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Abstract

We analyse the optimal design of unilateral climate policy in an open economy where the government is committed to a target for reduction of domestic CO2 emissions but where it is also concerned about carbon leakage. We highlight the importance of distinguishing between leakage at the extensive margin where firms relocate to a foreign country to avoid the domestic carbon tax, and leakage at the intensive margin where domestic firms lose world market shares to foreign competitors due to the tax. Assuming that the government cannot implement border carbon adjustments, we show that the optimal allocation can still be implemented through a combination of taxes on emissions, taxes on domestic consumption of energy and final goods, an output subsidy as well as a lump-sum location subsidy to leakage-exposed firms, subsidies to carbon capture, taxes on domestic production of fossil fuels, and a subsidy to domestic production of green energy. Simulation experiments indicate that the social welfare gain from implementing the optimal leakage-adjusted tax-subsidy scheme rather than a single uniform emissions tax could amount to 0.5 percent of national income. A location subsidy aimed at reducing leakage at the extensive margin contributes to reducing the welfare loss from leakage.

JEL-Codes: H210, H230, Q480, Q540.

Keywords: carbon leakage, optimal carbon taxation in an open economy.

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July 2021

OPTIMAL UNILATERAL CLIMATE POLICY WITH CARBON LEAKAGE AT THE EXTENSIVE AND THE INTENSIVE MARGIN

Peter Kjær Kruse-Andersen and Peter Birch Sørensen[1]

1. A diabolic problem in a world of uncoordinated climate policy: carbon leakage

Hoping to lead the world community by example, several European countries have adopted ambitious targets for reduction of greenhouse gas emissions that go well beyond their obligations towards the European Union, and some countries outside the EU and Europe have also unilaterally tightened their targets for emissions reduction in recent years. As Hoel (2012) and Greaker et al. (2019) explain, even a small country may affect the climate policies of other countries in a number of ways, e.g., via demonstration effects and development of green technologies. However, frontrunners in climate policy face the problem that domestic emissions may simply leak abroad to countries that are not committed to binding targets for emissions reduction. Although the rate of carbon leakage is usually estimated to be less than 100 percent,[2] it can still nullify a large part of the global impact of the climate policy effort of frontrunner countries.

To prevent carbon leakage, countries with ambitious climate policies could introduce a system of border carbon adjustments by imposing a tax on the estimated carbon content of imported goods and offering a rebate for (part of) the domestic carbon tax on the production of exported goods. Under such a system, analyzed by Hoel (1996), Böhringer et al. (2012), and Fischer and Fox (2012), among others, the international competitiveness of domestic producers could in principle be preserved. A related anti-leakage policy is to levy a carbon tariff on imported goods in proportion to their estimated carbon content, an idea recently promoted in the "European Green Deal" proposed by the European Commission (2019).

However, unless carefully designed, a system of border carbon adjustments may be challenged under WTO rules, as explained by Cosbey et al. (2019). At any rate, it involves the risk of triggering a trade war. Some authors have therefore analysed optimal unilateral climate policies when this policy instrument cannot be employed. Hoel (1996) finds that if carbon taxes are the only available instrument, the government should differentiate carbon tax rates across sectors to mitigate leakage. Kruse-Andersen and Sørensen (2019) show that the optimal climate policy for a small open economy wishing to reduce its contribution to global emissions without using tariffs or export subsidies involves a variety of instruments including a differentiated carbon tax,

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[2] See the survey by Carbone and Rivers (2017).

subsidies to renewable electricity production, and consumption taxes on internationally traded goods.

Related approaches to reducing leakage include so-called output-based allocation (OBA) where tradable CO_2 emission allowances are allocated freely to firms or where firms receive a tax refund in proportion to their output levels (Böhringer and Lange (2005); Neuhoff et al. (2016), Batini et al. (2020)); expenditure-based refunding where firms are given a tax refund in proportion to their expenditure on abatement equipment (Hagem et al. (2020)), and differentiated consumption taxes on goods produced by emission-intensive and trade-exposed industries (Böhringer et al. (2017); Kaushal and Rosendahl (2020)).

The analysis in this paper assumes that the country considered cannot unilaterally introduce a system of border carbon adjustment (BCA) but may otherwise adopt all of the policy instruments just mentioned. We describe a set of climate policy instruments that allows the government to implement the first-best allocation characterized by Hoel (1996), thus offering an alternative to BCA.

The paper contributes to the literature on carbon leakage in the following ways: 1) We present an integrated general equilibrium analysis of an optimal carbon tax scheme and an optimal support scheme for green energy that allows a frontrunner country to meet a target for domestic emissions reduction while counteracting carbon leakage in a systematic, cost-effective way. 2) We allow for the endogeneity of carbon leakage rates and derive the general equilibrium links between the various leakage rates that the optimal tax and subsidy rates must account for. 3) We allow for carbon leakage via international trade in fossil fuels and final goods as well as leakage at the "extensive" margin where firms may relocate to foreign jurisdictions. 4) We account for abatement of CO_2 emissions via substitution towards green energy sources as well as abatement through technologies for carbon capture. 5) We carry out simulations with a calibrated version of our theoretical model to indicate the likely magnitude of the social welfare loss incurred if domestic climate policy does not account for carbon leakage. Our simulations suggest that this welfare loss could be significant.

To our knowledge, this paper is the first one to distinguish between carbon leakage at the intensive and the extensive margin in an analysis of optimal carbon taxation. At the *intensive* margin emissions leak abroad as domestic firms react to a cost-increasing domestic carbon tax by reducing their output, thereby losing world market shares to competing foreign firms, and as the domestic carbon tax (slightly) reduces the international price of fossil fuel, thus inducing foreign firms and households to increase their fossil fuel use. At the *extensive* margin emissions leak from the domestic to the foreign economy as firms choose to relocate their entire production activity to foreign jurisdictions. As we shall see, leakage at the extensive margin has non-trivial implications for the optimal design of carbon taxation.

There is some evidence that cross-country differences in energy prices do in fact influence business location decisions. Using a sample of listed firms from 9 manufacturing sectors in 24 OECD countries over the period 1995-2008, Garsous et al. (2020) find a statistically significant effect of domestic energy prices on outward FDI, although the effect seems relatively small.

Koch and Mama (2016) and Borghesi et al. (2019) study whether the European Emissions Trading System has induced firms covered by the system to increase investment in plants in countries with weaker or no regulation. Both studies find some effect of the ETS on outward FDI. Ben-David et al. (2021) find that multinational firms relocate some of their carbon-intensive activities to other countries when their original host country tightens its environmental policy. All of these studies provide evidence of leakage at the external margin. Nevertheless, leakage at the extensive margin is absent in standard computable general equilibrium (CGE) models of climate policy. This is problematic, since countering leakage at the intensive and the extensive margins requires different policy instruments, as we shall demonstrate. Our CGE model in section 6 makes up for this shortcoming.

In line with the 2015 Paris Agreement according to which countries are responsible for greenhouse gas emissions from their own territory, our analysis assumes that the home country is committed to a binding target for reducing domestic emissions. The domestic social planner seeks to meet this target at a minimum welfare cost to the representative domestic citizen. This welfare cost includes a welfare loss from carbon leakage, reflecting that citizens do not only care about their country's formal international climate policy obligations, but also about the net impact of domestic policy on the global climate. Thus we focus on a situation where the domestic government wishes to demonstrate to the rest of the world how to meet an ambitious target for reduction of *territorial* emissions, as required by the Paris Agreement, but where the government is also concerned about carbon leakage. One real-world example illustrating the relevance of this assumption is the Danish Climate Act of 2020 which commits the Danish government to reducing domestic greenhouse gas emissions by 70 percent in 2030 relative to the 1990 emission level, but to do so in a way which ensures that "..Danish policy measures do not simply move all of the emissions beyond Danish borders", as stated in the preamble of the Act.

Our analysis takes a broad view of carbon leakage, considering changes in foreign emissions caused by policy-induced changes in domestic consumption of final goods as well as changes in foreign emissions generated by efforts to reduce domestic consumption and production of fossil fuel. This comprehensive definition of leakage seems logical if the policy concern about leakage is motivated by a concern about the effects of domestic policy on the global climate.

We describe a carbon tax system and an accompanying support scheme for green energy that allows a front-runner country to meet its target for domestic emissions reduction at the minimum possible social welfare cost, when social welfare is negatively affected by losses of consumer welfare and by carbon leakage. The optimal tax-subsidy scheme includes the following elements: i) A uniform tax on CO_2 emissions from all emitters who do not engage in carbon capture. ii) A reduced emission tax rate on emitters who undertake carbon capture. iii) A subsidy to investment in carbon capture. iv) An output subsidy combined with a lump-sum "location subsidy" to firms in leakage-exposed sectors. v) A consumption tax on carbon-intensive traded goods equal to the output subsidy granted to domestic producers of such goods. vi) A subsidy to domestic production of green energy. vii) A uniform tax on all domestic consumption of green energy. viii) A tax on domestic production of fossil fuels. The policy instruments ii) through viii) are all motivated by the desire to reduce carbon leakage. In the special case where the government only focuses on reducing emissions from domestic territory, our model generates the standard policy prescription that all domestic emissions should be subject to a uniform carbon tax in which case the policy instruments ii) through viii) are all redundant.[3]

One practical obstacle to the elaborate tax-subsidy scheme outlined above is that it may be difficult to administer. Policy makers may also lack the information needed to differentiate the tax and subsidy in accordance with differences in carbon leakage rates, and a system with differentiated taxes and subsidies may invite rent-seeking from powerful lobby groups. Because of these risks, it may be hard to justify efforts to implement the theoretically optimal leakage-adjusted tax-subsidy scheme unless it is likely to yield significant welfare gains in a Panglossian world where policy makers have all the necessary information, administrative costs are negligible, and lobbyism is absent. In the last part of our paper we therefore carry out simulations with a quantitative version of our model calibrated with plausible parameter values to investigate the likely magnitude of the social welfare gain attainable by a move from a simple uniform carbon tax to the optimal leakage-adjusted tax-subsidy scheme including all of the instruments i) through viii) mentioned above. Our analysis suggests that the welfare gain from implementing the theoretically optimal domestic climate policy could amount to around 0.5 percent of GDP in the case where policy makers care just as much about foreign as about domestic emissions.

The rest of the paper is organized as follows. Section 2 states our assumptions about tastes and technologies and how we model carbon leakage. Section 3 explains how to derive the socially optimal allocation of resources in the presence of carbon leakage, and section 4 describes the allocation emerging in the market economy. In section 5 we use insights from sections 3 and 4 to derive the set of tax and subsidy rates which will ensure that the market economy generates the socially optimal allocation. Section 6 sets up a calibrated version of our general equilibrium model, and section 7 carries out simulations with the model to estimate the welfare effects of implementing the optimal tax-subsidy scheme rather than sticking to uniform carbon taxation. Section 8 summarizes our main findings, and two technical appendices documents the details of our analysis.

2. Modelling carbon leakage

This section sets up a simple model of the international economy to illustrate the various channels through which carbon leakage can occur. We will use the model to derive exact formulas for the carbon leakage rates which are key to determining the optimal tax and subsidy scheme presented in section 5.

Our model divides the world economy into the domestic and the foreign economy. Each economy includes a household sector and a business sector encompassing a sector producing internationally tradable final goods, a sector producing a final good that is not traded across borders, a sector producing internationally traded fossil fuel, and a sector producing a "green"

[3] Our model abstracts from knowledge spillovers that might justify subsidies to investment in carbon capture and green energy.

energy input which is likewise traded internationally. An example of a green energy input could be wood-based biomass or electricity based on solar, wind or hydro energy, since biomass is traded extensively and electricity is also traded across borders in some parts of the world, e.g., in Northwestern Europe.[4] To account for carbon leakage at the extensive margin, our model allows for the possibility that some domestic firms in the sector for traded final goods may choose to relocate to the foreign economy by incurring a firm-specific mobility cost. Below we describe the model in detail.

2.1. Tastes and technology in the domestic economy

The preferences of the representative domestic household are given by the utility function

$$u = u\left(x^h, y^h, e^h(b^h, g^h)\right), \tag{1}$$

$$u_x \equiv \frac{\partial u}{\partial x^h} > 0,\ \ u_y \equiv \frac{\partial u}{\partial y^h} > 0,\ \ u_e \equiv \frac{\partial u}{\partial e^h} > 0,\ \ e_b^h \equiv \frac{\partial e^h}{\partial b^h} > 0,\ \ e_g^h \equiv \frac{\partial e^h}{\partial g^h} > 0,$$

where x^h and y^h are household consumption of traded and non-traded final goods, respectively, and e^h is household consumption of energy services which are produced with inputs of "black" (fossil) energy b^h and "green" energy g^h. The marginal utilities of all goods are assumed to be positive, but declining, just as the two energy inputs have positive but declining marginal productivities in the production of energy services. The two energy inputs may have a high degree of substitutability, but it may not be perfect. For example, fossil-fueled vehicles and electric vehicles powered by green electricity may not be perfect substitutes in the eyes of consumers. More generally, the intermittency of solar and wind power may require some kind of energy storage or back-up facilities that make this energy source an imperfect substitute for fossil fuel for some purposes.

The representative domestic firm in the sector for traded goods produces a quantity x using the labour input n^x and combining the inputs b^x and g^x of black and green energy to generate the energy services e^x in the production function

$$x = f\left(n^x, e^x(b^x, g^x)\right), \tag{2}$$

$$f_n \equiv \frac{\partial f}{\partial n^x} > 0, \quad f_e \equiv \frac{\partial f}{\partial e^x} > 0, \quad e_b^x \equiv \frac{\partial e^x}{\partial b^x} > 0, \quad e_g^x \equiv \frac{\partial e^x}{\partial g^x} > 0.$$

We assume that the marginal productivities of all inputs are positive, but declining. Behind the scene there is another production factor entering the production function (2) in a fixed amount. This factor could represent a specialized intangible asset that is needed for production and which can be utilized in a similar amount abroad if the firm decides to relocate its activity. Thus, while we assume that the production function $f(n^x, e^x)$ has decreasing returns to scale in the inputs n^x

[4] There is an ongoing debate on the extent to which biomass is a carbon-neutral source of energy. We leave this issue aside here, noting that under official accounting rules, biomass is considered to be a "green" source of energy.

and e^x, there may be constant returns to scale when the third factor is added, in line with the usual replication argument. The mobile third factor is a source of rents and provides an incentive for domestic firms to move abroad if they can earn a higher rent in the foreign economy, as we shall explain in detail later.

The representative domestic firm in the sector for non-traded goods uses the labour input n^y and the black and green energy inputs b^y and g^y to produce a quantity y of non-tradables by means of the following production function with positive but declining marginal products of all inputs:

$$y = y\big(n^y, e^y(b^y, g^y)\big), \tag{3}$$

$$y_n \equiv \frac{\partial y}{\partial n^y} > 0, \quad y_e \equiv \frac{\partial y}{\partial e^y} > 0, \quad e_b^y \equiv \frac{\partial e^y}{\partial b^y} > 0, \quad e_g^y \equiv \frac{\partial e^y}{\partial g^y} > 0.$$

Our theoretical analysis requires that the returns to scale in the production function $y(n^y, e^y)$ are non-increasing; in the numerical version of our model in section 6 we will assume decreasing returns, in parallel to our assumption regarding the tradable goods sector.

By normalization, the burning of one unit of fossil fuel emits one unit of CO_2 in the absence of abatement efforts, but when subject to regulation, firms can choose to install an end-of-pipe carbon-capture technology that prevents (part of) the CO_2 from escaping into the atmosphere. Capturing carbon is costly, requiring the input of some amount of the traded final good to make the abatement technology work. Since some firms have better technological opportunities for carbon capture than others, there are decreasing returns to this input at the sector level, and it may be prohibitively costly to capture all of the carbon released through the burning of fossil fuel. If $a_x(x_x^a)$ and $a_y\big(x_y^a\big)$ are the sector-specific fractions of CO_2 that are *not* captured, and x_x^a and x_y^a are the inputs in the carbon-capture technology in the two sectors, we therefore assume that

$$a_x(x_x^a) \in (0,1], \quad a_x' < 0, \quad a_x'' > 0, \quad a_x(0) = 1, \quad \lim_{x_x^a \to \infty} a_x(x_x^a) = 0, \tag{4a}$$

$$a_y\big(x_y^a\big) \in (0,1], \quad a_y' < 0, \quad a_y'' > 0, \quad a_y(0) = 1, \quad \lim_{x_y^a \to \infty} a_y\big(x_y^a\big) = 0. \tag{4b}$$

Reflecting that energy production is typically very capital-intensive, we simplify the exposition by assuming that the domestic production of black (fossil) energy inputs, b, and the domestic production of green energy inputs, g, only requires the inputs of the amounts x^b and x^g of the final tradable good. Because of natural resource scarcity there are decreasing returns in the provision of energy raw materials, reflected in the production functions

$$b = b(x^b), \quad b' > 0, \quad b'' < 0, \qquad g = g(x^g), \quad g' > 0, \quad g'' < 0. \tag{5}$$

This completes the description of tastes and technologies in the domestic economy. In section 4 we will add a description of domestic markets and market equilibria, but for the moment we assume that the domestic government has sufficient policy instruments to fully control the domestic allocation of resources. Section 4 will specify the set of taxes and subsidies that will enable the government to exercise such control.

2.2. World market equilibrium

The foreign economy is structured in the same way as the domestic economy. Utility and profit maximization implies that the foreign economic variables may be specified as functions of foreign and international prices and wages. We will use the internationally traded final good as numeraire. The relative price of the foreign non-traded final good is P^y, and the foreign wage rate is W. The international relative prices of black and green energy inputs are p^b and p^g, respectively. The domestic government cannot control economic activity abroad, so the use of black and green energy b^{xf} and g^{xf} in a domestically-owned firm operating abroad is determined by profit-maximizing behaviour, yielding energy demand functions $b^{xf}(p^b, p^g, W)$ and $g^{xf}(p^b, p^g, W)$. The number of domestically-owned firms in the tradable goods sector is normalized to 1, and the share of these firms operating in the domestic economy is denoted by s. For later purposes it will be convenient to specify the domestic impact on the world economy in terms of net import functions. We may then describe the equilibrium of the world economy by the following equations (to be explained below):

$$B^s(p^b, p^g, P^y, W) = (1-s)b^{xf}(p^b, p^g, W) + m^b, \qquad m^b = sb^x + b^y + b^h - b(x^b), \tag{6}$$

$$G^s(p^b, p^g, P^y, W) = (1-s)g^{xf}(p^b, p^g, W) + m^g, \qquad m^g = sg^x + g^y + g^h - g(x^g), \tag{7}$$

$$X^s(p^b, p^g, P^y, W) + (1-s)x^f(p^b, p^g, W) = m^x, \tag{8}$$

$$m^x = x^b + x^g + sx_x^a + x_y^a + x^h - sf\big(n^x, e^x(b^x, g^x)\big),$$

$$N^x(p^b, p^g, W) + N^y(p^b, p^g, P^y, W) + (1-s)n^{xf}(p^b, p^g, W) = \bar{\bar{N}}. \tag{9}$$

Eq. (6) is the condition for equilibrium in the world market for fossil fuel. The function $B^s(\circ)$ on the left-hand side of (6) is an excess supply function describing the excess supply of fossil fuel from the foreign to the domestic economy. The term $(1-s)b^{xf}$ on the right-hand side is the fossil fuel demand from domestically-owned firms operating abroad, and m^b is the net import of fossil fuel into the domestic economy, consisting of fossil fuel use in domestic firms and households minus domestic fossil fuel production.

Eq. (7) describes the equilibrium in the world market for green energy inputs, with $G^s(\circ)$ denoting the excess supply of green energy from the foreign to the domestic economy, and the right-hand side of (7) indicating the use of green energy in domestically-owned firms located abroad plus domestic net imports of green energy, m^g.

The international market for tradable final goods clears when condition (8) is met. $X^s(\circ)$ is the excess supply of these goods from foreign-owned firms, and $(1-s)x^f$ is the production of tradables in domestically-owned firms operating in the foreign economy. The right-hand side of (8) is the domestic net import of tradables which are used partly as inputs in domestic production $(x^b + x^g + sx_x^a + x_y^a)$ and partly for final household consumption (x^h). The need for net imports is reduced by the domestic production of tradables, $sf(n^x, e^x)$.

The functions $N^x(\circ)$ and $N^y(\circ)$ indicate the demand for labour from foreign firms in the tradables and non-tradables sector, and $(1-s)n^{xf}(\circ)$ is the labour demanded by domestically-owned firms operating abroad, while $\bar{N}$ is the exogenous total foreign labour supply. Thus eq. (9) states the condition for equilibrium in the foreign labour market.

As mentioned, the domestic government controls the domestic resource allocation given by the variables $s, b^x, b^y, b^h, g^x, g^y, g^h, x^b, x^g, x_x^a, x_y^a, x^h, n^x$. The four equations (6) through (9) then determine the four equilibrium prices p^b, p^g, P^y, W. In the next subsection we shall see how this general equilibrium system can illustrate the various channels for carbon leakage.

2.3. Deriving carbon leakage rates

Let E_0 denote the total CO_{2e} emissions from the foreign economy before the domestic government unilaterally introduces a carbon tax scheme that changes domestic resource allocation. With variables without 0-subscripts indicating the situation *after* the introduction of the domestic carbon tax, the total carbon leakage to the foreign economy can be written as

$$L = B^x(p^b, p^g, W) + B^y(p^b, p^g, P^y, W) + B^h(p^b, p^g, P^y, W)$$
$$+(1-s)b^{xf}(p^b, p^g, W) - E_0, \tag{10}$$

where B^x, B^y, and B^h are the total quantities of fossil fuel used in the foreign business and household sectors (and hence the total emissions from these sectors), and $(1-s)b^{xf}$ is fossil fuel use in (and emissions from) domestically-owned firms located on foreign territory. In our optimal tax analysis we shall need to know the *marginal* leakage rates indicating how changes in domestic economic variables affect the total leakage defined in (10). Eq. (10) makes clear that leakage (positive or negative) occurs either when firms change their location, implying a change in our variable s, or when a change in domestic economic activity induces changes in the international prices p^b, p^g, P^y, W. The general equilibrium system (6) through (9) shows that these prices must change whenever there is a change in s or in domestic net imports m^i, $i = b, g, x$, reflecting that the domestic and the foreign economy are linked via the relocation of firms and via trade in fossil fuel, green energy, and final goods. From (10) it follows that the impact on leakage of a change in net import category m^i is

$$\rho^i \equiv \frac{\partial L}{\partial m^i} = \overbrace{\left[B_{p^b}^x + B_{p^b}^y + B_{p^b}^h + (1-s)b_{p^b}^{xf}\right]\frac{\partial p^b}{\partial m^i}}^{Leakage\ via\ fossil\ fuel\ market} + \overbrace{\left[B_{p^g}^x + B_{p^g}^y + B_{p^g}^h + (1-s)b_{p^g}^{xf}\right]\frac{\partial p^g}{\partial m^i}}^{Leakage\ via\ market\ for\ green\ energy}$$

$$+ \quad \overbrace{\left[B_W^x + B_W^y + B_W^h + (1-s)b_W^{xf}\right]\frac{\partial W}{\partial m^i} + \left(B_{Py}^y + B_{Py}^h\right)\frac{\partial P^y}{\partial m^i}}^{Leakage\ via\ reallocation\ in\ foreign\ markets\ for\ labour\ and\ final\ goods} \quad , \qquad i = b, g, x, \tag{11}$$

where subscripts indicate partial derivatives of the fossil fuel demand functions with respect to the various prices. Eq. (11) illustrates the many possible channels of carbon leakage. The well-known fossil fuel market channel is captured by the term beneath the first curly bracket in (11): a fall in domestic net imports of fossil fuel (m^b) induced by the introduction of a domestic carbon tax will reduce the international fossil fuel price ($\frac{\partial p^b}{\partial m^i} > 0$), thereby stimulating foreign fossil fuel use $(B^x_{p^b} + B^y_{p^b} + B^h_{p^b} + (1-s)b^{xf}_{p^b} < 0)$. But via general equilibrium effects, a change in domestic net import of fossil fuel will also affect the international price of green energy and the foreign prices of labour and non-tradables, leading to further (positive or negative) leakage via the international market for green energy and the foreign product and labour markets, as indicated in (11). Eq. (11) shows that leakage (positive or negative) also occurs as a consequence of changes in domestic net import of green energy (m^g) and in domestic net import of final goods (m^x), so if a domestic carbon tax leads to an increase in m^g (due to substitution away from fossil energy) and an increase in m^x (due to reduced international competitiveness of domestic final goods producers), the resulting marginal leakage effects can be calculated from (11).

Going back to the world market equilibrium conditions (6), (7), and (8), we see that many domestic economic variables affect domestic net imports and thereby international prices in a parallel way. To design an optimal carbon tax scheme that accounts for carbon leakage, the government will need to know the marginal leakage rate associated with a change in each domestic economic variable affecting net imports as well as the leakage associated with a relocation of domestic firms. When stating the links between the various leakage rates, we will use the above notation $\rho^i \equiv \frac{\partial L}{\partial m^i}$, $i = b, g, x$, where ρ^i is given by (11). From the definitions of the three categories of net imports in (6), (7), and (8) and the fact that all variables with the same impact on net imports will also affect international prices and thereby carbon leakage in the same way, we obtain the following links between the marginal leakage rates associated with changes in domestic economic variables:

$$\frac{\partial L}{\partial b^y} = \frac{\partial L}{\partial b^h} = \rho^b, \tag{12}$$

$$\frac{\partial L}{\partial g^y} = \frac{\partial L}{\partial g^h} = \rho^g, \tag{13}$$

$$\frac{\partial L}{\partial x^h} = \frac{\partial L}{\partial x^a_y} = \rho^x, \qquad \frac{\partial L}{\partial x^a_x} = s\rho^x, \tag{14}$$

$$\frac{\partial L}{\partial x^b} = \rho^x - b'\rho^b, \qquad \frac{\partial L}{\partial x^g} = \rho^x - g'\rho^g, \tag{15}$$

$$\frac{\partial L}{\partial b^x} = s(\rho^b - f_e e^x_b \rho^x), \qquad \frac{\partial L}{\partial g^x} = s\big(\rho^g - f_e e^x_g \rho^x\big), \qquad \frac{\partial L}{\partial n^x} = -s f_n \rho^x, \tag{16}$$

$$\rho^s \equiv -\frac{\partial L}{\partial s} = b^{xf} - (x^f + x^a_x - x)\rho^x + (b^{xf} - b^x)\rho^b + (g^{xf} - g^x)\rho^g - n^{xf}\frac{\partial L}{\partial \bar{N}}. \tag{17}$$

A special feature not included in the previous literature on carbon leakage is the *leakage at the extensive margin* described by eq. (17) which measures the total increase in emissions from foreign territory occurring when a domestically-owned firm relocates to the foreign economy. The first term b^{xf} on the right-hand side of (17) is the emission arising directly from the firm's use of fossil fuel on foreign soil. The second term captures the reduction in global emissions arising as the relocation reduces the global excess demand for final goods by the amount $x^f - (x - x^a_x)$, since the relocating firm can attain a higher productivity abroad where there is no regulation. The third term is the fall in global emissions occurring as the additional fossil fuel use $b^{xf} - b^x$ in the relocating firm drives up the fossil fuel price a bit, while the fourth term captures the effect on emissions generated by the change in the price of green energy caused by the change $g^{xf} - g^x$ in the relocating firm's use of that source of energy. The final term on the right-hand side of (17) reflects that a firm moving abroad will need to employ foreign workers which will generate excess demand for foreign labour, thus working like a reduction in foreign labour supply ($\bar{N}$).

Analytically one cannot unambiguously determine the signs of the marginal leakage rates (12) through (17) from the complex general equilibrium system (6) through (10), but intuition suggests the following results which are confirmed by the calibrated version of our model in section 6. $\rho^b < 0$: Higher domestic net import of fossil fuel drives up the fuel price which reduces foreign use of fossil fuel, thereby reducing foreign emissions. $\rho^g > 0$: Higher domestic net import of green energy raises its price which induces foreign firms and households to substitute towards fossil fuel. $\rho^x > 0$: Higher domestic net import of final goods requires higher foreign output of final goods which in turn requires greater foreign use of fossil fuel inputs. $\rho^s > 0$: When a domestically-owned firm moves abroad, its use of fossil fuel on foreign territory raises foreign emissions.

In the following we will analyze how these leakage rates affect the optimal unilateral climate policy.

3. The social planning problem

We assume that the domestic government is concerned about the welfare $u(\circ)$ of the representative domestic citizen but also about carbon leakage. These concerns are reflected in the social welfare function

$$SW = u\Big(x^h, y^h, e^h(b^h, g^h)\Big) - \eta L, \qquad \eta \geq 0, \tag{18}$$

where L is the total carbon leakage specified in (10), and η is the shadow cost of leakage expressed in units of household utility, indicating the social weight assigned to (avoiding) leakage relative to the weight put on the consumer's economic welfare. In the borderline case $\eta = 0$ where there is no concern about leakage, eq. (18) collapses to a standard social welfare function in an economy with homogeneous consumers. The government's concern about emissions from *domestic* territory is captured by the climate policy constraint

$$sa_x(a_x^a)b^x + a_y\big(a_y^a\big)b^y + b^h = \bar{E}. \tag{19}$$

The terms $sa_x(x_x^a)b^x$ and $a_y\big(x_y^a\big)b^y$ are the total emissions from the tradables and the non-tradables sectors, respectively, accounting for the use of carbon capture. According to (19), the government is thus committed to reducing total domestic emissions to the level $\bar{E}$ which is assumed to be lower than the emissions generated by an unregulated domestic market economy.

In addition to (19), the government must respect the technology constraints (2) through (5) and the resource constraints

$$y^h = y\big(n^y, e^y(b^y, g^y)\big), \tag{20}$$

$$sn^x + n^y = \bar{n}, \tag{21}$$

$$sf\big(n^x, e^x(b^x, g^x)\big) - (x^b + x^g + sx_x^a + x_y^a + x^h) + p^b[b(x^b) - sb^x - b^y - b^h]$$
$$+p^g[g(x^g) - sg^x - g^y - g^h] + (1-s)\Pi^f = 0, \tag{22}$$

where Π^f is the net profit earned by a domestically-owned firm operating abroad. Eq. (20) reflects that domestic consumption of non-tradables must equal domestic production of these goods; (21) states that the total use of labour in the domestic economy (the left-hand side) must equal the exogenous domestic labour supply, $\bar{\bar{n}}$, and (22) says that the current account (the sum of net exports of all goods plus the net income $(1-s)\Pi^f$ from abroad) must balance.

We assume that firms that move abroad incur a firm-specific relocation cost which is distributed uniformly within the interval $[0, \bar{c}]$ across domestically-owned firms in the tradable goods sector. A rational social planner will always relocate the firm with the lowest relocation cost before proceeding to relocate any other firm. With a uniform distribution of relocation costs, and a total number $(1-s)$ of firms being relocated, this means that the 'last' (marginal) firm moved abroad will have a relocation cost equal to $\bar{c}(1-s)$, and that the average relocation cost across all firms moved abroad will be half this number, i.e., $\frac{\bar{c}}{2}(1-s)$. The average net profit of a domestic firm that has relocated to the foreign economy may therefore be specified as

$$\Pi^f = \pi^f(p^b, p^g, W) - \frac{\bar{c}}{2}(1-s), \tag{23}$$

$$\pi^f_{p^b} \equiv \frac{\partial \pi^f}{\partial p^b} = -x^{bf}, \qquad \pi^f_{p^g} \equiv \frac{\partial \pi^f}{\partial p^g} = -x^{gf}, \qquad \pi^f_W \equiv \frac{\partial \pi^f}{\partial W} = -n^{xf},$$

where x^{bf}, x^{gf}, and n^{xf} are the firm's foreign inputs of black energy, green energy, and labour, respectively, and the profit function $\pi^f(p^b, p^g, W)$ gives the maximum attainable profit from the use of these inputs at the prevailing factor prices.

When deciding on resource allocation, the domestic social planner is aware that a change in the domestic economic variable i will have the following terms-of-trade effect Δ_i on the current account (CA) of the balance of payments specified in (22), given the properties of the profit function stated in (23):

$$\Delta_i = \frac{\partial CA}{\partial p^b}\frac{\partial p^b}{\partial i} + \frac{\partial CA}{\partial p^g}\frac{\partial p^g}{\partial i} + \frac{\partial CA}{\partial W}\frac{\partial W}{\partial i}, \quad i = s, b^x, b^y, b^h, g^x, g^y, g^h, x^b, x^g, x_x^a, x_y^a, x^h, n^x, \tag{24}$$

$$\frac{\partial CA}{\partial p^b} = b - sb^x - (1-s)b^{xf} - b^y - b^h, \qquad \frac{\partial CA}{\partial p^g} = g - sg^x - (1-s)g^{xf} - g^y - g^h,$$

$$\frac{\partial CA}{\partial W} = -(1-s)n^{xf}.$$

With these preliminaries, we can state the social planning problem: The social planner must choose the domestic economic variables $s, b^x, b^y, b^h, g^x, g^y, g^h, x^b, x^g, x_x^a, x_y^a, x^h, n^x$ so as to maximize the social welfare function (18) subject to the technology constraints (2) through (5), the resource constraints (20) through (22), and the climate policy constraint (19), taking account of the leakage effects (12) through (17) and the terms-of-trade effects (24).[5] The first-order conditions for the solution to this problem are derived in Appendix A. The economic intuition behind these optimum conditions will become clear when we have used them to characterize the optimal tax-subsidy scheme that will internalize the various climate and leakage effects of a change in resource allocation. For this purpose we will now consider the allocation of resources in the market economy.

4. Resource allocation in the market economy

To implement the optimal resource allocation in the market economy the domestic government uses a number of tax and subsidy instruments and recycles the net revenue via a lump sum transfer T^h to the household sector. The government budget constraint is

$$T^h = \tau^{xh}x^h + \tau^{yh}y^h + \tau^{bh}b^h + \tau^{gh}g^h + s\left(\tau^{bx}a_x b^x + \tau^{gx}g^x - \tau^{ax}x_x^a\right)$$
$$+\tau^{by}a_y b^y + \tau^{gy}g^y - \tau^{ay}x_y^a + \tau^b b(x^b) - \tau^g g(x^g) - s(\tau^s x + T^x), \tag{25}$$

where τ^{ih} is a tax on household consumption item i, τ^{jx} and τ^{jy} are taxes on or subsidies to the use of input j in the production of final goods, τ^{ax} and τ^{ay} are subsidies to carbon capture, τ^b is a tax on fossil fuel production, τ^g is a subsidy to green energy production, and τ^s and T^x are, respectively, an output subsidy and a lump sum subsidy granted to tradable goods firms operating on domestic territory.

The representative consumer in the market economy maximizes the utility function (1) with respect to the consumption variables x^h, y^h, b^h and g^h subject to the budget constraint

$$(1+\tau^{xh})x^h + (p^y+\tau^{yh})y^h + (p^b+\tau^{bh})b^h + (p^g+\tau^{gh})g^h = w\bar{n} + \Pi + T^h, \tag{26}$$

where w is the domestic wage rate and Π is total household profit income. The household takes its total income as given, and the marginal utility of income (the Lagrange multiplier associated with the budget constraint (26)) is denoted by λ. We then get the following first-order conditions for utility maximization, stating that the consumer's marginal willingness to pay for the various goods must equal their tax-inclusive prices:

$$\text{Choice of } x^h: \qquad \frac{u_x}{\lambda} = 1 + \tau^{xh} \tag{27}$$

[5] Because several domestic economic variables affect net imports in a parallel way, the world market equilibrium conditions (6) through (9) imply a number of links between the terms-of-trade effects. These links are stated in Appendix A and are exploited in the derivation of the solution to the social planning problem.

$$\text{Choice of } y^h: \quad \frac{u_y}{\lambda} = p^y + \tau^{yh} \tag{28}$$

$$\text{Choice of } b^h: \quad \frac{u_e e_b^h}{\lambda} = p^b + \tau^{bh} \tag{29}$$

$$\text{Choice of } g^h: \quad \frac{u_e e_g^h}{\lambda} = p^g + \tau^{gh} \tag{30}$$

Turning to the business sector, we can use the production function (2) to write the total net profit Π^x of the representative domestically-owned firm in the tradable-goods sector in the following way:

$$\begin{aligned}\Pi^x = {} & s\{(1+\tau^x)f\big(n^x, e^x(b^x, g^x)\big) - wn^x - [p^b + \tau^{bx}a_x(x_x^a)]b^x \\ & - (p^g + \tau^{gx})g^x - (1-\tau^{ax})x_x^a + T^x\} \\ & + (1-s)\left[f\left(n^{xf}, e^x(b^{xf}, g^{xf})\right) - Wn^{xf} - p^b b^{xf} - p^g g^{xf} - \tfrac{\bar{\bar{c}}}{2}(1-s)\right]\end{aligned} \tag{31}$$

The expression in the curly bracket in (31) is the net profit from domestic operations, and the square bracket is the profit from a domestically-owned firm operating abroad, net of the mobility cost $\frac{\bar{\bar{c}}}{2}(1-s)$ of moving business abroad. The profit-maximizing choices of the inputs $n^{xf},\ b^{xf}$ and g^{xf} into foreign operations lead to the profit function (23), and the first-order conditions for the profit-maximizing use of inputs in domestic operations take the following form:

$$\text{Choice of } n^x: \quad (1+\tau^x)f_n = w \tag{32}$$

$$\text{Choice of } b^x: \quad (1+\tau^x)f_e e_b^x = p^b + \tau^{bx}a_x \tag{33}$$

$$\text{Choice of } g^x: \quad (1+\tau^x)f_e e_g^x = p^g + \tau^{gx} \tag{34}$$

$$\text{Choice of } x_x^a: \quad -\tau^{bx}a_x' b^x = 1 - \tau^{ax} \tag{35}$$

Eqs. (32) through (34) are conditions of the form *marginal revenue product = factor price* adjusted for taxes and subsidies, and eq. (35) says that the marginal benefit from abatement effort (the reduction in the emission tax bill on the left-hand side) should equal the marginal subsidy-adjusted cost of abatement (the right-hand side).

Defining $\pi^f \equiv x^f - Wn^{xf} - p^b b^{xf} - p^g g^{xf}$, we can also use (31) to obtain the first-order condition for the location of activities that will maximize the global net profit of the representative tradable goods firm:

Location choice (s):

$$\bar{c}(1-s) = \pi^f - [(1+\tau^x)x - wn^x - (p^b + \tau^{bx}a_x)b^x - (p^g + \tau^{gx})g^x - (1-\tau^{ax})x_x^a + T^x] \tag{36}$$

The left-hand side of (36) is the marginal mobility cost of relocating business activity from the domestic to the foreign economy, and the right-hand side is the marginal net profit gain from relocation, consisting of the difference between the profit π^f from foreign operations (once the mobility cost has been incurred), and the after-tax profit from domestic operations (the square bracket).

Given the production function (3), the profit π^y of the representative firm in the domestic sector for non-tradables is

$$\pi^y = p^y y\big(n^y, e^y(b^y, g^y)\big) - wn^y - \big[p^b + \tau^{by} a_y\big(x_y^a\big)\big]b^y - (p^g + \tau^{gy})g^y - x_y^a(1 - \tau^{ay}) \tag{37}$$

from which we obtain the following first-order conditions for profit maximization, analogous to conditions (32) through (35), except that the government does not grant any output subsidy to the sheltered sector for non-tradables:

$$\text{Choice of } n^y: \qquad p_y y_n = w \tag{38}$$

$$\text{Choice of } b^y: \qquad p^y y_e e_b^y = p^b + \tau^{by} a_y \tag{39}$$

$$\text{Choice of } g^y: \qquad p^y y_e e_g^y = p^g + \tau^{gy} \tag{40}$$

$$\text{Choice of } x_y^a: \qquad -\tau^{by} a_y' b^y = 1 - \tau^{ay} \tag{41}$$

The domestic energy-producing firms use the production technologies in (5) to generate the profits

$$\pi^b = (p^b - \tau^b)b(x^b) - x^b, \qquad \pi^g = (p^g + \tau^g)g(x^g) - x^g, \tag{42}$$

from which it follows that the profit-maximizing input choices are

$$\text{Choice of } x^b: \qquad (p^b - \tau^b)b' = 1 \tag{43}$$

$$\text{Choice of } x^g: \qquad (p^g + \tau^g)g' = 1 \tag{44}$$

This completes the description of resource allocation in the market economy. The next section presents the tax and subsidy rates which will ensure that the market economy generates the socially optimal allocation.

5. The optimal carbon tax and subsidy scheme

The optimal tax and subsidy scheme ensures that the first-order conditions for privately optimal behaviour coincide with the first-order conditions for the socially optimal allocation. The formulas for the optimal tax and subsidy rates are derived in Appendix A. The formulas include the shadow price τ indicating the Domestic Social Cost of Carbon (DSCC), defined as the marginal social cost of meeting the target (19) for CO_2 emissions from domestic territory, measured in units of the numeraire traded good. The formulas also include the marginal shadow cost of carbon leakage which we express as a fraction α of the DSCC,[6] where α measures the

[6] The marginal social cost of leakage is given by the parameter η in the social welfare function (18) divided by the Lagrange multiplier associated with the resource constraint (22) to convert it into units of the numeraire traded good.

marginal social gain from a cut in foreign emissions relative to the marginal social gain from a cut in domestic emissions. In the borderline case where $\alpha = 1$, the domestic government is solely concerned with reducing global emissions, whereas $\alpha = 0$ means that the government only focuses on cutting domestic emissions. Furthermore, our formulas for the optimal tax and subsidy rates include the marginal leakage rates specified in (12) through (17) and the terms-of-trade effects defined in (24).

To illustrate the role of the leakage rates and the terms-of-trade effects, we may consider the formulas for the optimal output subsidy to firms in the tradable goods sector and for the optimal tax on fossil fuel use in that sector:

$$\text{Optimal output subsidy:} \qquad \tau^x = \alpha\tau\rho^x - \Delta_x \tag{45}$$

$$\text{Optimal emission tax:} \qquad \tau^{bx} = \tau + \frac{\alpha\tau\rho^b}{a_x} - \frac{\Delta_b}{a_x} \tag{46}$$

The term $\alpha\tau\rho^x$ in (45) reflects that a unit increase in domestic output of tradable final goods and the concomitant drop in net imports crowds out some foreign output of tradables which reduces foreign emissions by the amount ρ^x. This generates a domestic social benefit $\alpha\tau\rho^x$ which justifies a corresponding subsidy to domestic output of tradables, ceteris paribus. Further, recall that Δ_x measures the terms-of-trade effect of a unit increase in the net import of tradable final goods. If the domestic economy is a net importer of these goods, we have $\Delta_x < 0$, since a rise in net imports will tend to raise the relative price of tradables, thereby eroding domestic real income. By reducing net imports, an increase in domestic output of tradables will then improve the country's terms of trade, thus justifying a further addition to the domestic output subsidy, as indicated in (45). By contrast, if the domestic economy is already a net exporter of traded final goods, we have $\Delta_x > 0$, in which case the output subsidy should be reduced to reflect that the terms of trade deteriorate when domestic net exports increase.

The optimal emission tax on CO_2 emissions from the tradable goods sector is given by (46) where the term τ captures the marginal social cost of a unit increase in emissions from domestic territory. The term $\frac{\alpha\tau\rho^b}{a_x}$ reflects that the emission tax should be reduced to the extent that domestic use of fossil fuel reduces emissions abroad. For example, suppose a tradable goods firm has installed a technology that allows it to capture 90 percent of the carbon released by the burning of a tonne of fossil fuel, implying that $\alpha_x = 0.1$. The firm then uses 10 tonnes of fossil fuel for every tonne of CO_2 actually emitted, so the resulting reduction in foreign emissions is $\left|\frac{\rho_b}{a_x} = 10 \cdot \rho_b\right|$, assuming that the unregulated foreign emitters do not capture their carbon and that $\rho_b < 0$, as argued in section 2. The domestic social benefit from this fall in foreign emissions is $\left|\frac{\alpha\tau\rho^b}{a_x}\right|$ which motivates a corresponding reduction of the domestic emission tax rate. On the other hand, the increase $\frac{1}{a_x}$ in domestic fossil fuel use that generates a unit increase in

At the optimal allocation this Lagrange multiplier equals the consumer's marginal utility of income, λ, so $\alpha \equiv \frac{\eta}{\lambda}$. See Appendix A for details.

emissions increases the net import of fossil fuel by a corresponding amount which drives up the world market price of fossil fuel, thereby worsening the terms of trade when the domestic economy is a net importer of fossil fuel (in which case $\Delta_b < 0$). As stated in (46), this unfavourable terms-of-trade effect justifies an addition $\left|\frac{\Delta_b}{a_x}\right|$ to the emissions tax rate, or a further tax reduction of this magnitude in the case $\Delta_b > 0$ where the domestic economy is a net exporter of fossil fuel.

Appendix A provides the complete list of formulas for the optimal values of the tax and subsidy instruments in our model, accounting for terms-of-trade effects as well as leakage effects. However, since policies aimed at improving the terms of trade are beggar-thy-neighbour policies, it does not seem logical that a government which worries about carbon leakage - presumably because it is concerned about *global* welfare - would want to systematically manipulate the terms of trade in its favour. Moreover, as shown in Appendix B, the terms-of-trade effects of domestic policies in a small open economy are likely to be quite small relative to the social welfare effects of the leakage generated by these policies, despite the fact that both types of effects are generated by (small) changes in international prices. Against this background, the analysis below will abstract from the impact of terms-of-trade changes on the optimal tax and subsidy rates and will focus only on the impact of leakage effects which are likely to be much more important for the optimal climate policy in a relatively small economy.

Near-optimal taxation neglecting terms-of-trade effects

The formulas (A.34) through (A.47) for the optimal tax-subsidy scheme stated in Appendix A include the terms-of-trade effects discussed above. Setting these effects to zero, we obtain the following formulas for the optimal tax and subsidy rates which will be "near-optimal" in a small open economy (see the interpretation below):

Emission tax rates

$$\tau^{bh} = \tau(1 + \alpha\rho^b), \qquad \tau^{bx} = \tau\left(1 + \frac{\alpha\rho^b}{a_x}\right), \qquad \tau^{by} = \tau\left(1 + \frac{\alpha\rho^b}{a_y}\right), \tag{50}$$

Tax treatment of tradable goods

$$\tau^x = \alpha\tau\rho^x, \qquad \tau^{xh} = \alpha\tau\rho^x, \tag{51}$$

Subsidies to carbon capture

$$\tau^{ax} = 1 - \frac{\tau^{bx}}{\tau}(1 + \alpha\tau\rho^x), \qquad \tau^{ay} = 1 - \frac{\tau^{by}}{\tau}(1 + \alpha\tau\rho^x), \tag{52}$$

Taxes on consumption of green energy

$$\tau^{gh} = \alpha\tau\rho^g, \qquad \tau^{gx} = \alpha\tau\rho^g, \qquad \tau^{gy} = \alpha\tau\rho^g, \tag{53}$$

Taxes on and subsidies to production of energy

$$\tau^b = \alpha\tau\left(\frac{\rho^x}{b'} - \rho^b\right), \qquad \tau^g = \alpha\tau\left(\rho^g - \frac{\rho^x}{g'}\right), \tag{54}$$

Location subsidy

$$T^x = \alpha\tau\rho^s - (\tau - \tau^{bx})a_x b^x + \tau^{gx} g^x - \tau^x x - \tau^{ax} x_x^a, \tag{55}$$

Consumption tax on non-tradable goods

$$\tau^{yh} = 0. \tag{56}$$

The tax rate τ^{bh} on household emissions of CO_2 stated in (50) equals the domestic social cost of carbon τ adjusted for the marginal social benefit $\alpha\tau\rho^b$ from the reduction of carbon leakage obtained when domestic households increase their consumption of fossil fuel (recall that $\rho^b < 0$). The formulas for the emission tax rates τ^{bx} and τ^{by} on domestic firms have a similar interpretation, but they account for the fact that when carbon capture technology reduces the CO_2 emission to the amount $a < 1$ per unit of fossil fuel burned, a unit increase in emissions is associated with an increase $1\ /\ a$ in the amount of fossil fuel consumption which generates a social benefit $|\alpha\tau\rho^b\ /\ a|$ from reduced carbon leakage. Note from (50) that when it is unprofitable for firms to install carbon capture technology, we have $a_x = a_y = 1$ in which case firms must pay the *same* emission tax rate as households,[7] whereas firms engaging in carbon capture are rewarded by a lower emission tax rate because the fossil fuel consumption underlying their remaining emissions makes a greater contribution to the reduction of carbon leakage. Notice also that while the tax rates in (50) are expressed per unit of *emissions*, the tax rates per unit of *fossil fuels* used by firms are $\tau^{bx}a_x = \tau a_x + \alpha\tau\rho^b$ and $\tau^{by}a_y = \tau a_y + \alpha\tau\rho^b$, respectively. These tax rates consist of penalties τa_x and τa_y for the domestic emissions generated by fossil fuel use in the two business sectors, and a reward $|\alpha\tau\rho^b|$ for the reduction in foreign emissions generated by domestic fossil fuel consumption.

The rationale for the output subsidy τ^x to firms in the tradable goods sector has already been explained above: a unit increase in the domestic output of tradable final goods crowds out some amount of foreign output which reduces foreign emissions by ρ^x units, generating a domestic social benefit $\alpha\tau\rho^x$. The output subsidy τ^x stated in (51) internalizes this external benefit from domestic production of tradables. By analogy, if domestic consumption of tradable final goods and thereby net imports of these goods goes up by one unit, the increase in foreign output needed to match the increase in domestic consumption drives up foreign emissions by the amount ρ^x which has a domestic social cost equal to $\alpha\tau\rho^x$. The consumption tax rate τ^{xh} in (51) internalizes this external cost. Note that if the increase in the domestic demand for tradables is fully matched by a corresponding increase in the domestic output of tradables, the net tax burden on tradables is unchanged, as the increased output subsidy to firms is fully offset by an increase

[7] A proviso: For a country participating in the European Emissions Trading System (ETS) it may be optimal to implement a reduced carbon tax rate for firms covered by the ETS since the leakage rate ρ^b in (50) is likely to be higher inside than outside the ETS. See Silbye and Sørensen (2019) and Beck and Kruse-Andersen (2020) for estimates of leakage rates within the ETS over different time horizons.

in the consumption tax on households (since $\tau^x = \tau^{xh}$). This constancy of the tax burden reflects that a parallel increase in domestic production and consumption leaves net imports and thereby foreign emissions unchanged. Note also that, in practice, the leakage rate ρ^x will vary across the different subsectors of the tradable-goods sector, so the output subsidy τ^x and the consumption tax rate τ^{xh} will have to be differentiated across subsectors to achieve optimality.

The subsidy to investment in carbon capture τ^{ax} in (52) includes the component $1 - \frac{\tau^{bx}}{\tau}$ which compensates for the fact that the emission tax rate τ^{bx} in (50) is lower than the domestic social cost of carbon, τ (since $\rho^b < 0$). The reduced emission tax rate weakens the incentive to invest in carbon capture, creating a need for an offsetting subsidy.

On the other hand, since the carbon capture technology uses tradable goods as an input, the emission tax factor $\frac{\tau^{bx}}{\tau}$ creates an incentive to increase the net import of tradables for the purpose of investing in carbon capture. This boost to net imports generates a social cost of leakage $\alpha\tau\rho^x$ per unit of investment that reduces the optimal size of the subsidy to carbon capture, as indicated in (52). The formula for the subsidy τ^{ay} to carbon capture in the non-tradables sector has a similar interpretation.

The uniform tax on all domestic consumption of green energy implied by (53) is motivated solely by the social concern about leakage. When the domestic consumption of green energy goes up by one unit, its world market price increases slightly, inducing a substitution towards fossil fuel abroad which increases foreign emissions by the amount ρ^g. To internalize the resulting marginal domestic social cost $\alpha\tau\rho^g$, an energy tax of this magnitude is needed.

The tax τ^b in (54) on domestic output of fossil fuel is likewise motivated by leakage concerns. The production of one more unit of fossil fuel requires an input of $\frac{1}{b'}$ units of tradable goods which generates a social cost of $\frac{\alpha\tau\rho^x}{b'}$ from the leakage caused by the additional net imports. In addition, the increase in domestic fossil fuel supply stimulates foreign fossil fuel consumption which generates a further domestic social cost of leakage equal to $-\alpha\tau\rho^b > 0$. Adding up these marginal social costs, we obtain the optimal tax on domestic fossil fuel production in (54). On the other hand, a unit increase in the domestic production of green energy puts downward pressure on the international price of green energy, thereby reducing foreign emissions by the amount ρ^g via substitution from fossil fuel towards green energy with an accompanying domestic social benefit $\alpha\tau\rho^g$. At the same time, increasing domestic output of green energy requires an additional input of $\frac{1}{g'}$ units of tradable goods with an associated social cost of leakage $\left|\frac{\alpha\tau\rho^x}{g'}\right|$, so the optimal unit subsidy to domestic production of green energy is only the difference between the marginal social benefit and the marginal social cost, as stated in the second equation in (54).

A special feature of our analysis is the lump-sum subsidy T^x in (55) which serves to ensure that firms make the socially optimal choice of location. When a firm chooses to stay at home rather than moving abroad, society avoids an increase ρ^s in foreign emissions which generates a social benefit $\alpha\tau\rho^s$, captured by the first term on the right-hand side of (55). However, to the extent that the domestic social cost of carbon (τ) is not fully internalized by the domestic emission tax τ^{bx}, there is a social cost $(\tau - \tau^{bx})a_x b^x$ associated with the domestic emissions from a tradable goods firm that chooses to stay at home, so the location subsidy T^x should be reduced by a corresponding amount, as indicated by the second term on the right-hand side of (55). Moreover, to avoid that the domestic taxes on green energy and the domestic subsidies to output and carbon capture distort the location decision, the location subsidy must also be adjusted by the (positive or negative) net fiscal burden $\tau^{gx} g^x - \tau^x x - \tau^{ax} x_x^a$, as reflected by the last terms in the formula for T^x.

In our analysis of firm behaviour, we assumed that the individual firm takes the subsidy T^x as given, whereas formula (55) apparently implies that the subsidy depends endogenously on the firm's current activity. This might seem as a contradiction that would make it impossible to implement the subsidy. However, it is possible to make T^x exogenous to the individual firm while setting it at a level which will affect the location decision in the same way as the subsidy in (55). To demonstrate this, we insert the expression (17) for the leakage rate ρ^s along with the above formulas for the optimal values of $\tau^{bx}, \tau^{gx}, \tau^x$ and τ_x^a in (55) and find that

$$T^x = \alpha\tau\left[(1+\rho^b)b^{xf} + \rho^g g^{xf} - \rho^x x^f - n^{xf}\frac{\partial L}{\partial \bar{N}}\right] - \left(\frac{\tau - \tau^{bx}}{\tau}\right)(1+\alpha\tau\rho^x)x_x^a \qquad (57)$$

Before it embarked on its unilateral policy, the domestic government pursued the same climate policy as other countries. If a domestic firm moves abroad, its optimal choices of the variables b^{xf}, g^{xf}, x^f and n^{xf} in (57) will therefore correspond to its optimal historical choices of the corresponding domestic variables b^x, g^x, x and n^x before the domestic government introduced its unilateral climate policy. These historical choices cannot be undone by the firm after the introduction of the new policy, and the other parameters in formula (57) are likewise exogenous to the firm, except x_x^a. The optimal choice of this variable will have to be estimated by the policy maker for an exact application of formula (57), but in practice this will be relevant only for a small number of firms for which carbon capture is a realistic option. If information on the output, energy use, and employment of firms prior to the introduction of the tax-subsidy scheme is available, and if this information is combined with the policy maker's estimate of the cost x_x^a of investment in carbon capture wherever this is a realistic possibility, a location subsidy T^x calculated from the formula (57) will thus be exogenous to the individual firm but should still give a good approximation to the optimal subsidy (55).[8]

Finally, eq. (56) states that a consumption tax on non-traded goods is redundant, since the consumption of these goods does not generate any leakage effects.

The previous literature on unilateral carbon taxation with carbon leakage abstracted from carbon capture and leakage at the extensive margin. If we left out those features, our optimal tax rules would no longer include (52) and (55), and in (50) we would have $a_x = a_y = 1$. Our optimal tax

[8] To maintain the exogenous character of the subsidy over time, its magnitude would have to remain tied to the firm's historical activity level, so the approximation to the optimal subsidy level would gradually become less accurate.

system consisting of (50), (51), (53) and (54) would then be equivalent to a system with the following ingredients: i) A uniform carbon tax on all fossil fuel use levied at the rate τ to internalize the domestic social cost of carbon. ii) A leakage-motivated tax on the net export of fossil fuel levied at the rate $-\alpha\tau\rho^b$. iii) A leakage-motivated tax on the net import of green energy levied at the rate $\alpha\tau\rho^g$. iv) A leakage-motivated tax on the net import of final goods for use in the business sector as well as in the household sector levied at the rate $\alpha\tau\rho^x$. Essentially this tax system is identical to the first-best optimal carbon tax scheme with border tax adjustment described by Hoel (1996), and the taxes on net exports and net imports could be implemented via the following instruments which eliminate the need for border tax adjustments: a) A tax on domestic fossil fuel production at the rate $-\alpha\tau\rho^b$ combined with a subsidy to domestic fossil fuel consumption granted at a similar rate. b) A tax on domestic consumption of green energy at the rate $\alpha\tau\rho^g$ combined with a corresponding subsidy to domestic production of green energy. c) A tax at the rate $\alpha\tau\rho^x$ on all business and household consumption of traded final goods combined with a corresponding subsidy to domestic production of such goods. This equivalence between border carbon adjustment and a combination of domestic consumption taxes and domestic output subsidies has previously been highlighted by Böhringer et al. (2017).

However, our analysis shows that when one allows for carbon capture (CC), the system of taxes and subsidies described above should be supplemented by subsidies to CC (even in the absence of learning-by-doing effects), and the emission tax on firms engaged in CC should be reduced. Moreover, when carbon leakage at the extensive margin is accounted for, the optimal policy also includes a location subsidy to leakage-exposed firms that choose not to relocate abroad.

Optimal carbon taxation in the absence of leakage concerns

As a benchmark, it is interesting to consider the special case where the domestic government is not concerned about carbon leakage, focusing only on meeting the target for reduction of emissions from domestic territory ($\alpha = 0$). In this case inspection of eqs. (50) through (56) reveals that the optimal domestic climate policy is:

$$\tau^{bh} = \tau^{bx} = \tau^{by} = \tau, \tag{58}$$

$$\tau^x = \tau^{xh} = \tau^{ax} = \tau^{ay} = \tau^{gh} = \tau^{gx} = \tau^{gy} = \tau^b = \tau^g = T^x = 0. \tag{59}$$

In the absence of leakage concerns, our model thus implies that the government should impose a uniform carbon tax on all domestic emissions to ensure a cost-effective domestic abatement effort, in line with conventional wisdom. With such a tax in place, there is no need to subsidize carbon capture and domestic production of green energy. Nor is there any need for an output subsidy and a location subsidy to firms in the tradable goods sector, or for taxes on consumption of tradables and green energy, or for a tax on production of fossil energy, since these subsidies and taxes can only be motivated by leakage concerns, as we have seen.

One more point is worth noting: The domestic social cost of carbon (τ) is endogenous and depends inter alia on the strength of the government's aversion to leakage (α). Hence one cannot conclude from a comparison of (50) and (58) and the observation that $\rho^b < 0$ that the general level of carbon taxation will be higher when the government does not care about leakage. On the contrary, our numerical simulations in section 7 indicate that the policy concern about leakage tends to drive up the average level of carbon taxation needed to attain the target for domestic emissions reduction, because some of the subsidies in the optimal policy package tend to increase domestic emissions.

Can the optimal leakage-adjusted tax-subsidy scheme be implemented?

The leakage-adjusted tax-subsidy scheme described by (50) through (55) may seem complex, and it is natural to ask if it can be implemented in practice? However, governments in advanced economies already levy taxes on fossil fuels and other forms of energy. They also offer subsidies to green energy production and various forms of greenhouse gas abatement, and they impose taxes such as royalties on fossil fuel production as well as taxes on final consumption goods. In short, most of the tax and subsidy instruments appearing in the government budget constraint (25) are already used in practice, and the output subsidy τ^x in (51) could be based on the firms' value-added as recorded in their VAT accounts. Presumably, the main implementation challenge would be to obtain reliable information on the various leakage rates needed to calibrate the optimal tax and subsidy rates. In principle, carbon leakage rates can be estimated via simulations with computable general equilibrium models, as explained by Carbone and Rivers (2017), and as illustrated in section 7 in this paper.[9] Inevitably, these estimates are only as reliable as the CGE models themselves. Moreover, an elaborate differentiated tax-subsidy scheme is administratively cumbersome and may invite rent-seeking from strong lobby groups.

Since any real-world system of differentiated taxes and subsidies is likely to deviate from the theoretical optimum due to administrative barriers, lack of information and political economy factors, it would only seem worthwhile to attempt to implement the tax-subsidy scheme (50) through (55) if there is reason to believe that such a system could generate substantial welfare gains compared to the much simpler system (58) and (59) with uniform taxation and no subsidies. In the remainder of this paper we will set up a calibrated version of our model to investigate the likely magnitude of the welfare gain from the theoretically optimal leakage-adjusted tax-subsidy scheme.

[9] Beck et al. (2020) have developed a sophisticated variant of this approach in which they apply a large scale detailed CGE model of the Danish economy to estimate the effects of domestic climate policy on the various components of Danish imports and exports. They then feed these changes in imports and exports into the global trade model GTAP-E (Burniaux and Truong, 2002) to estimate the resulting changes in foreign greenhouse gas emissions, accounting for all the general equilibrium effects and emission coefficients embodied in the GTAP-E model. This method has been used to estimate leakage rates in the recent report from The Danish Environmental Economic Council (2021).

6. A quantitative model

This section describes the key assumptions on tastes and technologies in the quantitative version of our model and how we have calibrated its key parameters. A detailed documentation of the model is provided in Kruse-Andersen and Sørensen (2021).

Technologies

Final goods producers work with the following Cobb-Douglas versions of the production functions (2) and (3),

$$x = A(n^x)^\gamma (e^x)^\beta, \qquad 0 < \gamma, \beta < 1, \quad \gamma + \beta < 1, \tag{60}$$

$$y = B(n^y)^\Upsilon (e^y)^\epsilon, \qquad 0 < \Upsilon, \epsilon < 1, \quad \Upsilon + \epsilon < 1, \tag{61}$$

where A and B are total factor productivities. The production functions for energy services are assumed to take the CES form

$$e^x = \left[(\psi_b^{ex})^{\frac{1}{\varepsilon_x^e}}(b^x)^{\frac{\varepsilon_x^e-1}{\varepsilon_x^e}} + (1-\psi_b^{ex})^{\frac{1}{\varepsilon_x^e}}(g^x)^{\frac{\varepsilon_x^e-1}{\varepsilon_x^e}}\right]^{\frac{\varepsilon_x^e}{\varepsilon_x^e-1}}, \quad 0 < \psi_b^{ex} < 1, \quad \varepsilon_x^e > 0, \quad \varepsilon_x^e \neq 1, \tag{62}$$

$$e^y = \left[\left(\psi_b^{ey}\right)^{\frac{1}{\varepsilon_y^e}}(b^y)^{\frac{\varepsilon_y^e-1}{\varepsilon_y^e}} + \left(1-\psi_b^{ey}\right)^{\frac{1}{\varepsilon_y^e}}(g^y)^{\frac{\varepsilon_y^e-1}{\varepsilon_y^e}}\right]^{\frac{\varepsilon_y^e}{\varepsilon_y^e-1}}, \quad 0 < \psi_b^{ey} < 1, \quad \varepsilon_y^e > 0, \quad \varepsilon_y^e \neq 1, \tag{63}$$

where ε_x^e and ε_y^e are constant elasticities of substitution between fossil and green energy inputs. The production functions for energy production in (5) take the simple constant elasticity form

$$b^s = k(x^b)^\varrho, \qquad k > 0, \quad 0 < \varrho < 1, \tag{64}$$

$$g^s = h(x^g)^\nu, \qquad h > 0, \quad 0 < \nu < 1, \tag{65}$$

and the technologies for carbon capture are modelled as

$$a_x = \frac{1}{1+\beta_x(x_x^a)^{\alpha_x}}, \qquad a_y = \frac{1}{1+\beta_y\left(x_y^a\right)^{\alpha_y}}, \qquad \alpha_x, \alpha_y, \beta_x, \beta_y > 0. \tag{66}$$

The specifications in (66) satisfy our general assumptions in (4) about the characteristics of carbon capture technologies.

Tastes

Household preferences have the following nested CES structure where U is aggregate household utility, C is aggregate household consumption of final goods, and σ and φ are constant elasticities of substitution:

$$U = \left[\delta^{\frac{1}{\sigma}} C^{\frac{\sigma-1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}} (e^h)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 0, \quad 0 < \delta < 1, \tag{67}$$

$$C = \left[(\psi_x^c)^{\frac{1}{\varphi}} (x^h)^{\frac{\varphi-1}{\varphi}} + (1-\psi_x^c)^{\frac{1}{\varphi}} (y^h)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}}, \qquad \varphi > 0, \quad 0 < \psi_x^c < 1. \tag{68}$$

By analogy to (62) and (63), household consumption of energy services, e^h, is a CES aggregate of the consumption of fossil and green energy inputs with substitution elasticity ϕ:

$$e^h = \left[\left(\psi_b^{eh}\right)^{\frac{1}{\phi}} (b^h)^{\frac{\phi-1}{\phi}} + \left(1-\psi_b^{eh}\right)^{\frac{1}{\phi}} (g^h)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \qquad \phi > 0, \quad 0 < \psi_b^{eh} < 1. \tag{69}$$

General equilibrium

Tastes and technologies in the representative foreign country take the same form as in the domestic economy. The foreign economies are assumed to be symmetric, and there is a total of z foreign countries. By varying z, we can vary the relative size of the domestic economy. Given the specifications (60) through (69), Kruse-Andersen and Sørensen (2021) derive output supply functions, factor demand functions, and household demand functions for final goods and energy inputs in the domestic and in the foreign economies, assuming maximization of utility and profits. From these functions one obtains fully specified conditions for a general equilibrium in the world economy corresponding to eqs. (6) through (9) which can be used to calculate the marginal rates of carbon leakage from the domestic economy. When these leakage rates are fed into the formulas (50) through (56) for the near-optimal tax and subsidy rates, we end up with a system of 99 equations in 99 endogenous variables from which we can calculate the general equilibrium of the domestic and international economy and its implications for domestic welfare.

Calibration and estimated leakage rates

Table 1 summarizes the calibration of the model which is documented in detail in Kruse-Andersen and Sørensen (2021). The non-policy parameters are calibrated to an initial equilibrium where all tax and subsidy rates equal zero. This reflects a situation where the emissions reduction target (19) has not yet been adopted, so that equation is left out from the model initially. With no domestic taxes and subsidies, and assuming symmetry between domestic and foreign producers of traded goods, we have $\pi^x = \pi^{xf}$, implying that $s = 1$ initially. The symmetry also implies that there is no net trade between the home and foreign economy initially. Intuitively, with similar production technologies and regulation schemes, there are no comparative advantages to exploit. We assume that the domestic economy is relatively small, constituting only one percent of the global economy, implying that $z = 99$.

The share parameters ψ_b^{ex} and ψ_b^{ey} determining the energy mix were set to ensure that the initial share of fossil fuel in total energy consumption is 80 percent, corresponding roughly to the current global energy mix, according to the International Energy Agency (2020). The parameters

β and Υ equal the energy expenditure shares of the tradable and non-tradable sector, respectively. We define the trade-exposed sector as manufacturing and agriculture, and we obtain a β value of 0.12 based on data from Statistics Denmark (2018). For manufacturing alone, the number is 0.11, so the inclusion of the agricultural sector has little effect. We calculate Υ for the remaining part of the economy to 0.04, yielding an energy expenditure share for the entire Danish economy of almost 0.05 for all production sectors which matches the energy share of income of around 5 pct. for the US found by André and Smulders (2014). The energy expenditure share for the household sector, $1-\delta$, is set equal to 4 percent, roughly in line with Danish as well as U.S. data. We set the share of tradable goods in non-energy consumption, ψ_x^c, to 0.30. For the Danish economy, the number is 20 percent, but our choice of ψ_x^c has been guided by the GTAP Data Base 10 which shows that agriculture and manufacturing play a larger role at the global level.

The calibrated labour income shares γ and ϵ imply that total labour income makes up around 65 percent of total GDP, in line with stylized facts from growth theory (see Jones 2016), and they also ensure that the traded and non-traded sectors are equally profitable initially, since we do not want our welfare results to be driven by a shift towards the more or less profitable sector.

The input elasticity ϱ in the production function for fossil fuel is chosen so as to imply a plausible price elasticity of fossil fuel supply. The price elasticities of natural gas, oil, and coal vary a lot. For instance, Burniaux and Martins (2012) use a supply elasticity of one for oil and 20 for coal. Our baseline calibration assumes that the price elasticity of fossil fuel supply is 4. This is within the range of supply elasticities for coal used by Graham et al. (1999), but above the price elasticity of oil supply of 1.9 estimated by Ringlund et al. (2008). Our calibration of ϱ implies that about 77 percent of a cut in domestic production of fossil fuel will be offset by an increase in foreign fossil fuel production. This leakage rate on the supply side of the fossil fuel market is fairly close to the estimated supply-side leakage rates in Fæhn et al. (2017) and Erickson and Lazarus (2018). Our input elasticity ν is chosen to imply a price elasticity of green energy supply equal to 1, in line with the supply elasticity for low-carbon energy used by Burniaux and Martins (2012).

Our calibration of the substitution elasticities ε_x^e , ε_y^e, and ϕ between fossil and green energy is based on our interpretation of the empirical study by Papageorgiou et al. (2017), discussed in Kruse-Andersen (2019). The elasticity of substitution between energy and non-energy consumption for households, σ, is set equal to 0.5, implying that energy and non-energy consumption are gross complements. Although this implies that households are reluctant to substitute away from energy, our assumption on the elasticity of substitution between fossil and non-fossil energy means that households have good substitution possibilities when it comes to choosing alternative energy sources.

We also assume that tradable and non-tradable goods are gross complements such that φ equals 0.5, but this parameter value is unimportant for our results, since changes in the relative price of non-tradables have little effect in our scenarios.

Table 1. Calibration overview

Parameters, exogenous variables, and leakage variables	Value	Calibration
Size of foreign economy:		
z	99	The domestic economy constitutes 1 pct. of the global economy.
Labour supply:		
$\bar{n}$	100	Labour supply normalized to 100 in home economy.
$\bar{N}$	100	Labour supply normalized to 100 in each foreign economy.
Production sector:		
γ	0.58	The labour share of income.
β	0.12	The energy expenditures share of trade-exposed sectors.
Υ	0.66	The labour share of income.
ϵ	0.04	The energy expenditures share of non-trade-exposed sectors.
ψ_b^{ex}	0.80	Global energy mix.
ψ_b^{ey}	0.80	Global energy mix.
ε_x^e	1.50	Considerations based on Papageorgiou et al. (2017).
ε_y^e	1.50	Considerations based on Papageorgiou et al. (2017).
A and A^f	5.90	Calibration procedure.
B and B^f	6.00	Calibration procedure.
α_x	0.80	CCS cost and potential in the industrial sector.
β_x	0.50	CCS cost and potential in the industrial sector.
α_y	0.80	CCS cost and potential in the non-tradable sector.
β_y	0.35	CCS cost and potential in the non-tradable sector.
$\bar{c}$	5.00	Based on the relocation effect under uniform taxation.
$\bar{E}$	6.04	66 pct. emission reduction based on Danish 2030 emission target.
Energy sector:		
k	1.92	Calibration procedure.
h	3.50	Calibration procedure.
ϱ	0.80	The price elasticity of coal.
ν	0.50	Burniaux and Martins (2012).
Household sector:		
δ	0.96	Share of household expenditures on energy.
σ	0.50	Energy and non-energy goods are gross complements.
ψ_x^c	0.30	Share of tradeable goods in gross value added.
φ	0.50	Tradeable and non-tradeable goods are gross complements.
ψ_b^{eh}	0.35	Calibration procedure.
ϕ	1.50	Considerations based on Papageorgiou et al. (2017).
Leakage effects: [1]		
ρ^b	-0.219	Marginal leakage effect of black energy consumption.
ρ^g	0.132	Marginal leakage effect of green energy consumption.
ρ^x	0.008	Marginal leakage effect of final good consumption.
ρ^s	2.526	Marginal leakage effect of fossil-fuel consumption.

(1): The marginal leakage effects are endogenous. The values shown in the table are equilibrium values under the optimal tax-subsidy scheme. These values vary little across our alternative scenarios.

Our parameters α_x and β_x determine the costs of reducing emissions through carbon capture (CC) technologies. The Danish Council on Climate Change (2020) estimates that it will cost about 134 euros to reduce emissions by 1 tonne of CO_2 via Carbon Capture and Storage (CCS). We assume that this is the average cost per ton of cutting emissions from the tradables sector by 10 percent. The total cost of cutting emissions by that amount via CCS will then be roughly 0.1 percent of Danish GDP. On this basis we have chosen values of α_x and β_x which imply that a 10 percent cut in emissions from the tradables sector through carbon capture technology generates a total cost equal to 0.1 percent of national income in the model. In the sector for non-tradables, there is a potential for applying CCS or Carbon Capture and Utilization (CCU) technologies in district heating and waste treatment. We choose values of α_y and β_y to reflect this potential (based on Danish data) as well as the above-mentioned estimated unit cost of cutting emissions via CC technologies.

We set the target $\bar{E}$ for domestic emissions to 66 percent of laissez-faire emissions. This corresponds to the relative difference between the expected emissions in Denmark in 2030 under a frozen policy scenario and the Danish climate target of a 70 percent emission reduction compared to 1990 (based on numbers from the Danish Energy Agency (2021)).

In our baseline scenario with optimal taxes and subsidies, the policy parameter α is set equal to 1, implying that policy makers are equally concerned about foreign and domestic emissions. The mobility cost parameter $\bar{c}$ is set equal to 5 in our central case. This results in a relocation of about 8.5 percent of firms in the tradable-goods sector under uniform taxation which we do not consider implausible. However, since the parameters α and $\bar{c}$ are essentially unknown, we will illustrate the sensitivity of our results to changes in their values.

The bottom part of Table 1 shows the magnitude of the marginal leakage effects implied by our baseline calibration, calculated at the general equilibrium with optimal policy. We see that all leakage effects have the theoretically expected signs explained at the end of Section 2. The leakage effects vary endogenously with the policy regime, but not very much, so we only report their values in the baseline scenario with optimal policy. The reason for the near-constancy of the leakage effects is that the domestic economy is small relative to the world economy (given our choice of z).

Welfare analysis

We measure changes in household welfare in monetary terms by means of the representative domestic household's expenditure function. As shown in Kruse-Andersen and Sørensen (2021), our quantitative model implies the expenditure function

$$I = p_H U, \tag{70}$$

where I is the minimum amount of income the consumer needs to be able to attain the utility level U, given the current consumer prices reflected in the overall consumer price index p_H

which takes a CES form. With this expenditure function the compensating variation measure of the household welfare loss HWL from a deviation from the optimal climate policy is

$$HWL = (p_H - p_{H,0})U_0 + I_0 - I, \tag{71}$$

where $p_{H,0}$ and p_H are the consumer price levels prevailing before and after the deviation from the optimal policy, respectively; U_0 is household utility under the optimal policy, and I_0 and I are the nominal household income levels before and after the deviation from the optimal policy. The compensating variation $(p_H - p_{H,0})U_0$ measures the additional income the consumer would need to be able to maintain the welfare level enjoyed under the optimal policy, given the new set of consumer prices. When the income loss $I_0 - I$ from the deviation from optimal policy is added, we obtain the total additional amount of income that would be necessary to compensate the consumer for the policy deviation. Note that since the socially optimal policy which maximizes the leakage-adjusted social welfare function (18) differs from the policy that maximizes household utility (67), the household welfare loss in (71) could be negative.

According to the social welfare function (18), the compensating (monetary) measure of the social welfare loss from the leakage caused by a deviation from optimal policy is

$$WLL = \frac{\eta}{\lambda}\Delta L, \tag{72}$$

where ΔL is the change in total carbon leakage generated by the policy change, and λ is the marginal utility of disposable household income, so $\frac{\eta}{\lambda}$ is the marginal social cost of leakage measured in monetary terms. The definitions of our policy preference parameter α and our tax level parameter τ explained in section 5 imply that $\frac{\eta}{\lambda} = \alpha\tau$, and when consumers maximize their utility we have $\lambda = \frac{1}{p_H}$. Exploiting these results, we see that a consistent measure of the marginal social aversion to leakage in the initial equilibrium is

$$\eta = \frac{\alpha\tau_0}{p_{H,0}}, \tag{73}$$

where the 0-subscripts indicate that we are using the values of τ and p_H prevailing in the initial equilibrium with (near)optimal policy. It follows that the monetary measure of the marginal social aversion to leakage in the initial equilibrium is $\frac{\eta}{\lambda_0} = \eta p_{H,0} = \alpha\tau_0$, so the social welfare loss in (72) from the additional leakage caused by a deviation from optimal policy is $WLL = \alpha\tau_0\Delta L$. Hence the total (monetary) social welfare loss SWL from implementing a suboptimal policy is

$$SWL = HWL + WLL = (p_H - p_{H,0})U_0 + I_0 - I + \alpha\tau_0\Delta L. \tag{74}$$

In the next section we will use the welfare measure (74) for quantitative policy evaluation.

7. Leakage and welfare effects of a unilateral climate policy accounting for carbon leakage

In Table 2 below we report the leakage and welfare effects of five different policy packages all of which ensure that domestic emissions are reduced to the target level. The four suboptimal policies in the last four columns of the table have roughly the same impact on household welfare, but they differ in their impact on carbon leakage. Since the welfare effects are measured relative to a scenario where all policy instruments have been set at their first-best level, the welfare effects in the first column of Table 2 are zero by construction. The total leakage rates are measured in the usual way as the increase in foreign emissions (relative to the initial laissez-faire equilibrium) in percent of the reduction in domestic emissions. The second row in the table measures the percentage of firms in the tradable-goods sector that choose to relocate in response to domestic climate policy. Since firms within the sector are symmetric in our model (except for differences in mobility costs), these numbers can also be interpreted as the percentage of initial tradable-goods output which is relocated via outward FDI.

Table 2. Leakage and welfare effects of deviating from the optimal tax-subsidy scheme*

	Scenario				
	FIRST-BEST[a]	UNIFORM[b]	SECOND[c]	THIRD[d]	FOURTH[e]
Total leakage rate (percent)	-7.9	28.2	21.6	24.0	27.9
Relocation $(1-s)\cdot 100^1$	0.2	8.5	0.8	0.2	7.7
Household welfare loss (percent of national income)	0	-0.39	-0.32	-0.35	-0.39
Welfare loss from leakage (percent of national income)	0	0.87	0.71	0.77	0.86
Social welfare loss (percent of national income)	0	0.48	0.39	0.42	0.47

* Baseline calibration as reported in Table 1, including $\alpha = 1$. 1. Percentage of firms in tradable goods sector that relocate. a. All policy instruments are set at their optimal levels. b. Uniform carbon tax on all domestic emissions, no other taxes and subsidies. c. $\tau^b = \tau^g = 0$. d. $\tau^b = \tau^g = \tau^{xh} = \tau^{ax} = \tau^{ay} = \tau^{gh} = \tau^{gx} = \tau^{gy} = 0$. e. $\tau^b = \tau^g = \tau^{xh} = \tau^{ax} = \tau^{ay} = \tau^{gh} = \tau^{gx} = \tau^{gy} = T^x = 0$.

From the first column in Table 2 we see that the first-best optimal unilateral climate policy actually *reduces* foreign emissions compared to the initial laissez-faire policy. The main reason is that the optimal policy involves a substantial tax on domestic fossil fuel production which makes the domestic economy a net importer of fossil fuel, despite the fall in domestic fossil fuel use caused by the domestic carbon taxes. This net import drives up the international fossil fuel price (a bit), thereby reducing foreign fossil fuel use and the concomitant emissions. We also see that the first-best optimal policy hardly generates any relocation of economic activity via the extensive margin, as only a tiny 0.2 percent of domestic firms move abroad. This is due to the domestic location subsidy that induces almost all mobile firms to stay at home.

The second column in Table 2 shows the implications of choosing a uniform carbon tax rate on all domestic emissions and setting all other taxes and subsidies equal to zero. Such a policy enables the government to meet the target for domestic emissions reduction in a cost-effective way, and the resulting efficiency gain generates an increase in household welfare amounting to almost 0.4 percent of initial national income. However, the uniform carbon tax with no supplementary measures also increases the total leakage rate by more than 30 percentage points (from minus 7.9 percent to plus 28.2 percent) compared to the first-best policy. Furthermore, this policy causes almost 9 percent of activity in the tradable-goods sector to be relocated via the extensive margin. In our baseline, foreign emissions generate the same social welfare cost as domestic emissions ($\alpha = 1$), and the welfare loss from leakage at the intensive and the extensive margin outweighs the conventional household welfare gain from improved cost-efficiency, generating a net social welfare loss of almost 0.5 percent of initial national income. Although our model is very stylized and there is great uncertainty about the magnitude of many parameter values, this result suggests that accounting for leakage in the design of unilateral climate policy could generate a non-negligible social welfare gain.

The remaining scenarios in Table 2 are meant to illustrate some intermediate cases where the government can use some but not all of the instruments in the first-best policy package. In the scenario "SECOND" in the third column of Table 2 we assume that the government does not intervene on the supply side of domestic energy production, setting the tax on fossil fuel production (τ^b) and the subsidy to production of green energy (τ^g) equal to zero while maintaining all other policy instruments at their first-best levels. This scenario confirms the importance for leakage of taxing fossil fuel production. The absence of such a tax combined with high carbon tax rates on domestic fossil fuel use turns the domestic economy into a net exporter of fossil fuel, thereby increasing foreign emissions, so the total leakage rate increases from roughly minus 8 percent under the first-best scenario to about plus 22 percent. While the withdrawal of taxes on and subsidies to energy production increases production efficiency to the benefit of household welfare, the massive increase in leakage means that society suffers a net welfare loss of about 0.4 percent of initial national income compared to the first-best policy.

The scenario "THIRD" in the fourth column of Table 2 assumes that the government only implements the first-best levels of the carbon taxes plus the first-best output subsidy and the first-best location subsidy to firms in the tradable-goods sector whereas all other tax and subsidy instruments are set to zero. The absence of taxes on green energy and final goods means that household welfare is now higher than under the first-best policy, but the failure to counter leakage at all relevant margins means that there is a net social welfare loss of 0.42 percent of income compared to the first-best scenario. The increase in the welfare loss from 0.39 percent to 0.42 percent of national income as policy makers move from the SECOND to the THIRD scenario reflects the combined welfare loss from ruling out consumption taxes on green energy and on final goods and not granting subsidies to carbon capture. It appears that the welfare impact of these instruments is rather small.

In the scenario "FOURTH" in the last column of Table 2 we assume that the government only implements the first-best carbon taxes and the first-best output subsidy to tradable-goods producers, but not the location subsidy T^x. By comparing the THIRD to the FOURTH scenario, we can thus illustrate the importance of leakage at the extensive margin. Without the location subsidy to mobile firms, the total leakage rate increases by almost 4 percentage points, and a significant 7.7 percent of activity in the tradable-goods sector is relocated at the extensive margin, compared to a relocation of only 0.2 percent of activity in the first-best and in the THIRD scenarios where the location subsidy is activated. As a result, the welfare loss increases by a further 0.05 percent of national income as society moves from the THIRD to the FOURTH policy scenario. At the macro level this is a modest welfare loss from failing to counter leakage at the extensive margin, but in particular carbon-intensive sectors of the economy relocation of production may have substantial effects on domestic activity. If society does not want to impose highly unequal adjustment burdens on different groups in society, it may therefore be important to address carbon leakage at the extensive margin.

Table 3 illustrates the sensitivity of our welfare analysis to changes in the two parameters about which little is known, i.e., the mobility cost parameter $\bar{c}$ and the social cost of foreign relative to domestic emissions, α. In the baseline calibration we assume $\bar{c} = 5$ and $\alpha = 1$. By setting $\bar{c} = 25{,}000$, we make it so costly for firms to relocate that we effectively eliminate all leakage at the extensive margin, thereby also eliminating the need for the location subsidy T^x. As a consequence, there is no longer any difference between the welfare effects of the scenarios THIRD and FOURTH, and the welfare loss from relying on uniform carbon taxation rather than on the first-best optimal policy package is reduced from 0.48 percent to 0.42 percent of national income. On the other hand, if the mobility cost is as low as $\bar{c} = 1$, leakage at the extensive margin becomes relatively important, and the negative welfare effect of not activating the location subsidy in the scenarios UNIFORM and FOURTH becomes larger. Overall, however, the welfare effects do not seem to be highly sensitive to reasonable variations in our mobility cost parameter.

Table 3. Social welfare loss in percent of national income when deviating from the optimal tax-subsidy scheme: Sensitivity analysis*

Scenario	Baseline calibration	Importance of mobility cost			Importance of leakage cost	
		$\bar{c} = 1$	$\bar{c} = 25$	$\bar{c} = 25{,}000$	$\alpha = 0.5$	$\alpha = 0.25$
UNIFORM	0.48	0.50	0.44	0.42	0.10	0.03
SECOND	0.39	0.39	0.39	0.39	0.07	0.02
THIRD	0.42	0.42	0.42	0.42	0.08	0.02
FOURTH	0.47	0.49	0.44	0.42	0.10	0.02

* In the first two columns α is set equal to its baseline value 1. In the last two columns $\bar{c}$ is set equal to its baseline value 5.

As one would expect, Table 3 shows that the welfare cost of deviating from the first-best leakage-adjusted tax-subsidy scheme falls when policy makers assign a lower marginal social cost to leakage (a lower α). Indeed, if α falls from 1 to 0.5, the welfare loss from sticking to a uniform carbon tax rather than pursuing the optimal climate policy falls from roughly 0.5 percent to only 0.1 percent of national income, and if α is only 0.25, the welfare loss falls to a modest 0.03 percent of national income. Thus the welfare loss from a suboptimal climate policy varies significantly and more than proportionately with the policy concern about leakage.

8. Conclusions

According to conventional wisdom, an efficient climate policy requires a uniform carbon price throughout the economy to equalize the marginal abatement cost across sectors. Previous studies have shown that uniform carbon taxation remains the optimal policy even when there is potential for carbon leakage, provided leakage can be countered via border carbon adjustments. When border adjustments are not feasible, the recent literature has shown that a similar effect can be achieved through a combination of domestic output subsidies and domestic consumption taxes. In this paper we have set up a general equilibrium model with carbon leakage that reproduces these results as a special case. We showed that when leakage does not only occur via international trade in energy and final goods, but also via the extensive margin where firms can relocate investment from one country to another, the optimal package of unilateral climate policy includes a "location subsidy" to mobile firms exposed to international competition. We sketched how this subsidy could be based on indicators of the historical activity level of individual firms, and our simulation results illustrated the contribution of such a subsidy to the reduction of carbon leakage and the associated welfare costs. We also showed that when firms engage in carbon capture (CC) and the government is concerned about carbon leakage, a uniform emission tax rate is not sufficient to ensure the correct first-best incentive to invest in CC; such investment should also receive a subsidy. Our quantitative analysis indicated that a unilateral climate policy which systematically counters carbon leakage could generate a non-negligible social welfare gain of about 0.5 percent of national income compared to a policy involving a single uniform carbon tax. It also suggested that a tax on domestic production of fossil fuel that reduces fossil fuel supply to the world market could be a main contributor to this potential welfare gain.

As a caveat, our analysis did not account for the facts that our optimal tax-subsidy scheme may be difficult to administer and may invite lobbyism and wasteful rent seeking by interest groups. In practice policy makers will have to strike a balance between these concerns and the desire to achieve the theoretical first-best optimum to counter leakage. Although pure theory dictates that all traded goods should be candidates for output subsidies and corresponding consumption taxes, albeit at different rates reflecting differences in marginal leakage rates, administrative and political economy considerations suggest that these instruments should only be applied to a limited set of goods where the risk of leakage is obvious and likely to be quantitatively significant.

Another limitation of our study is that it neglects the interaction between the carbon tax and existing non-environmental market distortions due, for example, to other pre-existing taxes.[10] However, Kaplow (2004, 2013) argues that when the government can flexibly adjust a non-linear income tax schedule to strike an optimal balance between equity and efficiency, the marginal

[10] See, e.g., Bovenberg and Goulder (1996), Parry (1997), and Goulder (2013) for analyses of such interactions.

cost of public funds does in fact become equal to one, as assumed in this paper. In any case, it seems likely that the mechanisms highlighted by the present analysis will remain important for optimal carbon tax design in a more realistic model of the economy that allows for other market distortions.

APPENDIX A
SOLUTION TO THE SOCIAL PLANNING PROBLEM

This appendix derives the first-order conditions for the solution to the social planning problem stated in section 3. Inserting the leakage function (10) in the social welfare function (18) and accounting for the climate policy constraint (19), the technology constraints (2) through (5), the resource constraints (20) through (22), and the foreign profit function (23), we can write the Lagrangian $\mathcal{L}$ corresponding to the social planning problem in the following way:

$$\mathcal{L} = u\Big(x^h, y^h, e^h(b^h, g^h)\Big)$$

$$-\eta\left[B^x(p^b, p^g, W) + B^y(p^b, p^g, P^y, W) + B^h(p^b, p^g, P^y, W) + (1-s)b^{xf}(p^b, p^g, W) - E_0\right]$$

$$+\kappa\left[\overline{E} - sa_x(a_x^a)b^x - a_y\big(a_y^a\big)b^y - b^h\right] + \mu\left[y\big(n^y, e^y(b^y, g^y)\big) - y^h\right] + \theta(\bar{n} - sn^x - n^y)$$

$$+\lambda\Big\{sf\big(n^x, e^x(b^x, g^x)\big) - (x^b + x^g + sx_x^a + x_y^a + x^h) + p^b\left[b(x^b) - sb^x - b^y - b^h\right]$$

$$+p^g[g(x^g) - sg^x - g^y - g^h] + (1-s)\left[\pi^f(p^b, p^g, W) - \tfrac{\bar{c}}{2}(1-s)\right]\Big\} \qquad \text{(A.1)}$$

The social planner controls the variables $s, b^x, b^y, b^h, g^x, g^y, g^h, x^b, x^g, x_x^a, x_y^a, x^h, n^x$ and knows the effects of changes in these variables on the international prices p^b, p^g, P^y, W and the resulting leakage effects given by (11) and (12) through (17) as well as the terms-of-trade effects stated in (24). The social planner is also aware that, because several domestic economic variables affect net imports in a parallel way, the world market equilibrium conditions (6) through (9) imply the following links between the terms-of-trade effects:

$$\Delta_{x^h} = \Delta_{x_y^a} = \Delta_x, \quad \Delta_{x_x^a} = s\Delta_x, \quad \Delta_{x^b} = \Delta_x - b'\Delta_b, \quad \Delta_{x^g} = \Delta_x - g'\Delta_g,$$

$$\Delta_{b^h} = \Delta_{b^y} = \Delta_b, \quad \Delta_{b^x} = s(\Delta_b - f_e e_b^x \Delta_x) \quad \Delta_{g^h} = \Delta_{g^y} = \Delta_g, \quad \Delta_{g^x} = s\big(\Delta_g - f_e e_g^x \Delta_x\big).$$

From (A.1) we then obtain the following first-order conditions for the solution to the social planning problem:

$$\frac{\partial \mathcal{L}}{\partial x^h} = 0 \quad \Rightarrow \quad u_x = \lambda(1 - \Delta_x) + \eta\rho^x \qquad \text{(A.2)}$$

$$\frac{\partial \mathcal{L}}{\partial y^h} = 0 \quad \Rightarrow \quad u_y = \mu \qquad \text{(A.3)}$$

$$\frac{\partial \mathcal{L}}{\partial b^h} = 0 \quad \Rightarrow \quad u_e e_b^h = \lambda(p^b - \Delta_b) + \kappa - \eta\rho^b \qquad \text{(A.4)}$$

$$\frac{\partial \mathcal{L}}{\partial g^h} = 0 \quad \Rightarrow \quad u_e e_g^h = \lambda\big(p^g - \Delta_g\big) + \eta\rho^g \qquad \text{(A.5)}$$

$$\frac{\partial \mathcal{L}}{\partial n^x} = 0 \;\; \Rightarrow \;\; \lambda f_n(1+\eta\rho^x) = \theta - \frac{\lambda\Delta_{nx}}{s} \tag{A.6}$$

$$\frac{\partial \mathcal{L}}{\partial b^x} = 0 \;\; \Rightarrow \;\; f_e e_b^x(\lambda+\eta\rho^x) = \lambda\left(p^b - \frac{\Delta_{bx}}{s}\right) + \kappa a_x + \eta\rho^b \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial g^x} = 0 \;\; \Rightarrow \;\; f_e e_g^x(\lambda+\eta\rho^x) = \lambda\left(p^g - \frac{\Delta_{gx}}{s}\right) + \eta\rho^g \tag{A.8}$$

$$\frac{\partial \mathcal{L}}{\partial x_x^a} = 0 \;\; \Rightarrow \;\; -\kappa a_x' b^x = \lambda(1-\Delta_x) + \eta\rho^x \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial n^y} = 0 \;\; \Rightarrow \;\; \mu y_n = \theta \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial b^y} = 0 \;\; \Rightarrow \;\; \mu y_e e_b^y = \lambda(p^b - \Delta_b) + \kappa a_y + \eta\eta^b \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial g^y} = 0 \;\; \Rightarrow \;\; \mu y_e e_g^y = \lambda\left(p^g - \Delta_g\right) + \eta\eta^g \tag{A.12}$$

$$\frac{\partial \mathcal{L}}{\partial x_y^a} = 0 \;\; \Rightarrow \;\; -\kappa a_y' b^y = \lambda(1-\Delta_x) + \eta\rho^x \tag{A.13}$$

$$\frac{\partial \mathcal{L}}{\partial x^b} = 0 \;\; \Rightarrow \;\; \lambda\left(p^b b' + \Delta_{x^b}\right) = \lambda + \eta(\rho^x - b'\rho^b) \tag{A.14}$$

$$\frac{\partial \mathcal{L}}{\partial x^g} = 0 \;\; \Rightarrow \;\; \lambda(p^g g' + \Delta_{x^g}) = \lambda + \eta(\rho^x - g'\rho^g) \tag{A.15}$$

$$\frac{\partial \mathcal{L}}{\partial s} = 0 \;\; \Rightarrow \;\; \lambda[x - p^b b^x - p^g g^x - x_x^a - \pi^f + \bar{c}(1-s) + \Delta_s] = \theta n^s + \kappa a_x b^x - \eta\rho^s \tag{A.16}$$

We now define the shadow prices

$$\tau \equiv \frac{\kappa}{\lambda}, \qquad w^s \equiv \frac{\theta}{\lambda}, \qquad p^{ys} \equiv \frac{\mu}{\lambda}, \tag{A.17}$$

and introduce the political preference α for foreign versus domestic emissions reductions:

$$\eta = \alpha\kappa, \qquad 0 \le \alpha \le 1. \tag{A.18}$$

Using (A.17) and (A.18) wherever relevant, we can restate the first-order conditions (A.2) through (A.16) in the following form:

Optimal consumption of x^h: $$\frac{u_x}{\lambda} = 1 - \Delta_x + \alpha\tau\rho^x \tag{A.19}$$

Optimal consumption of y^h: $$\frac{u_y}{\lambda} = p^{ys} \tag{A.20}$$

Optimal consumption of b^h: $$\frac{u_e}{\lambda} = \frac{p^b - \Delta_b + \tau\left(1+\alpha\rho^b\right)}{e_b^h} \tag{A.20}$$

Optimal consumption of g^h: $$\frac{u_e}{\lambda} = \frac{p^g - \Delta_g + \alpha\tau\rho^g}{e_g^h} \tag{A.21}$$

Optimal input of n^x: $$f_n(1 + \alpha\tau\rho^x - \Delta_x) = w^s \tag{A.22}$$

Optimal input of b^x: $$f_e e_b^x(1 + \alpha\tau\rho^x - \Delta_x) = p^b + \tau(a_x + \alpha\rho^b) - \Delta_b \tag{A.22}$$

Optimal input of g^x: $$f_e e_g^x(1 + \alpha\tau\rho^x - \Delta_x) = p^g + \alpha\tau\rho^g - \Delta_g \tag{A.23}$$

Optimal input of x_x^a: $$-\tau {a'}_x b^x = 1 + \alpha\tau\rho^x - \Delta_x \tag{A.24}$$

Optimal input of n^y: $$p^{ys} y_n = w^s \tag{A.25}$$

Optimal input of b^y: $$p^s y_e e_b^y = p^b + \tau\left(a_y + \alpha\rho^b\right) - \Delta_b \tag{A.26}$$

Optimal input of g^y: $$p^s y_e e_g^y = p^g + \tau\alpha\rho^g - \Delta_g \tag{A.27}$$

Optimal input of x_y^a: $$-\tau {a'}_y b^y = 1 + \alpha\tau\rho^x - \Delta_x \tag{A.28}$$

Optimal input of x^b: $$p^b b' = 1 + \alpha\tau(\rho^x - b'\rho^b) + b'\Delta_b - \Delta_x \tag{A.29}$$

Optimal input of x^g: $$p^b g' = 1 + \alpha\tau(\rho^x - g'\rho^g) + g'\Delta_g - \Delta_x \tag{A.30}$$

Optimal location (s):

$$x - p^b b^x - p^g g^x - x_x^a - \pi^f + \bar{c}(1-s) + \Delta_s = w^s n^x + \tau(a_x b^x - \alpha\rho^s) \tag{A.31}$$

The optimal tax and subsidy rates in the market economy are derived from (A.19) through (A.31) in the following way: Assume that the socially optimal market allocation has in fact been achieved. The market prices w and p^y will then correspond to the shadow prices w^s and p^{ys}, respectively, and the private marginal utility of income will be equal to the marginal shadow value of the numeraire final good, λ. Now choose the tax and subsidy rates introduced in Section 4 so as to ensure that the first-order conditions for the privately optimal choices of consumption, input levels and business location coincide exactly with the social planner's first-order conditions (A.19) through (A.31), given the equality between market prices and shadow prices. Assuming there is a unique solution to the social planning problem satisfying these first-order conditions, and a unique general equilibrium in the market economy, it must then be the case that the market equilibrium with the chosen tax and subsidy rates and the relative prices $w = w^s$ and $p^y = p^{ys}$ corresponds to the socially optimal allocation.

To illustrate our procedure, consider the first-order condition (32) for the profit-maximizing choice of labour input in the tradable goods sector:

$$(1 + \tau^x) f_n = w. \tag{32}$$

If the output subsidy rate is

$$\tau^x = \alpha\tau\rho^x - \Delta_x, \tag{A.32}$$

the private optimum condition (32) will coincide with the social planner's optimum condition (A.22) for $w = w^s$. Furthermore, the first-order condition (33) for the profit-maximizing choice of fossil fuel input in the tradable goods sector is

$$(1 + \tau^x) f_e e_b^x = p^b + \tau^{bx} a_x. \tag{33}$$

If the tax on emissions from tradable goods firms is

$$\tau^{bx} = \tau\left(1 + \frac{\alpha\rho^b}{a_x}\right) - \frac{\Delta_b}{a_x}, \tag{A.33}$$

we see by inserting (A.32) and (A.33) into (33) that the firm's profit-maximizing choice of fossil fuel input will coincide with the social planner's optimal choice given by (A.22).

Following this procedure, and using the private optimum conditions stated in section 4 plus the social planner's optimum conditions (A.19) through (A.31), we end up with the following optimal tax and subsidy rates:

Optimal tax on x^h: $$\tau^{xh} = \alpha\tau\rho^x - \Delta_x \tag{A.34}$$

Optimal tax on b^h: $\tau^{bh} = \tau(1 + \alpha\rho^b) - \Delta_b$ (A.35)

Optimal tax on g^h: $\tau^{gh} = \alpha\tau\rho^g - \Delta_g$ (A.36)

Optimal output subsidy: $\tau^x = \alpha\tau\rho^x - \Delta_x$ (A.37)

Optimal tax on b^x: $\tau^{bx} = \tau\left(1 + \frac{\alpha\rho^b}{a_x}\right) - \frac{\Delta_b}{a_x}$ (A.38)

Optimal tax on g^x: $\tau^{gx} = \alpha\tau\rho^g - \Delta_g$ (A.39)

Optimal subsidy to x_x^a: $\tau^{ax} = 1 - \frac{\tau^{bx}}{\tau} + \frac{\tau^{bx}}{\tau}(\Delta_x - \alpha\tau\rho^x)$ (A.40)

Optimal tax on b^y: $\tau^{by} = \tau\left(1 + \frac{\alpha\rho^b}{a_y}\right) - \frac{\Delta_b}{a_y}$ (A.41)

Optimal tax on g^y: $\tau^{gy} = \alpha\tau\rho^g - \Delta_g$ (A.42)

Optimal subsidy to x_y^a: $\tau^{ay} = 1 - \frac{\tau^{by}}{\tau} + \frac{\tau^{by}}{\tau}(\Delta_x - \alpha\tau\rho^x)$ (A.43)

Optimal tax on $b(x^b)$: $\tau^b = \frac{\alpha\tau(\rho^x - b'\rho^b) + b'\Delta_b - \Delta_x}{b'}$ (A.44)

Optimal subsidy to $g(x^g)$: $\tau^g = \frac{\alpha\tau(g'\rho^g - \rho^x) - g'\Delta_g + \Delta_x}{g'}$ (A.45)

Optimal tax on y^h: $\tau^{yh} = 0$ (A.46)

Optimal location subsidy:

$$T^x = (\tau^{bx} - \tau)a_x b^x - \tau^{ax}x_x^a + \tau^{gx}g^x - \tau^x x + \alpha\tau\rho^s + \Delta_s \quad \text{(A.47)}$$

Setting all the Δ's with terms-of-trade effects equal to zero in (A.34) through (A.47), we obtain the near-optimal tax and subsidy rates reported in eqs. (50) through (56) in Section 5.

APPENDIX B

TERMS-OF-TRADE EFFECTS VERSUS LEAKAGE EFFECTS

In a small open economy the terms-of-trade effects of domestic policies are likely to be quite small relative to the social cost of leakage generated by these policies. This may be illustrated by a simple example: Suppose the initial domestic net import of fossil fuel is m^b, and consider an increase in domestic fossil fuel consumption b^d that increases the relative world market price of fossil fuel by the amount $\frac{\partial p^b}{\partial b^d}$. The impact Δ_b on domestic real income of this change in the terms of trade will then be

$$\Delta_b = -m^b \cdot \frac{\partial p^b}{\partial b^d}, \tag{B.1}$$

and the leakage effect L_b on foreign emissions will be

$$L_b \equiv \frac{\partial L}{\partial b^d} = z \cdot \frac{\partial b^f}{\partial p^b} \cdot \frac{\partial p^b}{\partial b^d}, \tag{B.2}$$

where z is the number of foreign countries, and $\frac{\partial b^f}{\partial p^b} < 0$ is the impact of the change in the fossil fuel price on fossil fuel consumption in the representative foreign country. The domestic social cost of the leakage effect (B.2) is $\alpha \tau L_b$, so from (B.1) and (B.2) we can derive the following ratio between the real income effect of the change in the terms-of-trade and the social cost of the leakage effect, where B^f is the total foreign consumption of fossil fuel, ε_b^f is the foreign numerical own price elasticity of fossil fuel demand, and τ / ρ^b is the ratio of the domestic social cost of carbon to the fossil fuel price:

$$\frac{\Delta_b}{\alpha \tau L_b} = -\frac{m^b}{\alpha \tau z \frac{\partial b^f}{\partial p^b}} = \frac{m^b / B^f}{(\tau / p^b) \alpha \varepsilon_b^f}, \qquad B^f \equiv z b^f, \qquad \varepsilon_b^f \equiv -\frac{\partial b^f}{\partial p^b} \frac{p^b}{b^f} > 0. \tag{B.3}$$

In a small open economy the fraction m^b / B^f in the numerator of (B.3) will be very close to zero, and even in larger economies the ratio of domestic net imports of fossil fuel to total world consumption of fossil fuel will be a small number. For a realistic price elasticity of fossil fuel demand, the terms-of-trade effect of a change in domestic fossil fuel demand will therefore be quite small relative to the resulting social cost of leakage, unless the concern about leakage (α) is also close to zero.[11]

[11] In the short run, the price elasticity of fossil fuel demand is typically estimated to be quite low, but since our model focuses on a long run general equilibrium, our elasticity ε_b^f should be interpreted as a long run price elasticity. Given the possibility of substituting towards green energy and other production factors, the long run price elasticity of fossil fuel demand may be substantial.

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