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Collective Choice with Heterogeneous Time Preferences

Abstract

This paper reviews recent research on the aggregation of heterogeneous time preferences. Main results are illustrated in simple Ramsey models with two or three agents who differ in their discount factors. We employ an intertemporal view on these models and argue that preferences of a decision maker should be represented by a sequence of utility functions. This allows us to clarify the issue of dynamic inconsistency and relate it to simple properties of discounting. We distinguish between private and common consumption cases. In the private consumption case, we discuss the properties of sequences of Paretian social welfare functions and explain why the notion of Pareto optimality under heterogeneous time preferences becomes problematic. In the common consumption case, we focus on the problem of collective choice under heterogeneous time preferences, discuss the difficulties with dynamic voting procedures and review some ways to overcome them. We conclude by highlighting the implications of our discussion for the problem of choosing an appropriate social discount rate.

JEL-Codes: D150, D710, H430, O400.

Keywords: collective decisions, social welfare function, heterogeneous agents, time consistency, voting, social discount rate.

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1 Introduction

Time preference — the intrinsic propensity of an individual to postpone immediate gratification in exchange for larger but delayed rewards — lies at the core of economic analysis. Both theoretical and empirical contributions suggest that time preference (patience) is a fundamental factor influencing economic development. In many macroeconomic models, the rate of time preference often determines the most relevant variables, e.g., the long-run income distribution and growth.

A vast amount of literature on time preferences is devoted to the analysis of representative agent models (see, e.g., the review by Hamada and Takeda, 2009). However, the representative agent assumption is subject to serious criticism, as people in the real world differ in many characteristics affecting their economic decisions. Kirman (1992, p. 117) argues that

> this reduction of the behavior of a group of heterogeneous agents ... is not simply an analytical convenience as often explained, but is both unjustified and leads to conclusions which are usually misleading and often wrong.

While it is clear that there are many sources of heterogeneity (e.g., wealth or education), variation in time preferences is arguably the most important of them, as it underlies human behavior. It is generally acknowledged that people are not equally patient, and there is no convergence toward an agreed-on or unique rate of time preference. Hence an attempt to employ the representative agent assumption in growth models faces serious difficulty, as it is unclear, which rate of time preference should be used.

Despite a large empirical literature on discounting (see Frederick et al., 2002; Cohen et al., 2020, for extensive reviews), researchers have only recently emphasized the apparent heterogeneity in time preferences both at individual and country levels. A number of recent large-scale international surveys show that people in the real world value the future differently, and average patience plays a decisive role in the process of economic development.

Falk et al. (2018) analyze the results of the 2012 Global Preference Survey covering 80000 individuals from 76 countries. They find that average patience across countries varies by 1.7 of a standard deviation, and the within-country variation is much larger than the cross-country variation: the former amounts to 86.5% in the total individual-level variation in patience, while the latter explains only the remaining 13.5%. Moreover, in the world population as a whole, time preferences vary significantly with individual characteristics such as gender or age. Wang et al. (2016) report the results of a survey comprising about 7000 students from 53 countries. They also find that the measured level of patience is heterogeneous both at individual and country levels, which cannot be explained by differences in interest or inflation rates. Their estimations imply that the median annual discount rate among the considered countries is 100%, ranging from 14% in Australia to 1567% in Bosnia and Herzegovina.

International surveys also pave the way for econometric analysis of the links between time preferences and economic growth at the aggregate level. Dohmen et al. (2018) in a sample of 76 countries find that average degree of patience in a country is causally related to economic development. Patience alone explains about 40% of the variation in per capita income, and it strongly correlates with the accumulation of physical capital, quantity and quality of schooling, and productivity growth. Hübner and Vannoorenberghe (2015) obtain similar results in a panel of 89 countries. They show that average degree of patience has a strong positive impact on income per worker, total factor productivity and capital stock. In their sample, patience also explains about 40% of the cross-country variation in income. They report that increasing patience by one standard deviation raises per-capita income by between 43% and 78%. Thus empirical evidence unambiguously suggests that heterogeneity in time preferences should be explicitly taken into account in economic modeling.

The canonical theoretical framework for studying heterogeneity in time preferences was proposed in a seminal paper by Ramsey (1928). After developing a model of optimal capital accumulation which is now widely known as the optimal growth model (the Ramsey model), he also considered a model with many agents who differ in their time preferences. He conjectured that in a stationary equilibrium the whole capital stock belongs to the most patient agent in the society whose consumption is the largest. All other, less patient agents, consume only at a subsistence level necessary to support their lives. This property of an equilibrium in many-agent models is referred to in the literature as the Ramsey conjecture.[1]

Ramsey (1928) neither spelled out the details of his many-agent model nor

[1]The connection between intertemporal choice and economic growth has long occupied the minds of economists. Adam Smith (1776) saw the propensity to save and invest capital instead of spending it immediately as a virtue which leads to the accumulation of all types of capital and increases the wealth of nations. The problem of what determines this propensity was explored by John Rae (1834), who argued that there are differences in the strength of the desire to accumulate among different individuals. People whose desire to accumulate is low become poor, while people whose desire to accumulate is high become rich. Further, Irving Fisher (1907) developed a more precise notion of time preference and argued that it is ultimately differences in rates of time preference that drive the distribution of income and wealth. Thus, for historical reasons, it is more correct to refer to the Ramsey conjecture as the "Rae–Fisher–Ramsey conjecture".

gave a definition of equilibrium, and his insights were formalized much later. Bewley (1982) proposed an interpretation of the many-agent Ramsey model as a general equilibrium model with infinitely many commodities. He considered an economy with a complete system of financial markets populated by agents who differ in their time preferences. Bewley (1982) proved that there exists a competitive equilibrium, and any equilibrium allocation is Pareto-optimal. He studied the properties of Pareto-optimal allocations and showed that eventually only the most patient agent has positive consumption levels. All other, less patient agents, borrow against their wealth to ensure more consumption at earlier dates, have to repay their loans at later dates and hence drive their future consumption to zero.[2] Rather informally, this stunning result can be stated as follows: in any Pareto-optimal allocation all agents except the most patient one starve to death. This controversial conclusion goes far beyond a mere verification of the Ramsey conjecture, as it implies that even if two agents differ infinitesimally in patience, their eventual positions from a socially optimal point of view would differ dramatically.

Another interpretation of the many-agent Ramsey model was proposed by Becker (1980) who considered an economy with borrowing constraints. Agents can sell or accumulate capital, but cannot borrow, which implies that nobody consumes zero or asymptotically approaches zero: even the less patient agents always consume at least part of their labor income. Becker (1980) showed that there exists a unique stationary equilibrium in which the real interest rate is determined by the discount factor of the most patient agent. Therefore, all capital is owned by the most patient agent, which also verifies the Ramsey conjecture. Models of this type received reasonable attention, and many important properties of equilibria were established (see Becker, 2006, for a survey).

Since in the many-agent Ramsey model with borrowing constraints markets are incomplete, equilibrium allocations in this type of models are not Pareto-optimal.[3] The optimal allocations in the Bewley- and Becker-type models are the same, and, as we have seen, their properties suggest that the notion of Pareto optimality under heterogeneous time preferences becomes problematic.

The above discussion emphasizes that heterogeneous agents models differ significantly from representative agent models. The applied importance of this fact

[2]If felicity function $u(c)$ is such that $u'(0) < \infty$, then impatient agents have zero consumption after a finite time. For more general felicity functions, consumption of impatient agents asymptotically converges to zero.

[3]Nevertheless, equilibrium paths of aggregate capital and consumption are technologically efficient (see Becker and Mitra, 2012). This is a weaker condition which implies that there is no overaccumulation of capital and the aggregate consumption path is intertemporally efficient, but ignores the distribution of consumption among different agents.

can be seen by comparing policy implications. Many economic policies aimed at increasing aggregate income in representative agent models affect only the distribution of income in heterogeneous agents models, which is apparent even in models with two types of agents (e.g., Smetters, 1999; Mankiw, 2000; Palivos, 2005).

A natural way to circumvent the problem with standard social welfare criteria in heterogeneous agents models is to decentralize the decision-making process and let agents collectively determine optimal allocations. Thus the recognition of the fact that people discount the future differently leads to the problem of collective choice and aggregation of heterogeneous time preferences. How can a society make a collective decision when its members have different time preferences? This question is at the junction of economic growth theory and social choice theory, and has lately received considerable attention.

In this survey we review the main results on the aggregation of heterogeneous time preferences and discuss the problem of collective choice in many-agent growth models. To make the presentation as clear as possible, we employ a one-sector deterministic growth model with two or three agents whose preferences exhibit exponential discounting. Though there is vast evidence of time-declining discounting, we do not discuss departures from constant exponential discounting. Similarly, though discounting under uncertainty remains a topic of current interest, introduction of uncertainty complicates matters quite dramatically, so this survey deals only with deterministic models.[4] Our simple framework allows us to explain the main difficulties in an instructive manner and to highlight the role of discounting.

The survey is organized as follows. Sections 2 and 3 deal with social optima in the many-agent Ramsey model with private and common consumption respectively. Section 4 considers majority voting over common consumption streams. Section 5 discusses the implications for the problem of choosing a social discount rate. Section 6 concludes.

2 Social optima under private consumption

In this section we analyze the many-agent Ramsey model with private consumption from the social welfare perspective. We explore and discuss the main difficulties arising with the notion of Pareto optimality under heterogeneous time preferences. To highlight the role of discounting, we consider a simple one-sector two-agent model where agents differ only in their discount factors and are otherwise identical.

[4]Collective choice under quasi-hyperbolic discounting is studied, e.g., by Lizzeri and Yariv (2017); Drugeon and Wigniolle (2020). For the literature on discounting under uncertainty see, e.g., Gollier and Weitzman (2010); Traeger (2013).

2.1 Paretian social welfare functions: intertemporal view

Consider an economy with two agents. Preferences of agent i over private consumption streams $\mathbf{C}^i = \{c_t^i\}_{t=0}^{\infty}$ are given by the additively time-separable intertemporal utility function with constant exponential discounting:

$$U^i(\mathbf{C}^i) = \sum_{t=0}^{\infty} \beta_i^t u(c_t^i), \qquad i = 1, 2,$$

where β_i is the discount factor and $u(c)$ is an increasing and strictly concave felicity function. Assume, without loss of generality, that agent 1 is more patient than agent 2: $1 > \beta_1 > \beta_2 > 0$.

A single homogeneous good is produced. In each period t the available amount of the good is allocated between aggregate consumption $C_t = c_t^1 + c_t^2$ and capital k_{t+1} for use in the next period production: $C_t + k_{t+1} = f(k_t)$, where $f(k)$ is a neoclassical production function satisfying standard assumptions. Capital is assumed to depreciate completely within one period.

If there is only one consumer (a "representative agent") with the discount factor β and the intertemporal utility function of the form

$$\sum_{t=0}^{\infty} \beta^t u(C_t) = u(C_0) + \beta u(C_1) + \beta^2 u(C_2) + \ldots, \tag{1}$$

then the model at hand is the standard optimal growth model. Given an initial capital stock k_0, the representative agent solves the following problem:

$$\max_{C_t \geq 0,\ k_{t+1} \geq 0} \sum_{t=0}^{\infty} \beta^t u(C_t), \qquad \text{s. t.} \quad C_t + k_{t+1} = f(k_t), \quad t \geq 0. \tag{2}$$

A solution to problem (2), $\{C_t^*, k_{t+1}^*\}_{t=0}^{\infty}$, is an **optimal path** in the economy. As is well known, under standard assumptions there is a unique optimal path.

In the standard optimal growth model, the representative agent with the discount factor β determines the optimal path. However, when there are two different agents, each with her own discount factor β_i, it is not clear who should represent the society and determine the aggregate consumption path. A reasonable way to take into account preferences of both agents is to introduce a social welfare function (SWF henceforth) $W(\mathbf{C}^1, \mathbf{C}^2)$ which evaluates different consumption streams from the perspective of the society as a whole and is maximized by a social planner.

A widely accepted normative property of a SWF is **Pareto optimality**, i.e., the requirement to respect preferences of agents. Formally, $W(\mathbf{C}^1, \mathbf{C}^2)$ *is Pare-*

tian (Pareto-optimal) if for any consumption bundles $\{\mathbf{C}^1, \mathbf{C}^2\}$ *and* $\{\tilde{\mathbf{C}}^1, \tilde{\mathbf{C}}^2\}$, $W(\mathbf{C}^1, \mathbf{C}^2) \geq W(\tilde{\mathbf{C}}^1, \tilde{\mathbf{C}}^2)$ *whenever* $U^i(\mathbf{C}^i) \geq U^i(\tilde{\mathbf{C}}^i)$ *for both* $i = 1, 2$, *and the first inequality is strict whenever the second is strict at least for one* i.

In the two-agent Ramsey model with private consumption, a Paretian SWF naturally appears as a weighted sum of the intertemporal utilities of both agents:

$$\begin{aligned} W(\mathbf{C}^1, \mathbf{C}^2) &= \lambda \sum_{t=0}^{\infty} \beta_1^t u(c_t^1) + (1-\lambda) \sum_{t=0}^{\infty} \beta_2^t u(c_t^2) \\ &= \lambda u(c_0^1) + (1-\lambda) u(c_0^2) + \lambda \beta_1 u(c_1^1) + (1-\lambda)\beta_2 u(c_1^2) + \ldots, \end{aligned} \tag{3}$$

where λ and $1-\lambda$ are constant non-negative Pareto weights.

Given an initial capital stock k_0, the social planner maximizes SWF (3):

$$\begin{aligned} \max_{c_t^i \geq 0,\ k_{t+1} \geq 0} \quad & \lambda \sum_{t=0}^{\infty} \beta_1^t u(c_t^1) + (1-\lambda) \sum_{t=0}^{\infty} \beta_2^t u(c_t^2)\,, \\ \text{s. t.} \quad & c_t^1 + c_t^2 + k_{t+1} = f(k_t), \qquad t \geq 0\,. \end{aligned} \tag{4}$$

A solution to problem (4), $\{c_t^{1*}, c_t^{2*}, k_{t+1}^*\}_{t=0}^{\infty}$, is the **(Pareto-) optimal allocation** in the economy. Clearly, this solution depends on λ, and all Pareto-optimal allocations can be found by varying $0 \leq \lambda \leq 1$. Under some mild assumptions, problem (4) has a unique solution (see, e.g., Le Van and Vailakis, 2003).

Note that in both problems (2) and (4) the sum of discounted utilities is maximized once and for all at date 0. This is an atemporal view on the problems at hand. However, decisions in the real world can hardly be regarded as once-and-for-all choices among specific plans of actions. Instead, they are sequential step-by-step choices based on the presently available opportunities. To illustrate this point, Koopmans (1967, p. 12) uses the following metaphor:

> The problem takes on some of the aspects of the ascent of a mountain wrapped in fog. Rather than searching for a largely invisible optimal path, one may have to look for a good rule for choosing the next stretch of the path with the help of all information available at the time.

A typical difficulty with once-and-for-all choices is a problem of dynamic inconsistency. Optimality criteria in both problems (2) and (4) presume that a decision maker (representative agent or social planner) can credibly commit to implement her decisions in the future. However, if a decision maker cannot precommit her future behavior, then at each date τ ("today") she has to revise the optimal plan implemented at date $\tau - 1$ ("yesterday") and to solve a new problem. In general, a solution to a today's problem is not optimal from the yesterday's perspective, which means that decisions are dynamically inconsistent.[5] Therefore, it is important to recognize that each decision maker actually faces a **sequence** of problems, each problem at different date. This view on growth models, emphasized by Koopmans (1967), is essentially intertemporal.

However, the representative agent formulation overlooks and ignores this fact. Indeed, consider the sequence of problems of the form (2). At date 0 the representative agent solves problem (2) which yields the date-0 optimal path. At date 1, given the date-1 capital stock on the date-0 optimal path, the representative agent solves a new problem:

$$\max_{C_t\geq 0,\ k_{t+1}\geq 0} \sum_{t=1}^{\infty} \beta^{t-1} u(C_t), \qquad \text{s. t.} \quad C_t + k_{t+1} = f(k_t), \qquad t \geq 1.$$

The date-1 utility function, $u(C_1) + \beta u(C_2) + \beta^2 u(C_3) + \ldots$, has the same form as the date-0 utility function (1), because from the perspective of the decision maker timing is restarted. Moreover, being multiplied by β, the date-1 utility function is the truncation of the date-0 utility function starting at date 1. Hence the solution to the date-1 problem is the truncation of the date-0 optimal path starting at date 1, $\{C_t^*, k_{t+1}^*\}_{t=1}^{\infty}$. Thus for the representative agent whose utility function has the form (1), i.e., whose preferences exhibit constant exponential discounting, it makes no difference whether she can commit or not: the optimal path chosen at date 0 once and for all remains optimal at any future date.

This well-known observation creates a wrong impression that preferences of a decision maker can be described by a single utility function and obscures the fact that her preferences are given by a **sequence of utility functions**. This becomes clear in the social planner case when agents have heterogeneous time preferences.

Indeed, consider the sequence of problems of the form (4). At date 1, the social planner solves the following problem:

$$\max_{c_t^i\geq 0,\ k_{t+1}\geq 0} \lambda_1 \sum_{t=1}^{\infty} \beta_1^{t-1} u(c_t^1) + (1-\lambda_1) \sum_{t=1}^{\infty} \beta_2^{t-1} u(c_t^2)\,,$$
$$\text{s. t.} \quad c_t^1 + c_t^2 + k_{t+1} = f(k_t), \qquad t \geq 1\,.$$

where λ_1 and $1-\lambda_1$ are non-negative Pareto weights at date 1. Since this is a new problem at different date, *a priori* there is no reason to assume that the planner

[5]It is well-known that if a current decision explicitly affects future opportunity sets, then currently chosen plan of actions typically will not be followed in the future by the rational agent. The problem of dynamic inconsistency in the optimal growth framework has been studied by, e.g., Strotz (1955); Pollak (1968); Phelps and Pollak (1968); Peleg and Yaari (1973), and many of their insights are helpful in the collective choice framework.

weighs the utilities of agents with the same Pareto weights as at date 0.

The planner's date-1 SWF is given by

$$\lambda_1 u(c_1^1) + (1-\lambda_1)u(c_1^2) + \lambda_1\beta_1 u(c_2^1) + (1-\lambda_1)\beta_2 u(c_2^2) + \dots,$$

and it may differ from the truncation of the date-0 SWF (3) starting at date 1:

$$\lambda\beta_1 u(c_1^1) + (1-\lambda)\beta_2 u(c_1^2) + \lambda\beta_1^2 u(c_2^1) + (1-\lambda)\beta_2^2 u(c_2^2) + \dots.$$

Two cases may arise. If at date 1 the planner uses the same Pareto weights as at date 0, $\lambda_1 = \lambda$, then the date-1 SWF has the same form as the date-0 SWF, but it is not the truncation of the date-0 SWF starting at date 1. Hence the truncation of the date-0 optimal allocation is not the date-1 optimal allocation. On the contrary, if at date 1 the planner uses the adjusted weights, namely,

$$\lambda_1 = \frac{\lambda\beta_1}{\lambda\beta_1 + (1-\lambda)\beta_2}, \qquad 1-\lambda_1 = \frac{\lambda\beta_2}{\lambda\beta_1 + (1-\lambda)\beta_2},$$

then the date-1 SWF is the truncation of the date-0 SWF (up to the constant factor $1/\left(\lambda\beta_1 + (1-\lambda)\beta_2\right)$). In this case, the date-0 optimal allocation remains optimal at date 1. Therefore, the problem of whether the once-and-for-all optimal allocation is the step-by-step optimal allocation is actually the problem of which sequence of SWFs the social planner uses.

2.2 Time consistency, time invariance and stationarity

In order to clarify the notion of dynamic inconsistency, we have to consider **sequences of preferences** and distinguish between three different properties: time consistency, time invariance and stationarity. We will give formal definitions in terms of the sequences of SWFs below.[6] Here we provide three simple examples in terms of individual utility functions which illustrate these properties and explain the differences between them.

First, consider the sequence of utility functions $\{U_\tau^{TC}\}_{\tau=0}^{\infty}$ given by

$$\begin{array}{lcccccccccc} U_0^{TC} & = & u(c_0) & + & \alpha\beta u(c_1) & + & \alpha\beta^2 u(c_2) & + & \dots, & & \\ U_1^{TC} & = & & & u(c_1) & + & \beta u(c_2) & + & \beta^2 u(c_3) & + & \dots, \\ U_2^{TC} & = & & & & & u(c_2) & + & \beta u(c_3) & + & \beta^2 u(c_4) \; + \; \dots, \end{array}$$

[6]General definitions in terms of sequences of preference relations are provided by Halevy (2015). See also definitions in terms of history-dependent intertemporal utility functions in Millner and Heal (2018). Once these properties are defined in terms of sequences, the ideas lying behind these notions become clear and transparent.

and so on. Up to the constant factor $\alpha\beta^\tau$, the utility function U_τ^{TC} at any date τ is the truncation of U_0^{TC} starting at date τ. However, due to the presence of α, U_0^{TC} does not have the same form as U_τ^{TC}. Moreover, discounting in U_0^{TC} is not constant: the discount factor between periods 1 and 0 equals $\alpha\beta$, while it is equal to β between periods $t+1$ and t for all $t \geq 1$. In this case the sequence $\{U_\tau^{TC}\}_{\tau=0}^\infty$ is time-consistent, but not time-invariant and not stationary.

Second, consider the sequence of utility functions $\{U_\tau^{TI}\}_{\tau=0}^\infty$ given by

$$\begin{array}{lcl} U_0^{TI} &=& u(c_0) + \alpha\beta u(c_1) + \alpha\beta^2 u(c_2) + \ldots, \\ U_1^{TI} &=& u(c_1) + \alpha\beta u(c_2) + \alpha\beta^2 u(c_3) + \ldots, \\ U_2^{TI} &=& u(c_2) + \alpha\beta u(c_3) + \alpha\beta^2 u(c_4) + \ldots, \end{array}$$

and so on. The utility function U_0^{TI} has the same form as U_τ^{TI} at any date τ. However, U_τ^{TI} is not the truncation of U_0^{TI}, and again discounting in U_τ^{TI} is not constant. In this case the sequence $\{U_\tau^{TI}\}_{\tau=0}^\infty$ is time-invariant, but not time-consistent and not stationary.

Finally, consider the sequence of utility functions $\{U_\tau^{ST}\}_{\tau=0}^\infty$ given by

$$\begin{array}{lcl} U_0^{ST} &=& u(c_0) + \beta_0 u(c_1) + \beta_0^2 u(c_2) + \ldots, \\ U_1^{ST} &=& u(c_1) + \beta_1 u(c_2) + \beta_1^2 u(c_3) + \ldots, \\ U_2^{ST} &=& u(c_2) + \beta_2 u(c_3) + \beta_2^2 u(c_4) + \ldots, \end{array}$$

and so on, where all β_τ are different. For any τ, the discount factor in U_τ^{ST} is constant, and each U_τ^{ST} is an exponentially discounted utility function. However, U_τ^{ST} depends on τ and is not related to any other utility function in this sequence. In this case the sequence $\{U_\tau^{ST}\}_{\tau=0}^\infty$ is stationary, but not time-consistent and not time-invariant.

To define time consistency, time invariance and stationarity for the sequences of SWFs and to link these properties directly to discounting, we assume that the sequence of preferences over bundles of consumption streams $\mathbf{C} = \{\mathbf{C}^1, \mathbf{C}^2\}$ at different decision dates τ is given by the following sequence of SWFs:

$$W_\tau = \sum_{i=1}^{2} \lambda_\tau^i \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^i \, u(c_t^i) = \lambda_\tau^1 \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^1 \, u(c_t^1) + \lambda_\tau^2 \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^2 \, u(c_t^2), \qquad \tau \geq 0,$$

where λ_τ^1 and λ_τ^2 are non-negative Pareto weights at date τ, and $\{\gamma_{\tau,t}^i\}_{t=\tau}^\infty \equiv \Gamma_\tau^i$ is a **discount function** of agent i at date τ.[7] In other words, we assume that every

[7]The term “discount function” originally appeared in continuous-time models and may be somewhat misleading in a discrete-time framework. Though it is more correct to refer to Γ_τ^i as a discount sequence, we will continue to use the more familiar term discount function.

element in the sequence of preferences (SWF at some decision date) is a weighted sum of additively time-separable utility functions (individual utility functions). The sequence of individual utility functions has the form

$$U^i_\tau = \sum_{t=\tau}^{\infty} \gamma^i_{\tau,t}\, u(c^i_t), \qquad \tau \geq 0,$$

i.e., in this sequence the felicity function of agent i, $u(c)$, is the same in all U^i_τ (at all dates and for both agents), but discount functions of agent i, Γ^i_τ, may differ for different dates τ.

Note that for all τ, discount function Γ^i_τ and Pareto weights are defined up to a constant factor. Sequences $\{\gamma^i_{\tau,\tau}, \gamma^i_{\tau,\tau+1}, \gamma^i_{\tau,\tau+2}, \ldots\}$ and $\{1, \frac{\gamma^i_{\tau,\tau+1}}{\gamma^i_{\tau,\tau}}, \frac{\gamma^i_{\tau,\tau+2}}{\gamma^i_{\tau,\tau}}, \ldots\}$ determine the same utility function U^i_τ. Similarly, pairs $\{\lambda^1_\tau, \lambda^2_\tau\}$ and $\{\frac{\lambda^1_\tau}{\lambda^1_\tau+\lambda^2_\tau}, \frac{\lambda^2_\tau}{\lambda^1_\tau+\lambda^2_\tau}\}$ determine the same SWF W_τ. Without loss of generality we apply the following standard normalizations:

$$\gamma^i_{\tau,\tau} = 1, \quad \tau \geq 0, \quad i = 1, 2, \qquad\qquad \lambda^1_\tau + \lambda^2_\tau = 1, \quad \tau \geq 0.$$

Clearly, the sequence of SWFs $\{W_\tau\}_{\tau=0}^{\infty}$ is fully determined by the sequences of discount functions and Pareto weights $\{\Gamma^1_\tau, \Gamma^2_\tau; \lambda^1_\tau, \lambda^2_\tau\}_{\tau=0}^{\infty}$.

Now let us define three properties of the sequences of SWFs. **Time consistency**, introduced by Strotz (1955), requires that different elements of the sequence of preferences are consistent in the sense that SWFs at different dates preserve the preference order between any two consumption streams. Time consistency is a rationality criterion implying that the choice for any future date is independent of the decision date. If a sequence of planner's preferences is time-inconsistent, she may reverse a decision made at the earlier date, e.g., undertake certain expensive project this year, but cancel it halfway next year. Formally, *a sequence* $\{W_\tau\}_{\tau=0}^{\infty}$ *is time-consistent if for any* τ, $\tau' > \tau$, *and any two bundles of consumption streams* $\mathbf{C}$ *and* $\tilde{\mathbf{C}}$,

$$\sum_{i=1}^{2} \lambda^i_\tau \sum_{t=\tau'}^{\infty} \gamma^i_{\tau,t}\, u(c^i_t) \;\geq\; \sum_{i=1}^{2} \lambda^i_\tau \sum_{t=\tau'}^{\infty} \gamma^i_{\tau,t}\, u(\tilde{c}^i_t)$$
$$\Longleftrightarrow \sum_{i=1}^{2} \lambda^i_{\tau'} \sum_{t=\tau'}^{\infty} \gamma^i_{\tau',t}\, u(c^i_t) \;\geq\; \sum_{i=1}^{2} \lambda^i_{\tau'} \sum_{t=\tau'}^{\infty} \gamma^i_{\tau',t}\, u(\tilde{c}^i_t).$$

Time invariance, considered by Halevy (2015), requires that different elements of the sequence of preferences are the same. In other words, time invariance implies that preferences do not depend on calendar time, but otherwise can have arbitrary structure. Formally, *a sequence* $\{W_\tau\}_{\tau=0}^{\infty}$ *is time-invariant if for any* τ, $\tau' > \tau$, $\mathbf{C}$ *and* $\tilde{\mathbf{C}}$,

$$\sum_{i=1}^{2} \lambda_\tau^i \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^i \; u(c_t^i) \;\geq\; \sum_{i=1}^{2} \lambda_\tau^i \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^i \; u(\tilde{c}_t^i)$$
$$\Longleftrightarrow \sum_{i=1}^{2} \lambda_{\tau'}^i \sum_{t=\tau}^{\infty} \gamma_{\tau',\tau'+t-\tau}^i \; u(c_t^i) \;\geq\; \sum_{i=1}^{2} \lambda_{\tau'}^i \sum_{t=\tau}^{\infty} \gamma_{\tau',\tau'+t-\tau}^i \; u(\tilde{c}_t^i).$$

Stationarity, introduced by Koopmans (1960), requires that for a given decision date the preference order between any two consumption streams is preserved when the streams are postponed by the same amount of time. This property plays a key role in the Koopmans' axiomatization of discounted utilitarian time preferences. Formally, *a SWF* W_τ *is stationary if for any* $\tau' > \tau$, $\mathbf{C}$ *and* $\tilde{\mathbf{C}}$,

$$\sum_{i=1}^{2} \lambda_\tau^i \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^i \; u(c_t^i) \;\geq\; \sum_{i=1}^{2} \lambda_\tau^i \sum_{t=\tau}^{\infty} \gamma_{\tau,t}^i \; u(\tilde{c}_t^i)$$
$$\Longleftrightarrow \sum_{i=1}^{2} \lambda_{\tau}^i \sum_{t=\tau}^{\infty} \gamma_{\tau,\tau'+t-\tau}^i \; u(c_t^i) \;\geq\; \sum_{i=1}^{2} \lambda_{\tau}^i \sum_{t=\tau}^{\infty} \gamma_{\tau,\tau'+t-\tau}^i \; u(\tilde{c}_t^i).$$

A sequence $\{W_\tau\}_{\tau=0}^{\infty}$ *is stationary if* W_τ *is stationary for each* τ.

Note that stationarity is a property of preferences at a given decision date. Time consistency and time invariance deals with decisions made at different dates, and hence they are essentially applied to the sequences of preferences.[8]

The above definitions are very cumbersome. They are usually given in a general axiomatic framework, where they look even more obscure. However, they are related to simple properties of discounting, because time consistency, time invariance and stationarity of the sequence of SWFs imply special forms of the individuals' discount functions.

A sequence of SWFs is **time-consistent** if and only if for each agent at any date the Pareto weights are proportional to that at a previous date, and the discount functions are truncations of that at a previous date:

$$\lambda_{\tau'}^i = \frac{\lambda_\tau^i \gamma_{\tau,\tau'}^i}{\lambda_\tau^1 \gamma_{\tau,\tau'}^1 + \lambda_\tau^2 \gamma_{\tau,\tau'}^2}, \qquad \text{and} \qquad \gamma_{\tau',t}^i = \frac{\gamma_{\tau,t}^i}{\gamma_{\tau,\tau'}^i}, \quad t \geq \tau', \qquad \forall\; \tau,\; \tau',\; i = 1, 2.$$

A sequence of SWFs is **time-invariant** if and only if for each agent the Pareto

[8]These definitions can be easily generalized to the cases where any number of agents with arbitrary felicity functions have private or common consumption streams (see also Section 3).

weights and the discount functions at any date are the same:

$$\lambda^i_{\tau'} = \lambda^i_\tau, \qquad \text{and} \qquad \gamma^i_{\tau',\tau'+t-\tau} = \gamma^i_{\tau,t}, \quad t \geq \tau, \qquad \forall\ \tau,\ \tau',\ i = 1, 2.$$

This is the only way to ensure that a SWF W_τ does not depend on τ.

A SWF is **stationary** if and only if both individual utility functions exhibit constant exponential discounting with the same discount factor:

$$\gamma^i_{\tau,t} = \beta^{t-\tau}_\tau, \qquad t \geq \tau, \qquad \forall\ \tau,\ i = 1, 2.$$

This is the famous result of Koopmans (1960) in our setting. Whenever the discount factors of agents are different, the SWF which takes into account both of them is non-stationary.

It is clear that time consistency, time invariance and stationarity are closely interrelated. By combining the above conditions, it is easily seen that *a sequence of SWFs satisfies any two of these three properties if and only if for each agent the Pareto weights are the same at any date, and both agents have exponential discount functions with* ***the same discount factor*** β:

$$\lambda^i_\tau = \lambda^i, \qquad \text{and} \qquad \gamma^i_{\tau,t} = \beta^{\tau-t}, \quad t \geq \tau, \qquad \forall\ \tau, \quad i = 1, 2.$$

In other words, while time consistency, time invariance and stationarity are pairwise independent, any two properties imply the third. This is a general result of Halevy (2015) in our setting. Moreover, a sequence of SWFs $\{W_\tau\}_{\tau=0}^\infty$ satisfies any two of the considered properties if and only if it has the form

$$W_\tau \ = \ \sum_{i=1}^2 \lambda^i \sum_{t=\tau}^\infty \beta^{t-\tau} u(c^i_t) \ = \ \sum_{t=\tau}^\infty \beta^{t-\tau} \sum_{i=1}^2 \lambda^i u(c^i_t), \qquad \tau \geq 0.$$

2.3 Properties of social welfare functions

Let us discuss these properties in the two-agent Ramsey model with private consumption. The sequence of individual preferences naturally has the form

$$U^i_\tau \ = \ \sum_{t=\tau}^\infty \beta_i^{t-\tau} u(c^i_t), \qquad \tau \geq 0.$$

For each agent, this sequence is time-consistent, time-invariant and stationary. Which of these properties are inherited by the sequence of the planner's SWFs?

The first conclusion is that *a Paretian SWF which takes into account hetero-*

geneous preferences of both agents is necessarily non-stationary. At any date τ, a weighted sum of individual utilities,

$$W_\tau = \lambda_\tau \sum_{t=\tau}^{\infty} \beta_1^{t-\tau} u(c_t^1) + (1-\lambda_\tau) \sum_{t=\tau}^{\infty} \beta_2^{t-\tau} u(c_t^2),$$

is non-stationary unless the planner's preferences essentially depend on a single discount factor, i.e., either $\lambda_\tau = 1$ and W_τ coincides with U_τ^1; $\lambda_\tau = 0$ and W_τ coincides with U_τ^2; or $\beta_1 = \beta_2$ in which case agents are in fact homogeneous.

Moreover, as we have seen, only in these three cases the sequence of planner's preferences simultaneously satisfies time consistency, time invariance and stationarity. Thus a simple two-agent case shows that a key role is played by heterogeneity in discount factors. This conclusion is a particular case of a general result proved by Zuber (2011). In his framework, agents with arbitrary preferences choose arbitrary consumption streams. He studies the properties of utilitarian aggregations, i.e., SWFs in which individual utility functions get equal weights. In our terms, utilitarian aggregation combines Pareto optimality and time invariance. Zuber proves that the sequence of planner's preferences is Paretian, time-invariant, time-consistent and stationary if and only if all sequences of individual preferences exhibit constant exponential discounting with the same discount factor. Zuber (2011, p. 375) concludes that

> Although some people seem to be more patient than others, any departure from the homogeneous patience case would introduce non-stationarity in the planner's objective.

Consider the implications of non-stationarity. Rewrite the date-τ SWF as

$$W_\tau = \sum_{t=\tau}^{\infty} \beta_1^{t-\tau} \left\{ \lambda_\tau u(c_t^1) + \left(\frac{\beta_2}{\beta_1}\right)^{t-\tau} (1-\lambda_\tau) u(c_t^2) \right\}.$$

Since agent 1 is the most patient agent, the factor $(\beta_2/\beta_1)^{t-\tau}$ converges to zero as $t \to \infty$. Hence for $0 < \lambda_\tau < 1$, the relative weight associated with consumption of the less patient agent decreases and becomes arbitrarily small in the long run. The most patient agent eventually dominates in any SWF, and increasingly influences consumption decisions of the society.

Furthermore, since the most patient agent emerges as the dominant consumer in the SWF, her socially optimal level of consumption is always positive, and her share in aggregate consumption tends to 1. At the same time, the socially optimal level of consumption and the share in aggregate consumption of the less patient agent tend to 0. Informally speaking, in any Pareto-optimal allocation in the many-agent Ramsey model all the less patient agents eventually starve to death. Thus the major source of difficulties with standard social welfare criteria in heterogeneous agents models is precisely the non-stationarity of a Paretian SWF.

The non-stationarity of a social planner who aggregates heterogeneous time preferences was also established in many other settings. Gollier and Zeckhauser (2005) consider a continuous-time model where a social planner determines a Pareto-optimal allocation of an exogenously given flow of a single non-storable consumption good among agents. They construct a Paretian SWF and prove that the instantaneous discount rate of the social planner is a weighted mean of the agents' discount rates with weights being proportional to the corresponding agents' tolerances for consumption fluctuations. Whenever agents differ in their discount rates, a Paretian social planner has time-varying discount rate.[9] Under standard assumptions on individual preferences, the weights of the more patient agents are increasing over time, and the discount rate of a social planner tends to the discount rate of the most patient agent.[10]

The second conclusion is that *a non-stationary sequence of SWFs can be either time-consistent or time-invariant.* If the planner uses the same positive Pareto weights λ and $1-\lambda$ at each date, then the sequence of SWFs $\{W_\tau^{TI}\}_{\tau=0}^{\infty}$ given by

$$W_\tau^{TI} \;=\; \lambda \sum_{t=\tau}^{\infty} \beta_1^{t-\tau} u(c_t^1) + (1-\lambda) \sum_{t=\tau}^{\infty} \beta_2^{t-\tau} u(c_t^2), \qquad \tau \geq 0,$$

is time-invariant, though not time-consistent. However, if at date τ the planner assigns to agents the positive weights

$$\lambda_\tau^1 = \frac{\lambda \beta_1^\tau}{\lambda \beta_1^\tau + (1-\lambda)\beta_2^\tau}, \qquad \lambda_\tau^2 = 1 - \lambda_\tau^1 = \frac{(1-\lambda)\beta_2^\tau}{\lambda \beta_1^\tau + (1-\lambda)\beta_2^\tau}, \qquad 0 \leq \lambda \leq 1,$$

then the sequence $\{W_\tau^{TC}\}_{\tau=0}^{\infty}$ given by

$$W_\tau^{TC} \;=\; \lambda_\tau^1 \sum_{t=\tau}^{\infty} \beta_1^{t-\tau} u(c_t^1) + \lambda_\tau^2 \sum_{t=\tau}^{\infty} \beta_2^{t-\tau} u(c_t^2), \qquad \tau \geq 0,$$

[9] Hara (2019) shows that the more heterogeneous in the discount rates are agents, the higher is the degree of non-stationarity of a social planner (in terms of equivalence classes of decreasing impatience).

[10] Heal and Millner (2013) generalize the results of Gollier and Zeckhauser (2005) for the case where agents derive consumption from a commonly managed resource. They also find that the discount rate of a social planner converges to the discount rate of the most patient agent. Note that this observation for the rates of time preference is analogous to the results of Weitzman (1998, 2001) for the real interest rates under uncertainty.

is time-consistent, though not time-invariant.[11] Indeed, up to the constant factor $1/\left(\lambda\beta_1^\tau + (1-\lambda)\beta_2^\tau\right)$, the date-$\tau$ SWF has the form

$$W_\tau^{TC} = \lambda\beta_1^\tau u(c_\tau^1) + (1-\lambda)\beta_2^\tau u(c_\tau^2) + \lambda\beta_1^{\tau+1} u(c_{\tau+1}^1) + (1-\lambda)\beta_2^{\tau+1} u(c_\tau^2) + \dots,$$

and is the truncation of any previous date-t SFW W_t^{TC}.

The above observation shows that a **naive** social planner who recalculates the optimal allocation at each date is time-consistent only at the expense of time invariance — due to the adjusted Pareto weights. Another way to ensure time consistency is to consider a **sophisticated** social planner who is aware of her time-inconsistent preferences and chooses the best path among those she would actually follow.[12] Drugeon and Wigniolle (2016) study a sophisticated (time-consistent) optimal allocation in the many-agent Ramsey model with private consumption and show that it can be obtained as a once-and-for-all solution to a date-0 problem with some effective time-varying discount function. In the two-agent case, this problem takes the form

$$\max_{c_t^i \geq 0,\ k_{t+1} \geq 0} \sum_{t=0}^{\infty} \gamma_{0,t} \left(\lambda u(c_t^1) + (1-\lambda) u(c_t^2)\right), \quad \text{s. t.} \quad c_t^1 + c_t^2 + k_{t+1} = f(k_t), \quad t \geq 0,$$

for a properly chosen discount function $\{\gamma_{0,t}\}_{t=0}^\infty$. Drugeon and Wigniolle analyze the properties of this solution and show that if agents differ in their discount factors, but have identical felicity functions and get equal weights in the SWF ($\lambda = 1/2$), then the sophisticated optimal allocation implies the same consumption for both agents ($c_t^{*1} = c_t^{*2}$ for all t). The distribution of discount factors in the population determines only aggregate consumption level at different dates, but does not influence the distribution of consumption between agents.

The many-agent Ramsey model with private consumption considered above raises certain issues with the very notion of Pareto optimality. However, when consumption is private, any Pareto-optimal allocation can be decentralized as a competitive equilibrium, which creates an impression that all the difficulties with constructing a SWF are not particularly relevant. After all, an optimal allocation can be reached by establishing a complete system of financial markets in which consumers can lend or borrow against their wealth and letting markets do their job. In what follows we will see that this impression is somewhat misleading, because a general equilibrium approach does not apply when consumption is common.

[11]The properties of SWFs with general time-varying Pareto weights are studied by Alcalá (2016).

[12]Sophisticated paths, introduced by Strotz (1955) and Pollak (1968), can be described as Markov-perfect Nash equilibria in a dynamic game played among the planner's different temporal selves.

3 Social optima under common consumption

In this section we turn to the many-agent Ramsey model with common consumption, which resembles the model studied above, but in which collective choice plays a central role. Instead of independent private consumption streams, agents share a common consumption stream arising from a collectively consumed public good or a common property resource. Individual utilities are based on collective decisions, and an important question is how a common consumption path can be chosen.

3.1 Motivation and the model

Problems in which society should make a collective decision naturally arise in a wide range of settings related to common property resources. Examples include hunting for animals or grazing of cattle in a common land, pollution of the atmosphere or drilling for oil in the common underground reservoir.

Consider a village situated near a fishing ground which is self-managed by the citizens who differ in their time preferences. The fish stock exploitation is determined by the harvest rate collectively set by heterogeneous agents. If all village citizens were identical, then their common discount factor would determine the rate of resource exploitation. However, it is not clear, what is the harvest rate of the fish stock when there are many different discount factors.

The discussion of common property resource management was initiated by Hardin (1968) who noted that an open-access resource tends to be overexploited and referred to this situation as the "tragedy of the commons". The absence of property rights or difficulties with establishing them lead to the exploitation of the resource at an excessive rate (compared to a socially optimal one). A free access market equilibrium is not Pareto-optimal: in the aforementioned examples there is always excessive fishing, overgrazing or redundantly rapid depletion of oil.

A typical solution to the tragedy of the commons is to establish private property rights. If instead of the fishing ground there is a meadow near the village where citizens graze their cattle, this meadow can be divided into equal plots, and citizens can be assigned proprietary rights over these plots. The introduction of private ownership decentralizes the decision-making process, and the resource exploitation becomes optimal from the perspective of the society.[13] When citizens make individually rational decisions on how many cattle to graze, each citizen's choice does not affect the ability of others to graze, and there is no damage to the commons. Once property rights are established, each owner acts optimally

[13]Of course, for this result to hold, property rights should be established and enforced costlessly.

according to her own rate of time preference. Thus a socially optimal outcome can be successfully achieved by decentralization, which is similar to the private consumption case considered in Section 2.

The situation is different in the case of common property resources or public goods, e.g., fishing ground or Earth's climate. The non-excludability of these goods prevents the enforcement of suitable private property rights or makes the introduction of private ownership extremely costly. For instance, an obvious tendency of fish to migrate makes it impossible to define property rights over the fish stock or parcel the fishing ground into different plots.

A typical solution in these cases is to introduce a governmental or community resource ownership and then use quota and licensing systems. If the sum of quotas is equal to the socially optimal outcome, then the competitive price determined by supply and demand in the market for quotas ensures optimal resource exploitation. In particular, this idea underlies a system of tradable carbon permits (cap-and-trade) aimed at reducing carbon emissions and combating climate change. A problem with this approach is that there are no market forces to determine the socially optimal outcome. In particular, the socially optimal level of total catch depends on the harvest rate collectively set by the village citizens. Hence we are back to the central question of a common property resource management: what is the rate of resource exploitation in a heterogeneous society? This discussion provides a clear incentive to study decision-making process under heterogeneous time preferences in the presence of a common consumption stream.

As a natural framework for the analysis of collective intertemporal decisions, we employ a one-sector two-agent Ramsey model with common consumption. Again, to simplify the presentation, two agents differ only in their discount factors. As in Section 2, in order to discuss difficulties with collective intertemporal choice, we have to consider sequences of preferences.

Preferences of agent i over consumption streams $\mathbf{C} = \{c_t\}_{t=0}^{\infty}$ at decision date τ are given by the additively time-separable intertemporal utility function:

$$U_{\tau}^{i}(\mathbf{C}) \;=\; \sum_{t=\tau}^{\infty} \beta_i^{t-\tau} u(c_t), \qquad \tau \geq 0, \tag{5}$$

where β_i is the constant discount factor and $u(c)$ is the felicity function. As before, agent 1 is more patient than agent 2: $1 > \beta_1 > \beta_2 > 0$.

We assume that $\mathbf{C} = \{c_t\}_{t=0}^{\infty}$ is a common consumption stream. Consumption c_t can be thought of as the amount of the extracted renewable or exhaustible natural resource (public good). The increase in the resource stock, $k_{t+1} - k_t$, i.e., its regenerative capacity, is described by a regeneration function $g(k_t)$ which depends on the current size of the stock. The dynamics of the resource stock can be written as the usual resource constraint $c_t+k_{t+1} = f(k_t)$, where $f(k) = g(k)+k$. If a resource is exhaustible, then $f(k) = k$. If a resource is renewable, it is assumed that $f(k)$ satisfies the same properties as a neoclassical production function. This model resembles the many-agent Ramsey model with private consumption, but has a convenient interpretation as a common property resource model.

Each agent $i = 1, 2$ at each date τ can determine an optimal consumption path from her perspective, by solving the following problem:

$$\max_{c_t \geq 0,\ k_{t+1} \geq 0} \sum_{t=\tau}^{\infty} \beta_i^{t-\tau} u(c_t), \qquad \text{s. t.} \quad c_t + k_{t+1} = f(k_t), \quad t \geq \tau. \tag{6}$$

Since two agents differ in their time preferences, at each date there are two different optimal consumption paths, each path is optimal for different agent. Which path will be chosen by the society consisting of these agents?

It is natural to assume that each agent plays a role in collective decision making, and thus the question is how to aggregate heterogeneous preferences at each date, i.e., how to construct an appropriate SWF. There are two principal ways to do this. The first is to introduce a social planner with some SWF, and determine a common consumption path by maximizing this function. The second is to determine a common consumption path directly by some social choice procedure (e.g., majority voting), and SWF is induced by the outcome of this procedure. This section studies the first of these ways. For the discussion of the second way, see Section 4.

3.2 Properties of social welfare functions

Suppose that a common consumption path is determined by a social planner. At each particular date she maximizes a SWF (collective utility function) which is constructed from individual utility functions and aggregates the preferences of different agents. The (date-τ) SWF $W_\tau(\mathbf{C})$ evaluates different consumption streams from the perspective of the society.

An important property of a SWF is **Pareto optimality**. Formally, $W_\tau(\mathbf{C})$ *is Paretian (Pareto-optimal) if for any consumption streams* $\mathbf{C}$ *and* $\tilde{\mathbf{C}}$, $W_\tau(\mathbf{C}) \geq W_\tau(\tilde{\mathbf{C}})$ *whenever* $U_\tau^i(\mathbf{C}) \geq U_\tau^i(\tilde{\mathbf{C}})$ *for both* $i = 1, 2$, *and the first inequality is strict whenever the second is strict at least for one* i. Pareto optimality means that if all agents prefer $\mathbf{C}$ over $\tilde{\mathbf{C}}$, a social planner also prefers $\mathbf{C}$ over $\tilde{\mathbf{C}}$, i.e., a Paretian SWF respects unanimous preference of agents. This property is sometimes referred to as unanimity and is a minimum reasonable requirement for a SWF.

In the two-agent Ramsey model with common as well as with private consumption, a Paretian date-τ SWF naturally appears as a weighted sum of the individual date-τ utility functions:

$$W_\tau = \lambda_\tau^1 \sum_{t=\tau}^{\infty} \beta_1^{t-\tau} u(c_t) + \lambda_\tau^2 \sum_{t=\tau}^{\infty} \beta_2^{t-\tau} u(c_t) = \sum_{t=\tau}^{\infty} \left(\lambda_\tau^1 \beta_1^{t-\tau} + \lambda_\tau^2 \beta_2^{t-\tau}\right) u(c_t), \qquad (7)$$

where the Pareto weights λ_τ^1 and λ_τ^2 are non-negative and sum to one. Given an initial capital stock k_τ, a social planner determines the consumption path for the society at date τ by maximizing SWF (7):

$$\max_{c_t \geq 0,\ k_{t+1} \geq 0} \sum_{t=\tau}^{\infty} \left(\lambda_\tau^1 \beta_1^{t-\tau} + \lambda_\tau^2 \beta_2^{t-\tau}\right) u(c_t), \qquad \text{s. t.} \quad c_t + k_{t+1} = f(k_t), \quad t \geq \tau.$$

Note that Paretian date-τ SWF (7) resembles the date-τ individual utility function in (5), and is also additively time-separable. The sequence of SWFs has the form

$$W_\tau = \sum_{t=\tau}^{\infty} \gamma_{\tau,t} u(c_t), \qquad \tau \geq 0, \qquad (8)$$

where at each date τ the planner's discount function $\{\gamma_{\tau,t}\}_{t=\tau}^{\infty}$ is the weighted average of the individual discount functions with Pareto weights λ_τ^1 and λ_τ^2:

$$\gamma_{\tau,t} = \lambda_\tau^1 \beta_1^{t-\tau} + \lambda_\tau^2 \beta_2^{t-\tau}, \qquad t \geq \tau.$$

Due to heterogeneity in discount factors, the planner's discount function at date τ is not a geometric progression unless $\lambda_\tau^1 = 1$ (in which case $\gamma_{\tau,t} = \beta_1^{t-\tau}$) or $\lambda_\tau^1 = 0$ (in which case $\gamma_{\tau,t} = \beta_2^{t-\tau}$). Hence the problem of a social planner is in general a time-varying discounted optimal growth problem. Under reasonable assumptions on $u(c)$ and $f(k)$, this problem also has a solution.

Similarly to the results of Section 2, the sequence of SWFs (8) is **time-consistent** if and only if the discount function at any date is a truncation (up to a constant factor) of the discount function at a previous date: for all τ and $\tau' > \tau$,

$$\gamma_{\tau',t} = \frac{\gamma_{\tau,t}}{\gamma_{\tau,\tau'}}, \qquad t \geq \tau'.$$

This sequence is **time-invariant** if and only if the discount functions at any date are the same: for all τ and $\tau' > \tau$,

$$\gamma_{\tau',\tau'+t-\tau} = \gamma_{\tau,t}, \qquad t \geq \tau.$$

This sequence is **stationary** if and only if for all τ,

$$\gamma_{\tau,t} \;=\; \beta_\tau^{t-\tau}, \qquad t \geq \tau.$$

It is again clear that time consistency, time invariance and stationarity are interrelated. *A sequence of SWFs* (8) *satisfies any two of these three properties if and only if each SWF exhibits constant exponential discounting with* ***the same discount factor***. Thus a sequence of SWFs $\{W_\tau\}_{\tau=0}^{\infty}$ satisfies any two of these properties if and only if it is of the form

$$W_\tau \;=\; \sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t), \qquad \tau \geq 0,$$

and hence it necessarily satisfies the third property.

It is now easily seen that aggregation of heterogeneous time preferences in the common consumption case is subject to the same difficulties as in the private consumption case. The sequences of individual utility functions (5) are time-consistent, time-invariant and stationary, while the sequence of SWFs (8) is in general not. The sequence of SWFs is time-consistent, time-invariant and stationary if and only if one of the following conditions holds: either the weight of agent 1 is always equal to 1, $\lambda_\tau^1 = 1$, $\forall \tau$; the weight of agent 2 is always equal to 1, $\lambda_\tau^2 = 1$, $\forall \tau$; or both agents have the same discount factor, $\beta_1 = \beta_2$. In all these three cases, collective preferences essentially depend on a single discount factor.

This observation is a particular case of a general result obtained by Jackson and Yariv (2015). Their result can be stated in our setting as follows: a sequence of Paretian time-invariant SWFs is time-consistent if and only if each element of this sequence exhibits constant exponential discounting with the same discount factor which is **exactly that of some agent** i.[14] If Pareto weights of at least two agents with different discount factors are positive, then a time-invariant sequence of SWFs is not stationary and not time-consistent.

Jackson and Yariv (2015, p. 161) formulate their result as follows:

> If there is any heterogeneity in temporal preferences by way of differing discount factors, then the only well-behaved collective utility functions that are time consistent and respect unanimity are dictatorial: they ignore the preferences of all but one agent (or a group of agents who share the same exact preferences).

[14]The result of Jackson and Yariv (2015) is very similar to that of Zuber (2011), but obtained in a slightly different setting. Zuber studies independent and private consumption streams, while Jackson and Yariv consider common consumption streams.

This "impossibility result" deserves some discussion.

First, any heterogeneity in discount factors leads to non-stationarity of a Paretian SWF. The relative weight of the most patient agent increases, and the instantaneous discount factor of a social planner increases and converges to that of the most patient agent. This fact in the common consumption setting is noted by Nocetti et al. (2008), who also show that the more patient or the more heterogeneous are agents, the higher is the planner's discount factor. However, unlike the private consumption case, the non-stationarity of SWF does not cause the less patient agents to consume nothing, since agents have common consumption stream.

Second, it should be emphasized that Jackson and Yariv (2015) implicitly consider only time-invariant sequences of SWFs. Millner and Heal (2018) note that there is in fact a trade-off between time consistency and time invariance for a non-stationary sequence of SWFs. They show that if a planner assigns different Pareto weights at different dates, then time consistency can be achieved at the expense of time invariance.

This observation can be illustrated in our two-agent case. Suppose that at each date τ the planner assigns to agents the same positive weights, $\lambda_\tau^1 = \lambda$ and $\lambda_\tau^2 = 1-\lambda$. Then the sequence of SWFs (8) takes the form

$$W_\tau^{TC} \; = \; \sum_{t=\tau}^{\infty} \left(\lambda\beta_1^{t-\tau} + (1-\lambda)\beta_2^{t-\tau}\right) u(c_t), \qquad \tau \geq 0,$$

and this sequence is time-invariant, though not time-consistent. Instead, suppose that at each date τ the planner assigns to agents different weights, namely:

$$\lambda_\tau^1 = \frac{\lambda\beta_1^\tau}{\lambda\beta_1^\tau + (1-\lambda)\beta_2^\tau}, \qquad \lambda_\tau^2 = \frac{(1-\lambda)\beta_2^\tau}{\lambda\beta_1^\tau + (1-\lambda)\beta_2^\tau},$$

Then the sequence of SWFs takes the form

$$W_\tau^{TI} \; = \; \frac{1}{\lambda\beta_1^\tau + (1-\lambda)\beta_2^\tau} \sum_{t=\tau}^{\infty} \left(\lambda\beta_1^t + (1-\lambda)\beta_2^t\right) u(c_t), \qquad \tau \geq 0,$$

and this sequence is time-consistent, though not time-invariant. Millner and Heal (2018) conclude that the choice between these two properties of the planner's preferences is purely normative, and argue that time consistency may be more attractive for intragenerational choices, while time invariance is more suitable for intergenerational choices.

Third, a number of recent studies suggest certain ways to circumvent the non-stationarity of a utilitarian aggregation of heterogeneous time preferences by weakening the standard notion of utilitarianism. Chambers and Echenique (2018) show that a SWF is Paretian and stationary if and only if it is based on the maximin criterion (i.e., the society evaluates a consumption stream according to the agent that values it the least). In accordance with previous findings, whenever agents differ in their discount factors, any date-τ SWF based on the utilitarian criterion (weighted sum of individual utilities) necessarily violates stationarity.

Feng and Ke (2018) argue that in the context of intergenerational choice the notion of Pareto optimality is too strong. Consider our Ramsey model with common consumption as an intergenerational model.[15] Then the standard Pareto property is essentially "current-generation", because at each date τ the planner takes into account only the preferences of generation τ. However, all future generations are also affected by the decision made at date τ, and hence the planner should take into account their preferences as well already at date τ. This motivates the weaker "intergenerational Pareto" property: if one consumption stream is preferred over another by every agent from every generation, then the planner also prefers the former. Clearly, if the planner is current-generation Paretian, she is also intergenerationally Paretian, but not vice versa.

Recall that in our two-agent case the current-generation Paretian date-τ SWF is given by (7), for some weights λ_τ^1 and λ_τ^2. The intergenerationally Paretian date-τ SWF has the form

$$\begin{aligned} W_\tau^{IGP} &= \sum_{t=\tau}^{\infty} \left\{ \lambda_{\tau,t}^1 \sum_{s=t}^{\infty} \beta_1^{s-t} u(c_s) + \lambda_{\tau,t}^2 \sum_{s=t}^{\infty} \beta_2^{s-t} u(c_s) \right\} \\ &= \left(\lambda_{\tau,\tau}^1 + \lambda_{\tau,\tau}^2\right) u(c_\tau) + \left(\lambda_{\tau,\tau+1}^1 + \lambda_{\tau,\tau+1}^2 + \lambda_{\tau,\tau}^1 \beta_1 + \lambda_{\tau,\tau}^2 \beta_2\right) u(c_{\tau+1}) + \ldots, \end{aligned}$$

where the sequence of Pareto weights is such that $0 < \sum_{t=\tau}^{\infty} \left\{\lambda_{\tau,t}^1 + \lambda_{\tau,t}^2\right\} < \infty$.

Feng and Ke (2018) show that for any $\beta > \beta_2$, a date-τ SWF given by

$$W_\tau^{IGP} = \sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t),$$

is intergenerationally Paretian and strongly non-dictatorial, i.e., the planner at each date τ does not ignore the preferences of any agent from any generation.[16]

[15]The sequence of individual utility functions (5) is interpreted as a sequence of intertemporal utility functions of different agents from successive generations. Agent i from generation τ lives for one period, and has intertemporal utility function U_τ^i. Her offspring, agent i from generation $\tau + 1$, inherits her discount factor and felicity function, and has utility function $U_{\tau+1}^i$.

[16]Feng and Ke (2018) actually consider the finite horizon case, and Feng et al. (2020) generalize these results to the infinite horizon case.

In other words, there exist strictly positive Pareto weights such that for each τ,

$$\sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t) \;=\; \sum_{t=\tau}^{\infty} \left\{ \lambda_{\tau,t}^{1} \, \sum_{s=t}^{\infty} \beta_1^{s-t} u(c_s) + \lambda_{\tau,t}^{2} \, \sum_{s=t}^{\infty} \beta_2^{s-t} u(c_s) \right\}.$$

Thus if the current-generation Pareto property is replaced by the intergenerational Pareto property, then the sequence of planner's SWFs is time-consistent, time-invariant and stationary whenever the planner is more patient than the **least patient** agent.[17]

Fourth, the use of the term "dictatorial" in Jackson and Yariv (2015) deserves some clarification. They call "dictatorial" SWFs that coincide with the individual utility function of one of the agents. However, this is not the same as the usual definition of dictatorship in social choice theory. The usual definition can be referred to as *ex ante* dictatorship: a dictator is chosen to represent the preferences of a society regardless of agents' preferences. The definition of dictatorship given by Jackson and Yariv is different: they call an agent a dictator if she appears to represent the preferences of a society for some fixed preference profile. Their notion is essentially *ex post* dictatorship.[18] The difference between these two notions is substantial: for instance, an aggregation rule that chooses the preferences of the median agent in the society is *ex post* dictatorial (in the sense of Jackson and Yariv), while it is not *ex ante* dictatorial (in the usual sense of social choice theory), because the median agent clearly differs across preference profiles.

Finally, in a companion paper, Jackson and Yariv (2014) provide some experimental evidence on collective intertemporal choice. They ask 60 subjects to act as social planners and choose common consumption streams for a group of three other subjects. They show that almost all (59 of 60) social planners are not time-consistent, and 75% of them exhibit decreasing impatience.

The result of Jackson and Yariv (2015) stresses an important difficulty for aggregation of heterogeneous time preferences. Even though each agent with her own discount factor is time-consistent, the utilitarian planner assigning equal weights to agents with different discount factors, is not.[19] The only case where the sequence

[17] Feng and Ke (2018) also consider a general setting and prove that when agents differ in their felicity functions, the same result holds whenever the planner is more patient than the **most patient** agent. Their findings are very similar to those of Caplin and Leahy (2004) who argue that under the dynamic principle of revealed preference a social planner should be more patient than a representative agent from each generation.

[18] Gerber (2019) refers to this property as "local dictatorship", because the dictator may change with the preference profile.

[19] Anchugina et al. (2019) generalize this result and show that utilitarian aggregation of heterogeneous discount functions from the same equivalence class of decreasing impatience exhibits even more decreasing impatience.

of SWFs satisfies time consistency, time invariance and stationarity is when a single agent represents the society as a whole. This case may be realized not only by selecting one agent as a dictator, but also as an outcome of some social choice procedure. One of these procedures, namely, majority voting, we discuss further.

4 Voting over common consumption streams

An alternative way to obtain a social preference ordering is to abandon the idea of a social planner and instead acknowledge the fact that decisions of society are determined through a political process. In this section we consider majority voting over common consumption streams in the many-agent Ramsey model, highlight the main difficulties with this procedure and describe some possibility results.

4.1 Non-existence of a Condorcet winner

A natural way of aggregating heterogeneous preferences is majority voting: if a feasible plan is preferred to all other plans by a majority of individuals, then this is the plan a society chooses. This approach allows us to replace the unclear terms "social welfare function" and "discount rate of a social planner" by the more familiar terms "individual utility" and "individual discount rate".

Suppose that in the many-agent Ramsey model with common consumption agents choose their common consumption stream by voting over all feasible streams. A natural idea is to find a Condorcet winner, i.e., a consumption stream preferred by a majority of voters when pairwise compared to any other feasible consumption stream. In voting over one-dimensional choice space, under certain plausible assumptions a Condorcet winner exists and coincides with the optimal path of the "median voter". This outcome holds much favor in economic and political contexts, because the median voter effectively appears as a representative of the population whose preferences determine all decisions of a society.

However, there is an important difficulty known as a curse of dimensionality. Since agents choose their common consumption stream over an infinite horizon, the choice space consists of infinite consumption sequences and is multi-dimensional. Due to a high dimensionality of the choice space, a Condorcet winner in general does not exist. A majority rule in this case is generically intransitive, and one should expect the emergence of cycles: a majority of agents would prefer consumption stream $\mathbf{C}_2$ over $\mathbf{C}_1$, $\mathbf{C}_3$ over $\mathbf{C}_2$ and $\mathbf{C}_1$ over $\mathbf{C}_3$, which is known as the **Condorcet paradox**.

A number of studies (e.g., Plott, 1967; Davis et al., 1972; Kramer, 1973; Bucovetsky, 1990) derive different forms of necessary and/or sufficient conditions for the existence of a Condorcet winner in voting over multi-dimensional choice space. It is shown that any degree of preference heterogeneity leads to the intransitivity of majority rule. Moreover, De Donder et al. (2012) prove that there is in general no Condorcet winner even in voting over bidimensional choice space even if agents are heterogeneous only in one dimension. They also show that generically, for any proposal there is another feasible proposal favored by all voters except one.

Let us illustrate this difficulty in a three-agent three-period example of the many-agent Ramsey model with common consumption.[20] The production function is $f(k) = k$, and the initial stock of resource is k_0. Agent i has utility function $U^i = \ln c_0 + \beta_i \ln c_1 + \beta_i^2 \ln c_2$, i.e., all agents have the same logarithmic felicity function and differ only in their discount factors. We assume that $1 > \beta_1 > \beta_2 > \beta_3 > 0$, so agent 2 has the median discount factor.

Problem (6) for agent i at date 0 takes the form:

$$\max_{c_t \geq 0} \; \ln c_0 + \beta_i \ln c_1 + \beta_i^2 \ln c_2, \qquad \text{s. t.} \quad c_0 + c_1 + c_2 = k_0 \, .$$

Its solution, *the optimal path for agent* i, $\mathbf{C}^{i*} = \{c_0^{i*}, c_1^{i*}, c_2^{i*}\}$, is given by

$$c_0^{i*} = \frac{k_0}{1 + \beta_i + \beta_i^2}, \qquad c_1^{i*} = \frac{\beta_i k_0}{1 + \beta_i + \beta_i^2}, \qquad c_2^{i*} = \frac{\beta_i^2 k_0}{1 + \beta_i + \beta_i^2} \, .$$

Suppose that agents vote at date 0 over the whole consumption stream. The set of alternatives over which they vote is $\mathcal{C} = \big\{(c_0, c_1, c_2) \in \mathbb{R}_+^3 \mid c_0 + c_1 + c_2 = k_0\big\}$. In order to illustrate the Condorcet paradox, let us first show that if a Condorcet winner exists, then it should coincide with the optimal path for agent 2. Indeed, suppose the opposite, i.e., that a Condorcet winner is a consumption stream $\mathbf{C}^* = \{c_0^*, c_1^*, c_2^*\}$ from $\mathcal{C}$ which does not coincide with $\mathbf{C}^{2*}$. Then one of the following inequalities holds:

$$c_1^* \neq \beta_2 c_0^*, \qquad \text{or} \qquad c_2^* \neq \beta_2 c_1^*.$$

Let for definiteness $c_1^* > \beta_2 c_0^*$ (all other cases can be considered similarly). Note that in this case also $c_1^* > \beta_3 c_0^*$.

Consider the inner product of ∇U^i at (c_0^*, c_1^*, c_2^*) and the vector $z = (1, -1, 0)$:

$$\nabla U^i(c_0^*, c_1^*, c_2^*) \cdot z \;=\; \left(\frac{1}{c_0^*}, \frac{\beta_i}{c_1^*}, \frac{\beta_i^2}{c_2^*}\right) \cdot (1, -1, 0) \;=\; \frac{1}{c_0^*} - \frac{\beta_i}{c_1^*} \;=\; \frac{c_1^* - \beta_i c_0^*}{c_0^* c_1^*}.$$

It is positive for $i = 2$ and $i = 3$. Hence for a sufficiently small perturbation

[20] This example can be easily generalized to the infinite time horizon.

of (c_0^*, c_1^*, c_2^*) in the direction z (i.e., slightly increasing consumption at date 0 and decreasing consumption at date 1), there exists a consumption stream $\mathbf{C}' = \{c_0', c_1', c_2'\}$ from $\mathcal{C}$ which agents 2 and 3 prefer over $\mathbf{C}^*$. Since two agents out of three prefer $\mathbf{C}'$ over $\mathbf{C}^*$, the latter consumption stream is not a Condorcet winner. It follows that a Condorcet winner, if it exists, should coincide with the optimal consumption path for agent 2, $\mathbf{C}^{2*}$.

Now let us show that $\mathbf{C}^{2*}$ cannot be a Condorcet winner. Indeed, consider the inner product of ∇U^i at $(c_0^{2*}, c_1^{2*}, c_2^{2*})$ and the vector $z = (1, -2, 1)$:

$$\nabla U^i(c_0^{2*}, c_1^{2*}, c_2^{2*}) \cdot z = \left(1 - 2\frac{\beta_i}{\beta_2} + \frac{\beta_i^2}{\beta_2^2}\right) \frac{1 + \beta_2 + \beta_2^2}{k_0} = \left(1 - \frac{\beta_i}{\beta_2}\right)^2 \frac{1 + \beta_2 + \beta_2^2}{k_0}.$$

It is positive for $i = 1$ and $i = 3$. Hence for a sufficiently small perturbation of (c_0^*, c_1^*, c_2^*) in the direction z, there is a consumption stream $\tilde{\mathbf{C}}$ from $\mathcal{C}$ which agents 1 and 3 prefer over $\mathbf{C}^{2*}$. Stream $\tilde{\mathbf{C}}$ has more consumption at date 0 (to satisfy the impatient agent 3) and at date 2 (to satisfy the patient agent 1), and less consumption at date 1 (to make it feasible). Again, since two agents out of three prefer $\tilde{\mathbf{C}}$ over $\mathbf{C}^{2*}$, the latter consumption stream is not a Condorcet winner. This contradiction leads to the conclusion that a Condorcet winner does not exist.

Our simple example illustrates a general result of Boylan et al. (1996) who explicitly prove that there is no Condorcet winner in the many-agent Ramsey model with common consumption even if agents differ only in their discount factors. Discussing this result, Boylan and McKelvey (1995, p. 863) note:

> The above result may seem surprising . . . One might think that, when utility functions differ only by one parameter, the median voter theorem would apply, implying that the optimal plan for the voter with the median discount factor would be a majority core point. In fact, the optimal plan for the median-discount-factor voter is defeated by a plan supported by a coalition including patient and impatient voters.

Furthermore, Jackson and Yariv (2015) in the already mentioned paper provide another impossibility result, which implies that voting over consumption streams when agents have heterogeneous time preferences cannot lead to an unambiguous outcome. Any non-dictatorial voting rule (in particular, majority and weighted supermajority rules) is intransitive, unless the set of potential consumption streams is severely restricted. It follows from their result that preferences of any single agent, including the agent with the median discount factor, cannot determine a voting equilibrium (as it would have to be transitive).

Thus an attempt to aggregate heterogeneous time preferences by a political process faces a serious difficulty. Though the optimal path of the median agent seems to be a desirable outcome of majority voting, it is not clear whether there even exist political institutions that could support and justify the choice of the agent with the median discount factor.

4.2 Review of possibility results

However, there are some ways to overcome the above mentioned impossibility results. One of these ways was proposed by Beck (1978). Instead of the infinite-dimensional set of all feasible consumption streams, agents vote only over the set of individually optimal paths. This idea can be realized as a two-step procedure: each agent nominates a consumption stream for the society to follow, and then agents vote over each pair of nominated streams choosing a Condorcet winner. This procedure can be interpreted as majority voting over the set of discount factors, and then implementing the optimal path for the agent with the winning discount factor (see also Gerber, 2019). It is shown that in voting over all individually optimal paths there exists a Condorcet winner, which is the optimal path for the agent with the median discount factor.

In our example this procedure can be illustrated as follows. Instead of voting over the set of all feasible consumption streams $\mathcal{C}$, agents vote over the three-element set $\{\mathbf{C}^{1*}, \mathbf{C}^{2*}, \mathbf{C}^{3*}\}$, where $\mathbf{C}^{i*}$ is the optimal path for agent i. Consider a pairwise contest between $\mathbf{C}^{2*}$ and $\mathbf{C}^{1*}$. Clearly, agent 1 prefers her optimal path $\mathbf{C}^{1*}$, while agent 2 prefers her optimal path $\mathbf{C}^{2*}$. The decisive voter is agent 3, and it is easily checked that she prefers $\mathbf{C}^{2*}$ over $\mathbf{C}^{1*}$. Indeed, $c_0^{i*} = k_0/(1 + \beta_i + \beta_i^2)$ is decreasing in β_i, and thus $c_0^{3*} > c_0^{2*} > c_0^{1*}$. Since agent 2 prefers $\mathbf{C}^{2*}$ over $\mathbf{C}^{1*}$, the same should be true for the more impatient agent 3 who prefers more consumption at earlier dates even stronger. Similar argument can be applied to a pairwise contest between $\mathbf{C}^{2*}$ and $\mathbf{C}^{3*}$: only agent 3 prefers the path $\mathbf{C}^{3*}$, while both agents 1 and 2 prefer $\mathbf{C}^{2*}$. Thus $\mathbf{C}^{2*}$, the optimal path for the agent with the median discount factor, wins all pairwise contests, and is a Condorcet winner. However, while this voting procedure ensures the existence of a stable outcome, this result comes at the expense of a severely restricted choice set.

A different voting procedure in the many-agent Ramsey model with common consumption is proposed by Boylan et al. (1996). They consider a finite horizon model and introduce two additional agents ("political candidates"). In each period the candidates propose a consumption level for the agents and care only about being elected. Agents vote for the candidates and take into account only their own intertemporal utility of consumption. The consumption level proposed by the winning candidate is implemented and becomes the actual consumption for this period. Boylan et al. (1996) prove that for any finite horizon, the optimal path for the agent with the median discount factor is a unique subgame perfect Nash equilibrium in this noncooperative game. They also show that when the time horizon increases, the sequence of Nash equilibria converges to the optimal path for the agent with the median discount factor in the infinite horizon model. This voting procedure, though also yielding the intuitively appealing outcome, seems unnecessarily complex.

A natural idea to overcome curse of dimensionality is to convert an infinite-dimensional choice space, consisting of consumption streams, into a series of one-dimensional choice spaces. Agents vote separately in each period, and the resulting sequence of voting outcomes is an equilibrium in the Nash sense: each element is a one-dimensional Condorcet winner, given that all other elements are chosen similarly. This procedure was proposed independently by Kramer (1972) and Shepsle (1979), and its outcome is known as a **Kramer–Shepsle equilibrium**.

A Kramer–Shepsle procedure has many advantages in dynamic models. First, it has a convenient interpretation as dynamic (step-by-step) majority voting under perfect foresight about outcomes of future votes. This is well in line with the Koopmans (1967) intertemporal view on economic models discussed in Section 2. Second, the impossibility result of Jackson and Yariv (2015) does not hold here. In their framework, intransitivity arises when agents atemporally vote once and for all over the whole sequence, while a Kramer–Shepsle procedure implies that agents vote intertemporally, step by step. Third, a Condorcet winner in a multi-dimensional problem (if it exists) is always a Kramer–Shepsle equilibrium, but the converse need not to be true. Indeed, a Kramer–Shepsle equilibrium exists in more general circumstances (see Kramer, 1972).

Unfortunately, a direct application of the Kramer–Shepsle procedure in the many-agent Ramsey model leads to a new difficulty. Consider a Kramer–Shepsle equilibrium in our example. Suppose that at date 0 agents have some common expectations about future consumption, $\bar{c}_1$ and $\bar{c}_2$, and vote over the date-0 consumption c_0. The preferred date-0 consumption for agent i is a solution to the following problem:

$$\max_{c_0 \geq 0} \ \ln c_0 \,, \qquad \text{s. t.} \quad c_0 + \bar{c}_1 + \bar{c}_2 = k_0 \,.$$

Clearly, this optimization problem is degenerate. If future consumption is fixed, there is no trade-off between consumption today and consumption tomorrow, so the intertemporal resource constraint under given expectations fully predetermines the optimal value of c_0. Moreover, this value is the same for all agents: it is optimal for each agent to consume today as much as possible, given future consumption and the initial amount of resource. The same argument applies to voting over c_1 and c_2. It follows that any consumption stream $\{c_0, c_1, c_2\}$ such that $c_0 + c_1 + c_2 = k_0$, can be obtained as the outcome of step by step voting over consumption levels under perfect foresight. While a Condorcet winner fails to exist in our example, there is an uncountable number of Kramer–Shepsle equilibria.

Nevertheless, a stable outcome of dynamic voting can be obtained by applying a Kramer–Shepsle procedure in terms of consumption rates. Borissov et al. (2017) consider a simple voting procedure referred to as **intertemporal majority voting** based on three principles: *i*) agents vote step by step; *ii*) agents vote over relative values and not absolute values (consumption rates instead of levels); and *iii*) agents have perfect foresight about outcomes of future votes. It is proved that when agents have identical felicity functions, the outcome of intertemporal majority voting is the optimal path for the agent with the median discount factor.

Formally, at each date agents choose the current consumption rate by majority voting, given the current capital stock and some expectations about future consumption rates. There is a unique instantaneous Condorcet winner in this one-dimensional voting, which suggests that intertemporal majority voting is a well-defined institutional framework even when expectations are heterogeneous and incorrect. Moreover, if agents are heterogeneous only in one dimension, i.e., have the same expectations and differ only in their discount factors, then each instantaneous Condorcet winner is the preferred consumption rate for the agent with the median discount factor. The sequence of instantaneous Condorcet winners obtained under perfect foresight about outcomes of future votes, coincides with the optimal path for the agent with the median discount factor.

Let us illustrate intertemporal majority voting in our example. Consider problem (6) for agent i at date 0 in terms of consumption rates $e_0 = c_0/k_0$, $e_1 = c_1/k_1$ and $e_2 = c_2/k_2$:

$$\max_{0 \le e_t \le 1} \ln(e_0 k_0) + \beta_i \ln(e_1(1-e_0)k_0) + \beta_i^2 \ln(e_2(1-e_1)(1-e_0)k_0) \ ,$$

Its solution, the optimal path of consumption rates for agent i, $\mathbf{E}^{i*} = \{e_0^{i*}, e_1^{i*}, e_2^{i*}\}$, is given by

$$e_0^{i*} = \frac{1}{1+\beta_i+\beta_i^2}, \qquad e_1^{i*} = \frac{1}{1+\beta_i}, \qquad e_2^{i*} = 1\,.$$

Note that there is a one-to-one correspondence between optimal paths of consumption rates $\mathbf{E}^{i*}$ and optimal paths $\mathbf{C}^{i*}$. While the change of control variables from levels to rates does not affect optimization problems, it completely changes the situation in a voting setting. It turns out that when expectations are formed about consumption rates, there is a degree of freedom in voting given an intertemporal resource constraint and future decisions.

Suppose that at date 0 agents have expectations about future consumption rates, $\bar{e}_1$ and $\bar{e}_2$, and vote over the current consumption rate. The preferred date-0 consumption rate for agent i is a solution to the following problem:

$$\max_{0\leq e_0\leq 1} \ln(e_0 k_0) + \beta_i \ln(\bar{e}_1(1-e_0)k_0) + \beta_i^2 \ln(\bar{e}_2(1-\bar{e}_1)(1-e_0)k_0),$$

and it is clear that this solution is e_0^{i*}, the optimal date-0 consumption rate for agent i. Since preferences of agents in one-dimensional voting over e_0 are single-peaked and e_0^{i*} are decreasing in β_i, a unique Condorcet winner is the preferred date-0 consumption rate for the agent with the median discount factor, i.e., agent 2. Thus if agents form their expectations about consumption rates, there appears a trade-off between consumption today and consumption tomorrow.

Further, suppose that at date 1 agents vote over the current consumption rate given the expected date-2 consumption rate $\bar{e}_2$ and the realized date-0 consumption rate e_0^*. The preferred date-1 consumption rate for agent i solves the problem

$$\max_{0\leq e_1\leq 1} \ln(e_1(1-e_0^*)k_0) + \beta_i \ln(\bar{e}_2(1-e_1)(1-e_0^*)k_0),$$

and it can be easily checked that its solution is e_1^{i*}, the optimal date-1 consumption rate for agent i. By the median voter theorem, there is a unique Condorcet winner which is the preferred date-1 consumption rate for agent 2.

At terminal date 2, the preferred date-2 consumption rate for agent i solves

$$\max_{0\leq e_2\leq 1} \ln(e_2(1-e_1^*)(1-e_0^*)k_0),$$

and coincides with $e_2^{i*} = 1$. Agents vote unanimously, and a unique Condorcet winner is $e_2^* = 1$.

The outcome of intertemporal majority voting, i.e., the sequence of Condorcet winners in one-dimensional voting over current consumption rate at the corresponding date given expectations about future consumption rates, has the form $\mathbf{E}^* = \left\{\frac{1}{1+\beta_2+\beta_2^2}, \frac{1}{1+\beta_2}, 1\right\}$.[21] Clearly, it coincides with the optimal path of consumption rates for the agent with the median discount factor, $\mathbf{E}^* = \mathbf{E}^{2*}$, and uniquely

[21]Due to the simplicity of our example, a Condorcet winner at each date does not depend on expectations. However, this is not true under more general assumptions.

corresponds to her optimal path. Borissov et al. (2017) show that this result holds in more general settings, including the infinite horizon many-agent Ramsey model with common consumption.[22]

It is instructive to compare the outcome of intertemporal majority voting with the two impossibility results of Jackson and Yariv (2015). First, this voting procedure is well-defined (in particularly, transitive), and yields unambiguous outcome determined by the preferences of the median agent — a property which seemed improbable according to the previous findings. The key insight is the intertemporal nature of voting: when agents vote step by step, the choice space is one-dimensional and curse of dimensionality is avoided. Second, note that intertemporal majority voting indirectly determines a sequence of SWFs:

$$W_\tau^{IMV} = \sum_{t=\tau}^{\infty} (\beta_{med})^{t-\tau} u(c_t), \qquad \tau \geq 0\,.$$

In each W_τ^{IMV}, the Pareto weight of the agent with the median discount factor is equal to 1, and the weights of all other agents are equal to zero. Clearly, this sequence is time-consistent, time-invariant and stationary. Therefore, the outcome of intertemporal majority voting is both Pareto-optimal and time-consistent, and coincides with the result of the maximization of the SWF W_0^{IMV}. Again, this conclusion does not contradict the observation of Jackson and Yariv (2015). The sequence of Paretian SWFs $\{W_\tau^{IMV}\}_{\tau=0}^{\infty}$ is *ex post* dictatorial in terms of Jackson and Yariv (2015), in the sense that it coincides with the sequence of utility functions of a particular agent (the agent with the median discount factor). However, this "median voter's dictatorship" is not the same as the definition of dictatorship in, e.g., Arrow's theorem.

Thus, it can be argued that intertemporal majority voting provides an institutional microfoundation for the choice of the agent with the median discount factor as a representative of the population.

[22]For logarithmic felicity function and Cobb–Douglas production function, it is possible to find a closed-form solution for the preferred policy of each agent, and explicitly calculate the winner in intertemporal majority voting. Hence this approach to voting can be used in dynamic general equilibrium models with standard felicity and production functions where agents differ in their time preferences. Borissov et al. (2014) study voting over environmental maintenance tax, Borissov and Pakhnin (2018) consider voting over the rate of extraction of exhaustible resources, and Borissov et al. (2019) study voting over the shares of public goods in total output. In all cases, equilibrium policy sequences are determined by the agent with the median discount factor.

5 Social discount rate

The many-agent Ramsey model with common consumption has a natural interpretation as an intergenerational model. In this case, the problem of aggregation of heterogeneous time preferences becomes closely related to the normative question of how should we discount the future as a society. In this section we study the implications of the above discussion for the problem of choosing an appropriate social discount rate (SDR).[23]

There is no consensus on how a SDR should be determined. Some theorists argue that the SDR should be chosen based on a set of ethical principles. For instance, Ramsey (1928, p. 543) saw discounting as

> a practice which is ethically indefensible and arises merely from the weakness of the imagination.

He strongly advocated zero SDR: in order not to discriminate against future generations, all generations should be treated equally. However, not discounting is in fact discounting at 0%, which may also have its own unacceptable implications. This point is emphasized by Koopmans (1967) who coined the term "the paradox of the indefinitely postponed splurge": zero discounting may force current generation to consume nothing, because reinvesting the resources will always do more good for future generations.[24] Koopmans (1967, p. 9) argues that

> too much weight given to generations far into the future turns out to be self-defeating. It does nobody any good. How much weight is too much has to be determined in each case.

But if SDR is strictly positive, should it be low or high? This question is especially important in the cost-benefit analysis of environmental projects, and particularly in integrated assessment models of climate change which appear to be very sensitive to the choice of SDR. The disagreement about the value of the SDR is at the core of the famous Stern–Nordhaus debate.

Stern (2007) uses the ethical principle of "intergenerational equity" and argues that the only justification for discounting is the possibility that future generations might not exist. He applies a very low SDR of 0.1%, which results in the consumption discount rate, i.e., real interest rate, of around 1.4%.[25] According to Stern,

[23] See also Millner and Heal (2021) for a survey of different approaches to social discounting.

[24] This situation may arise in the standard Ramsey model when marginal utility of consumption and marginal productivity of capital are strictly positive.

[25] SDR is the social rate of time preference, i.e., utility discount rate (ρ). It is linked to the real interest rate, i.e., consumption discount rate (r), by the Ramsey equation: $r = \rho + \eta g$, where η is the consumption elasticity of marginal utility and g is the growth rate of per capita consumption.

the utility of generation living 100 years from now is discounted by the factor $1/(1+0.001)^{100} = 0.9$, i.e., by 10%. This leads to the conclusion that humanity should provide strong and immediate response to climate change.

At the same time, Nordhaus (2007) uses "consumer sovereignty" as the ethical principle. He believes that consumption discount rate should reflect real decisions of individuals and be based on the revealed preferences of the members of society. He takes the average real interest rate of 5.5% from the market data, and derives from it the SDR of 1.5%. Hence Nordhaus discounts the utility of generation living a century later by $1/(1+0.015)^{100} = 0.23$, i.e., the welfare of future generation is valued four times less than that of the present generation. Stern's conclusion no longer holds if costs and benefits from climate change are discounted at the market interest rate. Moreover, the disagreement about SDR leads to the drastically different policy recommendations of the global social cost of carbon that vary by an order of magnitude.[26] Thus ethics by itself does not suggest whether the SDR should be zero, small, or large.

Based on our previous discussion, we can assess several other approaches to determine the SDR in a heterogeneous society. First, from the economic perspective, the SDR is the rate of time preference of a social planner whose SWF is a utilitarian aggregation of individual utility functions. As we have seen in Sections 2 and 3, there arise certain difficulties with the construction of an appropriate SWF. Even when the preferences of agents are time-consistent, time-invariant and stationary, the preferences of the social planner are not. Moreover, when agents have constant and different discount rates, the planner's discount rate is non-constant and declines over time.

The declining discount rates also emerge in other settings, most importantly, when uncertainty about the future is taken into account (see, e.g., Pearce et al., 2003; Gollier and Weitzman, 2010). All these observations suggest that instead of a constant SDR, a declining discount rate should be used in cost-benefit analysis from the perspective of the society. However, at least in the deterministic case, the declining discount rate of a social planner leads to time-inconsistent preferences, which violates a generally accepted criterion of rationality.[27]

Second, from the political perspective, the SDR is a collective choice of heterogeneous agents determined by political institutions. As we have seen in Section 4, the common result in the literature is that in the presence of heterogeneous time

[26]Under Stern discounting, the estimated social cost of emissions in 2020 is $299.6, while it is only $35.3 under Nordhaus discounting (see Nordhaus, 2018).

[27]Newell and Pizer (2003) argue that under uncertainty the term "time inconsistency" is misleading, as it can be justified only when it is known with certainty that the today's optimal path will not be followed in the future.

preferences, multi-dimensional voting cannot lead to a stable outcome. However, there is at least one political procedure, intertemporal majority voting, which justifies the conventional wisdom that preferences of the agent with the median discount rate determine decisions of a society.

Intertemporal majority voting aggregates well-behaved preferences and yields the Pareto-optimal and time-consistent outcome. Therefore, it can be argued that this procedure is consistent with the consumer sovereignty principle, as preferences of heterogeneous agents are revealed by voting. However, in this case the SDR coincides with the median discount rate in the population, which is typically much higher than the discount rate of the most patient agent. Since in the many-agent Ramsey model the discount rate of the most patient agent coincides with the long-run real interest rate, this approach carelessly suggests valuing the welfare of future generations even less than the market-oriented approach.

This rather pessimistic conclusion is not supported by recent empirical evidence from experimental studies of collective intertemporal choice made by a group of people. A remarkably general result is that collective decisions are more patient than individual ones. For instance, Shapiro (2010) recruited 176 subjects who choose between financial rewards for different dates. He compares individual decisions with unanimous decisions of pairs and groups of four, and finds that groups are more patient than pairs who are in turn more patient than individuals. The elicited weekly discount factors of groups of four (resp. pairs) are 4% (resp. 2%) higher than individual discount factors.

The experiment of Denant-Boemont et al. (2017) includes 60 subjects and studies collective decisions based on majority voting in five-person groups. It is found that groups tend to make more patient and time-consistent choices than individuals. Only 42.3% of individual decisions were patient, while this share was 80.6% for groups. Moreover, individual decisions mostly exhibited decreasing impatience, while constant impatience was dominant for collective decisions. Glätzle-Rützler et al. (2019) obtain similar results when studying unanimous decisions in groups of three in the experiment with 555 subjects. Their design allowed to identify causal effects of heterogeneous discount factors on patience in collective decisions. They show that three-person groups behave more patiently than individuals.

The above empirical results cannot be directly compared to the outcomes of collective choice models, since group decisions in the experimental settings are essentially based on information exchange and require coordination across subjects. Therefore it is reasonable to assume that within a group agents do not vote according to their individual preferences — when people think about public goods or social values, they tend to be more patient.

This seeming paradox between private and social discounting may be resolved by the dual-rate discounting approach (see, e.g., Yang, 2003; Borissov and Shakhnov, 2011). The idea is that a social planner is assumed to have two different discount rates: high consumption discount rate used to discount utility from private consumption goods, and low environmental discount rate used to discount utility from public goods (e.g., environmental quality). Then the sequence of (utilitarian) SWFs takes the form

$$W_\tau \;=\; \sum_{t=\tau}^{\infty} \left\{ \beta_1^{t-\tau} v(q_t) + \beta_2^{t-\tau} u(c_t) \right\}, \qquad \tau \geq 0\,,$$

where $v(q_t)$ is the utility from environmental quality and $\beta_1 > \beta_2$.

However, as we have seen in Section 2, each SWF W_τ is non-stationary, which implies that in the long run only environmental quality determines the decisions of the society, while the weight of private consumption becomes negligible. Moreover, dual-rate discounting naturally leads to time-inconsistent preferences, which is also not normatively appealing. Thus it seems that the reasonable solution to the problem of determining the SDR must steer between the Scylla of time inconsistency and the Charybdis of median voter's dictatorship.

6 Concluding remarks

An increasing number of empirical studies show that different individuals discount the future differently, and this heterogeneity in time preferences should be taken into account in economic modeling. The associated problem of aggregation of heterogeneous time preferences is especially relevant in many-agent growth models, where economic growth theory meets social choice theory. In this survey we use simple one-sector Ramsey models with private and common consumption where agents differ in their discount factors to illustrate the main results about aggregation of heterogeneous time preferences, and discuss the main difficulties with collective choice in many-agent growth models.

We note that the notion of Pareto optimality under heterogeneous time preferences becomes problematic, and typical results about aggregation have the form of impossibility theorems. The sequence of preferences of a social planner satisfies reasonable conditions (Pareto optimality, time consistency, time invariance, stationarity) if and only if either all agents have the same discount factor and are thus identical or the preferences of a social planner coincide with the preferences of some agent and hence the planner ignores the preferences of all but one agent.

In particular, the non-stationarity of planner's preferences implies that the most patient agent eventually determines consumption decisions of the society, and in the private consumption case it follows that in any Pareto-optimal allocation all agents except the most patient one starve to death. In turn, time inconsistency of planner's preferences violates a generally accepted criterion of rationality and may lead to decision reversals.

An attempt to aggregate heterogeneous time preferences through some voting procedure also faces serious difficulties. A Condorcet winner generically fails to exist in voting over multi-dimensional choice space, and any non-dictatorial voting rule appears to be inherently intransitive. A stable outcome of dynamic voting can be obtained either at the expense of restricted choice spaces or by intertemporal majority voting. These voting procedures support the choice of the median agent as the representative of the society. However, in some applications, e.g., in determining the social discount rate, this approach may violate the ethical principle of intergenerational equity.

Nevertheless, we hope that this survey helps to disentangle connections between ethics, economics and politics, and shows that the problem of collective choice under heterogeneous time preferences is more tractable that it may seem.

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