$ close to zero so that, $\alpha\left(\omega\right) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$.*

Under Assumption 2, with a one-dimensional policy space, $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}$, and concave policy preferences $u(\cdot,\omega)$ for each type ω, it is straightforward to verify that $\Pi^1(\cdot, p^2) : p^1 \mapsto \Pi^1(p^1, p^2)$ is a globally concave function for every value of p^2, and that $\Pi^1(p^1, \cdot) : p^2 \mapsto \Pi^1(p^1, p^2)$ is globally convex for every value of p^1. Consequently, with concave policy preferences, the regularity conditions stated in Section 1.2 hold for linear income taxes and *CRP* schedules, ensuring the existence of a unique pure strategy equilibrium, that is, moreover, symmetric (see Proposition 2). The following Proposition extends this observation to fully non-linear income taxes.

Proposition 4. *Suppose that Assumption 2 holds. Let $\mathcal{P}$ be the space of non-negative, bounded and monotonic earnings functions. Suppose that the regularity conditions in* *part D.1 of the Online-Appendix are satisfied.*[30] *Also suppose that preferences are quasi-linear in consumption and that effort costs are iso-elastic. Then there is a unique pure strategy equilibrium. This equilibrium is symmetric.*

The proof involves the following steps. We first use functional derivatives to derive first-order conditions that characterize the parties' best responses. This gives us a candidate for a symmetric equilibrium in pure strategies. We then show that this equilibrium candidate also satisfies the second-order conditions. Thus, parties have no incentive to deviate locally. Finally, we invoke the contraction mapping theorem to show that, under Assumption 2, there is one and only one policy that is a best response to itself and that this policy coincides with the equilibrium candidate. Hence, there neither is an incentive to deviate to a policy that is not in a neighborhood of the equilibrium candidate.

For probabilistic voting models with exogenous turnout, it is known that pure strategy equilibria exist under regularity conditions on the distributions of idiosyncratic party preferences.[31] In particular, it is known that, with i.i.d. distributions of idiosyncratic party biases, probabilistic voting gives rise to an equilibrium outcome that maximizes a weighted utilitarian welfare function, see Banks and Duggan (2005). If those distributions are, moreover, uniform, there is a dominant strategy equilibrium in which the parties maximize an unweighted utilitarian welfare function.[32]

Proposition 4 shows that these findings extend to a framework with endogenous turnout. Our analysis, moreover, moves beyond the special case of i.i.d. and uniform distributions by means of a continuity argument: existence and uniqueness of equilibrium also holds for distributions that are sufficiently "close" to that benchmark. The assumption that the uniform distributions have a wide support implies that strategic substitutes and complements play a limited role. In the limit, i.e. for $\bar{\beta} = 0$, equilibria are in dominant strategies, and best responses no longer depend on the tax policy proposed by the other party. The assumption that the distributions are close to symmetric implies that the race between the two parties is close. Thus, in a

[30] These regularity conditions are familiar from the literature on optimal welfare-maximizing taxation. They ensure that it suffices to study a relaxed best response problem that does not explicitly impose a monotonicity constraint on earnings functions. If the regularity conditions are met, the solution to the relaxed problem is monotonic.

[31] See, for instance, the seminal paper by Lindbeck and Weibull (1987).

[32] This finding can be squared with a Mirrleesian model of taxation because a party's best response problem now coincides with a utilitarian problem of welfare-maximization; see e.g. Farhi et al. (2012) for an application of this insight.

neighborhood of such a symmetric, dominant strategy equilibrium we can be assured that an equilibrium exists, and is unique.

Affine and *CRP* tax schedules: pure strategies. With a one-dimensional policy space, a pure-strategy equilibrium can be shown to exist under conditions that are weaker than those invoked in Proposition 4.

Proposition 5. *Let $\mathcal{P}$ be a compact subset of $\mathbb{R}$. Suppose that $u : (p, \omega) \mapsto u(p, \omega)$ is, for all ω, a continuously differentiable function of p, and that every type ω has an ideal policy in the interior of $\mathcal{P}$. Assume moreover that preferences satisfy the single-crossing property, $u_{12}(p, \omega) \leq 0$ for all $p \in \mathcal{P}$ and $\omega \in \Omega$.*

1. *Suppose there are scalars a and $b > 0$, so that, for all $(p, p') \in \mathcal{P}^2$,*

$$\Pi^1(p, p') = a + b\,\Pi^2(p', p), \qquad \textit{where} \qquad \Pi^2(p', p) := 1 - \Pi^1(p', p). \tag{17}$$

   Then there is a symmetric equilibrium in pure strategies.

2. *Alternatively, suppose that utility functions are concave in p. Also suppose that for every p^2, there is at most one p^1 so that*

$$\Pi^1_1(p^1, p^2) = 0 \quad \textit{and} \quad \Pi^1_{11}(p^1, p^2) < 0\,, \tag{18}$$

   where Π^1_1 is the first and Π^1_{11} the second derivative of Π^1 with respect to p^1; and analogously for party 2. Then there is a unique equilibrium in pure strategies. This equilibrium is symmetric.

Proposition 5 provides different sufficient conditions for the existence of a symmetric pure-strategy equilibrium. Part 1. involves a condition of symmetry. To interpret condition (17), suppose first that $a = 0$ and $b = 1$. Then the condition becomes $\Pi^1(p, p') = \Pi^2(p', p)$; i.e., if the parties flip their policies, so do their winning probabilities. This condition holds, for instance, if all the distributions B in our model are symmetric. Condition (17) is a generalization of this case of perfect symmetry, allowing both for a fixed advantage ($a > 0$) or disadvantage ($a > 0$) of party 2 relative to party 1, as well as for the possibility that a platform change that increases the winning probability of party 1 would have less ($b < 1$) or more ($b > 1$) of an impact on party 2's winning probability.

Part 1. provides a condition for existence that is parsimonious in the sense that it does not involve any assumption on the curvature of the functions $\{B(\cdot \mid \omega) : \omega \in \Omega\}$ that describe the party preferences of different types, nor on the functions $\{u(\cdot, \omega) : \omega \in \Omega\}$ that describe the policy preferences of different types. The proof consists in showing that the best response function of party 1 has a fixed point by Brouwer's fixed point theorem, and then to show that, under condition (17), any such fixed point is a saddle point of Π^1. No curvature assumption is needed along the way.

Part 2. is parsimonious in a different way. It avoids any assumption of symmetry, but imposes an assumption on curvature. This assumption ensures that any policy that satisfies the first and the second-order condition of a best response problem is in fact the solution to this problem. Equipped with this regularity condition, we show that the best responses functions of parties 1 and 2 have identical fixed points and, exploiting the concavity of u, that there is only one such fixed point. As a Corollary we obtain the existence, uniqueness and symmetry of an equilibrium in pure strategies.

The content of the curvature assumption can be most easily demonstrated under the assumption of linear voting costs, or $\lambda = 1$, so that the objective of party 1 is to maximize $\Pi^1(p^1, p^2) = \frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}$. As we show in the Online-Appendix, (18) holds if, for all $(p^1, p^2) \in \mathcal{P}^2$

$$\frac{b(\Delta u(\cdot) \mid \omega)}{B(\Delta u(\cdot) \mid \omega)} \quad \leq \quad \mid \frac{u_{11}(p^1, \omega)}{u_1(p^1, \omega)} \mid, \tag{19}$$

whenever ω is such that $u_1(p^1, \omega) > 0$, and

$$\frac{b(\Delta u(\cdot) \mid \omega)}{1 - B(\Delta u(\cdot) \mid \omega)} \quad \leq \quad \mid \frac{u_{11}(p^1, \omega)}{u_1(p^1, \omega)} \mid, \tag{20}$$

whenever ω is such that $u_1(p^1, \omega) < 0$. We use $\Delta u(\cdot)$ as a shorthand for $u(p^1, \omega) - u(p^2, \omega)$. To interpret these conditions, consider a marginal increase in the policy p^1. For individuals who benefit from such a change, condition (19) relates the percentage change in political support for party 1 to the percentage change in marginal utility. For individuals who are harmed by the policy shift, condition (20) relates the percentage change in political support for party 2 to the percentage change in marginal utility. Both conditions require that the percentage change in political support must not be larger than the percentage change in individual welfare. Broadly, the effect that is driven by idiosyncratic party preferences must not outweigh the effect that is driven by policy preferences.

As a more specific example, suppose that idiosyncratic party biases are, for each type ω, uniformly distributed: $B(\Delta u(\cdot) \mid \omega) = \alpha(\omega) + \beta(\omega)\,\Delta u(\cdot)$. The wider the support of the uniform distribution, the lower is $\beta(\omega)$. Hence, if all distributions are "close to uniform over the reals", then the left hand sides of both (19) and (20) are "close to zero", with the implication that these inequalities hold.

The assumption of single-crossing preferences is indispensable for the existence of a pure strategy equilibrium. As is well-known, symmetric zero-sum games, such as "matching pennies", do not typically have equilibria in pure strategies. In our setting, the single-crossing property implies that equilibrium policies belong to an interior set of Pareto-efficient policies that coincides with the set of the different voter types' ideal polices.[33] This also implies that the equilibrium policy admits an interpretation as the ideal policy of a *decisive voter*.

Second, as an implication of the single-crossing property, the voters' policy preferences over the set of Pareto-effcient policies are single-peaked. Single-peakedness and single-crossing preferences are typically viewed as unrelated sufficient conditions that enable a proof of a median voter theorem in models without idiosyncratic party preferences. It is therefore interesting to note that, in our setting, the single-crossing property implies single-peakedness.

Non-linear income taxation: mixed strategies. The case of fully non-linear income tax schedules or, equivalently, of bounded and monotonic earnings functions, is a high-dimensional policy space. For such spaces, the existence of a pure-strategy equilibrium cannot generally be expected. We can, however, provide a generic existence proof for a mixed strategy equilibrium. *Suppose that $\mathcal{P}$ is the space of non-negative, bounded and monotonic functions in a compact space. Suppose that $p_a \to p_b$, in the sense of uniform convergence, implies that $u(p_a, \omega) \to u(p_b, \omega)$ for all $\omega \in \Omega$. Then there exists an equilibrium in mixed strategies.*

Our proof is based on Glicksberg's existence theorem for mixed-strategy equilibria of zero-sum games. The application of Glicksberg's theorem is not straightforward. The previous literature contains existence proofs for mixed strategy equilibria based on a multi-dimensional Euclidian policy spaces (see, e.g., Banks and Duggan (2005)) but not for the space of non-negative, bounded and monotonic functions. In the proof we verify that – with an appropriate notion of convergence applied to the space

[33] In games with strategic complementarities, a single-crossing property implies the existence of pure strategy equilibria, see e.g. Amir et al. (2008) and the references therein. Here this does not apply. In the game between the two parties such complementarities do not arise.

of non-negative, bounded and monotonic functions – the premises of Glicksberg's theorem hold.

As argued by Laslier (2000), mixed strategies need not literally be interpreted as randomization devices. During a campaign, politicians may talk differently to different audiences, and remain vague most of the time. For instance, they may say both "the hard-working middle-class people must not be left behind" and "we cannot afford to discourage the productive efforts of the most talented", etc. Presumably, they differ in emphasis or in how often they say one thing or the other depending on the audience they are addressing. Such a strategy can be interpreted as a mixed strategy.[34]

Concluding remarks

This paper presents an analysis of political competition with endogenous turnout. Two parties choose platforms to maximize their probability of winning an election and thereby take into account the implications of their platform choice for turnout. If a left-leaning party differentiates itself from a pro-market party and proposes an expansion of the welfare state or higher taxes on top incomes, this gives its supporters incentives to turn out to vote: after all, there is now a political alternative worth fighting for. However, the stakes are also higher for the supporters of the pro-market party. From their perspective, there is now an increased urgency to prevent the worst case, a victory of the socialists.

Our equilibrium analysis pins down the strengths of these forces when parties best reply to each other in an attempt to find the optimal trade-off between the turnout incentives for their own supporters and the supporters of the competing party. While both parties ultimately propose the same platform, there is an asymmetry in this trade-off when the race between the parties is unbalanced. Say that the party preferences are such that the pro-market party is the likely winner of the election. Then, in equilibrium, it assigns more weight to the demobilization of the left-leaning voters than to the mobilization of its own conservative base. The left-leaning party, by contrast, puts more weight on the mobilization of its own supporters. Thus, both parties arrive at the same conclusion, the equilibrium platform, but for different reasons.

[34] This also connects to a political science literature on the "blurring" of party positions, see Han (2020).

In part E of the Online-Appendix, we complement this equilibrium analysis with an analysis of the parties' best response problems. We hypothesize a situation in which a pro-market party is headed for reelection and, initially, puts insufficient weight on the demobilization of left-leaning voters. We show that an adoption of a more redistributive platform then implies that its winning probability goes up while overall turnout goes down. There is more abstention when the parties' proposals become more similar. Turnout of "the rich", however, goes down disproportionately less than turnout of the "the poor," so that this move pays off for the pro-market party. The Online-Appendix also contains a detailed case study of federal elections in Germany in the era of Angela Merkel. We show that the predictions of our model are in line with the consequences of Merkel's adoption of an asymmetric demobilization strategy in response to the experience of the 2005 campaign where she ran on a pro-market platform and almost lost an election that originally looked like a sure victory.

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Online Appendix for "Taxes and Turnout"

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A Proofs for Section 1

A.1 Derivation of equation (4)

The total number of votes for party 1 is a random variable equal to

$$\tilde{V}^1 = \mathbb{E}[\sigma^1 \tilde{q}^1(\omega) B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)] =: \sigma^1 \tilde{\mathbf{B}}^1(p^1,p^2). \tag{21}$$

Analogously, the total number of votes for party 2 equals

$$\tilde{V}^2 = \mathbb{E}[\sigma^2 \tilde{q}^2(\omega)(1 - B(u(p^1,\omega) - u(p^2,\omega) \mid \omega))] =: \sigma^2 \tilde{\mathbf{B}}^2(p^1,p^2). \tag{22}$$

Given two party proposals p^1 and p^2, and given the turnout for party 2, σ^2, the best response problem of the group-rule-utilitarian supporters of party 1 is to choose σ^1 so as to maximize the expected value of the following expression

$$\begin{aligned}
&\mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\} \mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^1,\omega)\right] \\
&+ \left(1 - \mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\}\right) \times \\
&\mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^2,\omega) + \textstyle\int_{-\infty}^{u(p^1,\omega)-u(p^2,\omega)} \varepsilon\, b(\varepsilon \mid \omega) d\varepsilon\right] \\
&-\; k(\sigma^1)\; \mathbb{E}[\tilde{q}^1(\omega) B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)].
\end{aligned}$$

In this expression, $\mathbb{I}$ is an indicator function and the product

$$\mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\} \mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^1,\omega)\right]$$

is utilitarian welfare realized by the supporters of party 1 in the event that their party wins. Analogously,

$$(1 - \mathbb{I}\{\cdot\})\,\mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^2,\omega) + \int_{-\infty}^{u(p^1,\omega)-u(p^2,\omega)} \varepsilon\, b(\varepsilon \mid \omega) d\varepsilon\right]$$

is utilitarian welfare realized by the supporters of party 1 in the event that party 2 wins, where the integral term in this expression is the sum of the gains (or losses) that the supporters of party 1 realize because of their idiosyncratic party preference.

Upon exploiting the linearity of the expectations operator and dropping terms that do not depend on σ^1, we can equivalently write this optimization problem as follows: choose $\sigma^1 \in [0,1]$ to maximize

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2)W^1(p^1,p^2) - k(\sigma^1)\mathbf{B}^1(p^1,p^2), \tag{23}$$

where $\pi^1(p^1,p^2,\sigma^1,\sigma^2)$ is the probability that $\tilde{V}^1 \geq \tilde{V}^2$, $W^1(p^1,p^2)$ is defined by (2) and captures the welfare gain that is realized by the supporters of party 1 if their party wins, and $\mathbf{B}^1(p^1,p^2)$ is defined by (1) and captures the expected value of the mass of the ethical supporters of party 1.

A.2 Proof of Proposition 1

Proof of Footnote 12. The arguments in the derivation of equation (5) imply that, for given p^1 and p^2, party 1's probability of winning the election is given by

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2) = F_\eta\left(\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right).$$

Given σ^2, the best response problem of party 1's ethical voters in (4) can therefore be written as follows: choose σ^1 to maximize

$$F_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right) W^1(p^1,p^2) - \kappa(\sigma^1)\,\mathbf{B}^1(p^1,p^2)$$

If the Inada conditions on the cost function hold, the derivative of the objective with respect to σ is strictly positive at $\sigma^1 = 0$ and strictly negative at $\sigma^1 = 1$. Thus, the best response is interior and characterized by a first-order condition. Given the concavity of F_η and the convexity of the cost function the solution is moreover unique. The same argument applies to the best response problem of party 2.

Equilibrium relative turnout. Using equations (21) and (22), the probability that party 1 wins the election is equal to the probability of the event

$$\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)} \geq \frac{\eta^2}{\eta^1}.$$

Denote by F_η the c.d.f. and by f_η the density of the random variable $\frac{\eta^2}{\eta^1}$. Thus,

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2) = F_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right). \tag{24}$$

We take the party platforms p^1 and p^2 as given and characterize equilibrium turnout. We say that the turnout game has an interior equilibrium if $0 < \sigma^{1*}(p^1,p^2) < 1$ and $0 < \sigma^{2*}(p^1,p^2) < 1$. An interior equilibrium is characterized by the first-order conditions

$$\pi^1_{\sigma^1}(\cdot)W^1(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^1\right)^{1/\lambda-1}\mathbf{B}^1(p^1,p^2) = 0, \tag{25}$$

and

$$-\pi^1_{\sigma^2}(\cdot)W^2(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^2\right)^{1/\lambda-1}\mathbf{B}^2(p^1,p^2) = 0. \tag{26}$$

Using equation (24), these first order conditions can also be written as

$$f_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\frac{1}{\sigma^1}W^1(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^1\right)^{1/\lambda-1}\mathbf{B}^1(p^1,p^2) = 0, \tag{27}$$

and

$$f_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\frac{1}{\sigma^2}W^2(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^2\right)^{1/\lambda-1}\mathbf{B}^2(p^1,p^2) = 0. \tag{28}$$

Equations (27) and (28) allow us to pin down the equilibrium value of relative turnout,

$$\frac{\sigma^{1*}(p^1,p^2)}{\sigma^{2*}(p^1,p^2)} = \left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^\lambda. \tag{29}$$

The left-hand side of this equation is a measure of party 1's turnout advantage: the larger σ^*_1/σ^*_2, the larger the number of ethical supporters who turn out to vote for party 1, relative to the number of supporters who turn out to vote for party 2. The right-hand side is a ratio of the welfare gains per capita, $W^j/\mathbf{B}^j$, that the supporters of both parties can realize in case of winning the election. Thus, according to equation (29), the relative turnout for party 1 is increasing in the relative amounts that its supporters and those of the competing party have at stake.

Derivation of equation (5). Under Assumption 1,

$$\tilde{\mathbf{B}}^1(p^1,p^2) = \eta^1\mathbf{B}^1(p^1,p^2) \quad \text{and} \quad \tilde{\mathbf{B}}^2(p^1,p^2) = \eta^2\mathbf{B}^2(p^1,p^2).$$

The probability that party 1 wins the election is therefore equal to the probability of the event

$$\sigma^1\eta^1\mathbf{B}^1(p^1,p^2) \geq \sigma^2\eta^2\mathbf{B}^2(p^1,p^2)$$

or, equivalently,

$$\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)} \geq \frac{\eta^2}{\eta^1}.$$

Let F_η be the c.d.f. of the random variable η^2/η^1. Then this probability can be written as

$$\Pi^1(p^1,p^2) = F_\eta\left(\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right).$$

Thus, the probability that party 1 wins the election is a non-decreasing function of

$$\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}.$$

Therefore, party 1's objective is to maximize this expression and party 2's objective is to minimize it.

Proof of Proposition 1. Party 1 seeks to maximize

$$\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}$$

and party 2 seeks so minimize this term. Using equation (29) to substitute for $\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}$ yields

$$\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}$$

as the objective. We may as well assume that party 1 seeks to maximize a monotone transformation of this expression, whereas party 2 seeks to minimize it. Using the logarithm function as the monotone transformation yields Proposition 1.

A.3 Proof of Proposition 2

We first prove Proposition 2 under the assumption that $\mathcal{P}$ is a one-dimensional policy space. We then generalize the argument to higher dimensional policy spaces. We also use the following shorthands $Q := \mathbb{E}[\bar{q}(\omega)]$, $H_B(x) = \ln(\frac{x}{Q-x})$, $H_S(x) = \ln x$, Finally, we denote the derivatives of the functions H_B and H_S by h_b and h_s, respectively.

A.3.1 One-dimensional policy space

Best responses. Suppose that $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}$. Fix p_2. The derivative of $\Pi^1(p^1,p^2)$ with respect to the first argument, henceforth denoted by Π^1_1, equals

$$\begin{aligned}\Pi^1_1(p^1,p^2) \;&=\; (1-\lambda)\, h_B(\mathbf{B^1}(p^1,p^2))\, \mathbb{E}[\bar{q}(\omega)\; b(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\; u_1(p^1,\omega)] \\ &\quad +\lambda\; h_S(W^1(p^1,p^2))\; \mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\; u_1(p^1,\omega)] \\ &\quad +\lambda\; h_S(W^2(p^1,p^2))\; \mathbb{E}[(1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega))\; u_1(p^1,\omega)]\;.\end{aligned}$$

After a rearrangement of terms and upon denoting

$$\gamma_B(p^1,p^2,\omega) = h_B(\mathbf{B^1}(p^1,p^2))\bar{q}(\omega)\; b(u(p^1,\omega)-u(p^2,\omega)\mid\omega)$$

and

$$\begin{aligned}\gamma_S(p^1,p^2,\omega) \;&=\; h_S(W^1(p^1,p^2))B(u(p^1,\omega)-u(p^2,\omega)\mid\omega) \\ &\quad +h_S(W^2(p^1,p^2))\,[1-B\,(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\;,\end{aligned}$$

we can also write this derivative as

$$\Pi^1_1(p^1,p^2) = \mathbb{E}\Big[\, \big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\, u_1(p^1,\omega)\Big]\ .$$

For later reference, recall that

$$\gamma_S(p^1,p^2,\omega) = \gamma^*_S(\omega) \quad \text{and} \quad \gamma_B(p^1,p^2,\omega) = \gamma^*_B(\omega)$$

whenever $p^1 = p^2$. By a symmetric argument, the derivative of $\Pi^1(p^1,p^2)$ with respect to the second argument p^2 equals

$$\Pi^1_2(p^1,p^2) = -\mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\, u_1(p^2,\omega)\right]\ .$$

Under the regularity assumptions made in the text, p^1 is a best response to p^2 if and only if

$$\Pi^1_1(p^1,p^2) = 0 \quad \Leftrightarrow \quad \mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\, u_1(p^1,\omega)\right] = 0\ . \tag{30}$$

Likewise, p^2 is a best response to p^1 if and only if

$$\Pi^1_2(p^1,p^2) = 0 \quad \Leftrightarrow \quad \mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\, u_1(p^2,\omega)\right] = 0\ . \tag{31}$$

Existence of a symmetric equilibrium. Consider the policy p^* which solves

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma^*_B(\omega) + \lambda\,\gamma^*_S(\omega)]\}\, u_1(p^1,\omega)\right] = 0\ .$$

This policy maximizes

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma^*_B(\omega) + \lambda\,\gamma^*_S(\omega)\}\, u(p,\omega)\right]$$

over the set $\mathcal{P}$. Moreover, the pair of policies $(p^1,p^2) = (p^*,p^*)$ satisfies the first order conditions of both parties' best response problems in (30) and (31), respectively, and is hence an equilibrium.

Uniqueness. It remains to be shown that there is no other equilibrium. Suppose, to the contrary, that there is an equilibrium (p^1,p^2) with $p^1 \neq p^*$ or $p^2 \neq p^*$. In the following, we assume without loss of generality that $p^1 \neq p^*$. Since the game under study is zero-sum, this implies that also (p^1, p^*) is a Nash equilibria, see e.g. Osborne and Rubinstein (1994). This contradicts the assumption that party 1 has a unique best response to any policy $p^2 \in \mathcal{P}$. Thus, the assumption that there is an alternative equilibrium leads to a contradiction and must be false.

A.3.2 Multi-dimensional policy space

Suppose that $\mathcal{P}$ is a compact set. Let p^1 be an interior policy and let $h \in \mathcal{P}$ be a conceivable direction in which party 1 can deviate from p^1. We assume that such a deviation takes the form

$$p^1 + \mu\, h\ ,$$

where μ is a non-negative scalar that measures the size of the deviation from p^1. We denote by

$$\delta u(p^1, h, \omega)$$

the (functional) derivative of $u(p^1, \omega)$ in direction h at p^1. Equipped with this notation, we can now generalize the arguments for the one-dimensional policy space in a straightforward way.

Best responses. Given p^2, a best response for party 1 is a policy so that, for any admissible direction h,

$$\mathbb{E}\left[\left\{(1-\lambda)\,\gamma_B(p^1, p^2, \omega) + \lambda\,\gamma_S(p^1, p^2, \omega)\right\}\delta u(p^1, h, \omega)\right] = 0\ . \tag{32}$$

Likewise, p^2 is a best response to p^1 if and only if, for any admissible direction h,

$$\mathbb{E}\left[\left\{(1-\lambda)\,\gamma_B(p^1, p^2, \omega) + \lambda\,\gamma_S(p^1, p^2, \omega)\right\}\delta u(p^2, h, \omega)\right] = 0\ . \tag{33}$$

Existence of a symmetric equilibrium. Consider the policy p^* which solves, for any admissible direction h,

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)\}\,\delta u(p^*, h, \omega)\right] = 0\ .$$

This policy maximizes

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)\}\,u(p, \omega)\right]$$

over the set $\mathcal{P}$. Moreover, the pair of policies $(p^1, p^2) = (p^*, p^*)$ satisfies the first order conditions of both parties' best response problems in (32) and (33), respectively, and is hence an equilibrium.

Uniqueness. Uniqueness follows from the same argument as above.

B Proofs for Section 2

B.1 Affine income taxes

Concave policy preferences. Consider affine income taxation with quasi-linear in consumption utility and isoelastic effort costs. Then

$$y^*(\tau, \omega) = (1-\tau)^{\varepsilon}\, \omega^{1+\varepsilon}$$

We can solve for tax revenue as a function of τ. This yields

$$r(\tau) = \tau(1-\tau)^{\varepsilon}\, \mathbb{E}\left[\omega^{1+\varepsilon}\right] \ ,$$

i.e.,

$$r'(\tau) = \left(1 - \frac{\tau}{1-\tau}\, \varepsilon\right) (1-\tau)^{\varepsilon}\, \mathbb{E}\left[\omega^{1+\varepsilon}\right]$$

and

$$r''(\tau) = -\varepsilon\, (1-\tau)^{\varepsilon-1} \left(2 + \frac{\tau}{1-\tau}(\varepsilon - 1)\right) \mathbb{E}\left[\omega^{1+\varepsilon}\right] \ .$$

Policy preferences are captured by the indirect utility function $u(\tau, \omega)$. By the envelope theorem,

$$u_1(\tau, \omega) = r'(\tau) - y^*(\tau, \omega)$$

The ideal policy for type ω, $\tau^*(\omega)$ solves

$$r'(\tau) - y^*(\tau, \omega) = 0\ ,$$

or, equivalently,

$$\frac{\tau^*(\omega)}{1-\tau^*(\omega)} = \frac{1}{\varepsilon}\left(1 - \frac{\omega^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]}\right) \ .$$

Obviously, $\tau^* : \omega \mapsto \tau^*(\omega)$ is a strictly decreasing, continuously differentiable function. The second derivative of u with respect to τ is given by

$$u_{11}(\tau, \omega) = r''(\tau) - y_1^*(\tau, \omega) \ .$$

Note that, since $y_1^*(\tau, \omega) < 0$, concavity of α is not enough to ensure that policy preferences are concave.

Lemma 1. *Consider the affine income taxation setting with quasi-linear in consumption utility and isoelastic effort costs. Consider the policy space* $\mathcal{T} = [\tau^*(\underline{\omega}), \tau^*(\overline{\omega})]$. *Then* $\varepsilon \leq \frac{1}{2}$ *implies that, for all* $\tau \in \mathcal{T}$ *and all* $\omega \in [\underline{\omega}, \overline{\omega}]$, $u_{11}(\tau, \omega) \leq 0$.

Proof. Straightforward computations yield

$$u_{11}(\tau, \omega) = -\varepsilon \, (1 - \tau)^{\varepsilon - 1} \left(\left(2 + \frac{\tau}{1 - \tau}(\varepsilon - 1) \right) \mathbb{E}\left[\omega^{1+\varepsilon}\right] - \omega^{1+\varepsilon} \right) .$$

We seek to show that, for all $\tau \in \mathcal{T}$ and all $\omega \in [\underline{\omega}, \overline{\omega}]$,

$$2 + \frac{\tau}{1 - \tau}(\varepsilon - 1) \geq \frac{\omega^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]} \ .$$

A (necessary and) sufficient condition is that

$$2 + \frac{\tau}{1 - \tau}(\varepsilon - 1) \geq \frac{\bar{\omega}^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]} \ ,$$

or using the first order condition characterizing $\tau^*(\overline{\omega})$,

$$2 + \frac{\tau}{1 - \tau}(\varepsilon - 1) \geq 1 - \frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})}\varepsilon \ .$$

Since $\tau^* : \omega \mapsto \tau^*(\omega)$ is a strictly decreasing, a (necessary and) sufficient condition is that

$$2 + \frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})}(\varepsilon - 1) \geq 1 - \frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})}\varepsilon \ .$$

Equivalently,

$$\frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})} \ (2\varepsilon - 1) \geq -1 \ .$$

Since

$$\frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})} < 0 \ ,$$

this inequality holds if $2\varepsilon - 1 \leq 0$, or, equivalently, if $\varepsilon \leq \frac{1}{2}$. □

Derivation of equation (11). It follows from Proposition 2 and equation (10) that τ^* maximizes

$$\mathbb{E}[\gamma^*(\omega)]r(\tau) + \mathbb{E}\left[\gamma^*(\omega)\left((1-\tau)y^*(\tau,\omega) - k(y^*(\tau,\omega))\right)\right] .$$

Using the envelope theorem, the first order condition can be written as

$$r'(\tau) - \mathbb{E}\left[\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}y^*(\tau,\omega)\right] = 0 ,$$

where

$$r'(\tau) = \mathbb{E}\left[y^*(\tau,\omega)\right] + \tau\mathbb{E}\left[y_1^*(\tau,\omega)\right] .$$

Rearranging terms yields

$$\begin{aligned} \tau\mathbb{E}\left[\frac{y_1^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right] &= \mathbb{E}\left[\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]} - 1\right)\frac{y^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right] \\ &= Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \frac{y^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right) . \end{aligned} \tag{34}$$

With isoelastic effort costs and quasi-linearity in consumption, the first-order condition of individual utility maximization, $1-\tau = k_1(y,\omega)$, yields $y^*(\tau,\omega) = (1-\tau)^e\omega^{1+e}$ and $y_1^*(\tau,\omega) = -e\frac{1}{1-\tau}y^*(\tau,\omega)$. Substituting these expressions into (34) yields (11).

B.2 *CRP* taxes

Concave policy preferences. Policy preferences captured by the indirect utility function

$$u(\tau,\omega) = \ln r(\tau) + (1-\tau)\ln y^*(\tau,\omega) - k(y^*(\tau,\omega),\omega) .$$

By the envelope theorem,

$$u_1(\tau,\omega) = \frac{r'(\tau)}{r(\tau)} - \ln y^*(\tau,\omega) .$$

With isoleastic effort costs, straightforward computations yield

$$u_1(\tau,\omega) = -\frac{\tau}{1-\tau}\frac{1}{1+\frac{1}{\varepsilon}} + \ln\left(\mathbb{E}\left[(\ln\omega)\omega^{1-\tau}\right]\right) - \ln\omega .$$

and

$$u_{11}(\tau,\omega) = -\frac{1}{1+\frac{1}{\varepsilon}}(1-\tau)^{-2} - \mathbb{E}[(\ln\omega)^2\,\omega^{1-\tau}]\ .$$

Clearly, for $\tau \in (0,1)$, $u_{11}(\tau,\omega) < 0$, for all ω.

Derivation of equation (13).

Individual utility-maximization. Under a CRP schedule, an individual with earnings y has a consumption level of $c = ry^{1-\tau}$. With log consumption utility and iso-elastic effort costs, an individual of type ω solves the following utility-maximization problem

$$\max_{y} \quad \ln r + (1-\tau)\ln y - \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}\ .$$

Utility-maximizing earnings are hence given by $y^*(\tau,\omega) = (1-\tau)^{\frac{e}{1+e}}\omega$.

Tax revenue. We use the government budget constraint, $\mathbb{E}[T(y^*(\tau,\omega))] = 0$, to solve for r as a function of τ, which yields $r(\tau) = \frac{\mathbb{E}[y^*(\tau,\omega)]}{\mathbb{E}[y^*(\tau,\omega)^{1-\tau}]}$, or, equivalently,

$$r(\tau) = (1-\tau)^{\tau\frac{e}{1+e}}\ \frac{\mathbb{E}[\omega]}{\mathbb{E}[\omega^{1-\tau}]}$$

Policy preferences. Policy preferences are therefore captured by the indirect utility function $u(\tau,\omega) = \ln r(\tau) + (1-\tau)\ln y^*(\tau,\omega) - \frac{1}{1+1/e}\left(\frac{y^*(\tau,\omega)}{\omega}\right)^{1+1/e}$, or, equivalently,

$$u(\tau,\omega) = \frac{e}{1+e}\ln(1-\tau) + \ln\left(\frac{\mathbb{E}[\omega]}{\mathbb{E}[\omega^{1-\tau}]}\right) + (1-\tau)\ln\omega - \frac{e}{1+e}(1-\tau)\ . \qquad (35)$$

The assumption that $\ln\omega$ is normally distributed with mean μ_ω and variance σ^2_ω, can be shown to imply that

$$\mathbb{E}[\omega] = \exp\left(\mu_\omega + \frac{1}{2}\sigma^2_\omega\right) \quad \text{and} \quad \mathbb{E}[\omega^{1-\tau}] = \exp\left((1-\tau)\mu_\omega + \frac{1}{2}(1-\tau)^2\sigma^2_\omega\right)\ .$$

Thus,

$$\begin{aligned} u(\tau,\omega) &= \tfrac{e}{1+e}\ln(1-\tau) - \tfrac{e}{1+e}(1-\tau) \\ &\quad + \Big(1-(1-\tau)\Big)\mu_\omega + \tfrac{1}{2}\Big(1-(1-\tau)^2\Big)\sigma_\omega^2 \\ &\quad + (1-\tau)\ln\omega \,. \end{aligned} \tag{36}$$

Equilibrium policy. By Proposition 2, the political equilibrium tax policy maximizes $\mathbb{E}[\gamma^*(\omega)u(\tau,\omega)]$, or, equivalently,

$$\begin{aligned} \mathbb{E}[\bar{\gamma}^*(\omega)u(\tau,\omega)] &= \tfrac{e}{1+e}\ln(1-\tau) - \tfrac{e}{1+e}(1-\tau) \\ &\quad + \Big(1-(1-\tau)\Big)\mu_\omega + \tfrac{1}{2}\Big(1-(1-\tau)^2\Big)\sigma_\omega^2 \\ &\quad + (1-\tau)\mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] \,, \end{aligned}$$

where $\bar{\gamma}^*(\omega) = \frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}$. It is convenient to think of this objective as a function of $(1-\tau)$ rather than τ. The first order condition characterizing the equilibrium value of $1-\tau$ is

$$\frac{e}{1+e}\frac{\tau}{1-\tau} - \mu_\omega - (1-\tau)\sigma_\omega^2 + \mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] = 0 \,.$$

Rewriting this equation, using that $\mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] - \mu_\omega = Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \ln\omega\right)$, yields equation (13) in the main text.

B.3 Non-linear income taxes

Suppose that the preferences of a type ω individual over (c,y)-pairs are represented by a quasi-linear in consumption utility function $c - k(y,\omega)$, with $k_1 > 0$, $k_{11} > 0$, $k_2 < 0$ and $k_{12} < 0$.

In the following we sketch the argument for why any tax system T can be represented by a non-decreasing earnings function $\mathbf{y} : \Omega \to \mathbb{R}_+$. Specifically, by the taxation principle, see e.g. Hammond (1979); Guesnerie (1995), an allocation $(\mathbf{c},\mathbf{y})$ consisting of a consumption schedule $\mathbf{c} : \Omega \to \mathbb{R}_+$ and an earnings schedule $\mathbf{y} : \Omega \to \mathbb{R}_+$ can be induced by an income tax if and only if it satisfies the resource constraint,

$$\mathbb{E}[\mathbf{y}(\omega)] \geq \mathbb{E}[\mathbf{c}(\omega)] \tag{37}$$

and incentive compatibility constraints: for all ω and ω',

$$u(\omega) \geq \mathbf{c}(\omega') - k(\mathbf{y}(\omega'), \omega) \ , \tag{38}$$

where

$$u(\omega) := \mathbf{c}(\omega) - k(\mathbf{y}(\omega), \omega) \tag{39}$$

gives the utility that a type ω individual realizes under allocation $(\mathbf{c}, \mathbf{y})$.

It is also well-known how to obtain a characterization of incentive-compatible allocations in models with quasilinear preferences, see e.g. Myerson (1981). The utility realized by any one type-ω individual can be written as a sum of two terms, the minimal level of utility that is realized by the "poorest" type and the extra utility realized by higher types. More formally, an application of the envelope theorem makes it possible to show that incentive compatibility holds if and only if two conditions are satisfied. First, for all ω,

$$u(\omega) = \underline{u} + \rho(\mathbf{y}, \omega) \quad \text{where} \quad \rho(\mathbf{y}, \omega) = -\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \ dz \ , \tag{40}$$

and $\underline{u} := u(\underline{\omega})$ is a shorthand for the lowest type's utility and $-\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \ dz$ is the *information rent* realized by a higher type $\omega > \underline{\omega}$ in the presence of incentive compatibility constraints.[35] Second, $\mathbf{y}$ is a non-decreasing function, i.e., individuals with higher productive abilities must not earn less than individuals with lower productive abilities.

We can use these insights to derive a representation of preferences over tax polices in a reduced form that only depends on the income function $\mathbf{y}$ and no longer involves a reference to the consumption function $\mathbf{c}$. This will enable us to represent a tax policy design problem as a problem that no longer involve resource and incentive constraints. Suppose that $(\mathbf{c}, \mathbf{y})$ is incentive compatible, then using (39), (40) and an integration by parts we obtain

$$\mathbb{E}[c(\omega)] = \underline{u} + \mathbb{E}\left[k(\mathbf{y}(\omega), \omega) - \frac{1 - F(\omega)}{f(\omega)} \ k_2(\mathbf{y}(\omega), \omega)\right] \ .$$

Plugging this expression into the public sector budget constraint $\mathbb{E}[\mathbf{y}(\omega)] - \mathbb{E}[\mathbf{c}(\omega)] = 0$

[35] This terminology reflects that private information on types is the impediment to first-best redistribution.

yields an expression for $\underline{u}$; it is equal to the *virtual surplus* that is associated with an earnings function y:

$$\underline{u} := s_v(\mathbf{y}) := \mathbb{E}\left[y(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] . \tag{41}$$

The virtual surplus is a surplus measure that takes account of the information rents that tax-payers realize and which reduces what is available for the lowest type. To arrive at the virtual surplus, the (non-virtual) surplus of aggregate output over costs of effort

$$s(\mathbf{y}) := \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega)\right]$$

is reduced by the aggregate information rent

$$-\mathbb{E}\left[\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z)\; dz\right] = -\mathbb{E}\left[\frac{1-F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] ,$$

where the equality follows from an integration by parts. Thus,

$$\underline{u} = s_v(\mathbf{y}) = \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] . \tag{42}$$

Indirect utility induced by an incentive compatible allocation can now be written as a sum of virtual surplus and information rents

$$u(\omega) := s_v(\mathbf{y}) + \rho(\mathbf{y}, \omega) \tag{43}$$

With this characterization, the utility realized by a type ω individual depends on the whole earnings schedule $\mathbf{y} : \Omega \to \mathbb{R}_+$ but no longer on the consumption schedule $\mathbf{c} : \Omega \to \mathbb{R}_+$.

For the representation of policy preferences, we make the dependence of u on the earnings function explicit and write $u(\mathbf{y}, \omega)$ rather than simply $u(\omega)$.

To summarize, for non-linear income taxation, the policy space is the set of all non-decreasing earnings functions. Any such function generates a payoff profile that is characterized by equations (40) and (42).

Derivation of equation (14). In part D.3 of the Online-Appendix we show that, in a symmetric pure strategy equilibrium,

$$\frac{T'(\mathbf{y}^*(\omega))}{1-T'(\mathbf{y}^*(\omega))} \quad = \quad -\frac{1-F(\omega)}{f(\omega)}\left(1-\Gamma^*(\omega)\right)\,\frac{k_{21}(\mathbf{y}^*(\omega),\omega))}{k_1(\mathbf{y}^*(\omega),\omega)}\ ,$$

see equation (71). With an isoelastic effort cost function we can substitute $-\left(1+\frac{1}{e}\right)\frac{1}{\omega}$ for $\frac{k_{21}(\mathbf{y}^*(\omega),\omega))}{k_1(\mathbf{y}^*(\omega),\omega)}$, which yields (14).

B.4 Comparative statics

B.4.1 Using political equilibrium weights to order equilibrium tax systems

Consider two specifications of the model's primitives giving rise to two different weighting functions that are respectively denoted by $\gamma_0^* : \omega \mapsto \gamma_0^*(\omega)$ and $\gamma_1^* : \omega \mapsto \gamma_1^*(\omega)$. Suppose that there is a decreasing function $\delta : \omega \mapsto \delta(\omega)$ with $\mathbb{E}[\delta(\omega)] = 0$ so that

$$\frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]} = \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]} + \delta(\omega)\ . \tag{44}$$

For ease of notation, let $\bar{\gamma}_1^*(\omega) := \frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]}$ and $\bar{\gamma}_0^*(\omega) := \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]}$, so that $\mathbb{E}[\bar{\gamma}_0^*(\omega)] = \mathbb{E}[\bar{\gamma}_1^*(\omega)] = 1$. In the following we show that, for all models of redistributive taxation that we consider, the equilibrium tax system associated with γ_1^* is more redistributive than the one associated with γ_0^*. First note that, for any increasing function $z(\cdot)$, we have

$$\mathbb{E}_\omega[\bar{\gamma}_1^*(\omega)\, z(\omega)] = \mathbb{E}_\omega[\bar{\gamma}_0^*(\omega)\, z(\omega)] + \mathbb{E}_\omega\left[\delta(\omega)\, z(\omega)\right],$$

where $\mathbb{E}_\omega\left[\delta(\omega)\, z(\omega)\right] = Cov\left(\delta(\omega), z(\omega)\right) < 0$. Therefore, we obtain

$$\mathbb{E}[\bar{\gamma}_1^*(\omega)\, z(\omega)] < \mathbb{E}[\bar{\gamma}_0^*(\omega)\, z(\omega)].$$

For $z(\omega) = \omega^{1+e}$, this inequality implies that

$$Cov(\bar{\gamma}_1^*(\omega), \omega^{1+e}) < Cov(\bar{\gamma}_0^*(\omega), \omega^{1+e})\ .$$

Thus, the equilibrium marginal tax rate in the affine taxation setting satisfies $\tau_1^* > \tau_0^*$. Analogously, applying this inequality to the function $z(\omega) = \ln \omega$ implies

$$Cov(\bar{\gamma}_1^*(\omega), \ln \omega) < Cov(\bar{\gamma}_0^*(\omega), \ln \omega) .$$

Thus, the equilibrium rate of progressivity in the CRP setting satisfies $\tau_1^* > \tau_0^*$. Finally, for unrestricted non-linear taxation, define the functions

$$\Gamma_0^* : \omega \mapsto \Gamma_0^*(\omega) = \mathbb{E}\left[\bar{\gamma}_0^*(s) \mid s \geq \omega\right] ,$$

and

$$\Gamma_1^* : \omega \mapsto \Gamma_1^*(\omega) = \mathbb{E}\left[\bar{\gamma}_1^*(s) \mid s \geq \omega\right] ,$$

and note that

$$\Gamma_0^*(\underline{\omega}) = \Gamma_1^*(\underline{\omega}) = 1 .$$

Moreover, for any ω,

$$\Gamma_1^*(\omega) = \Gamma_0^*(\omega) + \Delta(\omega) , \quad \text{where} \quad \Delta(\omega) := \mathbb{E}\left[\delta(s) \mid s \geq \omega\right] .$$

Note that $\mathbb{E}[\delta(\omega)] = 0$ implies that $\Delta(\underline{\omega}) = 0$, so that, since δ is decreasing, $\Delta(\omega) < 0$, for all $\omega > \underline{\omega}$. Thus,

$$\Gamma_1^*(\omega) < \Gamma_0^*(\omega) ,$$

for all $\omega > \underline{\omega}$. Equation (14) therefore implies that marginal tax rates for all types $\omega > \underline{\omega}$ are higher when political equilibrium weights are given by $\gamma_1^* : \omega \mapsto \gamma_1^*(\omega)$, as compared to the case where they are given by $\gamma_0^* : \omega \mapsto \gamma_0^*(\omega)$.

B.4.2 Increasing W^{1s}/W^{2s}: proof of Proposition 3

Consider a shift in idiosyncratic party preferences so that ratio $\frac{W^{1s}}{W^{2s}}$ increases from an initial value $a_0 \geq 1$ to a new value a_1. Also assume that this change does not affect the size of the parties' bases, $\mathbf{B}^{1s}$ and $\mathbf{B}^{2s}$, nor the within-base income distributions captured by $B^{1s} : \omega \mapsto B^{1s}(\omega)$.

The implications for relative turnout follows from equation (29) and Proposition 2. Together with the assumption of linear voting costs, $\lambda = 1$, they imply that

$$\frac{\sigma^{1*}}{\sigma^{2*}} = \frac{W^{1s} / \mathbf{B}^{1s}}{W^{2s} / \mathbf{B}^{2s}} \tag{45}$$

Thus, $a_1 > a_0$ implies that $\left(\frac{\sigma^{1*}}{\sigma^{2*}}\right)_1 > \left(\frac{\sigma^{1*}}{\sigma^{2*}}\right)_0$.

To see that the equilibrium tax system becomes more redistributive, note that

$$\begin{aligned} \bar{\gamma}_0^*(\omega) &:= \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]} \\ &= \frac{B^{1s}(\omega)+a_0(1-B^{1s}(\omega))}{E[B^{1s}(\omega)+a_0(1-B^{1s}(\omega))]} \\ &= \frac{a_0-(a_0-1)B^{1s}(\omega)}{a_0-(a_0-1)\mathbb{E}_\omega[B^{1s}(\omega)]} \ , \end{aligned}$$

and that

$$\begin{aligned} \bar{\gamma}_1^*(\omega) &:= \frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]} \\ &= \frac{a_1-(a_1-1)B^{1s}(\omega)}{a_1-(a_1-1)\mathbb{E}_\omega[B^{1s}(\omega)]} \ . \end{aligned}$$

With $a_1 > a_0 \geq 1$ and $B^{1s} : \omega \mapsto B^{1s}(\omega)$ non-decreasing, both $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$ are non-increasing functions. Moreover, $\bar{\gamma}_0^*(\underline{\omega}) = \bar{\gamma}_1^*(\underline{\omega}) = 1$, and, since $a_1 > a_1$,

$$| \frac{\partial}{\partial \omega} \bar{\gamma}_1^*(\omega) | \quad > \quad | \frac{\partial}{\partial \omega} \bar{\gamma}_0^*(\omega) | \ ,$$

for all $\omega > \underline{\omega}$. Hence,

$$\bar{\gamma}_1^*(\omega) \quad < \quad \bar{\gamma}_0^*(\omega) \ ,$$

for all $\omega > \underline{\omega}$.

These observations imply that there exists a decreasing function $\delta(\omega)$ with mean $\mathbb{E}[\delta(\omega)] = 0$ satisfying (15). Thus, the tax system associated with weighting function γ_1^* is more redistributive than the one associated with weighting function γ_0^*.

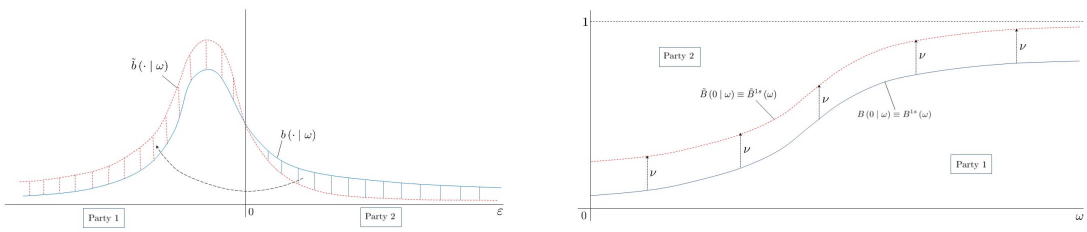
B.4.3 Increasing the base of party 1

Suppose that the base of party 1 rises uniformly: there is $\nu > 0$ such that, for all $\omega \in \Omega$, $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu$, where the functions B_0^{1s} and B_1^{1s} characterize party 1's base before and after the preference shift. The fact that the shift is uniform implies that it does not affect the density functions that describe the distribution of idiosyncratic party biases, see Figure 4, panel (b). We will now show that, in response to such a shift, the equilibrium tax schedule becomes more redistributive.

First note that this shift leads, mechanically, to an increase of the ratio $\frac{W^{1s}}{W^{2s}}$. The sum of the stakes of party 1's supporters goes up as more supporters are added. For

Figure 4: Comparative statics: uniform rise in political support

(a) Shift in the density $b(\varepsilon \mid \omega)$ for a fixed ω (b) Shift in the cdf $B(0 \mid \omega)$ as a function of ω



the analogous reason, the sum of the stakes of the supporters of party 2 goes down. As shown in the previous section B.4.2, this effect in isolation makes the equilibrium tax schedule more redistributive.

For the thought experiment considered here, the proof in section B.4.2 has to be adapted, though, to accommodate the change from B_0^{1s} to B_1^{1s}. This adjustment is straightforward, however, because a uniform shift is without consequence for the slope of the weighting functions $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$. Thus, the adaptation is line-by-line. We therefore omit the details.

B.4.4 Making party 1 more pro-market

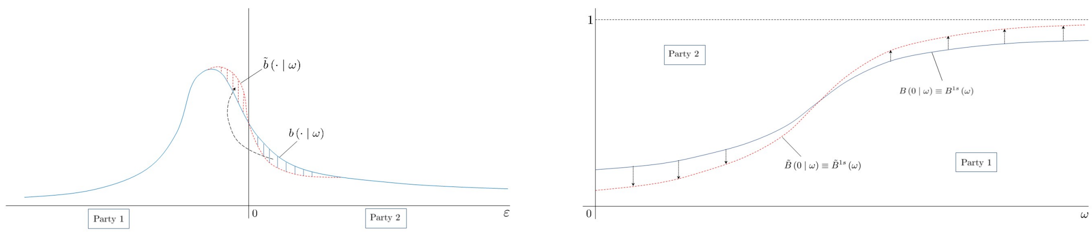
Consider a marginal shift in political biases such that $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu\beta(\omega)$, where $\beta(\cdot)$ is an increasing function with mean $\mathbb{E}[\beta(\omega)] = 0$. We, moreover, assume that the shift is concentrated on swing voters, i.e., on voters with party preferences ε close to zero.

To explain the nature of the thought experiment, fix ω so that $\beta(\omega) > 0$. We let voters with initial party preferences $\varepsilon_0 \in [0, \nu \cdot \bar{\varepsilon}(\omega)]$ in favor of party 2, swing to preferences $\varepsilon_1 \in [\nu \cdot \underline{\varepsilon}(\omega), 0]$ that favor party 1. Figure 5 provides an illustration. Panel (a) focuses on a high level of income ω: within this income group, party 1 is dominant, both before and (even more so) after the political preference shift. Panel (b) shows the resulting shift in party 1's base as a function of ω: both parties become stronger in the income groups where they were already strong.

In the following, to evaluate the consequences of such a shift, we look into the marginal changes of endogenous variables as $\nu \to 0$. Ultimately, we show that the equilibrium tax system becomes, at the margin, more redistributive. To this end, we

Figure 5: Comparative statics: rise in polarization

(a) Shift in the density $b(\varepsilon \mid \omega)$ for a large ω (b) Shift in the cdf $B(0 \mid \omega)$ as a function of ω



first show that, at $\nu = 0$, the marginal effect on the size of the parties' bases and the relative stakes of their supporters vanishes.

To see this, fix some ω so that $\beta(\omega) > 0$. The stakes of the supporters of party 2 go down by $\int_0^{\nu \cdot \bar{\varepsilon}(\omega)} \varepsilon b(\varepsilon \mid \omega)\, d\varepsilon$, an expression that is bounded from above by

$$\nu \cdot \bar{\varepsilon}(\omega) \int_0^{\nu \cdot \bar{\varepsilon}(\omega)} b(\varepsilon \mid \omega)\, d\varepsilon = \nu\, \bar{\varepsilon}(\omega) \left(B(\nu\, \bar{\varepsilon}(\omega) \mid \omega) - B(0 \mid \omega)\right),$$

implying that the marginal effect of a change in ν vanishes at $\nu = 0$. Since the same reasoning holds for any ω, the relative intensity of preferences W^{1s}/W^{2s} is not affected by the shift in political preferences. Moreover, $\mathbb{E}[B_1^{1s}(\omega)] = \mathbb{E}[B_0^{1s}(\omega)] =: \mathbb{E}[B^{1s}(\omega)]$ since the shift in political preferences satisfies $\mathbb{E}[\beta(\omega)] = 0$.

It remains to be shown that the tax system becomes more redistributive. To this end, we adapt the arguments in section B.4.2. Let $a = W^{1s}/W^{2s}$. The political equilibrium weights prior to the shift of preferences are given by

$$\begin{aligned} \bar{\gamma}_0^*(\omega) &:= \frac{\gamma_0^*(\omega)}{\mathbb{E}_\omega[\gamma_0^*(\omega)]} \\ &= \frac{a-(a-1)B_0^{1s}(\omega)}{a-(a-1)\mathbb{E}_\omega[B^{1s}(\omega)]}\,. \end{aligned}$$

After the shift they are equal to

$$\begin{aligned} \bar{\gamma}_1^*(\omega) &:= \frac{\gamma_1^*(\omega)}{\mathbb{E}_\omega[\gamma_1^*(\omega)]} \\ &= \frac{a-(a-1)B_1^{1s}(\omega)}{a-(a-1)\mathbb{E}_\omega[B^{1s}(\omega)]}\,. \end{aligned}$$

With $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu\beta(\omega)$, and β increasing, we have, for all $\nu > 0$,

$$\frac{\partial}{\partial\omega} B_1^{1s}(\omega) \quad > \quad \frac{\partial}{\partial\omega} B_0^{1s}(\omega) \; . \tag{46}$$

To complete the argument note that $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$ are non-increasing functions as both B_1^{1s} and B_0^{1s} are increasing by assumption. Moreover, $\bar{\gamma}_0^*(\underline{\omega}) = \bar{\gamma}_1^*(\underline{\omega}) = 1$, and, by (46),

$$|\frac{\partial}{\partial\omega} \bar{\gamma}_1^*(\omega)| \quad > \quad |\frac{\partial}{\partial\omega} \bar{\gamma}_0^*(\omega)| \; ,$$

for all $\omega > \underline{\omega}$. Hence,

$$\bar{\gamma}_1^*(\omega) \quad < \quad \bar{\gamma}_0^*(\omega) \; ,$$

for all $\omega > \underline{\omega}$.

These observations imply that there exists a decreasing function $\delta(\omega)$ with mean $\mathbb{E}[\delta(\omega)] = 0$ satisfying (15). Thus, the tax system associated with weighting function γ_1^* is more redistributive than the one associated with weighting function γ_0^*.

C Proofs of Section 3

C.1 Proof of Proposition 5

C.1.1 Preliminaries

Let $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}$. Let $p^*(\omega) = \text{argmax}_{p\in\mathcal{P}}\, u(p,\omega)$ be the ideal policy for voter type ω. The voters' ideal policies lie in the interior of $\mathcal{P}$ and satisfy the first order condition $u_1(p^*(\omega),\omega) = 0$. The single-crossing condition implies that $p^* : \omega \mapsto p^*(\omega)$ is non-increasing. Thus, for some $\varepsilon, \delta > 0$, $\underline{p} = p^*(\overline{\omega}) - \varepsilon$ and $\overline{p} = p^*(\underline{\omega}) + \delta$. The single-crossing property also implies that all types ω strictly prefer $p^*(\underline{\omega})$ over $\overline{p}$ and $p^*(\overline{\omega})$ over $\underline{p}$. Thus, $[p^*(\overline{\omega}), p^*(\underline{\omega})] \subset \mathcal{P}$ is the set of Pareto-efficient policies.

Assuming for simplicity that $\lambda = 1$, the objective of party 1 is to maximize

$$\Pi^1(p^1,p^2) = \frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}$$

and the objective of party 2 is to minimize this expression. Focusing on this case simplifies the exposition, but as we clarify below the argument does not depend on it and extends to any value of λ. Henceforth, we denote by Π_1^1 and Π_2^1 the partial derivatives of Π^1 with respect to p^1 and p^2, respectively.

Be reminded that $\Delta u(p^1,p^2,\omega) = u(p^1,\omega) - u(p^2,\omega)$, $W^1(p^1,p^2) = \mathbb{E}\left[G^1_W(\Delta u(\cdot) \mid \omega)\right]$ and $W^2(p^1,p^2) = \mathbb{E}\left[G^2_W(\Delta u(\cdot) \mid \omega)\right]$, where

$$G^1_W(x \mid \omega) := \int_{-\infty}^{x} (x - \varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon \; ,$$

and

$$G^2_W(x \mid \omega) := \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon \; .$$

The derivatives of the functions $G^1_W(\cdot \mid \omega)$ and $G^2_W(\cdot \mid \omega)$ are respectively given by

$$g^1_W(x \mid \omega) := B(x \mid \omega) \quad \text{and} \quad g^2_W(x \mid \omega) := -\left(1 - B(x \mid \omega)\right) \; .$$

Lemma 2 (**Best responses exist and are interior**)**.** *For any $p^2 \in \mathcal{P}$, there is a best response of party 1. Any best response of party 1 lies in the interior of $\mathcal{P}$ and satisfies the first order condition $\Pi^1_1(p^1,p^2) = 0$. Analogously, for any $p^2 \in \mathcal{P}$ there is a best response of party 2. Any best response of party 2 is interior and satisfies the first order condition $\Pi^1_2(p^1,p^2) = 0$.*

Proof. We only prove the statements referring to the best responses of party 1. For any p^2, the function $\Pi^1(\cdot,p^2)$ is continuous in p^1 and therefore attains a maximum on the compact policy space $\mathcal{P} = [\underline{p},\overline{p}]$. The function $\Pi^1(\cdot,p^2)$ is, moreover, differentiable. To prove that the maximum is interior and satisfies first-order conditions we show that, for any p^2,

$$\Pi^1_1(\underline{p},p^2) > 0 \quad \text{and} \quad \Pi^1_1(\overline{p},p^2) < 0 \; .$$

Given p^2, the derivative of $\Pi^1(\cdot,p^2)$ with respect to p^1 can be written as

$$\begin{aligned} \Pi^1_1(p^1,p^2) &= \frac{1}{W^2(p^1,p^2)} \mathbb{E}\left[g^1_W(\Delta u(\cdot) \mid \omega) u_1(p^1,\omega)\right] \\ &\quad - \frac{W^1(p^1,p^2)}{\left(W^2(p^1,p^2)\right)^2} \mathbb{E}\left[g^2_W(\Delta u(\cdot) \mid \omega) u_1(p^1,\omega)\right] \\ &= \frac{1+\Pi^1(p^1,p^2)}{W^2(p^1,p^2)}\; \mathbb{E}\left[\gamma^1(\omega \mid p^1,p^2)\; u_1(p^1,\omega)\right] \end{aligned}$$

where

$$\gamma^1(\omega \mid p^1,p^2) = \frac{1}{1+\Pi^1(p^1,p^2)}\; B(\Delta u(\cdot) \mid \omega) + \frac{\Pi^1(p^1,p^2)}{1+\Pi^1(p^1,p^2)}\; \left(1 - B(\Delta u(\cdot) \mid \omega)\right) .$$

Let $p^1 = \underline{p}$, then $u_1(p^1, \omega) > 0$ for all ω and hence $\Pi_1^1(p^1, p^2) > 0$. Analogously, if $p^1 = \overline{p}$, then $u_1(p^1, \omega) < 0$ for all ω and hence $\Pi_1^1(p^1, p^2) < 0$. □

Lemma 3 (**Best responses are continuous**). *For given p^2, let $p^{1*}(p^2)$ be a solution to the first order condition $\Pi_1^1(p^1, p^2) = 0$. Then p^{1*} is a continuous function.*

Proof. The implicit function theorem can be applied to the first-order condition for party 1 and implies that p^{1*} is a differentiable, hence continuous, function of p^2. □

Lemma 4 (**Existence of a fixed point**). *The function p^{1*} has a fixed point.*

Proof. The function p^{1*} is a continuous function from $\mathcal{P}$ to $\mathcal{P}$, where $\mathcal{P}$ is a non-empty, compact and convex set. Therefore, it has a fixed point by Brouwer's fixed point theorem. □

Lemma 5 (**Uniqueness of the fixed point**). *If utility functions are concave in p, $u_{11}(p, \omega) < 0$ for all p and ω, the function p^{1*} has only one fixed point.*

Proof. Let (p^1, p^2) be such a fixed point of the best response function p^{1*}. Such a fixed point satisfies $\Pi_1^1(p^1, p^2) = 0$, i.e.,

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\; u_1(p^1, \omega)\right] = 0 \ ,$$

and

$$p^1 = p^2 \ .$$

These two equations uniquely pin down p^1. To see this, note first that $p^1 = p^2$ implies $\Delta u(p^1, p^2, \omega) = 0$ for all ω and that $\gamma^1(\omega \mid p^1, p^2)$ depends on p^1 and p^2 only via $\Delta u(p^1, p^2, \omega)$. Let $\gamma(\omega)$ be the corresponding value of $\gamma^1(\omega \mid p^1, p^2)$, i.e.,

$$\gamma(\omega) := \frac{1}{1 + \Pi_*^1} B(0 \mid \omega) + \frac{\Pi_*^1}{1 + \Pi_*^1} \left(1 - B(0 \mid \omega)\right) , \quad \text{with} \quad \Pi_*^1 := \frac{\mathbb{E}\left[G_W^1(0 \mid \omega)\right]}{\mathbb{E}\left[G_W^2(0 \mid \omega)\right]} \ .$$

Then p^1 solves

$$\mathcal{A}(p^1) := \mathbb{E}\left[\gamma(\omega)\; u_1(p^1, \omega)\right] = 0 \ .$$

To see that this equation has a unique solution, note that the auxiliary function $\mathcal{A}(p^1)$ is differentiable, and decreasing as $\mathcal{A}'(p^1) = \mathbb{E}\left[\gamma(\omega)\; u_{11}(p^1, \omega)\right] < 0$. Moreover, $\mathcal{A}(\underline{p}) > 0$ and $\mathcal{A}(\overline{p}) < 0$. Thus, there is one and only one solution to the equation $\mathcal{A}(p^1) = 0$. □

C.1.2 Proof of Claim 1.

Suppose that

$$\begin{aligned} \Pi^1(p, p') &= a + b\, \Pi^2(p', p) \\ & \quad a + b\, (1 - \Pi^2(p', p)) \ . \end{aligned}$$

Hence,

$$\Pi^1_1(p, p') = -b\, \Pi^1_2(p', p) \ .$$

Now suppose that $(\hat{p}, \hat{p})$ is a fixed point of party 1's best response problem. Then,

$$\Pi^1(\hat{p}, \hat{p}) \geq \Pi^1(p, \hat{p}) \ ,$$

for all $p \in \mathcal{P}$. Further note that

$$\begin{aligned} \Pi^1(\hat{p}, \hat{p}) - \Pi^1(p, \hat{p}) &= \int_p^{\hat{p}} \Pi^1_1(s, \hat{p})\, ds \\ &= -b \int_p^{\hat{p}} \Pi^1_2(\hat{p}, s)\, ds \\ &= -b \Big(\Pi^1(\hat{p}, \hat{p}) - \Pi^1(\hat{p}, p) \Big) \\ &= b \Big(\Pi^1(\hat{p}, p) - \Pi^1(\hat{p}, \hat{p}) \Big) \ . \end{aligned}$$

Hence,

$$\Pi^1(\hat{p}, \hat{p}) \leq \Pi^1(\hat{p}, p) \ ,$$

for all $p \in \mathcal{P}$, implying that $(\hat{p}, \hat{p})$ is a saddle point of Π^1, and hence an equilibrium.

C.1.3 Proof of Claim 2.

Together Lemma 2 and the premise of Claim 2 imply that, for every p_2, there is a unique value of p^1 so that

$$\Pi^1_1(p^1, p^2) = 0 \quad \text{and} \quad \Pi^1_{11}(p^1, p^2) < 0 \ .$$

This value of p^1 is the unique best response of party 1 to policy p^2. Mutatis mutandis, the same holds true for party 2. Under these conditions the following Lemma holds true.

Lemma 6 (Identical fixed points). *Suppose that utility functions are concave in* p, $u_{11}(p,\omega) < 0$ *for all* p *and* ω. *Then, the best response functions* p^{1*} *and* p^{2*} *have the same fixed point.*

Proof. As argued above, if (p^1, p^2) is a fixed point of p^{1*} it satisfies

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\; u_1(p^1, \omega)\right] = 0 \tag{47}$$

and

$$p^1 = p^2 \; . \tag{48}$$

By Lemma 5, there is only one solution to this system of equations.

Analogously, given p^1, the best responses of party 2, $p^{2*}(p^1)$ is obtained as the solution to

$$\min_{p^2 \in \mathcal{P}} \quad \Pi^1(p^1, p^2)$$

and solves the first-order condition

$$\Pi^1_2(p^1, p^2) = -\frac{1 + \Pi^1(p^1, p^2)}{W^2(p^1, p^2)}\, \mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\; u_1(p^2, \omega)\right] = 0 \; .$$

Thus, a fixed point (p^1, p^2) of p^{2*} satisfies

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\; u_1(p^2, \omega)\right] = 0 \; , \tag{49}$$

and

$$p^1 = p^2 \; . \tag{50}$$

Hence, a fixed point of p^{2*} solves the same system of equations as a fixed point of p^{1*}. Thus, the two functions have the same fixed point. □

Therefore, if $\hat{p}$ is a fixed point of p^{1*}, then $(\hat{p}, \hat{p})$ is a symmetric equilibrium in pure strategies. By Lemma 4 such a fixed point exists.

It remains to be shown that there can be no other equilibrium. Suppose to the contrary that there is an alternative Nash equilibrium (p^1, p^2) with $p^1 \neq \hat{p}$ or $p^2 \neq \hat{p}$. Since the game under study is zero-sum, this implies that $(p^1, \hat{p})$ and $\hat{p}, p^2$ are also Nash equilibria, see e.g. Osborne and Rubinstein (1994). Suppose without loss of generality that $p^1 \neq \hat{p}$. Then, this implies that, for party 1, both $\hat{p}$ and p^1 are best responses to $\hat{p}^2$. This contradicts the assumption that party 1 has a unique best response to any policy $p^2 \in \mathcal{P}$. Thus, the assumption that there is an alternative equilibrium leads to a contradiction and must be false.

C.1.4 General Objective, $0 < \lambda < 1$.

The preceding argument uses that $W^1(\cdot)$ and $W^2(\cdot)$ depend on p^1 and p^2 only via $\Delta u(p^1, p^2, \omega) = u(p^1, \omega) - u(p^2, \omega)$ and the derivatives of the objective function can be written as a weighted sum of the different types' marginal utilities, where the weights are all positive. These properties remain intact with a more general objective function of the form

$$\Pi^1(p^1, p^2) = (1-\lambda) \ln \left(\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} \right) + \lambda \ln \left(\frac{W^1(p^1, p^2)}{W^2(p^1, p^2)} \right) .$$

For instance, the best response condition $\Pi^1_1(p^1, p^2) = 0$ for party 1 can then be written as

$$\mathbb{E}\left[\gamma^{1,\lambda}(\omega \mid p^1, p^2)\, u_1(p^1, \omega)\right] = 0 ,$$

where

$$\gamma^{1,\lambda}(\omega \mid p^1, p^2) \quad = \quad (1-\lambda)\, \bar{q}(\omega)\, b(\cdot \mid \omega) + \lambda \left(\tfrac{1}{\mathbf{B}^1(\cdot)} + \tfrac{1}{\mathbf{B}^2(\cdot)} \right)^{-1} \gamma^1(\omega \mid p^1, p^2).$$

C.2 Derivation of inequalities (19) and (20)

The objective of party 1 is to maximize

$$\Pi^1(p^1, p^2) = \frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}$$

and the objective of party 2 is to minimize this expression. For later reference, note that $W^1(p^1, p^2) = \mathbb{E}\,[G^1_W(\Delta u(\cdot) \mid \omega)]$ and $W^2(p^1, p^2) = \mathbb{E}\,[G^2_W(\Delta u(\cdot) \mid \omega)]$ where

$$G^1_W(x \mid \omega) := \int_{-\infty}^{x} (x - \varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon ,$$

and

$$G^2_W(x \mid \omega) := \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon .$$

The derivatives of the functions $G_W^1(\cdot \mid \omega)$ and $G_W^2(\cdot \mid \omega)$ are respectively given by

$$g_W^1(x \mid \omega) := B(x \mid \omega) \quad \text{and} \quad g_W^2(x \mid \omega) := -\left(1 - B(x \mid \omega)\right) .$$

Given a policy p^2, the first order condition of party 1's best response problem is

$$\Pi_1^1(\cdot) = \frac{1}{W^2(\cdot)^2}\left\{W_1^1(\cdot)W^2(\cdot) - W_1^2(\cdot)W^1(\cdot)\right\} = 0 . \tag{51}$$

The second derivative Π_{11}^1, evaluated at a policy that satisfies this first order condition, equals

$$\Pi_{11}^1(\cdot) = \frac{1}{W^2(\cdot)^2}\{W_{11}^1(\cdot)W^2(\cdot) - W_{11}^2(\cdot)W^1(\cdot)\} . \tag{52}$$

Thus, $\Pi_{11}^1(\cdot) < 0$ holds provided that

$$W_{11}^1(\cdot) = \mathbb{E}[b(\Delta u(\cdot) \mid \omega)u_1(p^1, \omega) + B(\Delta u(\cdot) \mid \omega)u_{11}(p^1, \omega)] \quad < \quad 0 , \tag{53}$$

and

$$W_{11}^2(\cdot) = \mathbb{E}[b(\Delta u(\cdot) \mid \omega)u_1(p^1, \omega) - (1 - B(\Delta u(\cdot) \mid \omega))u_{11}(p^1, \omega)] \quad > \quad 0 . \tag{54}$$

It is now straightforward to verify that the inequalities (19) and (20) stated in the main text imply that the inequalities (53) and (54) hold.

C.3 Existence of mixed-strategy equilibria

Glicksberg's existence theorem implies the existence of a mixed strategy equilibrium for a zero-sum game under the following conditions:

- *Pure strategy spaces are compact.*
- *The payoff function Π^1 is continuous in (p_1, p_2) for $(p_1, p_2) \in \mathcal{P}^2$.*

In the following, we introduce notions of compactness and continuity that can be applied to a policy space of non-negative, bounded and monotonic functions. Verifying that these properties indeed hold is then straightforward. The existence of a mixed strategy equilibrium then follows from Glicksberg's existence theorem.

Compactness. Let $\Omega = [\underline{\omega}, \overline{\omega}]$. Let the set of feasible earnings levels also be a compact subset of the reals and denote it by $\mathcal{Y} = [0, \overline{y}]$. An earnings function $\mathbf{y}$:

$\Omega \to \mathcal{Y}$ can be viewed as an element of a compact set $\Omega \times \mathcal{Y}$. Now consider a sequence of earnings functions $\{\mathbf{y}^k\}_{k=1}^{\infty}$ that converges to a limit function $\bar{\mathbf{y}}$ in the sense that, for every $\omega \in \Omega$, $\{\mathbf{y}^k(\omega)\}$ is a sequence in $\mathcal{Y}$ that converges to a limit point $\bar{y}(\omega)$.

The domain of all functions in the sequence is constant and equal to $\Omega = [\underline{\omega}, \overline{\omega}]$, which is also the domain of the limit function. Thus, we only need to worry about the convergence in the image of these functions. The image is an element of $\mathcal{Y}^{\#\Omega}$, a cartesian product of compact sets. By Tychonoff's theorem, a cartesian product of compact sets is a compact set. By assumption, the sequence $\{\mathbf{y}^k(\Omega)\}_{k=1}^{\infty}$, $\mathbf{y}^k(\Omega) \in \mathcal{Y}^{\#\Omega}$, converges to a limit $\bar{\mathbf{y}}$. Since $\mathcal{Y}^{\#\Omega}$ is a compact set it follows that $\bar{\mathbf{y}} \in \mathcal{Y}^{\#\Omega}$.

Continuity. A sketch of the argument suffices. What enters the parties' objective function Π^1 are averages, by type ω, of continuous functions (c.d.f.'s of party preferences) of utility differentials implied by policy differences. By hypothesis, $p_a \to p_b$, implies $u(p_a, \omega) \to u(p_b, \omega)$, for all $\omega \in \Omega$. Thus if the difference between two policies p_a and p_b vanishes in the sense of uniform convergence, then, for every type ω, the contribution to the objective under p_a converges to the contribution under p_b. This property survives continuous transformations and integration.

D Proof of Proposition 4

D.1 Regularity conditions

Optimal tax problems. We impose regularity conditions that are familiar from the literature on optimal taxation. As will become clear in the subsequent paragraph, these regularity conditions also facilitate the analysis of the parties' best responses.

As outlined in part B.3 of the Online-Appendix, a non-linear tax schedule can be represented by a non-negative, bounded and monotonic earnings function $\mathbf{y} : \Omega \to \mathbb{R}_+$. The policy preferences of a type ω individual are then represented by $u(\mathbf{y}, \omega)$. Social welfare S induced by an earnings function $\mathbf{y}$ is

$$S(\mathbf{y}) = \mathbb{E}\left[g(\omega)\, u(\mathbf{y}, \omega)\right] \ .$$

where $g : \Omega \to \mathbb{R}_+$ specifies the weights of different types in the welfare function. Without loss of generality we let $\mathbb{E}[g(\omega)] = 1$. The *full* optimal tax problem is to choose the earnings function $\mathbf{y}$ that maximizes this welfare objective over the set of non-decreasing functions. The *relaxed* problem is to choose the earnings function that maximizes this welfare objective over the set of all functions.

Assumption 3. *Suppose that the following conditions hold:*

1. *For any weighting function $g : \Omega \to \mathbb{R}_+$ with $\mathbb{E}[g(\omega)] = 1$, the above problem of welfare-maximization has a unique solution. Let $\mathbf{y}_g$ be the earnings function that solves this problem.*

2. *For any weighting function $g : \Omega \to \mathbb{R}_+$ with $\mathbb{E}[g(\omega)] = 1$, the relaxed problem of welfare-maximization has a unique solution. Let $\mathbf{y}_g^r$ be the earnings function that solves the this relaxed problem.*

3. *The function $\mathbf{y}_g^r$ satisfies Diamond's formula; i.e. for all ω,*

$$1 - k_1(\mathbf{y}(\omega), \omega) = -\frac{1 - F(\omega)}{f(\omega)}\ (1 - \mathcal{G}_g(\omega))\ k_{21}(\mathbf{y}(\omega), \omega)\ ,$$

   where $\mathcal{G}_g(\omega) := \mathbb{E}[g(z) \mid z \geq \omega]$. Moreover, $\mathbf{y}_g^r$ is the only function that satisfies Diamond's formula.

4. *For ω so that the monotonicity constraint on $\mathbf{y}_g$ is not binding, $\mathbf{y}_g^r(\omega) = \mathbf{y}_g(\omega)$.*

Assumption 3 is routinely invoked in models of optimal income taxation. The assumption can be justified. With an appropriate choice of the primitives, the solutions to the relaxed and the full problem of welfare-maximization can be shown to satisfy properties 1.– 4. We simply impose Assumption 3 as a shortcut.

To get from an earnings function $\mathbf{y}$ to the associated tax schedule T, one can use the first-order condition of the utility-maximization problem that individuals face in the presence of this tax system. If tax system T induces an incentive-compatible allocation $(\mathbf{c}, \mathbf{y})$, then

$$1 - T'(\mathbf{y}(\omega)) = k_1(\mathbf{y}(\omega), \omega)\ .$$

Hence, $1 - k_1(y(\omega), \omega)$ is interpreted as the marginal tax rate that type-ω agents face.

Best response problems. For ease of exposition, we focus on relaxed best response problems. Thus, given an earnings function $\mathbf{y}^2$ proposed by party 2, the problem of party 1 is to choose $\mathbf{y}^1$ so as to maximize $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$. This problem differs from the full best response problem that incorporates the constraint that $\mathbf{y}^1$ must be a non-decreasing function. Obviously, if the solution to the relaxed problem is non-decreasing then it is also a solution to the full problem. Otherwise, the solution of the full problem will give rise to bunching. While it is well-known how the analysis would have to be modified if bunching is an issue, see e.g. Hellwig (2007); Brett and Weymark (2016), the trade-offs that shape best responses are more easily exposed when focusing on the relaxed problem.

The equilibrium analysis that follows involves a characterization of best responses. As will become clear, the focus on relaxed best response problems in conjunction with Assumption 3 then implies that best responses are characterized by a version of Diamond's rule, albeit with a different weighting function.

D.2 Best responses: necessary conditions

For ease of exposition, we assume that $\lambda = 1$ so that

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^1, \mathbf{y}^2)}{W^2(\mathbf{y}^1, \mathbf{y}^2)} \ .$$

The best response of party 1 can be viewed as a compromise between maximizing the expression in the numerator and minimizing the expression in the denominator. We first look at each of these auxiliary objectives in isolation and then turn to the compromise.

Maximizing $W^1(\mathbf{y}^1, \mathbf{y}^2)$. Remember that $W^1(y^1, y^2) = \mathbb{E}[G^1_W(\Delta u(\cdot))]$ and that the derivative of $G^1_W(\cdot \mid \omega)$ equals $g^1_W(\cdot \mid \omega) = B\left(\Delta u(\cdot) \mid \omega\right)$. We write

$$\begin{aligned} \bar{g}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2) \ &:= \ \mathbb{E}[B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega') \mid \omega' \geq \omega] \\ &= \ \int_{\omega}^{\bar{\omega}} B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega') \tfrac{f(\omega')}{1-F(\omega)} d\omega' \end{aligned}$$

for the average value of $B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega')$ among individuals with a type ω' above some cutoff ω. To interpret these expressions, suppose that party 1 offers slightly more utility to type ω' individuals. Then $B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega')$ measures the extra gain that type ω'-supporters of party 1 realize in the event that party 1 wins rather party 2. Therefore, $\bar{g}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$ is the gain that party 1 can generate by offering all agents with types above ω slightly more utility. The gain that party 1 can generate by slightly raising everybody's utility is given by $\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)$ and the ratio

$$\mathcal{G}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2) := \frac{\bar{g}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}$$

relates the gain from making everybody with a type above ω better off to the gain from making everybody better off.

Lemma 7. *Given $\mathbf{y}^2$, the solution to $\max_{\mathbf{y}^1} W^1(\mathbf{y}^1, \mathbf{y}^2)$ is such that, for all ω,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \, \left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \, \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \, . \tag{55}$$

Proof. We begin by stating party 1's best response problem in a way that enables an analysis using a Gateaux differential. Let $\mathbf{y}^1 = \mathbf{y}^{1*} + \mu \, h^1$, be a perturbed version of party 1's best response $\mathbf{y}^{1*}$, in which μ is a scalar and $h^1 : \Omega \to \mathbb{R}$ is a function. If $\mathbf{y}^{1*}$ is a best response, then, for any perturbation (μ, h^1),

$$\mathbb{E}\left[G^1_W\left(u(\mathbf{y}^{1*}, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right] \geq \mathbb{E}\left[G^1_W\left(u(\mathbf{y}^{1*} + \mu \, h^1, \omega) - u(\mathbf{y}^2 \omega) \mid \omega\right)\right] \, . \tag{56}$$

Equivalently, using the characterization of policy preferences in part B.3 of the Online-Appendix, for any function h^1, $\mu = 0$ must be a maximizer of the auxiliary function

$$A(\mu \mid \mathbf{y}^{1*}, \mathbf{y}^2) = \mathbb{E}\left[G^1_W\left(s_v(\mathbf{y}^{1*} + \mu \, h^1) + \rho(\mathbf{y}^{1*} + \mu \, h^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right] \, .$$

In the following, we will characterize $\mathbf{y}^{1*}$ by analyzing the implications of the requirement that the derivative of this expression with respect to μ, evaluated at $\mu = 0$, is equal to zero. We express this condition as

$$A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = 0 \, , \tag{57}$$

for all functions h^1.[36] Using the characterization of information rents in part B.3 of the Online-Appendix, $\rho(\mathbf{y}, \omega) = -\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \, dz$, as well as the definition of the virtual surplus, $s_v(\mathbf{y}) = \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)} \, k_2(\mathbf{y}(\omega), \omega)\right]$, we note that

$$A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = \mathbb{E}\left[g^1_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) \, \left(s_{v,h^1}(\mathbf{y}^{1*}) - \int_{\underline{\omega}}^{\omega} h^1(z) \, k_{21}(\mathbf{y}^{1*}(z), z) \, dz\right)\right] \, ,$$

[36]Formally, $A_{h^1}(y^{1*}, y^2)$ is the Gateaux differential of $\mathbb{E}\left[G^1_W\left(s_v(\mathbf{y}^1) + \rho(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right]$ in direction h^1 evaluated at $\mathbf{y}^1 = \mathbf{y}^{1*}$.

where

$$g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) := b\left(s_v(\mathbf{y}^{1*}) - \int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}^{1*}(z), z)\, dz - u(\mathbf{y}^2, \omega) \mid \omega\right)$$

and

$$s_{v,h^1}(\mathbf{y}^{1*}) \;\; := \;\; \mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right]$$

is the Gateaux differential of the virtual surplus $s_v(\mathbf{y}^1)$ in direction h^1 evaluated at $\mathbf{y}^1 = \mathbf{y}^{1*}$. Thus, $A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)$ can now be rewritten as

$$\begin{aligned} A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) \;\; &= \;\; \bar{g}_W^1(\underline{\omega} \mid y^{1*}, y^2)\mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right] \\ &\quad -\mathbb{E}\left[g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) \; \textstyle\int_{\underline{\omega}}^{\omega} h^1(z)\, k_{21}(\mathbf{y}^{1*}(z), z)\, dz\right] \; , \end{aligned}$$

where, for any $\omega \in \Omega$, $\bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) := \mathbb{E}[g_W^1(\omega' \mid \mathbf{y}^{1*}, \mathbf{y}^2) \mid \omega' \geq \omega]$. Moreover, an integration by parts shows that

$$\begin{aligned} & \mathbb{E}\left[g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) \int_{\underline{\omega}}^{\omega} h^1(z)\, k_{21}(\mathbf{y}^{1*}(z), z)\, dz\right] \\ = \;\; & \mathbb{E}\left[h^1(\omega)\; \bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\; \frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right] \end{aligned} \tag{58}$$

so that condition (57) can equivalently be written as the requirement that, for all functions h^1,

$$\mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + (1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right] = 0 \; , \tag{59}$$

where $\mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) = \frac{\bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)}{\bar{g}_W^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)}$. Condition (59) can hold only if, for all ω,

$$1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + (1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega) = 0 \; , \tag{60}$$

or, equivalently, if

$$\frac{1 - k_1(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)} = -(1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\; \frac{1-F(\omega)}{f(\omega)}\; \frac{k_{21}(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)} \; . \tag{61}$$

Using $T'(\mathbf{y}^{1*}(\omega)) = 1 - k_1(\mathbf{y}^{1*}(\omega), \omega)$ we can rewrite this equation as

$$\frac{T'(\mathbf{y}^{1*}(\omega))}{1 - T'(\mathbf{y}^{1*}(\omega))} = -(1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)) \, \frac{1 - F(\omega)}{f(\omega)} \, \frac{k_{21}(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)} \,, \tag{62}$$

which is what had to be shown. □

Minimizing $W^2(\mathbf{y}^1, \mathbf{y}^2)$. The following Lemma describes the solution to another auxiliary problem for party 1, namely the problem to choose policy with the objective to minimize what is at stake for the supporters of party 2. We omit a proof and discussion of the Lemma as it would involve only a straightforward adjustment to those of Lemma 7. The Lemma involves a weighting function $\mathcal{G}_W^2$ for information rents that is derived from $W^2(\mathbf{y}^1, \mathbf{y}^2) = \mathbb{E}[G_W^2(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega)]$ along the same lines as $\mathcal{G}_W^1$ is derived from $W^1(\mathbf{y}^1, \mathbf{y}^2)$, where we now have:

$$g_W^2(\cdot \mid \omega) \equiv 1 - B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right).$$

Lemma 8. *Given $\mathbf{y}^2$, the solution to $\min_{\mathbf{y}^1} W^2(\mathbf{y}^1, \mathbf{y}^2)$ is such that, for all ω,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \, \left(1 - \mathcal{G}_W^2(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \, \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \,. \tag{63}$$

Party 1's best response. We introduce notation for a weighted average of $\mathcal{G}_W^1$ and $\mathcal{G}_W^2$. Let

$$\begin{aligned} \gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2) \quad &:= \quad \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \, g_W^1(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid y^1, y^2) \\ &\qquad + \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \, g_W^2(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid y^1, y^2) \\ &= \quad \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \, B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right) \\ &\qquad + \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \, \left(1 - B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right) \end{aligned}$$

and

$$\bar{\gamma}^1(\omega \mid y^1, y^2) \quad := \quad \mathbb{E}[\gamma^1(\omega' \mid y^1, y^2) \mid \omega' \geq \omega] \,.$$

Also define

$$\Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2) := \frac{\bar{\gamma}^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}.$$

Lemma 9. *Given* $\mathbf{y}^2$*, if* $\mathbf{y}^1$ *is a maximizer of* $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$ *then, for all* ω*,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \, \left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \, \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \; . \tag{64}$$

Proof. Given $\mathbf{y}^2$ we look at the problem to choose $\mathbf{y}^1$ with the objective to maximize

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^1, \mathbf{y}^2)}{W^2(\mathbf{y}^1, \mathbf{y}^2)} \; .$$

Suppose that $\mathbf{y}^{1*}$ is a solution to that problem. Then, it must also be that case that $\mu = 0$ solves the problem to choose a scalar μ with the objective to maximize

$$\Pi^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)}{W^2(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)} \; .$$

for any given but arbitrary function h^1. That is, we can characterize $\mathbf{y}^{1*}$ be the requirement that

$$\left. \frac{\partial \Pi^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)}{\partial \mu} \right|_{\mu=0} = 0 \; ,$$

or, equivalently, that

$$W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) \; W^2(\mathbf{y}^{1*}, \mathbf{y}^2) - W^1(\mathbf{y}^{1*}, \mathbf{y}^2) \; W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = 0 \; ,$$

where $W^j_{h^1}$ is the Gateaux differential of W^j in direction h^1. The following equations provide a characterization of $W^1_{h^1}$ and $W^2_{h^1}$ The equation follow from a straightforward adaptation of the arguments in the proof of Lemma 7. The Gateaux differential of W^1 in the direction h^1 evaluated at $(\mathbf{y}^{1*}, \mathbf{y}^2)$ equals

$$\begin{aligned} &W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = \bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2) \times \\ &\mathbb{E}\left[h^1(\omega) \left\{ 1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1 - F(\omega)}{f(\omega)} \left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega) \right\} \right] . \end{aligned} \tag{65}$$

The Gateaux differential of W^2 in the direction h^1 evaluated at (y^{1*}, y^2) equals

$$\begin{aligned} & W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = -\bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2) \times \\ & \mathbb{E}\left[h^1(\omega)\left\{1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right\}\right]. \end{aligned} \tag{66}$$

Straightforward algebra now yields the observation that the Gateaux differential of $\Pi^1 = W^1/W^2$ in the direction h^1 evaluated at $(\mathbf{y}^{1*}, \mathbf{y}^2)$ has the same sign as

$$\begin{aligned} & \frac{W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)\, W^2(\mathbf{y}^{1*}, \mathbf{y}^2) - W^1(\mathbf{y}^{1*}, \mathbf{y}^2)\, W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)(1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)) W^2(\mathbf{y}^{1*}, \mathbf{y}^2)} \\ = & \frac{1}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)}\left\{\frac{1}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) - \frac{\Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)\right\} \end{aligned}$$

Also notice that

$$\begin{aligned} & \frac{1}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} \bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) \\ & + \frac{\Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} \bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) \\ = & \bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \Gamma^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right). \end{aligned}$$

Therefore, expressions (65) and (66) imply that the Gateaux differential of Π^1 is given by

$$\mathbb{E}\left[h^1(\omega)\left\{1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right\}\right]. \tag{67}$$

□

For later reference, we state the analogues to the expressions in the proof of Lemma 9 for party 2's best response problem. The Gateaux differential of W^1 in the direction h^2 evaluated at $(\mathbf{y}^1, \mathbf{y}^{2*})$ equals

$$\begin{aligned} & W^1_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*}) = -\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*}) \times \\ & \mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right]. \end{aligned} \tag{68}$$

The Gateaux differential of W^2 in the direction h^2 evaluated at (y^1, y^{2*}) equals

$$W^2_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*}) = \bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*})\times$$
$$\mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right] . \tag{69}$$

The Gateaux differential of Π^1 in the direction h^2 evaluated at $(\mathbf{y}^1, \mathbf{y}^{2*})$ then has the same sign as

$$\begin{aligned} & \frac{W^1_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*})\, W^2(\mathbf{y}^1, \mathbf{y}^{2*}) - W^1(\mathbf{y}^1, \mathbf{y}^{2*})\, W^2_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*})}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*})(1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^{2*}))W^2(\mathbf{y}^1, \mathbf{y}^{2*})} \\ = \; & -\mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right] . \end{aligned}$$

Thus, we obtain an analogous characterization of party 2's best responses.

Lemma 10. *Given $\mathbf{y}^1$, if $\mathbf{y}^2$ is a minimizer of $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$ then, for all ω,*

$$\frac{T'(\mathbf{y}^2(\omega))}{1 - T'(\mathbf{y}^2(\omega))} \quad = \quad -\frac{1-F(\omega)}{f(\omega)}\;\left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right)\;\frac{k_{21}(\mathbf{y}^2(\omega), \omega))}{k_1(\mathbf{y}^2(\omega), \omega)} \; . \tag{70}$$

D.3 An equilibrium candidate

We hypothesize the existence of a symmetric equilibrium, $\mathbf{y}^1 = \mathbf{y}^2$. When both parties propose the same policies, $u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) = 0$, for all ω. Henceforth, we make use of the following shorthands: whenever $\mathbf{y}^1 = \mathbf{y}^2$ we write $\gamma^*(\omega)$ rather than $\gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$ and $\Gamma^*(\omega)$ rather than $\Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$.

If $\mathbf{y}^*$ is a symmetric equilibrium policy then, by Lemmas 9 and 10 it has to be such that

$$\frac{T'(\mathbf{y}^*(\omega))}{1 - T'(\mathbf{y}^*(\omega))} \quad = \quad -\frac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^*(\omega)\right)\;\frac{k_{21}(\mathbf{y}^*(\omega), \omega))}{k_1(\mathbf{y}^*(\omega), \omega)} \; . \tag{71}$$

Also note that, by Assumption 3, the function $\mathbf{y}^*$ is the unique candidate for a symmetric equilibrium.

The function $\mathbf{y}^*$ satisfies necessary conditions of both parties' best response problems: Given $\mathbf{y}^2 = \mathbf{y}^*$, $\mathbf{y}^1 = \mathbf{y}^*$ is a local extremum of the functional $\Pi^1(\cdot, \mathbf{y}^*) : \mathbf{y}^1 \to \Pi^1(\mathbf{y}^1, \mathbf{y}^*)$. Likewise, given $\mathbf{y}^1 = \mathbf{y}^*$, $\mathbf{y}^2 = \mathbf{y}^*$ is a local extremum of the functional $\Pi^1(\mathbf{y}^*, \cdot) : \mathbf{y}^2 \to \Pi^1(\mathbf{y}^*, \mathbf{y}^2)$.

D.4 Second order conditions / Saddle point

We now show that, under the premises of Proposition 4, the hypothetical equilibrium $(\mathbf{y}^1, \mathbf{y}^2) = (\mathbf{y}^*, \mathbf{y}^*)$ is a local saddle point of the function Π^1.

Lemma 11. *Suppose that the the premises of Proposition 4 are satisfied. Then, a pair of policies that satisfies (71) is a saddle point of the function* Π^1.

Proof. We seek to show that $(\mathbf{y}^*, \mathbf{y}^*)$ is a saddle point of the function

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\Pi^1(\mathbf{y}^1, \mathbf{y}^2))}{W^2(\Pi^1(\mathbf{y}^1, \mathbf{y}^2))} .$$

We now state this saddle point condition in a way that enables an analysis using functional derivatives. Let $\mathbf{y}^1 = \mathbf{y}^{1*} + \mu^1\ h^1$, be a perturbed version of $\mathbf{y}^{1*}$, in which μ^1 is a scalar and $h^1 : \Omega \to \mathbb{R}$ is a function. Analogously, let $\mathbf{y}^2 = \mathbf{y}^{2*} + \mu^2\ h^2$, be a perturbed version of $\mathbf{y}^2$. The local saddle point condition according to which, for all $(\mathbf{y}^1, \mathbf{y}^2)$ in neighborhood of $(\mathbf{y}^*, \mathbf{y}^*)$,

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^2)$$

can therefore be written as: for any pair of perturbations (μ^1, h^1) and (μ^2, h^2),

$$\Pi^1(\mathbf{y}^* + \mu^1\ h^1, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^* + \mu^2\ h^2) . \tag{72}$$

Equivalently, for all functions (h^1, h^2), the point $(\mu^1, \mu^2) = (0, 0)$ must be a saddle-point of

$$\Pi^1(\mathbf{y}^* + \mu^1\ h^1, \mathbf{y}^* + \mu^1\ h^2) = \frac{W^1(\mathbf{y}^1 + \mu^1\ h^1, \mathbf{y}^2 + \mu^2\ h^2)}{W^2(\mathbf{y}^* + \mu^1\ h^1, \mathbf{y}^2 + \mu^2\ h^2)} .$$

In the following, we use subscripts to indicate derivatives with respect to μ^1 and μ^2, respectively.

Having a saddle point requires that all entries of the Jacobi-matrix

$$J_\Pi(y^*, y^*) = \begin{pmatrix} \Pi^1_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*) \\ \Pi^1_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \end{pmatrix}$$

are equal to zero and that the Hessian

$$H_\Pi(y^*, y^*) = \begin{pmatrix} \Pi^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) & \Pi^1_{\mu^1,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \\ \Pi^1_{\mu^1,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) & \Pi^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \end{pmatrix}$$

is indefinite. As an implication of the necessary condition (71), all entries of the Jacobi-matrix are equal to zero. Hence, what remains to be shown is that $H_\Pi(\mathbf{y}^*, \mathbf{y}^*)$ is indefinite. To this end, it suffices to show that $\Pi^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0$, and $\Pi^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0$. These two inequalities can be shown to hold provided that

$$\frac{\partial}{\partial \mu^1} \left\{ W^1_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*) \right\} < 0 \;,$$

and

$$\frac{\partial}{\partial \mu^2} \left\{ W^1_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \right\} > 0 \;,$$

or, equivalently, if

$$W^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0 \;, \tag{73}$$

and

$$W^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0 \;. \tag{74}$$

Since $W^1(\mathbf{y}^*, \mathbf{y}^*) > 0$ and $W^2(\mathbf{y}^*, \mathbf{y}^*) > 0$ sufficient conditions for the validity of (73) and (74) are

$$W^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0 \quad \text{and} \quad W^2_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) > 0 \;, \tag{75}$$

and

$$W^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0 \quad \text{and} \quad W^2_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) < 0 \;. \tag{76}$$

We can now use the expressions for $W^1_{\mu^1}$, $W^2_{\mu^1}$, $W^1_{\mu^2}$ and $W^2_{\mu^2}$ (or, equivalently, the Gateaux differentials $W^1_{h^1}$, $W^2_{h^1}$, $W^1_{h^2}$ and $W^2_{h^2}$) derived above – see equations (65) (66), (68), and (69) – to compute $W^1_{\mu^1,\mu^1}$, $W^2_{\mu^1,\mu^1}$, $W^1_{\mu^2,\mu^2}$ and $W^2_{\mu^2,\mu^2}$.

Exploiting the assumption of unform party biases, so that $B\left(\Delta u(\cdot) \mid \omega\right) = \alpha\left(\omega\right) + \beta\left(\omega\right)\; \Delta u(\cdot)$, evaluating the resulting expressions in the limit case $\beta(\omega)$ close to zero and $\alpha(\omega)$ close to $\frac{1}{2}$, for all ω, one can verify that (75) and (76) indeed hold. For instance, we then find that

$$
\begin{aligned}
& W^1_{\mu^1,\mu^1}(\mathbf{y}^*,\mathbf{y}^*) \\
= \; & \bar{\alpha}(\underline{\omega})\mathbb{E}\left[h^1(\omega)^2\left(-k_{11}(\mathbf{y}^*(\omega),\omega)+\frac{1-F(\omega)}{f(\omega)}\left(1-\frac{\bar{\alpha}(\omega)}{\bar{\alpha}(\underline{\omega})}\right)k_{211}(\mathbf{y}^*(\omega),\omega)\right)\right],
\end{aligned}
$$

where $\bar{\alpha}(\omega) := \mathbb{E}\left[\alpha(\omega') \mid \omega' \geq \omega\right]$. With $\alpha(\omega) = \frac{1}{2}$, for all ω, it follows that $1-\frac{\bar{\alpha}(\omega)}{\bar{\alpha}(\underline{\omega})} = 0$, so that

$$
W^1_{\mu^1,\mu^1}(\mathbf{y}^*,\mathbf{y}^*) = \bar{\alpha}(\underline{\omega})\mathbb{E}\left[h^1(\omega)^2\left(-k_{11}(\mathbf{y}^*(\omega),\omega)\right)\right] < 0\;.
$$

□

D.5 Existence and uniqueness of equilibrium

We now show that the local saddle point characterized in the previous proof is indeed an equilibrium point. To this end, we need to show that it is a best response for party 1 to play the hypothetical equilibrium strategy – on the assumption that party 2 also plays this strategy. The results stated so far only imply that playing the hypothetical equilibrium strategy is a local best response for party 1. What remains to be shown is that this local best response is also the global best response and that there is no other global best response.

In the following, we will use the contraction mapping theorem to show that this is indeed the case. A symmetric argument then implies that it is a best response for party 2 to play the hypothetical equilibrium strategy provided that party 1 plays accordingly. To simplify the exposition, suppose moreover that the effort cost function is iso-elastic,

$$
k(y,\omega) = \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}\;.
$$

The condition characterizing party 1's best response $\mathbf{y}^{1*}$ to the hypothetical equilibrium strategy $\mathbf{y}^*$ then simplifies,

$$
\frac{1-k_1(\mathbf{y}^{1*}(\omega),\omega)}{k_1(\mathbf{y}^{1*}(\omega),\omega)} = \left(1+\frac{1}{e}\right)\frac{1-F(\omega)}{f(\omega)\,\omega}\left(1-\Gamma^1(\omega \mid \mathbf{y}^{1*},\mathbf{y}^*)\right)\;. \tag{77}
$$

or, equivalently,

$$
\omega^{1+\frac{1}{e}}\mathbf{y}^{1*}(\omega)^{-\frac{1}{e}} - 1 = \left(1+\frac{1}{e}\right)\frac{1-F(\omega)}{f(\omega)\,\omega}\left(1-\Gamma^1(\omega \mid \mathbf{y}^{1*},\mathbf{y}^*)\right)\;.
$$

Now, for an arbitrary earnings function $\mathbf{y}$ define $A(\mathbf{y}) = \{A(\omega, \mathbf{y})\}_{\omega\in\Omega}$ with

$$A(\omega, \mathbf{y}) = \left(1 + \left(1 + \frac{1}{e}\right) \frac{1 - F(\omega)}{f(\omega)\,\omega} \left(1 - \Gamma^1(\omega \mid \mathbf{y}, \mathbf{y}^*)\right)\right)^{-e} \omega^{1+e} \; .$$

Armed with this notation, we rewrite the previous equation one more time as

$$\mathbf{y}^{1*}(\omega) = A(\omega, \mathbf{y}^{1*}) \; ,$$

for all ω. We also know from the previous arguments that this equation is satisfied for $\mathbf{y}^{1*} = \mathbf{y}^*$.

It proves useful to interpret this equation as characterizing a fixed point in a functional space. Thus, given an arbitrary earnings function $\mathbf{y}$, first interpret $A(\cdot)$ as a functional of the earnings function $\mathbf{y}$ and then define by $\mathbf{y}^{A*}(A(\mathbf{y}))$ the earnings function that satisfies,

$$\mathbf{y}^{A*}(A(\mathbf{y}), \omega) = A(\omega, \mathbf{y}) \; ,$$

for all ω. By interpreting $\mathbf{y}^{A*}$ also as a function of $\mathbf{y}$, we can say that a fixed point of $\mathbf{y}^{A*}$ is an earnings function $\mathbf{y}^{fix}$ with the property that $\mathbf{y}^{A*}(A(\mathbf{y}^{fix})) = \mathbf{y}^{fix}$. By the previous arguments, we also know that $\mathbf{y}^*$ is such a fixed point.

Now, if $\mathbf{y}^*$ is not the best response of party 1, this implies that there must be another solution $\mathbf{y}^{fix} \neq \mathbf{y}^*$ to this fixed point equation. In the following we will rule out this possibility, by showing that, under the conditions of Proposition 4, $\mathbf{y}^{A*}$ is a contraction mapping and therefore has one and only one fixed point.

Consider a metric space of earnings functions equipped with the *sup* metric, i.e. for two earnings functions $\mathbf{y}_a$ and $\mathbf{y}_b$,

$$d\left(\mathbf{y}_a, \mathbf{y}_b\right) := \sup_{\omega\in\Omega} \; \mid \mathbf{y}_a(\omega) - \mathbf{y}_b(\omega) \mid \; .$$

To establish that $\mathbf{y}^{A*}(\cdot)$ is a contraction mapping, we need to show that, for any pair $(\mathbf{y}_a, \mathbf{y}_b)$,

$$d\left(\mathbf{y}^{A*}(A(\mathbf{y}_a)), \mathbf{y}^{A*}(A(\mathbf{y}_b))\right) \leq \delta \; d\left(\mathbf{y}_a, \mathbf{y}_b\right) \; , \tag{78}$$

for some $\delta \in (0, 1)$.

Remember that the analysis proceeds under the assumption that $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$ and $\beta(\omega) \leq \bar{\beta}$, for all ω. In the following, we show that an appropriate choice of $\bar{\alpha}$ and $\bar{\beta}$ ensures that, for any ω, $\mid \mathbf{y}^{A*}(\omega, A(\mathbf{y}_a)) - \mathbf{y}^{A*}(\omega, A(\mathbf{y}_b)) \mid$ becomes arbitrarily small. Note that

$$
\begin{aligned}
&\mid \mathbf{y}^{A*}(\omega, A(\mathbf{y}_a)) - \mathbf{y}^{A*}(\omega, A(\mathbf{y}_b)) \mid \\
&= \omega^{1+e} \mid \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1 - \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \\
&\qquad\qquad - \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1 - \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \mid \ .
\end{aligned}
$$

Moreover, by continuity,

$$\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \to \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)$$

implies

$$
\begin{aligned}
&\mid \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1 - \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \\
&\quad - \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1 - \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \mid \ \to \ 0 \ .
\end{aligned}
$$

Thus, it suffices to show that $\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*)$ is arbitrarily close to $\Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)$ for an appropriate choice of $\bar{\alpha}$ and $\bar{\beta}$.

Let $\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) = u(\mathbf{y}_a, \omega) - u(\mathbf{y}^*, \omega)$. Also let

$$\overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) = \max_{\omega \in \Omega} \ \Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \ ,$$

and

$$\underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) = \min_{\omega \in \Omega} \ \Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \ .$$

It is without loss of generality to assume that $\mathbf{y}_a$ is a Pareto-efficient earnings function which implies that

$$\overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) > 0 > \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) \ .$$

Using the notation introduced in Section D.2 above, we can write

$$\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \ = \ \lambda^1(\mathbf{y}_a, \mathbf{y}^*)\, \mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) + \lambda^2(\mathbf{y}_a, \mathbf{y}^*))\, \mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \ , \quad (79)$$

where

$$\mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = \frac{\int_\omega^{\overline{\omega}} B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega)\ d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega)\ d\omega} = \frac{\int_\omega^{\overline{\omega}} \{\alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \{\alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}$$

and

$$\begin{aligned}\mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) &= \frac{\int_{\omega}^{\overline{\omega}} (1 - B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega))\, d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} (1 - B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega))\, d\omega} \\ &= \frac{\int_{\omega}^{\overline{\omega}} \{1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \{1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}\end{aligned}$$

and, for any pair of earnings functions $(\mathbf{y}^1, \mathbf{y}^2)$

$$\begin{aligned}\lambda^1(\mathbf{y}^1, \mathbf{y}^2) &:= \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \frac{\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)} \\ \lambda^2(\mathbf{y}^1, \mathbf{y}^2) &:= \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \frac{\bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}\end{aligned}$$

so that

$$\lambda^1(\mathbf{y}^1, \mathbf{y}^2) + \lambda^2(\mathbf{y}^1, \mathbf{y}^2) = 1.$$

The assumptions that $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$ and $\beta(\omega) \leq \bar{\beta}$, for all ω, imply that, for all ω,

$$\begin{aligned}\tfrac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) &\geq \quad \alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \\ &\geq \quad \tfrac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)\ ,\end{aligned}$$

and

$$\begin{aligned}\tfrac{1}{2} + \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) &\geq 1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \\ &\geq \tfrac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)\ .\end{aligned}$$

Thus,

$$\begin{aligned}\frac{(1 - F(\omega))(\frac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)} &\geq \mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \\ &\geq \frac{(1 - F(\omega))(\frac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)}\ ,\end{aligned}$$

and

$$\begin{aligned}\frac{(1 - F(\omega))(\frac{1}{2} + \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)} &\geq \mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \\ &\geq \frac{(1 - F(\omega))(\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)}\ ,\end{aligned}$$

which implies that

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = \lim_{\bar{\alpha},\bar{\beta}\to 0} \mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = 1 - F(\omega) \; . \tag{80}$$

Hence, for all ω,

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = 1 - F(\omega) \; , \tag{81}$$

and, by a symmetric argument,

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*) = 1 - F(\omega) \; , \tag{82}$$

for all ω. Equations (79)-(82) imply that

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) - \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*) = 0 \; ,$$

for all ω. Thus, for any pair of functions $(\mathbf{y}_a, \mathbf{y}_b)$, $\bar{\alpha}, \bar{\beta} \to 0$ implies

$$\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \to \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)$$

Thus, for $\bar{\alpha}, \bar{\beta}$ sufficiently close to zero, $\mathbf{y}^{A*}$ is a contraction mapping.

E Case Study: Asymmetric Demobilization in the era of Angela Merkel

We use the theoretical framework in the body of the text for an analysis of German politics between 2005 and 2017. Merkel became the leader of the Christian democrats (CDU, center-right) in 2000 and successfully ran for the chancellory in 2005, 2009, 2013 and 2017. Her main competitor was the Socialdemocratic Party (SPD, center-left).

Policy space $\mathcal{P}$. We consider a policy space of linear income taxes and let the variable ω index an individual's position in the income distribution. One can interpret a redistributive policy platform τ^j of party j either narrowly or broadly. First, as in Meltzer and Richard (1981), the tax rate τ^j can be interpreted as the "size of the government" or of the welfare state, which includes income taxes and monetary transfers but also social insurance, public education, etc. Second, τ^j can be interpreted more broadly as an index of the party's position on the "left-right" axis, a higher value of τ^j corresponding to a more leftist platform. In addition to the previous variables, this broad index would also account for, e.g., the party's stance on the minimum wage, gay marriage, or nuclear energy. These policies also played an important role in Merkel's campaign strategy which is discussed in more detail below.

Party preferences and policy preferences. By convention and without loss of generality, we interpret smaller values of ε as more "conservative" preferences, and larger values of ε as more "liberal" preferences. Thus, we identify party 1 with the CDU, and party 2 with the SPD: given identical policy platforms $\tau^1 = \tau^2$, party 1 (resp., party 2) is overly supported by voters with conservative preferences $\varepsilon < 0$ (resp., liberal preferences $\varepsilon > 0$). In practice, these party preferences may be shaped by party identities, e.g., roots in the worker's movement or the Christian churches, or the cultural milieu from which parties recruit their members. Party preferences may also reflect fixed party positions that are not adjusted in the political campaign. For instance, a salient issue in German politics in the Merkel era was whether families should be supported by direct transfers, as advocated by the CDU, or by publicly-provided childcare, as preferred by the SPD.

E.1 The status quo ante

We assume that potential voters of the SPD have stronger preferences for redistributive policies than potential voters of the CDU.[37] Therefore, we suppose that the electorate of party 1 (CDU) is over-represented among the rich (i.e., high ω), while party 2 (SPD) is over-represented among the poor (i.e., low ω). Thus, for the analysis that follows, we take as a starting point policies (τ^1, τ^2) such that:

(i) The function $B(u(\tau^1, \omega) - u(\tau^2, \omega) \mid \omega)$ is increasing in ω.

[37] For instance, the election outcomes in 2009 and 2013 show the following pattern: the vote shares of SPD and CDU among public servants and white collar workers were, by and large, in line with the parties' overall vote shares, see Jung et al. (2010, 2015). Hence, in absolute numbers, the CDU got more votes from these groups than the SPD. In relative terms, the CDU was stronger among the self-employed and the SPD among workers. The CDU voters also tend to be older and more formally educated. Thus, SPD voters benefit to a larger extent from redistributive policies.

Status quo policy τ^1. The 2005 election was an early election called by Merkel's predecessor from the SPD, Gerhard Schröder. After an adoption of controversial labor market reforms, the SPD had lost various state elections. When the 2005 election was called, the CDU had a strong 21 percent lead over the SPD in opinion polls, and was expected to become by far the strongest party. The CDU decided to run on a pro-market platform, emphasizing the need for deregulation and lower taxes. Over the course of the election campaign, however, the SPD recovered and in the end the CDU won only by a tiny margin of victory: it was only 1 percent ahead of the SPD. Notice that this outcome is consistent with the implications of Proposition 2 discussed in the main text: because it was the clear front runner, the CDU would have maximized its chances of victory by focusing on demobilizing the opposition, rather than mobilizing its own electorate – i.e., by running on a redistributive platform more favorable to the SPD's core supporters rather than taking a fiscally conservative stance to benefit its own base. Therefore, the CDU's status quo policy τ^1 ahead of the 2009 election was "too far" to the right, i.e.:

(ii) $\tau^1 < \tau^{1*}$, where τ^{1*} denotes party 1's best response to τ^2, i.e. to the policy prosed by party 2.

One can show that this condition is satisfied if τ^1 is small enough, or if it is below and close enough to the best response $\tau^{1*}(\tau^2)$ to party 2's policy.

Odds of winning $\bar{\pi}^1(\tau^1, \tau^2)$. In 2009, the CDU was clearly headed for reelection. The polls estimated that the CDU would get 35 percent of the votes, against 25 percent for the SPD and less than 15 percent for all the other parties. A week before election day, Merkel traded at 1.08 (1/12) in the "next Chancellor" market on Betfair – i.e., party 2 was given a chance of winning $1-\bar{\pi}^1(\tau^1, \tau^2)$ of 8 percent. This motivates the following assumption:

(iii) Party 1 is the likely winner of the election, i.e., the probability that party 1 wins is $\bar{\pi}^1(\tau^1, \tau^2) > 1/2$.

E.2 CDU's *Asymmetric Demobilization* strategy

After the federal election in 2005 the CDU adopted the strategy of *asymmetric demobilization.* What defines this strategy is an avoidance of controversial positions or even an adoption of the rival's position in an attempt to lower the turnout of its potential voters. This strategy was successful and continued during the 2013 and 2017 campaigns. The clearest illustration is given by the 2013 official CDU program, which included many policies traditionally advocated by the SPD including the creation of a minimum wage, rent control in tight city areas, a financial transactions tax, a floor on pensions, or tax credits for families and single mothers. In addition, in 2011 Merkel had announced a plan to shut down all nuclear reactors by 2022, a measure traditionally favored by the left-leaning Green party. In 2017, the CDU avoided controversial topics on economic and social policy, and Merkel initiated a parliamentary decision on the question of gay marriage that her SPD opponent had made a central campaign issue – at the cost of alienating her own base. Narrative records of this strategy abound in the national and international press. While such journalistic documentation of the CDU's asymmetric demobilization strategy is overwhelming, it is also apparent in systematic quantitative analyses of party positions by political scientists, as we now describe.

Data sources. The *Manifesto Project*, see Volkens et al. (2018), provides a quantitative text analysis of party manifestos. The text is split into quasi-sentences, units of text that contain one political statement. Quasi-sentences are then assigned to categories such as *Free Market Economy*, *Market Regulation*, *Welfare State Expansion* or *Welfare State Limitation*.[38] Following our discussion of the policy space $\mathcal{P}$ above, we focus on two such indices. See Volkens et al. (2018) for a detailed description of the data set and the methodology.[39]

First, we use the *Welfare State* index, which corresponds to our narrower interpretation of a policy platform τ^j. This index aggregates all of the favourable mentions of the "need to introduce, maintain or expand any public social service or social security scheme ... for example: government funding of health care, child care, elder care and pensions, social housing"; and of "equality: concept of social justice and the need for fair treatment of all people". Second, we use the *Right-Left* index, which corresponds to our broader interpretation of a policy platform τ^j. This index positions a party

[38] The overall analysis is not restricted to economic policy dimensions, but also contains categories for positions on foreign policy, migration, political corruption and others.

[39] An alternative data source is the *Chapel Hill Expert Survey*, see Polk et al. (2017); Bakker et al. (2015). It also provides an analysis of party positions in various dimensions, including a left-versus-right positioning for economic policy issues. It differs from the *Manifesto Project* in that it is based on a survey of expert opinions as opposed to the text of party manifestos. This data set does not yet cover the most recent federal election in Germany in 2017. For the elections between 2002 and 2013 it shows the same pattern as the Party Manifesto data.

manifesto on a one-dimensional policy space by taking the share of quasi-sentences that are indicative of rightist positions (e.g., favorable mentions of military, freedom and human rights, constitutionalism, political authority, free market economy, incentives, economic orthodoxy, welfare state limitation, national way of life, traditional morality, law and order, civic mindedness) and substracts the share of quasi-sentences that are indicative of leftist positions (e.g., favorable mentions of anti-imperialism, peace, internationalism, democracy, economic planning, protectionism, nationalization, welfare state expansion, education expansion, labor groups).

Results. The table below describes how the positions of the CDU and the SPD evolved according to the two indices from the Manifesto Project for the federal elections since 2002. Both indices are normalized to 1 for the SPD in 2002. Larger (resp., smaller) values of the "Welfare State" index mean that the party's manifesto puts stronger (resp., weaker) emphasis on the expansion of the welfare state. Larger (resp., smaller) values of the "Right-Left" index mean that the party's manifesto is located further to the right (resp., left). This table shows clearly that the party positions diverged between the 2002 and 2005 elections. While the SPD reinforced its emphasis on welfare state expansion (the Welfare State index increased from 1 to 1.49) and overall moved further to the left (the Right-Left index decreased from 1 to -0.53), instead the CDU advocated a smaller welfare state (the corresponding index decreased from 0.85 to 0.58) and overall moved further to the right (the corresponding index increased from 5.06 to 6.25). From 2009 onwards, instead, the CDU moved to the left according to both indices: the welfare state index increased continuously from 0.58 in 2005 to 1.08 in 2017, and the right-left index decreased from 6.26 in 2005 to 0.67 in 2017. The two parties moved in parallel: when the SPD moved to the left, so did the CDU. Notice that according to both indices, the CDU was substantially more left-leaning in 2017 than the SPD was in 2002.

	Welfare State		**Right-Left**	
	SPD	**CDU**	**SPD**	**CDU**
2002	1	0.85	1	5.06
2005	1.49	0.58	-0.53	6.25
2009	1.76	0.74	-4.46	2.13
2013	2.14	0.83	-5.75	0.63
2017	1.83	1.08	-5.23	0.67

E.3 Analysis of turnout and election outcomes

In this section we analyze the impact of the CDU's asymmetric demobilization strategy on turnout rates and election results. Our goal is to confront the comparative statics predictions of our model with the outcomes of German elections from 2009 to 2017.

Comparative statics predictions. A major insight of our theoretical analysis is that a party that is leading in the polls has an incentive to adopt a platform that is appealing to the core supporters of its competitor. Thereby the potential voters of the competitor are demobilized. Proposition 6 below is an adaptation of this finding to the German context described above. For convenience, we invoke additional functional form assumptions.

Assumption 4.

a) *Voting costs are linear:* $\lambda = 1$.

b) *Party biases* ε *follow a uniform distribution at each income level: for any* $\omega \in \Omega$ *there exist* $\alpha(\omega) \in (0,1)$ *and* $\beta(\omega) > 0$ *such that* $B(x \mid \omega) = \alpha(\omega) + \beta(\omega)\, x$. *Moreover, the distributions have a wide support and are close to symmetric:*

   There exists $\bar{\beta}$ *close to zero so that* $0 < \beta(\omega) \leq \bar{\beta}$, *for all* ω.

   There exists $\bar{\alpha}$ *close to zero so that,* $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$.

c) *The random variables* η^1 *and* η^2, *defined in Assumption 1, are uniformly distributed on an interval* $[1-\delta, 1+\delta]$ *with* $\delta > 0$.

d) *Party 1 is more right-leaning than party 2:* $\tau^1 \leq \tau^2$ *implies that* $B(u(\tau^1,\omega) - u(\tau^2,\omega) \mid \omega)$ *is increasing in* ω, *and that* $B(u(\tau^1,\underline{\omega}) - u(\tau^2,\underline{\omega}) \mid \underline{\omega}) < \frac{1}{2}$, *i.e., among the very poor there is more support for party 2 than for party 1.*

As the proof below makes clear, Assumption 4 is sufficient, but by no means necessary, to obtain our next result.

Proposition 6. *Suppose that Assumption 4 holds. Consider a one-dimensional policy space and suppose that policy preferences satisfy the single crossing property and are concave. Suppose that* $\mathcal{P} = [\underline{\tau}, \overline{\tau}]$ *is a set of Pareto-efficient tax systems. Consider* $\tau^1, \tau^2 \in (\underline{\tau}, \overline{\tau})$ *with* $\tau^1 < \tau^2$, $\bar{\pi}^1(\tau^1, \tau^2) > \frac{1}{2}$, *and* $\tau^1 < \text{argmax}_{\tau}\ \bar{\pi}^1(\tau, \tau^2)$. *Then a marginal increase of party 1's tax rate has the following implications:*

1. *Party 1's probability of winning increases.*

2. *Party 1's expected vote share increases.*

3. *Overall turnout decreases.*

4. *The demobilization is asymmetric:* σ^{1*}/σ^{2*} *increases.*

A proof of Proposition 6 can be found in Section E.4 below. In the remainder of this section we confront the theoretical predictions in Proposition 6 with the election outcomes in Germany.

Empirical election outcomes. As we discussed above, the strategy of asymmetric demobilization was adopted in the 2009, 2013 and 2017 elections in response to the 2005 experience, in which Merkel learned that running on a platform that appeals to the core voters of her own party could jeopardize an almost sure victory. This strategy paid off: despite a similar lead in the polls in 2009, her margin of victory over the SPD increased from 1 percent in 2005 to more than 10 percent. Overall turnout (70.8 percent) went down by 6.9 percentage points compared to the 2005 election, and was at an all-time low. Crucially, turnout was lower among potential SPD voters than among potential CDU voters: 52 percent of the potential SPD voters indeed voted for the SPD, whereas 62 percent of the potential CDU voters voted for the CDU, see Jung et al. (2010); Forschungsgruppe Wahlen (2013b,a).[40]

In 2013, the CDU moved further left in parallel with the SPD. The election outcome was again a great success for the CDU: it gained 41.5 percent of the votes, was close to an absolute majority in parliament, and was 16 percent ahead of the SPD. Again, mobilization was asymmetric: turnout was 51 percent among the potential SPD voters and 69 percent among the potential CDU voters, see Forschungsgruppe Wahlen (2015). In 2017, the rise of a right-wing populist party implied large losses for the CDU relative to the 2013 election. The SPD also lost, however, and so the CDU stayed more than 12 percent ahead of the SPD. Moreover, it defended its dominant

[40]These numbers are obtained in the following way. The research institute Forschungsgruppe Wahlen runs a monthly survey with a representative sample of voters. The study is known as the *Politbarometer*. Shortly before an election it includes questions on prospective voting behavior. A person who plans to vote SPD or who includes the SPD in the set of conceivable parties is considered a potential SPD voter. Likewise for the CDU. The ratio of actual to potential voters then gives the numbers of 62 percent for the CDU and of 52 percent of the SPD. As a caveat, note that the Politbarometer is not a panel; i.e., it is not tracking the actual voting behavior of the participants in the survey.

position in the German party system: as the only party with more than 30 percent of the votes, every realistic option for government formation had the CDU in the leading role with Merkel as the chancellor. Again, turnout of potential CDU voters (60 percent) was much higher than the turnout of the potential SPD voters (44 percent), see Forschungsgruppe Wahlen (2018). Overall turnout was slightly higher in 2013 and somewhat higher in 2017 than it was in 2009, at 76 percent, but still lower than any turnout ratio observed prior to 2009.

These outcomes are all consistent with our theoretical comparative statics predictions of Proposition 6.

E.4 Proof of Proposition 6

Party 1's probability of winning. If policy preferences are concave, and if part b) of Assumption 4 is satisfied, then one can easily show that the function

$$\Pi^1(\cdot, \tau^2) : \tau^1 \mapsto \Pi^1(\tau^1, \tau^2) := \frac{W^1(\tau^1, \tau^2)}{W^2(\tau^1, \tau^2)} = \frac{\sigma^{1*}(\tau^1, \tau^2)\mathbf{B}^1(\tau^1, \tau^2)}{\sigma^{2*}(\tau^1, \tau^2)\mathbf{B}^2(\tau^1, \tau^2)}$$

is globally concave for every value of τ^2. Moreover, recall that party 1's probability of winning the election is an increasing function of $\Pi^1(\tau^1, \tau^2)$. Thus, for every value of τ^2, there is a unique best response and moving closer to that best response unambiguously increases the winning probability.

Party 1's expected vote share. The total number of votes for party j is equal to $\tilde{V}^j(\tau^1, \tau^2) = \sigma^{j*}(\tau^1, \tau^2)\tilde{\mathbf{B}}^j(\tau^1, \tau^2)$. Hence party 1's expected vote share is equal to

$$\begin{aligned}
\mathbb{E}_\eta \left[\frac{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)}{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^2(\tau^1,\tau^2)} \right] &= \mathbb{E}_\eta \left[\left(1 + \frac{\sigma^{2*}(\tau^1,\tau^2)}{\sigma^{1*}(\tau^1,\tau^2)} \frac{\eta^2 \mathbf{B}^2(\tau^1,\tau^2)}{\eta^1 \mathbf{B}^1(\tau^1,\tau^2)} \right)^{-1} \right] \\
&= \mathbb{E}_\eta \left[\left(1 + \frac{W^2(\tau^1,\tau^2)}{W^1(\tau^1,\tau^2)} \frac{\eta^2}{\eta^1} \right)^{-1} \right] \\
&= \mathbb{E}_\eta \left[\left(1 + \frac{1}{\Pi^1(\tau^1,\tau^2)} \frac{\eta^2}{\eta^1} \right)^{-1} \right] ,
\end{aligned}$$

where expectations are taken with respect to the distribution of $\frac{\eta^2}{\eta^1}$. With $\frac{\partial \Pi^1(\tau^1,\tau^2)}{\partial \tau^1} > 0$, this expression increases in τ^1.

Overall turnout. Expected overall turnout is equal to

$$\begin{aligned}
\Sigma &:= \mathbb{E}_\eta\left[\frac{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^2(\tau^1,\tau^2)}{\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\tilde{\mathbf{B}}^2(\tau^1,\tau^2)}\right] \\
&= \frac{\sigma^{1*}(\tau^1,\tau^2)\mathbf{B}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\mathbf{B}^2(\tau^1,\tau^2)}{\mathbb{E}\left[\bar{q}(\omega)\right]} \\
&= \frac{\sigma^{2*}(\tau^1,\tau^2)\mathbf{B}^2(\tau^1,\tau^2)}{\mathbb{E}\left[\bar{q}(\omega)\right]}\left(1+\frac{W^1(\tau^1,\tau^2)}{W^2(\tau^1,\tau^2)}\right) \\
&= \frac{f_\eta\left(\Pi^1(\tau^1,\tau^2)\right)\Pi^1(\tau^1,\tau^2)W^2(p^1,p^2)}{\kappa\mathbb{E}\left[\bar{q}(\omega)\right]}\left(1+\Pi^1(\tau^1,\tau^2)\right),
\end{aligned}$$

where f_η is the density of the random variable $\frac{\eta^2}{\eta^1}$ and the last equality follows from (28). Denoting by g_η the density of the random variable $\frac{\eta^1}{\eta^2}$, a change of variables implies that $f_\eta(x)\,dx = -g_\eta(\frac{1}{x})d(\frac{1}{x})$, that is, $f_\eta(x) = \frac{1}{x^2}g_\eta(\frac{1}{x})$. Therefore, we can rewrite the previous expression as

$$\Sigma := \frac{W^2(p^1,p^2)}{\kappa\mathbb{E}\left[\bar{q}(\omega)\right]}g_\eta\left(\frac{1}{\Pi^1(\tau^1,\tau^2)}\right)\left(1+\frac{1}{\Pi^1(\tau^1,\tau^2)}\right).$$

We now show that $\partial\Sigma/\partial\tau^1 < 0$. Since all the terms in the expression for Σ are positive, and since $\frac{\partial\Pi^1(\tau^1,\tau^2)}{\partial\tau^1} > 0$ as shown above, the result follows if both $g_\eta\left(1/\Pi^1(\tau^1,\tau^2)\right)$ and $W^2(\tau^1,\tau^2)$ are decreasing in τ^1.

We first show that $g_\eta\left(1/\Pi^1(\tau^1,\tau^2)\right)$ is decreasing in τ^1. Part (c) of Assumption 4 can be shown to imply that[41]

$$g_\eta(x) = \frac{1}{(\bar{\eta}-\underline{\eta})^2}\int \eta^2\mathbb{I}_{\{\eta^2\in[\underline{\eta},\bar{\eta}]\}}\mathbb{I}_{\{\eta^2\in[\underline{\eta}/x,\bar{\eta}/x]\}}d\eta^2\ ,$$

where $\underline{\eta} := 1-\delta$, $\bar{\eta} := 1+\delta$ and $\mathbb{I}$ is the indicator function. Note that if $x > 1$, we have $\underline{\eta}/x < \underline{\eta}$ and $\bar{\eta}/x < \bar{\eta}$, so that $[\underline{\eta}/x,\bar{\eta}/x]\cap[\underline{\eta},\bar{\eta}] = [\underline{\eta},\bar{\eta}/x]$. Conversely, if $x < 1$,

[41] To see this, note that

$$\begin{aligned}
G_\eta(x) &= \text{prob}(\tfrac{\eta^1}{\eta^2}\leq x) \\
&= \textstyle\int \text{prob}(\eta^1 \leq x\eta^2 \mid \eta^2)\mu^2(\eta^2)d\eta^2 \\
&= \textstyle\int \frac{x\eta^2-\underline{\eta}}{\bar{\eta}-\underline{\eta}}\mathbb{I}_{\{\eta^2\in[\underline{\eta}/x,\bar{\eta}/x]\}}\mu^2(\eta^2)d\eta^2
\end{aligned}$$

where $\mu^2(\eta^2) = \frac{\mathbb{I}_{\{\eta^2\in[\underline{\eta},\bar{\eta}]\}}}{\bar{\eta}-\underline{\eta}}$ is the density of η^2. Computing the derivative with respect to x yields the expression for $g_\eta(x)$ in the text.

we have $[\underline{\eta}/x, \bar{\eta}/x] \cap [\underline{\eta}, \bar{\eta}] = [\underline{\eta}/x, \bar{\eta}]$. Therefore,

$$g_\eta(x) = \frac{1}{(\bar{\eta}-\underline{\eta})^2} \begin{cases} \int_{\underline{\eta}/x}^{\bar{\eta}} \eta^2 d\eta^2 & \text{if } x \leq 1, \\ \int_{\underline{\eta}}^{\bar{\eta}/x} \eta^2 d\eta^2 & \text{if } x > 1. \end{cases}$$

We easily obtain that, for any $x \leq 1$,

$$g_\eta(x) = \frac{\bar{\eta}^2 - \underline{\eta}^2/x^2}{2(\bar{\eta}-\underline{\eta})^2},$$

which is increasing in x. It is, moreover, straightforward to verify that $G_\eta(1) = 1/2$. Now recall that party 1's probability of winning increases in response to the deviation considered in this proof, so that $\partial[1/\Pi^1(\tau^1,\tau^2)]/\partial\tau^1 < 0$. But party 2's probability of winning, which is the probability of the event $\frac{\sigma^2}{\sigma^1}\frac{\mathbf{B}^2(p^1,p^2)}{\mathbf{B}^1(p^1,p^2)} \geq \frac{\eta^1}{\eta^2}$, is equal to $\bar{\pi}^2(\tau^1,\tau^2) = G_\eta(1/\Pi^1(\tau^1,\tau^2))$. But $\bar{\pi}^2(\tau^1,\tau^2) = 1 - \bar{\pi}^1(\tau^1,\tau^2) < 1/2$ by assumption. Thus, we must have $1/\Pi^1(\tau^1,\tau^2) < 1$. Therefore, $g_\eta(1/\Pi^1(\tau^1,\tau^2))$ is locally decreasing in τ^1, and hence $\partial g_\eta(1/\Pi^1(\tau^1,\tau^2))/\partial\tau^1 < 0$ in response to party 1's deviation.

We now show that $W^2(\tau^1,\tau^2)$ is decreasing. First, recall that $\Pi^1(\tau^1,\tau^2) = \frac{W^1(\tau^1,\tau^2)}{W^2(\tau^1,\tau^2)}$, or, equivalently,

$$H_s(W^1(\tau^1,\tau^2)) - H_s(W^2(\tau^1,\tau^2))$$

increases in τ^1. Further note that

$$\frac{\partial W^1(\tau^1,\tau^2)}{\partial\tau^1} = \mathbb{E}\left[B\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) u_1(\tau^1,\omega)\right]$$

and

$$\frac{\partial W^2(\tau^1,\tau^2)}{\partial\tau^1} = -\mathbb{E}\left[\left(1 - B\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right)\right) u_1(\tau^1,\omega)\right]$$

Thus, $\Pi^1(\tau^1,\tau^2)$ increasing in τ^1 is equivalent to

$$\mathbb{E}\left[\left\{B(\Delta u(\omega) \mid \omega) + \frac{h_s(W^2)}{h_s(W^1)}(1 - B(\Delta u(\omega) \mid \omega))\right\} u_1(\tau^1,\omega)\right] > 0\ ,$$

or, once more, equivalently,

$$\mathbb{E}\left[\left\{(1-\lambda')B(\Delta u(\omega)\mid\omega)+\lambda'(1-B(\Delta u(\omega)\mid\omega))\right\}u_1(\tau^1,\omega)\right]>0\ , \tag{83}$$

where $\Delta u(\omega)$ is a shorthand for $u(\tau^1,\omega)-u(\tau^2,\omega)$, and λ' for $\frac{h_s(W^2)}{h_s(W^1)}\left(1+\frac{h_s(W^2)}{h_s(W^1)}\right)^{-1}$. By the single crossing property $u_1(\tau^1,\omega)$ is decreasing in ω. Also, since τ^1 is, by assumption, an interior policy, $u_1(\tau^1,\omega)$ is positive for small values of ω and negative for large values of ω.

We now proceed by contradiction. Suppose, contrary to what we seek to show, that $\frac{\partial W^2(\tau^1,\tau^2)}{\partial\tau^1}\geq 0$, or, equivalently, that

$$\mathbb{E}\left[(1-B(\Delta u(\omega)\mid\omega))\ \ u_1(\tau^1,\omega)\right]\leq 0\ . \tag{84}$$

Now compare the weighting functions

$$\gamma'(\omega):=(1-\lambda')B(\Delta u(\omega)\mid\omega)+\lambda'(1-B(\Delta u(\omega)\mid\omega)$$

and

$$\gamma(\omega):=1-B(\Delta u(\omega)\mid\omega)\ .$$

Since by hypothesis $B(\Delta u(\omega)\mid\omega)$ is increasing in ω, γ' puts less weight on low values of ω, corresponding to positive values of $u_1(\tau^1,\omega)$, and more weight on high values of ω, where $u_1(\tau^1,\omega)$ takes negative values. Therefore, (84) implies that

$$\mathbb{E}\left[\left\{(1-\lambda')B(\Delta u(\omega)\mid\omega)+\lambda'(1-B(\Delta u(\omega)\mid\omega))\right\}u_1(\tau^1,\omega)\right]<0\ ,$$

a contradiction (83). Thus, the assumption that $\frac{\partial W^2(\tau^1,\tau^2)}{\partial\tau^1}\geq 0$ has led to a contradiction and must be false.

Relative turnout. Finally, we show that the relative turnout σ^{1*}/σ^{2*} increases. We have

$$\frac{\sigma^{1*}(\tau^1,\tau^2)}{\sigma^{2*}(\tau^1,\tau^2)}=\Pi^1(\tau^1,\tau^2)\frac{\mathbb{E}\left[\bar{q}\left(\omega\right)\right]-\mathbf{B}^1(\tau^1,\tau^2)}{\mathbf{B}^1(\tau^1,\tau^2)}.$$

In response to party 2's deviation, we have

$$\frac{\partial \mathbf{B}^1(\tau^1,\tau^2)}{\partial \tau^1} = \mathbb{E}\left[\bar{q}(\omega)\, b\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) \frac{\partial u\left(\tau^1,\omega\right)}{\partial \tau^1}\right]$$

$$= \mathbb{E}\left[\bar{q}(\omega)\, b\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) \left(-y^1(\omega) + \mathbb{E}y^1 - \frac{\tau^1}{1-\tau^1} e\mathbb{E}\left[y^1(\omega)\right]\right)\right].$$

Part b) of Assumption 4 implies that this derivative is close to zero. Hence, the result follows immediately from the fact that $\Pi^1(\tau^1,\tau^2)$ increases in τ^1.

F Alternative Settings

F.1 A model that includes ethical voters who always vote

We now assume that the base of each party is split into three groups: a group that always votes, a group that always abstains, and a group of voters whose voting decision follows from a rule-utilitarian calculation. We denote by $\tilde{q}^{jv}(\omega)$ the fraction of definite voters among the type ω supporters of party j, by $\tilde{q}^{ja}(\omega)$ the fraction of definite abstainers and by $\tilde{q}^{ju}(\omega)$ the fraction of rule-utilitarian or ethical supporters, with $\tilde{q}^{jv}(\omega)+\tilde{q}^{ja}(\omega)+\tilde{q}^{ju}(\omega) = 1$. We assume that these are random quantities both from the perspective of parties when choosing platforms and from the perspective of voters when choosing whether or not to vote. We write $\tilde{q}^j = \{\tilde{q}^{jv}(\omega), \tilde{q}^{ja}(\omega), \tilde{q}^{ju}(\omega)\}_{\omega\in\Omega}$ for the collection of random variables that refer to party j. We denote the expected value of the random variable $\tilde{q}^{ju}(\omega)$ by $\bar{q}^{ju}(\omega)$. The total number of votes for party 1 is then a random variable equal to

$$\tilde{V}^1(p^1,p^2,\sigma^1,\tilde{q}^1) = \mathbb{E}[(\tilde{q}^{1v}(\omega) + \sigma^1\ \tilde{q}^{1u}(\omega))B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)]\ .$$

Analogously, the total number of votes for party 2 equals

$$\tilde{V}^2(p^1,p^2,\sigma^2,\tilde{q}^2) = \mathbb{E}[(\tilde{q}^{2v}(\omega) + \sigma^2\ \tilde{q}^{2u}(\omega))(1 - B(u(p^1,\omega)\ - u(p^2,\omega) \mid \omega))]\ .$$

We assume throughout that $\mu \to \infty$, so that the per capital cost of voting is equal to $\kappa\sigma^j$.

Assume furthermore that the random variables $\tilde{q}^1$ and $\tilde{q}^2$ are driven by aggregate shocks that affect the shares of definite and rule-utilitarian voters one the one hand and of definite abstainers on the other so that the following two properties are satisfied. First, the ratio of definite and rule-utilitarian voters is not subject to randomness; i.e., shocks affect the ratio of potential voters to definite abstainers without affecting the internal composition of the set of potential voters. Second, among the supporters of party j, the ratio of definite to rule-utilitarian voters is the same for all types.

Assumption 5. *There is a pair of independent random variables, η_1 and η_2, so that, for all ω,*

$$\tilde{q}^{1v}(\omega) = \bar{q}^{1v}(\omega)\ \eta_1 \quad \textit{and} \quad \tilde{q}^{1u}(\omega) = \bar{q}^{1u}(\omega)\ \eta_1$$

and

$$\tilde{q}^{2v}(\omega) = \bar{q}^{2v}(\omega)\ \eta_2 \quad \textit{and} \quad \tilde{q}^{2u}(\omega) = \bar{q}^{2u}(\omega)\ \eta_2.$$

In addition, there are numbers q^{1v}, q^{1u}, q^{2v} and q^{2u} so that, for all ω

$$\bar{q}^{1v}(\omega) = q^{1v} \quad \textit{and} \quad \bar{q}^{1u}(\omega) = q^{1u}$$

and

$$\bar{q}^{2v}(\omega) = q^{2v} \quad \textit{and} \quad \bar{q}^{2u}(\omega) = q^{2u}\ .$$

Under Assumption 5 the total number of votes for party 1 can be written as

$$\tilde{V}^1(p^1, p^2, \sigma^1, \tilde{q}^1) = \eta^1\ V^1(p^1, p^2, \sigma^1)$$

where $V^1(p^1, p^2, \sigma^1) := m^1(\sigma^1)\ \mathbf{B}^1(p^1, p^2)$ and $m^1(\sigma^1) := q^{1v} + \sigma^1\ q^{1u}$ is a multiplier that determines how party 1's base $\mathbf{B}^1(p^1, p^2)$ is transformed into actual votes. Analogously, the votes for party 2 are given by $\tilde{V}^2(p^1, p^2, \sigma^2, \tilde{q}^2) = \eta^2\ V^2(p^1, p^2, \sigma^2)$, where $V^2(p^1, p^2, \sigma^2) := m^2(\sigma^2)\ \mathbf{B}^2(p^1, p^2)$ and $m^2(\sigma^2) = q^{2v} + \sigma^2\ q^{2u}$. Armed with this notation, we can express the probability that party 1 wins as

$$\pi^1(p^1, p^2, \sigma^1, \sigma^2) = P\left(\frac{V^1(p^1, p^2, \sigma^1)}{V^2(p^1, p^2, \sigma^2)}\right) = P\left(\frac{m^1(\sigma^1)}{m^2(\sigma^2)} \frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}\right)\ , \tag{85}$$

where P is the *cdf* of the random variable η^2/η^1. Its density function is denoted by ρ. Note that imposing Assumption 5 implies a multiplicative separability between the term

$$R^\sigma(p^1, p^2) = \frac{m^1(\sigma^1(p^1, p^2))}{m^2(\sigma^2(p^1, p^2))}\ , \tag{86}$$

that is shaped by the rule-utilitarian voter's participation thresholds and the ratio of their bases

$$R^{\mathbf{B}}(p^1, p^2) = \frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}$$

so that we can write

$$\pi^1(p^1, p^2, \sigma^1, \sigma^2) = P\left(R^{\sigma}(p^1, p^2)\; R^{\mathbf{B}}(p^1, p^2)\right)\ . \tag{87}$$

Turnout. For now, we take the party platforms p^1 and p^2 as given and characterize the parties' equilibrium turnout. We say that the turnout game has an interior equilibrium if $0 < \sigma^{1*}(p^1, p^2) < 1$ and $0 < \sigma^{2*}(p^1, p^2) < 1$. If the function P is continuously differentiable then an interior equilibrium is characterized by the first order conditions

$$\pi^1_{\sigma^1}(\cdot)\; W^1 - \kappa\, q^{1u}\, \mathbf{B}^1 = 0\ , \tag{88}$$

and

$$-\,\pi^1_{\sigma^2}(\cdot)\; W^2 - \kappa\, q^{2u}\, \mathbf{B}^2 = 0\ . \tag{89}$$

Using Assumption 5 we can rewrite these conditions as

$$\frac{\rho(\cdot) R^{\sigma}(p^1, p^2)}{q^{1v} + \sigma^1\, q^{1u}}\; W^1 - \kappa\, \mathbf{B}^1 = 0\ , \tag{90}$$

and

$$\frac{\rho(\cdot) R^{\sigma}(p^1, p^2)}{q^{2v} + \sigma^2\, q^{2u}}\; W^2 - \kappa\, \mathbf{B}^2 = 0\ . \tag{91}$$

Equations (90) and (91) imply that

$$R^{\sigma}(p^1, p^2) = \frac{W^1/\kappa\, \mathbf{B}^1}{W^2/\kappa\, \mathbf{B}^2} = \frac{W^1/\mathbf{B}^1}{W^2/\, \mathbf{B}^2}\ , \tag{92}$$

which is the same expression as (29) in the body of the text.

Probability of winning. Suppose first that parties seek to maximize the probability of winning, i.e.,

$$P\left(R^{\sigma}(p^1, p^2)\; \frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}\right)$$

and party 2 seeks to minimize this expression. As P is a non-decreasing function we can as well assume that party 1 maximizes

$$R^{\sigma}(p^1, p^2)\, \frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}$$

or any monotone transformation of it such as, e.g.,

$$\ln\left(R^{\sigma}(p^1, p^2)\right) + \ln\left(\mathbf{B}^1(p^1, p^2)\right) - \ln\left(\mathbf{B}^2(p^1, p^2)\right) \ . \tag{93}$$

Remark 1. The "conventional" probabilistic voting model can be viewed as a special case of this that is defined by two properties. First, since turnout is exogenous and universal, $R^{\sigma}(p^1, p^2) = 1$ for all (p^1, p^2) and hence $\ln\left(R^{\sigma}(p^1, p^2)\right) = 0$. Second, and again for the reason that turnout is exogenous and universal, $V^1 = \mathbf{B}^1(p^1, p^2)$ and $V^2 = \mathbf{B}^2(p^1, p^2) = 1 - \mathbf{B}^1(p^1, p^2)$. In the probabilistic voting model, the objective of party 1 can therefore be taken to be $\ln\left(\mathbf{B}^1(p^1, p^2)\right) - \ln\left(1 - \mathbf{B}^1(p^1, p^2)\right)$ or simply $V^1 = \mathbf{B}^1(p^1, p^2)$. I.e., maximizing the probability of winning is the same as maximizing the number of votes.

Remark 2. With Nash equilibrium rather than subgame perfect equilibrium as the solution concept (or, equivalently, with $\mu = 0$), the parties view $R^{\sigma}(p^1, p^2)$ as exogenously fixed, albeit at the level that is induced by the equilibrium policies. Party 1 then seeks to maximize

$$\ln\left(\mathbf{B}^1(p^1, p^2)\right) - \ln\left(\mathbf{B}^2(p^1, p^2)\right)$$

and party 2 seeks to minimize this expression. Since $\mathbf{B}^2(p^1, p^2) = 1 - \mathbf{B}^1(p^1, p^2)$, party 1's objective can as well simply taken to be $\mathbf{B}^1(p^1, p^2)$ and $\mathbf{B}^2(p^1, p^2)$ can be taken to be the objective of party 2. Nash equilibrium then requires that p^1 solves $\max_{\hat{p}^1 \in P}\ \mathbf{B}^1(\hat{p}^1, p^2)$ and that p^2 solves $\max_{\hat{p}^2 \in P}\ \mathbf{B}^2(p^1, \hat{p}^2)$. Note that these equilibrium are also the equilibrium conditions in the "conventional" probabilistic voting model. Thus, equilibrium existence in the "conventional" probabilistic voting model implies the existence of a Nash equilibrium in the given setup.

If the turnout subgame has an interior equilibrium, then the probability of winning for party 1 can be written in a reduced form that no longer involves an explicit reference to the participation thresholds σ^1 and σ^2. Specifically, equation (92) implies that the winning probability in (87) becomes

$$\bar{\pi}^1(p^1,p^2) = P\left(\Pi^1(p^1,p^2)\right) \quad \text{for} \quad \Pi^1(p^1,p^2) := \frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\,. \tag{94}$$

Thus, as in the main body of the text (Proposition 1), under Assumption 5, if (p^1, p^2) is a pair of interior subgame perfect equilibrium policies, then it it is a saddle point of the function Π.

F.2 Public goods

Our framework for studying endogenous turnout and endogenous platforms in political competition is developed for a generic policy domain. We have emphasized that the set of non-linear income tax systems is a policy domain of particular interest. That said, our framework can also be applied to study the implications of endogenous turnout for political competition over other policy domains. In this section, we briefly summarize the results from such an analysis. Specifically, we report on the implications of our framework for public goods provision.

Individuals have preferences over public goods that are given by $u(\omega,p) = \omega\, p - k(p)$, where $p \in \mathbb{R}_+$ denotes the quantity of the public good, $\omega \in \Omega$ is an individual's public goods preference and the cost function k captures the per capita cost of public goods provision.[42] We begin with a characterization of the public good provision level that party 1 would choose if its sole objective was to mobilize its supporters. In this case, it would choose q^1 with the objective to maximize

$$W^1(p^1,p^2) = \mathbb{E}[G^1_W(\omega\, p^1 - k(p^1) - u(p^2,\omega) \mid \omega)]\,,$$

where we denote by

$$\begin{aligned} G^1_W(x \mid \omega) &:= \int_{-\infty}^{x} (x-\varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon \\ G^2_W(x \mid \omega) &:= \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon. \end{aligned}$$

Note that the derivatives of the functions $G^1_W(\cdot \mid \omega)$ and $G^2_W(\cdot \mid \omega)$ are respectively

[42]In an economy with a continuum of individuals and private information on public goods preferences, equal cost sharing is the only way of satisfying robust incentive compatibility, budget balance and anonymity, see Bierbrauer and Hellwig (2016).

given by

$$
\begin{aligned}
g_W^1(x \mid \omega) &:= B(x \mid \omega) \\
g_W^2(x \mid \omega) &:= -(1 - B(x \mid \omega)) .
\end{aligned}
$$

Given p^2, the first order condition characterizing the optimal choice of p^1 is

$$\mathbb{E}\left[\mathcal{G}_W^1(\omega \mid p^1, p^2)\, \omega\right] = k'(p^1)$$

where

$$\mathcal{G}_W^1(\omega \mid p^1, p^2) = \frac{g_W^1(u(p^1,\omega) - u(p^2,\omega) \mid \omega)}{\mathbb{E}[g_W^1(u(p^1,\omega) - u(p^2,\omega) \mid \omega)]} .$$

This first order condition is a political economy analogue to the Samuelson rule for first-best public good provision. For the given setup, the Samuelson rule stipulates that $\mathbb{E}[\omega] = k'(p)$, i.e., it requires equal weights for all public goods preferences. For the purpose of mobilizing its supporters, party 1 does not apply equal weights. Instead the public good preferences of different individuals are weighted according to the function $\mathcal{G}_W^1$. The public good provision level that party 1 would choose if it only wanted only to demobilize the supporters of party 2 is such that

$$\mathbb{E}\left[\mathcal{G}_W^2(\omega \mid p^1, p^2)\, \omega\right] = k'(p^1) ,$$

and the policy that maximizes $\frac{W^1(p^1 p^2)}{W^2(p^1,p^2)}$ satisfies

$$\mathbb{E}[\mathcal{G}_{SP}^1(\omega \mid p^1, p^2)\, \omega] = k'(p^1)$$

where

$$\mathcal{G}_{SP}^1(\omega \mid p^1, p^2) := \lambda^1(p^1,p^2)\, \mathcal{G}_W^1(\omega \mid p^1, p^2) + (1 - \lambda^1(p^1,p^2))\, \mathcal{G}_W^2(\omega \mid p^1, p^2)$$

and $\lambda^1(p^1, p^2)$ is defined by

$$\lambda^1(p^1,p^2) \;:=\; \frac{1}{1 + \Pi^1(p^1,p^2)} \frac{\bar{g}_W^1(\underline{\omega} \mid p^1, p^2)}{\mathbb{E}[\gamma^1(\omega \mid p^1, p^2)]}$$

with

$$\gamma^1(\omega \mid p^1, p^2) := \frac{1}{1+\Pi^1(p^1,p^2)} g_W^1(u^1(\omega) - u^2(\omega) \mid p^1, p^2) + \frac{\Pi^1(p^1,p^2)}{1+\Pi^1(p^1,p^2)} g_W^2(u^1(\omega) - u^2(\omega) \mid p^1, p^2).$$

Again, the party compromises between mobilizing its own supporters and demobilizing the supporters of the other party – with the weight on the own supporters being smaller if the party is more likely to win.

F.3 Alternative modelling choices for ethical voting

The ethical voter models by Feddersen and Sandroni (2006), on the one hand, and by Coate and Conlin (2004), on the other differ, in some aspects. For instance, Feddersen and Sandroni (2006) assume that the population consists of ethical voters and of non-ethical voters. Moreover, the fraction of ethical voters is a priori uncertain. Uncertainty over election outcomes in Feddersen and Sandroni (2006) is entirely due to this uncertainty about the fraction of ethical voters. Coate and Conlin (2004), by contrast, assume that all voters are ethical voters. Uncertainty over election outcomes in their framework is driven by uncertainty over the policy preferences of these ethical voters.

In this section of the Online-Appendix we show that these modelling choices are not essential for our main results. We could go either way. In the main text, we present an analysis that adopts the framework of Feddersen and Sandroni (2006). We show that we could as well work with the model of Coate and Conlin (2004) in Section F.3.2.

For tractability, our adaptation of Feddersen and Sandroni makes use of an assumption which implies that the parties' bases add up to a constant. An advantage is that it becomes transparent that the standard probabilistic voting model is nested as a special case of our analysis. In Section F.3.3 we present an extension that does not rest on this assumption. The extension shows that the parties' trade-off between attracting swing voters, mobilizing their own core voters and demobilizing the opponent's core voters is at the heart of our analysis, with or without the assumption that the parties' bases add up to a constant.

The bottom line is that what is really driving our results is the combination of probabilistic and ethical voting. We use the probabilistic voting model to determine preferences over policies and parties. We use a model of ethical voting to determine turnout. How exactly we model ethical voting is of secondary importance. Our analysis is robust to alternative specifications of ethical voting.

F.3.1 A general framework

We begin with a general framework for political competition that connects probabilistic and ethical voting. As will become clear, the general framework contains as special cases

- an environment where all voters are ethical voters and with uncertainty about policy preferences as in Coate and Conlin (2004),
- an environment where the population share of ethical voters is a random quantity as in Feddersen and Sandroni (2006).

Party and policy preferences. Consider a pair of policies p^1 and p^2 proposed by parties 1 and 2, respectively. As in the body of the text, a type ω-individual preferes a victory of party 1 if

$$u(p^1, \omega) - u(p^2, \omega) \geq \varepsilon \ ,$$

where the utility function u captures policy preferences and the variable ε captures idiosyncratic party preferences. We assume that, conditional on ω, ε is a random variable with a distribution that can be represented by a cumulative distribution function $\tilde{B}(\cdot \mid \omega, \eta)$. Thus,

$$\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)$$

is the fraction of type ω-voters who are better off if party 1 wins. The complement

$$1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)$$

is the fraction of type ω-voters who are better off if party 2 wins.

The formulation differs from the one in the main text in that we allow these distributions to be random objects themselves. The distributions of idiosyncratic party preferences now depend on the realization of a random variable η. Thus, we allow for uncertainty in policy preferences as in Coate and Conlin (2004).

Example: Aggregate uncertainty on preferences. At this stage there is no need to introduce more specific assumptions about η. Still, the following example might be helpful to get an idea of what the randomness in party and policy preferences might look like: For any type ω, there is a set of feasible distributions $\Phi(\omega)$, with generic entry $\tilde{B}(\cdot \mid \omega, \eta)$. Distributions in this set can be ordered according to first order stochastic dominance. Let this order be represented by a mapping from the unit interval to the set of feasible distributions. Also suppose that there is a random variable η_ω taking values in the unit interval, indicating which of these distributions materializes. Finally, let there be one such a random variable for each type ω. Then the random variable η that governs the *state of the system* is a stochastic process that can be written as $\eta = \{\eta_\omega\}_{\omega \in \Omega}$.

In Feddersen and Sadroni (2006), by contrast, party and policy preference are deterministically fixed once the alternatives p^1 and p^2 are given. The following assumption contains a more formal version of this statement.

Assumption 6 (**Feddersen and Sandroni: No aggregate uncertainty on preferences**). *For every type ω, there exists a cumulative distribution function $B(\cdot \mid \omega)$, so that, for all p^1 and p^2 and all possible realizations of η,*

$$\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta) = B(u(p^1, \omega) - u(p^2, \omega) \mid \omega) \ .$$

Ethical and non-ethical voters. A complete description of the state of the system also requires to specify, for each type, the fraction of ethical voters. Formally, let $\tilde{q}^1(\omega, \eta)$ be the fraction of ethical voters among those type ω-individuals who are better off if party 1 wins. Likewise denote by $\tilde{q}^2(\omega, \eta)$ be the fraction of ethical type ω-individuals who are better off if party 2 wins. In the approach of Feddersen and Sandroni these are random objects. Here, we capture this again, through the dependence on an aggregate shock, or, more specifically, the random variable η. By contrast, in the model of Coate and Conlin, $\tilde{q}^1$ and $\tilde{q}^1$ are set equal to one. For ease of reference, we also highlight this assumption.

Assumption 7 (**Coate and Conlin: All voters are ethical voters**). *For any ω, $\tilde{q}^1(\omega, \eta)$ and $\tilde{q}^2(\omega, \eta)$ are degenerate random variables so that*

$$\tilde{q}^1(\omega, \eta) = \tilde{q}^2(\omega, \eta) = 1$$

for all realizations of η.

The parties' bases. The potential voters of party 1 are those who vote for party 1 in case of turning out to vote. This mass of these voters is a random variable

$$\tilde{\mathbf{B}}^1(p^1, p^2, \eta) := \mathbb{E}\left[\tilde{q}^1(\omega, \eta)\ \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right]$$

Analogously, the mass party 2's potential voters is given by

$$\tilde{\mathbf{B}}^2(p^1, p^2, \eta) := \mathbb{E}\left[\tilde{q}^2(\omega, \eta)(1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta))\right]$$

We denote, respectively, by

$$\mathbf{B}^1(p^1, p^2) = \int \tilde{\mathbf{B}}^1(p^1, p^2, \eta)\ dP(\eta)$$

and

$$\mathbf{B}^2(p^1, p^2) = \int \tilde{\mathbf{B}}^2(p^1, p^2, \eta)\ dP(\eta)$$

the expected values of $\tilde{\mathbf{B}}^1(p^1, p^2, \eta)$ and $\tilde{\mathbf{B}}^2(p^1, p^2, \eta)$, where P is the cumulative distribution function of the random variable η. For brevity, we also refer to $\mathbf{B}^1(p^1, p^2)$ and $\mathbf{B}^2(p^1, p^2)$ as the parties' bases.

The turnout subgame. As in the main text, the ethical voters of party 1 choose σ^1 to maximize

$$\pi^1(\sigma^1, \sigma^2, p^1, p^2)\ W^1(p^1, p^2) - k(\sigma^1)\ \mathbf{B}^1(p^1, p^2)$$

and the ethical voters of party 2 choose σ^2 to maximize

$$(1 - \pi^1(\sigma^1, \sigma^2, p^1, p^2))\ W^2(p^1, p^2) - k(\sigma^2)\ \mathbf{B}^2(p^1, p^2)\ .$$

We have to adjust, however, our definitions of $W^1(p^1, p^2)$ and $W^2(p^1, p^2)$ so that they are consistent with the more general setup that we are currently exploring. We now let

$$\tilde{W}^1(p^1, p^2, \eta) = \mathbb{E}\left[\int_{\mathbb{R}} \max\left\{\ u(p^1, \omega) - [u(p^2, \omega) + \varepsilon]\ ,\ 0\right\}\ \tilde{b}(\varepsilon \mid \omega, \eta)\, \mathrm{d}\varepsilon\right]. \qquad (95)$$

denote the stakes for the ethical voters of party 1 in state η and let $W^1(p^1,p^2)$ be the expectation of $\tilde{W}^1(p^1,p^2,\eta)$, conditional on party 1 winning the election. We denote by $\tilde{b}(\cdot \mid \omega,\eta)$ the derivative of $\tilde{B}(\cdot \mid \omega,\eta)$, i.e. $\tilde{b}(\cdot \mid \omega,\eta)$ is the density of ε, conditional on type ω and state η. We define $\tilde{W}^2(p^1,p^2,\eta)$ and $W^2(p^1,p^2)$ analogously. Party 1 wins the election if

$$\sigma^1 \, \tilde{\mathbf{B}}^1(p^1,p^2,\eta) \geq \sigma^2 \, \tilde{\mathbf{B}}^2(p^1,p^2,\eta) \ ,$$

where σ^1 and σ^2 are the turnout rates of the potential voters of party 1 and party 2, respectively. Equivalently, party 1 wins if

$$\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1 \ .$$

The probability that party 1 wins the election is therefore given by

$$\pi^1(\sigma^1,\sigma^2) = \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1\right) \ .$$

For later reference, note that we can also write this winning probability as an average winning probability over the different states η, i.e. so that

$$\bar{\pi}^1(p^1,p^2) = \int \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1 \mid \eta\right) dP(\eta) \ . \tag{96}$$

Note that the turnout rates enter this expression only via the ratio $\frac{\sigma^1}{\sigma^2}$. This implies that our analysis of the turnout subgame – for given policies p^1 ans p^2 – does not depend on wether we adopt the Feddersen-Sandroni or the Coate-Conlin formulation. As a consequence, Lemma 1 in the main text goes through. Thus, irrespectively of whether Assumption 7 or Assumption 6 is imposed, in an equilibrium of the turnout subgame

$$\frac{\sigma^{1*}(p^1,p^2)}{\sigma^{2*}(p^1,p^2)} = \left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^{\lambda} \ . \tag{97}$$

F.3.2 Adopting the approach of Coate and Conlin: Only ethical voters

We now impose Assumption 7, i.e. the Assumption made by Coate and Conlin (2004) that there are only ethical voters. Thus, to have non-trivial winning probabilities, we must not at the same time impose Assumption 6. Put differently, we suppose that policy preferences are subject to aggregate shocks. We will establish two findings: First, our Proposition 1 in the main text rests on a simplifying assumption on the nature of aggregate uncertainty. The same assumption can be made in the Coate and Conlin version of our model and has the same effect. Proposition 1 is therefore robust to the way in which we model ethical voting. Second, the parties' bases add up to a constant. A model of ethical voting that does not share this property therefore requires to relax Assumption 7.

Recall equations (96) and (97), i.e. that taking the endogeneity of turnout into account, the probability of winning can be written as

$$\bar{\pi}^1(p^1, p^2) = \int \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1, p^2, \eta)}{\tilde{\mathbf{B}}^2(p^1, p^2, \eta)} \geq 1 \mid \eta \right) dP(\eta) \ ,$$

where

$$\frac{\sigma^1}{\sigma^2} = \left[\frac{W^1(p^1, p^2) \,/\, \mathbf{B}^1(p^1, p^2)}{W^2(p^1, p^2) \,/\, \mathbf{B}^2(p^1, p^2)} \right]^{\lambda} \ .$$

In principle, there is no problem to working directly with this objective, it gives raise to the same tradeoffs as those highlighted in our manuscript. For a tractable comparative statics analysis, we would, however, have to impose (possibly non-parametric) assumptions on how different realizations of the random variable η shift the distributions $\tilde{B}(\cdot)$. Our Assumption 1 in the main text is one conceivable way of doing this, a way that has the advantage of simplicity. The main text focuses on the Feddersen-Sandroni version of ethical voting and Assumption 1 is imposed in this context. As we will now explain, we can get to same conclusions also with a Coate-Conlin approach. To see this, consider the following assumption.

Assumption 8 (Multiplicative shocks I). *Suppose that $\eta = (\eta^1, \eta^2)$ is a pair of two random variables η^1 and η^2 so that*

$$\tilde{\mathbf{B}}^1(p^1, p^2, \eta) = \eta^1 \, \mathbf{B}^1(p^1, p^2) \tag{98}$$

and

$$\tilde{\mathbf{B}}^2(p^1, p^2, \eta) = \eta^2 \, \mathbf{B}^2(p^1, p^2) \ . \tag{99}$$

Under Assumption 8 the aggregate shocks η^1 and η^2 can be interpreted as percentage deviations of the random variables $\tilde{\mathbf{B}}^1(p^1, p^2, \eta)$ and $\tilde{\mathbf{B}}^2(p^1, p^2, \eta)$ from their respective means. To see this, suppose that the means of both η^1 and η^2 are equal to

1 and rewrite (98) and (99) as

$$\frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)-\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^1(p^1,p^2)}=\eta^1-1$$

and

$$\frac{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)-\mathbf{B}^2(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}=\eta^2-1$$

The left hand sides of these equations give the percentage deviation of the random variables $\tilde{\mathbf{B}}^1(p^1,p^2,\eta)$ and $\tilde{\mathbf{B}}^2(p^1,p^2,\eta)$ from their respective means. The right-hand sides give the deviations of η^1 and η^2 from their means.

Under Assumption 8 the expression for the probability of winning in (96) becomes

$$\bar{\pi}^1(p^1,p^2)=\text{prob}\left(\left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^{\lambda}\times\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\geq\frac{\eta^2}{\eta^1}\right) \tag{100}$$

Upon letting $\eta:=\frac{\eta^2}{\eta^1}$, we can write this as

$$\bar{\pi}^1(p^1,p^2)=P\left(\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\times\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}\right)\,. \tag{101}$$

P is a cumulative distribution function and hence a monotonic function. Maximizing (minimizing) $\bar{\pi}^1(p^1,p^2)$ is therefore equivalent to maximizing (minimizing) the argument of P,

$$\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\times\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}$$

or any monotone transformation of it such as

$$(1-\lambda)\,\log\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\;+\;\lambda\,\log\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\,.$$

We summarize these observations in the following Lemma.

Lemma 12. *Suppose that Assumptions 7 and 8 hold. Then party 1's objective is to maximize*

$$\Pi^1(p^1,p^2)\;\;:=\;\;(1-\lambda)\,\log\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\;+\;\lambda\,\log\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}, \tag{102}$$

and party 2's objective is to minimize it. Thus, if (p^{1*}, p^{2*}) is a pair of interior subgame perfect equilibrium policies, then it is a saddle point of the function $\Pi^1(p^1, p^2)$.

Note that the Lemma gives exactly the same conclusion as Proposition 1 in the body of the text. This shows that – even though Assumptions (98) and (99) may have a more plausible microfoundation in the Feddersen-Sadroni-model – our approach is essentially agnostic on the question how to best model ethical voting. We can work both with the Coate-Conlin formulation and with the Feddersen-Sandroni formulation.

The parties' bases add up to a constant. The following Lemma shows that the Coate-Conlin specification of ethical voting shares one property of the model that we present in the main text, namely that the parties bases add up to a constant. Thus, a change of policies that increases the base for, say, party 1 translates one-for-one into a decrease of the base of party 2.

Lemma 13. *Suppose that Assumption 7 holds. Then*

$$\mathbf{B}^1(p^1, p^2) + \mathbf{B}^2(p^1, p^2) = 1 \; .$$

Proof. If $\tilde{q}^1(\omega, \eta) = \tilde{q}^2(\omega, \eta) = 1$ for all realizations of η, we have

$$\begin{aligned}
\tilde{\mathbf{B}}^1(p^1, p^2, \eta) &:= \mathbb{E}\left[\tilde{q}^1(\omega, \eta)\; \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= \mathbb{E}\left[\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= 1 - \mathbb{E}\left[1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= 1 - \mathbb{E}\left[\tilde{q}^2(\omega, \eta)(1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta))\right] \\
&= 1 - \tilde{\mathbf{B}}^2(p^1, p^2, \eta) \; .
\end{aligned}$$

Hence, also

$$\begin{aligned}\mathbf{B}^1(p^1,p^2) &:= \textstyle\int \tilde{\mathbf{B}}^1(p^1,p^2,\eta)\; dP(\eta) \\ &= 1 - \textstyle\int \tilde{\mathbf{B}}^2(p^1,p^2,\eta)\; dP(\eta) \\ &= 1 - \mathbf{B}^2(p^1,p^2)\;.\end{aligned}$$

□

F.3.3 An alternative version of the Feddersen-Sandroni model in which the parties' bases do not add up to a constant

In the following, we consider an extension of our model in which the parties' bases do not add up to a constant. It follows from Lemma 13 that we cannot employ Assumption 7 according to which the electorate consists, in all states, entirely of ethical voters. For ease of exposition, we impose instead Assumption 6, due to Feddersen and Sandroni, so that there is no aggregate uncertainty in policy preferences. This has the expositional advantage that all the aggregate uncertainty in the model is due to the randomness of the share of ethical voters.

We seek to show that the parties' tradeoffs between attracting swing voters, catering to their own core voters in an attempt to mobilize them and catering to the rival's core voters with the intention to demobilize them does not rest on the assumption that the parties's bases add up to a constant. Recall that, in the main text, this property is implied by the assumption, that, for any type ω, the random variables $\tilde{q}^1(\omega,\eta)$ and $\tilde{q}^2(\omega,\eta)$ have the same mean

$$\bar{q}(\omega) \quad := \quad \int \tilde{q}^1(\omega,\eta)\; dP(\eta) \quad = \quad \int \tilde{q}^1(\omega,\eta)\; dP(\eta)\;.$$

Therefore,

$$\begin{aligned}
\mathbf{B}^1(p^1,p^2) &= \int \tilde{\mathbf{B}}^1(p^1,p^2,\eta)\, dP(\eta) r \\
&= \int \mathbb{E}[\tilde{q}^1(\omega,\eta)\; B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\, dP(\eta) \\
&= \mathbb{E}\left[\left(\int \tilde{q}^1(\omega,\eta)\; dP(\eta)\right) B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\right] \\
&= \mathbb{E}\left[\bar{q}(\omega)B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\right]\;,
\end{aligned} \tag{103}$$

and, by the same logic,

$$\mathbf{B}^2(p^1,p^2) \;=\; \mathbb{E}\left[\bar{q}(\omega)(1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega))\right]\;. \tag{104}$$

Obviously, equations (103) and (104) imply that

$$\mathbf{B}^1(p^1,p^2)+\mathbf{B}^2(p^1,p^2)=\mathbb{E}[\bar{q}(\omega)]$$

so that the two bases add up to an exogenous constant $\mathbb{E}[\bar{q}(\omega)]$, i.e. a term that does not depend on the policies that are proposed.

Example. As a simple case that avoids the property that the parties bases add up to a constant consider the following Assumption.

Assumption 9 (Party specific means). *There are numbers $\bar{q}^1$ and $\bar{q}^2$ so that, for all ω,*

$$\bar{q}^1 \;=\; \int \tilde{q}^1(\omega,\eta)\; dP(\eta) \quad \textit{and} \quad \bar{q}^2 \;=\; \int \tilde{q}^2(\omega,\eta)\; dP(\eta)\;.$$

The assumption says, all supporters of party 1 are, irrespective of their type ω, equally likely to be of the ethical type: For any supporter of party 1, this probability is equal to $\bar{q}^1$. Likewise, all supporters of party 2 are of the ethical type with probability $\bar{q}^2$.

An implication of this Assumption is that

$$\mathbf{B}^1(p^1,p^2)=\bar{q}^1\;\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]$$

and

$$\mathbf{B}^2(p^1,p^2)=\bar{q}^2\;\mathbb{E}[1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\;.$$

Hence,

$$\mathbf{B}^1(p^1,p^2)+\mathbf{B}^2(p^1,p^2)=\bar{q}^2+\left(\bar{q}^1-\bar{q}^2\right)\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\ ,$$

which implies that the bases add up to a quantity that depends on p^1 and p^2. The overall mass of potential voters therefore does depend on the policies that the parties. Also note that

$$\begin{aligned}\mathbf{B}^2(p^1,p^2) &= \bar{q}^2-\bar{q}^2\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \bar{q}^2-\tfrac{\bar{q}^2}{\bar{q}^1}\,\bar{q}^1\ \mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \bar{q}^2-\tfrac{\bar{q}^2}{\bar{q}^1}\,\mathbf{B}^1(p^1,p^2).\end{aligned}$$

Note that it is still the case that an increase of party 1's base implies a decrease of party 2's base – even though no longer one-by-one.

Henceforth and in parallel to our previous analysis we impose an assumption of multiplicative shocks. This assumption of multiplicative shocks is consistent with Assumption 9, i.e. both assumptions can hold simultaneously, but does not require it. That is, we can have multiplicative shocks without party specific means.

Assumption 10 (Multiplicative shocks II). *Let $\bar{q}^1(\omega):=\int\tilde{q}^1(\omega,\eta)dP(\eta)$ be the expected value of the random variable $\tilde{q}^1(\omega,\eta)$ for any ω. Analogously, let $\bar{q}^2(\omega):=\int\tilde{q}^2(\omega,\eta)dP(\eta)$ be the expected value of the random variable $\tilde{q}^2(\omega,\eta)$. Suppose that $\eta=(\eta^1,\eta^2)$ is a pair of two random variables η^1 and η^2 so that, for all ω,*

$$\tilde{q}^1(\omega,\eta)=\eta^1\ \bar{q}^1(\omega) \tag{105}$$

and

$$\tilde{q}^2(\omega,\eta)=\eta^2\ \bar{q}^2(\omega)\ . \tag{106}$$

Note the following implications of this Assumption:

$$\begin{aligned}\tilde{\mathbf{B}}^1(p^1,p^2,\eta) &= \mathbb{E}[\tilde{q}^1(\omega,\eta)B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \eta^1\mathbb{E}[\bar{q}^1(\omega)B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \eta^1\ \mathbf{B}^1(p^1,p^2)\end{aligned}$$

and, analogously,

$$\tilde{\mathbf{B}}^2(p^1, p^2, \eta) \;\;=\;\; \eta^2\, \mathbf{B}^2(p^1, p^2)\;.$$

This shows that equations (98) and (99) – imposed previously in our analysis of the Coate and Conlin model – also hold in the given context. An immediate implication is that Lemma 12 also extends to the given setup. This observation yields the following Corollary.

Corollary 1. *Suppose that Assumption 10 holds. Then party 1's objective is to maximize*

$$\Pi^1(p^1, p^2) \;\;:=\;\; (1-\lambda)\,\log\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} \;+\; \lambda\,\log\frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}, \qquad (107)$$

and party 2's objective is to minimize it. Thus, if (p^{1*}, p^{2*}) is a pair of interior subgame perfect equilibrium policies, then it is a saddle point of the function $\Pi^1(p^1, p^2)$.

The significance of the Corollary is to show that Proposition 1 in our main text also extends to a model in which the parties' bases do not add up to a constant. Thus, the tradeoffs that we highlight in our main text also extend to the given setup, albeit with some modifications. To understand these modifications, it is again instructive to look first at the polar cases $\mu = \infty$ and $\mu = 0$.

For $\mu = \infty$, the parties' bases do not matter at all for the probability of winning the election. The analysis therefore has exactly the same logic as the one presented in the body of the text: Party 1 focuses on maximizing

$$\frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}$$

and party 2 seeks to minimize this expression. From the perspective of party 1, the numerator $W^1(p^1, p^2)$ points to the political returns from increasing the stakes for its own core voters, the denominator points to the political returns from decreasing the stakes for party 2's core voters. Moreover, how these motives balance depends on the equilibrium value of $W^1(p^1, p^2)/W^2(p^1, p^2)$. The larger this quantity, the larger is party 1's equilibrium probability of winning and the more it has an incentive to focus on the demobilization of the potential voters of party 2.[43]

[43] Recall from the analysis in the main text that any equilibrium is symmetric and that this observation makes it possible to pin down the equilibrium value of $W^1(p^1, p^2)/W^2(p^1, p^2)$.

The case $\mu = 0$ is the exact mirror image. The stakes for the parties' core voters play no role, and all that matters is the ratio of the parties bases. Party 1 now seeks to maximize

$$\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}$$

while party 2 minimizes this expression. This problem of party 1 problem is – contrary to our analysis in the main text – not generally equivalent to maximizing $\mathbf{B}^1(p^1, p^2)$.

Remark 3. This equivalence holds, however, if we impose, in addition, Assumption 9. To see this, note that in this case,

$$\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} = \frac{\mathbf{B}^1(p^1, p^2)}{\bar{q}^2 - \frac{\bar{q}^2}{\bar{q}^1}\,\mathbf{B}^1(p^1, p^2)} \;,$$

which is an expression that is increasing in $\mathbf{B}^1(p^1, p^2)$.

If the equivalence does not hold, party 1 faces a tradeoff between maximizing $\mathbf{B}^1(p^1, p^2)$ and minimizing $\mathbf{B}^2(p^1, p^2)$. Maximizing $\mathbf{B}^1(p^1, p^2)$ would mean to cater primarily to those voters who are likely to swing into the base of party 1. Minimizing $\mathbf{B}^2(p^1, p^2)$ would give priority to those voters who swing out of the base of party 2 if party 1 offers a better deal. Since the bases do not add up to a constant, those who swing out of the base of party 2 do not automatically swing into the base of party 1. Thus, there is again a tradeoff between doing something that is good for the own vote share and doing something that is bad for the rival's vote share. How this tradeoff is resolved depends, again, on the equilibrium value of $\mathbf{B}^1(p^1, p^2)/\mathbf{B}^2(p^1, p^2)$. The larger this value the larger the weight on the minimization of the rival's base.

Obviously, for values of μ that are interior, $\mu \in (0, \infty)$ both forces are at play, and the parties consider the implications of their platforms choices both for their relative base advantage, as measured by $\mathbf{B}^1(p^1, p^2)/\mathbf{B}^2(p^1, p^2)$, and for their relative stake advantage, as measured by $W^1(p^1, p^2)/W^2(p^1, p^2)$.

To summarize this discussion we highlight two observations: First, a more general model in which the parties bases do not add up to a constant gives rise to the same tradeoffs as our analysis in the main text, but possibly with some modifications in the relevant formulas. Second, if we impose an additional assumption, Assumption 9, then no such modifications are needed and the analysis in the main text literally extends – even though the parties' bases do not add up to a constant.