CESifo WORKING PAPERS

9267
2021

August 2021

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9267*

Quality Differentiation and Optimal Pricing Strategy in Multi-Sided Markets

Abstract

This paper analyzes the generalized quality differentiation model in multi-sided markets with positive externalities, which leads to new insights into the optimal pricing structure of the firm. We find that quality differentiation for users on one side affects not only the side involving differentiation but also the other side due to cross-side network externalities, thereby affecting the pricing structure of multi-sided firms. In addition, quality differentiation affects the strategic relationships among all the choice variables for the platform, enabling the platform to strategically use quality differentiation to raise its profits.

JEL-Codes: D430, L110, L420.

Keywords: multi-sided market, quality differentiation, platform business strategies.

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We would like to thank Jay Pil Choi, Thomas Jeitschko, Kyoo il Kim, Hanzhe Zhang, and seminar participants at the Red Cedar Conference at Michigan State University for valuable discussions and comments.

1 Introduction

There has been an increasing shift toward multi-sided firms in many industries. The early pioneering models of multi-sided platforms were introduced by Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), and Rochet and Tirole (2003). As many previous studies emphasize, it is important to establish how multi-sided platforms differ from typical one-sided firms and how such differences lead to new business implications. Many markets that have traditionally featured one-sided firms now feature more two-sided firms due to advanced technology; for example, the taxi industry exclusively involved one-sided firms before Uber appeared. Given this shift toward multi-sided business, the main strategic choice that we analyze in this paper is the choice of the quality of interaction between users on two different sides. We find that apart from the price elasticity on the two sides, quality of interaction also influences the price ratio between the two sides. Additionally, if the platform uses the quality to price discriminate on one side, then the optimal price on the other side changes as well.

We focus on platforms that enable interaction between two sides, represented as users and developers. The term "interaction" covers various traditional interactions, including exchanges between application developers and application users on software platforms such as computers (e.g., Apple, Microsoft), mobile devices (e.g., iPhone, Samsung), and video games (e.g., Sony PlayStation, Xbox). Interaction also refers to the interactions observed in auction houses and those on internet sites in person-to-business transactions (e.g., Amazon). These markets usually have more than one type of quality access on the user's side. For example, in the video game industry, Sony launched PlayStation *Pro* (high quality) together with *Slim* (low quality). Amazon offers users two types of quality access: basic (low quality) access is free, while premium access (*Prime* membership, which is high quality) is the paid service. It is easy to see that the interactions or exchanges through the high-quality access have better quality—for example, PlayStation *Pro* supports 4K resolution, and *Prime* provides two-day shipping. Many ride-sharing services offer differentiated quality tiers for customers. For example, Uber provides riders with multiple types of quality services. Although each region may have different availability, there is basically a low-quality service at a less expensive price (e.g., *Uber X, Uber Pool*) and a high-quality service at a slightly more expensive price (e.g., *Uber XL, Uber Select, Uber Black*).

From these examples of quality differentiation by firms serving two sides, we can see that the platform's different quality provision on the user's side affects the per user profit for developers. For example, if Sony upgrades PlayStation in terms of display quality, those game console users become more likely to subscribe to video game providers, which implies that there are positive effects of Sony's quality improvement on game developers' revenues. In this regard, our paper provides the business implications of quality differentiation in the multi-sided platform market by building a simple model of two-sided monopolists. In the model, the quality access for one side also affects the utility derived by the other side, thereby extending Rochet and Tirole (2003) by accommodating the effect of quality on cross-side network externality.

We first emphasize that by providing more quality choices, even if the quality differentiation is small, the platform is able to obtain more profits. Given that users are heterogeneous in their valuation of product quality, there is an incentive for the platform to offer different levels of product quality at different prices to extract more rents from users. More importantly, we endogenize the impact of quality of the platform's product or service used by the user's side on the profit made by developers. We find that optimal pricing depends on the quality of the platform's product or service. In particular, we show that the quality of the platform changes the pricing ratio on the two sides due to the multi-sidedness of the platform. Based on this finding, we present managerial implications about the quality of the product to be provided such that access quality for users not only affects the price on the user's side but would also influence the price on developer side.

Another related main finding of the paper is that the platform can optimally charge a higher price for developers upon quality differentiation on the user's side. Specifically, we provide the condition under which quality differentiation on the user's side raises the price for developers: if quality differentiation on the user's side results in a greater profit increase on the developer's side than on the user's side, it is optimal that the price for developers is higher under the two-quality case than under the one-quality case because of sufficiently large network externalities such that the profit-increasing effects are greater on the other side than on the side involving differentiation.

Furthermore, we show that the price of the low-quality product on the user's side and the fee charged on the developer's side move in the opposite direction upon quality differentiation on the user's side. This finding suggests that the platform can optimally lower the basic quality product price for users, while raising the price for developers upon quality differentiation. Thus, the introduction of a high-quality product on one side alters the pricing dynamics between the two sides via network effects. Based on the model predictions, we discuss some business implications for how quality differentiation on one side of the market reshapes the platform's profit-maximizing strategies via cross-side externalities.

Related literature There is a broad literature on the corresponding problem of a monopolistic firm seeking to maximize profits by offering quality-differentiated products in one-sided standard markets. The seminal papers are Spence (1977), Mussa and Rosen (1978), Maskin and Riley (1984), and M. Itoh (1983). We generalize this problem by varying the number of sides served by the firm.

This paper is also related to the literature on pricing structure in markets with multi-sided firms (e.g., Rochet and Tirole, 2003; Parker, Geoffrey G., and Marshall W. Van Alstyne 2005; Armstrong, 2006b; Reisinger, 2010). A major difference in this paper is that we consider a three-way interaction among prices for users and developers and quality choice for the optimal pricing structure in multi-sided firms. In addition to being broadly related to the literature on pricing in multi-sided markets, papers on skewed pricing in multi-sided markets are closely related to our paper in terms of theoretical implications. Suarez and Cusumano (2008) discuss the platform's subsidy pricing strategy to attract greater user adoption, although they do not set up an economic model to confirm this strategy. Bolt and Tieman (2008), Schmalensee (2011), and Dou and Wu (2018) study skewed pricing strategies in two-sided markets, i.e., the subsidy and money sides. However, those papers do not consider forms of product differentiation, such as the quality differentiation examined in our paper, as a means of skewing prices. Additionally, Sridhar et al. (2011) focus on cross-market network effects in two-sided markets, as we do in this study; however, their focus is on empirical analysis of how the optimal marketing investment allocation is affected by cross-market effects.

Regarding markets with multi-sided firms, few papers have focused on firms considering quality of interaction on their platform. These papers study markets with negative network externalities and do not take into account how quality differentiation on one side affects the other side. Crampes and Haritchabalet (2009) examine the choice of offering a pay ads regime and no pay ads packages. Peitz and Valletti (2004) compare the advertising intensity when media operators offer free services and when the subscription price is positive. Viecens (2006) is an exception because she studies a setup with endogenous quality differentiation on two-sided platforms. However, the quality differentiation in her model takes a different form from ours in that she focuses on the quality provided by users on one side and not on the quality provided by the platform itself. Therefore, her results do not provide any implications for the platform's dynamic pricing structure.

Another quality-related aspect explored in the context of two-sided markets is the case in which users care about the quality of the other users with whom they interact, which is relevant for matching markets such as dating sites. Jeon et al. (2016) examine this problem in a platform setting. Renato and Pavan (2016) consider this problem in a matching setup. Hagiu (2012) studies a model in which users value the average quality of other users. The setup in these papers, however, is different from that in ours in that we focus on the quality of interaction and not on the quality of users.

Overall, the previous literature on quality discrimination focuses on the quality of product sold on the platform, which affects the interaction between users similarly to our case. However, those studies do not consider quality differentiation as a cross-market business strategy that the platform can implement. Our paper aims to provide a basic model to endogenize the effect of quality. Given that the quality of the platform is an important feature that is observed in many multi-sided markets, our approach also provides an opportunity for extending the model to diverse types of platform markets (e.g., ride-sharing, e-commerce, and game platforms) involving quality differentiation.

Our analyses are also related to the literature on product differentiation in multi-sided markets in that quality differentiation is one form of product differentiation. Smet and Cayseele (2010) focus on product differentiation in platform markets, which still differs from our paper in that they do not account for its consequences for optimal pricing strategies. Additionally, our model also considers how quality access affects the user's interaction with the other side, namely, cross-side network externalities.

In summary, the literature has not focused on the choice of network quality differentiation in multi-sided markets with positive cross-network externalities. Therefore, our results on how multi-sided markets are combined with quality differentiation on a platform, given that multiple quality access on one side affects the revenue-maximizing prices of both or multiple sides, provide new insights into the related business, which is underexplored in the literature.

2 Model Setup

We model the interaction between users and developers. Economic value is created through the interaction between these two sides. We consider the case of a single firm providing a platform for interactions between users and developers. For instance, Sony PlayStation as a game console provides interactions between game users and developers. In addition to charging an access fee to use the game platform, the firm can also control the quality of the interaction by launching a new generation or releasing multiple quality tiers, e.g., PlayStation *Pro* vs. *Slim*. Figure 1 displays the structure of the two-sided firm with quality differentiation on the user's side using PlayStation as an example. Users are able to access games only when they buy compatible devices, in this case, different generations of PlayStation; thus, purchasing a game console is equivalent to purchasing access to the game store. The two types of access quality offered on the user's side are PlayStation *Slim* (basic low quality) and *Pro* (high quality). For instance, if users purchase *Pro*, which provides 4K resolution, which does not come with *Slim*, *Pro* users obtain greater utility from playing the same games.

Quality provision We consider two scenarios: with and without quality differentiation implemented by the platform. In the one-quality model, as described in Section 2.1, the platform does not differentiate the quality level offered to users; one quality level in this model is denoted as $q \in [0, 1]$. In the two-quality model, as in Section 2.2, the platform introduces a higher quality product in addition

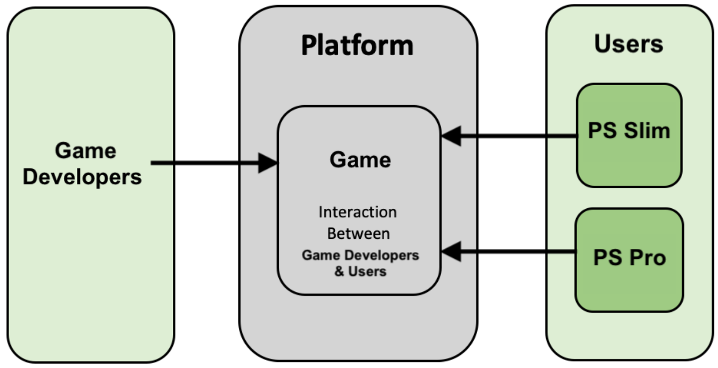

Figure 1: Two-sided firm with quality differentiation

to basic quality option. We denote basic quality as $q \equiv q_l$ and high quality as q_h. For instance, Sony operates the platform, i.e., PlayStation, to connect users and game developers: users join the platform by purchasing PlayStation (game console), whereas game developers sell games on the platform. If Sony offers PlayStation *Slim* only, i.e., the one-quality case, its users obtain utility from its quality measured by q. If Sony offers both (basic) PlayStation *Slim* and (upgraded) *Pro*, i.e., the two-quality case, the basic low quality of *Slim* is $q_l \equiv q$, which is the same as before, and the high quality of *Pro* is measured by $q_h > q = q_l$.

User's side Specifically, we model a two-sided monopolist firm that price discriminates on the user's side by offering two different types of access quality: $q_k \in \{q_l, q_h\}$, where k denotes the quality provision, either $l$ow or $h$igh. In this model, the user's gain from the platform is derived through interaction between users on two sides: users and developers. Both users and developers obtain utility from interacting with each other. The quality variable controls the gain from each such interaction. We assume that both users and developers are heterogeneous with respect to the per interaction or usage benefit. The usage or per interaction benefits are $b^u(q_k)$ for the user's side (for an individual user) and b^d for the developer's side, where superscripts u and d denote users and developers, respectively. For simplicity, we assume that $b^u(q_k) = \mathcal{U}\alpha^u q_k$, where $\mathcal{U}$ represents the basic benefit for every user. This benefit is dependent on the quality of interaction; thus, the monopolist can control the benefit by choosing whether to offer a higher quality access on the user's side. The term α^u denotes the heterogeneity among users; it follows a distribution function F^u with support on $[0, 1]$ and density f^u. Additionally, each user pays a price for using the platform, which is denoted P_k^u, as a fixed fee. For a user, the utility function is given as follows:

$$U_k^u = (\mathcal{U}\alpha^u q_k N^d) - P_k^u, \tag{1}$$

where $k \in \{l, h\}$. The user's utility is the benefit from each interaction with the other side, i.e., $\mathcal{U}\alpha^u q_k N^d$, subtracted by a fixed fee to access the platform P_k^u.[1] From the utility specification, it can be shown that the user with the highest benefit is that with $\alpha^u = 1$.

We assume that the access fee P_k^u is a fixed fee. For example, Sony charges a fixed fee as the price for PlayStation *Slim* or *Pro*. However, as shown in Appendix A.3, the model with a usage-based per transaction fee, such as Uber, yields the same results as in the model with a fixed fee.[2] Thus, according to the equivalence, we use the per transaction price $p_k^u \equiv \frac{P_k^u}{N^d}$ instead of the fixed fee P_k^u for the remainder of the analysis. The equation below shows the equivalence.

$$\begin{aligned} U_k^u &= (\mathcal{U}\alpha^u q_k N^d) - P_k^u \\ \Leftrightarrow U_k^u &= (\mathcal{U}\alpha^u q_k - p_k^u) N^d \end{aligned} \tag{2}$$

where $k \in \{l, h\}$. The user's utility is the net benefit from each interaction with the other side, i.e., $\mathcal{U}\alpha^u q_k - p_k^u$, multiplied by the number of interactions, which is denoted N^d.

Developer's side We assume that $b^d = \mathcal{D}\alpha^d$, where α^d represents heterogeneity among developers with respect to the per usage benefit, which is also distributed by a distribution function F^d with support on [0,1] and density f^d. For a developer, the utility function is given as follows:

$$U^d = \begin{cases} [g(q) \times (\mathcal{D}\alpha^d - p^d)] N^u, & \text{if one-quality case;} \\ [g(q) N_l^u + g(q_h) N_h^u] \times [(\mathcal{D}\alpha^d - p_{2Q}^d)] & \text{if two-quality case,} \end{cases} \tag{3}$$

where $g(q_h)$ (or $g(q)$, where $q \equiv q_l$) are increasing and concave in quality level and N_k^u denotes the number of users who demand products with quality k. This utility specification accommodates the effect of the platform's quality provision on the per user profit for developers. For example, if Sony upgrades PlayStation quality in terms of display quality, i.e., q increases, those game console users are more likely to make high in-game purchases or spend more on the game.[3] In this sense, $g(q)$ captures

[1] We assume here that every user interacts with every developer and that every developer interacts with every user. We can easily extend this to a model where the number of interactions is a function of the total number of developers, such as $g(N^d)$. The results are robust to such extensions.

[2] Note that the equivalence holds as users only extract benefit from the platform through interaction with the other side, which is the case in Uber or game consoles or Amazon

[3] Note that the model setting allows a general interpretation for why revenue might increase for developers if users access the higher-quality platform; for example, it accommodates cases where better quality access means that the

the positive effects of Sony's quality improvement on game developers' revenues. Additionally, $g(q)$ captures how much the platform can extract additional rents from game developers in the form of the platform access fee p^d by offering higher quality product to the user's side.

The platform access fee charged to developers p^d can be interpreted in different ways. For example, Microsoft as a game publisher/platform, which builds games for its own consoles, *Xbox*, often makes a contract with (second- or third-party) game developers. Upon a contract agreement, the publisher pays royalties to developers from sales of games. At the same time, since the publisher covers costs for development, it takes a certain share of the royalties, around 10-20%. The size of share that goes to the publisher can be regarded as the fee charged to developers in this case.

The developer's utility is the net benefit from each interaction with the other side, i.e., $g(q) \times (\mathcal{D}\alpha^d - p^d)$, multiplied by the number of interactions, which is denoted N^u.[4] All developers are charged p^d per interaction, so the price discrimination is only on the user's side.[5]

The platform Next, the cost of the two-sided monopoly firm depends on the quality provided. The total cost of a transaction is given by $c(q) = c(q_l) \geq 0$ for a transaction between a low-quality user and a developer and $c(q_h) \geq 0$ for a transaction between a high-quality user and a developer. In the PlayStation example, such cost differentiation captures the fact that better resolution for PlayStation *Pro* users is costlier than standard resolution for *Slim* users. We normalize the cost for developers $c^d = 0$. The cost function is assumed to be increasing and convex in quality ($c'(q) > 0$, $c''(q) > 0$). We analyze the nontrivial case in which $q_h > q_l = q$.

The demand on the user's and developer's sides is represented by D^u and D^d, respectively. In equilibrium, demand will be equal to the number of participants on each side, which means $D_k^u = N_k^u$, where $k \in \{l, h\}$ and $D^d = N^d$. Given the equilibrium demands, we turn to the monopolistic platform's problem. The monopoly platform's problem can be written as follows:

- If one type of quality is offered:

$$\max_{p^d, p^u} \Pi = [p^u - c(q) + g(q)p^d] D^u D^d. \tag{4}$$

- If two types of quality are offered:

$$\max_{p_{2Q}^d, p_l^u, p_h^u} \Pi = [p_l^u - c(q) + g(q)p_{2Q}^d] D_l^u D^d + [p_h^u - c(q_h) + g(q_h)p_{2Q}^d] D_h^u D^d, \tag{5}$$

developer can charge users a higher price for the game, that users buy more in-game items or that users become more likely to subscribe to video game providers. All of these cases can be accommodated in our given model.

[4] In this model, we assume that the developer's side is affected only by the total number of users, not the type of user with which a developer interacts. Thus, the total number of interactions for each developer is N^u, and the total number of interactions by developers is not affected by the type of user with which they interact.

[5] Note that the qualitative results still hold under a generalized setup with any function of $N^u N^d$.

where p^d_{2Q} denotes the price for developers when two quality levels are offered to users.

Note that we endogenize the platform's decision regarding whether to provide quality differentiation. As we will show, the platform prefers quality differentiation with high and low quality levels to users because it is more profitable than one-quality provision. Nevertheless, we analyze the case of one-quality provision in Section 2.1 for a benchmark.

Throughout the paper, we make the following assumptions.

Assumption 1. *The cost is increasing and convex in quality:* $c'(q) > 0$, $c''(q) > 0$.

This assumption suggests that it becomes increasingly costly to provide higher-quality service, which is a standard assumption.

Assumption 2. *The distribution functions for users and developers are increasing in type:* $(f^u)' \geq 0$ *and* $(f^d)' \geq 0$*, which implies that* $f^u(1) > 0$*, and the inverse hazard rate is non-increasing in user type:* $\frac{\partial \frac{1-F^u(\theta)}{f^u(\theta)}}{\partial \theta} \leq 0$ *and* $\frac{\partial \frac{1-F^d(\theta)}{f^d(\theta)}}{\partial \theta} \leq 0$.

Assumption 2 implies that we impose the standard monotone hazard rate condition on F. Additionally, it guarantees that as users attribute more value to the per usage benefit, there are more platform users than nonusers.

Assumption 3. $\frac{\partial U^u(\alpha^u,p^u,q)}{\partial p^u} < 0$, $\frac{\partial U^u(\alpha^u,p^u,q)}{\partial q} > 0$, $\frac{\partial U^d(\alpha^d_i,p^d)}{\partial p^d} < 0$.

Assumption 4. *The single-crossing property holds:* $\frac{\partial^2 U^u(\alpha_i,q(\alpha_i))}{\partial \alpha_i \partial q} > 0$.

This means that we can always distinguish high-type users from low-type users based on their α_i.

Timing of the game The timing of the game is set as follows.

0. Before the main game begins, the platform is assumed to provide one quality q to users at p^u, which respectively are the basic (low) quality level and the price. The price for developers is set at p^d.

1. The platform decides whether to provide two levels of quality to users. If engaging in quality differentiation, it determines the price levels for users (p^u_l and p^u_h), given the exogenously given quality levels (basic low $q_l = q$ and high q_h). It also optimally sets the price for developer p^d_{2Q}.

2. Buyers and developers make participation decisions.

The solution concept we use for this game is the Perfect Bayesian Nash Equilibrium (PBE) for multiperiod games with observed action, which can be solved by backward induction. PBE consists of a sequentially rational strategy profile for all players and a set of consistent beliefs about user's and developers' valuations with respect to benefits from using the platform.

2.1 Model with the one-quality case

As a benchmark, we begin by considering a model that omits the practice of quality differentiation and then modify the model to allow differentiation in Section 2.2. The two-sided monopolist offers a single quality of access in this case. The user's utility is given by Equation (2) with $q_k = q$ and $p_k^u = p^u$.

We first analyze the user's side (users and developers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u = Prob\,(U^u \geq 0) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u}{\mathcal{U}q}\right). \\ D^d = Prob\left(U^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{6}$$

Note that the equilibrium supply D^d is not affected by $g(q)$. This result comes from the assumption that the effect of quality, measured by $g(q)$, affects developers' revenues, namely, $\mathcal{D}\alpha_j^d$, and fees paid to the platform, namely, p^d, in the same manner. In most cases, the fees charged to developers are proportional to their revenues earned from the platform. For example, a video game platform takes a certain percentage of purchases made by users from games. High-quality users, such as those using *Xbox Series X*, are more likely to make additional purchase in game, especially physical editions of games, than those with low-quality consoles, such as *Xbox Series S* because the low-quality console, which has no optical drive, does not provide the physical edition of games; therefore, game developers can earn more revenues proportionally from high-quality provision. That is, if the effect of quality improvement on the user's side on developer's revenues is measured by $g(q)$, that on the fees can be represented by $\delta g(q)$, where $\delta \in (0, 1)$. For simplicity, we normalize δ to one: however, even if δ is in between zero and one, i.e., the effect of quality on the developers' revenues is different from that on the fees, the qualitative results would hold as long as the first effect dominates the second.

From the platform's profit maximization problem, as in Equation (4), we can solve for the optimal solutions for the prices on the user's and developer's sides as follows:[6]

$$\frac{p^u - c(q) + p^d}{p^u} = \frac{1}{\varepsilon^u}; \quad \frac{p^d - c(q) + p^u}{p^d} = \frac{g(q)}{\varepsilon^d}. \tag{7}$$

From Equation (7), we obtain the equilibrium condition that captures the trade-off between p^u

[6]The details are in Appendix A.1.

and p^d as follows.

$$\frac{p^u}{\varepsilon^u} = \frac{p^d \times g(q)}{\varepsilon^d} \tag{8}$$

Comparing Equation (8) to the equilibrium condition in the basic model of Rochet and Tirole (2003), we find that the quality of the platform changes the pricing ratio on the two sides; specifically, if the positive effect of quality improvement on the developer's side is greater than the relative price sensitivity between the two sides, then the platform charges a higher price for developers than for users.

Additionally, we find a more interesting interaction between the effect of quality on the developer's side and relative privacy sensitivity. Specifically, if the positive effect of quality improvement on the increase in the developer's side revenue, measured by $g(q)$, is greater (less) than the relative price sensitivity between users and developers, measured by $\frac{\varepsilon^d}{\varepsilon^u}$, the platform charges a lower (higher) fee to developers than to users.

Proposition 1. *The optimal relative price for users to that for developers depends not only on the relative price sensitivity between the two sides but also on the external effect of quality improvement on the user's side on the developer's side. Specifically, if the positive effect of quality improvement on the developer's side is less than the relative price sensitivity between the two sides, then the platform charges a higher price for developers than for users.*

$$g(q) \leq (\geq) \frac{\varepsilon^d}{\varepsilon^u} \iff p^u \leq (\geq) p^d$$

As mentioned above, Equation (8) is similar to the equilibrium of the two-sided market with a quality decision by the platform, as in Rochet and Tirole (2003). However, our equilibrium condition still differs from that in Rochet and Tirole (2003) insofar as we focus on a three-way trade-off that includes the exogenous quality variable in addition to a trade-off between prices on the two sides. Moreover, this interactive effect arises from the multi-sided nature of the platform such that the quality affects the user's willingness to pay for a better quality product, which in turn increases the revenues generated by developers. By placing the main focus on price variables only, Rochet and Tirole (2003) find that the price on the user's side would be lower if developers were more price elastic because the platform wants to balance the sizes of the two sides. Allowing the platform to provide differentiated quality tiers for the user's side creates more opportunities for optimal business strategies insofar as quality provision choice offers another level of flexibility in terms of profit maximization for the platform. Indeed, as we will show, the platform's optimal pricing decisions depend on its quality differentiation choice.

2.2 Model with two qualities case

As mentioned above, the platform's basic low-quality level is not varied when a high-quality service is introduced, i.e., $q = q_l$. Thus, the choice variables for the platform include p_l^u, p_h^u, and p_{2Q}^d.

We start by analyzing the user's side (users and developers) to identify the equilibrium demand. First, the users have two choices for accessing the platform. They can join the platform through either low-quality access or high-quality access. Given the two types of quality, high and low, the number of participants joining with low-quality access is determined by the number of users who satisfy the following two conditions:

1. (IR constraint) The user's utility from low-quality access is greater than zero: $Pr(U_l^u \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from low-quality access exceeds that from high-quality access: $Pr(U_l^u \geq U_h^u)$.

The two conditions jointly determine the proportion of low-type users.

$$D_l^u = \Pr\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)} \geq \alpha^u \geq \frac{p_l^u}{\mathcal{U}q}\right) = F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)}\right) - F^u\left(\frac{p_l^u}{\mathcal{U}q}\right), \tag{9}$$

where $D_l^u \equiv D^u(p_h^u, p_l^u, q_h, q)$. Similarly, the number of participants joining the high-quality service is given by the number of users who satisfy the following two conditions:

1. (IR constraint) The user's utility from the high-quality good is greater than zero: $Pr(U_h^u \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from the high-quality good exceeds that from the low-quality good: $Pr(U_h^u \geq U_l^u)$.

The IR condition is satisfied when the IC constraint of the high type and IR constraint of the low type hold.[7] Thus, the proportion of high-type users is given by the following:

$$D_h^u = \Pr\left(\alpha_i^u \geq \frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)}\right) = 1 - F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)}\right), \tag{10}$$

where $D_h^u \equiv D^u(p_h^u, p^u, q_h, q)$. Given the utility function for users, the total number of users joining the platform is given by the following:

[7] We maintain the standard single-crossing condition, which implies that higher types have greater willingness to pay (WTP) for quality at any price or that consumers may be ordered by their type.

$$
\begin{aligned}
& D^u = \Pr(U_l^u \geq 0) = \Pr(b_l^u \geq p_l^u) \\
\Leftrightarrow\ & D^u = \Pr(\mathcal{U}\alpha^u q \geq p_l^u) = \Pr(\alpha^u \geq \frac{p_l^u}{\mathcal{U}q}) = 1 - F^u\left(\frac{p_l^u}{\mathcal{U}q}\right),
\end{aligned} \tag{11}
$$

where $D^u \equiv D(p_l^u, q)$. Equation (11) shows us how the number of participants on the user's side depends only on the price and quality of the low-quality good. Although there are network externalities in the total utility derived from the platform or the gross transaction utility, the per unit transaction demand is not dependent on the participation rate on the other side.[8] This condition is necessary because the participation constraint (IR constraint) for the high-quality users is slack. This means that the participation of low-quality users guarantees the participation of high-type users. In other words, the users on the margin of joining the platform are low-quality users.

Next, the total number of developers who join the platform is given by the following:

$$
D^d = \Pr(U^d \geq 0) = 1 - F^d\left(\frac{p_{2Q}^d}{\mathcal{D}}\right), \tag{12}
$$

where $D^d \equiv D^d(p_{2Q}^d)$. Given the total number of users and developers, the equilibrium level of participation is the following:

$$
\begin{aligned}
& D^u = D(p_l^u, q) = 1 - F^u\left(\frac{p_l^u}{\mathcal{U}q}\right); \quad D^d = D^d(p_{2Q}^d) = 1 - F^d\left(\frac{p_{2Q}^d}{\mathcal{D}}\right). \\
& D_l^u = F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)}\right) - F^u\left(\frac{p_l^u}{\mathcal{U}q}\right); \quad D_h^u = D^u(p_h^u, p_l^u, q_h, q) = 1 - F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q)}\right).
\end{aligned} \tag{13}
$$

Given quality differentiation, the monopoly problem given by Equation (5) can be rearranged as follows:

$$
\max_{p_{2Q}^d, p_l^u, p_h^u} \Pi = (\pi_l D^u + \pi_{h-l} D_h^u) D^d, \tag{14}
$$

where $\pi_l \equiv p_l^u + g(q)p_{2Q}^d - c(q)$ and $\pi_{h-l} \equiv (p_h^u - p_l^u) + (g(q_h) - g(q))p_{2Q}^d - (c(q_h) - c(q))$. Note that π_l and π_{h-l} represent the platform's per transaction profit from low-quality access and from high-quality access, respectively. The following is the breakdown of the equilibrium prices and quality for users and developers.[9]

2.2.1 Price of low-quality access on the user's side

The price of low-quality access on the user's side can be obtained as follows:

[8] This setup has one restriction that we need to impose, which is that the proportion of low-type users has to be nonnegative: $D_l^u \geq 0$.

[9] The details are in Appendix A.2.

$$\pi_l = \frac{D^u}{(-D^u)'_{p^u_l}} = \frac{p^u_l}{\varepsilon^u}, \tag{15}$$

where $(D^u)'_{p^u_l}$ means that $\frac{\partial D^u}{\partial p^u_l}$, and the price elasticity of demand is represented by ε^u. The optimal price for low-quality access is determined at the level at which the per transaction profit from low-quality access is equal to an additional markup obtained from low-quality users, i.e., $\frac{p^u_l}{\varepsilon^u}$.

2.2.2 Price of high-quality access on the user's side

The price of high-quality access on the user's side can be obtained as follows:

$$\pi_{h-l} = \frac{D^u_h}{(-D^u_h)'_{p^u_h}} = \frac{p^u_h}{\varepsilon^u_h}, \tag{16}$$

where $(D^u_h)'_{p^u_h}$ means that $\frac{\partial D^u_h}{\partial p^u_h}$, and the price elasticity of demand for high-quality access is represented by ε^u_h. The optimal price for high-quality access is determined at the level at which the per transaction profit from high-quality access is equal to an additional markup obtained from high-quality users, i.e., $\frac{p^u_h}{\varepsilon^u_h}$.

2.2.3 Price for developers

We now turn our attention to the price for developers. For the ease of notation, let the ratio of consumers on the high-quality access be $\frac{D^u_h}{D^u} \equiv \lambda_h$.

$$\pi_l + \pi_{h-l}\lambda_h = \frac{p^d}{\varepsilon^d}\left\{g(q) + [g(q_h) - g(q)]\lambda_h\right\}. \tag{17}$$

Equation (17) implies that the optimal price charged to developers is set at the level at which the average per transaction profit from users is equal to an additional markup obtained from developers, i.e., $\frac{p^d}{\varepsilon^d}\left\{g(q) + [g(q_h) - g(q)]\lambda_h\right\}$. That is, the optimal p^d is set by optimally balancing the marginal revenues earned from two different sides: users and developers.[10]

3 Equilibrium Results

We derive several important implications from the model by showing how quality differentiation determined by the multi-sided platform affects its optimal use of the interaction between two sides, such as cross-subsidization, in the following subsections.

[10]Note that we have normalized the cost on the developer's side to zero, so $c^d = 0$.

3.1 Effects of quality differentiation on the platform and two sides

Before we proceed, we first show that the platform has an incentive to engage in quality differentiation in the first place: we find that quality differentiation leads to greater profit for the platform, as in Proposition 2. The detailed proof is in Appendix B.

Proposition 2. *The platform strictly prefers to price discriminate by quality on the user's side.*

Proposition 2 implies that a platform that provides only one type of quality is able to obtain more profit if it slightly differentiates product quality. Even a minor quality improvement with a small price increase can increase the platform's profit as long as it continues to provide differentiated products, such as low- and high-quality products. Thus, quality differentiation permits the platform to earn more profit by implementing premium service in addition to basic service, which not only expands the size of the total user market but also extracts more rents from the relatively high-quality type of users.

Next, we focus on the multi-sided platform case with the provision of two quality levels to investigate how quality differentiation on the user's side affects the price for developers, the price for users, and the interaction between the two sides. We first find that the price of low-quality product for users and the fee for developers move in the opposite direction as the platform differentiates the quality tier for users by introducing high-quality product. Lemma 1 summarizes this finding.

Lemma 1. *If the monopolist introduces a high-quality product, the price of low-quality product on the user's side and the fee charged on the developer's side move in the opposite direction.*

Unlike the one-sided case, the profit-maximizing strategy of a two-sided firm is modified by re-evaluating the relative prices on the two sides. Specifically, quality differentiation, which affects both the user's and developer's sides' pricing strategies, allows the platform to cross-subsidize the two sides by having the price on the two sides move in the opposite direction in order to maximize the gain from the quality differentiation.

Additionally, we find the cross-market externalities arising from quality differentiation on the user's side. That is, if the platform introduces a high-quality access on the user's side, such quality differentiation generates externalities on the developer's side's pricing. Whether the external effects of quality differentiation increase or decrease the price for developers depends on the relative size of the average profit increase on the two sides. Proposition 3 summarizes this finding.

Proposition 3. *Upon quality differentiation on the user's side, the price on the developer's side changes depending on the relative size between the average profit increase on the developer's side and* *that on the user's side.*

Specifically, if quality differentiation on the user's side results in a greater profit increase on the developer's side than on the user's side, then it is optimal that the price for developers is higher under the two-quality case than under the one-quality case. For example, when the platform releases a high-quality game console in addition to the basic-quality console, such as *Xbox Series X* (high quality) and *Xbox Series S* (basic quality), it can attract more users, and more users, in turn, attract more game developers. Thus, quality differentiation on the user's side increases the profit not only on that side but also on the other side, i.e., the developer's side, via cross-side network externalities. Proposition 3 states that if network externalities are sufficiently large such that the profit-increasing effects are greater on the other side than on the side involving differentiation, the platform optimally charges a higher price for developers upon quality differentiation, even if there is no quality improvement on the developer's side.

Combining Lemma 1 with Proposition 3, we further find that the relative price of basic product for users in the one-quality provision (i.e., p^u) case to that for the basic, low-quality, product in the two-quality provision case (i.e., p_l^u) depends on the relative size of the average per transaction profit from users and the average effect of the user's side's quality improvement on the developer's side. That is, the platform lowers the price for the basic-quality product if it earns more average additional profit from the developer's side than from the user's side. Thus, the introduction of a high-quality product in one side alters pricing dynamics between the two sides via network effects.

Next, we conduct further comparative statics analysis to determine how changes in the exogenous parameters affect the equilibrium outcomes. In particular, we are interested in how $\mathcal{U}$, which represents the basic benefit from the quality dimension for every user, affects consumer demand in the case of two qualities. Proposition 4 summarizes the result.

Proposition 4. *Consumer demand for the high-quality service always increases in the basic benefit from better quality ($\mathcal{U}$). Whether consumer demand for the low-quality service increases in $\mathcal{U}$ is ambiguous.*

In other words, $\frac{\partial D_h^u}{\partial \mathcal{U}}$ is always positive (where D_h^u is given by Equation (10)), whereas $\frac{\partial D_l^u}{\partial \mathcal{U}}$ is positive only if a certain condition is met (where D_l^u is given by Equation (9)). Specifically, $\frac{\partial D_l^u}{\partial \mathcal{U}}$ is positive if $\frac{p^u}{q} f^u \left(\frac{p^u}{\mathcal{U}q}\right) > \frac{p_h^u - p^u}{q_h - q} f^u \left(\frac{p_h^u - p^u}{\mathcal{U}(q_h - q)}\right)$ and negative otherwise. That is, when the basic benefit from the quality dimension increases, it can reduce the user's demand for low-quality access if the price-quality ratio for the quality difference (between high and low quality) is greater than that for low-quality service. As in Equation (9), users in the middle range of willingness to pay (i.e., those who are willing to pay for low-quality service but not for higher-priced high-quality service) demand low-quality service access. As $\mathcal{U}$ increases, we observe two simultaneous outcomes: (i) more users who were not in the market join the low-quality service (measured by $\frac{p^u}{q} f^u \left(\frac{p^u}{\mathcal{U}q}\right)$), and (ii) more users who previously used the low-quality service switch to the high-quality service (measured by $\frac{p_h^u - p^u}{q_h - q} f^u \left(\frac{p_h^u - p^u}{\mathcal{U}(q_h - q)}\right)$). If the latter effect is larger than the former, a greater $\mathcal{U}$ leads to fewer users for the low-quality service. According to Proposition 4, we can see that considering quality differentiation for the platform affects its strategic choices for the price and demand structure. If the platform overlooks the dynamic relationship between quality choice and optimal pricing, it could overlook potential better business strategies using the three-way interaction among quality, the user's side, and the developer's side.

3.2 The combined effects of openness and quality differentiation

Thus far, we have considered the platform to be a multi-sided firm and have not investigated how the effects of quality differentiation for the multi-sided platform are different from those for a one-sided firm. By comparing the effects of quality differentiation on a one-sided firm to those on a two- or multi-sided firm, we can see how serving more sides as the platform, which we call openness in the paper, is affected differently by quality differentiation through exploiting the interactions between different sides. In order to explore the difference between quality offered by a one-sided versus two sided platform, we generalize the model further to the case where the monopolist also optimizes the quality level. This implies that profit maximization occurs in terms of price and quality.

If the monopolist serves only one side of the market, say the user's side, its profit function is given by the following:

$$\max_{p^u, q} \Pi_{\text{one-sided monopolist}} = [p^u - c(q)]D^u, \tag{18}$$

in the case of one quality.

We find that the monopolistic firm provides higher quality to users for each dollar that they pay when it serves more sides of the market. As the monopolist opens more sides to serve, say from a one-sided firm serving users only to a two-sided firm as a platform serving both users and developers, such openness increases the quality per dollar offered to users. Mathematically, the quality per dollar is denoted as $\frac{q}{p^u}$. This finding is summarized in Proposition 5.

Proposition 5. *As a firm serves more sides of the market, it provides higher quality per dollar offered to users:* $\frac{q}{p^u}^{\text{One-sided firm}} \leq \frac{q}{p^u}^{\text{Two-sided firm}}$.

Intuitively, the monopolist serving multiple sides has more incentive to offer a better-quality price ratio to users because it now obtains more profit from the developer's side. If the two-sided firm offers a better-quality price menu to users, it attracts more of them. When it serves the developer's side at the same time, more demand from the user's side means that the developers on the other side earn more revenue. The platform can extract the additional revenues on the developer's side, which incentivizes it to offer a better-quality price ratio to users. In other words, opening the platforms from serving only one side of the user base to serving both sides increases the quality per dollar, which ultimately attracts more users than a one-sided platform.

Moreover, both the traditional one-sided firm and two-sided platforms offer differentiated quality tiers for users. When those two types of firms engage in quality differentiation on the user's side, the logic in Proposition 5 can be also interpreted as follows: servicing both sides makes the platform more willing to offer a better deal to low-type members by providing high quality per dollar in the basic level than a one-sided firm. For example, the average quality of services provided by taxi companies, as traditional one-sided firms, is known to be lower than that provided by ride-sharing platforms, such as Uber and Lyft. As Liu et al. (2019) show, taxi drivers are more likely to detour with non-local customers, which results in longer travel time. Such empirical evidence supports our finding that the platform's basic quality provision is better than that of a one-sided firm. The reason the platform provides a higher quality for basic service is that it is able to exploit such quality improvement on one side to encourage more participation from the other side, thereby raising its profits. As in the ride-sharing platform example, when such a platform provides higher basic quality to riders, thereby attracting more riders, it ultimately creates a stronger incentive for drivers to join, thereby allowing it to extract more rent from drivers. The underlying incentives for platforms to engage in higher service provision than a one-sided firm arise from this cross-subsidization motive. This finding is summarized in Corollary 1.

Corollary 1. *The platform in a two- or multi-sided market is more likely to offer a higher-quality basic service or product than a one-sided firm.*

4 Implications for the video game industry

The video game industry has multiple users who interact through a gaming platform. In particular, there are three main types of gaming platform: (i) open-source platforms accessed through a PC such as social platforms, e.g., Facebook; (ii) game consoles; and (iii) mobile gaming. For application of this model, a game console is the best fit as the only utility derived from the platform arises from playing games on it, whereas users of PC or mobile game platforms derive other benefits apart from using the platform as a gaming platform. Moreover, the game consoles need to have both users and developers on the platform to make a profit. Thus, the market generates positive cross-network externalities as the number of users using the console have a positive effect on games that are developed for the platform and vice versa.[11] Using the game console industry as an example, we summarize several managerial insights derived from our findings.

4.1 The interaction between quality access on the user's side and price ratio between the two sides.

As discussed above, the two sides in the game industry, i.e., the game users and developers, are interrelated by cross-network effects. In other words, the value derived from the platform for each side depends on the number of users on the other side. Under these conditions, does the quality of the platform (e.g., graphics, display of PlayStation) on one side (user's side) affect the other side (developer's side)? If yes, does the platform internalize this effect in the optimal prices? The first main finding of our paper answers these questions. By extending Rochet and Tirole (2003) to endogenize the impact of quality of the game console for the user's side on the profit made by developers, we find that the optimal two-sided pricing in the extended model is affected by the quality of the console. In particular, we find that the quality of game console changes the pricing ratio on the two sides.

Recall that quality differentiation for users affects both the user's side, because users are willing to pay a higher price for a better-quality console, and the developer's side via revenues generated by game developers. The former is the effect of price discrimination on profit margin incurred from the user's side, which is known in the literature. The latter is the effect of quality of the game console on the cross-side network externalities arising from the multi-sided nature: the level of quality of the game console would impact the time spent playing games by the user, which in turn impacts the profit earned by the developers. Proposition 1 states that if the positive effect of quality improvement on the developer's side is greater than the relative price sensitivities between the two sides, then the platform charges a higher price for developers than for users.

These findings also match the pricing decisions observed in this market. According to sector analysts, Microsoft sold its *Xbox 360* console at a price of at least \$125, which was under its marginal cost. Sony's PlayStation 2 console, which costs \$299 at the time of its release, was also sold at a loss.[12]

[11] For more details on platform dynamics in this market, refer to Shankar and Bayus (2003), Venkatraman and Lee (2004), Clements and Ohashi (2005), Prieger and Hu (2006), Nair (2007), Zhu and Iansiti (2007), Corts and Lederman (2008), Derbenger (2008, 2011), Lee (2009), Binken and Stremersch (2009), Liu (2010), Dubé et al. (2010), Chao and Derbenger (2011).

[12] See, for example, Herald News Service 30 March (2000, p. 56). and "Sony losing over US 300 on each 20GB PS3",

Such examples could indicate that the positive effects of the user's side's quality improvement on the developer's side is sufficiently large; thus, the platform extracts more rents from the developer's side via higher fees, whereas it lowers the price for users.[13] This finding has managerial implications about the quality of the product for one side of the market: the access quality for users not only affects the price on the user's side but also influences the price on the developer's side through cross-side network externalities.

Specifically, our theoretical prediction recommends that in order to maximize revenue, it is optimal for the game console platform to change the price/royalties charged on the developer's side when a new quality game console is introduced to users. This cross-side network externality drives the change in royalties paid by the developers when a new quality game console is introduced. In this regard, Lemma 1 and Proposition 3 show that allowing quality differentiation makes the platform consider relative profits between two sides when setting an optimal price. Specifically, if quality differentiation on the user's side leads to more average additional profit from the developer's side, then the platform lowers the price for the basic-quality product even if there is no quality degradation, whereas it raises a price for developers. Thus, quality differentiation can be one of the market aspects that the platform in a two-sided market takes into account to exploit cross-market externalities.

4.2 The optimal price for users of game consoles with two quality tiers and its openness

Again, our model focuses on the case in which the quality of the game console generates positive cross-side network externalities. Indeed, the quality differentiation of the game console (e.g., PlayStation *Pro* vs. *Slim*) can improve the gaming experience for users, thereby increasing the revenues for developer on the other side. Thus, unlike one-sided firms that decide the prices for game consoles with two different quality tiers on the basis of price elasticity on the user's side, the two-sided platform has an additional effect to consider, which is the effect of quality differentiation on cross-side network effects. Specifically, Proposition 5 shows that the price charged for the low-quality access (e.g., PlayStation *Slim*) by a gaming platform would be lower for a two-sided platform than for a one-sided firm. In terms of managerial implications, this means that managers need to account for the two-sided nature of the interaction when deciding the prices for different quality tiers of game consoles.

For example, Sony decreased the price for the PlayStation 2 (PS2) when a new generation with

http://www.gamesindustry.biz.

[13]Note that we interpret that below-cost pricing for the user's side indicates that the platform generates profit from the other side by means of a higher price for developers.

better-quality features, the PlayStation 3, was launched.[14] The lower price for the PS2 increased the market size for users, which had positive effects on the developer's revenue. In this case, our model suggests that one of the reasons for decreasing the price of the PS2 on the user's side could be to extract a higher profit from the developer's side by increasing the size of the royalty charged to the game developers.

Finally, it is worth noting that the basic intuition holds for other platforms that have two sides, such as mobile gaming as well social media platforms such as Facebook. For example, similar to our model, both Facebook and mobile app stores such as iOS charge fees to gaming apps for using their platforms. Therefore, we believe that the model can be extended to these markets as long as the platform charges fees to developers.

5 Conclusion

This paper analyzes a generalized version of quality differentiation by a monopolist in a multi-sided market. The main focus of this paper is the effect of the quality differentiation on the user's side on the optimal pricing strategy for the platform. We first showed that quality differentiation on the user's side can increase the price charged to developers but decrease the price charged to users. This finding suggests how a platform can exploit cross-market externalities by introducing quality differentiation on the other side.

We also found that quality differentiation, which leads to a lower price on the developer's side, ultimately increases the platform's profit. Thus, the platform can strategically use quality differentiation to raise its profits. Overall, this paper introduces a simple model that sheds light on the impact of platforms' quality differentiation on cross-side network externalities. Our findings suggest one plausible business strategy for the platform: how quality differentiation implemented by the platform can be used as an optimal business strategy. It would be interesting for future research to examine a dynamic setting in which users await a high-quality platform or buy it in the current period. Additionally, researchers could investigate how competition in the platform market alters our results. In this regard, a model with an asymmetric setup in the competitive market structure faced by the platform could determine the extent to which such asymmetry affects the platform's optimal business strategy.

[14] See Davidovici-Nora, M., Bourreau, M., and Libbrecht, E. (2012), Table 1 for details

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Appendix

A Further Discussion

A.1 Details of the equilibrium price and quality equations for one-quality case

If one type of quality is offered, the platform solves its profit maximization problem with respect to p^u and p^d, as given by Equation (4). The first order condition with respect to p^u is given as follows.

$$\begin{aligned} & [p^u + g(q)p^d - c(q)](D^u)'D^d + D^u D^d = 0 \\ \Leftrightarrow \quad & [p^u + g(q)p^d - c(q)] = \frac{D^u}{(-D^u)'} = \frac{p^u}{\varepsilon^u}, \end{aligned} \tag{19}$$

where $(D^u)'$ means that $\frac{\partial D^u}{\partial p^u}$.

The first order condition with respoect to p^d is given as follows.

$$\begin{aligned} & [p^u + g(q)p^d - c(q)](D^d)'D^u + g(q)D^d D^u = 0. \\ \Leftrightarrow \quad & [p^u + g(q)p^d - c(q)] = \frac{g(q)D^d}{(-D^d)'} = \frac{g(q)p^d}{\varepsilon^d}, \end{aligned} \tag{20}$$

where $(D^d)'$ means that $\frac{\partial D^d}{\partial p^d}$.

Using Equations (20) and (19), we obtain the following equation.

$$\frac{p^u}{\varepsilon^u} = \frac{g(q)p^d}{\varepsilon^d}.$$

A.2 Details of the equilibrium price and quality equations for two-quality case

If the platform differentiates its offered quality tiers, the platform solves its profit maximization problem with respect to p_l^u, p_h^u, and p_{2Q}^d, as in Equation (5). First, the first order condition with respect p_l^u is given as follows.

$$\begin{aligned} & (D^u - D_h^u)D^d + \pi_l (D^u)'_{p_l^u} D^d + \pi_{h-l} (D_h^u)'_{p_l^u} D^d \\ = \quad & D_l^u + \pi_l (D^u)'_{p_l^u} + \pi_{h-l} (D_h^u)'_{p_l^u} = 0, \end{aligned} \tag{21}$$

where $(D^u_k)'_{p^u_k}$ means that $\frac{\partial D^u_k}{\partial p^u_k}$ and $(D^u_k)'_{p^u_j}$ means that $\frac{\partial D^u_k}{\partial p^u_j}$, where $j \neq k$. Additionally, $[p^u_l + f(q_l)p^d_{2Q} - -c(q_l)] \equiv \pi_l$ and $[(p^u_h - p^u_l) + (f(q_h) - f(q_l))p^d_{2Q} - (c(q_h) - c(q_l))] \equiv \pi_{h-l}$. We also denote $\frac{D^u_h}{D^u} \equiv \lambda_h$.

The first order condition with respect to p^u_h is obtained as follows.

$$\begin{aligned} & \pi_{h-l}(D^u_h)'_{p^u_h} D^d + D^u_h D^d = 0. \\ \Leftrightarrow \quad & \pi_{h-l} = \frac{D^u_h}{(-D^u_h)'_{p^u_h}} = \frac{p^u_h}{\varepsilon^u_h}, \end{aligned} \tag{22}$$

where $(D^u_h)'_{p^u_h}$ means that $\frac{\partial D^u_h}{\partial p^u_h}$. By using Equation (22), we simplify Equation (21) as follows.

$$\begin{aligned} & D^u_l + \pi_l (D^u)'_{p^u_l} + \left[-\frac{D^u_h}{(D^u_h)'_{p^u_h}} \right] (D^u_h)'_{p^u_l} \\ = \quad & D^u_l + \pi_l (D^u)'_{p^u_l} + \left[\frac{D^u_h}{(-D^u_h)'_{p^u_h}} \right] (-D^u_h)'_{p^u_h} \\ = \quad & D^u_l + \pi_l (D^u)'_{p^u_l} + D^u_h = 0 \\ \Leftrightarrow \quad & \pi_l = \frac{D^u}{(-D^u)'_{p^u_l}} = \frac{p^u_l}{\varepsilon^u}, \end{aligned} \tag{23}$$

where the second equality comes from $(D^u_h)'_{p^u_l} = -(D^u_h)'_{p^u_h}$,

Lastly, the first order condition for p^d_{2Q} is given as follows.

$$(\pi_l D^u + \pi_{h-l} D^u_h)(D^d)'_{p^d_{2Q}} + g(q) D^u D^d + [g(q_h) - g(q)] D^u_h D^d = 0. \tag{24}$$

By using the definition of $\lambda_h \equiv \frac{D^u_h}{D^u}$, we can simplify Equation (24) as follows.

$$\begin{aligned} & (\pi_l + \pi_{h-l}\lambda_h)(D^d)'_{p^d_{2Q}} + \{g(q) + [g(q_h) - g(q)]\lambda_h\} D^d = 0. \\ \Leftrightarrow \quad & \pi_l + \pi_{h-l}\lambda_h = \frac{p^d}{\varepsilon^d} \{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\}. \end{aligned} \tag{25}$$

A.3 Proof of equivalence of the main model with the fixed fee case

Here we show that the setup with per trasaction fee is analog to the case where both fixed and transaction fee is charged on the user side.[15]

Without loss of generality, let us assume that the buyers' side charged a per transaction fee and a fixed fee, which implies that buyers have the following utility:

$$U^u_i = (\mathcal{U}\alpha^u_i q - p^u)N^d - P^u, \tag{26}$$

where P_b is the fixed fee charged to the buyers. The seller has the following utility:

[15]the change does not affect the model as long as there are no fixed benefits for the users.

$$U_j^d = (\mathcal{D}\alpha_j^d - p^d)N^u. \tag{27}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u &= Prob\,(U_i^u \geq 0) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u + \frac{P^u}{N^d}}{\mathcal{U}q}\right). \\ D^d &= Prob\left(U_j^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{28}$$

The monopoly problem can be written as follows:

$$\max_{p^d, p^u, q} \Pi = [p^u + p^d - c(q)]D^u D^d + P^u D^u. \tag{29}$$

Now, we modify the above case to show the equivalence. Let $p^u_{new} = p^u + \frac{P^u}{N^d}$ be the per transaction fee on the buyers' side and let the fixed fee be zero. Buyers have the following utility:

$$\begin{aligned} U_i^u &= (\mathcal{U}\alpha_i^u q - p^u)N^d - P^u. \\ &= (\mathcal{U}\alpha_i^u q - p^u - \frac{P^u}{N^d})N^d. \quad = (\mathcal{U}\alpha_i^u q - p^u_{new})N^d. \end{aligned} \tag{30}$$

Thus, this shows the utility is the same in the case of (i) usage fee p^u as the fixed fee, and (ii) usage fee of p^u_{new}. The seller has the following utility:

$$U_j^d = (\mathcal{D}\alpha_j^d - p^d)N^u. \tag{31}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u &= Prob\,(U_i^u \geq 0) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u_{new}}{\mathcal{U}q}\right). \\ D^d &= Prob\left(U_j^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{32}$$

The monopoly problem can be written as follows:

$$\begin{aligned} \max_{p^d, p^u, q} \Pi &= [p^u + p^d - c(q)]D^u D^d + P^u D^u \\ &= [p^u + p^d - c(q) + \frac{P^u}{D^d}]D^u D^d \\ &= [p^u_{new} + p^d - c(q)]D^u D^d \end{aligned}$$

Thus, as shown above, the profit function for the platform and the users utilities are the same in these two cases. □

B Proofs of Propositions

Proof of Proposition 1. Let us prove the first case. Assume $g(q) \leq \frac{\varepsilon^d}{\varepsilon^u}$. We use the above assumption and the equilibrium Equation (8) stated below:

$$\frac{p^u}{\varepsilon^u} = \frac{p^d \times g(q)}{\varepsilon^d}.$$

The assumption and above equation can hold if and only if $p^u \leq p^d$. Hence, the proof for the other case is analogous to above. □

Proof of Proposition 2. Let $(\bar{p}^u, \bar{q}, \bar{p}^d)$ be the profit maximization variable for the single-quality case. We prove whether the platform wants to set a nonzero demand for high-quality products at this price and quality level. The demand for high-quality products will be zero if

$$\frac{p_h^u - p^u}{\mathcal{U}(q_h - q)} = 1. \Leftrightarrow p_h^u - p^u = \mathcal{U}(q_h - q). \tag{33}$$

We choose (p_h^{u*}, q_h^*) such that Equation (33) is satisfied and then determine whether the first order condition on (p_h^u, q_h) shows that the platform will attempt to increase demand for the high-quality product above zero. The first order condition with respect to p_h^u at $(\bar{p}^u, \bar{q}, \bar{p}^d, p_h^{u*}, q_h^*)$ is given as follows:

$$\begin{aligned}
\Phi^{p_h^u} &= D^d \left\{ \{[p_h^{u*} - c(q_h^*)] - [\bar{p}^u - c(\bar{q})]\}(D_h^u)'_{p_h^u} + D_h^u \right\}. \\
&= D^d \left\{ \{[p_h^{u*} - c(q_h^*)] - [\bar{p}^u - c(\bar{q})]\} (-f^u(1)) \right\} \quad \text{as } \frac{p_h^u - \bar{p}^u}{\mathcal{U}(q_h - \bar{q})} = 1. \\
&= D^d \left\{ \{\mathcal{U}(q_h^* - \bar{q}) - [c(q_h^*) - c(\bar{q})]\} (-f^u(1)) \right\}. \\
&\leq 0 \quad \text{if } f^u(1) \neq 0 \text{ and } c'(\bar{q}) < \mathcal{U}.
\end{aligned} \tag{34}$$

In a similar manner, we can prove that at q_h^*, the first order condition is greater than zero. Thus, the platform will decrease p_h^u and increase q_h such that $D_h^u \neq 0$. Therefore, we see that the profit increases when offering two product qualities as long as $f^u(1) \neq 0$ and $c'(\bar{q}) < \mathcal{U}$. $c'(\bar{q}) < \mathcal{U}$ is true, as $c'(\bar{q}) < \mathcal{U}$ implies that $\frac{\bar{p}}{B\bar{q}} < 1$, which holds for nonzero demand. □

Proof of Lemma 1. Let the equilibrium values for the 1Q case be $(\tilde{p}^u, \tilde{p}^d)$ and those for the 2Q case be $(\bar{p}_l^u, \bar{p}_h^u, \bar{p}^d)$. Let us start with the case of $\bar{p}^d > \tilde{p}^d$. Using equilibrium conditions for p^u for both 1Q and 2Q, we obtain the following:

$$\underbrace{\frac{p^u}{\varepsilon_b} - (p^u - c(q))}_{2Q} > \underbrace{\frac{p^u}{\varepsilon_b} - (p^u - c(q))}_{1Q}.$$

As the above equation is decreasing in p^u, we obtain $\bar{p}^u < \tilde{p}^u$. Thus, $\bar{p}^u < (>)\tilde{p}^u$ if and only if $\bar{p}^d > (<)\tilde{p}^d$. □

Proof of Proposition 3. We will use Lemma (1) in this proof. Firstly, by Lemma (1), we know that the price on the developer's side and the user's side will move in opposite directions after the introduction of one more quality level. Thus, all we need to show is that the price on the developer's side increases if and only if the average profit increase on the developer's side is more than that on the user's side, which implies the following.

$$(p^d)^{1Q} \leq (p^d)^{2Q} \iff \frac{\text{Buyer's side per transaction profit}^{2Q}}{\text{Buyer's side per transaction profit}^{1Q}} \leq \frac{\text{developer's side per transaction profit}^{2Q}}{\text{developer's side per transaction profit}^{1Q}}. \tag{35}$$

Let us start with assuming $(p^d)^{1Q} \leq (p^d)^{2Q}$: in this case, Equation (35) will hold. The assumption implies implies the following:

$$\left(\frac{p^d}{\varepsilon^d}\right)^{1Q} \geq \left(\frac{p^d}{\varepsilon^d}\right)^{2Q} \iff \left(\frac{1}{\varepsilon^d}\right)^{1Q} \geq \left(\frac{1}{\varepsilon^d}\right)^{2Q}. \tag{36}$$

We will use the equilibrium equation for the developer's price for both the 1Q and 2Q case, reproduced below:

$$1\text{Q} \rightarrow \; p^u + g(q)p^d - c(q) = \frac{g(q)p^d}{\varepsilon^d} \tag{37}$$

$$2\text{Q} \rightarrow \; \pi_l + \pi_{h-l}\lambda_h = \frac{p^d}{\varepsilon^d}\left\{g(q) + [g(q_h) - g(q)]\lambda_h\right\}. \tag{38}$$

Using the above equilibrium conditions and Equation (36), we obtain the following.

$$\frac{\pi_l + \pi_{h-l}\lambda_h}{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}} \leq \frac{p^u + g(q)p^d - c(q)}{p^d g(q)}.$$
$$\iff \frac{\pi_l + \pi_{h-l}\lambda_h}{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}} - 1 \leq \frac{p^u + g(q)p^d - c(q)}{p^d g(q)} - 1.$$
$$\iff \frac{\pi_l + \pi_{h-l}\lambda_h - p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}}{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}} \leq \frac{p^u + g(q)p^d - c(q) - p^d g(q)}{p^d g(q)}.$$
$$\iff \frac{p_l^u - c(q_l) + [p_h^u - p_l^u - (c(q_h) - c(q_l))]\lambda_h}{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}} \leq \frac{p^u - c(q)}{p^d g(q).}$$
$$\iff \frac{p_l^u - c(q_l) + [p_h^u - p_l^u - (c(q_h) - c(q_l))]\lambda_h}{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}} \leq \frac{p^u - c(q)}{p^d g(q).}$$
$$\iff \frac{p_l^u - c(q_l) + [p_h^u - p_l^u - (c(q_h) - c(q_l))]\lambda_h}{p^u - c(q)} \leq \frac{p^d \{g(q) + [g(q_h) - g(q)]\lambda_h\}}{p^d g(q).}$$

Another way to see the inequality is

$$\iff \frac{\text{Buyer's side per transaction profit}^{2Q}}{\text{Buyer's side per transaction profit}^{1Q}} \leq \frac{\text{developer's side per transaction profit}^{2Q}}{\text{developer's side per transaction profit}^{1Q}}.$$

Hence, the proof is completed—similarly the reverse can be proved. □

Proof of Proposition 4. The proof is given in the paper.

Proof of Proposition 5. This proof shows that the platform offers a better price-quality ratio in the case of one quality offering. This implies

$$\underbrace{\frac{p^u}{q}}_{\text{Platform (two-sided)}} \leq \underbrace{\frac{p^u}{q}}_{\text{One-sided}}. \tag{39}$$

We start by comparing the profit functions for two different cases. For simplicity, we normalize c^d to zero.

$$\begin{aligned} \Pi_{\text{one-sided}} &= [p^u - c(q)]D^u \equiv \Psi(p^u, q). \\ \Pi_{\text{platform}} &= [p^u + p^d - c(q)]D^u D^d \equiv \Psi(p^u, q)D^d + p^d D^u D^d. \end{aligned} \tag{40}$$

The one-sided monopolist's profit maximization problem is given in Equation (18). Let the optimal solution be $\bar{x} = (\bar{p}^u, \bar{q})$ for the one-sided monopoly problem and $\widetilde{x} = \left(\widetilde{p}^u, \widetilde{q}, \widetilde{p}^d\right)$ for the two-sided platform. Given that the monopolist's optimal value for $p^d \neq 0$,

$$\Psi(\widetilde{p}^u, \widetilde{q})\widetilde{D}^d + \widetilde{p}^d \widetilde{D}^u \widetilde{D}^d \geq \Psi(\bar{p}^u, \bar{q}). \tag{41}$$

As $(\bar{p}^u, \bar{q})$ is the optimal solution $\Pi_{\text{one-sided}}$, this implies that $\Psi(\bar{p}^u, \bar{q}) \geq \Psi(\widetilde{p}^u, \widetilde{q}) \geq \Psi(\widetilde{p}^u, \widetilde{q})\widetilde{D}^d$, so Equation (41) holds only if

$$
\begin{aligned}
&\widetilde{D}^d\widetilde{p}^d(D^u)_{\text{at }\widetilde{x}} > \widetilde{D}^d\widetilde{p}^d(D^u)_{\text{at }\bar{x}}. \\
\Leftrightarrow\ & \underbrace{\frac{p^u}{q}}_{\text{platform}} < \underbrace{\frac{p^u}{q}}_{\text{one-sided}}.
\end{aligned}
\tag{42}
$$

Hence, the proof is complete. □

Proof of Corollary 1. It is guaranteed by Proposition 5.