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# The Focusing Effect in Negotiations <br> Andrea Canidio, Heiko Karle 

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# The Focusing Effect in Negotiations 


#### Abstract

Two players with preferences distorted by the focusing effect (Koszegi and Szeidl, 2013) negotiate an agreement over several issues and one transfer. We show that, as long as their preferences are differentially distorted, an issue will be inefficiently left out of the agreement or inefficiently included in the agreement whenever the importance of the other issues on the table is sufficiently large. Anticipating this possibility, the negotiating parties may negotiate in stages, by first signing an incomplete agreement and later finalizing the outcome of the negotiation. Negotiating in stages increases the efficiency of the negotiation, despite the fact that the players' preferences are distorted by the focusing effect also when negotiating the incomplete agreement.


JEL-Codes: C780, D030, D860, F510.
Keywords: salience, focusing effect, bargaining, negotiations, incomplete agreements.

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## 1 Introduction

Negotiations often begin with a preliminary negotiation round. For example, the 2012 peace negotiation between the FARC and the Colombian government was preceded by a preliminary negotiation round aimed at defining the issues to be discussed during the subsequent peace negotiation. ${ }^{1}$ Another well known historical example is the Panama Canal negotiation between the Panamanian and the American governments, that was preceded by the signing of some threshold agreements-i.e., agreements guaranteeing to the Panamanian government the achievement of minimum outcomes on some issues. ${ }^{2}$ Similarly, professional negotiators often recommend starting a negotiation by defining a framework agreement: a document in the form of an agreement, with blank spaces on each term to be resolved by the negotiation. ${ }^{3}$

According to practitioners, these preliminary rounds play a key role in determining the importance of the different issues on the bargaining table. For example, Fisher, Ury and Patton (1991) in their seminal textbook write: "working out a framework agreement, however detailed, will help ensure that important issues are not overlooked during the negotiation". ${ }^{4}$ Also, according to Raiffa (1982), page 216, "the importance of an issue might be lessened by the parties first narrowing the range of possible outcomes on that issue". Although, at a superficial level, these observations may sound quite intuitive, at a closer look they are rather intriguing. They imply that, absent a preliminary negotiation round, the parties may not assign the "right" importance to the various issues on the table. Even more intriguing, they also imply that the bargaining parties may agree on how to shape the importance of the various issues that will be discussed in subsequent rounds of negotiation.

Motivated by the above observations, in this paper we provide a theory of how the importance of the different issues on the bargaining table is determined. We do so by considering a bargaining problem in which the players' preferences are distorted

[^0]by the focusing effect (see the next paragraph). We compare the outcome of the negotiation when the players bargain in one step, to that of a two-step negotiation in which, first, the players can sign an incomplete agreement and, then, they finalize the outcome of the negotiation. We model incomplete agreements as constraints on the future bargaining set. As we will see, our model will largely rationalize the practitioner's view discussed above, and hence help us understand the role of preliminary negotiation rounds.

In choice problems, the focusing effect (or focusing illusion) occurs whenever a person places too much importance on certain aspects of her choice set (i.e., when certain elements are more salient than others). Intuitively, an agent's attention is unconsciously and automatically drawn toward certain attributes, which are therefore overvalued when making a choice. Kőszegi and Szeidl (2013) formalize this concept by assuming that agents maximize a focus-weighted utility

$$
\tilde{U}\left(x_{1}, x_{2}, . ., x_{n}\right)=\sum_{s=1}^{n} h_{s} u_{s}\left(x_{s}\right)
$$

where $x=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ is a given good with $n$ attributes. The focus weights $h_{s}$ are defined as:

$$
h_{s}=h\left(\max _{x \in C} u_{s}\left(x_{s}\right)-\min _{x \in C} u_{s}\left(x_{s}\right)\right)
$$

where $h()$ is the focusing function (assumed strictly increasing) and $C$ is the agent's consideration set, which is the set of outcomes considered as possible by the decision maker (see the next paragraph). In this formalization, an agent overweighs the utility generated by the attributes in which her options differ more, where these differences are measured in utility terms. ${ }^{5}$

We introduce the focusing effect in a bargaining problem in which the players negotiate over $n$ binary issues and a continuous issue which we interpret as a trans-

[^1]fer. The main challenge in doing so is defining the consideration set. In choice problems the consideration set typically coincides with the choice set and is therefore exogenous - in particular, prices are exogenously given. Bargaining, however, is precisely the process by which monetary transfers are determined. For this reason, we assume that the players' consideration set is given by the bargaining outcomes that satisfy the participation constraint of at least one player. Hence, in our model, the largest possible transfer a player is willing to make (and therefore the salience of transfers) is endogenous and depends on the importance this player assigns to the various issues on the negotiating table. Note that, as a consequence of this assumption, the players' participation constraints determine the players' focus weights, while, at the same time, the focus weights determine the players' preferences and their participation constraints. Hence, solving for the focus weights and the consideration set is a fixed-point problem for which we show existence and uniqueness.

We derive conditions under which, when the players negotiate in one step, the outcome of the negotiation is inefficient relative to a rational benchmark in which all focus weights are equal to 1 . We show that an inefficient outcome can occur if the players' preferences are differentially distorted by the focusing effect (i.e., the two players' focusing functions are not identical) and if there are at least 2 issues on the negotiating table. When these two conditions are satisfied, a given issue will be inefficiently exclude from the agreement (or inefficiently included into the agreement) whenever the value of this issue is sufficiently small relatively to the other issues on the table. It is even possible that the negotiation will inefficiently end without an agreement, in what we call a negotiation breakdown. ${ }^{6}$

Intuitively, by increasing the importance of some issues, the largest transfer that the players consider possible also increases. By the focusing effect, the players' attention is drawn more and more toward the transfer dimension, which therefore becomes more salient for both players. Crucially, if the players' focusing functions

[^2]differ, this increase will be stronger for one player than for the other, and therefore creating a form of disagreement. It is possible that the cost of making a transfer for a player increases faster than the benefit of receiving it for the other player, in which case an issue may be inefficiently left out from the final agreement. But it is also possible that the cost of making a transfer for a player increases slower than the benefit of receiving it for the other player, in which case an issue may be inefficiently included in the final agreement.

We then allow the players to negotiate in two stages, with preferences distorted by the focusing effect in both stages. In the first stage, the players can sign an incomplete agreement that imposes a constraint on the future bargaining set. We follow Kőszegi and Szeidl (2013) in assuming that players are consequential, in the sense that they evaluate actions based on their future consequences (cf. p. 73). In our environment, this implies that when a bargaining outcome is excluded from the bargaining set in the first stage, it does not affect the players' preferences anymore in the second stage. Similarly, off-equilibrium bargaining outcomes (which are never reached for any incomplete agreement the players may sign) do not affect the players' preferences when negotiating incomplete agreements at the first stage.

We show that the possibility of signing an incomplete agreement is welfare improving: if the players agree to either impose a bound on the transfer dimension or to restrict the set of issues that will be discussed in the future, then the efficiency of the negotiation will increase (relative to a one-step negotiation). The reason is that the set of bargaining outcomes achievable by imposing a specific incomplete agreement is, in general, strictly smaller than the full bargaining set. For example, very large transfers may be in the consideration set of the one-step negotiation but not in the consideration set when negotiating an incomplete agreement. Hence, even if not fully rational, the players' are "closer to rational" when negotiating the incomplete agreement than when negotiating in one step.

To illustrate this point, we fully solve an example with two issues, in which absent an incomplete agreement there is an inefficient breakdown of the negotiation. In this example, the players agree to restrict the set of issues that will be discussed in the final bargaining round. Interestingly, doing so reduces the future salience of the transfer
dimension, and hence leads to the inclusion of issues into the final agreement that would otherwise be excluded-in other words, the players prevent some issues from being overlooked (as in the above citation from Fisher et al., 1991). Furthermore, as in Raiffa (1982), the players will further reduce the salience on the transfer dimension by imposing bounds on future transfers. Therefore, in the example, by imposing an incomplete agreement the players are able to prevent an inefficient breakdown of the negotiation.

We structure the paper in the following way. In the remainder of this section we discuss the relevant literature. In Section 2 we introduce the model. In Section 3 we solve for the one-step negotiation. In Section 4 we introduce an additional bargaining round. In Section 5 we discuss the robustness of our results to changes in our assumption, such as, for example, using models of salience different from Kőszegi and Szeidl (2013). The last section concludes. Unless otherwise noted all proofs are relegated to the appendix.

### 1.1 Relevant Literature

There is a small literature studying the impact of behavioral biases and non-standard preferences on negotiations. For example, Bénabou and Tirole (2009) consider a bargaining model with self-serving beliefs and show that inefficient negotiation breakdown may occur. Inefficient breaksdowns and delays are also possible in models with heterogeneous beliefs (such as Yildiz, 2004 and Nageeb Ali, 2006). Inefficient delays also arise in Compte and Jehiel (2003), who introduce reference-dependent utility in a game of alternating offers. Because we consider a different behavioral mechanism, the conditions under which inefficiencies arise are different in our paper than in the papers mentioned above. Furthermore, novel with respect to the literature, we consider multiple issues and hence the possibility that issues are inefficiently included into or excluded from the final agreement. Also related is Shalev (2002), who shows that increasing the degree of loss aversion of a player worsens the bargaining outcome for this player. Here we are interested in the inefficiencies caused by the focusing effect, rather than in evaluating the bargaining outcome of a single player.

The literature on incomplete contracts has long argued that behavioral biases and cognitive limitations may explain why agreements are often incomplete (see, for example, Segal, 1999; Battigalli and Maggi, 2002; Bolton and Faure-Grimaud, 2010; Tirole, 2009; Hart and Moore, 2008; Herweg and Schmidt, 2015 and Herweg, Karle and Müller, 2018). However, to the best of our knowledge, all these explanations rely on the resolution of some uncertainty: after signing an incomplete contract, the arrival of new information makes the environment less complex, reduces the number of possible contingencies, and allows the contract to be either executed or, if renegotiation is allowed, completed in a final negotiating round. Instead, we consider a deterministic environment. This is justified by the observation that in many negotiations the start of a new bargaining round follows the end of the previous round and is not determined by the arrival of new information.

An exception are models based on unawareness, such as, for example, Von Thadden and Zhao (2012), Auster (2013) and Schumacher and Thysen (2017, forthcoming), in which an informed principal offers a contract to an unaware agent. This contract determines what the agent will be aware of when choosing an action, which implies that the principal may want to offer an incomplete contract so to leave the agent unaware of certain elements. This framework is, however, not directly applicable to the study of negotiations in which both players could be identical. Also related are papers showing that, if information is perfect but players must write an incomplete contract, then the players may decide to leave some potentially contractible aspects of an agreements unspecified, therefore making the incomplete contract "more incomplete" than strictly necessary (see Bernheim and Whinston, 1998, Battaglini and Harstad, 2016 and Harstad, 2007). In our paper, instead, the bargaining parties do not need to write an incomplete agreement, in the sense that they can decide to bargain over the entire bargaining set in the following period (with no cost of waiting).

Finally, in our model, all transfers and exchanges will realize conditional on reaching an agreement during the last round of negotiation. That is, there is a single agreement that could be reached via multiple negotiating rounds. This distinguishes us from Esteban and Sákovics (2008), who allow the negotiating parties to sign sep-
arate agreements on different parts of the surplus. It also distinguishes us from the literature on agenda setting in negotiations, in which the players can reach several, issue-specific agreements. ${ }^{7}$

## 2 The Model

Two players $a$ and $b$ bargain over $i \in\{1, \ldots, n\}$ binary issues and a continuous issue $t$. We interpret $t>0$ as a transfer $t$ from $b$ to $a$, while $t<0$ is a transfer $|t|$ from player $a$ to player $b$. On any given issue $i \leq n$, the players may agree to maintain the status quo, in which case the players' payoffs from issue $i$ are normalized to zero. But they also could decide to change the status quo, in which case player $a$ earns $\alpha_{i}$ while player $b$ earns $\beta_{i}$, where $\alpha_{i}$ and $\beta_{i}$ could be positive or negative real numbers. We call $q_{i} \in\{0,1\}$ the bargaining outcome with respect to issue $i$. If $q_{i}=0$, then the bargaining outcome maintains the status quo on issue $i$, in which case we say that issue $i$ was excluded from the final agreement. If instead $q_{i}=1$, then the bargaining outcome changes the status quo on issue $i$, in which case we say that issue $i$ was included in the final agreement. The bargaining set is therefore $X \equiv\{0,1\} \times \ldots \times\{0,1\} \times \mathbb{R}$, with $x=\left\{q_{1}, q_{2}, \ldots, q_{n}, t\right\} \in X$ a possible bargaining outcome.

We make the following parametric assumptions:

1. there is at least an issue $i$ such that $\alpha_{i} \cdot \beta_{i}<0$.
2. there are no issues $i \leq n$ such that $\alpha_{i}<0$ and $\beta_{i}<0$.

As it will become clear later (see Equation A.2), issues that generate costs to both players will never be included into an agreement, and issues that generate benefits to both players are always included into the final agreement. The first restriction therefore excludes from our analysis the uninteresting case in which, independently of the focusing effect, all issues are included into or excluded from the agreement.

[^3]The second restriction is instead without loss of generality. The reason is that issues that generate costs to both players are irrelevant in determining the shape of the consideration set (see Equations 3.1 and 3.2).

This environment nests several bargaining problems. For example, it corresponds to a standard buyer-seller negotiation whenever $\beta_{i}>0>\alpha_{i}$ for all $i \leq n$, so that all benefits accrue to player $b$ and all costs accrue to player $a$. Alternatively, we could have an international negotiation in which some benefits accrue to each player.

The negotiation. The players negotiate in stages. In the first stage, the players can sign an incomplete agreement that imposes a constraint on the future bargaining set. In the second period, they finalize the negotiation by choosing an outcome from the constrained bargaining set.

Definition 1 (Incomplete agreements). An incomplete agreement is a set $S \subset X$, assumed closed and bounded. A negotiation structure $\mathbb{S} \subset P(X)$ (where $P(X)$ is the power set of $X$ ) is the collection of $S$ that can be chosen in period 1 .

Hence, the negotiation structure is an exogenously given collection of subsets of $X$, where each of these subsets represents an incomplete agreement that the players may sign in period 1. Also, because there is no time discounting, a one-step negotiation is equivalent to a two-step negotiation in which the outcome of the period-1 negotiation is $S=X$.

Assumption 1 (Outside options). In period 1, each player can unilaterally decide to bargain over the unconstrained bargaining set $X$ in period 2. In period 2, each player can unilaterally end the negotiation, in which case the players utilities are zero.

Hence, $S=X$ constitutes the outside option of the first-stage negotiation. ${ }^{8}$ Note also that the above assumption implies that $X \in \mathbb{S}$, i.e., if each player can unilaterally decide to bargain over the unconstrained bargaining set $X$, then the players can also jointly decide to do so.

[^4]Finally, to better focus on the sequence of agreements, we solve each negotiating round via Nash bargaining. ${ }^{9}$

Assumption 2 (Bargaining solution). Each bargaining round is solved by Nash bargaining.

That is, we assume that, within each bargaining round and for given preferences, the players will achieve an agreement that is invariant to affine transformations, Pareto optimal, symmetric and satisfies independence of irrelevant alternatives (see Nash, 1953). Of course, whereas in Nash (1953) the players' preferences are one of the primitives of the model, here instead they are endogenous. Once the endogeneity of preferences is taken into account, irrelevant alternatives might affect the bargaining solution.

Players' preferences. In both periods, the players have context-dependent preferences à la Kőszegi and Szeidl (2013). ${ }^{10}$ We start by writing their preferences taking as given the consideration set which is the set of outcomes that each player considers possible. We later derive the consideration set endogenously.

Call $h_{a}()$ player $a$ 's focus function and $h_{b}()$ player $b$ 's focus function, both assumed strictly positive and strictly increasing, with $h_{a}(0)=h_{b}(0)>0$. The utility functions are

$$
\begin{aligned}
& U^{a}(x)=\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}(\bar{t}-\underline{t}) t, \\
& U^{b}(x)=\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}(\bar{t}-\underline{t}) t
\end{aligned}
$$

where, for all $i \leq n, h_{a}\left(\left|\alpha_{i}\right|\right), h_{a}(\bar{t}-\underline{t}), h_{b}\left(\left|\beta_{i}\right|\right)$, and $h_{b}(\bar{t}-\underline{t})$ are the players' focus weights. The focus weight on the transfer dimension depends on $\bar{t} \geq 0$ and $\underline{t} \leq 0$, which are the largest and smallest transfer from player $b$ to player $a$ in the consideration set. The focus weight on a given issue depends on the utility generated by including that issue in the final agreement. The focusing effect causes each player

[^5]to focus more on, and hence to overweight, the dimension of the bargaining problem with the largest difference in terms of possible bargaining outcomes.

For ease of notation, we define the focus wedge as

$$
\Delta(y) \equiv \frac{h_{a}(|y|)}{h_{b}(|y|)}
$$

which measures the distortion in player $a$ 's preferences relative to those of player $b$. In what follows, we consider three cases:

- $\Delta(y)=1$ for all $y$, which implies that $h_{a}()=h_{b}()$. That is: the players' preferences are equally distorted by the focusing effect. We also say that the players are "equally focused."
- $\Delta(y) \geq 1$ and strictly increasing for all $y>0$. That is: player $a$ is always "more focused" than player $b$, the more so the larger is $y$.
- $\Delta(y) \leq 1$ and strictly decreasing for all $y>0$. That is: player $b$ is always "more focused" than player $a$, the more so the larger is $y$.

Having specified the preferences for a given consideration set, we can now define the consideration set. Kőszegi and Szeidl (2013) argue that the consideration set should be equal to the agent's choice set, with the possible exclusion of options that are dominated (that is, very bad in all attributes; see Kőszegi and Szeidl, 2013, Section 2.2, remark 2). Here, no issue is "dominated" in the sense that all issues have positive value for at least one player. But some transfers may be "dominated" for a player, in the sense that, if included in an agreement, one player will always prefer his outside option no matter how the other $n$ issues are resolved. The following assumption implies that those transfers are not part of the players' consideration set.

Assumption 3 (Consideration set). Consider a bargaining outcome $x=\left\{q_{1}, q_{2}, \ldots, q_{n}, t\right\} \in$ $X$ with $t>0$ (respectively $t<0$ ). This bargaining outcome is in the consideration set if and only if $x \in S$ and $U^{b}(x) \geq 0$ (respectively $\left.U^{a}(x) \geq 0\right)$.

That is, a bargaining outcome is in the consideration set if and only if it satisfies any prior incomplete agreement and also satisfies the participation constraint of the
player making the transfer. Note also that the above assumption implies that prior incomplete agreements are binding. ${ }^{11}$

Rational benchmark. The focus-weighted utility introduced above is a decision utility, because it describes the decision maker's choice. We will contrast players' decision utility with their material utility corresponding to a rational benchmark in which all the focus weights are equal to one. We call the sum of the two material utilities the material welfare, and we say that an outcome is materially efficient (inefficient) if it maximizes (does not maximize) material welfare.

## 3 One-step negotiation

Suppose that no restriction on the bargaining set was imposed by the players in period 1, that is, $S=X$ and the players bargain in one step. By Assumption 3, in the one-step negotiation, the bounds on the transfer dimension $\bar{t}$ and $\underline{t}$ are given by the largest possible transfers that satisfy each player's participation constraint. To derive those bounds, we consider the best possible bargaining outcome for each player, which is that containing only the issues in which this player earns positive utility. We then consider the largest possible transfer that leaves a player indifferent between such an agreement and his outside option. Mathematically:

$$
\begin{align*}
& \sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}=-h_{a}(\bar{t}-\underline{t}) \underline{t}  \tag{3.1}\\
& \sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}=h_{b}(\bar{t}-\underline{t}) \bar{t} \tag{3.2}
\end{align*}
$$

The following lemma shows the existence of $\bar{t}$ and $\underline{t}$, and establishes an important preliminary result: absent a prior incomplete agreement, the salience of the transfer

[^6]dimension is increasing in the benefit generated by the issues under consideration and in the number of issues under consideration.

Lemma 1. $\bar{t}$ and $\underline{t}$ exist and are unique. Furthermore:

- $\forall i \leq n, \bar{t}-\underline{t}$ is strictly increasing in $\alpha_{i}$ if $\alpha_{i}>0$, and constant in $\alpha_{i}$ otherwise,
- $\forall i \leq n, \bar{t}-\underline{t}$ is strictly increasing in $\beta_{i}$ if $\beta_{i}>0$ and constant in $\beta_{i}$ otherwise,
- $\bar{t}-\underline{t}$ is strictly increasing in $n$.

Having characterized the players' focus weights, we can now derive the bargaining outcome. By Assumption 2, it is given by the Nash bargaining solution:

$$
\operatorname{argmax}_{x \in X} U^{a}(x) \cdot U^{b}(x),
$$

which, by normalizing each utility function by the focus weight on transfer, becomes: ${ }^{12}$

$$
\begin{equation*}
\operatorname{argmax}_{q_{1}, \ldots q_{n}, t}\left(\sum_{i=1}^{n} \frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})} \cdot q_{i}+t\right)\left(\sum_{i=1}^{n} \frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})} \cdot q_{i}-t\right) . \tag{3.3}
\end{equation*}
$$

Call the solution to the above problem $\left\{q_{1}^{*}, \ldots q_{n}^{*}, t^{*}\right\}$. For a given $\left\{q_{1}^{*}, \ldots q_{n}^{*}\right\}$ the transfer solving (3.3) is:

$$
\begin{equation*}
t^{*}=\frac{1}{2} \sum_{i=1}^{n} q_{i}^{*} \cdot\left(\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}-\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}\right) . \tag{3.4}
\end{equation*}
$$

In addition, a given issue $i$ is included in the agreement (and hence $q_{i}^{*}=1$ ) if and only if

$$
\begin{equation*}
\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}+\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}>0 . \tag{3.5}
\end{equation*}
$$

[^7]The left-had side of this expression is the focus-weighted surplus generated by issue $i$, that is, the sum of the benefits generated by including issue $i$ in the final agreement, expressed in monetary equivalent.

Hence, for a given issues included in the agreement, the focusing effect distorts the equilibrium transfer and how the resulting surplus is shared among the two players. More importantly, the focusing effect also determines which issue is included in the agreement and hence the efficiency of the negotiation. In particular, a given issue is included in the agreement if and only if its focus-weighted surplus is positive (see 3.5), but it is material efficient to do so if and only if $\alpha_{i}+\beta_{i}>0$. It is therefore possible that a given issue is inefficiently excluded from the agreement, or inefficiently included into the agreement.

We start the analysis of the efficiency properties of the equilibrium by presenting sufficient conditions guaranteeing that the outcome of the negotiation is efficient.

Lemma 2. If either $n=1$ or $\Delta(x)=1$ (i.e., the players are equally focused), then the negotiating outcome is materially efficient. That is, an issue $i \leq n$ is included in the final agreement if and only if $\alpha_{i}+\beta_{i}>0$.

Hence, a necessary condition for the outcome to be inefficient is that there are at least 2 issues on the table. This is similar to what is found by Kőszegi and Szeidl (2013) (cf. their rationality in balanced trade-offs result in Proposition 3, applied to two-attribute choices). However, contrary to Kőszegi and Szeidl (2013), inefficiencies emerge not because the players have preferences that are distorted by the focusing effect, but rather because players have preferences that are differentially distorted by the focusing effect. The next lemma shows that inefficiencies may emerge when the focus weight on the transfer dimension is sufficiently large.

Lemma 3. Assume $n \geq 2$, and consider an issue $i$ with $\alpha_{i} \cdot \beta_{i}<0$. Suppose that the player who benefits from including issue $i$ into the agreement (i.e. who would make a larger transfer if issue $i$ was included) is more focused. ${ }^{13}$

[^8]- If $\alpha_{i}+\beta_{i}<0$, then issue $i$ is always efficiently excluded from the agreement.
- If $\alpha_{i}+\beta_{i}>0$, then issue $i$ is efficiently included in the agreement for small $\bar{t}-\underline{t}$, but will be inefficiently excluded from the agreement whenever $\bar{t}-\underline{t}>Y_{i}$ where $Y_{i}$ is implicitly defined by:

$$
\Delta\left(Y_{i}\right)=\frac{h_{a}\left(\left|Y_{i}\right|\right)}{h_{b}\left(\left|Y_{i}\right|\right)}:=\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \cdot\left|\alpha_{i}\right|}{h_{b}\left(\left|\beta_{i}\right|\right) \cdot\left|\beta_{i}\right|} .
$$

Suppose instead that the player who bears the cost of including issue $i$ into the agreement (i.e. who would receive a larger transfer if issue i was included) is more focused. Then:

- If $\alpha_{i}+\beta_{i}>0$, then issue $i$ is always efficiently included into the agreement.
- If $\alpha_{i}+\beta_{i}<0$, then issue $i$ is efficiently excluded from the agreement for small $\bar{t}-\underline{t}$, but will be inefficiently included into the agreement whenever $\bar{t}-\underline{t}>Y_{i}$.

The above lemma shows that, if the player who should make the additional transfer required to include an issue into the agreement is more focused than the other, then it is possible that such issue is inefficiently excluded from the agreement. If instead the player who would receive the additional transfer required to include an issue into the agreement is more focused than the other, then it is possible that such issue is inefficiently included into the agreement. The inefficient outcome is more likely to occur whenever the salience of the transfer dimension is large. Intuitively, the willingness of the players to include an additional issue in the agreement depends on two elements: the perceived utility of receiving a transfer relative to the perceived disutility of making a transfer. Both of these elements increase when the salience of the transfer dimension increases. However, depending on who is more focused, one will grow faster than the other.

Note that $Y_{i}$ always exists when the surplus generated by issue $i$ is close to zero, i.e. $\left|\alpha_{i}\right| \approx\left|\beta_{i}\right|$ or $\alpha_{i}+\beta_{i} \approx 0$. Quite intuitively, if it is marginally efficient to include or to exclude an issue, then even a small distortion in the preferences may generate an inefficient outcome. $Y_{i}$ also always exists when $\Delta(y)$ is unbounded above (and
player $a$ is more focused) or converges to zero (and player $b$ is more focused). In such cases, the distortion in one player's preferences relative to that of the other player grows arbitrarily large with $\bar{t}-\underline{t}$. If instead $\Delta(y)$ is bounded above / converges to a strictly positive number, and at the same time $\left|\alpha_{i}\right|$ and $\left|\beta_{i}\right|$ are sufficiently far apart, then $Y_{i}$ may not exist. In such cases, the negotiating outcome is always efficient with respect to issue $i$.

The above lemma has an important implication that will play a central role in the next section: when the salience of the transfer dimension is reduced, the players are more likely to behave as rational. We summarize this observation in the following corollary.

Corollary 1. Suppose that, following a reduction in $\bar{t}-\underline{t}$, the focus-weighted surplus on issue $i$ turns negative (positive). Then it must be that $\alpha_{i}+\beta_{i}<0\left(\alpha_{i}+\beta_{i}>0\right)$.

The above two lemmas then deliver our main proposition for this section, which provides conditions on the primitives of the problem for an inefficient outcome to emerge.

Proposition 1. Suppose $n \geq 2$. Consider issue $i$ with $\alpha_{i} \cdot \beta_{i}<0$ such that $Y_{i}$ exists. Suppose that the player who should make (would receive) the additional transfer required to include that issue in the agreement is more focused than the other player and $\alpha_{i}+\beta_{i}>0\left(\alpha_{i}+\beta_{i}<0\right)$. Then the agreement will be inefficient with respect to issue $i$ whenever either

- some $\beta_{j}>0$ with $j \neq i$ is sufficiently large.
- some $\alpha_{j}>0$ with $j \neq i$ is sufficiently large.
- $n$ is sufficiently large.

Hence, the outcome on a given issue may be inefficient whenever the value of agreeing on some other issue is sufficiently large, or if there are sufficiently many issues on the table. The reason is that, as the value of an unrelated issue or the number of issues increases, then the salience of the transfer dimension increases (see

Lemma 1). If inefficiencies are possible for some focus weight on transfer (see Lemma 3 ), then the outcome of the negotiation will be inefficient. An important implication is that the players may inefficiently fail to agree on any issue, that is, an inefficient breakdown of the negotiation is possible. We illustrate this possibility in Section 4.1.

## 4 Two-step negotiation

In this section we show that material welfare is higher when the negotiation has two steps rather than a single step, despite the fact that preferences are distorted by the focusing effect in both periods. We illustrate this result by first fully solving an example with two issues, in which absent a prior incomplete agreement there is an inefficient negotiation breakdown. We show that the players will agree in period 1 to restrict their future bargaining set and to impose a bound on the transfer dimension. By doing so they will achieve the materially efficient outcome. We then consider the general model and show that when incomplete agreements are used, the material welfare of the negotiation increases relative to the one-step negotiation (cf. Proposition 2).

An important preliminary step in solving for the two-step negotiation is specifying a negotiation structure (see Assumption 1): i.e., what type of incomplete agreements the players can sign in period 1. Here we restrict our attention to incomplete agreements that:

- commit the players to include an issue in the final agreement.
- commit the players to exclude an issue from the final agreement.
- impose an upper bound $\bar{T} \geq 0$ and a lower bound $\underline{T} \leq 0$ to the transfers that can be made by the players.

We say that $Q_{i}=\{1\}$ if the players commit to include issue $i$ in the final agreement, $Q_{i}=\{0\}$ if the players commit to exclude issue $i$ from the final agreement, and $Q_{i}=\{0,1\}$ otherwise.

Restricting our attention to the above negotiation structure is with loss of generality because it does not allow the players to "bundle" issues, that is, the players cannot decide in period 1 that if their future selves agree on issue $i$ they must also agree on issue $j$ (or disagree on both issues, or agree on one and disagree on the other). The problem with bundling issues is that it is unclear how to write the period-2 focus weights. One could reasonably claim that if two issues become "bundled" then there should be a single focus weight on both issues (that is, they effectively become a single issue). We find this idea intriguing but prefer to leave this extension for future work.

Finally, an important observation is that, no matter what is agreed upon in period 1, in period 2, players may disagree, so that the bargaining outcome $\overline{0}=\left\{q_{1}=\right.$ $\left.0, \ldots, q_{n}=0, t=0\right\}$ is always in the consideration set. Therefore, agreeing to include an issue in the future agreement imposes a constraint on the future bargaining set without affecting the players' period-2 preferences. On the other hand, both agreeing to exclude an issue and imposing bounds on the transfer dimension might affect the period-2 preferences.

### 4.1 An example: using an incomplete agreement to avoid a negotiation breakdown

Suppose that there are only two issues with $\alpha_{1}+\beta_{1}>0$ and $\alpha_{2}+\beta_{2}<0$, so that material welfare is maximized by an agreement including only the first issue. Furthermore, for ease of exposition, assume $\alpha_{1}<0, \alpha_{2}<0, \beta_{1}>0$ and $\beta_{2}>0$, that is, all benefits accrue to player $b$ (that therefore can be seen as the buyer) and all costs accrue to player $a$ (that therefore can be seen as the seller). We also assume that player $b$ is more focused, i.e., $\Delta(y) \leq 1$ and $\Delta^{\prime}(y)<0$ for all $y>0$.

In this section, we consider a situation in which, absent an incomplete agreement, there is a negotiation breakdown: no issue is included in the final agreement. By Lemma 3, in a one-step negotiation issue 2 is always excluded from the final
agreement. Issue 1 is also excluded from the final agreement if and only if

$$
\begin{equation*}
\Delta(\bar{t}-\underline{t}) \leq \frac{h_{a}\left(\left|\alpha_{1}\right|\right)\left|\alpha_{1}\right|}{h_{b}\left(\left|\beta_{1}\right|\right) \beta_{1}} . \tag{4.6}
\end{equation*}
$$

By (3.1), (3.2), $\underline{t}=0$ and $\bar{t}$ is implicitly defined by:

$$
h_{b}\left(\left|\beta_{1}\right|\right) \beta_{1}+h_{b}\left(\left|\beta_{2}\right|\right) \beta_{2}=h_{b}(\bar{t}) \bar{t} .
$$

Hence, if $\beta_{2}$ is sufficiently large, then (4.6) holds and issue 2 is also excluded from the agreement, leading to a negotiation breakdown. Intuitively, despite the fact that issue 2 is never included in the final agreement, it may nonetheless generate a large benefit and hence be very salient. If it is sufficiently salient, issue 1 may be "overlooked" and hence excluded from the final agreement. We assume that this is the case: $\beta_{2}>\hat{\beta}_{2}$, where $\hat{\beta}_{2}$ is the value of $\beta_{2}$ such that (4.6) is satisfied with equality.

Consider now the two-step negotiation. First, suppose that, in period 1, the players exclude issue 2 by setting $Q_{2}=\{0\}$ but do not impose any other restriction on the future bargaining set. In this case, the period-2 focus weight on transfer is $\beta_{1}$ and issue 1 is included in the final agreement (see 3.1, 3.2, and Lemma 2, the case in which $n=1$ ).

Now suppose instead that the players set $Q_{2}=\{0\}$ and impose a cap on transfer $\bar{T}$. If $\bar{T} \geq \beta_{1}$, then the cap on transfer is irrelevant, because it is larger than the largest transfer player $b$ is willing to make in period $2 .{ }^{14}$ If instead $\bar{T}<\beta_{1}$, then the period-2 focus weight is precisely $\bar{T}$. In this case, it is possible that the cap on transfer is not binding - in the sense that the period-2 transfer for given $\bar{T}$ (as given by equation 3.4) is below $\bar{T}$. For $\bar{T}$ sufficiently low, however, $\bar{T}$ will be binding, in the sense that the period- 2 transfer will be exactly $\bar{T}$ if $\bar{T} \geq-\alpha_{1}$ (so that player $a$ can cover the cost of including issue 1) and 0 otherwise.

Mathematically, for given $Q_{2}=\{0\}, Q_{1} \neq\{0\}$, and $\bar{T} \in\left[-\alpha_{1}, \beta_{1}\right]$, the period-2

[^9]transfer is:
\[

$$
\begin{equation*}
t^{*}=\min \left\{\frac{1}{2}\left(\frac{h_{b}\left(\left|\beta_{1}\right|\right) \beta_{1}}{h_{b}(\bar{T})}-\frac{h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}}{h_{a}(\bar{T})}\right), \bar{T}\right\} \tag{4.7}
\end{equation*}
$$

\]

The first term of the minimum in the above expression is the period- 2 transfer when the cap $\bar{T}$ is not binding, as given by (3.4). It is decreasing in $\bar{T} \in\left[0, \beta_{1}\right]$, because as the salience of the transfer dimension decreases, the transfer that player $b$ is willing to make to player $a$ increases (remember that, in this example, $b$ is more focused). Furthermore, the difference between the two terms of the minimum in the above expression is positive at $\bar{T}=0$ and negative at $\bar{T}=\beta_{1}$. Hence, the largest possible transfer achievable via an incomplete agreement is implicitly defined by

$$
\hat{t}^{*} \equiv \bar{T}: \frac{1}{2}\left(\frac{h_{b}\left(\left|\beta_{1}\right|\right) \beta_{1}}{h_{b}(\bar{T})}-\frac{h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}}{h_{a}(\bar{T})}\right)=\bar{T}
$$

Consider now period 1. In constructing the period-1 context, we maintain Assumption 3: a feasible bargaining outcome is in the consideration set if and only if it satisfies one of the players' participation constraints. The key, however, is to realize that the bargaining outcomes that are feasible from period- 1 viewpoint are only those that can be achieved as an outcome of the negotiation for some incomplete agreement. ${ }^{15}$

Note that issue 2 will never be included in the final agreement for any incomplete agreement the players may sign. In this sense, issue 2 is not feasible from period-1 viewpoint, and hence it is not part of the period-1 consideration set. Issue 1, instead, may or may not be included in the final agreement depending on whether issue 2 is excluded, and therefore it is part of the period-1 consideration set. Furthermore, from period-1 viewpoint, the feasible transfers are those that solve (4.7). This implies that the largest possible transfer in period-1 consideration set is $\hat{t}^{*}$, which is therefore the period- 1 focus weight on transfers.

[^10]We can therefore write the period-1 bargaining problem as:

$$
\begin{aligned}
& \max _{Q_{1}, Q_{2}, \bar{T}}\left(\frac{h_{a}\left(\left|\alpha_{1}\right|\right)}{h_{a}\left(\hat{t}^{*}\right)} \alpha_{1} \cdot q_{1}^{*}+t^{*}\right)\left(\frac{h_{b}\left(\beta_{1}\right)}{h_{b}\left(\hat{t}^{*}\right)} \beta_{1} \cdot q_{1}^{*}-t^{*}\right) \\
& \text { s.t. } \begin{cases}q_{1}^{*}=1 \text { and } t^{*} \text { given by }(4.7) & \text { if } Q_{2}=\{0\}, Q_{1} \neq\{0\}, \bar{T} \in\left[-\alpha_{1}, \beta_{1}\right] ; \\
q_{1}^{*}=0 \text { and } t^{*}=0 & \text { otherwise }\end{cases}
\end{aligned}
$$

The focus-weighted surplus generated by issue 1 is

$$
\frac{h_{a}\left(\left|\alpha_{1}\right|\right)}{h_{a}\left(\hat{t}^{*}\right)} \alpha_{1}+\frac{h_{b}\left(\beta_{1}\right)}{h_{b}\left(\hat{t}^{*}\right)} \beta_{1},
$$

and is strictly positive because $\hat{t}^{*}<\beta_{1}$. Furthermore, it is easy to see that the transfer maximizing the period- 1 Nash product when issue 1 is included is $\hat{t}^{*}$. Hence, the players agree in period 1 to exclude issue 2 and to impose a cap on transfer $\bar{T}=\hat{t}^{*}$. As a consequence, in the following period, the players will include issue 1 in the agreement and agree on the transfer $\hat{t}^{*} .{ }^{16}$

In this example, therefore, the players will restrict both the number of issues to be discussed in the second round of negotiation, and also the range of possible transfers. They will do so to avoid an inefficient negotiation breakdown. As a consequence, the efficient bargaining outcome is achieved.

### 4.2 General Case

We now go back to our general model. We first characterize the period-2 solution for any given incomplete agreement. We then characterize the period-1 problem of choosing which incomplete agreement to sign.

[^11]
### 4.2.1 Period 2

For a given incomplete agreement, the set of feasible bargaining outcomes achievable in period 2 is $Q_{1} \times \ldots \times Q_{n} \times[\underline{T}, \bar{T}] \cup \overline{0}$. We start by defining $\tilde{t}$ and $\underset{\sim}{t}$ as the largest and smallest transfer in the consideration set in the absence of explicit bounds on the transfer dimensions $\bar{T}$ and $\underline{T}$ :

$$
\begin{align*}
& \sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\} \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=-h_{a}(\tilde{t}-\underset{\sim}{t}) \underset{\sim}{t}  \tag{4.8}\\
& \sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\} \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=h_{b}(\tilde{t}-\underset{\sim}{t}) \tilde{t} \tag{4.9}
\end{align*}
$$

where $\mathbb{1}\{$.$\} is the indicator function, so that \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=0$ whenever issue $i$ was excluded via an incomplete agreement, and $\mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=1$ otherwise.

Hence, whenever $\bar{T} \geq \tilde{t}$ the players are effectively negotiating without an upper bound on transfers (and similarly whenever $\underset{\sim}{t} \underset{\sim}{t}$ ). In this case, the salience of the transfer dimension depends on the issues that have not been excluded by a prior incomplete agreement. If instead either $\bar{T} \leq \tilde{t}$ or $\underline{T} \geq \underset{\sim}{t}$, the salience of the transfer dimension depends on the upper and lower bound on the transfer.

It follows that the solution to the period-2 bargaining problem is the bargaining outcome $x \in Q_{1} \times Q_{2} \times \ldots Q_{n} \times[\underline{T}, \bar{T}] \cup \overline{0}$ maximizing:

$$
\begin{equation*}
\left(\sum_{i=1}^{n} \frac{h_{a}\left(\left|\alpha_{i}\right|\right)}{h_{a}(\min \{\bar{T}, \tilde{t}\}-\max \{\underline{T}, t \in\})} \alpha_{i} \cdot q_{i}+t\right)\left(\sum_{i=1}^{n} \frac{h_{b}\left(\left|\beta_{i}\right|\right)}{h_{b}(\min \{\bar{T}, \tilde{t}\}-\max \{\underline{T}, t \in\})} \beta_{i} \cdot q_{i}-t\right) \tag{4.10}
\end{equation*}
$$

For a given $x^{*}=\left\{q_{1}^{*}, \ldots, q_{n}^{*}, t^{*}\right\} \neq \overline{0}$ solution to the above problem, the equilibrium
transfer is
$t^{*}=\max \left\{\min \left\{\frac{1}{2} \sum_{i=1}^{n} q_{i}^{*} \cdot\left(\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\min \{\bar{T}, \tilde{t}\}-\max \{\underline{T}, \underset{\sim}{t}\})}-\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\min \{\bar{T}, \tilde{t}\}-\max \{\underline{T}, \underset{t}{t}\})}\right), \bar{T}\right\}, \underline{T}\right\}$

With respect to the issues included in the agreement, also here an issue $i$ with $Q_{i}=\{0,1\}$ will be included only if it generates positive focus-weighted surplus (as in condition 3.5). Here, however, this condition is not sufficient to determine the issues included in the agreement. The reason is that the constraints on the transfer dimension may be binding and therefore affecting the ability of a player to compensate the other for the inclusion of an additional issue in the agreement, even if this issue generates positive focus-weighted surplus. ${ }^{17}$

### 4.2.2 Period 1

Call $\left(q_{1}^{O S}, \ldots, q_{n}^{O S}, t^{O S}\right)$ the bargaining outcome of a one-step negotiation, which constitutes the outside option in period 1 of a two-step negotiation. Call $\hat{t}^{*}$ and $t^{*}$ the largest and smallest transfer (as in Equation 4.11) that can be achieved via some incomplete agreement, and that determine the period-1 focus weight on transfer. The solution to the period-1 problem is the incomplete agreement $\left\{Q_{1}, \ldots, Q_{n}, \underline{T}, \bar{T}\right\}$ that maximizes:

$$
\left(\sum_{i=1}^{n} \frac{h_{a}\left(\alpha_{i}\right)}{h_{a}\left(\hat{t}^{*}-t^{*}\right)} \alpha_{i} \cdot\left(q_{i}^{*}-q_{i}^{O S}\right)+\left(t^{*}-t^{O S}\right)\right)\left(\sum_{i=1}^{n} \frac{h_{b}\left(\beta_{i}\right)}{h_{b}\left(\hat{t}^{*}-\underline{t}^{*}\right)} \beta_{i} \cdot\left(q_{i}^{*}-q_{i}^{O S}\right)-\left(t^{*}-t^{O S}\right)\right)
$$

subject to

$$
\begin{equation*}
\left\{q_{1}^{*}, \ldots, q_{n}^{*}, t^{*}\right\}=\operatorname{argmax}_{x \in\left\{Q_{1}, \ldots, Q_{n}, \underline{T}, \bar{T}\right\}} \tag{4.10}
\end{equation*}
$$

Note that the above problem is mathematically equivalent to the players choosing in period 1 a bargaining outcome among those that can be reached via an incomplete agreement.

[^12]The next proposition shows that the difference between $\hat{t}^{*}$ and $t^{*}$ is, in general, smaller than the difference between the smallest and largest transfer in the one-step negotiation. By Corollary 1, this implies that the players' preferences are "closer" to rational when bargaining over the incomplete agreement than in the one-step negotiation. It follows that, if the players impose an incomplete agreement, then the material efficiency of the negotiation increases.

Proposition 2. Suppose the players impose an incomplete agreement in period 1, and this incomplete agreement changes the issues included in the final agreement relative to the one-step negotiation. Then, the material welfare of the negotiation is strictly greater than in the one-step negotiation.

Hence, the possibility of jointly imposing a constraint on the future bargaining set is welfare increasing.

The proof of the above proposition derives two conditions under which the salience of the transfer dimension in the period-1 negotiation is strictly smaller than that in the one-step negotiation. The first of such conditions emerges when some issues cannot be included in the final agreement no matter what incomplete agreement the players impose. In such a case, the issues that are feasible from period-1 viewpoint are fewer than $n$, which is the number of issues feasible in the one-step negotiation (as in the example we solved in the Section 4.1 where $n=2$ but only issue 1 was feasible from a period-1 viewpoint). The second of such conditions emerges when there is no negotiation breakdown in the one-step negotiation. Remember that in period 1, bargaining outcomes are evaluated relative to what can be earned in the onestep negotiation (which is the players' outside option). Hence, if both players earn strictly positive utility from the one-step negotiation, then the largest transfer they are willing to make to secure a different agreement in period 1 is reduced (relative to the one-step negotiation), and with it the salience of the transfer dimension. When one (or both) of the above conditions are satisfied, then the players are strictly "more rational" than in the one-step negotiation and Corollary 1 applies. If instead both conditions are violated (i.e., there is a negotiation breakdown in the one-step negotiation and all $n$ issues are feasible from period- 1 viewpoint), we show that the
players are nonetheless weakly "more rational" in period-1 of the two-step negotiation than in the one-step negotiation.

Note that the reverse of the above proposition is not true: the players may not choose the materially efficient period-1 agreement, nor an incomplete agreement that increases the material welfare relative to the one-step negotiation. There are two reasons for this. First, the period-1 preferences are distorted by the focusing effect, which means that the players may prefer some other agreement to the most efficient incomplete agreement. Second, the period-1 negotiation is a NTU bargaining problem: there could be a bargaining outcome that is preferred by both players but nonetheless is not achievable via any incomplete agreement.

## 5 Discussion

We now discuss a number of variations to our basic assumptions.

### 5.1 The consideration set

We assumed that a bargaining outcome is in the consideration set if it satisfies the participation constraint of the player making the transfer. The alternative is to assume that a bargaining outcome is in the consideration set if it satisfies the participation constraint of both players. Here we show that, under this alternative assumption, if the consideration set exists and is unique, then our results are largely unchanged. Importantly, however, under this alternative assumption the consideration set may not be unique.

Assume that this alternative assumption holds. Assume also that the consideration set exists and is unique: issues $z \leq n$ are in the consideration set, with smallest and largest transfer in the consideration set $\underline{t}$ and $\bar{t}$. Similarly to what discussed earlier, when only the issues from which player $a$ benefits are included (out of the $z$
issues in the consideration set), player $a$ is willing to pay at most $-\underline{t}$ defined as

$$
\begin{equation*}
\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}=-h_{a}(\bar{t}-\underline{t}) \underline{t} \tag{5.12}
\end{equation*}
$$

Similarly, when only the issues from which player $b$ benefits are included (out of the $z$ issues in the consideration set), player $a$ is willing to pay at most $\bar{t}$ defined as

$$
\begin{equation*}
\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}=h_{b}(\bar{t}-\underline{t}) \bar{t} \tag{5.13}
\end{equation*}
$$

The above conditions are again (3.1) and (3.2), except that here the summation is over $z$ issues instead of $n$.

In this case, however, the participation constraint of the other player must also be satisfied. That is, when only the issues from which player $b$ benefits are included and transfers are $\bar{t}$, player $a$ 's participation constraint must be satisfied:

$$
\begin{equation*}
\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \mathbb{1}\left\{\beta_{1}>0\right\}+h_{a}(\bar{t}-\underline{t}) \bar{t} \geq 0 \tag{5.14}
\end{equation*}
$$

Similarly, when only the issues from which player $a$ benefits are included and transfers are $\underline{t}$, player $b$ 's participation constraint must also be satisfied:

$$
\begin{equation*}
\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \mathbb{1}\left\{\alpha_{1}>0\right\}-h_{b}(\bar{t}-\underline{t}) \underline{t} \geq 0 \tag{5.15}
\end{equation*}
$$

Consider now an increase in one of the benefits, that is an increase in a $\beta_{i}>0$ or an $\alpha_{i}>0$. Clearly, for given $z$ this increase does not affect (5.14) and (5.15), which continue to hold. Hence, identical to the case considered in the body of the text, the effect of an increase in one of the benefit is an increase in the salience of the transfer dimension via equations (5.12) and (5.13). In this case, however, an increase in $\beta_{i}>0$ or $\alpha_{i}>0$ may also lead to an increase in the number of issues in the consideration set $z$. That is, because the largest possible transfers increase,
there may be additional issues that now satisfy the participation constraint of the player bearing the cost (i.e., equations 5.14 and 5.15). Note, however, that this additional effect strengthens this result because, as $z$ increases, the salience of the transfer dimension increases. Overall, also here when a $\beta_{i}>0$ or an $\alpha_{i}>0$ increases Proposition 1 holds.

Consider now a change in one of the costs, that is a change in a $\beta_{i}<0$ or an $\alpha_{i}<0$. Suppose that one of these costs increases (in the sense that its absolute value increases). Again, if $z$ does not change, then the salience of the transfer dimension does not change, and Proposition 1 holds. In this case, however, it is possible that $z$ decreases, leading to a discontinuous drop in the salience of the transfer dimension. Hence, relative to the body of the text, the difference under this alternative assumption is that changes in a $\beta_{i}<0$ or an $\alpha_{i}<0$ may lead to step-wise changes in the salience of the transfer dimension (whereas under our working assumption, the salience of the transfer dimension is constant in a $\beta_{i}<0$ or an $\alpha_{i}<0$ ).

Finally, and perhaps most importantly, under this alternative assumption the consideration set may not be unique. To illustrate this problem, assume $n=2$, $\alpha_{1}<0, \beta_{1}>0, \alpha_{2}>0, \beta_{2}<0, \alpha_{1}+\beta_{1}>0, \alpha_{2}+\beta_{2}>0$. In this case, if $z=2$ then (5.12), (5.13), (5.14) and (5.15) become:

$$
\begin{gathered}
h_{a}\left(\alpha_{2}\right) \alpha_{2}=-h_{a}(\bar{t}-\underline{t}) \underline{t} \\
h_{b}\left(\beta_{1}\right) \beta_{1}=h_{b}(\bar{t}-\underline{t}) \bar{t} \\
h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}+h_{a}(\bar{t}-\underline{t}) \bar{t} \geq 0 \\
h_{b}\left(\left|\beta_{2}\right|\right) \beta_{2}-h_{b}(\bar{t}-\underline{t}) \underline{t} \geq 0
\end{gathered}
$$

It is possible to find conditions under which one of the above equations is violated, and therefore it is not possible to have $z=2 .{ }^{18}$ Following the same steps, we can show that if only issue 1 is in the consideration set, then $\bar{t}-\underline{t}=\beta_{1}$ and player $a$ 's

[^13]participation constraint is
$$
h_{a}\left(\beta_{1}\right) \beta_{1}-h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}>0
$$
which always holds under our assumptions. Hence, there is a consideration set that contains only issue 1 with $\bar{t}-\underline{t}=\beta_{1}$. However, the same steps show that there is also a consideration set containing only issue 2 with $\bar{t}-\underline{t}=\alpha_{2}$.

In this example, therefore, the players may enter the negotiation thinking that the only possible agreement is one that includes only issue 1 . But they may also think that the only possible agreement is one that includes only issue 2 . As a consequence, the bargaining problem has multiple equilibria: one in which the players agree only on issue 1 , the other in which the players agree to include only issue 2 . There is an intuitive sense in which, if the two issues differ in the surplus generated, the most reasonable equilibrium is the one in which the players agree on the most valuable issue. To formalize this intuition, an appropriate equilibrium selection criterion should be introduced.

To summarize: switching to this alternative assumption has costs and benefits. The cost is that, because the consideration set may not be unique, it requires the introduction of an appropriate equilibrium selection criterion. The benefit is that, in this case, also changes in the costs will affect the salience of the transfer dimension, although in a step-wise, discontinuous way. We believe the cost is larger then the benefit, which is why we developed our argument in the main text under Assumption 3.

### 5.2 The bargaining solution

We assumed that each bargaining round is solved via Nash bargaining. This assumption allowed us to focus on the sequence of agreements signed by the players, without explicitly modeling the sequence of offers and counteroffers within each negotiating round. Our results, however, extend to different bargaining solutions, including noncooperative ones.

With respect to period 2, we showed that an issue is included in the agreement if
and only if its focus weighted-surplus is positive. This is the case under most cooperative bargaining solution, with the difference being in how the resulting surplus is split. It is also the case if the period-2 negotiation is modeled as a game of alternating offers à la Rubinstein (1982). In that game, the benefit and cost of agreeing on a given issue are the same in every period of the negotiation conditionally on reaching such period (that is because the disutility of waiting is, at that point, sunk). Therefore, if the consideration set is defined as in Assumption 3, the consideration set and the players' preferences do not change over time. This implies that the solution to the game of alternating offers is again the Nash bargaining solution.

The period-1 negotiation, instead, involves a positive outside option and nontransferable utility. Hence the non-cooperative implementation of the Nash bargaining solution is somewhat non standard. ${ }^{19}$ Nonetheless, Proposition 2 is based on the observation that the range of possible outcomes from period- 1 viewpoint is smaller than the range of possible outcomes in the one-step negotiation. As a consequences, the players' preferences are "closer to rational" in period 1 than in the one-step negotiation. Hence, when they decide to restrict the future bargaining set, they push the bargaining solution closer to the solution that rational players would prefer. This remains true independently of the specific bargaining solution used.

### 5.3 The outside option

In the body of the text, we assumed that the outside option of the period-1 negotiation is the one-step negotiation. That is, each player can unilaterally decide to negotiate in one step. Here we consider the alternative assumption: if the players fail to find an agreement in period 1, then the negotiation ends. We maintain that $X \in \mathbb{S}$, that is, the players can agree to impose no constraint on the bargaining set and therefore negotiate in one step in the following period.

Interestingly, Proposition 2 continues to hold under this assumption as well. We show this formally at the end of its proof (see the paragraph "additional consid-

[^14]erations"). The reason is that, also in this case, the bargaining outcomes that are achievable from period-1 point of view are fewer than in the one-step negotiation. For example, there may be issues that cannot be included in the final agreement for any incomplete agreement the players may sign. In this case, the salience of the transfer dimensions of the period-1 negotiation will be strictly smaller than that of the one-step negotiation. As a consequence, the players' preferences are strictly "closer to rational" than in the one-step negotiation. If instead all issues can be included in the final agreement for some incomplete agreement, then period-1 preferences are identical to those in the one-step negotiation. In this case, the players' preferences are weakly "more rational" than in the one-step negotiation.

### 5.4 Non-binding incomplete agreements

In the text, we assumed that the players must respect a prior incomplete agreement. However, in most jurisdictions, when facing a sequence of agreements between two parties, in case a later agreement contradicts an earlier agreement courts typically enforce the most recent one. When courts are not available (for example, in the case of international negotiations), the same two parties are free to jointly ignore a previous agreement.

To rationalize these observations, here we introduce the following modification to our model: in period 2, the players can either comply with a prior incomplete agreement, or trigger a third round of negotiation (at no cost). During the third round of negotiation, they will be bargaining over an exogenously given set. This set could be equal to the entire bargaining set, but other possibilities are plausible. For example, in period 3 the players may bargain with the help of a mediator, and this somehow restricts the set of outcomes that are achievable. Depending on the situation, triggering this additional negotiation round could be a joint decision, or even a unilateral decision by a player. In either case, we say that incomplete agreements are non binding.

Non-binding incomplete agreements may nonetheless affect the outcome of the negotiation because they will affect the players' period-2 preferences. The difference
with the case considered in the main text is that, now, the bargaining solution to the third negotiating round must be an element of the period-2 consideration set. The presence of this element imposes a constraint on the way in which period-1 players can manipulate period-2 preferences. For example, call the transfer solving the period-3 bargaining problem $t_{3}^{*}$. If this transfer is positive, then effectively period-1 agents are restricted to choosing $\bar{T} \geq t_{3}^{*}$ because a lower cap will not have any effect on the period-2 consideration set. To summarize, the fact that incomplete agreements are non binding imposes an additional constraint on the period-1 problem, without, however, changing our results.

### 5.5 Other models of salience

We develop our argument using Kőszegi and Szeidl (2013), but other models of salience have been proposed. The most important ones are Bushong, Rabin and Schwartzstein (2021) and Bordalo, Gennaioli and Shleifer (2013).

Bushong, Rabin and Schwartzstein (2021) propose a model in which the salience of a dimension decreases with the range of possible options in that dimension. Mathematically, the model is identical to that of Köszegi and Szeidl (2013), except that the focus functions $h_{a}()$ and $h_{b}()$ are decreasing. For our purposes, the fact that the focus functions are decreasing implies that a cap on transfers will increase the salience of the transfer dimension relative to no cap. It follows that the results derived in Proposition 1 are now reversed. The salience of the transfer dimension increases whenever $\beta_{i}>0$ or $\alpha_{i}>0$ decreases. Hence, the inefficient outcomes on a given issue become more likely to occur when the value of reaching an agreement on other issues are small rather than large.

Applying Bushong, Rabin and Schwartzstein (2021) to our bargaining problem would lead to a number of counterfactual results. For example, imposing a cap on the transfer dimension in period 1 increases the salience of the transfer dimension in period 2. However, as already discussed in the introduction, practitioners believe that reducing the range of possible outcomes on a dimension reduces the importance of this dimension within a negotiation (see Raiffa, 1982, p. 216). Similarly, elim-
inating issues would decrease the salience of the remaining issues relative to that of the transfer dimension. Again, this seems to run against the practitioner's view that eliminating issues via a framework agreement reduces the possibility that the remaining issues are overlooked (see Fisher et al., 1991, p. 172).

Bordalo, Gennaioli and Shleifer (2013) propose a model of salience in which different options are evaluated relative to a reference point. This model could be applied to our framework by assuming that imposing bounds on transfers shifts the reference point of the transfer dimension. ${ }^{20}$ The effect of introducing these bounds will then depend on whether the transfer dimension becomes more or less salient. If its salience decreases, then we are back to a logic similar to Kőszegi and Szeidl (2013), leading to results qualitatively similar to the ones discussed in the body of the paper. If instead its salience increases, then we are back to a logic similar to Bushong, Rabin and Schwartzstein (2021), leading to results qualitatively similar to the ones discussed above. We will be in one or the other case depending on how exactly imposing a bound affects the reference point, which is an issue beyond the scope of this paper.

### 5.6 Additional continuous issue

We assumed that there is only one continuous issue, the transfer. Here we provide an example in which there is a second continuous issue.

Assume that issue 1 is also continuous: instead of agreeing on whether to include it or not, the players need to agree on its level $\gamma \in[0,1]$. When issue 1 is included in the agreement with level $\gamma$, then player $b$ 's utility is $\gamma \cdot \beta_{1}$, and player $a$ 's utility is $\gamma^{2} \cdot \alpha_{1}$. To avoid triviality, we also assume that $\beta_{1}>0$ and $\alpha_{1}<0 .{ }^{21}$

[^15]In the one-step negotiation, $\bar{t}$ and $\underline{t}$ are again given by (3.1) and (3.2). Hence, this modification to the baseline model does not affect that salience of the transfer dimension. The focus-weighted surplus generated by including issue 1 in the agreement, with level $\gamma$ is

$$
\frac{h_{a}\left(\left|\alpha_{1}\right|\right) \gamma^{2} \alpha_{1}}{h_{a}(\bar{t}-\underline{t})}+\frac{h_{b}\left(\left|\beta_{1}\right|\right) \gamma \beta_{1}}{h_{b}(\bar{t}-\underline{t})}
$$

The players will therefore agree on $\gamma$ such that

$$
\gamma^{*}= \begin{cases}1 & \text { if }-\frac{1}{2} \frac{h_{b}\left(\left|\beta_{\beta}\right| \mid\right) \beta_{1}}{h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}} \Delta(\bar{t}-\underline{t})>1 \\ -\frac{1}{2} \frac{h_{b}\left(\left|\beta_{1}\right|\right) \beta_{1}}{h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}} \Delta(\bar{t}-\underline{t}) & \text { otherwise }\end{cases}
$$

Note that, relative to a rational benchmark in which all focus weights are equal to 1 , the level of issue 1 is too high whenever $\frac{h_{b}\left(\left|\beta_{\beta}\right|\right)}{h_{a}\left(\left|\alpha_{1}\right|\right)} \Delta(\bar{t}-\underline{t})>1$ and too low if $\frac{h_{b}\left(\left|\beta_{1}\right|\right)}{h_{a}\left(\left|\alpha_{1}\right|\right)} \Delta(\bar{t}-\underline{t})<1$.

Hence, contrary to the model with only binary issues, here the outcome of the negotiation may be inefficient even if the two players are equally focused. It will also be inefficient if issue 1 is the only issue on the table, because in that case $\bar{t}-\underline{t}=\beta_{1}$. In both cases, the level of issue 1 is too high when $\beta_{1}>\left|\alpha_{1}\right|$, and is too low when $\beta_{1}<\left|\alpha_{1}\right|$. Hence, here Lemma 1 does not apply: the outcome on issue 1 will be inefficient also when both players are equally focused, and also when issue 1 is the only issue.

Furthermore (and similarly to what discussed in the main text) if the players are differentially focused and there is more than 1 issue, then the salience of the transfer dimension will also determine the intensity of issue 1 . If $a$ is more focused than $b$, as the salience of the transfer dimension increases, then the level of issue 1 increases. If instead player $b$ is more focused, then as the salience of the transfer dimension increases, the level of issue 1 decreases. The effect of changes in $\bar{t}-\underline{t}$ may, however, be non monotonic. That is: it is possible that there is an interior value of $\bar{t}-\underline{t}$ that maximizes material welfare, which however monotonically decreases for any larger or smaller $\bar{t}-\underline{t}^{22}$

[^16]To summarize: we have shown in the body of the text that, with binary issues, as the salience of the transfer dimension increases the material welfare of the negotiation decreases (in a sense made precise in Lemma 3). We have shown here that, for continuous issues, this is true only if the salience of the transfer dimension is already sufficiently large. This implies, for example, that if $n$ is sufficiently large, then as the importance of an issue $i>1$ increases, the outcome on issue 1 moves farther away from efficiency (as in Proposition 1). But if $n$ is small, then as the importance of an issue $i>1$ increases, the outcome on issue 1 may first become more efficient, and then moves farther away from efficiency.

### 5.7 Issue-specific transfers

We described the model as if there is a single transfer. However, introducing issuespecific transfers will not change our results. This is because in the focusing model by Kőszegi and Szeidl (2013), focus weights are attached to consumption goods. In our case, as long as all the transfers are received and consumed in the same period, the utility functions of the two players depend on "money" (received or spent) rather than separately on the different issue-specific transfers. As a consequence, in the utility function all these transfers are weighted by the same focus weight. Such model is therefore identical to the one presented.

## 6 Conclusion

We have studied a bargaining problem in which the players' preferences are distorted by the focusing effect. When the issues on the bargaining table are uneven in their importance to the players, some of them may be inefficiently included in or excluded from the final agreement. The emergence of an inefficient outcome depends on the strength of each players' bias relative to that of the other player, and on the number
are analogous). Then $\frac{h_{b}\left(\left|\beta_{1}\right|\right)}{h_{a}\left(\left|\alpha_{1}\right|\right)}>1$, which implies that at $\bar{t}-\underline{t}=0$ the level of issue 1 is too high. As $\bar{t}-\underline{t}$ increases, the level of issue 1 decreases, possibly reaching the efficient level. Further increases in $\bar{t}-\underline{t}$ decrease the level of issue 1 below its efficient level.
of issues on the negotiating table.
We also allowed the players to negotiate in stages, by first restricting their future bargaining set via an incomplete agreement, and then finalizing the negotiation outcome. We show that when players agree on an incomplete agreement, they always increase the material efficiency of the negotiation (relative to a one-step negotiation). However, due to an inherent non-transferable utility problem, there is no presumption that players will always manage to agree on an incomplete agreement, even if it is in principle possible to do so. There is also no presumption that they will agree on the most efficient incomplete agreement.

We believe our paper improves our understanding of preliminary negotiation rounds and preliminary agreements (also called framework agreements), which are widely used in negotiations. In line with the practitioners' view discussed in the introduction, our theory shows that preliminary agreements are an important determinant of the importance of the issues on the negotiating table and, as a consequence, of the outcome of negotiations.

## A Appendix

Proof of Lemma 1. (3.1) and (3.2) together yield

$$
\begin{equation*}
\bar{t}-\underline{t}=\frac{\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}}{h_{b}(\bar{t}-\underline{t})}+\frac{\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}}{h_{a}(\bar{t}-\underline{t})} . \tag{A.1}
\end{equation*}
$$

The LHS is strictly increasing in $\bar{t}-\underline{t}$; the RHS is strictly decreasing in $\bar{t}-\underline{t}$. Furthermore, when $\bar{t}-\underline{t}=0$ the RHS of the above expression is strictly positive (remember that, by our initial parametric restrictions, we rule out issues that generate cost for both players), and hence strictly above the LHS. At the same time, for $\bar{t}-\underline{t}$ sufficiently large the LHS must be above the RHS. Taken together, this implies that RHS and LHS always cross only once.

If $\bar{t}-\underline{t}$ exists and is unique, then by (3.1) and (3.2) also $\bar{t}$ and $\underline{t}$ exist and are unique. Also, the RHS of (A.1) is increasing in $\alpha_{i}$ if $\alpha_{i}>0$, in $\beta_{i}$ if $\beta_{i}>0$, and in $n$, which implies our statement.

Proof of Lemma 2. If $n=1$, then by our initial parametric restrictions, $\beta_{1} \cdot \alpha_{1}<0$. Suppose $\beta_{1}>0$ (the case $\alpha_{1}>0$ is analogous). By (3.1) and (3.2) we have $\bar{t}=\beta_{1}$ and $\underline{t}=0$. Hence, by (3.5), there is an agreement if and only if

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{a}\left(\left|\beta_{i}\right|\right) \beta_{i}>0
$$

Because $h_{a}(|x|)$ is increasing in $|x|$, the above condition is equivalent to $\alpha_{i}+\beta_{i}>0$. Hence, issue $i$ is included in the agreement if and only if it is efficient to do so.

Suppose now that $\Delta(x)=1$. In this case $h_{a}(\bar{t}-\underline{t})=h_{b}(\bar{t}-\underline{t})$ and hence, by (3.5), issue $i$ is included in the agreement if and only if

$$
h_{b}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}>0 .
$$

Because $h_{b}(|x|)$ is strictly increasing in $|x|$, the above condition is equivalent to $\alpha_{i}+\beta_{i}>0$. Hence, issue $i$ is included in the agreement if and only if it is efficient to do so.

Proof of Lemma 3. We limit our proof to the case that player $b$ is "more focused" than player $a$ (that is, $\Delta(x) \leq 1$ and strictly decreasing). The proof of the opposite case follows by interchanging cases A.) and B.), respectively.

As a preliminary step, remember that equation (3.5) establishes that an issue is included in the final agreement if and only if

$$
\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}+\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}>0 .
$$

This condition is equivalent to

$$
\begin{equation*}
\frac{\Delta\left(\alpha_{i}\right)}{\Delta(\bar{t}-\underline{t})} \alpha_{i}+\frac{h_{b}\left(\left|\beta_{i}\right|\right)}{h_{b}\left(\left|\alpha_{i}\right|\right)} \beta_{i}>0 \tag{A.2}
\end{equation*}
$$

The proof proceeds by considering different cases.
Case A.) $\alpha_{i}<0$ and $\beta_{i}>0$ : Suppose $\alpha_{i}+\beta_{i}<0$, that is, it is efficient not to include issue $i$ into the agreement. In case $\bar{t} \leq\left|\alpha_{i}\right|$, then the largest transfer
player $b$ is willing to make does not cover the cost for player $a$ of including issue $i$. This issue will, therefore, not be included in the agreement. If instead $\bar{t}>\left|\alpha_{i}\right|$, then the largest transfer player $b$ is willing to make covers the cost for player $a$ of including issue $i$. It follows that issue $i$ will be included into the agreement if the focus weighted surplus generated by issue $i$ is positive. Note that, in this case, it must be that $\bar{t}-\underline{t}>\left|\alpha_{i}\right|$. Together with the fact that player $b$ is more focused (i.e., $\Delta(x) \leq 1$ and decreasing) and with the fact that $\alpha_{i}<0$ implies that the the LHS of (A.2) is always smaller than

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}
$$

which is negative whenever $\alpha_{i}+\beta_{i}<0$. Hence, if $\alpha_{i}+\beta_{i}<0$, then (A.2) does not hold and the players do not want to include issue $i$ in the agreement.

Suppose next that $\alpha_{i}+\beta_{i}>0$, that is, it is efficient to include issue $i$ in the agreement. In this case, we have $h_{b}\left(\left|\beta_{i}\right|\right) \geq h_{a}\left(\left|\beta_{i}\right|\right)>h_{a}\left(\left|\alpha_{i}\right|\right)$ (where the first inequality follows from the fact that player $b$ is more focused), which implies that

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}>0
$$

If $\bar{t}-\underline{t} \rightarrow 0$ then $\Delta(\bar{t}-\underline{t}) \rightarrow 1$, which together with the above expression imply that (A.2) holds and the players want to include issue $i$ in the final agreement. However, if there exists a $Y$ such that

$$
\begin{equation*}
\Delta(Y)=-\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}} \tag{A.3}
\end{equation*}
$$

then for $\bar{t}-\underline{t}>Y$ issue $i$ will be inefficiently excluded from the agreement.
Case B.) $\alpha_{i}>0$ and $\beta_{i}<0$ : Suppose $\alpha_{i}+\beta_{i}>0$, that is, it is efficient to include issue $i$ into the agreement. This implies that $\left|\beta_{i}\right|<\alpha_{i}$, and $h_{b}\left(\left|\beta_{i}\right|\right)<h_{b}\left(\alpha_{i}\right)$. Furthermore, by definition of $\underline{t}$ (see equation 3.1), we have $h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \leq-h_{a}(\bar{t}-$ $\underline{t}) \underline{t}$. Therefore, $h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \leq h_{a}(\bar{t}-\underline{t})(\bar{t}-\underline{t})$ which implies that $\alpha_{i} \leq(\bar{t}-\underline{t})$ and that $\Delta\left(\alpha_{i}\right) \geq \Delta(\bar{t}-\underline{t})$. Taken together, the fact that $\alpha_{i}>0, \beta_{i}<0, \alpha_{i}+\beta_{i}>0$,
$h_{b}\left(\left|\beta_{i}\right|\right)<h_{b}\left(\alpha_{i}\right)$ and $\Delta\left(\alpha_{i}\right) \geq \Delta(\bar{t}-\underline{t})$ imply that (A.2) holds, and issue $i$ is included in the agreement.

Suppose next that $\alpha_{i}+\beta_{i}<0$. Then,

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}<0
$$

Again, if $\bar{t}-\underline{t} \rightarrow 0$ then $\Delta(\bar{t}-\underline{t}) \rightarrow 1$, which together with the above expression imply that (A.2) does not holds and the players will efficiently exclude issue $i$ in the final agreement. However, if $\bar{P}$ (as in A.3) exists, then $\bar{t}-\underline{t}>\bar{P}$ issue $i$ will be inefficiently included in the agreement.

Proof of Proposition 1. By Lemma 1, $\bar{t}-\underline{t}$ is

- increasing in $\beta_{j}$, strictly so for $\beta_{j}>0$, for all $j \leq n$.
- increasing in $\alpha_{j}$, strictly so for $\alpha_{j}>0$, for all $j \leq n$.
- increasing in $n$.

The statement then follows by Lemma 3.
Proof of Proposition 2. Without loss of generality, order the issues such that:

- issues $i \in\{1, \ldots, z\}$ are included in the final agreement of the one-step negotiation, (where $z \in\{0, \ldots, n\}$, with the convention that if $z=0$, then the one-step negotiation leads to no agreement),
- issues $i \in\{z+1, \ldots, w\}$ are excluded from the final agreement of the onestep negotiation, but can be included in the final agreement of the two-step negotiation provided that a given incomplete agreement is signed in period 1 (where $w \in\{z, \ldots, n\}$, with the convention that if $w=z$, then such issues do not exist).

Issues 1 to $w$ are therefore the feasible issues from period- 1 viewpoint. For future reference, remember that in the one-step negotiation instead all $n$ issues are feasible.

Call the smallest and largest $t^{*}$ that can be achieved as a function of an incomplete agreement (as in Equation 4.11) $t^{*}$ and $\hat{t}^{*}$, respectively. ${ }^{23}$ Note that, $t \in\left[t^{*}, \hat{t}^{*}\right]$ is the set of feasible transfers from period-1 viewpoint.

A feasible bargaining outcome $x$ satisfies the player's participation constraints if and only if

$$
\begin{aligned}
U^{a}(x)= & \sum_{i=1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}\left(\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{{\underset{v}{*}}^{*},{\underset{\sim}{t}}^{1}\right\}\right) t \\
& \geq \sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{a}\left(\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{t^{*},{\underset{\sim}{1}}^{1}\right\}\right) t^{O S} \\
U^{b}(x)= & \sum_{i=1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}\left(\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{t^{*},{\underset{\sim}{t}}^{1}\right\}\right) t \\
& \geq \sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}-h_{b}\left(\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{t^{*},{\underset{\sim}{t}}^{1}\right\}\right) t^{O S} .
\end{aligned}
$$

where $\tilde{t}^{1}$ and ${\underset{\sim}{t}}^{1}$ are the largest and lowest transfer in the period- 1 consideration set when the constraint $t \in\left[t^{*}, \hat{t}^{*}\right]$ is not binding. They are implicitly defined as the transfers leaving each player indifferent between an agreement including only the issues giving him positive utility and his outside option:

$$
\begin{equation*}
\sum_{i=1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}-\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}=-h_{a}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)\left({\underset{\sim}{t}}^{1}-t^{O S}\right) \tag{A.4}
\end{equation*}
$$

[^17]\[

$$
\begin{equation*}
\sum_{i=1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}-\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}=h_{b}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)\left(\tilde{t}^{1}-t^{O S}\right) \tag{A.5}
\end{equation*}
$$

\]

The next step is to show that $\tilde{t}^{1}$ and ${\underset{\sim}{t}}^{1}$ exist and are unique. To see this, note that (A.4) and (A.5) together yield

$$
\begin{align*}
\tilde{t}^{1}-{\underset{\sim}{t}}^{1} & =\frac{\sum_{i=1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}-\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}\left(\tilde{t}^{1}-t_{\sim}^{1}\right)} \\
& +\frac{\sum_{i=1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}-\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)} . \tag{A.6}
\end{align*}
$$

The argument is identical to the one presented in the proof of Lemma 1. The LHS is strictly increasing in $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$; the RHS is strictly decreasing in $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$, and they always cross only once. Hence, $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$ exists and is unique. By (A.4) and (A.5), then, also $\tilde{t}^{1}$ and ${\underset{\sim}{t}}^{1}$ exist and are unique.

We now compare $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$ with $\bar{t}-\underline{t}$, the focus weight on the transfer dimension of the one-step negotiation (as in (A.1), see the proof of Lemma 1). By simple algebra, we can show that the RHS of (A.6) is below the RHS of (A.1) at a given range of possible transfers $y$ whenever
$0<\frac{\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}+\sum_{i=w+1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}}{h_{b}(y)}+\frac{\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+\sum_{i=w+1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}}{h_{a}(y)}$
Note that

$$
\frac{\sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}+\frac{\sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}
$$

is the focus-weighted surplus generated by the agreement in the one-step negotiation. It is strictly positive whenever $z>0$ and zero when $z=0$. Hence, whenever either $z>0$ or $w<n$, then the RHS of (A.6) is below the RHS of (A.1) at $y=\bar{t}-\underline{t}$. This then implies that the solution to (A.6) must be below the solution to (A.1). If instead $z=0$ and $w=n$, then the two solutions coincide.

Hence, when either $z \geq 1$ or $w<n$, then $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}<\bar{t}-\underline{t}$. This immediately implies
that $\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{{\underset{\sim}{t}}^{*},{\underset{\sim}{t}}^{1}\right\}<\bar{t}-\underline{t}$. If instead $z=0$ and $w=n$, then $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}=\bar{t}-\underline{t}$. This implies that $\min \left\{\hat{t}^{*}, \tilde{t}^{1}\right\}-\max \left\{{\underset{\sim}{*}}^{*},{\underset{\sim}{t}}^{1}\right\} \leq \bar{t}-\underline{t}$. It follows that period- 1 preferences, although not rational, are "closer" to rational than the preferences in the one-step negotiation, strictly so if $z \geq 1$ or $w<n$. Hence Corollary 1 applies: because they "more rational" than in the one-step negotiation, if the players use incomplete agreements to change the issues included in the agreement (relative to the one-step negotiation), this change must increase material efficiency.

Additional considerations The above proof can be extended easily to the case in which the players outside option in the period-1 negotiation is no agreement. The only difference is that, in this case, we need to assume that $z=0$ from the outset.

Hence, if $w<n$, then the focus weight on transfer in the period-1 negotiation is strictly smaller than that of the one-step negotiation. If instead $w=n$, then the focus weight on transfer of the period-1 negotiation is weakly smaller than that of the one-step negotiation. Again, Corollary 1 applies.

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[^0]:    ${ }^{1}$ See https://noref.no/Publications/Regions/Colombia/Designing-peace-the-Colombian-peaceprocess, accessed on August 10th 2021.
    ${ }^{2}$ See Raiffa (1982), "The art and science of negotiation", p. 178.
    ${ }^{3}$ See, for example, Fisher et al. (1991) "Getting to yes", page 171.
    ${ }^{4}$ Ibid, p. 172.

[^1]:    ${ }^{5}$ There is ample empirical evidence for the focusing effect. See, for example, Schkade and Kahneman (1998), Kahneman, Krueger, Schkade, Schwarz and Stone (2006), and Dertwinkel-Kalt, Gerhardt, Riener, Schwerter and Strang (2017). There is also a recent literature studying contract and menu design when consumers' preferences are distorted by the focusing effect; see, for example, Apffelstaedt and Mechtenberg (2017) and Dertwinkel-Kalt, Köster and Peiseler (2019).

[^2]:    ${ }^{6}$ An example of negotiation breakdown which is related to our earlier discussion is that of the peace negotiation between ELN and Colombian government. While negotiating the peace agreement with the FARC, the government initiated peace talks with a smaller guerrilla group, the ELN. The parties agreed to an agenda for the peace talks but never reached a peace agreement. See https://colombiapeace.org/the-eln/\#Peace (accessed on August 18th 2021).

[^3]:    ${ }^{7}$ See Lang and Rosenthal (2001), Bac and Raff (1996), Inderst (2000), Busch and Horstmann (1999b), Busch and Horstmann (1999a), Flamini (2007), Chen and Eraslan (2017).

[^4]:    ${ }^{8}$ In Section 5.3, we discuss the alternative assumption that the outside option of the period-1 negotiation is no agreement. That is, if the players fail to agree in period 1 , then the negotiation ends. We show there that all our results continue to hold.

[^5]:    ${ }^{9}$ We argue in Section 5.2 that this modeling choice is without loss of generality.
    ${ }^{10}$ Note that other models of salience exist in the literature. We discuss them in Section 5.5.

[^6]:    ${ }^{11}$ We discuss in an extension (Section 5.4) the case of in which an incomplete agreement is binding only in the current round of negotiation but can be ignored in future rounds if no agreement is reached in the current round.

[^7]:    ${ }^{12}$ In this expression, we ignore the constraint that $\underline{t} \leq t \leq \bar{t}$. The reason is that this constraint is never binding. If it was, one of the two players would earn zero surplus, which clearly does not maximize the objective function.

[^8]:    ${ }^{13}$ That is, if $\alpha_{i}>0$ then suppose player $a$ is more focused, while if $\beta_{i}>0$ then suppose player $b$ is more focused.

[^9]:    ${ }^{14}$ By the same logic, if there is also a lower bound on the transfer dimension $\underline{T} \leq 0$, this lower bound is irrelevant, because the largest transfer player $a$ is willing to make is zero. This is why, in this example, we can ignore $\underline{T}$.

[^10]:    ${ }^{15}$ In other words, we assume that players are consequential. This assumption is already made in Koszegi and Szeidl (2013, p. 73) and it basically says that an agent's consideration set is determined by the set of future outcomes achievable as consequence of today's choices.

[^11]:    ${ }^{16}$ Clearly, with respect to $Q_{1}$, either $Q_{1}=\{1\}$ or $Q_{1}=\{0,1\}$ is a solution.

[^12]:    ${ }^{17}$ That is, we are dealing with a Non Transferable Utility (NTU) bargaining problem.

[^13]:    ${ }^{18}$ For example, suppose $h_{a}()=1$ (player $a$ is rational) while $h_{b}()>1$ and strictly increasing. Then, the above conditions imply $h_{b}\left(\beta_{1}\right) \beta_{1}=h_{b}\left(\bar{t}+\alpha_{2}\right) \bar{t}$ and $\bar{t} \geq-\alpha_{1}$. The first equality implies that $\bar{t}$ is strictly decreasing in $\alpha_{2}$. Hence, for $\alpha_{2}$ sufficiently large and/or $-\alpha_{1}$ sufficiently small the second inequality is violated.

[^14]:    ${ }^{19}$ It can be done, for example, via a game of alternating offers with an exogenous probability of breakdown of the negotiation as in Binmore, Rubinstein and Wolinsky (1986).

[^15]:    ${ }^{20}$ This is the case when the reference point is the average of all possibilities available, as in Bordalo, Gennaioli and Shleifer (2013). However, the reference point may also be defined in other ways. For example in Bordalo, Gennaioli and Shleifer (2020) the reference point is (at least partially) defined by what the agents remember about past interactions, in which case the reference point is not affected by bounds on transfers.
    ${ }^{21}$ The fact that the utility of player $b$ is linear in $\gamma$ while the utility of player $a$ is quadratic in $\gamma$ is simply to allow for interior solutions, in which the players agree to include issue 1 with $\gamma \in(0,1)$. If both utility functions were linear, then, in equilibrium, $\gamma$ would be either 0 or 1 , and issue 1 would be, essentially, binary again.

[^16]:    ${ }^{22}$ To see this, consider the case in which $\alpha_{1}+\beta_{1}>0$ and player $b$ is more focused (all other cases

[^17]:    ${ }^{23}$ The existence of $t^{*}$ and $\hat{t}^{*}$ follows from the fact that, as already discussed in Section 4.2.1, when $\bar{T}$ is above a certain threshold, then the period- 2 focus weights (and the period-2 solution as in Equation 4.11) do not depend on $\bar{T}$. The same happens when $\underline{T}$ is below a certain threshold. Without loss of generality, therefore, we can restrict $\bar{T}$ to belong to a closed interval (and similarly for $\underline{T}$ ). This, together with the fact that the objective function is continuous, guarantees that the maximum and minimum of (4.11) exist.

