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Fair Pension Policies with Occupation-Specific Aging

Abstract

We discuss public pension systems in a multi-period overlapping generations model with gerontologically founded human aging and a special focus on occupation-specific morbidity and mortality. We examine how distinct replacement rates for white-collar and blue-collar workers and early retirement policies could be designed to provide a fair and aggregate welfare-enhancing public pension system. Calibrating the model to Germany, we find that a pension system that equalizes relative pension contributions and the relative present-discounted value of expected benefits across occupational groups calls for a significant increase in replacement rates of blue-collar workers. If the statutory retirement age is sufficiently high or the life expectancy gap across occupations is sufficiently large, fair pensions raise aggregate welfare and should feature early retirement incentives.

JEL-Codes: H550, I140, I240.

Keywords: fair pensions, early retirement, occupation, health gradient, life expectancy, replacement rate.

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1 Introduction

Our study of fair public pension policies is motivated by two observations. The first observation is that of pronounced differences in remaining life expectancy at age 65 across lifetime earnings and occupational groups. This fact implies that pension systems typically do not achieve equivalence of payments. The internal rate of return of pension contributions increases with longevity and old-age inequality in lifetime pension benefits exceeds the inequality of lifetime earnings (Haan et al., 2020). Consequently, equity concerns call for pension reforms that raise the rate of return to pension contributions from individuals facing higher mortality risk.

The second observation is that of occupation-specific health and longevity (Case and Deaton, 2005; Marmot, 2015; Abeliansky and Strulik, 2020). Physically demanding or stressful blue-collar jobs imply faster aging than physically less-demanding and less stressful white-collar jobs. The issue that lower-paid workers typically have more health deficits at the statutory retirement age (SRA) and live shorter because of their occupations is at the center of the public debate on raising the retirement age to mitigate pension finance problems.

One possibility to design a pension system that accounts for the lower life expectancy of blue-collar workers is to raise the replacement rate for low-income earners, conditional on their early retirement choice. This includes to incentivize early retirement for low-income workers, thereby accounting for their typically high health burden of working that leads to high disutility of labor close to the SRA. In contrast to such equalizing reform, uniformly increasing the SRA, which has been prominently suggested to remedy funding problems of pension systems, is likely to increase old-age inequality. One reason for this conclusion is that the rate of health deficit accumulation on the job is typically higher for blue-collar workers than for white-collar workers. Increasing the SRA thus widens the occupation-specific gap in life expectancy, which causes larger differences in the present-discounted value (PDV) of pension benefits.

Here, we investigate how distinct replacement rates for white-collar and blue-collar workers and early retirement policies could be designed to account for occupation-specific health, aging, and longevity in order to provide a fair account for occupation-specific differences in the public pension system. For this purpose, we propose a multi-period overlapping generations model with an age-structured population, stochastic survival, and occupation-dependent morbidity and mortality. A fair pension system is conceptualized as an equal benefit-contribution ratio across occupations. The benefit-contribution ratio is defined as the PDV of pension payments expected at retirement age divided by lifetime contributions to the pension system. We consider the aging process of blue-collar workers and white-collar workers and how their health status determines labor supply at the intensive margin (hours supplied) and extensive margin (early retirement decision). Blue-collar workers are characterized by less steep age-earnings profiles earlier in their career, lower lifetime contributions to the pension system, and faster accumulation of health deficits on the job, implying higher mortality at any age and higher disutility of labor supply.

Our modeling of physiological aging in working age and retirement age builds on insights in gerontology. Aging is conceptualized as accumulation of health deficits and measured by the frailty index (see Searle, 2008, for methodological background). The most salient property of the frailty index is that it increases, on average, exponentially with age at a rate between 3 and 5 percent (e.g. Mitnitski et al., 2002a; Mitnitski and Rockwood, 2016; Abeliansky and Strulik, 2018). The convexity of the age–frailty nexus reflects the feature that existing health deficits are conducive to the development of new health deficits. This self-productive nature of health deficit accumulation implies that individuals employed in health-demanding occupations continue to age faster in retirement. The frailty index has high predictive power for death at the individual level, and for mortality at the group level (e.g. Mitnitski et al., 2002a, 2002b, 2005, 2006, 2007). Death is thus conceptualized as a condition-specific event (triggered by high frailty) rather than as an age-specific event. The frailty index as a measure of current health status allows us to conceptualize increasing disutility (pain) of work and the desire to retire early as motivated by deteriorating health rather than increasing chronological age.[1]

[1]The health deficit approach has been introduced into economics by Dalgaard and Strulik (2014) and has been applied to a wide array of research questions. See Strulik (2021) and Grossmann (2021) for overviews

After characterizing the feature of a fair pension system analytically, we continue with a calibration of the model for Germany. While the basic question of how to design a fair pension system is universal, Germany appears to be a particularly appropriate choice for the calibration because income of its elderly citizens relies heavily on the provision from public pension system (e.g. Börsch-Supan, 2000; Börsch-Supan et al., 2015) while, at the same time, there exists a private annuity market.

The benchmark calibration of the model provides the prediction that life expectancy of blue-collar workers is 3.3 years lower than that of white-collar workers. Given the longevity gap and the current pension system of a unique and occupation-independent replacement rate, the (lifetime) benefit-contribution ratio of blue-collar workers is 15 percent lower than that of white-collar workers. This difference increases to 21 percent when the SRA increases from 65 to 70 years. A fair pension system that equalizes the benefit-contribution ratio across occupations requires that the replacement rate for blue-collars workers exceeds the one of white-collar workers by a factor of 1.18 when the SRA is 65, conditional on not retiring early (which results as an equilibrium choice from early retirement disincentives in Germany). This factor increases to 1.27 for an SRV of 70. In the fair pension system, blue-collar workers supply less labor and experience higher welfare compared to the status quo system with equal replacement rates, whereas the opposite holds for white-collar workers. At the society level, implementing such fair replacement rates increases aggregate welfare if individuals enter the social welfare function with equal weights.

Fair pensions could also be implemented by reducing the early retirement penalty for blue-collar workers. Early retirement of blue-collar workers would reduce their lifetime pension contributions. This has large adverse effects on white-collar workers by reducing their pension benefits and also their hourly wage rate due to general equilibrium repercussions.

on the concept and applications. Dalgaard and Strulik (2017) use the model to explore how the long-run gains in labor productivity and medical progress explain the historical evolution of years spent in retirement. Grossmann and Strulik (2019) investigate the interaction between health and pension policy for health inequality and the effects of increased longevity on the optimal pension system and health spending. In contrast to the present study, they neither allow for heterogeneity in earnings nor for an occupation-health gradient. They also do not consider an age-structured population and do not focus on the fairness of pension systems.

The higher the SRA, the larger the welfare gain of blue-collar workers from switching to a fair pension system which incentivizes early retirement. Such a system could make both occupational groups better off compared to fair pensions without early retirement incentives. We investigate the sensitivity of these results with respect to increasing life expectancy in general as well as with respect to an increasing longevity gap between blue-collar and white-collar workers.

Our study contributes to the literature studying the interaction between health and/or longevity, labor supply, and pension systems in quantified life cycle models (e.g. French, 2005; Ludwig and Reiter, 2010; Fehr et al., 2013; Bloom et al., 2014; Haan and Prowse, 2014; Kuhn et al., 2015; see French and Jones, 2017, for a review). A couple of studies focussed, like us, on the effects of pension system on inequality when individuals differ in life expectancy according to their socio-economic status. Laun et al. (2019) investigate welfare and inequality effects of Norwegian pension system reforms when income and health shocks differ for educational groups. Sanchez-Romero and Prskawetz (2017) show that an increasing gap in life expectancy by education (ability) enhances the regressivity of the U.S. pension system and income inequality. In a similar framework, calibrated to the U.S. economy, Sanchez-Romero et al. (2020) evaluate pension reforms that mitigate the regressivity of pension systems from income-related life expectancy differences.

Our study differs from the literature in at least two important ways. Firstly, we use a methodology that captures health status, aging, and mortality risk in one unifying variable, the frailty index. It allows us to consider the feature that labor productivity and the disutitliy of work decline because of deteriorating health (rather than because of increasing chronological age), that the type of work affects individual health and aging, and that retirement has an important feedback mechanism on health since work-related health deficits are no longer accumulated. The self-productive nature of health-deficit accumulation, however, also implies that already present health deficits are conducive to the development of further health deficits such that a hazardous and/or stressful work life has a legacy in retirement. We thus consider a couple of new channels through which health affects labor supply and through which retirement affects health and welfare. Instead of stratifying society by education we focus on a stratification by occupation (blue vs. white collar) and focus on the health-burden of work in order to investigate the newly developed pathways between work, health, and retirement. Secondly, we consider a particular type of pension reforms. Specifically, we take the "equivalence principle" seriously and investigate the features of fair pension systems, i.e. pension systems that equalize the benefit-contribution ratio across occupational groups.

2 The Model

We set up a multi-period overlapping generations model with stochastic and health-dependent death, a public pension system, endogenous labor supply, and heterogeneity of workers in terms of earnings and occupation-specific aging and mortality. Individuals can freely lend and borrow at exogenous real interest rate $\bar{r} > 0$. We deliberately exclude a feedback channel from savings behavior to the interest rate in order to avoid that the implications of pension reforms in our calibrated model are contaminated by second-order effects. Such feedback effects appear to be negligible in view of the high degree of financial globalization of advanced economies. Goods and factor markets are perfectly competitive. There exists a perfect private annuity market in addition to the public pension system.

2.1 Government Policy Instruments

The pension system is financed by proportional contributions on earnings at time-invariant rate $\tau^p \in (0, 1)$. We start out with the ("status quo") case where relative annual pension benefits across surviving retirees within a cohort are proportional to their annual contributions during working life. The proportionality between annual benefits and contributions is characterized as "equivalence principle" in the German pension system (Eggert, 2021). We argue, however, that this term is misleading because life expectancy and thus the duration of benefit payments differs across occupational groups. This feature implies that, for occupational groups with high life expectancy, relative lifetime pension benefits exceed relative working-life contributions. As an alternative, we propose a fair pension system defined by proportionality between lifetime contributions and expected discounted lifetime benefits. The conceptualization of fairness is motivated solely by a strict interpretation of equivalence, considered in terms of lifetime contributions and benefits.

There is the possibility of receiving pension benefits although retiring before the SRA. In order to simplify the analysis, we consider a binary decision to retire early, i.e. whether or not to retire $\bar{E}$ years earlier than the SRA, $\bar{R}$. The focus on the SRA despite some flexibility to choose the retirement age in most pension systems is justified by the observation that entry in retirement is clustered at the statutory level (Seibold, 2021). We allow the pension reduction in case of early retirement to differ across occupational groups. For instance, it is conceivable that pension reductions are lower for blue-collar workers prone to higher mortality risk.

For simplicity, we assume that pension income and capital income are not taxed and that wages received in declared retirement are fully taxed such that retired individuals do not supply any labor. Before retirement, earnings are taxed at (time-invariant) rate $\tau^w \in (0, 1)$. Tax revenue is paid out lump sum in uniform fashion to the workers of the same period ("earned income tax credit"). This feature captures redistributive and progressive labor income taxation, which adds to the distortion of labor supply implied by pension contributions.

2.2 Individuals

Each period a unit mass of newly born individuals with stochastic lifetime enters the labor market, choosing their consumption path, the time path of working hours before retirement age, and whether or not to retire early. Individuals are either white-collar workers or blue-collar workers, indexed by $j \in \{W, B\}$. White-collar workers account for a share $\theta \in (0, 1)$ of the workforce. For simplicity, all individuals start economic life at the same age. The fact that (for educational reasons) the "representative" white-collar worker may fully enter the labor market at a later age than the "representative" blue-collar worker is captured in the calibrated model by a low early-in-life productivity of white-collar workers. Workers born into economic life in period v are said to belong to cohort v. An individual from cohort v and occupational group j retires at age

$$Z_v^j = \bar{R} - E_v^j, \tag{1}$$

where $E_v^j \in \{0, \bar{E}\}$ is the chosen number of years of early retirement (i.e. $v + Z_v^j$ is the first period in retirement).

2.2.1 Health Deficits, Mortality, and Survival

As explained in the Introduction, modern gerontology suggests to conceptualize (biological) aging as accumulation of health deficits. Health deficit accumulation is measured by the frailty index as the fraction of health deficits present in an individual out of a long list of potential health deficits. The empirical evidence suggests that the frailty index grows on average exponentially with age. Here, we additionally take into account that the speed of health deficit accumulation is occupation-dependent during working life. This feature captures differences in physical and mental health burden of the tasks performed in different occupations. Health deficit accumulation is known to be particularly fast when the job is physically demanding, involves stress or workplace hazards, and provides little autonomy in decision-making (Marmot and Wilkinson, 2005; Marmot, 2015; Abeliansky and Strulik, 2021). We thus assume faster health deficit accumulation for blue-collar workers (e.g. manufacturers, nurses, construction workers, delivery service workers).[2]

Formally, denote the frailty index of an individual from cohort v and occupation-group j in period t by $d_{v,t}^j$, where, initially, $d_{v,v}^W = d_{v,v}^B = d_{\min} > 0$. Health deficits grow at different

[2]Although we do not explicitly endogenize health-relevant behavior, our formulation also includes the possibility of occupation-specific unhealthy consumption, such as smoking, possibly induced by peer-group effects. The assumption that the average white-collar worker experiences less physical and psycho-social job burden does not well represent high-ranked manager or health professionals in the top earning classes who may experience higher stress levels than the average blue-collar worker. However, these high-ranked individuals are typically not covered in the public pension system or are only marginally relevant for the pension system.

rates during working life to capture the occupation-health gradient:

$$d^W_{v,t} = d_{\min}(1+\mu^W_v)^{(t-v)} \text{ for all } t \geq v, \tag{2}$$

$$d^B_{v,t} = \begin{cases} d_{\min}(1+\mu^B_v)^{(t-v)} & \text{for } v \leq t \leq v + Z^B_v - 1 \\ d^B_{v+Z^j_v-1}(1+\mu^W_v)^{t-v-(Z^j_v-1)}, & t \geq v + Z^B_v, \end{cases} \tag{3}$$

where the rates of aging fulfill $\mu^W_v < \mu^B_v$. The first line on the right-hand side of (3) refers to working life of blue-collar workers and the second line to their time spent in retirement where deficit accumulation slows down to that of white-collar workers. The feature that health deficits during retirement grow at the same rate for all workers captures the notion that the cause of health deficit accumulation is work-related. Blue-collar workers, however, enter retirement with a higher level of health deficits and will thus display more health deficits than white-collar workers at all ages.

The unconditional probability $S^j_{v,t}$ of an individual from occupational group j and cohort v to survive to age $t-v$ decreases in the frailty index $d^j_{v,t}$ of the individual at age $t-v$. Thus, life-expectancy is occupation-specific. Moreover, according to (1) and (3), early retirement raises life expectancy of blue-collar workers. Consistent with data from life tables, we assume that the survival probability follows a logistic function of the frailty index (Schuenemann et al., 2017):

$$S^j_{v,t} = S(d^j_{v,t}) = \frac{1+\omega}{1+\omega e^{\kappa d^j_{v,t}}}, \tag{4}$$

$\kappa > 0$, $\omega > 0$. The survival function assumes the value of one at the state of best health (for $d^j_{v,t} = 0$) and may be close to zero when the frailty index is high.

2.2.2 Preferences and Productive Endowments

Labor supply is considered as an endogenous choice variable in order to capture the distortions of pension finance. The disutility of work rises with health deficits, an assumption, which is particularly relevant in the context of early retirement. Specifically, an individual from occupational group j and cohort v faces expected lifetime utility

$$U_v^j = \sum_{t=v}^{v+T-1} \beta^{t-v} S(d_{v,t}^j) \left[\frac{(c_{v,t}^j)^{1-\sigma} - 1}{1-\sigma} - \frac{D(d_{v,t}^j)(\ell_{v,t}^j)^{1+1/\eta}}{1+1/\eta} + \bar{u} \right], \tag{5}$$

where $c_{v,t}^j$ denotes the consumption level of the final good in period t, $\ell_{v,t}^j$ the number of hours worked, $T > 0$ the maximum length of life, $\beta \in (0, 1]$ the discount rate, $\sigma > 0$ the degree of relative risk aversion, $\eta > 0$ the Frisch elasticity of labor supply, and $\bar{u} \geq 0$ is a constant base utility level ensuring that instantaneous utility (the term in squared brackets) is positive at all times (e.g. Hall and Jones, 2007). $D(d)$ is a non-decreasing function, capturing that more health deficits are associated with higher disutility from work. The function D is not explicitly age-specific but it depends indirectly on age and occupation through its relation to health deficits.

Denote the occupation-specific mortality rate between period $t-1$ and t by $m_{v,t-1}^j = -\frac{S_{v,t}^j - S_{v,t-1}^j}{S_{v,t-1}^j}$. We assume occupation-specific fair insurance within a cohort.[3] The interest factor of a group member j of cohort v in t between date t and $t+1$ then reads

$$1 + r_{v,t}^j = \frac{1+\bar{r}}{1 - m_{v,t-1}^j}. \tag{6}$$

An individual from cohort v and occupation-group j is endowed with productive ability $a_{v,t}^j$ in period t and supplies $a_{v,t}^j \ell_{v,t}^j$ efficiency units of labor. With wage rate w_t^j per efficiency unit of labor of type j in t, the hourly wage rate is given by $W_{v,t}^j = w_t^j a_{v,t}^j$ and earnings are $W_{v,t}^j \ell_{v,t}^j$.

In addition to factor income, individuals receive income from two kinds of public sources, the earned income tax credit, $\bar{I}_t$, a lump-sum payment during working life, and pension benefits after declared retirement. Pension benefits depend on the history of pension contri-

[3]This implies an annuity market where zero-profit insurance companies pay a rate of return above $\bar{r}$ and keep the wealth of the deceased. As survival rates are occupation-specific and possibly also cohort-specific, we assume that insurers pool individuals from the same occupation and the same cohort.

butions; more precisely, on "pension wealth", $P^j_{v,t}$, that accumulates according to

$$P^j_{v,t+1} - P^j_{v,t} = \tilde{r} P^j_{v,t} + \tau^p W^j_{v,t} \ell^j_{v,t} \tag{7}$$

prior to decleared retirement. The second term on the right-hand side are contributions in period t. We assume that contributions are not possible after declared retirement. The first term is the addition to pension wealth from attributing a (time-invariant) calculatory interest rate $\tilde{r} \geq 0$, set by the policy maker, to pension wealth. In a standard pay-as-you-go (PAYG) system, $\tilde{r} = 0$, which is the case we exclusively focus on.[4]

Income from public sources is then given by

$$I^j_{v,t} = \tilde{I}(P^j_{v,t}) \equiv \begin{cases} \bar{I}_t \text{ for } v \leq t \leq v + Z^j_v - 1, \\ s^j_t P^j_{v,t} \text{ for } v + Z^j_v \leq t \leq v + T - 1, \end{cases} \tag{8}$$

where s^j_t denotes the replacement rate, i.e. the ratio between per-period pension benefits and calculatory pension wealth.[5]

Individual financial wealth accumulates according to

$$k^j_{v,t+1} - k^j_{v,t} = r^j_{v,t} k^j_{v,t} + (1 - \tau^j_{v,t} - \tau^p) W^j_{v,t} \ell^j_{v,t} + I^j_{v,t} - c^j_{v,t}, \tag{9}$$

where $k^j_{v,t}$ denotes asset holdings and $\tau^j_{v,t}$ is the tax rate on labor income, with

$$\tau^j_{v,t} = \begin{cases} \tau^w \text{ for } v \leq t \leq v + Z^j_v - 1, \\ 1 - \tau^p \text{ otherwise.} \end{cases} \tag{10}$$

The feature that wages received after declared retirement are fully taxed (with deduction of

[4]Setting $\tilde{r} > 0$ would be a possibility to account for the typically less steep age-earnings profiles of blue-collar workers by weighing pension contributions in the distant past higher than those close to the retirement age. Such policy has been recently suggested by Richter and Werding (2021).

[5]The term replacement rate is sometimes used differently as the ratio between benefits and earnings in the final year before retirement starts. However, to employ the replacement rate as a policy instrument, its definition has to capture that pension benefits depend on the full earnings history.

pension contributions) implies that individuals supply no labor after retirement.

An individual from cohort v and occupational group j controls the time paths of both $c_{v,t}^j$ and $\ell_{v,t}^j$ to maximize (5) subject to (7), (8), (9), (10), initial condition $k_{v,v}^j = 0$ (as there are no bequests) and terminal condition $k_{v,v+T}^j \geq 0$. Notably, as individuals optimize subject to constraint (7), they understand that higher labor supply will raise their pension benefits. The occupation-specific processes of health deficit accumulation (as given by (2) and (3)) and its effect on survival rates (4) are also taken into account in intertemporal decision making.

The decision whether or not to retire early involves the comparison of indirect utility for $E_v^j = \bar{E}$ and $E_v^j = 0$ for given factor prices. As individuals have a mass of zero, all macro aggregates and thus factor prices remain unchanged when a single individual changes its early retirement decision and all individuals from the same cohort and occupational group make the same early retirement decision. In equilibrium, the choices $\{E_v^W, E_v^B\}_{v=0}^{\infty}$ are such that individuals cannot gain higher lifetime utility by deviating from their decisions.

2.3 Production

Final goods are produced with physical capital and an intermediate good. Aggregate output in period t is given by

$$Y_t = (K_t)^\alpha (X_t)^{1-\alpha}, \tag{11}$$

$\alpha \in (0,1)$, where K and X denote the input of physical capital and the intermediate good, respectively. Physical capital depreciates at rate $\delta \geq 0$.

White-collar and blue-collar workers are imperfectly substitutable in production of the intermediate good across occupations and perfectly substitutable within occupations. Input of H efficiency units of white-collar and L units of blue-collar labor produces intermediate good output

$$X_t = A_t \left[\chi (H_t^X)^\rho + (1-\chi)(L_t^X)^\rho\right]^{\frac{1}{\rho}}, \tag{12}$$

$\rho < 1$, where $A_t > 0$ and $\chi \in (0,1)$ are productivity parameters. The implied elasticity of substitution between the two types of labor is $\frac{1}{1-\rho}$. We assume that productivity A_t grows exponentially at rate $g \geq 0$.

2.4 Equilibrium

An equilibrium is reached when firms choose factor demand to maximize profits in a perfectly competitive environment, households maximize expected lifetime utility (choosing the time paths of consumption, labor supply at the intensive margin, and whether or not to retire early), factor markets clear, and the government balances its budgets of redistributive income taxation and the pension system. We focus on long-run equilibria. The equilibrium analysis is relegated to the appendix.

3 Fair Pension System

The PDV of expected pension benefits for a member of cohort v in occupational group $j \in \{W, B\}$ is calculated as:

$$B_v^j \equiv \sum_{t=v+Z_v^j}^{v+T-1} S_{v,t}^j \frac{I_{v,t}^j}{\prod_{\tau=v+Z_v^j}^{t}(1+r_{v,\tau}^j)}. \tag{13}$$

Total pension contributions until entering retirement are given by

$$C_v^j \equiv \sum_{t=v}^{v+Z_v^j-1} \tau^p W_{v,t}^j \ell_{v,t}^j. \tag{14}$$

As members of cohort v from group j fully retire after Z_v^j periods and $C_v^j = P_{v,v+Z_v^j-1}^j$ for $\tilde{r} = 0$, the pension benefit is given by

$$I_{v,t}^j = s_t^j C_v^j \text{ for } v + Z_v^j \leq t \leq v + T - 1, \tag{15}$$

according to (8). The status quo of the pension system is characterized by $s_t^W = s_t^B$. The condition is misleadingly referred to as "equivalence principle" since it seemingly suggests that benefits are equally proportional to contributions for all individuals (for both occupational groups). This characterization, however, ignores differences in life expectancy between occupational groups. We will compare outcomes of the status quo pension system with those of fair pension systems.

Definition 1. *A fair pension for members of cohort v with full retirement after Z_v^j for group j equalizes relative lifetime pension contributions and the relative PDV of expected pension benefits across occupational groups; i.e.,*

$$\frac{B_v^W}{B_v^B} = \frac{C_v^W}{C_v^B} \quad \Longleftrightarrow \quad \frac{B_v^W}{C_v^W} = \frac{B_v^B}{C_v^B}. \tag{16}$$

A fair pension system thus equalizes the benefit-contribution ratio across occupational groups.[6]

We next derive the ratio of occupation-specific replacement rates that implements the fair pension system. We focus on a replacement rate ratio (RRR) which is the same for each retirement period of a cohort. It is denoted by ξ_v for cohort v. We arrive at the following result.

Proposition 1. *In a fair pension system, for cohort v, the replacement rate of blue-collar workers exceeds that of white-collar workers by a factor*

$$\xi_v = \frac{\sum_{t=v+Z_v^W}^{v+T-1} S_{v,t}^W \frac{s_t^W}{\prod_{u=v+Z_v^W}^{t}(1+r_{v,u}^W)}}{\sum_{t=v+Z_v^B}^{v+T-1} S_{v,t}^B \frac{s_t^W}{\prod_{u=v+Z_v^B}^{t}(1+r_{v,u}^B)}}, \tag{17}$$

where the sequence of replacement rates for white-collar workers is implicitly given by

$$s_t^W = \frac{\tau^p \cdot \left(w_t^B L_t + w_t^W H_t\right)}{\theta \sum_{v=t-T+1}^{t-Z_v^W} S_{v,t}^W C_v^W + (1-\theta) \sum_{v=t-T+1}^{t-Z_v^B} S_{v,t}^B C_v^B \frac{\sum_{u=v+Z_v^W}^{v+T-1} S_{v,u}^W \frac{s_u^W}{\prod_{u=v+Z_v^W}^{t}(1+r_{v,u}^W)}}{\sum_{u=v+Z_v^B}^{v+T-1} S_{v,u}^B \frac{s_u^W}{\prod_{u=v+Z_v^B}^{t}(1+r_{v,u}^B)}}}. \tag{18}$$

[6]The definition is similar to Breyer and Hupfeld (2009) except that they do not consider time discounting of benefits. Breyer and Hupfeld (2009) provide empirical estimates on the relationship between annual earnings and life expectancy of German retirees with and without fair pensions rather than comparing outcomes in a calibrated overlapping generations model.

The proof is relegated to the Appendix. From (17) we find that in the case of long-run equilibria where $s_t^W = s^W$ and $s_t^B = s^B$ for all t, the RRR ξ_v does not depend on pension contributions. To see this, substitute $s_t^W = s^W$ into (17) to obtain

$$\xi_v = \frac{\sum_{t=v+Z_v^W}^{v+T-1} \frac{S_{v,t}^W}{\prod_{u=v+Z_v^W}^{t}(1+r_{v,u}^W)}}{\sum_{t=v+Z_v^B}^{v+T-1} \frac{S_{v,t}^B}{\prod_{u=v+Z_v^B}^{t}(1+r_{v,u}^B)}}. \tag{19}$$

We see that ξ_v depends only on the PDV of post-retirement survival probabilities (where the discount rates themselves depend on mortality rates, according to (6)). In the calibrated version of the model, we will calculate the RRR ξ_v with and without early retirement and analyze occupation-specific life-cycle behavior and welfare.

4 Calibration

In order to quantitatively analyze the effects of establishing a fair pension system, we calibrate the model to the case of Germany. Germany has long served as a prime example for a PAYG pension system (with $\tilde{r} = 0$) that is regressive due to its negligence of heterogeneity in mortality risk (e.g. Haan et al., 2020).[7]

We numerically solve the model with the relaxation algorithm by Timborn et al. (2008). We aim to capture a representative male white-collar worker and a male representative blue-collar worker in terms of their rates of health deficit accumulation, survival rates, labor supply, and earnings. Both groups start their working life at age 20, albeit with different age-earnings profiles. White-collar workers earn initially less (capturing in a stylized way

[7] We focus on a stylized pre-2021 pension system. In 2021, Germany has introduced a "basic pension" ("Grundrente") for individuals who would under the pre-reform system fall significantly (80 percent and lower) below the average pension level despite contributing for at least 33 years. The basic pension income component is about EUR 400,- at maximum and presumably well below EUR 100,- on average. The maximum applies for those that would have pre-reform annual pension benefits of 30 percent than the average and at least 35 contribution years. For those below the 30 percent mark, the "Grundrente" does not apply as there has been already an instrument in place that pays out a minimum benefit ("Grundsicherung im Alter"). Notably, the new basic pension is similar to an increase in the replacement rate, as we have defined it, for eligible groups rather than being a lump sum payment.

a longer education period) and face a considerably steeper age-earnings profile than blue-collar workers. For computational reasons and since most earnings data are presented in 5-year intervals, a model period comprises five years. We will nevertheless report calibrated parameters as if one period is one year to facilitate the interpretation of results. In our calibration strategy, we distinguish between externally set parameters and estimated parameters. Externally set parameter values are taken from other (comparable) studies or are directly observed in the data. Estimated parameters are simultaneously calibrated to fit the model's response to moments observed in the data.

4.1 Parametrization

We start by imposing functional forms on expressions that we kept general so far. For the deficit function, we assume $D(d^j_{v,t}) = d^j_{v,t}$. We further assume that the labor units per hour $a^j_{v,t}$ evolve over age according to $a^j_{v,t} = \vartheta^j_0 \exp[\vartheta^j_1(t-v) + \vartheta^j_2 d^j_{v,t}]$. With this formulation, we capture the notion that individual labor productivity exhibits positive returns to age (i.e. experience), $\vartheta^j_1 > 0$, and declines due to health deficit accumulation in the course of aging, $\vartheta^j_2 < 0$. Confronting our formulation with the Mincerian wage equation, we thus generate the observed diminishing labor productivity at the end of the working life through the deterioration of health rather than the mere advancement of chronological age. In contrast to the original Mincer equation but in line with intuition and recent empirical findings, we thus assume that chronological age (more experience) contributes positively to productivity at all ages, but there is a productivity-reducing impact of declining health for any given age (Dalgaard et al., 2021).

4.2 Externally Set Parameters

The preference parameter related to labor supply, η, equals the Frisch elasticity and is set to 0.5 in accordance with the estimates by Chetty et al. (2011). Moreover, we set $\sigma = 2$ which reflects a mean value of 0.5 for the intertemporal elasticity of substitution from the literature (Chetty, 2006; Havranek, 2015).

As far as occupation-specific aging is concerned, we rely on estimates by Abeliansky and Strulik (2020) that are conceptionally equivalent to the aging parameters in our model. The authors report aging parameters for blue- and white-collar workers before and after retirement using the Survey of Health, Aging, and Retirement in Europe (SHARE). The study finds that before retirement blue- and white-collar workers age at rate $\mu^B = 0.038$ and $\mu^W = 0.035$, respectively. In other words, blue-collar workers' health deficits grow by 3.8% per year while health deficits of white-collar workers grow by 3.5% per year. When retirement sets in and the work-related health burden comes to an end, blue-collar workers enjoy a drop in the rate of health deficit accumulation such that the rate of aging between blue- and white-collar workers equalizes after retirement. For the initial frailty index of individuals we set $d_0 = 0.0273$. This is in line with the estimate in Mitnitski et al. (2002a) who find by exploiting Canadian data that, out of a potential list of 38 health deficits, males at age 20 roughly have, on average, one deficit.

At the production side, we set the capital share to $\alpha = 0.37$ (Feenstra et al., 2015) and $\rho = 1/3$ such that the elasticity of substitution between high-skilled and low-skilled labor amounts to $\frac{1}{1-\rho} = 1.5$ (Johnson, 1997). For the depreciation rate we assume a value of $\delta = 0.058$ (Davis and Heathcote, 2005) and for the real interest rate a value of $\bar{r} = 0.03$ in line with recent evidence of Jorda et al. (2019) (rate of return between that on bonds and equity). In the model, wages grow at rate g in the long run. The average annual growth rate of real wages in Germany in the last decade is roughly 1% (BPB, 2020). We thus assume that total factor productivity A grows at rate $g = 0.01$ and normalize the initial level to $A_0 = 1$.

In the German pension system, pension contributions are proportional to labor income and amount to 18.6% of gross earnings (DRV, 2020). Therefore, we set $\tau^p = 0.186$. Regarding the income tax, we follow Grossmann and Strulik (2019) and set $\tau^w = 0.25$. Further, we set $\theta = 0.5$ implying that blue- and white-collar workers represent an equal share in the total labor force (Haipeter and Slomka, 2015, Figure 1). Finally, we set $\bar{R} = 45$ and $T = 80$. Since economic life in our model starts at the age of 20, this reflects an SRV of 65 (OECD, 2019) and a maximum life span of 100 years. Table 1 summarizes the externally set parameter values along with the associated source.

Table 1: Externally Set Parameters

Parameter	Value	Explanation	Source
η	0.5	Frisch elasticity of labor supply	Chetty et al. (2011)
$1/\sigma$	0.5	intertemporal elasticity of substitution	Chetty (2006), Havranek (2015)
μ^B	0.038	rate of aging of blue-collar workers	Abeliansky and Strulik (2020)
μ^W	0.035	rate of aging of white-collar workers	Abeliansky and Strulik (2020)
d_0	0.0273	initial deficit level	Mitnitski et al. (2002)
α	0.37	capital share	Feenstra et al. (2015)
$\frac{1}{1-\rho}$	1.5	elast. of subst. blue- and white-collar	Johnson (1997)
δ	0.058	depreciation rate	Davis and Heathcote (2005)
$\bar{r}$	0.03	(real) interest rate	Jorda et al. (2019)
A_0	1	initial productivity level	normalized
g	0.01	long run growth rate of wage rates	BPB (2020)
τ^p	0.186	pension contribution rate	DRV (2020)
τ^w	0.25	income tax rate	Grossmann and Strulik (2019)
θ	0.5	share of white-collar workers	Haipeter and Slomka (2015)
$\bar{R}$	45	length of working life until statutory retirement	OECD (2019)
T	80	maximum length of economically active life	convention

4.3 Calibrated Parameters

Given the directly observed aging parameters μ^B and μ^W, we calibrate survival function parameters ω and κ to obtain the weighted average of the occupation-specific life expectancies that matches a male life expectancy at age 20 of 59.2 years and at age 65 of 18.1 years – as reported in life tables of the World Health Organization (WHO, 2020) for the year 2019. As life expectancies depend on early retirement decisions in the model, we assume that there is no early retirement and show that this is an equilibrium outcome under the existing early retirement penalties in Germany.

Moreover, we normalize $\vartheta_0^B = 1$ (determining initial productive ability of blue-collar workers) and simultaneously calibrate the remaining parameters β, $\bar{u}$, γ, ϑ_0^W, ϑ_1^W, ϑ_1^B, ϑ_2^W, ϑ_2^B, and χ in the status quo case where $s^B = s^W$ to match the following cross-sectional data moments for males: (i) earnings differential between blue- and white-collar workers, (ii) the age profile of aggregate earnings , (iii) relative labor supply of blue- to white-collar workers at age 35-39, as well as the following longitudinal data moments: (iv) consumption profile over the life cycle, (v) occupation-specific evolution of earnings over the life cycle, and (vi) the ratio between the empirical value of a statistical life (VSL) and earnings at age 50.

Data for calibration target (i) comes from Haipeter and Slomka (2015) who find an earnings ratio of white- to blue-collar workers in the German manufacturing sector of 1.6. For calibration target (ii) we use data from Dossche and Hartwig (2019) on the cross-sectional age profile of earnings. For calibration target (iii), we assume that blue- and white-collar workers supply on average the same amount of labor at age 35-39. This is plausible as the vast majority of workers is healthy at that age and the fraction of part-time workers among males is low. For calibration target (iv), we match a stable consumption profile according to Browning and Ejrnaes (2009). The authors find that once family size is controlled for, the consumption trajectory over the life cycle becomes flat. With $\bar{r} = 0.03$, this implies $\beta = (1 + \bar{r})^{-1} = 0.97$. Calibration target (v) is included to capture occupation-specific differences in life-cycle earnings. According to Ruzik-Sierdzinska et al. (2013), earnings of white-collar workers increase at a higher pace at the beginning of life than those of blue-collar workers. Complementing this observation, Bönke et al. (2015) find that earnings of high-skilled workers start off lower than those of low-skilled workers, coincide between age 25-29, and than continue to grow at higher rate thereafter. Further, the findings by Ruzik-Sierdzinska et al. (2013) suggest that the earnings of both blue- and white-collar workers peak between 45-49. We aim to capture these stylized facts by matching occupation-specific earnings to those moments of the data. Finally, for calibration target (vi), we use the ratio between the VSL and earnings at age 50 of 100, as reported in Murphy and Topel (2006). Table 2 shows the calibration results.

Table 2: Calibration Results

Parameter	Value	Explanation
β	0.97	time preference
$\bar{u}$	5.78	utility constant
ϑ_0^W	0.49	level parameter white-collar ability
ϑ_1^B	0.32	experience coefficient blue-collar ability
ϑ_1^W	0.56	experience coefficient white-collar ability
ϑ_2^B	-22.0	health deficit coefficient blue-collar ability
ϑ_2^W	-43.0	health deficit coefficient white-collar ability
χ	0.68	white-collar weight in production
κ	16.29	survival function parameter
ω	0.0197	survival function parameter

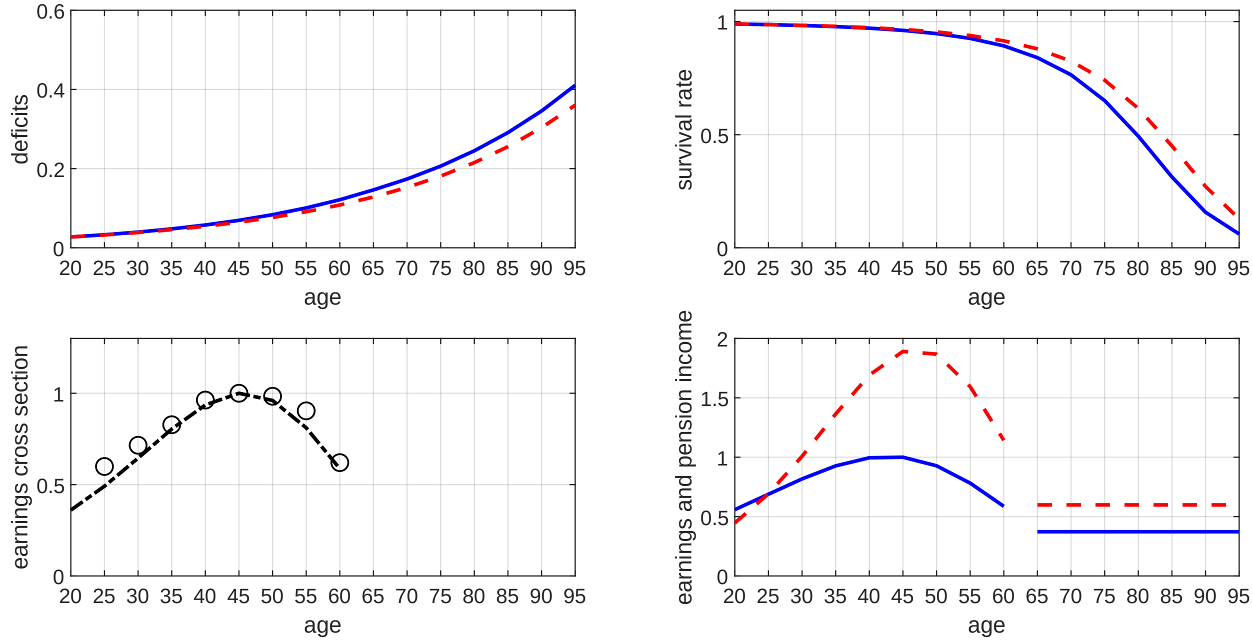
We normalized $\vartheta_0^B = 1$ (level parameter blue-collar ability).

Our estimates for the productivity parameters suggest that there are positive returns to chronological age (capturing experience) and negative returns to (physiological) aging due to health deficit accumulation. This pattern is reflected in positive estimates for ϑ_1^W and ϑ_1^B and negative estimates for ϑ_2^W and ϑ_2^B. The fact that $\vartheta_0^B > \vartheta_0^W$ and $\vartheta_1^B < \vartheta_1^W$ implies that earnings of white-collar workers start off lower but grow at a higher rate over age than those of blue-collar workers. Finally, the earnings ratio between blue- and white-collar workers of 1.6 together with a similar population share implies an earnings share of white-collar workers in total earnings of 0.62. This data moment is to a great extent reflected in our estimate for χ of 0.68.

4.4 Some Implications

Figure 1 illustrates implications for observables for the given set of parameters when $s^B = s^W$ in the case where all workers retire at age 65 (no early retirement). If not stated differently, we show results for a representative individual of each group of the same cohort over their life cycle. Blue (solid) lines represent blue-collar workers, red (dashed) lines represent white-collar workers, and black (dash-dotted) lines represent the model response for the weighted average between these two.

Figure 1: Model Results: Benchmark Run



Blue (solid lines): blue-collar workers. Red (dashed) lines: white-collar workers. Black (dash-dotted) lines: weighted average between blue- and white-collar workers. Dots indicate data points. Data for cross-sectional earnings are from Dossche and Hartwig (2019).

The upper left panel shows the evolution of health deficits over age. Due to the higher work-related health burden, deficits of blue-collar workers accumulate at a higher rate than those of white-collar workers. At retirement age 65, the health deficit trajectory of blue-collar workers exhibits a kink due to the drop in the rate of aging once those individuals retire, according to (3). Although blue- and white-collar workers face the same rate of aging after retirement, the deficit paths continue to diverge. The reason behind this observation is that the evolution of health deficits over age is path-dependent, meaning that the speed of deficit accumulation is positively affected by the amount of health deficits an individual has already accumulated. Since blue-collar workers have accumulated a higher amount of health deficits at the time of retirement, health deficit accumulation remains faster than that of white-collar workers.

The upper right panel shows the associated survival functions for the two groups, which imply that survival rates decline logistically with age. The higher health deficit levels of blue-collar workers result in a lower survival probability at any age, according to (4). The model predicts that life expectancy at age 20 is 3.3 years higher for white-collar workers than for blue-collar workers. Luy et al. (2015) report that in Germany male life expectancy at age 40 of employees and public servants exceeds that of manual workers by 3.2 to 3.7 years. The non-targeted prediction of the model accords well with these empirical observations.

The lower left panel illustrates the age profile of earnings in the cross section, i.e. across cohorts for a given point in time, relative to earnings at age 45-49. We calculate a weighted average between the earnings of blue- and white-collar workers according to their share in the population and their survival rate $S^j_{v,t}$. Dots indicate data points from Dossche and Hartwig (2019). As can be seen in the figure, the model predicts the typical hump-shaped pattern of the cross sectional earnings trajectory reasonably well.

The lower right panel shows occupation-specific earnings over the life-cycle. The reported numbers are computed relative to earnings of a blue-collar worker at age 45-49. As intended by our calibration strategy and consistent with empirical evidence, earnings of white-collar workers start off at a lower level, but grow at a higher rate and quickly surpass earnings of blue-collar workers. After a peak around age 45-49, earnings start to decrease as a result of growing health deficits that impair labor productivity and induce lower labor supply. Starting at age 65, individuals receive a per-period pension income proportional to their pension contributions as implied by the status quo pension system.

5 Implementing Fair Pensions

In the numerical analysis, we compare results of the status quo pension system with those of fair pension policies that fulfill Definition 1. The status quo pension system holds that, in absence of early retirement, replacement rates of both occupational groups coincide ($s^B = s^W$). In fact, there exists a substantial early retirement penalty in the status quo system

($s^B < s^W$) that disincentives early retirement. The fair pension policy is characterized by the fair RRR (Proposition 1), i.e. $\xi = s^B/s^W$ as given by (19). We allow for different replacement rates across occupational groups in two ways. First, we identify the fair RRR when early retirement disincentives are kept in place and therefore all workers work until the SRA. Second, we determine the fair RRR under the condition that blue-collar workers choose to retire early which requires a reduction of status quo early retirement penalties. We then compare the behavioral responses and welfare implications of the two policy alternatives.[8] Finally, we investigate how results change when life expectancy increases in general and when the gap between between blue- and white-collar life expectancy widens.

5.1 Fair Pensions Without Early Retirement

In this section we consider a fair pension system when both occupational groups work until the SRA (which is an equilibrium outcome for the status quo early retirement policy) and compare life-cycle behavior and welfare of the two occupational groups with that of status quo pensions. Using the calibrated survival functions in the expression for the fair replacement ratio (19), we compute $\xi = 1.18$. Accounting for occupation-specific mortality in a fair pension system thus implies that the replacement rate of blue-collar workers should exceed the one of white-collar workers by 18%. This result is shown in the first part of Table 3. Columns 2-4 show the ratio of replacement rates ($\frac{s^B}{s^W}$), pension contributions ($\frac{C^B}{C^W}$), and the PDV of expected pension benefits ($\frac{B^B}{B^W}$) between blue- and white-collar workers.

Line 1 of Table 3 shows the results for the status quo pension system when all workers work until the SRA of 65. As targeted, per-period pension benefits are directly proportional to pension contributions and thus the two occupational groups face the same replacement rate ($\xi = 1$). The contribution ratio coincides with the earnings ratio between the two groups, and is obtained as 0.61. The benefit ratio is 0.52 and thus 9 percentage points lower than

[8] We therefore ask how old-age inequality would have looked like under pension policies that accounted for occupation-specific differences in mortality rates. In a policy reform analysis, without anticipation of a reform, those who are non-retired would reoptimize after the reform is announced. This would make the analysis more complex.

required by a fair pension system. The implied benefit-contribution ratio is $0.52/0.61 = 0.85$, indicating that, in the status quo, blue-collar workers receive 15 percent less benefits from contributions than white-collar workers.

Line 2 of Table 3 shows results for the fair pension system without early retirement that, by design, equalizes the contribution ratio and the ratio of the PDV of expected pension benefits. We see that fair pensions require that the replacement rate of blue-collar workers need to exceed that of white-collar workers by a factor $\xi = 1.18$.

Table 3: Status Quo vs. Fair Pensions

Pension system	$\xi = \frac{s^B}{s^W}$	$\frac{C^B}{C^W}$	$\frac{B^B}{B^W}$
SRA of 65			
1) Status quo	1.00	0.61	0.52
2) Fair w/o early retirement	1.18	0.61	0.61
3) Fair with early retirement	0.94	0.59	0.59
SRA of 70			
4) Status quo	1.00	0.61	0.48
5) Fair w/o early retirement	1.27	0.61	0.61
6) Fair with early retirement	0.94	0.59	0.59

$\frac{s^B}{s^W}$: RRR between blue- and white-collar workers. $\frac{C^B}{C^W}$: pension contribution ratio between blue- and white-collar workers. $\frac{B^B}{B^W}$: ratio of the expected PDV of pension benefits between blue- and white-collar workers. Status quo (lines 1 and 4) refers to the case without early retirement.

We next show the effects of switching to the fair pension policy on occupation-specific life-cycle behavior and outcomes, and in particular on welfare. These results are shown in Table 4. The entries in the table indicate percentage deviations from the policy reform for lifetime labor supply of someone who survives until declared retirement, average hourly wages over the lifetime, the PDV of expected earnings, the PDV of expected pension income, as well as welfare for blue-collar and white-collar workers. Changes in welfare are measured in consumption equivalents. To this end, we identify the factor by which consumption in the status quo system would have to be multiplied each period such that the individual under consideration experiences the same lifetime utility as under the fair policy alternative. The computation of welfare changes in terms of consumption equivalents is explained in detail in the Appendix.

Table 4: Welfare Analysis of Fair Pension Systems

	$\Delta \ell$	ΔW	$\Delta W\ell$	ΔI	ΔU
SRA of 65					
W/o early retirement: $\xi = 1.18$					
1) blue-collar workers	-0.24	0.17	-0.11	11.0	0.99
2) white-collar workers	0.20	-0.10	0.10	-5.61	-0.69
With early retirement: $\xi = 0.94$					
3) blue-collar workers	-6.09	2.05	0.03	25.3	0.75
4) white-collar workers	-0.08	-1.24	-1.32	-8.27	-0.85
SRA of 70					
W/o early retirement: $\xi = 1.27$					
5) blue-collar workers	-0.34	0.25	-0.17	16.5	1.37
6) white-collar workers	0.27	-0.15	0.14	-7.87	-0.95
With early retirement: $\xi = 0.94$					
7) blue-collar workers	-5.30	1.24	0.10	36.9	1.89
8) white-collar workers	-0.07	-0.76	-0.81	-10.0	-0.74

We report changes in percent when deviating from the status quo system for lifetime labor supply conditional on survival until declared retirement (ℓ), average hourly wages over the lifetime (W), the PDV of expected earnings ($W\ell$), the PDV of expected pension benefits (I), and welfare (U). The effect on welfare is measured in consumption equivalents. ξ refers to the fair RRR between blue-collar and white-collar workers.

The upper part of Table 4 shows the effects of switching to the fair pension policy for an SRV of 65. Anticipating the higher replacement rate in old age, blue-collar workers slightly reduce average labor supply by 0.24% (line 1). Since blue-collar labor becomes more scarce, average hourly wages go up by 0.17%. Both effects combined result in an overall decline of the PDV of expected lifetime earnings of 0.11%. With fair instead of equal replacement rates, the PDV of expected lifetime pension income increases by 11.0%. With respect to welfare, the negative effect from (slightly) lower wage income is dominated by the positive effects of lower lifetime labor supply and higher pension income, leading to a welfare gain of 0.99% compared to the benchmark run.[9]

Behavior of white-collar workers is affected in the opposite direction. The lower replacement rate induces white-collar workers to work more (0.20%), resulting in a reduction in hourly wages of 0.10% (line 2). The net effect on earnings, however, is still positive (0.10%). Despite slightly higher earnings during working age, the drop in lifetime pension income of 5.61% leads to a welfare loss for white-collar workers of 0.69%. Recalling that blue- and white-collar workers contribute equal shares to the total population, we assign the same weight to both groups in the computation of total welfare and conclude that the fair pension system improves welfare compared to the status quo system (since $0.99 > 0.69$).

A popular theme in the discussion of pension system reforms is an extension of the retirement age. We thus also report results for a higher SRA of 70 in the bottom part of Table 3. Line 5 of Table 3 indicates that the ratio of the PDV of expected pension benefits amounts to 0.48 and is thus 13 percentage points lower than the contribution ratio. This implies that, under the status quo with equal replacement rates (given that there is no early retirement), blue-collar workers receive 21 percent ($= 1 - 0.48/0.61$) less benefits from contributions than white-collar workers. The difference is more pronounced than in the case where the retirement is 65 because blue-collar workers suffer 5 years more from

[9] In order to assess the welfare gain appropriately, it may be worthwhile to recall that a one percent welfare gain (from abolishing capital taxation) has been appraised as the largest gain that quantitative welfare economics can provide (Lucas, 1990) and that the welfare gained from eliminating all business cycles has been estimated to be somewhere between 0.008 and 0.1 percent (Krusell and Smith, 1999).

higher health-related job burden compared to white-collar workers, thereby reducing post retirement survival rates and thus life expectancy. Therefore, the fair RRR increases for the higher SRA to $\xi = 1.27$.

Since the required change in replacement rates is larger for higher retirement age, the effects on life-cycle behavior and welfare are quantitatively also more pronounced, but remain qualitatively unchanged compared to the case of retirement age of 65. According to Table 4, switching to the fair RRR increases welfare of blue-collar workers by 1.37% (line 5) while reducing welfare of white-collar workers by 0.95% (line 6). Assigning equal weights to both occupational groups in total welfare thus again implies an aggregate welfare gain for the economy.

5.2 Fair Replacement Rates With Early Retirement

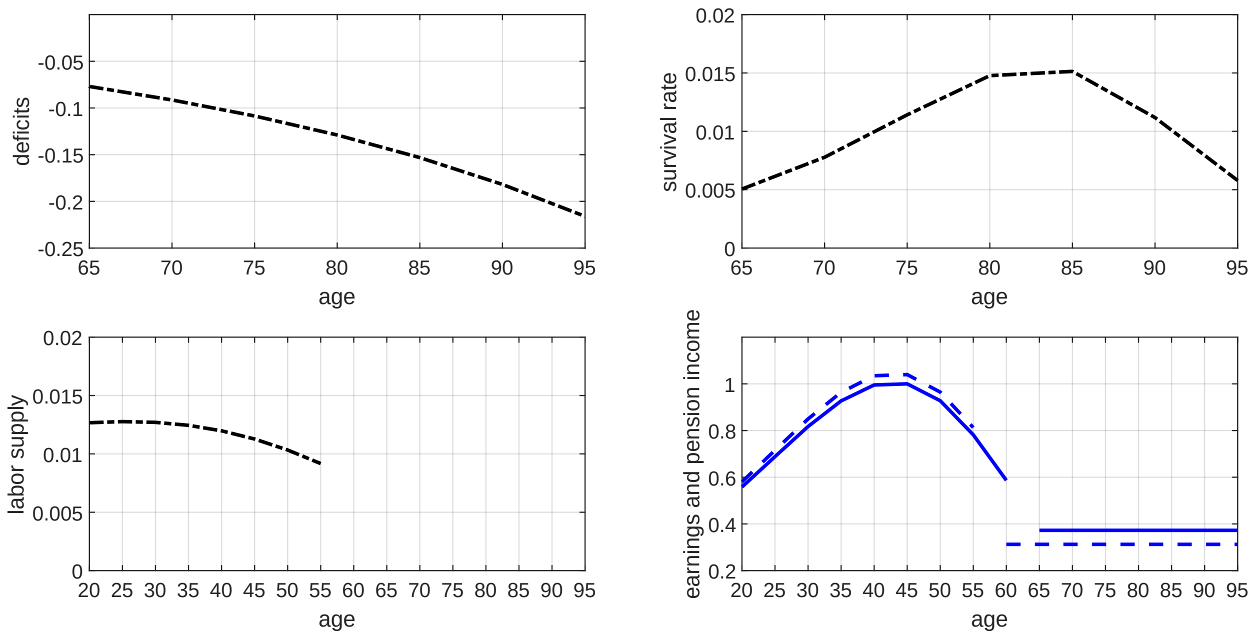
A natural recommendation to account for occupation-specific mortality and its implications for the pension system is to give blue-collar workers the opportunity to retire before the age of 65. In a PAYG pension system like the one of Germany, early retirement is heavily disincentivized through two channels. Early retirement reduces pension contributions and thus pension income due to the shortened work life. On top of that, pension law requires that pension income is reduced by 0.3% per month that the individual retires before the SRA. With our model at hand, we aim to causally identify the effect of early retirement with fair pensions on the reduction of old-age inequality, equivalence of the pension system, and welfare of the two occupational groups.

Early retirement has positive and negative effects on lifetime utility. On the one hand, early retirement shortens the length of the working period when supplying labor is particularly painful due to a relatively high number of health deficits (captured by function $D(d)$ in utility function (5)). Further, early retirement slows down the work-related accumulation of health deficits for blue-collar workers, according to (3). This results in higher survival rates after retirement (and thus higher life expectancy), implying higher expected utility. On the other hand, early retirement reduces the periods in which the individual contributes to the pension system and thus decreases annual pension benefits. In addition, under the status quo German pension system, retiring at age 60 instead of 65 would leave a worker with another 18% (0.3% per month times 60 months) penalty on pension benefits. In other words, the effective replacement rate would be only 82% of that of the white-collar worker. In our calibrated model, under the status quo pension system, early retiring individuals are worse off and thus would choose to work until they reach the SRA of 65.

We find that a fair pension system with early retirement of blue-collar workers would require a relative replacement rate of 0.94 instead of 0.82 (as shown in the third line of Table 3). In this case, blue-collar workers would indeed choose to retire early. The result implies that the monthly penalty of early retirement should lie around 0.1% rather than the current 0.3% to establish true equivalence.

We next compare certain life-cycle outcomes for blue-collar workers under the status quo system with those under fair pensions with early retirement. Results are shown in Figure 2 and lines 3 and 4 of Table 4. Black (dash-dotted) lines in the first three panels of Figure 2 show the difference across the two policy scenarios for health deficits, survival, and labor supply of blue-collar workers (normalized by the respective initial value under status quo policy). Until age 60, health deficits coincide under both pension policies. Since blue-collar workers retire at age 60 instead of 65, they enjoy five more years of leisure without accumulating work-related deficits. As a result, deficits accumulate at lower pace, which is reflected in the figure by a negative difference of deficits by age. Due to the self-productive nature of deficit accumulation, the health gain from early retirement increases with advancing age. At age 80, for example, blue-collar workers are predicted to display a 20 percentage points lower health deficit index. The gain in health translates into improved survival rates in old age, which are depicted in the upper right panel of Figure 2. The gain in survival first increases due to the self-productive nature of health deficits and then declines because in the end everybody dies. Remaining life expectancy of blue-collar workers at age 60 rises from 20.7 to 21.1 years (compared to 23.5 years for white-collar workers). The life expectancy gap between occupational groups at age 20 declines from 3.3 to 3.1 years. Improved survival leads to higher expected utility of blue-collar workers.

Figure 2: Blue-Collar Life-Cycle Outcomes: Status Quo vs. Fair Pension with Early Retirement



Blue (solid) lines: per-period earnings and per-period pension benefits under the status quo pension system. Blue (dashed) lines: per-period earnings and per-period pension benefits with a fair RRR and early retirement. Black (dash-dotted) lines: Difference in outcomes between status quo pension system and fair pensions with early retirement, normalized by the respective initial value under the status quo pension system.

Anticipating that working life is shorter and thus earnings and pension income will be lower, blue-collar workers increase labor supply at any age to partly compensate for this loss of income, as shown in the lower left panel of Figure 2. Increased labor supply translates into higher annual earnings. This can be seen in the lower right panel of Figure 2, in which blue solid lines represent the life-cycle trajectory of a blue-collar worker in the status quo pension system, while the blue dashed lines represent the outcome with a fair RRR. The combined effect of increased labor supply when working and early retirement is negative. As can be seen from Table 4, lifetime labor supply decreases by 6.09%, which leads to an increase in hourly wages by 2.05%. The PDV of expected earnings remains virtually unchanged, since the relatively large drop in earnings occurs at the end of the working life and is thus discounted more heavily than the increase in earnings earlier in life.

Lifetime pension income of blue-collar workers increases by 25.3%. The reason behind this observation is twofold. On the one hand, pension contributions decrease, leading to lower pension income. Furthermore, blue-collar workers face a penalty of 0.1% per month of early retirement. On the other hand, blue-collar workers collect pension income already five years earlier and, through better health and thus higher survival rates, exhibit a higher probability of collecting pension for a longer time in retirement. This, taken for itself, has a positive impact on lifetime pension income and is the dominating force in this experiment. Due to the higher pension income combined with better health and higher life expectancy and the reduction of labor supply (especially at the end of the working life when working is especially painful), welfare of blue-collar workers increases by 0.75%. Therefore, blue-collar workers would prefer early retirement with fair pensions over the status quo pension system.

The considerably lower labor supply of blue-collar workers reduces the production of the intermediate good and causes a decline of the marginal product of white-collar workers, which results in a drop of white-collar hourly wages by 1.24%. This leads to a reduction in their lifetime earnings by 1.32%. Since both blue-collar and white-collar workers contribute less to the pension system and white-collar workers have to cross-subsidize early retirement of blue-collar workers, the PDV of expected pension income of white-collar workers decreases substantially, by 8.27%. As both the lifetime earnings and lifetime pension income decrease, welfare of white-collar workers declines by 0.85%.

When assigning the same weight in total welfare to blue- and white collar workers, the policy reform would reduce total welfare (as $0.75 < 0.85$). In fact, a fair pension system without early retirement incentives, as discussed in the previous subsection, would be Pareto-improving, as welfare would be higher for both blue-collar ($0.99 > 0.75$) and white-collar occupations ($-0.69 > -0.85$). The main reason for the inferiority of the early retirement regime are the general equilibrium repercussions caused by the reduced output of intermediate goods in response to the reduced labor supply of blue-collar workers.

Matters change when we assume an SRV of 70 instead of 65. According to Table 3, the fair RRR with early retirement again roughly equals 0.94. Lifetime earnings of blue-collar workers increase somewhat more and those of white-collar workers decline by less compared to statutory retirement at age 65 (compare lines lines 7 and 8 with lines 5 and 6, respectively). The reason is that the decline of labor supply of blue-collar workers is smaller when the SRA is 70.

Since labor supply decreases for older ages because of deteriorating health, the reduction in hours worked caused by early retirement is smaller for ages 65-69 than for ages 60-64. This implies that the forgone pension contributions from early retirement are lower and – combined with the fact the RRR is slightly higher – the increase in the PDV of expected pension benefits for blue-collar workers is higher than for statutory retirement at age 65 (36.9%). Since more painful work at 65-69 can be avoided by retiring early, the welfare increase is higher than in the case of an SRV of 65 (1.89%).

Earnings of white-collar workers deteriorate by less because the production decline of the intermediate goods is weaker than for statutory retirement at age 65. As shown in Table 4, the welfare effect of switching to a fair pensions system with early retirement for white-collar workers is more moderate at -0.74%. Weighing both occupational groups equally, the policy reform thus entails an aggregate welfare gain. In fact, for the SRA of 70, moving from the fair pension scenario without early retirement incentives to one with early retirement incentives now entails a Pareto improvement ($1.89 > 1.37$, $-0.74 > -0.95$).

Summarizing results of the two pension reforms (fair pension system with and without early retirement incentives), we conclude that policy implications change for a higher SRA. For the case of an SRV of 65, it is more favorable for both groups to implement a fair pension system without early retirement incentives, while for an SRV of 70, it is more favorable to implement a fair system that incentivizes early retirement. The main reason behind this observation is that early retirement entails greater welfare gains for a higher SRA because working becomes more painful at higher ages. In addition, the general equilibrium repercussions through reduced output are smaller because the reduction in labor supply due to early retirement is less pronounced.

Table 5: Fair Pension Systems and Welfare Effects of Policy Switch: Sensitivity Analysis

	W/o early retirement			With early retirement		
	Fair RRR	ΔU^B	ΔU^W	fair RRR	ΔU^B	ΔU^W
SRA of 65						
1) benchmark	$\xi = 1.18$	0.99	-0.69	$\xi = 0.94$	0.75	-0.85
2) Higher LE	$\xi = 1.16$	0.89	-0.62	$\xi = 0.96$	0.04	-1.08
3) Higher LE Gap	$\xi = 1.34$	1.76	-1.19	$\xi = 1.03$	2.51	-1.00
SRA of 70						
4) benchmark	$\xi = 1.27$	1.37	-0.95	$\xi = 0.94$	1.89	-0.74
5) Higher LE	$\xi = 1.24$	1.22	-0.84	$\xi = 0.97$	1.57	-0.89
6) Higher LE Gap	$\xi = 1.54$	2.44	-1.64	$\xi = 1.08$	3.76	-1.16

ξ refers to the fair RRR. ΔU^B and ΔU^W are welfare changes in percent when switching from the status quo to the fair pension policy for blue- and white-collar workers, respectively. We distinguish fair pension policies without and with early retirement incentives. The effect on welfare is measured in consumption equivalents.

5.3 Increasing Life Expectancy

In this section, we investigate in comparative dynamic analysis the effects of higher life expectancy. To this end, we consider a lower rate of aging for both blue- and white-collar workers and redo the policy experiments from the last section. We consider an increase in life expectancy by three years for both occupational groups, which corresponds to the increase in male life expectancy observed over the last 20 years in Germany (World Bank, 2021). We thus follow evidence for Germany showing that differences in life expectancy between subpopulations, including different education strata, have remained stable in recent decades (Unger and Schulze, 2013). In model terms, we reduce the rates of aging from $\mu^B = 0.38$ to $\mu^B = 0.36$ for white-collar workers and from $\mu^W = 0.35$ to $\mu^W = 0.33$ for blue-collar workers.

The results are shown in Table 5. The table shows the fair pension policy with and without early retirement incentives along with the associated welfare changes for blue- and white-collar workers. The upper and the lower part of the table include results for an

SRV of 65 and 70, respectively. Lines 1 and 4 reiterate the results from our benchmark experiment. Lines 2 and 5 show results for higher life expectancy of blue- and white-collar workers. Without early retirement incentive, results are similar to those in the previous section. However, we observe that welfare changes become less favorable for both considered retirement ages in the early retirement regime. This finding holds for both blue-collar workers ($0.04 < 0.75$, $1.57 < 1.89$) and white-collar workers ($-1.08 < -0.85$, $-0.89 < -0.74$).

For blue-collar workers, early retirement becomes less favorable because better health leads to higher productivity and higher labor supply also in old age and thus the loss in earnings caused by early retirement is more pronounced. Moreover, early retirement is associated with a lower welfare gain since, with better health, working is less painful in old age. For white-collar workers, the early retirement of blue-collar workers is also less favorable when the aging process slows down, for two reasons. First, since blue-collar workers are more productive and able to supply more labor, the forgone earnings and thus pension contributions following early retirement reduce the pension base, which also hurts white-collar workers. Second, since for better health early retirement implies a greater reduction of the labor force, the effect on wages and thus the general equilibrium repercussions are stronger. While we saw in the previous subsection that, for an SRV of 70, moving from a fair pension system without early retirement incentive to a fair pension system that incentivizes early retirement entails a Pareto improvement, we do not see this pattern for the high life expectancy scenario. While blue-collar workers are better off with early retirement ($1.22 < 1.57$), this is not true for white-collar workers ($-0.84 > -0.89$).

5.4 Higher Life Expectancy Gap

In this subsection, we analyze how results change when the occupational gap in life expectancy is larger. This experiment is motivated by the observation that there exists considerable heterogeneity in longevity also within occupational groups. Luy et al. (2015) find that the difference in life expectancy at 40 between employees performing qualified tasks and public servants to that of *low-skilled* manual workers lies between 5.0 and 5.7 years.

Moreover, it may be possible that behavioral changes towards more unhealthy consumption particularly of blue-collar workers foster larger occupational difference in the health deficit accumulation growth rate (e.g. caused by obesity or drug abuse).

To discuss the effects, we now assume that the gap in life expectancy increases from 3.3 to 5.5 years, by raising the aging parameter of blue-collar workers from $\mu^B = 0.38$ to $\mu^B = 0.40$ while the white-collar aging parameter remains at its benchmark value of $\mu^W = 0.35$. The results are shown in lines 3 and 6 of Table 5. Compared to the benchmark calibration, a higher life expectancy gap naturally leads to a higher fair RRR. For an SRV of 65 and 70, the fair RRR without early retirement incentive increases from 1.18 to 1.34 and from 1.27 to 1.54, respectively. Since mortality rates of blue-collar workers exceed those of white-collar workers to a higher extent, establishing a fair pension system requires greater adjustments of relative replacement rates. This implies that the welfare effects are larger compared to the benchmark experiment.

With fair pensions conducive to early retirement, the fair RRR increases from 0.94 (0.94) to 1.03 (1.08) for an SRV of 65 (70). Thus, for a higher life expectancy gap, blue-collar workers have to be assigned a higher replacement rate than white-collar workers even when they are incentivized to retire early. Blue-collar workers find it more beneficial to retire early since working in old age becomes more painful and the loss in earnings is not as pronounced as in the benchmark experiment because productivity and labor supply of blue-collar workers decreases more rapidly with age. This results in a less pronounced loss of pension contributions of blue-collar workers and a less detrimental general equilibrium effect on wages than in the benchmark experiment.

Consequently, the fair pension system with early retirement incentive now is more favorable for both blue- and white-collar workers under both SRAs (and not just for SRA of 65 as under the benchmark calibration). Blue-collar workers gain 2.51% (3.76%) in welfare with early retirement incentive for SRA of 65 (70) compared to 1.76% (2.44%) without early retirement incentive. Moreover, white-collar workers only lose 1.00% (1.16%) instead of 1.19% (1.64%).

6 Concluding Remarks

We analyzed fair pension policies in a multi-period overlapping generations model with stochastic survival. The model captures the important feature that blue-collar workers have higher mortality risk than white-collar workers, associated with faster health deficit accumulation. We have derived occupation-specific replacement rates that equalize the ratio of the PDV of expected pension benefits and lifetime pension contributions across occupations and analyzed both allocative and welfare effects of implementing fair pensions. Calibrating the model to Germany showed that the (lifetime) benefit-contribution ratio of blue-collar workers is substantially lower than that of white-collar workers. The gap is increasing in the SRA. This calls for a significant increase in the replacement rates for blue-collar workers that is also increasing in the SRA. Implementing fair replacement rates would reduce labor supply of blue-collar workers and increase their welfare, while the opposite holds for white-collar workers.

The model also features an early retirement decision, where early retirement of blue-collar workers implies that work-related health deficit accumulation slows down, lowering the longevity gap to white-collar workers. We find that the replacement rate for blue-collar workers should be similar to that of white-collar workers even when they retire 5 years earlier, while currently the replacement rate is significantly reduced in proportion to each month an individual retires before the SRA. If the SRA or the longevity gap between occupations is high enough, then incentivizing early retirement is Pareto-improving compared to a fair pension system that is conditional to working until SRA. Under these conditions, implementing fair pensions also increases aggregate welfare.

Our framework could be extended in several directions in future research. First, it could be interesting to derive a more differentiated picture of a fair pension system by accounting for heterogeneity in health deficit accumulation also within occupations. This requires differentiated health deficit accumulation rates according to occupational subgroups for the model calibration, however. Second, instead to differentiating replacement rates according to occupation, our framework could be employed to investigate the policy alternative of attributing a calculatory interest rate on past contributions to benefit blue-collar workers that have relatively high earnings earlier in life (Richter and Werding, 2020). Finally, it could be interesting to endogenize health investments that affect the health deficit accumulation rates over and above the early retirement decision.

A Appendix

A.1 Equilibrium Analysis

A.1.1 Factor Prices and Aggregate Labor Supply

The user cost of capital, $\bar{r}+\delta$, is equal to its marginal product. According to (11), $\bar{r}+\delta = \alpha(X_t/K_t)^{1-\alpha}$. Similarly, denoting the price of the intermediate good by p, we have $p_t = (1-\alpha)(X_t/K_t^Y)^{-\alpha}$. Thus,

$$p_t = (1-\alpha)\left(\frac{\alpha}{\bar{r}+\delta}\right)^{\frac{\alpha}{1-\alpha}} \equiv \bar{p}. \tag{20}$$

The wage rate per unit of white-collar labor, w_t^W, also equals the value of its marginal product. According to (11), (12) and (20), we have

$$w_t^W = \bar{p}\chi(A_t)^\rho \left(\frac{X_t}{H_t^X}\right)^{1-\rho}. \tag{21}$$

Analogously, the wage rate per unit of blue-collar labor, w_t^B, reads as

$$w_t^B = \bar{p}(1-\chi)(A_t)^\rho \left(\frac{X_t}{L_t^X}\right)^{1-\rho}. \tag{22}$$

The total white-collar human capital level employed in period t, H_t, is given by the sum of the individual white-collar human capital levels of all individuals below retirement age (and alive in period t), i.e. [10]

$$H_t = \theta \cdot \sum_{v=t-Z_v^W+1}^{t} S_{v,t}^W a_{v,t}^W \ell_{v,t}^W. \tag{23}$$

Analogously, the total labor supply of blue-collar labor is given by

$$L_t = (1-\theta) \cdot \sum_{v=t-Z_v^B+1}^{t} S_{v,t}^B a_{v,t}^B \ell_{v,t}^B. \tag{24}$$

[10] Equation (23) can be understood as follows. An individual from cohort v and occupational group j supplies $a_{v,t}^j \ell_{v,t}^j$ efficiency units of occupation-specific labor in period t. Since a mass θ of white-collar workers enters the labor market each period, there are $\theta S_{v,t}^W$ of such workers left in period t.

A.1.2 Government Budget Constraints

Total labor income from the two types of labor is $w_t^B L_t + w_t^W H_t$. As the income tax revenue (at marginal tax rate τ_t^w) is used solely for redistributive purposes, a balanced budget requires that the earned income tax credit is a lump-sum payment. That is,

$$I_{v,t}^j = \bar{I}_t \equiv \frac{\tau_t^w \cdot \left(w_t^B L_t + w_t^W H_t\right)}{\theta \cdot \sum_{v=t-Z_v^W+1}^{t} S_{v,t}^W + (1-\theta) \cdot \sum_{v=t-Z_v^B+1}^{t} S_{v,t}^B} \text{ for all } j,\ t \in [v, v+Z_v^j - 1], \tag{25}$$

where the denominator is the total number of workers alive in t. A pure PAYG pension system is characterized by a balanced budget as well, implying

$$\tau_t^p \cdot \left(w_t^B L_t + w_t^W H_t\right) = \theta \cdot \sum_{v=t-T+1}^{t-Z_v^W} S_{v,t}^W I_{v,t}^W + (1-\theta) \cdot \sum_{v=t-T+1}^{t-Z_v^B} S_{v,t}^B I_{v,t}^B, \tag{26}$$

where the left-hand side of (26) equals aggregate contributions in period t and the right-hand side equals pension benefit payments for all retirees being alive in t.

A.1.3 Individual Optimization

According to (5)-(9), the intertemporal optimization problem of an individual from cohort v and occupation-group j conditional on the number of periods retiring early is given by

$$\max_{\{c_{v,t},\ell_{v,t},P_{v,t+1},k_{v,t+1}\}_{t=v}^{v+T-1}} \sum_{t=v}^{v+T-1} S_{v,t}^j \beta^{t-v} \left[\frac{(c_{v,t})^{1-\sigma} - 1}{1-\sigma} - D(d_{v,t}^j)\frac{(\ell_{v,t})^{1+1/\eta}}{1+1/\eta} + \bar{u}\right] \text{ s.t.} \tag{27}$$

$$P_{v,t+1} = (1+\tilde{r})P_{v,t} + \mathbf{1}_t(Z_v^j)\tau^p W_{v,t}^j \ell_{v,t}, \tag{28}$$

$$k_{v,t+1} - k_{v,t} = r_{v,t}^j k_{v,t} + (1 - \tau_{v,t}^j - \tau_t^p) W_{v,t}^j \ell_{v,t} + \tilde{I}(P_{v,t}) - c_{v,t}, \tag{29}$$

$\ell_{v,t} \geq 0$ for all $t \in \{v, ..., v+T-1\}$,[11] and boundary conditions $k_{v,v} = 0$ and $k_{v,v+T} \geq 0$. $\mathbf{1}_t(Z_v^j)$ is an indicator function that takes the value of one for $v \leq t \leq v + Z_v^j - 1$ (pre-retirement), where pension contributions are possible, and zero otherwise (post-retirement).

[11]The non-negativity constraints for labor supply are explicitly stated because of the possibility of corner solutions (in contrast to consumption).

Using $S^j_{v,v} = 1$, survival rates and mortality rates are related according to

$$S^j_{v,t} = \prod\nolimits_{u=v}^{t-1} (1 - m^j_{v,u}). \tag{30}$$

The hourly wage rate net of taxes and contributions reads as

$$\tilde{W}^j_{v,t} = \begin{cases} (1 - \tau^w - \tau^p) W^j_{v,t},\ v \le t \le v + Z^j_v - 1, \\ 0 \text{ otherwise}, \end{cases} \tag{31}$$

according to (10). Using (30) and (31) in (27), the Lagrangian $\mathcal{L}^j_v$ for the optimization problem of an individual from cohort v and occupation-group j can be written as

$$\begin{aligned} \mathcal{L}^j_v \;=\; & ... + \beta^{t-v} S^j_{v,t} \left[\frac{(c_{v,t})^{1-\sigma} - 1}{1-\sigma} - D(d^j_{v,t}) \frac{(\ell_{v,t})^{1+1/\eta}}{1 + 1/\eta} \right] + \\ & \beta^{t+1-v} S^j_{v,t+1} \left[\frac{(c_{v,t+1})^{1-\sigma} - 1}{1-\sigma} - D(d^j_{v,t+1}) \frac{(\ell_{v,t+1})^{1+1/\eta}}{1 + 1/\eta} \right] + ... + \\ & \lambda^j_{v,t} \left[(1 + r^j_{v,t}) k_{v,t} + \tilde{W}^j_{v,t} \ell_{v,t} + \tilde{I}(P_{v,t}) - c_{v,t} - k_{v,t+1} \right] + \\ & \lambda^j_{v,t+1} \left[(1 + r^j_{v,t+1}) k_{v,t+1} + \tilde{W}^j_{v,t+1} \ell_{v,t+1} + \tilde{I}(P_{v,t+1}) - c_{v,t+1} - k_{v,t+2} \right] + ... + \\ & \epsilon^j_{v,t} \left[(1 + \tilde{r}) P_{v,t} + \mathbf{1}_t (Z^j_v) \tau^p W^j_{v,t} \ell^j_{v,t} - P_{v,t+1} \right] + ..., \end{aligned} \tag{32}$$

where $\{\lambda^j_{v,t}\}_{t=v}^{v+T-1}$ denotes the sequence of shadow prices for capital holdings and $\{\epsilon^j_{v,t}\}$ is the sequence of shadow prices for the stock of pension contributions. At the end of life, either the pension stock or the shadow price of the pension stock must be equal to zero. Since the pension stock is always greater than zero, transversality condition $\epsilon^j_{v,v+T-1} = 0$ must hold. Moreover, as there is no bequest motive, capital holdings after death must be zero, $k^j_{v,v+T} = 0$.

The first-order conditions $\partial \mathcal{L}^j_v / \partial c_{v,t} = \partial \mathcal{L}^j_v / \partial c_{v,t+1} = \partial \mathcal{L}^j_v / \partial k_{v,t+1} = \partial \mathcal{L}^j_v / \partial P_{v,t+1} = \partial \mathcal{L}^j_v / \partial \ell_{v,t} = 0$ can be written as

$$\beta^{t-v} (c^j_{v,t})^{-\sigma} S^j_{v,t} = \lambda^j_{v,t}, \tag{33}$$

$$\beta^{t+1-v} (c^j_{v,t+1})^{-\sigma} S^j_{v,t+1} = \lambda^j_{v,t+1}, \tag{34}$$

$$\lambda^j_{v,t} = \lambda^j_{v,t+1} (1 + r^j_{v,t+1}), \tag{35}$$

$$(1 + \tilde{r}) \epsilon^j_{v,t+1} - \epsilon^j_{v,t} + \tilde{I}'(P^j_{v,t+1}) \lambda^j_{v,t+1} = 0, \tag{36}$$

$$\beta^{t-v} D(d^j_{v,t}) (\ell^j_{v,t})^{1/\eta} S^j_{v,t} \le \lambda^j_{v,t} \tilde{W}^j_{v,t} + \mathbf{1}_t (Z^j_v) \epsilon^j_{v,t} \tau^p W^j_{v,t}. \tag{37}$$

Evidently, (37) holds with equality for $v \leq t \leq v + Z_v^j - 1$,[12] whereas we get corner solutions $\ell_{v,t}^j = 0$ for $v + Z_v^j \leq t \leq v + T - 1$ (after decleared retirement), according to (31) and the fact that $\mathbf{1}_t = 0$ for $v + Z_v^j \leq t \leq v + T - 1$.

We can rewrite (36) to first-order difference equation $\epsilon_{v,t}^j = (1+\tilde{r})^{-1}(\epsilon_{v,t-1}^j - \tilde{I}'(P_{v,t}^j)\lambda_{v,t}^j)$, with solution

$$\epsilon_{v,t}^j = \frac{\epsilon_{v,v}^j}{(1+\tilde{r})^{t-v}} - \sum_{u=v+1}^{t} \frac{\tilde{I}'(P_{v,u}^j)\lambda_{v,u}^j}{(1+\tilde{r})^{t-u+1}}. \tag{38}$$

To obtain initial value $\epsilon_{v,v}^j$, note that (38) implies

$$\epsilon_{v,v+T-1}^j = \frac{\epsilon_{v,v}^j}{(1+\tilde{r})^{T-1}} - \sum_{u=v+1}^{t} \frac{\tilde{I}'(P_{v,u}^j)\lambda_{v,u}^j}{(1+\tilde{r})^{v+T-u}} \tag{39}$$

and use transversality condition $\epsilon_{v,v+T-1}^j = 0$ to find

$$\epsilon_{v,v}^j = \sum_{u=v+1}^{v+T-1} \frac{\tilde{I}'(P_{v,u}^j)\lambda_{v,u}^j}{(1+\tilde{r})^{v-u+1}}. \tag{40}$$

According to (8), $\tilde{I}'(P_{v,t}^j) = 0$ for $t \in \{v, ..., v + Z_v^j - 1\}$ and $\tilde{I}'(P_{v,t}^j) = s_t^j$ otherwise. Using this and inserting (40) into (38), we obtain

$$\epsilon_{v,t}^j = \begin{cases} \sum\limits_{u=v+Z_v^j}^{v+T-1} \frac{s_u^j \lambda_{v,u}^j}{(1+\tilde{r})^{t+1-u}} \text{ for } v \leq t \leq v + Z_v^j - 1, \\ \sum\limits_{u=t+1}^{v+T-1} \frac{s_u^j \lambda_{v,u}^j}{(1+\tilde{r})^{t+1-u}} \text{ for } v + Z_v^j \leq t \leq v + T - 1, \end{cases} \tag{41}$$

where $\lambda_{v,u}^j = \beta^{u-v}(c_{v,u}^j)^{-\sigma} S_{v,u}^j$, according to (33).

Combining (33)–(35) and using (30) leads to

$$\left(\frac{c_{v,t+1}^j}{c_{v,t}^j}\right)^\sigma = \beta(1 - m_{v,t}^j)(1 + r_{v,t+1}^j). \tag{42}$$

[12] In this case, (37) equates the marginal disutility of labor with the marginal labor income (converted into utils through shadow price λ) plus marginal pension contributions (converted into utils through shadow price ϵ) from working an additional hour.

Using $1+r^j_{v,t+1} = \frac{1+\bar{r}}{1-m^j_{v,t}}$ from (6) in (42) implies

$$c^j_{v,t+1} = \left[\beta(1+\bar{r})\right]^{\frac{1}{\sigma}} c^j_{v,t}. \tag{43}$$

By iterating we obtain for $t \geq v$ that

$$c^j_{v,t} = \left[\beta(1+\bar{r})\right]^{\frac{t-v}{\sigma}} c^j_{v,v}. \tag{44}$$

According to (29), $k_{v,v} = 0$ and $k_{v,v+T} = 0$, we find that the intertemporal budget constraint of a member of cohort v is given by

$$c^j_{v,v} + \sum_{t=v+1}^{v+T-1} \left(\frac{c^j_{v,t}}{\prod_{u=v+1}^{t}(1+r^j_{v,u})} \right) = \tilde{W}^j_{v,v}\ell^j_{v,v} + I^j_{v,v} + \sum_{t=v+1}^{v+T-1} \left(\frac{\tilde{W}^j_{v,t}\ell^j_{v,t} + I^j_{v,t}}{\prod_{u=v+1}^{t}(1+r^j_{v,u})} \right), \tag{45}$$

where $I^j_{v,t} = \tilde{I}(P^j_{v,t})$. Substituting $1+r^j_{v,u} = \frac{1+\bar{r}}{1-m^j_{v,u-1}}$ from (6) and (44) into the left-hand side of (45), we find

$$\begin{aligned} c^j_{v,v} + \sum_{t=v+1}^{v+T-1} \left(\frac{c^j_{v,t}}{\prod_{u=v+1}^{t}(1+r^j_{v,u})} \right) &= c^j_{v,v} + \sum_{t=v+1}^{v+T-1} \left(\frac{\left[\beta(1+\bar{r})\right]^{\frac{t-v}{\sigma}} c^j_{v,v} \prod_{u=v+1}^{t}(1-m^j_{v,u-1})}{(1+\bar{r})^{t-v}} \right) \\ &= c^j_{v,v} \left(1 + \sum_{t=v+1}^{v+T-1} \left(\frac{\beta}{(1+\bar{r})^{\sigma-1}} \right)^{\frac{t-v}{\sigma}} S^j_{v,t} \right), \end{aligned} \tag{46}$$

where we used (30) for the latter equation. Using (6) and (30), we also obtain

$$\sum_{t=v+1}^{v+T-1} \left(\frac{\tilde{W}^j_{v,t}\ell^j_{v,t} + I^j_{v,t}}{\prod_{u=v+1}^{t}(1+r^j_{v,u})} \right) = \sum_{t=v+1}^{v+T-1} S^j_{v,t} \frac{\tilde{W}^j_{v,t}\ell^j_{v,t} + I^j_{v,t}}{(1+\bar{r})^{t-v}}. \tag{47}$$

Recall that $I^j_{v,t} = \bar{I}_t$ for $v \leq t \leq v + Z^j_v - 1$, according to (8), and $I^j_{v,t} = s^j_t P^j_{v,t}$ otherwise, according to (15). Also recall that $\ell^j_{v,t} = 0$ for all $t \geq v + Z^j_v$. Substituting (46) and (47) in (45) and using (31) thus implies that the optimal initial consumption level, $c^j_{v,v}$, is given by

$$c^j_{v,v} = \frac{(1-\tau^w-\tau^p)W^j_{v,v}\ell^j_{v,v} + \bar{I}_v + \sum_{t=v+1}^{v+Z^j_v-1} S^j_{v,t} \frac{(1-\tau^w-\tau^p)W^j_{v,t}\ell^j_{v,t}+\bar{I}_t}{(1+\bar{r})^{t-v}} + \sum_{t=v+Z^j_v}^{v+T-1} S^j_{v,t} \frac{s^j_t P^j_{v,t}}{(1+\bar{r})^{t-v}}}{1 + \sum_{t=v+1}^{v+T-1} \left(\frac{\beta}{(1+\bar{r})^{\sigma-1}} \right)^{\frac{t-v}{\sigma}} S^j_{v,t}}. \tag{48}$$

$c^j_{v,v}$ as given in (48) gives us the consumption path by using (44) for a given labor supply path, a given path of income from public sources, and a given path of wage rates.

We finish the equilibrium analysis with the labor supply path. Using (31) in (37), we find that, for all $t \in \{v, ..., v + Z_v^j - 1\}$,

$$\ell_{v,t}^j = \left(\frac{\lambda_{v,t}^j (1 - \tau^w - \tau^p) + \epsilon_{v,t}^j \tau^p}{\beta^{t-v} D(d_{v,t}^j) S_{v,t}^j} W_{v,t}^j \right)^\eta, \tag{49}$$

with $\lambda_{v,t}^j = \beta^{t-v} (c_{v,t}^j)^{-\sigma} S_{v,t}^j$ and $\epsilon_{v,t}^j = \sum_{u=v+Z_v^j}^{v+T-1} \frac{s_u^j \beta^{u-v} (c_{v,u}^j)^{-\sigma} S_{v,u}^j}{(1+\tilde{r})^{t+1-u}}$ as given by (33) and (41).

A.2 Proof of Proposition 1

Using (13) in (16) and substituting both (15) and $s_t^B = \xi_v s_t^W$ confirms (17) by solving for ξ_v. To derive (18), insert both $I_{v,t}^W = s_t^W C_v^W$ and $I_{v,t}^B = \xi_v s_t^W C_v^B$ into (26) and use the expression for ξ_v in (17). This concludes the proof.

A.3 Welfare Comparisons

To enable welfare comparisions, we compute group-specific consumption equivalents with respect to alternative policy scenarios, i.e. compute the factor by which consumption in the benchmark scenario is multiplied each period such that an individual experiences the same utility under the benchmark policy $s_t^L = s_t^H$ and the considered policy alternative (*equivalent variation*).

Formally let superscripts "0" and "1" endogenous variables indicate the values in the benchmark and alternative policy scenario, respectively, and define

$$\tilde{U}_v^{j,k}(\phi) := \sum_{t=v}^{v+T-1} S_{v,t}^{j,k} \beta^{t-v} \left[\frac{(\phi \cdot c_{v,t}^{j,k})^{1-\sigma} - 1}{1 - \sigma} - D(d_{v,t}^{j,k}) \frac{(\ell_{v,t}^{j,k})^{1+1/\eta}}{1 + 1/\eta} + \bar{u} \right] \tag{50}$$

as the intertemporal utility of any individual from cohort v and occupational group j under pension policy $k \in \{0, 1\}$ when the optimally chosen consumption levels are multiplied by factor ϕ. The equivalent variation measure ϕ_v^j is then implicitly defined as

$$\tilde{U}_v^{j,0}(\phi) = \tilde{U}_v^{j,1}(1). \tag{51}$$

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