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Structural Transformation of Occupation Employment

Abstract

We use census data to show that structural transformation reflects a fundamental reallocation of labor from goods to services, instead of a relabelling that occurs when goods-producing firms outsource their in-house service production. The novelty of our approach is that it categorizes labor by occupations, which are invariant to outsourcing. We find that the reallocation of labor from goods-producing to service-producing occupations is a robust feature in censuses from around the world and different time periods. To understand the underlying forces, we propose a tractable model in which uneven occupation-specific technological change generates structural transformation of occupation employment.

JEL-Codes: O110, O140.

Keywords: biased technological change, occupations, outsourcing, structural transformation.

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1 Introduction

Growing economies undergo structural transformation, that is, they reallocate economic activity across broad sectors. A large body of recent literature shows that structural transformation is a crucial force behind the behavior of aggregate variables like hours worked, labor productivity, and the skill premium as well as behind regional convergence and urbanization. The common approach to measure economic activity in broad sectors is through sectoral labor. At the most basic level, the literature distinguishes between the share of employment in the goods sector, which produces tangible output, and the share of employment in the service sector, which produces intangible output.[1]

An important concern with measuring economic activity as sectoral labor is that structural transformation may then reflect the relabelling of employment that occurs when firms outsource what they used to produce in-house; see e.g. van Neuss (2019). To be concrete, consider the example of a manufacturing firm that lays off its cleaning staff and starts to purchase cleaning services from a contractor. Since the manufacturing firm is in the goods sector and the contractor is in the service sector, the resulting changes in sectoral employment are interpreted as structural transformation. More generally, if the firms which perform the outsourced tasks are in a different sector than the firms which outsource the tasks, then outsourcing changes the sectoral allocation of labor. Furthermore, if the firms which perform the outsourced tasks are in the service sector and the firms which outsource the tasks are in the goods sector, then outsourcing is interpreted as structural transformation. Since nothing fundamental has changed regarding the tasks performed, this "reallocation" of labor has no fundamental content or implications.

In this paper, we assess whether outsourcing was a major force behind structural transformation. To achieve this, we propose to measure employment at the occupation level instead of at the sector level. In particular, we tailor the categorization of occupations to the outsourcing issue and classify occupations as *goods occupations*, which produce, process or transform tangible value added, and *service occupations*, which produce or process intangible value added. Accordingly, farmers and miners are goods occupations whereas cleaners and managers are service occupations. To illustrate the advantage of our occupation classification in the context of outsourcing, it is useful to return to the example from above. The outsourcing of cleaning services from the manufacturing firm does not affect the employment of either goods or service occupations, as cleaners are a service occupation irrespective of the sector in which they work. Consequently, the measurement of labor at the occupation level is unaffected by outsourcing, implying that the reallocation from the goods and the service occupations does reflect structural transformation instead of outsourcing.

[1]Herrendorf et al. (2014) provide a review of the literature. Key contributions to it include Echevarria (1997), Laitner (2000), Caselli and Coleman (2001), Kongsamut et al. (2001), Gollin et al. (2007), Ngai and Pissarides (2007), Rogerson (2008), Duarte and Restuccia (2010), Buera and Kaboski (2012), Herrendorf et al. (2013), Boppart (2014), and Herrendorf et al. (2021).

We use a large sample of cross-country census data to establish two novel facts contradicting that outsourcing is a major force behind structural transformation. As countries develop, the share of service occupations in total employment increases; the share of service occupations in each sector's employment increases. If outsourcing was the major force behind structural transformation, then one would observe instead that the share of service occupations in total employment does not change much and the share of service occupations in goods-sector employment decreases. That we do not find these patterns is good news for the literature on structural transformation. To avoid misunderstandings, we emphasize that all we can say is that outsourcing was not a major driver behind structural transformation, but we cannot say how quantitatively important it actually was for structural transformation. In particular, that occupation labor is unaffected by outsourcing implies that the observed reallocation from goods occupations to service occupations reflects fundamental structural transformation, but not that there was no outsourcing in addition. Moreover, that we observe that the share of service occupations in good-sector employment increases implies that outsourcing cannot have been the main determinant of the goods sector's occupation composition, for if it had been the share of service occupations would have decreased, but there may still have been outsourcing that was offset by other forces.

To shed light on the economic forces behind the occupation patterns in the data, we propose a tractable model of structural transformation that features occupations and sectors. In our model, the main driver of structural transformation is uneven technological change, but in contrast to the traditional approach to structural transformation, technological change is specific to occupations and not to sectors. The tractability of the model allows us to establish that the model is qualitatively consistent with the novel patterns of the reallocation of occupation employment across and within sectors as well as with the standard patterns of structural transformation of sectoral employment.[2] While occupation-specific technological progress may seem unusual in the context of structural transformation, we emphasize that it is rather natural in the context of the task-based approach to labor market outcomes. Central to the task-based approach is the notion that technological progress affects tasks differently and that occupations perform different bundles of tasks. Occupation-specific technological change captures this notion in a reduced-form way.

We end with a quantitative analysis of the implications of our model. In particular, we calibrate the model to the post-war U.S. data and then study the quantitative properties of the equilibrium. We find that uneven occupation-specific technological change can generate most

[2]The standard patterns of structural transformation are: as GDP per capita increases, labor is reallocated from the goods sector to the service sector; the value added share of the service sector increases; the value added price of services relative to goods increases; labor productivity growth is faster in the goods sector than in the service sector. Note that the literature on structural transformation typically disaggregates goods further into agriculture and industry. We do not follow this practice here because the two-sector split into goods and services is sufficient to capture the effects of outsourcing.

of the observed employment reallocation between occupations and sectors in the post-war U.S.[3] We illustrate the usefulness of our model by showing that it also performs well along several non-targeted dimensions. To begin with, our model captures most of the reallocation of employment in the U.S. during 1850–1950 and in our sample of censuses from around the world. Moreover, our model is successful at making out-of-sample predictions about the changes in the composition of broad categories of sector and occupation employment in the U.S. during 1972–2014. Indeed, in most cases, the model forecasts outperform the BLS occupation forecasts, which are among the most downloaded statistics from the BLS website. This suggests that the BLS could improve its occupation forecasts by taking into account the forces behind structural transformation that our model highlights.

The paper is organized as follows. In the next section, we present the data and establish the stylized facts of structural transformation of occupation employment. Afterwards, we develop the theoretical model. In Section 4, we present our analytical results. The next section contains the quantitative results. Section 6 concludes. The proofs of our analytical results are in the Appendix.

2 Evidence

In this section, we establish stylized facts about occupation employment across countries and sectors that shed light on the question whether outsourcing was a major force behind structural transformation.

Our main data source is Minnesota Population Center (2015). IPUMS International provides census data for countries from all over the globe. Our sample consists of 182 censuses from 67 countries[4]. This includes 21 countries from America and the Caribbean, 19 from Africa (including many Sub-saharan countries), 14 from Europe, and 13 from Asia. In 1990, these countries represented more than two thirds of world output, they covered three quarters of the world population, and they included seven of the ten most populous countries (namely, China, India, U.S., Indonesia, Brazil, Pakistan, and Nigeria). Importantly, the countries in the sample are from all income levels and the largest income difference exceeds a factor fifty (the richest country is the U.S. in 2000 with $30,491 and the poorest country is Guinea in 1990 with $544, both in 1990 international $'s).

An invaluable feature of the data from IPUMS is that the information has been harmonized and is comparable across countries and across time. The harmonized information includes the sector and occupation identifiers which is crucial for our purpose. The sector (occupation)

[3]Building on our work, Bárány and Siegel (2020, 2021) found that sectoral differences in labor productivity growth are largely due to sectoral differences in the growth rate of routine-labor-augmenting technologies. This is consistent with our result.

[4]Our sample countries are listed in Appendix A.

classification in IPUMS distinguishes between 15 different sectors (10 occupations). In a first step, we follow the conventional approach and aggregate the 15 sectors into the two broad sectors, namely, goods and services. In particular:

- Goods sector: Agriculture, Fishing, and Forestry; Mining; Manufacturing; Construction; Electricity, Gas and Water.
- Service sector: Wholesale and Retail Trade; Transportation and Communications; Financial Services and Insurance; Real Estate and Business Services; Health and Social Work; Education; Hotels and Restaurants; Public Administration and Defense; Private Household Services; Other Services

Next, we aggregate the occupation groups into the broad categories of goods and service occupations. As mentioned above, we apply the principle that goods (services) occupations produce, process or transform predominantly tangible (intangible) output:[5]

- Goods occupations: Elementary Agricultural and Industry Occupations; Skilled Agricultural and Fishery Workers; Crafts and Related Trades Workers; Plant and Machine Operators and Assemblers.

- Service occupations: Armed Forces; Clerks; Elementary Service Occupations; Legislators, Senior Officials and Managers; Professionals; Service Workers and Shop and Market Sales; Technicians and Associate Professionals.

To establish the usefulness of our classification, we need to address several issues that might arise with it. To begin with, although IPUMS puts a great deal of effort into harmonizing the data, some occupation classifications may change over time. Since our analysis is at a very broad level with only two occupation classifications, this is unlikely to constitute a problem. Second, some broad categories of goods occupations may contain some service occupations and vice versa. We use the U.S. censuses during 1850–2010 to establish that this is not a quantitatively important issue. The detailed results are in Appendix A.

Some additional remarks about assigning the different occupations to the two categories are in order. First, it is natural to put occupations who have a unique sector in their name into the corresponding goods or service category. This applies to Elementary Agricultural Occupations, Skilled Agricultural and Fishery Workers, Elementary Industry Occupations in the goods occupations and to Elementary Service Occupations and Service Workers and Shop and Market Sales in the service occupations. Second, Crafts and Related Trades and Plant and Machine Operators and Assemblers are clearly goods occupations, as they predominantly produce, process or transform tangible output. Third, many occupations that we classified as services are

[5]Goods occupations are related to, but not equal to blue-collar or brawn-intensive occupations whereas service occupations are related to but not equal to white-collar or brain-intensive occupations.

indeed counted as part of the service sector when they are outsourced and provided by independent contractors. Examples include: outsourced cleaning or janitorial services performed by Elementary Service Occupations; outsourced accountancy and legal services performed by Clerks and Professionals; outsourced computer repair, maintenance, and programming services performed by Technicians and Professionals. Taken together, these arguments lend credibility to our classification of goods and service occupations.

Next, we document the empirical patterns of structural transformation of employment for our sample of countries for the working-age population (age 15–64). We first follow the conventional approach and compute the sectoral employment shares. We then adopt our new approach and measure employment at the occupation level by calculating the employment share of goods occupations and of service occupations for every census in our sample. Figure 1 plots the employment shares against GDP per capita, which is from Maddison's Groningen database and is in 1990 international $'s. The observations for the U.S., which span the period 1850–2010, are indicated by diamonds. The left panel shows that for sectoral employment the standard patterns of structural transformation hold both across countries and over time in the U.S. That is, an increase in GDP per capita is accompanied by a decline in the share of goods sector employment and an increase in the share of service sector employment. Quite strikingly, the right panel shows a very similar pattern also for occupation employment. The close similarity between the patterns of structural transformation for sector and occupation employment is remarkable in light of the fact that many occupations are not sector specific, but are used in both sectors (as we will show below). Table 1 concisely summarizes the patterns shown in Figure 1 by reporting how the composition of sector employment and occupation employment varies with GDP per capita.

An additional feature of Figure 1 deserves comment. The U.S. time series is similar to the patterns in the cross-country data. This provides support for the notion that when the U.S. was poor it had a similar sectoral composition as currently poor countries have. Many authors have conjectured that this is the case, and in fact several have made this assumption for lack of data from currently poor countries. However, to the best of our knowledge, we are the first to provide hard supporting evidence from high quality census data for a broad set of currently rich and poor countries that covers the vast majority of world population and world production.

In the next step, we provide evidence to substantiate that there is no one-to-one link between occupations and sectors but that instead goods and service occupations are employed in both sectors. To this end, we analyze the composition of occupation employment within each sector and how it changes with GDP per capita. Table 2 reports the shares of occupation employment in each sector for different levels of GDP per capita. One important observation stands out: As GDP per capita increases, the share of service occupation employment increases in both sectors. The implied reallocation to service occupations is particularly pronounced in the goods sector.

Figure 1: Structural Transformation in our Panel of Countries and in the U.S. Time Series
(black dots are country-year observations, blue diamonds are U.S. observations)

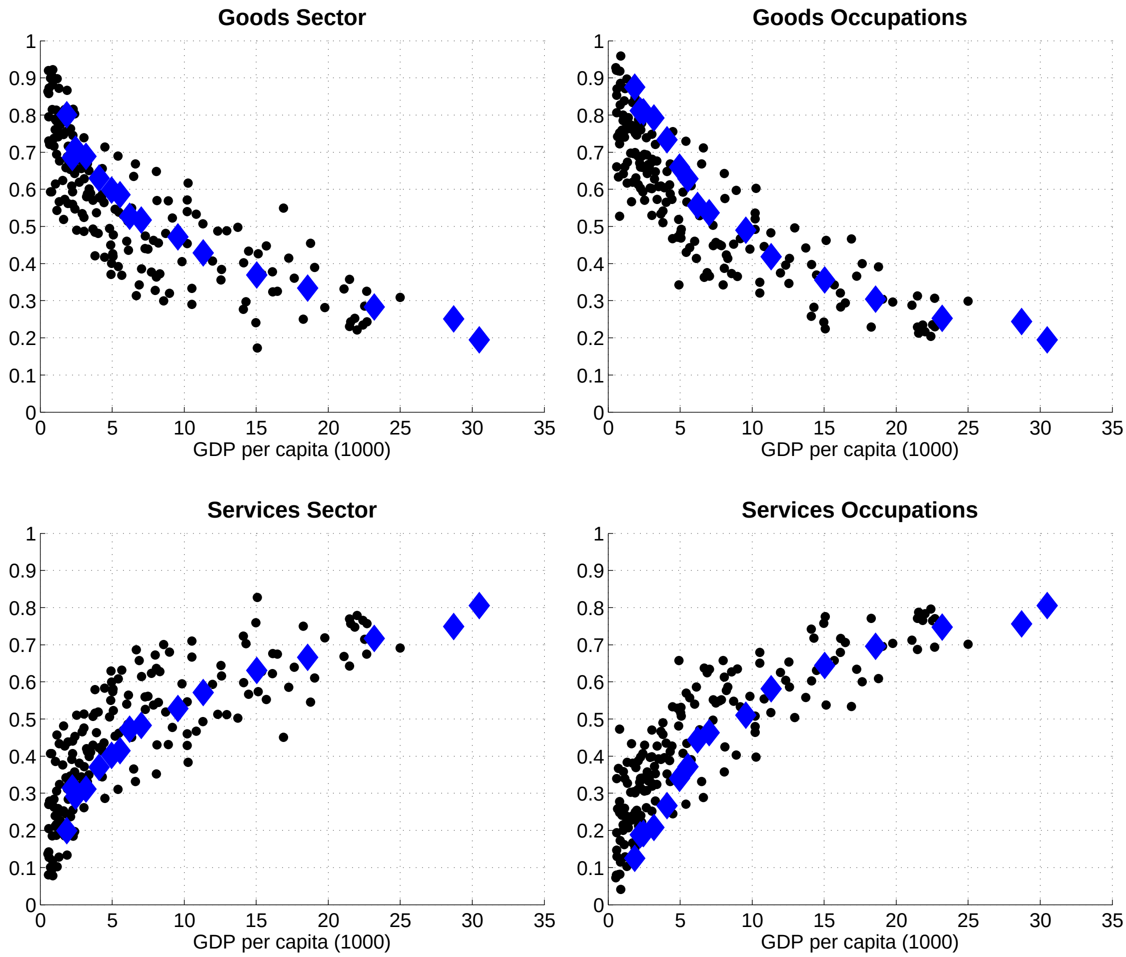


Table 1: Reallocation between sectors and occupations

GDP per capita (1990 int. $'s)	1,000	15,000	30,000
Share in total employment of			
goods sector	74.3	38.0	19.9
service sector	25.7	62.0	80.1
goods occupations	77.1	36.6	16.3
service occupations	22.9	63.4	83.7

[a] Shares are in percent and are from locally weighted splines (LOWESS) fitted through the data.

As a consequence, goods sector employment in rich countries consists predominantly of service occupations.[6]

Taken together, the evidence presented in Tables 1 and 2 shows the exact opposite of what outsourcing would imply. If structural transformation resulted just from outsourcing, then we should observe (1) a decrease in the share of service occupation employment in goods sector employment (due to goods firms laying-off in-house service workers), and 2) a constant share of service occupation employment in total employment (since total employment is unaffected by whether service workers are employed in the goods or in the service sector). Hence, our findings strongly support the notion that structural transformation indeed reflects a fundamental shift of economic activity across sectors.

Table 2: Reallocation of occupations within sectors

	Goods sector			**Service sectors**		
GDP per capita (1990 int. $'s)	1,000	15,000	30,000	1,000	15,000	30,000
Employment share of						
... goods occupations	97.3	73.5	42.1	18.7	13.8	9.9
... service occupations	2.7	26.5	57.9	81.3	86.2	90.1

[a] Shares are in percent and are from locally weighted splines (LOWESS) fitted through the data.

Our finding that outsourcing is not the main force behind structural transformation comes from a large number of rich and poor countries, and so it nicely complement some existing evidence for the U.S. Herrendorf et al. (2013) observed that outsourcing does not affect the composition of final expenditure. Since there is ST in final expenditure in the postwar U.S., it cannot be the case that all structural transformation is due to outsourcing. Berlingieri (2014)

[6]A shift-share analysis reveals that moving from GDP per capita of $\$1,000$ to GDP per capita of $\$30,000$, roughly half of the reallocation from goods-occupation labor to service-occupation labor is due to the reallocation of labor between the goods and the service sector and the other half is due to the reallocation of labor from the goods to the service occupations within the two sectors.

found for the postwar U.S. that changes in the input-output structure have increased service employment by 40%, with increases in business services being a crucial driver.

The stylized facts we have documented hold very broadly across countries and over time. This raises the question what common forces are behind them. Answering this question is not only interesting in its own right, but it also helps us understand what to expect about the future occupation composition. In what follows, we suggest a tractable model of structural transformation that has occupation-specific technological change as the driving force. We establish that our model can quantitatively generate the stylized facts. We also use our model to predict the employment shares of goods and service occupations for the U.S.

3 Model

Our model builds on the multi-sector framework developed by Ngai and Pissarides (2007). However, instead of assuming a homogeneous labor input, we introduce different types of occupations. Moreover, we consider technological change that is specific to occupations.

3.1 Environment

Time is discrete and runs forever. There are three sectors which produce investment X; consumption goods C_G; consumption services C_S. In each period, the investment good is the numeraire. The investment technology is of the AK form:

$$Y_{Xt} = A_X K_{Xt} \tag{1}$$

where A_X is the TFP of producing investment goods from capital K_X. We will use upper-case letters to index sectors and lower-case letters to index occupations. The consumption technologies are of the Cobb-Douglas form:

$$Y_{Jt} = K_{Jt}^{\theta} L_{Jt}^{1-\theta} \tag{2}$$

where $J \in \{G, S\}$ is the sector index; $\theta \in (0, 1)$ is the capital share parameter;[7] L_J is a CES aggregator of labor from the two occupations:

$$L_{Jt} = \left[(\alpha_J)^{\frac{1}{\sigma}} \Big(A_{gt} N_{Jgt} \Big)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_J)^{\frac{1}{\sigma}} \Big(A_{st} N_{Jst} \Big)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

[7]While Valentinyi and Herrendorf (2008) show that in the data $\theta_g \neq \theta_s$, Herrendorf et al. (2015) show that Cobb-Douglas production functions with equal θ do a reasonable job at capturing the technological forces behind the postwar structural transformation in the U.S. Acemoglu and Guerrieri (2008) explore what happens when θ_j are sector specific.

where $\alpha_J \in [0, 1]$ is the intensity of labor from the goods occupations; $j \in \{g, s\}$ is the occupation index; N_{Jj} is the labor from occupation j employed in sector J; A_j is occupation-specific labor-augmenting technological progress (which is not sector specific); $\sigma > 0$ is the elasticity of substitution between the two occupations. Note that the standard model of structural transformation in which labor is homogeneous is a special case for $\alpha_J = 0$, or for $\alpha_J = 1$, or for $\sigma = \infty$ together with $A_g = A_s$.

The way in which we model the aggregation of labor from different occupations has similarities to the canonical model of skill-biased technological change as described for example by Acemoglu and Autor (2011), which also assumes that technological progress is specific to broad categories of labor. The assumption that the intensity of occupation labor depends on the sector but labor-augmenting technological progress is independent of the sector can be viewed as a reduced-form of a more elaborate production process that involves three stages: value added in each sector is produced from different tasks and the intensity of each task differs across sectors; each tasks is produced from labor of different occupations and other inputs according to a technology that is common to all industries. Goos et al. (2014) develop an example of such a model. The way we model the aggregation of labor from different occupations also has similarities with Ngai and Petrongolo (2017) and Bárány and Siegel (2018). They embedded a Roy model into a structural transformation model, assuming that there are time-invariant CES functions that aggregate the different categories of labor to sector labor and that sector-specific technological progress augments all sector labor. The novelty of our paper is that labor-augmenting technological progress is occupation specific instead of sector specific. In a follow up paper to our work, Bárány and Siegel (2020) allowed for both occupation-specific and sector-specific technological change to cause sectoral differences in labor productivity growth. They found that sectoral differences in labor productivity growth are largely due to sectoral differences in the growth rate of routine-labor-augmenting technologies. That is consistent with our conclusion that occupation-specific technological change can go a long way to account for the patterns of structural transformation.

There is a continuum of measure one of identical households. The present discounted lifetime utility takes the standard separable form:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t)$$

where $\beta \in (0, 1)$ is the discount factor. C_t is a composite consumption good that consists of the consumption of goods and services:

$$C_t = \left[(\alpha_U)^{\frac{1}{\varepsilon}} \left(C_{Gt} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_U)^{\frac{1}{\varepsilon}} \left(C_{St} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\alpha_U \in [0, 1]$ is a relative weight and ε is the elasticity of substitution between consumption varieties. The representative household is endowed with a positive initial capital stock, $K_0 > 0$, which can be used in all sectors. Moreover, it is endowed with one unit of labor in each period, which can be used in both sectors and in both occupations. The usual assumption in the canonical model of structural transformation is that workers can use their labor endowment in all sectors, implying that in equilibrium real wages are equalized across sectors. We make the same assumption also for occupations, implying that in equilibrium real wages will also be equalized across occupations[8]. The resource constraints and market-clearing conditions are:

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + X_t \\
K_t &= K_{Xt} + (K_{Gt} + K_{St}) \\
N_{Jt} &\equiv N_{Jgt} + N_{Jst} \\
N_{jt} &\equiv N_{Gjt} + N_{Sjt} \\
1 &= N_t = N_{Gt} + N_{St} = N_{gt} + N_{st} \\
Y_{Xt} &= X_t, \quad Y_{Gt} = C_{Gt}, \quad Y_{St} = C_{St}
\end{aligned}
$$

The first equation is the standard law of motion for capital. The second equation is the adding up constraint for capital in each period. The third, fourth, and fifth equations are the adding up constraints for sectoral labor, occupation labor, and total labor. The last equations are the market clearing constraints for investment, consumption goods, and consumption services.

4 Analytical Results

4.1 Solving for the equilibrium

The household problem is:

$$
\begin{aligned}
&\max_{\{K_{t+1}, C_{Gt}, C_{St}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log\left(\left[(\alpha_U)^{\frac{1}{\epsilon}}(C_{Gt})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha_U)^{\frac{1}{\epsilon}}(C_{St})^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}\right) \\
&\text{s.t.} \quad p_{Gt}C_{Gt} + p_{St}C_{St} + K_{t+1} = (1 + r_t - \delta)K_t + w_t
\end{aligned}
$$

[8]For recent structural transformation models in which wages are not equalized, see Ngai and Petrongolo (2017), Bárány and Siegel (2018), and Buera et al. (2018).

The first-order conditions to the household problem are standard:

$$\frac{C_{t+1}p_{t+1}}{C_t p_t} = \beta(1 + r_{t+1} - \delta) \tag{4}$$

$$\lim_{t\to\infty} \beta^t \frac{K_{t+1}}{C_t p_t} = 0 \tag{5}$$

$$\frac{p_{St}C_{St}}{p_{Gt}C_{Gt}} = \frac{1-\alpha_U}{\alpha_U}\left(\frac{p_{St}}{p_{Gt}}\right)^{1-\epsilon} \tag{6}$$

where

$$p_t = \left[\alpha_U (p_{Gt})^{1-\epsilon} + (1-\alpha_U)(p_{St})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

The problem of the firm in the investment sector is:

$$\max K_{Xt}(A_X - r_t)$$

The first-order condition to the investment-sector firm problem is:

$$r_t = A_X \tag{7}$$

The problems of the firms in the consumption sectors are:

$$\max p_{Jt}\left(K_{Jt}\right)^{\theta}\left(L_{Jt}\right)^{1-\theta} - r_t K_{Jt} - w_t(N_{Jgt} + N_{Jst}) \tag{8}$$

$$\text{s.t.} \;\; L_{Jt} = \left[(\alpha_J)^{\frac{1}{\sigma}}(A_{gt}N_{Jgt})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_J)^{\frac{1}{\sigma}}(A_{st}N_{Jst})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

The first-order conditions to the problems of the consumption-sector firms imply:[9]

$$\frac{K_{Gt}}{N_{Gt}} = \frac{K_{St}}{N_{St}} \tag{9}$$

$$\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \left(\frac{L_{Gt}/N_{Gt}}{L_{St}/N_{St}}\right)^{1-\theta} \tag{10}$$

$$\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \frac{p_{St}}{p_{Gt}} \tag{11}$$

$$\frac{N_{Jst}}{N_{Jgt}} = \frac{1-\alpha_J}{\alpha_J}\left(\frac{A_{gt}}{A_{st}}\right)^{1-\sigma} \tag{12}$$

$$\frac{N_{Sst}}{N_{Gst}} = \frac{1-\alpha_S}{1-\alpha_G}\left(\frac{L_{Gt}/N_{Gt}}{L_{St}/N_{St}}\right)^{\sigma}\frac{L_{St}}{L_{Gt}} \tag{13}$$

(9) is the usual result that the capital-labor ratios are equalized if the sectoral production

[9]The derivations are in the Appendix.

functions are Cobb-Douglas with equal exponents. (10) shows that, as a result, the ratio of the labor productivities depends only on the ratio of the sector-labor aggregators per unit of sector-labor input.[10] (11) implies that the price of services relative to goods is inversely related to the relative sectoral labor productivities. (12) implies that changes in the within-sector allocation of labor between goods and service occupations are driven by the relative occupation technologies, together with the elasticity of substitution σ. For example, if both occupations are complements ($\sigma < 1$), then faster technological progress for goods occupation ($A_g/A_s \uparrow$) leads to a reallocation of labor from goods to service occupations in both sectors. Lastly, (13) describes how labor from service occupations is allocated between the two sectors.

4.2 Structural transformation along the balanced growth path

Since there is reallocation of labor between the consumption sectors, there is no balanced growth path along which all ratios are constant. Hence, we follow Kongsamut et al. (2001) and study a generalized balanced growth path (GBGP), which is an equilibrium path along which the real interest rate is constant while sectoral ratios may change. A GBGP trivially exists here because of the AK technology in the investment sector. We state that in a proposition:[11]

Proposition 1.
There is a unique GBGP iff $\gamma \equiv \beta(1 + A_x - \delta) > 1$. Along the GBGP, aggregate capital, capital in each sector, expenditure on total consumption, GDP, investment, and the wage all grow with factor γ.

Proposition 2.
If and only if (i) $\alpha_S < \alpha_G$; (ii) $\sigma < 1$; (iii) $\varepsilon < 1$; (iv) $A_{gt}/A_{st} \uparrow$, then the goods (service) sector is more intensive in the goods (service) occupations and as GDP per capita increases:

1. *labor is reallocated from goods to service occupations in both sectors;*
2. *labor productivity increases more in the goods sector than in the service sector;*
3. *the relative price of services increases;*
4. *the expenditure share of the service sector increases;*
5. *labor is reallocated from the goods sector to the service sector;*
6. *labor is reallocated from goods occupations to service occupations.*

Condition (i) says that the goods sector is more intensive in the goods occupation than the service sector and the service sector is more intensive in the service occupation than the goods sector. Condition (ii) says that the inputs into the production function are complements (they

[10] This shows that in terms of reallocation of labor our model behaves like a simpler model without capital.
[11] All proofs are in the Appendix.

are less substitutable than Cobb Douglas).[12] Condition (iii) says that the inputs into the utility function are complements.[13] Condition (iv) says that technological progress is faster for the goods than the service occupation. Proposition 2 establishes that our model is qualitatively consistent with the stylized facts of structural transformation although it does not feature technological progress at the sector level at all; it only needs uneven occupation-specific technological progress.

The intuition for Proposition 2 is as follows. 1. Labor gets allocated from the goods to the service occupations in each sector, because the occupations are complements in the production functions and the technological change augmenting the goods occupations grows relative to that augmenting the service occupations. 2. Labor productivity increases more in the goods sector than in the service sector, because the goods (service) sector is more intensive in the goods (service) occupations and the technological change augmenting the goods occupations grows relative to that augmenting the service occupations. 3. The relative prices of services to goods increases because it is inversely related to the relative productivities. 4. Expenditures get reallocated from the goods to the service sector because value added from the two sectors are complements in the utility function and the relative price of services increases. 5. Labor gets reallocated from the goods to the service sector because labor and expenditure shares move together. 6. Labor is reallocated from the goods to service occupations in the whole economy because that happens in each sector and also labor is reallocated from the goods sector, which is less intensive in service occupations, to the service sector, which is more intensive in service occupations.

In sum, two forces generate the reallocation of labor from the goods to the service occupations: substitution between occupations within each sector; substitution of labor between sectors. In our model, both effects result from uneven technological progress at the occupation level.

4.3 Discussion

An obvious question to ask at this point is whether there are plausible examples for the notion that technological change is occupation specific and uneven. A first supportive example comes from Goldin and Katz (2008) who pointed out that during the 19. century manufacturing technologies tended to replace skilled artisans. This is consistent with our model because skilled artisans are in the goods occupations. A second supportive example come from Baumol (1967) who argued that the increasing relative prices of many services (his famous "cost disease") is

[12]Note that this is different from the canonical model with unskilled and skilled labor inputs described by Acemoglu and Autor (2011), in which it is empirically plausible to choose an elasticity of substitution larger than one.

[13]The evidence from Herrendorf, Rogerson, and Valentinyi (2013) suggests that $\epsilon \approx 0$. In other words, imposing $\epsilon < 1$ is not restrictive.

related to the lack of technological progress in the production of these services. Specifically, on page 415 and following he wrote: "... economic activities can ... be grouped into two types: technologically progressive activities in which innovations, capital accumulation, and economies of large scale all make for a cumulative rise in output per man hour and activities which, by their very nature, permit only sporadic increases in productivity. ... The basic source of differentiation resides in the role played by labor in the activity. In some cases labor is primarily an instrument – an incidental requisite for the attainment of the final product, while in other fields of endeavor, for all practical purposes the labor is itself the end product. Manufacturing encompasses the most obvious examples of the former type of activity. ... On the other hand there are a number of services in which the labor is an end in itself, in which quality is judged directly in terms of amount of labor. Teaching is a clear-cut example, where class size (number of teaching hours expended per student) is often taken as a critical index of quality." Baumol's distinction between the two types of labor is closely related to our distinction between goods occupations and service occupations.

A third supportive example comes from the recent labor literature on job polarization. Autor et al. (2006) and Autor and Dorn (2013) documented that job polarization has happened in the non-farm sector of the U.S. since the 1980s: middle-wage occupations have experienced declines in their relative employment and relative wages compared to low-wage and high-wage occupations during recent decades. Goos et al. (2014) documented that the same phenomena happened in Western Europe during 1993–2000. These papers argue that the main force behind job polarization is routine-biased technological change, which has increasingly replaced routine tasks that tend to be produced by middle-wage occupations, but has hardly affected non-routine tasks that tend to be produced by low-wage and high-wage occupations. Routine-biased technological progress is one reason for why the goods occupations have experienced stronger labor-augmenting technological progress than the service occupation in recent decades. Specifically, while service occupations perform mostly non-routine tasks (e.g., managers) or routine tasks (e.g. clerks), goods occupations tend to perform mostly routine tasks. Hence, routine-biased technological change affects the goods occupations more strongly than the service occupations.

We conclude this subsection with a brief discussion of three additional aspects of what we have achieved thus far. First, although our model generates the qualitative patterns of *nominal* expenditure shares, it cannot generate the qualitative patterns of *real* expenditure. In the model the *real* share of services decreases whereas in the data it increases. This is a common problem with CES utility because it implies that the real expenditure shares move opposite to relative prices, except in the extreme Leontief case in which the real expenditure shares stay constant. The recent work of Boppart (2014) and Comin et al. (2018) uses non-homothetic CES utility functions that, under some restrictions, are able to account for the patterns of real shares. While that work is important for understanding the forces behind structural transformation that arise on the preference side, we use a homothetic CES utility function here because our focus is on the forces on the technology side.

Second, instead of occupation-specific technological progress, the literature assumes that sectoral labor is homogenous and uses sector-specific, labor-augmenting technological progress:

$$Y_{Jt} = K_{Jt}^{\theta}(A_{Jt}L_{Jt})^{1-\theta} \tag{14}$$

where :

$$L_{Jt} = \left[(\alpha_J)^{\frac{1}{\sigma}} N_{Jgt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_J)^{\frac{1}{\sigma}} N_{Jst}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{15}$$

It is easy to show that with sector-specific technological progress the model is consistent with stylized 2.–6. from Proposition 2 above if and only if Assumption (i) and (ii) hold and $A_{Gt}/A_{St} \uparrow$. Because N_{Jgt}/N_{Jst} is constant in equilibrium for each sector in the model used by the literature, it cannot match the within-sector reallocation of labor between occupations (1. from above). As we documented above, this reallocation is a quantitatively important part of the reallocation of occupation employment, and so we will focus our attention on occupation-specific technological change.

Third, whether technological progress happens at the sector level or at the occupation level has an important implication for the behavior of sectoral labor productivity growth, where labor productivity is defined as $Lp_{Jt} \equiv Y_{Jt}/N_{Jt}$. If we assume that technological progress happens at the sector level and sectoral technological progress grows at a constant rate, then sectoral labor productivity grows at constant rate. This is the common case analyzed in the literature. If, instead, we assume that technological progress happens at the occupation level and grows at a constant rate, then sectoral labor productivity will not grow at constant rates. The intuitive reason for this is that structural transformation implies a *non-linear* relationship between sectoral value added and the sectoral occupation-labor composition. The next Proposition formalizes this intuition.

Proposition 3.
If $0 < \alpha_S < \alpha_G < 1$*;* $0 < \sigma < 1$*;* $A_{gt}/A_{st} \uparrow$ *with constant growth factors* $\gamma_j \equiv A_{jt}/A_{jt-1}$*, then:*

- $\lim_{t\to\infty}[\Delta \log(Lp_{St}) - \Delta \log(Lp_{Gt})] = 0$*;*
- $\exists \bar{t} > 0\ \forall t > \bar{t}\ \Delta \log(Lp_{St}) - \Delta \log(Lp_{Gt}) \leq 0$ *and increases over time.*

While Proposition 3 implies that in at least one sector the growth rate of labor productivity changes over time, the quantitative analysis that follows next shows that the growth rates of both sectoral labor productivities change over time.

5 Quantitative Results

Proposition 2 specifies under what conditions our model is qualitatively consistent with the six stylized facts. In this section, we show that our model is successful quantitatively as well. To establish this, we calibrate it to the composition of occupation employment in the U.S. in 1950 and in 2000. We find that the model has no trouble matching this episode, and that it performs well along several dimensions that we have not targeted. We then show that the calibrated model accounts for most of the structural transformation of occupation employment in both the U.S. during 1850–1950 and in our sample of countries from around the world.

5.1 Calibration

We need to calibrate the following parameters: the discount factor β; the depreciation rate δ; the elasticities ϵ, σ, θ; the relative weights $\alpha_G, \alpha_S, \alpha_U$; the technological-progress parameters A_g, A_s, A_X. We choose standard parameter values to the extent possible and calibrate the remaining parameters jointly by matching salient features of the U.S. in 1950 and 2000.[14] Table 3 lists the resulting parameter values. Specifically, we choose the standard values $\beta = 0.96$ and $\delta = 0.05$. We choose a low value $\epsilon = 0.05$, which is based on the evidence provided by Herrendorf et al. (2013) that the elasticity of substitution between broad consumption categories is close to zero. We choose $\theta = 0.17$, which implies an aggregate capital share of $1/3$.[15] We choose $A_X = 0.10$, which implies a net real interest rate $r - \delta = 0.05$. We make the following normalizations: $A_{g,1950} = A_{s,1950} = K_{1950} = 1$. We jointly calibrate $\sigma, \alpha_G, \alpha_S, \alpha_U, A_{g,2000}, A_{s,2000}$ to match six U.S. targets: we use that, according to Maddison, real GDP per capita increased by a factor of three during 1950–2000; from the population censuses we use five shares in total employment: the goods occupations working in the goods sector in 1950 and in 2000, the service occupations working in the goods sector in 1950 and in 2000, the service occupations working in the service sector in 1950. Note that these targets imply that we implicitly also target the share in total employment of the service occupations working in the service sector in 1950; the shares in total employment of labor working in the two sectors in 1950 and 2000.

Table 4 shows that we match our targets well and that our model also performs reasonably well along several dimensions that we did not target, including the capital-output ratio, the investment-output ratio, the evolution of the labor productivity of goods relative to services, and the price of goods to services. Note that since the calibration procedure matches the shares in total employment of goods and service- occupation employment of the goods sector, it also matches the shares in total employment of goods-sector employment. Note too that the model

[14] Although we have data for 2010 we deliberately do not calibrate the model to 2010 because we want to avoid the Great Recession.

[15] Note that in our model the capital share parameter in the consumption sectors is lower than usual because the capital share in the investment sector equals one.

Table 3: Calibrated Parameters

β	0.96	α_U	0.47		**1950**	**2000**	
δ	0.05	α_G	0.80	A_g	1	20.15	(6.1% average growth p.a.)
ϵ	0.05	α_S	0.22	A_s	1	2.14	(1.5% average growth p.a.)
σ	0.56	θ	0.17	A_X	0.10	0.10	

Table 4: Targets (in boldface and blue) and Model Predictions

	Model		U.S. Data	
Increase in per capita GDP (in 1990 prices) 1950 to 2000	3		**3**	
Capital share in total income	1/3		**1/3**	
Capital-to-output ratio	3.33		~ 3	
Investment-to-output ratio	0.19		0.23	
	1950	**2000**	**1950**	**2000**
Share in total employment of service occ. in goods sector	0.10	0.10	**0.10**	**0.10**
goods occ. in goods sector	0.38	0.15	**0.38**	**0.15**
services occ. in service sector	0.41	0.68	**0.41**	0.66
goods occ. in service sector	0.12	0.07	0.12	0.08
Relative labor productivity of goods to services	1	2.8	1	2.2
price of goods to services	1	0.36	1	0.53
Nominal expenditure share of goods	47.2	25.3	60.8	36.1

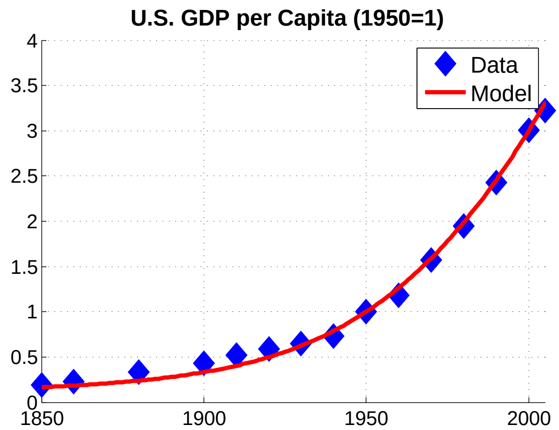
[a] Employment numbers are from population censuses. All other targets are standard parameter values or from the BEA.

does not closely match the nominal expenditure share of goods, which is not targeted. This is expected because homothetic utility function like our CES specification cannot capture that the income elasticities of broad sectors are not equal to one in the data; see for example Boppart (2014) and Comin et al. (2018). We nonetheless work with a homothetic CES utility function because it is analytically more tractable and performs well along the labor-allocation dimensions we are interested in.

5.2 Experiments

In this subsection, we use our model to conduct several experiments. First, we explore how well it accounts for the reallocation of occupation employment in cases that we have not targeted: the U.S. during 1850–1950 and our full sample of censuses from around the world. Then we explore how well it does if we calibrate it for subperiods and use it for out-of-sample predictions. Lastly, we document by how much the growth rates of sectoral labor productivity change

Figure 2: U.S. GDP per capita: Model versus Data



over time and compare it with the data.

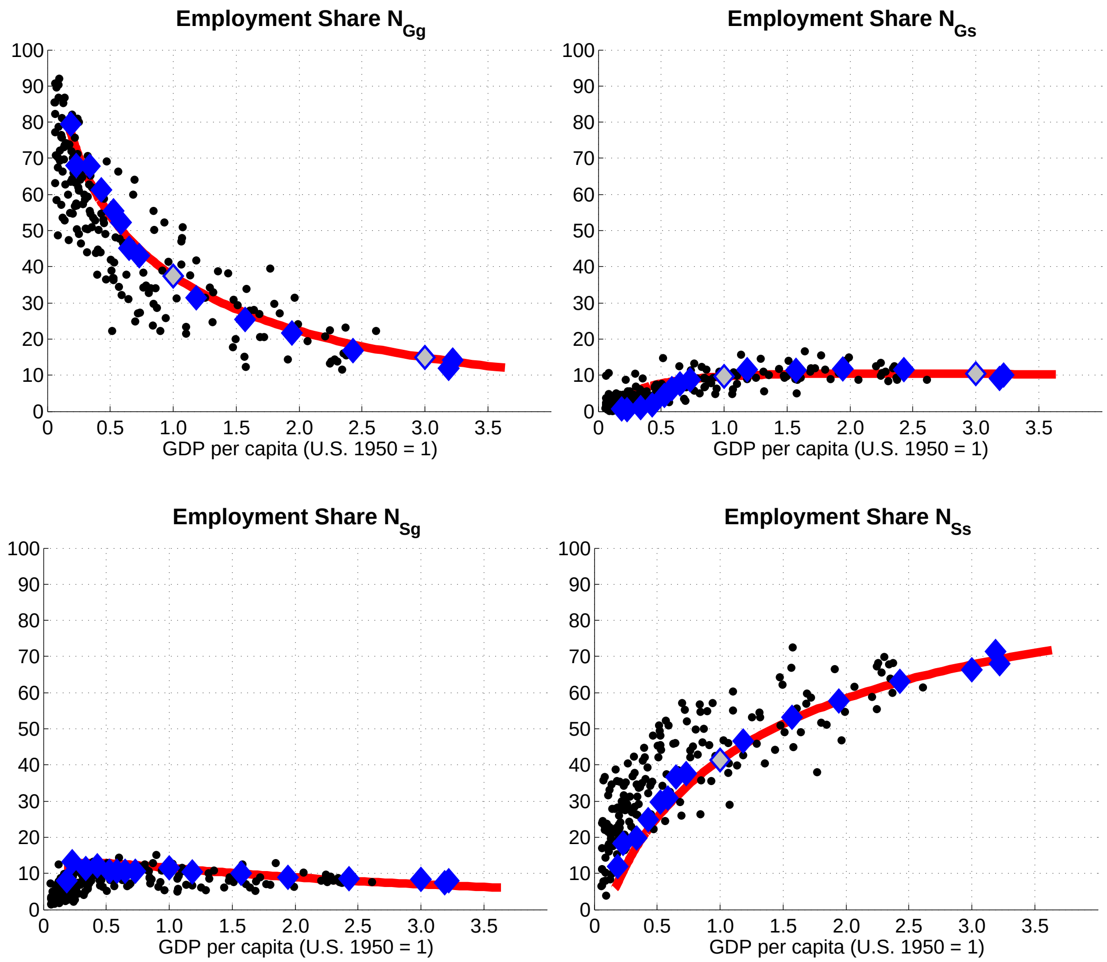
To obtain the model predictions for the U.S. during 1850–1950 and for our sample of censuses from around the world, we impose that technological progress grows at the constant average annual growth rates that we calibrated for the U.S. during 1950–2000, namely, $\Delta A_g/A_g = 6.1\%$ and $\Delta A_s/A_s = 1.5\%$. We then simulate the model over time while leaving all model parameters unchanged and report the labor allocations that it creates for each level of GDP er capita. We calculate the GDP per capita in the model in constant model prices, which we choose as the model prices from the U.S. in 1990.[16]

Figure 2 shows that the assumption of constant annual growth rates of A_{jt} yields a good time series fit of U.S. GDP per capita during 1850–2010. Figure 3 reports the relationship between GDP per capita and the shares of occupation employment by sector in total employment in the model and the data. We can see that our simple model captures the general trends in the evolution of the occupation shares for the U.S. during 1850–1950 as well as for our sample of censuses from around the world. We stress that this is not a foregone conclusion because we have imposed severe restrictions: the average growth rates of occupation-labor-augmenting technological progress that we calibrated from the U.S. during 1950–2000 also applies during 1840–1950 and to the sample of countries; the elasticity of substitution between goods- and service-occupation labor is the same in both sectors; there is no sector-specific, labor-augmenting technological progress, which features prominently in most models of structural transformation.

The previous results suggest that our model should do well at forecasting changes in the composition of broad industry and occupation categories. To establish that is the case, we

[16]The Groningen database calculates GDP in constant international dollars from 1990. These prices are not equal to those in the U.S., but since the U.S. has a large GDP weight, they are not far off either.

Figure 3: Structural Transformation of Occupation Employment: Model versus Data
(black dots are country-year data, blue diamonds are U.S. data, grey-blue diamonds are targets, red solid line is model)



compare forecasts from our model with actual data and with the forecasts published by the BLS for 1972–2014. We choose the BLS forecasts because they are among the most downloaded BLS statistics which are commonly used to form expectations about changes in the occupation composition.[17] While the BLS provides its forecasts at a fairly disaggregate level, one can aggregate them to the two broad occupation and industry categories that we have studied here. We focus on seven subperiods that have end dates five years apart; see Table 5.[18] To obtain model forecasts (abbreviated D-H below) for these subperiods, we calibrate our model to U.S. data from 1950 until the first year of the subperiod for which we forecast. We then simulate our model forward while keeping all parameters including the growth rates of occupation-labor-augmenting technological progress unchanged. We emphasize that this procedure implies that the calibration does not use data from the subperiod for which we wish to forecast.

Table 5 reports the results. We can see that our model does really well compared to the actual data. In fact, it actually outperforms the BLS forecasts except during the 1990s. This suggests that our model captures quantitatively important non-linearities of structural transformation that model-free forecasting techniques miss. While our model does not imply forecasts at the same disaggregate level as those provided by the BLS, we conjecture that the BLS could improve upon its disaggregate forecasts if it took the aggregate implications of our model into account.

Table 5: Data vs. Forecasts of Changes in Employment Shares of Goods Occupations and Goods Sector (in percentage points)

	72–85	**78–90**	**82–95**	**88–00**	**90–05**	**00–10**	**04–14**
	Change in employment share of goods occupations						
Data	-6.6	-6.2	-5.3	-3.2	-3.4	-3.7	-2.7
BLS forecast	-3.9	-1.8	-1.5	-3.0	-2.9	-1.0	-1.0
D-H forecast	-7.9	-6.6	-6.7	-5.7	-6.7	-3.8	-3.5
	Change in employment share of goods sector						
Data	-5.3	-5.8	-5.5	-4.0	-4.2	-4.1	-3.1
BLS	-3.4	-2.1	-0.4	-3.3	-4.2	-1.9	-2.4
D-H	-5.4	-4.6	-4.8	-4.2	-4.9	-3.1	-2.9

We conclude this subsection by returning to Proposition 3, in which we proved that at least one growth rate of sectoral labor productivity changes over time. The simplest way to assess the quantitative implications of our model in this regard is to regress the model-generated annual changes in sectoral labor productivity in constant prices on a constant and a linear time

[17] According to the BLS,"The Occupational Outlook Handbook is one of the nations most widely used sources of career information. It provides details on hundreds of occupations and is used by career counselors, students, parents, teachers, jobseekers, career changers, education and training officials, and researchers."

[18] The last subperiod is only four years long because we do not yet have data for 2015.

trend. This regression gives a time trend of −0.016 for the goods sector and −0.009 for the service sector. In comparison, for actual U.S. data 1950–2010 the coefficients are −0.027 for the goods sector and −0.014 for the service sector. All of these coefficient come out significant at the 5 or 1% level. These estimates suggest that our model offers a simple first step towards understanding why sectoral growth rates have declined. In a companion paper, Duernecker et al. (2019), we show that this has happened more widely across countries and make the connection to Baumol's Cost Disease, that is, the decline in aggregate growth that results when labor is reallocated to the service sector and service occupations, which both experience slower growth than the goods sector and the goods occupations.

6 Conclusion

We have used cross-country census data to show that structural transformation reflects a fundamental reallocation of labor from goods to services, and not mere relabelling of labor that occurs when firms in the goods sector outsource their in-house service production to the service sector. The key to this result is to measure labor at the level of occupations which are invariant to outsourcing. We have established two novel empirical facts that directly contradict that outsourcing is a mayor driver of structural transformation: as countries grow richer, the employment share of service occupations in total employment increases; the employment share of service occupations within each sector increase. To understand the forces behind these patterns, we propose a tractable model of structural transformation and show that uneven occupation-specific technological change is the key driver of the observed employment reallocation between occupations and sectors.

Our work is related to a large literature on labor-market polarization, which studied the fact that after 1980 employment and wages of low-wage and high-wage occupation increased relative to middle-wage occupation; see for example Autor et al. (2006), Autor and Dorn (2013), and Goos et al. (2014). Our model is consistent with employment polarization in that it implies an increase in the share of service occupations, which comprise most low-wage and high-wage occupations. Interestingly, several dimensions our work are broader than in the literature on labor-market polarization. To begin with, the literature focuses on non-farm employment of currently rich western countries in which agriculture plays a negligible role. In contrast, our sample of 67 countries covers many currently poor countries from Africa, Asia, and Latin America in which the agriculture sector is sizeable or even the largest sector. We have therefore included the agricultural sector in our analysis as part of the goods sector. With regards to the U.S., we find very similar reallocation patterns also for more than a century starting in 1850 when ICT technology advancements did not yet play a large role.[19]

[19]Note that our model has nothing to say about wage polarization because it assumes that wages are equalized

Bárány and Siegel (2018) and Lee and Shin (2015) also studied the relationship between labor-market polarization and structural transformation. The main difference between our work and Bárány and Siegel (2018) is that they focused on the interaction between sector-specific technological change and heterogenous individual abilities. In contrast, we focus on occupation-specific technological change when individual abilities are homogeneous, and we show that occupation-specific technological change is an important force behind structural transformation among broad occupation categories. The finding that occupation-specific technological change is an important driver of structural transformation links up nicely with the task-based approach to labor-market outcomes, which argues that different occupations perform different task bundles and that technological progress affects tasks differently. The main difference between our work and Lee and Shin (2015) is that they are focused on the U.S. and emphasized the role that managers play in the process of U.S. structural transformation. In contrast, we study a large sample of countries and treat all service occupations as one category that includes managers. An obvious and potentially fruitful extension of our model would be to disaggregate the service occupations into high-skilled and low-skilled service occupations. Managers would then be an important, albeit not the only, part of the high-skilled service category.

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Appendix A: Data

Census observations

Armenia (2011); Argentina (1970, 1980); Austria (1971, 1981, 1991, 2001); Bolivia (1976, 1992, 2001); Brazil (1960, 1970, 1980, 1990, 2000, 2010); Burkina Faso (1996); Cambodia (1998, 2008); Cameroon (2005); Canada (1971, 1981, 1991, 2001); Chile (1960, 1970, 1982, 1992, 2002); China (1982, 1990); Colombia (1964, 1973); Costa Rica (1973, 1984, 2000, 2011); Dominican Republic (1960, 1970, 1981); Ecuador (1962, 1982, 1990, 2001, 2010); Egypt (2006); El Salvador (1992); France (1962, 1968, 1975, 1982, 1990, 1999, 2006, 2011); West Germany (1970, 1987); Ghana (1984, 2000, 2010); Greece (1971, 1981, 1991, 2001); Guinea (1983); Haiti (1982, 2003); Hungary (2001); India (1983, 1987, 1993, 1999, 2004); Indonesia (1971, 1976, 1980, 1985, 1990, 1995); Iran (2006); Ireland (1971, 1981, 1991, 1996, 2002, 2006, 2011); Italy (1997); Jamaica (1982, 1991); Kyrgistan (2004); Liberia (2008); Malawi (1987, 1998, 2008); Malaysia (1970, 1980, 1991, 2000); Mali (1987, 1998, 2009); Mexico (1970, 1990, 1995, 2000, 2010); Mongolia (2000); Morocco (1982, 1994, 2004); Mozambique (1997, 2007); Netherlands (2001); Nicaragua (1971, 1995, 2006); Nigeria (2008, 2009, 2010); Pakistan (1973); Panama (1960, 1970, 1980, 1990, 2000, 2010); Paraguay (1962, 1972, 1982, 1992, 2002); Peru (1993, 2007); Philippines (1990); Portugal (1981, 1991, 2001, 2011); Puerto Rico (1990, 2000, 2005, 2010); Romania (1992, 2002); Rwanda (2002); Senegal (1998); Sierra Leone (2004); Slovenia (2002); South Africa (2007); Spain (1981, 1991, 2001); Sudan (2008); Switzerland (1970, 1980, 1990, 2000); Tanzania (2002); Turkey (1985, 1990, 2000); Uganda (2002); U.K. (1991, 2001); Uruguay (1963, 1996, 2006); U.S. (1960, 1970, 1980, 1990, 2000, 2010); Venezuela (1981, 1990, 2001); Vietnam (1999, 2009); Zambia (1990, 2000, 2010).

Elementary occupations

To determine the validity of our way of assigning elementary occupations to agriculture, industry, and service occupations, we use that U.S. census data have detailed occupation information that allows us to decompose the broad category elementary occupations into finer subcategories that we can assign to services and goods. The first two panels in Tables 6 and 7 compare the employment shares for the U.S. obtained from our approximation (upper panel) with the actual employment shares. By and large, the approximation is accurate and, more importantly, the trends in the data are fully preserved.

Table 6: Aggregate Employment Shares in the U.S. 1850–2000

Employment share of	1850	1900	1950	2000
	Baseline			
goods occupations	87.5	73.3	49.0	23.2
service occupations	12.5	26.7	51.0	76.8
	Re-classify elementary			
goods occupations	87.7	73.0	50.9	25.2
service occupations	12.3	27.0	49.1	74.8
	Re-classify all			
goods occupations	84.4	68.1	44.4	21.1
service occupations	15.6	31.9	55.6	78.9
service occupations re-classified as goods	0.0	0.0	0.0	0.0
goods occupations re-classified as services	3.1	4.6	6.5	4.0

Broad versus fine occupation groups

The information provided by IPUMS International about a worker's occupation is encoded in the variable OCCISCO. This variable captures 10 broad occupation groups. An important question is whether the classification that IPUMS applied to obtain the 10 groups is consistent with our classification of occupations into goods and services. For our purpose it is important that each of the 10 IPUMS groups are sufficiently homogenous and do not contain a mixture of service and goods occupations. We again use U.S. census data from IPUMS International to find out whether there are any occupations in the IPUMS goods occupation groups which are, in fact, service occupations and vice versa. The advantage of the U.S. censuses is that they come with a fine 3-digit occupation classification (in addition to broad IPUMS occupation classification), which allows us to determine the composition of each broad IPUMS occupation group.

Out of the 269 3-digit occupations in the U.S. censuses, we re-classify 25 occupations. All of them are service occupations that are part of a broad IPUMS group which we have classified as goods. The total employment share of the re-classified occupations is shown below in Table 8. The same table also shows the largest occupations (in terms of their 2000 employment share) which we re-classify. The lower panels in Tables 6 and 7 show the employment shares of goods and service occupations in the U.S. economy under the adjusted classification. As before, there are level effects but the trends are unaffected.

Table 7: Sectoral Employment Shares in the U.S. 1850–2000

Employment share of	Goods sector				Service sector			
	1850	1900	1950	2000	1850	1900	1950	2000
Baseline								
goods occupations	99.3	97.1	79.5	58.9	40.4	32.8	21.8	11.1
service occupations	0.7	2.9	20.5	41.1	59.6	67.2	78.2	88.9
Re-classify elementary occupations								
goods occupations	99.3	97.5	81.7	62.3	41.3	31.3	23.5	12.6
service occupations	0.7	2.5	18.3	37.7	58.7	68.7	76.5	87.4
Re-classify all								
goods occupations	99.3	97.0	77.7	58.5	24.7	19.1	14.7	8.5
service occupations	0.7	3.0	22.3	41.5	75.3	80.9	85.3	91.5

Table 8: Reclassified Occupations for the U.S. 1850–2000

	1850	1900	1950	2000
Truck and tractor drivers	0.70	1.90	2.41	2.36
Linemen and servicemen	0.00	0.05	0.39	0.39
Bus drivers	0.08	0.02	0.29	0.36
Laundry and dry cleaning operatives	0.00	0.35	0.75	0.22
Taxicab drivers and chauffeurs	0.08	0.23	0.36	0.18
Stationary engineers	0.00	0.13	0.38	0.15
Stationary firemen	0.01	0.09	0.22	0.04
Locomotive engineers	0.01	0.25	0.13	0.04
Deliverymen and routemen	0.02	0.18	0.42	0.04
Power station operators	0.00	0.00	0.04	0.03
Sailors and deck hands	1.54	0.21	0.08	0.02
Rest	0.66	1.19	1.02	0.22
Total	3.08	4.59	6.50	4.04

7 Appendix B: Proofs

First-Order Conditions to the Firm Problem

- We need to show (9)–(13).
- We drop the time indexes when this does not cause confusion.

- The first-order conditions to problem (8) are:

$$r = \theta p_J K_J^{\theta-1} L_J^{1-\theta} = \theta p_J \left(\frac{K_J}{N_J}\right)^{\theta-1} \left(\frac{L_J}{N_J}\right)^{1-\theta} \tag{16}$$

$$w = (1-\theta) p_J K_J^{\theta} L_J^{-\theta} L_J^{\frac{1}{\sigma}} \alpha_J^{\frac{1}{\sigma}} A_g^{\frac{\sigma-1}{\sigma}} N_{Jg}^{-\frac{1}{\sigma}} \tag{17}$$

$$w = (1-\theta) p_J K_J^{\theta} L_J^{-\theta} L_J^{\frac{1}{\sigma}} (1-\alpha_J)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Js}^{-\frac{1}{\sigma}} \tag{18}$$

- (12) follows by dividing (17) and (18) by each other.
- Multiplying (17)–(18) with the respective labor and adding up gives that:

$$w = (1-\theta) p_J \left(\frac{K_J}{N_J}\right)^{\theta} \left(\frac{L_J}{N_J}\right)^{1-\theta} \tag{19}$$

- Dividing (19) by (16), we find:

$$\frac{w}{r} = \frac{1-\theta}{\theta} \frac{K_J}{N_J} \tag{20}$$

- Hence,

$$\frac{K_G}{N_G} = \frac{K_S}{N_S} \tag{21}$$

  which was (9).
- (19) and (21) imply (10) and (11):

$$\frac{Y_G/N_G}{Y_S/N_S} = \left(\frac{L_G/N_G}{L_S/N_S}\right)^{1-\theta}$$
$$\frac{Y_G/N_G}{Y_S/N_S} = \frac{p_S}{p_G}$$

- It remains to show (13). Using (18) for $J = G, S$, we obtain:

$$p_G K_G^{\theta} L_G^{-\theta} L_G^{\frac{1}{\sigma}} (1-\alpha_G)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Gs}^{-\frac{1}{\sigma}} = p_S K_S^{\theta} L_S^{-\theta} L_S^{\frac{1}{\sigma}} (1-\alpha_S)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Ss}^{-\frac{1}{\sigma}}$$

  Using (11) and (21), this can be simplified to (13):

$$\frac{N_{Ss}}{N_{Gs}} = \frac{1-\alpha_S}{1-\alpha_G} \left(\frac{L_S}{L_G}\right)^{1-\sigma} \left(\frac{N_S}{N_G}\right)^{\sigma}$$

- QED

Proof of Proposition 1

- Need to show that there is a unique GBGP.
- (7) implies that $r_t = r = A_x$.
- (4) implies that
$$\frac{C_{t+1}p_{t+1}}{C_t p_t} = \gamma \equiv \beta(1 + A_X - \delta)$$
- $w_t = w_t N_t = (1-\theta) C_t p_t$ implies that w_t grows at the same factor as $C_t p_t$, i.e., γ.
- $r_t K_{Ct} = A_X K_{Ct} = \theta C_t p_t$ implies that K_{Ct} grows at the same factor as $C_t p_t$, i.e., γ.
- The consumer budget constraint can be rewritten to:
$$\frac{\theta C_t p_t}{K_t} + \frac{K_{t+1}}{K_t} = 1 + A_x - \delta$$
Hence, if K_t grows at a constant factor, then that factor must be γ.
- $K_t = K_{Xt} + K_{Ct}$ implies that K_{Xt} grows at factor γ too.
- QED

Proof of Proposition 2

The goods (service) sector is more intensive in goods (service) occupations if and only if Assumption (i) holds. This leaves to show that Assumptions (i)–(iv) imply claims 1.–6. and Claims 1.–6. imply assumptions (ii)–(iv).

Proof of "$\Longrightarrow$"

Claim 1.

- That N_{Js}/N_{Jg} increases follows directly from (12) and Assumptions (ii) and (iv).
- QED

Claim 6.

- We need to show that N_S/N_G increases.
- (12) implies:
$$N_J = N_{Jg} + N_{Js} = \left[\frac{N_{Jg}}{N_{Js}} + 1\right] N_{Js} = \left[\frac{\alpha_J}{1-\alpha_J}\left(\frac{A_g}{A_s}\right)^{\sigma-1} + 1\right] N_{Js}$$

- Hence, the ratio of sectoral labor satisfies:

$$\frac{N_S}{N_G} = \left[\frac{1 + \left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1 + \left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right] \frac{N_{Ss}}{N_{Gs}} \tag{22}$$

- Combining (6) and (11) and rearranging gives:

$$\frac{p_S}{p_G} = \left(\frac{\alpha_U}{1-\alpha_U}\frac{N_S}{N_G}\right)^{\frac{1}{1-\varepsilon}}$$

  Substituting this into (13), we obtain

$$\frac{N_{Ss}}{N_{Gs}} = \frac{1-\alpha_S}{1-\alpha_G}\left(\frac{1-\alpha_U}{\alpha_U}\right)^{\frac{1-\sigma}{(1-\varepsilon)(1-\theta)}}\left(\frac{N_S}{N_G}\right)^{1-\frac{1-\sigma}{(1-\varepsilon)(1-\theta)}}$$

- Substituting this into (22) gives:

$$\frac{N_S}{N_G} = \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}} \frac{1-\alpha_U}{\alpha_U}\left[\frac{1 + \left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1 + \left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}} \tag{23}$$

- Define

$$f(x) \equiv \left[\frac{1+\tilde{\alpha}_S x^{\sigma-1}}{1+\tilde{\alpha}_G x^{\sigma-1}}\right]^{\frac{1}{\sigma-1}}$$

  where $\tilde{\alpha}_J \equiv \alpha_J/(1-\alpha_J)$.
- It is straightforward to show that given Assumption (i) we have $f'(x) < 0$.
- Claim 6. now follows from (23), $f'(x) < 0$, and Assumption (iii) and (iv).
- QED

Claim 5.

- N_s/N_g increases.
- To see this, note that:

$$N_s = N_{Gs} + N_{Ss} = N_G\frac{N_{Gs}}{N_G} + N_S\frac{N_{Ss}}{N_S}$$

- Hence,

$$\Delta N_s = \Delta N_G \frac{N_{Gs}}{N_G} + N_G \, \Delta \frac{N_{Gs}}{N_G} + \Delta N_S \frac{N_{Ss}}{N_S} + N_S \, \Delta \frac{N_{Ss}}{N_S}$$

- Using that $N_S = 1 - N_G$, this becomes:

$$\Delta N_s = N_G \Delta \frac{N_{Gs}}{N_G} + N_S \Delta \frac{N_{Ss}}{N_S} + \Delta N_S \left(\frac{N_{Ss}}{N_S} - \frac{N_{Gs}}{N_G} \right)$$

- Claim 6. implies $\Delta N_S > 0$; Claim 1. implies $\Delta N_{Js}/N_J > 0$;
- Hence, the right-hand side is positive and N_s grows.
- Since $N_g = 1 - N_s$, this implies that N_g falls.
- QED

Claim 4.

- We need to show that $(p_S Y_S)/(p_G Y_G)$ grows.
- To see this, note that (11) implies that

$$\frac{Y_S \, p_S}{Y_G p_G} = \frac{N_S}{N_G} \tag{24}$$

- Claim 4. therefore follows from Claim 5., which we have just proved.
- QED

Claims 2. and 3.

- We need to show that p_S / p_G increases and $(Y_S / N_S)/(Y_G / N_G)$ decreases.
- (11) implies that either one of these statements is true iff the other one is true.
- We therefore only show that $(Y_S / N_S)/(Y_G / N_G)$ decreases.
- (10) implies that this is equivalent to showing that $(L_S / N_S)/(L_G / N_G)$ decreases.

- To see that $(L_S/N_S)/(L_G/N_G)$ decreases, rewrite (3) while using (12):

$$\begin{aligned}\frac{L_S}{L_G} &= \left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]^{\frac{1}{\sigma}}\left[(A_gN_{Sg})/(A_sN_{Ss})\right]^{\frac{\sigma-1}{\sigma}}}{1+\left[\alpha_G/(1-\alpha_G)\right]^{\frac{1}{\sigma}}\left[(A_gN_{Gg})/(A_sN_{Gs})\right]^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\frac{N_{Ss}}{N_{Gs}} \\ &= \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1+\left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}}\frac{N_{Ss}}{N_{Gs}}\end{aligned} \tag{25}$$

- Dividing this by (22) gives:

$$\frac{L_S/N_S}{L_G/N_G} = \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1+\left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{1}{\sigma-1}} \tag{26}$$

- Using Assumption (iv) and that $f'(x) < 0$, it follows that $(L_S/N_S)/(L_G/N_G)$ decreases.
- QED

Proof of "$\Longleftarrow$"

Recall that (i) is equivalent to the goods (service) sector being more intensive in goods occupations. Thus, we only need to show that 1.–6. imply (ii)–(iv).

- Assumption (iii): follows from (6), 3., and 4.
- Assumption (iv): (23) along with (i) and (iii) imply that if 5. holds then $(A_g/A_s)^{1-\sigma}$ must increase. (26), (i), and that $(L_S/N_S)/(L_G/N_G)$ decreases then imply that $\sigma < 1$.
- Assumption (ii): (12), (iv), and 1. imply (ii).
- QED

Proof of Proposition 3

We need to show that at least one of the two growth rates

$$\Delta \log(LP_{Jt}) \equiv \log\left(\frac{Y_{Jt}}{N_{Jt}}\right) - \log\left(\frac{Y_{Jt-1}}{N_{Jt-1}}\right)$$

changes over time. We are going to achieve this by showing that their difference changes over time. The difference of the two growth rates is given by:

$$\begin{aligned}
&\Delta \log(LP_{St}) - \Delta \log(LP_{Gt}) \\
&= \log\left(\frac{Y_{St}/N_{St}}{Y_{Gt}/N_{Gt}}\right) - \log\left(\frac{Y_{St-1}/N_{St-1}}{Y_{Gt-1}/N_{Gt-1}}\right) \\
&= \log\left(\frac{K_{St}/N_{St}}{K_{Gt}/N_{Gt}}\right)^{\theta}\left(\frac{L_{St}/N_{St}}{L_{Gt}/N_{Gt}}\right)^{1-\theta} - \log\left(\frac{K_{St-1}/N_{St-1}}{K_{Gt-1}/N_{Gt-1}}\right)^{\theta}\left(\frac{L_{St-1}/N_{St-1}}{L_{Gt-1}/N_{Gt-1}}\right)^{1-\theta} \\
&= (1-\theta)\left[\log\left(\frac{L_{St}/N_{St}}{L_{Gt}/N_{Gt}}\right) - \log\left(\frac{L_{St-1}/N_{St-1}}{L_{Gt-1}/N_{Gt-1}}\right)\right]
\end{aligned}$$

where we have used that in each period the capital-labor ratio is equalized across sectors. Recalling (35), we can rewrite the difference of the growth rates:

$$\begin{aligned}
&\Delta \log(LP_{St}) - \Delta \log(LP_{Gt}) \\
&= \frac{1-\theta}{\sigma-1}\left[\log\left(\frac{1+\frac{\alpha_S}{1-\alpha_S}\left(\frac{A_{gt}}{A_{st}}\right)^{\sigma-1}}{1+\frac{\alpha_G}{1-\alpha_G}\left(\frac{A_{gt}}{A_{st}}\right)^{\sigma-1}}\right) - \log\left(\frac{1+\frac{\alpha_S}{1-\alpha_S}\left(\frac{A_{gt-1}}{A_{st-1}}\right)^{\sigma-1}}{1+\frac{\alpha_G}{1-\alpha_G}\left(\frac{A_{gt-1}}{A_{st-1}}\right)^{\sigma-1}}\right)\right] \\
&= \frac{1-\theta}{\sigma-1}\left[\log\left(1+a_S\gamma^t\right) - \log\left(1+a_G\gamma^t\right) - \log\left(1+a_S\gamma^{t-1}\right) + \log\left(1+a_G\gamma^{t-1}\right)\right]
\end{aligned}$$

where $a_J \equiv (A_{g0}/A_{s0})^{\sigma-1}\alpha_J/(1-\alpha_J)$ and $\gamma \equiv (\gamma_g/\gamma_s)^{\sigma-1}$. After taking time derivatives and doing some tedious algebra, we obtain:

$$\begin{aligned}
&\frac{\partial}{\partial t}[\Delta \log(LP_{St}) - \Delta \log(LP_{Gt})] \\
&= \frac{\gamma^{t-1}\log(\gamma)(1-\theta)(1-\gamma)(a_G-a_S)}{1-\sigma}\,\frac{a_G a_S\gamma^{2t-1}-1}{(1+a_S\gamma^t)(1+a_G\gamma^t)(1+a_S\gamma^{t-1})(1+a_G\gamma^{t-1})}
\end{aligned}$$

Since $\gamma < 1$, the first ratio is negative and converges to zero. Since $\gamma < 1$, γ^{2t-1} is monotonically decreasing towards zero, the second ratio becomes negative to the right of a finite threshold value of t. Hence, the overall derivative is positive from that threshold value onwards. Going from $t-1$ to t to the right of the threshold value, the growth rate difference at t must be larger than the growth rate difference at $t-1$ to the right of the threshold value. Since the growth rate difference goes to zero that must mean that the growth rate difference is negative all the time while becoming less and less negative, approaching zero from below. QED

Appendix C: Calibration

The elasticity between goods and services ϵ is not identified from the employment shares. The calibration of ϵ requires information on real expenditure shares or the relative price of services.

We set $\epsilon = 0.05$, that is, goods and services enter as strong complements in the consumption basket. We normalize the initial relative technologies and set $A_g(0)/A_s(0) = 1$. In the next step, we calibrate α_G, α_S and α_U so that the model matches the employment shares $N_{Gg}(0) = 0.375$, $N_{Gs}(0) = 0.097$, $N_{Ss}(0) = 0.413$. We obtain: $\alpha_G = 0.795$, $\alpha_S = 0.218$, $\alpha_U = 0.472$. The model matches the data targets exactly.

Next, we calibrate the remaining parameters $A_g/A_s(1)$ and σ to match two targets: $N_{Gg}(1) = 0.149$, $N_{Gs}(1) = 0.104$. The model can match the targets exactly and we obtain: $A_g(1)/A_s(1) = 9.42$ and $\sigma = 0.557$.

Next, we calibrate the level of occupation-specific technology. First, we set $A_s(0) = 1$. Since, $A_g(0)/A_s(0) = 1$, we get that $A_g(0) = 1$. Then, we set $A_s(1)$ so that the increase in real GDP per capita between periods 0 and 1 that is implied by the model matches the increase in the data. In the data, GDP per capita has increased by a factor 3. In the model, we can derive neither the level of nor the change in real GDP without knowing the level of relative prices (p_G, p_S) and quantities (C_G, C_S, X). We proceed as follows to compute the levels:

- First, we normalize the aggregate capital stock in period 0 to 1 ($K(0) = 1$). Since the model is AK, this normalization does not result in a loss of generality.
- Given $K(0)$, we can use K_C/K from above to compute $K_C(0)$. Next, we use K_G/K_S from above and $K_C = K_G + K_S$ to compute $K_G(0)$ and $K_S(0)$. Since $K_X = X/A_X = K - K_C$, we also get $X(0)$.
- Using the period-0 labor allocation, the technology level $A_g(0) = A_s(0) = 1$ and initial capital $K_G(0), K_S(0)$, we can compute $C_G(0)$ and $C_S(0)$ from the sectoral production functions.
- Moreover, using $rK_G(0) = \theta p_G(0)C_G(0)$ and $rK_S(0) = \theta p_S(0)C_S(0)$, we compute $p_G(0)$ and $p_S(0)$.

Now we have everything to compute real GDP in period 0:

$$Y(0) = p_G(0)C_G(0) + p_S(0)C_S(0) + X(0)$$

Real U.S. GDP per capita increased by a factor 3 between 1950 and 2005. We know that the aggregate capital stock K grows at an annual rate equal to γ. Hence after 50 years, the capital stock is $K(1) = \gamma^{50}K(0)$. To derive γ, we assume that $\beta = 0.96$, $\delta = 0.05$ and $A_X = 0.10$. The implied net real interest rate is $A_X - \delta = 0.05$ and $\gamma = 1.008$. Moreover, we calibrate θ so that the capital share in the model is equal to 1/3. The implied $\theta = 0.17$. Once we have computed $K(1)$, we follow the same steps as above to obtain $C_G(1), C_S(1)$ and $X(1)$ and real per-capita GDP:

$$Y(1) = p_G(0)C_G(1) + p_S(0)C_S(1) + X(1)$$

Finally, we search for the value of $A_s(1)$ so that $Y(1)/Y(0) = 3$.