CESifo WORKING PAPERS

9115
2021

May 2021

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Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9115*

The Long-Term Effects of Capital Requirements

Abstract

We build a stylized dynamic general equilibrium model with financial frictions to analyze costs and benefits of capital requirements in the short-term and long-term. We show that since increasing capital requirements limits the aggregate loan supply, the equilibrium loan rate spread increases, which raises bank profitability and the market-to-book value of bank capital. Hence, banks build up larger capital buffers which (i) lowers the public losses in case of a systemic crisis and (ii) restores the banking sector's lending capacity after the short-term credit crunch induced by tighter regulation. We confirm our model's dynamic implications in a panel VAR estimation, which suggests that bank lending has even increased in the long-run after the implementation of Basel III capital regulation.

JEL-Codes: E210, E320, F440, G210, G280.

Keywords: bank capital requirements, credit crunch, systemic risk.

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March 08, 2021
We thank Toni Ahnert, Hengjie Ai, Philippe Bacchetta, Bruno Biais, Markus Brunnermeier, Peter DeMarzo, Jean-Paul Décamps, Sebastian Di Tella, Leonardo Gambacorta, John Geanakoplos, Sebastian Gryglewicz, Hans Gersbach, Zhiguo He, Christian Hellwig, Florian Hoffmann, Michael Magill, Semyon Malamud, David Martinez-Miera, Erwan Morellec, Alistair Milne, Henri Pagès, Bruno Parigi, Greg Phelan, Guillaume Plantin, Martine Quinzii, Rafael Repullo, Yuliy Sannikov, Joao Santos, Amit Seru, Hyun Shin, Felipe Varas, Dimitri Vayanos, Stéphane Villeneuve, and seminar participants at the Banque de France, 3rd Benelux Banking Research Day, BIS, CEMFI, CICF 2015 conference, Chicago Booth, DNBCEPR Conference on Bank Equity over the Cycle, the European Winter Financial Summit 2016, EPF Lausanne, ETH Zürich, FIRS 2016, IMF, MFM Winter 2016 Meeting, Princeton University, Stanford University, Toulouse School of Economics, the University of Bonn, the University of Nanterre, Yale University. Sebastian Pfeil thanks the University Research Priority Program FinReg of the University of Zurich for financial support. Jean Charles Rochet thanks the European Research Council (RMAC project) and the Swiss Finance Institute for financial support. An earlier version of the paper has been circulated under the title "Aggregate Bank Capital and Credit Dynamics."

1 Introduction

The 2007–2009 financial crisis has provoked a quite remarkable sequence of events. Initially, large losses have eroded the capital buffers of banking systems around the world. Although many banks were recapitalized by a combination of government interventions,[1] and capital injections from shareholders,[2] a severe credit crunch and reductions in real economic activity (a.k.a. the Great Recession) has followed.[3] Furthermore, regulators have responded by increasing capital requirements. This has sparked an ensuing debate in which capital requirements have been accused of reducing bank lending even further and provoking additional credit crunches.[4] At the center stage of current research and policy debates is the question of the extent to which the costs of higher capital requirements are offset by their financial stability benefits. In its comprehensive review of the literature, the Basel Committee on Banking Supervision (2019) concludes that while there is a consensus that the financial stability marginal benefits of the current level of capital requirements may exceed their marginal costs, there is still a large variation of measures of the cost-benefits trade-offs in current studies which warrants further investigation. Our model provides a framework for the quantitative evaluation of the medium to long-term impact of capital requirements.

We consider a stylized dynamic general equilibrium model of the banking sector, that yields implications very much in line with the above description of events. It also provides further implications on the impact of capital requirements on the evolution of the banking sector and aggregate lending. The motivation for capital requirements in our model is the risk of systemic crises, in which banks are bailed out by a tax-financed deposit insurance agency. We show that, while reducing the public losses in case of a systemic crisis, a tightening of capital requirements may indeed lead to a severe short-run credit crunch, which is in line with empirical evidence. However, our model emphasizes that in the long run, capital requirements have a rather small impact on lending. The reason is the following: Capital requirements reduce the total supply of bank loans, which raises the loan

[1]See e.g. Laeven and Valencia (2013).

[2]See e.g. Black et al. (2016) or Homar and van Wijnbergen (2017).

[3]See e.g. Aiyar et al. (2016); Fraisse et al. (2019); De Jonghe et al. (2020).

[4]While some authors argue that these changes have not gone far enough (Admati et al. (2013); Admati and Hellwig (2014)), others caution that increases in capital requirements penalize bank liquidity creation and impair the economic recovery (DeAngelo and Stulz (2015); Gorton and Winton (2017)).

rate above its competitive level. This makes banks' loan making business more profitable and, hence, increases the value of (equity) capital, with which additional loans can be financed. Banks, thus, accumulate higher capital buffers before making any payouts to shareholders. In addition, since banks' lending business becomes more profitable, they obtain higher earnings, which can in turn be converted into book equity. Taken together, this implies that with higher minimum capital ratios, banks tend to *voluntarily* build up higher capital buffers above and beyond the regulatory minimum thus leaving the banking sector better capitalized on average.[5] This effect helps to restore bank lending after the initial credit crunch, as a better capitalized banking sector is able and willing to provide more loans to the real sector in the medium- to long-term, as the economy moves to the new long-run equilibrium. However, the fact that banks hold higher capital buffers on average further reduces the expected public losses in case of a systemic crisis. Hence, the trade-off between aggregate lending and public losses may be severe in the short-run, but it is much less strenuous in the medium- to long-run.

To analyze the effect of capital requirements on aggregate lending, interest rates and capital buffers, we endogenize the dynamics of bank capital in a stylized general equilibrium model of banking. Banks perform the intermediary role of financing risky loans to the real sector by liquid deposits, which households want to hold for transaction purposes.[6] There are two frictions in the model: (i) households cannot invest directly in the real sector and (ii) banks incur a flotation cost when they issue equity. This implies that banks will retain earnings in order to cover future losses on their loan portfolios and save on refinancing costs.[7] Our model has a unique competitive Markov equilibrium in which individual banks' maximization problems are homogenous in equity and the equilibrium loan rate is a function of aggregate capital only. Furthermore, banks are subject to a simple capital requirement, according to which a minimum fraction of loans has to be financed by equity. When this constraint is binding, the aggregate supply of bank loans

[5]In practice, capital requirements are indeed rarely ever binding. With the exception of De Nicolò et al. (2014), most existing theories, however, are either static (such as Allen et al. (2011)), or assume that capital requirements are always binding (Van den Heuvel (2008)), thereby imposing a mechanical effect of capital requirements on bank lending.

[6]In contrast to existing models such as Brunnermeier and Sannikov (2014), in which financial firms themselves manage productive assets in the economy, we explicitly consider banks' decision to lend to the productive sector.

[7]While this resembles the liquidity management problems in partial equilibrium models such in Bolton et al. (2011) or Décamps et al. (2011), we do not assume any exogenous carry costs, but the costs (and benefits) to retain earnings are determined in general equilibrium.

is equal to a constant fraction of aggregate bank capital. However, also when the capital requirement does not bind, which is the case for sufficiently high levels of bank capital, the volume of loans supplied by the banking sector increases with aggregate bank capital.

A key empirical implication of our model is that the equilibrium loan rate is strictly decreasing in aggregate bank equity. To understand this relation, note that even though banks are perfectly competitive, they are willing to lend to firms only if the spread between the loan rate and their own refinancing costs is strictly positive. The required spread is proportional to the endogenous volatility of aggregate capital and to banks' effective risk-aversion.[8] More precisely, banks' shareholders behave as if they were risk-averse with respect to variation in capital, as the marginal (or market-to-book) value of individual bank capital decreases with *aggregate* bank capital. This is a result of the common shocks to banks' assets: when an individual bank retains profits, its capital increases. However, since the profits of all banks are positively correlated, other banks' capital increases as well. Higher aggregate bank capital implies a higher aggregate supply of bank loans and, thus, a lower market clearing loan rate. As banks use the additional capital to finance loans to the real sector, the marginal value of capital must decrease when banks earn profits.[9] By the same mechanism, losses on the loan portfolio, which reduce banks' capital, are followed by an increase in the marginal value of capital. Hence, as with a concave utility function, the marginal shareholder value attributed to profits is lower than that of equal losses and the loan rate spread contains a risk-premium. However, the risk-premium decreases if the threat of costly equity issuance becomes less acute (see also Décamps et al. (2011)). Hence, the equilibrium spread decreases with aggregate capital and eventually converges to zero when aggregate capital is high enough.

Bank strategies in our model resemble the optimal corporate policies in partial equilibrium models such as Bolton et al. (2011, 2013), Décamps et al. (2011), or De Nicolò et al. (2014), where issuance costs in combination with (exogenous) costs to hold liquidity lead to a decreasing marginal value of cash (i.e. firms become effectively risk averse). However, in our general equilibrium model, there exists a feedback loop between the optimal policies of individual banks and the dynamics of macroeconomic variables, in particular

[8]Note that while banks are risk-neutral in our setting, the existence of refinancing costs renders them effectively risk-averse. See for instance Décamps et al. (2011) for a related result in a partial equilibrium model.

[9]This marks a key difference also between our framework and the existing continuous time models of banking as Hugonnier and Morellec (2017).

the equilibrium interest rate for bank loans and the aggregate bank capital in the economy. That is, banks' issuance and payout policies depend on the incremental shareholder value of investing an additional unit of capital in the loan market, i.e. the marginal value of capital which, in turn, depends on the equilibrium loan rate. More precisely, when aggregate equity is sufficiently high, such that the increased aggregate loan supply has depressed the equilibrium loan rate sufficiently, the marginal loan has a NPV of zero and thus additional earnings should instead be distributed to shareholders. If, on the other hand, aggregate equity is sufficiently low, the aggregate loan supply is restricted by the binding capital requirement. This implies a high equilibrium loan rate, which makes lending to the real sector very profitable at the margin. Despite having to bear flotation costs, it is then optimal for banks to issue equity in order to finance additional loans to the real sector. Hence, bank policies follow a barrier strategy. At the lower, or issuance barrier, the market-to-book ratio reaches its maximum equal to the marginal issuance costs. At the upper, or dividend barrier, the market-to-book ratio reaches its minimum equal to shareholders' marginal utility of consuming the paid out earnings. In between the two barriers, all earnings are retained to build up capital buffers which, in return, are used to cover losses on the asset side. Our theory of bank capital therefore emphasizes its loss-absorbing role instead of its incentive effects.[10]

The crucial question is then, how capital requirements affect individual banks' policies and aggregate lending in equilibrium. Consider, first, a given level of aggregate capital at which the capital requirement is binding. As aggregate credit is completely determined by the binding capital requirement, a higher minimum capital ratio must lead to lower aggregate credit and, thus, a higher loan rate. Now consider a level of aggregate capital at which the capital requirement is slack. In this case, the equilibrium loan rate is determined by banks' binding individual rationality constraint, i.e., the loan rate spread equals the risk-premium required by banks. Tighter capital requirements increase banks' effective risk-aversion for any given level of aggregate capital. This reflects the fact that a tighter capital requirement increases the pressure of having to refinance the bank by costly equity issuance.[11] As a result, tighter capital requirements lead to a higher equilibrium loan rate

[10]We believe that this also captures regulators' motives to impose capital requirements that restrict banks' total equity (i.e. their capital structure). What matters for banks managers' risk-taking incentives, by contrast, is a bank's inside equity including compensation packages.

[11]A similar intuition implies that the effective risk-aversion with respect to liquidity in partial equilibrium models such as Décamps et al. (2011) is most severe when cash holdings are low and the threat of costly

and a lower volume of aggregate credit for any *given* level of aggregate capital.

However, banks also optimally adjust their issuance policies in response to the regulatory change, i.e., they accelerate recapitalization and postpone the distribution of dividends. Intuitively, if a larger fraction of a bank's assets has to be financed by equity, the marginal (or market-to-book) value of capital increases for any given level of aggregate capital. Hence, as capital becomes more valuable, banks become more willing to raise equity and more reluctant to distribute dividends. As a result, the lower boundary at which banks raise new capital, and the upper boundary at which banks distribute dividends, both increase. If the banking system is initially poorly capitalized, the reduction of aggregate credit that is induced by a tightening of capital requirements may therefore be attenuated by immediate recapitalization up to the new issuance boundary. More generally, since both boundaries increase with tighter capital requirements, banks tend to operate on overall higher capital levels. This effect is reinforced by banks' increased profitability due to higher equilibrium loan rates under tighter capital requirements. Hence, tighter capital requirements reduce the loss to the public in case of a systemic banking crisis. Furthermore, while tighter capital requirements cause a severe drop in lending in the short-run, this is not true in the long-run. While tighter capital requirements lead to a lower volume of aggregate credit for any *given* level of aggregate capital, it also makes it more profitable for banks to operate on high levels of capital which, in turn, tends to increase the amount of credit that can be supplied by the banking system.

The interplay between regulatory restrictions, optimal bank policies and the equilibrium spread described above implies three key testable implications of the model: spreads and market-to-book ratios are negatively related to aggregate bank equity, and aggregate lending is positively related to aggregate bank equity. Using two large data international databases for the 1990-2017 period, we find that these implications are broadly consistent with the data. Specifically, we find positive and significant correlations at a cross sectional level between aggregate bank equity and total lending, and a negative and significant correlation between aggregate bank equity, spreads and market-to-book ratios. Furthermore, we assess the consistency of the model's implications for the equilibrium dynamics by estimating a Panel VAR model which includes aggregate bank equity, total bank lending, spreads, and market to book ratios as the endogenous variables in the system. The Panel

refinancing, thus, very acute.

VAR estimation indicates that a *ceteris paribus* increase in aggregate bank equity *predicts* an increase in bank lending and a decline in spreads and market-to-book ratios. Moreover, a standard structural identification of a shock to aggregate bank equity generates a positive response of spreads and market to book ratios and a negative response of lending, with the adjustment of all variables to the long-term equilibrium occurring gradually.

Related Literature First and foremost, our paper is related to the academic literature that views capital requirements as a way to trade off the expected social cost of bank failures (which is not internalized by bankers) and the welfare reduction due to the limitations on banks' deposits and lending activities that capital requirements induce. Admati et al. (2013) argue that the first effect dominates and that capital requirements should be high. On the contrary, DeAngelo and Stulz (2015) consider that the second effect dominates and that high leverage is optimal for banks. They argue in particular that high capital requirements impede banks' provision of liquidity. Van den Heuvel (2008) was the first to develop a dynamic general equilibrium model allowing a quantitative assessment of this trade off. By calibrating his model on US data, he finds that the social cost of capital requirements amounts to a permanent reduction of aggregate consumption of 0.1 to 1 percent. Martinez-Miera and Suarez (2014) also analyse the impact of capital requirements on bankers' risk-taking incentives in a quantitative dynamic general equilibrium.

However in practice, banks typically maintain equity ratios well in excess of regulatory capital requirements,[12] which can only be understood as precautionary buffers against a future need for a costly issuance of new capital. Milne and Whalley (2001) were the first to explore a simple dynamic model with this feature. They show that in the long run, capital requirements have no impact on banks' risk taking. Allen et al. (2011) elaborate on the role of capital buffers by showing how they can allow banks to commit to monitoring loans, which ultimately benefits borrowers.

Following He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014), a new strand of the literature has developed dynamic macroeconomic models with financial frictions and shown that the (endogenous) capitalization of the financial sector was a crucial factor for explaining the performance of the economy. In Kondor and Vayanos (2019) arbitrageurs offer risk management services to hedgers. At equilibrium, the wealth

[12] Fonseca and González (2010) show how these capital buffer vary across countries.

of arbitrageurs is a priced risk factor that influences market risk aversion. In Phelan (2016), banks cannot issue new equity and can only rely on retained earnings to increase their capital buffer. As a result, aggregate outcomes depend on the endogenously determined total equity of the banks. In Bolton et al. (2020), banks cannot perfectly control their deposit flows. Precautionary equity buffers are needed to limit the risk that sudden inflows of deposits may force a bank to issue costly equity. Banks' risk aversion is also endogenously determined. Our model exhibits similar features: banks' risk aversion and credit spreads are endogenous functions of the total capitalization of the banking sector.

Our paper belongs to the recent literature that has developed dynamic general equilibrium models to examine the long term impact of capital requirements. Even if higher capital requirements certainly reduce bank lending in the short run (the famous "credit crunch") they may actually increase lending in the long run. Indeed, our empirical analysis suggests that this is the case. Begenau (2020) analyses the long term impact of capital requirements on bank lending in a quantitative general equilibrium model. She finds that imposing tighter capital requirements tends to reduce banks' demand for deposits, which drives down interest rates paid on deposits and, thus, banks' funding costs. As a result, bank lending can actually increase. We obtain a similar result, except that it operates through a decrease in competition for lending, which increases the price of credit and improves banks' return on assets. This is corroborated by the empirical analysis of Fonseca and González (2010): in countries where banks have more market power, they also have higher capital buffers. Similarly Bridges et al. (2014) use the fact that the Bank of England imposes capital requirements that are time varying and bank specific to assess the impact of increasing capital requirements on lending. They estimate that a 1 percent increase in capital requirement on a typical UK bank leads to a progressive increase in the actual capital ratio of the bank: 0.4 percent after one year, 0.9 percent after three years, and 1 per cent on the long run, implying a complete restoration of capital buffers. Using two large international databases, we provide evidence that strongly suggests a high probability that bank lending has increased in the long-run as a result of the new Basel III regulatory regime. Finally, our paper is also related to the recent literature that analyses the impact of new banking regulations such as liquidity requirements (Hugonnier and Morellec (2017)) or countercyclical capital requirements (Malherbe (2020)).

The remainder of the paper is organized as follows. Section 2 introduces the model and in Section 3 we solve for the competitive equilibrium. In Section 4 we discuss the properties of the equilibrium and show how it is affected by a tightening of the capital requirement. Section 5 presents an empirical analysis of our model's testable implications and Section 6 concludes. All proofs are relegated to the Appendix.

2 Model

Households, Banks, and Firms. We consider a stylized dynamic general equilibrium model that captures in the simplest possible way the role of banks in the economy. We model banks as intermediaries that provide payment services to households and extend loans to the real sector. Time is continuous with infinite horizon and there is a single physical good that can be consumed or invested. The economy is populated by a continuum of households that, for simplicity, are assumed to be risk-neutral. The deposit market is competitive, such that households are paid a deposit rate which equals their discount rate r.[13] Finally, to ensure that deposits perform their role as a medium of payment, they are insured by a deposit insurance agency, which is financed by taxes.

The economy is further populated by a continuum of banks, that are financed by equity and deposits. As is common for instance in the literature on corporate liquidity management, such as Décamps et al. (2011) or Bolton et al. (2011), we assume that the equity market is not perfectly efficient in the sense that when banks issue new equity, they incur a proportional flotation cost of γ. These costs may result from brokerage commissions or underwriting fees. The productive sector consists of a continuum of short-lived entrepreneurs, that demand bank loans for production. Due to technological or informational frictions, households cannot invest directly in the productive sector, but only save in deposits or invest in bank equity.[14] This highlights the intermediary role of banks to transform equity and deposits on the liability side of their balance sheets into loans to the real sector on the asset side. The considered frictions, i.e., the flotation costs γ, and the need for intermediation by banks, provides us with a parsimonious setting

[13] We further assume, as for instance in Brunnermeier and Sannikov (2014), that households can consume positive as well as negative amounts. Negative consumption can be interpreted as providing funds generated by an alternative source of income, often referred to as a "backyard-technology." Thus, at a deposit rate r, the market for deposits always clears.

[14] See e.g. Freixas and Rochet (2008), Chapter 2.

with non-trivial dynamics of bank capital and a role for building up equity buffers, i.e., operating at capital ratios above the regulatory minimum.

Dynamics of Bank Capital. Banks can only invest in one risky asset k_t, which can be interpreted as loans to the real sector. We denote by R_t the expected rate of return on assets net of the risk-free rate, i.e., the loan rate spread. Hence, the instantaneous return on a given bank's assets is given by:

$$(R_t + r)dt - \sigma dZ_t - \phi dN_t. \tag{1}$$

The Brownian increments dZ_t reflect a bank's asset risk due to aggregate (macro) shocks, where σ denotes the bank's exposure to the aggregate shocks.[15] The Poisson increments dN_t capture in a reduced form the (exogenous) risk of a systemic banking crisis.[16] We assume that a systemic crisis arrives with a Poisson intensity ζ and the associated loss $\phi < 1$ destroys a sufficiently large fraction of a bank's assets to wipe out all of its equity.

The bank thus defaults at the point in time at which the banking system is hit by a systemic crisis. Denote this point in time by

$$t_1 := \inf\{t : dN_t > 0\}. \tag{2}$$

If there is a systemic crisis, depositors would incur a loss of the size $e_{t_1} - k_{t_1}\phi < 0$. This loss is covered by a tax-financed deposit insurance agency. Banks are hence left with an equity value and deposit value of zero. As we show in Section 3, it is optimal from shareholders' perspective to inject fresh equity and refinance the bank after the deposit insurance agency has covered the excess losses.

The bank's liabilities consist of equity capital e_t and deposits d_t, so at any point in time, the following accounting identity has to hold

$$k_t = d_t + e_t. \tag{3}$$

The deposit market is competitive, such that the bank has to pay the risk-free rate r on its outstanding deposits. The amount of newly issued equity is given by di_t, while payouts to

[15] The Brownian shocks in (1) reflect changes in total factor productivity (see e.g. Brunnermeier and Sannikov (2014)).

[16] Note that we neglect idiosyncratic shocks to banks' loan portfolios.

the bank's shareholders are denoted by dc_t. The book value of bank capital, thus, evolves according to

$$de_t = re_t dt + k_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + di_t - dc_t. \tag{4}$$

The bank's capital grows at the risk-free rate, r, which equals the rate it has to pay on deposit financing. Capital also grows with retained and converted profits (if the second term in (4) is positive) and newly issued issued shares ($di_t \geq 0$). Capital decreases with payouts to shareholders in the form of dividends or stock repurchases ($dc_t \geq 0$) and with losses (if the second term in (4) is negative). This highlights the role of capital as a loss-absorbing buffer. Since issuing equity is costly, banks will in fact finance loans to the real sector foremost by retained earnings. Such internally generated equity allows banks to perform their role of financing risky assets (loans to firms) by liquid liabilities (deposits) while saving on the costs of issuing equity. Summing up the equity of all banks in the economy, we obtain the aggregate bank equity E_t, which evolves according to

$$dE_t = rE_t dt + K_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + dI_t - dC_t,$$

where K_t, dI_t and dC_t stand, respectively, for the aggregate volumes of lending, equity issuance and payments to shareholders at time t.

Banks are assumed to be run in their shareholders' interest and thus choose payouts dc_t, equity issuance di_t, and loan volume k_t, such as to maximize shareholder value. The latter is given by the present value of total payments to shareholders net of capital injections and flotation cost. Banks' maximization problem is subject to a regulatory capital requirement

$$l_t := \frac{k_t}{e_t} \leq \Lambda, \tag{5}$$

stipulating that a bank's leverage (asset-to-equity ratio) l_t may not exceed Λ. Or, equivalently, at least fraction $1/\Lambda$ of a bank's assets has to be financed by equity. The motivation for these capital requirements is to limit the loss to the public in case of a systemic crisis. As we show below, imposing capital requirements involves a trade-off between lending and public losses in case of systemic crises.[17]

[17] See Miles et al. (2013) for empirical evidence on the costs and benefits of capital requirements.

Demand for Bank Loans. We stipulate that the total volume of bank loans demanded by the real sector is a decreasing function of the loan rate spread $L(R)$, with $L'(R) < 0$. We denote the maximum demand $L(0) = \widehat{L}$, which corresponds to the loan demand in the frictionless benchmark. We show below that in the presence of financial frictions, banks require a lending premium in terms of a strictly positive loan rate spread, which results in an lending gap of size $\widehat{L} - K_t$.

3 Competitive Equilibrium

Equilibrium Conditions. A competitive equilibrium is defined by a map from shock histories $\{Z_s, N_s, s \in [0,t]\}$, to the loan rate spread and bank capital such that, given the spread, individual banks maximize their shareholder values and the loan market clears. Consider first banks' maximization problems, in which each individual bank's dynamic strategy consists of a lending policy, dividend distributions, and equity issuance as a function of the bank's individual capital e, and aggregate capital E_t. We focus on Markov equilibria, in which the equilibrium loan rate spread, R_t, is a deterministic function of the aggregate level of bank capital. Furthermore, we assume that banks' shareholders form rational expectations and, in particular, anticipate that the loan rate spread depends only on aggregate equity. We assume (and later verify) that individual banks' problems are homothetic in the level of their individual book equity e.

Note that while aggregate bank policies K_t, dC_t, and dI_t, are determined as the sums of banks' individual policies k_t, dc_t, and di_t, banks are competitive and take all aggregate variables as given. Furthermore, banks' lending policies have to satisfy the regulatory capital (or leverage) requirement (5). Banks maximize the market value of equity (i.e., the shareholder value), which is given by

$$v(e_t, E_t) = \max_{k_t \in [0, \Lambda e_t],\ dc_t \geq 0,\ di_t \geq 0} \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} \Big(dc_s - (1+\gamma) di_s\Big)\right], \tag{6}$$

where individual bank capital follows

$$de_t = re_t dt + k_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + di_t - dc_t, \tag{7}$$

and the dynamics of aggregate bank capital are given by

$$dE_t = rE_t dt + K_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + dI_t - dC_t. \tag{8}$$

The bank is run until the stochastic default time $\tau := \inf\{t : e_t \leq 0\}$, which denotes the first time when the book value of its equity falls to or below zero. At the equilibrium of our model, banks only default if there is a systemic crisis. Given that the flotation costs γ are not too high (which we assume implicitly), it will be optimal from shareholders' perspective to inject fresh equity before it falls to zero, such that

$$e_t > 0, \tag{9}$$

for all $t < t_1 := \inf\{t : dN_t > 0\}$. Hence, the time of default is given by $\tau = t_1$.

Definition 1. *A competitive Markov equilibrium is described by stochastic processes adapted to the filtered probability space defined by the Brownian motion $\{Z_t, t \geq 0\}$ and the Poisson process $\{N_t, t \geq 0\}$: the loan rate spread $\{R_t\}$, lending policies $\{k_t\}$, recapitalization policies $\{i_t\}$, and dividend distributions $\{c_t\}$; such that*

i) the equilibrium spread $\{R_t\}$ is a deterministic function of aggregate bank equity;

ii) banks maximize their shareholder values (6) subject to capital requirement (5) and taking as given $\{R_t\}$, as well as aggregate bank policies $\{I_t\}$, and $\{C_t\}$;

iii) individual and aggregate bank capital follow (7) and (8) with initial conditions e_0 and E_0, respectively;

iv) the market for bank loans clears: $K_t = L(R_t)$.

From Itô's Lemma, the change of variables formula for jump processes, and the dynamics of capital in (7) and (8), it follows that banks' shareholder values have to satisfy the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rv = \max_{k_t \in [0, \Lambda e],\ dc \geq 0,\ di \geq 0} & \big[re + Rk + di - dc\big] v_e + \big[rE + RK + dI - dC\big] v_E \\ & + [dc - (1+\gamma)di] + \big(k^2 v_{ee} + K^2 v_{EE} + 2kK v_{eE}\big)\frac{\sigma^2}{2} - \zeta v, \end{aligned} \tag{10}$$

where we use sub-indices to denote partial derivatives and omit all function arguments for brevity. The first line on the right hand side of (10) reflects the change in shareholder value induced by a change in the bank's own capital, e, and aggregate capital, E, respectively. The first term of the second line represents payments to the bank's shareholders net of newly raised equity. The second term captures the value impact of the variance of (and the covariance between) individual and aggregate bank capital. The last term in the second line of (10) reflects the loss of all bank capital in case of a systemic crisis.

Now note that individual banks' optimization problems can be greatly simplified by observing that the shareholder value function $v(e, E)$ is homogenous of degree one in individual bank capital e. That is, when we multiply the initial condition e_0 by some factor $n > 0$, it is clearly optimal for banks to follow a strategy that consists of equivalently scaled controls $ni, nc,$ and nk. Since both the feasible set of strategies and the objective function itself are homogenous, the shareholder value in (6) satisfies

$$v(ne, E) = nv(e, E). \tag{11}$$

We define the scale-adjusted version of the bank's policies, for $n = 1/e$, which can be interpreted as the bank's seasoned offerings relative to outstanding equity di/e, its dividend-to-equity ratio dc/e, and its leverage (asset-to-equity ratio) $l = k/e$, which is restricted by the capital requirement (5). Likewise, by using (11), define the bank's scaled shareholder value,

$$u(E) := v(1, E) = \frac{v(e, E)}{e}, \tag{12}$$

which can be interpreted as its market-to-book ratio of equity. A direct implication of the shareholder value being homogenous of degree one in e is that the market-to-book value of equity is the same for all banks, and a deterministic function of aggregate bank equity E. Intuitively, aggregate equity determines the total lending capacity of the banking sector, which shifts the aggregate loan supply and thus the equilibrium loan rate spread.

The observations above allow us to neglect the initial distribution in bank size e_0, and focus on equilibria in which all banks are homothetic. We thus consider only Markovian equilibria in which the loan rate spread is a deterministic function of aggregate equity. Hence we can reduce the dimensionality of banks' problem (6) by considering a "representative bank," that solves a scaled version of the original stochastic control problem with aggregate bank capital E as the single state variable.

Recapitalization and Dividend Policies. The simplification of banks' problem to one with a single state variable allows us to derive a simple characterization of their equity issuance and payout policies. If we take the first order condition for payouts in equation (10) and use the fact that by (12), $v_e(e, E) = u(E)$, it follows immediately that it is optimal to make payments to shareholders ($dc > 0$) if and only if $u(E) \leq 1$. That is, as long as the value of maintaining book equity inside the bank is higher than shareholders' marginal value of receiving a payout, banks retain profits to build up equity buffers. Similarly, from the first order condition for capital issuance in equation (10), it follows that raising new capital ($di > 0$) is optimal if and only if $u(E) \geq 1 + \gamma$. Hence, as long as the value of an additional unit of book equity is smaller than the marginal costs of raising new capital, only the internally generated equity buffer is used to absorb losses on the bank's loans.

These considerations give rise to "barrier-type" payout and recapitalization strategies: Namely, banks absorb losses by issuing new equity if aggregate capital reaches a lower bound $\underline{E}$, which satisfies

$$u\left(\underline{E}\right) = 1 + \gamma. \tag{13}$$

Similarly, banks pay out earnings to shareholders if aggregate equity reaches an upper bound $\overline{E}$, which is characterized by

$$u\left(\overline{E}\right) = 1. \tag{14}$$

Between the two boundaries, banks make no payments to shareholders and do not issue new capital.[18] To pin down the payout boundary, we invoke a standard no-arbitrage condition,

$$v(e - dc, \overline{E} - dC) + dc = v(e, \overline{E}). \tag{15}$$

That is, the ex-dividend equity value plus the dividend payment must equal the cum-dividend equity value. Applying a Taylor expansion to the left-hand side of (15), while

[18]It is important to stress that, in contrast to the partial equilibrium models featuring similar barrier-type recapitalization and dividend policies (see e.g. Bolton et al. (2011), Décamps et al. (2011), Hugonnier and Morellec (2017)), in our framework these policies depend on the aggregate, rather than the individual state.

using the homotheticity property $v(e, E) = eu(E)$, yields:

$$u(\overline{E})dc + eu'(\overline{E})dC = dc.$$

Since $u\left(\overline{E}\right) = 1$ by (14) and book equity e is always strictly positive by (9), it follows that, whenever any payouts are made ($dC > 0$), it must hold that

$$u'(\overline{E}) = 0. \tag{16}$$

A similar no-arbitrage condition has to hold at the recapitalization boundary:

$$v(e + di, \underline{E} + dI) - (1 + \gamma)di = v(e, \underline{E}),$$

which, after applying a Taylor expansion, yields

$$u(\underline{E})di + eu'(\underline{E})dI = (1 + \gamma)di. \tag{17}$$

Since e remains strictly positive by (9) it is immediate that also $\underline{E} > 0$. Furthermore, as $u(\underline{E}) = 1 + \gamma$ by boundary condition (13), this implies that whenever any new equity is issued ($dI > 0$), we must have that

$$u'\left(\underline{E}\right) = 0. \tag{18}$$

Since banks are homothetic, they all follow the same strategy and, such that aggregate payouts ($dC > 0$) cause aggregate capital to be reflected at $\overline{E}$. Likewise, banks' joint issuance strategies prevent aggregate capital from falling below $\underline{E}$. In between the two boundaries, aggregate capital follows

$$dE_t = rE_t dt + K_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big). \tag{19}$$

Intuitively, when the banking system is well capitalized, the aggregate loan supply is high, which makes loan banking relatively unprofitable. Hence, banks pay out profits to shareholders (*payout region*) instead of retaining them inside the bank. By contrast, banks issue new equity when the banking system is poorly capitalized and aggregate loan supply is so low that loan banking is highly profitable (*external financing region*). For intermediate levels of capitalization, banks retain profits and convert them into book equity which is used to finance loans to the real sector (*internal financing region*).

Market-to-Book Value and Equilibrium Loan Rate Spread. The fact that (i) banks follow a barrier strategy and (ii) individual banks' optimization problem is homothetic with respect to individual bank equity (see (12)) allows to significantly simplify equation (10), which determines banks' shareholder values. Since in the *internal financing region* $(\underline{E}, \overline{E})$ banks neither issue equity, nor pay out profits to shareholders, the market-to-book value of bank equity satisfies the following equation:

$$\begin{aligned}\zeta u(E) = &\Big[rE + R(E)L(R(E))\Big]u'(E) + \frac{\sigma^2 L(R(E))^2}{2}u''(E) \\ &+ \max_{l\in[0,\Lambda]} l\Big[R(E)u(E) + \sigma^2 L(R(E))u'(E)\Big].\end{aligned} \tag{20}$$

Since (20) is a second order ODE, it requires two boundary conditions, (13) and (14), to pin down a solution. In addition, we need to determine the two free boundaries, $\underline{E}$ and $\overline{E}$, for which we invoke the no-arbitrage conditions (16) and (18).

Because the value function in (20) is linear in leverage l, we have to consider two cases. First, if the term in square brackets in the second line of (20) is positive, banks' value is maximized at the upper bound of the admissible range $l = \Lambda$, meaning that the capital requirement (5) is binding. Second, if the term in square brackets is equal to zero, banks are indifferent with respect to leverage $l \leq \Lambda$, and the requirement is slack. As we show in the Proof of Proposition 1, the capital requirement will be binding for low levels of aggregate bank equity and it will be slack for high levels of aggregate equity.

If the capital requirement is binding, the term in square brackets can be interpreted as the "shadow costs" associated with the capital requirement. Notably, as the term in square brackets is positive in this case, the constraint *increases* the bank's market-to-book value, which might sound counterintuitive at first. This reflects the fact that a binding capital requirement, which restricts each individual bank's loan supply, puts upwards pressure on the equilibrium loan rate spread. This in turn increases banks' collective profits compared to the perfectly competitive, unregulated market outcome. Due to homotheticity, the constraint binds also on the aggregate level, and the aggregate supply of loans is given by $K = \Lambda E$. For the loan market to clear, this has to equal entrepreneurs' aggregate demand for bank loans given the equilibrium spread, $L(R(E))$. Inverting this market clearing condition yields the equilibrium spread in the case where the capital requirement is binding:

$$R(E) = L^{-1}(\Lambda E). \tag{21}$$

When the capital requirement is slack, there is no longer a "mechanical" relation between aggregate bank capital and the loan rate spread (like condition (21)). In particular, aggregate loan supply has to be consistent with banks' individually rational lending decisions under the prevailing loan rate spread. Recall that when the regulatory capital requirement is slack, the equilibrium interest rate spread is determined by setting the shadow costs attributed to the constraint to zero, which are given by the term in square brackets in the second line of (20):

$$R(E) = -\frac{u'(E)}{u(E)}\sigma^2 L(R(E)). \tag{22}$$

Since the market-to-book ratio of bank equity is decreasing in aggregate equity (as we show in the proof of Proposition 1), condition (22) implies that banks require a positive interest rate spread over the risk-less interest rate r, i.e., $R(E) \geq 0$. This reflects the following mechanism: when an individual bank makes profits, it knows that all other banks make profits. As banks retain these profits, the banking sector as a whole becomes better capitalized and the total lending capacity increases accordingly. This, however, intensifies competition among banks, which lowers the profits that can be made from issuing new loans (partly financed by equity). That is, the (market) value of the bank's original (book) profits decreases, which is reflected by a lower market-to-book ratio as $u(E)$ decreases in aggregate equity. While the equilibrium mechanism tends to reduce the value of banks' profits, it increases the value of its losses: When a bank makes losses, it knows that other banks make losses as well. As the capitalization of the banking system deteriorates, this reduces overall lending capacity and, therefore, softens competition. This in turn is reflected by a higher market-to-book ratio of equity.

Proposition 1. *There exists a unique competitive Markov equilibrium, which is characterized by two functions $u(E)$ and $R(E)$, and two boundaries $\underline{E}$ and $\overline{E}$, where $u(E)$ solves*

$$\begin{aligned} \zeta u(E) = & \Big[rE + R(E)L(R(E))\Big]u'(E) + \frac{\sigma^2 L(R(E))^2}{2}u''(E) \\ & + \max_{l\in[0,\Lambda]} l\Big[R(E)u(E) + \sigma^2 L(R(E))u'(E)\Big], \end{aligned} \tag{23}$$

subject to the four boundary conditions

$$u\left(\underline{E}\right) - 1 = u\left(\overline{E}\right) - (1+\gamma) = u'\left(\underline{E}\right) = u'\left(\overline{E}\right) = 0. \tag{24}$$

Furthermore, there exists a threshold E^* such that for $E \leq E^*$, the regulatory capital requirement (5) binds and the equilibrium loan rate spread is given by (21). For $E_t > E^*$, the regulatory capital requirement (5) is slack and the equilibrium loan rate spread is given implicitly by (22). The threshold E^* is uniquely defined by the property that $R(E)$ is continuous at E^*.

Note that all equilibrium objects are indeed deterministic functions of aggregate bank capital E, the single state variable. Likewise, the market value of a unit of individual book equity (the market-to-book ratio) $u(E)$, is a deterministic function of aggregate equity as well. The market-to-book ratio satisfies the Hamilton-Jacobi-Bellman equation (20) subject to boundary conditions (24), implying that the equity market is arbitrage-free. The equilibrium loan rate is either given by the binding capital requirement (21) or, when the requirement is slack, by banks' individual rationality condition (22). The latter can be substituted in (23) to eliminate the market-to-book ratio $u(E)$ and its derivatives. Therefore, the equilibrium spread satisfies the following first-order differential equation:[19]

$$R'(E) = -\left(\frac{1}{\sigma^2}\right)\frac{2\zeta\sigma^2 + R(E)^2 + 2r\frac{E}{L(R(E))}R(E)}{L(R(E)) - L'(R(E))R(E)}, \tag{25}$$

for $E \in [E^*, \overline{E}]$, subject to the boundary condition:

$$R(\overline{E}) = 0. \tag{26}$$

[19]This expression turns out to be extremely useful for computing the numerical solution, as described in the Proof of Proposition 1.

Hence, at the payout barrier $\overline{E}$, where $u' = 0$ by boundary condition (18), the interest rate spread vanishes. Competition on the loan market has become so intense that loan making becomes unprofitable at the margin and banks thus distribute profits to shareholders instead of retaining them to finance new loans.

4 Equilibrium Analysis

4.1 Banks' Policies and Equity Value

We have shown existence and uniqueness of a competitive Markov equilibrium by establishing a solution to the boundary value problem given by (23) and (24) and the loan rate spread (21) and (22). We now analyze its properties. The equilibrium evolution of aggregate bank capital follows from banks' individual issuance and payout policies, which in turn are individually optimal under the boundary conditions (24). That is, arbitrage-freeness of the equity market implies that individual banks cannot increase their shareholder value by deviating from the barrier strategy to raise new equity ($di > 0$) at $\underline{E}$ and to distribute dividends ($dc > 0$) at $\overline{E}$.

Corollary 1. *In the unique competitive Markov equilibrium, characterized in Proposition 1, the level of aggregate bank capital evolves according to*

$$dE_t = rE_t dt + L\big(R(E_t)\big)\big(R(E_t)dt - \sigma dZ_t - \phi dN_t\big), \tag{27}$$

for $E_t \in \big(\underline{E}, \overline{E}\big)$ *and it is reflected at* $\underline{E}$ *by* $dI_t > 0$ *and at* $\overline{E}$ *by* $-dC_t < 0$;

When aggregate capital is low (i.e., for $E \leq E^*$), the total supply of bank loans is determined by the binding regulatory capital requirement (5). Equilibrium lending in this region is given by $K(E) = \Lambda E$, such that the loan rate spread equals $R(E) = L^{-1}(\Lambda E)$. Hence, when the capital requirement is binding, the spread decreases in aggregate equity. The same holds also in the region where the constraint is slack (i.e., for $E > E^*$), as can be seen from differential equation (25). Since the loan rate spread $R(E)$ is positive and aggregate demand is strictly decreasing in the spread, i.e., $L'(R) < 0$, the denominator of (25) is positive as well. Hence, the right hand side of (25) is strictly negative, implying that the equilibrium spread decreases in aggregate equity also in the region where the constraint is slack. The intuition behind this result is the same as in the region where

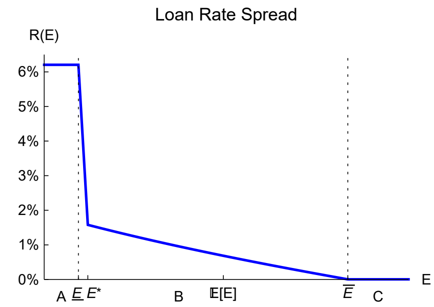

Figure 1: This figure illustrates the loan rate spread for a maximum leverage of $\Lambda = 25$, which corresponds to a minimum regulatory capital ratio of 4%. Interval A corresponds to the *external financing region*, interval B to the *internal financing region*, and interval C to the *payout region*, where the loan rate spread equals zero. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

the constraint is binding: When the banking sector is better capitalized, it can supply a larger volume of loans, which leads to a lower equilibrium spread $R(E)$. As is illustrated in Figure 1, the spread is highest in the external financing region and it falls to zero in the payout region.

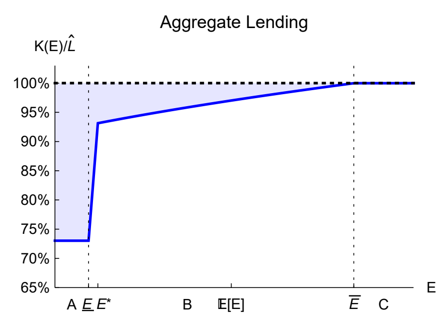

Figure 2: This figure illustrates aggregate lending for a maximum leverage of $\Lambda = 25$, which corresponds to a minimum regulatory capital ratio of 4%. Aggregate lending is expressed relative to the level of aggregate lending in the friction-less benchmark. Hence, the blue shaded area denotes the lending gap. Interval A corresponds to the *external financing region*, where banks are most severely constraint, leading to the largest lending gap. In the *internal financing region* B, the lending gap decreases with aggregate capital and in interval C, the *payout region*, the lending gap disappears. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

Next, the total lending volume of $K(E)$ is strictly increasing in aggregate equity. To see this, recall that total demand for loans is strictly decreasing in the spread, i.e., $L'(R) < 0$, and the spread is strictly decreasing in aggregate equity, $R'(E) < 0$. By the chain rule, it follows that the aggregate loan volume is strictly increasing in aggregate equity: $K'(E) = L'(R(E))R'(E) > 0$. As is illustrated in Figure 2, the lending gap, i.e., the percentage of potential lending $\widehat{L}$, that is lost due to the presence of financial frictions, is largest in the external financing region.

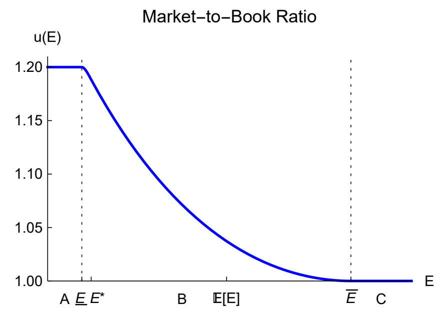

Figure 3: This figure illustrates the market-to-book value for a maximum leverage of $\Lambda = 25$, which corresponds to a minimum regulatory capital ratio of 4%. Interval A corresponds to the *external financing region*, where the market-to-book ratio equals $1+\gamma$, i.e., the total costs to raise one unit of book equity. Interval B denotes the *internal financing region*, and interval C the *payout region*, where the market-to-book ratio equals one, i.e, the marginal value that shareholders attribute to payouts. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

Finally, since the equilibrium loan rate spread is decreasing in aggregate bank equity, loan-making becomes less profitable as the banking sector becomes better capitalized. Since loans are partially financed by equity, the value of an additional unit of individual book equity, i.e., the market-to-book ratio of of equity, $u(E)$, is decreasing in aggregate equity as well. As illustrated in Figure 3, it obtains its maximum in the external financing region, where book equity is so valuable that banks raise new capital. The market-to-book ratio is strictly decreasing in the internal financing region and obtains its minimum in the payout region. Here, the value of book equity is so low, that banks distribute earnings to shareholders instead of retaining them.

These testable implications are summarized in the following Corollary:

Corollary 2. *At the competitive Markov equilibrium characterized in Proposition 1, it holds that*

i) aggregate loan volume, $K(E)$, is strictly increasing in aggregate equity;

ii) the loan rate spread, $R(E)$, is strictly decreasing in aggregate equity;

iii) the market-to-book ratio of equity, $u(E)$, is strictly decreasing in aggregate equity.

As a final remark, note that the equilibrium spread in (22) bears some similarity with a standard risk-premium. However, it is rather a hedging term, as it contains the *cross-derivative* of the bank's shareholder value with respect to individual and aggregate equity, i.e., $v_{eE}(e, E)/v_e(e, E)$, and not its second derivative, as in a measure for risk aversion.[20] The demand for hedging in the form of building up book equity buffers (beyond the required minimum level), is thus determined by the market-to-book ratio, which reflects the profitability of loan making, i.e., the equilibrium loan rate spread. Hence, a capital requirement in our model induces banks to hold larger capital buffers even when it is not binding. Through its effect on the equilibrium loan rate spread, it makes it more profitable for banks to retain earnings and build up capital buffers when the constraint is slack.

4.2 The Impact of Tightening Capital Requirements

Having established the competitive equilibrium and its properties, we now ask how it is affected by capital requirements. We first show how capital requirements affect banks' issuance and payout policies. Next, we analyze the impact of tightening capital requirements on equilibrium lending in the short-run as well as in the long-run. For the following analysis we consider an unexpected, substantial increase in the maximum leverage from $\Lambda_{old} = 25$ ("old regulatory regime") to $\Lambda_{new} = 12.5$ ("new regulatory regime"). This corresponds to an increase in the minimum capital ratio (k/e) from 4% to 8%.

Banks' Issuance and Payout Policies. As we have shown in Section 3, banks follow a barrier strategy by making payouts to shareholders when aggregate capital is sufficiently high and issuing new equity when aggregate bank capital is sufficiently low. How does a tightening of the capital requirement affect banks' payout and issuance policies? For the issuance boundary $\underline{E}$, the mechanism is straightforward: Since the capital requirement binds at $\underline{E}_{old}$, a reduction of the maximum leverage reduces the total supply of loans and, thus, increases the equilibrium loan rate spread. A higher spread makes lending more profitable, which would, all else equal, increase the market-to-book ratio to a value greater than $1 + \gamma$. By continuity, $\underline{E}_{new}$ thus has to be higher than $\underline{E}_{old}$, which is illustrated in Figure 4. Intuitively, when regulatory capital requirements are tightened, banks should

[20] A similar mechanism drives the intertemporal hedging demand in the dynamic model of liquidity provision by Kondor and Vayanos (2019).

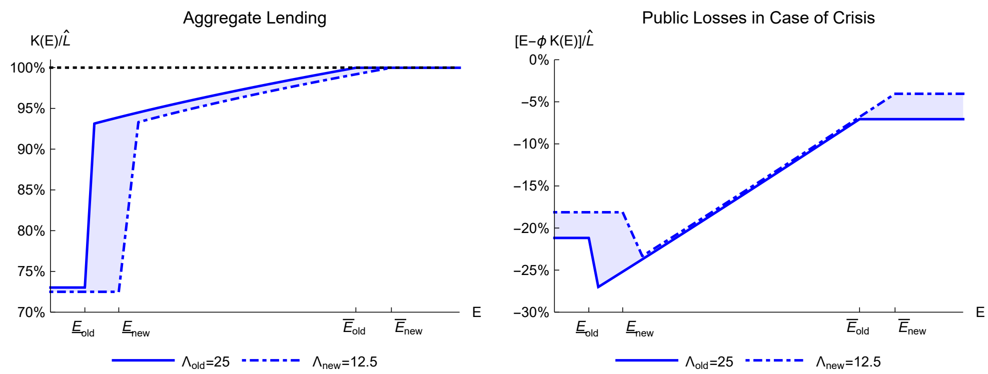

Figure 4: This figure illustrates aggregate lending and public losses as a function of aggregate capital for two regulatory regimes $\Lambda_{old} = 25$ and $\Lambda_{new} = 12.5$, which corresponds to an increase in the minimum regulatory capital ratio from 4% to 8%. Both, aggregate lending and the public loss in case of a systemic crisis are expressed relative to the level of aggregate lending in the friction-less benchmark. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$, $\phi = 1/3$.

have been already issuing equity at $\underline{E}_{old}$. The same reasoning applies to the critical level of aggregate capital E^*, where the minimum capital requirement becomes slack. Note that under the initial capital requirement, the constraint was just marginally binding in E^*_{old}. However, all else equal, a tighter capital requirement turns the constraint into a strictly binding one at the initial value E^*_{old}. Again, by continuity, it has to hold that $E^*_{new} > E^*_{old}$. Finally, also the payout boundary $\overline{E}$, increases when capital requirements are tightened. To see this, note that tighter capital requirements soften competition, which makes loan banking more profitable, even at levels of aggregate capital where they are slack. That is, even though the constraint has no direct effect on loan supply when it is slack, it drives up the market-to-book ratio of bank equity $u(E)$ at any given E, reflecting the fact that the constraint will bind eventually and then banks will earn a higher return on each unit of book capital that they can use to finance loans. This implies that, at the initial payout boundary $\overline{E}_{old}$, loan making is still profitable, so that banks optimally retain earnings, instead of making payouts to shareholders. Hence, as illustrated in Figure 4, also the payout boundary has to increase: $\overline{E}_{new} > \overline{E}_{old}$.

Short-Term Impact on Lending and Public Losses. Having understood the impact of capital requirements on banks' issuance and payout policies, we now ask how they affect bank lending and public losses in case of a systemic crisis in the short run. Figure 4 shows the costs and benefits of tightening capital requirements as a function of the level of aggregate capital that prevails at the time at which the tightening occurs. The costs of sacrificing lending (left panel) and the benefits of reducing public losses in case of a crisis (right panel) are tightly linked because the public losses depend on both the equity buffer as well as the banking sector's exposure to systemic risk implied by the equilibrium volume of lending. Hence, the trade-off between the two can look quite differently depending on how well the banking sector is capitalized at the time when the regulatory change becomes effective.

Consider first the case that aggregate capital is in the external financing region under the old regulatory regime, $E < \underline{E}_{old}$. As illustrated in the left panel of Figure 4, lending is then barely affected by the regulatory change as (i) the lending gap was already large under the old regulatory regime and (ii) the regulatory change triggers a sizable recapitalization to $\underline{E}_{new}$.[21] As the immediate recapitalization tops up banks' loss absorbing capital, the benefits of tighter capital requirements in terms of reducing the public losses in a systemic crisis, are sizable in this region (see the right panel of Figure 4).

Next, assume that, at the time when the regulatory change becomes effective, capital is in the internal financing region under the old regulatory regime, but in the external financing under the new one, i.e., $\underline{E}_{old} < E \leq \underline{E}_{new}$. Since for $E > \underline{E}_{old}$, the lending gap under the old regulatory regime is smaller than in the case considered above, the drop in lending due to the tighter capital requirements is even larger now. As illustrated in the left panel of Figure 4, this may lead to a spectacular credit crunch with a quintupling of the lending gap. What does this imply for the public losses in case of a systemic crisis? As shown by the right panel of figure 4, tighter capital requirements reduce the public losses more significantly than in the first case, even though the immediate recapitalization $\underline{E}_{new} - E$, is now smaller. However, the sizable reduction in lending leaves the banking sector less exposed to systemic risk, which overcompensates the first effect. Hence, tightening capital requirements induces the highest costs in terms of reduced lending, but also brings about the most significant reduction of public losses in case of a systemic crisis.

When the level of capital lies in the internal financing region under the old and the

[21] Yet, lending still decreases slightly, since when the capital requirement is tightened, the increase in $\underline{E}$ is not sufficient to compensate the reduction in Λ. Hence, the total loan volume at the dividend boundary decreases, i.e., $\Lambda_{old}\underline{E}_{old} > \Lambda_{new}\underline{E}_{new}$. As a result, the spread at the respective issuance boundaries increases in response the tightened capital requirements: $L^{-1}(\Lambda_{old}\underline{E}_{old}) < L^{-1}(\Lambda_{new}\underline{E}_{new})$. Intuitively, the higher loan rate spread increases banks' profitability and ensures that the market-to-book value still satisfies $u(\underline{E}_{new}) = 1 + \gamma$, despite banks' policies being more severely restricted under the new regulatory regime.

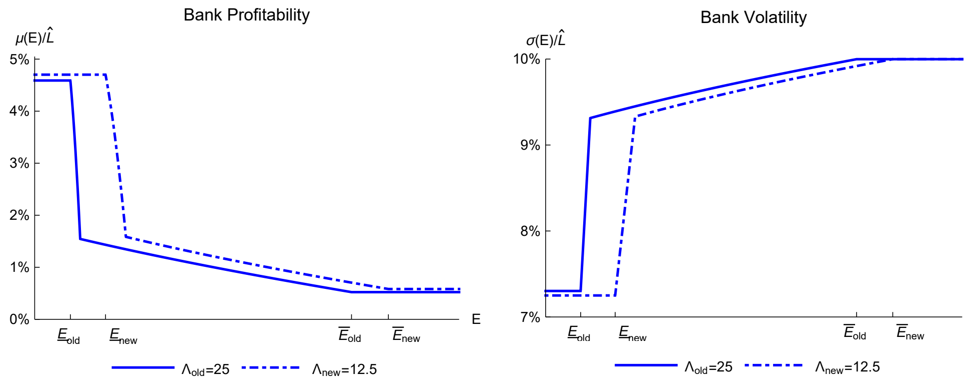

Figure 5: This figure illustrates bank profitability and bank volatility as a function of aggregate capital for two regulatory regimes $\Lambda_{old} = 25$ and $\Lambda_{new} = 12.5$, which corresponds to an increase in the minimum regulatory capital ratio from 4% to 8%. Both, profitability and volatility are expressed relative to the level of aggregate lending in the friction-less benchmark. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

new regime, i.e., $\underline{E}_{new} < E < \overline{E}_{old}$, the regulatory change no longer prompts an immediate recapitalization. However, as illustrated in the right panel of Figure 4, tightening the capital requirements still significantly reduces the public losses if it is introduced at a level of capital for which the new constraint is binding. (i.e., $E < E^*_{new}$). The reason is that in this case, the volume of aggregate lending and thus banks' exposure to systemic risk is significantly reduced by the tighter regulation (see the left panel of Figure 4). This is no longer the case when the new regulation is imposed at a level of capital at which neither the old nor the new capital requirement binds (i.e., $E \geq E^*_{new}$). In this case, the reduction in lending is rather mild (see the left panel of Figure 4), such that the reduction in public losses is also very small (see the right panel of Figure 4).

Finally, consider the case where capital regulation changes when capital is in the payout region under the old regime, $E > \overline{E}_{old}$. In this region, the impact on lending is even smaller (see left panel of Figure 4). However, as tighter regulation makes bank equity more valuable, it still reduces the public losses in case of a systemic crisis because banks postpone planned payouts to their shareholders and instead build up larger capital buffers (see right panel of Figure 4).

Long-Term Impact on Lending and Public Losses. We have seen in the last paragraph that the short-term impact of a change in regulation on lending and public losses varies to a large extent with the banking sector's capitalization at the time it is introduced. We now turn to the long-term impact of tighter capital regulation, which depends

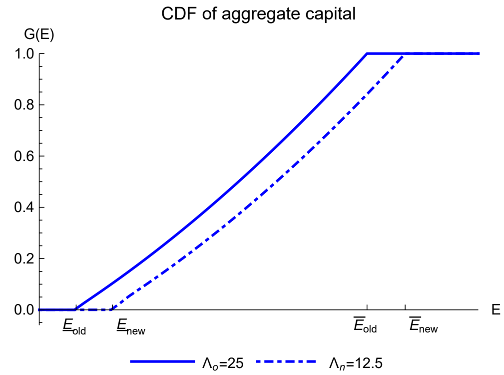

Figure 6: This figure illustrates the cumulative distribution function of aggregate capital for two regulatory regimes $\Lambda_{old} = 25$ and $\Lambda_{new} = 12.5$, which corresponds to an increase in the minimum regulatory capital ratio from 4% to 8%. Aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$ and parameter values are: $r = 0.02$, $\zeta = 0.03$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

on the *expected* changes in lending and capital buffers. Because banks tend to hold more capital when capital requirements are stricter, the loss absorbing capital buffer increases accordingly, which reduces reduces expected public losses. On the other hand, since aggregate lending is increasing in aggregate capital, also banks' exposure to systemic risk increases with the higher expected capitalization. Hence, to grasp the total impact of capital requirements on expected lending and public losses, we need to understand how the distribution of aggregate bank capital changes in response to tighter capital requirements.

For $t < t_1$ and a given level of capital requirements, the evolution of aggregate bank equity in the internal financing region $(\underline{E}, \overline{E})$, is given by

$$dE_t = \mu(E_t)dt - \sigma(E_t)dZ_t, \tag{28}$$

where the drift rate $\mu(E) := rE + K(E)R(E)$ reflects the profitability of banks and banks' volatility is given by $\sigma(E) := \sigma K(E)$. The profitability of banks is illustrated in the left panel of Figure 5. At any given level of bank capital, the profitability increases in response to tighter capital requirements since the total revenue from lending $K(E)R(E)$ increases. Hence, banks not only issue equity earlier, but the higher profitability of their loan making business also facilitates the accumulation of capital buffers. The right panel of Figure 5 illustrates the volatility of aggregate bank capital $\sigma(E)$, which reflects the banking sector's exposure to macro shocks under the equilibrium volume of aggregate lending $K(E)$. It is immediate that tighter capital requirements make aggregate bank capital less volatile, in particular at low levels of aggregate capital. This last effect, however, can render

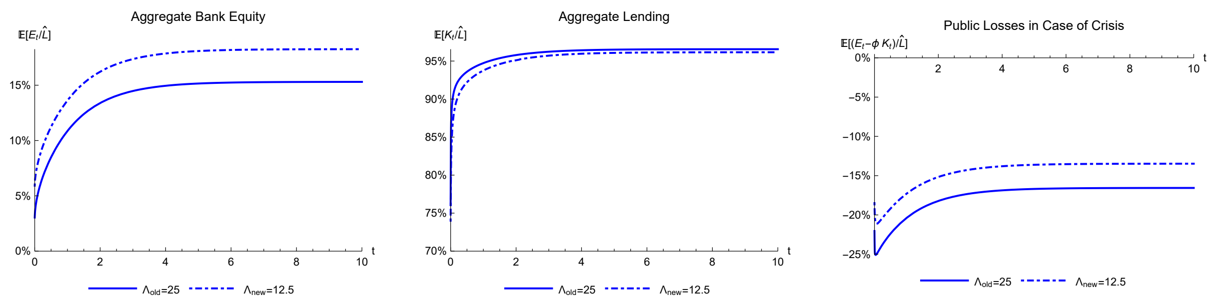

Figure 7: This figure illustrates the average bank capitalization, the lending gap, and the public losses in case of a systemic crisis as a function of the time that has passed after recapitalizing the banking sector ($E_0 = \underline{E}$), for two regulatory regimes $\Lambda_{old} = 25$ and $\Lambda_{new} = 12.5$, which corresponds to an increase in the minimum regulatory capital ratio from 4% to 8%. The respective averages of equity, aggregate lending and the public loss in case of a systemic crisis are computed according to (B.4) and expressed relative to the level of aggregate lending in the friction-less benchmark. The aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$. Parameter values are: $r = 0.05$, $\zeta = 0.03$, $\alpha = 0.25$, $\beta = 1$, $\sigma = 0.1$, $\gamma = 0.2$, $\phi = 1/3$.

low capitalization states "adhesive" and, thus, cause the banking sector to end up poorly capitalized for an extended period of time.[22] Still, as is illustrated in Figure 6, tighter capital requirements shift the equilibrium distribution of aggregate capital in a first-order stochastic dominance sense. States of the world in which the banking system is highly capitalized will thus become more likely as capital requirements are tightened.

This effect on banks' average capitalization can also be seen in the left panel of Figure 7. It shows for the two regulatory regimes how average bank capital evolves after the banking sector has been recapitalized. The difference in average capital at the time of recapitalization, $t = 0$, reflects the difference in issuance boundaries (i.e. $\underline{E}_{new} > \underline{E}_{old}$). As time goes by and the average capitalizations converge to their respective long-term values, the difference becomes even more pronounced. This reflects the first order stochastic dominance shift illustrated in Figure 6. The middle panel in Figure 7 confirms our intuition that the quite severe immediate drop in lending is attenuated by the fact that banks will optimally accumulate higher capital buffers, which in turn increases the overall loan supply. Finally, the right panel of Figure 7 shows that the initial reduction in public losses due to lower exposure from lending is amplified in the long-run by an increase in the banking system's average capital buffers. Hence, in the long-run, tightening capital regulation leads to a significant reduction in public losses, while the costs in terms of lost lending are minimal.

[22]See e.g. Brunnermeier and Sannikov (2014) or Klimenko et al. (2017).

5 From the Model to the Data

In this section we empirically assess the implications of our model for the dynamics of bank lending and bank capitalization. A novel feature of our analysis is the use of panel VAR (pVAR) models estimated with data from two large international panel datasets at an annual frequency covering publicly quoted banks (Dataset 1) and aggregate country data (Dataset 2).

5.1 Data and descriptive statistics

Dataset 1 is taken from the Worldscope database, retrieved from Datastream, which contains consolidated accounts and market data for a large number of publicly quoted banks worldwide. Dataset 1 covers data for 1,316 banks in 39 countries during the period 1990-2017, including 629 U.S. Bank Holding Companies (BHCs), 304 European banks, 192 Asia (developed) banks, and 191 banks operating in countries classified as emerging. This panel dataset is unbalanced due to mergers and acquisitions, but all banks active in each period are included in the sample to avoid survivorship bias.

The variables in Dataset 1 include the log of bank (common) equity ($lequity$), the log of *aggregate* bank equity (lE) by country, the log of bank loans ($lloans$), the interest spread on bank loans ($spread$), the bank market-to-book ratio (mtb), and the bank (common) equity-to-asset ratio (ea). The variables in levels are all expressed in US\$. The interest rate spread on bank loans is computed as the difference between the loan rate and the cost of funding, where the cost of funding is the weighted average of the cost of deposits and market sources of funding. We use the bank (common) equity-to-asset ratio rather than a regulatory capital ratio, as bank coverage of the latter is very limited in this database.[23]

Dataset 2 is taken from the World Bank Financial Structure Database, which assembles financial and bank data from a wide array of international databases. Dataset 2 covers 120 countries during the period 1998-2017, including 47 high income countries and 73 middle-to-low income countries, as per the income classification of the World Bank.

The variables in Dataset 2 include country aggregates of the log of bank regulatory

[23]The country aggregate of the log of bank loans is denoted by lL. The country averages of bank spreads and market-to-book ratios are denoted by $SPREAD$ and MTB respectively. The data points in the database used to construct our variables are: total assets (WC02999), total loans (WC02771), total liabilities (WC03999-WC03501), common equity (WC03501), total deposits (WC03019), loan rate (WC01007/WC02271), total interest expenses (WC01075), and market capitalization (WC08001).

capital (RC), the log of bank loans (L), the spread between lending and deposit rates ($SPREAD$), and the bank regulatory capital ratio (RCR), measured by the ratio of regulatory capital to risk weighted assets. As in Dataset 1, the variables in levels are all expressed in US$.[24]

We use these two large datasets to maximize the robustness of the empirical assessment of our model. Dataset 1 allows us to explore the implications of our model for market valuation. However, the banks included in this dataset do not represent the entire banking system in a country, although they capture a significant proportion of total assets of each country's banking system. Dataset 2 complements Dataset 1 by including data for entire banking systems, with a country coverage significantly larger than that of Dataset 1. Importantly, Dataset 2 includes medium-to-low income countries where banks are the predominant vehicles in the provision of credit, as in our model.

A preliminary assessment of the implications of our model is provided by looking at simple correlations between its key variables shown in Figure 8.

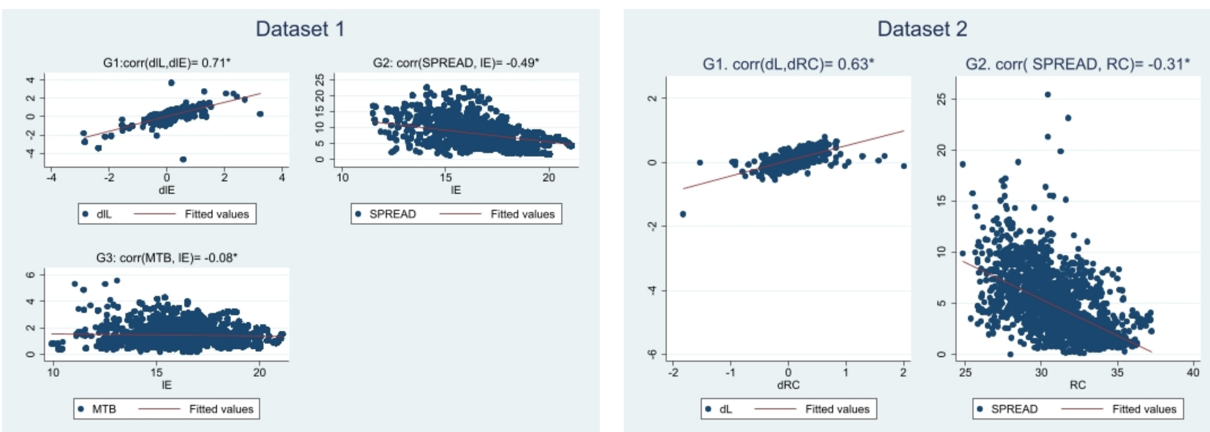

Figure 8: Correlations of the Δ(log) of aggregate bank (common) equity with Δ(log) of total bank loans (Dataset 1, Graph G1, and Dataset 2, Graph G1), and correlation of the (log) of aggregate bank (common) equity with the interest spread on bank loans (Dataset 1, Graph G2, and Dataset 2, Graph G2), and with the market-to-book ratio (Dataset 1, Graph G3). The * denotes significance at a 5% level.

Our model predicts that aggregate bank equity is positively correlated with aggregate bank lending, and negatively correlated with loan spreads and market-to-book ratios (Corollary 2). Figure 8 illustrates scatter plots and correlations of the time series of the

[24]The data points in the database used to construct our variables are: Bank regulatory capital to risk-weighted assets (GFDD.SI.05), the bank lending-deposit spread (GFDD.EI.02), Bank regulatory capital to total assets (GFDD.SI.03), Deposit money banks' assets to GDP (GFDD.DI.02), and GDP in current US$ (NY.GDP.MKTP.CD)

(log) difference of aggregate bank (common) equity with the (log) difference of total bank loans (as both series trend upward), and the correlations of aggregate bank equity with interest spreads and market-to-book ratios. The signs of these correlations, which are all statistically significant, match the predictions of our model.

5.2 The pVAR

We capture the dynamics of our model by working with a k-variate pVAR(1) model of the following form:

$$Y_{it} = AY_{it-1} + \alpha_i + \eta_{it}; \;\; i \in \{1, 2, .., N\}; \;\; t \in \{1, 2, .., T_i\}; \tag{29}$$

where Y_{it} is a $(k \times 1)$ vector of endogenous variables and A a $(k \times \; k)$ matrix of coefficients. Cross sectional heterogeneity is captured by the $k \times 1$ fixed effects vector α_i. The $k \times 1$ innovation vector satisfies $E(\eta_{it}) = 0$, $E(\eta'_{it}\eta_{it}) = \Sigma$, and $E(\eta'_{it}\eta_{is}) = 0$ for all $t > s$. Following Holtz-Eakin et al. (1988), estimation of all pVAR(1) models described below is carried out by standard GMM methods, and they all fulfill the standard stability conditions.

The vector Y_{it} includes the key variables of our model. We consider two pVAR models as applied to our two datasets, denoted by p1VAR and p2VAR respectively. The vector Y_{it} in p1VAR includes the log of aggregate bank equity (by country), the bank equity-to-asset ratio. bank total loans, bank spreads, and bank market to-book ratios. i.e. $Y_{it} = (lE_{ct}, ea_{it}, lloans_{it}, spread_{it}, mtb_{it})$, where subscript c denotes a country, and subscript i denotes a bank. The vector Y_{it} in p2VAR includes the log of aggregate regulatory capital, the aggregate regulatory capital ratio, country bank loans, and country averages of bank spreads. i.e. $Y_{it} = (RC_{it}, RCR_{it}, L_{it}, SPREAD_{it})$, where subscript i denotes a country.[25]

[25]Recall that in our model we impose a regulatory restriction on capital by an upper limit on the leverage ratio, defined as the ratio of loans over equity. The inverse of this ratio is an equity-to-asset ratio where total assets are equated to total loans. The pVARs include equity-to-asset and regulatory capital ratios as bank capital and buffers are determined relative to either total assets, which include investments in securities, or risk weighted assets. Interestingly, all results reported in the sequel are qualitatively identical when we use the empirical counterpart of the model definition of the leverage ratio in the pVARs.

5.3 Dynamics

As detailed in the previous section, the dynamic interactions of our model are driven by banks' optimal equity issuance and payout policies and credit market equilibrium. A better capitalized banking sector can support higher lending associated with lower bank spread, and the relevant decline in profitability results in lower market-to-book ratios.

We capture these dynamic interactions with the impulse response functions (IRFs) of total loans, bank spreads and market-to-book ratios to a negative (unexpected) change in aggregate bank equity, which captures the response of the system to a shock possibly leading to a banking crisis. The IRFs of the pVARs are generated by a Cholesky decomposition where the "structural" shock to aggregate bank equity is not affected by other shocks at impact, and it is ordered first in the system.

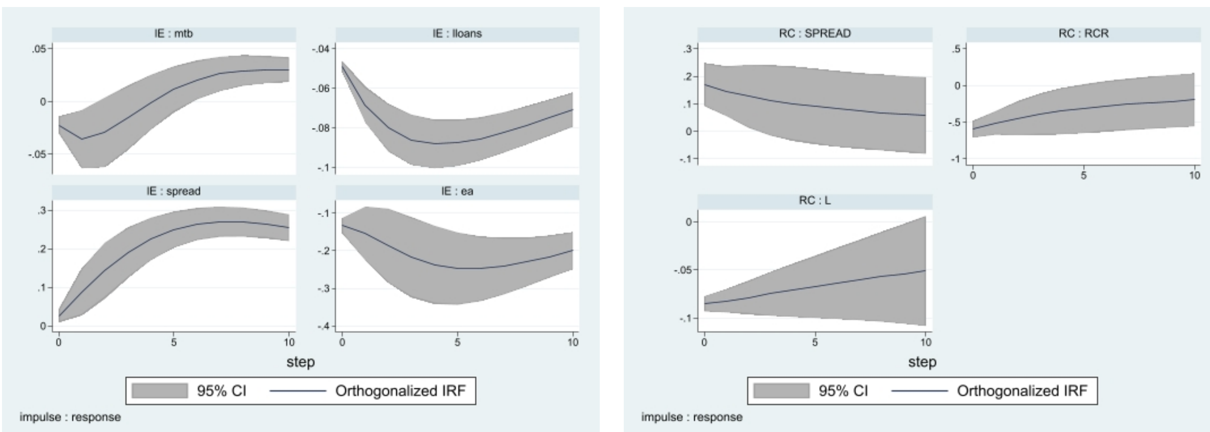

Figure 9: Impulse Response Functions (IRFs) of bank loans, spreads, and market-to book ratios in p1VAR (left panel), and of banking system loans, aggregate regulatory capital ratio and average spreads in p2VAR (right panel) to a negative shock to aggregate bank equity.

Figure 9 reports the Impulse Response Functions (IRFs) resulting from a *negative* shock to aggregate bank equity obtained with both the p1VAR and p2VAR estimates, where responses are tracked for a 10 periods (years) horizon. Figure 10 illustrates the dynamics implied by our theory model. I.e. we consider the respective Impulse Response Functions under the equilibrium characterized in Section 4, when we impose a similar *negative* shock to aggregate equity as in Figure 9. Comparing Figures 9 and 10, we obtain four results. First, the sign of the responses to a negative shock to aggregate bank equity are broadly consistent with the implications of our model. Namely, lending declines and spreads increase using both datasets. The market-to-book ratio appears to decline slightly

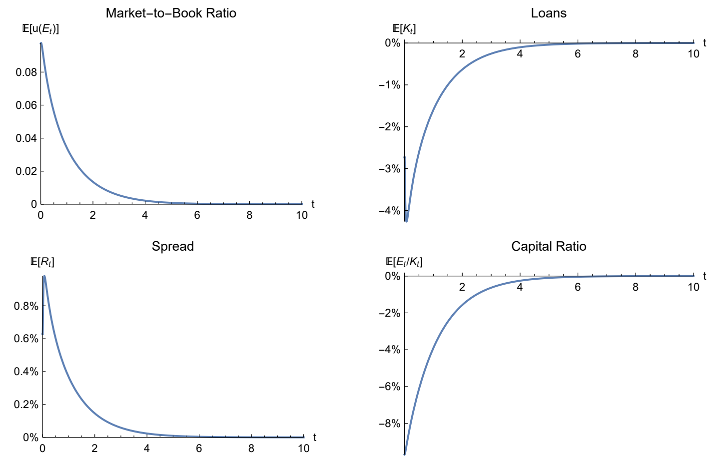

Figure 10: This figure illustrates the impulse response functions after a negative shock to aggregate equity of the market-to-book ratio, aggregate lending, the loan rate spread (in percentage points) and banks' capital ratio implied by our model. The prevailing maximum regulatory leverage equals $\Lambda = 25$, which corresponds to a minimum regulatory capital ratio of 4%. The respective averages are computed according to (B.4). Loans and aggregate capital are expressed relative to the level of aggregate lending in the frictionless benchmark $\widehat{L}$. The aggregate loan demand is given by $L(R) = \widehat{L}(1 - R/\widehat{R})^{\beta}$. Parameter values are: $r = 0.05$, $\zeta = 0.03$, $\alpha = 0.25$, $\beta = 1$, $\sigma = 0.1$, $\gamma = 0.2$.

at impact, but increases afterwards. Second, since lending declines and spreads increase, the negative shock to aggregate bank equity can be viewed as primarily a negative supply shock, as it can be ascertained considering a simple loan demand-supply diagram. Third, the response of both the equity-to asset ratio (Dataset 1) and the regulatory capital ratio (Dataset 2) are negative, indicating that system capitalization relative to overall bank activities is worsened by a decline in aggregate bank equity. Fourth, a negative shock to aggregate equity is highly persistent, as all variables appear to return to the long-term equilibrium slowly. This result suggests that the adjustment of capital buffers is gradual, as found in several contributions of the literature.

5.3.1 The short-term impact of tightening capital requirements on lending

The short-term impact of a tightening of a capital requirement following a negative banking system shock in our model depends on whether banks are either in the external financing or the internal financing region prior to the tightening, which may determine different dynamics as related to the implied profitability trade-offs. Regardless the initial condition prior to a regulatory change, our model implies that lending unambiguously declines in the short-term.

We assess the impact of a tightening of capital requirements by examining the IRFs of total lending to a positive shock to capital ratios. In doing so, we make a standard identification assumption common to several previous studies (see e.g. Berrospide and Edge (2010)), where the IRFs are generated by a Cholesky decomposition where the "structural" shock to either the equity-to-asset ratio (Dataset 1) or the regulatory capital ratio (Dataset 2) is not affected by other shocks at impact, and it is ordered first in the pVARs.

Figure 11 reports the Impulse Response Functions (IRFs) of total lending and aggregate bank equity resulting from a *positive* shock to capital ratios, where responses are tracked for a 10 periods (years) horizon.

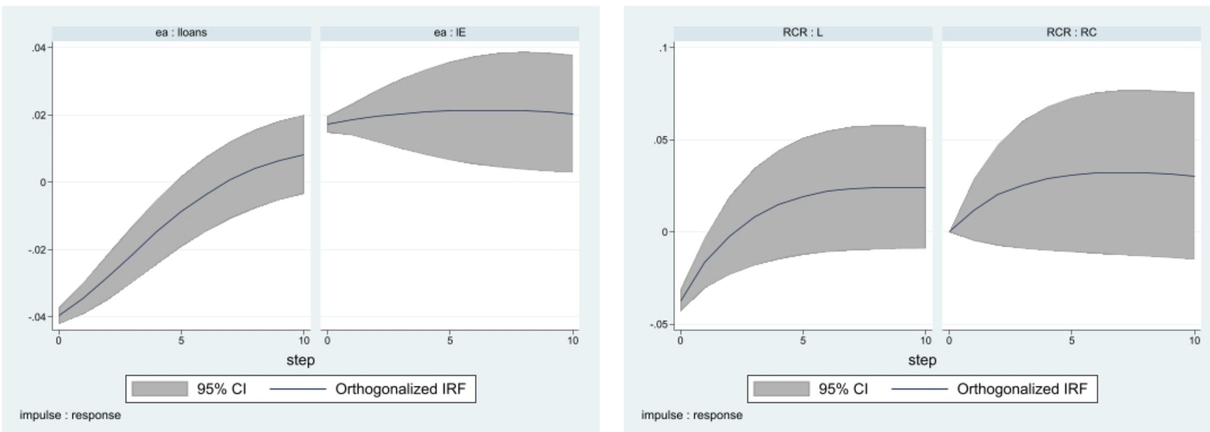

Figure 11: Impulse Response Functions (IRFs) of lending and aggregate bank equity to a positive shock to the equity-to-asset ratio (p1VAR, left panel) and to a positive shock to the regulatory capital ratio (p2VAR, right panel)

The results support the key mechanics embedded in our model concerning the negative effect of a tightening of capital requirements on lending. Lending declines at impact, and slowly recovers. Notably, at some point in time lending recovers above its long-run-equilibrium value in the *medium-run* due to the increase in aggregate bank capital. Thus, lending increases relative to its steady state value in the *medium-run*. As shown in Figure 11, as capital requirements tighten, bank loans decline at impact and slowly recover, remaining below the long-run equilibrium level (the value of 0 on the vertical axis) for about 7 years (Dataset 1) and about 3 years (Dataset 2), but thereafter they increase above long-run equilibrium values, driven by the positive impact of the increase of aggregate bank equity on lending. The existence of these lending dynamics in response to *temporary* increase of capital requirements begs the question on which effect might dominate in determining lending in the long-run under a *permanent* rise in capital requirements, to which we now turn.

5.3.2 The long-term impact of bank capitalization on lending

According to our model, the restoration of capital buffers following an adverse shock and a concomitant tightening in capital requirements may lead to either a *decline* in bank lending in the long-run, as illustrated by the numerical example in the previous section, or an *increase* in bank lending in the long-run, depending on the strength of the positive impact on lending of an increase in aggregate bank equity.[26] Here we assess whether the worldwide evidence during the past two decades seems consistent with either conclusion regarding the long-term relationship between bank capitalization and lending.

As documented in World Bank (2020), *minimum* capital requirements have steadily increased worldwide since 2008. The evidence shown in Figure 12 suggests that regulatory *regimes* such as Basel III might have been important in shifting the dynamics of bank lending and bank capitalization choices following the 2007-2009 financial crisis, as actual bank regulatory capital ratios have increased significantly following the full implementation of Basel III regulation, especially in high income countries.

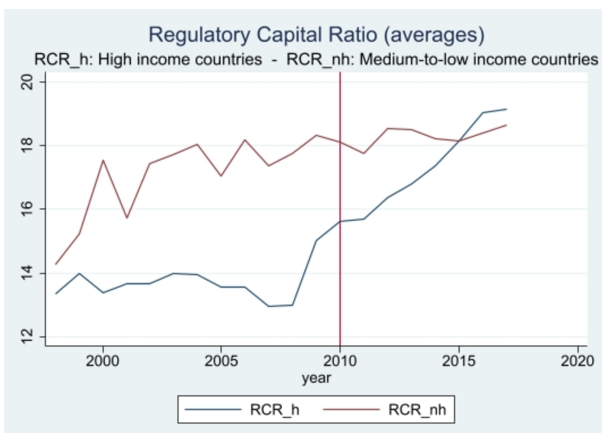

Figure 12: Evolution of the regulatory capital ratio (1998-2017)

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To assess the long-term impact of changes in bank capitalization on lending, we compute the cross-sectional distribution of the ratio of the expected steady state values of the variables in the vector Y_{it} implied by our pVARs estimates relative to the sample averages

[26]The extensive calibration exercise by Begenau (2020) identifies some parameter configurations that can produce an increase in lending in the transition to higher capital requirements. However, that study does not establish whether this potential increase would be sustained in the form of a higher level of lending in the long-run, since the calibrated steady state is by assumption fixed by the moments of the data, as it is typical in exercises focusing on business cycle frequencies.

of these variables during the entire estimation period, called a *steady state ratio*. Since our estimation period includes significant increases in regulatory capital ratios shown in Figure 12, gauging the implied steady state values of lending can be suggestive of the likely impact of increases in bank capitalization prompted by a tightened regulatory regime in the long-run.

Our *steady state ratio* metric is given by

$$Y_i^{ss} = \frac{(I - \bar{A})^{-1}\bar{\alpha}_i^s}{T^{-1}\sum_{i=1}^{T} Y_{it}} \tag{30}$$

where $\bar{A}$ denotes the estimated matrix of coefficients in a pVAR, $\bar{\alpha}_i^s$ denotes the estimated panel fixed effects, and $T^{-1}\sum_{i=1}^{T} Y_{it}$ is the average of Y_{it} over the period $[1, T]$. Values of Y_i^{ss} greater than 1 (less than 1) can be interpreted as measuring the steady state increase (decrease) of a variable relative to its average over the estimation period. This average can be viewed as a proxy measure of either the estimation period's "initial" condition, or the realized long-term value of the variable, as typically assumed in standard calibration exercises.

Considering first the results using our bank level dataset, Figure 13 reports the distributions of steady state ratios of bank lending and capitalization implied by the p1VAR. As shown in Graph G1, the distribution of the steady state ratio of bank lending indicates

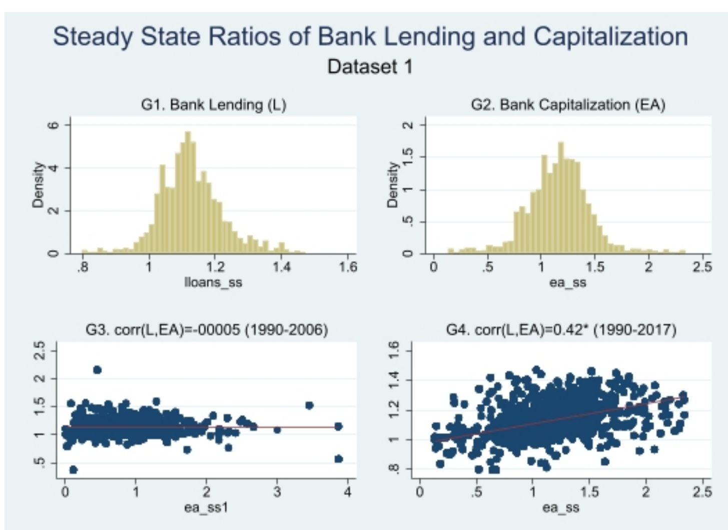

Figure 13: Distributions of steady state ratios ($Y_i^{ss} = \frac{(I-\bar{A})^{-1}\bar{\alpha}_i^s}{T^{-1}\sum_{i=1}^{T} Y_{it}}$) of bank lending (Graph G1), equity-to-asset ratios (Graph G2), and plots of steady state ratios of lending vs. capital ratios pre-crisis (Graph G3) and for the full period (Graph G4).

that the number of banks with values greater than 1 is predominant, accounting for 96% of the sample, even though the fraction of banks having equity-to-asset ratios less than 1 is about 25% (Graph G2). Importantly, the correlation of the steady state ratio of bank lending with bank capitalization is negative and significant pre-crisis (Graph G3),[27] but becomes positive and significant when the post-crisis period is included in the estimation (Graph G4). This result suggests that the post-crisis regime of tightened capital requirements may have played a role in increasing aggregate bank equity to levels sufficient to support a higher steady state level of bank lending.

Turning to the results using our country dataset, Figure 14 reports the distributions of steady state ratios of lending and capitalization implied by the p2VAR, together with a plot of regulatory capital ratios vs. aggregate regulatory capital (left panel), as well as the cross sectional relationship between steady state ratios of lending and regulatory capital ratios pre-crisis[28] and for the full period (right panel).

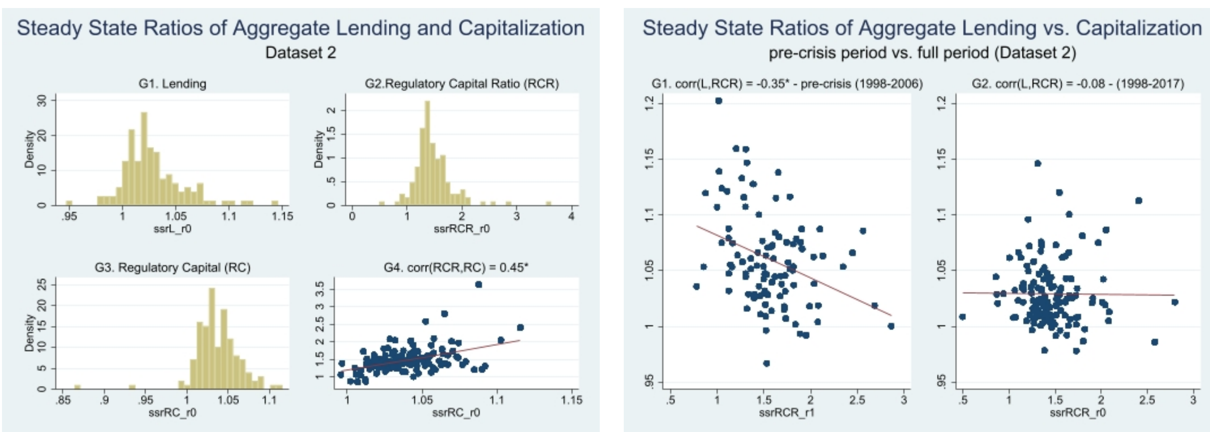

Figure 14: Distributions of steady state ratios ($Y_i^{ss} = \frac{(I-\bar{A})^{-1}\bar{\alpha}_i^s}{T^{-1}\sum_{i=1}^{T} Y_{it}}$) ad regulatory capital ratio vs. aggregate regulatory capital (left panel), and aggregate lending vs. regulatory capital ratio pre-crisis and full period (right panel).

The steady state ratios of lending and aggregate regulatory capital are larger than 1 in about 95% of countries (left panel, Graph G1), the percentage of countries for which the steady state ratio of the regulatory capital ratio is greater than 1 is about 67% (left panel, Graph G2), the steady state ratio of aggregate bank equity is greater than 1 in

[27] Pre-crisis steady state ratios are obtained by estimating the p1VAR using data for the sub-period 1990-2006.

[28] Pre-crisis steady state ratios are obtained by estimating the p2VAR using data for the sub-period 1998-2006.

almost all countries (left panel, Graph G3), and the cross-sectional relationship between steady state ratios of lending and aggregate bank capital is positive (left panel, Graph G4). Importantly, and consistently with the bank level evidence, the cross-sectional correlation of steady state lending with steady state regulatory capital ratios is negative during the pre-crisis period, but becomes close to nihil when the post-crisis period is added to the estimation sample. This results suggests that the tighter capital regulation post-crisis may have been an important factor in determining the documented shift of the relationship between banking system capitalization and lending in the long-run.

In conclusion, the above evidence suggests that the impact of tightened capital requirements on lending might have been positive in the long-run.

5.4 Summary

In this section we have used pVARs specified consistently with our model to disentangle the dynamics of bank lending and capitalization. We have documented that the key mechanisms of our model are consistent with evidence based on two large international databases. We have further provided evidence of the negative impact of tightened capital requirements in the short-run, consistent with many contributions of the literature.

However, our impulse response analysis has also detected a reversal of the sign of tightened capital requirements on lending, suggesting that the restoration of capital buffers following tightened capital requirements may support a resumption of lending above steady state values in the medium term through the attendant increase in aggregate bank equity.

Lastly, we constructed a simple steady state metric aimed at assessing how the steady state variables implied by our pVAR models differ from their average values measured in the past 20 years typically associated with realized long-term realization in standard calibration exercises. The results of this assessment suggest that, conditional on the past 20 years experience of many banks and countries, there is a high probability that lending has increased in the long-run as a result of the new Basel III regulatory regime.

6 Conclusion

This paper analyzes the costs and benefits of bank capital requirements in the short-run and in the long-run. We develop a stylized dynamic general equilibrium model of the banking sector in which banks finance risky loans to the real sector by safe and liquid deposits. A crucial assumption is that banks incur flotation costs when issuing equity. To save on issuance costs, banks build up capital buffers and require a strictly positive loan rate spread even though the lending market is perfectly competitive.

Capital requirements restrict the total supply of loans, which increases the spread and thus the profitability of banks' loan making business. Since loans are partly financed by equity, the marginal (or market-to-book) value of bank capital increases accordingly. Furthermore, higher profits facilitate the accumulation of internally generated equity. Hence, while tighter capital requirements depress lending in the short-term ("credit crunch"), they induce banks to accumulate larger capital buffers. This reduces the public losses in case of a systemic banking crisis and, at the same time, helps to restore the banking sector's lending capacity. Hence, in the medium- to long-run, the trade-off between costs and benefits of capital requirements tilts more heavily in favor of the benefits.

Our model emphasizes the importance of aggregate bank capital which determines the banking sector's lending capacity and hence, the equilibrium loan rate spread. Key testable implications of our model are that aggregate lending increases in aggregate capital, while the loan rate spread and the market-to-book ratio decrease in aggregate capital. Using two large international databases, we find that these implications are broadly consistent with the data in the cross-section. Moreover, using a panel VAR estimation, we show that a shock to aggregate bank equity (akin to a systemic crisis) generates an initial response and a gradual adjustment of all variables to the long-term equilibrium, that are in line with our model's predictions, and provide evidence suggesting that bank lending has increased in the long-run as a result of the new Basel III regulatory regime.

References

Admati, A. and Hellwig, M. (2014). *Bankers' New Clothes: What's Wrong with Banking and What to Do about It*, revised edn, Princeton Univers. Press, Princeton.

Admati, A. R., DeMarzo, P. M., Hellwig, M. F. and Pfleiderer, P. C. (2013). Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Socially Expensive, *Working Paper Stanford University.*

Aiyar, S., Calomiris, C. W. and Wieladek, T. (2016). How does credit supply respond to monetary policy and bank minimum capital requirements?, *European Economic Review* **82**: 142–165.

Allen, F., Carletti, E. and Marquez, R. (2011). Credit Market Competition and Capital Regulation, *The Review of Financial Studies* **24**(4): 983–1018.

Basel Committee on Banking Supervision (2019). The costs and benefits of bank capital - a review of the literature, *BCBS Working Paper No. 37, June.*

Begenau, J. (2020). Capital requirements, risk choice, and liquidity provision in a business-cycle model, *Journal of Financial Economics* **136**(2): 355–378.

Berrospide, J. M. and Edge, R. M. (2010). The Effects of Bank Capital on Lending: What Do We Know, and What Does It Mean?, *International Journal of Central Banking* **6**(34): 1–50.

Black, L., Floros, I. V. and Sengupta, R. (2016). Raising Capital When the Going Gets Tough: U.S. Bank Equity Issuance from 2001 to 2014, *Federal Reserve Bank of Kansas City Working Paper No. 16-05.*

Bolton, P., Chen, H. and Wang, N. (2011). A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management, *The Journal of Finance* **66**(5): 1545–1578.

Bolton, P., Chen, H. and Wang, N. (2013). Market timing, investment, and risk management, *Journal of Financial Economics* **109**(1): 40–62.

Bolton, P., Li, Y., Wang, N. and Yang, J. (2020). Dynamic Banking and the Value of Deposits, *Fisher College of Business Working Paper No. 2020-03-013*.

Bridges, J., Gregory, D., Nielsen, M., Pezzini, S., Radia, A. and Spaltro, M. (2014). The impact of capital requirements on bank lending, *Bank of England Working Paper No. 486, January.*

Brunnermeier, M. and Sannikov, Y. (2014). A Macroeconomic Model with a Financial Sector, *American Economic Review* **104**(2): 379–421.

De Jonghe, O., Dewachter, H. and Ongena, S. (2020). Bank capital (requirements) and credit supply: Evidence from pillar 2 decisions, *Journal of Corporate Finance* **60**: 101518.

De Nicolò, G., Gamba, A. and Lucchetta, M. (2014). Microprudential Regulation in a Dynamic Model of Banking, *Review of Financial Studies* **27**(7): 2097–2138.

DeAngelo, H. and Stulz, R. M. (2015). Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks, *Journal of Financial Economics* **116**(2): 219–236.

Décamps, J.-P., Mariotti, T., Rochet, J.-C. and Villeneuve, S. (2011). Free Cash Flow, Issuance Costs, and Stock Prices, *The Journal of Finance* **66**(5): 1501–1544.

Fonseca, A. R. and González, F. (2010). How bank capital buffers vary across countries: The influence of cost of deposits, market power and bank regulation, *Journal of Banking & Finance* **34**(4): 892–902.

Fraisse, H., Lé, M. and Thesmar, D. (2019). The Real Effects of Bank Capital Requirements, *Management Science* **66**(1): 5–23.

Gorton, G. and Winton, A. (2017). Liquidity Provision, Bank Capital, and the Macroeconomy, *Journal of Money, Credit and Banking* **49**(1): 5–37.

Hanson, F. B. (2007). *Applied Stochastic Processes and Control for Jump Diffusions: Modeling, Analysis, and Computation*, illustrated edition edn, Society for Industrial and Applied Mathematics, Philadelphia, PA.

He, Z. and Krishnamurthy, A. (2012). A Model of Capital and Crises, *The Review of Economic Studies* **79**(2): 735–777.

Holtz-Eakin, D., Newey, W. and Rosen, H. S. (1988). Estimating Vector Autoregressions with Panel Data, *Econometrica* **56**(6): 1371–1395.

Homar, T. and van Wijnbergen, S. J. G. (2017). Bank recapitalization and economic recovery after financial crises, *Journal of Financial Intermediation* **32**: 16–28.

Hugonnier, J. and Morellec, E. (2017). Bank capital, liquid reserves, and insolvency risk, *Journal of Financial Economics* **125**(2): 266–285.

Klimenko, N., Pfeil, S. and Rochet, J.-C. (2017). A simple macroeconomic model with extreme financial frictions, *Journal of Mathematical Economics* **68**: 92–102.

Kondor, P. and Vayanos, D. (2019). Liquidity Risk and the Dynamics of Arbitrage Capital, *The Journal of Finance* **74**(3): 1139–1173.

Laeven, L. and Valencia, F. (2013). The Real Effects of Financial Sector Interventions during Crises, *Journal of Money, Credit and Banking* **45**(1): 147–177.

Malherbe, F. (2020). Optimal Capital Requirements over the Business and Financial Cycles, *American Economic Journal: Macroeconomics* **12**(3): 139–174.

Martinez-Miera, D. and Suarez, J. (2014). Banks' endogenous systemic risk taking, *Manuscript, Center for Monetary and Financial Studies*.

Miles, D., Yang, J. and Marcheggiano, G. (2013). Optimal Bank Capital, *The Economic Journal* **123**(567): 1–37.

Milne, A. and Whalley, A. E. (2001). Bank Capital and Incentives for Risk-Taking, *WBS Finance Group Research Paper No. 15*.

Phelan, G. (2016). Financial Intermediation, Leverage, and Macroeconomic Instability, *American Economic Journal: Macroeconomics* **8**(4): 199–224.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements, *Journal of Monetary Economics* **55**(2): 298–320.

World Bank (2020). *Bank Regulation and Supervision a Decade after the Global Financial Crisis*, Washington D.C.

Appendix A Omitted Proofs

Proof of Proposition 1. In the region where the capital requirement is binding, we denote the market-to-book ratio and the equilibrium interest rate by $u_b(\cdot)$ and $R_b(\cdot)$, respectively, and in the region where it is slack by $u_s(\cdot)$ and $R_s(\cdot)$. For future reference, define

$$\begin{aligned} A(E) &:= -\frac{u_b'(E)}{u_b(E)}, \text{ and} \\ B(E) &:= \frac{L^{-1}(\Lambda E)}{\Lambda E \sigma^2}. \end{aligned} \tag{A.1}$$

We first establish the properties of $u(E)$ and $R(E)$ for a region in which the constraint (5) is slack.

Lemma A.1. *Assume that the constraint is slack, i.e., $A(E) \geq B(E)$, over a region $E \in [E^*, \overline{E}]$ with $E^* > \underline{E}$, then it holds for $E \in [E^*, \overline{E})$ that*

i) $u_s'(E) < 0$,

ii) $R_s(E) > 0$,

iii) $u_s''(E) > 0$.

Proof. Note first that substituting boundary conditions (14) and (16) into HJB (20) implies that u_s'' is positive at the top:

$$u_s''(\overline{E}) = \frac{2}{\sigma^2 \widehat{L}^2} \zeta > 0. \tag{A.2}$$

By continuity, it must therefore hold that $u_s'(\overline{E} - \epsilon) < 0$ for a small ϵ. Now assume that u_s' changes sign and let $\hat{E} := \sup\{E < \overline{E} : u_s'(E) > 0\}$. By continuity it holds that $u_s'(\hat{E}) = 0$ and $u_s''(\hat{E}) < 0$, implying that $u_s(\hat{E}) = \frac{\sigma^2 L^2}{2\zeta} u_s''(\hat{E}) < 0$. This is a contradiction since $u_s(\overline{E}) = 1$ and $u_s'(E) < 0$ for $E \in (\hat{E}, \overline{E})$. The first claim of Lemma A.1 thus follows. The second claim follows then immediately from (22). The third claim follows immediately from (20) together with the first two claims. ■

We next establish an auxiliary result analogous to Lemma A.1 for a region where the constraint is binding.

Lemma A.2. *Consider the region $E \in [\underline{E}, \widetilde{E}]$ with $\widetilde{E} \leq \overline{E}$, such that $A(E) < B(E)$, i.e. constraint (5) is binding.*

i) It then holds that $R_b(E) > 0$ *for* $E \in [\underline{E}, \widetilde{E}]$*.*

ii) If $\widetilde{E} < \overline{E}$*, it holds that* $u_b'(E) < 0$ *for* $E \in [\underline{E}, \widetilde{E}]$*. If* $\widetilde{E} = \overline{E}$ *it holds that* $u_b'(E) < 0$ *for* $E \in [\underline{E}, \widetilde{E})$ *and* $u_b'(\widetilde{E}) = 0$*.*

Proof. To prove the first claim, we establish that

$$R_b(E) = L^{-1}(\Lambda E) > 0 \; \forall E \in [\underline{E}, \widetilde{E}]. \tag{A.3}$$

This follows from $\partial L^{-1}(\Lambda E)/\partial E < 0$ together with the fact that $R_b(\widetilde{E}) > 0$. That is, if the constraint binds globally, i.e., $\widetilde{E} = \overline{E}$, then $R_b(\widetilde{E}) = L^{-1}(\Lambda \overline{E}) > 0$ since $u_b'(\overline{E}) = 0$ by boundary condition (16). If the constraint becomes slack at $\widetilde{E} = E^*$, it follows from Lemma A.1 that $R_b(\widetilde{E}) = R_s(E^*) > 0$.

In order to prove the second claim, we first establish that

$$u_b''(\underline{E}) < 0 < u_b''(\widetilde{E}). \tag{A.4}$$

If the constraint binds only over $E \in [\underline{E}, \widetilde{E}]$ with $\widetilde{E} = E^* < \overline{E}$, condition (A.4) follows immediately from Lemma A.1. If the constraint binds globally, i.e., $\widetilde{E} = \overline{E}$, assume to the contrary that $u_b''(\underline{E}) > 0$, which implies that $u_b'(\underline{E} + \epsilon) > 0$ and, thus, $u_b(\underline{E} + \epsilon) > 1 + \gamma$, a contradiction. Similarly, $u_b''(\overline{E}) < 0$ would imply that $u_b'(\overline{E} - \epsilon) > 0$ and, thus, $u_b(\overline{E} - \epsilon) < 1$, a contradiction.

We can now establish that $u'(E) < 0$ for $E \in (\underline{E}, \widetilde{E})$. Assume to the contrary that there exists a $\hat{E} < \widetilde{E}$ such that $u_b'(\hat{E}) \geq 0$. Since $u_b'(\underline{E}) = 0$, this would imply that there exists $\hat{E}_1 < \hat{E}$, at which $u_b''(E)$ becomes positive. But from (A.4) and the fact that $u_b'(\widetilde{E}) \leq 0$, there has to exist a $\hat{E}_2 > \hat{E}$ where $u_b''(E)$ turns negative. Furthermore, as $u_b''(\widetilde{E}) > 0$ by (A.4), there would exist another critical level $\hat{E}_3 > \hat{E}_2$, where $u_b''(E)$ becomes positive again. Evaluating equation (20) in $\hat{E}_2$ and $\hat{E}_3$ yields

$$\begin{aligned} \left(\zeta - \Lambda R_b(\hat{E}_2)\right) u_b(\hat{E}_2) &= \left[r\hat{E}_2 + R_b(\hat{E}_2)\Lambda \hat{E}_2 + \sigma^2 \Lambda \hat{E}_2\right] u_b'(\hat{E}_2), \\ \text{and } \left(\zeta - \Lambda R_b(\hat{E}_3)\right) u_b(\hat{E}_3) &= \left[r\hat{E}_3 + R_b(\hat{E}_3)\Lambda \hat{E}_3 + \sigma^2 \Lambda \hat{E}_3\right] u_b'(\hat{E}_3), \end{aligned} \tag{A.5}$$

respectively. From (A.3), the terms in square brackets on the RHS of (A.5) are strictly positive. Since $u_b'(\hat{E}_2) > 0 > u_b'(\hat{E}_3)$, we would, thus, have that

$$\left(\zeta - \Lambda R_b(\hat{E}_2)\right) > 0 > \left(\zeta - \Lambda R_b(\hat{E}_3)\right),$$

which is a contradiction as $R_b'(E) = \partial L^{-1}(\Lambda E)/\partial E < 0$ and $\hat{E}_2 < \hat{E}_3$. Hence, it must hold that $u_b'(E) < 0$ for $E \in (\underline{E}, \widetilde{E})$ and the second claim follows. ■

Finally, we are ready to piece together the two regions characterized in Lemma A.1 and Lemma A.2 and show that there can exist at most two regions, i.e., if the constraint becomes slack at some E^*, it is slack for all $E \in [E^*, \overline{E}]$. Note that E^*, the lowest point at which the constraint is not strictly binding, is characterized by $A(E^*) = B(E^*)$. Note further that in E^* it has to hold that $R_s = R_b$, i.e., using (21) and (22),

$$\frac{\Lambda E}{L^{-1}(\Lambda E)} = -\frac{1}{\sigma^2}\frac{u_s(E)}{u_s'(E)}. \tag{A.6}$$

Now note that for the constraint to become binding again at some $E_1^* > E^*$, (A.6) would have to hold with equality at E_1^* as well. Differentiating the LHS of (A.6) yields

$$\Lambda \frac{L^{-1}(\Lambda E) - \Lambda E \frac{\partial L^{-1}(\Lambda E)}{\partial E}}{(L^{-1}(\Lambda E))^2} > 0,$$

which follows from $L'(R) < 0$ and (A.3). Differentiating the RHS of (A.6) yields

$$-\frac{u_s'(E)^2 - u_s'(E)u_s''(E)}{(\sigma u_s'(E))^2} < 0,$$

which follows from Lemma A.1. Hence, (A.6) cannot be satisfied for any other value $E_1^* \neq E^*$.

After having established the above regularities, we now show how to construct the equilibrium. To solve for the equilibrium couple $u(E)$ and $R(E)$ in this case, we first consider a candidate value for the recapitalization barrier, $\underline{E}_c$, and solve (20) subject to boundary conditions (13) and (18), i.e.,

$$u_b(\underline{E}_c; \underline{E}_c) - (1 + \gamma) = \frac{\partial}{\partial E} u_b(\underline{E}_c; \underline{E}_c) = 0.$$

Here, we adopt the notation $u_b(E; \underline{E}_c)$ to emphasize that the market-to-book ratio is a function of E and parameterized by the candidate value $\underline{E}_c$, for which the remaining boundary conditions are not necessarily satisfied. We can then determine, also parameterized by $\underline{E}_c$, the critical level of aggregate equity,

$$E_c^* := E^*(\underline{E}_c),$$

at which the constraint imposed by (5) becomes slack, i.e.,

$$-\frac{\frac{\partial}{\partial E}u_b(E_c^*;\underline{E}_c)}{u_b(E_c^*;\underline{E}_c)} = B(E_c^*). \tag{A.7}$$

Note that through the respective boundaries, also the equilibrium spread is parameterized by the candidate value $\underline{E}_c$:

$$R_b\left(E;\underline{E}_c\right) = L^{-1}\left(\Lambda E\right),\ E \in \left[\underline{E}_c, E_c^*\right]. \tag{A.8}$$

Turning next to the region where the regulatory constraint is slack, we can determine the equilibrium spread $R_s(E;\underline{E}_c)$ by solving (25) subject to the following boundary condition

$$R_s(E_c^*;\underline{E}_c) = R_b(E_c^*;\underline{E}_c), \tag{A.9}$$

which ensures continuity of the spread at the point E_c^*. By substituting $R_s(E;\underline{E}_c)$ into the first order condition for banks' leverage (22), we can compute the the market-to-book ratio in the region where the constraint is slack:[29]

$$u_s(E;\underline{E}_c) = u_s(E_c^*;\underline{E}_c) \times \exp\Big(-\int_{E_c^*}^{E} \frac{R_s(q;\underline{E}_c)}{\sigma^2 L\big(R_s(q;\underline{E}_c)\big)}dq\Big), \tag{A.10}$$

It is important to stress that $u_s(E;\underline{E}_c)$ is parameterized by the candidate $\underline{E}_c$ first, through the spread from (A.9) and, second, by imposing value-matching at E_c^* in (A.10), i.e.,

$$u_s(E_c^*;\underline{E}_c) = u_b(E_c^*;\underline{E}_c).$$

Next, we determine — also parameterized by the candidate $\underline{E}_c$ — the dividend boundary

[29]Note that this expression allows us to explicitly derive the dynamics of $R(E)$ in the benchmark case without capital regulation by setting $E_c^* = \underline{E} = 0$ and $E = \overline{E}$, such that $u_s(E;\underline{E}_c) = 1$ and $u_s(E_c^*;\underline{E}_c) = 1+\gamma$.

$\overline{E}(\underline{E}_c)$ by using the boundary condition (16):

$$\frac{\partial}{\partial E} u_b\big(\overline{E}(\underline{E}_c); \underline{E}_c\big) = 0.$$

Finally, note that we have constructed a continuous, piecewise function

$$u(E; \underline{E}_c) = \begin{cases} u_b(E; \underline{E}_c) & \text{if } E \leq E_c^*, \\ u_s(E; \underline{E}_c) & \text{if } E > E_c^*. \end{cases}$$

The same applies to $R(\cdot)$. Since all endogenous objects are parameterized by the candidate value $\underline{E}_c$, it remains to pin down the latter by the remaining boundary condition (14):

$$u(\overline{E}(\underline{E}_c); \underline{E}_c) = 1.$$

■

Appendix B Equilibrium Dynamics

In this appendix we derive the stationary density of aggregate capital which is implied by the equilibrium dynamics of aggregate capital (28). We define $G(E, t; E_0)$ as the cumulative probability density, i.e. the probability that aggregate bank capital is smaller than $E \in [\underline{E}, \overline{E}]$ at time t conditional on the initial value $E_0 \in [\underline{E}, \overline{E}]$ and the regulatory capital requirement Λ.

Proposition B.1. *The transition density of E can be computed as $g(E, t; E_0) = \frac{\partial G(E,t;E_0)}{\partial E}$, where the cumulative density $G(E, t; E_0)$ satisfies*

$$\frac{\partial G(E, t; E_0)}{\partial t} = \frac{1}{2}\sigma^2(E)\frac{\partial^2 G(E, t; E_0)}{\partial E^2} + \frac{\partial G(E, t; E_0)}{\partial E}\Big(\sigma'(E)\sigma(E) - \mu(E)\Big) - \zeta G(E, t; E_0), \tag{B.1}$$

with the boundary conditions $G(\underline{E}, t; E_0) = 0$, $G(\overline{E}, t; E_0) = 1$ and $G(E, 0; E_0) = H_{E_0}(E)$ where $H_{E_0}(.)$ is a Heaviside step function.

Proof. Given the equilibrium law of motion of E in the internal financing region $\big[\underline{E}, \overline{E}\big]$, the transition probability density $g(E, t; E_0)$, should satisfy the forward Kolmogorov equation (see Theorem 7.5 in Hanson (2007)):

$$\frac{\partial g(E,t;E_0)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial E^2}\big(\sigma^2(E)g(E,t;E_0)\big) - \frac{\partial}{\partial E}\big(\mu(E)g(E,t;E_0)\big) \\ + \zeta\big(g(E-\phi K(E),t;E_0) - g(E,t;E_0)\big). \tag{B.2}$$

Rearranging the terms in the right-hand side of (B.2) yields

$$\frac{\partial g(E,t;E_0)}{\partial t} = \frac{\partial}{\partial E}\left[\frac{1}{2}\sigma^2(E)\frac{\partial g(E,t;E_0)}{\partial E} - \big(\sigma'(E)\sigma(E) - \mu(E)\big)g(E,t;E_0)\right] \\ + \zeta\big(g(E-\phi K(E),t;E_0) - g(E,t;E_0)\big). \tag{B.3}$$

Next, we integrate (B.3) over E and use that $G(E-\phi K(E),t;E_0) = G(\underline{E},t;E_0) = 0$ because $\phi K(E) > E$, $\forall E$. This yields the second-order partial differential equation for $G(E,t;E_0)$ in (B.1), which can be solved numerically under the initial boundary condition $G(E,0;E_0) = H_{E_0}(E)$, where $H_{E_0}(.)$ is a Heaviside step function. The two additional boundary conditions implied by the notion of probability are $G(\underline{E},t;E_0) = 0$ and $G(\overline{E},t;E_0) = 1$. The transition density then can be computed by differentiating the obtained solution of (B.1) with respect to E. ■

We can then compute the expected value at time t of any function $\psi(E)$ – such as for instance equilibrium spread, $R(E)$ – given that the capitalization in $t=0$ equals E_0, by

$$\mathbb{E}_t[\psi(E)|E_0] = \int_{\underline{E}}^{\overline{E}} \psi(E)g(E,t;E_0)dE. \tag{B.4}$$

As a final remark, for t going to infinity, we can directly obtain the stationary cumulative distribution function,

$$G(E) = \lim_{t\to\infty} G(E,t;E_0),$$

as the solution to the stationary version of (B.1), i.e., with the time-derivative on the left hand side set equal to zero. The stationary transition probability density is then, as before, obtained by differentiating the c.d.f.: $g(E) = G'(E)$. Intuitively, it conveys information on how frequently each state of the support $[\underline{E},\overline{E}]$ is visited in the long run.