

Breaking Bad: Supply Chain Disruptions in a Streamlined Agent Based Model

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Abstract

We explore the macro-financial consequences of the disruption of a supply chain in an agent based framework characterized by two networks, a credit network connecting banks and firms and a production network connecting upstream and down-stream firms. We consider two scenarios. In the first one, because of the lockdown all the upstream firms are forced to cut production. This generates a sizable down-turn during the lockdown due to the indirect effects of the shock (network based financial accelerator). In the second scenario, only those upstream firms located in the “red zone” are forced to contract production. In this case the recession is milder and the recovery begins earlier. Upstream firms hit by the shock, in fact, will be abandoned by their customers who will switch to suppliers who are located outside the red zone. In this way firms endogenously reconstruct (at least in part) the supply chain after the disruption. This is the main determinant of the mitigated impact of the shock in the “red zone” type of lockdown.

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1 Introduction

International supply chains – also known as Global Value Chains (GVCs) – are key drivers of the present stage of globalization, which goes under the name of Great Convergence (Baldwin (2016)) or hyper-globalization (Rodrik (2011)). According to Antras (2020), p.3: “A global value chain consists of a series of stages involved in producing a product or service that is sold to consumers, with each stage adding value, and with at least two stages being produced in different countries.” A stylized GVC consists of a downstream sector populated by firms located in an advanced economy and producing final goods and an upstream sector populated by firms located in an emerging country which supply intermediate inputs to downstream firms. GVCs therefore are first and foremost *production networks*.

However, there is more to a supply chain than a productive and organizational arrangement. Upstream suppliers are also lenders as they extend trade credit to downstream firms. Moreover, both types of firms need external finance, which is provided by financial intermediaries (banks for short). Hence GVCs are also *financial networks*. In this paper, therefore, conceive GVCs as networks of productive and financial interlinkages between U-firms, D-firms and banks.

Being complex webs of trade-credit-logistic arrangements, GVCs are vehicles for the transmission and amplifications of shocks. Real or financial shocks hitting a stage of production yield a *disruption* of the supply chain which reverberates on the other stages via backward and forward linkages. This is the *ripple effect*. Due to the interlinkages of a myriad of spatially dispersed heterogeneous firms international trade has become a complex network characterized by high systemic risk (Goldin and Mariathan (2014)).

For instance, an earthquake or the outbreak of a pandemic in an emerging country forces upstream firms to close down. Hence downstream firms face capacity constraints, being short of intermediate goods: the shock trickles down through productive edges. In parallel it percolates along firm-to-firm financial edges as the network of trade credit unravels. Last but not least the shock jeopardizes banks-firms financial relationships. The disruption of a supply chain is indeed a new type of *financial risk*. Firms experience liquidity shortfalls, leverage shoots up, banks will record non-performing loans and in the end there is a high risk of outright bankruptcy.

In this paper we explore the macro-financial consequences of the disruption of a supply chain in a *minimalistic* macroeconomic agent based framework based on Delli Gatti et al. (2006, 2010) characterized by two networks, a credit network connecting banks and firms and a production

(and trade credit network) connecting upstream and downstream firms. We deem agent-based models particularly apt to explore supply chain disruptions, as they provide a natural framework to encompass productive and financial interactions among heterogeneous firms. Our agent based model is minimalistic, however, because we abstract from a number of relevant real world features (which are dealt with properly in more sophisticated macroeconomic agent based frameworks).¹ First of all we focus only on firms and banks, deliberately downplaying the role of households. Second, we abstract from the multi-stage input-output structure of real world supply chains, considering only two stages/sectors (downstream and upstream). Third, we assume that financial factors play an essential role in production decision at the downstream end of the supply chain and that the upstream sector accommodates the demand for intermediate inputs coming from downstream firms. As a consequence, in normal times firms in the downstream sector do not face demand constraints or capacity constraints.

In this setting, a low frequency/high impact disruptive event such as Covid-19 and the associated lockdown in the upstream end of the chain forces downstream firms to face a sudden capacity constraint, with relevant macro-financial repercussions. We consider two scenarios. In the first one we assume that all the upstream firms are forced to cut production down with respect to the pre-pandemic level for a given time interval. This generalized contraction of the supply of intermediate inputs generates a huge downturn due to the direct and indirect effects of the shock. In fact, both upstream and downstream firms experience a contraction of profits and net worth while banks experience an increase in non-performing loans. The economy recovers only when the lockdown is lifted and goes back rapidly approximately to the pre-shock level of activity.

In the second scenario, we assume that only a fraction of upstream firms are forced to contract production, i.e., firms located in the “red zone”, at the centre of the epidemic. In this second scenario the recession is milder and less persistent than in the first one and the recovery begins earlier. In a sense, this is obvious since the localized shock is, by construction, less pervasive than the generalized one. The most interesting feature of the localized scenario, however, is the change in production interlinkages among downstream and upstream firms. Downstream firms usually supplied by firms located in the red zone, in fact, will switch to suppliers outside the red zone. This *diversification effect* is the main determinant of the mitigated impact of the shock

¹See for instance Assenza et al. (2015, 2018). For an exhaustive survey of macroeconomic agent based model see Dawid and Delli Gatti (2018).

in the localized type of lockdown. In this way in fact firms endogenously reconstruct (at least in part) the supply chain after disruption. In the managerial literature, it is often claimed that supply chains with “backups” may be less efficient but are more resilient to shocks than “lean” chains.² Our analysis, confirms this conjecture. Contrary to a generalized lockdown, red zoning allows downstream firms to find alternative suppliers and relax the capacity constraint due to the shock.

The paper is organized as follows. Section 2 is a concise overview of the literature. In section 3 we present the model, describing the behavioural rules followed by each class of agents. In section 4 we discuss the mechanism for the selection of partners in the production and financial networks. We then pause briefly in section 5 to wrap up and describe the interrelation of markets at a given point in time. Section 6 is devoted to the definition of the law of motion of net worth – which drives the dynamics of the model – for each class of agents. In section 7 we present and discuss the results of simulations in the baseline (pre-pandemic) scenario. Section 8 is devoted to the discussion of the consequences of the pandemic shock to the upstream end of the supply chain under two scenarios: generalized lockdown and red zone. Section 9 concludes.

2 A concise review of the literature

GVCs have been extensively studied in the last decade. The canonical Krugman-Melitz framework used in this literature is characterized by monopolistic competition, cost differences across countries (affected by factor endowments), firms’ heterogeneity (in terms of productivity), scale economies due to fixed cost of offshoring, imperfection/incompleteness of contracts; vertical and horizontal interactions shaping the configuration of the supply chain. The current literature conveys the following basic message: (a) only the most productive downstream firms are able to incur the fixed cost of outsourcing; (b) these firms outsource to upstream suppliers located in countries where the variable cost of production is lower; (c) they keep the upstream stages of production within the firm boundaries if transaction costs are high, i.e., if the quality of market institutions in the destination country is low.³

Two (relatively) under-researched aspects of GVCs are worth exploring further: (i) the role of financial constraints in GVC participation; (ii) the macroeconomic repercussion of (changes in

²See for instance Dolgui et al. (2018).

³For a insightful exhaustive overview of this literature, see Antras (2016) and the references therein.

the) organization of GVCs following a disruptive event. The straightforward effect of the event is a sizable shortening of the GVC. This reshaping has profound and persistent consequences for the macroeconomic performance of countries participating in the GVC which must be duly recognized and explored.

There is already a wide range of models which analyze the effect of supply chain disruptions. We can group them in two classes: agent based input-output models (AB-IO) and production network models.

To the best of our knowledge, the first AB-IO model for this purpose was proposed by Hallegatte and co-authors a decade ago. Fanny et al. (2011) and Hallegatte (2014) employ a dynamic I-O framework in which firms follow adaptive rules to carry out production tasks (order, production, inventories) to study the effects of catastrophic events. Inoue and Todo (2019a,b, 2020) apply Hallegatte's framework to track the effects of supply chain disruptions in Japan. Pichler et al. (2020) employ an AB-IO model augmented with an epidemiological SIR component to assess demand and supply effects of Covid-19.

Production network models spring from the literature on the granular origins of macroeconomic fluctuations (Gabaix (2011)). Starting from an optimizing conceptual framework, Barrot and Sauvagnat (2016); Baqaee and Fahri (2020); Carvalho et al. (2020) employ I-O techniques to model a production network and study the effects of disruptive events. The analysis of the interlinkages between real and financial shocks is still in an early phase (Bigio and LaO (2016); Luo (2020); Altinoglu (2018)).

Central to both AB-IO models and models of production networks are the productive interlinkages among firms. A satisfactory mapping of the production network to the financial network associated to the supply chain (trade credit and finance) is still lacking.

As anticipated in the introduction we adopt an agent based perspective but propose a streamlined model with only downstream and upstream firms. We go (fairly) granular, however, in describing the productive and financial sides of GVCs. We adapt the approach of Delli Gatti et al. (2006, 2010) to GVCs.⁴

⁴See Battiston et al. (2007, 2012) for previous work on the production/trade credit network among firms and for contagion in financial networks. Riccetti et al. (2013) have enriched the framework put forward by Delli Gatti et al. (2010) with a more sophisticated theory of leverage determination.

3 The Model

3.1 The environment

The economy under scrutiny is populated by four classes of agents: downstream and upstream firms (D-firms and U-firms hereafter), banks and households. Agents interact on five markets: intermediate goods, consumption goods, labour, credit and deposits. On the market for consumption goods, D-firms (indexed by $i = 1, 2, \dots, N_D$) sell final goods to households. To produce consumption goods, D-firms purchase intermediate goods from U-firms (indexed by $j = 1, 2, \dots, N_U$). Banks (indexed by $z = 1, 2, \dots, N_B$) extend credit to firms and receive deposits from households. Finally, on the labour market firms hire workers.

3.2 Households

Households supply labour to firms, earn wages and spend on consumption goods. We assume that households play a passive role in the economy. They purchase all the output of D-firms, accommodating the supply of consumption goods. Hence the production decisions of D-firms are not constrained by households' consumption decisions. This (admittedly strong) assumption allows to focus on the financial determinants of D-firms' production. On the labour market households supply labour inelastically. We assume that labour supply is always abundant. Therefore, firms can obtain all the labour they need (at the given wage) to produce either intermediate or final goods. In a sense, labour supply always accommodates the demand. Labour shortages are ruled out by assumption.

3.3 Downstream firms

3.3.1 Technology and costs

In the following, in order to save on notation, all undated variables are referred to period t (the present). We will introduce the time suffix in section 6 when we will discuss the dynamics of net worth.

In order to produce, the generic i -th D-firm needs labour (N_i) and intermediate inputs (Q_i). For simplicity, the production function is of the Leontief type:

$$Y_i = \min \left(\frac{1}{\gamma_D} N_i, \frac{1}{q} Q_i \right) \quad (1)$$

The coefficients γ_D and q measure the labour and the intermediate input requirements (per unit of output). Since labour supply is “abundant” (labour shortages are ruled out by assumption), employment at firm i is linearly increasing in the scale of activity:

$$N_i = \gamma_D Y_i \quad (2)$$

Thanks to complementarity, also the intermediate input is linearly increasing with output:

$$Q_i = q Y_i \quad (3)$$

We denote the real wage in the D-industry with w_D and the (real) price of intermediate inputs sold by the j -th supplier to the i -th firm with p_j^i . Both w_D and p_j^i are ratios of the corresponding nominal variable (the nominal wage and the price of intermediate input respectively) to the GDP deflator.

As we will show in section 3.4, the price of the intermediate input p_j^i has a “fundamental” component p_U – uniform across firms – and a firm-specific component which in turn will be affected by the interest rate on trade credit and by an idiosyncratic shock. We assume that the firm-specific component will be revealed *ex post*, i.e., after the firm has set the price and the quantity (see assumption 1 below). Therefore, when the i -th D-firm sets the quantity, it takes only the fundamental component of the price of intermediate inputs into account.

Thanks to Leontief technology, total real *ex ante* operating costs are: $TC_i = c_D Y_i$ where

$$c_D := w_D \gamma_D + p_U q \quad (4)$$

is the *ex ante* marginal operating cost.⁵

3.3.2 The financially constrained output function

For a given technology, the scale of activity of a firm can be constrained by: (i) the availability of productive inputs; (ii) the demand for the goods the firm produces; (iii) the availability of finance to purchase inputs and carry out production. In this model, for simplicity, we abstract from constraints (i) and (ii). In fact we assume that inputs are available in unlimited supply

⁵The *ex post* marginal cost for each D-firm will be based on the actual price of intermediate inputs the firm will pay, which has a firm-specific component. See subsection 3.4.

and households consume all the firm’s output. We focus instead on constraint (iii). To simplify matters we will consider the following timing.

Assumption 1 *Intra-period timing* *At the beginning of period t , the firm’s financial condition proxied by net worth A_i is made public. On this basis, financial resources $F_i = A_i + L_i$ available to fund production are determined, with $L_i \geq 0$ the amount of bank loans. We assume that D -firms have market power. The firm sets the price and the optimal scale of activity “ex ante”, i.e., before the actual marginal cost will be revealed. In setting the price and the quantity, the firm employs the ex ante marginal cost. Once revealed, the actual marginal cost will determine ex post gains or losses.*

We show the sequence of events within period t in figure 1.

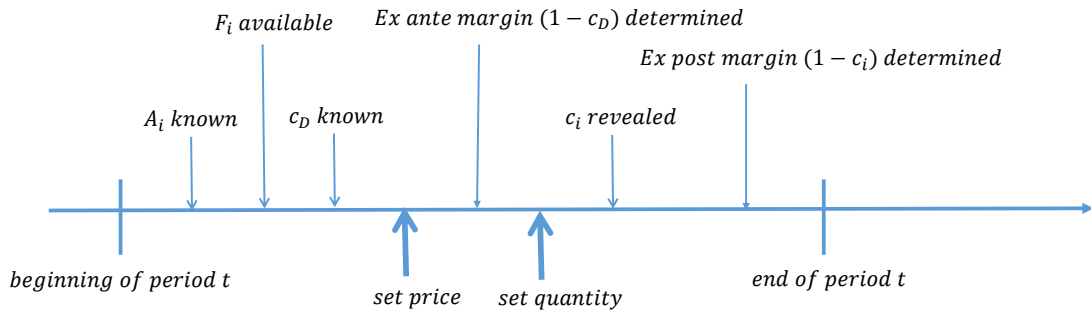


Figure 1: Timing

Assumption 2 *Pricing (D-firms)* *As far as pricing is concerned, ex ante firms are identical as they have the same technology and the same marginal cost c_D . Hence all the D -firms will set the same price. We assume they adopt a simple markup pricing rule: $P_D = C_D(1 + \mu)$ where P_D is the price of D -goods (which coincide with the GDP deflator), C_D is nominal (ex ante) marginal cost and μ is the markup. Since all the firms set the same price, the relative price – i.e., the ratio of the individual price to the average price level – will be equal to 1. Dividing both*

sides of the pricing rule by the GDP deflator we get

$$1 = c_D(1 + \mu) \quad (5)$$

where $c_D = \frac{C_D}{P_D}$ is the real ex ante marginal cost. Using equation (4) we can rewrite (5) as follows:

$$\frac{1}{1 + \mu} = w_D \gamma_D + p_U q \quad (6)$$

As a consequence, the ex ante price-cost margin is:

$$m := 1 - c_D = \frac{\mu}{1 + \mu} \quad (7)$$

From the previous assumption follows that the operating profit is an increasing linear function of output: $\pi_i = \frac{\mu}{1 + \mu} Y_i$. In order to set production, the firm maximizes operating profits subject to the financial constraint: total costs must be covered by the liquidity F_i available to the firm. In symbols:

$$\begin{aligned} \max_{Y_i} \pi_i &= \frac{\mu}{1 + \mu} Y_i \\ \text{s.t. } c_D Y_i &\leq F_i \end{aligned}$$

Thanks to the linearity of the profit function, in order to maximize profits the firm must maximize output. Since output is bounded by the availability of finance, the firm employs all the financial resources to produce. In other words, the financial constraint is binding: $c_D Y_i = F_i$. The (corner) solution of the problem is

$$Y_i = \frac{F_i}{c_D} = (1 + \mu) F_i \quad (8)$$

Let's note now that the firm can either be self-financed or in need of external finance. We assume that, due to asymmetric information, sources of funds can be ordered in a financing hierarchy, in which internal finance A_i ranks first, i.e., it has the lowest cost.⁶

⁶The opportunity cost of internal finance is the risk free interest rate r which will be introduced below, see section 3.5.

If in need of external finance, the firm applies to a bank to obtain a credit line. The bank employs human and computational resources to screen the applicant – i.e., to go through the books and collect data to assess the creditworthiness of the firm – in order to decide the size of the loan. There are benefits and costs of screening. We posit that the benefit – i.e., the accuracy and reliability of the evaluation of the firm’s creditworthiness – is linear in the size of the loan L_i while the cost is quadratic. The objective function of the bank in screening the applicant is $Z^i = z(A_i)L_i - \frac{1}{2}L_i^2$ where $z(A_i)$ is the (average and) marginal benefit of screening, which is a function of the borrower’s net worth. The bank maximizes this function with respect to L_i . The first order condition yields the optimal loan size: $L_i = z(A_i)$.

Let’s suppose that the marginal benefit of screening is a hump shaped function : $z(A_i) = aA_i^\beta - A_i$ where $a > 0$ and $0 < \beta < 1$. The marginal benefit of screening is low when net worth is low because it is difficult to obtain reliable information on the financial conditions of a small firm. Reliability increases with size but beyond a certain threshold it decreases because big firms can obscure or hide important financial information. Substituting this marginal benefit equation in the optimal loan size, we get

$$L_i = aA_i^\beta - A_i \tag{9}$$

The optimal size of the loan is therefore also hump shaped. Thanks to this assumption, total funds available to the borrowing firm are

$$F_i = L_i + A_i = aA_i^\beta \tag{10}$$

Total finance is an increasing concave function of net worth. Substituting this expression in (8) we get

$$Y_i = \alpha A_i^\beta \tag{11}$$

with $\alpha = \frac{a}{c_D} = a(1 + \mu)$. This is the *financially constrained output function* (hereafter FY) for a firm in need of external finance. The FY function is increasing and concave on the (A_i, Y_i) plane.⁷ Delli Gatti et al. (2010) postulate a similar relationship but they do not provide a

⁷If the marginal benefit of screening were linearly increasing with size, e.g. $z = \lambda A_i$, the first order condition would read $L_i = \lambda A_i$. This rule can be interpreted as follows: the bank sets the optimal size of the loan by applying a leverage target λ to the firm’s net worth. In this case $F_i = (1 + \lambda)A_i$. Substituting this expression in (8) we get the FY function: $Y_i = (1 + \mu)(1 + \lambda)A_i$. In this case the FY function of the

microeconomic foundation.⁸

If, on the contrary, the firm is self-financed, then $L_i = 0$. The firm becomes self-financed when $F_i = A_i$. Substituting this expression in (8) we get

$$Y_i = (1 + \mu)A_i \quad (12)$$

This is the FY function for a self-financed D-firm. In this case, the FY function is linearly increasing on the (A_i, Y_i) plane.

We can compute a cut-off value of net worth \hat{A} such that the firm can be either in need of external finance if it is relatively “poor” – i.e., if its net worth is smaller than the threshold – or self-financed if it is relatively “wealthy”, i.e. with net worth bigger than (or equal to) the threshold. This cut-off value is determined by plugging (10) into the condition $F_i = A_i$. We get

$$\hat{A} := a^{\frac{1}{1-\beta}} \quad (13)$$

This threshold allows to divide the set Φ_D of D-firms into two subsets. The subset of self-financed D-firms is $\Phi_{SD} = \{i \in \Phi_D | A_i \geq \hat{A}\}$. As a consequence $\Phi_{BD} = \Phi_D - \Phi_{SD} = \{i \in \Phi_D | A_i < \hat{A}\}$ is the set of borrowing D-firms.

To sum up, the FY function can be written as follows :

$$Y_i = \begin{cases} \alpha A_i^\beta & \text{if } i \in \Phi_{BD}, \\ (1 + \mu)A_i & \text{if } i \in \Phi_{SD} \end{cases} \quad (14)$$

borrowing firm would be linear.

⁸An alternative simple microfoundation of the concave FY function can be found following the approach pioneered by Greenwald and Stiglitz (1993). The problem of the firm consists in maximizing expected profits net of bankruptcy costs. Suppose that there are two states of the world. In the positive or favourable state – which occurs with probability $(1 - \phi_i^b)$ – the firm earns operating profits $\pi_i^s = \frac{\mu}{1+\mu}Y_i$. In the negative or unfavourable state of the world – which occurs with probability ϕ_i^b – the firm goes bankrupt and earns profits $\pi_i^b = \pi_i^s - C_i^b$ where $C_i^b = \frac{1}{2}Y_i^2$ is the cost of bankruptcy. The firm maximizes the expected value of profits:

$$\max_{Y_i} V_i = \pi_i^s - C_i^b \phi_i^b = \frac{\mu}{1+\mu}Y_i - \frac{\phi_i^b}{2}Y_i^2$$

The (closed form) solution to this problem is $Y_i = \frac{\mu}{(1+\mu)\phi_i^b}$. Let's assume now that the probability of bankruptcy is a decreasing (convex) function of financial robustness captured by net worth: $\phi_i^b = A_i^{-\beta}; A_i \geq 1$. Substituting this expression into the solution above we get the FY function $Y_i = \frac{\mu}{(1+\mu)}A_i^\beta$. Also in this setting the FY function is increasing and concave.

Equation (14) can be interpreted in two ways. According to the first interpretation, it shows the optimal levels of output *an individual firm* should set depending on the levels of net worth it has. If the firm is relatively “poor” ($A_i < \hat{A}$), output is an increasing concave function of net worth; if it is “wealthy” ($A_i \geq \hat{A}$), output increases linearly with net worth.

Alternatively, the FY function can be interpreted as the optimal levels of output generated by *different firms*, with different levels of net worth. In this case, the domain of the function coincides with the support of the distribution of firms’ net worth.

In figure 2 we draw the relationship between total financial resources available to the firm and net worth. It is the kinked bold line denoted with F_i . When the firm is not wealthy enough – i.e., when its net worth is lower than the cut off value – total finance is bigger than net worth because the firm resorts to bank loans. This is captured by the concave section of the line. When the firm is wealthy enough, it is self-financed and the line coincides with the 45 degree line. The kink is located at the threshold value of net worth. Therefore also the FY function is kinked. If the firm is borrowing, the FY function is represented by the concave section, while if it is wealthy, it is linear and steep. Therefore, we can define the demand for loans expressed by

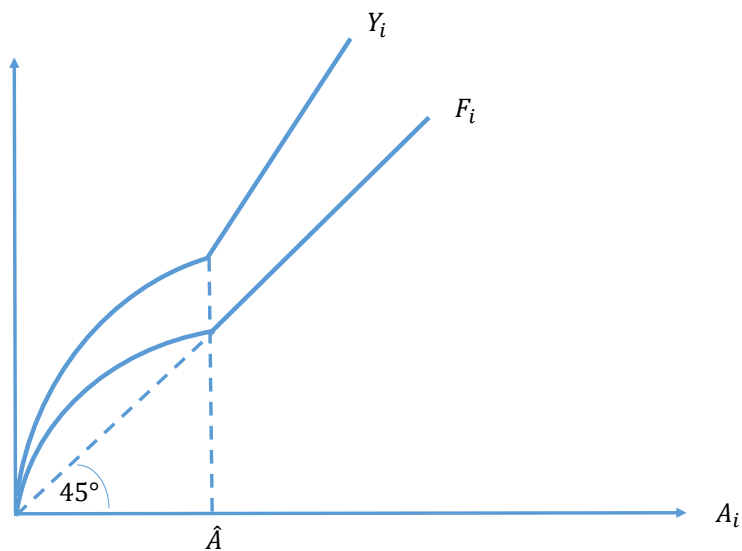


Figure 2: D-firms: output, finance and net worth

the generic i -th firm as:

$$L_i = \mathbf{1}_{\Phi_{BD}} \left(aA_i^\beta - A_i \right) \quad (15)$$

where $\mathbf{1}_{\Phi_{BD}}$ is an indicator function which takes value 1 if the firm is borrowing (i.e., $A_i < \hat{A}$), 0 if the firm is self-financed. The leverage ratio of the i -th D-firm will be defined as the ratio of loans to net-worth. Rearranging one gets:

$$\lambda_i = \mathbf{1}_{\Phi_{BD}} \left(\frac{a}{A_i^{1-\beta}} - 1 \right) \quad (16)$$

Notice that the leverage of the D-firm is decreasing with its net worth.

Using (14) and (2) we can express total labour requirement as follows:

$$N_i = \begin{cases} \gamma_D \alpha A_i^\beta & \text{if } i \in \Phi_{BD}, \\ \gamma_D (1 + \mu) A_i & \text{if } i \in \Phi_{SD} \end{cases} \quad (17)$$

Analogously, using (14) and (3) the total requirement of intermediate inputs is:

$$qY_i = \begin{cases} q\alpha A_i^\beta & \text{if } i \in \Phi_{BD}, \\ q(1 + \mu) A_i & \text{if } i \in \Phi_{SD} \end{cases} \quad (18)$$

Hence also the input requirements are kinked functions of net worth.

Let's wrap up: the quantity produced by a generic D-firm is constrained and determined entirely by its financial robustness. Total production (GDP) will be $Y = \sum_{i=1}^{N_D} Y_i$. We assume that households absorb production. Hence $Y = C$.

3.4 Upstream firms

N_U U-firms produce intermediate goods on demand. We assume an asymmetric structure of the customer-supplier relationship. While each D-firm is attached to a single supplier, a U-firm can have more than one customer. We denote the set of downstream customers of the j -th U-firm with Φ_j . The scale of production of the j -th U-firm Q_j is demand constrained, i.e., it is determined only by the demand for intermediate goods expressed by partner D-firms. In

symbols:

$$Q_j = \sum_{i \in \Phi_j} qY_i = q \left(\sum_{i \in \Phi_j | A_i < \hat{A}} \alpha A_i^\beta + \sum_{i \in \Phi_j | A_i > \hat{A}} (1 + \mu) A_i \right) \quad (19)$$

From (19) follows that the financial conditions of the downstream customers determine the size of the upstream supplier.

In order to produce, the j -th U-firm employs only labour. For simplicity, the production function is linear: $Q_j = \frac{1}{\gamma_U} N_j$ where γ_U is the labour requirement per unit of U-output. Hence employment at firm j is linearly increasing in the scale of activity: $N_j = \gamma_U Q_j$. From (19) follows that also employment at the U-firm is determined by the financial conditions of its D-customers. Hence the financial conditions of D-firms are the drivers of production and employment not only in the downstream sector but also in the upstream sector.

Assumption 3 Pricing (U-firms) *The contract between the j -th supplier and its i -th customer envisages either payment “on delivery” at the so called cash price $p_{j,c}^i$ or payment one period after delivery, at the “post-shipment” price p_j^i . These prices⁹ are connected as follows*

$$p_j^i = (1 + r_j^i) p_{j,c}^i \quad (20)$$

where r_j^i is the interest rate on trade credit.¹⁰ We postpone the discussion of the determinants of the interest rate on trade credit (see equation (30)). As to the determination of the cash price, we assume that U-firms do not have market power. Therefore, the cash price must be equal to the marginal cost: $p_{j,c}^i = c_j$. Moreover, we assume that the marginal cost is $c_j = (1 + u_j) w_U \gamma_U$ where w_U is the real wage in the U-industry and u_j captures the idiosyncratic component of the marginal cost, $u_j \sim \mathcal{U}(-0.5 : +0.5)$. In the end:

$$p_{j,c}^i = (1 + u_j) p_U \quad (21)$$

with $p_U = c_U = w_U \gamma_U$. In words: the cash price has a fundamental deterministic component p_U – equal to the average marginal cost, which in turn is equal to the unit labour cost in the

⁹The prices in question are real prices, i.e., ratios of the corresponding nominal price of intermediate inputs to the GDP deflator.

¹⁰In real world contracts, if the customer pays on delivery, she will get a discount: $p_{j,c}^i = d_j^i p_j^i$ with $0 < d_j^i < 1$. It is straightforward to interpret the discount as the reciprocal of the gross interest rate on trade credit.

upstream sector – and a random idiosyncratic component u_j .¹¹ This essentially means that $p_{j,c}^i$ is distributed as a uniform with expected value equal to p_U . In order to make the argument simple and clear, we assume that all the transactions between D-firms and their U-suppliers are carried out at the post shipment price. Hence, using (21) and (20) the market price will be

$$p_j^i = (1 + r_j^i)(1 + u_j)p_U \quad (22)$$

The shock to the marginal cost of the upstream supplier translates into an unexpected change of the price of intermediate inputs and therefore generates a shock to the marginal cost of the downstream customer. Having specified the average price of intermediate inputs, equation (6) should be rewritten as follows.

$$\frac{1}{1 + \mu} = w_D \gamma_D + w_U \gamma_U q \quad (23)$$

We will use this parameter restriction in the calibration of the model (see section 7).

The firm is self-financed if $A_j > W_j$ where $W_j = w_U \gamma_U Q_j$ is the wage bill and Q_j is determined by the financial conditions of D-customers as in (19). The wage bill is the cut-off value of net worth: $\hat{A}_j := w_U \gamma_U Q_j$. Notice that, contrary to the case of D-firms, the cut-off value of net worth is different from one U-firm to another (because the set of D-partners is different) and is time-varying because the set of D-customers changes over time (as we will show in section 4) and because the net worth of each D-firm also changes (see section 6).

In figure 3 the x-axis measures the net worth of the j-th U-firm. Output Q_j and the wage bill $w_U \gamma_U Q_j$ are represented by horizontal lines (being determined by the net worth of D-customers). The cut-off value of net worth is determined at the intersection of the wage bill line with the 45 degree line. When the firm is not wealthy enough – i.e., when its net worth is lower than the cut off value – the firm resorts to bank loans. When the firm is rich, it is self-financed. Figure 3 depicts the situation of *an individual firm*. It cannot be interpreted as the levels of output associated to *different firms*, with different levels of net worth. While the relationship between the distribution of net worth and the distribution of output across D-firms is nicely and neatly determined by the upward sloping FY function, we cannot posit

¹¹The fundamental component of the cash price of intermediate inputs has been already introduced above. It contributes to the determination of the ex ante marginal cost for D-firms (see assumption 2).

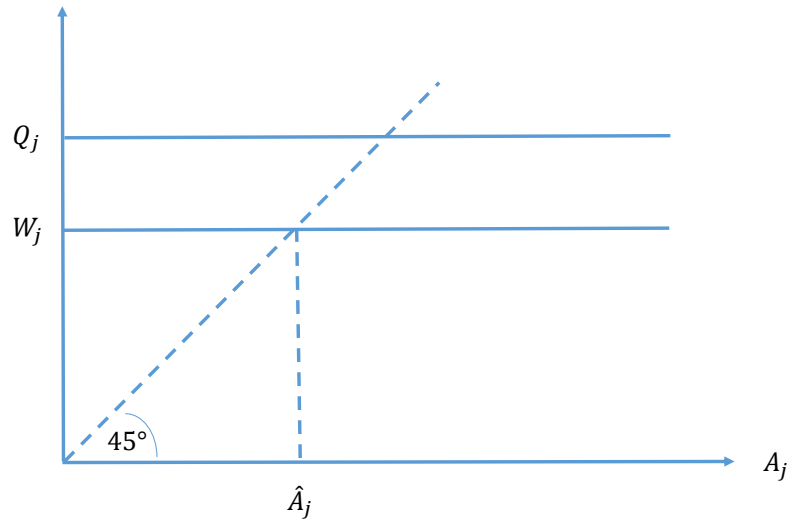


Figure 3: Output, the wage bill and net worth of the j-th U-firm.

an analogous relationship for U-firms. The distribution of output across U-firms, in fact, is determined by the demand of intermediate inputs on the part of D-customers and is therefore disconnected from the distribution of net worth across U-firms.¹²

We can define the demand for loans expressed by the generic j-th U-firm as:

$$L_j = \mathbf{1}_{\Phi_{BU}}(w_U \gamma_U Q_j - A_j) \quad (24)$$

where $\mathbf{1}_{\Phi_{BU}}$ is an indicator function which takes value 1 if the firm is borrowing (i.e., $A_j < \hat{A}_j$), 0 if the firm is self-financed. Graphically, the size of the loan asked for by the firm when its net worth is at a given level is measured by the vertical distance between the wage bill line and the 45 degree line at the given level of net worth.

¹²The relationship could even be “downward sloping”. Consider, for instance, two U-firms, say U1 and U2, with $A_1 \ll A_2$. Suppose the relatively “poor” U1 has many and/or rich D-customers such that Q_1 is “high” while the relatively rich U2 has few and/or poor D-customers so that Q_2 is “low”. Hence it could be the case that $Q_1 > Q_2$.

The leverage ratio of the j -th U-firm is:

$$\lambda_j = \mathbf{1}_{\Phi_{BU}} \left(w_U \gamma_U \frac{Q_j}{A_j} - 1 \right) \quad (25)$$

Notice that the leverage of the U-firm is decreasing with its net worth and increasing with output. Output in turn is increasing with the net worth of D-customers. Hence, the higher the latter, the higher will be the output of the U-supplier and its leverage. In an expansion in which profits and net worth tend to increase in both sectors, leverage will decrease for D-firms (see equation (16)) but the effect on the leverage of U-firms is ambiguous: both output and net worth of the U-firm, in fact, will increase.

3.5 Banks

We assume that the banking sector is willing to provide all the liquidity that firms need. In other words, we rule out the possibility of credit rationing. This simplifying assumption aims at clearing the picture of unnecessary complications in order to focus on the dynamics of net worth as the driver of economic activity. We hypothesize an asymmetric structure of the firms-banks network: a single bank can be linked to many firms (both upstream and downstream), while each firm can ask loans to one bank only. The bank therefore should adapt the interest rate it charges to a specific firm to the latter's financial characteristics. The size of the loan extended by the z -th bank to the i -th D-firm is determined according to rule (9) while the loan received by the j -th U-firm is determined by (24).

Let's now turn to interest rates. We index the firms with $f = 1, 2, \dots, N_D, N_D + 1, \dots, N_F$ where $N_F = N_D + N_U$.¹³ The rule adopted by the z -th bank to set the interest rate for firm f is:

$$r_z^f = r + \rho \left(A_z^{-\rho} + \lambda_f^\rho \right) \quad (26)$$

where $\rho > 0$. Taking into account the definition of leverage for D-firms and U-firms (equations

¹³In words: firms indexed with $f \in [1, N_D]$ produce D-goods; firms indexed with $f \in [N_D + 1, N_F]$ produce intermediate goods.

(16) and (25), we can write:

$$r_z^f = \begin{cases} r + \rho[A_z^{-\rho} + (aA_f^{-(1-\beta)} - 1)^\rho] & \text{if } f \in [1, N_D], \\ r + \rho[A_z^{-\rho} + (w_U \gamma_U \frac{Q_f}{A_f} - 1)^\rho] & \text{if } f \in [N_D + 1, N_F] \end{cases} \quad (27)$$

The interest rate on bank loans has 3 components: (i) the risk free interest rate r (this is the instrument of monetary policy under the control of the central bank); (ii) a bank-specific component, $A_z^{-\rho}$ which is decreasing with the financial soundness of the bank represented by its net worth; (iii) a firm-specific component, λ_f^ρ which is increasing with the borrower's financial fragility.

Component (iii) is the *external finance premium*.¹⁴ When the bank lends money to a firm, it requires a risk premium which depends on the leverage of the firm. Notice that the leverage of the borrowing D-firm is a function only of its net worth. On the contrary, the leverage of the borrowing U-firm is a function also of the net worth of the customer D-firms, which determine the scale of activity for the U-supplier. When the net worth of D-customers increases, they increase demand for intermediate inputs, borrowing U-firms increase the demand for loans, their leverage increases and leads to an increase of the interest rate they face. Hence the interest rate charged to the U-firm is decreasing with the net worth of the U-firm and increasing with the net worth of the D-customers.

Component (ii) is based on the notion that a financially robust bank will be eager to extend credit at favourable terms, in order to increase its market share. This component is key in shaping the credit network as we will show in the next section.

4 Networks

The macro-economy under scrutiny is characterized by two networks, a credit network connecting banks and firms and a production network connecting U-firms and D-firms. We assume an asymmetric structure of both networks. In the credit network, each bank – say the z -th bank – can be linked to several firms, which are the elements of the set of borrowers Φ_z , while each firm, either downstream or upstream, can ask for loans to one bank only.

¹⁴We adopt the wording of the literature on financial frictions. For the pioneering work, see Bernanke et al. (1999).

In the production network, each upstream firm – say the j -th firm – can serve several downstream firms, which are the elements of the set of customers Φ_j , while a single D-firm buys intermediate goods from one supplier only.

The credit and production networks interact as shown in the upper and intermediate layers of figure 4 where U_1 is an upstream firm, B_1 is a bank, D_i ($i=1,2,3$) are D-firms and H_h ($h=1, \dots, 5$) are households. Links among agents are constantly changing, due to the partner’s

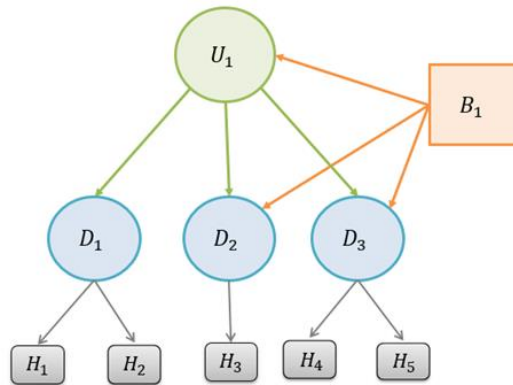


Figure 4: Networks

selection mechanism. In every period, in fact, on either network, agents can switch from one partner to another. On the credit network (downstream and upstream) firms look for the bank which is able to extend credit at the lowest interest rate. On the credit network, downstream firms search for the supplier (upstream firm) which can sell intermediate goods at the lowest price.

4.1 Partner selection in the credit network

Consider the credit network. At period zero (the initial condition), the credit network is randomly determined, i.e., the links among firms and banks are casually established. Following the “Strategic Link Formation” approach (Jackson (2008)), we assume that in each period $t > 0$ each borrower will consider the opportunity of changing partner (lender) by comparing interest rates. Firm f changes the lending bank in period t with a probability of switching ϕ_f^z which

is an increasing function of the difference between the interest rate set by the previous bank partner z_0 , $r_{z_0}^f$, and the interest rate set by the potential new partner z_1 , chosen at random, $r_{z_1}^f$. We denote the ratio of the two with $R_{0,1}^z = r_{z_0}^f/r_{z_1}^f$. We postulate the following law governing the probability of switching:

$$\phi_f^z = \begin{cases} 1 - e^{1-R_{0,1}^z} & \text{if } R_{0,1}^z > 1, \\ 0 & \text{if } R_{0,1}^z \leq 1 \end{cases} \quad (28)$$

This law states that the firm keeps the old borrowing/lending relationship if the new partner charges an interest rate higher than (or equal to) the old one (i.e., if $R_{0,1}^z < 1$), while it may switch to the new partner if the latter charges an interest rate lower than the old one (i.e., if $R_{0,1}^z > 1$), with the probability of switching increasing with the ratio and tending asymptotically to 1.

Notice now that, recalling (26) we can write:

$$R_{0,1}^z = \frac{r_{z_0}^f}{r_{z_1}^f} = \frac{r + \rho \left(A_{z_0}^{-\rho} + \lambda_f^\rho \right)}{r + \rho \left(A_{z_1}^{-\rho} + \lambda_f^\rho \right)} \quad (29)$$

It is easy to infer that $R_{0,1}^z > 1$ if $A_{z_1} > A_{z_0}$. In words: the firm will consider to switch to the new partner if the latter is more financially robust than the old partner. Absent component (ii) of the interest rate (see again equation (26)), the old and new potential partners would be equivalent in the eyes of the firm. The bank-specific component of the interest rate on loans, therefore, plays a crucial role in partner selection on the credit network.

The number of links among firms and banks changes over time, due to this partner selection mechanism, so that the topology of the network is constantly changing. By construction, however, the total number of nodes is constant. The network shows a pattern of increasing polarization because the above mentioned approach to link formation leads to Preferential attachment (Barabási (1999)). The most “prosperous” banks attract an ever increasing number of firms, increasing their profits. A self-reinforcing mechanism is at work: the more profitable a bank is, the higher will be the number of relationships it has and the more eager the bank will be to charge lower interest rates, attracting even more customers.

4.2 Partner selection in the production network

There is an analogous rule for partner selection in the production network. Notice that the production network is also a trade credit network. We assume in fact that the j -th supplier sells intermediate goods to its D-customers allowing payment either on delivery at a lower price or post shipment at a higher price. The difference between the two is the interest payment on trade credit, see equation (20). We assume that the interest rate on trade credit is:

$$r_j^i = \rho \left(A_j^{-\rho} + \lambda_i^\rho \right) \quad (30)$$

Similarly to the interest rate on bank loans, the interest rate on trade credit is decreasing with the lender's net worth and increasing with the borrower's leverage, where the U-supplier is the lender and the D-customer is the borrower.

The i -th D-firm changes the U-partner with a probability of switching ϕ_i^j which is increasing with the difference between the post shipment price of the previous supplier/partner, p_{j0}^i , and decreasing with that of the potential new partner, p_{j1}^i . We denote the ratio of the two with $R_{0,1}^j$. In symbols:

$$\phi_i^j = \begin{cases} 1 - e^{1-R_{0,1}^j} & \text{if } R_{0,1}^j > 1, \\ 0 & \text{if } R_{0,1}^j \leq 1 \end{cases} \quad (31)$$

with $R_{0,1}^j = \frac{p_{j0}^i}{p_{j1}^i}$. Recalling (22) and (30), with a little algebra we can rewrite this expression as follows

$$R_{0,1}^j = \frac{A_{j0}^{-\rho} + \lambda_i^\rho}{A_{j1}^{-\rho} + \lambda_i^\rho} \times \frac{1 + u_{j0}}{1 + u_{j1}} \quad (32)$$

where u_{j0} and u_{j1} are realizations of the random variable u_j . The D-firm may switch to the new U-partner if the latter is more financially robust than the old partner (because in this case the new partner will charge a lower post shipment price).

5 Intermezzo

Let's now pause briefly to summarize the main features of the model. In this section, we "take a picture" of the macro-economy, as described by the model, at a certain point in time. By definition, aggregate GDP is equal to the aggregate production of D-firms and to the aggregate

consumption of households. Since households absorb all the output of D-firms, the market for consumption goods is always in equilibrium. The aggregate production of D-firms is determined by the financial conditions of D-firms, captured by their net worth. Hence the fluctuations in aggregate output will be driven mainly by the variation of D-firms' net worth, as we will show in section 7.

Since U-firms produce on demand, the scale of activity of D-firms determines the amount of intermediate goods produced by U-firms. There will not be involuntary inventories of intermediate inputs because we assume that the U-supplier produces “just in time”. In normal times – i.e., in the absence of disruptive events – the demand for intermediate inputs will be satisfied and D-firms will not face capacity constraints. Hence, also the output of U-firms depends ultimately on the net worth of D-firms.

On the market for labour, the demand coming from U-firms and D-firms is satisfied by assumption. Labour supply, in fact, does not constrain the employment decisions of firms. Hence aggregate employment, once again, ultimately depends on the net worth of D-firms. Due to real wage stickiness, there can be (and generally there is) involuntary unemployment: the market for labour, in other words, is characterized by persistent excess supply.

On the market for credit, banks accommodate demand, extending loans to relatively “poor” D-firms and to U-firms which register a financing gap. There will not be credit rationing.

This is simply a time frame of a “movie” which goes on during the simulation interval. In the following section we will describe the engine of change built in the model that allows the movie to proceed from one time frame to the following one, namely the accumulation of net worth.

6 Profits and the accumulation of net worth

In this section we describe the determination of profits and the law of motion of net worth for the banks and the different categories of firms. For each category of firms, retained profits are the difference between operating profits (i.e., earnings before interest and dividends) and the sum of interest payments (if the firm is borrowing) and dividends. Retained profits are employed to accumulate net worth.

6.1 Downstream firms

At this stage of the analysis we introduce the time index. The operating profit of the self-financed i -th D firm (in real terms) is the difference between revenue Y_{it} and operating cost $c_{it}Y_{it}$ where c_{it} is the actual (ex post) marginal cost: $c_{it} := w_D\gamma_D + p_{jt}^i q$. Therefore the actual operating profit is $\pi_{it} = (1 - c_{it})Y_{it}$. By definition, self-financed D-firms do not have debt commitments. However, they pay out dividends. We assume that dividends are a fraction $(1 - \theta_D)$ (the dividend payout ratio) of operating profits, where $0 < \theta_D < 1$ is the retention ratio. Therefore, recalling that the FY function for self-financed D-firms is linear – see equation (12) – for each $i \in \Phi_{SD}$ retained profits are:

$$\pi_{it}^{SD} = \theta_D(1 - c_{it})Y_{it} = \theta_D(1 - c_{it})(1 + \mu)A_{it} \quad (33)$$

Retained profits are re-invested in the firm. Net worth in period $t+1$ therefore will be.

$$A_{it+1} = (1 - \delta)A_{it} + \pi_{it}^{SD} \quad (34)$$

where $0 < \delta < 1$ is the fraction of net worth that is appropriated by shareholders who “exit” and liquidate their shares. We introduce this parameter for a technical reason. In the absence of exit, net worth would grow “too fast” and make the model uninteresting because all the firms would become super-wealthy over time. In many financial friction models, therefore, it is assumed that a fraction of the population of entrepreneurs “exits”. For example, in Bernanke et al. (1999) wording δ would be the “death rate” of entrepreneurs.

Recalling the definition of marginal cost, after substitution the law of motion of net worth becomes:

$$A_{it+1} = [1 - \delta + \theta_D(1 - w_D\gamma_D - p_{jt}^i q)(1 + \mu)] A_{it} \quad (35)$$

where $p_{jt}^i = (1 + r_{jt}^i)(1 + u_{jt})p_U$ and u_{jt} is the realization in period t of the random variable $u_j \sim \mathcal{U}[-0.5 : 0.5]$. Notice that the accumulation of net worth of the self-financed D-firm is (i) non-linear because the leverage of the firm enters the interest rate on trade credit and (ii) affected by the evolution over time of the financial conditions of the U-supplier, because the net worth of the latter enters the definition of the interest rate on trade credit. The higher the net worth of the U-supplier, the lower the interest rate on trade credit and the higher the future net worth of the D-firm.

If the firm is borrowing, to determine retained profits we must subtract from operating profits not only dividends but also interest payments $r_{zt}^i L_{it}$. Recalling (14) and (9), for each $i \in \Phi_{BD}$ the retained profit of the borrowing D-firm is:

$$\pi_{it}^{BD} = \theta_D(1 - c_{it})Y_{it} - r_{zt}^i L_{it} = [\theta_D(1 - c_{it})\alpha - r_{zt}^i a] A_{it}^\beta + r_{zt}^i A_{it} \quad (36)$$

Hence the law of motion of net worth is

$$A_{it+1} = A_{it}(1 - \delta + r_{zt}^i) + [\theta_D(1 - w_D \gamma_D - p_{jt}^i q)\alpha - r_{zt}^i a] A_{it}^\beta \quad (37)$$

In the case of the borrowing D-firm, the law of motion of net worth is

- non-linear because of (i) the non-linearity of the FY function and (ii) the impact of the firm's leverage on the interest rates on loans and trade credit;
- coupled with the evolution of the net worth of the z-th bank and the net worth of the j-th U-supplier, which affect the interest rates.

The firm goes bankrupt (in period t) if assets turn out to be lower than liabilities, i.e., if net worth becomes negative: $A_{it} < 0$. The bankrupt firm exits. It is insolvent, i.e., it will not reimburse bank loans and will not pay intermediate inputs. Hence the bank will register non performing loans on its balance sheets. Analogously, the upstream supplier will register a loss on its balance sheet. Bankrupt firms are replaced one-to-one by new entrants so that the total population of firms does not change. New entrants are relatively "small": they are endowed with initial net worth drawn from a uniform distribution with support (0,1].

6.2 Upstream firms

The revenue of the generic j-th U-firm is equal to the sales of intermediate goods to all the solvent D firms in its production network. We will denote with Φ_j^s the subset of Φ_j consisting of solvent customers. Hence the complement of Φ_j^s is the set of defaulting customers: $\Phi_j^b = \Phi_j - \Phi_j^s = \{i \in \Phi_j | i \notin \Phi_j^s\}$. Operating costs consist of the wage bill only. Suppose firm j is self-financed: $j \in \Phi_{SU}$. In this case the firm does not have debt commitments. The firm, however, distribute as dividend the fraction $(1 - \theta_U)$ of operating profits. Hence retained profits

are

$$\pi_{jt}^{SU} = \theta_U \left(\sum_{i \in \Phi_j^s} p_{jt}^i q Y_{it} - w_U N_{jt} \right) \quad (38)$$

The U-firm (as a lender) has to take into account also the loss due to insolvent D-firms (non-performing loans or bad debt). Hence net worth of the j -th U-firm is defined as follow:

$$A_{jt} = A_{jt-1}(1 - \delta) + \pi_{jt}^{SU} - NP_{jt} \quad (39)$$

where

$$NP_{jt} = \sum_{i \in \Phi_j^b} q Y_{it} \quad (40)$$

Consider now firm $j \in \Phi_{BU}$. In the case of borrowing U-firms, we must consider also the interest paid on bank loans. Hence retained profit will be

$$\pi_{jt}^{BU} = \theta_U \left(\sum_{i \in \Phi_j^s} p_{jt}^i q Y_{it} - w_U N_{jt} \right) - r_{zt}^j L_{jt} \quad (41)$$

Adopting the same approach we used in the previous section, *mutatis mutandis*, we obtain the following law of motion of net worth:

$$A_{jt} = A_{jt-1}(1 - \delta) + \pi_{jt}^{BU} - NP_{jt} \quad (42)$$

where NP_{jt} is defined as in (40).

An upstream firm goes bankrupt if $A_{jt} < 0$. Bankrupt firms exit. They are insolvent, i.e., they will not reimburse bank loans. Hence banks will record non-performing loans. Bankrupt firms are replaced one-to-one by new entrants so that the total population of U-firms does not change. New entrants are relatively “small” (we adopt the same rule applied to D-firms).

6.3 Banks

Consider the generic z -th bank. Let Φ_z denote the set of borrowing firms and Φ_z^s the subset of solvent (non-bankrupt) firms: $\Phi_z^s = \{f \in \Phi_z | A_f \geq 0\}$. The revenue of the z -th bank is equal to interest payments made by solvent firms. Costs consists of (i) interest payments on deposits (which are remunerated at the risk free rate) and (ii) operating costs, which are proportional to the size of the bank (measured by total assets). The second component comes from the

“industrial organization approach to banking”, i.e., the conception of the bank as a firm. In order to manage a given balance sheet the bank must incur operating costs (e.g., the cost of clerical workers), which are increasing with the “size” of the balance-sheet itself. The latter, in turn, is measured by total assets. In our setting the bank’s assets consist of loans and bank reserves and are equal, by accounting identity, to the sum of deposits and net worth. Hence the profit of the bank is equal to:

$$\pi_{zt} = \sum_{f \in \Phi_z^s} (1 + r_{zt}^f) L_{ft} - r D_{zt} - c_B (D_{zt} + A_{zt}) \quad (43)$$

where D_{zt} are deposits and $c_B > 0$ is the marginal cost of banking activity. The sum of deposits and net worth is equal – by accounting identity – to total assets.

Assuming that the bank does not distribute dividends, the law of motion of the net worth is:

$$A_{zt} = A_{zt-1}(1 - \delta) + \pi_{zt} - NP_{zt} \quad (44)$$

where non-performing bank loans are

$$NP_{zt} = \sum_{f \in \Phi_z^b} L_{ft} \quad (45)$$

and $\Phi_z^b = \Phi_z - \Phi_z^s = \{f \in \Phi_z | A_f < 0\}$. On the asset side of the balance sheet, the bank records loans extended to D-firms and U-firms. Bank reserves, equal to a fraction (the reserve coefficient $0 < d_R < 1$) of deposits, are liquid assets. Hence the balance sheet identity can be written as follows:

$$\sum_{f \in \Phi_z^s} L_{f,t} = (1 - d_R) D_{z,t} + A_{z,t} \quad (46)$$

Bad debts are an important channel of financial contagion in the network. When a firm f goes bankrupt, there will be a negative shock to the bank’s net worth. The deterioration of bank’s net worth makes the interest rate, set by the z -th bank, increase to all its borrowers, so that the financial conditions of the latter will deteriorate. Through this channel, the insolvency of a firm can affect also other firms not directly linked to the defaulted one.

A bank goes bankrupt if $A_{zt} < 0$. Bankrupt banks exit. They will be replaced one-to-one by new entrants so that the total population of banks does not change. New entrants are relatively “small” (we adopt the same rule applied to D-firms and U-firms).

Table 1: Numerical values of parameters

Parameter	Description	Value
N_D	Number of D-firms	500
N_U	Number of U-firms	250
N_B	Number of banks	100
a	Multiplicative parameter (financially constrained output function)	3
β	Exponent (financially constrained output function)	0.4
γ_D	Labour requirement per unit of output (D-firms)	1/2
γ_U	Labour requirement per unit of output (U-firms)	3/4
q	Intermediate input requirement per unit of output (D-firms)	1/2
w_D	Real wage D-firm	1.3
w_U	Real wage U-firm	2/3
p_U	average price of intermediate inputs	1/2
u_j	Idiosyncratic component of the price of j-th U-firm	$\mathcal{U}[-0.5 : 0.5]$
r	Policy rate (risk-free interest rate)	0.01
ρ	Interest rate setting parameter	0.01
δ	Exit rate	0.1
θ_D	Retention ratio D-firms	0.7
θ_U	Retention ratio U-firms	1
d_R	Reserve requirement (per unit of deposit)	0.05
c_B	Bank's cost parameter	0.1

7 Simulations: the baseline scenario

In order to run the baseline version of the model we employ the numerical values of the parameters shown in table 1.

The economy is populated by households (not explicitly modelled) and three groups of heterogeneous agents: $N_D = 500$ downstream firms, $N_U = 250$ upstream firms and $N_B = 100$ banks. We ran 50 Montecarlo simulations over a time span of $T=1000$ periods. In the following we omit the initial 100-period interval, which represents the transient phase. Hence we show the time series generated by the simulations on the interval $1 < t < 900$, where the 1st period of this interval is the 101st period of the simulation.

We assume that when a firm or a bank goes bankrupt – i.e, when net worth becomes negative – it is replaced by another one. Therefore the size of each group is time invariant. We assume, moreover, that entrants are “small” relative to the average size of the incumbent in each group. In order to capture a well know empirical stylized fact concerning GVCs, we assume that labour productivity and the real wage in the upstream sector (which is usually located in emerging countries in the real world) are remarkably smaller than the corresponding parameters in the downstream sector (located in advanced countries). With the current calibration, the unit labour cost in the upstream sector $w_U\gamma_U = 0.5$ is approximately 75% of the unit labour cost in

the downstream sector.

As argued above, the fundamental real price of intermediate inputs p_U is equal to the unit labour cost in the upstream sector. Therefore p_U turns out to be one half of the average price of D-goods (i.e., of the GDP deflator). With this calibration, from the parameter restriction (23) follows that the mark up in the downstream sector is $\mu = 1/9$. For each unit produced, the D-firm gets 1 unit of revenue and incurs average costs of intermediate inputs equal to 0.25 and labour cost equal to 0.65. This means that the wage bill represents 2/3 of GDP, an empirically plausible figure.

We assume “normal times”. Hence the risk free interest rate is set at $r = 1\%$, slightly above the Zero Lower Bound. The reserve coefficients for banks is set at 5% to take into account not only mandatory but also free bank reserves. For simplicity we assume that all profits are retained in the upstream sector while in the downstream sector firms distribute 30% of their profits as dividends.

In figure 5 we show the output of one (representative) simulation concerning the downstream sector. The top left panel shows aggregate D-output (which coincides with GDP), computed “from the bottom up”: $Y_t = \sum_{i=1}^{N_D} Y_{it}$ with $t = 1, \dots, 900$. GDP fluctuates irregularly around a “long run mean” which we can characterize as a *quasi-equilibrium*. This pattern can be observed in all the Montecarlo simulations. In table 2 we report the long run mean and standard deviation computed on the average of the Montecarlo simulations.¹⁵ The long run mean of GDP is approximately equal to $Y = 2137$, with very small volatility, captured by the standard deviation $\sigma^Y = 12$ i.e., 0.6% of the mean.

As we repeatedly said, since the scale of production of D firms is financially constrained, net worth is the main driver of output fluctuations. In fact, aggregate output fluctuates in synch with aggregate net worth $A_t^D = \sum_{i=1}^{N_D} A_{it}$ (see middle left panel). Table 3 shows the correlation index for a number of selected variables. Not surprisingly, the correlation index between D-firms aggregate output and net worth is close to 1 as shown in table 3.¹⁶

By construction – see equation (16) – the leverage of D-firms is a decreasing function of net worth and therefore it is countercyclical (middle right panel). The interest rate on loans charged by banks to D-firms is increasing with leverage, and therefore it shows the same countercyclical

¹⁵For each variable, (i) we apply the HP filter to the time series generated by each simulation, (ii) we compute the average of the filtered time series and (iii) we use the averaged time series to compute the long run mean and standard deviation.

¹⁶The correlation indexes are computed on the same filtered and averaged data used for table 2.

dynamic pattern (bottom left panel). Notice however that the range of oscillation of the interest rate is very small. The long run mean of the interest rate on loans is 3% (see table 2), so that on average the external finance premium is around 2%. As expected, the correlation between the interest rate on loans to D-firms and their net worth is negative and high in absolute value (see table 3). On the basis of equation (26) we expected the correlation between the interest rate on loans and banks' net worth to be negative. In the simulated data, however, it is very low in absolute terms.

Given the numerical values of a and β (see table 1), the cut-off value of net worth for D-firms is $\hat{A} = 6.2$. We have calibrated the model so as to generate a population of D-firms consisting primarily of borrowing firms. In fact, the average net worth of D-firms over the chosen time horizon is $\bar{A} = 1088/500 = 2.2$. Only a small subset of firms are rich enough to be self-financed as shown by the bottom right panel of figure 5. Not surprisingly, the number of self-financed D-firms is of procyclical.

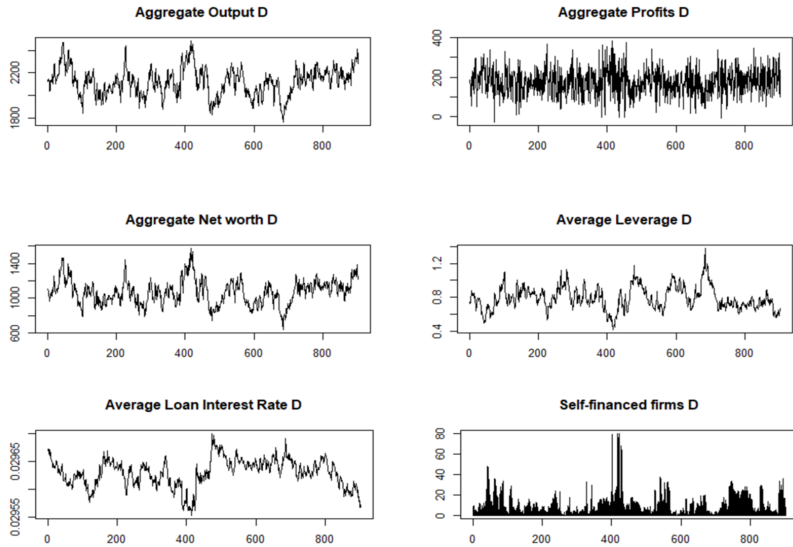


Figure 5: Baseline: D-firms (aggregate variables)

In figure 6 we show the time series generated by the same simulation concerning the upstream sector. The top left panel shows aggregate output of U-firms $Q_t = \sum_{j=1}^{N_U} Q_{jt}$. By construction, the production decisions of D-firms determine the output of U-firms, since the latter produce on demand. Hence $Q_t = qY_t$ where, in the current calibration, $q = 1/2$. Therefore aggregate production of U-firms has the same dynamic pattern of GDP. The long run mean of Q in table 2

is one half of the production of D-firms and half the volatility (so that the coefficient of variation is the same). Hence Q has the same correlations of Y .

The aggregate net worth of U-firms (middle left panel) is increasing with profits which scale up with production and therefore with GDP. In fact, as shown in table 3, the net worth of U-firms is strongly and positively correlated with the net worth of D-firms and GDP.

The interest rate on loans charged by banks to U-firms (bottom left panel) is pro-cyclical while the interest rate charged to D-firms is countercyclical (as shown in the previous figure). In fact, the interest rate on U-loans is increasing with U-output which is pro-cyclical by construction. Notice however that – as in the case of D-firms – the range of oscillation of the interest rate charged to U-firms is very small.

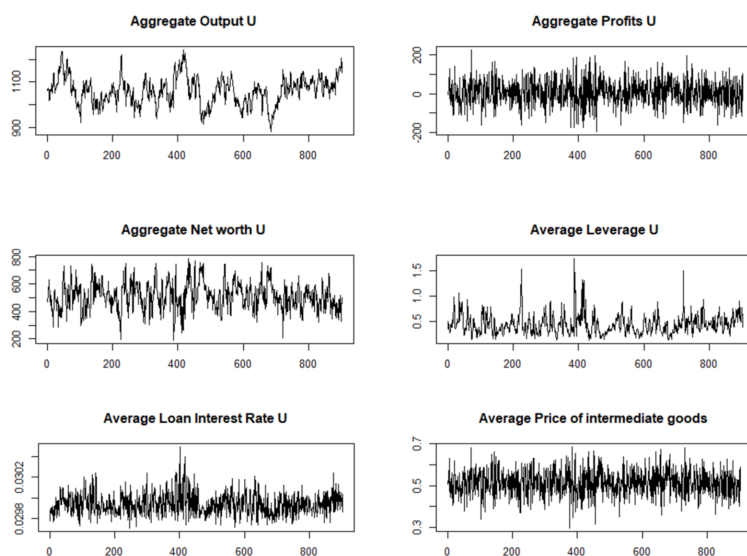


Figure 6: Baseline: U-firms (aggregate variables)

In the baseline scenario, only a small fraction of firms goes bankrupt, most of them in the upstream sector. On average, only 7 D-firms per period go bankrupt (out of 500) and 20 U-firms (out of 250). Overall around 3.5% of the firms are exiting and replaced. As expected, the number of defaults is strongly counter-cyclical.

Table 2: **Baseline: long run mean and standard deviation**

	Mean	St. dev.
Y	2137,28	12,26
A^D	1087,83	14,57
π^D	178,72	2,67
Q	1068,64	6,13
A^U	505,33	6,54
π^U	4,15	1,78
A^B	2389,37	10,04
r^D	0,03	0,00
r^U	0,03	0,00
L^D	827,91	4,01
L^U	211,48	3,60

Note: We run 50 Montecarlo simulations over the interval (1-900). Then we apply the HP filter to the resulting time series and take the average of the filtered time series. Finally we use this series to compute the long run mean and standard deviation. **Legenda:** Y =aggregate production of D-firms (GDP); A^D =aggregate net worth of D-firms; π^D =aggregate profit of D-firms; Q =aggregate production of U-firms; A^U =aggregate net worth of U-firms; π^U =aggregate profit of U-firms; A^B =net worth of banks; r^D =average interest rate on loans to D-firms; r^U =average interest rate on loans to U-firms; L^D =bank loans extended to D-firms; L^U =bank loans extended to U-firms.

Table 3: **Correlations**

	Y	A^D	A^U	A^B	r^B	r^U	L^D	L^U
Y	1,000							
A^D	0,999	1,000						
A^U	0,881	0,879	1,000					
A^B	0,083	0,081	0,269	1,000				
r^D	-0,858	-0,853	-0,773	-0,028	1,000			
r^U	0,712	0,710	0,531	-0,445	-0,866	1,000		
L^D	-0,985	-0,990	-0,897	-0,088	0,835	-0,690	1,000	
L^U	0,778	0,783	0,419	-0,308	-0,696	0,788	-0,746	1,000

Note: The correlation indexes are computed on artificial data filtered and averaged as explained in table 2. Symbols represent aggregate variables (see legenda of table 2).

8 Supply chain disruptions

The production network which connects D-firms and U-firms can be conceived of as a web of supply chains. Each firm on the downstream side of the chain is linked, by assumption, to a single supplier (of intermediate inputs) on the upstream side. In the baseline model, the mechanism driving output fluctuations (for both D-firms and U-firms) is the accumulation of D-firms' net worth. Hence, a negative financial shock hitting the downstream side of a chain – e.g., a sudden increase of the interest rate charged to D-firms – would have repercussions on the upstream side and generate a contraction of the scale of activity of both the downstream firm and its upstream supplier. This transmission mechanism is straightforward and predictable.

In this section we analyse the macroeconomic effects of a disruption of the supply chain which occurs upstream because of a shock which impairs the capability of U-suppliers to respond to the demand for inputs coming from their downstream customers. A straightforward example comes to mind. Consider for instance a Global Value Chain whose downstream side is located in an advanced country – say, the United States – and the upstream side is located in an emerging country, China. If an epidemic erupts in China and U-firms are (temporarily) shut down during the ensuing lockdown, D-firms located in the USA will be unable to carry on production because of the interruption of the supply of intermediate inputs, independently of their net worth. In other words, D-firms will hit a sudden capacity constraint.

This shock may be captured in our model by assuming that U-firms suddenly contract their production at the moment of the lockdown, which will be lifted after a given number of periods. In the simulations discussed in this section, we divide the time horizon (900 periods) in three intervals: the pre-shock and pre-lockdown phase (phase 0) consists of the time interval [1-400); the lockdown phase (phase 1) consists of the interval [400-450) in which the lockdown is enforced; the post-lockdown phase (phase 2) consists of the interval [450-900) in which the lockdown is lifted. In the following subsections, we will consider two types of upstream-driven supply chain disruptions: (i) a generalized forced contraction of U-firms' production capabilities (generalized lockdown) and (ii) a targeted forced contraction which applies only to a subset of U-firms (localized lockdown or “red zone”).

8.1 Generalized Lockdown

For our first experiment, we assume that, due to the lockdown, there is a temporary shock that takes the form of a forced 30% reduction of the scale of activity of each and every U-firm which starts in period $T_0 = 400$ and lasts for 50 periods. In figure 7 we show artificial time series generated by one (representative) simulation concerning D-firms. Before the lockdown, the macro-economy behaves as in the baseline model. GDP fluctuates irregularly (top left panel) around a mean (computed over the interval of phase zero) which we can characterize as the *pre-shock* quasi-equilibrium. The mean of GDP in phase zero is $Y_0 = 2131$ (see table 4).

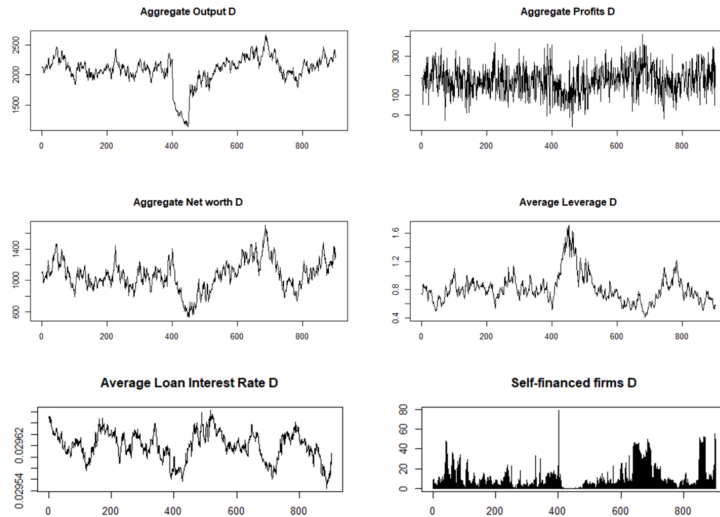


Figure 7: Generalized lockdown: D-firms

Macroeconomic volatility, captured by the standard deviation of GDP, is $\sigma_0^Y = 38$ i.e., 1.8% of the mean. Figure 8 shows the time series generated by the same simulation for U-firms. By construction, since $q = 1/2$, the average production of U-firms and volatility in phase zero is half the size of the corresponding indicators for D-firms: $Q_0 = 1065$ and $\sigma_0^Q = 19$.

Immediately after the shock the production of U-firms (see top left panel of figure 8) drops by one third, from a pre-shock level of 1200 to 800, and slides down further in the subsequent periods reaching a trough of less than 600 at the end of phase 1. From the pre-shock level to the trough, U-production halved. On average (see table 4), during the entire phase 1, U-production falls to $Q_1 = 623$ (40% less than the pre-shock level) and volatility shoots up to $\sigma_1^Q = 80$. Also net worth shrinks approximately by the same amount. The overall reduction of production due

to the lockdown is much bigger than the contraction forced on U-firms at the beginning of the lockdown (-30%). Not surprisingly, the fall of GDP is of the same size as that of U-production (top left panel of figure 7).

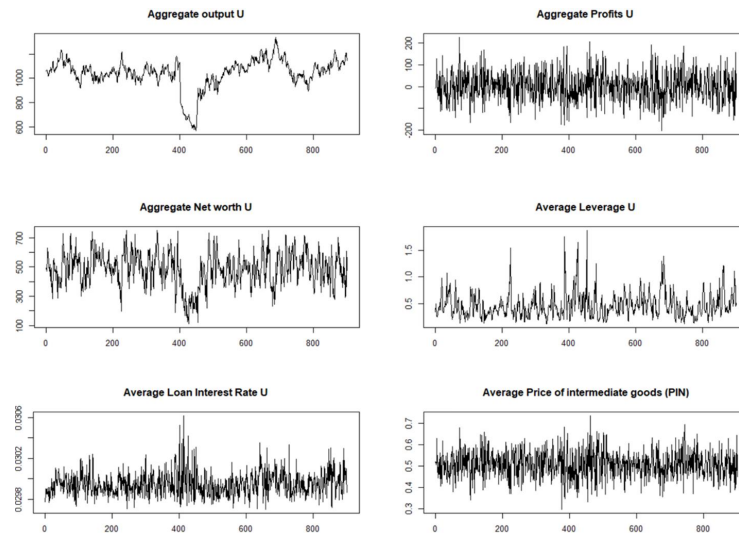


Figure 8: Generalized lockdown: U-firms

The magnification of the impact of the shock is due to the indirect effects of the lockdown. The reduction of upstream supply forces D-firms to cut back on production and sales. Therefore, also D-firms will experience a contraction of profits and net worth. As a consequence, D firms face an increase of their debt, which determines the rise of leverage and of the risk premium on loans. Self-financed D-firms almost disappear during the lockdown. The initial contraction in D-profits, therefore, will trigger further contractionary effects on D-firms' net worth.

Some firms will go bankrupt. In the pre-lockdown phase (as in the baseline), on average 7 D-firms and 20 U-firms per period go bankrupt. Overall around 3.5% of the firms are exiting and replaced. In the lockdown phase the total number of firms which go bankrupt is the same, with a slight change in composition: 9 D-firms and 18 U-firms per period go bankrupt.

After the removal of the lockdown the economy rebounds. The after-shock levels of GDP, production of U-firms, net worth of D-firms and U-firms are slightly below the pre-shock levels (see figures 7 and 8 and the average values for phase 2 as compared with phase 1 in table 4).

Table 4: **Generalized lockdown: mean of selected variables**

	Pre-lockdown (0)	Lockdown (1)	Post-lockdown (2)
Y	2130,80	1247,67	2101,20
A^D	1085,62	635,61	1050,63
π^D	178,08	92,25	174,02
Q	1065,40	623,83	1050,60
A^U	506,10	326,89	499,35
π^U	4,18	2,60	5,15
A^B	2388,35	2430,85	2406,21
r^D	0,03	0,03	0,03
r^U	0,03	0,03	0,03
L^D	827,88	893,81	835,35
L^U	219,25	113,73	207,04

Note: Mean of selected variables over three phases: Pre-lockdown interval (phase 0): periods 1-399; Lockdown (phase 1): periods 400-449; Post-lockdown (phase 2): periods 450-900. Each mean is computed on data filtered and averaged as explained in the note of table 2. Symbols represent aggregate variables (see legenda of table 2).

8.2 Red zone

In this section, we explore the consequences of a different type of lockdown, namely a contraction of production forced on a subset of upstream firms. This is the shock that occurs when the lockdown is imposed in a specific area (“red zone”) which is at the centre of the epidemic. Only firms within the red zone are subject to the lockdown. We assume that 100 upstream firms (out of 250) are forced to contract production by 75% for 50 periods (phase 1). In Figures 9 and 10 we show artificial time series generated by one (representative) simulation concerning D-firms and U-firms respectively in this scenario.

Before the lockdown, the macro-economy behaves as in the pre-shock period in the previous experiment. We remind that the mean of GDP is $Y_0 = 2131$ so that the corresponding descriptive statistics for U-firms is $Q_0 = 1065$. After the shock, the production of U-firms goes down (top left panel of figure 10). The mean of U-production during the lockdown is $Q_1 = 811$ (see table 5): in the red zone case aggregate U-production goes down by approximately one fourth. The fall of GDP is of the same size as that of U-production. Also net worth of D-firms shrinks approximately by the same amount. The reduction of upstream supply, in fact, forces D-firms to cut back on production and sales.

Notice that, contrary to the generalized lockdown scenario, after the shock U-production does not keep falling until the removal of the lockdown but it begins to increase in the middle of the lockdown phase. In the red zone case, therefore, the magnitude of the macroeconomic

downturn and its shape are markedly different from those of the generalized lockdown scenario: the economy takes less time to come back to the pre-shock level, after the removal of the supply restriction.

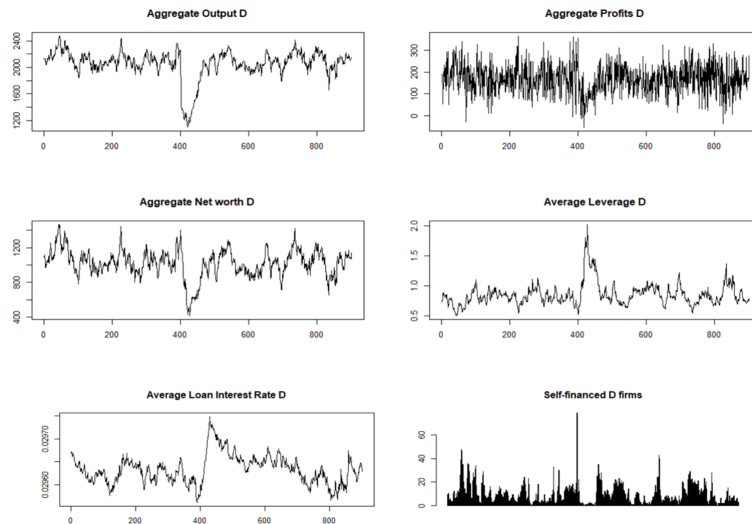


Figure 9: Red zone: D-firms

In part, this is obvious since the majority of firms are not subject to forced downsizing. But this is not the end of the story. In the localized case, in fact, the macroeconomic impact of the shock is affected by the change in production interlinkages among D-firms and U-firms.

U-firms hit by the shock will experience a contraction of their net worth. The interest rate on trade credit extended by these firms therefore will go up. As a consequence, their D-customers will switch to suppliers who can sell at more favourable terms, i.e. U-firms located outside the red zone. In figure 11 we show the number of downstream customers for each U-supplier (numbered from 1 to 250 on the x axis) in a given period during the localized lockdown, compared with the same period in the baseline.

The upper panel takes a picture of the situation at the end of pre-lockdown phase while the lower panel takes a picture at the end of the same phase. After the shock, red zone U-firms (the first 100 firms) lose customers in favour of U-firms that are not subject to the lockdown. This diversification effect of the lockdown on the portfolio of suppliers available to D-firms is the main determinant of the mitigated impact of the shock in the red zone case.

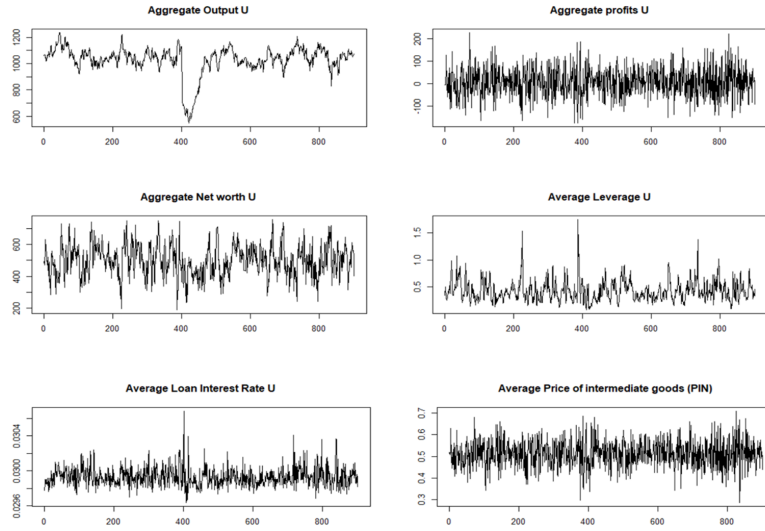


Figure 10: Red zone: U-firms

Table 5: Red zone: mean of selected variables

	Pre-lockdown (0)	Lockdown (1)	Post-lockdown (2)
Y	2130,80	1622,19	2124,83
A^D	1085,62	847,56	1075,33
π^D	178,08	131,72	176,99
Q	1065,40	811,09	1062,42
A^U	506,10	387,39	504,03
π^U	4,18	1,39	4,62
A^B	2388,35	2286,28	2392,93
r^D	0,03	0,03	0,03
r^U	0,03	0,03	0,03
L^D	827,88	815,92	829,69
L^U	219,25	163,50	210,14

See note of table 4.

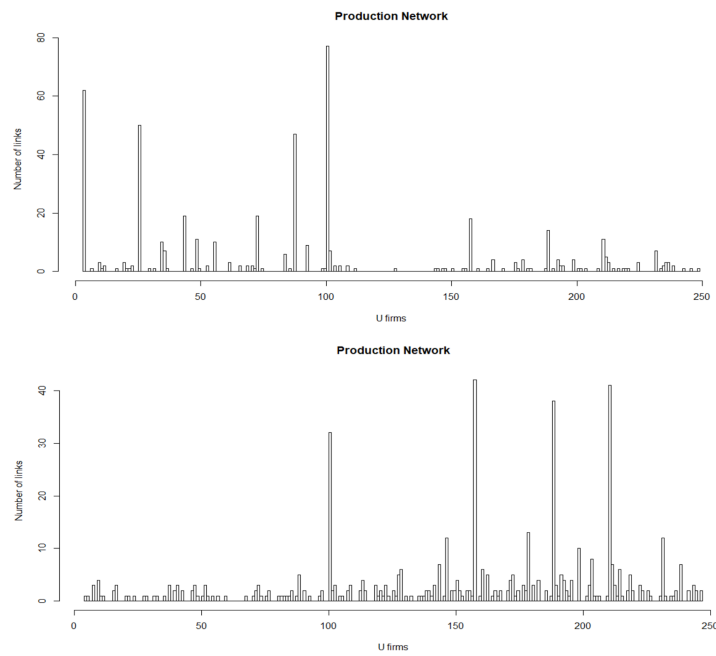


Figure 11: Number of links links for each U-firm: pre-pandemic period (upper panel) and localized lockdown period (lower panel)

9 Conclusion

The disruptions of supply chains is generally recognized as one of the key economic consequences of Covid-19. In this paper we have investigated the macroeconomic and financial consequences of the disruption of a supply chain in a streamlined agent based framework characterized by a credit network connecting banks and firms and a production network connecting upstream and downstream firms. These networks propagate shocks through financial contagion.

We have experimented with two types of supply chain disruptions. In the first scenario we have assumed that the lockdown takes the form of a generalized contraction of the supply of upstream firms. This shock engenders a deep recession. The sudden reduction in the supply of intermediate goods leads to a generalized contraction of profits and net worth.

In the second scenario only some upstream firms are forced to cut on production, namely those located in the “red zone”. In the localized lockdown case, the recession is less persistent and the recovery begins earlier because downstream firms can switch to suppliers who are located outside the red zone. Hence they can limit the contraction of their production. The overall impact of the shock in the “red zone” case is therefore mitigated.

Our experiments shed light on the trade-off between lean production and resilience to shocks in GVCs. In normal, pre-pandemic times, since U-firms produce on demand, the scale of activity of the associated D-firm determines the amount of intermediate goods produced by U-firms. There will not be involuntary inventories of intermediate inputs because the U-supplier produces “just in time”. In normal times therefore, GVCs are “lean” and D-firms will not face capacity constraints. In the generalized lockdown case, the interruption of production at the upstream level leaves the downstream firms with no alternatives to a sizable downsizing. In the redzone case, on the contrary, the shock is mitigated because U-suppliers located outside the redzone operate as “backups”. From the managerial point of view GVCs with backups are less efficient than lean GVC. However when the likelihood of a sizable disruptive event is high – as in the current situation – managers and policy makers should put more emphasis on resilience than on “leanness” of the production network and provide with the necessary alternative sources of inputs.

We are aware of the limitations of the model (highlighted in the discussion of the assumptions). These limitations notwithstanding, we deem these first results encouraging. The model can be further exploited to answer a wide range of related questions. For example, it would be inter-

esting to study ways of re-organizing supply chains after the disruption or ways of responding by means of monetary/fiscal macro-stabilization tools. We leave these issues to future research.

References

- ALTINOGLU, L. (2018): “The origins of aggregate fluctuations in a credit network economy,” *Finance and Economics Discussion Series 2018-031*.
- ANTRAS, P. (2016): *Global Production. Firms, Contracts and Trade Structure*, Princeton: Princeton University Press.
- (2020): “Conceptual Aspects of Global Value Chains,” *Harvard University, mimeo*.
- ASSENZA, T., P. COLZANI, D. DELLI GATTI, AND J. GRAZZINI (2018): “Does fiscal policy matter? Tax, transfer, and spend in a macro ABM with capital and credit,” *Industrial and Corporate Change*, 27, 1069–1090, <https://doi.org/10.1093/icc/dty017>.
- ASSENZA, T., D. DELLI GATTI, AND J. GRAZZINI (2015): “Emergent dynamics of a macroeconomic agent based model with capital and credit,” *Journal of Economic Dynamics and Control*, 50, 5–28, <https://doi.org/10.1016/j.jedc.2014.07.001>.
- BALDWIN, R. (2016): *The Great Convergence. Information Technology and the New Globalization*, Cambridge, Mass.: Harvard University Press.
- BAQAEE, D. AND E. FAHRI (2020): “Supply and Demand in Disaggregated Keynesian Economies,” *NBER Working Papers 27152*.
- BARABÀSI, A. (1999): “Emergence of scaling in random networks,” *Nature*, 286, 509–512.
- BARROT, J. AND J. SAUVAGNAT (2016): “Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks,” *Quarterly Journal of Economics*, 131, 1543–1592.
- BATTISTON, S., D. D. GATTI, M. GALLEGATI, B. GREENWALD, AND J. STIGLITZ (2007): “Credit chains and bankruptcy propagation in production networks,” *Journal of Economic Dynamics and Control*, 31, 2061 – 2084.
- (2012): “Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk,” *Journal of Economic Dynamics and Control*, 36, 1121–1141.
- BERNANKE, B., M. GERTLER, AND S. GILCHRIST (1999): “The financial accelerator in a quantitative business cycle framework,” in *Handbook of macroeconomics*, ed. by J. Taylor and M. Woodford, Amsterdam: North-Holland, 1341–1393.
- BIGIO, S. AND LAO (2016): “Financial frictions in production networks,” *NBER Working Paper 22212*.
- CARVALHO, V., M. NIREI, Y. SAITO, AND A. TAHBAZ-SALEHI (2020): “Supply chain disruptions: Evidence from the great East Japan earthquake,” *Quarterly Journal of Economics*.
- DAWID, H. AND D. DELLI GATTI (2018): “Agent-based macroeconomics,” in *Handbook of Computational Economics, volume 4*, ed. by C. Hommes and B. LeBaron, Amsterdam: North-Holland, 63–156.
- DELLI GATTI, D., M. GALLEGATI, B. GREENWALD, A. RUSSO, AND J. STIGLITZ (2006): “Business fluctuations in a credit-network economy,” *Physica A*, 370, 68–74.
- (2010): “The financial accelerator in an evolving credit network,” *Journal of Economic Dynamics and Control*, 34, 1627 – 1650.

- DOLGUI, A., D. IVANOV, AND B. SOKOLOV (2018): “Ripple effect in the supply chain: an analysis and recent literature,” *International Journal of Production Research*, 56, 414–430.
- FANNY, H., S. HALLEGATTE, AND L. TABOURIER (2011): “Firm-network characteristics and economic robustness to natural disasters,” *Journal of Economic Dynamics and Control*, 36, 150–167.
- GABAIX, X. (2011): “The granular Origins of Aggregate Fluctuations,” *Econometrica*, 3, 733–772.
- GOLDIN, I. AND M. MARIATHASAN (2014): *The Butterfly Defect: How Globalization Creates Systemic Risks, and What to Do about It*, Princeton: Princeton University Press.
- GREENWALD, B. AND J. STIGLITZ (1993): “Financial market imperfections and business cycles,” *Quarterly Journal of Economics*, 108, 77–114.
- HALLEGATTE, S. (2014): “Modeling the Role of Inventories and Heterogeneity in the Assessment of the Economic Costs of Natural Disasters,” *Risk Analysis*, 34, 152–167.
- INOUE, H. AND Y. TODO (2019a): “Firm-level propagation of shocks through supply-chain networks,” *Nature Sustainability*.
- (2019b): “Propagation of negative shocks across nation-wide firm networks,” *PLoS ONE*, 14.
- (2020): “The propagation of economic impacts through supply chains: The case of a mega-city lockdown to prevent the spread of COVID-19,” *PLoS ONE*, 15.
- JACKSON, M. (2008): *Social and economics networks*, Princeton: Princeton University Press.
- LUO, S. (2020): “Propagation of financial shocks in an input-output economy with trade and financial linkages of firms,” *Review of Economic Dynamics*, 36, 246–269.
- PICHLER, A., M. PANGALLO, M. DEL RIO-CHANONA, F. LAFOND, AND J. FARMER (2020): “Production networks and epidemic spreading: How to restart the UK economy?” *INET Oxford Working Paper*, 12.
- RICCETTI, L., A. RUSSO, AND M. GALLEGATI (2013): “Leveraged network-based financial accelerator,” *Journal of Economic Dynamics and Control*, 37, 1629 – 1640.
- RODRIK, D. (2011): *The Globalization Paradox. Why Global Markets, States and Democracy Can’t Coexist*, Cambridge, Mass.: Harvard University Press.