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Abstract

We study the consequences of a working time reduction (WTR hereafter) in a growth model with efficiency wages and an essential natural resource (natural capital). Considering that technical progress cannot reduce the resource content of final production to zero, we show that the effects of a WTR on (un)employment depend on the abundance of natural capital. If it is unlimited, the economy converges toward a balanced growth path and a WTR lowers output, employment and wage levels along this path. With finite natural capital, the economy converges toward a stationary state. A WTR then increases the hourly wage and employment if natural capital is scarce enough, which is necessarily the case if technical progress on produced capital and labour is unbounded. The long-term elasticity of employment (resp., of the hourly wage) to the cut in hours is larger (resp., smaller) when natural capital is scarcer. A numerical analysis of the transitory impacts of a WTR confirms that when natural capital is scarcer, it increases employment more and the hourly wage less, with a less negative initial impact on output.

JEL-Codes: J680, O440, Q570.

Keywords: unemployment, fair wage, work sharing, (limits to) growth, natural capital.

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1 Introduction

Unemployment remains chronically or structurally present all over the globe. Its causes and the alternative policy interventions to curb it have been analysed by many scholars. Among these policies, the reduction of working time (WTR hereafter) has sometimes been advocated and implemented, typically with mixed results. In particular, according to the theoretical literature, a positive effect of a WTR on employment is at best a short run result that does not hold in the long run, i.e. once capital accumulation (or entry of firms) is taken into account (see Section 2). Existing modelling of the effects of a WTR however neglects environmental constraints[1] and overlooks the fact that human production consumes natural resources. This paper shows that if these resources are an essential productive input, their abundance is a key determinant of the effect of a WTR policy on employment. If they become scarce enough, a WTR favours employment both in the short and long runs.

In a very aggregate model, the term "(natural) resource" must be understood in a broad sense: it is an aggregate of all the useful and available resources. As in e.g. van Geldrop and Withagen [2000] and Smulders et al. [2014], the resource we consider here is *natural capital*. The OECD Glossary of Statistical Terms (http://stats.oecd.org/glossary) defines it as all the "natural assets in their role of providing natural resource inputs and environmental services for economic production. (...) Examples are mineral deposits, timber from natural forests, and deep sea fish." Even though descriptive realism might justify distinguishing different resource types (like energy and non energy resources and/or renewable and non-renewable resources), restricting ourselves to a unique resource aggregate amounts to assuming that all resource types are perfect substitutes. This assumption is, admittedly, optimistic but it certainly does not exacerbate the constraints imposed by the finite nature of resources and may therefore be seen as a conservative choice in this respect.

We develop, calibrate and simulate a one-sector deterministic Ramsey growth model with unemployment and natural capital. Unemployment is endogenous in the tradition of the efficiency wage literature, and more specifically the gift-exchange version of it (Akerlof, 1982). Firms set their production plan, wages, hours worked, headcount employment and capital use, while workers choose their work effort, consume and save. Natural capital is exploited and transformed by produced capital and labour (agent inputs for short)[2]. An exogenous technical progress makes the production process less resource intensive but its potential is limited: the resource intensity of output decreases through time but cannot become

[1] Jackson and Victor [2011], Victor [2012]) have nonetheless analysed WTR policies in a context of low growth or degrowth. But in their approach, the aggregate number of hours worked does no react to a change in working time.

[2] We use here the taxonomy of Anderson (1987). "The agent inputs are the actual performers of the production, or the actors of the process" (p.1) whereas material and energy inputs are transformed during this process.

zero. Another exogenous technical progress increases the productivity of agent inputs.

We first study the long-run properties of this economy successively without and with resource limits. With unlimited natural capital, the long-run equilibrium is a balanced growth path when technical progress on agent inputs is unlimited. A WTR would decrease the levels of output and employment along this path. With finite natural capital, the long-run equilibrium is a steady-state with a finite output level that depends on the available resource stock. In this case, a WTR leads to a less severe economic contraction (if any) than in an economy where the resource constraint is totally relaxed; as a corollary, its effect on employment is more favorable and positive if the output effect is weak enough. With unlimited technical progress on agent inputs, the steady-state level of output is unaffected by the duration of working time. A WTR then necessarily reduces the long-run unemployment rate and does more so when natural capital is scarcer: with a lower stock of natural capital, the employment rate is lower and a WTR increases the hourly wage less and is more favourable to employment. A WTR is furthermore welfare enhancing. If technical progress on agent inputs is instead bounded, a WTR increases employment if natural capital is scarce enough, i.e. if its exploitation rate is high enough.

We analyse the transitory dynamics of the model and the impact of a WTR numerically. In a benchmark case calibrated on world data and with unlimited technical progress on agent inputs, a WTR has a positive effect on employment during the whole transitory dynamics and this effect is stronger in periods where the employment rate is lower. A sensitivity analysis confirms that when natural capital is scarcer, a WTR increases employment more with a less negative initial impact on output.

The rest of the paper is organised as follows. Section 2 summarises the literature about the effects of a WTR. Section 3 presents the theoretical framework, and Section 4 its solution. The latter section also presents the properties of a WTR in a steady state. Section 5 develops the numerical analysis of the dynamic adjustment of the modelled economy. Section 6 concludes.

2 Literature Review on Working Time Reduction

Since the eighties, the impacts of the regulation of working time has been studied in a range of theoretical set-ups without natural resource and most often without produced capital (or without capital adjustments). In the efficiency wage literature, firms choose labour demand and the wage level subject to a relationship describing in-work effort or quit behaviour as a function of some model-specific determinants. In chapter 10 of their book on unemployment, Layard et al. [1991] develop a streamlined model with homogeneous workers and firms where these determinants are the unemployment rate and an indicator of the generosity of the pay policy of the firm. Layard et al. [1991] assume that this generosity is measured by the ratio between the real wage paid by the firm and the average wage in the economy (supposed to be the reference level for the workers). In a symmetric equilibrium, this ratio is equal to one, which implies that the in-work effort level is independent of the wage level. This property makes, by construction, the equilibrium unemployment rate independent of working time (see Layard et al., 1991, p. 503). The key assumption that underlies this result can however be questioned. For instance, Danthine and Kurmann [2004] argue that "the positive incentive effect of a larger own wage is stronger than the negative effect of a higher comparison wage" (p. 112). It can be checked that in a model à la Layard et al. with an effort function implying a positive effect of the wage level on the in-work effort *in equilibrium,* work-sharing can reduce unemployment. Before Layard et al., Hoel and Vale [1986] studied the effects of a WTR in an efficiency wage model with quit behaviour and training costs. The assumptions they make on the quit rate function are *mutatis mutandis* equivalent to those made by Layard et al. on their effort function: the quit rate depends negatively on both the unemployment rate and the ratio between the wage paid by the firm and the average wage in the economy. Consequently, the quit behaviour of workers in a symmetric equilibrium depends on the unemployment rate but not on the wage level. Because work-sharing increases training costs, the authors then conclude that a WTR affects (un)employment unfavourably. In the final discussion of their paper, they however consider an alternative specification for the quit behaviour, which leads them to conclude their analysis cautiously.

In a model without capital, Rocheteau [2002] mixes a search and matching framework and a moral hazard problem (workers having the possibility to shirk on the job). He shows that the impact of a WTR depends on whether efficiency wage considerations matter or not: a WTR stimulates employment when the no-shirking-constraint is binding (when unemployment is high enough); but it always worsens the labour market situation when unemployment is initially low enough.

The literature on collective bargaining has also analysed the consequences of work-sharing. In the partial equilibrium settings (see e.g. Calmfors, 1985, Booth and Schiantarelli, 1987, and Booth and Ravaillon,1993), a WTR affects employment negatively or at best ambiguously. In a highly stylised model of a unionised economy, Layard et al. [1991] conclude in the same way as in their efficiency wage model: a WTR does not affect the equilibrium unemployment rate (see p. 503-4). In a much more general and dynamic framework with endogenous labour market participation and an endogenous number of firms, Cahuc and Granier [1997] obtain a possible positive effect of work-sharing on employment at given number of firms but reach the same overall conclusion as Layard et al. [1991] once the number of firms has adjusted. In a search and matching model with bargaining over wage and hours, Marimon and Zilibotti

[2000] conclude that a sufficiently small reduction in working hours below its *laissez-faire* value may have a favourable effect on equilibrium employment at given capital stock. But it is no longer the case once capital becomes endogenous. Hence, for the authors, “the positive employment and welfare effects which may materialise in the short run are likely to vanish as firms adjust their productive capacity.” (p. 1310.)

To sum up, the theoretical literature provides mixed conclusions about the effect of a WTR on (un)employment. Moreover, when it prevails, the result of a positive employment effect of work-sharing typically appears as a short-term one: it is obtained in frameworks in which produced capital is neglected or taken as given but it is not confirmed once produced capital (or the number of firms) becomes endogenous as in the last two quoted papers. We show that the presence of an essential natural resource may lead to a different conclusion. More precisely, when the resource is scarce enough, a WTR favours employment both in the short and long runs.

Several reforms have historically cut working hours in order to hopefully curb a rise in or a high level of unemployment. All in all, the empirical evaluation literature[3] indicates that reductions of the standard working time are often, by law or not, accompanied by increases in hourly wages and their net impact on employment is typically gloomy except in specific contexts (strong recessions) or when they are accompanied by other reforms. This empirical literature is obviously exploiting data of periods where the environmental constraints were less acute than currently and in the future. Hence, in the context of the present study, its conclusions are at best weakly informative.

3 The Economy

This section develops a growth model of a world economy with natural resource. As mentioned in the introduction, we interpret the resource as natural capital. Unless explicitly stated otherwise, we use the terms “natural capital” and “resource” interchangeably. The impact of resource finiteness on the evolution of aggregate economic activity has been the subject of numerous theoretical contributions. They initially focused on the problem raised by exhaustible resources[4] and then broadened it to include renewable resources.[5] In a nutshell, long-run economic growth has been shown to be possible in a world with finite resources if agent inputs are sufficiently good substitutes for those resources[6] and/or if the potential

[3]See e.g. Hunt [1999], Crépon and Kramarz [2002], Skuterud [2007], Raposo and van Ours [2010].

[4]Many articles followed the special 1974 issue of the Review of Economic Studies on this subject. Among recent references, see Mitra et al. [2013] and Section 2 of the contribution of Hassler et al. [2016] to the Handbook of Macroeconomics.

[5]See e.g. van Geldrop and Withagen [2000] and Akao and Managi [2007].

[6]In endogenous growth models with natural resources, the assumption of good substitution possibilities between man-made inputs and resources is sometimes hidden in the assumption that the innovation or accumulation process responsible for growth is not resource consuming, contrary to the other production sectors of the economy. See for instance Bretschger and Smulders [2012] or Peretto and Valente [2015] where the production of innovations responsible for long term growth

of resource saving technical progress is not limited. While it may be(come) possible to replace some resources by others (notably non-renewables by renewables), several authors have however put forward that both the possibilities of substitution of natural resources by man-made inputs and the potential of resource saving technical progress are bounded by the physical laws that govern any transformation of matter or energy, including the production of goods and services.[7] To put it in a non-technical way, these bounds mean that human productions cannot become completely dematerialised and non-energy consuming. Even in an idealized world where certain types of production are stripped of all material content and where economic growth would only be based on the production of these immaterial services, any additional production of these services would remain energy consuming.[8] Our model takes the two above limits into account. In a framework with a very aggregate resource concept, these limits mean that the resource content of one unit of output cannot become nil or, equivalently, that the marginal productivity of the resource cannot become infinite. With a finite stock of natural capital, the economy then tends towards a stationary state with a finite income level per capita (see Section 4).

The economy we model consists of three markets: perfectly competitive good and capital markets and an imperfectly competitive labour market. The final good is produced by identical firms which exploit and transform natural capital. These operations require agent inputs (for the record, labour and produced capital). As in a.o. Dasgupta and Heal [1979], these operations become more intensive in agent inputs and therefore more costly when the exploitation rate of the stock of natural capital is higher.

The production process is improved through time by two forms of exogenous technological progress: one increases the productivity of agent inputs; the other one makes final production less resource intensive, i.e. reduces the quantity of natural capital needed to produce one unit of final good. This resource saving technical progress is however bounded for the reasons detailed above.

Labour is homogeneous but working time, work effort and headcount are distinguished. Firms decide over hours worked,[9] labour demand and wages. Wage setting is modelled in the tradition of the efficiency wage literature, and more specifically the so-called gift-exchange version of it (Akerlof, 1982): the employees of a firm choose their (non-contractible) work effort. The latter depends on the firm's wage policy and on working conditions in the rest of the economy. Three considerations motivate this model-

does not consume any resource unit (either directly or indirectly).

[7] See a.o. Anderson [1987]. Considering production functions combining agent and material resource inputs, Baumgärtner [2004] shows that functions that verify the Inada conditions (and so make possible an infinite resource productivity) are inconsistent with the law of mass conservation. Complementarily, Meran [2019] shows that thermodynamic constraints imply that the productivity of energy and the energy saving technical progress are necessarily bounded.

[8] Note furthermore that this growing immaterial production is likely to consume material resources indirectly because the development of the infrastructures of production or distribution of these services cannot itself be totally dematerialized.

[9] Mishra and Smyth [2013], Pencavel [2016], Pencavel [2018] and Bell and Blanchflower [2021], among others, provide supporting evidence of the key role of employers in the choice of working hours.

ling choice. First, various experiments provide support for the gift-exchange mechanism.[10] Next, several contributions to the efficiency wage literature have analysed the effects of a WTR (see section 2) and provide an interesting point of comparison. Finally as we model a world economy, we avoid introducing unions and labour market institutions (such as employment protection legislation, minimum wages,...) that are present in some rich countries but are not generalised worldwide. The imperfectly competitive wage setting leads to a positive equilibrium unemployment rate, which is affected by the resource scarcity through its impact on economic activity and labour demand.

If our model describes economic activity as a consumer of natural capital, we however disregard pollution although it is the other channel through which economic activity deteriorates natural capital. Our purpose is not to downplay this problem but to put forward that the effect a WTR can have on employment in an economy with finite resource holds true even if the impacts of pollution are ignored.

3.1 Families of Consumers-Workers

Since our focus is not on income inequality, we model the worker-consumer side of the economy in a way that neutralises the problem of income inequality between employed and unemployed people (see e.g Danthine and Kurmann, 2004). We consider that the economy is populated by M families, each sharing the income of its n_t infinitely-lived members in any period t. The population size (understood as the labour force) is therefore $P_t = M\, n_t$. When employed, a family member supplies h_t work hours and devotes an in-work effort level e_t; she is paid at an hourly real wage w_t, with h_t and w_t chosen by firms (see later). When unemployed, she earns no income. We assume that unemployment is equally spread across families so that the proportion of unemployed in a family is equal to the aggregate unemployment rate. In each period, the number of workers by family is therefore given by $L_t/M \leq n_t$ where L_t is the aggregate employment level, with $L_t \leq P_t$.

Decisions are taken at the family level. A family consumes and saves by accumulating produced capital. The depreciation rate of produced capital is unitary because the length of a time period is long enough (15 years in the numerical section below).[11] The capital stock $\tilde{k}_t$ accumulated by a family at the end of period $t-1$ is rent to firms during period t at a real rental price v_t. In period t, a family enjoys a capital income $v_t\widetilde{k}_t$ and a labour income $w_t h_t L_t/M$. If the final good is chosen as *numéraire*, the budget

[10] Laboratory experiments generally find that on average workers exert additional effort when they are offered a higher wage rate (see e.g. Fehr et al. [1993] and Brandts and Charness [2004]). Field experiments lead to more mixed conclusions. For instance, Gneezy and List [2006] conclude that this effect is present but only temporary. However, Cohn et al. [2015] find strong and lasting support for the gift-exchange hypothesis.

[11] The same assumption is made e.g. by Hassler et al. [2016], p. 1908.

constraint of a family in period t is given by

$$c_t + \widetilde{k}_{t+1} = v_t \widetilde{k}_t + w_t h_t \frac{L_t}{M}, \qquad \forall t = 1, ..., T \tag{1}$$

where c_t is the consumption level of the family and $\tilde{k}_{t+1}$ is the capital accumulated at the end of t.

A family chooses c_t, $\tilde{k}_{t+1}$ and the work effort of its employed members, e_t, so as maximise the discounted sum of the instantaneous utility of its average member:

$$\max_{\{c_t, \tilde{k}_{t+1}, e_t\}_{t \geq 1}} \sum_{t=1}^{\infty} \beta^t \left[\ln\left(\frac{c_t}{n_t}\right) - \left[\left[e_t - g\left(w_t, \overline{w}_t, u_t\right)\right]^2 + d(h_t) \right] \frac{L_t}{P_t} \right], \tag{2}$$

subject to (1) where k_1 is given and β ($\in]0,1[$) is the discount factor. The multiplicative term in front of L_t/P_t measures the disutility of work hours and effort for an employed member of the family. Along the idea of Akerlof [1982], workers have a socially-built norm of a fair level of non contractible effort. Collard and de la Croix [2000], Danthine and Kurmann [2004], Danthine and Kurmann [2007] and de la Croix et al. [2009] among others integrated this mechanism in the analysis of business cycles. We follow their specification by assuming in (2) that the utility level reached in each period is affected by a quadratic loss function of the discrepancy between the actual work effort and the effort level that is considered as fair, namely the function $g(.)$. Following Akerlof [1982], this fair effort level is rising in the gap between the returns proposed by the firm (its wage) and "the returns to other persons in the workers' reference sets" (p. 557). These returns increase with the average wage $\bar{w}_t$ in the economy and with the chance of being employed, i.e. 1 minus the unemployment rate $u_t = 1 - L_t/P_t$. Function $d(h_t)$ measures the disutility of work hours. It is increasing and convex: $d'(.) > 0, d''(.) \geq 0$. Specification (2) implies that the utility function is separable in consumption and effort as well as working time.[12]

The optimality conditions of problem (2) straightforwardly lead to

$$\frac{c_{t+1}}{c_t} = \beta v_{t+1} \tag{3}$$

$$e_t = g\left(w_t, \overline{w}_t, u_t\right). \tag{4}$$

Condition (3) describes the standard consumption smoothing behaviour, the rental price of capital v_{t+1} being equivalent (when the depreciation rate is unitary) to 1 plus the real interest rate. Combining (3) and (1) gives the evolution of the capital stock, which must also verify the transversality condition. The optimal effort level is given by (4) and has the properties of function $g(\cdot)$.

[12] We discuss in section 4.3.3 the consequences of assuming that the fair effort level $g(\cdot)$ also depends on working time.

3.2 Final good sector

The final good sector consists of N perfectly competitive and identical firms. In order to keep the model as simple as possible, we model the production technology in a way that does not require to describe the allocation of agent inputs to the resource exploitation on the one hand and to the resource transformation into final production on the other hand. The production process is described by two relationships: 1) In order to produce y_t units of final good, a firm needs x_t units of natural capital, with x_t given by

$$x_t = \mu_t y_t \tag{5}$$

where $\mu_t > 0$ is the resource content of one unit of output and is exogenous at the firm level. 2) In order to exploit and transform x_t units of natural capital, a firm also needs agent inputs, i.e. produced capital k_t and labour. The labour input is three-dimensional: the number of employed workers, l_t, their effort at work e_t and the length of the working time h_t. We assume that the exploitation of natural capital is characterised by increasing marginal costs: it becomes increasingly intensive in agent inputs when the exploitation rate of natural capital, E_t, rises. Variable E_t is defined as

$$E_t =_{\text{def}} X_t/R_t \quad \text{with} \quad 0 \leq E_t \leq 1, \tag{6}$$

where variable R_t is the stock of natural capital at the beginning of period t and variable X_t is the aggregate consumption of this stock in t.

With the technology $\mathcal{F}_t$ available in period t, a combination of agent inputs (k_t, l_t, h_t, e_t) allows the representative firm to capture and transform x_t units of natural capital, with x_t given by

$$x_t = \frac{\mathcal{F}_t(k_t, l_t, h_t, e_t)}{B(E_t)}. \tag{7}$$

Function $\mathcal{F}_t$ is strictly increasing and concave in its arguments. Function $B(E_t)$ is strictly increasing in E_t and its presence in equation (7) reflects that the exploitation of natural capital becomes increasingly intensive in agent inputs when E_t rises: with a higher E_t, a given combination of agent inputs (k_t, l_t, h_t, e_t) captures less units of natural capital. More formally said, we assume that $B(0) = 1$, $B'(E) > 0$, $B''(E) > 0$ and $B(1) \to +\infty$.

Appendix A.1 rationalises the following functional form for $\mathcal{F}_t$:

$$\mathcal{F}_t(k_t, l_t, h_t, e_t) = \eta_t U(h_t) k_t^{\alpha} \left[e_t l_t\right]^{1-\alpha} \quad \text{with} \quad 0 < \alpha < 1 \tag{8}$$

where $\eta_t > 0$ is the exogenous total productivity factor of agent inputs. Function $U(h)$ is increasing in h. As Appendix A.1 shows, it reflects the positive effect of working time on both the quantity of labour input (at given e_t and l_t) and the use of productive capital.

As E_t is a macroeconomic variable that each firm perceives as independent of its own decisions, (7) and (8) imply that returns-to-scale are constant at the micro level. The optimal size of a final firm is therefore indeterminate. Each firm minimises the cost of a given output level and chooses accordingly (i) its capital and resource requirements, (ii) its employment level, (iii) the length of the working time and (iv) the hourly wage it offers so as to induce the appropriate work effort. Given (5), the choice of x_t is tight to the output level y_t and optimisation bears on the choice of k_t, l_t, h_t and w_t. Combining (5) and (7) leads to the following relationship:

$$y_t = \frac{\eta_t}{\mu_t B(E_t)} U(h_t) k_t^{\alpha} \left[e_t l_t\right]^{1-\alpha}. \tag{9}$$

A parallel can be drawn between (9) and the technological relationship in Hassler et al. [2016] (e.g. p. 1938-9). In both cases but via a different mechanism, a stronger pressure of human activities on the Earth's capacity, either as a resource provider (in our case) or as a receptacle of pollution (in Hassler *et al*), has a negative impact on the productivity of agent inputs.

In each period t, the representative firm therefore chooses k_t, l_t, w_t, h_t so as to minimise $v_t k_t + w_t h_t l_t$ under constraint (9) and (4). Appendix A.2 shows that the optimality conditions with respect to k_t, l_t, w_t and h_t can be written respectively as:

$$v_t k_t = \alpha\, y_t \tag{10}$$

$$w_t h_t l_t = [1-\alpha]\, y_t \tag{11}$$

$$\frac{w_t}{e_t} \frac{\partial g\left(w_t, \bar{w}_t, u_t\right)}{\partial w_t} = 1 \tag{12}$$

$$h_t \frac{U'(h_t)}{U(h_t)} = 1-\alpha. \tag{13}$$

Conditions (10) and (11) imply that capital and labour shares stay constant. Condition (12) gives the optimal wage policy and is a modified Solow condition (Solow, 1979). Condition (13) has a unique solution $\bar{h}$ if, as we henceforth assume, the elasticity of $U(h)$ with respect to h is decreasing in h from a value larger than $1-\alpha$ at $h=0$ toward 0 when $h \to +\infty$.[13] The choice of a constant value of $h_t = \bar{h}$ follows from the

[13] This assumption is compatible with various functional forms for U, including the case of an always concave function or of a function first convex, next concave for sufficiently large values of h. It excludes an isoelastic function but this exclusion is only necessary in the case of a Cobb-Douglas assumption for $\mathcal{F}_t$.

Cobb Douglas assumption in (9) and can be understood by considering the marginal choice made by a firm between the number of work hours and the number of workers. Increasing hours (resp. employment) by 1% raises the wage bill by 1% and output by a percentage equal to the elasticity of $\mathcal{F}_t(k_t, l_t, h_t, e_t)$ with respect to hours (resp. employment). At the lowest cost, hours and employment must therefore be chosen in a way such that the elasticity of $\mathcal{F}_t$ with respect to hours is equal to the elasticity of $\mathcal{F}_t$ with respect to employment. Since the latter elasticity is equal to $1-\alpha$ in the Cobb-Douglas case, the former has to be equal to the same constant.

A resource saving technological progress reduces μ_t over time with, for the reasons explained in the introduction to this section, a lower bound:

$$\lim_{t \to +\infty} \mu_t = \underline{\mu} > 0. \tag{14}$$

A second form of technological progress raises the productivity of agent inputs: $\eta_t \leq \eta_{t+1}, \forall t$. If nothing guarantees that such productivity gains could go on without limit, their boundedness is not explicitly rooted in physical laws and is not systematically assumed in our analysis.

3.3 Macroeconomic Equilibrium

At the aggregate level, final output, resource consumption, employment and produced capital are respectively given by $Y_t = Ny_t$, $X_t = Nx_t = N\mu_t y_t = \mu_t Y_t$, $L_t = Nl_t$ and $K_t = Nk_t$. Given (5), the aggregation of the technological relationship (7) leads to the following macro relationship

$$Y_t = \frac{\eta_t}{B(E_t)\mu_t} U(h_t) K_t^{\alpha} \left[e_t L_t\right]^{1-\alpha} \tag{15}$$

At the macroeconomic level, (10) and (11) become:

$$\begin{aligned} v_t K_t &= \alpha Y_t && (16) \\ w_t h_t L_t &= [1-\alpha] Y_t && (17) \end{aligned}$$

The capital market clearing requires that $K_t = Nk_t = M\widetilde{k}_t$. On the final good market,

$$Y_t = C_t + K_{t+1} \tag{18}$$

where $C_t = Mc_t$ is aggregate private consumption and K_{t+1} is investment (recall the assumptions of a unitary depreciation rate and of a period time to build). Furthermore C_t must verify (3):

$$\frac{C_{t+1}}{C_t} = \beta v_{t+1}. \tag{19}$$

As in de la Croix et al. [2009], we assume that the fair effort norm $g(.)$ is given by

$$g\left(w_t, \overline{w}_t, 1 - \frac{L_t}{P_t}\right) = \frac{1}{\psi}\left[\phi_1 w_t^{\psi} - \phi_2 \overline{w}_t^{\psi} - \phi_3 \left[1 - \frac{L_t}{P_t}\right]^{-\psi\phi_4}\right] \tag{20}$$

with, according to the earlier discussion about the determinants of the norm g, $\phi_1, \phi_2, \phi_3, \phi_4 > 0$ and $\psi > 0$. As Danthine and Kurmann [2004] write, "intuition also suggests that (...) the positive incentive effect of a larger own wage is stronger than the negative effect of a higher comparison wage" (p. 112), which means $\phi_1 > \phi_2$. Firms manipulate the wage to elicit the level of effort they prefer. Since with (20), the wage elasticity of the effort function is equal to $\phi_1 {w_t}^{\psi}/e_t$, the optimality condition (12) becomes:

$$e_t = \phi_1 w_t^{\psi}. \tag{21}$$

This prefered effort level needs to be compatible with the value $g(.)$ considered as fair by the workers. In macroeconomic equilibrium, the individual and the average wage coincide ($w_t = \overline{w}_t$). So, by (4), (20) and (21), the fair effort level is

$$e_t = \frac{\phi_1 - \phi_2}{\psi} w_t^{\psi} - \frac{\phi_3}{\psi}\left[1 - \frac{L_t}{P}\right]^{-\psi\phi_4}. \tag{22}$$

Combining (21) and (22) gives an increasing relationship between the hourly wage and employment:

$$w_t = w(L_t) =_{\mathrm{def}} \left[\frac{\phi_3}{[1-\psi]\phi_1 - \phi_2}\right]^{1/\psi}\left[1 - \frac{L_t}{P_t}\right]^{-\phi_4}, \quad \text{with } w(0) > 0. \tag{23}$$

Since $\phi_3 > 0$[14] and $w_t > 0$, the following parametric restriction must be imposed: $\psi < 1 - \phi_2/\phi_1$. Function $w(L_t)$ admits a vertical asymptote in case of full employment: $L_t = P_t$.

3.4 Natural capital dynamics

At the beginning of any period t, the economy is endowed with a stock of natural capital R_t. It is depleted by the resource consumption linked to human activity X_t ($= \mu_t Y_t$). It is replenished by the inflow F_t (≥ 0),

[14] A higher unemployment rate deteriorates the position of the "workers' reference sets" and induces more in-work effort.

which is the amount of renewable resource produced by the biosphere, i.e. the biocapacity. Natural capital therefore evolves as follows:

$$R_{t+1} - R_t = F_t - X_t. \tag{24}$$

This representation is consistent with the interpretation of natural capital R_t as an aggregate of renewable and non-renewable resources even though F_t can only be a flow of renewables.[15] This renewable flow F_t is supposed to be exogenous. Equation (24) then corresponds to the accumulation equation assumed by Dasgupta and Heal [1974] when they consider the simultaneous presence of an exhaustible resource and of a renewable resource which they describe as a "perfectly durable commodity (e.g. an energy source)".[16] If the assumption of a resource flow independant of human action is appropriate for some renewable resources (such as solar energy), others (e.g. wild vegetation and animals) have a regenerative or reproductive capacity that depends on the resource stock and therefore on human activity through its impact on this stock. The assumption of an exogenous flow F_t is however made for simplicity but is not essential to the validity of our results as discussed in section 4.3.3.

4 Model solution

4.1 Dynamic system

Equations (16), (18) and (19) lead to the property of a constant savings rate.[17] Note first that (16) and (19) imply that $C_{t+1}/C_t = \alpha\beta\, Y_{t+1}/K_{t+1}$ or $K_{t+1}/C_t = \alpha\beta\, Y_{t+1}/C_{t+1}$. Given (18), this equality may be rewritten as $[Y_t/C_t] - 1 = \alpha\beta\, [Y_{t+1}/C_{t+1}]$, i.e. as a first-order difference equation in Y_t/C_t with constant coefficients. Solving it forward over an infinite horizon leads to the following solution:

$$\frac{Y_t}{C_t} = \frac{1}{1-s} \qquad \text{or} \qquad C_t = [1-s]\, Y_t, \quad \text{with} \quad s = \alpha\beta, \quad \forall t \geq 1. \tag{25}$$

[15] If we use the superscript r for renewables and nr for non renewables, the stock of renewables R^r evolves as $R^r_{t+1} = R^r_t + F_t - X^r_t$ where X^r_t is the consumption of renewables; the stock of non-renewables R^{nr} evolves as $R^{nr}_{t+1} = R^{nr}_t - X^{nr}_t$ where X^{nr}_t is the consumption of non-renewables. The flow F only consists of renewables as there is no natural replenishment of the stock of non-renewables on a time scale like that of human history. If the two stocks are measured in the same units, the total resource stock (natural capital) in t is $R_t = R^r_t + R^{nr}_t$ and the total resource consumption in t is $X_t = X^r_t + X^{nr}_t$. The sum of the two accumulation equations then leads to (24). Furthermore, if both types of resources are perfect substitutes, (24) provides a sufficient description of the dynamics of the resource endowment of the economy.

[16] See the introductory paragraphs of section 1 (p.8) in Dasgupta and Heal [1974] and their resource equation (1.3) where the exogenous flow of "durable commodity" is furthermore assumed constant.

[17] As outlined by Fagnart and Germain [2011], this result follows from the combination of three assumptions (logarithmic instantaneous utility of consumption, unitary depreciation rate and Cobb Douglas relationship between agent inputs) and it only holds if agents face an infinite horizon. See also Hassler et al. [2016].

The higher β and α, the higher the savings rate s. Given (25), (18) leads to

$$K_{t+1} = \alpha\beta Y_t. \tag{26}$$

Given the constant value $h_t = \bar{h}$ given by (13), the dynamic model can be summarized by the system of equations (6), (15), (17), (23), (24) and (26) where $K_{t+1}, Y_t, E_t, L_t, R_{t+1}, w_t, e_t$ $(t \geq 1)$ are the unknowns and R_1 and K_1 are initial conditions. Variables η_t, μ_t, F_t and P_t are exogenous; $\alpha, \beta, \phi_1, \phi_2, \phi_3, \phi_4$ and ψ are parameters. This system can be iteratively solved forward. With its solution, one can then compute C_t via (18), X_t via $X_t = \mu_t Y_t$ and v_t via (16).

4.2 Long-run impacts of a WTR with unlimited natural capital

Under assumption (14) of a bounded resource-saving technical progress, this section and the next one analyse the long run properties of the economy, respectively in the absence and in the presence of an upper limit on the stock of natural capital. The impact of a working time reduction policy (a.o. on wages and employment) turns out to be very different in the two cases.

If natural capital is unlimited and η_t grows unboundedly at constant rate $g_\eta > 0$, the economy admits a balanced growth path along which output grows at a constant rate. With infinite values of F_t and R_t, equation (24) disappears and (6) implies that $E_t = 0, \forall t \geq 1$ (even for $t \to \infty$). Consequently, $B(E_t) = B(0) = 1$, $\forall t \geq 1$. Natural capital never constrains the economy and, in particular, does not affect the features/properties of its balanced growth path. It is easily shown that the growth rate of output is then proportional to the growth rate of technical progress $g_\eta > 0$ and does not depend on any endogenous variable, and in particular not on $\bar{h}$.

Hereafter we call "a working time reduction policy" (in short 'WTR') an exogenous and finite cut $\Delta h < 0$ evaluated at the level $\bar{h}$ firms would have freely chosen. If $\bar{h}$ does not impact the growth rate of output along the BGP, it influences the levels of output, employment and hourly wage along this path:

Proposition 1 *In an economy with unlimited natural capital but a bounded resource-saving technical progress, implementing a WTR lowers the levels of output, employment and wage along the balanced growth path.*

Proof. See Appendix A.3. ■

4.3 Long-run properties of an economy with finite natural capital

This subsection considers successively that the technical progress on agent inputs is unbounded and bounded. It ends with a discussion of alternatives to our setting.

4.3.1 Stationary state effects of a WTR with unbounded technical progress on agent inputs

If natural capital is finite and η_t grows unboundedly at constant rate $g_\eta > 0$, the economy reaches a stationary state where the output level is finite. The stationary expression of (24) leads to

$$Y = \frac{F}{\underline{\mu}}, \tag{27}$$

where we use uppercase letters without time subscript to indicate the stationary-state value of the corresponding variable. Long-run output is proportional to the resource inflow F and inversely proportional to the resource content of one unit of output $\underline{\mu}$. Note that this property does not depend on the particular technological assumption we made. It would hold with any production function for which the average productivity of the resource is bounded (e.g a CES production function between the resource and the agent inputs with an elasticity of substitution strictly lower than 1). The resource exploitation rate E then tends to 1[18]: $X = R$ and the stationary expression of (24) therefore implies that $R = F$. Intuitively said, when the productivity of agents inputs grows endlessly, firms are ultimately led and able to fully use natural capital in spite of the growing cost of doing so; the stock of natural capital then tends toward the value of the resource flow F.

Given expression (17), which implies that w, h and L are chosen so as to maintain the labour share equal to $1 - \alpha$, expression (27) implies the following stationary state relationship between the hourly wage, total hours worked $\bar{h}L$ and the abundance of natural capital measured in efficient units, i.e. $F/\underline{\mu}$:

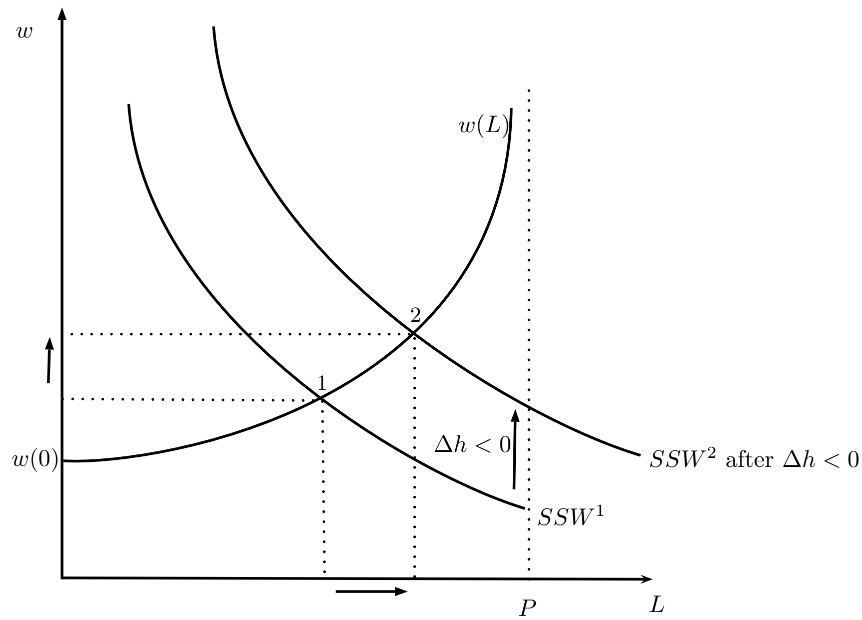
$$SSW \equiv w = \frac{1-\alpha}{\bar{h}L}\frac{F}{\underline{\mu}}. \tag{28}$$

SSW is a pseudo aggregate labour demand, which negatively links the hourly wage w and employment L, given $F/\underline{\mu}$ and the optimal value of $\bar{h}$. In Figure 1, the curve SSW^1 represents this pseudo labour demand which is a branch of hyperbola with a vertical asymptote at $L = 0$. The upward-sloping wage-setting relationship (23) and SSW^1 jointly determine the stationary state equilibrium of the labour market with

[18]With a finite Y, (26) implies a constant K and (17) and (23) jointly determine constant levels of L and w. Via (21), effort is constant as well. On the left-hand side of (15), η_t tends towards infinity while all the endogenous variables reach stationary values. Hence, equality (15) can only be verified if its right-hand side also tends towards infinity, i.e. if $E_t \to 1$.

h given by 13. It is necessarily unique and corresponds to point 1 in Figure 1. The following proposition shows how the labour market equilibrium is affected by the abundance of natural capital $F/\underline{\mu}$.

Figure 1: Stationary state equilibrium of the labour market



Point 1 (resp. Point 2) is the equilibrium before (resp. after) a WTR

Proposition 2 *In a world with finite natural capital (i.e. finite $F/\underline{\mu}$) and unlimited technical progress on agent inputs, a scarcer natural capital, i.e. a smaller value of $F/\underline{\mu}$, leads to lower steady-state levels of output Y, employment L and earnings per worker wh.*

This is obvious for Y given (27) and easy to see for wh and L. Given (28), a lower $F/\underline{\mu}$ decreases w at given L. In Figure 1, it implies a downward shift of SSW and so lower equilibrium levels of L, w and earnings (since h remains unchanged). Intuitively said, when natural capital is scarcer, its exploitation gets more intensive in agent inputs; hence, a lower $F/\underline{\mu}$ implies a lower labour productivity, which decreases labour demand and thereby the equilibrium wage, employment and output levels.

The effects of a WTR in this stationary economy are very different from those described in section 4.2 as the following proposition establishes.

Proposition 3 *In a world with finite natural capital and unlimited technical progress on agent inputs,*

1. *A WTR increases the stationary state levels of employment L and the wage rate w. The positive effect of a WTR on employment (resp. hourly wage) is stronger (resp. weaker) when natural capital*

is scarcer i.e. when $F/\underline{\mu}$ is smaller.

2. *A WTR reduces the total number of hours worked hL and earnings per worker wh. When $F/\underline{\mu}$ is smaller, it decreases hL less and wh more.*

3. *A WTR improves the stationary state level of the lifetime discounted utility of a family.*

Proof. See Appendix A.4 ■

The positive impacts of a WTR on L and w are graphically illustrated in Figure 1. As (28) shows, a WTR implies an increase in w at given L, i.e. an upward shift of the pseudo labour demand from SSW^1 to SSW^2. As a WTR leaves the output level unchanged, it indeed leads firms to substitute working hours by other determinants of the labour input: at the initial employment level, they are pushed to offer a higher hourly wage so as to stimulate work effort e (see (21)). As the wage-setting relationship (23) does not depend on h, the upward shift in the labour demand shifts the equilibrium from point 1 to point 2 in Figure 1, with an increase in both w and L. As (28) shows at given $F/\underline{\mu}$, the product wL increases by the same percentage as the decrease in h. But the respective increases in w and L depend on the slope of the wage setting relationship (23) at the initial equilibrium and thereby on the abundance of natural capital: if $F/\underline{\mu}$ is smaller, the output and employment levels before WTR are lower and the labour market equilibrium lies in a flatter part of (23). Hence the WTR increases the wage rate less and employment more in an economy where natural capital is less abundant (i.e. if $F/\underline{\mu}$ is smaller).

The second point of the proposition follows from the fact that w and L both increase in response to the cut in h, while whL is a given share of aggregate output and the latter is not affected by a WTR. Hence, both hL and wh decrease. The decrease in hL (resp. in wh) is however smaller (resp. larger) when natural capital is scarcer since L (resp. w) increases more (resp. less) in this case (see above).

Moreover, as a WTR does not affect the savings rate of households, it does not change the stationary state level of consumption. Therefore, it only changes the stationary lifetime discounted utility of a family through its impact on the disutility of work $d(\bar{h})L$. Since $d(h)$ is increasing and convex in h, a WTR of $x\%$ decreases the disutility per employed family member by more than $x\%$. As it stimulates employment by less than $x\%$, it necessarily decreases $d(h)L$ and enhances the stationary state level of the lifetime discounted utility of a family.

4.3.2 Stationary state effects of a WTR with bounded technical progress

If now all forms of productivity gains are bounded (i.e. if $\eta_t \to \bar{\eta} < \infty$ and $\mu_t \to \underline{\mu} > 0$), the steady-state exploitation rate of natural capital is strictly smaller than 1 ($E < 1$). Appendix A.5 presents this case extensively and establishes the results summarized in the following proposition.

Proposition 4 *In a world with a finite resource flow F where both technical progresses are bounded,*

1. *A WTR stimulates the stationary state levels of the hourly wage and employment if natural capital is scarce enough, i.e. if $F/\underline{\mu}$ is small enough. It decreases the stationary state level of output if $F/\underline{\mu}$ is large enough.*

2. *When a WTR stimulates employment and the hourly wage, its positive effect on employment (resp. the hourly wage) is stronger (resp. weaker) when $F/\underline{\mu}$ is smaller.*

Proof. See Appendix A.5. ■

As far as the impact of a WTR on output and employment is concerned, case 4.3.2 appears as an intermediary situation between the one developed in Subsection 4.2 (where the resource constraint is totally relaxed) and the one discussed in Subsection 4.3.1 (where, in the long run, the resource is fully used). In this intermediary case, a long run equilibrium with $E \to 0$ prevails if $F/\underline{\mu}$ is large enough, i.e. such that the economy consumes less resource than F (R_t tending toward infinity). When $E \to 0$, a WTR necessarily decreases output, wage and employment. This remains the case if E is sufficiently small. But if E is high enough, a WTR raises employment for reasons explained in Subsection 4.3.1.

4.3.3 Discussion of alternative assumptions

This section discusses the consequences of alternative assumptions about three features of the model, namely the possible endogeneity of the resource flow F_t and the specifications of the utility function and the production function.

As we already mentioned in section 3.4, the resource inflow F_t could be made endogenous. It could in particular be assumed to be an endogenous function of the existing stock of natural capital as e.g. in Smulders et al.. In Appendix A.6, we show that the steady-state results put forward in Section 4.3 also hold when F_t is endogenized in this way.

We have assumed that working time enters additively in households' preferences. In particular, the fair level of effort $g(.)$ does not depend on the number of worked hours. Working hours are however a determinant of working conditions and might thereby also influence the level of effort that workers consider fair. We have explored this possibility by considering a function $g(w_t, \bar{w}_t, u_t, h_t, \bar{h}_t)$ where g is a simple generalization of (20), decreasing in the hours worked by the employee of a given firm, h_t, but increasing in the average working time in the other firms, $\bar{h}_t$.[19] In a symmetric macro equilibrium ($\bar{w}_t = w_t$ and $\bar{h}_t = h_t$), the optimal number of hours freely chosen by firms is then no more constant but increasing in the wage level and the unemployment rate. Taking into account this relationship between hours, the hourly wage and employment, one still obtains an increasing (but modified) wage setting relationship $w(L_t)$ if the elasticity of g with respect to working time is not too large. The steady-state labour market equilibrium remains then graphically described by a figure similar to Fig. 1. But a WTR then shifts the modified $w(L_t)$ curve rightward on top of shifting the decreasing SSW curve upwards. These two shifts lead to an unambiguous increase in employment as with the initial effort function.

Turning to technological assumptions, for analytical tractability, we have assumed a strict complementarity between agent and resource inputs at a given point in time. Let us make clear that our results about the (un)employment effect of a WTR are not rooted in this assumption. Any (combination of) assumption(s) that imply that the marginal productivity of the finite resource is bounded opens the way to a positive employment effect of a WTR. For the sake of illustration, we consider two alternative assumptions. First suppose the case of a technology with two embedded Cobb-Douglas functions: final production is described by a Cobb-Douglas function of capital, labour and the resource, the resource exploitation technology being itself a Cobb-Douglas function of agent inputs. In this embedded Cobb-Douglas case, all forms of technical progress (resource saving versus labour-and-capital saving) are somehow equivalent in the sense that technical progress on agent inputs also increases the productivity of the resource. If this technical progress went on endlessly, the marginal productivity of the resource would go to infinity. The economy would then admit a balanced growth path even though the resource inflow is finite and a WTR would produce a negative effect on employment as in section 4.2. But when Cobb-Douglas functions are assumed, the physical laws which limit the marginal productivity of the resource require to suppose that all forms of technical progress are bounded. In this case, a WTR stimulates employment when, in relative terms, natural capital is scarce enough as in Proposition 4.

An intermediary case between our initial assumption and the Cobb-Douglas case is a CES technology between a mix of agent inputs on the one hand and natural capital on the other hand, with an elasticity of substitution smaller than 1 between the two types of inputs. With an elasticity of substitution below 1, the marginal productivity of the resource remains finite even with an unbounded technical progress on

[19]More precisely, $g\left(w_t, \overline{w}_t, u_t, h_t, \bar{h}_t\right) = 1/\psi \left[\phi_1 w_t^{\psi} - \phi_2 \overline{w}_t^{\psi} - \phi_3 \left[u_t\right]^{-\psi\phi_4} - \phi_5 h_t^{\psi} + \phi_6 \overline{h}_t^{\psi}\right]$, with $\phi_5 > \phi_6 > 0$, all other parameters keeping the same sign.

agent inputs. With an unbounded (resp. bounded) technical progress on agent inputs, a WTR can be shown to affect employment as in Proposition 3 (resp. Proposition 4).

5 Numerical Analysis

In Subsection 5.1, we calibrate the model of the world economy in which the labour-and-capital-saving technical progress is unbounded while the resource-saving one is not. In the long-run, it therefore admits a stationary state equilibrium whose properties have been studied in subsection 4.3.1. Subsection 5.2 is devoted to simulations of the dynamics of the modelled economy. First we display and comment the time profile of a range of outcomes in the benchmark environment (i.e. the one that has been calibrated). Next we numerically analyse the transitory dynamics following a WTR which takes the form of an exogenous permanent shock to the individual duration of working time. The impacts of this policy are also studied in a number of alternatives to the benchmark case where either the natural resource endowment is modified in various ways or the responsiveness of the wage to the unemployment rate varies.

5.1 Calibration of the Model

This subsection explains how initial values of endogenous variables and parameter values are fixed. It is necessary to consider particular functional forms for $B(E_t)$ and $U(h_t)$. For $B(E_t)$, we suppose

$$B(E_t) = \frac{1}{1 - E_t}. \tag{29}$$

This specification verifies all the assumptions made when introducing the congestion effect in (7). For $U(h)$, we assume (see Appendix A.7):

$$U(h) = \tilde{\lambda}\, h^{\alpha} \left[1 - \exp(-\xi \cdot h)\right]^{1-\alpha} \quad \tilde{\lambda}, \xi > 0, \tag{30}$$

which is a strictly increasing and unbounded function of working time h_t. Under this assumption,

$$h_t \frac{U'(h_t)}{U(h_t)} = \alpha + [1 - \alpha] \frac{\xi\, h_t}{\exp(\xi h_t) - 1}, \tag{31}$$

which is decreasing in h_t, from 1 when $h_t \to 0$ to α when $h_t \to +\infty$. The optimal value of $h_t = \bar{h}$ (which verifies (13)) is the strictly positive root of

$$\exp(\xi \bar{h}) = 1 + \frac{1-\alpha}{1-2\alpha} \xi \bar{h}. \tag{32}$$

For this equation to admit a solution, $1 - 2\alpha$ must be positive, which implies the restriction $\alpha < 1/2$. Given specification (30) for $U(h)$, we need to fix parameters $\alpha, \beta, \xi, \tilde{\lambda}, \phi_1, \phi_2, \phi_3, \phi_4$, and ψ. In addition, assumptions need to be made about the exogenous trajectories of $\{P_t, \eta_t, \mu_t, F_t\}$. A unit of time is assumed to last 15 years. The initial period $t = 0$ covers the years 2000 - 2014. For the world economy, Table 1 summarises the observable initial conditions.

		2000-2014 ($t = 0$)
Y_0 (Trillions 2010USD)	Sum	921.222
Labour force aged 15+ P_0 (persons)	Annual Mean	3,068,093,042
Employment 15+ L_0 (persons)	Annual Mean	2,891,079,080
Productivity Y_0/L_0 (2010USD/head)	Annual Mean	318,643
Unemployment Rate $1 - (L_0/P_0)(\%)$	Annual Mean	5.77
Savings Rate s (%)	Annual Mean	25

Table 1: Initial conditions (`https://data.worldbank.org/indicator`) and own calculation.

The values of s and Y_0 in Table 1 yield the initial condition K_1. Initial conditions regarding the working hours and labour share at the world level cannot be obtained from the World Bank database and are computed from the Penn World Table (PWT).[20] For 69 countries over 169, PWT provides average working hours. For instance, in 2014, total employment in these 69 countries represented 81% of aggregate employment in the 169 countries. On the basis of this information, we obtain a weighted[21] average working time $\bar{h} = 30,930$ hours (over 15 years).

Under the assumed technology, the labour share is $1 - \alpha$. The Penn World Table (PWT) computes the labour share for a range of countries. The online appendix of Feenstra et al. [2015] explains the corrections introduced to get what the authors call the "best estimate" labour share. On the basis of 127 country data, the average labour share is equal to 0.52 in 2005, well below the standard 0.66 - 0.7 benchmark range of values (see Feenstra et al., 2015, p. 3178). Under Specification (30), we have seen that the optimality condition (13) can only be solved if one imposes $\alpha < 0.5$. Therefore, given the PWT evidence, we set $\alpha = 0.45$. This choice and the above initial values have several consequences. First, since

[20] See `https://www.rug.nl/ggdc/productivity/pwt/`. We use version 9.0 of the data base.

[21] The weights are the employment share in aggregate employment for the countries where working hours are available. equal to 2062 hours/year for $t = 0$. The order of magnitude we get is compatible with the weekly actual working time values computed by Bick et al. [2018] extrapolated on a yearly basis.

the model's savings rate $s = \alpha\beta$, combining the observed value of s at time zero (in Table 1) and the chosen value for α yields $\beta = 0,555$ (i.e. a yearly discount rate of 4%). Next, turning to the parameters of $U(h)$, given the values adopted above, the solution to (32) is $\xi = 6.16\,10^{-5}$. Because of the Cobb-Douglas specification, we can normalise $\tilde{\lambda}$ to 1. In addition, since at any time t, $w_t\bar{h} = [1-\alpha]Y_t/L_t$, we deduct that at time 0, the average world hourly wage w_0 amounts to 5.67 2010USD/hour.[22]

Turning to the parameters of (23), notice that this equality relates the level of the wage rate to the unemployment rate. So, it can be called a "wage curve" (Blanchflower and Oswald, 1994). Wage curves have been estimated in many countries and the finding of a negative correlation of about -0.1 is fairly robust. We thus set ϕ_4 in (23) to 0.1. We furthermore assume that $\phi_3 = \phi_1 - \phi_2$.[23] At time 0, given w_0 and the unemployment rate in Table 1, the following equality holds: $4.65^{\psi} = {}^{[\phi_1-\phi_2]}/_{[[1-\psi]\phi_1-\phi_2]}$. We arbitrarily set $\psi = 0.25$ and $\phi_2 = 0.1$ and checked that the simulation results are not much affected by this choice. This implies that $\phi_1 = 0.46$ and leads to $e_0 = 0.46\,w_0^{0.25} = 0.71$.

As far as the resource is concerned and consistently with our interpretation of R_t as the natural capital and F_t as the biocapacity, we refer to the data collected by the Global Footprint Network (GFN hereafter) and interpret the resource consumption X_t as the Ecological Footprint, which measures the human demand on natural capital. The GFN estimates the biocapacity and the ecological footprint on a yearly basis (see `https://data.footprintnetwork.org`). By summing these yearly data for the biocapacity and the ecological footprint over the 2000-2014 period, we obtain the initial values F_0 and X_0 respectively. Given this value of X_0 and that of Y_0 in Table 1, we obtain μ_0 as the ratio X_0/Y_0. Making an assumption on the unobserved E_0, (6) then allows us to calculate the initial resource stock R_0 simply as to $\mu_0 Y_0/E_0$. In the reference scenario, we set E_0 to 0.7, which leads to a monotonic convergence of output toward its steady state. A sensitivity analysis will later be proposed, in which a lower E_0 (or equivalently a larger R_0) will be considered. Data collected by the GFN show that the world Biocapacity has been increasing since the sixties. The gain is however declining. Given the observed increasing but slowing trend in the evolution of biocapacity over the last decades, we assume that F_t will temporarily continue to grow starting from the initial value F_{-1} (the sum of the yearly biocapacity taken over the 1985-1999 period) up to a finite stationary value F. In the reference scenario, F is set to $1.3\,F_0$ but given the uncertainty on this limit value, a sensitivity analysis will be conducted in the sequel. We assume that

[22] To the best of our knowledge, it is hard to find some benchmark information to which this value could be compared.

[23] Function g given by (20) is then linear in the logarithms of its arguments in the limit case where $\psi \to 0$, which corresponds to the effort function in Danthine and Kurmann [2004].

F_t is an increasing and concave relationship converging to F, namely

$$F_t = F + \frac{F_0 - F}{F_{-1} - F}[F_{t-1} - F], \; t \geq 1.$$

The assumption of an increasing F_t, which is dictated by the observed evolution of F_t in the data, cannot be suspected of strengthening the problem of resource scarcity and can therefore be seen as a conservative assumption as far as the employment impact of a WTR is concerned: keeping F constant at its initial value would strengthen the resource constraint when time passes, which, given our analytical results, would reinforce the positive impact of a WTR on employment.

As far as the evolution of the resource content of a unit of output μ_t is concerned, we assume a 50% scope of gain compared to the initial value μ_0, i.e. $\underline{\mu} = \mu_0/2$. As in the case of F, the uncertainty about this limit value will lead us to conduct a sensitivity analysis with respect to the potential of resource saving technical progress. Computing X_{-1} and Y_{-1} over the years 1985 to 1999 by summing the annual observations for X and Y over these years, we deduct $\mu_{-1} = X_{-1}/Y_{-1}$. The time path of μ_t toward $\underline{\mu}$ is assumed to describe the following declining and convex trajectory:

$$\mu_t = \underline{\mu} + \frac{\mu_0 - \underline{\mu}}{\mu_{-1} - \underline{\mu}}[\mu_{t-1} - \underline{\mu}], \; t \geq 1.$$

We assume that η_t increases at an exogenous growth rate g_η: $\eta_t = \eta_0 g_\eta^t, t \geq 1$. Given the observed evolution of GDP between period "-1" (covering the years 1985 to 1999) and period 0, we set $g_\eta = 1.084$. Parameter η_0 is obtained by solving (15) at time 0.

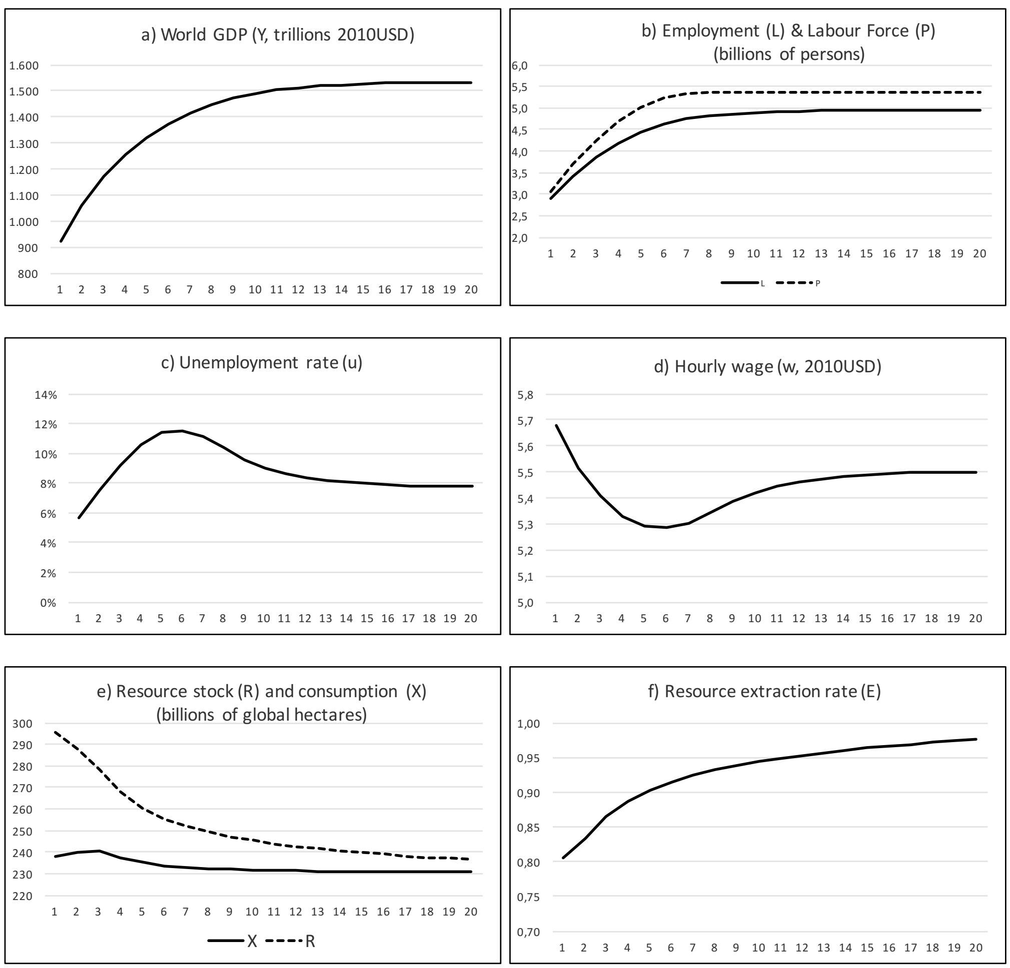
The exogenous trajectory of the workforce P_t for $t \geq 1$ is based first on the ILO Labour Market Projections.[24] This source provides labour participation rates and size of the labour force estimates in 2016 and 2022 on average in the world. The world average participation rate among the 15+ population declines from 62.1% in 2016 to 61% in 2022. This tendency is assumed to continue because of ageing and the increasing length of education. We set the participation rate at 58% in 2100 and interpolate this rate during the intermediate period. Population Division of the United Nations proposes population predictions until 2100.[25] The evolution of the population aged 15 or more is taken from this source. The size of the workforce is then simply given by the product of the trajectory of this population size and the one assumed for the participation rate. The size of the workforce reaches a plateau around the year 2100 (about 5,371,200 thousands people). We keep the level of the workforce unchanged beyond 2100.

[24] ILO Modelled estimates available in May 2018. See `https://ilostat.ilo.org/data/`.
[25] Probabilistic Population Projections based on the World Population Prospects: The 2017 Revision.

5.2 Simulation Results

Thanks to the result of an endogenously constant saving rate (see section 4.1), the model can be numerically solved forward starting with initial conditions on K_1 and R_1 (and given the calibrated values of the model parameters and exogenous variables). All the simulation exercices we did (in the benchmark case and in numerous variants of which only a few are presented here) show that the transitory dynamics of the economy converges toward its stationary state. All these numerical experiments allow us to confidently conclude that the stationary state is stable even though this has not been analytically established.

Figure 2: The reference scenario V0 before working time reduction



The simulation period is indicated on the horizontal axis of each panel.

Simulations are conducted over the period $t = 1, ..., T = 20$ i.e. from the period 2015-2029 to 2300-2314. The reference scenario, "V0", is illustrated by Figure 2, which shows, for $t \geq 1$, the evolutions of Y_t, L_t, u_t, w_t, X_t, R_t and E_t governed by the system (15)-(21) and the initial conditions. Our comments focus more on qualitative considerations than quantitative ones (e.g. the absolute values reached by output and the unemployment rate). As the main purpose of our simulation exercise is to illustrate the dynamic effects of a WTR and what influences them, the reference scenario (before WTR) first serves as a benchmark in which the evolution of unemployment (higher than at the very beginning of the simulation period) makes the very question of a WTR meaningful.

As the two upper panels of Figure 2 show, V0 is characterised by a monotonic evolution of output (capital) and employment toward their respective steady-state value. This economic growth process is accompanied by a progressive rise in the exploitation rate of natural capital. This is the consequence of a steady decline in the stock of natural capital even though the resource saving technical progress leads to a decrease in natural capital consumption after the first simulation periods (see Panels 2.e, 2.f).

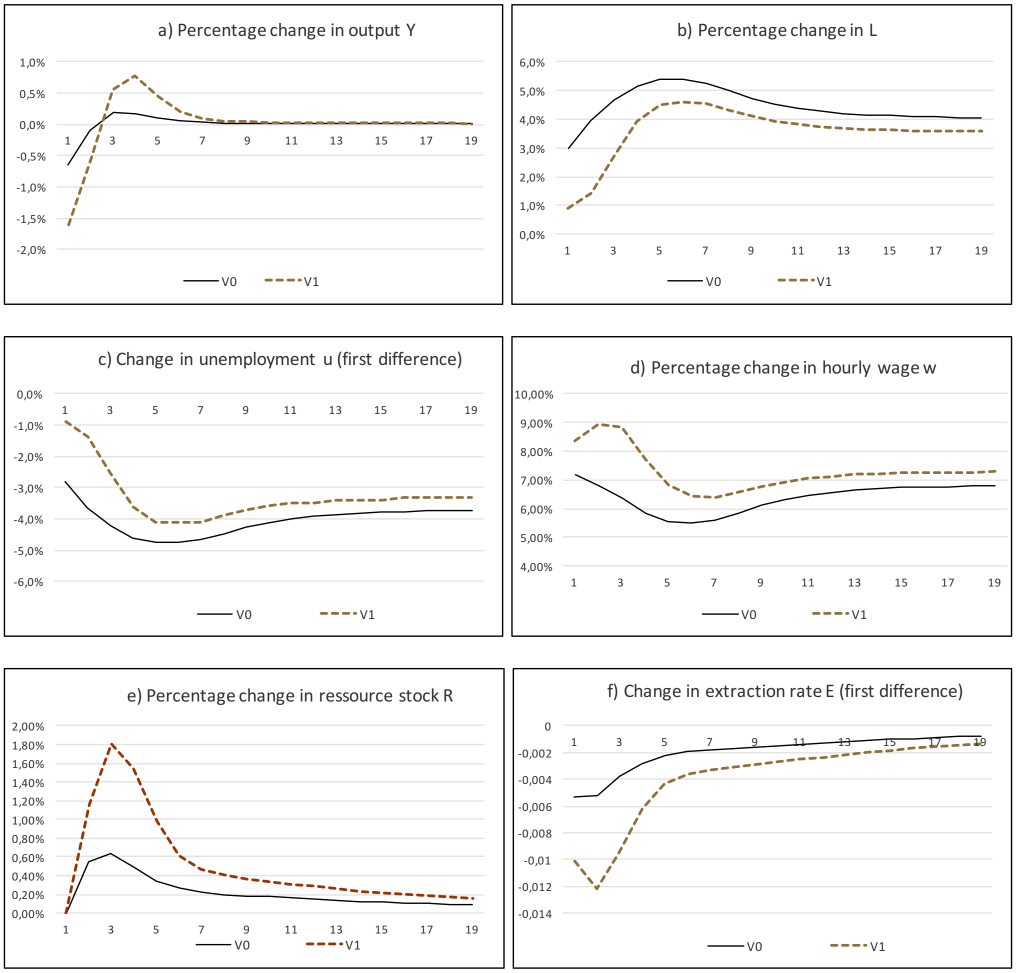
Even though the growth potential of the world economy is initially important (World GDP increasing by about 70% over the simulation period), the increase in employment is initially lower than the rise in the labour force and the world unemployment rate rises and peaks at a value close to 12% (see Panel 2.c). Later, as employment keeps on increasing after the stabilisation of the labour force, the unemployment rate decreases and tends toward a value of about 8%, higher than in the first simulation period. Given (23), the non-monotonic evolution of the unemployment rate explains the U-shaped evolution of the hourly wage (see Panel 2.d) and of the individual wage income (as $\bar{h}$ is kept constant by firms).

5.2.1 A Working Time Reduction Policy in the reference scenario

The continuous line in the six panels of Figure 3 illustrates the impact of a WTR equal to 10 % of $\bar{h}$ in scenario V0, the other curve corresponding to a first variant discussed later. Panels 3.a, 3.b, 3.d and 3.e display the percentage change in the variable of interest with respect to the reference scenario without WTR. Panels 3.c and 3.f display the absolute change in u_t and E_t.

When technical progress on agent inputs is unbounded, Section 4.3.1 has shown that a WTR has no long run impact on output (Y), the exploitation rate of natural capital (E) and its use and stock (X and R). During the transitory dynamics however, output first decreases (Panel 3.a) since the change in the agent input mix of firms raises their production cost. Accordingly, the economy first consumes less natural capital and its stock (Panel 3.e) slightly increases. Its exploitation rate therefore falls and will remain below its value in the reference scenario during the whole transition dynamics (Panel 3.f). This lower E_t contributes to lowering the resource exploitation cost, which progressively compensates the initial cost increase following the cut in h. From period 3 onward, it even allows the economy to reach an output level slightly larger than in scenario V0 during the rest of the transition towards the steady state. Accordingly, from this period onward, the consumption of natural capital is higher after a WTR.

Figure 3: Impact of a 10% working time reduction in reference scenario V0 versus V1



Each panel displays the impact of a permanent 10% WTR on the variable of interest in the reference scenario V0 and the alternative scenario V1 where the resource stock is initially more abundant (1.4 greater than in V0).

As Panel 3.b (resp. 3.c) illustrates, a WTR has a positive (resp. negative) impact on employment (resp. unemployment) all along the transitory dynamics. The highest gain is achieved after six periods with a rise of aggregate employment of about 5% and a drop in unemployment of almost 5 percentage points. Section 4.3.1 has already explained why the long-run effect of the WTR on employment is positive. But the size of the (un)employment effect of the policy however evolves non monotonically. The comparison between Figures 2 and 3 shows that the (un)employment impact of the WTR is the largest during the periods where unemployment peaks in the absence of a WTR; the opposite is true for the hourly wage that the WTR increases more in periods where the employment rate is higher. These observations extend to transitory dynamics the first steady-state property of Proposition 3: a WTR creates more jobs and increases less the hourly wage when the employment rate is lower (or the unemployment rate higher).

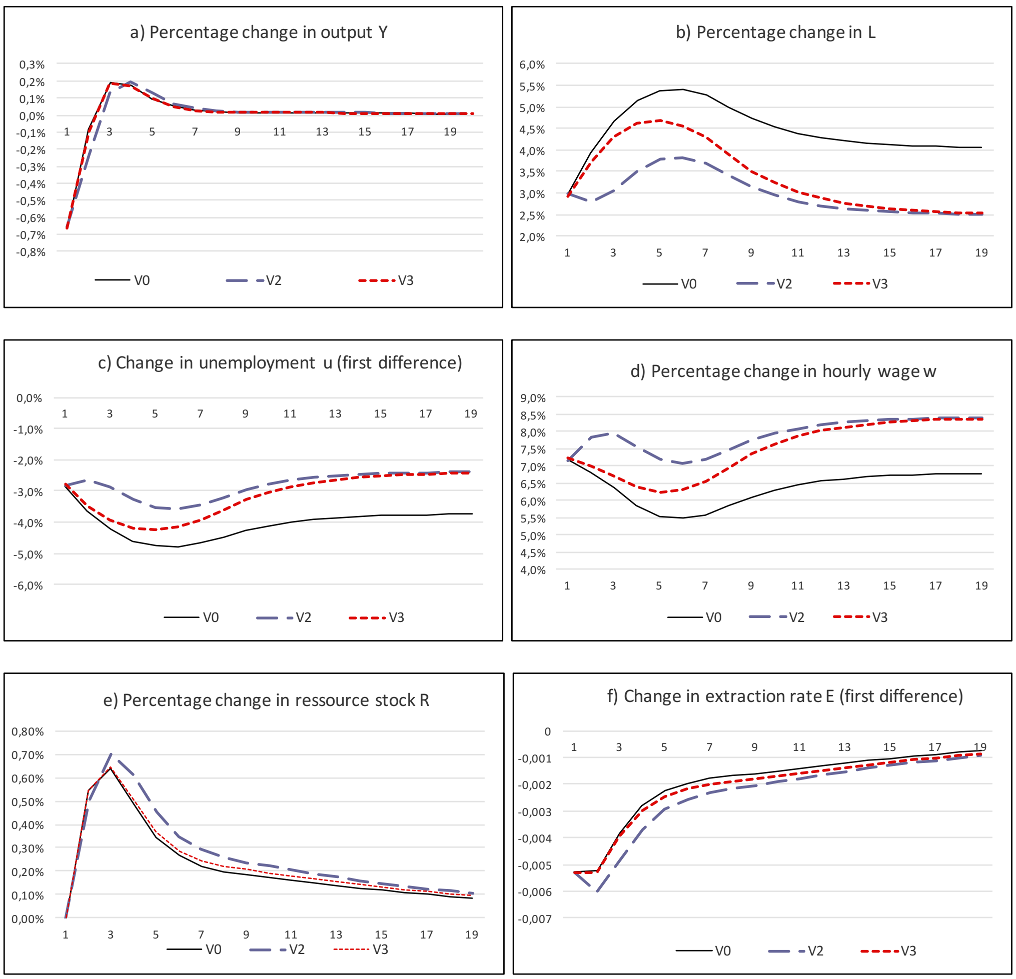
Given the observed transitory dynamics, the total effect of the WTR on the lifetime discounted utility of a family is a bit less clear-cut than at the stationary state. Two channels are at work: consumption per head and the disutility of work. Consumption per head is proportional to Y_t/P_t. Since the time path for P_t is the same with or without the WTR, only the time path of Y_t matters. Panel 3.a shows that the output effect of the 10% WTR is initially negative but becomes transitorily positive as of the third period. The second channel materialises via the product $d(h_t)L_t$. For the reason already explained in the proof of the third point of Proposition 3, a WTR decreases $d(h_t)L_t$, which enhances the lifetime discounted utility of a family. In sum, a 10% WTR has an ambiguous impact on the instantaneous utility level at the very beginning of its implementation but an unambiguous positive effect later on (a property that remains true even in the limit case where $d(h)$ is uniformly nil).

5.2.2 A Working Time Reduction Policy in alternative scenarios

We now turn to the impacts of a 10% WTR in three alternative scenarios where the value of an initial condition or a parameter is modified. When hours are freely chosen by firms, these four scenarios lead to simulation outcomes different from the reference scenario V0. However, for the sake of brevity, the presentation of the simulations of these variants focuses only on the changes induced by the 10% WTR and compares them to the changes obtained in V0.

The first alternative scenario V1 (also presented in Figure 3) describes an economy that differs from V0 only by an initial condition, namely: the initial stock of natural capital R_0 is 1.4 times larger in V1 than in V0 so that its initial exploitation rate is lower: $E_0 = 0.5$ in V1 instead of $E_0 = 0.7$ in V0. As the model parameters are identical in V0 and V1, both of them tend toward the same stationary state with identical long-run impacts of a WTR. But natural capital remains temporarily more abundant in V1 so that, before the introduction of a WTR, the employment rate is larger in V1 than in V0 during the transitory dynamics. Consequently, the transitory impacts of a WTR in V0 and V1 are different. The comparison between V0 and V1 in Figure 3 generalises to transitory dynamics the stationary state result of Proposition 3 regarding the employment and wage effects of a WTR: as natural capital is temporarily more abundant and the employment rate higher in V1, the (un)employment effect of the WTR is temporarily much weaker in V1 (Panels 3.b-c): the increase in employment is initially three times larger (+3% versus +1%) in V0 where natural capital is relatively scarcer, three quarters of the long run employment impact of the WTR (about + 4%) being reached from the start while it is only one quarter of it in V1. Symmetrically, the 10%-WTR leads to a larger (percentage) change in the hourly wage (Panel 3.d) in V1 than in V0. The initial negative effect of the WTR on output is initially much sharper in V1 (see Panel 3.a) but the subsequent recovery of output is also stronger. As a corollary, the transitory positive effect of the WTR on the stock of natural capital (panel 3.e) is stronger in V1. As time passes, V0 and V1 behave more and more similarly.

Figure 4: Impact of a 10% working time reduction in reference scenario versus V2 and V3



In scenario V2, the resource inflow F is 10% larger than in scenario V0.
In scenario V3, the lower bound for the resource intensity of output $\underline{\mu}$ is 10% smaller than in V0.

In the two next variants, labelled V2 and V3, the long-run value of $F/\underline{\mu}$ is 10% larger than in V0 so that natural capital is, in the long run, more abundant than in V0. In variant V2, F is increased by 10% with respect to V0; in variant V3, the lower bound $\underline{\mu}$ is decreased by 10% with respect to V0. The impact of a WTR in V2 and V3 is displayed in Figure 4 and compared to its impact in V0 (continuous line for V0, long dashed blue line for V2, short dashed red line for V3).

As $F/\underline{\mu}$ is greater in V2 and V3, a WTR is less effective in the long run than in V0 (see Section 4.3.1 and Panels 4.b-c)). Moreover, since, by construction, $F/\underline{\mu}$ is the same in V2 and V3, they are characterised by the same stationary values of variables Y, K, w, L, e, E and a WTR has identical long-run effects in both variants. The transitory dynamics however differs: in V2, a greater F makes natural capital more abundant from the start; in V3, the trajectory of μ_t toward a lower limit value departs only gradually from its trajectory in V0. Therefore, with regard to the effect of the WTR, the trajectories in V3 initially resemble those observed in V0 but gradually tend towards those observed in V2. As E_t is initially lower in V2 (and will remain so during the transition dynamics as Panel 4.f shows), employment (resp. hourly wage) initially increases less (resp. more) in V2 than in V3.

6 Conclusion

This paper reconsiders the consequences of a working time reduction (WTR) on (un)employment in a dynamic growth model with efficiency wages and an essential natural resource (natural capital). So far, the theoretical literature has analyzed WTR policies in (often static) models without natural resources. A survey of this literature shows that a positive effect of a WTR on employment appears at best a short-run result that does not hold once the accumulation of produced capital is endogenised. If this result would also hold in our model if natural capital was unlimited, we show that a WTR can be sustainably conducive to more employment (or less unemployment) in the short and long runs if natural capital is finite and scarce enough. This is in particular necessarily the case if technological progress on agent inputs (capital and labour) is unlimited, a case in which a WTR also increases the stationary value of the lifetime discounted utility of workers. The long-run increase in employment (resp., the hourly wage) is furthermore larger (resp. smaller) if natural capital is scarcer, i.e. if the renewable resource inflow or biocapacity is smaller and/or if the resource saving technical progress is more limited. Numerical simulations in the case of an unlimited technical progress on agent inputs show that a WTR has a positive effect on employment during the whole transitory dynamics and this effect is stronger in periods where the employment rate is lower. A sensitivity analysis confirms that when natural capital is scarcer, a WTR increases employment more and the hourly wage less with a less negative initial impact on output.

In our analysis, the modeling of natural capital is limited to one aggregate of renewable and non-renewable resources. This amounts to assuming that all resource types are perfect substitutes in the use human production makes of them. This is an optimistic hypothesis, but not one that is essential to our results as to the positive impact of a reduction in working time on employment. They ultimately rest on the constraint, imposed by the physical laws, that the natural resource content of one output unit cannot become null. This also means that the resource saving technical progress is necessarily bounded. If several types of resources were explicitly distinguished, this constraint would take a more complex form than in the case of a generic resource type.

A further analysis should take into account that labour skills and tasks are heterogeneous and that the implementation of a WTR becomes more complicate in the presence of a costly matching process between workers and jobs. It could also distinguish formal from informal firms: if a WTR could only be enforced in the formal sector, it would presumably induce changes in the composition of aggregate output. Such changes would impact employment and wages differently if technologies differ in the formal and informal sectors. Finally, demography and participation decisions could be made endogenous. Since the lifetime discounted utility of labour market participants improves after a WTR (at least in the medium run according to our simulations), the participation rate could eventually rise.

A Appendix

A.1 Functional form for $\mathcal{F}_t(k_t, l_t, h_t, e_t)$

A time period is divided in elementary units of time (hours). A pool of workers l_t consists of λ shifts or teams of $\tilde{l}_t$ workers per hour. The production made by a shift of $\tilde{l}_t$ workers which operate k_t units of produced capital depends both on the effort level e_t of these workers and on the number of hours they work h_t. Per worked hour in period t, we write the output of the combination $(k_t, \tilde{l}_t)$ as a function $f_t(k_t, e_t D(h_t)\tilde{l}_t)$, where $e_t D(h_t)\tilde{l}_t$ represents the total labour input. Function f_t is strictly increasing and concave in its two arguments and exhibits constant returns to scale in $(k_t, e_t D(h_t)\tilde{l}_t)$. A shift of workers who work h_t hours on produced capital k_t therefore produces $h_t f_t(k_t, e_t D(h_t)\tilde{l}_t)$. If there are $\lambda \geq 1$ shifts of workers (where λ will be considered as exogenous), total output in period t is equal to:

$$\lambda h_t\, f_t\left(k_t,\, e_t\, D(h_t)\, \tilde{l}_t\right) \qquad \text{or} \qquad f_t\left(\lambda\, h_t\, k_t,\, e_t\, h_t\, D(h_t)\, \lambda\, \tilde{l}_t\right),$$

since f is homogenous of degree 1. Let us define $d(h_t) \equiv \lambda\, h_t$, $\mathcal{D}(h_t) \equiv h_t D(h_t)$ and $l_t \equiv \lambda\, \tilde{l}_t$. The output in t can then be written as $\mathcal{F}_t\left(k_t, l_t, h_t, e_t\right) \equiv f_t\left(d(h_t)\, k_t, e_t\, \mathcal{D}(h_t)\, l_t\right)$, where function $d(h_t)$ captures the effect of the total working time on the use of capital and $\mathcal{D}(h_t)$ measures the effect of the working hours of a shift of workers on the effective labour units. If function f_t is of the Cobb-Douglas type, i.e. if

$$f_t\left(d(h_t)\,k_t, e_t\,\mathcal{D}(h_t)\,l_t\right) = \eta_t\left[d(h_t)\,k_t\right]^{\alpha}\left[e_t\,\mathcal{D}(h_t)\,l_t\right]^{1-\alpha} \quad \text{with} \quad \eta_t > 0,$$

we can further write $\mathcal{F}_t\left(k_t, l_t, h_t, e_t\right)$ as in (8) with $U(h_t) \equiv [d(h_t)]^{\alpha}\cdot[\mathcal{D}(h)]^{1-\alpha}$ and $U'(h) > 0$.

A.2 Cost Minimisation of a Producer of Final Good

Let $\lambda_t \geq 0$ be the Lagrangian multiplier associated to constraint (9) and $\mathcal{L}_t$ be the Lagrangian of the firm's problem in t. The first-order optimality conditions (or FOC hereafter) are respectively:

$$\begin{aligned}
\frac{\partial\mathcal{L}_t}{\partial k_t} &= v_t - \lambda_t\alpha\frac{y_t}{k_t} = 0\\
\frac{\partial\mathcal{L}_t}{\partial l_t} &= w_t h_t - \lambda_t[1-\alpha]\frac{y_t}{l_t} = 0\\
\frac{\partial\mathcal{L}_t}{\partial w_t} &= w_t h_t - \lambda_t[1-\alpha]\frac{y_t}{l_t}\frac{w_t}{e_t}\frac{\partial e_t}{\partial w_t} = 0 \quad \text{with} \quad \frac{\partial e_t}{\partial w_t} = \frac{\partial g(w_t, \overline{w}_t, u_t)}{\partial w_t}\\
\frac{\partial\mathcal{L}_t}{\partial h_t} &= w_t l_t - \lambda_t\frac{y_t}{U(h_t)}U'(h_t) = 0\\
\frac{\partial\mathcal{L}_t}{\partial \lambda_t} &= y_t - \frac{\eta_t}{B(E_t)\mu_t}U(h_t)k_t^{\alpha}\left[e_t l_t\right]^{1-\alpha} = 0
\end{aligned}$$

The FOC on h_t may be rewritten as $w_t h_t = \lambda_t\frac{y_t}{l_t}h_t\frac{U'(h_t)}{U(h_t)}$, which, in combination with the FOC on l_t, leads to (13). With constant returns-to-scale, a firm makes zero profits and the value added by a given output level y_t is shared between labour income and capital rent. With the final good chosen as *numéraire*, one therefore has $v_t k_t + w_t h_t l_t = y_t$. As the sum of the FOCs on k_t and l_t leads to $v_t k_t + w_t h_t l_t = \lambda_t\, y_t$, we obtain $\lambda_t = 1$. The optimality conditions can then straightforwardly be rewritten as in the main text.

A.3 Proof of Proposition 1

After using (26), (17), (21) and $B(0) = 1$, (15) can be rewritten as

$$\frac{\eta_t}{\mu_t}\frac{U(\bar{h})}{\bar{h}^{1-\alpha}}\left[\alpha\beta\right]^{\alpha}\left[\phi_1\left[1-\alpha\right]\right]^{1-\alpha}w_t^{[\psi-1][1-\alpha]} = \left[\frac{Y_t}{Y_{t-1}}\right]^{\alpha}.$$

Along a BGP characterised by a constant output growth rate g_Y, one has $Y_t = \exp(g_Y)Y_{t-1}$ and $[Y_t/Y_{t-1}]^\alpha = \exp(\alpha g_Y)$. Therefore, the previous equation becomes

$$\frac{\eta_t}{\mu_t} V(\bar{h}) \frac{[\alpha\beta]^\alpha [\phi_1 [1-\alpha]]^{1-\alpha}}{\exp(\alpha g_Y)} = w_t^{[1-\psi][1-\alpha]} \quad \text{where} \quad V(\bar{h}) =_{\text{def}} \frac{U(\bar{h})}{\bar{h}^{1-\alpha}}. \tag{33}$$

As g_Y is independent of $\bar{h}$, (33) shows that a WTR will lead to an increase or a decrease in hourly wage w_t according to whether $V(\cdot)$ is decreasing or increasing at values of h lower than $h = \bar{h}$. Let us determine it by analysing the sign of the elasticity of $V(\cdot)$, which is equal to

$$h\frac{V'(h)}{V(h)} = h\frac{U'(h)}{U(h)} - [1-\alpha]. \tag{34}$$

By condition (13) characterising the choice of h, this elasticity is nil at $h = \bar{h}$. Let us then consider a finite WTR, $\Delta h < 0$, which decreases h to the value $\bar{h} + \Delta h < \bar{h}$. As the elasticity of function U is assumed to be decreasing in h, this elasticity is larger than $1 - \alpha$ for any $h \in \left]0, \bar{h}\right[$. Given (34), the elasticity of V is therefore strictly positive for any $h \in \left]0, \bar{h}\right[$. Consequently, $V(\bar{h} + \Delta h) < V(\bar{h})$ if $\Delta h < 0$. Given (33), a WTR therefore reduces w_t (since $1 - \psi > 0$). Via (23), a decrease in w_t also means a decrease in L_t. Since h (now $= \bar{h} + \Delta h$), w_t and L_t all decrease, (17) implies that Y_t decreases.

A.4 Proof of Proposition 3

The differential of the ln of (28) with respect to a change in h (evaluated at $\bar{h}$) and L is

$$\frac{dw}{w} = -\frac{dh}{\bar{h}} - \frac{dL}{L}. \tag{35}$$

The differential of the ln of (23) can then be written as

$$\frac{dw}{w} = \phi_4 \frac{\ell}{1-\ell}\frac{dL}{L} \quad \text{with} \quad \ell =_{\text{def}} \frac{L}{P}. \tag{36}$$

The differential of the ln of (17) implies that the relative changes in L, Y, w and $\bar{h}$ are linked by $dL/L = dY/Y - dw/w - dh/\bar{h}$. Introducing this expression into (36) gives

$$\frac{dw}{w} = \phi_4 \frac{\ell}{1-\ell}\left[\frac{dY}{Y} - \frac{dw}{w} - \frac{dh}{\bar{h}}\right] \qquad \text{or} \qquad \frac{dw}{w} = -\frac{\phi_4 \ell}{1-\ell+\phi_4\ell}\frac{dh}{\bar{h}}, \tag{37}$$

since a change in h does not affect Y (see (27)). Combining (35) and (37) leads to

$$\frac{dL}{L} = -\frac{1-\ell}{1-\ell+\phi_4\ell}\frac{dh}{\bar{h}}. \tag{38}$$

Equations (37) and (38) show that a decrease in h leads to an increase in both w and L. Moreover, (37) shows that the absolute value of the elasticity of w with respect to h is increasing in ℓ, from 0 (when $\ell = 0$) to 1 (when $\ell = 1$). Conversely, (38) shows that the absolute value of the elasticity of L with respect to h is decreasing in ℓ, from 1 (when $\ell = 0$) to 0 (when $\ell = 1$). If $F/\underline{\mu}$ is smaller, Y and L are smaller too (see Proposition 2). So is ℓ, which completes the proof of the first point of the proposition.

The second point of the proposition follows from (17): since a WTR increases L without affecting Y (see (27)), wh necessarily decreases. Since w increases, (17) furthermore implies a reduction in hL. Given the first point of the proposition, the size of the changes in wh and hL depends on the value of ℓ and thereby that of $F/\underline{\mu}$ as stated in the second point of the proposition. The elasticity of hL to h ranges from 0 when $\ell = 0$ to 1 when $\ell = 1$; that of wh ranges from 1 when $\ell = 0$ to 0 when $\ell = 1$.

The third point of Proposition 3 follows from the fact that a WTR leaves the stationary consumption level of a family unchanged but decreases the disutility of work $d(\bar{h})L$. Consumption is unchanged because a WTR does not change either the stationary state output level nor the savings rate. Since $d(h)$ is increasing and convex in h, a WTR of x% decreases $d(h)$ by more than x%. As it increases L by less than x%, it unambiguously decreases $d(h)L$.

A.5 Proof of Proposition 4

When η tends to $\overline{\eta} < +\infty$, $E < 1$ in the stationary state and equation (15) becomes

$$\overline{\eta}\, U(\bar{h}) \left[\frac{K}{Y}\right]^{\alpha} \left[e\frac{L}{Y}\right]^{1-\alpha} = \underline{\mu} B(E) \tag{39}$$

After replacing K/Y, L/Y and e by the stationary state expressions obtained from (26), (17) and (21), we can rewrite (39) as follows:

$$w^{[1-\psi][1-\alpha]} = [\alpha\beta]^{\alpha} \left[\phi_1[1-\alpha]\right]^{1-\alpha} \frac{\overline{\eta}}{\underline{\mu}} \frac{V(\bar{h})}{B(E)}, \tag{40}$$

with $V(\bar{h})$ defined in (33). At given $\bar{h}$, (40) implies that w is decreasing in E since $B'(E) > 0$.

The proof successively considers three cases, which differ as far as the impact of a WTR is concerned. In the first case, $E = 0$ before a WTR. In the second case, $E > 0$ before a WTR and remains so after it. A boundary case is finally considered where $E > 0$ before a WTR but becomes nil as a result of it.

a) When F is finite, a stationary state equilibrium with $E = 0$ corresponds to a situation where, in the long run, the economy consumes less resource than F (i.e. $\underline{\mu} Y < F$) so that asymptotically $R_t \to \infty$. When $E = 0$, $B(E) = B(0) = 1$ and (40) determines w as a function of $\bar{h}$ (and model parameters):

$$w = \left[[\alpha\beta]^{\alpha} \left[\phi_1 [1-\alpha]\right]^{1-\alpha} \frac{\overline{\eta}}{\underline{\mu}} V(\bar{h}) \right]^{\frac{1}{[1-\psi][1-\alpha]}}. \tag{41}$$

Using this value of w, one obtains L (via (23)) and e (via (21)), which are both increasing in w. Together, w, L and $\bar{h}$ then determine, via (17), the stationary state level of output that we note as follows:

$$Y = Y^*(\bar{h}), \tag{42}$$

which is the equilibrium level if $Y^*(\bar{h}) < F/\underline{\mu}$. Since $V(h)$ is increasing in h for any $h \in]0, \bar{h}[$, a finite reduction in h below $\bar{h}$ decreases $V(h)$ and therefore w (via (41)) and L (via (23)). Lower values of h, w and L imply, via (17), a decrease in the output given by (42): with $\bar{h}' =_{def} \bar{h} + \Delta h < \bar{h}$, $Y^*(\bar{h}') < Y^*(\bar{h})$.

b) Let us now consider a WTR in a stationary state with $0 < E < 1$. When $E > 0$, the resource stock tends toward a finite stationary state level and (24) implies that $Y = F/\underline{\mu}$, which does not depend on $\bar{h}$. Substituting L by (17) into (23) and using $Y = F/\underline{\mu}$ then lead to

$$w = \left[\frac{\phi_3}{(1-\psi)\phi_1 - \phi_2} \right]^{1/\psi} \left[1 - \frac{1-\alpha}{w\bar{h}} \frac{F/\underline{\mu}}{P} \right]^{-\phi_4}. \tag{43}$$

This equation implicitly determines w as a decreasing function of $\bar{h}$ and an increasing function of $F/\underline{\mu}$. With this value of w, one obtains L (via (23)) and e (via (21)). Like w, they are decreasing in $\bar{h}$ and increasing in $F/\underline{\mu}$. Using (40), one can also determine E: it is decreasing in w and therefore increasing in h and decreasing in $F/\underline{\mu}$. The obtained values for E and Y then determine R (via (6)). As equation (43) implies a decreasing relationship between w and $\bar{h}$, a WTR increases w which, using (23), also means an increase in L. The expression of the elasticity of L (resp. w) with respect to h is easily shown to be the same as in the proof of point 1 of Proposition 3 (see expression (38) (resp. (37))).

c) Let us finally consider the boundary case in which a WTR decreases E from a strictly positive value to 0. Comparing $Y = F/\underline{\mu}$ and (42) shows that $E > 0$ and $Y = F/\underline{\mu}$ if $F/\underline{\mu} < Y^*(\bar{h})$ but $E = 0$ and $Y = Y^*(\bar{h})$ if $F/\underline{\mu} \geq Y^*(\bar{h})$. Because a WTR from $\bar{h}$ to $\bar{h}'$ decreases the level of output given by (42) from $Y^*(\bar{h})$ to $Y^*(\bar{h}') < Y^*(\bar{h})$, it also decreases the threshold value of $F/\underline{\mu}$ above which $E = 0$.

Three intervals of the possible values of $F/\underline{\mu}$ must therefore be distinguished:

- if $F/\underline{\mu} \in [Y^*(\bar{h}), \rightarrow [$, a WTR decreases L, Y and w as analysed in paragraph a);
- if $F/\underline{\mu} \in]0, Y^*(\bar{h}')]$, a WTR increases L and w without affecting Y as explained in paragraph b);
- if $F/\underline{\mu} \in]Y^*(\bar{h}'), Y^*(\bar{h})[$, a WTR reduces Y but all the less so as $F/\underline{\mu}$ is close to $Y^*(\bar{h}')$. Therefore if $F/\underline{\mu}$ is sufficiently close to the lower bound $Y^*(\bar{h}')$, a WTR necessarily decreases L; but it decreases L if $F/\underline{\mu}$ is sufficiently close to the upper bound $Y^*(\bar{h})$.

The claim in the third bullet hereabove relies on a continuity argument. If $F/\underline{\mu} > Y^*(\bar{h}')$, a WTR implies a fall in output from $F/\underline{\mu}$ to $Y^*(\bar{h}')$: the more $F/\underline{\mu}$ exceeds $Y^*(\bar{h}')$, the larger the output fall (with a maximum effect observed when, before WTR, $F/\underline{\mu} = Y^*(\bar{h})$). If $F/\underline{\mu}$ is sufficiently close to $Y^*(\bar{h}')$, a WTR reduces Y to a rather small extent and therefore still allows for an increase in L (which is all the smaller as $F/\underline{\mu}$ exceeds $Y^*(\bar{h}')$). At the opposite, when $F/\underline{\mu}$ is sufficiently close to $Y^*(\bar{h})$, the negative output effect of a WTR is strong enough to imply a negative effect on L. There is therefore a value of $F/\underline{\mu}$ between $Y^*(\bar{h}')$ and $Y^*(\bar{h})$ (say $\widehat{F/\underline{\mu}}$) for which the negative output effect of a WTR is just strong enough to imply that L and w remain unchanged. To this value $\widehat{F/\underline{\mu}}$, we can associate a certain level $\hat{E} > 0$ of the resource exploitation rate before WTR. For any lower value of $F/\underline{\mu}$ (or equivalently if $E > \hat{E}$ before WTR), a WTR increases L but all the less so as $F/\underline{\mu}$ is close to $\widehat{F/\underline{\mu}}$. The employment effect of a WTR is just nil if $F/\underline{\mu} = \widehat{F/\underline{\mu}}$ (or equivalently if $E = \hat{E}$ before WTR). If $F/\underline{\mu}$ is strictly larger than $\widehat{F/\underline{\mu}}$ (or equivalently if $E < \hat{E}$ before WTR), a WTR decreases L and all the more so as $F/\underline{\mu}$ is large (with a maximum negative effect reached when $F/\underline{\mu}$ is equal to $Y^*(\bar{h})$).

A.6 Generalization to the case of an endogenous flow of renewable resource

In this appendix, we consider that the flow F_t of renewable resource is endogenous and function of the stock of natural capital: $F_t = \mathbf{F}(R_t)$. This assumption on F_t takes into account the fact that some renewable resources are characterized by a regeneration process that depends on the existing resource stock. The equation of natural capital accumulation (24) then becomes $R_{t+1} - R_t = \mathbf{F}(R_t) - X_t$ and is the same as that assumed in e.g. Smulders et al. [2014] (p. 428). Using (6), it can be rewritten as

$$R_{t+1} - R_t = \left(\frac{\mathbf{F}(R_t)}{R_t} - E_t \right) R_t.$$

In a stationary state where $R_{t+1} = R_t = R$, it implies $E = \frac{\mathbf{F}(R)}{R} =_{def} \mathcal{E}(R)$. Furthermore, if $\eta \rightarrow +\infty$, we have seen that $E = 1$: so R is such that $\mathcal{E}(R) = 1$, i.e. $R = \mathcal{E}^{-1}(1)$. As $X = R$ when $E = 1$,

(27) is replaced by $Y = \mathcal{E}^{-1}(1)/\underline{\mu}$.[26] This difference does not alter the reasoning behind Propositions 2 and 3 and their proof, which remains valid *mutatis mutandis*: in section 4.3.1, the term F must be substituted by the term $\mathcal{E}^{-1}(1)$ (the abundance of natural capital (in efficient units) now being measured by $\mathcal{E}^{-1}(1)/\underline{\mu}$) but the wording of the propositions and theirs proofs are otherwise identical.

A.7 Functional form for $U(h_t)$ in numerical experiments

Obtaining a specific functional form for $U(h_t)$ requires an assumption on the function $\mathcal{D}(h_t)$ introduced in Appendix A.1. We opt for the following reduced form: $\mathcal{D}(h_t) = A \cdot [1 - \exp[-\xi \cdot h_t]]$ with $A,\ \xi > 0$. In this case, $U(h_t) = [\lambda \cdot h_t]^{\alpha} \left[A \cdot [1 - \exp[-\xi \cdot h_t]]\right]^{1-\alpha}$, which leads to (30) where $\tilde{\lambda} =_{\text{def}} \lambda^{\alpha} A^{1-\alpha}$.

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[26] For instance, in the standard case of the regeneration function $\mathbf{F}(R_t) = \chi R_t \left[1 - R_t/Q\right]$, with $0 < \chi < 1$ and $Q > 0$, (24) describes a logistic growth equation of natural capital (in the absence of human consumption of it), in which Q determines a ceiling for the value of R (also called the carrying capacity). In this case, $\mathcal{E}(R) = \chi \cdot [1 - R/Q]$ and $R = \mathcal{E}^{-1}(1)$ becomes $R = [\chi^{-1} - 1]Q$, which leads to $Y = [\chi^{-1} - 1]Q/\underline{\mu}$.

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