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Public Debt and the Political Economy of Reforms

Abstract

We develop a two-period model of redistributive politics in which two politicians compete in an election in each period. In the first period, the politicians propose both whether to experiment with an efficient reform with uncertain benefits and choose the amount of public debt. Politicians also allocate pork-barrel spending to voters in each period. We show that allowing politicians to raise debt ensures that the reform is always implemented when the reform's ratio of private good to public good gains exceeds a threshold, i.e. the reform generates enough private good benefits. This is not the case when the reform's ratio of private good to public good gains is below this threshold. We also examine hard and a soft debt limits, and find that both limits reduce the political success of the reform. However, at moderate debt levels soft limits dominate hard limits with respect to equilibrium efficiency of reform provision.

JEL-Codes: C720, D720, D780, H600.

Keywords: political competition, public debt, reforms, redistributive politics, debt and spending limits.

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1 Introduction

What determines whether efficient reforms are implemented in the political process? This is an evergreen question in academic and policy spheres.[1] A key to explaining the political decision to reform is to understand under which circumstances electoral incentives can stand in the way of reforms. Electoral competition occurs to a considerable degree through targeting electoral favors to subsets of voters in order to gain their support. Since many reforms imply a shift of resources across time, the decision to reform should be influenced by the incentives to target resources to voters. The second important dimension that determines the allocation of resources across time is the decision to raise public debt. Therefore, in order to understand incentives for reform it is important to investigate how the decisions to reform *and* to raise public debt interact with political competition. This paper is the first political economy analysis where both the choice of debt and reform are decided in the political equilibrium.

We develop a two-period model of redistributive politics that builds on Lizzeri (1999).[2] Two politicians compete for election in each period. They do so by targeting available resources to subsets of voters at the expense of others. This tactical redistribution does not imply any efficiency gain. In the first period, politicians also choose the level of public debt *and* whether to experiment with a reform with uncertain benefits. The reform is efficient in the sense that it costs resources in the first period but yields higher expected benefits in the second period. The introduction of a reform decision at the time that the decision to raise debt is made is our main contribution.

A first main insight from our analysis is that the ability to raise public debt to target current voters can help sustain the efficient reform in political outcomes. The argument is the following: resources left in the future cannot be targeted

[1]See, for instance, Rodrik (1996), Persson and Tabellini (2000), or Drazen (2000).

[2]Using the model of Lizzeri (1999) is particularly compelling for our analysis: (1) the model shows the effect of electoral competition on policy outcomes without any pre-imposed heterogeneity, (2) it derives political turnover endogenously as the outcome of the electoral game, and (3) there are no *ad hoc* assumptions on the shape of the pork-barrel distributions.

to specific voters due to endogenously arising electoral uncertainty between the two periods. Without public debt, politicians have access only to first-period resources to target voters. This implies a disadvantage for a reforming candidate, who loses a potentially big proportion of these targetable resources through the first-period reform cost. In contrast, the use of public debt allows politicians to also compete on targeting future resources. This gives a competitive edge to a reforming candidate, since her advantage lies in the future where the benefits of the reform occur. Indeed, our results show that the reform will always be implemented in political equilibrium when the use of public debt allows a reformer to make up for her loss in targeting capacity in the first period. On the other hand, the efficient reform will not be implemented with probability one if the reform's benefits are primarily of a non-targetable public good nature or if the use of public debt is heavily restricted. Both aspects hinder a reformer's ability to compensate for her first-period targeting disadvantage. However, even if the reform corresponds to investing in a pure public good, we show that putting a more restrictive limit on public debt will still decrease the probability of reform. The driving force behind this result is the following: restricting public debt means reducing the amount of targetable resources on which electoral competition occurs. A given amount of reform costs therefore creates a relatively bigger disadvantage in terms of targeting capacity. These results highlight a new view on the tradeoff between targeted pork-barrel spending, which does not increase aggregate welfare, and efficient policies such as investing in a beneficial reform: as long as efficient policies create benefits in future electoral cycle, allowing enough debt-related targeted spending might be necessary to incentivize investing in these policies.

On the nature of the reform. More precisely, the first part of our analysis focus on how the nature of the reform affects the reform and debt decisions. We impose no restrictions on debt except for the natural debt limit. Implementing the reform increases the natural debt limit by the induced increase in the second-period endowment.

We show that if the proportion of reform benefits that increases the endowment is high enough, then both politicians will choose to implement the reform with probability one. The intuition behind this result is the following: due to endogenously arising electoral uncertainty, resources in the second period cannot be targeted to specific voters. This gives both candidates the incentive to transfer as many resources as possible to the first period by debt in order to target them to specific voters. Since debt repayment capacity increases by the reform-induced increase in the endowment, a reformer can raise higher debt than a non-reformer. This allows a reformer to compensate for the disadvantage of losing targetable resources through the reform costs. In contrast, when the reform benefits are mainly of a public good nature and do not increase the endowment much, the result is overturned. In that case, a reformer cannot raise much more debt than a non-reformer. If the major part of the reform benefits has the character of a public good, then this part is non-targetable by nature and also cannot be made targetable through the use of public debt. Therefore, by saving on the costs of the reform, a non-reforming candidate has more targetable resources and she can use this advantage to compensate at least a majority of voters for missing out on the net gain that the reform creates. Due to this efficiency gain the reform will still be implemented with positive probability, but it will no longer be implemented with certainty. For reforms that create mainly public good benefits, we therefore get a failure of the political process to implement the efficient policy.

On the availability of the debt channel. Constitutional limits on debt and spending limits are a popular response to debt crisis and are present in many jurisdictions.[3]

First, we show that an exogenous restriction on public debt that prevents a reformer from raising more debt than the non-reforming candidate gives these po-

[3]For instance, most U.S. states have a balanced-budget rule and the Stability Pact in the European Union limits gross government debt to sixty percent of GDP. Germany adopted in 2009 a constitutional rule referred to as the debt brake that requires the federal and state governments to run balanced budgets from 2016 and 2020 onwards respectively (see Janeba (2012) for details). See Rose (2010) and Schaechter, Kinda, Budina and Weber (2012) for reviews of balanced-budget rules and debt limits.

tentially targetable benefits the character of non-targetable public good benefits. From the point of view of first-period voters, future reform benefits that cannot be transferred to the present have the character of providing a public good that promises higher utility for everyone, but whose benefits cannot be targeted to specific voters. In our first set our results, this public good character was given through the nature of the reform. Now, it is artificially created through the debt limit. We show that if the debt limit becomes too stringent, the efficient reform is no longer implemented with probability one. However, even if the reform only creates non-targetable public good benefits, we show that putting a more restrictive exogenous limit on public debt will still decrease the probability of reform. This probability is lowest if public debt is not allowed at all. If there is no public debt, electoral competition is restricted to the allocation of the present resources only. The disadvantage of having to finance the first-period reform costs is then relatively bigger. In other words, allowing public debt gives politicians the opportunity to also compete on targeting the pie of future resources. Although such targeting does not create any efficiency gain itself, by putting a reformer in a relatively better position it incentivizes spending on efficient policies whose benefits only occur in the next electoral cycle. This gives us a new view on the effects of targeted spending which until now has mainly been shown to disincentive efficient spending on public goods in the same electoral cycle.

Second, we examine debt and spending limits. Given that, in the first-period, the reform's benefits are uncertain, it is natural to consider debt and spending limits that are *ex ante* binding, i.e. hold in expectation, and limits that are *ex post* binding, i.e. hold with probability one. We refer to the former as soft limits and the later as hard limits. We compare debt limits and spending limits and show that in equilibrium both hard and soft variations of these limits reduce the success of the reform in the political process. Furthermore, we find that it is possible to map any combination of a type of limit, debt or spending, and variation of the limit, hard or soft, into any other combination of a type of limit and variation of the limit. That is, no combination dominates the others with regard to equilibrium efficiency of reform provision.

Outline. The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the formal framework. Our main results are presented in Section 4, where we solve for the equilibrium of the game, and in Section 5 where we study the implications of constitutional limits on debt and spending. The last section contains concluding remarks. We relegate the proofs to the Online-Appendix.

2 Related literature

Our paper relates to several streams of literature on the political economy of public debt and reforms.

Our work builds on the game-theoretic literature on the divide-the-dollar game. Following Myerson (1993), this literature features models of political competition in which a policy proposal specifies how a cake of a given size should be distributed among voters.[4] Our model differs from these models in that policy proposals affect the size of the cake that is available for redistribution.[5]

Lizzeri and Persico (2001; 2005) extend the framework of Myerson (1993) by characterizing political equilibria under the assumption that politicians face a choice between an efficient public good and pork-barrel redistribution. In their static framework, they show that targeted pork-barrel spending stands against the efficient policy of providing a public good that creates a net gain in utility.[6] In contrast, we consider an efficient policy that is of a dynamic nature in the

[4]Contributions to this literature include Laslier and Picard (2002), Roberson (2006), Sahuguet and Persico (2006), Carbonell-Nicolau and Ok (2007), Kovenock and Roberson (2008; 2009), or Eguia and Nicolò (2019). See Kovenock and Roberson (2012) for a review.

[5]Some related papers that endogenize the size of the redistributive pie are Ueda (1998), Bierbrauer and Boyer (2016), and Boyer, Konrad and Roberson (2017), however these papers are static and do not study the interaction between debt and reforms.

[6]Roberson (2008) adds the possibility to provide different public good to different districts. Crutzen and Sahuguet (2009) allow for inefficiencies in the process of collecting resources.

sense that its benefits only occur in the next electoral cycle. For such policies, we show that allowing more debt-related targeted spending can actually increase the probability of implementing the efficient policy.

The first extension of Myerson (1993)'s setup to a dynamic model was done by Lizzeri (1999). Lizzeri (1999) shows that in a two-period model of divide-the-dollar electoral competition, candidates will always raise the maximal debt, because it allows them to better target the pool of resources to voters. Our analysis builds on Lizzeri (1999) by studying the interaction between debt and reform in such a redistributive politics setup. This setup allows to distill the pure effect of electoral competition on policy outcomes, because it does not impose any exogenous heterogeneity on politicians or voters. Furthermore, it derives political turnover endogenously as the outcome of the electoral game. In contrast, the literature on strategic debt has derived the tendency of the political process to accumulate debt from partisan preferences combined with the exogenously imposed threat that a currently ruling government is replaced in the future. Alesina and Tabellini (1990) show that a currently ruling party that has different spending objectives than a potential future incumbent uses debt to tie its successor's hands.[7] Recently there has been a revival of the literature on the political economy of public debt.[8] Battaglini and Coate (2008) introduce Barro (1979)'s tax smoothing setup of public debt into an infinite horizon model of legislative bargaining. They show that, when an electoral district is not sure to remain in the governing coalition, the incentive of politicians to spend pork on their own district leads to the use of public debt even when this means accepting higher tax distortions in the future.[9]

[7] Other pioneer papers in this line of research are Persson and Svensson (1989), Aghion and Bolton (1990) and Tabellini and Alesina (1990). See Martimort (2001) for an extension of these models to an optimal income taxation setup.

[8] See Yared (2019), Alesina and Passalacqua (2016), and Battaglini (2011) for recent reviews of this literature.

[9] See Barseghyan and Battaglini (2016) for a recent application of the legislative bargaining model investigating public debt in a growth setup. Further papers with different setups are Yared (2010), Drazen and Ilzetzki (2011), Song, Storesletten and Zilibotti (2012), Maskin and Tirole (2019), Azzimonti, de Francisco and Quadrini (2014), and Müller, Storesletten and Zilibotti

In that sense, investing into low public debt is an efficient dynamic policy whose benefits only occur in the future. By its very nature, it is the only such policy that cannot be incentivized by a higher use of public debt. We add an important aspect to this literature by establishing this incentivizing effect of public debt for all other efficient dynamic policies that have costs today and benefits in the future.

We also complement the existing literature on political economy of reforms. In this paper, we shut down the channels that the previous literature has identified as impediments to reform.[10] Our objective is to show how efficient reforms and public debt interact in a setup of electoral competition, absent all the previously identified channels. For observers of the public policy debates throughout the Great Recession, understanding the intertwined relationship between debt and reform is crucial for policy issues.[11] The only previous papers that have looked at public debt in combination with reforms do not model electoral competition. Specifically, Beetsma and Debrun (2004; 2007) rely on the assumption of an exogenous probability of change in political power. They do not consider a feedback of the decisions on debt and reform on the electoral outcome. As we show in our

(2016).

[10]In contrast to Fernandez and Rodrik (1991) and Cukierman and Tommasi (1998), our analysis does not link the benefits and costs of reform to specific voters. Consequently, we also do not consider problems of asymmetric information in compensating losers of the reform as in Grüner (2002). Furthermore, we have no uncertainty regarding appropriate timing of the reform as in Laban and Sturzenegger (1994a; 1994b) and Mondino, Sturzenegger and Tommasi (1996). Reforms do not fail because of insufficient technical knowledge by decision makers as in Caselli and Morelli (2004) and Mattozzi and Merlo (2015). We also exclude powerful vested interest that could block reform as in Olson (1982), Benhabib and Rustichini (1996) and Gehlbach and Malesky (2010). There is no conflict between different groups about who will bear the costs of reform as in Alesina and Drazen (1991), Drazen and Grilli (1993) and Hsieh (2000). Finally, the success of the reform does not depend on the competence of politicians as in Prato and Wolton (2014) or Bowen, Chan, Dube and Lambert (2016). Inefficiencies of the political process to pursue efficient investment have been investigated in several setups, see, e.g., Besley and Coate (1998), Battaglini and Coate (2007), Azzimonti, Sarte and Soares (2009), Battaglini, Nunnari and Palfrey (2012), and Azzimonti (2015).

[11]Müller, Storesletten and Zilibotti (2015) provide an important analysis of positive and normative implications of interacting sovereign debt dynamics and structural reforms.

model, these forces of political competition are crucial to understanding the interaction between debt and reforms. Ribeiro and Beetsma (2008) is an important first step towards endogenizing political turnover. However, they still need to add a final period with exogenous probability of change in power. Furthermore, one politician is forced to run a reform platform and she cannot decide not to reform, while her opponent is exogenously set to run a no-reform platform. This precludes to see the workings of the forces of political competition that we establish in this paper.[12]

Recent papers using alternative models make progress on our understanding of the role of fiscal rules constraining debt on the outcome of the political process.[13] Azzimonti, Battaglini and Coate (2016) analyze the impact of a balanced budget rule that requires that legislators do not run deficits in the setup of Battaglini and Coate (2008).[14] They show that imposing a balanced budget rule reduces existing debt levels with beneficial long run effects because it reduces the revenues that must be devoted to servicing the debt. Cunha and Ornelas (2018) investigate the tradeoff between intense political turnover and unrestricted access to debt. In particular they show that strict limits on government borrowing can exacerbate political economy distortions by making a political compromise unsustainable. Piguillem and Riboni (2020) and Coate and Milton (2019) study the implications for fiscal policy if it is possible for politicians to override the rules with enough supports among elected politicians or in the electorate. Finally, Bouton, Lizzeri and Persico (2020) are interested in the interaction between debt and entitlements. One of their main results is to show that it may be beneficial to relax a constraint on debt, and always to limit but not eliminate entitlements.

[12]The importance of considering these forces can be seen in Esslinger and Mueller (2015) (see also Chapter 4 of Esslinger (2016, University of Mannheim)) who do not model electoral competition either. They show how the interaction between future investments and public debt can be impaired when the forces of electoral competition are taken out of the picture.

[13]For the optimality of rules see, e.g., Amador, Werning and Angeletos (2006), Halac and Yared (2014), or Halac and Yared (2019).

[14]Halac and Yared (2018) study the design of fiscal rules in a global economy in which individual rules affect global interest rates.

In our context, first-period debt limits decrease the maximum level of targetable resources in the first period, and consequently increase the maximum level of targetable resources in the second period. As the maximum level of targetable resources in the first period decreases, the opportunity cost of implementing the policy in the first period increases. Thus, we find that the introduction of first-period debt limits always decreases the equilibrium probability with which the efficient policy is implemented. As a result, in political competition in which reform is of a dynamic nature, allowing enough debt-related pork-barrel spending may be necessary to incentivize candidates to choose the reform policy.

With uncertain policy benefits, a natural issue that arises is how soft debt limits, in which the constraint holds only in expectation across the set of possible policy states, compare with hard debt limits, in which the constraint holds for each realized state of policy benefits. In a related application of soft and hard budget constraints, Hwang, Koh and Lu (2021) examine a two-player strategic-form contest involving a continuum of component contests. Rather than endowing the players with an exogenous budget, in their model, the players' budgets are endogenous. In the baseline case, the players face a "soft" budget constraint on the average amount of resources that may be allocated across the set of component contests. They also examine an extension in which the players face a "hard" constraint on the maximum level of resources that a player may allocate to each of the component contests. In applying their model to the redistributive politics framework of Myerson (1993), Hwang et al. (2021) find an equivalence between the "soft" budget constraint on the average transfers and the "hard" budget constraint on the maximum transfer to any individual voter. In contrast to a constraint on the transfers to individual voters, our formulation of hard and soft constraints are with respect to the uncertain policy state. In this context, we find that both variations of debt limits reduce the equilibrium probability with which the efficient policy is implemented. However, there exists a portion of the parameter space, with sufficiently moderate debt levels, in which soft constraints dominate hard constraints with respect to equilibrium efficiency of policy provision.

3 The model

Consider a two-period redistributive politics game with policy investment that is described as follows.

The electorate. There are two periods and a continuum of voters of measure one. All voters are ex-ante homogeneous. They are risk-neutral, live for the two periods, and have a discount factor equal to 1. There are two goods, money and a public good. Voters have linear utility over both goods and the marginal utility of money is normalized to one.[15] In each period, each voter is endowed with one unit of money which is perfectly divisible.

Political process. In each of the two periods, denoted $t \in \{1, 2\}$, there is an election in which voters choose between two candidates. The set of candidates is the same for both periods. One candidate is denoted by A, the other by B. Each candidate $i \in \{A, B\}$ is purely office-motivated and maximizes their vote share in each period.

Platforms. In each period, each candidate announces a binding platform involving transfers and, in the first period, there is the possibility of experimenting with a policy with uncertain benefits. If the policy is implemented, then the second-period policy costs are incurred and the value of the (uncertain) second-period policy benefits are realized.

The policy's second-period benefits feature a mix of pure-public good benefits and private-good spillovers: a portion of the reform benefits consist of an increase in the endowment of the economy and the remaining portion of the benefits are of a public good nature. By way of illustration, starting from a situation with deficient enforcement of property and civil rights, consider a reform of the legal system that ensures efficient and universal enforcement of these rights. This is

[15] Our main results extend directly to the case of a quasi-linear utility function that is concave in public good consumption.

what is usually termed establishing the rule of law.[16] By decreasing uncertainty for investors, such a reform will lead to an increase in the economy's GDP,[17] which in our case corresponds to an increase in the endowment of the economy. Besides that there will be a general increase in well-being beyond the increase in the endowment. For instance, everybody will feel more safe in such a functioning legal environment. This second kind of benefit has the properties of a public good in the sense that it is non-rival and non-excludable.[18] When the benefits result in an increase in the endowment that can be taxed, the benefits can potentially be redistributed to specific voters. In the case where the benefits have a public good nature, a politician that decides to do the reform cannot shuffle the benefits derived by the voters from it. In line with the political economy literature we assume that benefits that have a private good nature can be targeted to individual voters whereas targeting is precluded for the public good part of the reform (see, e.g., Lizzeri and Persico (2001)).

Formally, a fraction $\lambda \in [0, 1]$ of the policy benefits are in the form of private-good benefits. The remaining part $(1 - \lambda)$ of the benefits are in the form of pure public-good benefits. Hence, for $\lambda = 0$, we have the case of a pure-public good. For $\lambda = 1$ on the other hand, the policy benefits are in the form of a private good and increase the second-period per-capita endowment of the economy. Note that it is impossible for politicians to affect the distribution, across voters, of the fraction $(1 - \lambda)$ of policy utility derived from the public-good component of the gains. This is often referred to as the non-targetable part of the policy in the redistributive politics literature. In contrast, the fraction λ of policy utility from the private goods component of the gains is targetable and can be redistributed among voters in the political process. Because the proportion λ of policy utility may, potentially, be redistributed across voters, we refer to λ also as the degree of

[16]See, for instance, La Porta, de Silanes and Shleifer (2008), Besley and Persson (2011), and Acemoglu and Robinson (2012).

[17]Rodrik, Subramanian and Trebbi (2004) and Djankov, McLiesh and Shleifer (2007) provide empirical support for this claim.

[18]Excluding some people from access to the legal system would mean a failure to establish the rule of law.

targetability of policy benefits.

Candidate i's first-period platform p_1^i has three elements: a possibly random decision of whether or not to enact the policy,[19] a level of public debt, and promises of taxes and transfers to each individual voter, and we examine each of these three components of the first-period platform in further detail below. In the case that the policy is implemented, both the level of public debt and the promises of taxes and transfers may be contingent on the realized state of the uncertain policy benefits. Conditional on the observable outcome of the first-period's election and resulting policy benefits and debt level, candidate i's second-period platform p_2^i consists of promises of taxes and transfers to each individual voter.

1. Policy. We denote by c the per capita cost and by e the realization of the per capita benefit from the policy, where the discrete random variable $\widetilde{e}$ is distributed according to a probability mass function Γ_e with the set of possible values $\mathcal{E}$, a finite subset of $\mathbb{R}_+$.[20] We use the notation $e = \emptyset$ to denote that the policy was not implemented, and we focus on the case that the parameters $E_{\Gamma_e}(e)$ and c satisfy the following two conditions:

$$1 > E_{\Gamma_e}(e) - c > 0, \tag{$A1$}$$

$$1 > c. \tag{$A2$}$$

Assumption $(A1)$ states that the average net policy benefits $E_{\Gamma_e}(e) - c$ are large enough that the policy should always be implemented from an *ex-ante* efficiency perspective. Furthermore, $(A1)$ states that the average net policy benefits $E_{\Gamma_e}(e) - c$ are less than the (per period) endowment of the economy. Thus, our focus is on policies with net benefits that are, independently of redistributive politics considerations, neither so high that they would always be provided in the political process nor so low that they would never be provided in the political process.

[19] A mixed strategy in this game could in principle be a very complicated object. We focus on the case that candidates only mix over the decision to implement the reform which generates an associated debt level and distribution of transfers. This convention follows Lizzeri and Persico (2001), and as we show, contingent on the reform choice, debt is always deterministic in equilibrium.

[20] Note that the case of certain policy benefits is a special case of our model.

Assumption ($A2$) ensures that there is enough first-period endowment to finance the policy, i.e. implementing the policy does not require a second-period debt obligation.

Let $\iota_i \in \{0, 1\}$ be a policy position indicator function, where $\iota_i = 1$ if candidate i implements the policy. In the following, we let $\beta_i \in [0, 1]$ denote the probability that candidate i implements the policy. Finally, let $\iota(e)$ denote the first-period policy choice resulting from the first-period's political process and realization of $e \in \mathcal{E} \cup \emptyset$, where $\iota(\emptyset) = 0$ and $\iota(e) = 1$ for all $e \in \mathcal{E}$.

2. Debt. Government debt is financed by borrowing from abroad and there is no possibility of default.[21] The size of the deficit in the first period is interpreted as the fraction of the average voter's second-period resources that is pledged to the repayment of the debt.[22]

The natural limit on debt corresponds to the total resources that can be mobilized to repay debt. Let $\delta_i(e)$ denote the debt level resulting from candidate i's first-period platform when the realized policy benefit level is $e \in \mathcal{E} \cup \emptyset$. If candidate i implements the policy ($\iota_i = 1$) and the realized policy benefit level is $e \in \mathcal{E}$, then the maximal amount of resources that can be transferred from the second period to the first period increases by λe, and feasibility of the debt level requires that $\delta_i(e) \in [-1 + c, 1 + \lambda e]$. If candidate i does not implement the policy ($\iota_i = 0$ and $e = \emptyset$), then feasibility of the debt level requires that $\delta_i(\emptyset) \in [-1, 1]$. Given the outcome of the first-period's political process, it will also be useful to let $\delta(e)$ denote the realized debt level of the economy conditional on the realization of policy benefits e generated by the winning candidate's first-period policy position and to let $\mathcal{S}_{pd}$ denote the set of feasible policy and debt states $(e, \delta(e))$:

$$\mathcal{S}_{pd} = \{(e, \delta(e)) | e \in \mathcal{E} \cup \emptyset \ \& \ \delta(e) \in [-1 + \iota(e)c, 1 + \iota(e)\lambda e]\} .$$

3. Redistribution. In the analysis that follows, we focus on the voters' endow-

[21] The implications of considering the distortions generated by a default on debt in a similar setup are treated in Chapter 3 of Esslinger (2016, University of Mannheim). An overview of key issues in the economics of sovereign debt is provided by Aguiar and Amador (2014).

[22] We also allow for the possibility that the government runs a surplus which will, however, never occur in equilibrium.

ments of the private good net any taxes or transfers in each period $t \in \{1, 2\}$, which we refer to as the period t *net endowment* and which must be weakly positive. Note that, because each voter is endowed with one unit of money in each period, a period t net endowment in the interval $[0, 1]$ corresponds to a tax on the voter's endowment of one unit of money, and a net endowment greater than 1 corresponds to a positive transfer to a voter.

We follow Myerson (1993) and assume that, conditional on the policy state $e \in \mathcal{E} \cup \emptyset$, the period t net endowments that candidate i offers to different voters are *i.i.d.* random variables distributed according to the cumulative distribution functions $F_{i,1}(\cdot|e)$ and $F_{i,2}(\cdot|e, \delta(e))$, in periods 1 and 2 respectively. We appeal to the law of large numbers for large economies and interpret $F_{i,1}(x|e)$ and $F_{i,2}(x|e, \delta(e))$ not only as the probability that any particular individual receives an offer weakly smaller than x, but also as the population share of voters who receive such an offer.

Because there are $|\mathcal{E}|$ possible policy states and first-period redistribution may be contingent on the realized policy state, each candidate i's first-period net endowment offer to an arbitrary voter is a random $(|\mathcal{E}| + 1)$-tuple, denoted by $\{\widetilde{x}_{i,1}(e)\}_{e \in \mathcal{E} \cup \emptyset}$ for candidate i. For any policy state $e \in \mathcal{E} \cup \emptyset$, $\widetilde{x}_{i,1}(e)$ denotes the random variable corresponding to candidate i's first-period net endowment offer to an arbitrary voter in policy state e. Let $P_{i,1}$ denote the joint distribution of candidate i's first-period state-contingent net endowment offers, with the set of univariate marginal distributions $\{F_{i,1}(x|e)\}_{e \in \mathcal{E} \cup \emptyset}$ where $F_{i,1}(x|e)$ denotes candidate i's cumulative distribution of first-period net endowment offers conditional on the policy state e.

Let $P_{i,1}^{\mathcal{E}}(\mathbf{x})$ denote the $|\mathcal{E}|$-variate marginal distribution of $P_{i,1}(\mathbf{x})$ corresponding to the state-contingent net endowment offers for the policy states in $\mathcal{E}$. At times we will be interested in the random variable formed by taking the expectation with respect to the policy state e of a random draw of an $|\mathcal{E}|$-tuple, $\{\widetilde{x}_{i,1}(e)\}_{e \in \mathcal{E}}$, from $P_{i,1}^{\mathcal{E}}(\mathbf{x})$, which we denote by $\widetilde{x}_{i,1}^{\Gamma_e}$ where

$$\widetilde{x}_{i,1}^{\Gamma_e} := \sum_{e \in \mathcal{E}} \Gamma_e(e) \widetilde{x}_{i,1}(e). \tag{1}$$

Note that the cumulative distribution of $\widetilde{x}_{i,1}^{\Gamma_e}$, denoted $F_{x_{i,1}^{\Gamma_e}}(x)$, is calculated as the measure of the support of $P_{i,1}^{\epsilon E}$ below the hyperplane defined by $\sum_{e\in\mathcal{E}} \Gamma_e(e)\widetilde{x}_{i,1}(e) \leq x$.

Given the policy and debt state $(e, \delta(e)) \in \mathcal{S}_{pd}$, each candidate i's second-period net endowment offer to an arbitrary voter is a random variable $\widetilde{x}_{i,2}(e, \delta(e))$. Let $F_{i,2}(\cdot|e, \delta(e))$ denote candidate i's cumulative distribution of second-period net endowment offers contingent on the state $(e, \delta(e))$. It will also be useful to define the complete set of candidate i's second-period net endowment offers to an arbitrary voter for all possible realizations of $(e, \delta(e)) \in \mathcal{S}_{pd}$ as the parametric family of distribution functions $\{F_{i,2}(\cdot|e, \delta(e))\}_{(e,\delta(e))\in\mathcal{S}_{pd}}$.[23]

Feasible platforms. Recall that each candidate i's first-period platform p_1^i consists of a possibly random decision of whether or not to enact the policy, $\beta_i \in [0, 1]$, and, contingent on the realization of the policy state e, a level of public debt, $\{\delta_i(e)\}_{e\in\mathcal{E}\cup\emptyset}$, and net endowment offers for each voter, $\{\widetilde{x}_{i,1}(e)\}_{e\in\mathcal{E}\cup\emptyset}$ with joint distribution $P_{i,1}$. Hence,[24]

$$p_1^i := \{\beta_i, \{\widetilde{x}_{i,1}(e), \delta_i(e)\}_{e\in\mathcal{E}\cup\emptyset}\}.$$

Given the policy and debt state $(e, \delta(e)) \in \mathcal{S}_{pd}$, candidate i's second-period platform $p_2^i(e, \delta(e))$ is a random variable $\widetilde{x}_{i,2}(e, \delta(e))$ with conditional cumulative distribution function $F_{i,2}(\cdot|e, \delta(e))$. It will also be useful to define the complete set of candidate i's second-period platforms for all possible realizations of $(e, \delta(e)) \in \mathcal{S}_{pd}$ as

$$p_2^i := \{\widetilde{x}_{i,2}(e, \delta(e))\}_{(e,\delta(e))\in\mathcal{S}_{pd}}.$$

[23]Alternatively, a complete set of second-period net endowment offers could be specified by the random field $\{\widetilde{x}_{i,2}(e, \delta(e)) : (e, \delta(e)) \in \mathcal{S}_{pd}\}$. However, the correlation structure of the second-period net endowment offers across states is not payoff relevant.

[24]Note that because it is always optimal for each candidate to choose budget-balancing platforms, it follows that in all equilibria we know that for each realization of the policy benefit e the debt level $\delta(e)$ follows directly from ι_i and $F_{i,1}(x|e)$. However, it is possible that a candidate does not choose a budget-balancing platform and thus, we include the debt level $\delta(e)$ as part of the first-period platform.

Platforms are feasible if they satisfy the following budget constraints. For all $e \in \mathcal{E} \cup \emptyset$, the first-period budget constraint is:

$$\int_0^{+\infty} x dF_{i,1}(x|e) = E_{F_{i,1|e}}(x) \leq 1 + \delta_i(e) - \iota_i c. \tag{2}$$

Given the outcome of the first-period's political process, i.e. $(e, \delta(e)) \in \mathcal{S}_{pd}$, the second-period budget constraint is:

$$\int_0^{+\infty} x dF_{i,2}(x|e, \delta(e)) = E_{F_{i,2|e,\delta(e)}}(x) \leq 1 + \iota(e)\lambda e - \delta(e). \tag{3}$$

In the first period, the additional resources that can on average be given by candidate i to each voter depend on the endowment, the resources transferred from the future by debt $\delta_i(e)$, and the costs $\iota_i c$ that have to be paid in the case that the policy is implemented ($\iota_i = 1$). Given the outcome of the first-period's political process, the realized debt level of the economy $\delta(e)$ – which is conditional on the level of policy benefits e – must be repaid in the second period. However, when the policy is implemented, the portion of the policy benefits that are in the form of private-good benefits, $\iota(e)\lambda e$, increase the amount of resources that can be redistributed across voters. In the second period, each voter also receives utility $\iota(e)(1 - \lambda)e$ from the public-good component of policy benefits.

Timing. The timing of the game is summarized as follows:

Period 1:

Stage 1 Each vote-share maximizing candidate $i = \{A, B\}$ announces a first-period platform p_1^i.

Stage 2 Each voter observes each candidate i's realized policy position ι_i. If $\iota_i = 0$, then each voter also observes: (i) candidate i's debt level $\delta_i(\emptyset)$ and (ii) a first-period net endowment offer $x_{i,1}(\emptyset)$. Otherwise, if $\iota_i = 1$, then each voter observes: (i) an $|\mathcal{E}|$-tuple of state-contingent debt levels $\{\delta_i(e)\}_{e \in \mathcal{E}}$ and (ii) an $|\mathcal{E}|$-tuple of policy state-contingent net endowment offers $\{x_{i,1}(e)\}_{e \in \mathcal{E}}$. Each voter casts a first-period vote for the candidate that provides the higher first-period expected continuation utility, with ties broken by fair randomization. The candidate with the higher first-period vote share wins the first-period election.

Stage 3 The platform of the winner of the first-period election is implemented. In the event that the winner of the first-period election chose to enact the policy, the value of the policy benefit $e \in \mathcal{E}$ is observed, and the winning candidate's first-period state-contingent transfers are made. If the winner of the the first-period election chose not to enact the policy, then the state is $e = \emptyset$ and the corresponding transfers are made.

Given the observable state of policy and debt $(e, \delta(e)) \in \mathcal{S}_{pd}$ from the first-period's political process, there are two stages in period 2:

Period 2:

Stage 1 Each candidate $i \in \{A, B\}$ announces a second-period platform $p_2^i(e, \delta(e))$.

Stage 2 Each voter observes, for each candidate i, a second-period net endowment offer $x_{i,2}(e, \delta(e))$ and then votes for the candidate that provides the higher second-period local utility, with ties broken by fair randomization. The candidate with the higher second-period vote share wins the second-period election.

Note that in this two-period redistributive-politics model with policy investment, for each candidate i a strategy, which is denoted by $\{p_1^i, p_2^i\}$, consists of the combination of a first-period platform p_1^i and the complete set of candidate i's second-period platforms p_2^i, which specifies a second-period platform $p_2^i(e, \delta(e))$ for each possible realization of $(e, \delta(e)) \in \mathcal{S}_2$. Before moving on to the vote-share calculations, we provide a brief example of a strategy.

Example 1 *Consider an example strategy for candidate i, $\{p_1^i, p_2^i\}$, described as follows. In the first period $\beta_i = 1$, $\delta(e) = 1 + \lambda e$, and $P_{i,1}(\mathbf{x}) = F_{i,1}(x_\emptyset|\emptyset) \cdot \min_{e \in \mathcal{E}}\{F_{i,1}(x_e|e)\}$ where for all $e \in \mathcal{E}$, $F_{i,1}(x_e|e)$ is uniformly distributed on*

$[0, 2+2\delta(e)-2c]$ *and thus satisfies first-period budget balance as defined by equation (2).*[25]

Note that $P^{\mathcal{E}}_{i,1}(\mathbf{x}) = \min_{e\in\mathcal{E}}\{F_{i,1}(x_e|e)\}$. *Given the form of* $P^{\mathcal{E}}_{i,1}(\mathbf{x})$, *it can be shown*[26] *that the random variable* $\widetilde{x}^{\Gamma_e}_{i,1}$ *is uniformly distributed on* $[0, 2+2E_{\Gamma_e}(\delta(e))-2c]$.

Vote shares. We begin with the voters' second-period local utilities and the candidates' second-period expected vote shares. Then, we move back through the game tree to the calculation of the voters' first-period continuation utilities and the candidates' first-period expected vote shares.

Beginning in the second-period with any policy and debt state $(e, \delta(e)) \in \mathcal{S}_{pd}$ generated by the first-period's political process, in the event that candidate $i \in \{A, B\}$ wins the second-period election the second-period local utility for a generic voter z who, at the end of the second period, receives, from candidate i, the transfer $x_{i,2}(e, \delta(e))$ is:

$$u_{z,2}(x_{i,2}(e,\delta(e))|e) = x_{i,2}(e,\delta(e)) + \iota(e)(1-\lambda)e. \tag{4}$$

Note that the term $\iota(e)(1-\lambda)e$ in equation (4) depends only on the policy state e and not a candidate identity.

Voter z casts a second-period vote for candidate i over candidate j if

$$u_{z,2}(x_{i,2}(e,\delta(e))|e) > u_{z,2}(x_{j,2}(e,\delta(e))|e) \quad \Longleftrightarrow \quad x_{i,2}(e,\delta(e)) > x_{j,2}(e,\delta(e))$$

with ties broken by fair randomization. At the beginning of the second period candidate i's net endowment offer of $x_{i,2}(e, \delta(e))$ to voter z is still a random variable, denoted $\widetilde{x}_{i,2}(e, \delta(e))$, that is distributed according to $F_{i,2}(\cdot|e, \delta(e))$. Given the

[25] Because $\beta_i = 1$, $F_{i,1}(x_\emptyset|\emptyset)$ is not payoff relevant and $F_{i,1}(x_\emptyset|\emptyset)$ may be any feasible budget-balancing distribution of first-period net endowments.

[26] At each $|\mathcal{E}|$-tuple in the support of $P^{\mathcal{E}}_{i,1}(\mathbf{x})$, $F_{i,1}(x_e|e) = F_{i,1}(x_{e'}|e')$ for all $e, e' \in \mathcal{E}$. That is, at each $|\mathcal{E}|$-tuple in the support of $P^{\mathcal{E}}_{i,1}(\mathbf{x})$ there exists an $\omega \in [0, 1]$ such that each state $e \in \mathcal{E}$ net endowment offer may specified by $x_1(e, \omega) = [2 + 2\delta(e) - 2c]\,\omega$. Furthermore, taking a random draw of an $|\mathcal{E}|$-tuple from $P^{\mathcal{E}}_{i,1}(\mathbf{x})$ is equivalent to taking a draw of a univariate random variable $\widetilde{\omega}$ that is uniformly distributed on $[0, 1]$ and forming the $|\mathcal{E}|$-tuple $\{x_{i,1}(e, \widetilde{\omega})\}_{e\in\mathcal{E}}$. It follows directly that the random variable $\widetilde{x}^{\Gamma_e}_{i,1}$ is uniformly distributed on $[0, 2 + 2E_{\Gamma_e}(\delta(e)) - 2c]$.

state of the policy and debt $(e, \delta(e)) \in \mathcal{S}_{pd}$ generated by the first-period's political process, candidate A's second-period expected vote share is calculated as,

$$S_2^A(p_2^A(e,\delta(e)), p_2^B(e,\delta(e))|e,\delta(e)) = \text{Prob}\,(\widetilde{x}_{A,2}(e,\delta(e)) > \widetilde{x}_{B,2}(e,\delta(e))) \\ + \frac{1}{2}\text{Prob}\,(\widetilde{x}_{A,2}(e,\delta(e)) = \widetilde{x}_{B,2}(e,\delta(e))) \quad (5)$$

with $S_2^B(p_2^B(e,\delta(e)), p_2^A(e,\delta(e))|e,\delta(e))$ analogously defined.

Moving back to the first period, we now construct the voters' first-period continuation utilities at the end of the first period in the event that candidate $i \in \{A, B\}$ wins the first-period election. Given that candidate i has won the first-period election and that the policy state is $e \in \mathcal{E} \cup \emptyset$, the first-period local utility for a generic voter z who, at the end of the first period, receives, from candidate $i \in \{A, B\}$, the net endowment offer $x_{i,1}(e)$ is:

$$u_{z,1}(x_{i,1}(e)) = x_{i,1}(e).$$

Recall from equation (3) that second-period budget balancing requires that for each candidate i,

$$E(x_{i,2}(e,\delta(e))) = 1 + \iota(e)\lambda e - \delta(e).$$

If for each state $(e, \delta(e)) \in \mathcal{S}_{pd}$ both candidates use second-period budget-balancing platforms,[27] then from equations (3) and (4) it follows that in policy state $e \in \mathcal{E} \cup \emptyset$ the continuation utility for a generic voter z who, at the end of the first period, receives a transfer of $x_{i,1}(e)$ from the candidate i that won the first-period election with a realized policy position of ι_i and debt level of $\delta_i(e)$ is:

$$U_z(x_{i,1}(e), \iota_i, \delta_i(e)|e) := x_{i,1}(e) + 1 + \iota_i e - \delta_i(e).$$

If there exists at least one candidate i with $\iota_i = 1$, then the draw of the policy state e from Γ_e is payoff relevant, and when voters cast their first-period votes they

[27] Given our focus on subgame perfect Nash equilibrium, we focus here on the case that both candidates use second-period budget balancing platforms. However, it is straightforward to extend the continuation utilities to the case that one or both of the candidates do not use second-period budget-balancing platforms.

do not know the policy state $e \in \mathcal{E}$. Let $E_{e|\iota_i}\left(U_z(x_{i,1}(e), \iota_i, \delta_i(e)|e)\right)$ be defined as follows:

$$E_{e|\iota_i}\left(U_z(x_{i,1}(e), \iota_i, \delta_i(e)|e)\right) = \begin{cases} x_{i,1}(\emptyset) + 1 - \delta_i(\emptyset) & \text{if} \quad \iota_i = 0 \\ E_{\Gamma_e}\left(x_{i,1}(e) + 1 + e - \delta_i(e)\right) & \text{if} \quad \iota_i = 1 \end{cases}$$

where $E_{e|\iota_i}\left(U_z(x_{i,1}(e), \iota_i, \delta_i(e)|e)\right)$ denotes the expected continuation utility for a generic voter z who receives a net endowment offer of $x_{i,1}(\emptyset)$ from candidate i in the case that $\iota_i = 0$ and receives an $|\mathcal{E}|$-tuple of net endowment offers $(\{x_{i,1}(e)\}_{e\in\mathcal{E}})$ from candidate i in the case that $\iota_i = 1$. Voter z casts a first-period vote for candidate i over candidate j if

$$E_{e|\iota_i}\left(U_z(x_{i,1}(e,), \iota_i, \delta_i(e)|e)\right) > E_{e|\iota_j}\left(U_z(x_{j,1}(e), \iota_j, \delta_j(e)|e)\right),$$

with ties broken by fair randomization.

At the beginning of the first period, each candidate i announces a first-period platform of p_1^i and the expected continuation utility $E_{e|\iota_i}\left(U_z(x_{i,1}(e), \iota_i, \delta_i(e)|e)\right)$ that candidate i provides to an arbitrary voter z is a random variable, denoted $\widetilde{U}_z(p_1^i)$, where,

$$\widetilde{U}_z(p_1^i) := \beta_i\left(\widetilde{x}_{i,1}^{\Gamma_e} + 1 + E_{\Gamma_e}(e - \delta_i(e))\right) + (1-\beta_i)\left(\widetilde{x}_{i,1}(\emptyset) + 1 - \delta_i(\emptyset)\right), \quad (6)$$

where $\widetilde{x}_{i,1}^{\Gamma_e}$ denotes the random variable corresponding to candidate i's average, with respect to the policy state e, first-period net endowment offer for an arbitrary $|\mathcal{E}|$-tuple drawn from $P_{i,1}^{\mathcal{E}}(\mathbf{x})$.

In period 1, we denote by $S_1^A(p_1^A, p_1^B)$ the first-period vote share that candidate A receives when she chooses the first-period platform p_1^A and candidate B chooses the first-period platform p_1^B, and both candidates use second-period budget-balancing platforms. Hence,

$$S_1^A(p_1^A, p_1^B) = \text{Prob}\left(\widetilde{U}_z(p_1^A) > \widetilde{U}_z(p_1^B)\right) + \frac{1}{2}\text{Prob}\left(\widetilde{U}_z(p_1^A) = \widetilde{U}_z(p_1^B)\right) \quad (7)$$

and $S_1^B(p_1^B, p_1^A)$ is analogously defined.

4 Equilibrium characterization

In this two-period redistributive-politics game with policy investment, a subgame perfect Nash equilibrium is characterized by a pair of platforms for each candidate, $\{p_1^i, p_2^i\}_{i=A,B}$, such that in all subgames the restriction of the strategy profile to the subgame is a Nash equilibrium. In Theorem 1 we characterize the subgame perfect equilibrium strategies of this two-period game (with a chance move). In the statement of Theorem 1, it will be useful to define H as $H := 2c-(1+\lambda)E_{\Gamma_e}(e)$. Note that H is a function of λ, c, and $E_{\Gamma_e}(e)$. Furthermore, holding c and $E_{\Gamma_e}(e)$ constant, if the fraction λ of private policy benefits is sufficiently high, then $H \leq 0$. Similarly, if the fraction λ of private policy benefits is sufficiently low, then $H > 0$.

Theorem 1 *The set of subgame perfect equilibrium is completely characterized as follows.*

First Period

*In the first period, there are two cases labeled **(I.)** and **(II.)**.*

(I.) *If $H \leq 0$, then in any subgame perfect equilibrium both candidates choose a first-period platform p_1^* that implements the policy with probability $\beta^* = 1$ and for each realization of the policy state $e \in \mathcal{E}$:*

- **(i)** *announce the maximum feasible debt: $\delta^*(e) = 1 + \lambda e$, and*
- **(ii)** *choose an $(|\mathcal{E}| + 1)$-variate joint distribution $P_1^*(\mathbf{x})$ of first-period net endowments such that the random variable $\widetilde{x}_1^{\Gamma_e}$ is uniformly distributed on the interval $[0, 4 + 2\lambda E_{\Gamma_e}(e) - 2c]$ and for each possible policy state e the random variable $\widetilde{x}_1^*(e)$ satisfies first-period budget balancing as defined in equation (2).*[28]

[28]Because $e = \emptyset$ arises with probability 0 when $\beta^* = 1$, in case (I.) any feasible specification of first-period transfers may be used to complete the specification of a strategy for the policy state $e = \emptyset$.

(II.) *If $H > 0$, then in the unique subgame perfect equilibrium both candidates choose a first-period platform p_1^* that implements the policy with probability $\beta^* = 1 - \frac{1}{2}H(< 1)$ and for each realization of the policy state $e \in \mathcal{E} \cup \emptyset$:*

(i) *announce the maximum feasible debt: $\delta^*(e) = 1 + \iota(e)\lambda e$, and*

(ii) *choose an $(|\mathcal{E}| + 1)$-variate joint distribution $P_{i,1}^*(\mathbf{x})$ of first-period net endowments such that:*

$$F_1^*(x|e = \emptyset) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{1}{2}\left(\frac{x}{H}\right), & \text{if } 0 \leq x \leq H, \\ \frac{1}{2}, & \text{if } H \leq x \leq 4 - H, \\ \frac{1}{2}\left(1 + \frac{x-4+H}{H}\right), & \text{if } 4 - H \leq x \leq 4, \\ 1, & \text{if } x \geq 4. \end{cases} \tag{8}$$

and for $e \neq \emptyset$, the random variable $\widetilde{x}_1^{\Gamma_e}$ is uniformly distributed on the interval $[0, 4 + 2\lambda E_{\Gamma_e}(e) - 2c]$ such that for each possible policy state e the random variable $\widetilde{x}_1^*(e)$ satisfies first-period budget balancing as defined in equation (2).

Second Period

Given any second-period state $(e, \delta(e)) \in \mathcal{S}_{pd}$, the unique subgame perfect second-period local equilibrium is for each candidate to choose the second-period platform $p_2^*(e, \delta(e))$ that uniformly distributes net endowments on the interval $[0, 2(1+\iota(e)\lambda e - \delta(e))]$.

Along any equilibrium path, the equilibrium debt level is $\delta^*(e) = 1 + \iota(e)\lambda e$ and the equilibrium distribution of second-period net endowments is degenerate with all mass placed on the net endowment 0.

We provide an intuition for the underlying interactions between the decision to reform and to raise public debt, and the implications for the chance of a beneficial reform going through the political process.

Decision to reform. When only a share λ of the reform benefits translates into an increase in the second-period endowment, then the natural debt limit under reform increases only by λe compared to no-reform. This means that through using public debt, the reformer can only make the part $\lambda E_{\Gamma_e}(e)$ of expected reform benefits targetable to first-period voters. For the remaining part, she is forced through the public good nature of these benefits to offer them equally across all voters. However, Part (I.) in Theorem 1 covers the case where targetability λ is high enough so that the expected additional debt that a reformer can raise covers the disadvantage coming from the reform costs: the reformer has more to offer in total even if she is partly forced to distribute this bigger pie in an egalitarian way. The question then is, if the efficiency gain combined with increased debt capacity is enough to compensate the first-period cost savings of the non-reformer.

For the case $\lambda E_{\Gamma_e}(e) < c$, a no-reform candidate has more resources available in the first period for targeting voters. Specifically, the additional per-capita amount available to him equals the difference between the reform costs and the part of the future benefits that can be transferred to the present through debt, $c - \lambda E_{\Gamma_e}(e)$. On the other hand, in case of reform everyone expects a boost in future utility through the public good benefits of the reform. More specifically, each voter expects additional utility $e - \lambda E_{\Gamma_e}(e)$ in case of reform. Since the reform is efficient in the sense that the expected benefits $E_{\Gamma_e}(e)$ are greater than costs c, the additional public good utility, $E_{\Gamma_e}(e) - \lambda E_{\Gamma_e}(e)$, surmounts the loss in targetable resources in the first period, $c - \lambda E_{\Gamma_e}(e)$. However, these public good benefits cannot be targeted. Hence the additional public good utility must be high enough so that the non-reformer cannot convince a majority to vote for her. In particular, she should not be able through her advantage in targetability to make at least half of the voters as well off as under reform. This is exactly the condition of Theorem 1: $E_{\Gamma_e}(e) - \lambda E_{\Gamma_e}(e) \geq 2(c - \lambda E_{\Gamma_e}(e)) \Leftrightarrow H \leq 0$. The factor “2” on the right hand side of this inequality is explained by the fact that in order to win a majority through targeting, a candidate can promise very low offers to $\frac{1}{2}$ of the voters in order to offer attractive benefits the other half. If the condition $E_{\Gamma_e}(e) - \lambda E_{\Gamma_e}(e) > 2(\bar{c} - \lambda E_{\Gamma_e}(e))$ is fulfilled as in Part (I.) of Theorem 1, the efficiency gain of the reform is thus high enough to trump the targetability advantage of the non-reformer.

In Part (II.) of Theorem 1, the additional public good utility under reform is not enough to compensate for the fact that a no-reform candidate has more targetable resources in the first period. We therefore interpret $H = 2(c - \lambda E_{\Gamma_e}(e)) - (E_{\Gamma_e}(e) - \lambda E_{\Gamma_e}(e)) > 0$ as the net targeting advantage of *not* doing the reform. If $H > 0$, the additional targetable resources of a non-reformer is enough to outweigh the efficiency gains from reform and the reform cannot be offered with probability 1 in equilibrium. This means that we get a failure of the political process to deliver the efficient outcome. Notice that even with a net targeting advantage of no-reform, the reform will still be offered with positive probability in equilibrium as long as it is efficient in expectation, i.e. $E_{\Gamma_e}(e) - c > 0$. The reason for this will be discussed when we interpret the equilibrium transfer distributions below.

Decision to raise debt. The fact that both candidates raise the maximum debt follows the political forces highlighted in Lizzeri (1999). Whatever amount of resources is left in the future is not targetable to first-period voters. A candidate that does not run the maximal debt is therefore forced to offer an egalitarian distribution for the resources that she leaves in the future. This goes against the incentive to skew the distribution of resources in order to gain the electoral support of the voters that are treated favorably in the process of redistribution. The electoral uncertainty is not an artefact of the assumption that politicians are unable to commit about second-period transfers. Lizzeri (1999) shows that allowing candidates to commit does not change the electoral incentives to run debt: a candidate who commits to future transfers can only make promises about her own future behaviour, not about ones made by the other candidate. This implies that if a candidate does not run the maximal deficit, there is still an element of redistributive uncertainty concerning the second period outcome. This uncertainty is enough to implies that voters' views on the outcome of the future elections are relatively egalitarian.

An important insight from Theorem 1 is that the ability to raise higher debt under reform ensures the implementation of the reform with certainty only when the benefits of the reform are mainly of a private good nature. In the opposite case, when the nature of the reform is such that only a small part of the reform benefits have a private good aspect, a large share of the reform benefits are non-targetable to begin with cannot be targeted to first-period voters through the use of debt. Therefore, we are getting into the trade-off between efficient (non-targetable) public good spending and targetable transfer spending. This trade-off is at the core of the static setup of Lizzeri and Persico (2001) which we discuss below.

Second Period equilibrium transfer distributions. In the second period, all that candidates can compete over is redistributing all available *targetable* resources. The amount of targetable resources is increased by the reform benefits with private good character, in the case that the reform was implemented in the first period, and it is decreased by any debt that has to be repaid. Therefore, in the second period we are back to a static version of the divide-the-dollar game where the average resources available for making transfer offers are given by the resources left. If all second-period targetable resources are necessary for debt repayment, both candidates' offer distribution are degenerated on the net endowment 0. If some resources are left, the equilibrium offer distribution is uniform on distributes net endowments on the interval $[0, 2(1 + \iota(e)\lambda e - \delta(e))]$.

The crucial feature of the second-period election is the uncertainty for voters regarding the outcome of the process of redistributive politics. Given a uniform distribution on $[0, 2(1 + \iota(e)\lambda e - \delta(e))]$, in period 2 each voter expects to get the average of such distribution in the case of no-reform. In case of reform, each voter expects on top the transfer offer the public good utility $(1 - \lambda)e$. For the analysis of the first period, this expectation about future utility fully captures how a voter evaluates the future effects of a proposed policy. However, the equilibrium distribution implies that some voters are treated very well and others are treated very badly. The politicians have an incentive to "cultivate favored minorities" as in Myerson (1993). This uncertainty is the driving force behind the electoral incentives to do the reform and accumulate debt in the first period.

First Period equilibrium transfer distributions. When the reform has mainly private good benefits, the candidates can target these benefits to particular voters in the first period: the reform is implemented with certainty and maximal debt is raised. Therefore, both candidates compete on redistributing the same amount of resources in the first period. The form of the transfer distribution follows the insight of Myerson (1993).

When the reform has mainly public good benefits, the efficiency gain of doing the reform cannot compensate for the targeting disadvantage of having to cover the reform costs. Nevertheless, the reform will still be implemented with positive probability. The underlying mechanism has been analyzed by Lizzeri and Persico (2001) in a static setup. By still playing the reform strategy with some probability, a candidate can use the efficiency gain of the reform to force her opponent to concentrate half of her offers on relatively "expensive" voters: these voters can be convinced to vote against the reform by receiving at least a transfer that fully cover the utility loss from the no-reform decision plus an additional transfer by the reforming candidate. This will give the reforming candidate an advantage if her opponent were to never offer the reform. As can be seen from Theorem 1, the distribution offered in case of no-reform $F_1^*(x|e=\emptyset)$ has a disconnected support with an upper and a lower part. The upper part starts where any transfer on this part will ensure the vote of any voters and corresponds to the offers made to the expensive voters: the expected utility loss $(1-\lambda)E_{\Gamma_e}(e)$ from not implementing the reform plus the best transfer offered by the reforming candidate $4-2(c-\lambda E_{\Gamma_e}(e))$.

When the net targeting advantage H of the non-reformer decreases, the probability of reform goes up. Ceteris paribus, H decreases when the targetability λ of reform benefits goes up. That is, the more private good aspects a reform has, the more public debt can help in overcoming the reformer's targeting disadvantage from financing the reform costs and the higher the chance of the reform to be implemented in electoral competition. On the other hand, if a reform has a high share of public good benefits, then public debt, which can only transfer the private good aspects to the present, cannot overcome the targeting disadvantage of the reformer completely. For the same efficiency gain, such reform will therefore be implemented with lower probability as an electoral outcome.

5 Budget constraint equilibrium characterization: hard and soft constitutional limits on debt

We begin this section by examining how exogenous restrictions on the amount of first-period debt that may be incurred by politicians change the nature of the political competition and the resulting probability that the efficient reform is adopted. We interpret such restrictions as constitutional limits on debt. In subsection 5.1, we focus on "hard" debt limits which must be satisfied with probability one. Once we have characterized equilibrium for the case of a hard debt limit, we then examine the remaining case of a soft debt limit in subsection 5.2, where a "soft" limit which must only hold on average across the set of policy states. The soft and hard variations of the debt constraint are formally defined as follows. Recall that each candidate i's first-period platform p_1^i specifies candidate i's level of public debt contingent on the realization of the policy state $\{\delta_i(e)\}_{e\in\mathcal{E}\cup\emptyset}$ and consider the case that debt is constrained to be below a level of $\overline{\delta}$. A hard debt limit of $\overline{\delta} > 0$ requires that for each player i and each policy state $e \in \mathcal{E} \cup \emptyset$

$$\delta_i(e) \leq \overline{\delta}$$

whereas a soft debt limit of $\overline{\delta} > 0$ requires that for each player i

$$\delta_i(\emptyset) \leq \overline{\delta} \quad \text{and} \quad E_{\Gamma_e}(\delta_i(e)) \leq \overline{\delta}.$$

5.1 Hard constitutional limit on debt

Suppose that debt is constrained to be below a level of $\overline{\delta} > 0$. For a hard debt limit, the maximum feasible debt for any $e \in \mathcal{E} \cup \emptyset$ is:[29]

$$\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \iota(e)\lambda e\}. \tag{9}$$

In the case that the candidates utilize the maximum feasible debt, let B^d_{NP} denote the first-period budget when the policy is not implemented, let $B^d_P(e)$ denote the first-period budget when the policy is implemented and the policy state is $e \in \mathcal{E}$, and let B^d_P denote the expectation of the first-period budget when the policy is implemented, where:

$$B^d_{NP} = 1+\widehat{\delta}^d(\emptyset), \quad B^d_P(e) = 1+\widehat{\delta}^d(e)-c, \quad B^d_P = E_{\Gamma_e}(B^d_P(e)) = 1+E_{\Gamma_e}(\widehat{\delta}^d(e))-c. \tag{10}$$

Note that because $\widehat{\delta}^d(\emptyset) = \min\{\overline{\delta}, 1\}$, it follows that $B^d_{NP} = 1 + \min\{\overline{\delta}, 1\}$. Similarly, because $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \lambda e\}$ for all $e \in \mathcal{E}$, it follows that $B^d_P = 1 + E_{\Gamma_e}(\min\{\overline{\delta}, 1 + \lambda e\}) - c$.

With a hard debt limit of $\overline{\delta}$,[30] the first-period budget constraint for a candidate i with the maximum feasible debt, i.e. $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \iota(e)\lambda e\}$, is modified as follows. For all $e \in \mathcal{E} \cup \emptyset$:

$$\int_0^{+\infty} x dF_{i,1}(x|e) = E_{F_{i,1|e}}(x) \leq \iota_i B^d_P(e) + (1 - \iota_i)B^d_{NP}. \tag{12}$$

[29]Note that if $\overline{\delta} > 1 + \lambda\overline{e}$ then it follows that the hard debt limit is non-binding.

[30]Although our focus is on debt limits, it is straightforward, for a given a hard debt limit of $\overline{\delta} \leq 1 + \lambda\overline{e}$, to construct an equivalent hard spending limit. Consider the policy-dependent hard spending limit $\overline{S}(e)$ which for each $e \in \mathcal{E} \cup \emptyset$ is defined as

$$\overline{S}(e) := 1 + \overline{\delta} - \iota(e)c. \tag{11}$$

Note that for the policy-dependent hard spending limit $\overline{S}(e)$, the first-period budgets with the hard spending limit are the exact same as the corresponding first-period budgets with the hard debt limit, B^d_{NP} and $B^d_P(e)$ respectively. Thus, it follows that the set of subgame perfect equilibria with the hard spending limit are characterized by Theorem 2 and the success of the reform in the political process is equally likely with the hard debt limit $\overline{\delta}$ as with the corresponding policy-dependent hard spending limit $\overline{S}(e)$ defined in equation (11). In the case of a soft debt limit, a similar extension applies to the construct of an equivalent soft spending limit.

For Theorem 2, we also define

$$\widehat{H}^d := 2B^d_{NP} - 2B^d_P - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e)). \tag{13}$$

Theorem 2 *Given a hard debt constraint of $\overline{\delta} > 0$, the set of subgame perfect equilibria is completely characterized as follows.*

First Period

*In the first period, there are two cases labeled **(I.)** and **(II.)**.*

(I.) *If $\widehat{H}^d \leq 0$, then in any subgame perfect equilibrium both candidates choose a first-period platform p^*_1 that implements the policy with probability $\beta^* = 1$ and for each realization of the policy state $e \in \mathcal{E}$:*

- **(i)** *announce the maximum feasible debt: $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \lambda e\}$, and*
- **(ii)** *choose an $(|\mathcal{E}| + 1)$-variate joint distribution $P^*_1(\mathbf{x})$ of first-period net endowments such that the random variable $\widetilde{x}^{\Gamma_e}_1$ is uniformly distributed on the interval $[0, 2B^d_P]$ and for each possible policy state e the random variable $\widetilde{x}^*_1(e)$ satisfies first-period budget balancing as defined in equation (12).*[31]

(II.) *If $\widehat{H}^d > 0$, then in the unique subgame perfect equilibrium both candidates choose a first-period platform p^*_1 that implements the policy with probability $\beta^* = 1 - \frac{\widehat{H}^d}{B^d_{NP}} (< 1)$ and for each realization of the policy state $e \in \mathcal{E} \cup \emptyset$:*

- **(i)** *announce the maximum feasible debt: $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \iota(e)\lambda e\}$, and*
- **(ii)** *choose an $(|\mathcal{E}| + 1)$-variate joint distribution $P^*_{i,1}(\mathbf{x})$ of first-period net endowments such that:*

[31] Because $e = \emptyset$ arises with probability 0 when $\beta^* = 1$, in case (I.) any feasible specification of first-period transfers may be used to complete the specification of a strategy for the policy state $e = \emptyset$.

$$F_1^*(x|e=\emptyset) = \begin{cases} 0, & \textit{if } x \leq 0, \\ \frac{1}{2}\left(\frac{x}{\widehat{H}^d}\right), & \textit{if } 0 \leq x \leq \widehat{H}^d, \\ \frac{1}{2}, & \textit{if } \widehat{H}^d \leq x \leq 2B_{NP}^d - \widehat{H}^d, \\ \frac{1}{2}\left(1+\frac{x-2B_{NP}^d+\widehat{H}^d}{\widehat{H}^d}\right), & \textit{if } 2B_{NP}^d - \widehat{H}^d \leq x \leq 2B_{NP}^d, \\ 1, & \textit{if } x \geq 2B_{NP}^d. \end{cases} \tag{14}$$

and for $e \neq \emptyset$*, the random variable* $\widetilde{x}_1^{\Gamma_e}$ *is uniformly distributed on the interval* $[0, 2B_p^d]$ *such that for each possible policy state* e *the random variable* $\widetilde{x}_1^*(e)$ *satisfies first-period budget balancing as defined in equation (12).*

Second Period

Given any second-period state $(e, \delta(e)) \in \mathcal{S}_{pd}$*, the unique subgame perfect second-period local equilibrium is for each candidate to choose the second-period platform* $p_2^*(e, \delta(e))$ *that uniformly distributes net endowments on the interval* $[0, 2(1+\iota(e)\lambda e - \delta(e))]$.

Along any equilibrium path, the equilibrium debt level is $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1+\iota(e)\lambda e\}$ *and the second-period local equilibrium net endowments are uniformly distributed on the interval* $[0, 2(1+\iota(e)\lambda e - \widehat{\delta}^d(e))]$.

We have seen in Theorem 1 that the nature of reform benefits and the availability of public debt are crucial determinants of the success of reforms in the political process. For intuition on Theorem 2, the following Corollary examines how the debt limit interacts with the likelihood of the policy being offered in the special case that the policy has only private-good benefits (i.e. $\lambda = 1$).

Corollary 1 *Suppose that the reform benefits are of a private good nature, i.e.* $\lambda = 1$*, in which case* $\widehat{H}^d = 2\left(c - \overline{\delta} + 1\right) - \left(E_{\Gamma_e}(e) - \overline{\delta} + 1\right)$.

(I.) *When the hard debt limit is such that* $\widehat{H}^d \leq 0$*, then in any subgame perfect equilibrium both candidates choose a first-period platform* p_1^* *that implements the policy with probability* $\beta^* = 1$ *and announce the maximal feasible debt.*

(II.) *When the hard debt limit is restrictive enough such that* $\widehat{H}^d > 0$*, then in the unique subgame perfect equilibrium both candidates choose a first-period platform* p_1^* *that implements the policy with probability* $1 - \frac{1}{2}\widehat{H}^d (< 1)$ *and announce the maximal feasible debt.*

Comparing these results to Theorem 1, we see that restricting public debt has a similar effect as increasing the proportion of reform benefits with public good nature. In particular, even if now the full reform benefits are potentially targetable, any reform benefits that have to be left in the future due to the debt limit acquire the characteristics of a non-targetable public good from the point of view of first-period voters. The amount $E_{\Gamma_e}(e) - \overline{\delta} + 1$ corresponds to the part of future reform benefits that cannot be transferred to the present. Since the outcome of future redistribution is uncertain, these resources cannot be skewed to specific voters. On the contrary, each voter expects the same amount $E_{\Gamma_e}(e) - \overline{\delta} + 1$ of additional second-period transfers under reform. The part $E_{\Gamma_e}(e) - \overline{\delta} + 1$ of the reform benefits that has to be left in the future is just like a public good whose benefits cannot be targeted. The difference to Theorem 1 is that if more debt was allowed, this part could also be targeted to first period voters. For the case of Theorem 1, the public good characteristic was given through the nature of the reform and could not be changed. Here, in contrast, it is created through the debt limit combined with future electoral uncertainty.

Through this analogy we get the same case distinction as before. If the hard debt limit is not too restrictive such that enough future reform benefits can be targeted to first period voters, the cost-saving advantage of the non-reformer is overcome and reform is implemented with certainty in the political equilibrium. On the other hand, if the hard debt limit becomes too restrictive, the no-reformer has a net targeting advantage. Due to its efficiency gain the reform will still be implemented with some probability for the same reason as discussed for Theorem

1. However, the reform will no longer be implemented with certainty and the political process fails to deliver the efficient outcome.

To sum up, the results in this subsection point to a new view on the trade-off between targeted pork-barrel spending and efficient spending decisions, like the financing of a beneficial reform. In particular, when the reform is of a *dynamic* nature, allowing enough debt-related pork-barrel spending might be necessary to incentivize the reform in political competition. It opens the question on the benefits or costs of relaxing the debt limit. As the following Corollary shows the answer depends on how restrictive initially the debt limit is.

Corollary 2 **(I.)** *When the hard debt limit is such that* $0 < \overline{\delta} < 1$*, then an increase in* $\overline{\delta}$ *weakly decreases the probability with which the policy is implemented* $(1 - \frac{1}{2}\widehat{H}^d)$*.*

(II.) *When the debt limit is such that* $1 \leq \overline{\delta} < 1+\lambda\overline{e}$*, then an increase in* $\overline{\delta}$ *weakly increases the probability with which the policy is implemented* $(1 - \frac{1}{2}\widehat{H}^d)$*.*

The proof of Corollary 2 follows directly from the expression for $\widehat{H}^d$ in equation (13), and is thus, omitted. In the next subsection, we examine the relationship between a hard debt limit and a soft debt limit.

5.2 Soft constitutional limit on debt

Suppose that debt is constrained to be below a level of $\overline{\delta} > 0$ and recall that a soft debt limit requires that for each candidate i

$$\delta_i(\emptyset) \leq \overline{\delta} \quad \text{and} \quad E_{\Gamma_e}(\delta_i(e)) \leq \overline{\delta}. \tag{15}$$

For example, consider the case that in the event that candidate i implements the policy, candidate i's set of policy-state contingent public debt levels $\{\widehat{\delta}_i^\eta(e)\}_{e\in\mathcal{E}}$ bring forward a constant fraction $\eta \in (0,1)$ of the second-period endowment and realized policy benefits, subject to feasibility with respect to the soft debt limit. In this case, candidate i's policy-state contingent public debt levels may be defined, for each $e \in \mathcal{E} \cup \emptyset$ as:

$$\widehat{\delta}_i^{\eta}(e) = [1 - \iota(e)] \min\left\{\overline{\delta}, \eta\right\} + \iota(e)\eta\,[1 + \lambda e] \tag{16}$$

where $E_{\Gamma_e}(\widehat{\delta}_i^{\eta}(e)) \leq \overline{\delta}$.

If η is given by $\eta^* := \frac{\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\}}{1+\lambda E_{\Gamma_e}(e)}$, then it follows that candidate i's set of policy-state contingent public debt levels $\{\widehat{\delta}_i^{\eta*}(e)\}_{e\in\mathcal{E}\cup\emptyset}$ defined by equation (16) satisfy the soft debt limit condition in equation (15), $E_{\Gamma_e}(\widehat{\delta}_i^{\eta*}(e)) \leq \overline{\delta}$. In particular, if $\overline{\delta} < 1 + \lambda E_{\Gamma_e}(e)$ then $\eta^* = \frac{\overline{\delta}}{1+\lambda E_{\Gamma_e}(e)} < 1$ and $E_{\Gamma_e}(\widehat{\delta}_i^{\eta*}(e)) = \overline{\delta}$, but if $\overline{\delta} \geq 1 + \lambda E_{\Gamma_e}(e)$ then $\eta^* = 1$ and $E_{\Gamma_e}(\widehat{\delta}_i^{\eta*}(e)) \leq \overline{\delta}$.

Next, note that the set of policy-state contingent public debt levels $\{\widehat{\delta}_i^{\eta*}(e)\}_{e\in\mathcal{E}}$ may not be feasible under the corresponding hard debt limit. That is, with a soft debt limit the candidates may be able to smooth the debt constraint $\overline{\delta}$ over the set of policy states in ways that are not feasible with a hard debt constraint. From the maximum feasible debt expression in equation (9) the hard debt limit is binding for policy states $e > \frac{\overline{\delta}-1}{\lambda}$ and non-binding for policy states $e < \frac{\overline{\delta}-1}{\lambda}$. Thus, if the debt constraint $\overline{\delta}$ satisfies $\overline{\delta} \in (1 + \lambda E_{\Gamma_e}(e), 1 + \lambda\overline{e})$, then $\eta* = 1$ and for each realization of the policy state $e \in \mathcal{E}$ such that $e \in (\frac{\overline{\delta}-1}{\lambda}, \overline{e})$ it follows that $\widehat{\delta}_i^{\eta*}(e) > \overline{\delta}$.

In the case that the candidates use platforms in which (i) when the policy is not implemented the soft debt limit is binding for all $\overline{\delta} < 1$ and (ii) when the policy is implemented the soft debt limit is binding for all $\overline{\delta} < 1 + \lambda E_{\Gamma_e}(e)$, let B_{NP}^{sd} denote the first-period budget when the policy is not implemented and let B_P^{sd} denote the expectation of the first-period budget when the policy is implemented, where:

$$B_{NP}^{sd} = 1 + \min\{\overline{\delta}, 1\} \quad \text{and} \quad B_P^{sd} = 1 + \min\{\overline{\delta}, 1 + \lambda E_{\Gamma_e}(e)\} - c. \tag{17}$$

Given $\overline{\delta}$, the constraint on the average first-period budget for a candidate i is:

$$(1 - \iota_i)E_{F_{i,1|e=\emptyset}}(x) + \iota_i E_{\Gamma_e}[E_{F_{i,1|e}}(x)] \leq (1 - \iota_i)B_{NP}^{sd} + \iota_i B_P^{sd}. \tag{18}$$

and the corresponding expression for $\widehat{H}^d$ becomes

$$\widehat{H}^{sd} := 2B_{NP}^{sd} - 2B_P^{sd} - 1 - E_{\Gamma_e}(e) + \min\{\overline{\delta}, 1 + \lambda E_{\Gamma_e}(e)\}. \tag{19}$$

Theorem 3 *Given a soft debt constraint of* $\overline{\delta} > 0$*, the set of subgame perfect equilibria is completely characterized as follows.*

First Period

In the first period, there are two cases labeled ***(I.)*** *and* ***(II.)***.

(I.) *If* $\widehat{H}^{sd} \leq 0$*, then in any subgame perfect equilibrium both candidates choose a first-period platform* p_1^* *that implements the policy with probability* $\beta^* = 1$ *and:*

- **(i)** *announce the maximum feasible average debt:* $\min\{\overline{\delta}, 1 + \lambda E_{\Gamma_e}(e)\}$*, and*
- **(ii)** *choose an* $(|\mathcal{E}| + 1)$*-variate joint distribution* $P_1^*(\mathbf{x})$ *of first-period net endowments such that the random variable* $\widetilde{x}_1^{\Gamma_e}$ *is uniformly distributed on the interval* $[0, 2B_P^{sd}]$ *and satisfies the constraint on the average first-period budget as defined in equation (18).*[32]

(II.) *If* $\widehat{H}^{sd} > 0$*, then in the unique subgame perfect equilibrium both candidates choose a first-period platform* p_1^* *that implements the policy with probability* $\beta^* = 1 - \frac{\widehat{H}^{sd}}{B_{NP}^{sd}} (< 1)$ *and:*

- **(i)** *announce the maximum feasible average debt:* $\min\{\overline{\delta}, 1 + \iota(e)\lambda E_{\Gamma_e}(e)\}$*, and*
- **(ii)** *choose an* $(|\mathcal{E}| + 1)$*-variate joint distribution* $P_{i,1}^*(\mathbf{x})$ *of first-period net endowments such that:*

$$F_1^*(x|e = \emptyset) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{1}{2}\left(\frac{x}{\widehat{H}^{sd}}\right), & \text{if } 0 \leq x \leq \widehat{H}^{sd}, \\ \frac{1}{2}, & \text{if } \widehat{H}^{sd} \leq x \leq 2B_{NP}^{sd} - \widehat{H}^{sd}, \\ \frac{1}{2}\left(1 + \frac{x - 2B_{NP}^{sd} + \widehat{H}^{sd}}{\widehat{H}^{sd}}\right), & \text{if } 2B_{NP}^{sd} - \widehat{H}^{sd} \leq x \leq 2B_{NP}^{sd}, \\ 1, & \text{if } x \geq 2B_{NP}^{sd}. \end{cases}$$

[32]Because $e = \emptyset$ arises with probability 0 when $\beta^* = 1$, in case (I.) any feasible specification of first-period transfers may be used to complete the specification of a strategy for the policy state $e = \emptyset$.

$$(20)$$

and for $e \neq \emptyset$, the random variable $\widetilde{x}_1^{\Gamma_e}$ is uniformly distributed on the interval $[0, 2B_p^{sd}]$ and satisfies the constraint on the average first-period budget as defined in equation (18).

Second Period

Given any second-period state $(e, \delta(e)) \in \mathcal{S}_{pd}$, the unique subgame perfect second-period local equilibrium is for each candidate to choose the second-period platform $p_2^*(e, \delta(e))$ that uniformly distributes net endowments on the interval $[0, 2(1+\iota(e)\lambda e - \delta(e))]$. Along any equilibrium path, the equilibrium average debt level is $\min\{\overline{\delta}, 1 + \iota(e)\lambda E_{\Gamma_e}(e)\}$.

The proof of Theorem 3 follows along the same lines as the proof of Theorem 2, with the caveat that unlike the case of a hard debt limit, when the policy is implemented a soft debt limit does not directly impose conditions on the first-period budget for each of the individual policy states $e \in \mathcal{E}$. Instead the soft debt limit only imposes a constraint on the expectation of the first-period budget, across the set of policy states, when the policy is implemented, B_P^{sd}. Thus, the set of policy-state contingent public debt levels $\{\widehat{\delta}_i^{\eta *}(e)\}_{e\in\mathcal{E}}$, given by equation (16) with η equal to η^*, provide one set of equilibrium policy-state contingent public debt levels, but the equilibrium debt level for each policy state is not pinned down by the soft debt constraint.

The following result compares the efficiency of policy provision with soft and hard debt limits.

Corollary 3 **(I.)** *For all debt constraints $\overline{\delta} > 0$, the equilibrium probability that the policy is implemented under the soft debt limit is at least as high as under the hard debt limit.*

(II.) *For any $\overline{\delta} > 0$ such that $\widehat{H}^{sd} > 0$ the equilibrium probability that the policy is implemented under the soft debt limit is strictly higher than under the hard debt limit if and only if $\min\{\overline{\delta}, 1 + \lambda E_{\Gamma_e}(e)\} > E_{\Gamma_e}(\min\{\overline{\delta}, 1 + \lambda e\})$.*

Given that (i) $\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\} \geq E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\})$ for all $\overline{\delta} > 0$ and (ii) from equations (10) and (17) we know that $B^d_{NP} = B^{sd}_{NP} = 1 + \min\{\overline{\delta}, 1\}$, it follows directly from parts (I.) and (II.) of Theorems 2 and 3 that the equilibrium probability that the policy is implemented under the soft debt limit is at least as high as under the hard debt limit if and only if $\widehat{H}^{sd} \leq \widehat{H}^d$ for all $\overline{\delta} > 0$. Then, from the definitions of $\widehat{H}^d$ and $\widehat{H}^{sd}$ in equations (13) and (19) respectively and recalling that $B^d_P = 1 + E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\}) - c$, it follows that $\widehat{H}^{sd} \leq \widehat{H}^d$ requires that

$$\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\} - E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\}) \leq \\ 2\left(\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\} - E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\})\right) \quad (21)$$

Because $\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\} \geq E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\})$ for all $\overline{\delta} > 0$, it follows that $\widehat{H}^{sd} \leq \widehat{H}^d$ for all $\overline{\delta} > 0$, and thus, the equilibrium probability that the policy is implemented under the soft debt limit is at least as high as under the hard debt limit.

For part (II.) of Corollary 3, note that because $\widehat{H}^{sd} \leq \widehat{H}^d$ for all $\overline{\delta} > 0$ it follows that if $\widehat{H}^{sd} > 0$ then $\widehat{H}^d > 0$, and, as a result, the equilibrium probabilities of the policy being implemented under the heard debt limit and the soft debt limit are specified in part (II.) of Theorems 2 and 3, respectively. Then, recalling that $B^d_{NP} = B^{sd}_{NP}$, it follows from part (II.) of Theorems 2 and 3, that the equilibrium probability that the policy is implemented under the soft debt limit is strictly higher than under the hard debt limit if and only if $0 < \widehat{H}^{sd} < \widehat{H}^d$, which from equation (21) requires that $\min\{\overline{\delta}, 1+\lambda E_{\Gamma_e}(e)\} > E_{\Gamma_e}(\min\{\overline{\delta}, 1+\lambda e\})$. Note that this correspondes exactly to the portion of the parameter region in which the debt constraint $\overline{\delta}$ satisfies $\overline{\delta} \in (1+\lambda E_{\Gamma_e}(e), 1+\lambda\overline{e})$ and the set of policy-state contingent public debt levels $\{\widehat{\delta}^{\eta *}_i(e)\}_{e\in\mathcal{E}}$ were feasible under the soft debt limit but not under the hard debt limit.

6 Concluding remarks

In this paper we show that the decision to raise public debt is decisive in shaping the electoral incentives for implementing a reform. We prove that the reform is always implemented when sufficient debt can be raised. This is the case if enough reform benefits are of a private good nature that translate into an increase in the future endowment and can potentially be transferred to the present by debt. We also show that restricting the use of public debt hampers the chances of a reform to go through the political process. Our results point towards a new evaluation of the trade-off between targeted spending and efficient spending decisions: enough debt-related targeted spending might be necessary to incentivize efficient spending on dynamic policies whose benefits only accrue in the next electoral cycle. This result implies that constitutional restrictions on public debt and spending might be a hurdle for the implementation of reforms by politicians.

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Online Appendix

Proof of Theorem 1

We begin in the second period with any state $(e, \delta(e)) \in \mathcal{S}_{pd}$ and show that in the subgame arising in state $(e, \delta(e))$ the corresponding Theorem 1 second-period local strategies form a second-period local equilibrium and, furthermore, establish that this second-period local equilibrium is unique. Then, given the second-period local equilibrium strategies we move back through the game tree to the first period and characterize the unique subgame-perfect equilibrium first-period local strategies.

Second Period

In the second period with any state $(e, \delta(e))$, it follows from the second-period expected vote share calculation given in equation (5) that candidate A's second-period expected vote share,

$$S_2^A(p_2^A(e, \delta(e)), p_2^{*B}(e, \delta(e))|e, \delta(e)),$$

from using the arbitrary second-period local strategy $p_2^A(e, \delta(e))$, given that candidate B uses the equilibrium second-period local strategy $p_2^{*B}(e, \delta(e))$ is:

$$S_2^A(p_2^A(e, \delta(e)), p_2^{*B}(e, \delta(e))|e, \delta(e)) = \int_{\mathrm{Supp}F_{A,2|e,\delta(e)}} F_2^*(x|e, \delta(e)) dF_{A,2}(x|e, \delta(e)). \tag{22}$$

In any best response, it is clear that candidate A does not provide a voter z with a second-period utility level that is strictly greater than $2(1 + \iota(e)\lambda e - \delta(e))$. Thus, from equation (3)'s second-period budget-balancing condition (i.e. $E_{F_{A,2|e,\delta(e)}}(x) = 1 + \iota(e)\lambda e - \delta(e))$ it follows from equation (22) that A's second-period expected vote share satisfies

$$\int_{\mathrm{Supp}F_{A,2|e,\delta(e)}} \frac{x}{2(1 + \iota(e)\lambda e - \delta(e))} dF_{A,2}(x|e, \delta(e)) \leq \frac{1 + \iota(e)\lambda e - \delta(e)}{2(1 + \iota(e)\lambda e - \delta(e))} = \frac{1}{2}.$$

To complete the proof that for all states $(e, \delta(e)) \in \mathcal{S}_{pd}$ the Theorem 1 second-period local strategies form a second-period local equilibrium, observe that candidate A receives $\frac{1}{2}$ of the second-period vote share from any budget-balancing second-period local strategy $F_{A,2}(x|e,\delta(e))$ with $\text{Supp}\,(F_{A,2}|e,\delta(e)) \subseteq [0, 2(1 + \iota(e)\lambda e - \delta(e))]$ and that candidate A has no profitable deviations. Because the second-period subgame for each state $(e,\delta(e)) \in \mathcal{S}_{pd}$ involves only redistribution, the proof of uniqueness of the second-period local equilibrium strategies follows from standard results on Myerson's formulation of the relaxed Colonel Blotto game (a.k.a. the General Lotto game, for further details see Kovenock and Roberson (2020)).

First Period

Given the second-period local equilibrium strategies specified by Theorem 1, we now move back through the game tree to the first period and characterize the unique subgame-perfect equilibrium first-period local strategies. We begin by examining the first-period vote share calculation. Then, we turn to the proof that in part (I.) the Theorem 1 first-period local strategies form a first-period local equilibrium. Next, we perform the corresponding analysis for part (II.). The proof that the first-period local strategies are unique is given in the appendix.

For the first-period vote share calculation, suppose, without loss of generality, that candidate A uses an arbitrary first-period local strategy p_1^A. Given that candidate B uses the equilibrium first-period platform p_1^*, candidate B's expected promise of continuation utility for an arbitrary voter z is the random variable $\widetilde{U}_z(p_1^*)$ defined by equation (6) as:

$$\widetilde{U}_z(p_1^*) = \beta^* \left(\widetilde{x}_1^{\Gamma_e} + 1 + E_{\Gamma_e}(e - \delta_i(e))\right) + (1-\beta^*)\left(\widetilde{x}_1^*(\emptyset) + 1 - \delta^*(\emptyset)\right). \quad (23)$$

For $u \in [0,4]$, let $G^*(u)$ denote the distribution of the random variable $\widetilde{U}_z(p_1^*)$, which we will examine in more detail below for cases (I.) and (II.) of Theorem 1. Similarly, let $G_{p_1^A}(u)$ denote the distribution of the random variable $\widetilde{U}_z(p_1^A)$ generated by the first-period platform p_1^A via equation (6).

The probability that candidate A wins voter z's first-period vote when A provides voter z with a first-period continuation utility of $U_z(p_1^A)$ is $G^*(U_z(p_1^A))$. Thus, candidate A's first-period expected vote share when using an arbitrary first-period local strategy p_1^A and candidate B is using the the first-period platform p_1^* is

$$S_1^A(p_1^A, p_1^*) = \int_{\text{Supp}G_{p_1^A}} G^*(u) dG_{p_1^A}(u) \tag{24}$$

We now use the equation (24) first-period vote share calculation in the proof that in part (I.) of Theorem 1 – where $H = 2c - (1+\lambda)E_{\Gamma_e}(e) \leq 0$ – the Theorem 1 first-period local strategies form a first-period local equilibrium. Given that candidate B is using the first-period local equilibrium strategy p_1^* specified by part $(I.)$ of Theorem 1, it follows that the random variable $\widetilde{U}_z(p_1^*)$ is distributed according to[33]

$$G^*(u) = \begin{cases} 0, & \text{if } u \leq (1-\lambda)E_{\Gamma_e}(e), \\ \frac{u-(1-\lambda)E_{\Gamma_e}(e)}{4+2\lambda E_{\Gamma_e}(e)-2c}, & \text{if } (1-\lambda)E_{\Gamma_e}(e) \leq u \leq 4-H, \\ 1, & \text{if } u \geq 4-H. \end{cases}$$

In any best-response, it is clear that candidate A does not provide voter z with a utility level $U_z(p_1^A)$ that is strictly greater than $4-H$.[34] Thus, given that B is using the first-period local equilibrium strategy p_1^*, it follows from equation (24) that candidate A's first-period expected vote share in state e, from an arbitrary first-period local strategy p_1^A is

$$S_1^A(p_1^A, p_1^*) = \int_{\text{Supp}G_{p_1^A}} \frac{u-(1-\lambda)E_{\Gamma_e}(e)}{4+2\lambda E_{\Gamma_e}(e)-2c} dG_{p_1^A}(u). \tag{25}$$

First we consider the case that $\iota_A = 1$. From equation (6) with $\iota_A = 1$, it follows that

$$\widetilde{U}_z(p_1^A) = \widetilde{x}_{A,1}^{\Gamma_e} + 1 + E_{\Gamma_e}(e - \delta_A(e)). \tag{26}$$

Then, from equation (1) we know that

$$E_{G_{p_1^A}}(\widetilde{x}_{A,1}^{\Gamma_e}) = E_{G_{p_1^A}}\left(\sum_{e\in\mathcal{E}} \Gamma_e(e)\widetilde{x}_{i,1}(e)\right) = \sum_{e\in\mathcal{E}} \Gamma_e(e) E_{F_{A,1|e}}(\widetilde{x}_{i,1}(e)) \leq 1 + E_{\Gamma_e}(\delta_A(e)) - c$$

[33]Note that because $H = 2c - (1+\lambda)E_{\Gamma_e}(e)$ and $\delta^*(e) = 1 + \lambda e$ when $\beta^* = 1$, it follows that $4 - H = 2 + 2E_{\Gamma_e}(\delta^*(e)) - 2c + (1 + E_{\Gamma_e}(e - \delta^*(e)))$ and $(1 + E_{\Gamma_e}(e - \delta^*(e))) = (1-\lambda)E_{\Gamma_e}(e)$.

[34]Note that because $\delta^*(e)$ is the maximum level of debt, $\delta_A(e) \leq \delta^*(e)$ and $1 + e - \delta_A(e) \geq 1 + e - \delta^*(e)$. That is, if candidate A chooses $\iota_A = 1$, then candidate A is unable to provide voter z with a continuation utility below $1 + e - \delta^*(e)$.

$$(27)$$

where the last inequality in equation (27) follows from the first-period budget constraint given in equation (2). Inserting, equations (26) and (27) into equation (25) we see that

$$S_1^A(p_1^A, p_1^*) \leq \frac{2 + \lambda E_{\Gamma_e}(e) - c}{4 + 2\lambda E_{\Gamma_e}(e) - 2c} = \frac{1}{2}. \tag{28}$$

To summarize, if $H = 2c - (1+\lambda)E_{\Gamma_e}(e) \leq 0$ and candidate B uses the first-period local equilibrium strategy p_1^{*B} specified in part $(I.)$ of Theorem 1, then candidate A's first-period expected vote share from any arbitrary first-period platform p_1^A with $\iota_A = 1$ is less than or equal to $\frac{1}{2}$, where equation (28) holds with equality if candidate A's strategy is first-period budget balancing as specified by equation (2).

To complete the proof of existence for part $(I.)$ of Theorem 1, consider the remaining case in which candidate A chooses an arbitrary first-period strategy in which $\iota_A = 0$ with strictly positive probability. We now show that candidate A's payoff from a first-period platform with $\iota_A = 0$ is strictly less than if $\iota_A = 1$. Therefore, in any best response candidate A chooses $\iota_A = 1$ with probability one. From equation (6) with $\iota_A = 0$ and the first-period budget constraint given in equation (2), it follows that

$$E_{G_{p_1^A}}(\widetilde{U}_z(p_1^A)) = E_{F_{A,1|\emptyset}}(\widetilde{x}_{A,1}(\emptyset)) + 1 - \delta_A(\emptyset) \leq 2. \tag{29}$$

From equations (25) and (29), candidate A's first-period expected vote share, from such a strategy, is

$$S_1^A(p_1^A, p_1^*) \leq \frac{2 + \lambda E_{\Gamma_e}(e) - E_{\Gamma_e}(e)}{4 + 2\lambda E_{\Gamma_e}(e) - 2c} < \frac{1}{2} \tag{30}$$

where the strict-inequality in equation (30) follows from assumption (A1). Thus, candidate A's first-period expected vote share from deviating to any arbitrary first-period strategy with $\iota_A = 0$ is less than or equal to $\frac{1}{2}$. This completes the proof of part (I.) of Theorem 1.

We now examine part (II.) of Theorem 1, in which $H = 2c - (1+\lambda)E_{\Gamma_e}(e) > 0$. Given that candidate B is using the first-period equilibrium strategy specified by part (II.) of Theorem 1, it follows that $\beta_B = \beta^* = 1 - \frac{1}{2}H < 1$ and for each realization of the policy benefit $e \in \mathcal{E} \cup \emptyset$ the debt is $\delta^*(e) = 1 + \iota(e)\lambda e$. In the event that $\iota_B = 0$, the random variable $\widetilde{x}_{B,1}(\emptyset)$ is distributed according to

$$F_1^*(x|e=\emptyset) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{1}{2}\left(\frac{x}{H}\right), & \text{if } 0 \leq x \leq H, \\ \frac{1}{2}, & \text{if } H \leq x \leq 4-H, \\ \frac{1}{2}\left(1 + \frac{x-4+H}{H}\right), & \text{if } 4-H \leq x \leq 4, \\ 1, & \text{if } x \geq 4. \end{cases} \tag{31}$$

and in the event that $\iota_B = 1$, the random variable $\widetilde{x}_1^{\Gamma_e}$ is uniformly distributed on the interval $[0, 4 + 2\lambda E_{\Gamma_e}(e) - 2c]$. Because $\iota_B = 1$ and $\iota_B = 0$ are mutually exclusive events, the random variable $\widetilde{U}_z(p_1^*)$ is distributed according to[35]

$$G_1^*(u) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{u}{4}, & \text{if } 0 \leq u \leq H, \\ \frac{H}{4}, & \text{if } H \leq u \leq (1-\lambda)E_{\Gamma_e}(e), \\ \frac{H}{4} + \left(1 - \frac{H}{2}\right)\left(\frac{u-(1-\lambda)E_{\Gamma_e}(e)}{4+2\lambda E_{\Gamma_e}(e)-2c}\right), & \text{if } (1-\lambda)E_{\Gamma_e}(e) \leq u \leq 4-H, \\ \frac{u}{4}, & \text{if } 4-H \leq u \leq 4, \\ 1, & \text{if } x \geq 4. \end{cases} \tag{32}$$

If candidate A chooses a first-period platform p_1^A with $\iota_A = 1$ and $\text{Supp}(G_{p_1^A}(u)) \in [(1-\lambda)E_{\Gamma_e}(e), 4-H]$, then candidate A's expected vote share in state e, from such a strategy is

$$S_1^A(p_1^A, p_1^*) = \frac{H}{4} + \left(1 - \frac{H}{2}\right)\int_{\text{Supp}G_{p_1^A}} \frac{u-(1-\lambda)E_{\Gamma_e}(e)}{4+2\lambda E_{\Gamma_e}(e)-2c} dG_{p_1^A}(u) \tag{33}$$

Inserting, equations (26) and (27) into equation (33), we have that

$$S_1^A(p_1^A, p_1^*) \leq \frac{H}{4} + \left(1 - \frac{H}{2}\right)\frac{2+\lambda E_{\Gamma_e}(e) - c}{4+2\lambda E_{\Gamma_e}(e)-2c} = \frac{1}{2} \tag{34}$$

[35]Note that when $\iota_B = 1$, $\delta^*(e) = 1 + \lambda e$ and, thus, $4 - H = 2 + 2E_{\Gamma_e}(\delta^*(e)) - 2c + (1 + E_{\Gamma_e}(e - \delta^*(e)))$ and $(1 + E_{\Gamma_e}(e - \delta^*(e))) = (1-\lambda)E_{\Gamma_e}(e)$.

Thus, candidate A's expected vote share from any first-period platform p_1^A with $\iota_A = 1$ and $\text{Supp}(G_{p_1^A}(u)) \in [(1-\lambda)E_{\Gamma_e}(e), 4-H]$ is less than or equal to $\frac{1}{2}$.

We now show that, given that candidate B is using the first-period equilibrium platform p_1^* specified by part (II.) of Theorem 1, in any best-response by candidate A with $\iota_A = 1$ it must be the case that $\text{Supp}(G_{p_1^A}(u)) \in [(1-\lambda)E_{\Gamma_e}(e), 4-H]$. First, if candidate A uses a strategy with $\iota_A = 1$, then candidate A provides each voter with an expected utility of at least $(1-\lambda)E_{\Gamma_e}(e)$. Next, note that it is clearly suboptimal for candidate A to ever provide utility levels $U_z(p_1^A)$ above 4. The only remaining case with $\iota_A = 1$ is that there exists a measurable subset of $\text{Supp}(G_{p_1^A}(u))$ in the interval $[4-H, 4]$.

Because $\iota_A = 1$ all voters have a continuation utility offer of at least $(1-\lambda)E_{\Gamma_e}(e)$ from candidate A. Let M_1 denote the average of the continuation utility offers that candidate A makes in the interval $[(1-\lambda)E_{\Gamma_e}(e), 4-H]$, where $M_1 \geq (1-\lambda)E_{\Gamma_e}(e)$ and $G_{p_1^A}(4-H)$ voters receive such offers. Similarly, let M_2 denote the average of the continuation utility offers that candidate A makes in the interval $[4-H, 4]$, where $M_2 \geq 4-H$ and $1-G_{p_1^A}(4-H)$ voters receive such offers. From equations (26) and (27) it follows that

$$G_{p_1^A}(4-H)M_1 + (1-G_{p_1^A}(4-H))M_2 \leq 2 + E_{\Gamma_e}(e) - c. \tag{35}$$

Note that because $M_1 \geq (1-\lambda)E_{\Gamma_e}(e)$ and $M_2 \geq 4-H = 4-2c+(1+\lambda)E_{\Gamma_e}(e)$, it follows from equation (35) that $G_{p_1^A}(4-H) \geq 1/2$, i.e. candidate A can offer at most half of the voters net endowments such that their continuation utility is at or above $4-H$.

Returning to candidate A's first period expected vote share which is given by:

$$S_1^A(p_1^A, p_1^*) = G_{p_1^A}\left[(4-H)\left(\frac{H}{4}\right) + \left(1-\frac{H}{2}\right)\frac{(M_1-(1-\lambda)E_{\Gamma_e}(e))}{4+2\lambda E_{\Gamma_e}(e)-2c}\right] + \frac{(1-G_{p_1^A}(4-H))M_2}{4}. \tag{36}$$

Because $\frac{\left(1-\frac{H}{2}\right)}{4+2\lambda E_{\Gamma_e}(e)-2c} > \frac{1}{4}$, it follows that, for any $G_{p_1^A}(4-H) \geq 1/2$, candidate A's first period expected vote share in equation (36) increases as M_2 decreases towards its lower bound of $4-H$ and M_1 increases subject to the constraint in equation (35). This completes the proof that in any best-response by candidate A with $\iota_A = 1$ it must be the case that $\text{Supp}(G_{p_1^A}(u)) \in [(1-\lambda)E_{\Gamma_e}(e), 4-H]$.

For the case that candidate A chooses a first-period platform p_1^A with $\iota_A = 0$ and $\text{Supp}(G_{p_1^A}(u)) \in [0, H] \cup [4-H, 4]$, it follows from equation (32) that candidate A's expected vote share is

$$S_1^A(p_1^A, p_1^*) = \int_{\text{Supp}G_{p_1^A}} \frac{u}{4} dG_{p_1^A}(u) \tag{37}$$

Given budget feasibility with $\iota_A = 0$, see equation (2), it follows from equation (37) that candidate A's expected vote share from any such a strategy p_1^A is less than or equal to $1/2$, which holds with equality if p_1^A is budget balancing.

In the case of a strategy p_1^A with $\iota_A = 0$, it is clearly not payoff increasing for candidate A to offer continuation utilities in the interval $[H, 4-H]$. For the remaining case that of $\iota_A = 0$ with continuation utility offers in the interval $[4-H, 4]$, let $\widehat{M_1}$ denote the average of the continuation utility offers that candidate A makes in the interval $[0, H]$, where μ_1 voters receive such offers. Let $\widehat{M_2}$ and $\widehat{M_3}$ be similarly defined for the average of the continuation utility offers that candidate A makes in the intervals $[(1-\lambda)E_{\Gamma_e}(e), 4-H]$ and $[4-H, 4]$ respectively, where μ_2 and μ_3 voters receive such offers, respectively. From equations (26) and (27) it follows that

$$\mu_1 \widehat{M_1} + \mu_2 \widehat{M_2} + \mu_3 \widehat{M_3} \leq 2 \tag{38}$$

where $\mu_1 + \mu_2 + \mu_3 = 1$.

Candidate A's first period expected vote share, with $\iota_A = 0$ and $\mu_2 \geq 0$ is given by:

$$S_1^A(p_1^A, p_1^*) = \frac{\mu_1 \widehat{M}1 + \mu_3 \widehat{M_3}}{4} + \mu_2 \left[\frac{H}{4} + \left(1 - \frac{H}{2}\right) \frac{(\widehat{M_2} - (1-\lambda)E_{\Gamma_e}(e))}{4 + 2\lambda E_{\Gamma_e}(e) - 2c} \right]. \tag{39}$$

Inserting the constraint in equation (38) into equation (39), we have

$$S_1^A(p_1^A, p_1^*) \leq \frac{2 - \mu_2 \widehat{M_2}}{4} + \mu_2 \left[\frac{H}{4} + \left(1 - \frac{H}{2}\right) \frac{(\widehat{M_2} - (1-\lambda)E_{\Gamma_e}(e))}{4 + 2\lambda E_{\Gamma_e}(e) - 2c} \right]. \tag{40}$$

It follows from equation (40) that $S_1^A(p_1^A, p_1^*)$ is strictly decreasing in μ_2,

$$\frac{\partial S_1^A(p_1^A, p_1^*)}{\partial \mu_2} = \left(\frac{H}{4}\right) + \left(1 - \frac{H}{2}\right) \frac{(\widehat{M_2} - (1-\lambda)E_{\Gamma_e}(e))}{4 + 2\lambda E_{\Gamma_e}(e) - 2c} - \frac{\widehat{M_2}}{4} < 0 \tag{41}$$

where the strict inequality in equation (41) follows from the combination of $\widehat{M}_2 \in [(1-\lambda)E_{\Gamma_e}(e), 4-H]$ and $H < 2$. This completes the proof that in any best-response by candidate A with $\iota_A = 0$ it must be the case that $\text{Supp}(G_{p_1^A}(u)) \in [0, H] \cup [4-H, 4]$, and, thus, completes the proof of part (II.) of Theorem 1. ■

Proof of Theorem 2: Hard limit on debt

With a few modifications, the proof of Theorem 2 follows along the lines of the proof of Theorem 1. Beginning in the second period with any state $(e, \delta(e)) \in \mathcal{S}_{pd}$, note that borrowing is not possible in the second period and so the debt limit $\overline{\delta}$ does not change the Theorem 1 second-period local equilibrium strategies. Given the second-period local equilibrium strategies, we move back through the game tree and examine the effect of the debt limit on the first-period local equilibrium strategies. We begin by examining the first-period vote share calculation. Then, we turn to the proof that in part (I.) the Theorem 2 first-period local strategies form a first-period local equilibrium. Next, we perform the corresponding analysis for part (II.).

We now use the equation (24) first-period vote share calculation in the proof that in part (I.) of Theorem 2 – where $\widehat{H}^d \equiv 2B_{NP}^d - 2B_P^d - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e))$ – the Theorem 2 first-period local strategies form a first-period local equilibrium. Given that candidate B is using the first-period local equilibrium strategy p_1^* specified by part $(I.)$ of Theorem 1, it follows that the random variable $\widetilde{U}_z(p_1^*)$ is distributed according to

$$G^*(u) = \begin{cases} 0, & \text{if } u \leq 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), \\ \frac{u - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e))}{2B_P^d}, & \text{if } 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)) \leq u \leq 2B_P^d + 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), \\ 1, & \text{if } u \geq 2B_P^d + 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)). \end{cases}$$

In any best-response, it is clear that candidate A does not provide voter z with a utility level $U_z(p_1^A)$ that is strictly greater than $2B_P^d + 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e))$. Thus, given that B is using the first-period local equilibrium strategy p_1^*, it follows from equation (24) that candidate A's first-period expected vote share in state e, from an arbitrary first-period local strategy p_1^A with $\iota_A = 1$, is

$$S_1^A(p_1^A, p_1^*) = \int_{\text{Supp}G_{p_1^A}} \frac{u - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e))}{2B_P^d} dG_{p_1^A}(u). \tag{42}$$

First we consider the case that $\iota_{A,1} = 1$. From equation (6) with $\iota_{A,1} = 1$, it follows that

$$\widetilde{U}_z(p_1^A) = \widetilde{x}_{A,1}^{\Gamma_e} + 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)). \tag{43}$$

Then, from equations (1) and (12) we know that

$$E_{G_{p_1^A}}(\widetilde{x}_{A,1}^{\Gamma_e}) \leq B_P^d \tag{44}$$

where the inequality in equation (44) follows from the first-period budget constraint given in equation (12). Inserting, equations (43) and (44) into equation (42) we see that

$$S_1^A(p_1^A, p_1^*) \leq \frac{B_P^d}{2B_P^d} = \frac{1}{2}. \tag{45}$$

To summarize, if $\widehat{H}^d \leq 0$ and candidate B uses the first-stage local equilibrium strategy p_1^{*B} specified in part $(I.)$ of Theorem 2, then candidate A's first-period expected vote share from any arbitrary first-period platform p_1^A is less than or equal $\frac{1}{2}$, where equation (45) holds with equality if candidate A's strategy is first-period budget balancing as specified by equation (12).

The proof of the remaining case in which candidate A chooses an arbitrary first-period strategy in which $\iota_A = 0$ with strictly positive probability, follows along the lines of the corresponding part of the Theorem 1 proof.

We now examine part (II.) of Theorem 1, in which $\widehat{H}^d > 0$. Given that candidate B is using the first-period equilibrium strategy specified by part (II.) of Theorem 2, it follows that $\beta_B = \beta^* = 1 - \frac{1}{2}\widehat{H}^d < 1$ and for each realization of the policy benefit $e \in \mathcal{E} \cup \emptyset$ the debt is $\widehat{\delta}^d(e) = \min\{\overline{\delta}, 1 + \lambda e\}$. In the event that

$\iota_B = 0$, the random variable $\widetilde{x}_{B,1}(\emptyset)$ is distributed according to

$$F_1^*(x|e=\emptyset) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{1}{2}\left(\frac{x}{\widehat{H}^d}\right), & \text{if } 0 \leq x \leq \widehat{H}^d, \\ \frac{1}{2}, & \text{if } \widehat{H}^d \leq x \leq 2B_{NP}^d - \widehat{H}^d, \\ \frac{1}{2}\left(1 + \frac{x - 2B_{NP}^d + \widehat{H}^d}{\widehat{H}^d}\right), & \text{if } 2B_{NP}^d - \widehat{H}^d \leq x \leq 2B_{NP}^d, \\ 1, & \text{if } x \geq 2B_{NP}^d. \end{cases} \quad (46)$$

and for $e \neq \emptyset$, the random variable $\widetilde{x}_1^{\Gamma_e}$ is uniformly distributed on the interval $[0, 2B_p^d]$. Because $\iota_B = 1$ and $\iota_B = 0$ are mutually exclusive events, the random variable $\widetilde{U}_z(p_1^*)$ is distributed according to

$$G_1^*(u) = \begin{cases} 0, & \text{if } x \leq 0, \\ \frac{u}{2B_{NP}^d}, & \text{if } 0 \leq u \leq \widehat{H}^d, \\ \frac{\widehat{H}^d}{2B_{NP}^d}, & \text{if } \widehat{H}^d \leq u \leq 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), \\ \frac{\widehat{H}^d}{2B_{NP}^d} + \left(1 - \frac{\widehat{H}^d}{B_{NP}^d}\right)\left(\frac{u - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e))}{2B_P^d}\right), & \text{if } 1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)) \leq u \leq 2B_{NP}^d - \widehat{H}^d, \\ \frac{u}{2B_{NP}^d}, & \text{if } 2B_{NP}^d - \widehat{H}^d \leq u \leq 2B_{NP}^d, \\ 1, & \text{if } x \geq 4. \end{cases} \quad (47)$$

If candidate A chooses a first-period platform p_1^A with $\iota_A = 1$ and $\text{Supp}(G_{p_1^A}(u)) \in [1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), 2B_{NP}^d - \widehat{H}^d]$, then candidate A's expected vote share in state e, from such a strategy is

$$S_1^A(p_1^A, p_1^*) = \frac{\widehat{H}^d}{2B_{NP}^d} + \left(1 - \frac{\widehat{H}^d}{B_{NP}^d}\right) \int_{\text{Supp}G_{p_1^A}} \frac{u - 1 - E_{\Gamma_e}(e - \widehat{\delta}^d(e))}{2B_P^d} dG_{p_1^A}(u) \quad (48)$$

Inserting, equations (43) and (44) into equation (48), we have that

$$S_1^A(p_1^A, p_1^*) \leq \frac{\widehat{H}^d}{2B_{NP}^d} + \left(1 - \frac{\widehat{H}^d}{B_{NP}^d}\right) \frac{B_P^d}{2B_P^d} = \frac{1}{2} \quad (49)$$

Thus, candidate A's expected vote share from any strategy with $\iota_A = 1$ and $\text{Supp}(G_{p_1^A}(u)) \in [1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), 2B_{NP}^d - \widehat{H}^d]$ is less than or equal to $\frac{1}{2}$. The proof that in any best-response by candidate A with $\iota_A = 1$ it must be the case that $\text{Supp}(G_{p_1^A}(u)) \in [1 + E_{\Gamma_e}(e - \widehat{\delta}^d(e)), 2B_{NP}^d - \widehat{H}^d]$ follows along the same lines as the corresponding part of the proof of Theorem 1.

For the case that candidate A chooses a first-period platform p_1^A with $\iota_A = 0$ and $\text{Supp}(G_{p_1^A}(u)) \in [0, \widehat{H}^d] \cup [2B_{NP}^d - \widehat{H}^d, 2B_{NP}^d]$, it follows from equation (47) candidate A's expected vote share is

$$S_1^A(p_1^A, p_1^*) = \int_{\text{Supp}G_{p_1^A}} \frac{u}{2B_{NP}^d} dG_{p_1^A}(u) \tag{50}$$

Given budget feasibility with $\iota_A = 0$, see equation (12), it follows from equation (50) that candidate A's expected vote share from any such a strategy p_1^A is less than or equal to $1/2$, which holds with equality if p_1^A is budget balancing. The proof that in any best-response by candidate A with $\iota_A = 0$ it must be the case that $\text{Supp}(G_{p_1^A}(u)) \in [0, \widehat{H}^d] \cup [2B_{NP}^d - \widehat{H}^d, 2B_{NP}^d]$ follows along the same lines as the corresponding part of the proof of Theorem 1. ■