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Albert Jan Hummel

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Abstract

This paper studies optimal income taxation in an environment where matching frictions generate a trade-off for workers between high wages and low unemployment risk. A higher marginal tax rate shifts the trade-off in favor of low unemployment risk, whereas a higher tax burden or unemployment benefit has the opposite effect. Changes in unemployment generate fiscal externalities, which modify optimal tax formulas. I show that optimal employment subsidies (such as the EITC) phase in with income and that the provision of unemployment insurance justifies a positive marginal tax rate even without income heterogeneity. A calibration exercise to the US economy suggests that optimal transfers for low-income individuals are larger if unemployment risk is taken into account.

JEL-Codes: H210, J640, J650, J680.

Keywords: directed search, optimal taxation, unemployment insurance.

Albert Jan Hummel
University of Amsterdam / The Netherlands
a.j.hummel@uva.nl

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1 Introduction

An extensive literature studies optimal income taxation when individuals supply labor on the intensive (hours, effort) margin (Mirrlees (1971), Diamond (1998), Saez (2001)), the extensive (participation) margin (Diamond (1980)), Choné and Laroque (2011)), or both (Saez (2002), Jacquet et al. (2013), Hansen (2021)). A common assumption in this literature is that there is no unemployment: all individuals who wish to work are employed. Consequently, income taxes only affect incentives to supply labor. There is, however, ample theoretical and empirical evidence that unemployment responds to taxation as well. This paper asks: what are the implications of unemployment for the optimal design of income taxes?

To study this question, I set up a directed search model where individuals differ in terms of their skills and participation costs. They supply labor on the intensive (hours, effort) and extensive (participation) margin to identical firms, which post vacancies to attract applicants. Matching frictions generate unemployment risk, which is not privately insurable. The government observes individuals' labor earnings and employment status, but not their labor effort and search behavior. It provides partial insurance and redistributes income by setting a uniform unemployment benefit and a non-linear income tax. I use the model to (i) study how the tax-benefit system affects labor-market outcomes and (ii) derive optimal policy rules. Furthermore, I explore the quantitative implications of unemployment for the optimal tax-benefit system by calibrating the model to the US economy. My main findings are the following.

First, an increase in the marginal tax rate reduces unemployment, whereas an increase in the tax burden or the unemployment benefit has the opposite effect. I label the first of these the employment-enhancing effect (EEE) and the second the employment-reducing effect (ERE) of taxation. Both effects originate from a trade-off individuals face between high wages and low unemployment risk: applying for a job that pays a higher wage reduces the likelihood of being matched. A higher *marginal* tax rate makes individuals care less about the wage, whereas a higher tax burden (i.e., a higher *average* tax rate) or unemployment benefit makes individuals care less about finding a job. Consequently, a local increase (decrease) in the marginal (average) tax rate lowers wages, which leads firms to hire more workers and unemployment to decline. Moreover, because a local increase in the marginal tax rate mechanically increases the tax burden at higher income levels, lowering the unemployment rate at some point in the skill distribution through an increase in the marginal tax rate comes at the costs of raising the unemployment rate among individuals with higher skills.

Second, I derive sufficient-statistics optimal tax formulas that clearly illustrate how unemployment responses should be taken into account when designing the tax-benefit system. Because search is competitive, unemployment is constrained efficient. However, changes in unemployment generate fiscal externalities as unemployed workers receive benefits and do not pay income taxes. These fiscal externalities call for intuitive adjustments of standard optimal tax formulas. Whether optimal marginal tax rates are higher or lower once unemployment risk is taken into account depends on the responsiveness of unemployment to the marginal tax rate and the tax burden and on the employment tax (i.e., the sum of the income tax and the unemployment benefit) at different points in the income distribution.[1] In the typical case that the

[1]The literature typically uses the term participation tax to refer to the sum of the income tax and the

employment tax is positive, the reduction in unemployment through the EEE raises the optimal marginal tax rate, whereas the increase in unemployment among individuals with higher skills through the ERE has the opposite effect.

Third, optimal employment subsidies phase in with income. Put differently, if the *level* of the employment tax for low-income workers is negative, these workers should also face a negative *marginal* tax rate. My model thus provides a rationale for the phase-in region of the EITC. This result complements those from Diamond (1980) and Saez (2002). They show that it is optimal to subsidize employment if the government cares sufficiently about low-income workers and labor-supply responses are mostly concentrated on the extensive margin. An employment subsidy at the bottom, however, induces low-skilled workers to apply for jobs that pay inefficiently low wages. A negative marginal tax rate alleviates this distortion by making it more attractive to apply for jobs that pay a higher wage. The reduction in the employment rate positively affects government finances if employment for low-skilled workers is subsidized.

Fourth, I show that the provision of unemployment insurance (UI) justifies a positive marginal tax rate even without heterogeneity in labor earnings. To see why, suppose all individuals are *ex ante* identical, but face uninsurable unemployment risk. In this case, the tax-benefit system is merely used for insurance (and not redistributive) purposes. As in the framework of Baily (1978) and Chetty (2006), the optimal employment tax is positive and balances the insurance benefits against the costs of upward distortions in unemployment. I complement this result by showing that the optimal *marginal* tax rate is positive as well. Intuitively, raising the marginal tax rate puts downward pressure on wages. The associated increase in employment (through the EEE) alleviates the upward distortion in unemployment generated by UI. The optimal marginal tax rate satisfies a simple inverse-elasticity rule and increases in the size of the employment tax and the elasticity of the employment rate with respect to the marginal tax rate. This result also makes clear that, contrary to what is typically assumed, financing UI benefits through lump-sum or proportional taxes is generally sub-optimal.

Finally, a calibration exercise to the US economy suggests that optimal transfers for low-income individuals are larger if unemployment risk is taken into account. Specifically, in the baseline calibration both individuals who are not employed or employed at low earnings receive a transfer that is approximately $400 per year larger than in the competitive benchmark without unemployment. Despite this, the impact of unemployment risk on optimal marginal tax rates and employment taxes appears to be modest.

1.1 Related literature

Taxation in imperfect labor markets

Theory – Many papers study the impact of the tax-benefit system on labor-market outcomes in imperfectly competitive labor markets. See Bovenberg and van der Ploeg (1994) and Picard and Toulemonde (2003) for overviews. A robust finding is that, for a given tax burden, an increase in the marginal tax rate reduces unemployment. Conversely, for a given marginal tax rate, an increase in the tax burden or unemployment benefit raises unemployment. These results are

unemployment benefit. However, as pointed out by Kroft et al. (2020), the term employment tax is more appropriate if there are individuals who participate (i.e., who look for a job), but nevertheless remain unemployed.

obtained in union bargaining models (Hersoug (1984), Lockwood and Manning (1993), Koskela and Vilmunen (1996)), in matching models with individual bargaining (Pissarides (1985, 1998)) and in models where firms pay efficiency wages (Pisauro (1991)). This paper contributes to this literature by showing the results also hold in a directed search environment where firms post wages in order to attract applicants. I refer to the negative impact of the marginal tax rate on unemployment as the employment-enhancing effect (EEE) of taxation and to the positive impact of the tax burden or unemployment benefit on unemployment as the employment-reducing effect (ERE) of taxation.

Evidence – There exists ample evidence that benefit generosity positively affects unemployment and unemployment duration, in line with the ERE (see, e.g., Meyer (1990), Chetty (2008) and Card et al. (2015)). Moreover, many macro-empirical studies document a positive effect of income taxes on unemployment (see, e.g., Nickell and Layard (1999), Blanchard and Wolfers (2000), Daveri and Tabellini (2000), Griffith et al. (2007), Bassanini and Duval (2009)). However, because these studies do not distinguish between marginal and average tax rates, the results are not very informative about the EEE and ERE. The only study I know which empirically separates these channels is a recent paper by Lehmann et al. (2016). Using data on a panel of OECD countries, they find that average tax rates positively affect unemployment, whereas marginal tax rates have the opposite effect. Similarly, Manning (1993) shows that a single index of tax progressivity (which increases in the marginal tax rate and decreases in the average tax rate) is negatively associated with unemployment in the UK.

More indirect evidence comes from studies that analyze the impact of income taxes on wages. A typical finding is that an increase in the average tax rate is associated with an increase in the hourly wage, whereas an increase in the marginal tax rate has the opposite effect (see, e.g., Malcomson and Sartor (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995)). Using micro-level data, Blomquist and Selin (2010) exploit a series of Swedish tax reforms and find a strong negative effect of marginal tax rates on wages, in line with the EEE. Schneider (2005) and Rattenhuber (2012) obtain similar results using data on German workers. The results from these studies are consistent with the predictions from my model.

Optimal taxation with intensive and extensive labor-supply responses

This paper builds on a literature that studies optimal income taxation when individuals supply labor both on the intensive (hours, effort) and on the extensive (participation) margin. In an influential paper, Saez (2002) shows that if labor supply is most responsive on the intensive margin, the optimal policy features a Negative Income Tax (NIT), i.e., a substantial guaranteed income that is quickly phased out. By contrast, if labor-supply responses are mostly concentrated along the extensive margin, the optimal tax schedule more closely resembles that of an Earned Income Tax Credit (EITC) with a low guaranteed income and substantial in-work benefits. Jacquet et al. (2013) generalize this framework and derive conditions under which marginal tax rates and employment taxes are positive. Hansen (2021) analyzes under which conditions marginal tax rates and employment taxes are negative at low levels of income. Jacobs et al. (2017) use a similar framework to derive optimal tax formulas in terms of sufficient statistics.

I contribute to this literature by analyzing the implications of unemployment for optimal tax design. As such, my optimal tax formulas generalize those of Jacobs et al. (2017).[2] Moreover, I show that if unemployment responses are taken into account, it is optimal to let employment subsidies (such as the EITC) phase in with income.

Optimal taxation and search

This paper is closely related to a literature that studies optimal taxation with search frictions. Boone and Bovenberg (2004, 2006) analyze a model where workers engage in costly search effort. More recently, Sleet and Yazici (2017) study optimal income taxation in a framework similar to that of Burdett and Mortensen (1998) with on and off the job search. In these papers unemployment is not affected by income taxes. By contrast, the interaction between unemployment and the design of the tax-benefit system lies at the heart of this paper.

Hungerbühler et al. (2006) study optimal income taxation in a model where wages are determined through Nash bargaining, but labor supply is exogenous. As in my model, an increase (decrease) in the marginal (average) tax rate lowers wages, which reduces unemployment. Hungerbühler and Lehmann (2009), Lehmann et al. (2011) and Jacquet et al. (2014) extend the framework to study minimum wages, endogenous participation and alternative bargaining structures. Golosov et al. (2013) use a similar matching model to analyze the optimal redistribution of residual wage inequality (i.e., wage inequality among equally skilled workers). In concurrent work, da Costa et al. (2019) characterize constrained efficient allocations in a directed search environment that is similar to the one analyzed in the current paper. I contribute to this literature in two ways. First, I study a model where individuals supply labor both on the intensive and extensive margin. Second, and more importantly, rather than deriving properties of constrained efficient allocations using tools from mechanism design, I analyze tax reforms to derive optimal tax rules in terms of the income distribution and behavioral responses ("sufficient statistics" in the terminology of Chetty (2009)). These formulas clearly illustrates how unemployment responses should be taken into account when designing the tax-benefit system.

Optimal unemployment insurance

Finally, this paper relates to a literature on the optimal provision of unemployment insurance (UI). In an early contribution, Baily (1978) shows that optimal UI policy balances insurance gains against the adverse effects of UI on job search. Chetty (2006) generalizes this framework and derives a sufficient-statistic formula for the optimal UI benefit which holds in a wide class of models. The framework I present differs in a number of ways from these and most other models used in the literature. First, I do not restrict UI payments to be financed through lump-sum or proportional taxes on labor income. Second, I abstract from dynamic considerations but allow UI provision to not only affect unemployment but also wages (as in Acemoglu and Shimer (1999) and Fredriksson and Holmlund (2001)) and labor supply. Consequently, UI

[2] Hummel and Jacobs (2018) and Kroft et al. (2020) also derive optimal tax formulas in a model where wages and unemployment respond to taxation. These studies feature a discrete set of occupations and the government sets the tax liability in each occupation separately. By contrast, in my model there is a single labor market and a continuum of skill types. A direct consequence is that marginal and average tax rates cannot be set independently.

affects government finances through both unemployment and earnings responses. This calls for an intuitive adjustment of the Baily-Chetty formula. Furthermore, I show that the provision of UI justifies a positive marginal tax rate on labor earnings even without income heterogeneity. An immediate implication is that financing UI payments through lump-sum or proportional taxes on labor income, as is commonly assumed in the literature, is generally sub-optimal.

The remainder of this paper is organized as follows. Section 2 presents a directed search model of the labor market. Section 3 discusses the efficiency properties and analyzes how the tax-benefit system affects labor-market outcomes. Section 4 characterizes the optimal tax-benefit system. Section 5 numerically illustrates the results by calibrating the model to the US economy. Section 6 concludes and discusses directions for future research. All proofs and additional details on the numerical analysis can be found in the appendices.

2 A directed search model of the labor market

This section presents the model that is used in the remainder of the analysis. The main ingredients are the following. There is a continuum of individuals who are heterogeneous in terms of their ability and costs of participating in the labor market, both of which are private information. They supply labor on the intensive (hours, effort) and on the extensive (participation) margin to homogeneous firms. Firms post vacancies in order to attract applicants and continue to do so until expected profits are zero. Matching frictions generate unemployment, and the risk of becoming unemployed is not privately insurable. The government has a preference for redistribution but faces asymmetric information regarding individuals' types and their labor supply and search behavior. It only observes an individual's labor earnings and whether or not she is employed. Consequently, it can levy a non-linear income tax and provide a uniform unemployment benefit. The latter is paid both to non-participants and to individuals who decided to search but nevertheless remain unemployed. I first characterize the equilibrium for a given tax-benefit system and analyze the optimal tax problem separately in Section 4.

Individuals

The economy is populated by a unit mass of individuals, who are also referred to as workers. They differ both in terms of their ability (or skill), and in their costs of participating in the labor market. Ability is denoted by $n \in [n_0, n_1]$ and participation costs by $\varphi \in [\varphi_0, \varphi_1]$. $F(\varphi, n)$ describes the joint distribution and $f(\varphi, n)$ the corresponding density.

Individuals derive utility from consumption c and disutility from exerting labor effort ℓ. To produce y units of output, an individual with ability n must exert $\ell = y/n$ units of effort. Each individual can be in three states: employment, unemployment or non-participation. If employed, an individual consumes her earnings z net of taxes $T(z)$. If not, she consumes an unemployment benefit b. Hence, the government does not distinguish between individuals who chose not to participate and those who are (involuntarily) unemployed. Moreover, unemployment risk is not privately insurable.[3] I denote by e the employment rate conditional on participation, which,

[3] Both the assumption that unemployment risk is not privately insurable and that the government cannot

given that the model is static, can also be interpreted as the matching probability or the job-finding rate. The unemployment rate is then given by $1-e$. As will be made clear below, in equilibrium labor earnings, output and the employment rate all vary with ability n.

The expected utility of an individual (n,φ) who applies for a job where she earns income z, produces output y and becomes employed with probability e, equals:

$$U(n,\varphi) = e\left[u(z-T(z)) - v\left(\frac{y}{n}\right)\right] + (1-e)u(b) - \varphi. \tag{1}$$

Here, sub-utility over consumption $u(\cdot)$ is assumed to be strictly increasing and weakly concave, and disutility of labor effort $v(\cdot)$ is strictly increasing and strictly convex. The value of $v(0)$ is normalized to zero. Note that participation costs are incurred irrespective of whether or not an individual finds a job. Hence, they can equivalently be interpreted as the costs of searching.

Firms

Firms are homogeneous and post vacancies at unit cost $k>0$. A vacancy (y,z) specifies (i) how much output y a potential employee is expected to produce, and (ii) the income z she receives as compensation. Because a vacancy specifies output (and not effort), the only source of uncertainty which matters for the firm is *whether* the vacancy is filled, not *by whom*. The firm is therefore indifferent as to whether the vacancy is filled by a high-skilled worker who exerts little effort or by a low-skilled worker who has to work harder to produce the same output.

Government

There is a government that has a preference for redistribution. Its objective is formally defined in Section 4. The government cannot observe individuals' types nor their labor supply or search behavior. Instead, it can only observe an individuals' labor earnings and whether or not she is employed. Consequently, the government can levy a non-linear tax $T(z)$ on labor income to finance a (uniform) benefit b paid to both non-participants and the involuntary unemployed and some exogenous spending G.

Matching

Unemployment results from matching frictions in the labor market. Frictions are captured in a reduced-form way through a matching function. The latter maps the number of job-seekers and vacancies in a particular sub-market into matches. As will be made clear below, in equilibrium labor markets are going to be perfectly segmented by skill type. Within each sub-market, there is a common matching function that features constant returns to scale. Consequently, the job-finding (or employment) rate depends only on labor market tightness, i.e., the ratio of vacancies to job-seekers in a particular sub-market. I denote the inverse of this relationship by $\theta=\theta(e)$. Hence, $\theta(e)$ measures how many vacancies relative to job-seekers must be posted for a fraction e

distinguish between non-participants and unemployed workers can be micro-founded by assuming an individual's application strategies are private information. See Boadway and Cuff (1999) and Boadway et al. (2003) for an analysis of optimal income taxation if the government can distinguish between non-participants and the involuntary unemployed, e.g., through costly monitoring.

of job-seekers to find employment in a given sub-market. The probability that a firm is matched is then given by $e/\theta(e)$. The function $\theta(e)$ fully captures the presence and severity of matching frictions. I assume it is strictly increasing, strictly convex, and satisfies $\theta(0) = 0$. Furthermore, I assume that the elasticity $\theta'(e)e/\theta(e)$ is non-decreasing.[4]

2.1 Equilibrium

Firms continue to post vacancies until profits are zero in expectation. Therefore, if a vacancy (y, z) is posted in equilibrium, the following must hold:

$$k = \frac{e}{\theta(e)}(y - z). \tag{2}$$

Free entry ensures that the cost of opening a vacancy equals the probability that a vacancy is filled, multiplied by the profit margin. If a posted vacancy (y, z) implies a high profit margin, free entry ensures it is filled with low probability. The conditions on the matching function then imply that workers are matched with a high probability if they apply for such a vacancy. Intuitively, the matching probability of workers is high if there are many vacancies relative to job-seekers, and *vice versa* for firms. However, a vacancy that specifies a high profit margin (and hence a high matching probability for workers) implies a low wage per unit of effort.[5] When deciding where to apply, each worker thus faces a trade-off between a high wage and low unemployment risk. This trade-off between 'prices and probabilities' is typical in models where search is directed (see Wright et al. (2021)) and plays a crucial role in what follows.

Individuals maximize their expected utility (1) by optimally choosing (i) whether to apply and, conditional on participation, (ii) where to apply. Starting with the first, suppose an individual of type (n, φ) decides to participate. After incurring participation costs φ, her expected utility is:

$$\mathcal{U}(n) \equiv \max_{y,z,e} \left\{ e\left[u(z - T(z)) - v\left(\frac{y}{n}\right)\right] + (1 - e)u(b) \ \text{ s.t. } \ k = \frac{e}{\theta(e)}(y - z) \right\}. \tag{3}$$

Because participation costs are sunk, individuals with the same ability n make the same choices conditional on participation. Hence, their expected utility net of participation costs only depends on n. The decision whether or not to participate is then a very simple one: an individual of type (n, φ) participates if and only if her participation costs φ are below the threshold

$$\varphi(n) \equiv \mathcal{U}(n) - u(b). \tag{4}$$

I denote by $\pi(n)$ the participation rate of individuals with ability n. The latter is given by:

$$\pi(n) = \frac{\int_{\varphi_0}^{\varphi(n)} f(\varphi, n)d\varphi}{\int_{\varphi_0}^{\varphi_1} f(\varphi, n)d\varphi}. \tag{5}$$

[4]This condition holds for virtually all commonly employed matching functions, including the micro-founded ones. The assumption is made for technical convenience as it ensures labor earnings are monotone in ability n.

[5]To see why, let $w = z/(y/n)$ denote the wage per unit of effort (say, hours). For given ability and output, a high profit margin $y - z = y(1 - w/n)$ then corresponds to a low wage.

When considering where to apply, the zero-profit condition (2) implies each individual faces a trade-off not only between income and leisure, but also between a high wage and a low probability of becoming unemployed (i.e., between prices and probabilities). Combining the first-order conditions with respect to output y and earnings z gives

$$u'(z - T(z))n(1 - T'(z)) = v'\left(\frac{y}{n}\right). \tag{6}$$

This condition equates the marginal utility costs of exerting one more unit of effort (on the right-hand side) to the marginal benefits in the form of higher income (on the left-hand side). Equation (6) coincides with the standard labor-supply equation in a competitive equilibrium without frictions, where individuals optimally choose their working hours and the costs of opening a vacancy are zero (in which case free entry implies $z = y$).

In addition to the standard trade-off between consumption and leisure, individuals also face a trade-off between high wages and low unemployment risk. Combining the first-order conditions with respect to earnings z and the probability e of finding a job gives

$$eu'(z - T(z))(1 - T'(z)) = \frac{e}{(\theta'(e) - \theta(e)/e)k}\left[u(z - T(z)) - v\left(\frac{y}{n}\right) - u(b)\right]. \tag{7}$$

The left-hand side multiplies the employment rate by the marginal utility of income. As such, it gives the marginal benefits of applying for a job that pays a higher wage. The right-hand side captures the marginal costs of doing so. It equals the product of two terms. The first term measures by how much the job-finding probability decreases if an individual applies for a job which specifies higher earnings (i.e., a higher wage). This reduction is multiplied by the (opportunity) costs of not finding employment, as given by the utility difference between employment and unemployment.

Combined, the two first-order conditions (6)–(7) and the zero-profit condition (2) constitute a system of three equations in three unknowns. Because individuals with the same ability but different participation costs make the same choices (conditional on participation), I denote the solution by $z(n)$, $y(n)$ and $e(n)$, respectively. The next section discusses in detail how these outcomes are affected by the tax-benefit system. For now, I complete the characterization of the equilibrium by requiring the government budget constraint is satisfied:

$$\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} e(n)(T(z(n)) + b)dF(\varphi, n) = b + G. \tag{8}$$

The employment tax $T(z(n)) + b$ measures the change in government revenue if an individual with ability n moves from non-participation or unemployment to employment. It consists of both income taxes and the foregone payments in unemployment benefits. The revenues from collecting employment taxes from all employed individuals must be sufficient to finance benefit payments b and public consumption G.

3 Efficiency and comparative statics

This section discusses the efficiency properties of the equilibrium and analyzes how the labor-market outcomes are affected by the tax-benefit system. Both turn out to be crucial for understanding how unemployment should be taken into account when the government optimally designs the tax-benefit system.

3.1 Efficiency

In an environment where search is directed, the allocation of resources is typically efficient in the absence of government intervention (see, for instance, Moen (1997)). The reason is that wages are posted in advance and as such, play an allocative role similar to that of prices in a frictionless economy. In Appendix A I show that this result also holds in my model where workers are heterogeneous in two dimensions and supply labor on the intensive and extensive margin, *provided* they are risk-neutral (i.e., $u(c) = c$). With a linear utility function and without a tax-benefit system, condition (7) simplifies to:

$$\left(\theta'(e(n)) - \frac{\theta(e(n))}{e(n)}\right) k = z(n) - v\left(\frac{y(n)}{n}\right). \tag{9}$$

The above relationship equates two externalities associated with posting a vacancy. On the one hand, if a firm posts a vacancy, it does not take into account that it becomes more difficult for other firms to fill theirs. This "business-stealing externality" is captured on the left-hand side and equals the difference between the social costs $\theta'(e)k$ and the private costs $\theta(e)k/e$ of getting a vacancy filled. *Ceteris paribus*, this externality induces excessive vacancy creation. On the other hand, if a firm posts a vacancy, it also does not take into account the utility gain a potential employee experiences if she finds a job, as captured by the right-hand side. *Ceteris paribus*, this "thick-market externality" induces too little vacancy creation. Vacancy creation is efficient if both externalities are exactly off-setting. In models with random search and *ex post* wage bargaining, this property only holds if the bargaining power of workers and firms equals the elasticity of the matching function with respect to their input, cf. Hosios (1990). By contrast, if search is directed, the condition for efficiency is endogenously satisfied.

In Appendix A I also show that the allocation is Pareto inefficient if individuals are risk-averse. This is because unemployment risk is not privately insurable. Consequently, it is possible to increase all individuals' *expected* utilities by transferring income from the state of employment to the state of unemployment. However, it is not possible to increase an individual's *realized* utility without lowering that of someone else. The allocation without government intervention is therefore said to be constrained efficient, meaning there is no scope for an *ex post* Pareto improvement. As will be made clear in Section 4, the fact that unemployment is constrained efficient has an important implication for policymakers: the tax-benefit system is only used for providing insurance and redistributing income, *not* for correcting market failures.

3.2 Comparative statics

I now turn to study how changes in the tax-benefit system affect labor-market outcomes. Unlike the unemployment benefit b, tracing out the effects of income taxes requires changing a function $T(z)$ as opposed to a parameter. In a recent paper, Golosov et al. (2014) propose a method for doing so. I follow their approach and define a function

$$T^*(z, \kappa) = T(z) + \kappa R(z). \tag{10}$$

Here, $T^*(z, \kappa)$ is the tax function individuals face if the tax function $T(z)$ is perturbed in the direction $R(z)$ by a magnitude κ. Hence, $R(z)$ can be interpreted as a reform to the current system. The impact of a tax reform $R(z)$ on the labor-market outcomes can then be derived in two steps. The first is to characterize the equilibrium for a given tax system $T^*(z, \kappa)$. This simply requires replacing $T(z)$ by $T^*(z, \kappa)$ in the individual optimization problem (3). The second step is to determine the impact of the reform parameter κ on the equilibrium outcomes and evaluate the result at the reform of interest. See also Gerritsen (2016) and Jacquet and Lehmann (2017).

As can be seen from equations (5)–(7), earnings, participation and unemployment are affected by changes in the marginal tax rate $T'(z)$, the tax liability $T(z)$ and the benefit b. I therefore consider a local increase in each of these, holding the others fixed. This can be done by setting $R(z) = z - z(n)$ and $R(z) = 1$. The first reform raises the marginal tax rate while leaving the tax liability (and hence the average tax rate) at income level $z(n)$ unaffected. The second reform increases the tax liability (and hence, the average tax rate) while holding the marginal tax rate fixed. The next Proposition summarizes how these changes in the tax-benefit system affect labor-market outcomes.

Proposition 1. *Table 1 shows how a local increase in the marginal tax rate, the tax burden and the unemployment benefit affect equilibrium participation, earnings and unemployment.*

Table 1: Comparative statics

	Participation	Earnings	Unemployment
Marginal tax rate	=	−	−
Tax burden	−	+	+
Unemployment benefit	−	+	+

Note: A +/=/− indicates that the row variable has a positive/zero/negative impact on the column variable.

The first row from Table 1 shows the impact of locally increasing the marginal tax rate on participation, earnings and unemployment. Because the reform only increases the marginal tax rate (and not the tax liability), the participation rate remains unaffected. Earnings decline in response to the reform, for two reasons. First, as in Mirrlees (1971), a higher marginal tax rate reduces effort. Second, a higher marginal tax rate also reduces the wage per unit of effort. As explained earlier, individuals face a trade-off not only between consumption and leisure but also between high wages and low unemployment risk. An increase in the marginal tax rate makes individuals care less about a higher wage. Therefore, a higher marginal tax rate induces them to apply for a job that specifies a lower wage. This encourages vacancy creation, which reduces unemployment. I label this the employment-enhancing (EEE) effect of taxation:

Definition 1. ***Employment-enhancing effect (EEE)*** *For a given tax burden, an increase in the marginal tax rate raises employment and hence, reduces unemployment.*

It is worth pointing out that the EEE has important implications for the elasticity of taxable income (ETI). This statistic of central interest in the public economics literature, because it serves as a sufficient statistic for calculating the the efficiency costs of taxation in a wide class of models. See Gruber and Saez (2002) and Saez et al. (2012) for reviews. The ETI measures the percentage increase in earnings following a one-percent increase in the net-of-tax rate. As pointed out by Hungerbühler et al. (2006), both the canonical labor-supply model and models with matching frictions and wage bargaining are consistent with a positive ETI. However, the mechanisms which drive the ETI are very different. In the labor-supply model, earnings decline because a higher marginal tax rate lowers effort. By contrast, in models with wage bargaining, a higher marginal tax rate lowers earnings as it makes individuals less interested in higher wages. The model presented here captures both these mechanisms: a higher marginal tax rate reduces both effort as well as the wages posted by firms. While the focus of the optimal tax literature has been almost exclusively on the first of these, Blomquist and Selin (2010) find that a substantial share of the ETI can be attributed to wage (as opposed to hours) responses – at least in the short run. Furthermore, as will be made clear below, if both these forces are present, the ETI is no longer a sufficient statistic to calculate the efficiency costs of taxation.

Turning to the second row of Table 1, consider an increase in the tax liability which leaves the marginal tax rate unaffected (i.e., $R(z) = 1$). Such a tax reform reduces the benefits of working and hence, lowers participation. Earnings increase in response to the reform, again for two reasons. First, a higher tax bill raises labor effort if there are income effects in labor supply. Second, a higher tax bill reduces the utility gain of finding a job. This induces individuals to apply for a job that pays a higher wage, thereby accepting an increase in the probability of remaining unemployed. I label this effect the employment-reducing effect (ERE) of taxation:

Definition 2. ***Employment-reducing effect (ERE)*** *For a given marginal tax rate, an increase in the tax burden reduces employment and hence, raises unemployment.*

The last row of Table 1 shows the impact of increasing the unemployment benefit. By raising the value of non-employment, this reform lowers participation. In addition, conditional on participation, a higher unemployment benefit reduces the utility gain of finding employment. Following an increase in the unemployment benefit, individuals apply for higher-wage jobs which reduces their matching probability. Hence, both earnings and unemployment increase. This second effect is very similar to the ERE discussed above.

4 Optimal taxation

I now turn to study how unemployment affects the optimal design of the tax-benefit system. To do so, I assume the government maximizes the following social welfare function:

$$\mathcal{W} = \int_{n_0}^{n_1} \int_{\varphi(n)}^{\varphi_1} \Psi(u(b)) dF(\varphi, n) + \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n)} \Psi(\mathcal{U}(n) - \varphi) dF(\varphi, n). \tag{11}$$

Here, $\Psi(\cdot)$ is strictly increasing and weakly concave, and $\mathcal{U}(n)$ and $\varphi(n)$ are as defined in equations (3) and (4). Note that the government maximizes a concave transformation of expected, as opposed to realized, utilities. As such, it respects individual preferences. Strict concavity in either $\Psi(\cdot)$ or $u(\cdot)$ generates a motive for redistribution, which is absent only if the marginal utility of consumption is constant (i.e., $u''(\cdot) = 0$) and if the government attaches the same weight to each individual's expected utility (i.e., $\Psi''(\cdot) = 0$).

4.1 Variational approach

The government chooses the tax function $T(\cdot)$ and the unemployment benefit b to maximize social welfare (11) subject to the budget constraint (8), taking into account the behavioral responses as summarized in Proposition 1. I solve this problem using the variational approach. See, e.g., Saez (2001), Golosov et al. (2014) and Gerritsen (2016). This approach differs from the classic mechanism-design approach introduced in Mirrlees (1971) by relying directly on perturbations (i.e., reforms) of the tax system.[6] These reforms generate welfare-relevant effects. Optimal policy rules are then derived from the notion that if the tax-benefit system is optimal, these welfare-relevant effects must sum to zero. The reforms I consider are exactly the ones analyzed in Proposition 1: a (local) increase in (i) the marginal tax rate, (ii) the tax burden and (iii) the unemployment benefit.

Figure 1 graphically illustrates how the variational approach can be used to derive optimal policy rules. It shows the welfare-relevant effects associated with an increase in the marginal tax rate in the (small) interval $[Z, Z + \omega]$. The dashed and solid line show the tax schedule before and after the reform, respectively. The increase in the marginal tax rate generates behavioral responses. According to Proposition 1, it reduces earnings and, through the EEE, unemployment for individuals with earnings potential between Z and $Z + \omega$.[7] Because a local increase in the marginal tax rate does not affect the expected utility of individuals who are affected, I label these substitution (or compensated) effects. By the envelope theorem, the behavioral responses have no first-order effect on individuals' expected utility. However, changes in earnings and employment do affect government finances. These so-called fiscal externalities are relevant for welfare and should be taken into account when considering such a reform.

The reform increases the tax liability for individuals with earnings above $Z + \omega$. This generates three types of welfare-relevant effects. First, there is a mechanical welfare effect as the

[6] Instead, the classic mechanism-design approach proceeds by first deriving the allocation that maximizes welfare subject to resource and incentive constraints, and then deriving the tax system which implements the allocation. See Jacquet and Lehmann (2021) for a comparison of both approaches.

[7] The individuals who are affected by the marginal tax rate are those who apply for jobs which pay an income in the interval $[Z, Z + \omega]$. Since not all applicants are successful, I refer to these workers as those with earnings potential (rather than those with earnings) between Z and $Z + \omega$.

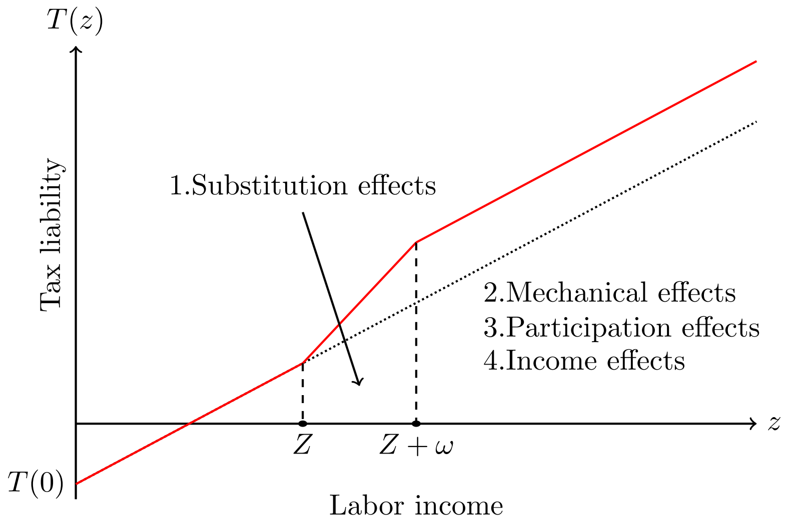

Figure 1: Variational approach

reform transfers income from these individuals to the government budget. Second, a higher tax burden reduces participation among individuals with earnings potential above $Z+\omega$. Because only individuals who are indifferent between participation and non-participation change their participation decision following an increase in the tax liability, these responses have no direct utility effect. However, participation responses do affect government finances. Third, a higher tax burden also generates income effects in earnings and unemployment, cf. Proposition 1. In particular, individuals with earnings potential above $Z+\omega$ decide to search for higher-wage jobs which, through the ERE, raises unemployment. Again, by the envelope theorem these behavioral responses do not affect individuals' expected utilities. However, because earnings and employment are taxed they do generate welfare-relevant fiscal externalities.

4.2 Sufficient statistics

An important advantage of the variational approach is that it allows for a derivation of optimal tax formulas that can easily be interpreted, as they are expressed in terms of sufficient statistics. See also Chetty (2009) for a discussion. In the current framework, these sufficient statistics are the income distribution, behavioral responses and social welfare weights.

Income distribution

I denote the income distribution by $H(z)$, and the corresponding density by $h(z)$. Moreover, let $z_0 = z(n_0)$ and $z_1 = z(n_1)$ denote the lowest and highest level of positive earnings. The (observable) income distribution $H(z)$ is related to the (unobservable) distribution of types $F(\varphi, n)$ through

$$H(z(n)) = \int_{n_0}^{n_1}\int_{\varphi(m)}^{\varphi_1} f(\varphi,m)d\varphi dm + \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(m)} (1-e(m))f(\varphi,m)d\varphi dm + \int_{n_0}^{n}\int_{\varphi_0}^{\varphi(m)} e(m)f(\varphi,m)d\varphi dm. \tag{12}$$

Here, $f(\cdot)$ denotes the density of the type distribution, $e(m)$ the employment rate for individuals with ability m and the participation thresholds $\varphi(m)$ are as defined in equation (4). The fraction of individuals with income below $z(n)$ equals the sum of the non-participants (first term), the unemployed (second term) and the employed individuals with ability below n (third term). Because there is non-participation and unemployment, the income distribution has a mass point at zero. The fraction of individuals with zero income equals $H(z_0)$.

Behavioral responses

In addition to the income distribution, a second main ingredient in the optimal tax formulas are the behavioral responses with respect to changes in the tax-benefit system. With a slight abuse of notation, I denote by $x_{T'} = \mathrm{d}x/\mathrm{d}T'$ and $x_T = \mathrm{d}x/\mathrm{d}T$ the equilibrium changes in $x \in \{\pi, z, e\}$ following a (local) increase in the marginal tax rate and the tax burden, respectively.[8] For instance, $\mathrm{d}z/\mathrm{d}T'$ measures the impact of a local increases in the marginal tax rate on income, holding the tax liability (i.e., the average tax rate) fixed. Conversely, $\mathrm{d}z/\mathrm{d}T$ measures the change in earnings following an increase in the tax liability, holding the marginal tax rate fixed.

For future references I introduce the following elasticities:

$$\varepsilon_{zT'} = -\frac{\mathrm{d}z}{\mathrm{d}T'}\frac{1-T'}{z}, \quad \varepsilon_{eT'} = \frac{\mathrm{d}e}{\mathrm{d}T'}\frac{1-T'}{e}. \tag{13}$$

Both elasticities are positive according to Proposition 1. The first is the elasticity of taxable income (ETI). As discussed before, the ETI captures both labor-supply and wage responses. The second measures the percentage decrease in the employment rate following a one-percent increase in the net-of-tax rate. This elasticity therefore quantifies the employment-enhancing effect (EEE) of taxation.

Welfare weights

The final ingredients in the optimal tax formulas are the social welfare weights, which summarize the government's preferences for redistribution. I denote the welfare weight for individuals with zero earnings by $g(0)$ and the welfare weight of individuals with earnings z by $g(z)$. Both are formally defined in Appendix C. In words, $g(0)$ measures the average, monetized increase in social welfare if an individual with zero labor earnings (i.e., the non-participants and the involuntarily unemployed) receives an additional unit of income. Among other things, $g(0)$ depends on the marginal utility of consumption $u'(b)$ for the non-participants and the unemployed. The welfare weight $g(z)$, in turn, measures the average, monetized increase in social welfare if an individual with labor earnings z receives an additional unit of income. Naturally, it depends on the marginal utility of consumption $u'(z - T(z))$ for individuals who are employed at earnings z. In the typical case where the government wishes to redistribute from individuals with high to individuals with low income and from employed individuals to those with zero labor earnings, $g(z)$ is declining in labor earnings and $g(0)$ exceeds $g(z)$ on average.

[8] These behavioral responses capture the (total) equilibrium changes along the actual budget curve, and not along a linearized budget curve, as in Saez (2001). Hence, they account for the non-linearity of the tax schedule. See Jacquet et al. (2013) for a discussion of this issue.

4.3 Optimal tax formulas

The next Proposition characterizes the optimal tax-benefit system in terms of the aforementioned sufficient statistics.

Proposition 2. *If the tax-benefit system is optimal, the following condition must hold for all $z \in [z_0, z_1]$:*

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon_{zT'}} \int_z^{z_1} \left[1 - g(z') + \frac{\pi_T}{\pi}(T(z') + b) + z_T T'(z')\right] \frac{dH(z')}{zh(z)} + \frac{\varepsilon_{eT'}}{\varepsilon_{zT'}} \frac{(T(z)+b)/z}{1-T'(z)} + \frac{1}{\varepsilon_{zT'}} \int_z^{z_1} \frac{e_T}{e}(T(z')+b)\frac{dH(z')}{zh(z)}. \tag{14}$$

In addition, the following conditions must hold at the optimum:

$$\int_{z_0}^{z_1} \left[1 - g(z) + \frac{\pi_T}{\pi}(T(z)+b) + z_T T'(z)\right] dH(z) + \int_{z_0}^{z_1} \frac{e_T}{e}(T(z)+b)dH(z) = 0, \tag{15}$$

$$(g(0)-1)H(z_0) + \int_{z_0}^{z_1} \frac{\pi_b}{\pi}(T(z)+b)dH(z) + \int_{z_0}^{z_1} \left[z_b T'(z) + \frac{e_b}{e}(T(z)+b)\right] dH(z) = 0. \tag{16}$$

These optimal tax formulas are obtained from considering a local increase in (i) the marginal tax rate at earnings z, (ii) the tax burden and (iii) the unemployment benefit, respectively. I now discuss each of these formulas in turn.

Optimal marginal tax rates

Equation (14) gives the optimality condition from considering a local increase in the marginal tax rate around income level z, as graphically illustrated in Figure 1. The formula clearly demonstrates how unemployment responses should be taken into account when deciding what marginal tax rate to set. Without unemployment responses (i.e., $\varepsilon_{eT'} = e_T = 0$), both terms in the second line would cancel. The resulting optimal tax formula then corresponds to those derived in Saez (2002), Jacquet et al. (2013) and Jacobs et al. (2017), who analyze a model with labor-supply responses on the intensive and extensive margin but abstract from unemployment. For a detailed explanation of this formula, see their papers.

When matching frictions generate unemployment, the expression for the optimal marginal tax rate is modified in two ways: see the second line of equation (14). The first modification results from the employment-enhancing effect (EEE) of taxation. Intuitively, an increase in the marginal tax rate at income level z reduces wage pressure around this income level, as the reform makes individuals care less about higher wages. This leads firms to hire more workers, which increases the employment rate of individuals with earnings potential z. By how much depends on the elasticity of the employment rate with respect to the net-of-tax rate, as captured by $\varepsilon_{eT'}$. This term is multiplied by the fiscal externality $T(z) + b$ of lowering unemployment to capture the impact on government finances. In the typical case that the employment tax is positive, the EEE calls for a *higher* marginal tax rate.

The second modification occurs because an increase in the marginal tax rate at income level

z mechanically raises tax liabilities further up in the income distribution (see Figure 1). In response, individuals with higher earnings potential choose to apply for higher-wage jobs and accept a decrease in the probability of finding employment. The magnitude of this employment-reducing effect (ERE) of taxation is captured by the responsiveness of the employment rate with respect to the tax liability, as given by e_T. As before, the behavioral response is multiplied by the employment tax. If the product of these terms averaged over all individuals with earnings above z is negative (which is to be expected, as $e_T < 0$ and typically $T(z') + b > 0$), the ERE calls for a *lower* marginal tax rate.

Which of these forces dominates depends critically on two types of statistics: (i) the responsiveness of the employment (or unemployment) rate with respect to the marginal and average tax rate (holding the other fixed) and (ii) the relative hazard rate of the income distribution. To see this, multiply the expression for the optimal marginal tax rate (14) by $\varepsilon_{zT'}$. The last two terms on the right-hand side can then be written as:

$$\frac{e_{T'}}{e}\left(\frac{T(z)+b}{z}\right) + \mathbb{E}\left[\frac{e_t}{e}\left(\frac{T(z')+b}{z'}\right)\middle| z' > z\right]\frac{1-H(z)}{zh(z)}. \tag{17}$$

Here, e_t measures the impact of raising the average tax rate $t = T(z)/z$ on the employment rate, holding the marginal tax rate fixed. In addition to the fiscal externalities associated with changes in unemployment (as captured by $(T(z) + b)/z$), equation (17) depends on the semi-elasticity of the employment rate with respect to the marginal and average tax rate (as captured by $e_{T'}/e$ and e_t/e) and the relative hazard rate $zh(z)/(1 - H(z))$ of the income distribution. The latter is critical for quantifying the EEE and ERE, as it determines how many people are affected by an increase in the marginal tax rate at income level z compared to the number of people who see their tax liability increase, i.e., those with earnings above z. If employment taxes are positive, unemployment is more likely to reduce optimal marginal tax rates if the hazard rate of the income distribution is low.

At high levels of earnings, the relative hazard rate is approximately constant (see, e.g., Saez (2001)). This implies the top of the income distribution is well approximated by a Pareto distribution. Equation (14) can then readily be manipulated to obtain an expression for the optimal top rate (see Appendix D for details).

Corollary 1. *If incomes at the top are Pareto distributed with tail parameter a and if the elasticities of earnings, employment and participation with respect to (one minus) the marginal and average tax rate converge, the optimal top rate is given by:*

$$T'(z) = \frac{1-g(z)}{1-g(z)+a(\varepsilon_{zT'}-\varepsilon_{eT'})+\varepsilon_{\pi t}+\varepsilon_{zt}+\varepsilon_{et}}, \tag{18}$$

where ε_{xt} denotes the elasticity of $x \in \{\pi, z, e\}$ with respect to one minus the average tax rate.

Equation (18) generalizes the results of Jacquet et al. (2013) and Jacobs et al. (2017), who abstract from unemployment responses and who assume away income effects for high-income earners (in which case $\varepsilon_{zt} = 0$). Whether the optimal top rate is higher or lower if unemployment is taken into account depends on a very simple condition. In particular, for a given welfare weight, earnings and participation elasticities for top-income earners, the optimal top rate is higher compared to a setting without unemployment if and only if the following condition holds:

$$a\varepsilon_{eT'} > \varepsilon_{et}. \tag{19}$$

This condition is intuitive. If the employment tax is positive (as is always the case at the top), the employment-enhancing effect raises the optimal top rate, whereas the employment-reducing effect does the opposite. The optimal top rate with unemployment is therefore higher if the responsiveness of employment with respect to the marginal tax rate is high relative to the average tax rate (i.e., if $\varepsilon_{eT'}$ is high relative to ε_{et}) and if the tail of the income distribution is thin (i.e., if a is high). A high Pareto parameter a implies that an increase in the marginal tax rate reduces the employment prospects of only only a few people further up in the income distribution. While the presence of unemployment leads to an intuitive adjustment of the expression for the optimal top rate, it should be noted that the quantitative implications are likely to be small if unemployment is not an important margin for individuals with high ability.

Employment taxes and the optimality of an EITC

Equation (15) gives the optimality condition from considering a uniform increase in the tax burden for all employed individuals. Compared to the frictionless benchmark without unemployment, the only modification of the optimal tax formula (15) is due to the employment-reducing effect (ERE) of taxation, as captured by the second term. Intuitively, a higher tax burden induces individuals to look for higher-wage jobs. In response, firms post fewer vacancies and unemployment increases. Provided the employment tax is positive on average, the increase in unemployment has a negative impact on government finances. For a given distribution of income, welfare weights, and behavioral responses, the optimal tax liability (and hence the employment tax) is therefore lower if unemployment responses are taken into account.

Combined, equations (14) and (15) have an important implication for the optimal design of employment subsidies, such as the EITC.

Proposition 3. *If employment is subsidized for low-income workers, the optimal marginal tax rate at the bottom is negative. Put differently, it is optimal to let employment subsidies (such as the EITC) phase in with income.*

In two influential papers, Diamond (1980) and Saez (2002) show that employment for low-income workers is optimally subsidized if labor-supply responses are concentrated (mostly) on the extensive margin and if the government cares sufficiently about the working poor. These papers thus explain why the *level* of the employment tax can be negative for low-income workers. Proposition 3 complements this result by showing that if the employment tax of low-skilled workers is negative (i.e., if employment of low-skilled workers is subsidized), these workers should also face a negative *marginal* tax rate. Put differently, employment subsidies such as the EITC should phase in with income. Intuitively, employment subsidies induce individuals to apply for jobs that pay inefficiently low wages in order to increase their likelihood of being matched. This leads to an upward distortion in employment. A negative marginal tax rate alleviates this distortion by making it more attractive for individuals to apply for jobs that pay a higher wage, which induces firms to hire fewer workers. The associated reduction in the employment rate for low-skilled workers positively affects government finances if the employment tax for these workers is negative.

To see how Proposition 3 and the results from Diamond (1980) and Saez (2002) are related, consider a reform which decreases the marginal tax rate around a low income level z combined with an increase in the intercept of the tax function which ensures the net income of individuals with earnings above z is unaffected. The optimality condition associated with this reform can be obtained by combining equations (14) and (15). This gives

$$\begin{aligned}\varepsilon_{zT'}\frac{T'(z)}{1-T'(z)}zh(z) = &\int_{z_0}^{z}\left[g(z')-1-\frac{\pi_T}{\pi}(T(z')+b)+z_T T'(z')\right]dH(z') \\ &+\varepsilon_{eT'}\frac{T(z)+b}{1-T'(z)}h(z)-\int_{z_0}^{z}\frac{e_T}{e}(T(z')+b)dH(z'). \end{aligned} \tag{20}$$

Equation (20) illustrates that it is optimal to subsidize employment (i.e., to set $T(z')+b<0$) if the government cares a lot about the working poor (i.e., if $g(z')>1$ at low levels of income) and if earnings and unemployment responses are absent (i.e., if $\varepsilon_{zT'}=z_T=\varepsilon_{eT'}=e_T=0$). This finding goes back to Diamond (1980), who was the first to provide a rationale for employment subsidies (such as the EITC) in an optimal-tax framework. Saez (2002) generalizes this result to an environment where individuals also supply labor on the intensive margin, in which case $\varepsilon_{zT'}$ and z_T are not equal to zero.

The (un)employment responses on the second line of equation (20) highlight why, if employment subsidies are in place, it is generally optimal to let them phase in with income (i.e., why the optimal marginal tax rate on the left-hand side is negative). First, a negative marginal tax rate raises wages and reduces employment around income level z through the EEE. If employment is subsidized, this generates a positive fiscal externality. Second, the reduction in the marginal tax rate at earnings z allows the government to increase the intercept of the tax schedule. Through the ERE, this lowers the employment rate among individuals with earnings potential below z.[9] The associated reduction in employment again generates a positive fiscal externality if employment is subsidized. Hence, if the employment rate for low-skilled workers is subsidized, the fiscal externalities of the EEE and ERE go hand in hand and it is generally optimal to let employment subsidies phase in with income.

Optimal unemployment insurance

The results from Proposition 2 are also related to those from Baily (1978) and Chetty (2006). They study the optimal provision of unemployment insurance in a model where (identical) risk-averse individuals face an uninsurable risk of becoming unemployed. The government optimally provides UI payments, which are financed through a lump-sum or proportional tax on labor income. The optimal unemployment benefit trades off the insurance gains against the distortionary costs of UI on job search. To see how my results are related to theirs, suppose all individuals are identical and decide to participate (i.e., $n_0=n_1$ and $\varphi_0=\varphi_1$ sufficiently low).

[9] Recall: the joint reduction in the marginal tax rate around income z and the increase in the intercept leaves individuals with earnings potential above z unaffected.

Equations (15) and (16) then simplify to:[10]

$$e(1-g(z)) + e\left[z_T T'(z) + \frac{e_T}{e}(T(z)+b)\right] = 0, \tag{21}$$

$$(1-e)(g(0)-1) + e\left[z_b T'(z) + \frac{e_b}{e}(T(z)+b)\right] = 0. \tag{22}$$

If individuals are risk-averse, the marginal utility of consumption for the unemployed exceeds the marginal utility of consumption for the employed. Consequently, $g(0) > g(z)$ and the government optimally provides unemployment insurance. As in the Baily-Chetty framework, optimal UI provision balances the insurance gains against the distortionary costs of raising unemployment, as captured by e_b and e_T. I show in Appendix F how the Baily-Chetty formula (see Proposition 1 in Chetty (2006)) is recovered if UI payments are financed by lump-sum taxes, in which case $T'(z) = 0$. If this is not the case and if UI also affects wages (as is the case in my model), the Baily-Chetty formula is modified to take into account the fiscal externalities associated with earnings responses as well. These responses are captured by z_b and z_T, respectively, and multiplied by the marginal tax rate.

The fact that I do not restrict UI payments to be financed by lump-sum or proportional taxes on labor income has another implication.

Proposition 4. *Suppose all individuals are identical and decide to participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). If the government provides unemployment insurance, the optimal marginal tax rate is positive and satisfies the following inverse-elasticity rule:*

$$\frac{T'(z)}{(T(z)+b)/z} = \frac{\varepsilon_{eT'}}{\varepsilon_{zT'}}. \tag{23}$$

Moreover, financing UI payments through lump-sum or proportional taxes on labor income is generally sub-optimal.

A common assumption in the literature on unemployment insurance is that benefits are financed either by lump-sum or proportional taxes on labor income. See, e.g., Baily (1978), Flemming (1978), Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Acemoglu and Shimer (1999), Chetty (2006, 2008). According to Proposition 4, doing so is generally sub-optimal even if individuals are *ex ante* identical and there is no income heterogeneity. To understand why, suppose individuals are risk-averse and the risk of becoming unemployed is not privately insurable. In this case, the government optimally provides an unemployment benefit, which is financed through a positive *average* tax rate on labor earnings. Hence, the employment tax is positive. As in the framework of Baily (1978) and Chetty (2006), a positive employment tax generates an upward distortion in unemployment. The government can alleviate this distortion by setting a positive *marginal* tax rate on labor earnings. A positive marginal tax rate makes it less attractive for individuals to apply for high-wage jobs. This puts downward pressure on wages, which leads firms to hire more workers and unemployment to decline. According to equation (23), the optimal marginal tax rate satisfies a simple, inverse-elasticity rule. Naturally, it is increasing in the employment tax and the responsiveness of the employment rate with respect

[10]The income distribution has a mass point at zero. Consequently, $H(z_0) = 1-e$ and the income distribution between z_0 and z_1 integrates to e: $H(z_1) - H(z_0) = e$.

to the marginal tax rate. The downside of a positive marginal tax rate is that it decreases incentives to earn labor income. Therefore, the optimal marginal tax rate is decreasing in the elasticity of taxable of income (ETI).

The above discussion makes clear that marginal and average tax rates play a distinct role when it comes to UI provision. The *average* tax rate is used to finance unemployment benefits, whereas the *marginal* tax rate is used to alleviate upward distortions in unemployment generated by UI provision. The optimal marginal tax rate is therefore positive but not the same as the average tax rate, except in special cases. Financing UI payments through lump-sum or proportional taxes, as is commonly assumed, is therefore generally sub-optimal.

5 Numerical illustration

This section explores the quantitative implications of unemployment for the (optimal) design of the tax-benefit system. The purpose is twofold. The first is to calculate the fiscal externalities associated with unemployment responses when considering reforms to the *current* tax-benefit system. The second is to characterize the *optimal* tax-benefit system if unemployment responses are taken into account.

Unfortunately, to the best of my knowledge there are no direct estimates of the separate effects of the marginal tax rate and the tax burden on unemployment rates that can be used to quantify the employment-enhancing and employment-reducing effects of taxation. What makes estimating these effects challenging is that one requires data on unemployment rates across the earnings (or skill) distribution, which cannot readily be observed. Moreover, because unemployment rates vary with skills, the effects are likely to vary across the income distribution as well. For these reasons, rather than attempting to implement optimal tax formulas in terms of sufficient statistics, I calibrate a structural version of the model. The calibrated model can then be used to study both tax reforms and the optimal tax-benefit system.

5.1 Calibration

The model is calibrated to the US economy. The primary data source is the March release of the 2018 Current Population Survey (CPS). This data set provides detailed information on earnings, taxes and benefits for a large sample of individuals. Importantly, it also provides information on individuals' employment status (i.e., employment, unemployment, or not in the labor force). The participation rate is 86.6% and the unemployment rate conditional on participation is 4.1%. The final sample includes individuals between 25 and 65 years, who work full-time and whose hourly wage is at least half the federal minimum wage $7.25. Appendix H provides additional details on the sample selection.

In the model, earnings, participation and unemployment all vary with ability n. Naturally, in the data I only observe individual earnings and their employment status, but not their ability or probability of having found employment. To get an estimate of these, I invert the first-order conditions (6) and (7) for each individual with positive earnings. This gives a distribution of abilities that is consistent with the empirical income distribution. Doing so requires specifying the current tax-benefit system, functional forms for the utility and matching function, and a value for the costs of opening a vacancy k. Moreover, to get an estimate of the participation rate at different ability levels, I require an empirical counterpart of the distribution of participation costs as well. I discuss each of these in turn.

Tax-benefit system

As in Saez (2001) and Sleet and Yazici (2017), I approximate the current US tax schedule by regressing total taxes paid on taxable income. This gives an estimate of a (constant) marginal tax rate of 33.1% and a negative intercept of \$4,691. Figure 5 in Appendix H plots the actual and fitted values for annual earnings up to \$200,000. The value for the unemployment benefit is set at \$5,319, which equals the average income from unemployment compensation for individuals who received such income and whose reported labor force status is unemployment. Combined with data on the aggregate participation and unemployment rate, the government budget constraint (8) then implies a revenue requirement of $G = \$13,233$. The latter corresponds to approximately 20.2% of average earnings.

Utility function

The utility function is assumed to take the following form:

$$u(c) - v(\ell) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{\ell^{1+1/\varepsilon}}{1+1/\varepsilon}. \tag{24}$$

Here, σ and ε govern the curvature in the sub-utility over consumption and labor effort, respectively. The parameter σ determines, among other things, income effects in labor supply, how much individuals value unemployment insurance and the government's preference for redistributing from individuals with high to individuals with low or zero labor income. The baseline calibration employs a value of $\sigma = 0.5$, but the Appendix also plots results for $\sigma = 1$ (logarithmic utility). The value for the intensive-margin elasticity of labor supply is set at $\varepsilon = 0.33$, as suggested by Chetty (2012).

Matching function and the costs of opening a vacancy

For the matching function, I use the commonly employed Cobb-Douglas specification

$$\theta(e) = Be^{\lambda}, \tag{25}$$

where the employment rate (which varies with ability) is capped at a maximum value of one. The parameter B is related to the efficiency of the matching technology. As it turns out, with the above specification of the matching function, equilibrium employment and earnings only depend on the product Bk, and not B and k separately. I therefore harmlessly normalize $B = 1$. The parameter λ, in turn, is related to the matching elasticity, i.e., the elasticity of the job-finding rate with respect to the ratio of vacancies to job-seekers. The parameters k and λ are calibrated jointly to match the unemployment rate of 4.1% observed in the data and an estimate of the average recruitment costs. Regarding the latter, Villena-Roldán (2012) reports that firms spend approximately 2.5% of their total labor costs on recruiting activities. Matching these targets gives $k = \$6,895$ and $\lambda = 42.70$. The costs of opening a vacancy appear very high, but this figure also includes the impact of unobserved matching efficiency, as equilibrium outcomes only depend on the product Bk. In a dynamic context, the calibrated value of λ is consistent with a matching elasticity of approximately 0.57.[11] This value is slightly above the range of 0.3–0.5 suggested by Petrongolo and Pissarides (2001).

Distribution of ability and participation costs

Ability and participation costs and are assumed to be independently distributed. To obtain the ability distribution, I first invert the first-order conditions (6) and (7) and calculate the ability and employment rate for each individual with positive earnings. As in Saez (2001), this gives an ability distribution that is consistent with the empirical income distribution.[12] I subsequently smooth the ability distribution by estimating a kernel density. Figure 6 in Appendix H plots both the observed and the estimated kernel density for ability levels that correspond to earnings below \$200,000. Next, consistent with the observation that incomes at the top are approximately Pareto distributed, I append a right Pareto tail starting at \$350,000 in annual earnings. The value of the Pareto coefficient of the *ability* distribution is set in such a way that the Pareto coefficient of the *income* distribution is $a = 1.6$, cf. Saez and Stantcheva (2018). Finally, the scale parameter of the Pareto distribution is set to ensure the density is continuous at the point where the Pareto tail starts.

Turning to the distribution of participation costs, I assume the the latter is such that the participation rate is iso-elastic with respect to the *utility* difference:[13]

$$\pi(n) = A\varphi(n)^{\eta}. \tag{27}$$

As with the employment rate, the participation rate is capped at a maximum value of one. Besides its simplicity, an additional benefit of using this functional form is that, in line with empirical evidence, it generates a decreasing pattern of participation elasticities (i.e., the percentage increase in the participation rate if the *consumption* difference increases by 1%). See Hansen (2021) for references. The parameters A and η are calibrated jointly to match an aggregate participation rate of 86.6% observed the data and an average participation elasticity

[11] Because the model I use is static, there is no distinction between the job-finding and the employment rate. Empirically, however, the matching elasticity is typically estimated by regressing the job-finding rate within a particular period (and *not* the employment rate) on the number of vacancies relative to job-seekers, both in logarithms. To get a sense of how the elasticity of the employment rate and the elasticity of the job-finding rate with respect to the ratio of vacancies to job-seekers are related, consider a dynamic version of the model with a constant job-destruction rate δ. As in the DMP model, the steady state employment rate is $e(\theta) = p(\theta)/(p(\theta)+\delta)$, where $p(\theta)$ is the job-finding rate. Differentiating both sides with respect to θ gives, after simplifying,

$$\frac{\partial e(\theta)}{\partial \theta}\frac{\theta}{e(\theta)} = \frac{\partial p(\theta)}{\partial \theta}\frac{\theta}{p(\theta)}(1 - e(\theta)). \tag{26}$$

The left-hand side is the inverse of λ. With a steady-state unemployment rate of $1 - e(\theta) = 0.041$, the implied matching elasticity is approximately $\frac{\partial p(\theta)}{\partial \theta}\frac{\theta}{p(\theta)} = \frac{1}{42.70}\frac{1}{0.041} \approx 0.57$.

[12] Strictly speaking, this approach ignores that high-skilled individuals are more likely to show up in the sample of employed individuals, as they are more likely to participate and, conditional on participation, less likely to be unemployed. Accounting for these features slightly affects the calibrated ability distribution.

[13] This expression for the participation rate is obtained if the lowest value for the participation costs is $\varphi_0 = 0$ and the (conditional) density is $\eta A \varphi^{\eta-1}$.

of 0.15. The latter is below the one reported in Chetty et al. (2011), who base their estimate of 0.25 on an extensive meta analysis. However, in the analysis I focus on full-time employees, who are typically found to be less responsive. Moreover, the value of 0.15 masks heterogeneity: the implied participation elasticity is around 0.23 at the bottom of the income distribution, and 0 at income levels above $126,000.

Overview

Table 2 summarizes the baseline calibration. Here, $\tilde{F}(n)$ denotes the conditional distribution of ability. While the table assigns to each parameter a single target, it is to be understood that, as described in the main text, the parameters λ and k are calibrated jointly, and the same is true for A and η.

Variable	Value	Target
σ	0.5	External
ε	0.33	External
B	1	Normalization
λ	42.70	Unemployment rate
k	$6,895	Average recruitment costs
A	0.33	Participation rate
η	0.18	Average participation elasticity
G	$13,233	Government budget constraint
b	$5,319	Unemployment benefit
$T(z)$	Figure 5	Tax liability
$\tilde{F}(n)$	Figure 6	Income distribution

Table 2: Calibration

Figures 7 and 8 in Appendix H plot the participation rate and the employment rate (i.e., one minus the unemployment rate) for individuals with ability levels corresponding to earnings below $200,000. Figure 7 shows that the participation rate increases monotonically from approximately 60.9% for the least-skilled workers to 100% for those with earnings potential above $126,000. The aggregate participation rate is 86.6%, as in the data. Turning to the employment rate, Figure 8 shows that individuals with higher skills are less likely to become unemployed. In particular, conditional on participation, the employment rate for the least-skilled workers is approximately 90.2%. Hence, the unemployment rate for these workers is 9.8%. The unemployment rate is subsequently declining in earnings and reaches zero at an ability level corresponding to earnings slightly above $200,000. In line with the CPS data, the aggregate unemployment rate is 4.1%. For comparison, Figures 7 and 8 also plot the (non-targeted) profile of participation and employment rates against average earnings for individuals with different educational attainments.[14] In line with the model, the data shows an increasing relationship between earnings and participation and between earnings and employment.

[14] The categories are the following: less than high school, high school, some college, college, advanced.

5.2 Fiscal externalities due to unemployment responses

The welfare effects due to unemployment responses depend critically on the fiscal externalities associated with the EEE and ERE, cf. Proposition 2. To calculate these, I study the effects of a one percentage point increase in the marginal tax rate in the interval $[z, z+\Delta]$, starting from the *current* tax-benefit system. To facilitate the interpretation, I set $\Delta = \$100$. Consequently, the reform raises the tax burden of individuals with earnings above $z+\Delta$ by exactly \$1. The unemployment responses to this tax reform affect government finances in two ways. First, a higher marginal tax rate reduces wage pressure in the interval $[z, z+\Delta]$. The associated increase in employment has a positive impact on government finances. This is shown by the top line from Figure 2. It plots, for each income level z below \$200,000, the fiscal externality due to the EEE when the marginal tax rate at income z is locally increased. Because the impact on government finances is proportional to the number of individuals for whom the marginal tax rate is increased, the shape is similar to that of the skill, or income distribution: see Figure 6. Second, an increase in the marginal tax rate in the interval $[z, z+\Delta]$ increases the tax burden for individuals with earnings potential above $z+\Delta$. A higher tax burden induces them to apply for higher-wage jobs, which leads firms to post fewer vacancies and unemployment to increase. The bottom line of Figure 2 shows the negative impact on government finances. It plots, for each income level z below \$200,000, the fiscal externality due to the ERE when the marginal tax rate is locally increased. Because only individuals with earnings above $z+\Delta$ see their tax burden increase, the shape is more similar to that of the cumulative income distribution.

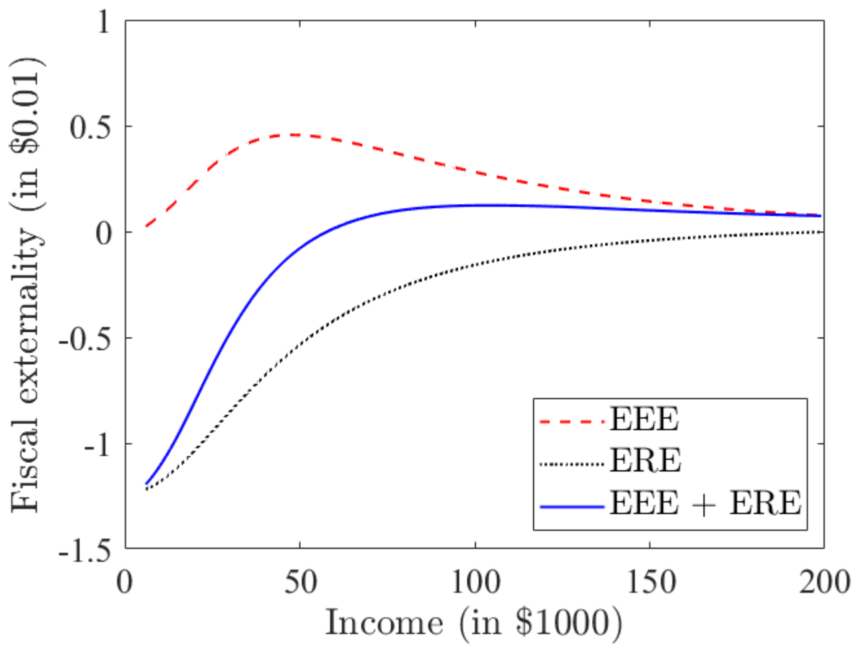

Figure 2: Budgetary effects

Figure 2 also plots the sum of both effects. Clearly, in the calibrated economy the net revenue effect due to unemployment responses is virtually zero when considering a local increase in the marginal tax rate at earnings levels above \$50,000. This suggests that unemployment responses are most relevant when considering tax reforms at low income levels. Furthermore, at low levels of income, the negative fiscal externality due to the ERE is much larger than the positive fiscal externality due to the EEE. The reason is that the hazard rate of the income distribution is very small at low levels of income. Consequently, a local increase in the marginal tax rate at low earnings improves the employment prospects of only a few individuals (through the EEE), whereas the implied increase in the tax burden raises the unemployment rate at virtually all income levels (through the ERE). Naturally, the net impact on government finances is negative. It should be noted, however, that the revenue effect is modest. In particular, the government loses a bit over \$0.01 due to unemployment responses if it collects \$1 from almost all employed individuals by raising the marginal tax rate at the bottom of the income distribution.[15] Figure 9 shows that this number is considerably smaller than the fiscal externalities associated with earnings and participation responses, which traditionally have been the focus in the optimal tax literature. The finding that the fiscal externalities due to unemployment responses are modest also has implications for the optimal tax-benefit system, which is discussed next.

5.3 The optimal tax-benefit system

To study the optimal tax-benefit system, I numerically solve the government's optimization problem: see Appendix H for details. Doing so requires specifying a social welfare function, cf. equation (11). In what follows, I assume the government has a utilitarian objective: $\Psi(\mathcal{U}) = \mathcal{U}$. Consequently, the government's desire to redistribute from individuals with high to individuals with low or zero income is driven solely by the fact that the marginal utility of consumption is declining: $u''(\cdot) < 0$. It is possible to allow for concavity in the social welfare function $\Psi(\cdot)$ as well, but doing so does not fundamentally affect the results.

Figure 3 plots the optimal marginal tax rates $T'(z)$. For ease of interpretation, all results are plotted against *current* labor earnings. As can be seen from the figure, optimal marginal tax rates start out very low, then increase rapidly to around 43% before staying relatively flat up to approximately \$100,000 in annual earnings. After that, they start slowly increasing again up to \$350,000, the point where the Pareto tail is pasted, and remain at a constant level of around 65% (not plotted). Given the current specification of preferences, the optimal marginal tax rate averages at 43.2%, compared to a constant rate of 33.1% in the baseline economy (see Section 5.1). The higher marginal tax rates are used to finance both a larger unemployment benefit as well as larger transfers to low-income workers. In particular, at the optimal tax-benefit system the value for $b = \$10,890$, more than twice its value of \$5,319 in the calibrated economy. The negative intercept of the tax function also increases substantially. At the lowest level of positive earnings, individuals receive \$8,869 in transfers at the optimal tax-benefit system. The corresponding figure in the baseline economy is \$2,795.[16]

Figure 4 plots the optimal employment taxes $T(z) + b$, which capture the net impact on government finances if an individual with earnings potential z moves from non-participation or unemployment to employment. The optimal employment tax for the least-skilled workers with

[15] Because a local increase in the marginal tax rate at the lowest income level raises the tax burden for almost all employed individuals by \$1, the mechanical revenue gain of this reform is $0.866 \times (1 - 0.041) \times \$1 = \$0.83$.

[16] This figure is calculated as $4,691 - 0.331 \times \$5,720 = \$2,795$. Here, \$4,691 is the negative intercept of the tax function (see Section 5.1) and \$5,720 is the lowest level of positive earnings when restricting the sample to full-time workers whose hourly wage exceeds half the federal minimum wage.

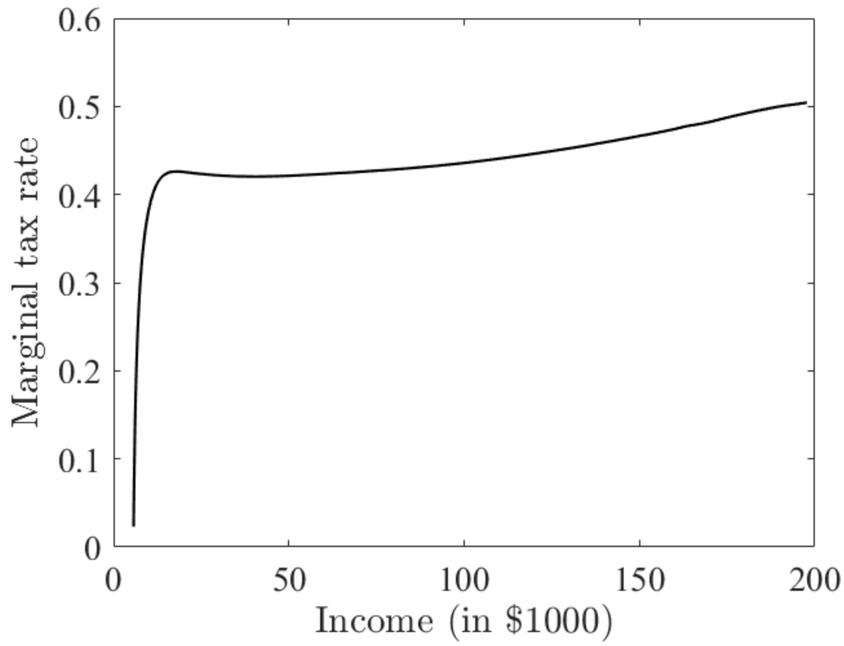

Figure 3: Optimal marginal tax rates

positive earnings is $-\$8,869 + \$10,890 = \$2,021$. This is fairly close to the employment tax these workers face in the calibrated economy, as given by $-\$2,795 + \$5,319 = \$2,524$. After that, for a large part of the earnings distribution, optimal employment taxes increase almost linearly at a rate somewhat above 40%. This is in line with the pattern of optimal marginal tax rates shown in Figure 3. As the average marginal tax rate exceeds the current rate of 33.1%, the employment tax for high-skilled workers at the optimal tax-benefit system significantly exceeds the employment tax for these workers in the baseline calibration. Specifically, for individuals whose current labor earnings are slightly below \$200,000 the employment tax at the optimal tax-benefit system $\$71,220 + \$10,890 = \$82,109$. The corresponding figure in the calibrated economy is $\$61,595 + \$5,319 = \$66,914$.[17]

It is worth pointing out that the employment tax at the optimal tax-benefit system is positive for *all* workers, including the low-skilled. As a result, employment for these workers is not subsidized on a net basis: given the current specification of preferences, individuals with low but positive earnings do not receive a transfer that exceeds the unemployment benefit. Despite the fact that low-income workers receive substantial transfers, subsidizing employment through an EITC is therefore not part of an optimal policy. The finding that the *level* of the employment tax for low-income workers is positive also explains why the *marginal* tax rate at the bottom of the income distribution is positive, see Figure 3. Intuitively, a positive employment tax for low-skilled workers generates a downward distortion in employment and hence, an upward distortion in unemployment. A positive marginal tax rate alleviates this distortion by making it less attractive for workers to apply for jobs that pay higher wages. The reduction in wages leads firms to hire more workers, which generates a positive fiscal externality as the employment

[17] This figure is calculated as the sum of the unemployment benefit \$5,319 and the tax burden at \$200,000 in annual earnings, as given by $\$200,000 \times 0.331 - \$4,691$ (see Section 5.1).

Figure 4: Optimal employment taxes

tax is positive. The optimal marginal tax rate at the very bottom of the income distribution is therefore not zero, but positive at around 2.3%.

Comparison to the optimal tax-benefit system without unemployment

To study how unemployment responses affect the optimal tax-benefit system, I also calculate the optimal tax-benefit under the assumption that labor markets are perfectly competitive. Hence, individuals supply labor on the intensive (hours, effort) and extensive (participation) margin, but do not face the risk of becoming unemployed. To do so, I first recalibrate the model assuming there is no unemployment. Hence, all (involuntary) unemployed individuals are classified as (voluntary) non-participants. The costs of opening a vacancy are zero and there are no matching frictions. Consequently, the employment rate is $e(n) = 1$ for all n. The current tax-benefit system, the revenue requirement G and the utility function are the same as before. The ability distribution as well as the parameters A and η are then recalibrated to match the empirical income distribution, a participation rate of $0.866 \times (1 - 0.041) = 0.830$ and an average participation elasticity of 0.15, see also Table 2. I subsequently solve the optimal tax problem using the same welfare function as before: $\Psi(\mathcal{U}) = \mathcal{U}$.

Comparing the optimal tax-benefit system with and without unemployment, the most significant difference is that transfers for individuals with zero or low income are larger if unemployment risk is taken into account. Specifically, the optimal benefit is $b = \$10,466$ if labor markets are competitive, which is approximately \$423 smaller than the optimal benefit if matching frictions generate unemployment risk. Similarly, individuals at the lowest level of positive earnings receive a transfer of \$8,454 if labor markets are competitive, which is approximately \$415 smaller than the transfer they receive if there are matching frictions. Apart from these differences, the optimal tax-benefit system with and without unemployment responses look very similar. Figures 10 and 11 in Appendix H illustrate this point by plotting the optimal marginal tax rates and employment taxes under perfect competition and under the assumption that matching frictions generate unemployment risk (as shown in Figures 3 and 4). Apart from the larger transfers individuals with zero or low earnings receive if there are matching frictions, the only noticeable difference concerns the marginal tax rate at the bottom of the income distribution. If labor markets are perfectly competitive, the optimal marginal tax rate faced by individuals at the lowest level of positive earnings is zero. This is a well-known result in optimal tax theory that goes back to Seade (1977), and is confirmed by Figure 10. By contrast, as explained earlier, the optimal marginal tax rate at the bottom of the income distribution is positive at around 2.3% if unemployment responses are taken into account.

As a final exercise, I calculate the optimal tax-benefit system with and without unemployment responses under the assumption that sub-utility over consumption is logarithmic: $\sigma = 1$. Figures 12 and 13 in Appendix H plot the optimal marginal tax rates and employment taxes. To calculate these, the model with and without matching frictions is first recalibrated in exactly the same way as described above. As before, both optimal marginal tax rates and employment taxes with and without unemployment responses look very similar. Comparing the results for different values of σ, optimal marginal tax rates are larger if the utility function is more concave – irrespective of whether there are matching frictions. To see this, compare Figures 10 and 12. Intuitively, the government has a stronger desire to redistribute from individuals with high to individuals with zero or low earnings if sub-utility over consumption is more concave. Consequently, individuals with zero or low earnings receive larger transfers as well. If $\sigma = 1$ and matching frictions generate unemployment risk, the optimal benefit is $b = \$13,494$ whereas individuals who are employed at the lowest earnings receive a transfer of \$11,766 (compared to \$10,890 and \$8,869, respectively if $\sigma = 0.5$). As before, these transfers are smaller if labor markets are perfectly competitive. In that case, the optimal unemployment benefit is $b = \$13,160$ and the transfer received by those employed at the lowest earnings is \$11,584.

6 Conclusion

This paper characterizes optimal income taxation in a directed search model where matching frictions generate unemployment risk. Individuals differ in terms of their ability and participation costs and supply labor on the intensive and extensive margin. In addition to the standard trade-off between consumption and leisure, they face a trade-off between high wages and low unemployment risk. The government affects this trade-off and hence, unemployment by altering the costs and benefits of searching. On the one hand, an increase in the marginal tax rate raises employment as it lowers the benefits of looking for higher-wage jobs. On the other hand, an increase in the tax burden or unemployment benefit reduces employment as it lowers the benefits of finding a job. I label the first of these the employment-enhancing effect (EEE) and the second the employment-reducing (ERE) effect of taxation.

Because an unemployed worker receives unemployment benefits and does not pay income taxes, changes in unemployment generate fiscal externalities. These call for intuitive adjustments of standard optimal tax formulas. The latter are used to obtain the following insights.

First, whether unemployment responses lead to higher or lower marginal tax rates depends on the responsiveness of unemployment to the marginal tax rate and the tax burden and on the employment tax (i.e., the sum of income taxes and unemployment benefits) at different points in the income distribution. Second, if it is optimal to subsidize employment, the optimal marginal tax rates at the bottom of the income distribution is negative. My model thus provides a rationale for the phase-in region of the EITC. Third, the optimal provision of unemployment insurance (UI) justifies a positive marginal tax rate even without heterogeneity in labor income. As a result, financing UI payments through lump-sum or proportional taxes on labor income, as is commonly assumed in the literature, is generally sub-optimal. Finally, a calibration exercise to the US economy suggests that transfers for individuals with zero or low income are somewhat larger if unemployment risk is taken into account.

The analysis from this paper can be extended in a number of directions. First, in my model unemployment is constrained efficient. As a result, changes in unemployment only generate fiscal externalities. It would be interesting to allow for further departures from efficiency. Second, I have assumed wages can freely adjust in response to changes in the tax-benefit system. This may not be the case if there is a binding minimum wage. How the tax-benefit system and minimum wages should jointly be optimized is an interesting and policy-relevant question. Third, I have abstracted from dynamic considerations. As a result, individuals cannot insure their unemployment risk through precautionary savings and UI payments cannot be conditioned on past earnings. Extending the model to include these features would both make it more realistic and significantly enrich the set of policy questions it can address. Finally, this paper can serve as a motivation for future empirical work. There is little evidence about the separate effects of the marginal tax rate and the tax burden on unemployment. My analysis indicates that both responses generate very different welfare effects.

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A Constrained efficiency

This appendix formally demonstrates that the allocation of resources in the absence of government intervention is Pareto efficient (from an *ex ante* perspective) if and only if individuals are risk-neutral. To do so, I first show that if individuals are risk-neutral the *laissez-faire* allocation maximizes the sum of expected utilities subject to the aggregate resource constraint. Then, I show that if individuals are risk-averse, there exists a Pareto-improving and resource-feasible perturbation of the equilibrium allocation.

If individuals are risk-neutral and if there are no taxes and benefits (i.e., $u(c) = c$, $T(z) = b = 0$ and hence $G = 0$), the equilibrium satisfies the following conditions:

$$\theta(e(n))k = e(n)(y(n) - z(n)), \tag{28}$$

$$\varphi(n) = e(n)\left(z(n) - v\left(y(n)/n\right)\right), \tag{29}$$

$$n = v'\left(y(n)/n\right), \tag{30}$$

$$\big(\theta'(e(n)) - \theta(e(n))/e(n)\big)\, k = z(n) - v\left(y(n)/n\right). \tag{31}$$

These correspond to the zero-profit condition (2), the participation decision (4) and the first-order conditions (6) and (7). To see why the implied allocation $[z(n), y(n), e(n), \varphi(n)]_{n_0}^{n_1}$ is Pareto efficient, suppose the government chooses the allocation which maximizes the sum of all individuals' expected utilities subject to the resource constraint. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = & \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} \left[e(n)\left(z(n) - v\left(\frac{y(n)}{n}\right)\right) - \varphi\right] f(\varphi, n)d\varphi dn \\ & + \lambda\left(\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} \Big[e(n)(y(n) - z(n)) - \theta(e(n))k\Big] f(\varphi, n)d\varphi dn\right).\end{aligned} \tag{32}$$

Here, $\mathcal{U}(n) - \varphi = e(n)(z(n) - v(y(n)/n)) - \varphi$ denotes the expected utility of an individual of type (n, φ) who participates. Utility of the non-participants equals $u(0) = 0$. Moreover, λ denotes the multiplier on the aggregate resource constraint. The resource constraint is obtained by integrating the zero-profit condition over all participants. The first-order conditions associated with the optimization problem (32) are:

$$z(n): \quad (1 - \lambda)\int_{\varphi_0}^{\varphi(n)} e(n) f(\varphi, n)d\varphi = 0, \tag{33}$$

$$y(n): \quad \int_{\varphi_0}^{\varphi(n)} e(n)\left(-\frac{v'(y(n)/n)}{n} + \lambda\right) f(\varphi, n)d\varphi = 0, \tag{34}$$

$$e(n): \quad \int_{\varphi_0}^{\varphi(n)} \Big[z(n) - v(y(n)/n) + \lambda(y(n) - z(n) - \theta'(e(n))k)\Big] f(\varphi, n)d\varphi = 0, \tag{35}$$

$$\varphi(n): \quad \left(e(n)\left(z(n) - v\left(\frac{y(n)}{n}\right)\right) - \varphi(n)\right) + \lambda\Big(e(n)(y(n) - z(n)) - \theta(e(n))k\Big) = 0. \tag{36}$$

To verify that the equilibrium allocation (as implicitly defined by equations (28)–(31)) satisfies these conditions, first observe that equation (33) implies $\lambda = 1$. Then, equations (30) and (34) coincide. Similarly, setting $\lambda = 1$ and using equation (28) to substitute out for $z(n)$ in equation

(31) implies equation (35) holds as well. Finally, equations (36) and (28) are equivalent because the zero-profit condition (29) implies the second term in equation (36) is zero. The *laissez-faire* allocation thus maximizes the sum of expected utilities subject to the aggregate resource constraint. This implies there exists no Pareto improvement if individuals are risk-neutral.

Conversely, if individuals are risk-averse, there exists a Pareto-improving and resource-feasible perturbation of the equilibrium allocation. To see why, note that in the absence of taxes and benefits an individual with ability n who decides to participate, solves:

$$\mathcal{U}(n) \equiv \max_{y,z,e} \left\{ e\left[u(z) - v\left(\frac{y}{n}\right)\right] + (1-e)u(0) \;\; \text{s.t.} \;\; k = \frac{e}{\theta(e)}(y-z) \right\}. \tag{37}$$

Denote the solution of the above maximization problem by $(y(n), z(n), e(n))$. Hence, a fraction $e(n)$ of the participants with ability n becomes employed, produces $y(n)$ and consumes $z(n)$, whereas a fraction $1-e(n)$ remains unemployed and does not consume at all. Now, consider a perturbation where the consumption of individuals with ability n in the state of unemployment is marginally raised by $\mathrm{d}c_u > 0$ and consumption in the state of employment is reduced by $\mathrm{d}c_u(1-e(n))/e(n)$. Such a perturbation is resource feasible as it does not affect aggregate consumption of individuals with ability n. The impact on expected utility is:

$$d\mathcal{U}(n) = (1-e(n))(u'(0) - u'(z(n)))dc_u. \tag{38}$$

The latter is strictly positive whenever $u(\cdot)$ is strictly concave. Since the perturbation raises the expected utility of type n individuals without decreasing the expected utility of any other type, the laissez-faire equilibrium is not Pareto efficient if individuals are risk-averse. Note, however, that the reform only raises the *expected* (i.e., *ex ante*) utility of individuals. It does not generate an increase in *realized* (i.e., *ex post*) utilities, because the reform lowers the utility in the state of employment.

B Comparative statics

This appendix studies how a local increase in the marginal tax rate, the tax burden or the unemployment benefit affects equilibrium earnings, (un)employment and participation. Unlike the unemployment benefit, changing the marginal tax rate or tax burden requires changing a function (and not just a parameter). To do so, I use the variational approach introduced in Golosov et al. (2014). This approach can be implemented by, first, characterizing the equilibrium outcomes under the assumption that individuals face a "perturbed" tax schedule $T^*(z,\kappa) = T(z) + \kappa R(z)$, where $R(z)$ is a reform function (to be specified below). After that, one can use implicit or total differentiation to determine the impact of the reform parameter κ on the labor-market outcomes, evaluated at the reform of interest.

Suppose an individual with ability n faces the tax schedule $T^*(z,\kappa) = T(z) + \kappa R(z)$. Conditional on participation, she solves

$$\max_{z,e} \; L(z,e;n) = (1-e)u(b) + e\Big[u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right)\Big], \tag{39}$$

where $L(\cdot)$ denotes the objective and I used the zero-profit condition (2) to substitute out for output $y = z + \theta(e)k/e$. The first-order necessary conditions are

$$L_z(z,e;n) = e\left[u'(z - T(z) - \kappa R(z))(1 - T'(z) - \kappa R'(z)) - \frac{1}{n}v'\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right)\right] = 0, \quad (40)$$

$$L_e(z,e;n) = u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) - u(b)$$
$$- \frac{1}{n}v'\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right)\left(\theta'(e) - \frac{\theta(e)}{e}\right)k = 0. \quad (41)$$

These conditions are sufficient provided $L_{ee} < 0$ and $L_{ee}L_{zz} - L_{ze}^2 > 0$, which I henceforth assume.[18] Combined, equations (40) and (41) pin down the equilibrium earnings $z(n)$ and employment rate $e(n)$ for individuals with ability n as a function of the reform parameter κ and the unemployment benefit b.

To study how a change in the reform parameter κ or the unemployment benefit b affects equilibrium earnings and employment, totally differentiate equations (40)–(41). Ignoring function arguments to save on notation, this gives

$$L_{zz}\mathrm{d}z + L_{ze}\mathrm{d}e + L_{zx}\mathrm{d}x = 0, \quad (42)$$

$$L_{ez}\mathrm{d}z + L_{ee}\mathrm{d}e + L_{ex}\mathrm{d}x = 0, \quad (43)$$

where $x \in \{\kappa, b\}$. The impact of x on earnings and employment is then given by

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{L_{ee}L_{zx} - L_{ex}L_{ze}}{L_{ze}^2 - L_{ee}L_{zz}}, \quad (44)$$

$$\frac{\mathrm{d}e}{\mathrm{d}x} = \frac{L_{ex}L_{zz} - L_{ze}L_{zx}}{L_{ze}^2 - L_{ee}L_{zz}}. \quad (45)$$

The second-order condition for utility maximization implies the denominator on the right-hand side of equations (44)–(45) is negative. Furthermore, differentiating L_z and L_e with respect to earnings and employment gives

$$L_{zz} = e\left[u''(1 - T' - \kappa R')^2 - u'(T'' + \kappa R'') - \frac{v''}{n^2}\right] < 0, \quad (46)$$

$$L_{ze} = L_{ez} = -\frac{v''}{n^2}\left(\theta' - \frac{\theta}{e}\right)k < 0, \quad (47)$$

$$L_{ee} = -\frac{1}{e}\frac{v''}{n^2}\left(\theta' - \frac{\theta}{e}\right)^2 k^2 - \frac{v'}{n}\theta'' k < 0. \quad (48)$$

The signs of L_{ee} and L_{ze} follow from the assumptions on $v(\cdot)$ and $\theta(\cdot)$. The sign of L_{zz}, in turn, follows from the assumption that the second-order condition for utility maximization is satisfied: $L_{ee}L_{zz} - L_{ze}^2 > 0$.

The impact of a local increase in the marginal tax rate on earnings and employment can be

[18] As shown below, the first of these requirements follows from the assumptions on $v(\cdot)$ and $\theta(\cdot)$. By contrast, the second requirement depends on properties of the endogenous tax schedule. In what follows, I assume this condition is always satisfied and verify numerically this is indeed the case.

found by working out $\mathrm{d}z/\mathrm{d}\kappa$ and $\mathrm{d}e/\mathrm{d}\kappa$, evaluated at the reform

$$R(z) = z - z(n). \tag{49}$$

This reform increases the marginal tax rate, while leaving the tax burden at income level $z(n)$ unaffected (recall: $T^*(z,\kappa) = T(z) + \kappa R(z)$). Working out the effect of this reform on L_z and L_e gives, with a slight abuse of notation,

$$L_{zT'} = \left.\frac{\partial L_z}{\partial \kappa}\right|_{R(z)=z-z(n)} = -eu' < 0, \tag{50}$$

$$L_{eT'} = \left.\frac{\partial L_e}{\partial \kappa}\right|_{R(z)=z-z(n)} = 0. \tag{51}$$

The impact of a local increase in the marginal tax rate on earnings and employment is then given by (again, with a slight abuse of notation):

$$\frac{\mathrm{d}z}{\mathrm{d}T'} = \frac{L_{ee}L_{zT'} - L_{eT'}L_{ze}}{L_{ze}^2 - L_{ee}L_{zz}} = \frac{L_{ee}L_{zT'}}{L_{ze}^2 - L_{ee}L_{zz}} < 0, \tag{52}$$

$$\frac{\mathrm{d}e}{\mathrm{d}T'} = \frac{L_{eT'}L_{zz} - L_{ze}L_{zT'}}{L_{ze}^2 - L_{ee}L_{zz}} = \frac{-L_{ze}L_{zT'}}{L_{ze}^2 - L_{ee}L_{zz}} > 0, \tag{53}$$

where the signs follow immediately from the discussion above.

Turning to the impact of a local increase in the tax burden, consider the reform

$$R(z) = 1. \tag{54}$$

This reform increases the tax burden, but leaves the marginal tax rate unaffected. Working out the effects on L_z and L_e,

$$L_{zT} = \left.\frac{\partial L_z}{\partial \kappa}\right|_{R(z)=1} = -eu''(1-T') \geq 0, \tag{55}$$

$$L_{eT} = \left.\frac{\partial L_e}{\partial \kappa}\right|_{R(z)=1} = -u' < 0 \tag{56}$$

The impact of a local increase in the tax burden on earnings and employment is then given by

$$\frac{\mathrm{d}z}{\mathrm{d}T} = \frac{L_{ee}L_{zT} - L_{eT}L_{ze}}{L_{ze}^2 - L_{ee}L_{zz}} > 0, \tag{57}$$

$$\frac{\mathrm{d}e}{\mathrm{d}T} = \frac{L_{eT}L_{zz} - L_{ze}L_{zT}}{L_{ze}^2 - L_{ee}L_{zz}} < 0. \tag{58}$$

Again, the signs follow directly from the discussion above.

Next, consider an increase in the unemployment benefit b. The impact on L_z and L_e is

$$L_{zb} = 0, \tag{59}$$

$$L_{eb} = -u'_u < 0, \tag{60}$$

where $u'_u = u'(b)$ is the marginal utility of consumption for the unemployed. The impact on earnings and employment is

$$\frac{\mathrm{d}z}{\mathrm{d}b} = \frac{L_{ee}L_{zb} - L_{eb}L_{ze}}{L_{ze}^2 - L_{ee}L_{zz}} = \frac{-L_{eb}L_{ze}}{L_{ze}^2 - L_{ee}L_{zz}} > 0, \tag{61}$$

$$\frac{\mathrm{d}e}{\mathrm{d}b} = \frac{L_{eb}L_{zz} - L_{ze}L_{zb}}{L_{ze}^2 - L_{ee}L_{zz}} = \frac{L_{eb}L_{zz}}{L_{ze}^2 - L_{ee}L_{zz}} < 0. \tag{62}$$

To sum up, a local increase in the tax burden or unemployment benefit has a positive impact on labor earnings, whereas a local increase in the marginal tax rate reduces labor earnings. By contrast, a local increase in the tax burden or unemployment benefit negatively affects the employment rate, whereas a local increase in the marginal tax rate positively affects the employment rate. Naturally, the impact of any of these instruments on the unemployment rate $1 - e$ is opposite the impact on the employment rate. The second and third column of Table 1 summarize these results.

Finally, to analyze the impact of the tax-benefit system on the participation rate, note that the participation threshold satisfies:

$$\varphi(n) = \mathcal{U}(n) - u(b) = \max_{z,e} \left\{ e \left[u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) - u(b) \right] \right\}. \tag{63}$$

Differentiating (63) with respect to κ and b, evaluated at the reforms of interest, gives:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}T'} = \left.\frac{\mathrm{d}\varphi}{\mathrm{d}\kappa}\right|_{R(z)=z-z(n)} = 0, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}T} = \left.\frac{\mathrm{d}\varphi}{\mathrm{d}\kappa}\right|_{R(z)=1} = -eu' < 0, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}b} = -eu'_u < 0. \tag{64}$$

Because the participation rate (5) increases in the threshold (63), a local increase in the tax burden or unemployment benefit reduces participation. Furthermore, a local increase in the marginal tax rate does not affect the participation rate. These effects are stated in the first column of Table 1.

C Optimal tax-benefit system

To derive the optimal policy rules from Proposition 2, write the Lagrangian associated with the government's optimization problem as follows:

$$\begin{aligned} \mathcal{L} = & \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \Psi\left(\mathcal{U}(n,\kappa,b) - \varphi\right) f(\varphi,n) d\varphi dn + \int_{n_0}^{n_1} \int_{\varphi(n,\kappa,b)}^{\varphi_1} \Psi(u(b)) f(\varphi,n) d\varphi dn \\ & + \lambda \left[\int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n,\kappa,b)\left(T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b\right) f(\varphi,n) d\varphi dn - b - G \right]. \end{aligned} \tag{65}$$

Here, λ is the multiplier on the government's budget constraint and the tax function is given by $T(z) + \kappa R(z)$. The notation (n, κ, b) is used to denote which variables depend on ability n, the reform parameter κ and the unemployment benefit b. The expected utility of an individual with ability n who participates is given by:

$$\mathcal{U}(n,\kappa,b) = \max_{z,e} \left\{ (1-e)u(b) + e\left[u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) \right] \right\}, \tag{66}$$

where I used the zero-profit condition (2) to substitute out for $y = z + \theta(e)k/e$. Moreover, the participation threshold satisfies:

$$\varphi(n,\kappa,b) = \mathcal{U}(n,\kappa,b) - u(b) \tag{67}$$

To derive equation (16), differentiate the Lagrangian with respect to b, taking into account the impact on $\mathcal{U}(n,\kappa,b)$ and $\varphi(n,\kappa,b)$. The first-order condition is given by:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}b} &= \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)} (1-e(n,\kappa,b))\Big[\Psi'(\mathcal{U}(n,\kappa,b)-\varphi)u'(b)-\lambda\Big] f(\varphi,n)d\varphi dn \\
&+ \int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1} \Big[\Psi'(u(b))u'(b)-\lambda\Big] f(\varphi,n)d\varphi dn \\
&+ \lambda \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}z}{\mathrm{d}b}\Big[e(n,\kappa,b)(T'(z(n,\kappa,b))+\kappa R'(z(n,\kappa,b)))\Big] f(\varphi,n)d\varphi dn \\
&+ \lambda \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}e}{\mathrm{d}b}\Big[T(z(n,\kappa,b))+\kappa R(z(n,\kappa,b))+b\Big] f(\varphi,n)d\varphi dn \\
&+ \lambda \int_{n_0}^{n_1} \frac{\mathrm{d}\varphi}{\mathrm{d}b}\Big[e(n,\kappa,b)(T(z(n,\kappa,b))+\kappa R(z(n,\kappa,b))+b)\Big] f(\varphi(n),n)dn = 0.
\end{aligned} \tag{68}$$

The first two lines give the mechanical welfare effects of transferring income to the unemployed (first line) and the non-participants (second line). The mass of these individuals equals the fraction of the population with zero income:

$$H(z_0) = \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)} (1-e(n,\kappa,b))f(\varphi,n)d\varphi dn + \int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1} f(\varphi,n)d\varphi dn. \tag{69}$$

Denote their average welfare weight by:

$$\begin{aligned}
g(0) = \frac{1}{\lambda H(z_0)} \Big[&\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)} (1-e(n,\kappa,b))\Psi'(\mathcal{U}(n,\kappa,b)-\varphi)u'(b)f(\varphi,n)d\varphi dn \\
&+ \int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1} \Psi'(u(b))u'(b)f(\varphi,n)d\varphi dn\Big],
\end{aligned} \tag{70}$$

which measures the monetized increase in social welfare if individuals with zero income receive an additional unit of income. Using this notation, the first two terms of equation (68) can be written as:

$$\lambda(g(0)-1)H(z_0). \tag{71}$$

To write the remaining terms of equation (68) also in terms of the income distribution, note that the relationship between the income and type distribution implies

$$h(z(n))z'(n) = \int_{\varphi_0}^{\varphi(n)} e(n)f(\varphi,n)d\varphi, \tag{72}$$

which is obtained by differentiating equation (12) with respect to n. Upon changing variables and evaluating the reform at $\kappa = 0$, the second and third line from equation (68) are:

$$\lambda \int_{z_0}^{z_1} \left[z_b T'(z) + \frac{e_b}{e}(T(z) + b) \right] h(z) dz. \tag{73}$$

The final term can be simplified as follows. First, note that equation (5) implies:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}b} f(\varphi(n), n) = \frac{\mathrm{d}\pi/\pi}{\mathrm{d}b} \int_{\varphi_0}^{\varphi(n)} f(\varphi, n) d\varphi. \tag{74}$$

The last line of equation (68) then simplifies to (using the same change of variables):

$$\int_{z_0}^{z_1} \frac{\pi_b}{\pi}(T(z) + b) h(z) dz. \tag{75}$$

Equation (16) is obtained by setting the sum of equations (71), (73) and (75) equal to zero, divide the resulting expression by λ and rearrange.

The procedure for deriving equation (15) is very similar. As a first step, maximize the Lagrangian with respect to κ, evaluated at the reform $R(z) = 1$. The first-order condition is:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}T} = & \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n,\kappa,b) \\
& \times \left[\lambda - \Psi'(\mathcal{U}(n,\kappa,b) - \varphi) u'(z(n,\kappa,b) - T(z(n,\kappa,b)) - \kappa R(z(n,\kappa,b))) \right] f(\varphi, n) d\varphi dn \\
& + \lambda \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}z}{\mathrm{d}T} \left[e(n,\kappa,b)(T'(z(n,\kappa,b)) + \kappa R'(z(n,\kappa,b))) \right] f(\varphi, n) d\varphi dn \\
& + \lambda \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}e}{\mathrm{d}T} \left[T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b \right] f(\varphi, n) d\varphi dn \\
& + \lambda \int_{n_0}^{n_1} \frac{\mathrm{d}\varphi}{\mathrm{d}T} \left[e(n,\kappa,b)(T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b) \right] f(\varphi(n), n) dn = 0.
\end{aligned} \tag{76}$$

The first two lines capture the mechanical welfare effect of transferring income from all employed individuals to the government budget. To write this in terms of the income distribution and welfare weights, denote the average welfare weight of individuals with earnings $z(n)$ as:

$$g(z(n)) = \frac{\int_{\varphi_0}^{\varphi(n)} e(n) \Psi'(\mathcal{U}(n) - \varphi) u'(z(n) - T(z(n))) f(\varphi, n) d\varphi}{\int_{\varphi_0}^{\varphi(n)} e(n) f(\varphi, n) d\varphi}. \tag{77}$$

Here, the welfare weight is evaluated at $\kappa = 0$ and I suppress the dependency on the policy parameters (κ, b). The first two lines of equation (76) can now be written as:

$$\lambda \int_{z_0}^{z_1} (1 - g(z)) h(z) dz. \tag{78}$$

The last three lines of equation (76) can be simplified in exactly the same way as was done with the first-order condition with respect to the benefit b, i.e., equation (68). The welfare effect associated with the participation, earnings and unemployment responses is:

$$\lambda \int_{z_0}^{z_1} \left[\frac{\pi_T}{\pi}(T(z') + b) + z_T T'(z') + \frac{e_T}{e}(T(z') + b) \right] dH(z'). \tag{79}$$

Equation (15) from Proposition 2 is then obtained by setting the sum of equations (78) and (79) equal to zero, and dividing the resulting expression by λ.

Finally, to derive equation (14) consider the following reform:

$$R(z) = \begin{cases} 0 & \text{if } z \le z(m), \\ z - z(m) & \text{if } z \in (z(m), z(m) + \omega], \\ \omega & \text{if } z > z(m) + \omega. \end{cases} \tag{80}$$

For ω small, this reform corresponds to a local increase in the *marginal* tax rate around income level $z(m)$ (see Figure 1). Clearly, the reform does not affect individuals with ability below m. For individuals with earnings potential above $z(m) + \omega$, the reform increases the tax liability. The welfare effects are therefore the same as before (see equations (78) and (79)), and given by:

$$\omega \times \lambda \int_{z(m)+\omega}^{z_1} \left[1 - g(z') + \frac{\pi_T}{\pi}(T(z') + b) + z_T T'(z') + \frac{e_T}{e}(T(z') + b) \right] h(z') dz' \tag{81}$$

This equation is obtained by summing equations (78) and (79) and replacing the lower bound of the integral by $z(m) + \omega$. The multiplication with ω is because the increase in the tax liability for individuals with earnings above $z(m) + \omega$ is equal to ω.

The reform also affects individuals with earnings potential in the interval $[z(m), z(m) + \omega]$. The corresponding interval of the ability distribution is $[m, m + \omega/z'(m)]$. The welfare effect is obtained by differentiating the Lagrangian (65) with respect to κ, evaluated at the reform $R(z) = z - z(m)$:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}T'} &= \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n,\kappa,b) \times \\
&\Big[\lambda - \Psi'(\mathcal{U}(n,\kappa,b) - \varphi) u'(z(n,\kappa,b) - T(z(n,\kappa,b)) - \kappa R(z(n,\kappa,b)))\Big] (z(n) - z(m)) f(\varphi, n) d\varphi dn \\
&+ \lambda \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}z}{\mathrm{d}T'} \Big[e(n,\kappa,b)(T'(z(n,\kappa,b)) + \kappa R'(z(n,\kappa,b))) \Big] f(\varphi, n) d\varphi dn \\
&+ \lambda \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}e}{\mathrm{d}T'} \Big[T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b \Big] f(\varphi, n) d\varphi dn \\
&+ \lambda \int_m^{m+\frac{\omega}{z'(m)}} \frac{\mathrm{d}\varphi}{\mathrm{d}T'} \Big[e(n,\kappa,b)(T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b) \Big] f(\varphi(n), n) dn
\end{aligned} \tag{82}$$

To proceed, divide the resulting expression by ω and take the limit as $\omega \to 0$. The mechanical welfare effect then cancels (see the first line).[19] Moreover, by Proposition 1, $\mathrm{d}\varphi/\mathrm{d}T' = 0$ and hence the final term cancels as well. Finally, using the property (72) we can simplify the

[19]To see why, note that ω shows up in the upper bound of the integral and $z(n) - z(m) \le \omega$.

right-hand side of equation (82) to:

$$\lambda\left[z_{T'}T'(z(m))+\frac{e_{T'}}{e}(T(z(m)+b)\right]h(z(m)). \tag{83}$$

To obtain equation (14), also divide equation (81) by ω and take the limit as $\omega \to 0$. Add the resulting expression to equation (83) and set the sum equal to zero. Finally, use the definitions of the elasticities (13) and replace the point where the marginal tax rate is increased $z(m)$ by z. Rearranging gives equation (14).

D Optimal top rate

The expression for the optimal top rate (18) is obtained in a number of steps. First, define by

$$\varepsilon_{\pi t}=-\frac{\mathrm{d}\pi}{\mathrm{d}t}\frac{1-t}{\pi},\quad \varepsilon_{zt}=-\frac{\mathrm{d}z}{\mathrm{d}t}\frac{1-t}{z},\quad \varepsilon_{et}=-\frac{\mathrm{d}e}{\mathrm{d}t}\frac{1-t}{e} \tag{84}$$

the elasticity of participation, earnings, and employment with respect to one minus the *average* tax rate (i.e., holding the marginal tax rate fixed). The term $\frac{\mathrm{d}x}{\mathrm{d}t}$ is related to $\frac{\mathrm{d}x}{\mathrm{d}T}$ through $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}(T/z)}=\frac{\mathrm{d}x}{\mathrm{d}T}z$ for $x\in\{\pi,z,e\}$.

Next, consider equation (14). If the marginal tax rate converges, $\lim_{z\to\infty}(T(z)+b)/z=T'(z)$. For high levels of income, equation (14) can then be rewritten as:

$$\frac{T'(z)}{1-T'(z)}(\varepsilon_{zT'}-\varepsilon_{eT'})=\mathbb{E}\left[1-g(z')+\frac{T'(z')}{1-T'(z')}\left(\varepsilon_{\pi t}+\varepsilon_{zt}+\varepsilon_{et}\right)\middle|z'>z\right]\frac{1-H(z)}{zh(z)}. \tag{85}$$

If incomes at the top are Pareto distributed with tail parameter a, $zh(z)/(1-H(z))=a$. Moreover, if the welfare weight and elasticities for top-income earners converge, condition (85) simplifies to:

$$\frac{T'(z)}{1-T'(z)}a(\varepsilon_{zT'}-\varepsilon_{eT'})=1-g(z)+\frac{T'(z)}{1-T'(z)}\left(\varepsilon_{\pi t}+\varepsilon_{zt}+\varepsilon_{et}\right), \tag{86}$$

Equation (18) is then obtained by collecting terms and solving the above expression for $T'(z)$.

E Proof Proposition 3

The result from Proposition 3 follows directly from Propositions 1 and 2. To see this, evaluate equation (14) in Proposition 2 at $z=z_0$ and combine the result with equation (15) to find:

$$\frac{T'(z_0)}{(T(z_0)+b)/z_0}=\frac{\varepsilon_{eT'}}{\varepsilon_{zT'}}. \tag{87}$$

Since $\varepsilon_{eT'},\varepsilon_{zT'}>0$ (see Proposition 1), the marginal tax rate is negative at the bottom (i.e., $T'(z_0)<0$) if and only if the employment tax for individuals with the lowest skills is negative (i.e., $T(z_0)+b<0$). Hence, it is optimal to let employment subsidies (such as the EITC) phase in with income.

F Relation to Baily-Chetty formula

This appendix demonstrates the link between my results for the optimal provision of unemployment insurance and those from Baily (1978) and Chetty (2006). To do so, suppose all individuals are identical and participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently small). Moreover, assume unemployment benefits b are financed by a lump-sum tax T on workers and set the revenue requirement $G = 0$. The government's budget constraint then reads $eT = (1-e)b$.

For a given tax-benefit system (T, b), individuals solve:

$$V(T,b) = \max_{z,e} \left\{ (1-e)u(b) + e\left(u(z-T) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k \right) \right) \right) \right\}. \tag{88}$$

The government then optimally chooses T and b to maximize $V(T,b)$ subject to the budget constraint. Upon substituting $T = (1-e)b/e$, the first-order condition is:

$$\frac{\partial V}{\partial b} = (1-e)(u'(b) - u'(z-T)) + \varepsilon_{eb} u'(z-T) = 0, \tag{89}$$

where ε_{eb} is the elasticity of the employment rate with respect to a budget-neutral increase in the unemployment benefit.[20] Next, divide the above equation by $u'(z-T)$ and use a first-order Taylor approximation to write:

$$\frac{u'(b) - u'(z-T)}{u'(z-T)} = -\left(\frac{u''(z-T)(z-T)}{u'(z-T)} \right) \left(\frac{z-T-b}{z-T} \right). \tag{90}$$

The first term on the right-hand side is the coefficient of relative risk aversion and the second measures the percentage drop in consumption due to unemployment. Substituting this result in equation (89) gives the same result as in Chetty (2006), Proposition 1. The main difference between this result and equations (21)–(22) is that the latter are obtained from considering a separate perturbation of the tax liability $T(z)$ and the benefit b. By contrast, Baily (1978) and Chetty (2006) consider a joint, budget-neutral increase in the benefit and the tax liability. Moreover, I do not assume UI payments are financed by lump-sum taxes on labor. As a result, also the fiscal externalities due to the wage responses show up in equations (21) and (22), which are absent in Baily (1978) and Chetty (2006).

G Derivation inverse-elasticity rule (23)

This appendix derives the inverse-elasticity rule from Proposition 4. To do so, assume there is no heterogeneity and all individuals decide to participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). The government chooses the tax function $T(z)$ and unemployment benefit b to maximize social welfare. The Lagrangian is given by:

$$\mathcal{L} = \mathcal{U}(\kappa, b) + \lambda \Big[e(\kappa, b)(T(z(\kappa, b)) + \kappa R(z(\kappa, b)) + b) - b - G \Big]. \tag{91}$$

[20] The employment rate corresponds to one minus the unemployment duration in the Baily-Chetty framework.

Here, the expected utility is:

$$\mathcal{U}(\kappa, b) = \max_{z,e} \left\{ (1-e)u(b) + e\left(u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right)\right)\right\}. \tag{92}$$

Now, consider a local increase in the marginal tax rate. This can be done by setting $R(z) = z - z(\kappa, b)$, where $z(\kappa, b)$ denotes equilibrium income. A local increase in the marginal tax rate does not affect individual's expected utility (see Appendix C). Hence, an increase in the marginal tax rate only affects welfare through fiscal externalities. The first-order condition is:

$$\frac{\partial \mathcal{L}}{\partial T'} = \lambda \left[\frac{\mathrm{d}e}{\mathrm{d}T'}(T(z(\kappa, b) + \kappa R(z(\kappa, b)) + b) + \frac{\mathrm{d}z}{\mathrm{d}T'} e(\kappa, b)(T'(z(\kappa, b)) + \kappa R'(z(\kappa, b)))\right] = 0. \tag{93}$$

To obtain the inverse-elasticity rule (23), divide equation (93) by λ and evaluate at $\kappa = 0$. Next, use the definitions of $\varepsilon_{zT'}$ and $\varepsilon_{eT'}$. Rearranging gives equation (23).

The above result immediately implies that financing UI payments through lump-sum taxes is sub-optimal. Moreover, financing them through proportional taxes (i.e., $T(z) = tz$) is also generally sub-optimal, as it requires marginal and average tax rates to be the same. The reason why these are generally not identical at the optimum is that the *marginal* tax rate is set to maximize $e(T(z) + b)$ in equation (91). The *average* tax rate, by contrast, is set to finance UI payments. Hence, the optimal marginal and average tax rate coincide only in special cases.

H Numerical analysis

Sample selection

The 2018 March release of the Current Population Survey (CPS) can be freely downloaded in Stata format from `http://ceprdata.org/cps-uniform-data-extracts/march-cps-supplement/march-cps-data/`. The unemployment rate is calculated as the number of individuals whose reported labor-force status is unemployed as a fraction of all individuals whose reported labor-force status is either employed or unemployed. This gives an (aggregate) unemployment rate of 4.1%. To calculate the participation rate, I keep individuals who listed inability to find work or taking care of home and family as reasons for not working and drop those who listed disability, going to school, retirement or other reasons. This gives an (aggregate) participation rate of 86.6%.

I measure labor earnings as the income from wage and salary payments. To align the model with the data, the final sample includes individuals between 25 and 65 years who worked full-time (i.e., who were working at least 45 weeks for on average 35 hours per week or more). Moreover, I drop all individuals who received an hourly wage below half the federal minimum wage of \$7.25. Finally, I multiply the incomes of individuals who are top-coded by a factor of 8/3, which is consistent with a Pareto parameter of $a = 1.6$ (Saez and Stantcheva (2018)).[21] The number of observations I use in the final analysis is 66,614.

For each individual, I calculate the tax liability as the sum of state and federal taxes af-

[21] If incomes at the top are Pareto distributed the average income for individuals above some threshold z^* equals $\mathbb{E}[z|z \geq z^*] = \frac{a}{a-1}z^*$.

ter credits. The latter is regressed on taxable income to approximate the current US tax schedule. The unemployment benefit is calculated as the average income from unemployment compensation for individuals who received such income and whose reported labor force status is unemployment.

Optimization

To numerically solve the government's optimization problem, I formulate it as an optimal control problem where the government directly optimizes over the the unemployment benefit b and the allocation variables $\mathcal{U}(n)$, $\ell(n)$, $e(n)$ for each n. These stand for expected utility, labor effort and the employment rate, respectively. The output $y(n)$, labor earnings $z(n)$ and participation rate $\pi(n)$ of individuals with ability n follow directly from these.

The objective function is given by equation (11). The differential equation which serves as the incentive constraint in the optimization problem is obtained by differentiating expected utility (3) with respect to ability n:

$$\mathcal{U}'(n) = e(n)v'(\ell(n))\frac{\ell(n)}{n}. \tag{94}$$

Moreover, combining the household's first-order conditions (6) and (7) leads to the following implementability constraint on the allocation variables:

$$n\left(\mathcal{U}(n) - u(b)\right) = v'(\ell(n))\left(\theta'(e(n))e(n) - \theta(e(n))\right)k, \tag{95}$$

which must hold for all n. Finally, the aggregate resource constraint is obtained by using the definition of expected utility (3) and the zero-profit condition (2) to substitute out for the tax liability in the government budget constraint (8).

For the numerical optimization I use the GPOPS-II software package. For a detailed description, see Patterson and Rao (2014). The parameterization of the utility function, matching function and the joint distribution of ability and participation costs are as described in the main text. The values for n_0 and n_1 are set at the ability level of the individual with the lowest and highest positive earnings, respectively. All programs are available upon request.

Additional graphs

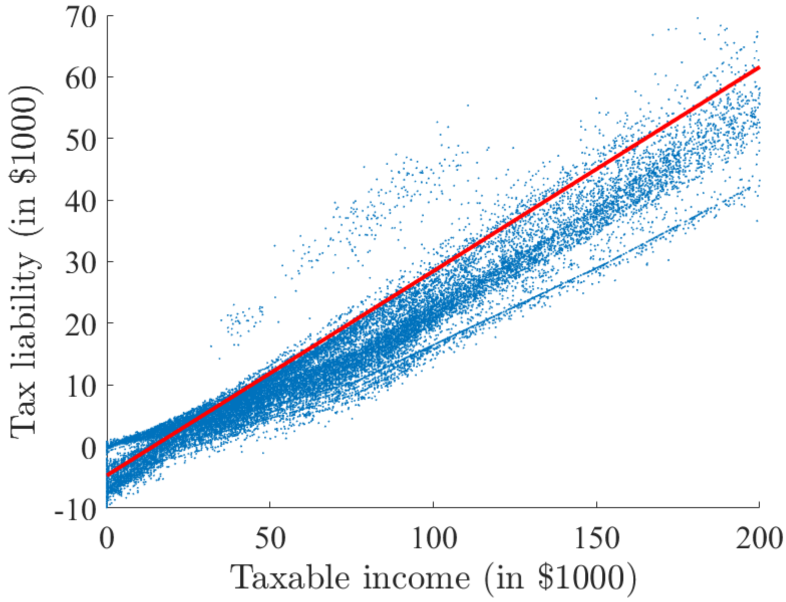

Figure 5: Current tax schedule

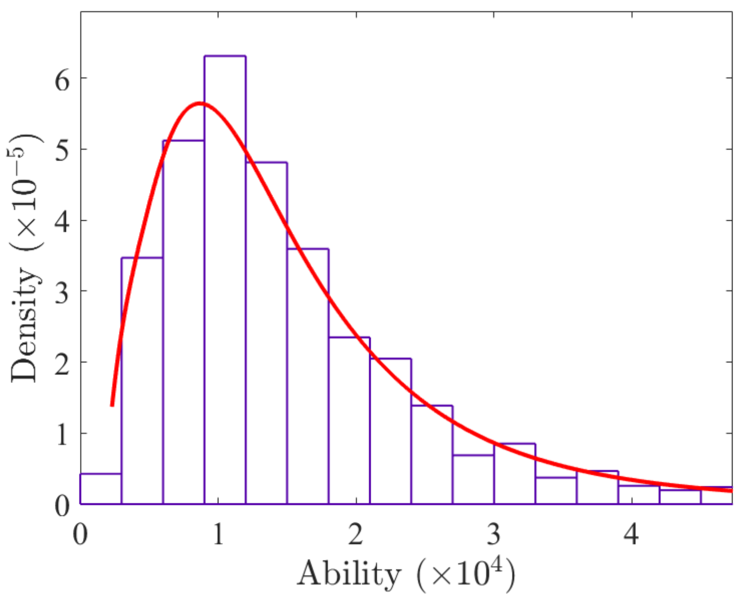

Figure 6: Ability distribution

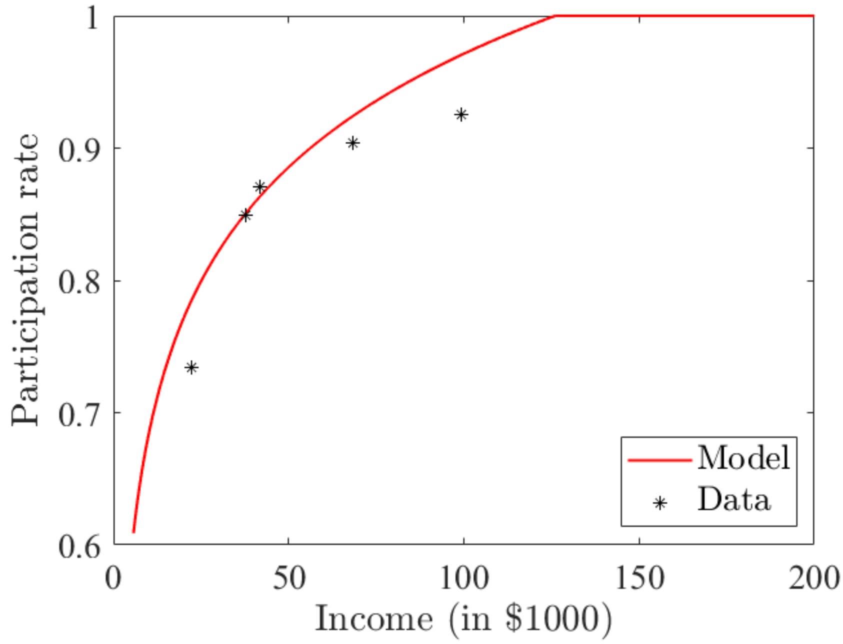

Figure 7: Participation rates by income

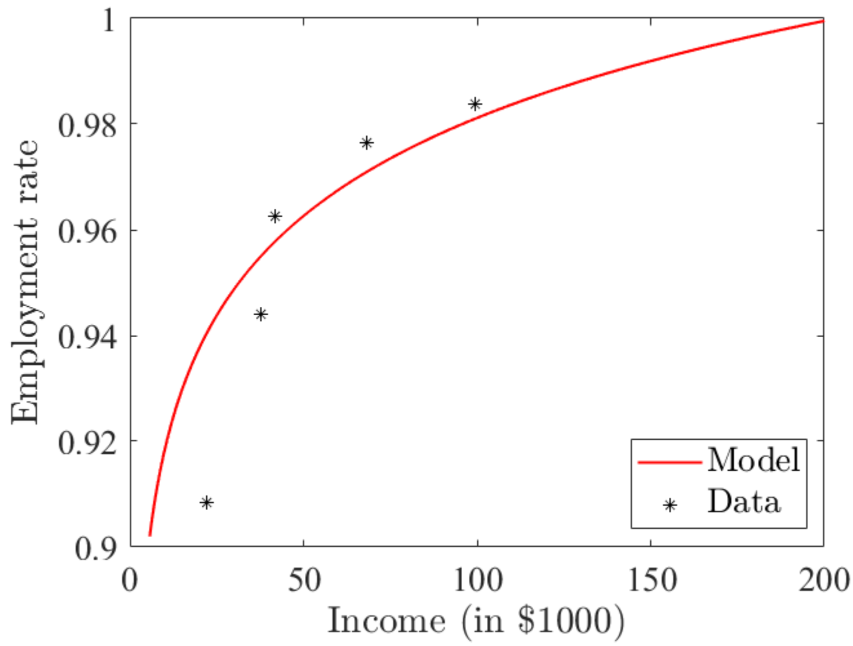

Figure 8: Employment rates by income

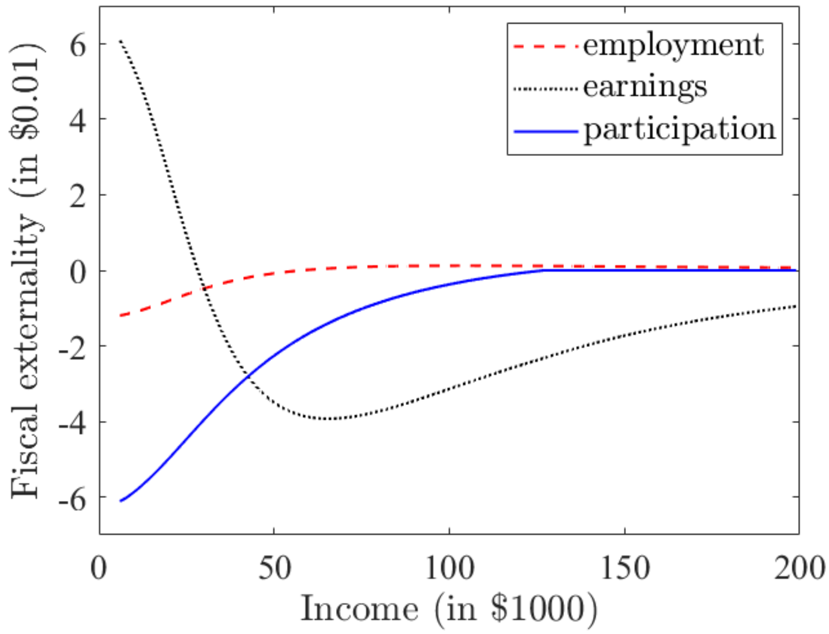

Figure 9: Comparison budgetary effects

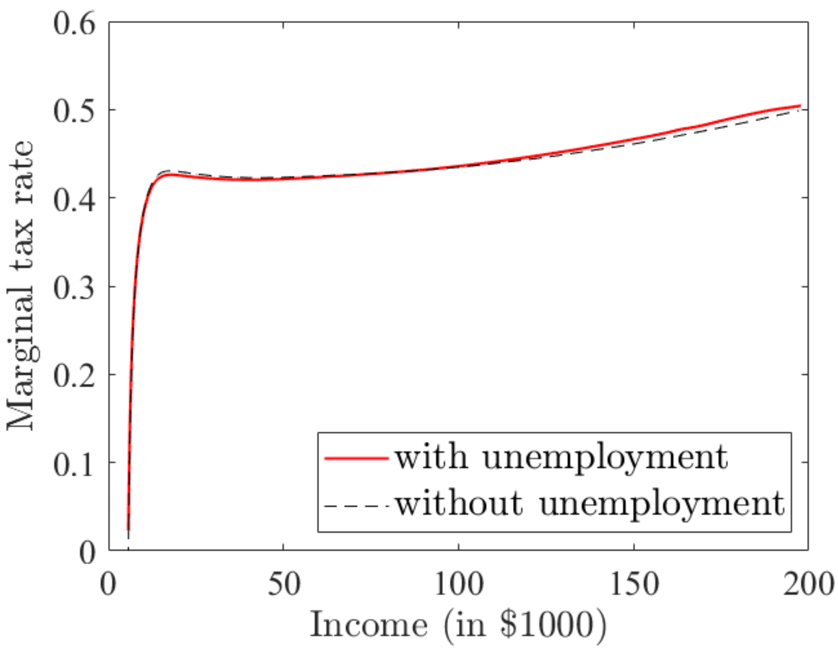

Figure 10: Comparison optimal marginal tax rates

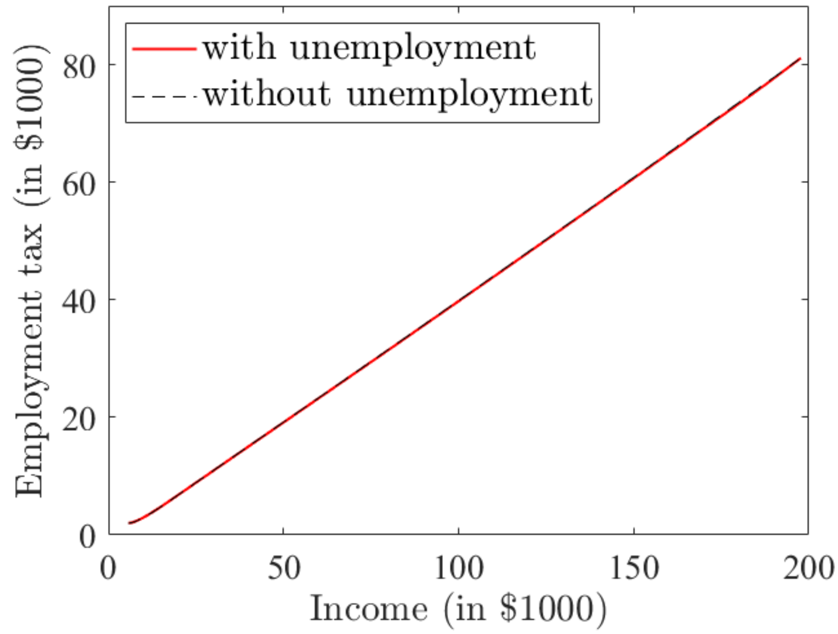

Figure 11: Comparison optimal employment taxes

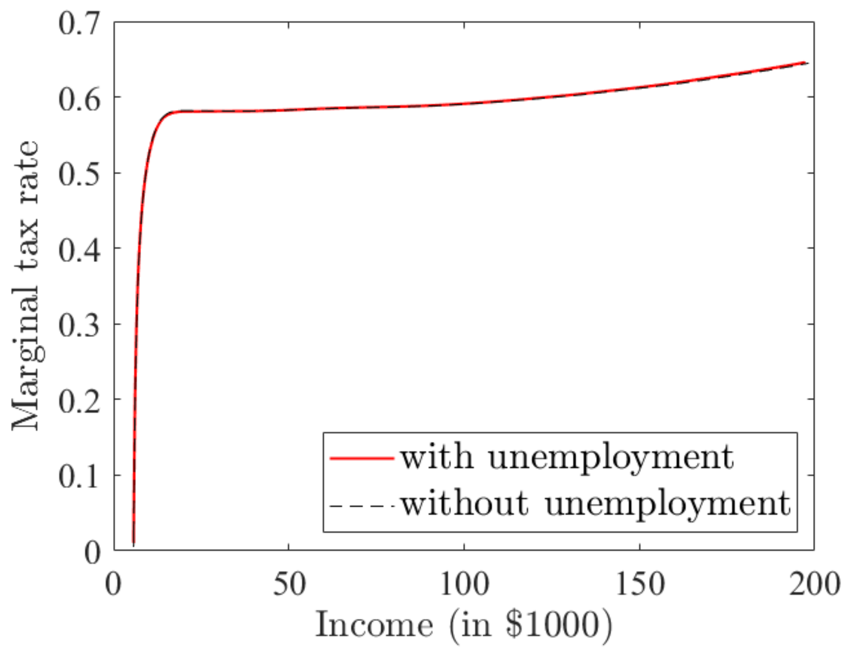

Figure 12: Comparison optimal marginal tax rates ($\sigma = 1$)

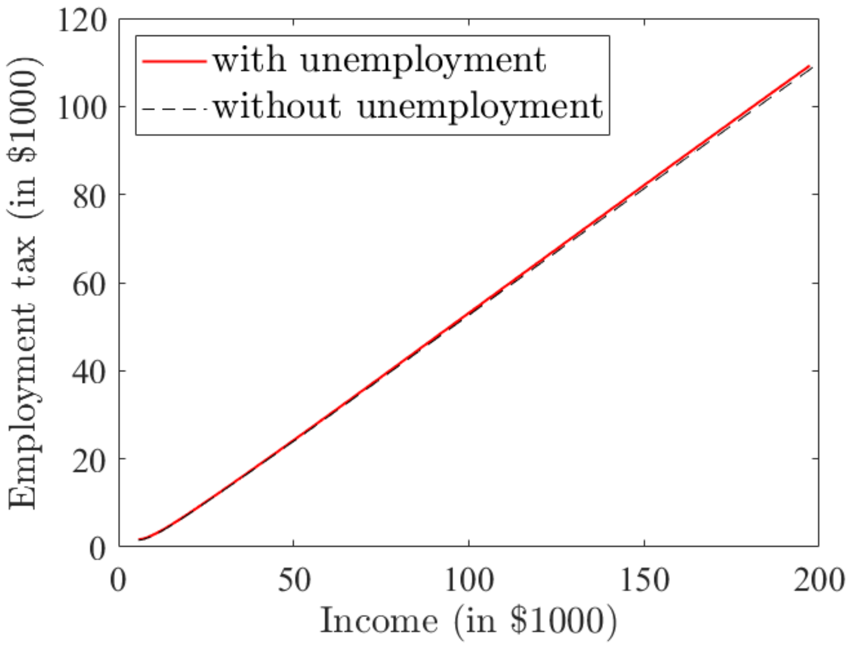

Figure 13: Comparison optimal employment taxes ($\sigma = 1$)