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9293
2021

September 2021

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9293*

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Abstract

What is a good reduced-form representation of Ramsey-Cass-Koopmans. (RCK) model? Solow's model (despite non-optimizing agents) provides predictions largely consistent with a closed-economy RCK but fundamentally differs regarding open-economy income convergence. Where RCK predicts partial income and consumption convergence between open economies Solow predicts full convergence. This paper presents, by a small modification of the savings behavior in the Solow model, a framework that matches RCK's properties in closed and open economies. The model, labeled rSolow, is analytically tractable, allowing closed-form solutions of all variables, thus makes several explicit and novel predictions. This includes how income and inequality depend on country size; that income growth will be a U-shaped function of initial income thus creating differentiated convergence; and that poor countries bene.t from higher saving but rich countries may not.

JEL-Codes: E100, E210, F210, F430, O110.

Keywords: convergence, Ramsey, Solow, inequality, growth.

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September 2021
I am grateful to Nils Gottfries, Georg Graetz, Paul Klein, Hannes Malmberg, David Weil, Erik Öberg and seminar participants at Uppsala University and JIBS for valuable comments. Funding from FORMAS grant number 2020-00371 is greatly appreciated.

1 Introduction

Everything should be made as simple as possible, but not simpler – Albert Einstein

This paper applies the pursuit of simplicity to the Ramsey-Cass-Koopmans model by asking how it can be best represented in reduced form.

As is well-known, the Solow (1956) model is largely consistent with the convergence properties of the Ramsey-Cass-Koopmans model of a closed economy (Ramsey, 1929; Cass, 1965; Koopmans, 1965; henceforth RCK) despite the Solow model having non-optimizing agents. However, in one central aspect, the Solow model differs substantially from RCK, namely, regarding income convergence between open economies.

In the open-economy RCK model, capital mobility ensures that output and wages converge instantaneously. However, consumption and income do not converge, since a higher initial capital level translates into a higher net foreign asset position in the long run (Chatterjee, 1994; Caselli and Ventura 2000). An open-economy Solow model, on the other hand, predicts that income and consumption *will* converge over time unless exogenous parameters differ between countries (Milbourne, 1997; Benge and Wells, 2002).[1] For open economies the Solow model is thus not a functioning reduced-form representation of RCK.

This paper shows that the equivalence between RCK and Solow can be reestablished if saving in the Solow model is not a share of income but of capital. This revised Solow model (henceforth rSolow) is inspired by the properties of RCK. In an open-economy RCK, all countries, endogenously in the steady state, save an identical share of their own capital income (Caselli and Ventura, 2000).[2] In the reduced-form RCK presented here, this property is assumed to hold already initially.

The rSolow model qualitatively matches, in a reduced-form, RCK with respect to the dynamics and steady-state properties of both closed and open-economies. The simplicity of rSolow buys a lot in terms of tractability, transparency and novel insights. All key properties can be attained in closed-form,[3] and one can illustrate the short-run gain and long-run loss of opening up for foreign capital early in the development process. It also illustrates the importance of distinguishing between GDP (i.e., within-country production) which theoretically does converge over time and GNI (total income of the citizens in a country) which does not (see Klein and Ventura, 2020, for differences found empirically).[4] Further, the free flow of capital in the model provides a quick boost to labor

[1] See e.g., equation (11) in Benge and Wells (2002) where steady-state wealth in a country is independent of the country's initial wealth. Steady-state wealth is instead determined by exogenous parameters, the world interest rate and capital placed in the country. Thus, if all countries have identical parameters, they will also have identical wealth in steady state.

[2] The intuition for this is that in steady state all countries have to replace the depreciated part of the capital they own which is proportional to the size of their owned capital stock). Since all countries face the same interest, also their capital income is proportional to their owned stock. Hence saving must be proportional to the owned stock. See Caselli and Ventura (2000) for formal results.

[3] While it it possible to analyze some properties of the steady-state distribution of income in RCK, it is, to my knowledge, not analytically possible to derive it in closed form which makes the usage of comparative statics difficult or impossible in some instances. The reason is that, when considering an arbitrary number of countries, all with different levels of initial capital, the transition path for any single country is well defined short of the initial choice of consumption and the steady-state level of consumption. Since the steady-state level of consumption in a single country depends on the full distribution between countries, it is not trivial to solve for the level of consumption and income initially and in the steady state in closed form.

[4] In official data GDP and GNI do not differ markedly for most countries, with some exceptions, such as Ireland, where GDP has been 10-20% higher than GNP (Klein and Ventura, 2020). The lack of difference in most of the data is a puzzle that can possibly be explained by dividend flows being muddled by FDI (which flows the other way) and, perhaps more importantly, by hidden flows through tax havens. For instance, Zucman (2013) finds that around 6-8%

income while a country's accumulation of wealth is more gradual. It thus illustrates that initially poor economies will see a higher growth in their labor income than their capital income (as shown empirically by Caselli and Feyrer, 2007).

A number of novel and surprising results can be easily derived and understood. One such result is how country size affects income and inequality. When the initially poor countries are large, their long run per-capita income is larger, but between-country inequality is (somewhat counterintuitively) exacerbated. Another result is the creation of differentiated convergence: over time poor countries' income will converge towards the income of middle-income countries, but rich countries' income will diverge. Finally, rSolow is similar to RCK with respect to optimal (golden rule) saving and global consumption. But rSolow further shows that the poor countries always gain if global saving goes up but that the rich countries may loose.

The model is also easily quantifiable without resorting to computer simulations. A simple such exercise, using standard macro-economic parameters and data on current capital ownership between countries, is provided at the end. The exercise illustrates the emergence of small but extremely rich economies (or individuals) whose income may be an order of magnitude larger than the world average, derived almost exclusively from capital income (e.g., Luxembourg).

The main motivation for the revised saving behavior in rSolow is technical: it matches the steady-state savings behavior of the RCK and it enables matching other key properties of RCK. But the savings assumption can also be empirically and conceptually motivated. It is well established empirically that the rich, and in particular capital owners, save a larger share of their income than others and that there is wealth concentration (Bernheim and Scholtz, 1993, Beverly 1995; Browning and Lussardi, 1995; Dynan et al.., 2004; Diamond & Hausmann 1984; Gentry & Hubbard, 2004; Quadrini, 2000; Alan et al. 2015; Dupas and Robinson, 2015; Gandelman 2017). One interpretation of the saving assumption is that each country consists of two agents, a capitalist and a worker, where the former is the main one accumulating capital, in line with the ideas of Karl Marx and as modeled by, e.g., Judd (1985), Doepke & Zillibotti (2008), Galor & Moav (2006) and Chakrabarty et al. (2008).

Naturally, by exchanging the word country for the word agent, rSolow can be readily reinterpreted as a single economy consisting of agents with different initial wealth levels. The results of unequal long-run income thus taps into, for instance, Krusell and Smith's (2015) discussion about Piketty's (2014) second fundamental law of capitalism – whether capital's share of income can be expected to increase thus drive inequality.[5] To illustrate their points they, just like this paper, utilize the Solow model. This paper shows that even if capital's share of income is constant, there can be substantial income inequality driven by heterogeneity in initial capital ownership.[6] In fact, rSolow shows that inequality can be exacerbated over time as capital-rich agents attain an increasing income from capital; and this is true even without any imbalances due to an increasing capital's share of income as suggested by Piketty's (2014) famous r-g. More precisely, the rSolow model predicts that the income of the poorest agents and the richest agents will grow the fastest in an open economy, while initial middle-income agents' income will grow slower. This result aligns with the finding of Lakner and Milanovich (2013).

of capital flows are hidden which explains why many rich countries appear to be lending more than many poor.

[5] The implicit assumption in their discussion is that capital owners and workers are two different groups. Thus capital's share of income becomes informative of inequality.

[6] Piketty (2015) and Krusell and Smith (2015) highlight that whether saving is based on income *net* of capital depreciation makes an important difference. This paper shows that whether saving is based on total income (Solow) or capital income (RCK in steady state) is important.

2 Closed economy model

Consider the basic Solow (1956) model but with the following change: the population saves a share s of its capital income.[7] This assumption is inspired by the endogenous long-run properties of the open-economy RCK model where agents (or countries) save a constant share of their capital income in steady state (Caselli and Ventura, 2000) and, as mentioned in the introduction, can be motivated both conceptually and empirically. Thus investment is

$$I_t = sK_t r_t$$

where K_t is capital and r_t is the return to capital which with a Cobb-Douglas production function

$$Y = AK^{\alpha}L^{1-\alpha}$$

is

$$r_t\left(K_t\right) = \frac{dY_t}{dK_t} = \alpha A K_t^{\alpha-1} L_t^{1-\alpha}.$$

A is a technological parameter which is deliberately chosen to be constant in order to highlight the convergence properties. Assume constant labor $L = 1$, implying

$$r_t = \alpha A K_t^{\alpha-1}. \tag{1}$$

In the closed economy, capital accumulates according to

$$\begin{aligned} K_{t+1} &= K_t\left(1-\delta\right) + I_t \\ &= K_t\left(1-\delta\right) + sK_t r_t \end{aligned}$$

where δ is the depreciation rate. Just like the standard Solow model we get convergence to a steady state (subscript ss). Here it is given by equating $K_{t+1} = K_t$ in the previous expression and using (1) for r:

$$K_{ss} = \left(\frac{s\alpha A}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

This result is qualitatively the same as the standard Solow model[8] and the RCK model: all countries will eventually converge to the same steady-state capital intensity and income which in the model here simply is

$$Y_{ss} = AK_{ss}^{\alpha} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

and consumption

$$\begin{aligned} C_{ss} &= Y_{ss} - I_{ss} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} - sK_{ss}r_{ss} \\ &= A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}\left(1-s\alpha\right). \end{aligned}$$

[7] In the standard Solow model the population saves a share of the *total* income, not only from its capital income.

[8] In the Solow model, steady state capital is $k_{ss} = (\frac{As}{\delta})^{1/(1-\alpha)}$, that is, the same expression short of α in the parenthesis.

Since $L = 1$, all variables can be interpreted as per capita. Throughout I will refer to the revised Solow model presented in this paper as rSolow.

Thus, just like in closed-economy versions of RCK and Solow, also rSolow has capital, income and consumption converging to a steady state which is independent of the initial capital level. Put differently, closed countries with identical preferences and fundamentals, converge to an identical steady state. For closed economies the precise saving assumption is thus not very important.

3 Open economies

Now suppose the economy is open. More precisely, a country indexed i is one out of J open economies. The countries are identical in *all* respects except their population size and initial level of total capital. Each country i consists of a representative agent of size n_i consuming, working and owning capital. $N = 1$ is the total world population. Since the economies are open we need to keep track of which country owns capital where. Let $K_{t,i,j}$ denote capital owned by country i placed in country j (in period t). As before, country i saves and thus invests a share of its total capital income

$$I_{t,i} = s \sum_{j=1:J} K_{t,i,j} r_{t,j}, \tag{2}$$

where i's capital income depends on the returns it gets in the countries j it has placed its ownership in. Country i produces

$$Y_{t,i} = A \left(\sum_{j=1:J} K_{t,j,i} \right)^{\alpha} n_i^{1-\alpha}. \tag{3}$$

Note, importantly, that the subscripts i and j swap places in (3) compared to (2). This is since country i's production depends on the amount of capital that all countries have placed in i.

With open economies, capital can flow between the countries. Naturally investors will seek the highest returns. Since there are decreasing returns to capital within a country this means capital will flow from richer to poorer countries until all capital intensities equalize. Like is standard in the literature on open economies, this equalization process is assumed to happen instantaneously.[9] However, while capital intensities in the countries equalize, ownership does not. This will be examined next. I will throughout the paper use owned capital and wealth interchangeably.

Since the capital intensity equalizes immediately between the countries, the returns to capital are the same in all the countries: $r_{t,j} = r_t$ for all j. Hence, wealth held by country i, $K_{t,i} \equiv \sum_{j=1:J} K_{t,i,j}$, accumulates according to

$$K_{t+1,i} = K_{t,i} (1 - \delta) + s K_{t,i} r_t.$$

From this follows that

$$\frac{K_{t+1,i}}{K_{t,i}} = (1 - \delta) + s r_t.$$

The right-hand side of this expression is void of the index i, hence follows that the *growth rate* of capital owned by each country is identical – wealth accumulates proportionally.

Since capital intensity equalizes between all countries, the world interest rate, r_t, is given by the

[9] See, e.g., Pellegrino et al. (2021) for a recent paper where this is not the case.

returns to capital in the global economy. Let $\bar{K}_t \equiv \sum_{i \in J} K_{t,i}$ and $\bar{Y}_t \equiv \sum_{i \in J} Y_{t,i}$. Then

$$\begin{aligned} r_t &= \frac{d\bar{Y}_t}{d\bar{K}_t} = \alpha A \bar{K}_t^{\alpha-1} N^{1-\alpha}, \\ w_t &= \frac{d\bar{Y}_t}{dN} = (1-\alpha) A \bar{K}_t^{\alpha} N^{-\alpha}. \end{aligned}$$

As the growth rate of wealth is the same in all countries and r is decreasing in $\bar{K}$, follows that the global economy will converge to a steady state level of capital where $\bar{K}_{t+1} = \bar{K}_t = \bar{K}_{ss}$. At that point the total capital will obey

$$1 = \frac{\bar{K}_{t+1}}{\bar{K}_t} = \frac{K_{t+1,i}}{K_{t,i}} = (1-\delta) + s r_{ss}\left(\bar{K}_{ss}\right)$$

that is a unique

$$\bar{K}_{ss} = \left(\frac{s\alpha A}{\delta}\right)^{\frac{1}{1-\alpha}}. \tag{4}$$

This expression tells us what the world's capital stock will be in steady state. But what is the division of wealth in steady state between the countries? Let $\gamma_{t,i} \equiv K_{t,i}/\bar{K}_t$ be country i's share of total wealth in a period. Since wealth accumulates proportionally in all countries $\gamma_{ss,i} = \gamma_{t,i} = \gamma_{0,i} = K_{0,i}/\bar{K}_0$ implying

$$K_{ss,i} = \gamma_{0,i}\bar{K}_{ss}, \tag{5}$$

that is, the share of total wealth in any period is the same as in the period before – wealth differences persist. In per-capita terms this implies

$$k_{ss,i} \equiv K_{ss,i}/n_i = \gamma_{0,i}\bar{K}_{ss}/n_i = \frac{\gamma_{0,i}}{n_i}\left(\frac{s\alpha A}{\delta}\right)^{\frac{1}{1-\alpha}}, \tag{6}$$

where (4) has been used.

In total we get that when starting from closed economies with countries opening up with different levels of initial capital (in period 0), capital intensity drops in high-income countries and increases in the low-income countries. This will lower the labor wage in high-income countries and increase the wage in the low-income countries (this can be interpreted as a temporary loss for the working class in rich countries and a gain for the working class in poor countries). Capital income increases in the high-income countries (since they now have higher returns to their investments) but drops in the low-income countries (a gain for the capital class in the rich country and a loss for the capital class in the poor country). The countries then emerge on a joint growth path: capital *income* is now growing more quickly in high-income countries than if economies were closed and vice versa for low-income countries. Eventually they will converge to a steady state.

In the steady state, domestic production is the same in all countries since capital intensity is equalized, but the income of the representative citizen is higher in initially capital-rich countries than in initially capital-poor countries. For this reason their consumption will differ. Denote national income in i by $G_{t,i}$. It consists of i's labor income (which depends on the population size) and i's capital income. In steady state

$$G_{ss,i} = n_i w_{ss} + r_{ss} K_{ss,i}.$$

Since $K_{ss,i} = \gamma_{0,i}\bar{K}_{ss}$

$$\begin{aligned} G_{ss,i} &= n_i w_{ss} + r_{ss}\gamma_{0,i}\bar{K}_{ss}. \\ &= n_i (1-\alpha) A \left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} + \gamma_{0,i}\alpha A \left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Let $g_{t,i} \equiv G_{t,i}/n_i$. Then in per-capita terms income is

$$g_{ss,i} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(1-\alpha + \frac{\gamma_{0,i}}{n_i}\alpha\right). \tag{7}$$

For consumption per capita we get

$$\begin{aligned} c_{ss,i} &= g_{ss,i} - \delta k_{ss,i} = ... \\ &= A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) + \frac{\gamma_{0,i}}{n_i}\alpha A \left(\frac{\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(s^{\frac{\alpha}{1-\alpha}} - s^{\frac{1}{1-\alpha}}\right) \end{aligned} \tag{8}$$

which is increasing in $\gamma_{0,i}$ since $s \in (0,1)$ and $\alpha \in (0,1)$.

On the global level (subscript W) total income and production (and per capita since $N = 1$) are

$$Y_{ss,W} = G_{ss,W} = N w_{ss} + r_{ss}\bar{K}_{ss} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{9}$$

Proposition 1 *[Open economies] In the standard Solow model with open economies there is convergence to a steady state where per-capita levels of wealth, income and consumption are the same in all countries. In the RCK and rSolow models with open economies there is convergence to a steady state where per capita levels of wealth, income and consumption are higher in initially rich countries.*

Proof. *The statement on the Solow model can be easily derived noticing that, when saving is proportional to income, then in ss wealth obeys* $1 = \frac{k_{t+1,i}}{k_{t,i}} = 1-\delta + sg_{t+1,i}/k_{t,i} = 1-\delta + sw_{ss,i}/k_{ss,i} + sr_{ss,i}k_{ss,i}/k_{ss,i}$. *Since* $w_{ss,i}$ *and* $r_{ss,i}$ *are the same in all countries, also* $k_{ss,i}$ *is the same in all countries . Hence, when capital ownership* $k_{ss,i}$ *is the same in all countries, so is* $y_{ss,i}$ *and* $c_{ss,i}$. *For the statement on RCK see, e.g., Caselli and Ventura (2000). For the statement on rSolow see derivations above and in particular (6) for wealth (7) for income and (8) for consumption in steady state. These expressions are all increasing in* $\frac{\gamma_{0,i}}{n_i}$ *i.e. in initial income.* ■

4 Income in rSolow

The results underlying Proposition 1, specifically equations (6)-(8) can be used for comparative statics. I first look at long-run income. Consider a given wealth per capita both in country i $k_{0,i} = K_{0,i}/n_i$ and in the rest of the world $k_{0,ROW} = K_{0,ROW}/(1-n_i)$ implying a world average of (recall $N = 1$) $\bar{K}_0 = \bar{k}_0 = k_{0,i}n_i + k_{0,ROW}(1-n_i)$ at the onset. Using

$$\gamma_{0,i} = K_{0,i}/\bar{K}_0 = \frac{k_{0,i}n_i}{k_{0,i}n_i + k_{0,ROW}(1-n_i)}$$

in (7) gives

$$g_{ss,i} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(1-\alpha + \frac{k_{0,i}}{k_{0,i}n_i + k_{0,ROW}(1-n_i)}\alpha\right). \tag{10}$$

In the following corollaries a country is called initially "poor" if $k_{0,i} \leq k_{0,ROW}$ and initially "rich" otherwise.

Corollary 1 *[Long-run income in rSolow]*

- *Every country's long-run income per capita is increasing in the saving rate (s).*
- *Every country's long-run income per capita is increasing in its initial capital intensity ($k_{0,i}$).*
- *Holding $k_{0,i}$ and $k_{0,ROW}$ fixed, the long run income per capita of i as a function of its size (n_i) is increasing if i was initially poor and decreasing if i was initially rich.*

Proof. *First and second bullets: follow trivially from differentiating (10) w.r.t. s and $k_{0,i}$ while holding all else fixed. Third bullet: holding all else fixed and using (10), $\frac{dg_{ss,i}}{dn_i} \geq 0$ iff $k_{0,i} \leq k_{0,ROW}$.*
■

Like in the standard Solow model, also rSolow highlights (first bullet) the importance of saving for long-run income in all countries, rich and poor. The second bullet highlights that being ahead at one point in time is positive for a country's long-run income. Perhaps most interestingly, the third bullet of the corollary says population size has different effects on initially rich and poor countries. It suggests that being the only rich country in a "vast sea of poverty" enables large riches in the long run. As an illustration, if capital in the rest of the world is very scarce ($k_{0,ROW} \to 0$) then long run income in an initially-rich country converges to $g_{ss,i} = A\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\alpha+\frac{\alpha}{n_i}\right)$ which can become arbitrarily large when the own country's share of the world population is very small. This can explain why some countries that manage to accumulate a lot of capital fast, which is then invested internationally (e.g., Norway), may display an income several times larger than the remaining world. The reason is that a small rich country will have a small effect on total capital accumulation in any one period (it will have small general-equilibrium effects) hence can count on high returns on its investments for long which also enables vast accumulation of capital. Conversely, a country that is poor will have a lower income in the long run if it is small compared to the rest-of-the-world. The reason is that then, when other countries invest in that country, its within-country capital stock will rise quickly not enabling it to accumulate that capital itself.[10]

5 Income inequality in rSolow

Now consider income *inequality* in the long run. Let

$$\triangle g_{ss,i} \equiv \ln\left(g_{ss,i}/g_{ss,ROW}\right) \tag{11}$$

denote the relative per capita income difference between i and the rest of the world. The absolute value of $\triangle g_{ss,i}$ is a measure of income inequality. To see this, note that when $\triangle g_{ss,i} < 0$ (i is poorer than ROW), then decreasing $\triangle g_{ss,i}$ further implies making i relatively poorer, i.e., enlarging income differences between i and the rest of the world. On the other hand, when $\triangle g_{ss,i} > 0$ (i is richer than

[10] A larger poor country implies a smaller rich ROW. As per the corollary, the larger the initially poor country is the higher long-run income it will have. Likewise, the smaller the initially rich ROW is the higher its long-run income will be. Does that mean that the global average long-run income increases? No, at the global level the average is fixed. What happens when the poor country is larger is that it also gets a higher weight in the global average precisely counteracting that income per capita in each country is larger.

ROW), a *larger* $\triangle g_{ss,i}$ implies larger income differences between i and ROW. Using (10) in (11) we get

$$\triangle g_{ss,i} = \ln\left(\frac{1-\alpha+\frac{k_{0,i}}{k_{0,i}n_i+k_{0,ROW}(1-n_i)}\alpha}{1-\alpha+\frac{k_{0,ROW}}{k_{0,i}n_i+k_{0,ROW}(1-n_i)}\alpha}\right). \tag{12}$$

Corollary 2 *[Long-run income inequality in rSolow]*

- *Long run income inequality ($|\triangle g_{ss,i}|$) as a function of i's initial capital intensity ($k_{0,i}$) is decreasing if i was initially poor and increasing if i was initially rich.*
- *Long run income inequality as a function of i's size (n_i) is increasing if i was initially poor and decreasing if i was initially rich.*

Proof. *First bullet: holding all else fixed, differentiating $g_{ss,i}/g_{ss,ROW}$ shows it is increasing in $k_{0,i}$ hence so is $\triangle g_{ss,i}$. Since $\triangle g_{ss,i}=0$ when $g_{ss,i}/g_{ss,ROW}=1$, $|\triangle g_{ss,i}|$ is decreasing iff $k_{0,i}\leq k_{0,ROW}$. Second bullet: holding all else fixed (including $k_{0,i}$ and $k_{0,ROW}$), differentiating $g_{ss,i}/g_{ss,ROW}$ shows it is decreasing in n_i hence so is $\triangle g_{ss,i}$. Since $\triangle g_{ss,i}=0$ when $g_{ss,i}/g_{ss,ROW}=1$, $|\triangle g_{ss,i}|$ is increasing in n_i iff $k_{0,i}\leq k_{0,ROW}$.* ■

The first bullet of the corollary is essentially a restatement of Proposition 1, that initial capital differences will increase also long-run inequality. The second bullet, how income inequality depends on population size, is possibly less intuitive. Despite its seeming inconsistency with Corollary 1, it actually follows the same mechanism as for how population size affects the income level. It says that if initially poor countries are large and initially rich countries are small, then income inequality will be large. Why? If an initially poor country is large, then the rich remaining world is "small" allowing those rich countries to accumulate a large capital ownership in the poor country making them rich. This creates cross-country inequality. At the same time, the initially poor country's income *increases* in its population size as per Corollary 1. In other words, population size makes the poor richer, yet inequality increases. Corollaries 1 and 2 thus jointly highlight an important tension between long run income (welfare) and inequality when comparing countries. Inequality across countries can seem very large (third bullet in Corollary 2) when a few small and super-rich countries (e.g., the Emirates, Norway, Luxembourg) accumulate a large capital ownership while income of the poorest is in fact higher (second bullet in Corollary 1). This tension, or artefact in measurement when looking across countries, thus weighs in on the discussion on whether global income should be measured by comparing countries or by comparing individuals (see, e.g., Lakner and Milanovich 2013). The analysis here suggests that, if the richest countries are small, then inequality as measured across countries will be inflated.

6 Income growth in rSolow

Another way to approach income and inequality is to analyze how they are changing over time. Suppose i starts from being closed, at time zero, but then opens for capital flows. The initial income of i only depends on its own capital level

$$g_{0,i} = An_i^{1-\alpha}K_{0,i}^{\alpha}/n_i = Ak_{0,i}^{\alpha}.$$

Slightly rewriting (7) for income in the, open, steady state an expression for income growth is attained:

$$D \equiv \frac{g_{ss,i}}{g_{0,i}} = \frac{\left(\frac{s\alpha A}{\delta}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\alpha+\frac{k_{0,i}}{\bar{k}_0}\alpha\right)}{k_{0,i}^{\alpha}}.$$

Differentiating this function with respect to $k_{0,i}$ (and assuming i is sufficiently small so that the effect of $n_i k_{0,i}$ on $\bar{k}_0$ is second order) shows D is decreasing in k_i iff $k_{0,i} \leq \bar{k}_0$. This means income growth is a U-shaped function of the initial capital level.

Hence, a corollary follows.

Corollary 3 *[Income and inequality growth in rSolow] Consider small countries that open up for capital movements.*

- *Income growth is highest in initially capital poor and initially capital rich countries and lowest in initially middle-income countries.*

Proof. *Follows from $dD/dk_{0,i} \leq 0$ iff $k_{0,i} \leq \bar{k}_0$.* ■

The corollary suggests a form of differentiated convergence. Low-income countries will converge towards the middle-income countries (a form of convergence club) but top-income countries will diverge. The result is somewhat surprising, given that labor income growth is lower the richer the country initially is while the growth rate of capital is constant across all countries. But the free flow of capital enables precisely this U-shaped pattern of income growth. It is explained by the top-income countries disproportionately increasing their income from capital. For them, the potential growth in labor income is dwarfed by the increase in capital income. For initially poorer countries on the other hand the accumulation of wealth is not the main driver of income growth, rather it is the labor income.

Interpreting i as an individual instead of as a country this result has bearing on the observed divergence between the top income class and the average. It largely aligns with the finding of U-shaped growth across income percentiles found in Lakner and Milanovich (2016), bar for the left part, in their Figure 1a. The result also has bearing on the debate on inequality growth and Piketty's (2014) famous claim on r-g driving inequality, questioned by, e.g., Krusell and Smith (2015). The model here shows that even without any technical change (the g-term in Piketty, 2014), top income can diverge from the rest due to capital ownership.

7 The golden rule in rSolow

Next we analyze how consumption in the steady state depends on saving. Using (8) and differentiating yields

$$\frac{dc_{ss,i}}{ds} = \frac{1}{1-\alpha}\frac{(\alpha A)^{\frac{1}{1-\alpha}}}{s}\delta^{\frac{-\alpha}{1-\alpha}}\left[(1-\alpha)\, s^{\frac{\alpha}{1-\alpha}} + \frac{\gamma_{0,i}}{n_i}\left(\alpha s^{\frac{\alpha}{1-\alpha}} - s^{\frac{1}{1-\alpha}}\right)\right]. \tag{13}$$

A number of results can be derived using this expression.

First off, note that if $\frac{\gamma_{0,i}}{n_i} \to 0$ then $\frac{dc_{ss,i}}{ds} = \frac{1}{1-\alpha}\frac{(\alpha A)^{\frac{1}{1-\alpha}}}{s}\delta^{\frac{-\alpha}{1-\alpha}}(1-\alpha)\, s^{\frac{\alpha}{1-\alpha}} > 0$. By continuity of $\frac{dc_{ss,i}}{ds}$ in $\frac{\gamma_{0,i}}{n_i}$, it thus holds that for sufficiently (initially) poor countries no golden rule exists, that is, they gain from high saving rates. The reason for this is that initially poor countries have only a small effect on global capital. The brunt of their income, which comes from labor, is dependent on capital accumulated by other countries. Thus they unequivocally gain when saving increases.

Next, on the global level ($\frac{\gamma_{0,W}}{n_W} = 1$),

$$\frac{dc_{ss,W}}{ds} = \frac{1}{1-\alpha} \frac{(\alpha A)^{\frac{1}{1-\alpha}}}{s} \delta^{\frac{-\alpha}{1-\alpha}} s^{\frac{\alpha}{1-\alpha}} [1-s]$$

which is strictly positive for $s < 1$. Thus, at the global level (equivalently for a closed economy), the rSolow model is closer to RCK in the sense that more saving (in RCK emanating from a higher discount factor) implies higher long-run consumption. As is well known, in the standard Solow model, more saving can hurt consumption. The reason for rSolow and RCK not having this feature, is that when saving is a share of the capital income (in rSolow assumed, in RCK a result in the steady state) the agent always leaves the labour income for consumption. This result aligns with Phelps (1961) who shows that in the Solow model the golden rule is for saving to equal capital's share of income.

Finally, define $h(s) \equiv \alpha s^{\frac{\alpha}{1-\alpha}} - s^{\frac{1}{1-\alpha}}$ (i.e., the last parenthesis in (13)) and note that $\lim_{s\to 0} h(s) = 0$, $\lim_{s\to 1} h(s) < 0$ and

$$h' = \frac{\alpha}{1-\alpha} \alpha s^{\frac{\alpha}{1-\alpha}-1} - \frac{1}{1-\alpha} s^{\frac{\alpha}{1-\alpha}}$$

which is strictly positive for sufficiently small but positive s.[11] This means that $h(s)$ goes from positive to negative in the range $s \in (0,1)$. $\frac{\gamma_{0,i}}{n_i} h(s)$ captures how net income from capital is affected by saving. Put together with the other, strictly positive, part of (13) implies that if $\frac{\gamma_{0,i}}{n_i}$ is sufficiently large there *does* exist a golden rule of saving – sufficiently rich countries can oversave. The intuition for this is that very rich countries get a disproportional part of their income from returns to the capital they own. If they save much of that, consumption is reduced. The following corollary summarizes these results. Subscript GR stands for golden rule.

Corollary 4 *[Golden rule in rSolow]*

- *In a closed (global) economy, saving is strictly below the golden rule ($s_{GR} = 1$).*
- *For a sufficiently rich open economy, $s_{GR} \in (0,1)$.*
- *For a sufficiently poor open economy, saving is strictly below the golden rule ($s_{GR} = 1$).*

Proof. *Follows from (8) and the derivations above.* ■

An additional interpretation can be made from the corollary. Saving in open economies is a form of public good. The average country (i.e., the global and closed economy) and particularly the poorest countries gain in consumption if all increase the saving rate . But the richest countries do not necessarily gain from this.

[11] To see this note that $\lim_{s\to 0} = \frac{\frac{\alpha}{1-\alpha}\alpha s^{\frac{\alpha}{1-\alpha}-1}}{\frac{1}{1-\alpha}s^{\frac{\alpha}{1-\alpha}}} = \alpha^2 s^{-1} \to \infty$. Thus the second term approaches zero faster than the first term.

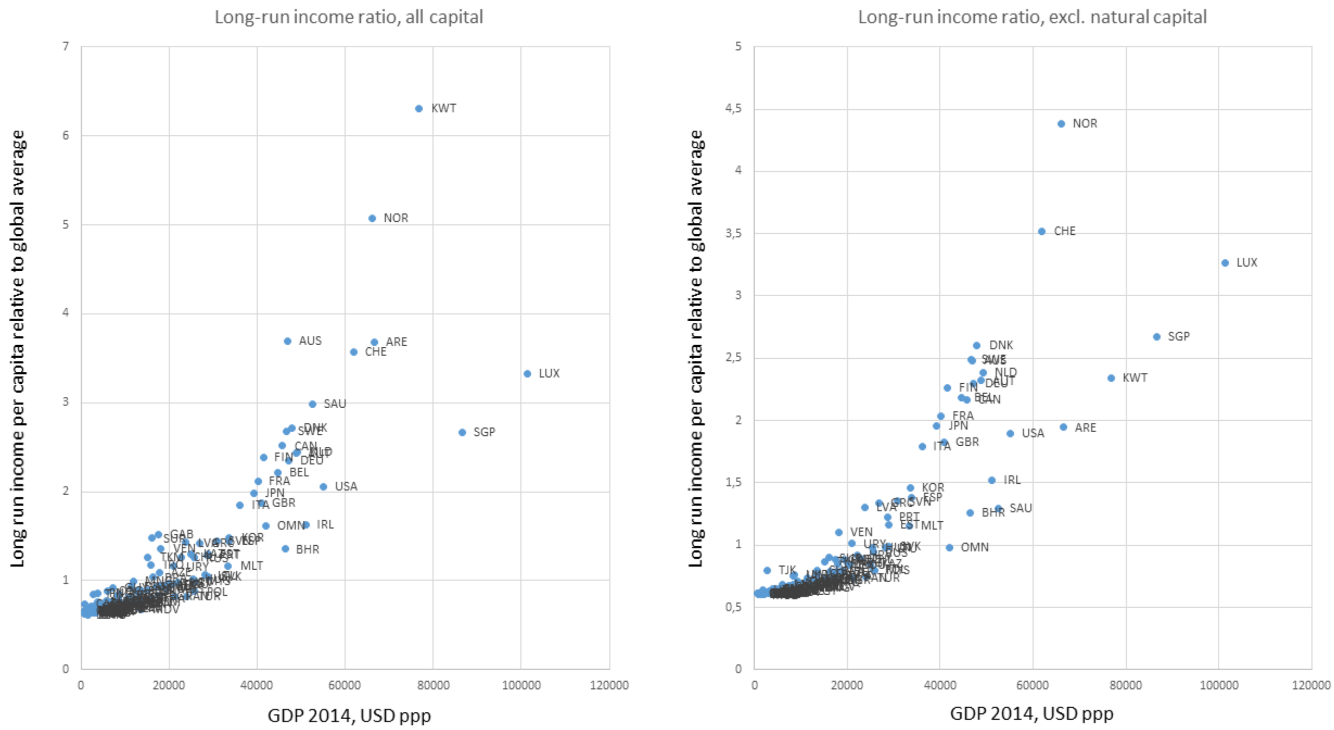

Figure 1: Long-run income relative to world average. Data from World Development Indicators. Left: capital ownership based on all capital. Right: capital ownership excl. natural capital.

8 Quantitative illustration: long-run income differences

Some further insights can be illustrated by a straightforward quantitative exercise. By using (7) and (9)

$$\frac{g_{ss,i}}{g_{ss,W}} = \left(1 - \alpha + \frac{K_{0,i}/\bar{K}_0}{n_i/N}\alpha\right)$$

provides a quantitative prediction for a country's long-run income relative to the world average given its capital stock ($K_{0,i}$), population size (n_i), the world's capital stock ($\bar{K}_0$) and population (N) and capital's share in income (α). Using the standard value of $\alpha = 0.4$ (to take account of income from capital and natural resources) and data from the Worldbank on capital ownership and population size in 2014, Figure 1 shows the relative income per capita in the long run as predicted by the rSolow model. Importantly, this data includes net foreign assets, so captures ownership in other countries as well. This is what explains why GDP in 2014 (x-axis) isn't alone determining long-run income (y-axis); some countries owned more assets relative to their GDP in 2014, so will have higher long-run income. As can be seen, long-run income differences can be substantial. Small countries that currently own large capital stocks (such as Kuwait, Norway, Switzerland and Luxembourg) can in the long run have an income which is about four to six times higher than average and seven to ten times higher than that of the poorest. Notably, these differences are much larger than simply capital's share of income. The reason for this is best illustrated by looking at the highest and lowest long-run incomes in the figure. Since the location of capital equalizes, labor income will be the same in all countries. This means that the lowest long-run per-capita income a country can have is $1 - \alpha$ of the global average person, in case the country owns no capital to begin with. This can be seen in the figure where there is a cluster of currently poor countries (which own nearly no capital) that have a long-run income of around 0.6 of global average. At the other extreme, the richest countries can have an income that is many times (here up to four to six) higher than average. The reason is of course that there is no bound on how much capital a single country can own. If a country with a small population owns a lot of capital initially, while the others have very little, then that small country can have an income from capital that is much larger than its income from labor. Income inequality between the richest and poorest will be even larger. In the left panel, Kuwait has around ten times higher income than the poorest and, in the right panel, Norway has around seven times higher income than the poorest. In essence, this illustrates quantitatively Corollaries 1 and 2.

9 Concluding remarks

This paper suggests a small revision to the canonical Solow model, making it a better reduced-form representation of the Ramsey-Cass-Koopmans model of open economies. The model is as simple and tractable as the canonical model and allows for capturing important phenomena such as long-run income differences analytically. The purpose of this reduced-form model is not to replace the RCK model. Rather it may provide a simple testing laboratory for ideas and extensions to the open-economy RCK and other models and a tool for presenting intuition and results in a very simple framework. Standard textbook treatments of convergence do not discuss the difference in predictions between open and closed economies, possibly because of the inconsistency between the open-economy Solow model and more micro-oriented models such as RCK (see, e.g., Perkins et al., 2013; Romer, 2018; Doepke et al., 1999; Barro and Sala-i-Martin, 2004). The rSolow allows for simple presentation of what a micro-oriented model predicts regarding convergence between open economies.

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