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Abstract

Circular Economy literature recommends longer lasting products, in order to reduce pollution from extraction, production, and disposal. Our economic analysis finds conditions where consumers choose lives that are too short – a “durability gap”. Then policies targeting durability raise welfare. While externalities are corrected by Pigovian taxes that ignore durability, raising the output tax nonetheless induces consumers to pay more for goods that last longer. Second, if the tax is suboptimal, a durability mandate raises welfare. Third, internalities have ambiguous effects. Fourth, a social discount rate less than private discount rate is the strongest case for policy to favor durability.

JEL-Codes: H210, H230, Q580.

Keywords: Pigovian taxes, first-best policy, externalities, internalities.

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Excess waste might be created by a "linear economy" that proceeds from resource extraction to production, use, and landfill disposal. In contrast, a "circular economy" (CE) could extract less, design products to last longer, design products to be recycled, and then encourage recycling. These ideas were introduced by an architect (Stahel 1982) and design engineers (*e.g.* Hendrickson *et al*, 1998). Inter-disciplinary CE literature reviewed in Stahel (2016) advocates for policies to encourage repair and reuse of products with longer lives – to reduce pollution from extraction through disposal. Such regulations are discussed in both the E.U. and U.S.[1] Ironically, circular economy has attracted little interest within economics. Here, we begin economic analysis of product durability, but we omit recycling because it is already well covered (*e.g*., Fullerton and Kinnaman, 1996, or Palmer *et al* 1997). We ask whether and when economic welfare can be improved by policies that target consumer choice of product lifetime.

Economists since Pigou (1932) show that optimal corrective taxes apply directly to externality-generating activities at a rate equal to marginal external damages (MED), at least under "perfect market assumptions": perfect competition, full information, perfect enforcement, certainty, and many identical consumers who are perfectly rational (*e.g*., Baumol and Oates, 1988). Producers pay the MED per ton of emissions, and consumers pay the MED per ton of disposal. Many subsequent papers relax each of those assumptions (as reviewed in Bovenberg and Goulder, 2002). But no economics literature asks about optimal corrections for production and disposal externalities when consumers can choose product durability. Perhaps economists thought corrective policy would be unnecessary, since the choice of durability itself does not generate an externality. Here, however, we ask what circumstances can justify calls in the interdisciplinary CE literature for policies that target durability.

Our questions about the choice of durability are analogous to those about the energy efficiency gap, defined as "a wedge between the cost-minimizing level of energy efficiency and the level actually realized" (Allcott and Greenstone 2010, p.4). This gap might be driven partly by externalities that prevent minimizing social costs and partly by "internalities" such as consumers' mistakes about energy cost (*e.g*., present bias or inattention; see Gerarden *et al* 2017). Thus, consumers may spend less money now on greater energy efficiency, even if it would save money on electricity in the long run. Analogously here, we define a "durability gap" as a wedge between the cost-minimizing level of durability and the level actually realized. This gap can also be driven partly by externalities and partly by internalities. Consumers might not pay more now for a product that lasts longer, even if it saves money in the long

[1] The E.U. notes that durability can increase GDP and environmental benefits (Montalvo *et al* 2016, pp.10-12). They suggest regulating product lifetimes. Richter *et al* (2019) calculate optimal durability of LED light bulbs and find that "longer lifetimes … in the E.U. could be appropriate" (p.107). They then discuss minimum durability standards and labelling requirements. In the U.S., the Federal Trade Commission (2021) discusses possible regulations.

run. If so, then policies can help consumers maximize their own welfare. In simple terms, making products that last twice as long can cut in half the repeated costs of extraction and disposal.[2]

These gaps are analogous but different. The energy efficiency gap focusses on policies to fix externalities from energy use during a product's lifetime – ignoring the choice of product durability.[3] In contrast, we focus on policies to fix externalities from production and disposal when consumers can *choose* durability – ignoring energy efficiency and externalities from use of the product.

We start with all the perfect market assumptions. Perfect competition means that firms cannot limit durability, plan obsolescence, or prevent repairs. Instead, selling at cost, firms offer product varieties with any combination of characteristics desired by consumers. Thus, we focus on consumer choice of durability. We ask whether an externality or internality creates a "durability gap", that makes chosen durability suboptimal. When we find a durability gap, we ask what policy could optimally fix it.

We use the present value of all private costs over a product's lifetime to derive annualized cost to consumers, and we use all social costs to derive annualized cost to society. We then solve for the optimal tax upon purchase that would induce buyers to minimize social costs. If that optimal tax depends on product lifetime, then we say that a "durability gap" can be corrected by a policy that explicitly targets the choice of lifetime. Given that standard Pigovian analysis has ignored the choice of durability, we ask whether our new analysis validates claims in the CE literature by finding results that are fundamentally any different from the standard Pigovian analysis. We allow for three categories of market failure: (1) externalities from extraction, production, maintenance, repair, or disposal; (2) internalities from mistakes, biases, inattention, or irrational beliefs about durability; and (3) a social planner's rate of discount different from consumers' private rate of discount. These market failures are kept separate until the analysis tells us which ones have similar or different impacts on any durability gap.[4] We solve for first-best policies that correct all of these market failures. We also show welfare effects of small changes from non-optimal policies (but we do not solve for second-best policies).

We demonstrate four main results. First, with only externalities in the first category, the optimal

[2] Future empirical work can address questions about the durability gap that are similar to those already addressed about the energy efficiency gap (Gillingham *et al*, 2009). Are the true long-run costs of durability known to the consumer and incorrectly measured by the analyst, or the other way around? Are consumers making rational decisions or not? Are external social cost being ignored? What is the size of each component of the gap?

[3] For example, Heutel (2015) studies policies to address an energy efficiency gap from "present bias" internalities, using a model with a single durable good that has a fixed lifespan and that causes externalities from its use of energy. He shows that the optimum requires both a Pigovian tax and another policy to address the internality.

[4] Other market failures are possible. Firms may have market power (Bernard 2019 or Kinokuni *et al* 2019), or the market may fail to provide a signal indicating product recyclability or durability (Eichner and Runkel, 2003). Also, long-lasting products may be more or less recyclable, or those made from recycled materials may be less durable.

tax does not favor products with longer lifetimes. Instead, we show that any increase in a per-unit tax itself induces consumers to pay more for extra durability. The reason is that buying a longer-lasting product delays the time that the tax must be paid to buy its replacement. Second, if the tax is lower than marginal external damage per unit output, then a durability subsidy or mandate can raise welfare. Third, an error or internality that under-estimates a product's life raises its perceived annual cost, which offsets part of any external production costs and thus reduces the optimal tax (and over-estimating a product's life reduces perceived annual cost and increases the optimal tax). A large underestimate can make the optimal tax negative. But this mistake has ambiguous effects on how optimal taxes depend on product life, so it conveys no clear message about a durability gap to be fixed by targeting durability. Fourth, when the social rate of discount is less than the private rate, then the optimal tax *does* explicitly encourage durability. The logic here is that private decisions favor cheaper and less-durable products, failing to account adequately for future excess social costs of disposal or of producing replacements.

1. Consumer Choice of Durability

We consider consumer choice from among a set of products that each can provide a given stream of services. A set of cars can provide transportation, a set of washing machines provide washing services, and a set of phones can provide calling services. We assume consumers face enough varieties to be able to acquire any combination of other desired characteristics, allowing us to focus on the choice among products that differ only by durability. We consider consumers with an arbitrarily long horizon who must choose one product from the relevant set (ignoring the opt-out decision).

Analysis requires a specific definition of durability. It could be based on economic depreciation, the annual fall in market value, but the resale price is not relevant for our many identical consumers who would have no interest in trading used appliances. In normal parlance, a more durable product might be one that requires less maintenance and repair. Our first model below allows consumers to choose among products with different repair cost schedules, where costs rise with product age. A vehicle can be repaired, repeatedly, but eventually the repair cost becomes high enough that the owner chooses to dispose of it and buy a replacement. Sometimes, however, a more durable product is simply one that lasts longer. A light bulb or a toaster-oven is a "one-hoss-shay" investment with no repair costs at all, and a constant service flow, until the product fails entirely. Our second model below explores this case where each product has no maintenance or repair but instead a fixed product lifetime.

We start with the consumer's choice among products indexed by $j = 1, \dots, N$. Products differ by purchase price P_j, final disposal cost D_j, and maintenance or repair expenses $m_j(t)$ that depend on age t. The function $m_j(t)$ is continuous, twice differentiable, and strictly convex ($m_j{}'(t) \equiv \partial m_j/\partial t > 0$).

These characteristics of each product are known and fixed. Before choosing their purchase, fully informed consumers plan the retirement date ($0 < T_j < \infty$) for each product j by balancing the rising maintenance and repair cost against costs of disposal and replacement. Then they compare the N products to choose the one with the lowest annualized cost. A longer-lasting product may cost more initially, all else equal, but have a lower long-run cost.[5]

Consumers also face various taxes that can be used to address our three categories of market failure. One category includes multiple externalities. First, the social cost of production P_j^* could exceed the sales price P_j if producers are not forced to cover all social costs during each phase from mineral extraction to final sale. Hence, we add a tax τ_j^P upon purchase. Second, social costs of disposal D_j^* could exceed private disposal cost D_j if disposal companies cover only their own costs.[6] Indeed, many disposal companies or cities charge a fixed monthly fee to collect garbage. Ignoring scrap value, then the private marginal cost D_j is zero.[7] Thus, a tax τ_j^D applies to disposal. Third, we allow for maintenance and repair to have their own social cost, $m_j^*(t)$, which is continuous, twice differentiable, and strictly convex ($m_j^{*\prime}(t) > 0$). We add a continuing tax $\tau_j^m(t)$ on maintenance. Any tax can be positive or negative.

Consumers face a discrete choice among products, however, so the optimal tax combination is not unique (any high tax on one product can shift choice to some other product). Thus, we only solve for one set of taxes that are *sufficient* to make consumers match the social planner's choices. To find tax rates that achieves optimality, our strategy is to set standard Pigovian taxes on maintenance and disposal and then solve for a value of τ_j^P that is sufficient to induce consumers to make the same choices as a planner who accounts for other market failures added below. That is, suppose maintenance taxes are $\tau_j^m(t) = m_j^*(t) - m_j(t)$, so consumers actually face the social costs of use.[8] The disposal tax is $\tau_j^D = D_j^* - D_j$, so consumers also face social cost of disposal.[9] We later confirm optimality.

Using continuous time, define PV_j as the present value of all *private* costs of product j, including

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[5] For example, Miotti *et al* (2016) calculate the present value of all ownership costs for 125 light-duty vehicles (and find that electric vehicles have higher initial costs but lower annualized costs than most fossil-fuel-powered cars).

[6] Social costs D_j^* include methane and leachate emissions from landfills as well as litter and noise from collection. These costs increase with toxicity, especially for products dumped improperly. A known disposal method allows the tax to be collected at purchase, but we see below whether P_j^* and D_j^* are separate terms in the optimal tax.

[7] At this point, a clever reader (referring to his own comments) said we have "free disposal".

[8] Ongoing costs m_j and m_j^* *could* represent private and social costs of the energy chosen to operate the product. We omit that discussion here because the extensive energy efficiency gap literature already studies those issues, and because those costs in our model are fixed. We also omit the possibility that usage affects product life.

[9] Equivalently, through "Extended Producer Responsibility", policy could require firms to bear the social cost of post-consumer waste. If so, then the competitive purchase price would include the present value of D_j^*.

taxes on maintenance and disposal but not including the purchase tax:

$$PV_j \equiv P_j + \int_0^{T_j} m_j^*(t)e^{-rt}dt + D_j^* e^{-rT_j} \tag{1}$$

where r is the private discount rate.[10] Then we set all consumer costs $\tau_j^P + PV_j$ equal to the present value of a flat cost c_j that the consumer could equivalently pay instead:

$$\tau_j^P + PV_j \equiv \int_0^{T_j} c_j\, e^{-rt} dt \tag{2}$$

We then solve for c_j:

$$c_j = \frac{r}{1 - e^{-rT_j}} (\tau_j^P + PV_j) \tag{3}$$

This annualized cost c_j is the “user cost” for comparison with an asset’s return, or the “rental rate” that a renter would pay to an owner who just breaks even when the owner pays for purchase and disposal.

The N products have different chosen lifetimes because of different costs of purchase, repair, and disposal. Annualized cost c_j is U-shaped across product age, first falling as purchase price is spread over more years of use, and then rising as repair costs rise with product age. Fully informed consumers with no liquidity constraints essentially differentiate (3) to find the age T_j at which $\partial c_j / \partial age$ is zero (where repair costs are rising with age enough for c_j to have a minimum at finite T_j).

After choosing the age of retirement for each product, the consumer then chooses the product with the lowest c_j. We assume all product attributes are constant over time, and that consumers have an arbitrarily long horizon, so they make the same choice again at the time of replacement.[11]

2. The Social Planner’s Choice of Durability

The consumer’s choices may not match the socially optimal choices once we consider all three categories of market failure. First, we already introduced externalities from extraction and production ($P_j^* > P_j$), product maintenance or use ($m_j^* > m_j$), and disposal ($D_j^* > D_j$). A second category is that consumers could make errors from inattention, bias, imperfect information, or irrational behavior. Later, using our model with one-hoss-shay products that have fixed lifetimes, we suppose that consumers

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[10] We ignore heterogeneity, including consumer discount rates (e.g., preferences, credit constraints, or tax rates). On heterogeneity, see Heutel (2015). If some consumers cannot borrow enough to pay the higher up-front cost of the product with optimal durability, then then the taxes considered here cannot achieve first-best outcomes.

[11] An extension might consider technical progress that reduces the future cost of a replacement purchase, but this extension would preclude the simple comparison of annualized costs used here. Even if all products have the same rate of technical progress, a consumer might rationally choose a product with higher annualized cost if its lifetime is short enough to take advantage of changes in technology that offer rapidly falling replacement cost, or expansion of services, or changes that enhance the durable’s other characteristics.

could mis-estimate those product lifetimes. Third, we now introduce a possible divergence between the social rate of discount ρ and the private rate of discount r.

We cannot review here the large literature on discount rates, but we summarize one prominent debate. Discussing climate policy, the Stern Review (2007) argues for a low social discount rate of 1.4%, partly on the grounds that the social welfare function should weight all generations equally and not under-weight consumption of future generations. In response, Nordhaus (2007) argues for a higher social discount rate of 4.3% based on market interest rates, because future generations could gain more by investing additional capital at market interest rates. This debate is explained by Goulder and Williams (2012) as a debate about two different concepts, rather than about two different values for a single "social rate of discount". The two concepts are blurred in models of Nordhaus and others that use a representative, infinitely-lived consumer, since intertemporal choice then reflects both utility maximization and social welfare maximization. This type of model has only one discount rate to be used both for private behavior and for social welfare. In contrast, for example, an overlapping generations model might have individuals who underweight future generations. If so, then current generations can affect future generations without fully taking their welfare into consideration.[12]

We focus on this market failure as an externality imposed by current consumers on future generations, but the difference between private and social rates of discount could reflect other market failures such as tax distortions, liquidity constraints, or even an internality. For example, Heutel (2015) models "present bias" as quasi-hyperbolic discounting, where consumers' short-run discount rate is too high relative to their own long-run rate. He assumes the social planner uses the "long-run criterion", employing only the lower long-run discount rate. Thus, our model with his *internality* would yield results similar to results here with our *externality*. Our numerical illustrations simply use a market interest rate of 4% for private optimization, while we vary the social discount rate from 4% down to 3%, 2%, or 1%.

To find choices that minimize social costs of providing a product's service flow, a planner would use the social discount rate ρ to calculate the present value of all social costs PV_j^* for each product, and then set that PV_j^* equal to the present value of the equivalent flat social cost flow c_j^*:

$$PV_j^* \equiv P_j^* + \int_0^{T_j^*} m_j^*(t)\, e^{-\rho t} dt \ + D_j^* e^{-\rho T_j^*} = \int_0^{T_j^*} c_j^*\, e^{-\rho t} dt \tag{4}$$

Next, solve for c_j^* as:

[12] The debate about discounting also inevitably involves uncertainty, irrational behavior, and short-run versus long-run discounting. Goulder and Williams (2012) argue that uncertainty would affect both the private and social discount rates in the same direction, though not necessarily by the same amount. In the second model below, we model consumer mistakes about perceived product lifetimes rather than mistakes about their own discount rate.

$$c_j^* = \frac{\rho}{1 - e^{-\rho T_j^*}} PV_j^* \tag{5}$$

Finally, using (5), the planner chooses lifetime T_j^* that minimizes c_j^* for each product, and then chooses the product with the minimum annualized cost.[13]

Given these behaviors, we want to find the product-specific tax τ_j^P such that private costs c_j in equation (3) exactly match social costs c_j^* in equation (5) for every product.[14] If so, then $c_j = c_j^*$, and the consumer's choice of lifetime T_j and product with minimum c_j will exactly match the planner's choices. We solve $c_j = c_j^*$ for the optimal tax as:[15]

$$\tau_j^P = \frac{\rho}{r} \cdot \frac{1 - e^{-rT_j}}{1 - e^{-\rho T_j^*}} \left(P_j^* + \int_0^{T_j^*} m_j^*(t)\, e^{-\rho t} dt + D_j^* e^{-\rho T_j^*} \right) - \left(P_j + \int_0^{T_j} m_j^*(t)\, e^{-rt} dt + D_j^* e^{-rT_j} \right) \tag{6}$$

As confirmed by substituting τ_j^P from (6) into c_j in (3), the resulting formula for c_j matches exactly the formula for c_j^* in (5). This equivalence has two implications. First, when the consumer and planner minimize their annual cost of each product, they choose the same lifetime. Thus we use T_j^* in place of T_j to write (6) simply as: $\tau_j^P = \frac{\rho}{r} \cdot \frac{1-e^{-rT_j^*}}{1-e^{-\rho T_j^*}} PV_j^* - PV_j$. Second, since each $c_j = c_j^*$, informed consumers choosing the product that minimizes their own c_j would also minimize social cost c_j^*.

Policymakers cannot exactly implement such specific and detailed optimal tax rates, since they would require much information about ρ, r, and every variable in (6) with a subscript. The point here is to understand where the concept of Pigovian taxation still pertains, or if rates must depend on product lifetimes – that is, cases where a "durability gap" leads to suboptimal durability.

3. Results from the First Model: Maintenance Costs Affect Consumer Choice of Lifetime

We use the model above to derive a series of analytical results. The Appendix shows derivations.

First, how do maintenance costs affect choices? We expect an increase in maintenance costs to induce earlier product retirements. To demonstrate this result, the Appendix introduces a shift parameter γ to the repair schedules: $\gamma m_j(t)$ and $\gamma m_j^*(t)$. The *partial* derivative $\partial \tau_j^P / \partial \gamma$ would not

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[13] The consumer in equation (3) takes the purchase price P_j and other costs as fixed, but a welfare-maximizing planner in (5) would take account of supply-side determination of costs. Thus, our simple model employs a flat supply curve, which would occur even in a general equilibrium model with perfect competition, constant returns to scale, and a single primary factor of production that also serves as numeraire.

[14] Actually, the minimum costs c_j and c_j^* need not match, as long as consumers make the socially optimal choice. Also, the choice of product j is a discrete choice, so other tax rates may lead consumers to the socially optimal choice. Thus, our solution shows a set of τ_j that is sufficient but not necessary.

[15] This policy induces the first-best choice of durability as long as other distortions are corrected elsewhere, but it is not the second-best optimal tax in a world with distortions in other markets that are not considered here.

account for the consumer's new choice for age of the product at retirement while facing τ_j^P, so we first derive $\partial c_j/\partial age$, the U-shaped curve, and then differentiate again to get $\partial^2 c_j/\partial age \partial\gamma$. This cross-partial is unambiguously positive, which means that γ raises the slope $\partial c_j/\partial age$. At the initial lifetime chosen, where this slope was zero, the slope becomes positive. Since c_j is strictly convex, the new minimum of c_j must shift leftward to a shorter lifetime. Thus, higher maintenance costs induce shorter product lifetimes (and lower γ induce longer lifetimes). Later, we show how T_j affects τ_j^P.

Second, what if the social discount rate ρ matches the private rate of discount r? This simplification yields four specific results.

(A.) Using $\rho = r$ and $T_j = T_j^*$, the optimal tax in (6) reduces to $\tau_j^P = P_j^* - P_j$. This standard Pigovian tax is the social cost of extraction and production not already covered in the purchase price. With this τ_j^P and the two other Pigovian taxes ($\tau_j^m(t)$ and τ_j^D), consumer decisions are socially optimal. None of these taxes depend on product lifetime, so this case has no durability gap. This result may not surprise economists, but it helps set the stage for other results below where policy *can* improve welfare by inducing or requiring longer product lives. Our results therefore suggest a change in focus, for those who recommend regulations on product durability.

(B.) Still assuming $\rho = r$, suppose a unit tax τ_j is less than optimal ($\tau_j < \tau_j^P = P_j^* - P_j > 0$). Then the chosen life T_j is not the optimal T_j^*, but an increase in this sub-optimal tax will increase chosen lifetimes. Our Appendix uses τ_j in place of τ_j^P in the U-shaped annualized cost c_j function in equation (3). It finds $\partial c_j/\partial age$ and differentiates again to find $\partial^2 c_j/\partial age \partial\tau_j$. That cross-partial is clearly negative, so a higher unit tax changes the slope of c_j from zero to negative at the original lifetime. Since $c_j(age)$ is strictly convex, consumers then choose a longer lifetime. Intuitively, the higher unit tax makes the consumer want to delay additional taxes imposed on disposal and on buying a replacement.

(C.) With the same conditions ($\rho = r$ and $\tau_j < P_j^* - P_j$, so $T_j < T_j^*$), then a small increase in product lifetimes raises social welfare. We define the annual welfare cost $WC(T_j)$ as the extent to which the annual social cost exceeds private cost (at any T_j). Using $\rho = r$ in equations (3) and (5), we have:

$$WC(T_j) \equiv c_j^*(T_j) - c_j(T_j) = \frac{\rho}{1 - e^{-\rho T_j}}[EC_j - \tau_j] \tag{7}$$

where $EC_j \equiv P_j^* - P_j$ is the external cost per unit of output. This welfare cost is zero if $\tau_j = EC_j$. If $\tau_j < EC_j$, then WC is positive. The Appendix differentiates (7) to find the condition under which $WC'(T_j) < 0$, namely, that any positive value of $EC'(T_j)$ must be less than $\frac{[EC(T_j)-\tau_j]\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$ (which is positive). With this condition, then an increase in product life reduces the welfare cost (raises social welfare).[16]

(D.) Notice that lifetime T_j (for each product) is the only relevant choice variable. Therefore, in this model with $\rho = r$ and $\tau_j < EC(T_j)$, the welfare-raising increase in product lives can be achieved by any of three different policies. First, we just showed that increasing the tax induces longer product lives (and then we showed that higher T_j raises welfare). Second, if τ_j cannot be raised, then a small subsidy to maintenance/repair can lengthen T_j (and increase welfare). A reduction in repair costs can be represented by a reduction of γ in $\gamma m_j(t)$, shown in the first result above to increase chosen product lives. That subsidy also raises welfare (assuming no economic distortions other than the market failures considered here). Third, similarly, the same welfare gain can be achieved by introducing a marginally binding mandate on firms to design longer lasting products.

In summary, when $\rho = r$, standard Pigovian taxes correct all externalities with no role for policy to favor long-lived products. The interdisciplinary CE literature is not necessarily wrong to call for durability regulations, but doing so could require showing that externality corrections are permanently sub-optimal. If so, then welfare can be raised by a durability mandate or a subsidy for repairs.

Third, what if $\rho < r$? Then the case for durability policy is even stronger, as the optimal tax is lower on products that last longer. Write the tax formula as $\tau_j^P = R \cdot PV_j^* - PV_j$, where $R \equiv \frac{\rho(1-e^{-rT_j^*})}{r(1-e^{-\rho T_j^*})}$. In the limit as T_j^* approaches zero, the ratio R approaches 1. Intuitively, discounting is irrelevant when T_j^* = 0. For positive T_j^*, our Appendix shows the full derivative of τ_j^P with respect to ρ. It's complicated, because ρ appears throughout PV_j^*. Its sign cannot be demonstrated analytically, but a grid search over parameters indicates that the sign is dominated by the derivative of R, which is unambiguously positive. Thus, the optimal tax falls as the social discount rate falls. The key to a durability gap, however, is whether the tax depends on product lifetime. As T_j^* increases, then ρ/r remains less than one, while the rest of R declines toward one. Thus, even starting with Pigovian taxes on all externalities, $\rho < r$ means that the optimal tax declines below the Pigovian tax on products that consumers choose to keep longer.

4. Effects of Internalities: The Second Model with Fixed Product Lifetimes

A more durable product might be one that requires less repair, or one that simply lasts longer. We now use the latter model where maintenance and repair costs are zero, lifetime T_j is fixed, and consumers just choose the product. Examples include a lightbulb or toaster oven: a product that stops

[16] Note that $EC'(T_j)$ in this condition can be either sign. An increase in durability (T_j) may require more materials, raise prices, and even raise external costs. Thus, it could raise the slope EC'. Sufficient for this condition is that P_j^* and P_j rise at the same rate (so that $EC' = 0$). But the weaker condition is just that $EC' > 0$ is not too large.

working goes to disposal. We retain assumptions of perfect information and certainty, but this model can consider all three categories of market failure (externalities, internalities, and divergence between social and private discount rates). In other words, we now add internalities such as inattention, bias, or irrational behavior.[17] Specifically, we suppose that identical consumers face tax τ_j^P upon purchase of product j while thinking that lives are δT_j. They minimize their own annual cost by choosing the product with minimum c_j in:

$$\left(P_j + \tau_j^P\right) + (D_j + \tau_j^D)e^{-r\delta T_j} = \int_0^{\delta T_j} c_j\, e^{-rt} dt = \frac{1 - e^{-r\delta T_j}}{r} c_j \tag{8}$$

Again we assume $\tau_j^D = D_j^* - D_j$, so consumers face social costs of disposal. Next, the socially optimal choice of durability is one that minimizes c_j^* in:

$$P_j^* + D_j^* e^{-\rho T_j} = \int_0^{T_j} c_j^*\, e^{-\rho t} dt = \frac{1 - e^{-\rho T_j}}{\rho} c_j^* \tag{9}$$

We solve for c_j and c_j^* and set them equal to each other. Then we solve for the tax rate on consumers that ensures $c_j = c_j^*$ for each product:

$$\tau_j^P = \frac{\rho}{r} \cdot \frac{1-e^{-r\delta T_j}}{1-e^{-\rho T_j}} (P_j^* + D_j^* e^{-\rho T_j}) - \left(P_j + D_j^* e^{-r\delta T_j}\right) \equiv \frac{\rho}{r} \cdot \frac{1-e^{-r\delta T_j}}{1-e^{-\rho T_j}} PV_j^* - PV_j \tag{10}$$

Facing this tax, consumers who minimize c_j also minimize c_j^*. The multiple terms in (10) enable τ_j^P and τ_j^D to fix all three types of market failures. Interestingly, if $\delta = 1$, then this model becomes a special case of the earlier model – assuming repair costs $m_j(t)$ and $m_j^*(t)$ are zero until time T_j when the product stops working. Thus, if perceived lifetimes are correct, then all results above also hold here.

With if $\delta \neq 1$, however, then we lose the result that a lower social discount rate necessarily implies a lower tax on products that last longer. The Appendix differentiates the tax formula (10) with respect to T_j and finds that the sign is ambiguous. In one numerical example below, the optimal tax *rises* with durability (though policy implications in that example are unclear).

What are the effects of internalities (δ)? We focus on the case where consumers underestimate product lives ($\delta < 1$), consistent with some labeling evidence.[18] We retain production and disposal externalities but again assume $\rho = r$. Then from (10), we have:

[17] With behavioral effects, results may depend on whether the tax is paid by the buyer or the seller.

[18] For example, Dupre *et al* (2016, p.12) state that “lifespan labeling has an influence on purchasing decisions in favor of products with longer lifespans” by an average of 13.8%. They find “significant influence on purchasing decisions in eight of nine product categories tested”, including *e.g.* suitcase (+23.7%), printer (+20.1%), trousers (+15.9%), or smartphone (+11.4%). Those examples explain our reduction of δ from 1.0 by as much as 20%.

$$\tau_j^P = \frac{1 - e^{-\rho\delta T_j}}{1 - e^{-\rho T_j}} \left(P_j^* + D_j^* e^{-\rho T_j}\right) - \left(P_j + D_j^* e^{-\rho\delta T_j}\right) \tag{11}$$

Taking the limit as T_j approaches zero, we find that τ_j^P approaches $\delta(P_j^* + D_j^*) - \left(P_j + D_j^*\right)$. Thus, $\delta < 1$ means the optimal τ_j^P is less than the standard Pigovian tax ($P_j^* - P_j$). For non-zero T_j, the Appendix shows the derivative of (11) with respect to δ is unambiguously positive. Thus, any perceived lifetime δT_j substantially below the actual lifetime T_j means that the optimal tax is reduced. If consumers *overestimate* the product lifetime, then τ_j^P is raised above $P_j^* - P_j$.

Why does the optimal tax fall as δ falls? A consumer who thinks product life is only δT_j must also think initial payment of P_j will yield services over a shorter life. Thus, a lower δ raises the perceived annual cost c_j. The consumer thinks services cost more per year relative to true cost c_j^*. This mistake can be corrected by a policy that *reduces* perceived annual cost via subsidy. The planner fixes production and disposal externalities with taxes but also can fix the internality by reducing τ_j^P. If the internality is large enough, the externality tax is more than offset, making τ_j^P negative.[19]

While this intuition explains the odd result, it is not ready for policy implementation. First, this simple model omits too many other relevant variables. Second, despite simplifications, the optimal tax in (10) has complicated components. Third, more evidence is needed to know whether consumers under-estimate or over-estimate product lives. Instead, the purpose here is to derive initial conceptual results as a framework for further analysis.

The optimal tax is reduced by $\delta < 1$, but then how does the tax relate to durability? And how does that relationship depend on changes in the social rate of discount? We address these questions using numerical illustrations that vary all parameters and all market failures.

5. Simple Numerical Illustrations

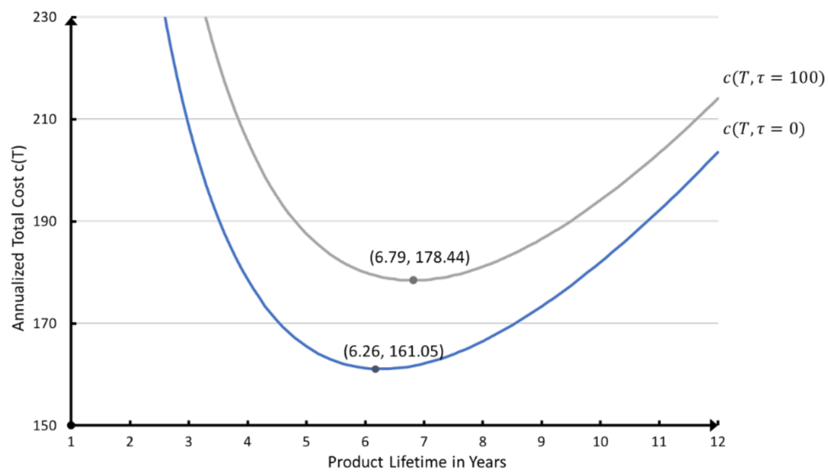
Using this second model, we can show exactly how optimal taxes change with durability for alternative parameter values – considering a continuum of products that allows a continuous choice of lifetime T. Without rising repair costs, we ensure the choice of a product with a finite T by assuming that production cost is $P(T)$ with $P'(T) > 0$. This assumption is plausible, both because increased durability can increase production cost and because consumers are often willing to pay extra for products that last longer. In fact, we assume $P'(T) > 0$ is high enough that both $c(T)$ and $c^*(T)$ are continuous, strictly convex, and twice differentiable – each with a minimum at finite T.

With $\delta = 1$, and $\rho = r = 0.04$, suppose consumers face social costs of disposal. Quadratic

[19] If the internality $\delta < 1$ is the *only* market failure, then the optimal tax rate is unambiguously negative.

production cost is $P(T) = 500 + 10T^2$, and social cost is $P^*(T) = 600 + 10T^2$. Thus, the optimal tax on purchase is the external cost, \$100 (regardless of T). First, to illustrate consumers choice of product, Figure 1 shows the annualized cost curves.[20] Untaxed consumers choose a lifetime just over six years, where the annual welfare cost is (179.09 – 161.05) = \$18.04 per year (per product). Facing a tax of \$100, consumers choose the optimum T of almost seven years, and the welfare cost is zero.

Figure 1: How Annualized Costs Depend on Product Lifetime



Next, we illustrate how the optimal tax depends on durability for alternative parameters. Assuming consumers pay social costs of disposal, Figure 2 shows how the purchase tax depends on product lifetime when $(P_j^* - P_j)$ = \$100. Figure 2A still assumes $P(T)$ is quadratic in T. Initially, however, look at only the top four curves (omitting any internality, $\delta = 1$). The top-most curve with $\rho = r = 0.04$ shows the case above where the optimal tax is a flat \$100 and does not depend on durability. In fact, all curves with $\delta = 1$ and $r = 0.04$ start at \$100, but any $\rho < 0.04$ yields optimal taxes that decline with T. This decline gets even steeper in the other three curves, as ρ is reduced from 0.03 toward 0.01.

Why is the subsidy to durability enlarged by reductions in the social discount rate? In our model, the consumer's choice of durability is repeated indefinitely, so longer-lasting products reduce the frequency that external costs are imposed – both from disposal and from production of a replacement. When $\rho = r$, private decisions about such timing are optimal, but $\rho < r$ means that consumers overly discount those future costs and choose sub-optimal durability.

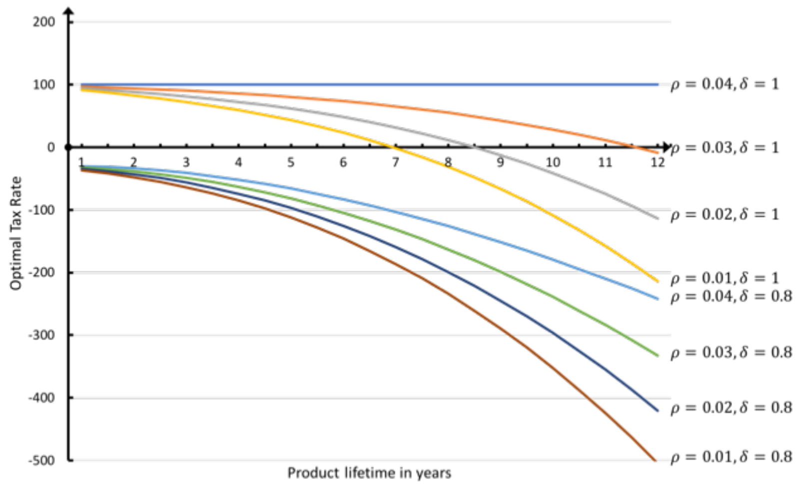
Next, with all market failures simultaneously, consider $\delta < 1$. That case is shown in equation (10), where the derivative of tax with respect to lifetime can be either sign. In Figure 2A, the four curves

[20] This illustration abstracts from many complications mentioned throughout, including the possibility that durability is correlated with other attributes like product recyclability.

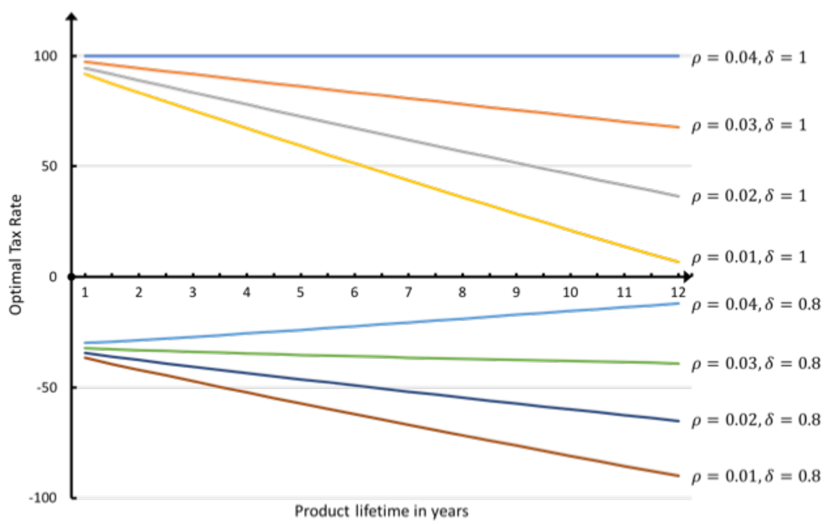
for $\delta = 0.8$ are similar to those for $\delta = 1$, where reductions in the social discount rate always decrease the level and slope of the curve. Optimal policy with $\rho < r$ favors durability.

Figure 2: How the Optimal Tax Rate on Purchase Depends on Product Lifetime

Panel A: Quadratic Prices (r = 0.04, $P(T) = 500 + 10T^2$, and $P^*(T) = 600 + 10T^2$)



Panel B: Non-Quadratic Prices (r = 0.04, $P(T) = 500 + 10T^{0.5}$, and $P^*(T) = 600 + 10T^{0.5}$)



Illustrations so far use quadratic prices (where $P(T) = 500 + 10T^2$ and $P^*(T) = 600 + 10T^2$). Figure 2B uses prices that rise much less steeply: $P(T) = 500 + 10T^{0.5}$ and $P^*(T) = 600 + 10T^{0.5}$. Overall, the array of eight curves in 2B are qualitatively similar to those in 2A, emphasizing our main result that lower ρ make optimal taxes decline with durability. Despite that similarity, one difference is that quadratic prices lead to nonlinear curves in 2A, while curves in 2B are almost linear. A second difference is that the optimal tax in 2B can rise with durability (when $r = \rho = 0.04$ and $\delta = 0.8$). In fact, prices in Figure 2B are not chosen to be realistic, but only to find a case where that slope is positive.

Since this anomaly occurs only when $P'(T)$ is very small, and only when $r = \rho$, its relevance is small.

Nonetheless, why would optimal policy ever discourage durability? The internality δ enters (10) in a ratio, $(1 - e^{-r\delta T_j})/(1 - e^{-\rho T_j})$, where only the *private* discount rate r is multiplied by $\delta < 1$. In other words, thinking that the product life is only 80% of its true life operates effectively like having a *lower* private discount rate. Consumers are buying a product that lasts longer than they anticipate, which means they effectively give more weight to the future than intended. If $r = \rho$, they weight the future by *more* than socially optimal. The policy corrects that internality by discouraging durability.

While we de-emphasize this case where the optimal tax rises with durability, the important point is that the internality can interact with discount rates and affect this slope in either direction. The consumer mistake here is a fixed proportional reduction in perceived life, from T_j to δT_j. If instead δ were to *vary* with T_j, then the optimal tax could vary correspondingly with T_j. We cannot analyze all models of internalities, so we make no general claims about how internalities affect the way optimal taxes relate to durability. Among others, Heutel (2015) discusses various models of present-bias internalities and policies to correct them.

The key unambiguous result is that $\rho < r$ leads to an optimal tax that falls with durability. In both Figures 2A and 2B, and with all market failures, the slope of the tax against T is always more negative as social discount rates fall from 0.04 to 0.01. Thus, a larger durability gap is generated as ρ falls, providing the most support for those who think optimal policy would encourage durability.

6. Conclusion

We thus arrive at four major conclusions. First, the choice of durability itself does not generate an externality that requires direct intervention. Consumers already make socially optimal choices about durability if they are informed and rational optimizers facing all social costs of production and disposal (*e.g.*, facing Pigovian taxes). Second, if the tax is less than marginal external damage per unit output, then raising the tax increases chosen durability. If the tax remains too low, then a durability subsidy or mandate can increase welfare. Third, if consumers understate a product's lifetime, then they overstate the annual cost of its services. A subsidy can correct that problem, offsetting the Pigovian tax, but that subsidy does not systematically relate to durability. Fourth, a social rate of discount less than the private rate does cause a durability gap – so the optimal tax explicitly encourages durability.

Yet much work remains. We introduce a "durability gap" and begin analysis, but extensions would make the model more useful. One extension could consider uncertainty about product lifetimes, with or without risk aversion of consumers. Another extension is technological progress that reduces the expected price to be paid to replace the product now being purchased. Other extensions might consider market power or other market imperfections, second-best considerations, general equilibrium effects, or consumer heterogeneity.

The "Circular Economy" is a popular topic in the interdisciplinary literature, but economic analysis is lacking. Further analysis can extend not only to the durability issues just listed, but also to other CE issues about how to delay and reduce creation of waste, how durability relates to recyclability, and how to encourage the conversion of waste into valuable inputs that can re-enter production.

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Online Appendix for:
Do Market Failures Create a 'Durability Gap' in the Circular Economy?
by Don Fullerton and Shan He (June 2021)

This Appendix demonstrates a series of results from the two models in sections 3 and 4 of the text.

3. Results from the First Model – Maintenance Costs Affect Consumer Choice of Lifetime

First, how do maintenance costs affect choices? We expect an increase in maintenance costs to induce earlier product retirements. To demonstrate this proposition, we introduce a shift parameter γ to the repair schedules: $\gamma m_j(t)$ and $\gamma m_j^*(t)$. The *partial* derivative $\partial\tau_j^P/\partial\gamma$ would not account for the consumer's new choice for age of product j at its retirement, while facing τ_j^P, so we first we derive $\partial c_j/\partial age$, the U-shaped curve in Figure 1. When consumers face the optimal tax, their cost c_j matches the planner's cost c_j^* (and thus $T_j = T_j^*$). Thus, we can differentiate equation (5) in the text to get:

$$\frac{\partial c_j}{\partial age} = \frac{\partial c_j^*}{\partial age} = -\frac{\rho^2 e^{-\rho T_j^*}}{(1-e^{-\rho T_j^*})^2}P_j^* - \left[\frac{\rho^2 e^{-2\rho T_j^*}}{(1-e^{-\rho T_j^*})^2} + \frac{\rho^2 e^{-\rho T_j^*}}{1-e^{-\rho T_j^*}}\right]D_j^*$$
$$+\left[\frac{\rho(1-e^{-\rho T_j^*})}{(1-e^{-\rho T_j^*})^2}\gamma m_j^*(T_j^*)e^{-\rho T_j^*} - \frac{\rho^2 e^{-\rho T_j^*}}{(1-e^{-\rho T_j^*})^2}\gamma\int_0^{T_j^*} m_j^*(t)\,e^{-\rho t}dt\right]$$

On the U-shaped c_j curve, that derivative is first negative and then zero and positive. Differentiate again:

$$\frac{\partial^2 c_j}{\partial age\partial\gamma} = \frac{\rho}{(1-e^{-\rho T_j^*})^2}\left[e^{-\rho T_j^*}(1-e^{-\rho T_j^*})m_j^*(T_j^*) - \rho e^{-\rho T_j^*}\int_0^{T_j^*} m_j^*(t)\,e^{-\rho t}dt\right]$$

This cross-partial is positive, so long as $m_j^*(T_j^*) > \frac{\rho}{1-e^{-\rho T_j^*}}\int_0^{T_j^*} m_j^*(t)\,e^{-\rho t}dt$. This condition says that the maintenance cost *flow* at the end of product life (T_j^*) is larger than the average annualized maintenance cost over the product life. It must hold, because maintenance cost is rising over time, $m_j^{*\prime}(t) > 0$. Thus, the increase in γ raises $\frac{\partial c_j}{\partial age}$, the slope of $c_j(age)$. At the initial chosen lifetime T_j^*, the slope was zero, so it rises to positive. Since c_j is strictly convex, the new minimum of c_j must shift left to a shorter life.

Second, what if the social discount rate ρ matches the private rate of discount r?

(A.) Using $\rho = r$ and $T_j = T_j^*$, the optimal tax in (6) reduces to $\tau_j^P = P_j^* - P_j$. With all three Pigovian taxes (this τ_j^P as well as $\tau_j^m(t)$ and τ_j^D), all consumer decisions are socially optimal. None of these taxes depend on T_j, so this case has no 'durability gap'.

(B.) Still assuming $\rho = r$, suppose a unit tax τ_j is less than optimal ($\tau_j < \tau_j^P = P_j^* - P_j > 0$). Then the chosen life T_j is *not* the optimal T_j^*, but we can demonstrate that an increase in this sub-optimal tax τ_j will increase chosen lifetimes T_j. We use τ_j in place of τ_j^P in the U-shaped annualized cost c_j function in equation (3) to derive:

$$\frac{\partial c_j}{\partial age} = -\frac{r^2 e^{-rT_j}}{\left(1 - e^{-rT_j}\right)^2}\left(\tau_j + P_j\right) - \left[\frac{r^2 e^{-2rT_j}}{\left(1 - e^{-rT_j}\right)^2} + \frac{r^2 e^{-rT_j}}{\left(1 - e^{-rT_j}\right)^2}\right] D_j^* +$$
$$\left[\frac{r(1 - e^{-rT_j})}{(1 - e^{-rT_j})^2} m_j^*(T_j) e^{-rT_j} - \frac{r^2 e^{-rT_j}}{(1 - e^{-rT_j})^2}\int_0^{T_j} m_j^*(t)\, e^{-rt} dt\right]$$

Then, we differentiate again to get:

$$\frac{\partial^2 c_j}{\partial age \partial \tau_j} = -\frac{r^2 e^{-rT_j}}{\left(1 - e^{-rT_j}\right)^2} < 0$$

Because that cross-partial is negative, a higher unit tax τ_j reduces the slope of c_j from zero to negative at the original lifetime. Since $c_j(age)$ is strictly convex, consumers then choose a longer lifetime.

(C.) With the same conditions ($\rho = r$ and $\tau_j < P_j^* - P_j$, so $T_j < T_j^*$), then a small increase in product lifetimes raises social welfare. We define the annualized welfare cost $WC(T_j)$ as the extent to which the annual social cost exceeds private cost (at any T_j). Using $\rho = r$ in equations (3) and (5), we have (7) in the text:

$$WC(T_j) \equiv c_j^*(T_j) - c_j(T_j) = \frac{\rho}{1 - e^{-\rho T_j}}\left[EC_j - \tau_j\right] \tag{7}$$

where $EC_j \equiv P_j^* - P_j$ is the external cost per unit of output. This welfare cost is zero if $\tau_j = EC_j$. If $\tau_j < EC_j$, then WC is positive. Next, we differentiate (7) to get:

$$WC'(T_j) = EC'(T_j)\frac{\rho}{1 - e^{-\rho T_j}} + \left[\tau_j - EC(T_j)\right]\frac{\rho^2 e^{-\rho T_j}}{\left(1 - e^{-\rho T_j}\right)^2}$$

and we find conditions under which an increase in T_j reduces welfare cost (*i.e., $WC'(T_j) < 0$*). The second term is negative because $\tau_j < EC(T_j)$. But $EC'(T_j)$ in the first term can be either sign. Thus, for WC' to be negative, any positive value of $EC'(T_j)$ must be less than $\frac{[EC(T_j) - \tau_j]\rho e^{-\rho T_j}}{1 - e^{-\rho T_j}}$ (which is positive). With this condition, then an increase in product life reduces welfare cost (raises social welfare).

Third, what if $\rho < r$? Write the tax formula (6) as $\tau_j^P = R \cdot PV_j^* - PV_j$, where $R \equiv \frac{\rho(1 - e^{-rT_j})}{r(1 - e^{-\rho T_j^*})}$. In the limit as T_j^* approaches zero, the ratio R approaches 1. Intuitively, discounting is irrelevant when T_j^* = 0. For positive T_j^*, we take the derivative of τ_j^P with respect to ρ, which is complicated (because ρ appears throughout PV_j^*):

$$\partial \tau_j^P / \partial \rho = \frac{1 - e^{-rT_j}}{r\left(1 - e^{-\rho T_j^*}\right)}\left(1 - \frac{\rho T_j^* e^{-\rho T_j^*}}{1 - e^{-\rho T_j^*}}\right)\left(P_j^* + \int_0^{T_j^*} m_j^*(t)\, e^{-\rho t} dt + D_j^* e^{-\rho T_j^*}\right)$$
$$-\frac{\rho}{r} \cdot \frac{1 - e^{-rT_j}}{1 - e^{-\rho T_j^*}}\left(\int_0^{T_j^*} t m_j^*(t)\, e^{-\rho t} dt + T_j^* D_j^* e^{-\rho T_j^*}\right)$$

Its sign cannot be demonstrated analytically, but a grid search over parameters indicates that the sign is dominated by the derivative of R – which is unambiguously positive. In the simple case with no disposal costs, we get a definitive result. Then $\tau_j^P = \frac{\rho}{r} \cdot \frac{1-e^{-rT_j}}{1-e^{-\rho T_j^*}} P_j^* - P_j$ and the derivative is:

$$\partial \tau_j^P / \partial \rho = \frac{1-e^{-rT_j}}{r\left(1-e^{-\rho T_j^*}\right)} \left(1 - \frac{\rho T_j^* e^{-\rho T_j^*}}{1-e^{-\rho T_j^*}}\right) P_j^* > 0$$

This derivative is unambiguously positive. The ratio inside the parentheses is zero when T_j is zero, and it approaches 1 as T_j goes to infinity, so the ratio must be between zero and one. The optimal tax τ_j^P rises with ρ, which means that the tax falls as the social discount rate falls further below the private rate.

4. Results from the Second Model – Consumer Choice Among Fixed Product Lifetimes

This second model with $\delta = 1$ is a special case of the first model. Thus, if perceived lifetimes are correct, then all results from section 3 also hold here. With if $\delta \neq 1$, however, then we lose the result that a lower social discount rate necessarily implies a lower tax on products that last longer. When we differentiate the tax formula (10) with respect to T_j, the sign of the derivative is ambiguous:

$$\frac{\partial \tau_j^P}{\partial T_j} = \frac{\rho}{r} \cdot \frac{r\delta e^{-r\delta T_j}\left(1-e^{-\rho T_j}\right) - \rho e^{-\rho T_j}\left(1-e^{-r\delta T_j}\right)}{(1-e^{-\rho T_j})^2} \cdot \left(P_j^* + e^{-\rho T_j} D_j^*\right)$$
$$-\frac{\rho}{r} \cdot \frac{1-e^{-r\delta T_j}}{1-e^{-\rho T_j}} \cdot \left(\rho e^{-\rho T_j} D_j^*\right) + r\delta e^{-r\delta T_j} D_j$$

Keeping production and disposal externalities, but with $\rho = r$, then the tax is shown in (11). When $\delta < 1$, this optimal τ_j^P is less than the standard Pigovian tax ($P_j^* - P_j$). For non-zero T_j, the derivative of (11) with respect to δ is:

$$\frac{\partial \tau_j^P}{\partial \delta} = \frac{\rho T_j e^{-\rho\delta T_j}}{1-e^{-\rho T_j}} \left(P_j^* + D_j^* e^{-\rho T_j}\right) + \rho T_j D_j^* e^{-\rho\delta T_j} > 0$$

Since this derivative is unambiguously positive, any perceived lifetime δT_j substantially below the actual lifetime T_j means that the optimal tax is reduced (possibly becoming a subsidy).