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Abstract

In recent years, a significant problem with the carbon credit market has been the higher than initially predicted price volatility. It is essential to study the market in a repeated-period dynamic setting to identify the factors enabling high fluctuations in prices. In this paper, we examine the dynamic auction design and propose a method to curb price volatility through a flexible supply cap. The equilibrium analysis shows that modifying the cap on per period supply can decrease price fluctuations. Currently, the government or the auctioneer sets a per-period limit on the supply, which reduces at a fixed rate over time. However, this paper suggests that a flexible cap on the per-period supply would be a better alternative. Specifically, we show that correlating the supply rate with expected future demand results in a more stable price.

JEL-Codes: D430, L110, L420.

Keywords: dynamic mechanism design, auctions, emissions permits, environmental regulation, climate change.

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1 Introduction

One of the most popular market-based solutions to limit greenhouse gas (GHG) emissions is to use cap-and-trade schemes. Under these schemes, firms use emission credits to pay for GHG emissions. The largest share of GHGs is CO2; thus, we focus on the case of carbon emission allowances, which are called carbon credits.[1]

The carbon credit market has historically exhibited high price fluctuations.[2] Demand shocks are one of the major factors that cause price fluctuations. For example, Figure 2 shows volatility in the California cap-and-trade program with a drastic decrease during 2016 and 2017. While price fluctuations do not obstruct the operation of cap-and-trade markets and price movements are part of the auction mechanism, large-scale price volatility may impede the reduction of carbon emissions. Sharp price increases can increase the cost of compliance for firms, and sharp price decreases can eliminate the incentive to invest in low-carbon technology.[3] Thus, attenuating large fluctuations in carbon prices in the short run can ensure environmental effectiveness. In this paper, we propose adopting a flexible supply cap that depends on the expected future demand as a possible solution to curb price volatility.

Carbon credit markets have responded to extreme fluctuations by using fixed prices and quantitative caps in the emissions trading system and carbon credit auctions. For example, in the cap-and-trade program of California, quarterly carbon credit auctions have a price ceiling and a reserve price, which escalate over time.[4] Similarly, in the European Union Emission Trading System (EU ETS), a "Market Stability Reserve" has been created, which releases permits when there are very few in circulation and withdraws them when there are considerably high amounts in circulation.[5] These fixed cap methods perform well under certainty; however, as noted by [Ellerman and Wing, 2003], in the presence of uncertainty, flexible caps and fixed caps lead to different results. Our method suggests that in the presence of demand uncertainty, a more flexible

[1] This model is also applicable to other markets for GHG emissions.

[2] For example, Nordhaus (2007) showed that the demand for allowances was likely inelastic in the short run, which caused high price volatility. Dutta (2018), Wang (2017), and Zhang and Sun (2016) also provided evidence of price volatility in carbon credit markets.

[3] For details, see [Köppl et al., 2011] and [Borenstein et al., 2019]

[4] Additionally, manufactures which are trade-sensitive and energy-intensive have an output-based updating allocation system, whereby free allowances are conveyed in proportion to production levels in previous periods. For details, refer to [Borenstein et al., 2019]

[5] For further details, refer to *https://ec.europa.eu/clima/policies/ets/reform_en*

limit on supply can decrease price volatility. Specifically, the cap on allowances available for sale should be structured to be directly proportionate to the intensity of the demand shock.[6]

This paper considers the primary market where the government or social planner sells carbon credits through an auction. To examine the price volatility, we set up a dynamic model where carbon credits are sequentially sold using a uniform auction design.[7] This is the most common auction format used by markets such as RGGI carbon dioxide (CO2) and the EU ETS (CO2).[8] Additionally, our model attempts to capture essential features of the market, such as differences in demand urgency across firms and stochastic demand. We capture these features within the auction design to create a more realistic market design. We allow firms to differ in their urgency of obtaining carbon credits; this is modeled as firms having different deadlines. The firm-specific deadlines for buying carbon credits may be due to the vintage of the carbon credit (i.e., expiry of the credit) or the current portfolio of credits owned by the firm. This feature helps us capture the heterogeneity among firms in terms of their demand duration.

In this setting, we derive the equilibrium bid and show that the bid is truthful and ex post incentive compatible. The optimal bidding strategy indicates that buyers consider their future-period payoffs when deciding the expected value of winning in the current period. Thus, the outside option value is endogenously determined through the expected payoff of auctions in future periods.[9]

In a dynamic setting, the introduction of demand uncertainty changes the optimal supply rate. In particular, expectations about future supply and demand impact the equilibrium price and increase price fluctuations. Consequently, we show how relating the supply with the expected future demand can decrease the impact of the demand shocks on price.

[6] For literature on the comparison of various price control instruments, see [Newell and Pizer, 2008], [Newell and Pizer, 2003], [Pizer, 2002], [Roberts and Spence, 1976], [Weitzman, 1978], [Weitzman, 1974], [Yohe, 1978], and [Stavins, 1996]

[7] We refer to the dynamic setting as the case in which a finite number of goods are sold to buyers that arrive over time. There are two types of dynamic setting in the single-good case. The first type holds the set of buyers fixed and changes their types over time as a function of allocations selected in earlier periods (for example, [Athey and Segal, 2013], [Eso and Szentes, 2007]). In the second type, a finite number of goods are sold to buyers that arrive over time. This paper considers the second type of dynamic setting, which we refer to as the changing buyer case. The term "changing buyer type" is taken from the dynamic mechanism design literature review [Vohra, 2012]

[8] For details, see [Lopomo et al., 2011]

[9] Apart from carbon credit auctions, this model can be applied to any market where the bidders differ in their valuation of the object and urgency of acquiring the good, including spectrum auctions (for wireless networks) or electricity markets with different delivery dates.

The comparative statistics section analyzes how demand shocks and supply rate impact the equilibrium price for carbon credits. The results show that an increase (decrease) in future supply decreases (increases) the current price. The change in future supply affects the current price by increasing the opportunity cost of winning for the firm. A raise in the future supply would increase the probability of winning carbon credits in future periods, thereby increasing the opportunity cost of winning in the current period. Meanwhile, we find that an increase (decrease) in expected future demand causes an increase (decrease) in the current price. The intuition behind the rise in the current price is that a rightward shift in future demand makes the firm demand more in the current period because the carbon credits can be stored and used in future periods. The outward shift of the demand curve consequently increases the equilibrium price. Therefore, this paper suggests that the future supply should be a function of the expected future demand. In particular, the government can decrease the supply whenever the expected future demand decreases and vice versa. We show that such a policy can reduce price fluctuations in the market.

Related literature This paper is related to the literature on price and supply restrictions imposed in the emissions market.[10] Earlier work such as [Weitzman, 1974] recognized that the optimal instrument would be a contingency message that provided instructions according to the state of the world.

The current literature has introduced new models that help us understand climate policy responsiveness. They suggest various proposals for the structure of carbon pricing instruments. This literature includes papers on the index regulation that suggests that the emission cap should be proportionate to an index such as the GDP or output. For example, [Jotzo and Pezzey, 2007] assess how well intensity targets indexed to future realized GDP can handle uncertainties in international GHG emissions trading. Additional papers that consider indexing in the emissions market are [Quirion, 2005], [Sue Wing et al., 2006], [Newell and Pizer, 2008], [Branger and Quirion, 2014], and [Ellerman and Wing, 2003]. The literature has investigated flexible caps, and it has considered the net benefit of applying a flexible cap using non-auction models such as cost-benefit analysis and emissions prediction & policy analysis.[11]. This paper extends this literature by analyzing how

[10] The early work on this topic was conducted by [Roberts and Spence, 1976] and [Weitzman, 1978]. They considered price ceilings and floors under demand and supply uncertainty in a static model.

[11] An emissions prediction and policy analysis (EPPA-EU) model is used in [Ellerman and Wing, 2003] and

a flexible cap affects the strategic bidding decision of a firm, which affects the final price in the primary sale (during the auction stage).

Apart from indexing, other papers in this literature study how pricing and quantitative instruments can be more responsive to economic fluctuations. [Heutel, 2012] finds that the optimal policy will accommodate the procyclical behavior of carbon emissions. The cap on the emissions trading system in this paper is reduced during recessions and increased during booms. [Doda, 2016] provides a comprehensive review of the literature comparing fixed and responsive caps in an emissions trading system. Our work extends the above literature by introducing a flexible cap in carbon credit auction design. The previous literature focused on using market equilibrium as the outcome of demand and supply in the emissions trading market. In this study, we focus on examining how the flexible cap interacts with cap-and-trade auctions. This paper aims to model the responsive supply cap as part of the auction mechanism by including dynamic features in the market, modeling the uncertainty of future demand, and demonstrating how the firm's bid and equilibrium auction price are affected by the flexible supply cap. We consider a dynamic model, which also accounts for heterogeneity in permit lifetime and compliance deadlines.

This paper is also related to the literature on the dynamic mechanism design with multi-dimensional private information (see the survey by [Bergemann and Said, 2010] for details).[12] It extends this literature to investigate the optimal auction for an auctioneer with an unknown number of buyers and sellers in each period. This paper is also related to the literature on efficient sequential auction with impatient buyers and those on stochastic auctions.[13]

The remainder of the paper is structured as follows: Section (2) and section (3) describe the

[Sue Wing et al., 2006]. The cost-benefit analysis was introduced by [Weitzman, 1974] and used in [Quirion, 2005] and [Newell and Pizer, 2008]

[12] The most relevant paper for our work is [Pai and Vohra, 2013], who consider the optimal auction for a single seller selling multiple units to stochastically arriving bidders; they also allow the arrival time to be private information. The main difference in our paper is that we consider the auctioneer's problem of identifying an efficient mechanism with stochastically arriving sellers. Furthermore, [Pai and Vohra, 2013] assumes perfectly patient bidders and focuses on the optimal auction to maximize the seller revenue. Another related paper is [Mierendorff, 2013], which investigates a revenue-maximizing mechanism for a seller selling a single good in a dynamic environment with buyers having multi-dimensional private information. The main difference in our work is that we consider an efficient mechanism in a dynamic environment with multiple and stochastically arriving sellers.

[13] The most relevant paper in the literature on sequential auction with impatient buyers is [Gershkov and Moldovanu, 2010]. They examine the allocation of a set of durable goods to a dynamic buyer population. In their setting, objects are durable, whereas in this paper, objects are non-durable, and the total supply in every period is stochastic. For the literature on stochastic auctions, refer to [McAfee and McMillan, 1987], [Mierendorff, 2013], [Said, 2012] and [Jeitschko, 1999]

generalized model setup and derive the equilibrium bid. Section (4) applies the model to an emissions trading market, and section (5) concludes the paper.

2 Model setup

Consider a sequential auction in infinite, discrete-time period model, $t \,\epsilon\, \{0, 1, ...\infty\}$. In each period multiple units of a homogeneous good are auctioned. Buyers with single unit demand arrive over time. Buyer i has a valuation v_i, which is an i.i.d random draw from the distribution $F(.)$ on $[\underline{v}, \bar{v}]$. The demand for each buyer lasts for multiple periods. In particular, the buyer has an arrival time a_i and a demand duration k_i. This implies that the demand lasts for all $t \,\epsilon\, \{a_i, ..., (a_i + k_i)\}$. Thus apart from heterogeneous valuation, a buyer also differs in terms of their demand lifetime. The type of a buyer is a triplet consisting of his valuation, arrival time and demand duration, $x = (v_i, a_i, k_i)$; and the type space is given as $X = [\underline{v}, \bar{v}] \times [\underline{a}, \bar{a}] \times [1, \bar{k}]$. A buyer's type is an i.i.d random draw from a commonly known distribution $\prod_{i=\{v,a,k\}} F_i = F_v \times F_a \times F_k$ over X. We assume that the three components of the type space, i.e. valuation, arrival time and demand duration are independent.

In our model, in period t, an agent of type $x_i = (v_i, a_i, k_i)$ who faces a possible payment z_t derives the following instantaneous utility:

$$U_{i,t}(v_i, k_i) = \begin{cases} (v_i - z_t), & \text{if he wins the auction} \\ 0, & \text{otherwise.} \end{cases}$$

The buyer's arrival rate is stochastic; In any period t, n_t new buyers arrive, where n_t, is an i.i.d random draw from the distribution F^n on $\{1, ..., \bar{n}\}$. The total number of potential buyers is given by $N \,\epsilon\, \mathbb{N}$ such that $N \geq \bar{n}\bar{k}$.[14] Additionally, for each buyer the probability of his demand surviving in the next period is τ, where $\tau \,\epsilon\, [0, 1]$. The parameter τ captures the future demand uncertainty that the buyer might have due to external economic reasons.

Sellers, with single period supply, arrive over time. Seller j has a per-unit value v^j, which is assumed to be the same for all sellers and normalized to zero, $v^j = 0 \;\forall\; j \,\epsilon\, S$.[15] We allow the sellers

[14] Recall that $\bar{n}$ is the upper bound on the new buyers arriving each period and $\bar{k}$ is the upper bound on the number of periods a demand can last for a buyer

[15] This assumption is made so that we can concentrate on the buyers' side in this paper

to have multiple units to sell but assume that sale decisions are independent of each other; thus this is equivalent to each seller selling single unit item.

Similar to the buyers, the sellers also arrive scholastically. However, unlike the buyers, sellers are active only for one period. The units auctioned by the sellers are identical, and these units remain valid only for one period. Each unit is considered for sale independent of its provider, as sellers offer homogeneous goods. In any period t, m_t new units are available in the market. It is assumed that for each t^{th} period, m_t is an i.i.d random draw from distribution F^m on $[\underline{m}, \bar{m}]$. The total number of potential seller is given by $M \in \mathbb{N}$ such that $M \geq \bar{m}$.

Information Structure: The distribution of buyer's type space, the distribution of buyers and sellers arrival rate, and seller's valuation for the object are assumed to be common knowledge. The buyer has private knowledge about his type, composed of his valuation and demand lifetime. He does not have knowledge of the exact number of other buyers and sellers, their types, or bid reports. The arrival rate of buyers and the number of items in each period are stochastic. Thus, the exact per period demand and supply is unknown for future periods.

The timeline of the game is as follows. In each period, t, the active buyers report a bid for a single unit to the auctioneer. The strategy set of the buyers in period t compromises of their bid in period t, i.e. $b_t(v_i, r_i)$, where v_i is the value and r_i is the number of active periods left at t^{th} period. The active buyers consist of new buyers arriving in the current period and the existing buyers who are still active and have unfulfilled demand. The active sellers report the items available for sale. After receiving bids from the active buyers and an estimate of the number of items to be sold from the active sellers, the auctioneer holds an auction to decide the number of items to be traded, denoted as S_t, and the clearing price denoted as z_t.

We now give a detailed description of the auction mechanism. The auctioneer holds a series of static uniform price auctions. Bidders simultaneously submit sealed bids for the item, and sellers report the units available for sale. The auctioneer calculates the total number of items to be traded, S_t, by equating the demand and supply in the current period. If the number of items traded in the market is equal to the available units, i.e. $S_t = m_t$, the auctioneer accepts all the active sellers' sale requests. On the other hand, if the number of items traded is less than available units i.e. $S_t < m_t$, the auctioneer accepts the sale request of randomly selected S_t number of sellers. The buyers are ranked in ascending order of their bids, and S_t highest bids are accepted to allocate the item. Each winning bidder gets one unit of the item and pays the price equal to the highest losing bid.

To formally determine the clearing price, we denote the order statistic for bid as $b^{(l)}$, which represents the l^{th} highest order statistic of the bidding values. Using the ordered bids, the auctioneer selects S_t highest bids in period t to trade. So the bid of a buyer is accepted for trade if $b_t(v_i, r_i)$ ϵ $\{b^{(1)}, b^{(2)}...........b^{(S_t)}\}$. Every winning bidder pays the bid of the highest losing bidder, i.e. $z_t = b^{(S_t+1)}$. The price paid by the winning buyers, in any period t, is equal to the $(S_t + 1)^{th}$ highest order statistic. All buyers who bid above z_t win the item at a price z_t and each accepted sale request yields a payment equal to z_t for sellers.

We solve the repeated period problem using Subgame Perfect Bayesian Nash equilibrium as the solution concept. In each period, the buyers report a bid for a single unit of the item to the auctioneer. The equilibrium defines a set of strategies and beliefs, such that given the opponents' strategies, the expected payoff of every buyer is maximized in each period.

Observation 1. *Notice that, although the arrival time a_i is private knowledge, the buyer does not have any incentive to lie about the arrival period. This is due to the independence of the buyer's payoff function and the arrival time a_i. Thus the relevant private information of buyer i is (v_i, k_i).*

3 Symmetric Equilibrium

Bidder i's bidding strategy is defined as $b^i = \{b_1(v_i, r_i), b_2(v_i, r_i), ...b_\infty(v_i, r_i)\}$, where $b_t(v_i, r_i)$ denotes the bid in the t^{th} period given that the bidder's value is v_i and demand will last for r_i periods after period t.[16] Note that the bidders arrive and leave at different time periods, thus the subscript on period t_{r_i}, i.e r_i, denotes the number of active periods for bidder i in period t_{r_i}. As this is a dynamic setting, we start by defining the state variables. Let $\sigma_t = \{m_t, n_t\}$ denotes the auction "state" in period t. Here n_t denotes the set of active buyers in period t; n_t is calculated as $n_t = \sum_{x=0}^{\bar{k}} n_{t,x}$, where $n_{x,t}$ denotes the number of bidders with x periods of active demand in period t. Also, recall m_t represents the number of sale units available in period t.

Let us now consider the maximizing objective function for the buyer which is the total lifetime

[16] All this assumes, of course, that he particular bidder has not already won an object so is still active in period t

payoff after realization of demand and supply, denoted by $V(v_i, r_i|\sigma)$

$$V(v_i, r_i|\sigma_t) \equiv \Bigg\{ Pr\Big(b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big|\sigma_t\Big)\mathbb{E}\Big[v_i - b^{(m_t+1)}(v_i, r_i)\Big|b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big] + \Big[1 - Pr\Big(b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big|\sigma_t\Big)\Big]\tau\int_{\sigma_{t+1}} V(v_i, r_i - 1|\sigma_{t+1})\Bigg\} \tag{1}$$

The first term represents the expected payoff from auction in the first period and the second term is the future period payoff integrated over possible arrival rate σ_{t+1}. The future payoff is equal to the total payoff for value v_i and $(r_i - 1)$ period left. Now we define the total payoff ex-ante, i.e., before the realization of the state variable 'σ_t'

$$W(v_i, r_i) \equiv \int_{\sigma_{t_{r_i}}} \tau\Bigg\{ Pr\Big(b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big|\sigma_{t_{r_i}}\Big)\mathbb{E}\Big[v_i - b^{(m_t+1)}(v_i, r_i)\Big|b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big] + \Big[1 - Pr\Big(b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big|\sigma_t\Big)\Big]W(v_i, r_i - 1)\Bigg\} \tag{2}$$

Using the total payoff ex-ante, we can rewrite the ex-post per period payoff in eqn(1) as

$$V(v_i, r_i|\sigma_{t_{r_i}}) \equiv \Bigg\{ Pr\Big(b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big|\sigma_{t_{r_i}}\Big)\mathbb{E}\Big[v_i - W(v_i, r_i - 1) - b^{(m_t+1)}(v_i, r_i)\Big|b(v_i, r_i) > b^{(m_t)}_{j\neq i}\Big] + W(v_i, r_i - 1)\Bigg\} \tag{3}$$

Pseudo type for each period:

Observe that we have the probability of winning in each period defined in terms of the bid, in order to define that in term of the bidder's type, we will define a per period pseudo type for each period t and bidder i as $\eta_{i,t}$. Let $G_t(.)$ be the distribution for $\eta_{t,i}$. The pseudo type is defined as follows :

$$\eta_{i,t} = \begin{cases} v_i - \sum_{l=t+1}^{k_i} \int_{\sigma_l} \tau^{l-t} G_l^{(m_l)}(\eta_{i,l}|\sigma_l)\Big(\eta_{i,l} - \mathbb{E}[(\eta^{(m_l)}_{j,l})_{j\neq i}|\eta_{i,l} > \eta^{(m_l)}_{j,l}]\Big), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise.} \end{cases}$$

The pseudo type helps simplify the payoff function as well as the bid. Following lemma shows that relation between pseudo type and the payoff function

Lemma 1. *The equilibrium bid is increasing in pseudo type and the total payoff in period t can be* *rewritten as follows :*

$$W(\eta_{i,t}) = \int_{\sigma_t} \tau G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \tag{4}$$

$$V(\eta_{i,t}) = G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \tag{5}$$

We can use this equation in order to determine equilibrium bid functions, as demonstrated in the following result. We focus on symmetric Bayesian Nash equilibrium.

Theorem 1. *If auction in every period is vickery auction then the equilibrium bidding strategy in the dynamic auction game is given as:*

$$b_i(\eta_{i,t}) = v_i - W(\eta_{i,t+1})$$

or

$$b_i(\eta_{i,t}) = \eta_{i,t}$$

4 Applications and Comparative Statics : Emission market

Here we look at the case of fixed decreasing supply with an uncertainty of demand in each period. In the emission auction markets, the goal is to reduce the supply of emission credits, which reduces the total emission. This is usually done by deciding in advance the decreasing rate of supply of emission credits. The general model, introduced in the previous section, is modified in terms of supply rate to fit the market better. For this section, instead of having a stochastic supply, we have the supply decreasing at a constant rate.[17] Let the rate at which supply decreases be denoted by $\lambda \ \epsilon \ [0, 1]$. This changes the state variable $\sigma_t = (\lambda m_t, n_t)$. Additionally, the probability of active demand τ will affect the future as well as the current value for the object. This is because any change in demand for credit in the future affects current demand as well. Everything else is the same. This section looks at comparative statics in terms of how λ and τ, which gives insight into

[17] To focus on the effect of supply and demand dynamics we will be abstracting away from the multi-unit demand nature of this market

how the supply rate and the demand uncertainty affect bidder behavior. The first two propositions look at how the supply rate, i.e. λ affects the market variables. These propositions enable us to evaluate how the supply rate can be strategically used to stabilize the price.

Proposition 1. *The bidder's expected payoff is increasing with the increase in the future supply rate.*

$$\frac{d(V(\eta_{i,t}|\sigma_t))}{d(\lambda)} \geq 0 \tag{6}$$

The above proposition shows that the expected payoff in each period t is positively correlated with the future periods' supply rate. The intuition behind the result is that as the supply increases, the chances of acquiring credits in the future increase, thereby increasing the total expected payoff of the buyer.

To understand the impact of supply rate on equilibrium market outcomes, the next proposition focuses on the supply rate's effect on the bid and the equilibrium price.

Proposition 2. *The bid and price are decreasing with the increase in the future supply rate.*

$$\frac{d(b(\eta_{i,t}))}{d(\lambda)} \leq 0$$
$$\frac{d\mathbb{E}(P_t)}{d(\lambda)} \leq 0$$

The price and the bid in the current period are negatively affected by an increase in the supply rate. Thus, even though the total payoff increases for the bidder, they will still decrease the current period's bid. The intuition behind the increase in the current period bid is that an increase in the supply rate increases the payoff in the future periods, which increases the opportunity cost of winning in the current period (t^{th} period). Thus, the buyer's bid and the auction price are negatively correlated to the supply rate. We have now established the effect of the supply rate on market variables. However, to see how the supply rate can stabilize the effect of demand uncertainty on the price, we also need to look at the effect of demand uncertainty on market outcomes. We now look at how do market variables react to demand uncertainty.

Proposition 3. *The bidder's expected payoff is decreasing in the uncertainty of demand.*

$$\frac{d(V(\eta_{i,t}|\sigma_t))}{d(\tau)} \geq 0 \tag{7}$$

Recall that τ represents the probability that bidders will have demand for the credits in the next period.[18] Thus, $1 - \tau$ represents the probability of demand reduction in the next period. For this study, we take $1 - \tau$ as a proxy for expectations about future demand uncertainty. Proposition(3) shows that the payoff is negatively affected when the demand uncertainty increases in the market.

Proposition 4. *The current bid and price increase with increase in expected future demand.*

$$\frac{d(b(\eta_{i,t}))}{d(\tau)} \geq 0$$
$$\frac{d\mathbb{E}(P_t)}{d(\tau)} \geq 0$$

This shows that increase(decrease) in expected future demand increases(decrease) the price.

Now that we have established the individual effects of supply rate and uncertainty on price, we will look at how to utilize it to stabilize the price path. From propositions 2 and 4 we see that λ and τ have an opposite effect on the price. Thus, an optimal strategy will negatively correlate the future supply rate and expected demand. Let the proposed supply rate be defined as follows :

$$\lambda_{new} = f(\tau) \tag{8}$$

$$f'(\tau) \geq 0 \tag{9}$$

The new supply rate changes with the change in demand. The next proposition proves that the change in price due to change in demand uncertainty would be less in case of the new supply rate.

Proposition 5. *In the even of change in expected future demand, the change in price would be lower in case of the new supply rate.*

The above proposition implies that the expected decrease in future demand will have a lower impact on price fluctuation in the flexible cap is adopted as proposed in equation(8). To understand

[18] this is assuming bidder has active demand next period and is between his entry and deadline period.

the motivation for this result, we should first analyze the impact on price in the absence of the adjustment in the supply rate. In this case, impact of change in future demand on expected price is given by $\frac{d\mathbb{E}(P_t|\lambda)}{\delta(\tau)}$. This is true because only τ changes as future demand changes; On the other side, in the new supply rate case, with change in future demand the expected price will be affected in two ways. Firstly, it will change due to change in τ; additionally, it will change due to change in supply. Mathematically this would look like

$$\text{Case1: fixed rate} \rightarrow \frac{d\mathbb{E}(P_t|\lambda)}{\delta(\tau)} \tag{10}$$

$$\text{Case2: proposed flexible rate} \rightarrow \frac{\delta\mathbb{E}(P_t|\lambda)}{\delta(\lambda)}\frac{\delta(\lambda)}{\delta\tau} + \frac{\delta\mathbb{E}(P_t|\lambda)}{\delta(\tau)} \tag{11}$$

Note that from proposition(4) we know that the expected price and future demand are positively correlated. Additionally, from the above two equations it is evident that the effect in the two cases differ due to the first term in equation(11), which is negative due to proposition 2 and equation(8). Thus, the change in price due to demand shock is lower in case of the proposed supply rate. □

4.1 Discussion: The practical implications of the results

This section provides data evidence that confirms the equilibrium bidding behavior derived in our model. Additionally, we look at price movements in the emission markets with fixed caps and identify potential residual fluctuations in the price due to short-term shocks. The evidence of price fluctuation in the markets with fixed caps further motivates an examination of flexible caps.

Recall that the optimal bidding strategy indicates that the buyers consider their future period payoffs when deciding the value of winning in the current period. This consideration would result in a correlation between current price and advance price of carbon credits. The dependence of current prices on the expected outcome of future auctions is evident if we compare the current and future prices in the California cap-and-trade market. Figure(2) plots the prices for current and advance prices, which appear highly correlated. Apart from the auctions held in the early periods, the current price closely follows the future price, which indicates that the current bid also accommodates the effect of future prices.

As noted by [Nordhaus, 2015] ,[Dutta, 2018], and [Zhang and Sun, 2016], high volatility in price is a problem in the carbon credit market. Even after the price caps, there can be high variance in price within the bounded prices. Let us use the example of the RGGI carbon credit market. Figure(1) shows the price path with significant events during the timeline. Here, we observe that the price fluctuates with changes in demand. For example, 2016 saw a substantial fall in price when the demand uncertainty increased because the supreme court halted the Environmental Protection Agency's Clean Power Plan.[19] This example shows that demand shocks can cause sharp price increases. In our model, the proposed flexible supply rate would have temporarily adjusted, thereby reducing the intensity of price drops from the temporary demand shock.

Additionally, in the European Union Emissions Trading System (EU ETS), shocks such as technological progress, weather conditions, and prices in related industries have caused a high degree of price fluctuation in the market. For example, EU allowances saw a drastic increase in prices in January 2005. According to the Carbon Market Monitor 2005 Review at PointCarbon, the sharp price change was due to high gas and oil prices (specifically observed in the UK), low coal prices, and the onset of cold weather.[20] More recently, the economic downturn caused by COVID-19 has caused a drop in carbon credit prices. For example, the EU ETS allowance prices decreased in the first quarter of 2020 to €17/tCO2e (US$19/tCO2e) compared to approximately €25/tCO2e (US$27/tCO2e) over 2019.[21]

Thus, past data shows the necessity of introducing responsive quantity caps that responds to uncertainty in the market. As discussed in the literature, multiple papers have suggested such a mechanism; our paper contributes by suggesting a demand-dependent cap at the auction stage. This cap may also have economic appeal. Adjusting the cap according to demand shocks can decrease the expected costs incurred for reaching a particular environmental target. This work shows that linking the supply rate to changing market factors will stabilize the price. Further, empirical analysis is required to estimate the exact functional form of the flexible cap.

[19] For details, see "Opinion: Supreme Court puts the brakes on the EPA's Clean Power Plan" - February 9, 2016, Washington Post.

[20] For details, see Carbon Market Monitor 2005 Review, Jan. 2006, available at: http://www.pointcarbon.com/research/carbonmarketresearch/monitor/ and [Mason, 2009]

[21] For details, refer to https://openknowledge.worldbank.org/handle/10986/33809

5 Conclusion

In this paper, we analyzed a dynamic auction setting with the stochastic arrival of bidders and multi-dimensional bidder's type. We derived the BNE bid in the repeated game and applied this setting to the cap-and-trade scheme for the auction of carbon credits. The setup was used to understand the fluctuation of price in this market. Price uncertainty is a significant concern in the cap-and-trade market because firms need a more stable short-term supply of carbon credits to change to more renewable energy. The paper identifies two factors that affect the price fluctuation: the rate of supply and uncertainty in future demand. The uncertainty in future demand is not in the control of the auctioneer (or the government in this case). However, the rate of supply is decided by the government. Thus, the suggested policy in this paper is that the government should correlate the supply rate with the uncertainty in the market. Specifically, they should decrease the future supply rate when the future demand uncertainty in the market increases. In other words, increase the supply rate as the future demand expectation improve. The results show that this policy will result in a more stable price over time. The paper also analyzed the general model setup, which can be applied to other markets. One key feature of the paper is that we looked at the multi-dimensional type of bidders. Thus, this model can fit any market where the bidders differ by more than only the object's value. Future works can extend this setting to multi-unit demand and examine double auction settings, which can further generalize the auction design.

This work shows that linking the supply rate to demand shocks can stabilize the price. Furthermore, empirical analysis is required to understand the implementation of such policy. [Borenstein et al., 2019] conducted an extensive study on how different factors affected the price fluctuation and provided simulations. Since they looked at the secondary trading market, similar studies are required to analyze the price fluctuation in the primary market of selling carbon credits through auction.

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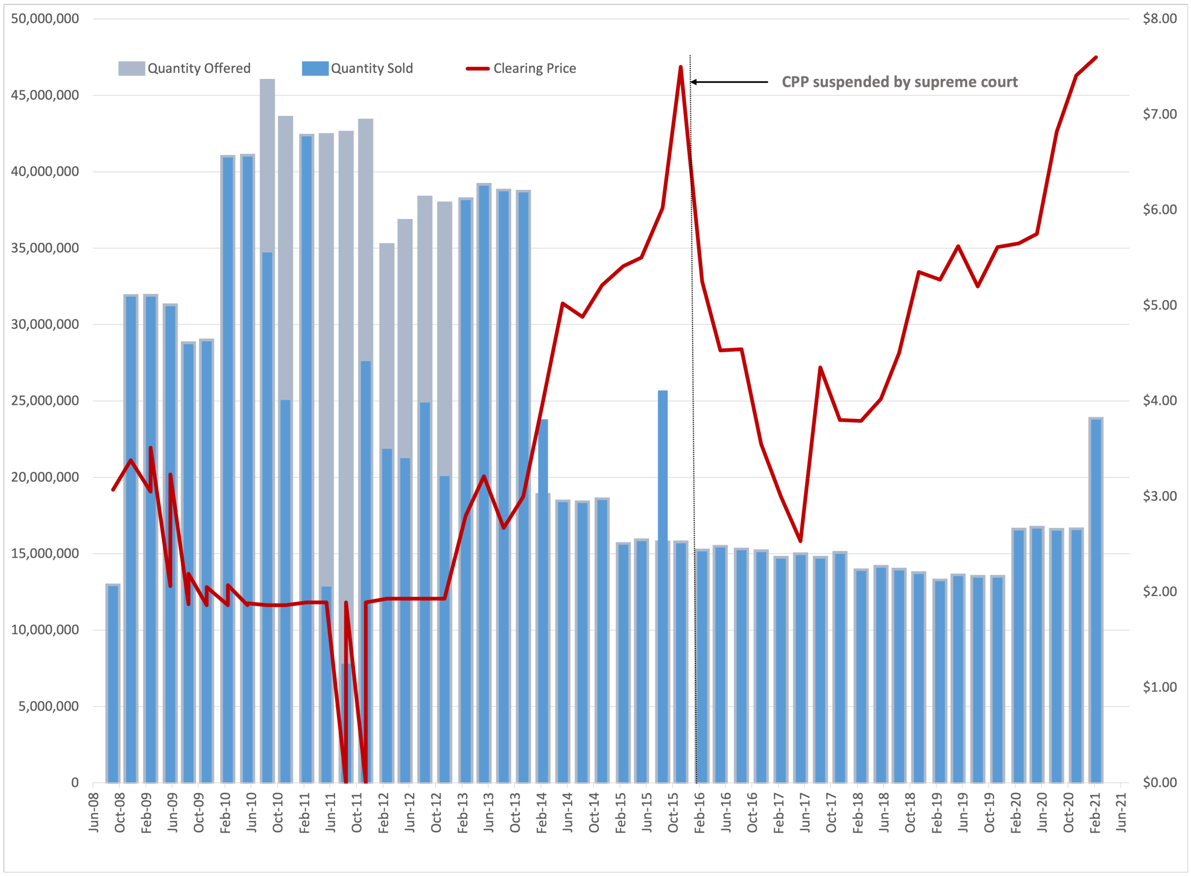
6 Appendix

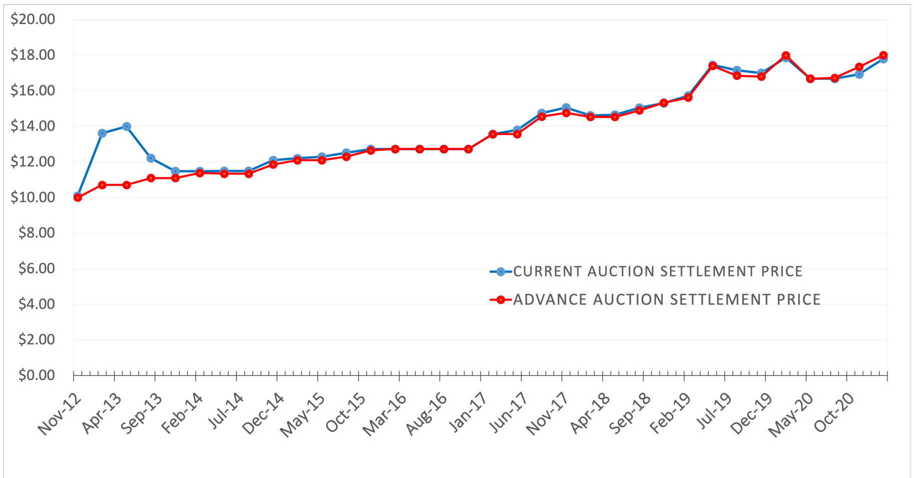
6.1 Figures

Figure 1: California Quarterly Auction Revenue Since 2018



Source: The Regional Greenhouse Gas Initiative (RGGI) — www.rggi.org

Figure 2: cap-and-trade auction curretn and advance prices in California



Source: The California Air Resources Board (CARB)

6.2 Proof

Proof of Lemma(1) Recall the definition of η is as follows:

$$\eta_{i,t} = \begin{cases} v_i - \sum_{l=t+1}^{k_i} \int_{\sigma_l} \tau^{l-t} G_l^{(m_t)}(\eta_{i,l}|\sigma_l)\Big(\eta_{i,l} - \mathbb{E}[(\eta_{j,l}^{(m_t)})_{j\neq i}|\eta_{i,l} > \eta_{j,l}^{(m_t)}]\Big), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise} \end{cases}$$

Through recursive addition and subtraction, it is easy to see that the above is equivalent to the following :

Replacing probability of bid with probability of pseudo type we can rewrite the payoff functions as

$$W(\eta_{i,t}) = \int_{\sigma_t} \tau G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + (1 - G_t^{(m_t)}(\eta_{i,t}|\sigma_t))W(\eta_{i,t+1}) \quad (12)$$

$$V(\eta_{i,t}) = G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + (1 - G_t^{(m_t)}(\eta_{i,t}|\sigma_t))W(\eta_{i,t+1}) \quad (13)$$

This can be rewritten as

$$W(\eta_{i,t}) = \int_{\sigma_t} \tau G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - W(\eta_{i,t+1}) - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \quad (14)$$

$$V(\eta_{i,t}) = G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - W(\eta_{i,t+1}) - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \quad (15)$$

Now, we rewrite the pseudo type in terms of the payoff function. Using addition and subtraction and using the definition of η we can rewrite the definition of η as :

$$\eta_{i,t} = v_i - \int_{\sigma_{t+1}} \tau G_t^{(m_t)}(\eta_{i,t}|\sigma_l)\Big(v_i - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big)$$
$$+ \sum_{l=t+2}^{k_i}\int_{\sigma_l}\Big(\prod_{q=1}^{l-(t+1)}(1-\int_{\sigma_q} G_q^{m_q}(\eta_{i,q}))\Big)\tau^{l-t}G_l^{(m_l)}(\eta_{i,l}|\sigma_l)\Big(v_i - \mathbb{E}[(\eta_{j,l}^{(m_l)})_{j\neq i}|\eta_{i,l} > \eta_{j,l}^{(m_l)}]\Big), \quad \text{if } t \geq a_i \text{ or } t \leq k_i$$

Note that the second term in the equation above is equal to $W(\eta_{i,t+1})$. Thus we have

$$\eta_{i,t} = \begin{cases} v_i - W(\eta_{i,t+1}), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise} \end{cases}$$

Thus, equation(14) and (15) as

$$W(\eta_{i,t}) = \int_{\sigma_t} \tau G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \tag{16}$$
$$V(\eta_{i,t}) = G_t^{(m_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_{j,t}^{(m_t)})_{j\neq i}|\eta_{i,t} > \eta_{j,t}^{(m_t)}]\Big) + W(\eta_{i,t+1}) \tag{17}$$

□

Proof of theorem(1)

The symmetric Bayesian Nash equilibrium bid in period t maximizes the following payoff of bidder t:

$$V(\eta_{i,t}) = \Big\{G_l^{(m_t)}(\eta_{i,t}|\sigma_l)\mathbb{E}\Big[n_{i,t} - b^{(m_t)}(n_{i,t})\Big|b(n_{i,t}) > b_{j\neq i}^{(m_t)}\Big] + W(\eta_{i,t+1})\Big\}$$

Notice that $W(\eta_{i,t})$ in the above expression is merely an additive constant. we will use this and backward induction to solve for the equilibrium bidding function.

First from the structure of $V(\eta_{i,t}|\sigma_t)$, it is clear that after the last active period k_i, the buyer's equilibrium bid will be equal to zero, i.e. $b_t^i = 0 \ \forall \ t > k_i$. This is because the buyer is only active till period k_i and would earn a negative profit from winning if he is active after the actual deadline. Thus we can rewrite the equilibrium bidding strategy as a set of finite bids given as b^i $= \{b(\eta_{i,1}), b(\eta_{i,2}), ...b(\eta_{i,k_i})\}$. Now we will show that in the last active round of bidder i's lifetime i.e. k_i^{th} period, bidder bids their pseudo valuation, so $b(\eta_{i,k_i}) = \eta_{i,k_i}$. Note that in the last period pseudo type is equal to value of the bidder, i.e. $\eta_{i,k_i} = v_i$

- If $b' < v_i$.

  In cases where the price for the object is in-between $b(v_i, k_i)$ and v_i, i.e. $b(v_i, k_i) < z_{t_{k_i}} < v_i$, the current period discounted utility from winning is positive i.e $(v_i - z_{t_{k_i}}) > 0$ but the buyer does not win. Thus this is not optimal

- If $b' > v_i$.

  In cases where the price for the object is in-between $b(v_i, k_i)$ and v_i, i.e. $b(v_i, k_i) > z_{t_{k_i}} > v_i$, the current period discounted utility from winning is negative i.e $(v_i - z_{t_{k_i}}) < 0$. Thus this is not optimal.

From above we get that any other bid than $b(\eta_{i,k_i}) = v_i = \eta_{i,k_i}$ would decreases buyers payoff. Thus $b(v_i, k_i) = \eta_{i,k_i}$ is an optimal bid in the last active period (k_i^{th} period) for bidder i.

Next we prove reporting bid equal to $b(\eta_{i,t})$ is optimal in an arbitrary t during the active demand period, i.e., $a_i < t\ k_i$, assuming it is optimal in all period after t. Recall that equilibrium bid maximizes $V(\eta_{i,t})$

$$V(\eta_{i,t}) = \left\{ G_l^{(m_t)}(\eta_{i,t}|\sigma_l)\mathbb{E}\left[n_{i,t} - b^{(m_t)}(n_{i,t}) \middle| b(n_{i,t}) > b_{j\neq i}^{(m_t)} \right] + W(\eta_{i,t+1}) \right\}$$

Here the first term represents the expected current period discounted utility and the second term represents the expected utility from future periods if he loses the current period auction . Notice that the second term is independent of the bid in period t_r. Thus, this is equivalent to the bid maximizing the first term.

Note that $\eta_{i,t} = v_i - W(\eta_{i,t+1})$ represents the adjusted value for bidder i in period t.We will now show $b(\eta_{i,t}) = \eta_{i,t}$ maximizes eqn(6). Consider any arbitrary $b' \neq b(\eta_{i,t})$.

- If $b' < b(\eta_{i,t})$

  In cases where the price for the object is in-between b' and $b(\eta_{i,t})$, i.e. $b' < z_t < b(\eta_{i,t})$, the current period discounted utility from winning is positive i.e $v_i - W(\eta_{i,t+1}) - z_{t_r} > 0$ but the buyer does not win. Thus this is not optimal.

- If $b' > b(\eta_{i,t})$

  In cases where the price for the object is in-between b' and $b(\eta_{i,t})$, i.e. $b' > z_t > b(\eta_{i,t})$, the current period discounted utility from winning is negative i.e $v_i - W(\eta_{i,t}) - z_t < 0$. Thus this is not optimal.

Which gives the optimal bidding strategy as, $b(\eta_{i,t}) = v_i - W(\eta_{i,t+1})$. □

Proof for proposition(1) First we look at how the rate of future supply effects the equilibrium payoff of the bidder . The period t bid maximizes the following

$$b(\hat{\eta}) \epsilon \operatorname{Arg}\max_{\hat{\eta}} G^{m_t}(\hat{\eta}) \mathbb{E}\left[v_i - b^{(m_t)}(\eta_{j,t})_{j \neq i} \middle| b(\hat{\eta}) > b^{(m_t)}(\eta_{j,t}) \right] + (1 - G^{m_t}(\hat{\eta})) W(\eta_{i,t+1}|\lambda) \tag{18}$$

Note that the bid is made after the supply and demand realization, so the supply rate λ only affects payoff from future periods. Also, $W(\eta_{i,t+1})$ is dependent on the supply and demand distributions as well as λ. Usually we suppress this dependence for easier notation Here we reintroduce its dependence on λ as it is critical here. Using integral-form Envelop theorem we get :

$$\frac{\delta(V(\eta_{i,t}|\sigma_t))}{\delta(\lambda)} = \left(1 - G_t^{(m_t)}[\eta_{i,t}|\sigma_t]\right) \frac{\delta(W(\eta_{i,t+1}|\lambda))}{\delta(\lambda)} \tag{19}$$

Now we use equation(8) and backward induction to derive the derivative

Let us derive this in the last period of active demand. Recall that the equilibrium bid in last period will be $b_i(\eta(v_i, r_i)) = \eta(v_i, 1) = v_i$ and $W(\eta_i(v_i, 0) = 0$, thus

$$\frac{\delta(W(\eta_i(v_i, 1)))}{\delta(\lambda)} \geq 0$$

Now let us assume for any arbitrary t' period of active demand left, i.e. assume $\frac{\delta(W(\eta_{i,t'})}{\delta(\lambda)} \geq 0$, we show that this hold in $t'+1$ too. Rewriting equation (4) for $t'+1$

$$\frac{\delta(W(\eta_i(v_i,r'+1)))}{\delta(\lambda)} = \frac{\delta\int_{\sigma_{t+1}}\tau G^{\lambda m_{t+1}}(\eta_{i,t+1})\bigg(\eta_{i,t+1}-\mathbb{E}[(\eta^m_{j,t+1})_{j\neq i}|\eta_{i,t+1}>\eta^m_{j,t+1}]\bigg)+W(\eta_{i,t+1})}{\delta\lambda}$$

Using envelop theorem and $t+1$ maximization equation we get

$$= \int_{\sigma_{t+1}}\tau\frac{\delta G^{\lambda m_{t+1}}(\eta_{i,t+1})}{\delta\lambda}\bigg(\eta_{i,t+1}-\mathbb{E}[(\eta^m_{j,t+1})_{j\neq i}|\eta_{i,t+1}>\eta^1_{j,t+1}]\bigg)$$

$$-G^{\lambda m_{t+1}}(\eta_{i,t+1})\frac{\delta\mathbb{E}[(\eta^m_{j,t+1})_{j\neq i}|\eta_{i,t+1}>\eta^m_{j,t+1}]\bigg)}{\delta\lambda}+\frac{\delta W(\eta_{i,t+1})}{\delta\lambda}$$

using the assumption $\frac{\delta(W(\eta_{i,t'})}{\delta(\lambda)}>0$, we get

$$\geq 0$$

Thus, $\frac{\delta(W(\eta_{i,t})}{\delta(\lambda)}\geq 0\ \forall t$ □

Proof for proposition(2) first we show bid is decreasing in supply rate:

$$\frac{\delta(b(\eta_{i,t})}{\delta(\lambda)} = \frac{\delta(v_i-W(\eta_{i,t+1}|\lambda))}{\delta\lambda} \tag{20}$$

$$-\frac{\delta(W(\eta_{i,t+1}|\lambda))}{\delta\lambda} \tag{21}$$

$$\leq \text{as a result of proposition(1)} \tag{22}$$

Notice that this also implies $\frac{\delta(\eta_{i,t})}{\delta(\lambda)}\leq 0$, thus we have

$$\mathbb{E}(P_t) = \mathbb{E}\bigg(\eta_t^{(\lambda * m_t)}\bigg)$$

$$\rightarrow \frac{\delta\mathbb{E}(P_t)}{\delta(\lambda)} = \frac{\delta(\mathbb{E}[\eta_t^{(\lambda*m_t)}]}{\delta(\lambda*m_t)}m_t+\frac{\delta(\mathbb{E}[\eta_t^{(\lambda*m_t)}]}{\delta(\eta_t)}\frac{\delta(\eta_t)}{\delta(\lambda)}\leq 0$$

□

Proof for proposition(3) First we look at how the rate of future supply effects the equilibrium payoff of the bidder . The period t bid maximizes the following

$$b(\hat{\eta})\epsilon \operatorname*{Arg\,max}_{\hat{\eta}} G^{m_t}(\hat{\eta})\mathbb{E}\left[\tau v_i - b^{(m_t)}(\eta_{j,t})_{j\neq i}\middle| b(\hat{\eta}) > b^{(m_t)}(\eta_{j,t})|\tau\right] + (1 - G^{m_t}(\hat{\eta}))W(\eta_{i,t+1}|\tau) \quad (23)$$

Note that the bid is made after the supply and demand realization, so the supply rate τ only affects payoff from future periods. Also, $W(\eta_{i,t+1})$ is dependent on the supply and demand distributions as well as τ. Usually we suppress this dependence for easier notation Here we reintroduce its dependence on τ as it is critical here. Using integral-form Envelop theorem we get :

$$\frac{\delta(V(\eta_{i,t}|\sigma_t))}{\delta(\tau)} = V(\eta_{i,t}|\sigma_t) > 0 \quad (24)$$

□

proof of proposition(4) first we show bid is decreasing in uncertainty:

$$\frac{\delta(b(\eta_{i,t})}{\delta(\tau)} = \frac{\delta(\tau v_i - W(\eta_{i,t+1}|\tau))}{\delta\tau} \quad (25)$$

$$v_i - \frac{\delta(W(\eta_{i,t+1}|\tau))}{\delta\tau} \quad (26)$$

$$\geq 0 \quad (27)$$

Notice that this also implies $\frac{\delta(\eta_{i,t})}{\delta(\tau)} \geq 0$, thus we have

$$\mathbb{E}(P_t) = \mathbb{E}\left(\eta_t^{(m_t)}\right)$$

$$\rightarrow \frac{\delta\mathbb{E}(P_t)}{\delta(\tau)} = \frac{\delta(\mathbb{E}[\eta_t^{(m_t)}]}{\delta(\eta_t)}\frac{\delta(\eta_t)}{\delta(\tau)} \geq 0$$

□

Proof of proposition(5) Proof in the text □