CESifo WORKING PAPERS

9346 2021

October 2021

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Didier Laussel, Ngo Van Long, Joana Resende

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
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CESifo *Working Paper No. 9346*

Dynamic Monopoly and Consumers Profiling Accuracy

Abstract

Using a Markov-perfect equilibrium model, we show that the use of customer data to practice intertemporal price discrimination will improve monopoly profit if and only if information precision is higher than a certain threshold level. This U-shaped relationship lends support to a popular view that knowledge is good only if it is sufficiently refined. When information accuracy can only be achieved through costly investment, we find that investing in profiling is profitable only if this allows to reach a high enough level of information precision. Consumers expected surplus being a hump-shaped function of information accuracy, we show that consumers have an incentive to lobby for privacy protection legislation which raises the cost of monopoly's investment in information accuracy. However, this cost should not dissuade firms to collect some information on customers' tastes, as the absence of consumers' profiling is actually detrimental to consumers.

JEL-Codes: C730, D420, L120, L150.

Keywords: consumers profiling, endogenous investment in profiling capability, dynamic monopoly, consumers‘ collective action on privacy protection legislation.

Didier Laussel
Aix-Marseille University / France
didier.laussel@outlook.fr

Ngo Van Long
Department of Economics
McGill University / Montréal / Canada
ngo.long@mcgill.ca

Joana Resende
CEF.UP
University of Porto / Portugal
jresende@fep.up.pt

This work was supported by the French National Research Agency Grant ANR-17-EURE-0020, and by the Excellence Initiative of Aix-Marseille University - A*MIDEX.
This research has been financed by projects NORTE-01-0145-FEDER-028540 and POCI-01-0145-FEDER-006890.

1 Introduction

Recent advances in digital technologies have enabled firms to collect, store and process "big data" in order to gain information on customers and thus enhance their profits by treating different types of customers differently.[1] The collection of huge amounts of consumer-specific information together with the use of complex algorithms may partly or even totally reveal individual customers' preferences, allowing firms to engage in increasingly complex price discrimination strategies. As an illustrative instance of this reality, we may cite Uber's route-based pricing which tries to implement a new fare system that charges customers what it predicts they are willing to pay.[2]

While in some cases, firms will only get coarse information on customers' characteristics[3] (for instance, the date when she first bought the good or service), in other cases firms have access to an impressive set of information[4], which may include customers' web browsing history, demographics, real time location, online purchasing/ consumption habits, social network activities and connections, and so on as illustrated with the Uber example above, sometimes information becomes so precise that firms are able to get precise estimates of consumers' willingness to pay for the commodity.[5]

As a matter of fact, the information that firms are able to collect on consumers may be more or less fine depending on a wide variety of factors, such as (i) the state of the information technologies, (ii) the amount the firms invest in their consumers' profiling capability (e.g., buying the hardware and software and/or hiring data analysts), and (iii) the state of legislation on consumers data protection (such as the EU General Data Protection Regulation (GDPR) of 2016 or the California Consumer Privacy Act of 2018), which may itself be influenced by consumers organizations' lobbying activities.

In all cases, taking advantage of a more or less perfect "consumer addressability" (Blattberg and Deighton, 1991), firms may be able to engage in sophisticated price discrimination/ market segmentation strate-

[1] As defined in the Cambridge Dictionary, big data is "very large sets of data that are produced by people using the internet, and that can only be stored, understood, and used with the help of special tools and methods".

[2] "Uber Starts Charging What It Thinks You're Willing to Pay", https://www.bloomberg.com/news/articles/2017-05-19/uber-s-future-may-rely-on-predicting-how-much-you-re-willing-to-pay

[3] This does not require very sophisticated or very costly means of information collection, storage and treatment and may only need the firm to keep a record by hand.

[4] With the use of tracking tools such as cookies, web beacons, or Etags.

[5] This requires the use of digital technologies and big data.

gies. More precisely, firms may target pricing, marketing and even product characteristics to groups of consumers or even to individual ones. In this paper, we identify conditions under which firms and consumers may benefit or, on the contrary, are hurt by the firms' capability to collect information on their customers and by the precision of such information. These variables critically determine the scope of firms' market segmentation strategies, which may vary from no segmentation at all (when firms do not collect any information on their customers) to hyper market segmentation (where each individual customer emerges as an independent market segment, allowing for personalized interaction with the firm). In between, we may find a wide diversity of cases (e.g. in some circumstances firms are only able to recognize the consumers' identity without getting any specific information on their preferences; in other cases firms may be able to reckon the preferences of a subset of customers only, etc.). For the sake of tractability, the degree of precision of information is defined here, conditional on the fact that a given customer's exact willingness to pay (WTP) has not been uncovered yet, by the probability that it will be discovered over the next short time interval.[6]

Identifying the conditions under which firms and consumers may benefit (or be hurt) by their capability to gather and make use of increasingly accurate customer information is a prerequisite to the analysis of the profitability of firms investment in profiling and of the motivation and effects of consumers organizations' lobbying activities to influence the scope of data protection policies. These two questions will also be investigated in this paper within our game theoretic framework.

The key features of our model are as follows. A monopolist produces a non-durable homogenous good (or service) which must be consumed instantaneously. Consumers are heterogeneous in terms of their WTPs for the good, which are initially private information. The monopolist makes pricing decisions at discrete points in time. We refer to the time interval between two consecutive price offers as the contractual commitment period. As in Laussel, Long and Resende (2020a), we assume that the present monopolist cannot prevent his future selves from exploiting the customers information that they will then have at hand. Our modelling

[6]The polar case where this probability is zero corresponds to the main model analyzed by Laussel, Long and Resende (2020a), who specified that the monopolist can never discover a consumer's exact WTP, and thus can only classify customers according to their first date of purchase, i.e., customers with different WTPs who make their first purchase on the same date are bunched together. The other polar case obtains under perfect information acquisition: as soon as a customer has made her first purchase, her exact WTP is instantaneously discovered through the analysis of big data.

framework constitutes a very stylized representation of firm's consumer profiling decisions within a dynamic setting. Admittedly, we propose a very stylized learning process on consumers' behavior as it allows us to identify different economic mechanisms shaping the monopolist's pricing and market expansion strategy (within a tractable setting). Despite the simplifying nature of our assumptions, they allow us to show that the abovementioned economic mechanisms may go in opposite directions, allowing us to make the point on the ambiguous profit and consumer welfare effects resulting from changes in firms' customer profiling accuracy. Indeed, even in a simplifying setting like ours, it is unclear that firms/ consumers always benefit/lose from improvements in firms' ability to recognize the individual preferences of their customers

The timing of the game we study is as follows. In the first stage of the game, the consumers collectively lobby to influence the toughness of privacy protection legislation in order to undermine the efficiency of firms' investment in profiling. In the second stage, the monopolist considers whether to invest or not in its big data capability, and, if yes, solves for the optimal amount it invests in profiling accuracy, thus endogenously determining the degree of information precision. The third stage of the game is an infinite-horizon dynamic game between the (partially) informed monopolist and the consumers, in which the former chooses in each period the prices offered to new and old customers while consumers decide, based on their type and their rational expectations of future prices, when to make their first purchase.

We solve the game by backwards induction. Thus, we first demonstrate that, in stage three, monopoly profit is a U-shaped function of the degree of information precision while the expected consumer's surplus is a hump-shaped function of it. These shapes follow from two conflicting effects of consumers profiling on firms' profits. On the one hand, there is a positive direct effect since, other things equal (i.e., keeping unchanged the firm's future strategies), intra-period (finer) price discrimination allowed by more accurate consumers' profiling would increase profits. On the other hand, a negative strategic effect arises as having more information on customers leads to lower introductory prices (targeted to new customers) and lower profits. Such downward pressure on firms' introductory prices tends to be beneficial to the consumers. However, increased precision of firm's information on consumer preferences also enables firms to extract more consumer surplus in future purchases, leading to the consumers' detrimental effect described above. This allows us to make the point that an increase in the precision of firms' customers profiling has ambiguous profit and consumer surplus effects and therefore consumers (firms) may not necessarily gain (lose) from tighter data protection regulation.

Indeed, when we look at the second-stage of our game, we find that the firm's investment must exceed a critical level amount in order for information collection to be worthwhile. This implies that a monopolist should never spend just enough money to obtain very coarse information on rational customers. Moreover, it may not pay off to invest in fully accurate customer profiling technologies, with firms optimally choosing not to disclose the WTP of all customers when this becomes excessively costly.

Finally, when looking at the first stage of the game, we show that, from the consumers' perspectives, the best protective legislation always should reduce firms' investment in profiling but should not always preclude it. When consumers are able to influence firms' consumer profiling ability (e.g., through tighter or more relaxed data protection rules) and they strategically anticipate firms' optimal investment decisions in the subsequent stage, consumers tend to favor milder data protection regulation: on the one hand, the total prohibition of data collection would actually hurt them (as it would eliminate pricing pressure from the monopolist's future selves); on the other hand, it is in their interest to keep a sufficiently large fraction of returning customers untraceable (to avoid the detrimental effects associated with the use of such information to engage in first-degree price discrimination over returning customers).

In order to clarify these effects, let us elaborate on the third stage interaction between the firm and the customers, taking as given the degree of precision of customer profiling. Given that the monopolist has chosen to collect and process customer information, in each of the future periods he classifies returning customers according to their first period of purchase ("their vintage"). Moreover, given any vintage n of former customers, the monopolist can identify at each point of time the exact WTP of *a fraction* of them. The remaining fraction corresponds to untraceable customers (in the sense that the firm recognizes them as returning consumers but has no information about their preferences). The fraction of identified customers depends on the degree of accuracy of the monopolist's customer profile. Moreover, for any given vintage, the size of the unidentified fraction gets smaller and smaller over time, thanks to the firm's perseverance in collecting and processing data.[7] Accordingly, the monopolist is able to engage in different types of price discrimination

[7] While there is inherent randomness concerning the time of discovery of the WTP of any given former customer, in the aggregate, as there is a continuum of customers, one can suppose that on average, the fraction of unidentified customers of any given vintage declines at a constant exponential rate, which in our model will be denoted by μ (please see Section 3 for more details).

among returning customers. Those include: (i) third-degree price discrimination with respect to the former untraceable customers, and (ii) first-degree price discrimination with respect to "identified customers" (the ones whose exact WTP have been discovered). At the beginning of any given period n, the firm faces n groups of former customers and decides on the size of the current period's new customers (i.e., the new market segment, denoted as the $(n+1)th$ market segment) by offering them an attractive "introductory offer" price.

The size of the new market segment is determined by the attractiveness of the new introductory price, which influences the decision of each potential new customer on whether to make her first purchase in that period, or delay it until the next period. Her decision depends on her expectations regarding next period's introductory price and the price she would have to face in the future as a former customer. The firm is fully aware of this. Their strategic interactions lead to the dynamic game played between firm and customers in the third stage. We characterize the Markov Perfect Equilibrium (MPE) of this stage game for any given value of the information precision parameters (which is endogenously chosen in previous stages). The monopolist's equilibrium profit turns out to be U-shaped in the degree of precision of his information about former customers. We show that if the firm's precision of information is high enough such that the rate of decline of the unidentified fraction exceeds a threshold level, its profit will be higher than what is obtained under the no information benchmark (where the static equilibrium is repeated indefinitely and therefore the monopolist avoids profit losses due to price competition from his future selves). Beyond this threshold, greater precision leads to higher profit. At the same time, we also find that the aggregate consumers surplus is a non-monotone, hump-shaped, function of the degree of precision of the firm's information about former customers, evolving in a way opposite to profits.[8]

Our analysis also unveils that whereas it is true that eventually the whole market is covered (in sharp contrast with the static equilibrium outcome), there is an overall positive Coasian correlation between the length of the commitment period and aggregate profit only when the firm's precision of information is very small, the reverse occurring when this precision is very high. Intuitively, a shortening of the commitment interval generates two opposite effects, whose relative importance depends on the degree of precision of the information. The first one is the well-known Coasian mechanism: the potentially new consumers, knowing that the introductory price will go down very quickly, have an incentive to delay their first purchase, triggering an even quicker price decrease for

[8] In particular, it takes its maximum value when profits take their minimum value.

new customers. The second, which goes the other way, is a "front loading effect"[9]: when the commitment period is shorter, the monopolist is able to extract sooner the surplus of those former customers whose exact WTP are discovered. The second effect globally dominates the first one if the rate of decline of the fraction of untraceable returning customers is sufficiently high, namely when it is greater than the rate of interest. The reverse occurs, and the dynamics are globally Coasian, in the opposite case. Looking at the limit value of the aggregate profit when the length of the commitment period becomes infinitesimal, we show that it is equal to zero if and only if the monopolist's information is the coarsest one and that, more generally, in this limiting case, aggregate profit is increasing in the degree of precision of the information.

Having understood the main findings concerning the stage-three equilibrium, the results for stage two and stage one of the game are intuitively plausible. In the second stage, the firm chooses, given the state of the legislation on consumers data protection, whether to invest or not in profiling and, when investing, the amount to invest (or, equivalently, the degree of accuracy of information the firm is able to collect and manipulate to engage in intra-temporal price discrimination). The outcome follows from the shape of the (third stage) profit function with respect to the monopolist's level of information and from the parameters of the investment function. Were information collection costless, the firm would always choose to learn perfectly the preferences of its old customers. With costly investment in information collection, it never chooses an investment level that would result only in a "coarse" information level, since not investing at all would then be more profitable. In the very first stage of the game, what is the best protective legislation from the consumers' point of view? This follows from the shape of the consumers surplus function with respect to the degree of information precision (which is optimally chosen by the firm in the second stage). The unconstrained maximum for CS obtains when the monopolist invests just enough to collect the coarsest possible information. This unconstrained maximum is however unattainable since the firm would always avoid that level of investment (and strategic consumers rationally anticipate this outcome). We show that the best legislation from the consumers' point of view does not necessarily preclude investments in profiling: when the cost of acquiring the coarsest information is low (which is a rather likely scenario), the monopolist will collect information beyond the coarsest level, but a less precise one than without any legislation.

[9] **We are grateful to an anonymous referee for suggesting us this expression.**

The rest of the paper is organized as follows: Section 2 presents an overview of the related literature. Section 3 introduces the model and the timing of the game, which is solved by backwards induction. Section 4 analyzes the third-stage of the game, investigating firm's price discrimination strategies for a given information precision degree. Section 5 characterizes the MPE of the dynamic game played in the third-stage. Section 6 presents the profit and consumers surplus effects of exogenous changes in firms' profiling ability. Section 7 endogenizes firms' decisions on profiling accuracy by looking at the monopolist's optimal investment decision in the second-stage of the game. Section 8 deals with the first stage of the game, allowing consumers to strategically influence firms' optimal profiling decisions. Finally, Section 9 concludes.

2 Related Literature

There is a stream of literature, developed over the past two or three decades, that attempts to assess whether, in the context of oligopolistic competition, better information on customers leads to higher profits for firms. The answer is not obvious, because while better information enables firms to practice price discrimination among their customers, competition among firms becomes fiercer.[10] The early papers focus on the case of a duopoly where information acquisition is exogenous. Corts (1998) for instance focused on the comparison between no discrimination and third-degree price discrimination, without assuming the Hotelling spatial structure. He found that, even under best response asymmetry[11], either both prices decrease ("all-out competition") or they both increase ("all-out price gouging") under price discrimination, depending on the relative importance of the two markets, and thus profits may increase or decrease. Thisse and Vives (1988) compared, within the Hotelling spatial model, the outcomes under no discrimination ("mill prices") and under first-degree price discrimination ("delivered prices"), concluding that equilibrium profits are smaller under the latter.[12] Liu and Serfes (2004) also used the Hotelling framework with exogenous information acquisition, but made the quality of information a continuous variable, with two-groups and perfect discrimination being the two extreme situations. They concluded that "equilibrium profits exhibit a U-shaped relationship with respect to information quality" (p. 671).

[10] The direct effect on profits (i.e; for given rivals' prices) is always positive while the strategic one is negative due to the increased competition.

[11] This is the case where the strong market of one firm is the weak market of the other.

[12] Bearing in mind that the Hotelling model satisfies best-response asymmetry assumed in Corts (1998), it appears that the robustness of this result is not clear.

A more recent strand of literature has considered endogenous information acquisition when firms are able to learn about consumers' purchase history and/or their personal tastes. This behavior-based price discrimination literature (e.g., Fudenberg and Tirole, 2000; Chen, 1997; Choe et al., 2018)[13] shares with our model the feature that firms learn about consumers' preferences by accessing data on their purchase history and use this information to practice price discrimination (see Fudenberg and Villas-Boas, 2006, and Stole, 2007, for surveys). Following the seminal works by Fudenberg and Tirole (2000) and Chen (1997), a number of authors have investigated the competitive effects of behavior-based price discrimination. See, for example, Chen and Pearcy (2010), Esteves (2009, 2010), Gherig et al. (2011, 2012), and Villas Boas (1999), just to mention a few. Most of these articles focus on duopolistic competition in a two-period setting and, with some exceptions, they tend to conclude that behavior-based pricing heightens competition and is detrimental to profits. Some of the two-period models, e.g., Acquisti and Varian (2005, page 7), allow new cohort of consumers in period 2. There are also models with an OLG structure, e.g., Villas-Boas (1999). Our modelling framework excludes learning within an OLG structure.

Compared with the above literature, our model has a number of novel features. First, instead of considering competition among contemporaneous oligopolists, we consider a current information-seeking monopolist who cannot prevent his future selves from offering a sequence of lower and lower introductory prices, which makes it difficult for him to induce potential consumers to make their first purchases in the current period. Similar to the oligopoly case, the question is whether the negative competition effect outweighs or does not outweigh the positive direct effect of price targeting but here it is the competition between the current and the future incarnations of the monopolist that reduces his market power. This is reminiscent of the literature on durable good monopoly, starting from Coase (1972), according to which a monopolist who is unable to fully commit to future prices would lose all his monopoly power if the time interval between two consecutive price offers goes to zero.[14] The intuition behind this result is that some customers would delay their purchase rather than accepting the durable-good monopolist's current price because they rationally anticipate a future price decrease. Unlike

[13]Fudenberg and Tirole (2000) considered a two-period model of a Hotelling duopoly. Chen (1996) studied the case of duopoly in the presence of consumers' switching costs. Choe et al. (2018) showed the multiplicity of equilibria under the assumption that the duopolists do not have the same information.

[14]See Stokey, 1981; Bulow, 1982; Gul et al., 1986; Bond and Samuelson, 1987; Kahn, 1987; Hart and Tirole, 1988; Karp, 1996; Laussel et al., 2015; Correia-da-Silva, 2019; and Long, 2015, for a survey.

the Coasian durable-good model where there is no role for acquisition of information on former customers, Laussel, Long and Resende (2020a) showed that a non-durable-good monopolist that acquires such information will in the future make use of it to exercise price discrimination between returning former customers of different vintages. In this setting, they found that if the information is very coarse (in particular, their baseline model focuses on the case in which the monopolist's information pertains only to customers' first date of purchase) then a similar Coasian mechanism is at work and the monopolist's aggregate profit is eroded, vanishing in the limit when length of the commitment period goes to zero.[15] In this paper, we allow the degree of precision of information to take any value between zero and infinity. We find that the adverse effect of information on profit no longer holds when the monopolist's precision of information on former customers exceeds a critical value. Beyond this threshold, profit is increasing in the quality of information.[16] The paper also enriches the extant literature on price discrimination based on customers' recognition by endogenizing firms' incentives to make upfront investments to improve the accuracy of their customer profiling technologies and by analyzing the effects of allowing for consumers' collective action on privacy protection legislation.

3 The Model

A monopolist produces a non-durable good (or a service). For simplicity, we set the marginal cost at zero. The firm faces a continuum of infinitely-lived consumers that differ from one another with respect to their willingness to pay (WTP), denoted by θ. At each instant, a customer may buy (and instantaneously consume) at most one unit of the good. If a type θ consumer buys a unit of the good at the price p, her instantaneous net utility is $\theta - p$. We assume that θ is uniformly distributed over the unit interval, $[0, 1]$. The fraction of the customer base whose type belongs to any interval $[\theta_a, \theta_b]$ where $0 \leq \theta_a < \theta_b \leq 1$ is simply $\theta_b - \theta_a$.

[15] In a companion paper, Laussel et al. (2020b) considered the case of a monopolist producing vertically differentiated products. The focus of that paper is on the existence of multiple Markov-perfect equilibria.

[16] As a referee points out, there is a parallel between the two types of price discrimination in this paper (one harmful to profit, while the other, based on exact WTP, increases it) and the idea that oligopoly price discrimination is harmful to industry profit when it is based on consumers' differences in cross-firm price elasticity but is beneficial to profit when it is based on consumers' differences in valuation (or price elasticity) for the product. See, e.g., Holmes (1989) and Chen (1999). It would also be worthwhile to extend the literature on behavior-based price discrimination to situations where oligopolists can also uncover some consumers' WTP.

The firm enters the market at time $t = 0$. It announces its prices (which may be consumer-specific, or group-specific) at discrete points in time, $\{t_n\}$, where $n = 0, 1, 2, ...$ For all n, the distance between t_{n+1} and t_n is Δ, an exogenous constant. We refer to Δ as the firm's "commitment interval". Over each commitment interval, the prices are unchanged. The interval $[n\Delta, (n+1)\Delta)$ of the time line is called "period n". The initial period is period zero. In each period, the firm sets targeted prices, that include: (i) an introductory price targeted to new consumers; (ii) a set of personalized prices, targeted to returning traceable customers, whose WTP is known to the firm; and (iii) a set of group prices targeted to cohorts of untraceable customers whose WTP is unknown to the firm (thus it is only able to cluster those customers according to the moment of their first purchase). The relative weight of returning traceable vs untraceable customers depends on the degree of precision of the monopolist's customer profiling. This degree will be an endogenous decision of the monopolist, who needs to do an up-front investment[17] in tracking/ profiling technologies before playing the dynamic pricing game with his customers.

More precisely, the timing of the game we study is as follows. In the first stage of the game, the consumers collectively lobby to influence the toughness of privacy protection legislation in order to undermine the efficiency of firms' investment in profiling, with μ denoting the degree of accuracy of firms' profiling capabilities (later on, we shall provide a more rigorous definition of μ). In the second stage, the monopolist considers whether to invest or not in its big data capability, and, if yes, he solves for the optimal amount he invests in profiling. The third stage of the game is an infinite-horizon dynamic game between the (partially) informed monopolist and the consumers, in which the former chooses in each period the prices offered to new and old customers while consumers decide, based on their type and their rational expectations of future prices, when to make their first purchase.

The game is solved by backwards induction and therefore we start by solving the third-stage of the game, taking μ as given. Customers who purchase the good for the first time in period i are called vintage i consumers. The set of all customers of vintage i is called the market segment i. We assume that, thanks to information technology, the monopolist can at least classify former customers according to their vintage. Moreover, making use of its "big data" capability, as soon as a new cus-

[17] This investment captures the costs to obtain the hardware, software and human resources (e.g. data analysts) that will be necessary to subsequently engage in price discrimination. The specifcation of such costs will be presented in more detail in Section 7.

tomer has made her first purchase, the firm is able to begin the process of trying to uncover the exact willingness to pay of returning customers. However, the process has only limited success since firms can hardly get to know the exact preferences of all their returning customers. To capture this feature, we assume that for any non-negative integer i, at any time $t \geq i$, there is a fraction of former customers of vintage i whose exact WTP remains unknown to the firm. Because the firm perseveres in its effort to uncover former customers' WTP, this fraction falls at a constant rate μ per unit of time.[18] In other words, the parameter μ reflects the constraints imposed by the state of information technology and consumers protection legislation (which are to be endogenized in the second and the third stage of the game; more generally, improvements in information technology will increase μ, whereas a tightening of legislations will reduce μ).

Among all individuals who make their first purchases at the beginning of period i, by the time period $i+1$ begins, the firm has managed to discover the exact WTP of a fraction $(1-e^{-\mu\Delta})$ of them. In all periods $i+j$ where $j=1,2,3,...$, the firm is able to perfectly discriminate these "identified customers" by charging them a personalized price equal to their WTP. This means that those consumers end up with zero surplus. In contrast, in dealing with the fraction $e^{-\mu\Delta}$ of vintage i customers whose WTP remain unidentified at the beginning of period $i+1$, the firm offers all of them a common returning-customer vintage-specific price, denoted by $p^u(i,i+1)$, where i refers to the vintage, and $i+1$ refers to the period. (The subscript u indicates that this price applies only to customers whose exact WTP remains unidentified.) More generally, in any period $n \geq i+1$, a vintage-specific price $p^u(i,n)$ is offered to the (dwindling) fraction $\left(e^{-\mu\Delta}\right)^{n-i}$ of vintage i customers whose exact WTP remain unidentified.

Given any finite period length $\Delta > 0$, it is useful to define $\delta \equiv e^{-\mu\Delta}$. We refer to δ as "*the degree of coarseness*" of the monopolist's information. The polar case $\delta = 1$ (i.e., $\mu = 0$) corresponds to the scenario where the monopolist is unable to uncover the exact WTP of any of his former customers. We refer to this as "*the coarsest information case*", because in this case all vintage i customers will forever remain tagged as "*vintage i customers with unidentified WTP*." At the opposite extreme, $\delta = 0$ (i.e., μ is infinite) means that the monopolist's information is extremely fine: the monopolist knows the exact WTP of each returning

[18] We suppose that, conditional on the fact that a given customer's exact WTP has not been uncovered at time t, the probability that it will be discovered over the next short time interval dt is μdt, where μ is a positive constant. In other words, μ is the arrival rate of a Poisson process.

customer (engaging in first-degree price discrimination).

For any $n = 0, 1, 2, 3, ..,$ it will be convenient to denote by $p^u(n, n)$ the "*introductory price*" offered to new customers in period n, i.e., those who have never bought the good in earlier periods. A new customer of type θ who buys one unit of the good at each instant of time in this period pays $p^u(n, n)$ per unit at each instant. If we denote by $r > 0$ the constant instantaneous discount rate, it follows that the customers' resulting discounted stream of net utility for that period is

$$v(\theta, n, n) = (\theta - p^u(n, n)) \int_0^\Delta e^{-r\tau} d\tau = (\theta - p^u(n, n)) \frac{1 - \beta}{r}$$

where we have defined $\beta \equiv e^{-r\Delta}$.[19] Concerning former customers belonging to any given vintage $i < n$, at the beginning of period n, the monopolist partitions them into two groups. The first group, which accounts for a fraction $(1 - \delta^{n-i})$ of them, consists of all those whose exact WTP has been discovered. Each of them is charged a personalized price (equal to their WTP), leaving them with zero net utility. The second group, which accounts for the remaining fraction, δ^{n-i}, faces a common price $p^u(i, n)$. We refer to members of this group as "*unidentified customers*" of vintage i. Clearly, the members of this group who have a high enough WTP, $\theta \geq p^u(i, n)$, will purchase the good and obtain $v(\theta, i, n) = (1 - \beta)(\theta - p^u(i, n))/r$, whereas those with $\theta < p^u(i, n)$ will not purchase the good.

4 Third-stage: Price Discrimination under Non-commitment

We begin our analysis by focusing on the monopolist's price discrimination among former customers, given that the firm has learnt the exact WTP of some of them while it only knows the date of first purchase of the others. The firm is able to exercise first-degree discrimination to identified former customers, while for unidentified former customers, it is only able to exercise third-degree price discrimination, setting one price for each vintage: $p^u(i, n)$ is generally different from $p^u(j, n)$, for $j \neq i$. After determining these prices, we will consider the monopolist's optimal pricing for new customers. Our main focus is on the non-commitment

[19] Notice that both δ and β increase when Δ decrease. They both tend toward 1 when $\Delta \to 0$. When $\Delta \to \infty$, they both tend toward 0, and by L'Hospital's rule, the ratio $\frac{\delta}{\beta} \to \frac{\mu}{r}$. In order to ascertain the effects of a changing degree of the coarseness of information on the monopolist's profits, we shall have to consider how these profits change when δ changes for a fixed value of β : this amounts to analyze the effect of changes in μ for a fixed length Δ of the period of commitment.

equilibria of the model, i.e; we assume that in any period the monopolist is unable commit to prices it will set for future periods.

4.1 Price discrimination among former customers

Let us begin with the following useful observation. Consumers being rational, if in equilibrium a consumer of type θ' finds it optimal to become a first-time customer in period $i \geq 0$, then it must hold that any consumer of type $\theta'' > \theta'$ also finds it optimal to be a first-time customer in some period i', where $i' \leq i$. At the beginning of any period $n \geq 0$, there exist n known cut-off types $1 \geq \theta_1 \geq \theta_2 \geq ... \geq \theta_n$, and the monopolist makes a decision on market expansion, i.e., on the optimal θ_{n+1}.[20] As a result, new customers in period n are of type $\theta \in (\theta_{n+1}, \theta_n]$. A customer of type θ_{i+1} is called a vintage-i *marginal customer*. By construction, such a customer is indifferent between making her first purchase in period i or in period $i+1$. As mentioned in the previous section, at the beginning of period n, the monopolist has discovered the exact WTP of a fraction $1-\delta^{n-i}$of vintage i customers. As to the remaining fraction δ^{n-i} of them, the only information the monopolist has is that they belong to vintage i. At the beginning of period n, all consumers whose types belong to $(\theta_n, 1]$ have already purchased the good at least once in previous periods. The monopolist does not offer former customers the introductory price that it proposes to new customers: although the monopolist is not able to identify the WTP of such customers, he is still able to identify their vintage, which translates into the possibility of engaging in third-degree price discrimination among returning customers that belong to different vintages.

In light of this, we have that, in any period n, the firm offers to former fully recognizable/identified customers (whose WTP has been identified) a type-dependent personalized price $p^I(\theta)$, equal to their WTP, thus extracting their whole surplus, i.e., $p^I(\theta) = \theta$.[21] The profits that the monopolist makes in period n over the set of identified vintage-i former customers equal

$$\left(1-\delta^{n-i}\right)\int_{\theta_{i+1}}^{\theta_i} \theta d\theta = \left(1-\delta^{n-i}\right)\left(\frac{\theta_i^2}{2} - \frac{\theta_{i+1}^2}{2}\right).$$

In dealing with vintage i former customers whose WTP have not been discovered, the monopolist offers a vintage-dependent price, $p^u(i,n)$, for each vintage $i < n$ $(i = 0, 1, 2, .., n-1)$, so as to maximize the profits it makes from this market segment. Given our assumption that θ is

[20] Since $\theta \in [0, 1]$, we set $\theta_0 = 1$.

[21] The superscript I in $p^I(\theta)$ refers to identified customers.

uniformly distributed over the interval $[0,1]$, the fraction of the customer base whose type belongs to the interval $[\theta_{i+1}, \theta_i]$ where $0 \leq \theta_{i+1} < \theta_i \leq 1$ is simply $\theta_i - \theta_{i+1}$. It follows that the population size of customers that belong to vintage i and whose exact WTP remain unknown to the firm at the beginning of period n is $\delta^{n-i}(\theta_i - \theta_{i+1})$. Clearly, offering these customers any $p^u(i,n) > \theta_i$ would be a dominated strategy because it would result in zero demand from that market segment. Thus, in period n, for these unidentified customers of vintage i (where $i < n$), the monopolist will offer a price $p^u(i,n) \leq \theta_i$. Clearly, the quantity $Q_{i,n}$ sold in period n to these unidentified customers is given by

$$Q_{i,n} = \begin{cases} \delta^{n-i}(\theta_i - p^u(i,n)) \text{ if } \theta_{i+1} \leq p^u(i,n) \leq \theta_i, \\ \delta^{n-i}(\theta_i - \theta_{i+1}) \text{ if } 0 \leq p^u(i,n) \leq \theta_{i+1}. \end{cases}$$

Let $\pi(i,n)$ denote the profit function from sales in period n to vintage i former customers, where $i \leq n-1$. It corresponds to the weighted sum of the profits over unidentified and identified customers, with weights δ^{n-i} and $(1-\delta^{n-i})$ respectively; i.e., for $i < n$,

$$\pi(i,n) = \frac{1-\beta}{r} \begin{bmatrix} \delta^{n-i} \left[\theta_i - \max\{\theta_{i+1}, p^u(i,n)\}\right] p^u(i,n) + \\ (1-\delta^{n-i}) \left(\frac{\theta_i^2}{2} - \frac{\theta_{i+1}^2}{2}\right). \end{bmatrix} \tag{1}$$

Straightforwardly, given θ_i and $\theta_{i+1} < \theta_i$, in any period $n > i$, the maximization of $\pi(i,n)$ with respect to $p^u(i,n)$ yields

$$p^{u*}(i,n) = \max\left\{\frac{\theta_i}{2}, \theta_{i+1}\right\}. \tag{2}$$

If $(1/2)\theta_i \leq \theta_{i+1} < \theta_i$, i.e., if θ_{i+1} is not too far below θ_i, then the optimal price $p^{u*}(i,n)$ is equal to θ_{i+1} (the WTP of the lowest-valuation member of the vintage i), and all of them will purchase the good and enjoy a non-negative surplus when they return after their first-purchase. In contrast, if $(1/2)\theta_i > \theta_{i+1}$, then the optimal price is set at $(1/2)\theta_i$ and thus not all unidentified customers in vintage i will buy the good. Indeed, in those circumstances, customers with $\theta \in [\theta_{i+1}, (1/2)\theta_i]$ will not buy the good. Notice that in either case, vintage i *marginal* customers (those whose type is θ_{i+1}) will have zero surplus in all periods $n \geq i+1$.

Accordingly, substituting (2) into eq. (1) yields the profit earned in period n from former customers of vintage i, where $i \leq n-1$,

$$\pi^F(i,n) = \frac{1-\beta}{r}\begin{bmatrix} \delta^{n-i}\left(\frac{\theta_i^2}{4}\right) + \\ (1-\delta^{n-i})\left(\frac{\theta_i^2}{2} - \frac{\theta_{i+1}^2}{2}\right) \end{bmatrix} \text{ if } \frac{\theta_i}{2} \geq \theta_{i+1}, \tag{3}$$
$$= \frac{1-\beta}{r}\begin{bmatrix} \delta^{n-i}(\theta_i - \theta_{i+1})\theta_{i+1} + \\ (1-\delta^{j-i})\left(\frac{\theta_i^2}{2} - \frac{\theta_{i+1}^2}{2}\right) \end{bmatrix} \text{ if } \frac{\theta_i}{2} \leq \theta_{i+1}.$$

where the superscript F is meant to clarify that we are concentrating on the profits that the monopolist makes on former customers (in period i).

The monopolist's profit in period n over all the former customers of all vintages, $i = 0, 1, 2, .., n-1$, is obtained by summing over i:

$$\Pi_n^F = \sum_{i=0}^{n-1} \pi^F(i,n). \tag{4}$$

Note that Π_n^F is a function of $\theta_0, \theta_1, \theta_2 ..., \theta_n$, where $\theta_0 = 1$.

4.2 Monopoly pricing for first-time customers

We now consider period n new customers. Let us denote by $p^u(n,n)$ the introductory price offered to new customers in period n. Let $U_n(\theta_{n+1})$ denote the intertemporal net utility of a period-n *marginal* new customer.[22] By definition, she is indifferent between (i) becoming a first-time customer in period n, buying at the introductory price $p^u(n,n)$, and (ii) becoming a first-time customer in period $n+1$, paying a lower introductory price $p^u(n+1, n+1)$ intended for period $(n+1)$ customers, but at the cost of foregoing a net utility $\theta_{n+1} - p^u(n,n)$ at each instant of time for the entire time interval Δ.

Under rational expectations, any period n first-time customer of type $\theta \in [\theta_{n+1}, \theta_n]$ anticipates correctly that in all later periods $j > n$, she will either (i) be offered an instantaneous price $p^{u*}(n,j) = \max\left\{\frac{\theta_n}{2}, \theta_{n+1}\right\}$ if her WTP remains unknown to the firm, or (ii) be charged the personalized price $p^I(\theta) = \theta$ if the monopolist has learnt her exact WTP. Consequently the period n *marginal* customer expects zero surplus in all later periods, implying that $U_n(\theta_{n+1})$ is equal to her net utility in period n, i.e., $U_n(\theta_{n+1}) = \frac{1-\beta}{r}(\theta_{n+1} - p^u(n,n))$. It is convenient to express this equality as

$$p^u(n,n) = \theta_{n+1} - \frac{r}{1-\beta}U_n(\theta_{n+1}). \tag{5}$$

[22] We will interchangeably refer to $U_n(\theta_{n+1})$ as the informational rent of the period n marginal new customer.

The monopolist's profit earned in period n from the period n first-time customers is

$$\Pi_n^N = \frac{1-\beta}{r}\left(\theta_n - \theta_{n+1}\right) p^u(n,n),$$

where the superscript N captures the fact that we are focusing on the profit made in period n over New customers. Using eq. (5) to substitute for $p^u(n,n)$, we obtain

$$\Pi_n^N = \frac{1-\beta}{r}\left(\theta_n - \theta_{n+1}\right)\left(\theta_{n+1} - \frac{r}{1-\beta}U_n(\theta_{n+1})\right). \tag{6}$$

If the firm wants to induce a new customer with $\theta \in [\theta_{n+1}, \theta_n]$ to buy in period n it must ensure that she would not be better off waiting until period $n+1$. This participation constraint may be written as

$$U_n(\theta) \geq \beta U_{n+1}(\theta) \text{ for all } \theta \in \left[\theta_{n+1}, \theta_n\right]. \tag{7}$$

However, since $U_n(\theta) \geq U_n(\theta_{n+1})$ for all $\theta \in [\theta_{n+1}, \theta_n]$, constraint (7) is redundant if the constraint $U_n(\theta_{n+1}) \geq \beta U_{n+1}(\theta_{n+1})$ is satisfied. This condition holds with equality because, by definition, a marginal customer is indifferent between being the first time customer in period n and being a first time customer in period $n+1$.[23] It is thus equivalent to:

$$U_n(\theta_{n+1}) = \beta\left[\frac{(1-\beta)\left(\theta_{n+1}-\theta_{n+2}\right)}{r(1-\delta\beta)} + U_{n+1}(\theta_{n+2})\right] \text{ if } \theta_{n+2} \geq \frac{\theta_{n+1}}{2}, \tag{8}$$

$$U_n(\theta_{n+1}) = \beta\left[\begin{array}{c}\frac{1-\beta}{r}\left(\theta_{n+1} - \theta_{n+2} + \frac{(\delta\beta)\theta_{n+1}}{2(1-\delta\beta)}\right) \\ +U_{n+1}(\theta_{n+2})\end{array}\right] \text{ if } \theta_{n+2} \leq \frac{\theta_{n+1}}{2}.$$

The proof is in the Appendix.

5 Third-stage: The Markov Perfect Equilibrium

We are dealing with a dynamic game between the monopolist and the consumers. We denote the state variable of our model by $\Theta(n)$, where $\Theta(0) = \theta_0 = 1$, and $\Theta(n) \equiv 1 - X(n)$, with $X(n) \in [0,1]$ denoting the fraction of the total population that has purchased the good prior to period n. Because of the uniform distribution in $[0,1]$, $X(n) = 1 - \theta(n)$. It follows that $\Theta(n) = \theta(n)$. The real interval $(\Theta(n), 1]$ represents the set of customers who have not purchased the good prior to period n.

Since the firm's optimal pricing for former customers has already been solved, the monopolist's dynamic optimization problem reduces

[23] The RHS of the first line (respectively, second line) of eq. (8) is equal to $\beta U_{n+1}(\theta_{n+1})$ in Case A (respectivly, Case B).

to choosing how to expand its market coverage (or equivalently how to optimally price the good for first-time customers). The firm's Markovian strategy consists of a cut-off rule ψ, such that $\Theta(n+1)$ is given by $\Theta(n+1) = \psi(\Theta(n)) \leq \Theta(n)$.

Concerning consumers, those who have not purchased the good prior to period n must decide on the optimal period to enter the market. We assume that consumers have a Markovian expectations rule $\Omega(.)$ that predicts the life-time rent of the marginal first-time customer in period n, i.e., $U_n(\theta_{n+1})$. Under the rational expectations hypothesis, their predictions are always correct, i.e.,

$$\Omega(\Theta(n)) = U_n(\theta_{n+1}), \tag{9}$$

where, using (8), $U_n(\theta_{n+1})$ may be expressed as a function of the whole path of the state variable induced by the monopolist's strategy.

A Markovian strategy $\psi(.)$ chosen by the monopolist is a best reply to the consumer expectations function $\Omega(.)$ if it yields a sequence of cut-off values θ_{n+1} that maximizes profits, starting from any pair $(n, \Theta(n))$, and condition (9) is satisfied by such a sequence. Given the uniform distribution assumption and the general structure of the problem, we conjecture that in equilibrium, the monopolist's optimal cut-off rule is a linear,

$$\Theta\,(n+1) = \psi(\Theta(n)) = \gamma\Theta(n), \text{ with } 1 \geq \gamma \geq 0, \tag{10}$$

and consumers' expectations function is also linear:[24]

$$\Omega(\Theta(n)) = \lambda\Theta(n). \tag{11}$$

In what follows, we will to determine the pair (γ^*, λ^*) such that the cut-off rule

$$\psi(\Theta(n)) = \gamma^*\Theta(n)$$

is the best reply to the expectations rule

$$\Omega(\Theta(n)) = \lambda^*\Theta(n)$$

and vice-versa.

The profit obtained in period n from old customers is given by equation (4). Corresponding to equation (6), the profit obtained in period n from first-time customers in that period is

$$\Pi_n^N\left(\Theta(n), \Theta(n+1), \lambda\right) = \left(\Theta(n) - \Theta(n+1)\right)\left(\frac{1-\beta}{r}\Theta(n+1) - \lambda\Theta(n)\right) \tag{12}$$

[24] We could use a more general conjectured linear-quadratic expectations functions, and show that consistency reduces to the linear rule.

where we have substituted $\lambda\Theta(n)$ for $U_n(\theta_{n+1})$ because of the rational expectations requirement.

It follows that the Bellman equation for the monopolist is[25]

$$V(\Theta(n)) = \max_{\Theta(n+1)} \{\Pi_n^F + \Pi_n^N (\Theta(n), \Theta(n+1), \lambda) + \beta V(\Theta(n+1))\}. \quad (13)$$

The results in Lemma 1 below show that the consumers' best reply to the monopolist's cut-off exhibits two regimes, depending on the value of γ. In what follows, it is useful to refer to $1 \geq \gamma \geq 1/2$ as the case of slow market expansion or "*market expansion by small steps*" (as it implies that θ_{n+2} is not too far below θ_{n+1}). Analogously, $\gamma < 1/2$ may be called the case of "*market expansion by large steps.*"

Lemma 1 (Consumers equilibrium expectations). *Given any pair (β, δ) where $0 \leq \beta \leq 1$ and $0 \leq \delta \leq 1$, in any MPE in which the monopolist's optimal cut-off rule and consumers' expectations rule are both linear in the state variable, it holds that the consumers' best reply to the monopolist's cut-off choice γ is given by*

$$r\lambda = \Lambda(\gamma, \beta, r) \text{ if } 1 \geq \gamma \geq \frac{1}{2}, \quad (14)$$

$$r\lambda = \Lambda(\gamma, \beta, r) - \frac{\beta^2(1-\beta)\gamma\delta\,(1-2\gamma)}{2(1-\beta\gamma)(1-\beta\delta)} \text{ if } \gamma \leq \frac{1}{2},$$

with $\Lambda(\gamma, \beta, \delta)$ given by

$$\Lambda(\gamma, \beta, \delta) \equiv \frac{\beta\gamma(1-\beta)(1-\gamma)}{(1-\beta\gamma)(1-\beta\delta)}. \quad (15)$$

Proof. See the Appendix. ■

Simple computations show that, for a given market expansion strategy of the monopolist, the expected lifetime utility of the marginal first time customers is increasing in δ; meaning that it is the greater the more slowly the firm learns the preferences of its past customers. The intuition for this result is rather clear. The higher is δ, the greater is the expected surplus that a consumer may obtain by delaying her purchase to the next period and hence the higher the informational rent she must be offered in order to buy in the present period. This is because the probability of retaining a positive surplus in all future periods by

[25] $V(\Theta(n))$ depends also on $\Theta(n-1)$ and all the other past values of the state variable. This is omitted for the sake of notational simplicity. Note that in period n the historical values of the state variable (namely $\Theta(n-1)$, $\Theta(n-2), ...\Theta(0)$), have no relevance for the decision on the optimal $\Theta(n+1)$. Only $\Theta(n)$ is relevant. Note also that Π_n^F depends on $\Theta(n)$ and on earlier values.

delaying one's first purchase is increasing in δ. Indeed the more slowly the firm learns about its past customers' preferences, the smaller is the fraction of customers against which it will be able to exercise first-order price discrimination and the greater the fraction who will be offered a vintage-dependent price (third-order price discrimination).

Taking the expectations parameter λ as given, the monopolist chooses the optimal cut off rule.

Lemma 2 (Monopolist's Optimal Market Expansion). *Given any pair* (β, δ) *where* $0 \leq \beta \leq 1$ *and* $0 \leq \delta \leq 1$*, in any MPE in which the monopolist's optimal cut-off rule and consumers' expectations rule are both linear in the state variable, it holds that the monopolist's cut-off choice* γ *is a best reply to the consumers' expectation rule iff the following conditions are satisfied:*

$$
\begin{aligned}
h(\gamma, \beta, \delta) &= 0 \text{ if } 1 \geq \gamma \geq \frac{1}{2}, \\
k(\gamma, \beta, \delta) &= 0 \text{ if } \gamma \leq \frac{1}{2},
\end{aligned} \tag{16}
$$

where $h(\gamma, \beta, \delta)$ *and* $k(\gamma, \beta, \delta)$ are defined by equations (45) and (46) in the Appendix.

Proof: See the Appendix.■

Lemma 2 implies that, for given consumers' expectations (i.e., a given value of λ), at least in the case of expansion by small steps $(1 \geq \gamma \geq 1/2)$, market expansion is going to be quicker when the monopolist learns more quickly the preferences of its past customers.[26] This is simply that because it is generally more profitable to expand quickly the market when the proportion of customers who become identified is greater at any point in the future. The exception arises only when the market expands fast (the monopolist expands it in large steps) and γ is too small. In that case, there is possibly a countervailing effect since the proportion of unidentified customers of each vintage who don't return buying may be large.

The following result follows immediately:

Proposition 1 *Given any pair* (β, δ) *where* $0 \leq \beta \leq 1$ *and* $0 \leq \delta \leq 1$*, there exists an MPE in which the monopolist's optimal cutoff rule and*

[26] This follows from $h(.)$ being increasing in δ so that the equibrium value of γ is increasing in δ. Notice that $k(.)$ is clearly increasing in δ as well, when γ is not too small.

consumers' expectations rule are both linear in the state variable, where the equilibrium parameters (γ^e, λ^e) are such that:

(i) The monopolist's equilibrium cut-off value $\gamma^e(\beta, \delta)$ is either as a root of equation (47) in the Appendix, with $1 \geq \gamma \geq 1/2$ or a root of equation (48) in the Appendix, with $1/2 \geq \gamma \geq 0$,

(ii) $\lambda^e(\beta, \delta)$ is given by equation (35) in the Appendix.

Proposition 1 allows us to investigate how the nature of the market expansion depends on the three parameters μ, r and Δ, because the equilibrium values of γ and λ can be equivalently expressed as $\gamma^e(r, \mu, \Delta)$ and $\lambda^e(r, \mu, \Delta)$, due to the definitions $\beta \equiv e^{-r\Delta}$ and $\delta \equiv e^{-\mu\Delta}$. When we focus on the effects of a variation in the degree of precision (or coarseness) of information, it will be convenient to study the effects of variations in the value of μ, for given values of r and Δ. This is equivalent to study the effect of variations of δ for a given value of β. On the contrary, when we try to ascertain the effects of a variation in the length Δ of the period of contractual commitment, we shall consider fixed values of r and μ (i.e., a given degree of precision of the monopolist's information).

It is also important to note that, as shown in Lemma 1, Lemma 2 and Proposition 1, the economic properties of the monopolist's optimal expansion strategy and consumers' expectations are indeed different, depending on whether, in equilibrium, γ^e is above or below $\frac{1}{2}$. Thus it is useful to try to characterize the circumstances leading to the razor edge case in which the equilibrium γ^e is exactly equal to $\frac{1}{2}$, which defines the threshold determining the monopolist's optimal decision to go for a slow or a fast market expansion regime. Remarks 1 and 2 below provide additional insights on this matter.

Remark 1: (i) For any given β in $(0, 1)$, the monopolist will choose to expand the market in large steps (i.e., $\gamma < 1/2$) when the degree of precision of information on customers is high enough (i.e., when δ is small enough),

(ii) There exists a value $\overline{\delta} = \Phi(\beta) < 1$ such that, for any δ in $(0, \overline{\delta})$, the monopolist will choose to expand the market in large steps (i.e., $\gamma < 1/2$) if β is big enough (i.e., if r or Δ is small enough).

Proof: See Appendix.

Remark 2: For any pair $(r, \mu) > (0, 0)$, in the limiting case where Δ becomes arbitrarily small, the whole market is covered instantaneously (Figure 3 below confirms this finding).

Proof: See Appendix.

In Figure 1 below, we have plotted the equilibrium value γ^e of the cutoff parameter as a function of the degree δ of coarseness of information for $\beta = 0.8$.[27] The green part of the curve corresponds to values of $\delta \in [0, 0.12848]$ which ensures that $\gamma^e(0.8, \delta) \leq 1/2$ (market expansion by large steps). It turns out that γ^e decreases when δ decreases, i.e., the equilibrium market expansion proceeds in bigger steps if the precision of information is higher (as mentioned in (i) of Remark 1). This is an intuitively plausible result for two different reasons. On one hand, as noticed after Lemma 2, the faster the fraction of unidentified former customers shrinks, given any consumers' expectations rule, the more the monopolist is interested in expanding quickly its market (given the prospects of additional profits resulting from first-degree price discrimination over a considerable set of customers). On the other hand, as noticed after Lemma 1, the greater the probability that the firm learns its customers' exact WTP, the smaller the rents they require not to delay their first purchases and, accordingly, the greater the incentive for the firm to expand quickly its market.

(PLEASE PLACE FIGURE 1 ABOUT HERE)

Figure 2 illustrates the results in part (ii) of Remark 1. It depicts the relationship between γ^e and β for two different values of the coarseness parameter, $\delta = 0.8$ (the upper curve) and $\delta = 0.1$ (the lower curve). As shown in this Figure, if the information is coarse enough (large values of δ), then γ^e is greater than $1/2$ (market expansion by small steps) for all values of β. For small values of δ, γ^e becomes smaller than $1/2$ if β is sufficiently large (the green part of the lower curve).

(PLEASE PLACE FIGURE 2 ABOUT HERE)

Let us now consider the relationship between the speed of market expansion and the length of the period of contractual commitment. Recall that the monopolist adopts a linear market expansion rule: in any period n, the fraction of the customer base that is not served is $(\gamma^e(\beta, \delta))^{n+1}$. The measure of customers that have been served by the end of period n is $(1 - (\gamma^e)^{n+1})$. For any time $t = n\Delta$, we have $(\gamma^e)^{n+1} = (\gamma(\Delta))^{\frac{t}{\Delta}+1}$. The following result, stated as Claim 1, has a Coasian flavor, as it shows that, whatever the value of $\mu > 0$, in the limit when $\Delta \to 0$, the market is covered instantaneously.

Claim 1: *When the length of time Δ between two different proposals to two consecutive sets of new customers becomes infinitesimal, the market is covered instantaneously. For any given time $t > 0$, as the*

[27] Notice that this is to be interpreted as following from an underlying relationship between γ and μ for a given value of Δ since $\delta = e^{-\mu\Delta}$.

length of the period of commitment Δ *tends to* 0 *the fraction of the customer base that has been served up to that time tends to* $1 - e^{\gamma'(0)t}$ *where* $\gamma'(0) \to -\infty$

$$\lim_{\Delta \to 0} (\gamma(\Delta))^{\frac{t}{\Delta}+1} = e^{\gamma'(0)t} = 0,$$

so that the market is covered by the monopolist in a twinkling of an eye at the beginning of the game.

Proof. See the Appendix. ■

The result in Claim 1 is driven by the firm's interest in expanding the market as fast as possible when Δ tends to zero, as in standard Coasian dynamics. In that limiting case, we have $\delta \to 1$, regardless of the value of μ. Thus, $\delta > \Phi(\beta)$ for very small Δ, implying that $\gamma^e > 1/2$. Even though the market expands by small steps in each period, since the periods are becoming very short when Δ tends to zero, it holds that, between any two given points of time t_1 and $t_2 > t_1$, the number of customers served by the firms is larger, the smaller is Δ. Even if, in each period the firm may not get information on a large fraction of customers, as periods last for infinitesimally short amount of time, the firm is able to get much information on customers' WTP in a short amount of time. Thus, the firm can quickly engage in personalized prices that allow the it to extract, for each individual, all the consumer surplus generated by each transaction. This means that, in the MPE, the firm expands very fast when Δ is very small, leading to instantaneous market coverage in the limit case where $\Delta \to 0$.

In Figure 3 below we picture γ^e as a function of the length Δ of the period of contractual commitment, where we have set $r = 1$ and $\mu = 0.5$. Notice that in Figure 3, for all $\Delta > 1$, we have $\gamma^e > 1/2$ (expansion by small steps). Consistent with Claim 1 above, this Figure shows that the slope of this curve tends to minus infinity as Δ tends to zero.

(PLEASE PLACE FIGURE 3 ABOUT HERE)

6 Economic effects of customers' profiling

6.1 Profits and information precision: a U-shaped relationship

In order to identify the profit effects of using customers' data to engage in sophisticated price discrimination practices (including first-degree price discrimination over a subset of customers, whose WTP is uncovered by the monopolist), we now consider the relationship between the monopolist's life-time profit, denoted by Π^*, and the parameters β and δ, where

$$\Pi^* \equiv \sum_{n=0}^{\infty} \beta^n \left[\Pi_n^F + \Pi_n^N\right],$$

and Π_n^F and Π_n^N are defined by (6) and (4), and where $\theta_{n+1} = \gamma^e(\beta,\delta)\theta_n$. Letting $\gamma = \gamma^e(\beta,\delta)$ we obtain the monopolist's life-time profit,

$$\Pi^*(\beta,\delta) = \frac{(1-\gamma)\left[2\gamma + \beta^2\gamma\left(1+\gamma(\delta-1)+\delta\right) - \beta\left(\gamma\left(3+\delta\right)+\delta-1\right)\right]}{2r(1-\beta\gamma)(1-\beta\gamma^2)(1-\beta\delta)}, \tag{17}$$

for all (β,δ) such that $\gamma^e(\beta,\delta) \geq 1/2$, and

$$\Pi^*(\beta,\delta) = \frac{\begin{array}{c} 2(1-\gamma)\left\{2\gamma + \beta\left[1+\beta\gamma(1-\gamma)-3\gamma\right]\right\} + \\ \beta\delta\left\{-1-\beta+\left[\beta(7-\beta)-4\right]\gamma + 2\left[3+(\beta-3)\beta\right]\gamma^2 - 2\beta\gamma^3\right\}\end{array}}{4r(1-\beta\gamma)(1-\beta\gamma^2)(1-\beta\delta)} \tag{18}$$

for all (β,δ) such that $\gamma^e(\beta,\delta) \leq 1/2$. (Clearly, the two expressions are equal at $\gamma = 1/2$**,** so that the monopolist's life-time profit is a continuos function of γ).

The relationship between the monopolist's life-time profit and the degree of coarseness of information is drawn in Figure 4(a) below for $\beta = 0.8$ (and $r = 1$).[28] Cases $\beta = 0.5, r = 1$ and $\beta = 0.2$, $r = 1$, are pictured in Figures 4(b) and 4(c)**.** Please note that Figures 4(a), 4(b) and 4(c) shows the same qualitative relationship between profit and δ (interpreted here as the degree of coarseness of the firm's customer information). When δ approaches 1, (i.e., the information on previous customers is very coarse, revealing only the date that a consumer makes her first purchase and the monopolist can never discover former consumers' exact willingness to pay), the monopolist is able to engage in third-degree price discrimination among different vintages of customers but profit is at its minimum. If the monopolist has strictly no information on its former customers (not even when or whether they have purchased the good), the only possible equilibrium is the repetition of the static monopoly equilibrium, leading to equilibrium profit equal to $0.25/r$. This obviously implies that the monopolist's life-time profit is non-monotonic with respect to the degree of information it can obtain

[28] The green part of the curve corresponds to the values of δ such that $\gamma^e(0.8,\delta) \leq 1/2$ (market expansion by large steps). This occurs when δ is small enough. Here, we have used the formula for $\Phi(\beta)$ in Remark 1.

on its former customers: the monopolist's profit jumps down when it begins to collect a minimal information ($\delta = 1$ or, equivalently, $\mu = 0$) and then continuously increases when the degree of precision of the collected information increases, taking its maximum value when $\delta = 0$ ($\mu = +\infty$).

PLEASE PLACE FIGURES 4(a), 4(b), AND 4(c) ABOUT HERE

In the scenario in which it cannot obtain any information at all about former customers, the monopolist's optimal pricing is to charge a time-invariant price, $p = 1/2$ and only $1/2$ of the market is served, leading to a life-time profit of $0.25/r$. When the technology enables the collecting and processing of information on former customers, the monopolist, being unable to commit on future prices, has the incentive to practice intertemporal price discrimination, offering successively lower and lower introductory prices to previously unserved customers, hoping to gain information about them once they have bought the good. If this information is coarse, i.e., δ is close to 1, meaning that after each period, the fraction of former customers whose exact WTP cannot be identified is near 1 (as in the baseline model of Laussel, Long and Resende, 2020a), the monopolist's life-time profit becomes strictly lower than $0.25/r$, because consumers, expecting lower and lower future introductory prices, prefer to postpone their purchases unless they are offered significant informational rents. This Coasian-like rationale leads firms to get lower profits than the ones obtained under no information on customers' preferences at all (in which the static equilibrium would be repeated indefinitely). Life-time profit is greater than $0.25/r$ only if the consumers' profiling technology is sufficiently advanced, allowing δ to fall below a critical threshold level δ_c. (For example, in Figure 4(a), $\delta_c \simeq 0.84$). Profit attains its global maximum at the polar case $\delta = 0$, in which the monopolist gets information on customers' WTP (and price discriminates accordingly), immediately after the first purchase.

These qualitative results appear to be robust with respect to the value of β. In Figures 4(b) and 4(c), the same qualitative relationship is obtained for some other values of β. There is always a threshold value δ_c (which depends on β), such that if $\delta \in (\delta_c, 1]$ then life-time profit is below $0.25/r$. This is true for all values of $\beta \in (0, 1]$. As is obvious from Figures 4(b) and 4(c), the curve is more and more horizontal for smaller and smaller values of β, converging to the horizontal dashed line as $\beta \to 0$. When $\beta = 0$ (which means $\Delta = \infty$) the "first commitment period" lasts for ever, and hence, not surprisingly, the equilibrium life-time profit is the same as that obtained in the case of infinite repetition of the static equilibrium.

Let us now consider the special case where $\Delta = 0$ (so that $\beta = \delta = 1$).

For this purpose, let us express life-time profits as a function of Δ for a given pair (r, μ), $\Pi^*(\Delta; r, \mu) = \Pi^*(e^{-r\Delta}, e^{-\mu\Delta})$ where Δ is the length of the period of contractual commitment. Claim 2 below shows the limiting behavior of profit as $\Delta \to 0$.

Claim 2: *When the length of time Δ between two different proposals to two consecutive sets of new customers becomes infinitesimal, $\Pi^*(\Delta; r, \mu) \to \frac{1}{2r}\frac{\mu}{r+\mu}$. Accordingly $\Pi^*(0; r, \mu)$ is increasing in the degree μ of precision of information, varying from $\Pi^*(0; r, \mu) = 0$ when $\mu = 0$ (i.e., when the only information which the monopolist gets on his former customers is their date of first purchase) to $\Pi^*(0; r, \mu) = \frac{1}{2r}$ when $\mu \to \infty$ (i.e., when the monopolist learns instantaneously the exact WTP of his consumers). Notice that $\frac{1}{2r}$ is the monopolist's life-time profit when he can fully discriminate among all consumers (first-degree price discrimination).*
Proof. See the Appendix. ■

Using Claim 2, we can prove the following Corollary, which compares the monopolist's life-time profit in the limiting case $\Delta \to 0$ with that in the limiting case $\Delta \to \infty$, for given r and μ. It turns out that the former is greater than the latter if $\mu < r$ and the opposite holds if $\mu > r$.

Corollary 1: *The monopolist's life-time profit when $\Delta \to \infty$ is greater (smaller) than when $\Delta \to 0$ iff r is greater (smaller) than μ.*

$$\text{SIGN}\left[\Pi^*(+\infty; r, \mu) - \Pi^*(0; r, \mu)\right] = \text{SIGN}\left[r - \mu\right].$$

Proof. It suffices to notice that: (i) from Claim 2, we have $\Pi^*(0; r, \mu) = \frac{1}{2r}\frac{\mu}{r+\mu}(\gtrless \frac{1}{4r} \iff \mu \gtrless r)$,
and that (ii) from (17) or (18) and $\gamma^e(0, 0) = 1/2$, we have $\Pi^*(+\infty; r, \mu) = \frac{1}{4r}$ for all $\mu \geq 0$. ■

Corollary 1 states a striking result: the dynamics of this model is globally Coasian if $r > \mu$ whereas it is globally non-Coasian if $r < \mu$. In the first case, the firm's life-time profit when the length of the commitment period tends to 0 is smaller than its life-time profit when this length is infinite; that is, if information is coarse (small μ), the monopolist would be better off by making an initial fixed price offer that lasts for ever (selling only to the top half of the customer base) than offering a sequence of prices (each lasting just an instant) which only helps discover very slowly customers' exact WTPs. This is because the future gains in profits from first-degree price discrimination grow only slowly and are heavily discounted (high r).

In the second case, i.e., $\mu > r$, the reverse holds: offering a sequence of prices (each lasting just an instant) helps discover rather quickly (thanks to a large μ) customers WTP, and the future gains are not heavily discounted (thanks to a low r), leading to a larger life-time profit than that obtained by making an initial fixed price offer that lasts for ever. Notice however that Corollary 1 compares the values of the profit for the two extreme values of Δ; and in general $\Pi^*(\Delta; r, \mu)$ is not a monotonic function of Δ. [29]

This shows that in this model there are two effects working in opposite directions. The first one (which is the stronger whenever $r > \mu$) is the classical Coasian effect: when introductory prices are fixed for a shorter period (Δ is smaller), consumers expect lower future introductory prices to occur more quickly, and at the same time they bear a lower waiting cost so that they prefer to postpone their purchases, implying that the prices decrease even more quickly. The second effect (which is the stronger whenever $r < \mu$) is a "front-loading effect"[30]: when Δ is smaller, the monopolist is able to extract sooner the surplus of traceable former customers whose exact WTP becomes known to the monopolist.

6.2 Consumers surplus and information precision: a hump-shaped relationship

In this subsection, we investigate how the degree of accuracy on consumers' profiling impacts social welfare and consumer surplus.

Let $w(n)$ denote the social welfare at the MPE in period n. As usual, we define social welfare as the sum of consumer surplus and profit. Consumer surplus is gross utility minus the payment made to the firm. Profit is the sum of consumers' payments to the firm (because we assume that the cost of production is zero). Therefore welfare in any period is simply the gross utility of consumers who are served in that period. Whenever $\gamma^e(\beta, \delta) \geq 1/2$, all customers whose type $\theta \geq \theta_{n+1}$ are served in period n. Thus the social welfare that accrues in period n is

$$w(n) = \frac{1-\beta}{2r}(1-\theta_{n+1}^2) = \frac{1-\beta}{2r}\left(1-\left(\gamma^{n+1}\right)^2\right). \qquad (19)$$

When $\gamma^e(\beta, \delta) \leq 1/2$, things are slightly more complicated, as not all new customers whose type belongs to the interval $(\theta_{n+1}, \theta_n]$ are served in period n. Consider indeed vintage i customers, $i < n$. A fraction $\left(1-\delta^{n-i}\right)$ of them are served whatever their type, because their individual WTP have been exactly identified and the monopolist is able to

[29] For instance $\Pi^*(+\infty; r, r) = \Pi^*(0; r, \mu) = \frac{1}{4r}$ but $\Pi^*(\Delta; r, r)$ is first increasing and then decreasing in Δ.

[30] We thank an anonymous referee for suggesting us this term.

make a personalized (limit) offer to each of them. For the remaining fraction δ^{n-i}, only the types between $\frac{\theta_i}{2}$ and θ_i are served as the monopolist prefers to leave some of the consumers in this vintage unserved (in order to extract more surplus from returning shoppers). Thus the social welfare that accrues in period n is, in that case:

$$w(n) = \frac{1-\beta}{2r}\left[\left(\theta_n^2 - \theta_{n+1}^2\right) + \sum_{i=0}^{n-1}\left(\left(1-\delta^{n-i}\right)\left(\theta_i^2 - \theta_{i+1}^2\right) + \delta^{n-i}\frac{3}{4}\theta_i^2\right)\right]. \tag{20}$$

The overall aggregate social welfare is the sum of the present values of $w(n)$ over all n, i.e.;

$$W(\beta,\delta) = \sum_{n=0}^{\infty}\beta^n w(n) = \frac{1}{2r}\frac{1-\gamma^2}{1-\beta\gamma^2}, \tag{21}$$

when $\gamma = \gamma^e(\beta,\delta) \geq 1/2$ and

$$W(\beta,\delta) = \sum_{n=0}^{\infty}\beta^n w(n) = \frac{4+(\beta-5)\beta\delta - 4\gamma^2(1+(\beta-2)\beta\delta)}{8r(1-\beta\gamma^2)(1-\beta\gamma)}, \tag{22}$$

when $\gamma = \gamma^e(\beta,\delta) \leq 1/2$.

Notice that the two values are equal whenever $\gamma^e(\beta,\delta) = 1/2$. Very intuitively, social welfare is greater, the finer is the monopolist's information about consumers' preferences. This is quite obvious in the case $\gamma^e(\beta,\delta) \geq 1/2$. Indeed, from (21), social welfare is the greater the quicker is the market expansion (i.e.; the smaller is $\gamma^e(\beta,\delta)$) and we have already seen, as illustrated in Figure 1, that market expansion proceeds by larger steps (smaller γ) the more precise is the firm's information (i.e., the smaller is δ). This holds true as well in the case $\gamma^e(\beta,\delta) < 1/2$. Figure 5 illustrates this point, showing social welfare as a function of δ (for $\beta = 0.8$ and $r = 1$).

PLEASE PLACE FIGURE 5 ABOUT HERE

Notice that if the monopolist does not obtain any information on previous customers and repeats forever the static monopoly equilibrium, then social welfare equals $\frac{3}{8r}$ which is greater than $W(\beta,1)$ but smaller than $W(\beta,0)$. Thus the relationship between social welfare and the degree of precision of information is also non-monotone. Notice that in the limit case when $\Delta \to 0$, social welfare $\to \frac{1}{2r}$ whenever the firm has some information about its former customers, whatever the degree μ of precision of this information. It is accordingly always better in that case, from the social welfare point of view, that the monopolist be informed, even in the coarsest possible way.

What about the aggregate consumers' surplus $CS(\beta, \delta)$? Since it equals the difference between social welfare and profits, we easily obtain it as

$$CS(\beta, \delta) = \frac{(1-\beta)(1-\gamma)^2(1+\beta\gamma)}{2r(1-\beta\gamma)(1-\beta\gamma^2)(1-\beta\delta)}, \tag{23}$$

when $\gamma = \gamma^e(\beta, \delta) \geq 1/2$ and

$$CS(\beta, \delta) = \frac{\begin{array}{c}(1-\beta)(4(1-\gamma)^2(1+\beta\gamma))- \\ \beta(2\gamma-1)\left\{\gamma\delta\left[2+\beta(2\gamma-1)-3\right]-3\delta\right\}\end{array}}{8r(1-\beta\gamma)(1-\beta\gamma^2)(1-\beta\delta)}, \tag{24}$$

when $\gamma = \gamma^*(\beta, \delta) \geq 1/2$.

The relationship between CS and δ is pictured in Figure 6 for $\beta = 0.8$ and $r = 1$.

PLEASE PLACE FIGURE 6 ABOUT HERE

Without any information about former customers, the infinite repetition of the static monopoly equilibrium leads to a consumer surplus equal to $\frac{1}{8r}$. When the firm acquires the coarsest possible information (corresponding to $\delta = 1$), the consumers surplus jumps to a greater level before decreasing when δ decreases. The relationship between CS and the degree of precision of information turns out to be in the opposite of the relationship between monopoly profit and information precision: it is hump-shaped, with a maximum value attained when the firm's information is the coarsest one ($\delta = 1 \Leftrightarrow \mu = 0$) This was to be expected since the firm uses information on consumer's profiling to set an individual price that coincides with the consumers' WTP, fully extracting the associated consumer surplus. When firms' ability to segment consumers according to their tastes remains rather coarse, consumers actually benefit from price discrimination (at least those whose preferences remain uncovered) as they are able to extract information rents for successive periods (until their preferences get disclosed).

In the limit case when $\Delta \to 0$, CS tends toward $\frac{1}{2r}\frac{r}{r+\mu}$. In that case, it is interesting to notice that, when the firm is able to obtain some information about its old customers, the social welfare is constant with μ, with profit and consumer surplus effects offsetting each other: the share of profits is increasing in μ from 0 to 1, and the share of consumers surplus is decreasing in μ from 1 to 0.

7 Second-stage: Optimal Investment in Customers Profiling

In the previous sections, we analyzed the MPE of the game for a given value of the profiling accuracy μ, showing that the greater is the profiling accuracy the greater are the equilibrium profits, keeping in mind that the profits in the coarsest information case ($\mu = 0$) are smaller than in the no information case. However, the accuracy of the information obtained by the firm is neither exogenous nor costless. It clearly depends on the amount the firm is willing to invest to collect and exploit consumers data in order to acquire the useful hardware and software, to hire good programmers, chief data officers and so on. Let us then look at the second-stage of our game, where we endogenize firm's decision about the degree of information precision on customers tastes. We simply assume that the firm's investment in big data is an increasing function $F(\mu)$ of the information accuracy which it wants to reach and that this investment must take place, once and for all, at the beginning the first period of the game (this assumption intends to capture that customers' profiling activities often require up-front investment in hardware and software whose costs are independent of the subsequent level of firms' engagement in customers profiling). We suppose in addition that $F(0) \geq 0$, $F'(\mu) \geq 0$ and $F''(\mu) \geq 0$, meaning that (i) collecting even the coarsest information may be costly, (ii) the amount invested is increasing in the accuracy of information and (iii) it is increasing at an increasing rate.

Accordingly the monopolist's net profits, when it chooses to invest in information collection, is the following function of μ :

$$\Pi^*(\Delta; \mu, r) - F(\mu),$$

where $\Pi^*(\Delta; \mu, r)$ follows from (17) and (18), with $\beta = e^{-r\Delta}$, $\delta = e^{-\mu\Delta}$.

When choosing to engage in customers profiling, the monopolist selects the value of μ in order to maximize its net profit, i.e.,

$$\mu^* = \arg\max_{\mu} [\Pi^*(\Delta; \mu, r) - F(\mu)], \tag{25}$$

which is equivalently expressed as

$$\delta^* = \arg\max_{\delta \in [0,1]} \left[\Pi^*(\beta, \delta) - F(-\frac{1}{\Delta} \ln \delta) \right].$$

As is well known, according to the Maximum Theorem, a continuous function always has a maximum over a closed interval.

Then, the firm chooses to become informed iff

$$\Pi^*(\Delta;\mu^*,r) - F(\mu^*) > \frac{1}{4r}, \tag{26}$$

i.e., iff this yields profits which are greater that the profits which it would earn by choosing not to collect any information at all on its customers.

Given the complexity of the profit function, it is impossible to derive in the general case analytical results on the optimal value of μ and on the precise conditions which ensure that the monopolist chooses to invest in information collection. Nevertheless it is straightforward to see that when the firm does invest in customer profiling, it must always target an information accuracy above a lower bound value $\underline{\mu}$ which is > 0 whenever $\Delta < +\infty$.

Claim 3: *If one defines* $\underline{\mu}$ *as the value of* μ *which solves* $\Pi^*(\Delta;\mu,r) = \frac{1}{4r}$*, it must be that* $\mu^* \geq \underline{\mu}$*.*

Proof. It is enough to remember that $\Pi^*(\Delta;\mu,r)$ is increasing in μ and that $F'(\mu) \geq 0$.

The intuition behind Claim 3 is simply that, even if collecting information is costless, the monopolist should optimally abstain from investments that allow information collection if such investment does not permit it to reach at least a minimum information accuracy level $\underline{\mu}$, which assures that the monopolist gets information on the true WTP of a minimal critical mass of customers, so that the additional future profits made on those customers compensate not only the investments in tracking and profiling technologies (which would depend on the upfront investments on hardware, software and human resources) but also the profit losses resulting from the monopolist's inter-temporal price discrimination incentives.

In what follows, for tractability, let us assume that the investment cost function takes the form $F(\mu) = f + k\mu$, where $k > 0$ and $f \geq 0$.[31] We call k the marginal investment cost of informational accuracy. If $f > 0$, then to obtain even the coarsest level of information, $\mu = 0$, which allows only purchase history information, the firm must incur a strictly positive fixed investment cost f (e.g. when $\mu = 0$, this parameter captures the costs to obtain and maintain a dataset where the firm registers each customer and the time of the first purchase). On the other hand, if the firm chooses not to invest at all, then the cost f is avoided, and the firm will have no information whatsoever about individual customers (as mentioned earlier, in this case, we have a repetition of the static solution as all customers are perceived by the firm as first-time

[31] It is equally easy to deal with a quadratic cost function.

shoppers, regardless of whether they had actually bought the product in the past or not). The optimal information accuracy, conditional on the monopolist investing in big data, is then $\mu^*(k, \Delta, r)$, which is decreasing in k. Let us denote by $PR(\Delta; \mu^*(k, \Delta, r), r, k, f)$ the monopolist's net profits conditional on investing in big data. From the Envelope Theorem, we have $dPR/dk = -\mu$, thus we obtain

$$PR(\Delta; \mu^*(k, \Delta, r), r, k, f) = PR(\Delta; \mu^*(0, \Delta, r), r, 0, f) - \int_0^k \mu^*(\kappa, \Delta, r) d\kappa.$$

Notice that if $k = 0$, then, conditional on investing, the optimal level of informational accuracy is $\mu^* = \mu^*(0, \Delta, r) = +\infty$, which implies $\delta = 0$. Thus $PR(\Delta; \mu^*(0, \Delta, r), r, 0, f)$ equal $\Pi^*(\beta, 0) - f$, where $\Pi^*(\beta, 0)$ are the firm's gross profits under full information acquisition. It follows that

$$PR(\Delta; \mu^*(k, \Delta, r), r, k, f) = \Pi^*(\beta, 0) - f - \int_0^k \mu^*(\kappa, \Delta, r) d\kappa.$$

Recall that $\Pi^*(\beta, 0) > 1/4r$, provided that $\beta > 0$. Clearly, if $f > \Pi^*(\beta, 0) - 1/4r$, then the right-hand side of the above equation is negative, and the firm would be better off by not investing in big data at all. Therefore, for investment to take place, it is necessary that $f \leq \Pi^*(\beta, 1) - 1/4r$, and we will focus on this case. Then, for each f in the interval $[0, \Pi^*(\beta, 0) - 1/4r]$, there is a corresponding marginal investment cost parameter value $\overline{k}(f) \geq 0$ such that $PR(\Delta; \mu^*(\overline{k}, \Delta, r), r, \overline{k}, f) = 0$, implying that the monopolist would find it profitable to invest in big data iff $k \leq \overline{k}(f)$.[32]

Since μ^* is a decreasing function of k, there is a critical value $\mu^{Crit}(f) \equiv \mu^*(\overline{k}(f), \Delta, r)$ such that whenever the monopolist finds it strictly profitable to invest in data collection (i.e., whenever $k < \overline{k}(f)$), its optimal accuracy level, $\mu^*(k, \Delta, r)$, is greater than $\mu^{Crit}(f)$. Thus we have proved the following claim:

Claim 4: *Profitable investments in data collection requires to invest an amount of money greater than some critical value* $F(\mu^{Crit}(f)) > 0$.

The following simple example, when $\Delta \to 0$, may be instructive.

Example 1: Let $\Delta = 0$, so that $\Pi^*(0; \mu, r) = \frac{1}{2r} \frac{\mu}{r+\mu}$. Straightforwardly $\underline{\mu} = r$ and $\mu^* = \max\{\frac{1}{\sqrt{2k}} - r, 0\}$. Clearly, the firm cannot benefit from collecting information if $r \geq \frac{1}{\sqrt{2k}} \Leftrightarrow k \geq \frac{1}{2r^2}$.[33] Consider then the

[32] Straightforwardly $\overline{k}(f) \to 0$ as $f \to \Pi^*(\beta, 0) - 1/4r$.

[33] Indeed when $\mu^* = 0$, the monopolist's net profit when collecting the coarsest information is negative whenever f is positive.

case where $k < \frac{1}{2r^2}$. Then $\Pi^*(0;\mu^*, r) - F(\mu^*) = \frac{1}{2r} + rk - \sqrt{2k} - f$. Condition (26) is then equivalent to

$$f < \frac{1}{4r} + rk - \sqrt{2k}, \tag{27}$$

or, equivalently,

$$k \leq \overline{k}(f) = \frac{3 + 4rf - 2\sqrt{2 + 8rf}}{4r^2}. \tag{28}$$

Since k cannot be negative, equation (28) means that the maximum possible value of f that is consistent with profitable investment is $\frac{1}{4r}$.It is easy to see that $\overline{k}(f) < \frac{1}{2r^2}$ so that (28) is necessary and sufficient to ensure that the monopolist invests in profiling.

Notice that, in order to profitably invest in data collection, the monopolist must aim at reaching an accuracy level greater than some critical value. Let us find what this minimum value is. Remembering now that $\mu^* = \frac{1}{\sqrt{2k}} - r$ and using (28) we find that when a profitable investment in big data takes place μ^* has to be greater than

$$\mu^{Crit} = r\left(\frac{\sqrt{2}}{\sqrt{3 + 4rf - 2\sqrt{2 + 8rf}}} - 1\right). \tag{29}$$

This is an increasing function of f which equals $r\left(\sqrt{\frac{2}{3-2\sqrt{2}}} - 1\right)$ when $f = 0$ and tends toward ∞ when $f \to \frac{1}{4r}$. It is in any case strictly greater than $\underline{\mu} = r$.

8 First-stage: Strategic consumers

Consumers' equilibrium surplus (CS) is impacted by the information which the monopolist is able to collect on its previous customers. We have seen that CS equals $1/8r$ when the firm has no information whatsoever about its previous customers, and that CS jumps to its maximum value when the firm obtains the coarsest possible information and then decreases when the information becomes more and more precise. Obviously the consumers have an incentive to influence the degree and the precision of the information which the monopolist can collect.

We now look at the first stage of the game, where consumers can take collective actions in order to maximize expected CS, through lobbying and/or political action which influences legislation on consumers data protection, thereby restraining firms' ability to collect more and more information on customers' preferences and willingness to pay (e.g.

through tighter RGPD legislation).[34] This action, which we assume for the sake of simplicity to be costless, takes place in the initial stage of the game, before the monopolist chooses the amount of money to invest in customers profiling. More precisely, we consider that consumers' action can raise the monopolist's cost of reaching any given level of information accuracy. Considering the simple case of the previous section where $F(\mu) = f + k\mu$, let us assume that in the absence of consumers' collective action, f and k take the value f_A and k_A. (The subscript A means "in the absence of protective legislation"). Consumers' action may result in $f \geq f_A$ and $k \geq k_A$.

The problem would be trivial if it is not profitable to invest in data profiling even in the absence of consumers data protection, i.e., if $k_A > \overline{k}(f)$. So consider the non-trivial case where investing in data profiling is optimal absent any legislation. What can a protective legislation achieve? What equilibria can be reached by appropriately choosing the values of f and k above their natural values f_A and k_A? Given the results in the previous section, possible equilibria include the no information one (yielding $CS = 1/8r$), which may be implemented by enacting a very restrictive data protection legislation (high enough values of f and k), and equilibria where the monopolist invests in profiling. Among the latter, for a given value of f, the consumer surplus is increasing in k (since a higher k implies a smaller $\mu^*(k)$). The highest value of k for which the firm finds it profitable to invest in profiling is $\overline{k}(f)$. This is accordingly the best "information equilibrium" from the consumers' point of view. The expected consumer surplus at this equilibrium is $CS((\Delta; \mu^{Crit}(f), r)$.

Claim 5: *Suppose that* $k_A \leq \overline{k}(f)$, *so that investing in data collection is profitable in the absence of a protective legislation. Then:*

(i) When $1/8r \geq CS((\Delta; \mu^{Crit}(f), r)$, *the best consumers' strategy is to favor a data protection legislation which dissuades any data collection by the monopolist;*

(ii) When $1/8r < CS((\Delta; \mu^{Crit}(f), r)$, *the best consumers' strategy is to favor a value of* $k = \overline{k}(f) \geq k_A$ *such that the monopolist invests in profiling but the information accuracy remains low, with* $\mu^{Crit}(f) < \mu^*(k_A)$; *in that case the protective legislation does not prevent data collection but reduces the precision of the information which is collected.*

Example 1 (continued): Assume for simplicity that the legisla-

[34] An alternative interpretation of the model is that, instead of having a continuum of consumers, there is one consumer whose type is a random variable which is realised only after the legislation on data protection is enacted.

tion can only impact k but not f. As already shown, when $\Delta \to 0$, $CS \to \frac{1}{2r}\frac{r}{r+\mu}$. Given (29), the condition $1/8r < CS((\Delta;\mu^{Crit}(f),r)$ is equivalent to $f < \frac{1}{32r}$. On the other hand, given the constraint $k \geq k_A$, there is a positive investment in data profiling by the monopolist only if $k_A < \frac{3+4rf-2\sqrt{2+8rf}}{4r^2}$. For an equilibrium with strategic consumers to admit positive investment in data collection it is then necessary and sufficient that $f < \frac{1}{32r}$ and $k_A < \frac{3+4rf-2\sqrt{2+8rf}}{4r^2}$. Below, in Figure 7, we picture (for $r = 1$), the areas in the (f,k)-space where the firm invests in profiling. The colored areas (blue and green) correspond to the couples of values of f and k_A for which the monopolist collects consumers information when consumers are passive, the blue area corresponds to the couples of values of f and k_A for which it invests in profiling when consumers are actively engaged in lobbying for protective data protection laws (strategic customers).

(PLEASE PLACE FIGURE 7 ABOUT HERE)

It turns out that consumers' lobbying activity reduces the scope for information collection while not necessarily eliminating it. It can indeed be argued that f being the cost to be incurred simply to collect and to keep the coarsest information (when customers buy the good for the first time), which can be done very simply by hand, f is not likely to be very great. So the most usual effect of protective legislations is probably to reduce the accuracy of the firm's information on its old customers, rather than preventing it from collecting some information.

9 Conclusion

In this paper we have analyzed a dynamic model in which the accuracy of the information that a non-durable good monopolist obtains on its previous customers depends on the amount that it invests in profiling, whereas the consumers influence the efficiency of this investment via lobbying over data protection legislation. Given the precision of information the firm obtains, a dynamic game takes place between it and the consumers.

At one extreme, we have the case of absence of learning: while the monopolist knows the distribution of the willingness to pay (WTP) of his customer base, he does not know the WTP of any particular customer, he cannot tell one consumer from another and does not even know if a given customer has purchased from him before. At the other extreme, we have the case of a monopolist who can discover the customers' exact WTP immediately after they have made their first purchase. In intermediate cases, the monopolist can recognize former customers and remember their first date of purchase, and at any time, the fraction of former customers whose exact WTP remains unknown to him (called the unidentified fraction) is a declining function of the distance between that time and the time of their first purchases. The rate of decline of this fraction is a measure of the precision of the monopolist's information (or, equivalently, the accuracy of his customer profiling technology). If this rate of decline is zero, the monopolist only knows the date of first purchase of each of his former customers. If the rate of decline is infinite, the monopolist discovers each customer's WTP immediately, and is able to extract all their surplus when they return in future periods. If the rate of decline is strictly positive and finite, the monopolists extracts all the surplus of the old customers whose WTP have become known while proposing a vintage-specific price to the other ones.

We have shown that the Markov-Perfect Equilibrium of this model is such that the profit is non-monotone in the degree of precision of the firm's information. Starting from a scenario of absence of learning, profits suffer a downward jump when the monopolist becomes able to group his former customers according to their date of first purchase in order to practice price discrimination. We show that there exists a unique threshold level of precision of information such that as long as the ability to learn about customers' WTP is below that threshold, profit remains inferior to the static monopoly profit. Above this threshold, profit increases when the degree of precision of the information increases, reaching its greatest level when the firm learns instantaneously the WTP of his customers. Interestingly, the aggregate consumers surplus varies exactly in the opposite way. Accordingly, it is not clear that customers' profiling is always good for firms (or bad for consumers).

The outcome of the comparison depends crucially on the accuracy of the technologies used by the firms for consumers' profiling, as well as on the extent of restrictions introduced by consumer protection legislation (which, by limiting the extent of consumer profiling, may shape in important ways the profit, consumer surplus and welfare effects of price discrimination strategies relying on consumers' profiling). We have shown that, in order to be beneficial, the monopolist's investment in data collection, and the resulting degree of precision of its information, must be large enough. In equilibrium the firm never invests in order to obtain only a very coarse information on its old customers. In addition, when the customers influence, via lobbying, the shape of the data protection legislation, we have shown that this may lead to no investment in profiling but that, most likely, it is in consumers' interest to assure that the firm is still interested to invest in customer profiling, though less than the firm would do in the presence of inactive consumers, who do not make any effort to influence the firm's ability to engage in customer recognition.

In this paper, we have studied how pricing dynamics may be affected by firms' ability to uncover some consumers' WTP. In our monopoly setting, this ability allows the monopolist to engage in two types of price discrimination: one based on consumers' differences in preferences (or the cost) for delaying purchases, and the other based directly on a consumer's willingness to pay. It seems natural to look for these possibilities in oligopoly settings, extending the literature on competitive behavior-based price discrimination to situations where firms can also uncover some consumers' WTP (or price elasticity) for the product. This could be especially relevant now as firms in oligopoly markets improve their information technology and acquire data beyond coarse information such as consumer's vintage. While this seems to favor consumers' welfare in a static setting (as already shown by Choe et al, 2018), the consideration of dynamic effects (and the underlying possibilities regarding intertemporal price discrimination) may open the door to new countervailing effects.

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APPENDIX

Proof of the participation constraint (8)

Consider a non-marginal new customer in period n, i.e., a customer whose type is θ, where $\theta_n \geq \theta > \theta_{n+1}$. What is the difference between her expected intertemporal utility and that of the marginal customer θ_{n+1}? Let us denote this difference by

$$D_n(\theta, \theta_{n+1}) \equiv U_n(\theta) - U_n(\theta_{n+1}) \tag{30}$$

To calculate this difference, let us recall that, for any integer $j > n$, the number δ^{j-n} is the probability of the event that the WTP of a period n first-time customer remains undiscovered at the beginning of period j. There are two cases to be considered: Case A (characterized by $\theta_{n+1} \geq (1/2)\theta_n$) and Case B (characterized by $\theta_{n+1} \leq (1/2)\theta_n$). Case A (respectively, B) means that the market expansion in period n is small (resp., big).

From equation (2), we know that in Case A the price charged (in any future period $j > n$) to unidentified former customers of vintage n is $p^{u*}(n, j) = \theta_{n+1}$ and consequently, compared with type θ_{n+1}, all those with WTP $\theta > \theta_{n+1}$ enjoy an excess surplus of $\left(\frac{1-\beta}{r}\right)(\theta - \theta_{n+1})$ per period.[35] Summing across periods (including period n when all new customers are charged a common introductory price) with proper accounting for time and for the probability of becoming identified in the future, we obtain for Case A,

$$D_n(\theta, \theta_{n+1}) = \left(\frac{1-\beta}{r}\right)(\theta - \theta_{n+1})\left(1 + \delta\beta + \delta^2\beta^2 + \delta^3\beta^3 + ...\right) \tag{31}$$

$$= \frac{(1-\beta)(\theta - \theta_{n+1})}{r(1-\delta\beta)} \tag{32}$$

In Case B, the price offered in any future period $j > n$ to unidentified vintage n customers is $p^{u*}(n, j) = (1/2)\theta_n > \theta_{n+1}$, and consequently, whereas in the period n the excess surplus of type θ over type θ_{n+1} is $\left(\frac{1-\beta}{r}\right)(\theta - \theta_n)$ as they are offered an attractive introductory price $p^u(n, n) < \theta_{n+1}$, in later periods $j > n$, the excess surplus is only

[35] In period n, all new customers are offered the same introductory price $p^u(n, n)$ and therefore the difference in surpluses gained by type θ and type θ_{n+1} is also $\left(\frac{1-\beta}{r}\right)(\theta - \theta_{n+1})$.

$\left(\frac{1-\beta}{r}\right)\max\left\{\theta-\frac{\theta_n}{2},\theta\right\}$. Summing across periods, we get for Case B,

$$D_n(\theta,\theta_{n+1})=\left(\frac{1-\beta}{r}\right)(\theta-\theta_{n+1})+ \tag{33}$$
$$+\left[\left(\frac{1-\beta}{r}\right)\max\left\{\theta-\frac{\theta_n}{2},\theta\right\}\right]\sum_{k=1}^{\infty}(\delta\beta)^k$$
$$=\left(\frac{1-\beta}{r}\right)\left[\theta-\theta_{n+1}+\frac{\delta\beta}{1-\delta\beta}\max\left\{\theta-\frac{\theta_n}{2},\theta\right\}\right]$$

Using equations (31) and (33), and the definition (30), we obtain the life-time net utility, $U_n(\theta)$, of a type θ customer (where $\theta\geq\theta_{n+1}$) who makes her first purchase in period n, as equal to the life-time utility of the marginal period n new customer, plus an expected utility differential between the two types over an infinite horizon. The life-time net utility $U_{n+1}(\theta)$ of any type θ who deviates to buy for the first time in period $n+1$ can be defined in a similar way.

Now $U_{n+1}(\theta_{n+1})=\beta(U_{n+1}(\theta_{n+2})+D_{n+1}(\theta_{n+1},\theta_{n+2}))$ implies (8). ■

Proof of Lemma 1. Given the monopolist's cut-off parameter γ, using our conjectured expectations rule (11), the rational expectations requirement, with the help of equation (8), can be rewritten as

$$\lambda\theta_n=\beta\left[\frac{(1-\beta)(\theta_{n+1}-\theta_{n+2})}{r(1-\delta\beta)}+\lambda\theta_{n+1}\right]\text{ if }\theta_{n+2}\geq\frac{\theta_{n+1}}{2}, \tag{34}$$
$$\lambda\theta_n=\beta\left[\frac{1-\beta}{r}\left(\theta_{n+1}-\theta_{n+2}+\frac{(\delta\beta)\theta_{n+1}}{2(1-\delta\beta)}\right)+\lambda\theta_{n+1}\right]\text{ if }\theta_{n+2}\leq\frac{\theta_{n+1}}{2}.$$

Using the conjectured equilibrium cut-off rule, we have $\Theta(n+1)=\gamma\Theta(n)$ and $\Theta(n+2)=\gamma^2\Theta(n)$, then equation (34) becomes

$$\lambda\theta_n=\beta\left[\frac{1-\beta}{r(1-\delta\beta)}\left(\gamma\theta_n-\gamma^2\theta_n\right)+\lambda\gamma\theta_n\right]\text{ if }\gamma\geq\frac{1}{2},$$
$$\lambda\theta_n=\beta\left[\frac{1-\beta}{r}\left(\gamma\theta_n-\gamma^2\theta_n+\frac{(\delta\beta)\gamma\theta_n}{2(1-\delta\beta)}\right)+\lambda\gamma\theta_n\right]\text{ if}\gamma\leq\frac{1}{2}.$$

from which we obtain the "*best response*" of consumers expectations to the monopolist's γ :

$$r\lambda=\frac{\beta\gamma(1-\beta)(1-\gamma)}{(1-\beta\gamma)(1-\beta\delta)}\text{ if }1\geq\gamma\geq\frac{1}{2}, \tag{35}$$
$$r\lambda=\frac{\beta\gamma(1-\beta)(1-\gamma+\frac{\delta\beta}{2(1-\delta\beta)})}{(1-\beta\gamma)}\text{ if }\gamma\leq\frac{1}{2}.$$

For later use, it is convenient to define the function $\Lambda(\gamma, \beta, \delta)$ by

$$\Lambda(\gamma, \beta, \delta) \equiv \frac{\beta\gamma(1-\beta)(1-\gamma)}{(1-\beta\gamma)(1-\beta\delta)} \tag{36}$$

Then the consumers' best reply λ to the monopolist's cut off choice γ can be expressed as follows:

$$\begin{aligned} r\lambda &= \Lambda(\gamma, \beta, r) \text{ if } 1 \geq \gamma \geq \frac{1}{2}, \qquad (37)\\ r\lambda &= \Lambda(\gamma, \beta, r) - \frac{\beta^2(1-\beta)\gamma\delta\,(1-2\gamma)}{2(1-\beta\gamma)(1-\beta\delta)} \text{ if } \gamma \leq \frac{1}{2}. \blacksquare \end{aligned}$$

Proof of Lemma 2.

Maximizing the right-hand side of the Bellman equation (13) with respect to $\Theta(n+1)$, one obtains the first-order condition[36]

$$(1-\beta)(\Theta(n) - 2\Theta(n+1)) + r\lambda\Theta(n) + r\beta V'(\Theta(n+1)) = 0, \tag{38}$$

where V' denotes the derivative of the value function.

From eq. (13), we obtain

$$\begin{aligned} &V(\Theta(n+1)) = \qquad (39)\\ &= \max_{\Theta(n+2)} \{\Pi^F_{n+1} + \Pi^N_{n+1}\,(\Theta(n+1), \Theta(n+2), \lambda) + \beta V(\Theta(n+2))\}. \end{aligned}$$

Differentiating equation (39) with respect to $\Theta(n+1)$, we get

$$V'(\Theta(n+1)) = \frac{\partial \Pi^F_{n+1}}{\partial \Theta(n+1)} + \frac{\partial \Pi^N_{n+1}}{\partial \Theta(n+1)}$$

where

$$\begin{aligned} \frac{\partial \Pi^F_{n+1}}{\partial \Theta(n+1)} &= \frac{\partial \pi^*(n, n+1)}{\partial \Theta(n+1)} \\ &= \begin{cases} -\frac{1-\beta}{r}\left[(1-\delta)\theta_{n+1} + \delta(2\theta_{n+1} - \theta_n)\right] & \text{if } \theta_{n+1} \geq \frac{\theta_n}{2} \\ -\frac{1-\beta}{r}(1-\delta)\theta_{n+1} & \text{if } \theta_{n+1} \leq \frac{\theta_n}{2} \end{cases} \end{aligned}$$

and, from $\Pi^N_{n+1} = [\theta_{n+1} - \theta_{n+2}]\left[\frac{1-\beta}{r}\theta_{n+2} - \lambda\theta_{n+1}\right]$,

$$\frac{\partial \Pi^N_{n+1}}{\partial \Theta(n+1)} = \left(\frac{1-\beta}{r}\right)\theta_{n+2} - \lambda\theta_{n+1} - \lambda\left[\theta_{n+1} - \theta_{n+2}\right]$$

[36]Note that Π^F_n is independent of $\Theta(n+1)$.

Thus we obtain

$$\begin{aligned} V'(\Theta(n+1)) &= -\lambda\left[2\Theta(n+1)-\Theta(n+2)\right]+ \\ \frac{1-\beta}{r}&\left[\Theta(n+2)-(1-\delta)\Theta(n+1)+\delta\left(\Theta(n)-2\Theta(n+1)\right)\right] \end{aligned} \tag{40}$$

if $\Theta(n+1) \geq \frac{1}{2}\Theta(n)$ and

$$\begin{aligned} V'(\Theta(n+1)) = &-\lambda\left[2\Theta(n+1)-\Theta(n+2)\right]+ \\ &\frac{1-\beta}{r}\left[\Theta(n+2)-(1-\delta)\Theta(n+1)\right] \end{aligned} \tag{41}$$

if $\Theta(n+1) \leq \frac{1}{2}\Theta(n)$.

Notice that $V'(\Theta(n+1))$ is continuous at $\Theta(n+1) = \frac{1}{2}\Theta(n)$. Moreover, the RHS of (40) takes on a smaller value than the RHS of (41) when $\Theta(n+1) \geq \frac{1}{2}\Theta(n)$, and conversely, when $\Theta(n+1) \leq \frac{1}{2}\Theta(n)$, the RHS of (41) takes on a smaller value than the RHS of (40). This allows us to express $r\beta V'(\Theta(n+1))$ as

$$\begin{aligned} &-r\lambda\beta\left[2\Theta(n+1)-\Theta(n+2)\right] \\ &+\beta\left(1-\beta\right)\left[\Theta(n+2)-(1-\delta)\Theta(n+1)\right] \\ &+\beta(1-\beta)\delta\min\left\{0,\Theta(n)-2\Theta(n+1)\right\} \end{aligned} \tag{42}$$

Substituting the above expressions for $r\beta V'(\Theta(n+1))$ into the first-order condition (38), we obtain the Euler equation

$$\begin{aligned} 0 = &\left(1-\beta\right)\left[\left(\Theta_n-2\Theta_{n+1}\right)+\beta\left(\Theta_{n+2}-(1-\delta)\Theta_{n+1}\right)\right]+r\lambda\Theta_n \\ &+\beta\delta(1-\beta)\min\left\{0,\Theta_n-2\Theta_{n+1}\right\}-r\lambda\beta\left[2\Theta_{n+1}-\Theta_{n+2}\right] \end{aligned} \tag{43}$$

It is convenient to define the following functions $H(.)$ and $K(.)$ (which apply for the case $\Theta_n - 2\Theta_{n+1} \leq 0$ and $\Theta_n - 2\Theta_{n+1} \geq 0$, respectively):

$$\begin{aligned} H(.) \equiv &\left(1-\beta\right)\left[\left(\Theta_n-2\Theta_{n+1}\right)+\beta\left(\Theta_{n+2}-(1-\delta)\Theta_{n+1}\right)\right]+r\lambda\Theta_n \\ &-r\lambda\beta\left[2\Theta_{n+1}-\Theta_{n+2}\right]+\beta\delta(1-\beta)\left[\Theta_n-2\Theta_{n+1}\right] \end{aligned}$$

and

$$K(.) \equiv H(.) - \beta\delta(1-\beta)\left[\Theta_n - 2\Theta_{n+1}\right]$$

Then the Euler equation may be written as

$$0 = \min\left\{H(\Theta_n,\Theta_{n+1},\Theta_{n+2}), K(\Theta_n,\Theta_{n+1},\Theta_{n+2})\right\}. \tag{44}$$

That is, the function $H(.)$ applies when $\Theta_{n+1} \geq (1/2)\Theta_n$ (the market is expanded by small steps) and the function $K(.)$ applies when $\Theta_{n+1} \leq (1/2)\Theta_n$ (the market is expanded by large steps).

Under the linear cut off rule, the function $H(.)$ applies iff $\gamma \geq 1/2$, in which case we have, from eq. (35), the following value of $r\lambda$:

$$r\lambda = \Lambda(\gamma, \beta, r) = \frac{\beta\gamma(1-\beta)(1-\gamma)}{(1-\beta\gamma)(1-\beta\delta)}$$

Substituting this $r\lambda$ into eq. (44), and replacing $\Theta_{n+1} = \gamma\Theta_n$ and $\Theta_{n+2} = \gamma^2\Theta_n$, we can see that when $H(.)$ applies, we have

$$\begin{aligned} h(\gamma,\beta,\delta) = {} & (1-\beta)\left\{(1-2\gamma) + \beta\gamma\left[\gamma - (1-\delta)\right] + \beta\delta\,(1-2\gamma)\right\} \\ & +(1+\beta\gamma(2-\gamma))\Lambda(.) \end{aligned} \tag{45}$$

Similarly, the function $K(.)$ applies iff $\gamma \leq 1/2$, in which case we have, from eq. (35) the following value of $r\lambda$:

$$r\lambda = \Lambda(\gamma, \beta, \delta) - \frac{\beta^2(1-\beta)\gamma\delta\,(1-2\gamma)}{2(1-\beta\gamma)(1-\beta\delta)}$$

Substituting this $r\lambda$ into eq. (44), and using $\Theta_{n+1} = \gamma\Theta_n$ and $\Theta_{n+2} = \gamma^2\Theta_n$, we we can see that when $K(.)$ applies, we have

$$\begin{aligned} & k(\gamma,\beta,\delta) = h(\gamma,\beta,\delta) - \\ & (1+\beta\gamma(2-\gamma))\left[\frac{\beta^2(1-\beta)\gamma\delta\,(1-2\gamma)}{2(1-\beta\gamma)(1-\beta\delta)}\right] - \beta\delta(1-\beta)\left[1-2\gamma\right] \blacksquare \end{aligned} \tag{46}$$

Proof of Proposition 1.

Lemma 2 tells us that given (β, δ), $k(.)$ as function of γ applies in the range $0 \leq \gamma \leq 1/2$, whereas $h(.)$ as function of γ applies in the range $1/2 \leq \gamma \leq 1$. Comparing (45) with (46) we see that $k(\gamma,\beta,\delta) \geq h(\gamma,\beta,\delta)$ if $\gamma \in [1/2, 1]$ and $k(\gamma,\beta,\delta) \leq h(\gamma,\beta,\delta)$ if $\gamma \in [0, 1/2]$, with $k(1/2,\beta,\delta) = h(1/2,\beta,\delta)$.[37] Furthermore, for all $(\beta,\delta) \in (0,1) \times (0,1)$, we have $h(\gamma,\beta,\delta) > k(\gamma,\beta,\delta)$ if $\gamma \in [0, 1/2)$ and $h(\gamma,\beta,\delta) < k(\gamma,\beta,\delta)$ if $\gamma \in (1/2, 1]$. This shows that, for any MPE in which the monopolist's optimal cut-off rule and consumers' expectation rules are mutual best-replies, either we have (i) $h(\gamma,\beta,\delta) = 0$ where $0 \leq \gamma \leq 1/2$, or we have (ii) $k(\gamma,\beta,\delta) = 0$ where $1/2 \leq \gamma \leq 1$, but not both.

Substituting $r\lambda = \Lambda(\gamma, \beta, r) = \frac{\beta\gamma(1-\beta)(1-\gamma)}{(1-\beta\gamma)(1-\beta\delta)}$ into eq. (45), we get the following fourth-order polynomial in γ

[37] This analytical result is confirmed when using the ManipulatePlot command in Mathematica to plot the RHS of both equations as functions of γ for all possible values of $(\beta,\delta) \in [0,1]^2$.

$$h(\gamma,\beta,\delta) = 1-\beta^2\delta^2-\gamma^4\beta^2+\gamma^3\beta^2(2+\beta\delta)+ \gamma^2\beta\left(2-\beta-2\beta\delta-\beta^2\delta-\beta^2\delta^2\right)+ \gamma\left(-2-\beta+\beta\delta+\beta^2\delta+\beta^2\delta^2+\beta^3\delta^2\right)=0 \tag{47}$$

Then, given any pair (β,δ) where $0\le\beta\le 1$ and $0\le\delta\le 1$, any real root $\gamma^*\in[1/2,1]$ of (47) is the monopolist's equilibrium cut off rule, and the consumers best response to γ^* is given by $r\lambda^*=\Lambda(\gamma^*,\beta,\delta)$.

Similarly, iff $\gamma\le 1/2$, we have, from eq. (35) :

$$r\lambda=\Lambda(\gamma,\beta,\delta)-\frac{\beta^2(1-\beta)\gamma\delta\,(1-2\gamma)}{2(1-\beta\gamma)(1-\beta\delta)}$$

Substituting this $r\lambda$ into eq. (46), we obtain the following fourth-order polynomial in γ:

$$k(\gamma,\beta,\delta) = \frac{1}{2}[2-2\beta\delta-\gamma^4\beta^2(2-2\beta\delta)+\gamma^3\beta^2(4-3\beta\delta)- \gamma^2\left(-4\beta+2\beta^2+6\beta^2\delta-2\beta^3\delta^2\right)- \gamma\left(4+2\beta-6\beta\delta-3\beta^2\delta+2\beta^2\delta^2\right)]=0 \tag{48}$$

Then, given any pair (β,δ) where $0\le\beta\le 1$ and $0\le\delta\le 1$, any real root $\gamma^{**}\in[0,1/2]$ of (48) is the monopolist's equilibrium cut off rule, and the consumers best response to γ^{**} is given by

$$r\lambda^{**}=\Lambda(\gamma^{**},\beta,\delta)-\frac{\beta^2(1-\beta)\gamma^{**}\delta\,(1-2\gamma^{**})}{2(1-\beta\gamma^{**})(1-\beta\delta)}\blacksquare$$

Proof of Remark 1:

We can use the equation $h(1/2,\beta,\delta)=0$ to characterize the subset S of points (β,δ) in the unit square $[0,1]\times[0,1]$, such that $\gamma(\beta,\delta)=1/2$. It is easy to verify that the set S can be represented by a curve $\delta=\Phi(\beta)$, where (i) $\Phi(\beta)\equiv\frac{4-\beta^2-\sqrt{16(1-\beta^2)+\beta^2(4+\beta)}}{4\beta(2-\beta)}$ for $\beta\in(0,1]$, (ii) $\Phi'(\beta)>0$, (iii) $\lim_{\beta\to1}\Phi(\beta)=(3-\sqrt{5})/4\equiv\overline{\delta}<1$, and (using L'Hospital's rule) $\lim_{\beta\to0}\Phi(\beta)=0$. It follows that $\gamma(\beta,\delta)<1/2$ (i.e., the market expands in large steps) for all points (β,δ) that lie below the curve, and $\gamma(\beta,\delta)>1/2$ for all points (β,δ) that lie above the curve.[38] $\blacksquare$

[38]It obviously follows from Remark 1 that $\gamma^e\left(e^{-r\Delta},e^{-\mu\Delta}\right)\to1/2$ as $\Delta\to+\infty$.

Proof of Remark 2.

In the limiting case when $\Delta \to 0$, we have $\delta \to 1$ and $\beta \to 1$; thus, by continuity, it follows that $\delta > \Phi(1) \geq \Phi(\beta)$ if Δ is small enough. We conclude that, for any given pair $(r, \mu) > (0, 0)$, we have $\gamma^e(r, \mu, \Delta) \geq 1/2$ (the market expands in small steps) for sufficiently small values of Δ. ■

Proof of Claim 1.

The monopolist's equilibrium cut off strategy implies a market expansion rule: by the end of period n, where $n = 0, 1, 2, ...$, the fraction of the customer base that has not been served is $(\gamma^e(\beta, \delta))^{n+1}$. The measure of customers that have been served by the end of period n is $(1 - (\gamma^e)^{n+1})$. Given r and μ, we consider β and δ as functions of Δ, hence we can define the function $\gamma(\Delta) = \gamma^e(\beta(\Delta), \delta(\Delta))$. Let $t = n\Delta$. Then $(\gamma^e(\beta, \delta))^{n+1} = (\gamma(\Delta))^{\frac{t}{\Delta}+1})$. Since $\lim_{\Delta \to 0} \gamma(\Delta) = 1$ (using eq.(47)), we have $\lim_{\Delta \to 0}(\gamma(\Delta)^{\frac{t}{\Delta}+1}) = \lim_{\Delta \to 0}\left(\gamma(\Delta)^{\frac{t}{\Delta}}\right)$. Notice that $\ln \gamma(\Delta)^{\frac{t}{\Delta}} = \frac{t}{\Delta} \ln \gamma(\Delta)$. In order to determine the limit value of $\frac{\ln \gamma(\Delta)}{\Delta}$ when $\Delta \to 0$, we have to use L'Hospital's rule and evaluate the ratio of the derivatives of the numerator and the denominator at $\Delta = 0$. This ratio turns out to be equal to $\gamma'(0)$. It follows that $\lim_{\Delta \to 0}\left(\gamma(\Delta)^{\frac{t}{\Delta}}\right) = e^{\gamma'(0)t}$.

Now let us determine $\gamma'(0)$. Write the Euler equation (47) as

$$E(\Delta, \gamma(\Delta)) = 0.$$

Differentiating this equation (an identity in Δ) with respect to Δ, we obtain $E'_\Delta(\Delta, \gamma(\Delta)) + E'_\gamma(\Delta, \gamma(\Delta))\gamma'(\Delta) = 0$. Remembering that $\gamma(0) = 1$, it turns out that the ratio $-\frac{E'_\Delta(\Delta, \gamma(\Delta))}{E'_\gamma(\Delta, \gamma(\Delta))} \to -\infty$ as $\Delta \to 0$.[39] It follows that $\lim_{\Delta \to 0}(\gamma(\Delta)^{\frac{t}{\Delta}+1}) = 0$.■

Proof of Claim 2.

From Remark 2, we only need to consider the value of profits as given by (17) (where, of course, $\beta = e^{-r\Delta}$ and $\delta = e^{-\mu\Delta}$). Since $\lim_{\Delta \to 0} \Pi^*(\Delta; r, \mu)$ is undefined, we have to use L'Hospital's rule and evaluate the ratio of the derivatives of the numerator and the denominator at $\Delta = 0$. This ratio equals

$$\frac{\gamma'(0)\left[-r(r+\mu) + \mu\gamma'(0)\right]}{r(r+\mu)(r - 2\gamma'(0))(r - \gamma'(0))}$$

which itself equals $\frac{\mu}{2r(r+\mu)}$ since, from Claim 1, $\gamma'(0) = -\infty$. ■

[39] More precisely this ratio tends toward N/D where $D \equiv (1 - \gamma(0))(4\gamma(0) - 1)$ and $N \equiv -\mu\left[2 + (\gamma(0) - 4)\right]\gamma(0) + r(\gamma(0) - 1)[2 + \gamma(0)(2\gamma(0) - 3)]$.

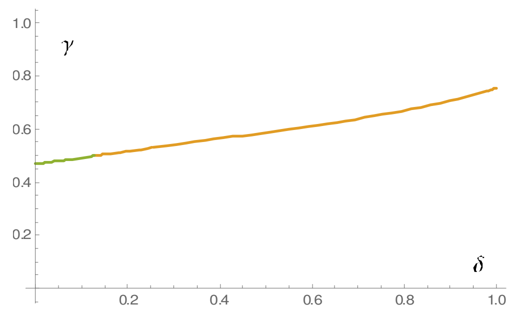

Figure 1: $\gamma^*(0.8, \delta)$

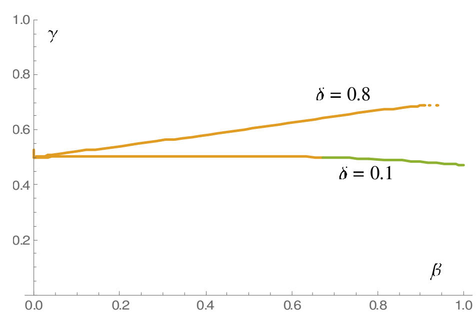

Figure 2: γ^* as a function of β

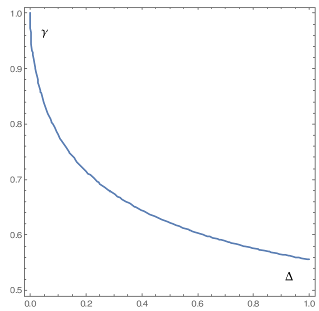

Figure 3: γ^* as a function of Δ

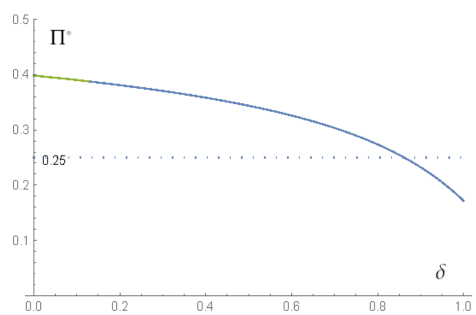

Figure 4a: $\Pi^*(0.8, \delta)$

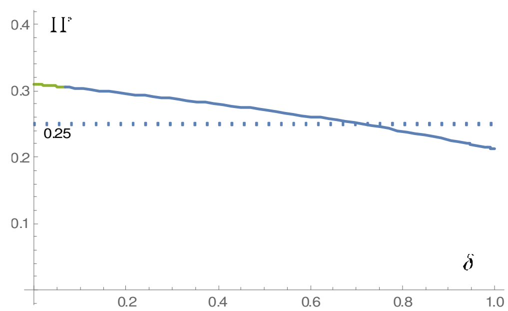

Figure 4b: $\Pi^*(0.5, \delta)$

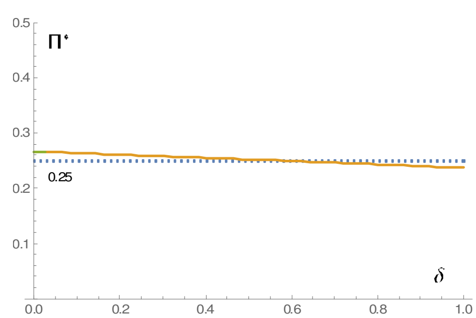

Figure 4c: $\Pi^*(0.2, \delta)$

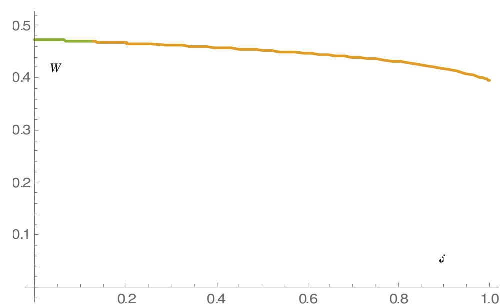

Figure 5: $W(0.8, \delta)$

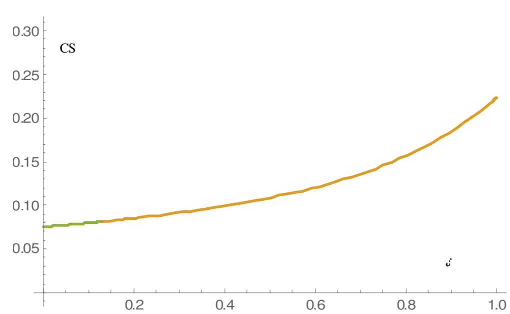

Figure 6: $CS(0.8, \delta)$

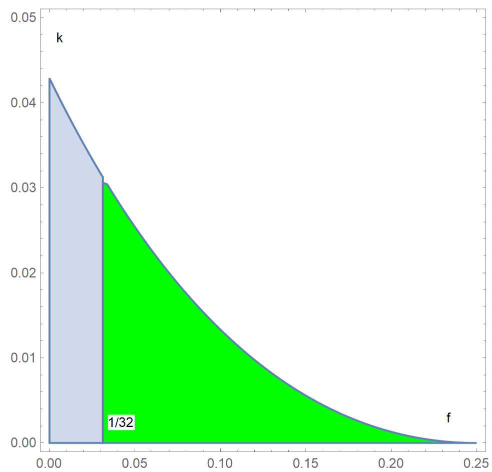

Figure 7: The positive profiling investment area with and without strategic customers