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Optimal Taxation of Capital in the Presence of Declining Labor Share

Abstract

We analyze the implications of the decline in labor's share in national income for optimal Ramsey taxation. It is optimal to accompany the decline in labor share by raising capital taxes only if the labor share is falling because of a decline in competition or other mechanisms that raise the share of pure profits. This result holds under various alternative institutional arrangements that are relevant for optimal taxation of capital income. A quantitative application to the U.S. economy shows that soaring profit shares since the 1980's can justify a significantly increasing path of capital income taxes.

JEL-Codes: E600, E610, E620.

Keywords: capital income tax, labor share, profit share, market power.

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1 Introduction

Governments use income taxes and debt to finance their spending. A central question in macroeconomics is: How should governments allocate the tax burden between two main tax bases, capital and labor income, over time? An influential literature - dating back to the original contribution of Ramsey (1927) - provides a key insight regarding this question: it is optimal to set the capital tax rate to zero. Judd (1985) and Chamley (1986) were the first ones to show that capital taxes should be zero in the long run in the neoclassical growth model.[1] If one is willing to assume that preferences belong to a class that is standard in the macroeconomics literature, then the long-run result holds in the short run as well: except for a few initial periods, it is optimal to set taxes on capital income to zero and put all the burden of taxation on the labor income tax base.

In this paper, we analyze optimal capital and labor income taxation in an economy where the labor tax base is shrinking. This is a very relevant issue from the perspective of practical policymaking since there is a broad consensus emerging among economists that the labor's share in national income (labor share hereafter) has been declining in many developed economies.[2] Figure 1 below depicts the labor share for the US economy for the post war era. The labor share is stable until the early 1980's and has been falling at a considerable rate since then. This trend implies that a government that, by following the suggestions of the Ramsey tax theory, sets capital income taxes to zero or at least keeps them low relative to labor income taxes, would experience a decline in overall tax revenue. Such a government would have to reform its tax system in order to make up for this decline. In an interview with Quartz in 2017, Bill Gates famously addressed this issue by stating that "Robots who take human jobs should pay taxes!" Since robots are part of the capital stock, his suggestion could be interpreted as raising the capital tax rate. Interestingly, the academic literature has remained silent on this important, policy-relevant question. In this paper, we take a

[1]Straub and Werning (2020) provide a set of conditions under which the optimality of zero taxes on capital in the long run does not hold. Chari, Nicolini, and Teles (2020) show that with a richer set of tax instruments and under the assumption that initial confiscation of wealth is restricted, one recovers the long-run optimality of zero capital taxes. See Chari and Kehoe (1999) for a thorough discussion of the earlier contributions to the Ramsey tax literature. The New Dynamic Public Finance literature, which has followed the seminal contribution of Golosov, Kocherlakota, and Tsyvinski (2003), also investigates optimal capital taxation in dynamic Mirrlesian private information models with idiosyncratic labor income shocks.

[2]See Elsby, Hobijn, and Sahin (2013), Karabarbounis and Neiman (2014), Autor, Dorn, Katz, Patterson, and Reenen (2017), Barkai (2020), De Loecker, Eeckhout, and Unger (2020), Acemoglu and Restrepo (2019), and Farhi and Gourio (2018).

first step in this direction and investigate the optimal way of reforming the tax system in the presence of a decline in the labor share using a standard macroeconomic model.

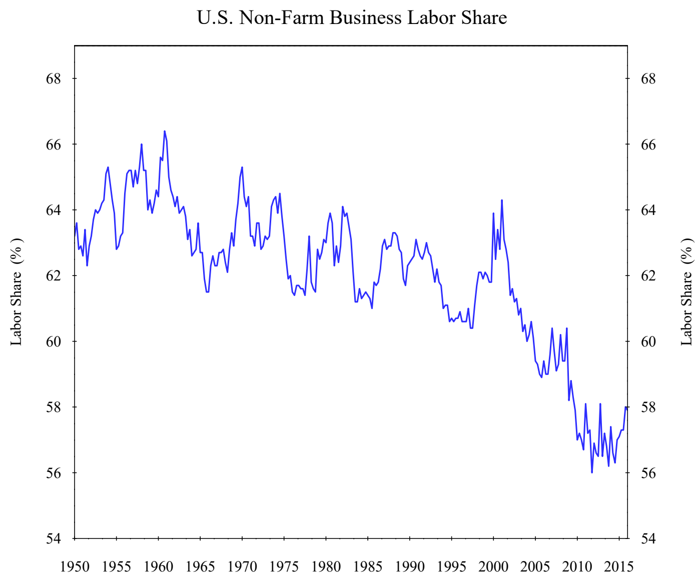

Figure 1: U.S. Non-Farm Business Labor Share

This figure depicts the evolution of U.S. non-farm labor share and is calculated using Bureau of Economic Analysis National Income and Product Accounts Tables.

We set up a representative agent neoclassical growth model in which there is a government that needs to finance an exogenous stream of expenditures using linear taxes on capital and labor income. We focus on a representative agent framework intentionally in order to avoid issues of inequality and redistribution. We allow for the possibility that firms may earn pure economic profits by modelling monopolistic competition in the product market. There is diverse opinion in the literature on the mechanisms that are responsible for the decline in labor share. Because we do not want to take a stance on which mechanisms are more important before deriving qualitative results, our model incorporates virtually all of them.

Observe that the following equality holds in any economy in any given year: $Y = I_L + I_K + I_\Pi$, where Y is national income, I_L is the aggregate labor income received by workers, I_K is the aggregate capital income received by those who own the capital stock, and I_Π is the aggregate (pure) profit income received by firm owners. The profit income is defined as the earnings of firms in excess of all production costs. This equality implies that if there is a decline in labor share, this must be happening due to a rise in capital share or a rise in profit share, or both. We categorize the theories of the decline in labor share proposed in the literature into two groups depending on whether they involve a rise in capital or profit share of income. Rise in automation, capital augmenting technical change, decline in capital prices and offshoring of labor-intensive production are all theories of rise in capital share whereas declining competition is a theory of increasing profit share.[3]

Our main qualitative finding is that the nature of the optimal tax reform for an economy that experiences a decline in labor share depends on whether this decline is accompanied by a rise in capital or profit share. If labor share is declining because production is becoming more capital intensive, say due to a rise in automation or cheapening of capital, then it is optimal to increase labor income taxes to make up for the loss in tax revenue. If, on the other hand, the decline in labor share coincides with a rise in profit share, say due to declining competition in product markets, then it is optimal to increase the tax rate on capital income.

Intuitively, whenever an economy is generating pure profits, it is optimal for the government to tax them away fully since taxing pure profits is non-distortionary. If taxing profit income at 100% is not an option for the government, an assumption maintained in the current paper, then it is optimal to tax factors that contribute to profit creation as this provides an *indirect* way of taxing profits. Obviously, capital is one such factor as it contributes to production, and hence, profit generation. Motivated by the observation that it is in general quite hard for governments to distinguish between capital and profit income, in our baseline optimal tax analysis we assume that government chooses a uniform tax rate that applies to both capital and profit income. This introduces another reason to tax capital since, under this assumption, a tax on capital income acts *directly* as a tax on profit income as well.

In line with the discussion above, the optimal long-run capital tax formula consists of two components which reflect the indirect and the direct profit tax revenue benefits of capital taxation. The formula also reveals that the optimal tax rate on capital income is proportional to the share of profits in national income. Additionally, we derive an alternative expression for the optimal capital tax formula, and use it to deliver a simple expression for a lower bound on the optimal long-run capital tax rate which depends on empirically estimable quantities. The lower bound increases with the profit share and the relative social value of public funds, while it decreases with the elasticity of national income with respect to the retention rate

[3] It is important to stress that we define profit income as firm earnings in excess of *all* production costs including fixed costs of production. For instance, positive net markups arising only from the presence of fixed capital costs, would not generate pure economic profits. Therefore, we would categorize a theory that explains declining labor share using rising fixed capital costs of production as a rising capital share theory of declining labor share.

(one minus the tax rate) on capital income.

When profits are generated due to the presence of market power as in our model, firms display inefficiently low investment and labor demand, which implies there is also a motive to subsidize investment and employment. In our baseline implementation, we assume that these inefficiencies are dealt with at the firm level, where they originate, through product market interventions.[4] We also consider an alternative implementation of the Ramsey allocation in which the government does not have access to such product market policies, and show that, in this case, in addition to the direct and indirect tax motives explained above, an additional Pigouvian subsidy term appears in the optimal capital income tax formula.

On the quantitative side, we calibrate our model to the decline in labor share observed in the US economy. Specifically, we calibrate the initial steady state of our model economy to the early 1980's U.S. economy where, in line with the empirical observations, we assume that the labor share was stable at two thirds and the profit share was around zero. We then calibrate the evolution of the model economy to match the empirical evolution of income shares in the US since the early 1980's. Following Barkai (2020) and De Loecker, Eeckhout, and Unger (2020), we target a 15% increase in profit share during this period. We assume that profit share remains around this level in the long run.

We first consider a hypothetical tax reform which the government carries out in the early 1980's foreseeing the upcoming trends in the underlying factors that give rise to the changes in income shares. In the baseline implementation, the optimal tax rate on capital income starts low, rises to about 36% by 2021, and stabilizes at around 37.6% in the long run whereas the optimal labor tax rate is virtually flat around 33%. The optimal capital tax rate is low early on because the profit share is low in the 1980's. After that, the optimal capital tax rate closely follows the pattern of the rising profit share. We then consider an optimal tax reform in 2021. We find that the optimal capital tax rate starts from about 34% and rises to 38.3% in the long run. The optimal labor tax rate is again smooth over time, now around 35%. The optimal tax rates are higher in the 2021 reform because the initial public debt is higher at the time of the 2021 reform. The optimal capital tax rate is lower but still significant in the alternative implementation without product market policies: in the early 1980's and the

[4]This implementation is in line with a growing literature that studies how to design product market policies in correcting inefficiencies stemming from market power. See, among others, Edmond, Midrigan, and Xu (2018), Atkeson and Burstein (2019), and Boar and Midrigan (2020). Our paper differs from these papers in the sense that while our focus is on the optimal financing of government spending using income taxation they focus on correcting inefficiencies stemming from product market distortions.

2021 reforms, it settles down at 8% and 10% in the long run, respectively.

We also consider two extensions of our main framework. First, we investigate optimal taxation in an economy where the government does not correct product market distortions, and show that the optimal capital tax formula for the resulting inefficient economy is virtually the same as the one in the baseline analysis in which distortions are corrected via product market subsidies. Second, we consider an environment in which the government is allowed to tax capital and profit income at different rates, but there is an exogenous upper bound on the profit tax rate. We find that the optimal capital tax formula in this case only features the indirect profit tax revenue component. In both of these alternative scenarios, our quantitative analyses show that the optimal taxes on capital income are qualitatively in line with the quantitative findings in our baseline scenario: optimal capital taxes increase as labor share declines, are significantly positive by 2021, and remain so in the long run.

It is important to stress that in this model the only reason for taxing capital is financing government spending. In reality, there may be other reasons for taxing capital such as redistribution. In this regard, the optimal capital tax rates we compute here should be seen as informative about how strong our mechanism is for capital taxation and not as a prescription for actual capital tax rates.

Related Literature. Our paper is related to several strands of literature, though, to the best of our knowledge, no other paper analyzes optimal tax design in face of declining labor share. First, in our environment, the optimality of increasing capital taxes in response to declining labor share comes from the rise in profit share. In this regard, an influential backdrop to our paper is Dasgupta and Stiglitz (1971) who show that, when there are pure profits due to decreasing returns to scale in production and profits cannot be taxed at 100%, it is optimal to tax intermediate inputs since taxing these inputs provide an indirect tax on profits. Jones, Manuelli, and Rossi (1997) show that this logic implies the optimality of taxing capital in the long run in the neoclassical growth model. Judd (2002) shows that, when profits exist due to monopolistic competition in the product market, the optimal long-run capital wedge is negative, calling for capital subsidies. Guo and Lansing (1999) and Coto-Martínez, Garriga, and Sánchez-Losada (2007) question the generality of Judd (2002)'s subsidy result by considering restricted government policies and different economic environments.[5] Our paper differs from this literature as it analyzes the optimal reform of the tax system in response to declining labor share. Our contribution lies in providing both qualitative lessons as to when capital taxes become an important part of such a reform and a quantitative analysis of how strong the capital tax response should be. Our analysis also incorporates various institutional arrangements that are currently debated in policy circles, such as the use of product market policies, to the taxation of capital and labor income.

Second, there is a burgeoning literature that makes a case for taxation of robots and automation technologies. Following the skill premium literature, Slavik and Yazici (2014) assume a machine-skill complementarity. This implies that machines raise the marginal product of the skilled relative to the unskilled, and this increases inequality. It is, thus, desirable to deter the accumulation of machines from the perspective of a redistributive government. Using a quantitative model that features technical progress in automation and endogenous skill choice, Guerreiro, Rebelo, and Teles (2021) show that a similar argument implies the optimality of taxing robots when a fraction of the workers currently active in the labor force is locked into routine occupations. Once these workers retire, optimal robot taxes are zero.[6] In all these papers taxing robots is socially desirable because it is redistributive while we argue that taxing robots (or capital in general) provides a more *efficient* way of financing government's budget when capital accumulation contributes to creation of pure economic profits.[7] An exception is Acemoglu, Manera, and Restrepo (2020) who investigate optimal taxation in a task-based framework of automation assuming a representative agent. While we allow for the possibility of different mechanisms behind the decline of labor share, they focus on automation. Their quantitative analysis shows that the optimal capital tax rate is larger than the actual tax rate on capital, implying that the US tax system is biased in favor of capital.[8]

[5] There is also a new set of papers that analyze optimal redistributive labor income taxation in the presence of market power. See, for example, Eeckhout, Fu, Li, and Weng (2021).

[6] See also Thuemmel (2018) for a similar argument for taxation of robots. Costinot and Werning (2018) show that a similar rationale implies taxing robots and trade in a static model with a more general production structure.

[7] There is also a growing literature on the optimal redistributive taxation of capital using quantitative models with rich heterogeneity and uninsured income risk. See, among others, Domeij and Heathcote (2004) and Conesa, Kitao, and Krueger (2009). See also Saez and Stantcheva (2018) who develop a theory of optimal redistributive capital taxation that expresses optimal tax formulas in sufficient statistics.

[8] It is optimal to tax capital in Acemoglu, Manera, and Restrepo (2020) because the authors assume that the government should balance its budget period by period. If one instead allows for a, perhaps more standard, intertemporal government budget, then one recovers the optimality of zero capital taxes in the long run in their environment. This is in line with our result that the capital-intensive theories of the decline

Finally, our modelling of the rise in market power as the main driving force behind the decline in labor share builds on the seminal works of Philippon (2019), Barkai (2020) and De Loecker, Eeckhout, and Unger (2020). Caballero, Farhi, and Gourinchas (2017), Eggertsson, Robbins, and Wold (2018), and Farhi and Gourio (2018) argue that rising market power also explains other key macroeconomic trends that occurred in the U.S. economy around the same time period. Our calibration echoes these arguments: we find that the rise in market power also explains the bulk of the rise in the divergence of the returns to productive capital and the risk-free rate that is observed in the U.S. economy since the 1980's. More generally, the current paper complements the positive findings of this literature by taking a normative perspective and analyzing optimal taxation under key recent macroeconomics trends.

The organization of the article is as follows. Section 2 presents the model. Section 3 and Section 4 present the theoretical characterizations of optimal taxes for the implementations without and with product market policies, respectively. Section 5 presents the quantitative results. Section 6 and Section 7 analyze optimal taxation in the inefficient economy and under exogenous upper bound on profit taxes, respectively. Finally, Section 8 concludes.

2 Model

Consider a neoclassical growth model in which there is a representative consumer who lives forever. There are also firms that produce and sell intermediate and final goods. All firms are owned by the representative consumer. We introduce profits into our environment in the simplest possible manner: Dixit-Stiglitz monopolistic competition. This is the key departure of our model from the neoclassical growth model.[9] Finally, there is a benevolent government that needs to finance a given stream of public spending.

Final Good Producers. Firms that produce the final good are perfectly competitive and operate a constant elasticity of substitution (CES) production function that combines a unit measure of intermediate goods $y_{i,t}$. Taking prices of intermediate goods, $\xi_{i,t}$, as given, the

in labor share such as automation do not per se justify taxing capital income.

[9] Although our exposition uses monopolistic competition to generate pure profits, the results of our model are more general. As we shall see in Section 3, independent of the microfoundation behind it, as long as labor share declines due to rising profit share, it is optimal to have rising capital income taxes.

problem of the representative final good producer is:

$$\max_{y_{i,t}} y_t - \int_0^1 \xi_{i,t} y_{i,t} di \qquad \text{s.t.} \qquad y_t = \left(\int_0^1 y_{i,t}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}.$$

The first-order optimality condition of this problem with respect to $y_{i,t}$ gives the demand as a function of price for each intermediate good $y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}$.

Intermediate Good Producers. Each intermediate good producer is a monopolistic competitor. Producer of intermediate good $y_{i,t}$ uses a CES technology, F_t, to combine capital and labor to produce the intermediate good. This firm solves:

$$\pi_{i,t} = \max_{\xi_{i,t}, y_{i,t}, k_{i,t}, l_{i,t}} \xi_{i,t} y_{i,t} - r_t k_{i,t} - w_t l_{i,t} \qquad \text{s.t.}$$

$$y_{i,t} = F_t(k_{i,t}, l_{i,t}), \tag{1}$$

where r_t and w_t represent the real rental rate of capital and real wage rate, respectively.

The problem of the intermediate good producer can be solved in two steps. In the first step, for a given marginal cost of producing the intermediate good, $m_{i,t}$, the firm chooses its price to maximize profits:

$$\max_{\xi_{i,t}} \xi_{i,t} y_{i,t} - m_{i,t} y_{i,t} \quad s.t. \quad y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}. \tag{2}$$

The solution to this problem implies a constant markup over marginal cost

$$\xi_{i,t} = m_{i,t} \frac{\varepsilon_t}{(\varepsilon_t - 1)}. \tag{3}$$

We focus on the symmetric equilibrium of the model in which all intermediate goods firms make identical choices of inputs and prices. This implies $y_{i,t} = y_t$ and $\xi_{i,t} = 1$ for all $i \in [0, 1]$. We, therefore, have the optimal marginal cost of producing one more intermediate good equals $m_{i,t} = m_t = 1 - \frac{1}{\varepsilon_t}$ for all firms.

In the second step, each firm chooses capital and labor to minimize the cost of production. The firms also make same input choices in the symmetric equilibrium, so we have $k_{i,t} = k_t$ and $l_{i,t} = l_t$. The marginal cost of producing one more unit of the intermediate good using capital or labor at the optimum equals $\frac{r_t}{F_{k,t}} = \frac{w_t}{F_{l,t}} = 1 - \frac{1}{\varepsilon_t}$, where $F_{k,t}$ is short-hand notation for $\frac{\partial F_t(k_l, l_t)}{\partial k_t}$ and $F_{l,t}$ is defined analogously. Therefore, the rental and wages rate are given by

$$r_t = \left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} \qquad \text{and} \qquad w_t = \left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t}. \tag{4}$$

As long as ε_t is finite, the intermediate goods producers possess market power. This allows them to keep their sale prices above marginal cost by producing at below the socially efficient level, which gives rise to inefficiently low demand for investment and labor. This is why the rental rates of capital and labor are below the corresponding marginal products.

In our baseline implementation laid out in Section 4, the government corrects inefficiencies coming from market power by encouraging investment and employment at the level of intermediate goods producers via product market policies that take the shape of sales subsidies. We do not introduce product market policies in the current section because, for expositional purposes, we find it more convenient to first present the implementation without product market subsidies (Section 3).

Income shares. Notice that since intermediate goods are used up in production, total income is given by the production of the final goods firm, y_t. Plugging (3) into (2) in the symmetric equilibrium implies that the total profit income generated by intermediate goods producing firms equals $\pi_t = \frac{1}{\varepsilon_t} y_t$. Thus, the share of profit income in total income in period t, denoted by $S_{\pi,t}$, equals

$$S_{\pi,t} \equiv \frac{\pi_t}{y_t} = \frac{1}{\varepsilon_t}. \tag{5}$$

Using the rental rates given by (4) to compute the income shares of capital and labor renders:

$$S_{k,t} \equiv \frac{r_t k_t}{y_t} = \left(1 - \frac{1}{\varepsilon_t}\right) \frac{F_{k,t} k_t}{y_t} \qquad \text{and} \qquad S_{l,t} \equiv \frac{w_t l_t}{y_t} = \left(1 - \frac{1}{\varepsilon_t}\right) \frac{F_{l,t} l_t}{y_t}.$$

Representative consumer. There is a unit measure of identical consumers who live forever. Each consumer is born in period one with $k_1 > 0$ units of physical capital and b_1 units of government debt. Taking prices as given, consumers decide on their consumption, labor, and saving allocations every period. Furthermore, they decide on how to allocate their saving between buying physical capital, government bonds, and private claims. The period utility of an individual who consumes c units of consumption and supplies l units of labor equals $u(c, l)$. The utility function satisfies standard assumptions, $u_c, -u_{cc}, -u_l, -u_{ll} > 0$, where u_c and u_{cc} are short-hand notation for $\frac{\partial u(c,l)}{\partial c}$ and $\frac{\partial^2 u(c,l)}{\partial c^2}$, respectively, and u_l and u_{ll} are defined similarly. We also assume that utility is separable between consumption and labor, that is

$u_{cl} = 0$. The separability assumption is not important for the main results of this paper, and is adopted merely to make the derivations of the optimal tax formulas simpler. People discount future with a factor $\beta \in (0,1)$. Taking prices $\{p_t, r_t, w_t\}_{t=1}^{\infty}$, taxes $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, and $k_1 > 0$ and b_1 as given, an individual chooses an allocation $(c,k,l) \equiv \{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ to solve the following problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.}$$

$$\sum_{t=1}^{\infty} p_t \left(c_t + q_t k_{t+1}\right) \leq \sum_{t=1}^{\infty} p_t \left(w_t l_t (1-\tau_{l,t}) + \bar{r}_t k_t + \pi_t (1-\tau_{\pi,t})\right) + p_1 b_1, \tag{6}$$

where p_t is the period t price of the consumption good, q_t is the relative price of investment good in terms of the consumption good in period t, and $\bar{r}_t = q_t + (r_t - q_t\delta)(1-\tau_{k,t})$ is the after-tax gross rate of return to capital. Looking at the right-hand side of the budget constraint above, we notice that the consumer has three sources of income: labor, capital and profit, each taxed at linear rates. Following the convention in actual tax systems, we allow for capital depreciation to be deducted from capital income tax base.

The first-order optimality conditions of the consumer's problem are:

$$\frac{\beta u_{c,t+1}}{u_{c,t}} = \frac{p_{t+1}}{p_t}, \tag{7}$$

$$p_t q_t = p_{t+1} \bar{r}_{t+1}, \tag{8}$$

$$\frac{u_{l,t}}{u_{c,t}} = -w_t (1-\tau_{l,t}). \tag{9}$$

Combining (7) and (8) with the rental rate of capital given by (4), we see that in equilibrium:

$$u_{c,t-1} q_{t-1} = \beta u_{c,t} \left[q_t + \left(\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t \right) (1-\tau_{k,t}) \right]. \tag{10}$$

Similarly, combining (9) with the wage rate given by (4), we have in equilibrium:

$$\left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t} (1-\tau_{l,t}) u_{c,t} = -u_{l,t}. \tag{11}$$

Conditions (10) and (11) are going to be useful when defining optimal taxes in Section 3.2.

Government budget balance. Government uses capital, labor, and profit income taxes $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$ to finance an exogenous stream of spending $\{g_t\}_{t=1}^{\infty}$ and initial debt b_1.

$$\sum_{t=1}^{\infty} p_t g_t + p_1 b_1 \leq \sum_{t=1}^{\infty} p_t \left(w_t l_t \tau_{l,t} + (r_t - q_t \delta) k_t \tau_{k,t} + \pi_t \tau_{\pi,t} \right). \tag{12}$$

Resource feasibility. Aggregate resource feasibility requires that for all $t \geq 1$

$$c_t + q_t k_{t+1} + g_t = F_t(k_t, l_t) + (1 - \delta) q_t k_t. \tag{13}$$

Tax-Distorted Market Equilibrium. *Given (k_1, b_1) and $\{g_t\}_{t=1}^{\infty}$, a tax-distorted market equilibrium is a policy $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ and a price system $\{p_t, r_t, w_t\}_{t=1}^{\infty}$ such that:*

1. *Given policy and prices, allocation solves representative consumer's problem;*
2. *All firms maximize profits;*
3. *Markets for final and intermediate goods, capital, and labor clear;*
4. *Government's budget constraint is satisfied.*

3 Optimal Tax Analysis

Consider the problem of a government who needs to finance a given stream of public spending. We assume that there is an institution or a commitment technology through which the government can bind itself to a particular sequence of policies once and for all in period one. Once the policy is chosen, consumers and firms interact in capital, labor and goods markets according to the equilibrium defined earlier. The government is sophisticated enough that it predicts that different government policies lead to different behavior of economic agents, which then leads to different market equilibria. There are possibly many policy sequences that can finance a given stream of government spending. Among these, the benevolent government chooses the one that maximizes the representative consumer's welfare.

It is well-known that in optimal tax problems of this sort the government would like to set the tax rate on capital income in the very first period as high as possible since this tax is effectively a lump-sum tax on first period capital income. To make the problem interesting, we follow the literature and set the initial capital tax rate to an exogenous value, $\bar{\tau}_{k,1}$.

Another assumption that we make about the set of tax instruments available to the government is that the tax rate on capital and profit income are the same in every period, i.e., $\tau_{\pi,t} = \tau_{k,t}$ for all t. We believe that this assumption is a reasonable one as it is hard for governments to distinguish profit income from capital income. This assumption is also broadly in line with actual tax policy practices in the United States and the developed economies.[10] Therefore, the government chooses $\tau = \{\tau_{k,t+1}, \tau_{l,t}\}_{t=1}^{\infty}$. Formally, given (k_1, b_1) and $\{g_t\}_{t=1}^{\infty}$, the optimal tax policy, τ^*, solves the following optimal tax problem:

$$\max_{\tau} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t)$$

subject to the fact that the tax system $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, the allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ and the price system $\{p_t, r_t, w_t\}_{t=1}^{\infty}$ constitute a market equilibrium. Following the literature, in what follows, we often refer to the optimal tax problem as the Ramsey problem.

3.1 Ramsey Allocation

The Ramsey problem defined above is a fairly hard problem to solve directly as the constraint set involves endogenous prices, and consumer and firm maximization problems. Instead of attacking this problem directly, we are going to follow the primal approach which is a common way of solving optimal tax problems in the literature. In this approach, we solve the Ramsey problem in three steps. First, we show that the Ramsey problem is equivalent to a planning problem where the government chooses an allocation subject to a number of conditions that summarize all the restrictions that are implied on allocations by the tax-distorted market equilibrium. Second, we characterize the Ramsey allocation, namely, the allocation that solves the Ramsey problem, by a set of optimality conditions. Finally, we back out optimal tax rates by comparing optimality conditions that come out of the Ramsey problem and the tax-distorted market equilibrium. The following proposition deals with the first step of the primal approach. It establishes that the resource feasibility constraint together with two other constraints characterizes the tax-distorted market equilibrium completely.

Proposition 1. *If an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is part of a tax-distorted market equilibrium, then it satisfies the resource feasibility constraint (13), and the constraints (14) and (15) below. Conversely, suppose an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ satisfies (13), (14) and (15). Then, we can construct prices and taxes such that this allocation together with constructed prices*

[10] Although business income is taxed according to various tax laws according to the U.S. tax code - mainly the corporate, dividend and capitals gains taxes - none of these taxes treat capital income and economic profits as separate tax bases.

and taxes constitute an equilibrium allocation.

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t}c_t + u_{l,t}l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t}\pi_t(1-\tau_{\pi,t}) + u_{c,1}(\bar{r}_1 k_1 + b_1), \tag{14}$$

$$\tau_{\pi,t} = 1 - \frac{\frac{u_{c,t-1}q_{t-1}}{\beta u_{c,t}} - q_t}{\left(1-\frac{1}{\varepsilon_t}\right)F_{k,t} - \delta q_t}, \quad \forall t \geq 2. \tag{15}$$

Proof. Relegated to Appendix A.1. □

The constraint (14) is called the implementability constraint, and is a standard constraint and a version of this is present in all Ramsey tax problems. The constraint (15) represents the restriction that profit and capital income tax rates are equal, and follows from (10).

Ramsey problem. Given (k_1, b_1), initial policies $\tau_{\pi,1} = \tau_{k,1} = \bar{\tau}_{k,1}$, and a sequence of government spending, $\{g_t\}_{t=1}^{\infty}$, the government chooses allocation (c, k, l) to solve the problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{16}$$

$$c_t + q_t k_{t+1} + g_t \leq F_t(k_t, l_t) + (1-\delta)q_t k_t, \quad \text{for all } t,$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t}c_t + u_{l,t}l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t}(1-\tau_{\pi,t})\pi_t + u_{c,1}(\bar{r}_1 k_1 + b_1),$$

where $\pi_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$ and $\tau_{\pi,t}$ is given by (15).

The first term on the right-hand-side of the implementability constraint, which involves terms related to profits, is the main difference between our problem and the standard Ramsey problem without profits. It is equal to the net-present-value of after-tax profit income. In the solution, the implementability constraint binds in the direction that the left-hand side should be greater than the right-hand side. In fact, an explicit derivation of this constraint from the government's budget constraint would reveal that the left-hand side corresponds to the revenue side of government's budget while the right-hand side corresponds to its spending. As such, the net-present-value of after-tax profits appear as a cost in the Ramsey problem. Intuitively, since taxing pure profits is not distortionary, the Ramsey planner would like to confiscate them fully. When this is not possible, this is identical to a case where the government taxes profits at 100% but needs to rebate $(1-\tau_{\pi,t})$ back to consumers.

Letting $\beta^{t-1}\mu_t$ and λ be the Lagrangian multipliers on period t feasibility constraint and the implementability constraint, respectively, and the star allocation denote the Ramsey allocation, the first-order optimality condition for capital in any period $t \geq 2$ is:

$$\begin{aligned}(k_t) : -\mu^*_{t-1} q_{t-1} + \beta\mu^*_t \left(F^*_{k,t} + (1-\delta)q_t\right) \\ - \lambda^*\beta u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial \pi^*_t}{\partial k_t} + \frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t}\pi^*_t\right] = 0.\end{aligned} \tag{17}$$

The two terms in the first line of (17) are standard. The first term represents the period $t-1$ physical cost of investing in period t capital stock whereas the second term represents the period t physical return to that investment. The existence of profits in the implementability constraint, (14), introduces two new terms into the first-order condition of capital presented in the second line of (17). We now discuss these terms.

Indirect tax on profit income. The first term is $-\lambda^*\beta u^*_{c,t}(1-\tau^*_{\pi,t})\frac{\partial \pi^*_t}{\partial k_t}$. Increasing capital in period t increases profits in the same period, and hence, increases the net-present value of after-tax profits. This tightens the implementability constraint, and as such, represents an additional cost of increasing capital. The rise in the net-present-value of after-tax profits equals the rise in after-tax profits in period t, $(1-\tau_{\pi,t})\frac{\partial \pi_t}{\partial k_t}$, times the shadow price of consumption $\beta u^*_{c,t}$. The multiplier on the implementability constraint, λ^*, measures the social value of an additional unit of public funds. This additional cost term implies a tax on capital income. Intuitively, since taxing profits is a lump-sum tax, government would like to tax profits away completely. When this is not possible, it is optimal to tax intermediate goods, capital in this case, since it acts as an *indirect* tax on profit income.

Direct tax on profit income. The second term is $-\lambda^*\beta u^*_{c,t}\frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t}\pi^*_t$. It follows from (15) that a higher level of capital stock is consistent with a lower profit tax rate in equilibrium since $\frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t} > 0$. This implies that increasing capital has an additional cost of increasing the net-present-value of after-tax profits by increasing after-tax profits in period t. The term $\lambda^*\beta u^*_{c,t}$ again translates this cost into the social value of public funds. This additional social cost of increasing capital introduces another reason for its taxation. Intuitively, since capital and profit income are taxed at the same rate, taxing capital income provides a *direct* way of taxing profit income, and this is beneficial since taxing profits is non-distortionary.

The first-order optimality condition for labor in any period $t \geq 2$ is:

$$(l_t) : u^*_{l,t} + \mu^*_t F^*_{l,t} + \lambda^* \left[u^*_{ll,t} l_t + u^*_{l,t}\right] - \lambda^* u^*_{c,t} \left[(1-\tau^*_{\pi,t})\frac{\partial \pi^*_t}{\partial l_t} + \frac{\partial(1-\tau^*_{\pi,t})}{\partial l_t}\pi^*_t\right] = 0. \tag{18}$$

The first three terms in (18) are standard. The last term is new and appears due to the presence of the net-present-value of after-tax profit income on the right-hand-side of the implementability constraint, (14). It consists of two terms. The first term is negative, and is analogous to the indirect tax on profit income term for capital: increasing labor raises profits, and so has an additional social cost. The second term is positive, which means there is an additional benefit of increasing labor. Since $F_{kl} > 0$, increasing labor is consistent with a higher tax on capital, and hence, profits in equilibrium. The first term calls for an additional tax while the second one calls for a subsidy on labor income.[11]

3.2 Optimal Taxes

In this section, we provide formulas for optimal taxes that implement the Ramsey allocation in the market equilibrium defined in Section 2.

Defining optimal taxes. Using (10) and (11), we define optimal taxes as the optimal distortions that implement Ramsey allocation in the tax-distorted market equilibrium:

$$1-\tau^*_{k,t} = \frac{\frac{u^*_{c,t-1}q_{t-1}}{\beta u^*_{c,t}} - q_t}{\left(\left(1-\frac{1}{\varepsilon_t}\right)F^*_{k,t} - \delta q_t\right)}, \tag{19}$$

$$1-\tau^*_{l,t} = \frac{-u^*_{l,t}}{u^*_{c,t}F^*_{l,t}\left(1-\frac{1}{\varepsilon_t}\right)}. \tag{20}$$

Steady-state tax formulas. We focus on steady-state tax formulas as the optimal tax formulas along the transition are complicated and do not add much to our understanding of the forces behind optimal taxes. Suppose the Ramsey allocation converges to a steady state. Let variables with no time subscript denote steady-state variables. Using the first-order optimality conditions of the Ramsey problem (16) and the definition of optimal taxes (19) and (20) at the steady state, one obtains the following optimal long-run tax formulas.

[11] The first-order optimality condition for labor in period 1 is different from (18) since the profit tax rate is exogenous in period 1. We provide this condition in Appendix A.2, along with the full characterization of the Ramsey allocation, which includes the first-order optimality condition with respect to consumption.

Proposition 2. *The long-run optimal tax rate on capital and labor income are given by*

$$\tau_k^* = \frac{1}{\left(1-\frac{1}{\varepsilon}\right)F_k^* - \delta q}\left(-\frac{1}{\varepsilon}F_k^* + \chi^*\left[\frac{\partial \pi^*}{\partial k}(1-\tau_\pi^*) + \frac{\partial(1-\tau_\pi^*)}{\partial k}\pi^*\right]\right) \tag{21}$$

and

$$\tau_l^* = 1 - \frac{1+\lambda^*\left(1+u_{cc}^* c^*/u_c^*\right)}{1+\lambda^*\left(1+u_{ll}^* l^*/u_l^*\right)}\left(1-\chi^*\frac{1}{F_l^*}\left[\frac{\partial \pi^*}{\partial l}(1-\tau_\pi^*) + \frac{\partial(1-\tau_\pi^*)}{\partial l}\pi^*\right]\right)\frac{1}{1-\frac{1}{\varepsilon}}, \tag{22}$$

where $\chi^* = \frac{\lambda^* u_c^*}{\mu^*}$ *is the relative social value of public funds at the steady state.*

Proof. Relegated to Appendix A.2. □

Interpretting the capital tax formula. The optimal capital tax rate given by (21) is the multiplication of two terms. Recall that the government taxes capital income net of depreciation expenses, which equals $\left(\left(1-\frac{1}{\varepsilon}\right)F_k - \delta q\right)k$. The first term then is proportional to the inverse of the capital income tax base. Intuitively, all else equal, we need a higher tax rate to generate a given (optimal) distortion if the tax is applied to a smaller income base.

The second term is given by the summation of the two terms in the parenthesis. The first term equals $-\frac{1}{\varepsilon}F_k^*$. This term is negative and as such calls for a subsidy on capital income. Recall that the equilibrium of the growth model introduced in this paper features inefficiently low investment demand due to monopolistic distortions in the product market. Whenever the government cannot correct these distortions directly at the firm level via product market interventions, it is optimal to boost capital accumulation indirectly by subsidizing consumer savings. As we will see in Section 4, when distortions are corrected on the production side of the economy where they originate, this term disappears from the optimal tax formula.

The second term in the parenthesis represents the profit tax revenue benefit of taxing capital. First, when we increase taxes on capital income, this reduces profits, and as such, provides an *indirect* tax on profit income. The term $\frac{\partial \pi^*}{\partial k}(1-\tau_\pi^*)$ represents this benefit. Second, since profit and capital income are taxed at the same rate, increasing taxes on capital income also increases the tax rate on profit income, and as such, acts as a *direct* tax on profit income. The term $\frac{\partial(1-\tau_\pi^*)}{\partial k}\pi^*$ represents this benefit. Both the direct and the indirect profit tax revenue benefit of capital taxation accrue in terms of higher government revenues. As such, this benefit must be weighted by the social value of public funds, $\lambda^* u_c^*$. On the other hand, the cost of taxing capital, which is the deadweight loss associated with slowing down capital accumulation, accrues in terms of lower output. The social cost of a unit decline in output equals the multiplier on the resource constraint, μ^*. The term, χ^*, which we call the relative social value of public funds, translates the revenue benefit into the same unit as the deadweight loss, that is foregone output.

It is important to note that the subsidy term, $-\frac{1}{\varepsilon}F_k^*$, would disappear if the profits accrued within a competitive framework, say, due to presence of factors of production that are in fixed supply. On the other hand, the indirect and direct tax terms remain intact as long as profits are pure economic rents, independent of the source of profit generation.

Interpretting the labor tax formula. The optimal labor tax rate given by (22) consists of three components. The first component, given by $\frac{1+\lambda^*(1+u_{cc}^*c^*/u_c^*)}{1+\lambda^*\left(1+u_{ll}^*l^*/u_l^*\right)} < 1$, is the standard Ramsey labor tax component which is present irrespective of the existence of profits in equilibrium. The second component, given by $\left(1-\chi^*\frac{1}{F_l^*}\left[\frac{\partial\pi^*}{\partial l}(1-\tau_\pi^*)+\frac{\partial(1-\tau_\pi^*)}{\partial l}\pi^*\right]\right)$, represents the profit tax revenue effects of labor taxes. Notice that whether this term calls for a tax or a subsidy on labor income is ambiguous since $\frac{\partial(1-\tau_\pi^*)}{\partial l} > 0$. The third term, $\frac{1}{1-\frac{1}{\varepsilon}} > 1$, represents the Pigouvian subsidy that is in place to correct for the underemployment arising from inefficiently low labor demand due to monopolistic competition.

4 Optimal Taxes with Product Market Interventions

The equilibrium of the growth model introduced in this paper features inefficiently low investment and labor demand due to the presence of monopolistic distortions in the product market. The inefficiently low capital and labor demand imply rental and wage rates that are lower than the corresponding marginal products, which, if not confronted with policy, gives rise to an equilibrium that features too little capital stock and labor. The analysis so far has not allowed the government to use product market interventions to correct distortions arising from monopolistic competition. In the absence of these policies, as we have seen in Section 3.2, there is a motive to subsidize consumer's capital and labor income in order to drive the equilibrium level of capital and labor up toward efficient levels.

In this section, we lay out our baseline implementation of the Ramsey allocation characterized in Section 3.1. In this decentralization, the problem of insufficient demand for inputs to production is dealt with via product market policies. This institutional design, where inefficiencies due to monopolistic competition are corrected at the product market, is a natural one since the insufficient demand originates in the product market.[12] The use of product market policies is also the topic of a growing literature that studies how to design product market policies in correcting inefficiencies stemming from market power.[13]

Product market policies. Consider a decentralization of the Ramsey allocation defined by the solution to (16) in which the government subsidizes the sales of intermediate goods producers at a flat rate. The level of the sales subsidy is set exogenously to correct the lack of demand for investment and employment that stem from monopolistic distortions. For reasons that will become clear soon, intermediate goods firms also face an exogenously set lump-sum tax. Formally, the problem of an intermediate goods firm which faces a sales subsidy $\tau_{s,t}$ and a lump-sum tax T_t in period t is given by:

$$\pi_{i,t} = \max_{\xi_{i,t}, y_{i,t}, k_{i,t}, l_{i,t}} (1+\tau_{s,t})\xi_{i,t} y_{i,t} - r_t k_{i,t} - w_t l_{i,t} - T_t \quad \text{s.t.} \quad y_{i,t} = F_t(k_{i,t}, l_{i,t}).$$

The subsidy rate is set to $\tau_{s,t} = \frac{1}{\varepsilon_t - 1}$ in any period t. This guarantees that the rental rates on capital and labor equal their marginal products in equilibrium. The presence of such a subsidy also changes the level of the profit income generated by firms, which then would alter the Ramsey problem the government faces since, as it is evident from the implementability constraint (14), after-tax profit income enters the Ramsey problem. The level of the lump-sum tax is set exogenously to ensure that the Ramsey problem remains identical to (16). Put differently, the value of the lump-sum tax is set exogenously to offset the budgetary effect of the product market subsidy and to guarantee that the government's financing needs in equilibrium remain unchanged relative to the original optimal tax problem. Appendix A.3 provides a full description of the product market policy and formally proves the claim that it allows for a decentralization of the Ramsey allocation in a market environment where the interest and the wage rates equal the marginal products of capital and labor.

[12] Correcting monopolistic distortions at the firm level may also be desirable since in reality there may be heterogeneity in market power among firms, and targeting firms with firm-specific corrective subsidies would not be possible by subsidizing consumers' savings.

[13] See, among others, Atkeson and Burstein (2019), Edmond, Midrigan, and Xu (2018), Boar and Midrigan (2020). The focus of the current paper is quite different from these papers since while they investigate correcting inefficiencies stemming from product market distortions we analyze optimal financing of government spending through income taxation.

Defining optimal taxes in the presence of product market policies. The optimal capital and labor income tax rates that implement the Ramsey allocation in the market equilibrium with product market policies are given by:

$$1-\tau_{k,t}^{*} = \frac{\frac{u_{c,t-1}^{*}q_{t-1}}{\beta u_{c,t}^{*}} - q_t}{F_{k,t}^{*}-\delta q_t}, \tag{23}$$

$$1-\tau_{l,t}^{*} = \frac{v_{l,t}^{*}}{u_{c,t}^{*}F_{l,t}^{*}}. \tag{24}$$

The optimal tax definitions reflect the fact that, in the presence of product market policies, the rental and wage rates equal the corresponding marginal products. The first-order conditions of (16) and the definition of taxes given by (23)-(24) render the following proposition.

Proposition 3. *The long-run optimal tax rate on capital and labor income are given by*

$$\tau_k^{*} = \frac{1}{F_k^{*}-\delta q}\chi^{*}\left[\frac{\partial \pi^{*}}{\partial k}(1-\tau_\pi^{*}) + \frac{\partial(1-\tau_\pi^{*})}{\partial k}\pi^{*}\right] \tag{25}$$

and

$$\tau_l^{*} = 1 - \frac{1+\lambda^{*}\left(1+u_{cc}^{*}c^{*}/u_c^{*}\right)}{1+\lambda^{*}\left(1+u_{ll}^{*}l^{*}/u_l^{*}\right)}\left(1-\chi^{*}\frac{1}{F_l^{*}}\left(\frac{\partial \pi^{*}}{\partial l}(1-\tau_\pi^{*}) + \frac{\partial(1-\tau_\pi^{*})}{\partial l}\pi^{*}\right)\right). \tag{26}$$

Proof. Relegated to Appendix A.4. □

The main difference of the optimal capital tax rate with product market interventions given by (25) relative to the optimal capital tax rate without such policies given by (21) is that in the latter formula there is an additional term, $-\frac{1}{\varepsilon}F_k^{*}$, that calls for a subsidy on capital income. This component is absent in (25) since in this case monopolistic distortions are already dealt with at the firm level where they originate via product market subsidies. For this reason, the optimal capital tax rate is higher in the case with product market policies.[14] For the same reason, the optimal labor tax formula in the case without product market subsidies, (22), features an additional subsidy term, $\frac{1}{1-\frac{1}{\varepsilon}}$, that is not present in (26).

[14]There is an additional, more subtle difference between the two optimal capital tax formulas. Notice that the first term on the right-hand-side of (21) has an additional $\left(1-\frac{1}{\varepsilon}\right)$ multiplying F_k^{*}. The difference is due to the fact that the rental rate on capital equals $\left(1-\frac{1}{\varepsilon}\right)F_k^{*}$ in the environment without product market policies while it is equal to F_k^{*} in the environment with.

4.1 Lower Bound on the Optimal Capital Tax Rate

In this section, we derive an alternative version of the optimal capital tax formula, which allows us to provide a lower bound on the optimal tax rate that depends on empirically estimable parameters. By taking the right-hand side of (25) into $(1-\tau_\pi^*)$ parenthesis and using $\tau_\pi^* = \tau_k^*$, we obtain the following formula for the optimal long-run capital tax rate

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{F_k^*}{F_k^* - \delta q}\chi^* S_\pi \left[1 + \frac{\mathcal{E}_{1-\tau_k,k}^*}{\mathcal{E}_{y,k}^*}\right], \tag{27}$$

where $\mathcal{E}_{1-\tau_k,k}^* = \frac{d\ln(1-\tau_k)}{dk}|_{k=k^*}$ is the elasticity of the retention rate on capital income with respect to the equilibrium level of capital stock and $\mathcal{E}_{y,k}^* = \frac{d\ln y}{dk}|_{k=k^*}$ is the elasticity of national income with respect to the capital stock. When $\mathcal{E}_{1-\tau_k,k}^*$ is larger, the retention rate is more sensitive to the equilibrium level of capital stock. In other words, a marginal decrease in the capital stock is consistent with a larger decrease in the retention rate of capital (and profit) income in equilibrium. This means that the direct profit tax revenue benefit of taxing capital is larger. This is why higher $\mathcal{E}_{1-\tau_k,k}^*$ implies a higher optimal capital tax rate. The tax rate is decreasing in $\mathcal{E}_{y,k}^*$ because the deadweight loss of raising capital taxes, and hence, reducing capital stock depends on the elasticity of output with respect to the capital stock.

Notice that $\frac{\mathcal{E}_{1-\tau_k,k}^*}{\mathcal{E}_{y,k}^*} = \frac{1}{\mathcal{E}_{k,1-\tau_k}^*\mathcal{E}_{y,k}^*} = \frac{1}{\mathcal{E}_{y,1-\tau_k}^*}$, where $\mathcal{E}_{y,1-\tau_k}^*$ is the elasticity of national income with respect to the tax rate on capital income. Plugging this back into (27), we obtain

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{F_k^*}{F_k^* - \delta q}\chi^* S_\pi \left[1 + \frac{1}{\mathcal{E}_{y,1-\tau_k}^*}\right]. \tag{28}$$

This optimal tax formula reveals that when the elasticity of national income with respect to the retention rate is larger, the optimal capital tax rate is lower. In other words, when national income is more sensitive to the tax rate on capital, the latter ought to be lower.

Corollary 1. *The optimal long-run tax rate on capital income satisfies:*

$$\tau_k^* > \frac{1}{1 + (\chi^* S_\pi)^{-1}(1 + \mathcal{E}_{y,1-\tau_k}^{*-1})^{-1}}. \tag{29}$$

Corollary 1 follows from (28) since $\delta q > 0$. The lower bound on the optimal capital tax is increasing in the relative social value of public funds, χ^*, and the profit share, S_π. These are intuitive: (i) a higher relative value of social funds means the revenue benefit of taxation is weighted more than its resource cost; (ii) both the direct and the indirect profit tax revenue benefits of capital taxation are proportional to the profit share. Moreover, the lower bound is decreasing in the sensitivity of national income with respect to the capital tax rate.

Corollary 1 is useful because, given the empirical knowledge of the relative social value of public funds, the profit share, and the elasticity of national income with respect to the capital tax rate (at the optimum), it provides a lower bound on the optimal long-run capital tax rate without requiring any knowledge of the details of the other structural parameters of the model. An alternative approach is to make further structural assumptions, which helps us to reduce the lower bound on the capital tax rate to the structural parameters of the model, and then use estimates of these parameters to gauge a lower bound (at the expense of making the model more specific). This is what the next proposition does.

Proposition 4. *Suppose* $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$*, where* $v', v'' > 0$*, and* $F = Ak^{\alpha}l^{1-\alpha}$*. The optimal long-run tax rate on capital income satisfies:*

$$\tau_k^* > \frac{1}{1 + (\lambda^{*^{-1}} + 1 - \sigma)\varepsilon\alpha}. \tag{30}$$

Proof. Relegated to Appendix A.5. □

All the parameters in this lower bound, with the exception of λ^*, the social value of public funds, are structural. Therefore, the proposition gives us a lower bound on the optimal long-run capital tax rate provided that we have an estimate of the social value of public funds.[15]

4.2 Zero Profit Income Benchmark

Notice that since π^* and $\frac{\partial \pi^*}{\partial k}$ are proportional to the share of profits in national income, the optimal long-run tax rate on capital income given by (25) is also proportional to the profit share. In particular, whenever the profit share is zero, so is the optimal tax on capital income in the steady state. The optimal long-run labor income tax rate reduces to the standard Ramsey component in this case. The following corollary summarizes this result.

Corollary 2. *If* $S_\pi = 0$*, then* $\tau_k^* = 0$ *and* $\tau_l^* = 1 - \frac{1+\lambda^*(1+u_{cc}^* c^*/u_c^*)}{1+\lambda^*\left(1+u_{ll}^* l^*/u_l^*\right)} > 0$.

The following proposition establishes a more general result about the optimality of not taxing capital that holds along the transition as well.

[15] An analogous expression for the lower bound on the long-run optimal capital tax rate can be provided for the implementation without product market subsidies. This is found in Appendix A.9. There, we also provide a lower bound for the optimal capital tax rate for the inefficient economy introduced in Section 6.

Proposition 5. *Suppose* $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$, *where* $v', v'' > 0$. *If in some period* $t \geq 3$, *we have* $S_{\pi,t-1} = S_{\pi,t} = S_{\pi,t+1} = 0$. *Then,* $\tau^*_{k,t} = 0$ *and* $\tau^*_{l,t} = 1 - \frac{1+\lambda^*(1+u^*_{cc}c^*/u^*_c)}{1+\lambda^*\left(1+u^*_{ll}l^*/u^*_l\right)} > 0$.

Proof. Relegated to Appendix A.6. □

In particular, the proposition implies a classical finding in the Ramsey literature (see, Chari and Kehoe (1999), for instance) that in an economy where the share of profits in national income is zero, the optimal tax rate on capital is also zero in the short and the long run.[16] An important implication of this result is that if we live in a competitive economy and the decline in labor share is occurring due to a rise in capital share, then the lessons from the classical Ramsey tax theory apply: it is optimal to set capital tax rate to zero and finance government spending with (higher) taxes on labor income.[17] Notice that this conclusion is independent of the exact mechanism behind the rise in capital share.

5 Quantitative Analysis

This section provides a discussion about the model calibration and the simulation results.

5.1 Calibration: Initial Steady State

We choose the parameters of the model so that the initial steady state of the model economy matches the early 1980's U.S. economy along selected key moments. The model is calibrated annually and the full set of parameters, targets and sources are summarized in Table 1.

Preferences. The discount factor β is set to 0.96 so that the model implied interest rate is equal to 4.1% (Atkeson and Kehoe (2005)). This implies a capital-output ratio of 2.5. The momentary utility function of the household takes the form

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{l^{1+\phi}}{1+\phi}.$$

[16] The proposition states that the capital tax rate is zero from third period onward. Recall that the tax rate on the first period capital is a lump-sum tax, and hence, is exogenously set. It is a standard result in optimal Ramsey tax theory that the tax rate on the second period capital is qualitatively different from the tax rate on future periods, and is generally very high when there is no upper bound on capital income taxes.

[17] Although we derive these two results on the optimality of not taxing capital income for the implementation with product market interventions, the results also hold for the institutional setup of Section 3 which implements the Ramsey allocation without product market policies.

The constant elasticity of intertemporal substitution (CEIS) coefficient σ is set to 1. The labor supply parameter ϕ is set to 1.33, which implies a Frisch elasticity of aggregate hours of 0.75 as in Chetty, Guren, Manoli, and Weber (2011). The parameter that captures the disutility of hours worked ψ is calibrated so that one third of available time is spent at work.

Production. The production function operated by the intermediate goods producers is given by

$$F_t(k_t, l_t) = (\alpha_{k,t}(A_{k,t}k_t)^\rho + \alpha_{l,t}(A_{l,t}l_t)^\rho)^{1/\rho} . \tag{31}$$

The elasticity of substitution between capital and labor, captured by the parameter ρ, is set to 0.20 as in Karabarbounis and Neiman (2014). The capital-augmenting and labor-augmenting technology parameters, A_k and A_l, are normalized to one, without loss of generality. The capital depreciation rate δ is set to 0.072, which is equal to its value leading to 1982 (over the period 1970-1982), calculated from Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) and BEA Fixed Asset Tables (FA). The parameter that governs the elasticity of substitution between intermediate inputs ε is set to 100, implying a profit share of 1% which is equal to the profit share reported by Barkai and Benzell (2018) for the early 1980's, and is also in line with the findings of findings of De Loecker, Eeckhout, and Unger (2020). The production function parameter α_k controls how the remaining 99% income share is divided between capital and labor, and is calibrated to match the observed labor share for the U.S. non-farm business sector over the period of 1947-1982, which we calculated using Bureau of Labor Statistics (BLS) data. The parameter α_l is normalized to $1 - \alpha_k$. The relative price of investment q is normalized to one in the initial steady state.

Government policy. The tax rates for the initial steady state are taken from McGrattan and Prescott (2010). In line with our baseline model, we assume that the tax rate on profit income is equal to the capital income tax rate observed in the data. Accordingly, we set the uniform tax rate on capital and profit income $\tau_k = \tau_\pi$ and labor income τ_l equal to 40% and 29%, respectively. The level of government expenditure is calibrated to match a government expenditure to GDP ratio of 0.20, which is equal to its observed level in the early 1980's calculated by using the St. Louis FED FRED data.

Table 1: Benchmark Calibration

Parameter	Symbol	Value	Source/Target
Preferences			
Discount factor	β	0.96	Risk-free rate = 4.1%
CEIS parameter	σ	1.00	-
Labor supply elasticity	ϕ	1.33	Chetty et al. (2011)
Disutility of hours worked	ψ	9.65	Labor supply = 1/3
Production			
Elasticity of substitution btwn. capital and labor	ρ	0.20	KN
Depreciation rate	δ	0.072	BEA
Production function parameter	α_k	0.295	Labor share = 0.64 (BLS)
Elasticity of substitution btwn. intermediate inputs	ε	100	Profit share = 0.01 (BB)
Government policy			
Tax rate on labor income	τ_l	29%	MP
Tax rate on capital income	$\tau_k = \tau_\pi$	40%	MP
Government expenditure	g/y	0.20	FRED
Government debt	b/y	0.31	FRED

The table reports the calibration of the model parameters to the early 1980's U.S. economy. The acronyms BB, BEA, BLS, FRED, KN, and MP stand for Barkai and Benzell (2018), Bureau of Economic Analysis, Bureau of Labor Statistics, Federal Reserve Bank of St. Louis FRED database, Karabarbounis and Neiman (2014), and McGrattan and Prescott (2010).

5.2 Calibration: The Evolution of the Economy

There are four time-varying parameters in the model - q_t, $\tau_{k,t}$, ε_t and $\alpha_{k,t}$ (or $1 - \alpha_{l,t}$). In this section, we discuss how they are calibrated. Figure 2 displays the externally calibrated parameters, q_t and $\tau_{k,t}$. The capital tax series is taken from McGrattan and Prescott (2010) while the price of equipment is computed directly from the data.[18] Because we want to abstract from business cycle variations, the capital tax series is smoothed with a piecewise linear function. Similarly, we fit a smooth polynomial form to match the change in price q_t over the period of interest, which captures the fact that (i) the decline in relative price of investment starts in 1983 and (ii) the rate of decline slows down through the end, which implicitly implies that the declining trend in q_t is expected to vanish before 2050 (Figure 2).

The remaining two time-varying parameters - ε_t and $\alpha_{k,t}$ - are calibrated internally. Figure 3 displays the evolution of these two calibrated series used in our simulations. The inverse of the parameter ε_t equals the profit share in the model economy. To calibrate the evolution of this parameter since the early 1980's, we follow the findings in the literature regarding the

[18]As in Karabarbounis and Neiman (2014), we construct the relative price of investment series by calculating the ratio of the investment price deflator to the consumption price deflator for the U.S. economy over the post-war period. Taking a close look at this series, we see that, in line with Karabarbounis and Neiman (2014), the relative price of investment is fairly stable until 1982 after which it starts to decline.

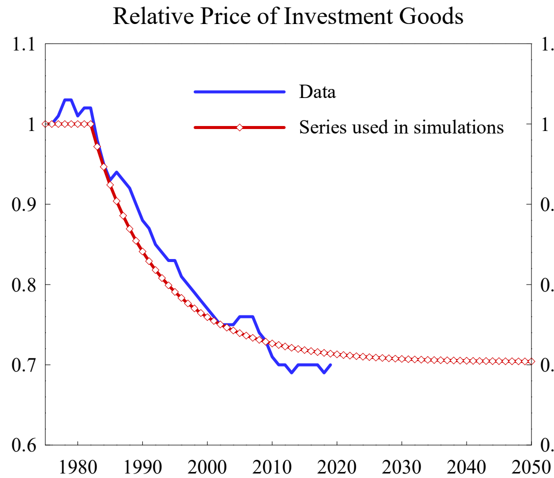

Figure 2: (a)

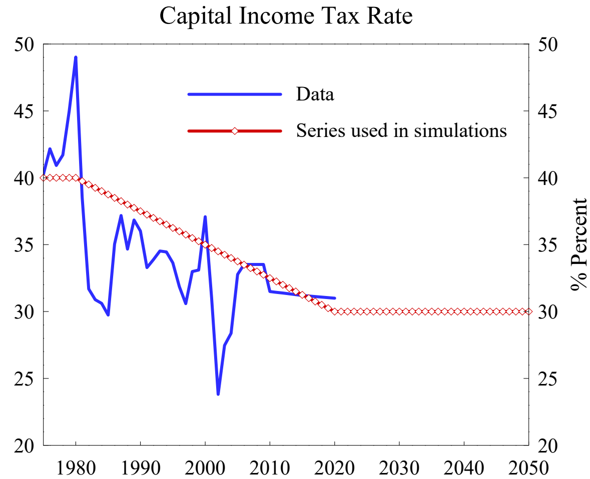

Figure 2: (b)

The figure depicts the relative price of investment goods (a) and the capital income tax rate (b) for the U.S. economy over time.

share of profits in national income. In a recent paper, Barkai and Benzell (2018) documents that the profit share increased roughly from a level of 1% to 15% over the period of interest. Similarly, Barkai (2020) finds that the profit share in the U.S. economy increased roughly by 13.5 percentage points during 1984-2014. These findings seem to be in line with the findings of other notable contributions in the literature. According to De Loecker, Eeckhout, and Unger (2020), the profit share for the U.S. economy increased approximately from 2% in the early-1980s to a level of 16% in the late-2010s. Eggertsson, Robbins, and Wold (2018) argues that the profit share, which was roughly zero in the early 1980's, increased to a level of 17% by 2015.[19] In line with these studies, we calibrate the change in ε_t so that (i) the profit share in the simulated economy increases from 1% to 15% since the early 1980's and (ii) the time-series dynamics of simulated profit share series tracks the one observed in the data.[20] The second time-varying input of our model - $\alpha_{k,t}$ - is calibrated to track the observed change in the labor share since early 1980's.[21]

[19]Similarly, Karabarbounis and Neiman (2018) document that while the labor share and the capital share summed up to 1 in the early 1980's, this number fell down to 0.85 over the period of interest.

[20]The data on the evolution of profit shares is from Karabarbounis and Neiman (2018). The time series for ε_t is calibrated specifically to ensure that the profit share (i) changes following a smooth monotonic polynomial trend, (ii) reaches its long-run level roughly by the end of 2020's, and (iii) matches its actual level in the mid-way of transition in 1999.

[21]The evolution of U.S. non-farm labor share is calculated from Bureau of Economic Analysis (BEA) NIPA Tables. The decline in $\alpha_{k,t}$ is captured by a smooth monotonic polynomial function and is calibrated so that (i) the simulated labor share equals its value in the data in 1982 and 2016 and (ii) the average values of the simulated labor shares for the first and second halves of the period 1983 to 2016 match their data counterparts.

Figure 4 displays the simulated time-series for profit share and labor share, as well as their counterparts in data, and reveals that the simulated series capture the long-term trends in labor and profit shares.

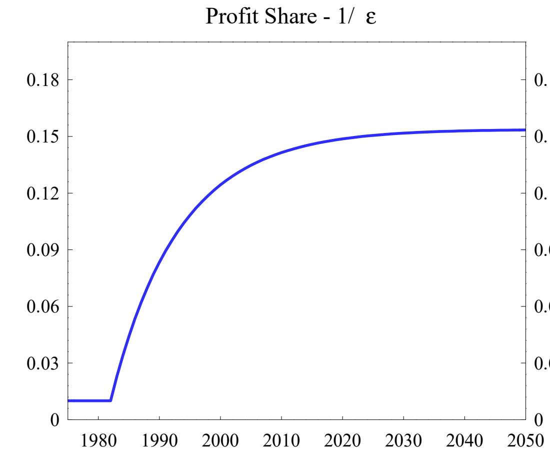

Figure 3: (a)

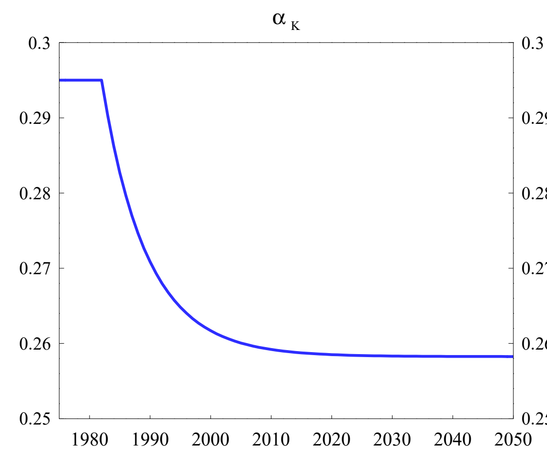

Figure 3: (b)

This figure depicts the calibrated time series of the model parameters ε (a) and α_k (b).

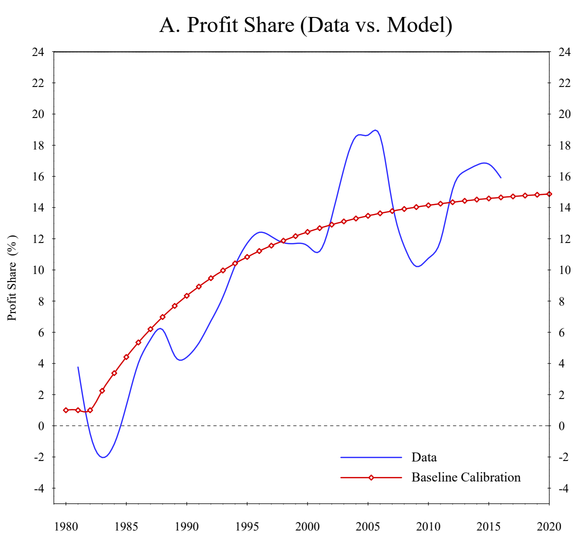

Figure 4: (a)

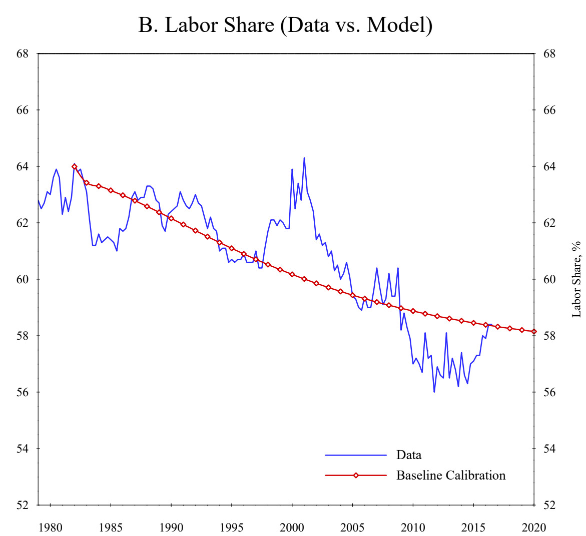

Figure 4: (b)

This figure depicts the time series of the observed and the model implied profit share (a) and labor share (b).

APK-$\tilde{R}$. Caballero, Farhi, and Gourinchas (2017), among others, document that a key macroeconomic trend in the US economy observed during the period of interest has been the growing divergence between the return on productive capital and the return on safe assets.

More specifically, they show that the difference between the Average Product of Capital (APK) and the return on government bonds ($\tilde{R}$) has increased significantly since the 1980's. In this section, we test the validity of our calibration by investigating how our calibrated model performs in terms of matching this untargeted data moment. The difference between these two variables is given by the following expression in the model:

$$APK_t - \tilde{R}_t = \frac{q_{t-1} - q_t}{q_{t-1}} + \frac{Y_t \frac{1}{\varepsilon_t}(1 - \tau_{k,t})}{q_{t-1} K_t}. \tag{32}$$

We can see from (32) that, among other factors, higher profit share leads to an increase in $APK - \tilde{R}$. In fact, our simulations show that the share of profits in the economy is the dominant factor in determining $APK - \tilde{R}$. Table 2 shows that while $APK - \tilde{R}$ was about -0.4% in 1982, it reached roughly to a level of almost 7.5% in 2020 in the data.[22] In the model economy, $APK - \tilde{R}$ increases from 0.2% to 5.4% over the same period. We conclude that our calibrated model performs well in tracking the rise in the divergence of return to productive capital and returns to safe assets.[23] One way to interpret this finding is that the rise in market power by itself can be a major factor in explaining this divergence, which is is in line with Eggertsson, Robbins, and Wold (2018) and Farhi and Gourio (2018).

Table 2: $APK - \tilde{R}$

	1982	2020
Data	-0.4%	7.5%
Model	0.2%	5.4%

This table reports the difference between average product of capital (APK) and the risk-free rate in the model and in the data.

5.3 Optimal Taxes with Product Market Policies

This section reports the optimal capital and labor income taxes for the baseline implementation with product market policies. We consider two distinct optimal tax reforms. In the first

[22] The data on APK series are generated for the US corporate sector based on BEA NIPA and Fixed Asset Tables and the methodology closely follows Caballero, Farhi, and Gourinchas (2017). Our baseline measure of $\tilde{R}$ is the nominal rate on 10-year U.S. Treasuries minus 5-year moving average of realized inflation that proxies expected inflation, as in Karabarbounis and Neiman (2018).

[23] We also conduct an alternative calibration in which the observed decline in labor share is due to an increase in capital share only (no change in profit share), and find that $APK - \tilde{R}$ remain roughly constant at the level of 0.2% over the period of interest. This could be interpreted as an additional support for the rise in market power in explaining the decline in labor share within the context of our model.

one, the government reforms the tax system now (in year 2021).[24] This exercise, which we refer to as the "2021 reform", aims to inform the policymakers about the following question: given the observed decline in labor share and the tax policies in place up until now, what is the current optimal policy response? In our second exercise, we assume that the tax reform is carried out in 1983. This analysis, which, in short, we refer to as the "1983 reform", informs us about what the optimal reaction to declining labor share would have been had the government anticipated the decline in labor share and reformed the tax system in 1983. A comparison of the two reforms will be informative about the cost of postponing the optimal tax reform. In all exercises, the series of government spending is set exogenously so that it roughly equals 20% of output in all periods in the Ramsey allocation. The initial debt level in both the 1983 and the 2021 reforms are taken from data.[25]

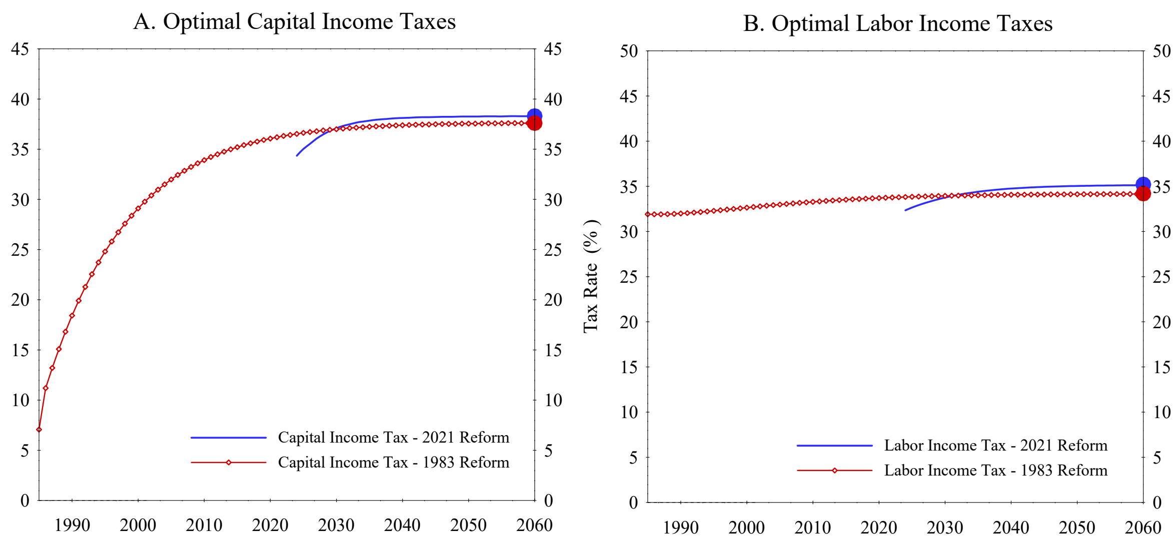

Figure 5: Optimal Tax Rates - Baseline Implementation

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the baseline implementation with product market subsidies.

Figure 5A below illustrates the time path of the optimal capital income taxes. The solid blue line depicts the 2021 reform whereas the red line with diamonds depicts the 1983 reform.[26] In the 2021 reform, the optimal capital tax rate starts from about 34%, increases

[24]To be more precise, we roll the economy out from the early 1980's steady state and the government introduces a one-time, unannounced tax reform in 2021.

[25]Using the St. Louis FED FRED data, the total federal debt to GDP ratio is calculated to be 0.31 and 1.03 in 1982 and late 2010's, respectively.

[26]Recall that the optimal capital tax rate in the first two periods is qualitatively different from the tax rate on future periods in Ramsey tax models. For this reason, in line with the literature, we skip displaying the capital income taxes for the first few periods.

over time, and converges to its long-run level of about 38.3%. In the 1983 reform, the optimal capital tax rate starts from a smaller level of 7% and over time it converges to a long-run steady state level of 37.6%.

These quantitative findings have three key implications. First, and foremost, in both reforms, the optimal tax rate on capital income is positive and large both in the short run and in the long run. Recall from the discussion in Section 3.1 that taxes on capital income are desirable because they act as indirect and direct taxes on profit income. That the optimal capital tax rates are significant implies that the tax on profits channel is quantitatively significant. Second, the optimal capital tax rate in the 1983 reform start low and rise with time following the time path of the profit share. This is expected since, as (17) reveals, the strength of both the indirect and the direct profit tax channels depend on the profit share. Third, the optimal long-run capital tax rate in the 2021 reform is somewhat larger than that in the 1983 reform even though the long-run profit shares are identical in the two cases. This is because the government has to finance a higher initial debt at the time of the 2021 reform. A glance at the long-run tax formula given by (25) shows that higher revenue requirement for the government imply higher optimal capital taxes via the term χ^*, which represents the relative social value of public funds. One way to interpret this finding is that the cost of delaying the optimal tax reform is forever higher capital income taxes.

Figure $5B$ shows that optimal labor income taxes are fairly smooth around 33% and 35% in the 1983 and the 2021 reforms, respectively. This is in line with the standard labor tax smoothing results of Barro (1979) and Lucas and Stokey (1983). Figure $5B$ also reveals that delaying the reform leads to higher optimal labor taxes as well.

5.4 Optimal Taxes without Product Market Policies

Figure $6A$ illustrates the path of the optimal capital income taxes for the alternative implementation without product market policies. In the 2021 reform, the optimal capital tax rate is around its long-run level of 10% throughout the period of interest. In the 1983 reform, the optimal capital income tax rate starts low again and then increases to a long-run steady state level of 8%. Like in the baseline case, delaying the reform implies higher capital taxes in the long run. Notice that the optimal capital tax rates are much lower than those in the case with product market interventions. A comparison of the steady-state optimal capital tax formula in the absence of product market policies, given by (21), with the formula for

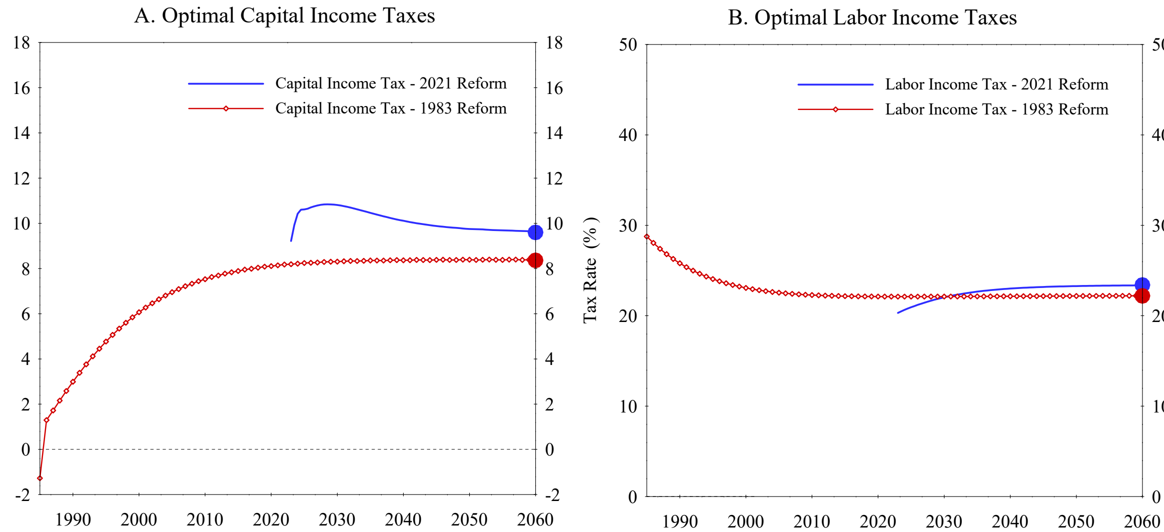

Figure 6: Optimal Tax Rates without Product Market Policies

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the implementation in which government is not allowed to use product market subsidies.

the case with product market policies, given by (25), reveals the reason. In the former case, there is an additional term, $-\frac{1}{\varepsilon}F_k^*$, that calls for a subsidy on capital income. This term is absent in the implementation with product market policies since in this case monopolistic distortions are dealt with at the firm level where they originate. It is important to stress that the optimal capital taxes are still significantly positive, which indicates that the presence of profits makes a significant case for capital taxation in this implementation as well.

Figure 6B shows that, in line with standard labor tax smoothing motivations, the optimal tax rate on labor income seems to be roughly constant at around 23% for both Ramsey problems, with only a slight exception in the 1980's and early 1990's during when optimal labor taxes are slightly above their long-run level.

6 Optimal Taxation in an Inefficient Economy

The optimal tax analysis so far has been carried out under the assumption that the government corrects monopolistic distortions that slow down capital accumulation and cause underemployment, be it via subsidizing capital and labor income or via product market policies. However, in reality, structural problems such as product market distortions might be hard to solve for a variety of reasons.[27] Alternatively, one may think that correcting product market distortions is the concern of other regulatory bodies within the government, and an all-encompassing reform that includes changing tax rates and regulating the product market distortions at the same time might be hard to achieve. In either case, there is a motive to analyze an optimal tax reform in a world in which product market distortions are not corrected. This section addresses this issue by analyzing optimal Ramsey taxation in a world with inefficiently low levels of capital accumulation and employment resulting from monopolistic distortions. To do so, we modify the Ramsey problem as follows.

Ramsey problem. Given (k_1, b_1), initial policies $\tau_{\pi,1} = \tau_{k,1} = \bar{\tau}_{k,1}$, a sequence $\{\tilde{\pi}_t\}_{t=1}^{\infty}$ and a sequence of government spending $\{g_t\}_{t=1}^{\infty}$, government chooses allocation (c, k, l) to solve the following problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{33}$$

$$c_t + q_t k_{t+1} + g_t \leq \left(1 - \frac{1}{\varepsilon_t}\right) F_t(k_t, l_t) + \tilde{\pi}_t + (1-\delta) q_t k_t, \quad \text{for all } t,$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t} c_t + u_{l,t} l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} (1 - \tau_{\pi,t}) \pi_t + u_{c,1} (\bar{r}_1 k_1 + b_1),$$

where $\pi_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$ and $\tau_{\pi,t}$ is given by (15), and in each period t, exogenously given profit income coincides with the profit income that the Ramsey problem generates: $\tilde{\pi}_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$.

To understand this problem, observe that in the equilibrium of our growth model with distortions, the private marginal returns to capital and labor faced by the intermediate goods producers are $\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t}$ and $\left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t}$, respectively. The social marginal returns, which can be observed from the Ramsey planning problem given by (16), on the other hand, equal $F_{k,t}$ and $F_{l,t}$. Technically, it is these wedges between the private and the social returns that make it optimal to introduce corrective subsidies in Section 4 (or a motive to subsidize capital and labor income in Section 3.2). The planning problem (33) modifies the planning problem (16) to ensure that the marginal returns to capital and labor perceived by the Ramsey planner are equal to the private returns firms face in equilibrium. This is achieved

[27] One such reason, for instance, might be that the degree of monopolistic distortions are heterogenous across firms and depend on unobservable firm characteristics. In such a world, it would be difficult to fully correct monopolistic distortions as the magnitude of Pigouvian corrections would depend on characteristics privately observed by firms. See Boar and Midrigan (2020) for an analysis of optimal regulation of monopolistic distortions in the presence of informational frictions.

by revising the resource constraint as in (33) by writing output as a summation of two parts: $\left(1-\frac{1}{\varepsilon_t}\right)F(k_t,l_t)$ and $\tilde{\pi}_t$. The first part ensures that the Ramsey planner perceives the same returns as the private agents. By setting the second part to $\tilde{\pi}_t=\frac{1}{\varepsilon_t}F(k_t,l_t)$, we ensure that, as in the market equilibrium, total output equals $F_t(k_t,l_t)$. This formulation guarantees that monopolistic distortions are not corrected in the solution to the Ramsey problem (33).

The first-order optimality conditions of (33) provided in Appendix A.7 and the optimal tax definitions (19)-(20) evaluated at the steady state imply the following proposition.

Proposition 6. *The long-run optimal tax rate on capital and labor income are given by*

$$\tau_k^*=\frac{1}{\left(1-\frac{1}{\varepsilon}\right)F_k^*-\delta q}\chi^*\left[\frac{\partial\pi^*}{\partial k}(1-\tau_\pi^*)+\frac{\partial(1-\tau_\pi^*)}{\partial k}\pi^*\right] \tag{34}$$

and

$$\tau_l^*=1-\frac{1+\lambda^*\left(1+u_{cc}^*c^*/u_c^*\right)}{1+\lambda^*\left(1+u_{ll}^*l^*/u_l^*\right)}\left(1-\chi^*\frac{1}{F_l^*\left(1-\frac{1}{\varepsilon}\right)}\left(\frac{\partial\pi^*}{\partial l}(1-\tau_\pi^*)+\frac{\partial(1-\tau_\pi^*)}{\partial l}\pi^*\right)\right). \tag{35}$$

Proof. Relegated to Appendix A.7. □

Notice that the optimal capital tax formula given by (34) is quite similar to the optimal capital tax formula for the baseline case in which distortions are corrected via product market policies given by (25). Specifically, the capital subsidy term, $-\frac{1}{\varepsilon}F_k^*$, which is present in the optimal capital tax formula in the case without product market distortions, (21), is absent from both the optimal tax formulas (25) and (34). The reason for why this term is absent differs across the two environments, however. While product market interventions eliminate the distortions and, therefore, the need for a subsidy on capital income in (25), the assumption that the Ramsey planner does not correct monopolistic distortions imply that there is no motive for a subsidy in (34). It is only the first terms, $\frac{1}{F_k^*-\delta q}$ vs. $\frac{1}{(1-\frac{1}{\varepsilon})F_k^*-\delta q}$, that differ across the capital tax formulas (25) and (34). This reflects the fact that the interest rate is inefficiently low in the inefficient economy due to monopolistic distortions.

6.1 Quantitative Analysis

Figure 7 below displays the paths of the optimal capital and labor income taxes over time under the assumption that monopolistic distortions are not corrected. The optimal labor income taxes are smooth over the periods of interest in both reforms. In the 2021 reform, the optimal tax rate on capital is positive and roughly constant around its long-run level of 21% in all calendar years. In the 1983 reform, the optimal capital tax rate starts low, increases over time, and converges to a long-run level of 22%. The main message to take away from Figure 7A is that the optimal capital taxes are still positive and significant when product market distortions are not corrected. The capital taxes are higher in the 1983 reform because, χ^*, which measures the social value of public funds relative to the social cost of distorting capital accumulation, is higher in the 1983 reform despite the fact that initial public debt, and hence, the government's revenue need is larger in the 2021 reform. This is because the social cost of distorting capital accumulation is higher in the 2021 reform since in this reform the economy starts already with a larger degree of monopolistic distortion whereas the 1983 experiences reform an initial transition of few decades of low distortions.

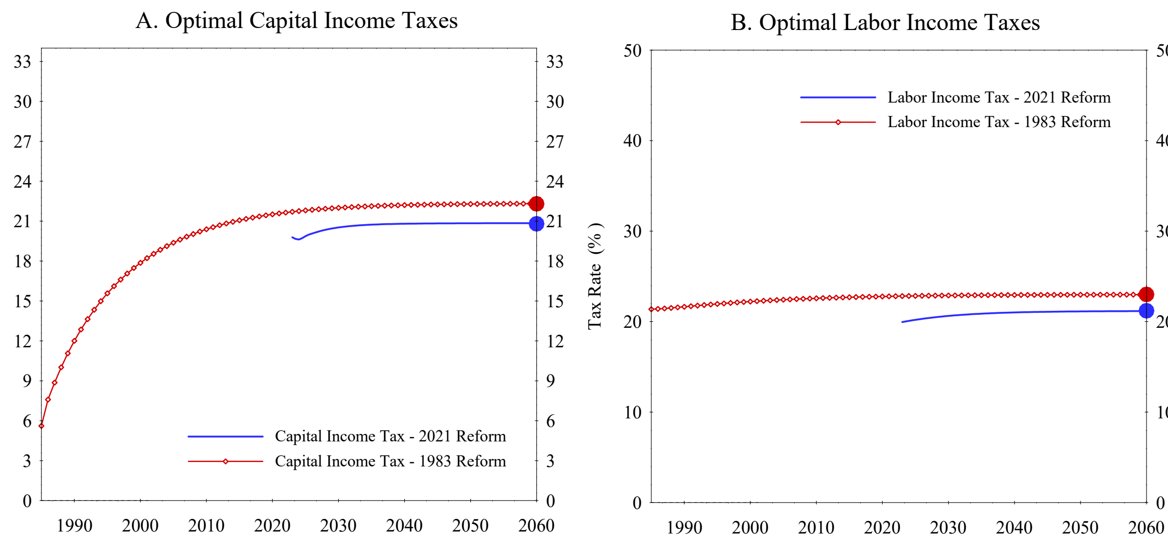

Figure 7: Optimal Tax Rates in the Inefficient Economy

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the economy in which government cannot correct monopolistic distortions.

Comparing Figure 5A and Figure 7A, we see that the optimal taxes in the inefficient economy are lower compared to the case where monopolistic distortions are corrected with product market subsidies. This may be surprising given that the tax base in the inefficient economy, $\frac{1}{(1-\frac{1}{\varepsilon})F_k^*-\delta q}$, is smaller than the tax base in the economy with corrective subsidies, $\frac{1}{F_k^*-\delta q}$, which implies that, all else equal, the tax rate should be higher in the inefficient economy. The opposite is true because all else is not equal across the two cases: namely, the relative social value of public funds, χ^*, is smaller in the inefficient economy. Recall that χ^*

represents the value of an additional dollar in government's pocket relative to the social cost of distorting capital accumulation. This is lower for the inefficient economy because, while the value of an additional dollar for the government is the same in the two cases, the social cost of distorting capital accumulation is higher in the inefficient economy since it is further away from its production possibilities frontier.[28]

7 Optimal Taxation with Exogenous Profit Taxes

The optimal tax analysis so far assumed that the tax rate on capital and profit income has to be the same, perhaps because it is hard to separate the two types of income. In this section, we derive optimal tax formulas for the case where profit income is taxed at a different rate than capital income. If we allow the government to choose the tax rate on profit income freely, it would choose to confiscate the profits fully since, as previously argued, profit tax is a lump-sum tax in our environment. In reality, however, taxing profits away at 100% may not be desirable or feasible for reasons that are not captured by our model such as the possibility that firms may be able to hide part of their profits. For this reason, we are going to set an exogenous upper limit on the profit tax rate. We maintain the assumption that the government uses product market policies to correct monopolistic distortions.[29]

The Ramsey problem for the case with exogenous profit taxes is identical to (16) except that now the profit tax is exogenously given. As a result, the problem does not have the constraint (15) which defines the profit tax rate in (16). The explicit statement of the Ramsey problem is deferred to Appendix A.8. The first-order optimality conditions of the Ramsey problem provided also in Appendix A.8 and the definition of tax rates given by (19)-(20) at the steady state deliver the following optimal tax formulas.

Proposition 7. *The long-run optimal tax rate on capital and labor income are given by*

$$\tau_k^* = \frac{1}{F_k^* - \delta q} \chi^* \frac{\partial \pi^*}{\partial k} (1 - \bar{\tau}_\pi) \tag{36}$$

[28]Technically, the social value of an additional dollar of government revenue is measured by the tightness of the implementability constraint in the solution, and the implementability constraints are identical across the two Ramsey problems (16) and (33). The value of an additional dollar to the society, on the other hand, is measured by the tightness of the feasibility constraint, and the feasibility constraint in the Ramsey problem (33) displays an inferior production function compared to the former Ramsey problem given by (16).

[29]The optimal long-run capital tax rate is shown to be negative for the case without product market policies and exogenous profit taxes in Judd (2002).

and

$$\tau_l^* = 1 - \frac{1+\lambda^*\left(1+u_{cc}^*c^*/u_c^*\right)}{1+\lambda^*\left(1+u_{ll}^*l^*/u_l^*\right)}\left(1-\chi^*\frac{1}{F_l^*}\frac{\partial\pi^*}{\partial l}(1-\bar{\tau}_\pi)\right). \tag{37}$$

Proof. Relegated to Appendix A.8. □

The main difference between the optimal capital tax formula given by (36) and the optimal tax formula in the baseline case with uniform tax rate on capital and profit income given by (25) is that in the former the direct taxation of profits term is absent. This is intuitive: a rise in capital income tax does not act as a rise in profit income tax since the two are separate. The only way in which taxing capital acts as a tax on profit income is through the indirect tax channel, the magnitude of which is now controlled by the exogenous tax rate on profit income, $\bar{\tau}_\pi$. smaller in the

7.1 Quantitative Analysis

In the case with exogenous profit taxes, the sequence of tax rates on profits is an additional parameter in the Ramsey planning problem. Following the calibration in Section 5.2, we set the profit income tax sequence to the sequence of status-quo capital income taxes.[30]

Figure 8*A* illustrates the time path of optimal capital income taxes. We find that in the 2021 reform, the optimal tax rate starts roughly from a level of 7%, increases over time and converges to a level of 9% in the long run. In the 1983 reform, the tax rate is again increasing through time converging to a long-run level of 7%. The first take-away message from the figure is that the optimal capital taxes are positive and significant under both reforms. Second, the optimal capital tax rates are smaller than the ones in the baseline case where capital and profit income are taxed at the same rate, given by Figure 5*A*. This follows from the comparison of the optimal long-run tax formulas (25) and (36) in the previous section: when capital and profit taxes are set separately, there is no direct profit tax revenue benefit coming from raising the capital tax rate.[31]

[30] The profit tax rate decreases from 40% to 30% following the observed capital income tax series in the calibration of Section 5.2. As a sensitivity analysis, we also consider an alternative calibration of the profit tax sequence in which it is set to equal the observed tax rate on distributions (see McGrattan and Prescott 2010), decreasing from a level of 40% to 15% over the period of interest. We find that the optimal capital income taxes are again positive, and somewhat higher than the ones implied by the benchmark calibration. The details of this alternative calibration procedure and the optimal tax results are given in Appendix B.

[31] Looking at Figure 8*A*, one may wonder as to why the optimal capital tax rate is negative in the first few periods following the 1983 reform. A detailed discussion of why it may be optimal to subsidize capital income in the short run in the model with exogenous profit taxes and why this never occurs in the long run is provided in Appendix A.8.

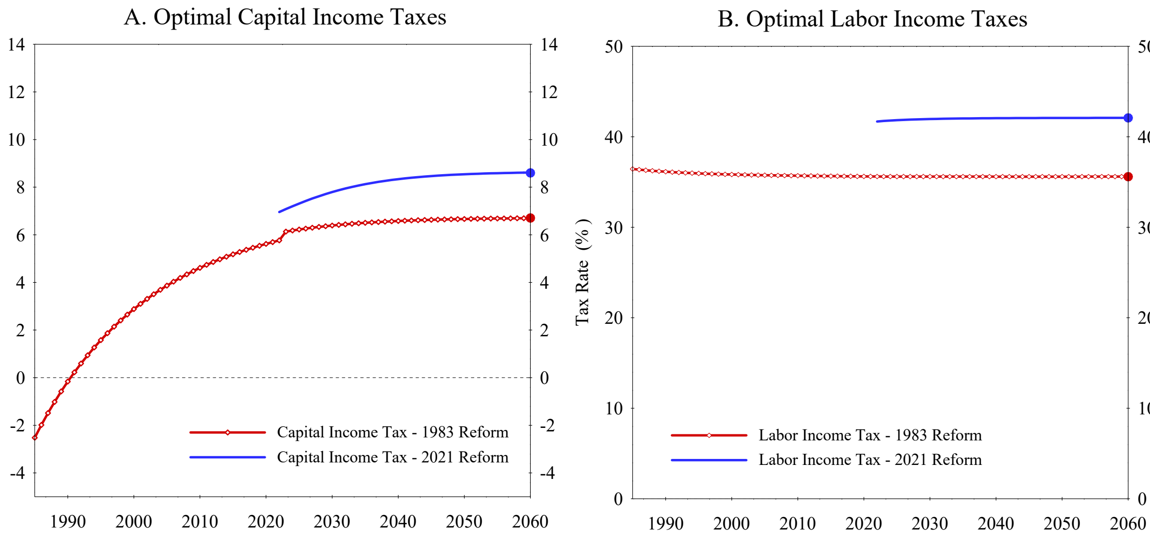

Figure 8: Optimal Tax Rates under Exogenous Profit Taxes

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the case in which government can differentiate between capital and profit taxes and the latter is exogenously fixed.

8 Conclusion

Numerous recent studies have documented that the labor's share in national income has been declining at a considerable rate since the early 1980's. In this paper, we analyze the implications of this decline for tax policy from the perspective of a government that needs to finance spending. We find that the optimal tax implications of the decline in the labor share depend on the mechanism responsible for it. In particular, if the labor share has declined due to a decline in competition or other mechanisms that raise the share of profits in national income, then it should optimally be accompanied with a rise in capital income taxes. If, on the other hand, the labor share has declined because of the rise in automation or other mechanisms that make the production more capital intensive, then it has no bearing on optimal capital income taxation. A quantitative application shows that soaring profit shares since the 1980's can justify significant tax hikes on capital income for the U.S. economy.

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Appendix - For Online Publication

A Proofs

A.1 Proof of Proposition 1

Proof. We first show that equilibrium allocation satisfies (13), (14), and (15). The fact that equilibrium allocation satisfies (13) follows from the fact that equilibrium allocation satisfies (6) and (12) with equality.

To see (14), first notice that if we plug (8) into (6) and use the transversality condition $\lim_{t\to\infty} p_t q_t k_{t+1} = 0$, we achieve

$$\sum_{t=1}^{\infty} p_t c_t = \sum_{t=1}^{\infty} p_t \left(w_t l_t (1-\tau_{l,t}) + \pi_t (1-\tau_{\pi,t}) \right) + p_1 b_1, \tag{38}$$

Normalizing $p_1 = 1$, plugging in (7) and (9) into (38) and, multiplying both sides by $u_{c,1}$, we prove that the allocation satisfies (14).

When we combine the first-order optimality conditions of the consumer, (7) and (8), with the equilibrium rental rate of capital given by (4), we see that in equilibrium:

$$u_{c,t-1} q_{t-1} = \beta u_{c,t} \left[q_t + \left(\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t \right) (1-\tau_{k,t}) \right]. \tag{39}$$

Deriving $1-\tau_{k,t}$ from (39) and recalling $\tau_{\pi,t} = \tau_{k,t}$ proves that equilibrium allocation satisfies (15).

Now, we prove the other direction. Suppose an allocation together with initial policies satisfies (13), (14), and (15). We will show that this allocation, with properly constructed prices and taxes, constitutes a tax-distorted equilibrium. First, use (4) and $\pi_t = \frac{1}{\varepsilon_t} y_t$ to construct factor prices and profit income every period. Normalize $p_1 = 1$ and use (7) to set p_t. Use (39) to construct capital (and profit) income taxes for periods $t \geq 2$ and and (9) to construct labor income taxes for periods $t \geq 1$. Given this constructions, the allocation satisfies consumer and firm optimality conditions. Using the constructed prices and taxes and transversality condition in (14), we obtain that the allocation satisfies consumer budget constraint (6). Combining (6) with the resource constraint (13) gives the government budget constraint (12), which completes the proof. $\square$

A.2 Proof of Proposition 2

Proof. We first provide the complete set of the first-order optimality conditions of (16).

First-order optimality conditions of Ramsey Problem (16). Although the first-order optimality conditions for capital and labor are provided in the main text, we report them here again for completeness. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the full set of first-order conditions are as follows. For $t \geq 2$:

$$(k_t): -\beta^{t-2}\mu^*_{t-1}q_{t-1}+\beta^{t-1}\mu^*_t\left(F^*_{k,t}+(1-\delta)q_t\right) \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial\pi^*_t}{\partial k_t}+\frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t}\pi^*_t\right]=0, \tag{40}$$

$$(l_t): \beta^{t-1}u^*_{l,t}+\lambda^*\beta^{t-1}\left[u^*_{ll,t}l_t+u^*_{l,t}\right] \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial\pi^*_t}{\partial l_t}+\frac{\partial(1-\tau^*_{\pi,t})}{\partial l_t}\pi^*_t\right]+\beta^{t-1}\mu^*_tF^*_{l,t}=0, \tag{41}$$

$$(c_t): \beta^{t-1}u^*_{c,t}+\lambda^*\beta^{t-1}\left[u^*_{cc,t}c^*_t+u^*_{c,t}\right]-\lambda^*\beta^{t-1}u^*_{cc,t}(1-\tau^*_{\pi,t})\pi^*_t \\ -\lambda^*\beta^{t-1}\left[u^*_{c,t}\frac{\partial(1-\tau^*_{\pi,t})}{\partial c_t}\pi^*_t+\beta u^*_{c,t+1}\frac{\partial(1-\tau^*_{\pi,t+1})}{\partial c_t}\pi^*_{t+1}\right]-\beta^{t-1}\mu^*_t=0. \tag{42}$$

The first-order optimality conditions for consumption and labor are different for $t=1$:

$$(c_1): \quad u^*_{c,1}+\lambda^*\left[u^*_{cc,1}c^*_1+u^*_{c,1}-u^*_{cc,1}(1-\tau^*_{\pi,1})\pi^*_1-\beta u^*_{cc,2}\frac{\partial(1-\tau^*_{\pi,2})}{\partial c_1}\pi^*_2-u_{cc,1}A_1\right]-\mu^*_1=0,$$
$$(l_1): \quad u^*_{l,1}+\lambda^*\left[u^*_{ll,1}l^*_1+u^*_{l,1}-u^*_{c,1}(1-\bar{\tau}_{\pi,1})\frac{\partial\pi^*_1}{\partial l_1}\right]+\mu^*_1F^*_{l,1}=0,$$

where $A_1=\bar{r}_1k_1+b_1$ is the real value of initial assets.

At the steady state (40) becomes

$$(k): -\mu^*q+\mu^*\left(F^*_k+(1-\delta)q\right)-\lambda^*u^*_c\left[(1-\tau^*_\pi)\frac{\partial\pi^*}{\partial k}+\frac{\partial(1-\tau^*_\pi)}{\partial k}\pi^*\right]=0. \tag{43}$$

Combining (43) with the steady-state version of (19), and rearranging gives the capital tax formula.

At the steady state (41) and (42) become

$$(l) : u_l^* + \lambda^* \left[u_{ll}^* l + u_l^* \right] - \lambda^* u_c^* \left[(1 - \tau_\pi^*) \frac{\partial \pi^*}{\partial l} + \frac{\partial (1 - \tau_\pi^*)}{\partial l} \pi_t^* \right] + \mu^* F_l^* = 0 \tag{44}$$

and

$$(c) : u_c^* + \lambda^* \left[u_{cc}^* c^* + u_c^* \right] - \mu^* = 0. \tag{45}$$

Combining (44) and (45) with the steady-state version of (20), and rearranging gives the labor tax formula. □

A.3 Implementation with Product Market Policies

The product market policies we consider are of the form: for all t,

$$\hat{\tau}_{s,t} = \frac{1}{\varepsilon_t - 1},$$
$$\hat{T}_t = y_t \left(\frac{1}{\varepsilon_t - 1} - \frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t} \right). \tag{46}$$

In this appendix, we show that under these product market policies (i) the equilibrium rental rates on capital and labor equal their marginal products; (ii) the set of equilibrium allocations that are attainable without product market policies is equivalent to the set of equilibrium allocations that are attainable with them. The latter implies that introducing product market policies do not alter the set of allocations available to the government, and hence, the Ramsey Problem given by (16) still characterizes this set under product market policies.

Before establishing these claims, we first describe how the existence of the product market policies described by (46) affects market equilibrium. An intermediate good producer's problem becomes:

$$\max_{\xi_{i,t}} (1 + \hat{\tau}_{s,t}) \xi_{i,t} y_{i,t} - m_{i,t} y_{i,t} - \hat{T}_t$$

subject to the demand for that intermediate good. The presence of product market policies also alter the government's budget constraint as follows:

$$\sum_{t=1}^{\infty} p_t \left(g_t + \tau_{s,t} y_t\right) + p_1 b_1 = \sum_{t=1}^{\infty} p_t \left(w_t l_t \tau_{l,t} + (r_t - q_t \delta) k_t \tau_{k,t} + \pi_t \tau_{\pi,t} + T_t\right). \tag{47}$$

The definition of equilibrium with product market policies is identical to the definition of market equilibrium given in Section 2 except that the intermediate goods producers' problem and the government budget constraint are modified as above.

The following proposition establishes claims (i) and (ii).

Proposition 8. *Given* (k_1, b_1) *and* $\{g_t\}_{t=1}^{\infty}$*, suppose the allocation* $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$*, together with prices* $\{p_t, r_t, w_t\}_{t=1}^{\infty}$*, profits* $\{\pi_t\}_{t=1}^{\infty}$*, and taxes* $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$ *constitute a tax-distorted market equilibrium without product market policies. Then,* $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ *is also an equilibrium allocation under product market policy given by (46) with appropriately constructed prices and taxes. Moreover, in this equilibrium, the factor prices are given by* $\hat{r}_t = F_{k,t}$, $\hat{w}_t = F_{l,t}$*. Conversely, for any tax-distorted equilibrium allocation under product market policies, we can construct prices and taxes so that this allocation is an equilibrium allocation without product market policies.*

Proof. We show that the allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is consistent with firm optimization, consumer optimization, consumer budget constraint, and government budget constraint in the decentralization with product market policies under appropriately defined prices and taxes. First, define product market policy as in (46). Define taxes as follows. For all $t \geq 1$:

$$1 - \hat{\tau}_{k,t} = \frac{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t}{F_{k,t} - \delta q_t} (1 - \tau_{k,t}) \tag{48}$$

and

$$1 - \hat{\tau}_{l,t} = \left(1 - \frac{1}{\varepsilon_t}\right)(1 - \tau_{l,t}). \tag{49}$$

We do not need to define profit income tax rate since it is equal to the tax rate on capital income.

Next, define prices as follows. For all $t \geq 1$:

$$\hat{p}_t = p_t, \tag{50}$$

$$\hat{r}_t = F_{k,t}, \tag{51}$$

$$\hat{w}_t = F_{l,t}. \tag{52}$$

We begin with the production side of the economy. The final good producer's problem is unchanged, so it still implies the same demand function:

$$y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}. \tag{53}$$

Intermediate goods producers solve:

$$\hat{\pi}_{i,t} = \max_{\xi_{i,t}, y_{i,t}, k_{i,t}, l_{i,t}} (1+\hat{\tau}_{s,t})\xi_{i,t} y_{i,t} - \hat{r}_t k_{i,t} - \hat{w}_t l_{i,t} - \hat{T}_t \tag{54}$$

$$s.t.$$

$$y_{i,t} = F_t(k_{i,t}, l_{i,t}) = \Big(\alpha_{k,t}(A_{k,t}k_{i,t})^{\rho} + \alpha_{l,t}(A_{l,t}l_{i,t})^{\rho}\Big)^{1/\rho}. \tag{55}$$

The intermediate good firm's problem can be solved in two steps. In the first step, for a given marginal cost of producing the good, $m_{i,t}$, the firm chooses price to maximize its profits:

$$\max_{\xi_{i,t}} (1+\hat{\tau}_{s,t})\xi_{i,t} y_{i,t} - m_{i,t} y_{i,t} - \hat{T}_t \quad s.t. \quad (53). \tag{56}$$

The solution to this problem implies a constant markup over marginal cost

$$(1+\hat{\tau}_{s,t})\xi_{i,t} = m_{i,t}\frac{\varepsilon_t}{\varepsilon_t - 1}. \tag{57}$$

In the symmetric equilibrium of the model, all varieties have the same production function and all intermediate goods firms make identical choices of inputs and prices. This implies $y_{i,t} = y_t$ and $\xi_{i,t} = 1$ for all $i \in [0,1]$. We therefore have the optimal marginal cost of producing one more intermediate good equals for all firms $m_{i,t} = M_t = \frac{\varepsilon_t - 1}{\varepsilon_t}(1+\hat{\tau}_{s,t}) = 1$ under the sales subsidy specified in (46).

In the second step, each firm chooses capital and labor to minimize the cost of producing intermediate good. The firms also make same input choices in the symmetric equilibrium, so we have $k_{i,t} = k_t$ and $l_{i,t} = l_t$. Marginal cost of producing one more unit using capital or labor at the optimum gives

$$\frac{\hat{r}_t}{F_{k,t}} = \frac{\hat{w}_t}{F_{l,t}} = m_t = 1, \tag{58}$$

which gives

$$\hat{r}_t = F_{k,t} \tag{59}$$

and

$$\hat{w}_t = F_{l,t}, \tag{60}$$

in line with the constructed factor prices in (51) and (52).

Using (56) and the constructed value of lump-sum tax (46), we can calculate

$$\hat{\pi}_t = (1 + \hat{\tau}_{s,t})\xi_{i,t}y_{i,t} - m_{i,t}y_{i,t} - \hat{T}_t = y_t \frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t}. \tag{61}$$

Now, we turn to the consumer side. We know that the allocation being part of an equilibrium without product market policies implies that for all $t \geq 1$:

$$u_{c,t}q_t = \beta u_{c,t+1} \left[q_{t+1} + \left(\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t+1} - \delta q_{t+1} \right) (1 - \tau_{k,t+1}) \right]. \tag{62}$$

This condition, together with the definition of capital taxes given by (48) imply

$$u_{c,t}q_t = \beta u_{c,t+1} \left[q_{t+1} + \left(F_{k,t+1} - \delta q_{t+1} \right) (1 - \hat{\tau}_{k,t+1}) \right]. \tag{63}$$

Similarly, the allocation being part of an equilibrium without product market policies implies that for all $t \geq 1$:

$$-u_{l,t} = \left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t}(1 - \tau_{l,t})u_{c,t}. \tag{64}$$

This condition, together with the definition of labor taxes given by (49) imply

$$-u_{l,t} = F_{l,t}(1 - \hat{\tau}_{l,t})u_{c,t}. \tag{65}$$

(63) and (65) together imply that the original allocation satisfies consumer's intertemporal and intratemporal optimality condition when he faces newly constructed prices and taxes, (50)-(52) and (48)-(49). Next we show that consumer's budget constraint holds with equality

under the original allocation and newly constructed prices and taxes. Since the original allocation is an equilibrium allocation without product market policies, it satisfies consumer's budget constraint in the no product market policies environment. That is,

$$\begin{aligned}
&\sum_{t=1}^{\infty} p_t \left(c_t + q_t k_{t+1}\right) \\
&= \sum_{t=1}^{\infty} p_t \left(w_t l_t (1-\tau_{l,t}) + [q_t + (r_t - q_t\delta)(1-\tau_{k,t})]k_t + \pi_t(1-\tau_{\pi,t})\right) + p_1 b_1. \qquad (66)
\end{aligned}$$

Using (66), the definitions of intertemporal prices and rental and wage rates (50)-(52), and the definition of taxes (48)-(49), it follows that

$$\begin{aligned}
&\sum_{t=1}^{\infty} \hat{p}_t \left(c_t + q_t k_{t+1}\right) \\
&= \sum_{t=1}^{\infty} \hat{p}_t \left(\hat{w}_t l_t (1-\hat{\tau}_{l,t}) + [q_t + (\hat{r}_t - q_t\delta)(1-\hat{\tau}_{k,t})]k_t + \hat{\pi}_t(1-\hat{\tau}_{\pi,t})\right) + \hat{p}_1 b_1.
\end{aligned}$$

So, consumer budget is satisfied.

Next, we need to show that the government budget constraint is satisfied under newly defined prices and taxes and the original allocation. In the original equilibrium, we have:

$$\sum_{t=1}^{\infty} p_t g_t + p_1 b_1 = \sum_{t=1}^{\infty} p_t \left(w_t l_t \tau_{l,t} + (r_t - q_t\delta)k_t\tau_{k,t} + \pi_t\tau_{\pi,t}\right). \qquad (67)$$

First, notice that, in every period $t \geq 1$, the product market policy brings the government an additional fiscal burden equal to

$$\hat{T}_t - \hat{\tau}_{s,t} y_t = -\frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t} y_t. \qquad (68)$$

Therefore, we need to show that

$$\sum_{t=1}^{\infty} \hat{p}_t \left(g_t + y_t \frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t} \right) + \hat{p}_1 b_1 = \sum_{t=1}^{\infty} p_t \left(\hat{w}_t l_t \hat{\tau}_{l,t} + (\hat{r}_t - q_t\delta)k_t\hat{\tau}_{k,t} + \hat{\pi}_t\hat{\tau}_{\pi,t}\right). \qquad (69)$$

One can show that, by construction of new taxes and prices

$$(\hat{r}_t - q_t\delta)k_t\hat{\tau}_{k,t} - (r_t - q_t\delta)k_t\tau_{k,t} = \frac{1}{\varepsilon_t}F_{k,t}k_t \tag{70}$$

and

$$\hat{w}_t l_t \hat{\tau}_{l,t} - w_t l_t \tau_{l,t} = \frac{1}{\varepsilon_t}F_{l,t}l_t. \tag{71}$$

Furthermore,

$$\hat{\pi}_t\hat{\tau}_{\pi,t} - \pi_t\tau_{\pi,t} = \frac{1}{\varepsilon_t}\left(\frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right)F_{k,t} - \delta q_t} - 1\right)y_t. \tag{72}$$

Plugging (70)-(72) into (67), and using the fact that F is a constant returns to scale production function, we see immediately that (69) holds with equality. Finally, market clearing is implied by the fact that this allocation is an equilibrium allocation without product market policies. We have shown that the original allocation constitutes an equilibrium with proposed product market policies and under the newly constructed prices and taxes. □

A.4 Proof of Proposition 3

Proof. Combining (43) with the steady-state version of (23), and rearranging gives the capital tax formula. Combining (44) and (45) with the steady-state version of (24), and rearranging gives the labor tax formula. □

A.5 Proof of Proposition 4

Proof. First, it follows from the first-order optimality condition of the Ramsey problem with respect to consumption, given by (42), that under the preference structure assumed in this proposition, at a steady state, we have

$$\mu^* = u_c^*(1 + \lambda^*(1 - \sigma)), \tag{73}$$

which implies that at a steady state, we have

$${\chi^*}^{-1} = \frac{\mu^*}{\lambda^* u_c^*} = {\lambda^*}^{-1} + 1 - \sigma. \tag{74}$$

Second, under the Cobb-Douglas production function, one can show that

$$\mathcal{E}^*_{y,1-\tau_k} < \frac{\alpha}{1-\alpha}. \tag{75}$$

Plugging $S_\pi^{-1} = \varepsilon$, and (74) and (75) into the lower bound formula (29) in Corollary 1 proves the proposition. □

A.6 Proof of Proposition 5

Proof. Suppose $S_{\pi,t-1} = S_{\pi,t} = S_{\pi,t+1} = 0$ for some $t \geq 3$. That $S_{\pi,t} = 0$ implies that the first-order condition for capital given by (40) becomes

$$(k_t) : -\mu^*_{t-1} q_{t-1} + \mu^*_t \left(F^*_{k,t} + (1-\delta) q_t\right) = 0. \tag{76}$$

Together with the assumption on preferences, that $S_{\pi,t} = S_{\pi,t+1} = 0$ implies that the first-order optimality condition for consumption for period t given by (42) becomes

$$(c_t) : \beta^{t-1} u^*_{c,t} \left[1 + \lambda^*(1-\sigma)\right] - \mu^*_t = 0. \tag{77}$$

That $S_{\pi,t-1} = S_{\pi,t} = 0$ implies that the first-order optimality condition for consumption for period $t-1$ becomes

$$(c_{t-1}) : \beta^{t-2} u^*_{c,t-1} \left[1 + \lambda^*(1-\sigma)\right] - \mu^*_{t-1} = 0. \tag{78}$$

Combining (76), (77) and (78), we get

$$u^*_{c,t-1} q_{t-1} = \beta u^*_{c,t} \left(F^*_{k,t} + (1-\delta) q_t\right). \tag{79}$$

Plugging (79) into (23) gives $\tau^*_{k,t} = 0$. That $S_{\pi,t} = 0$ implies that the first-order condition for labor given by (41) becomes

$$(l_t) : \beta^{t-1} u^*_{l,t} + \lambda^* \beta^{t-1} \left[u^*_{ll,t} l_t + u^*_{l,t}\right] + \mu^*_t F^*_{l,t} = 0. \tag{80}$$

Plugging (77) and (80) into (24) gives the result. □

A.7 Proof of Proposition 6

Proof. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the first-order conditions of Ramsey problem (33) for $t \geq 2$ are:

$$\begin{aligned}(k_t) : -\beta^{t-2}\mu^*_{t-1}q_{t-1} + \beta^{t-1}\mu^*_t \left(\left(1 - \frac{1}{\varepsilon_t}\right) F^*_{k,t} + (1-\delta)q_t \right) \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial \pi^*_t}{\partial k_t} + \frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t}\pi^*_t\right] = 0,\end{aligned} \tag{81}$$

$$\begin{aligned}(l_t) : \beta^{t-1}u^*_{l,t} + \lambda^*\beta^{t-1}\left[u^*_{ll,t}l_t + u^*_{l,t}\right] \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial \pi^*_t}{\partial l_t} + \frac{\partial(1-\tau^*_{\pi,t})}{\partial l_t}\pi^*_t\right] + \beta^{t-1}\mu^*_t\left(1 - \frac{1}{\varepsilon_t}\right)F^*_{l,t} = 0,\end{aligned} \tag{82}$$

$$\begin{aligned}(c_t) : \beta^{t-1}u^*_{c,t} + \lambda^*\beta^{t-1}\left[u^*_{cc,t}c^*_t + u^*_{c,t}\right] - \lambda^*\beta^{t-1}u^*_{cc,t}(1-\tau^*_{\pi,t})\pi^*_t \\ - \lambda^*\beta^{t-1}\left[u^*_{c,t}\frac{\partial(1-\tau^*_{\pi,t})}{\partial c_t}\pi^*_t + \beta u^*_{c,t+1}\frac{\partial(1-\tau^*_{\pi,t+1})}{\partial c_t}\pi^*_{t+1}\right] - \beta^{t-1}\mu^*_t = 0.\end{aligned} \tag{83}$$

Combining the steady-state versions of (81) and (19), and rearranging gives the capital tax formula. Combining the steady-state versions of (82) and (83), and (20), and rearranging gives the labor tax formula. □

A.8 Optimal Taxation with Exogenous Profit Taxes

Ramsey problem. Given (k_1, b_1), initial capital levy $\tau_{k,1} = \bar{\tau}_{k,1}$, the sequence of profit taxes $\{\bar{\tau}_{\pi,t}\}_{t=1}^{\infty}$, and a sequence of government spending $\{g_t\}_{t=1}^{\infty}$, government chooses allocation (c, k, l) to solve the following problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1}u(c_t, l_t) \qquad \text{s.t.} \tag{84}$$

$$c_t + q_t k_{t+1} \leq F_t(k_t, l_t) + (1-\delta)q_t k_t, \quad \text{for all } t,$$

$$\sum_{t=1}^{\infty} \beta^{t-1}\left(u_{c,t}c_t + u_{l,t}l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1}u_{c,t}(1-\bar{\tau}_{\pi,t})\pi_t + u_{c,1}(\bar{r}_1 k_1 + b_1),$$

where $\pi_t = \frac{1}{\varepsilon_t}F_t(k_t, l_t)$.

Proof of Proposition 7.

Proof. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the first-order conditions of Ramsey problem (84) for $t \geq 2$ are:

$$(k_t) : -\beta^{t-2}\mu^*_{t-1}q_{t-1} + \beta^{t-1}\mu^*_t \left(F^*_{k,t} + (1-\delta)q_t\right) - \lambda^*\beta^{t-1}u^*_{c,t}(1-\bar{\tau}_{\pi,t})\frac{\partial \pi^*_t}{\partial k_t} = 0, \tag{85}$$

$$(l_t) : \beta^{t-1}u^*_{l,t} + \lambda^*\beta^{t-1}\left[u^*_{ll,t}l_t + u^*_{l,t}\right] - \lambda^*\beta^{t-1}u^*_{c,t}(1-\bar{\tau}_{\pi,t})\frac{\partial \pi^*_t}{\partial l_t} + \beta^{t-1}\mu^*_t F^*_{l,t} = 0, \tag{86}$$

$$(c_t) : \beta^{t-1}u^*_{c,t} + \lambda^*\beta^{t-1}\left[u^*_{cc,t}c^*_t + u^*_{c,t}\right] - \lambda^*\beta^{t-1}u^*_{cc,t}(1-\bar{\tau}_{\pi,t})\pi^*_t - \beta^{t-1}\mu^*_t = 0. \tag{87}$$

Combining the steady-state versions of (85) and (19), and rearranging gives the capital tax formula. Combining the steady-state versions of (86) and (87), and (20), and rearranging gives the labor tax formula. □

Discussion on capital subsidies in the short run. Recall that the Ramsey planner wants to minimize the net-present value of after-tax profits that appears on the right-hand-side of the implementability constraint in (84). A subsidy on period t capital income increases savings into period t, which increases consumption, and hence, decreases the equilibrium price of consumption in that period. Since period t profits accrue in period t prices, this decreases the net-present value of period t profits. This introduces a motive to subsidize period t capital income. Similarly, a tax on period t capital income increases consumption in $t-1$, and hence, reduces the value of $t-1$ after-tax profits, which introduces a motive to tax capital. The magnitude of these forces are proportional to the magnitude of after-tax profit income in each period. The subsidy motive dominates the tax motive early on, and we get capital subsidies to be optimal. The capital income tax turns positive after a while because the profit share grows large enough that the aforementioned price effects of taxing capital become too small relative to the indirect profit tax revenue benefit of capital taxation. We do not get negative taxes to be optimal even early on in the baseline case with uniform tax on capital and profit income because in that case the presence of the additional direct profit tax revenue effect of capital income taxation dominates the price effects from the start of the reform. Finally, the price effects do not appear in the steady-state formulas because the aforementioned subsidy and tax motives exactly offset each other in a steady state.

A.9 Lower Bounds for Optimal Capital Taxes

Lower Bound in the Implementation without Product Market Policies.

Proposition 9. *Suppose* $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$*, where* $v', v'' > 0$*, and* $F(k,l) = Ak^{\alpha}l^{1-\alpha}$*. The optimal long-run tax rate on capital income satisfies:*

$$\tau_k^* > \frac{1 - ({\lambda^*}^{-1} + 1 - \sigma)\alpha}{1 + ({\lambda^*}^{-1} + 1 - \sigma)\varepsilon\alpha}. \tag{88}$$

Proof. By taking the right-hand side of (21) into $(1 - \tau_\pi^*)$ paranthesis and using $\tau_\pi^* = \tau_k^*$, we obtain

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{F_k^*}{(1-\frac{1}{\varepsilon})F_k^* - \delta q} S_\pi \left[-\frac{1}{1-\tau_k^*} + \chi^* \left(1 + \frac{1}{\mathcal{E}_{y,1-\tau_k}^*} \right) \right]. \tag{89}$$

Using the fact that $F_k^* > (1 - \frac{1}{\varepsilon})F_k^* - \delta q$ and $S_\pi^{-1} = \varepsilon$, and plugging (74) and (75) into (89) proves the proposition. □

Lower Bound in the Inefficient Economy.

Proposition 10. *Suppose* $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$*, where* $v', v'' > 0$*, and* $F(k,l) = Ak^{\alpha}l^{1-\alpha}$*. The optimal long-run tax rate on capital income satisfies:*

$$\tau_k^* > \frac{1}{1 + ({\lambda^*}^{-1} + 1 - \sigma)(\varepsilon - 1)\alpha}. \tag{90}$$

Proof. By taking the terms inside the bracket on the right-hand side of (34) into $(1 - \tau_\pi^*)$ parenthesis and using $\tau_\pi^* = \tau_k^*$, we obtain the following formula for optimal capital income tax rate

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{F_k^*}{\left(1-\frac{1}{\varepsilon}\right)F_k^* - \delta q} \chi^* S_\pi \left[1 + \frac{1}{\mathcal{E}_{y,1-\tau_k}^*} \right]. \tag{91}$$

It follows from the first-order optimality condition of the Ramsey problem with respect to consumption, given by (83), that under the preference structure assumed in this proposition, at a steady state, we have $\mu^* = u_c^*(1 + \lambda^*(1-\sigma))$, which implies that at a steady state, we have

$${\chi^*}^{-1} = \frac{\mu^*}{\lambda^* u_c^*} = {\lambda^*}^{-1} + 1 - \sigma. \tag{92}$$

Under the Cobb-Douglas production function, one can also show that

$$\mathcal{E}^{*}_{y,1-\tau_k} < \frac{\alpha}{1-\alpha}. \tag{93}$$

Plugging $S_{\pi}^{-1} = \varepsilon$, and (92) and (93) into equation (91) proves the proposition. $\square$

B Alternative Calibration of Exogenous Profit Taxes

Recall that, in the benchmark calibration of Section 5.2, the profit tax rate τ_π decreases from 40% to 30% following the observed capital income tax series. As a sensitivity analysis, we also consider an "alternative calibration" in which the time-series for τ_π is equalized to the observed tax rate on distributions (see McGrattan and Prescott 2010), decreasing from a level of 40% to 15% over the period of interest.

Calibration. Figure B.1. shows the τ_π time-series used in our benchmark and alternative parameterizations. We recalibrate the model under this time series for profit taxes using the same methodology as in Section 5.2. We also report the $APK - \tilde{R}$ implication of the alternative calibration and compare it with the one implied by the benchmark calibration. As Table B.1 shows, we find that the alternative calibration leads to a higher increase in $APK - \tilde{R}$ (6.5%) compared to the one in benchmark calibration (5.4%). In this sense, the alternative calibration provides an $APK - \tilde{R}$ series that is more in line with the data. This is a direct implication of the fact that lower levels of τ_π implies higher levels of $APK - \tilde{R}$ as observed in equation (32). Now we report optimal tax analysis results for both calibrations.

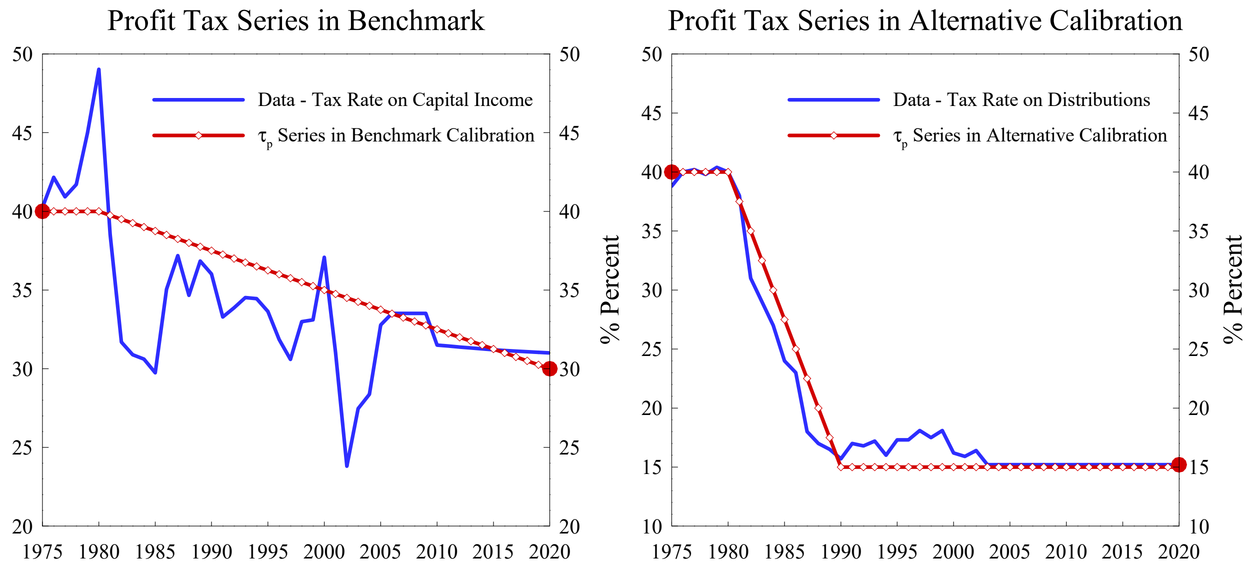

Figure B.1. Profit Tax Series - Benchmark vs. Alternative Calibration

Table B.1. APK-r (Data vs. Simulations)

Benchmark vs. Alternative Calibration

	1982	2020
Data	-0.4%	7.5%
Benchmark	0.2%	5.4%
Alt. Calibration	0.2%	6.5%

Figure B.2. illustrates the time-path of optimal capital income taxes. Under alternative calibration, we find that optimal capital income taxes are again positive and somewhat higher than the ones implied by the benchmark calibration. We also observe a similar pattern for the optimal labor income taxes under the alternative calibration. To sum up, our quantitative findings verify that, under alternative assumptions on the evolution of actual τ_π series, the optimal capital income taxes are still significantly positive in case of exogenous profit taxes.

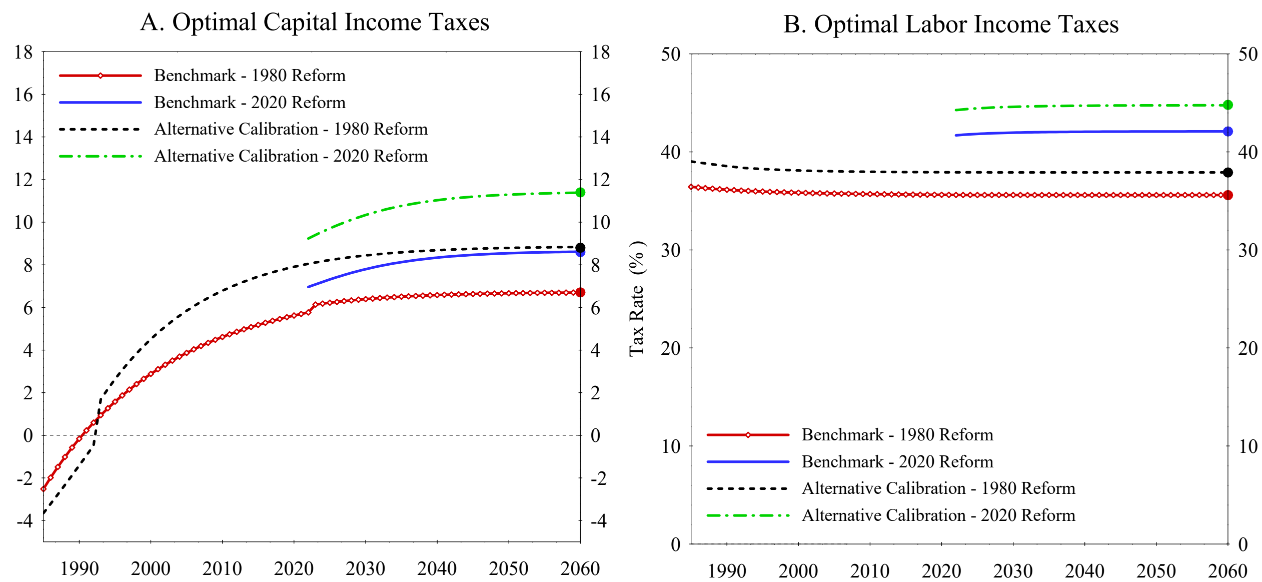

Figure B.2. Optimal Ramsey Taxes : Product Market Interventions - τ_p fixed