CESifo WORKING PAPERS

9375
2021

October 2021

Efficiency and Equity: A General Equilibrium Analysis of Rent-Seeking

Ben J. Heijdra, Pim Heijnen

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 9375*

Efficiency and Equity: A General Equilibrium Analysis of Rent-Seeking

Abstract

We study the rent-seeking phenomenon using a simple, static general equilibrium model. The economy consists of two sectors, both employing a constant returns-to-scale technology with labor as its sole input. One of the sectors is a monopoly, where a continuum of agents compete for a share of monopoly profits (i.e. rent). Agents are heterogeneous in labor productivity and rent-seeking ability: they face a choice between engaging in (productive) work or vying for a share of the rent (i.e. a contest against other rent-seekers). At the aggregate level, rent-seeking reduces the available amount of labor in the economy and thereby lowers output and welfare (rent-seeking is inefficient). At the individual level, rent-seeking shifts income towards rent-seekers. Consequently, an economy with few rent-seekers tends to have high income inequality: an effect that is exacerbated by the fact that rent is decreasing in the number of rent-seekers (low levels of rent-seeking increase inequity). This tradeoff between efficiency and equity is the primary focus of this paper. We investigate how the distribution of rent-seeking ability and the correlation between labor productivity and rent-seeking ability shape this tradeoff.

JEL-Codes: D620, D630, D720, E130.

Keywords: rent-seeking, economic waste, inequality, monopolization, contest.

Ben J. Heijdra
Faculty of Economics and Business
University of Groningen
P.O. Box 800
The Netherlands – 9700 AV Groningen
b.j.heijdra@rug.nl

*Pim Heijnen**
Faculty of Economics and Business
University of Groningen
P.O. Box 800
The Netherlands – 9700 AV Groningen
p.heijnen@rug.nl

*corresponding author

October 19, 2021

1 Introduction

We examine a general equilibrium model where (potentially) a monopolist is active in one sector of the economy. This raises the question of how monopoly profit, or rent, is distributed among agents.[1] In our setting agents are endowed with time, which they either spend on a productive activity (labor) or on rent-seeking (competing against other agents for a share of the rent). Rent-seeking is modelled along the lines of the classic Tullock contest: only relative effort matters. Consequently, agents spend too much time on rent-seeking in a futile attempt to outdo the others. Moreover, in a general equilibrium setting, the size of the rent is endogenously determined. Since rent-seeking leads to a lower work force, it has an impact on the 'real' economy: an increase in rent-seeking effort decreases the size of the rent.

Under the assumption that agents have identical labor productivity, but are heterogeneous in their rent-seeking aptitude, we establish a tradeoff between aggregate income and income inequality. If high aggregate income is seen as a desirable outcome, then it is best if there is a small group of agents who are extraordinarily good at rent-seeking. Faced with this kind of competition, most agents will abstain from rent-seeking. This will lead to high aggregate income, but also high monopoly profits. Since profit is distributed among a small group of rent-seekers, income inequality will be high.[2]

We show how aggregate income and income inequality depend on the distribution of rent-seeking aptitude. We investigate the possibility that a tax on non-labor income can lead to welfare gains for all agents, i.e. the conditions under which a redistributive tax leads to a Pareto-improvement. Finally, we introduce heterogeneity in labor productivity and argue that strong negative correlation between labor productivity and rent-seeking aptitude can lead to an equal distribution of income.

Obviously we are not the first to investigate the phenomenon of rent-seeking. Tullock (1967) already observed that the welfare loss of monopoly is larger than the Harberger triangle (Harberger, 1954) since effort is exerted in establishing or maintaining the monopoly position. While Tullock surmised that the total cost of effort could exceed monopoly profit, a formal game-theoretic foundation, in the form of contest theory, did not emerge until the 1980s: with a Tullock contest success function, rent dissipation can be substantial but the total cost of effort does not exceed monopoly profit.[3] See Konrad (2009) or Corchón (2007) for an overview of contest theory.

One of the downsides of contest theory is the fact that it is a partial equilibrium analysis. Both the cost of effort, presumably the opportunity cost of not engaging in productive economic activities, and monopoly profit are exogenously given. Brooks and Heijdra (1988) is an early, but unsatisfactory attempt to endogenize these quantities. In their setting some consumers are

[1]We will exclusively deal with the case where rent is created through market power. There are of course other sources of rent such as landholding, access to scarce resources or political power.

[2]If equity is the desired outcome, then this reasoning runs in the opposite direction.

[3]In two-player contests, the Tullock contest success function is

$$p_i = \frac{x_i^R}{x_i^R + x_j^R},$$

where p_i is the probability that player $i = 1, 2$ receives the monopoly profit, x_i is the cost of effort for player i, $i \neq j$ and $R > 0$. Then the total cost of effort as a fraction of monopoly profit is $\min\{R/2, 1\}$. See Ewerhart (2015) for a full analysis.

regulators with the power to grant monopoly rights to one sector of the economy. The aspiring monopolist bribes some of the regulators into taking this decision (monopoly profit is essentially distributed between the regulators). Since the income of the regulators increases, the demand curve shifts outward and monopoly profit increases, i.e. the size of the rent is endogenized. However, the rent-seeking process is modelled in an ad hoc manor: the agents have a fixed role and full rent dissipation is imposed. This is our main contribution: we embed the standard Tullock contest in a general equilibrium model in such a manner that agents actively choose whether and to what extent they rent-seek.

There is recent literature on (optimal) taxation (Lockwood, Nathanson, Weyl, 2017; Rothschild and Scheuer, 2016) that investigates the allocation of workers to jobs in the presence of externalities. Rothschild and Scheuer, in particular, introduce a rent-seeking sector whose size negatively influences productivity in the other sector. Without taxation, an abundance of workers end up in this sector. Rothschild and Scheuer determine the optimal taxation scheme to correct for this externality. The general equilibrium closure amounts to redistribution of the proceeds of the tax. It is a purely a model of occupational choice where the wage rate is exogenously determined. While we do not look at optimal taxation, we show that under certain circumstances, a tax on non-labor income can be Pareto-improving.

There have been a couple of recent attempts to incorporate rent-seeking in a full model of the macroeconomy (Barelli and Pessõa, 2012; Brou and Rutan, 2013). Barelli and Pessõa study a Ramsey model with two sectors (productive and non-productive). The non-productive sector appropriates part of the output of the productive sector. Consequently, rent-seeking is purely a macro-phenomenon and there is only welfare loss in the aggregate. We add detail at the micro-level and show that there are also effects on the distribution of income.

Brou and Rutan investigate an economy with many sectors, where is each sector is characterized by oligopolistic competition and free entry. Firms use labor for three different purposes: production, R&D and lobbying/rent-seeking (i.e. competing for lump-sum subsidies). Lobbying is completely independent of the firm's other activities. As a result, subsidies are not distortionary. The main question addressed by Brou and Rutan is the effect of rent-seeking on economic growth. In the companion paper, Heijdra and Heijnen (2021), we analyze a three-sector macroeconomic general equilibrium model featuring labour-using rent-seeking activities (with diminishing returns to rent-seeking time), overlapping generations, and endogenous growth originating from capital-based external effects. We find that, depending on the life-cycle payoff structure of rent-seeking activities, economic growth may be harmed or promoted by rent-seeking efforts. The intertemporal dimension is thus shown to play a crucial role in determining the ultimate effects of rent-seeking.

Finally, there is a large literature in institutional economics that deals with corruption and its implications for economic development. Examples include Acemoglu and Verdier (1998) and Murphy, Shleifer and Vishny (1993).[4] Typically, this literature is mainly concerned with (the optimal level of) property rights, whereas we assume that property rights are well-defined. In our setting, the goal of rent-seeking is essentially to obtain those property rights.

The paper is structured as follows. Section 2 contains the description of the model. Section 3 focuses on the simplest version of the model where there are constant returns to scale to rent-seeking and one of the sectors is monopolized with certainty. In that context, we study how the

[4]Note that Murphy et al. (1993) refer to corruption as a form of rent-seeking.

distribution of rent-seeking ability affects aggregate income and income inequality. In Section 4, we investigate the effects of a redistributive tax. Section 5 contains two robustness checks (non-constant returns to scale, monopolization with probability less than one) and we introduce heterogeneity in labor productivity. While most of our results continue to hold, we show that strong negative correlation between labor productivity and rent-seeking aptitude leads to an outcome where everyone specializes in their strongest profession. Finally, Section 6 concludes. All proofs are relegated to the appendix.

2 Model

Although rent seeking can take place in many different settings we stay as close as possible to the original idea of Tullock (1967) by assuming that the objective of rent seekers is to monopolize one of the productive sectors in the economy. The model has the following key features. There is a continuum of agents that are identical in all aspects except for their aptitude for rent-seeking, η. Assume that η has support $[\eta_0, \eta_1]$, where $\eta_1 > \eta_0 > 0$, a cumulative density function F and a probability density function f.[5] To avoid cluttering the notation we normalize the size of the population to unity. Each individual has one unit of time which they can use for productive purposes by working in one of the sectors in the economy and for 'lobbying' activities. We denote the rent-seeking time of individual η by $e(\eta) \in [0, 1]$. There are two production sectors in the economy, producing the distinct commodities X_1 and X_2. A rent-seeking 'success' consists of the monopolization of the sector producing X_1.[6] If monopolization takes place, profits in sector 1 will be equal to Π_1^m of which individual η will receive a share $s(\eta)$:

$$s(\eta) = \frac{\eta e(\eta)^{\varepsilon}}{R}, \tag{1}$$

where $\varepsilon \in (0, 1]$ is a parameter. Note that $\eta e(\eta)^{\varepsilon}$ denotes the *effective* rent-seeking effort and R is the total amount of lobbying taking place:

$$R = \int_{\eta_0}^{\eta_1} \eta e(\eta)^{\varepsilon} dF(\eta). \tag{2}$$

The key features of this specification of the reward function are as follows. First, because the share is strictly concave in rent-seeking effort when $\varepsilon < 1$, there are decreasing returns to scale to rent-seeking time.[7] Second, the entire profit is passed on to rent seekers, i.e. $\int_{\eta_0}^{\eta_1} s(\eta) dF(\eta) = 1$.

We model the rent seeking process as a two-stage game. In the first stage each individual chooses how much to engage in lobbying activities. In the second stage uncertainty is resolved (and monopolization of sector 1 does or does not occur) and consumption and production takes place. We solve the model by using backward induction.

[5] We allow for the possibility that $\eta_1 = +\infty$. Since only relative rent-seeking aptitude matters, we typically set the value of η_0 equal to one.

[6] There is no specific reason why sector 1 is vulnerable to monopolization, but implicitly we are assuming that it is commonly know that only this sector will produce rent. So, there are no coordination issues, where some agents exert effort to obtain a fraction of the non-existent profit in sector 2.

[7] When $\varepsilon = 1$, there are constant returns to scale to rent-seeking time: as we will see, in this case individuals will either specialize in rent-seeking or devote their time solely to productive work (in the spirit of the Roy model of occupational choice; Roy, 1951; Rothschild & Scheuer, 2016).

2.1 Stage 2: production and consumption

At the start of stage 2 each individual knows his/her income level $I(\eta)$. The utility function of an agent of type η is given by:

$$U(\eta) \equiv \left[\alpha x_1(\eta)^{1-1/\sigma} + (1-\alpha)x_2(\eta)^{1-1/\sigma}\right]^{1/(1-1/\sigma)}, \tag{3}$$

where $x_i(\eta)$ is the demand for good i by an agent of type η, α is a share parameter $(0 < \alpha < 1)$, and σ is the substitution elasticity between the two goods. We assume that $\sigma > 1$ so that, in case sector 1 is monopolized, the profit function of the firm in that sector is concave in its own price.

Note that agents have identical preferences over the two goods, but they differ in their ability to generate income so that the budget constraint is given by:

$$P_1 x_1(\eta) + P_2 x_2(\eta) = I(\eta), \tag{4}$$

where P_i is the price of good i. The demand functions for the two goods are given by:

$$x_1(\eta) = \frac{\alpha^\sigma P_1^{-\sigma}}{\alpha^\sigma P_1^{1-\sigma} + (1-\alpha)^\sigma P_2^{1-\sigma}} I(\eta), \tag{5}$$

$$x_2(\eta) = \frac{(1-\alpha)^\sigma P_2^{-\sigma}}{\alpha^\sigma P_1^{1-\sigma} + (1-\alpha)^\sigma P_2^{1-\sigma}} I(\eta). \tag{6}$$

The indirect utility function thus takes the following form:

$$V(\eta) = \frac{I(\eta)}{P_V}, \tag{7}$$

where P_V is the true price index:

$$P_V \equiv \left[\alpha^\sigma P_1^{1-\sigma} + (1-\alpha)^\sigma P_2^{1-\sigma}\right]^{1/(1-\sigma)}. \tag{8}$$

An individual η who spent $e(\eta)$ units of time on lobbying in the first stage will express a labour supply equal to $1 - e(\eta)$ in the second phase and earn a wage income equal to $W\left[1 - e(\eta)\right]$. The nominal wage is used as the numeraire, i.e. $W = 1$. It follows that the aggregate supply of labour in the second stage, L, is given by:

$$L = 1 - \int_{\eta_0}^{\eta_1} e(\eta)dF(\eta). \tag{9}$$

The production function in industry i features constant returns to scale, i.e. $X_i = L_i$ where L_i is the labour input used in sector i. Sector 2 is characterized by perfect competition so that marginal cost pricing results in a competitive price equalling:

$$P_2^c = W = 1. \tag{10}$$

Next we consider the monopolist in sector 1. Using equation (5), integrating over η, and noting that the price in sector 2 is equal to one, we obtain the demand curve:

$$X_1 = \frac{\alpha^\sigma P_1^{-\sigma}}{\alpha^\sigma P_1^{1-\sigma} + (1-\alpha)^\sigma} I, \tag{11}$$

where I is aggregate income:

$$I \equiv \int_{\eta_0}^{\eta_1} I(\eta) dF(\eta). \tag{12}$$

The monopolist takes aggregate income as given and chooses P_1 such that profit, $\Pi_1^m = X_1(P_1 - 1)$, is maximized. The resulting monopoly price is a markup on marginal cost:

$$P_1^m = \frac{\varepsilon_d^m}{\varepsilon_d^m - 1}, \tag{13}$$

where the demand elasticity resulting from the monopolist's pricing decision is defined (in absolute value) as:

$$\varepsilon_d^m \equiv -\frac{\partial x_1}{\partial P_1}\frac{P_1}{x_1} = \frac{\alpha^\sigma \left(P_1^m\right)^{1-\sigma} + \sigma(1-\alpha)^\sigma}{\alpha^\sigma \left(P_1^m\right)^{1-\sigma} + (1-\alpha)^\sigma} > 1. \tag{14}$$

Solving (13) and noting (14) we find a unique monopoly price for good 1 as a function of the structural parameters α and σ.

If the monopolization takes place aggregate income equals:

$$I = \int_{\eta_0}^{\eta_1} \left[1 - e(\eta) + s(\eta)\Pi_1^m\right] dF(\eta) = L + \Pi_1^m, \tag{15}$$

so that we obtain the following expression for monopoly profit:

$$\Pi_1^m = \frac{\alpha^\sigma \left(P_1^m\right)^{-\sigma} \left(P_1^m - 1\right)}{\alpha^\sigma \left(P_1^m\right)^{1-\sigma} + (1-\alpha)^\sigma} \left(L + \Pi_1^m\right). \tag{16}$$

Monopoly profit appears on the right-hand side of this expression because rent seekers ultimately capture the entire profit. Solving equation (16) for Π_1^m we find:

$$\Pi_1^m = \omega L, \tag{17}$$

where ω is a positive constant:

$$\omega = \frac{\alpha^\sigma \left(P_1^m\right)^{-\sigma} \left(P_1^m - 1\right)}{\alpha^\sigma \left(P_1^m\right)^{-\sigma} + (1-\alpha)^\sigma}. \tag{18}$$

Intuitively, ω represents the monopoly profit that would materialize in the *hypothetical* case in which monopolization occurs without any rent-seeking during the first stage (i.e. when no labour is wasted and $L = 1$).

The model has two features that greatly simplify the analysis of the model. First, the individual demand functions are linear in income and therefore the monopoly price is independent of the distribution of income.[8] Consequently, the analysis of the model is not complicated by distortionary price effects. Second, indirect utility is linear in the income of each individual agent. This implies that, when sector 1 is monopolized, we can use aggregate income I as a measure of welfare. In fact, from (15) and (17), we see that $I = (1+\omega)L$. So, if we take aggregate income as the measure for welfare, then welfare decreases as agents leave the labor force to engage in rent-seeking.

2.2 Stage 1: rent seeking

In stage 1 each individual makes the lobbying decision. With probability κ monopolization takes place, the monopoly price for good 1 is P_1^m, the competitive price of good 2 is P_2^c, and the income of a rent-seeking agent equals $I(\eta) = 1 - e(\eta) + s(\eta)\Pi_1^m$. With probability $1-\kappa$, however, monopolization does not occur, prices are given by $P_1^c = P_2^c = 1$, and income is equal to $I(\eta) = 1 - e(\eta)$. It follows that expected utility of the agent from the perspective of stage 1 is given by:

$$\mathbb{E}\left[V(\eta)\right] = \kappa \frac{1 - e(\eta) + s(\eta)\Pi_1^m}{P_V^m} + (1-\kappa)\frac{1-e(\eta)}{P_V^c}, \tag{19}$$

where P_V^m and P_V^c denote, respectively, the true price index with and without monopolization (see (8) above). Agent η chooses $e(\eta) \in [0,1]$ in order to maximize expected utility, $\mathbb{E}\left[V(\eta)\right]$, noting the share function (1) and taking as given the total amount of effective rent-seeking, R, the true price indices, P_V^m and P_V^c, and the total amount of monopoly profit, Π_1^m.

3 Benchmark

To analyze the incentives to engage in rent-seeking, we start with the benchmark case where $\varepsilon = 1$ (constant returns to rent-seeking) and $\kappa = 1$ (monopolization always happens). Then (19) reduces to:

$$\mathbb{E}\left[V(\eta)\right] = \frac{1 - e(\eta) + s(\eta)\Pi_1^m}{P_V^m}, \tag{20}$$

and the agent's goal is to maximize income, i.e.

$$1 - e(\eta) + s(\eta)\Pi_1^m = 1 - e(\eta) + \frac{\eta e(\eta)}{R}\Pi_1^m. \tag{21}$$

Since this expression is linear in $e(\eta)$, income is maximized by either $e(\eta) = 0$ (working) or $e(\eta) = 1$ (rent-seeking). From (21), we see that income from working is 1 for everyone, while income from rent-seeking is $\eta\Pi_1^m/R$ is increasing in rent-seeking aptitude. This strongly suggests that low-aptitude individuals specialize in working, while high-aptitude individuals devote all their time to rent-seeking. The indifferent individual $\hat{\eta}$ receives the same income for each activity:

$$1 = \frac{\hat{\eta}}{R}\Pi_1^m, \tag{22}$$

[8] A necessary and sufficient condition for a demand function to be linear in income is that the indirect utility function is of the Gorman form. This class includes CES-preferences.

or, after plugging in the various definitions and rewriting the expression slightly,

$$\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega\hat{\eta}F(\hat{\eta}) = 0. \tag{23}$$

Let $g(\hat{\eta})$ denote the right-hand side of (23) as function of $\hat{\eta}$. Note that $g(\eta_0) = \mathbb{E}\eta > 0$, $g(\eta_1) = -\omega\eta_1 < 0$ and $g'(\hat{\eta}) = -(1+\omega)\hat{\eta}f(\hat{\eta}) - \omega F(\hat{\eta}) < 0$. This establishes that there is a unique indifferent agent $\hat{\eta} \in (\eta_0, \eta_1)$ such that

$$e(\eta) = \begin{cases} 0 & \text{if } \eta < \hat{\eta} \\ 1 & \text{if } \eta \geq \hat{\eta} \end{cases} \tag{24}$$

The fraction of rent-seekers is $\hat{q} = 1 - F(\hat{\eta})$.

In the remainder of this section we investigate how the fraction of rent-seekers, aggregate income and income inequality depend on the distribution of rent-seeking aptitude. In particular, we investigate what happens when the right-tail of the distribution of η becomes thinner as this intuitively seems to capture the idea of increasing concentration of rent-seeking aptitude. This only leads to unambiguous comparative statics for the location of $\hat{\eta}$. While the location of the indifferent agent moves to the left in terms of absolute rent-seeking aptitude, in relative terms the agent may increase or decrease in strength. This is due to the fact that if we make the tails thinner, then some probability mass is inevitably pushed to the left. However once we know whether the fraction of rent-seekers $\hat{q}$ is either increasing of decreasing, then we see the tradeoff between efficiency and inequality:

Proposition 1. *Suppose*

$$\frac{\partial f}{\partial \alpha}(\eta, \alpha) \geq 0 \iff \eta \leq \eta^*(\alpha), \tag{25}$$

for some function $\eta^* : (\alpha_0, \alpha_1) \to (\eta_0, \eta_1)$. Then

(i) the location of the marginal rent-seeker $\hat{\eta}$ is decreasing α.

Moreover, if the fraction of rent-seekers consequently increases [decreases]

(ii) aggregate income decreases [increases] in α,

(iii) income inequality, as measured by the Gini-coefficient, decreases [increases] in α.

Note that we use the Gini-coefficient as measure for income inequality. However the result holds for any measure of inequality that is invariant to scale and that satisfies the (Pigou-Dalton) principle of transfers. Below are two examples to show that both possibilities can occur.

Example 1: Pareto distribution

The cumulative distribution function and the probability function are given by

$$F(\eta) = 1 - \eta^{-\alpha} \text{ and } f(\eta) = \alpha\eta^{-(\alpha+1)} \tag{26}$$

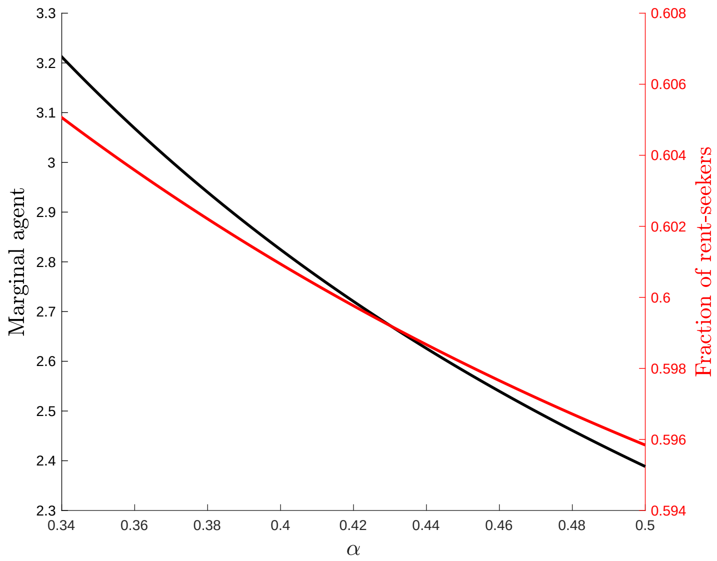


Figure 1: *The black line represents $\hat{\eta}$ as function of α, the red line $\hat{q}$ as function of α. The value of ω was chosen such that at $\alpha = {}^5\!/_{12}$, the fraction of rent-seekers is 60%.*

respectively. This distribution satisfies condition (25) (see Lemma A.2 in the Appendix). Straightforward algebra shows that the solution to (23) is given by:

$$\hat{\eta} = \sqrt[\alpha]{\frac{\omega(\alpha - 1) + \alpha}{\omega(\alpha - 1)}} \tag{27}$$

and the fraction of rent-seekers is

$$\hat{q} = \frac{\omega(\alpha - 1)}{\omega(\alpha - 1) + \alpha}. \tag{28}$$

One can verify that $\hat{q}$ is increasing in α.

Example 2: Gamma distribution

The cumulative distribution function and the probability function are given by

$$F(\eta) = \Gamma(\eta - 1, 1/\alpha) \text{ and } f(\eta) = \frac{(\eta - 1)^{1/\alpha - 1} e^{-(\eta - 1)}}{\Gamma(0, 1/\alpha)} \tag{29}$$

respectively. Note that (1) Γ is the incomplete gamma function, (2) the support has been shifted to $[1, \infty)$ (compared to the normal gamma distribution whose support is $[0, \infty)$ and (3) the parameter $\alpha \in (0, 1)$ is such that condition (25) is satisfied (compared to the normal way to specify the gamma distribution we have taken the reciprocal of that parameter). Unfortunately, there is no closed form solution for $\hat{\eta}$ or $\hat{q}$. In fact, even (25) could only be verified numerically. Figure 1 shows that $\hat{\eta}$ and $\hat{q}$ are both decreasing in α.

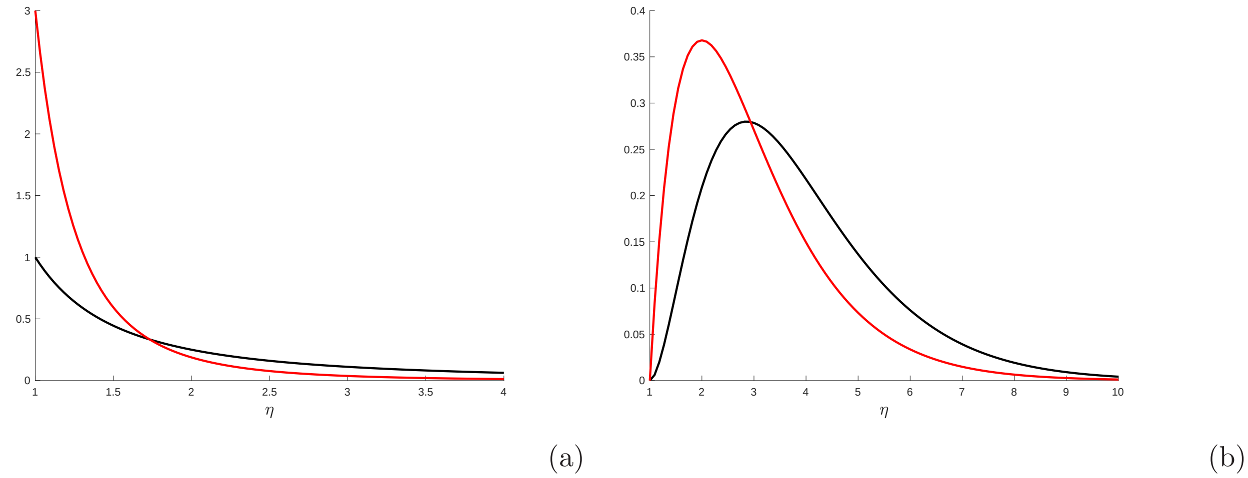


Figure 2: *Panel (a) shows the Pareto distribution and panel (b) the gamma distribution, in both cases for two different values of* α*, with the red line representing a higher value of* α *than the black line: for the Pareto distribution:* $\alpha \in \{1, 3\}$*, for the gamma distribution:* $\alpha \in \{0.35, 0.5\}$*.*

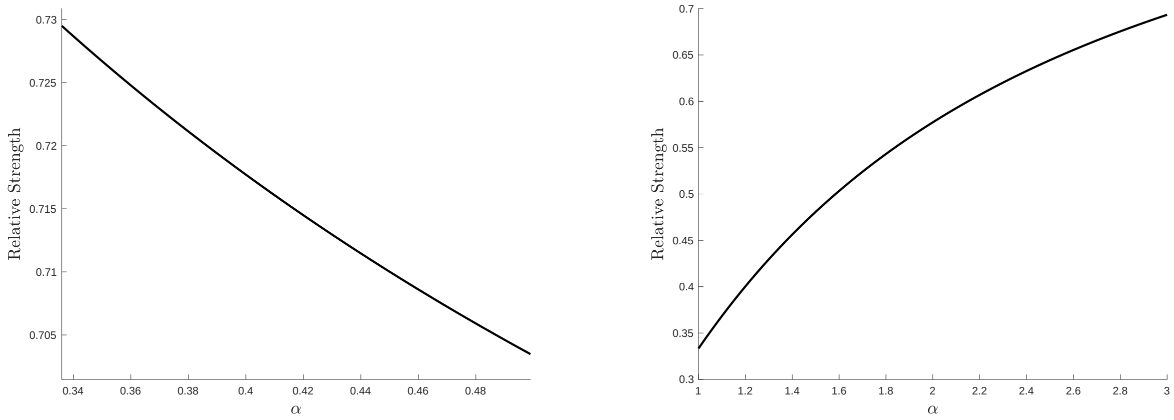
Comparison of examples

Figure 2 shows a side-by-side comparison of how the α-parameter changes the probability distribution function. Panel (a) shows the Pareto distribution and panel (b) the gamma distribution. The most striking difference is that the mode of the gamma distribution is not equal to the lower bound of the support, but in the rent-seeking decision the left tail of the aptitude distribution is irrelevant: each agent compares their skills against the best rent-seekers in the population and the change in the right tail is optically very similar.

The incentive to rent-seek depends on relative aptitude. So far, we have not been very specific how the individual agent's aptitude changes when the distribution of η changes. The idea is that the position of the agent in the distribution stays the same: an agent at the p-th percentile stays there for each value of α, while their (and other's) η may change. Let $\bar{\eta}_p(\alpha) = F^{-1}(p, \alpha)$ denote the rent-seeking aptitude of the agent at the p-th percentile as a function of α. If the agent at the p_0-th percentile gains strength compared to the agents at percentile $p > p_0$, then this increases the incentive to rent-seek. Hence, we expect that if for any $0 \leq p_0 < p_1 \leq 1$ we have

$$\frac{d}{d\alpha} \frac{\bar{\eta}_{p_0}(\alpha)}{\bar{\eta}_{p_1}(\alpha)} > 0 \tag{30}$$

then the fraction of rent-seekers will increase in α. (We refer to $\bar{\eta}_{p_0}(\alpha)/\bar{\eta}_{p_1}(\alpha)$ as the relative strength.) The exponential distribution gives a strong clue why this statement may be true.



(a) (b)

Figure 3: *Relative strength as a function of α: panel (a) is the gamma distribution and panel (b) the Pareto distribution. In both cases the agent at percentile* $p_0 = 0.7$ *is compared to the agent at percentile* $p_1 = 0.9$.

Example 3: Exponential distribution

Recall that the cumulative and probability distribution function for the exponential distribution are given by

$$F(\eta, \alpha) = 1 - e^{\alpha x} \text{ and } f(\eta, \alpha) = \alpha e^{\alpha x} \tag{31}$$

respectively. This distribution satisfies condition (25) (see Lemma A.3 in the Appendix). Note that for the exponential distribution, we get $\bar{\eta}_p(\alpha) = \log(1-p)/\alpha$. Consequently

$$\frac{d}{d\alpha}\frac{\bar{\eta}_{p_0}(\alpha)}{\bar{\eta}_{p_1}(\alpha)} = \frac{d}{d\alpha}\frac{\log(1-p_0)}{\log(1-p_1)} = 0. \tag{32}$$

Moreover, straightforward calculations reveal that the fraction of rent-seekers is independent of α.

Figure 3 plots the relative strength for the gamma- and the Pareto distribution as function of α. This further confirms our suspicion: if relative strength is increasing (decreasing), then rent-seeking increases (decreases) as well.[9]

We leave the formal proof to the appendix, but our intuition is spot on:

Proposition 2. *If*

$$\frac{d}{d\alpha}\frac{\bar{\eta}_{p_0}(\alpha)}{\bar{\eta}_{p_1}(\alpha)} > 0 \tag{33}$$

[9]It is tempting to relate this to the hazard rate which is constant for the exponential distribution, decreasing for the Pareto distribution and increasing for the gamma distribution. However, if we shift the support of the exponential distribution from $[0, \infty)$ to $[1, \infty)$, then the hazard rate is still constant, but the relative strength now depends on α.

for all $p_0, p_1 \in [0,1]$ *such that* $p_0 < p_1$, *then*

$$\frac{\partial \hat{q}}{\partial \alpha} > 0, \tag{34}$$

i.e. if the distribution of rent-seeking aptitude changes such that the relative strength of agents increases vis-à-vis more able rent-seekers, then rent-seeking becomes more prevalent.

Example 1 (cont.): Pareto distribution

Note that for the Pareto distribution we have:

$$\bar{\eta}_p(\alpha) = (1-p)^{-1/\alpha} \tag{35}$$

and

$$\frac{d}{d\alpha}\frac{\bar{\eta}_{p_0}(\alpha)}{\bar{\eta}_{p_1}(\alpha)} = \frac{d}{d\alpha}\left(\frac{1-p_1}{1-p_0}\right)^{1/\alpha} = -\frac{1}{\alpha^2}\left(\frac{1-p_1}{1-p_0}\right)^{1/\alpha}\log\left(\frac{1-p_1}{1-p_0}\right) > 0 \tag{36}$$

since $(1-p_1)/(1-p_0) < 1$. Applying Proposition 2, we conclude that rent-seeking increases in α.

4 A tax on non-labor income

In this section, we explore the effects of a redistributive tax. We assume that the government cannot distinguish between individual agents, but it can distinguish between different sources of income, in particular labor income (wages) and non-labor income (rent). The government sets an *ad valorem* tax on non-labor income equal to τ and then redistributes the proceeds equally among the agents as a lump-sum payment T.

Throughout this section, the dependency of the share of the rent that an agent receives and (total) monopoly profit on $\hat{\eta}$ is made explicit, i.e. $s(\eta, \hat{\eta})$ and $\Pi_1^m(\hat{\eta})$. Moreover, $\hat{\eta}_0$, R_0 and L_0 denote the indifferent agent, the total amount of lobbying and the aggregate supply of labor in absence of taxation.

Equation (23), that determines the indifferent agent, is easily adapted to include taxation:

$$\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega(1-\tau)\hat{\eta}F(\hat{\eta}) = 0 \tag{23'}$$

Note that the lump-sum payment does not appear in (23') since both workers and rent-seekers receive it. Moreover, since the introduction of the tax reduces (after-tax) income, rent-seeking is less profitable and $\hat{\eta}$ is decreasing in τ. Due to the inelastic labor supply, the tax only has redistributive effects, which implies that it increases aggregate income (as there is less rent-seeking) and it reduces inequality (since the income disparity between workers and rent-seekers is diminished). The only thing which is not immediately clear is the effect of the redistributive tax on (former) rent seekers since the increase in labor supply leads to higher monopoly profits that are shared among a smaller group of rent-seekers. This can potentially offset the negative effect of the tax for the most capable rent-seekers.

The income of a rent-seeker of type $\eta \geq \hat{\eta}$ is

$$I^R(\eta) = s(\eta, \hat{\eta})(1-\tau)\Pi_1^m(\hat{\eta}) + T = s(\eta, \hat{\eta})(1-\tau)\Pi_1^m(\hat{\eta}) + \tau\Pi_1^m(\hat{\eta}), \quad (37)$$

where $T = \tau\Pi_1^m(\hat{\eta})$ since all tax revenues are distributed. The change in income compared to the situation without taxation is:

$$\Delta I^R(\eta) = s(\eta, \hat{\eta})(1-\tau)\Pi_1^m(\hat{\eta}) + \tau\Pi_1^m(\hat{\eta}) - s(\eta, \hat{\eta}_0)\Pi_1^m(\hat{\eta}_0) \quad (38)$$

The change in income can be split into three main components:

$$\Delta I^R(\eta) = (s(\eta, \hat{\eta}) - s(\eta, \hat{\eta}_0))\Pi_1^m(\hat{\eta}_0) + s(\eta, \hat{\eta})(\Pi_1^m(\hat{\eta}) - \Pi_1^m(\hat{\eta}_0)) - (s(\eta, \hat{\eta}) - 1)(\Pi_1^m(\hat{\eta}))\tau, \quad (39)$$

where the first term is the increase in income due to the decrease in competition for the prize, the second term is the increase in income due to the increase in monopoly profit, and the third term is the net tax payment. Dividing each of these terms by $s(\eta, \hat{\eta})\Pi_1^m(\hat{\eta})$, we obtain the percentage increase compared to the hypothetical benchmark where the indifferent agent is the same as in the case with taxation but absent any transfers:[10]

1. The first term becomes $(R_0 - R)/R$, which is positive and the same for each rent-seeker.
2. The second term becomes $(L - L_0)/L$, which is positive and the same for each rent-seeker.
3. The third term becomes $-\frac{\eta - R}{\eta}\tau$, which is decreasing in η. Combined with the fact that the tax is budget neutral, it follows that the agents with the highest rent-seeking aptitude must be net tax payers.

This suggests the possibility of a redistributive tax that is a Pareto improvement. Obviously, agents with low rent-seeking aptitude benefit from the tax: either they now receive a lump-sum transfer in addition to their wage or they previously were marginal rent-seekers whose rent-seeking "premium" is low enough to be compensated by the transfer. The surprise is that at the top end of the distribution a smaller group of rent-seekers is now sharing higher monopoly profits (since the productive work force has expanded). This effect may be larger than their net tax payment. A necessary condition is that the distribution of η has a finite upper bound as the absolute value of third term in (39) is increasing in η and will eventually outweigh the other two terms. Figure 4 shows an example of a Pareto-improving redistributive tax when η is uniformly distributed on $[0, 1]$.

5 Extensions

The first two extensions just serve as a robustness check. In the first robustness check, rent-seeking becomes less effective as agents spend more time rent-seeking, i.e. the activity has decreasing returns to scale. As we shall see, effectively, this reduces the return on rent-seeking

[10] It turns out that this yields the cleanest expressions, qualitatively nothing changes when we choose a more obvious benchmark such as "no taxation" or "taxation with transfers".

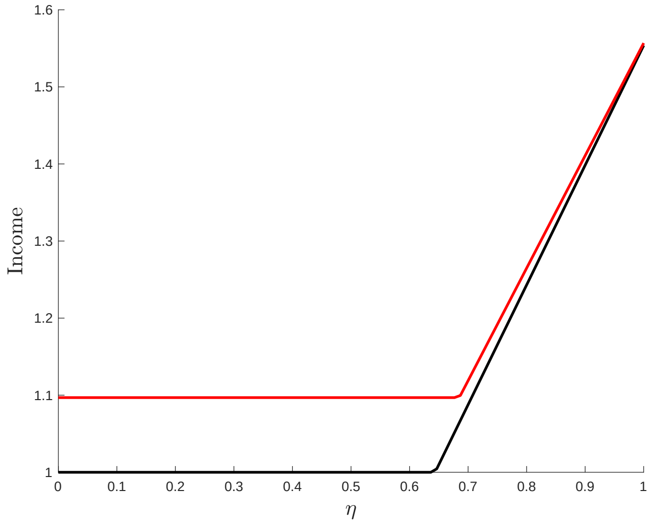


Figure 4: *The black line is income without tax as function of* η*, the red line income with a redistributive tax as function of* η*. Note that* η *is uniformly distributed on* $[0, 1]$*,* $\omega = \frac{1}{2}\sqrt{2}$ *and* $\tau = 0.2$*. Values of* ω *and* τ *chosen arbitrarily.*

and, in aggregate, there will be less rent-seeking. Since the tradeoff between income and inequality is unaltered, the effect is that aggregate income increases but is less evenly divided. The second robustness check, where monopolization occurs with probability less than one (or might even depend on the total amount of rent-seeking) stresses the same point. In the final extension, we introduce heterogeneity in labor productivity. We show, by example, that (substantial) negative correlation between labor productivity and rent-seeking aptitude can mitigate the negative effects of rent-seeking.

5.1 Decreasing returns to scale

This is the case where $\varepsilon < 1$, but we still have $\kappa = 1$. The price index is still irrelevant and an agent of type η choose $e \in [0, 1]$ to maximize:

$$1 - e + \frac{\eta e^{\varepsilon}}{R} \cdot \omega L \tag{40}$$

(cf. eq. 21). It follows that the interior solution (if it exists) is given by

$$e(\eta) = \left(\frac{\varepsilon \omega \eta L}{R}\right)^{1/(1-\varepsilon)} \tag{41}$$

Note that $e(\eta) > 0$ since the return on the first unit of rent-seeking is infinitely high for all agents. However at the top end of the distribution, agents may still choose to specialize fully in rent-seeking: with slight abuse of notation, suppose that $e(\eta) = 1$ if and only if $\eta > \hat{\eta}$.

First, suppose that $\hat{\eta} = \eta_1$ and there is an interior solution for every agent. Recall the definitions of R and L:

$$R = \int_{\eta_0}^{\eta_1} \eta e(\eta)^{\varepsilon} dF(\eta) \tag{42}$$

$$L = 1 - \int_{\eta_0}^{\eta_1} e(\eta) dF(\eta) \tag{43}$$

Plugging the expression for $e(\eta)$ into (42) and (43) we obtain:

$$R = \left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)} \int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) \tag{44}$$

$$L = 1 - \left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)} \int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) \tag{45}$$

Note that (44) and (45) can be solved to get two expressions for the integral.

$$\int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{R}{\left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}} \tag{46}$$

$$\int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{1-L}{\left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)}} \tag{47}$$

Obviously the right-hand side of (46) should be equal to the right-hand side of (47). If we equate both right-hand sides, then we see that R cancels out and we can solve it to obtain an expression for L. This leads to the following result:

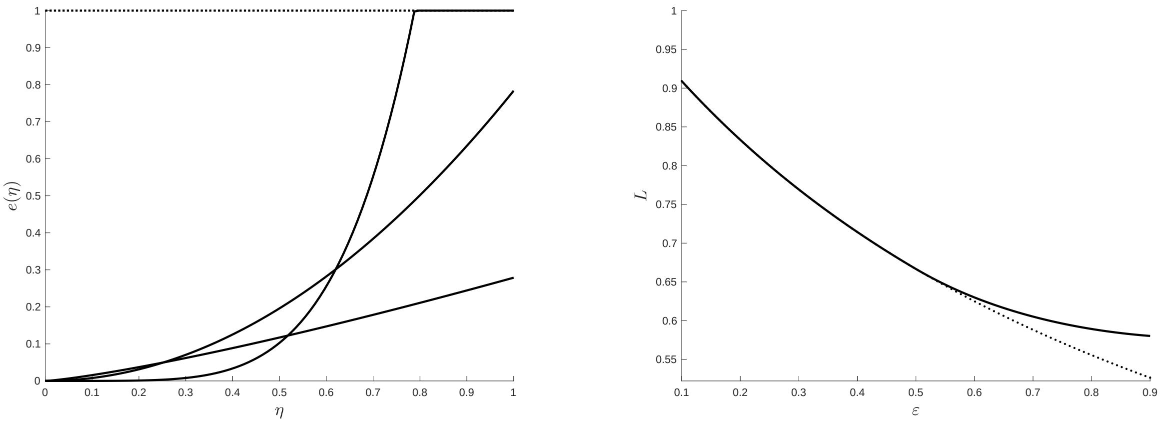
Proposition 3. *If $e(\eta) \in (0,1)$ for all η, then the size of the labor force is independent of the distribution of rent-seeking aptitude and given by:*

$$L = \frac{1}{1+\varepsilon\omega} \tag{48}$$

Since L is decreasing in ε, we see that the closer we get to constant returns, the smaller the labor force. The question is whether this conclusion also holds when $\hat{\eta} < \eta_1$.

Figure 5 shows a numerical example where η is uniformly distributed on the unit interval. Panel (a) depicts the rent-seeking effort as function of η for three different values of ε. It is clear that the effect of increasing ε on rent-seeking effort differs between types. At the top end of the distribution, it increases. At the bottom end, it decreases. For agents in the middle of the distribution, the effect is even non-monotonic. Note that, as ε increases, each incremental unit rent-seeking yields higher returns. So, *ceteris paribus*, rent-seeking is more profitable. However, on the whole, this reduces the productive labor force L (and monopoly profit) and increases the total amount of rent-seeking R. Both reduce the returns on rent-seeking, but the effect is larger for agents with low rent-seeking aptitude. In equilibrium, 'high η'-agents start to specialize in rent-seeking, while 'low η'-agents only spend a small fraction of their time on rent-seeking.

The net effect on L, for the case of the uniform distribution curve, is the area above the curve but below the $e = 1$-line in panel (a). It is clear that L decreases in ε: this is confirmed in panel (b), which shows L as function of ε. Note that the dashed line is the graph of equation (48): the reduction of the labor force is less severe than Proposition 3 predicts.



(a) (b)

Figure 5: *Panel (a) shows time allocated to rent-seeking as function of type for different values of ε and panel (b) aggregate labor as function of ε (solid line), the dashed line shows aggregate labor for the case $\hat{\eta} = \eta_1$ when $L = 1/(1+\varepsilon\omega)$. Note that η is uniformly distributed on* $[0, 1]$ *and* $\omega = \frac{1}{2}\sqrt{2}$.

The following Proposition confirms that our example generalizes to other distributions:

Proposition 4. *Suppose $\hat{\eta} < \eta_1$. As ε increases and the rent-seeking technology gets closer to constant returns to scale, more agents become full time rent-seekers (i.e. $\hat{\eta}$ is decreasing in ε). However, for agents with low rent-seeking aptitude, this means increased competition for monopoly profit and their response is to work more. Consequently, the comparative statics of ε on L are ambiguous.*

5.2 Probability of monopolization

We return to the case where $\varepsilon = 1$, but now we have $\kappa < 1$. As before, there is an indifferent agent $\hat{\eta}$ whose return to rent-seeking is the same as the wage rate: only those agents whose rent-seeking aptitude exceeds $\hat{\eta}$ specialize in rent-seeking. The agent with type $\hat{\eta}$ is indifferent between working and rent-seeking:

$$\kappa \frac{1}{P_V^m} + (1-\kappa)\frac{1}{P_V^c} = \kappa \frac{\hat{\eta}\omega L}{P_V^m R}, \tag{49}$$

where the left-hand side is obtained by evaluating (19) at $e = 0$ and the right-hand side by evaluating (19) at $e = 1$. Observe that there is a price effect where the higher prices in case of monopolization decrease indirect utility. Rearranging (49), we obtain:

$$1 + \frac{1-\kappa}{\kappa} \cdot \frac{P_V^m}{P_V^c} = \frac{\hat{\eta}\omega L}{R}. \tag{50}$$

The right-hand side is now identical to the benchmark, but the left-hand side shows a premium for workers equal to the odds against monopolization times the benefit of avoiding the monopoly price increase. This is not that surprising: when monopolization does not happen, workers unlike rent-seekers have a source of income and, as an additional bonus, prices are lower. The overall effect is less rent-seeking.[11]

This result continues to hold if the likelihood of monopolization depends on the aggregate amount of rent-seeking, i.e. $\kappa = \kappa(R)$. In particular, suppose that more rent-seeking increases the probability of monopolization: $R' > R$ implies $\kappa(R') > \kappa(R)$. We can think of it as a politician being persuaded by the shear amount of rent-seeking to grant monopoly rights in sector 1. For the individual agents, who take R as given, the incentives do not change drastically: in (50), κ simply becomes a function of R. However, at the macrolevel, there is a multiplier where rent-seeking begets more rent-seeking. This is a second-order effect that is dominated by the overall effect that monopolization no longer happens with certainty. Formally:

Proposition 5. *Let κ be a continuous function of R with the properties that $\kappa(0) = \kappa_0 \in (0, 1)$, $\kappa(\mathbb{E}\eta) = 1$, and for all $R \in (0, \mathbb{E}\eta)$ we have $\kappa(R) \in [\kappa_0, 1)$. Then, in every equilibrium, rent-seeking is reduced compared to the benchmark where $\kappa \equiv 1$.*

5.3 Heterogenous labor productivity

Throughout this paper, we have maintained the assumption each agent had the same marginal productivity of labor, normalized to one. In this Section we will introduce heterogeneity in labor productivity (in the benchmark case where $\varepsilon = 1$ and $\kappa = 1$).

When agents differ in labor productivity, the time allocation decision depends on rent-seeking aptitude relative to labor productivity: agents, whose labor productivity is (relatively) high compared to their rent-seeking aptitude, will become workers, and if labor productivity is relatively low, then becoming a rent-seeker is a more attractive option.

This makes the following scenario a possibility. Suppose that labor productivity and rent-seeking aptitude are negatively correlated. If the correlation is strong enough, then agents can be categorized either as good rent-seekers or as efficient workers and they will specialize in one of those professions. This implies that the loss in (effective) labor is limited as rent-seekers are inefficient workers. The drop in aggregate income is, consequently, limited as well. At the same time, workers earn higher wages than before (since the wage rate is equal to their high marginal productivity of labor). Previously we have demonstrated that high average income is linked to high income inequality, but here that link may be broken. We shall show by means of an example that this is indeed the case.

An agent is now characterized by their labor productivity $\xi \in [\xi_0, \xi_1]$ (with wage rate equal to ξ) and a rent-seeking aptitude $\eta \in [\eta_0, \eta_1]$. The probability density function is denoted by $f(\eta, \xi)$. Assume that $f(\eta, \xi) > 0$ for all $(\eta, \xi) \in [\eta_0, \eta_1] \times [\xi_0, \xi_1]$.

Adjusting the wage rate from one to ξ in (22) and observing that $\Pi_1^m = \omega L$, we obtain the

[11] The effect is equivalent to a tax on non-labor income pf

$$\hat{\tau} = \frac{(1-\kappa)P_V^m}{\kappa P_V^c + (1-\kappa)P_V^m}$$

(cf. eq. 23'). Since $\hat{\tau}$ is decreasing in κ, the result follows immediately.

following expression for the indifferent agent(s):

$$\xi = \frac{\eta \omega L}{R}. \tag{51}$$

While the calculation of L and R will be more involved, we immediately see that the indifferent agent can be expressed in terms of $z \equiv \eta/\xi$. An agent (η, ξ) will specialize in rentseeking if and only if $\eta/\xi \geq \hat{z}$ where $\hat{z}$ satisfies:

$$1 = \frac{\hat{z} \omega L}{R}. \tag{52}$$

The corresponding formulas for L and R as function of $\hat{z}$ are

$$L(\hat{z}) = \int_{\xi_0}^{\xi_1} \int_{\eta_0}^{\Delta(\hat{z}\xi)} \xi f(\eta, \xi) d\eta d\xi, \tag{53}$$

$$R(\hat{z}) = \int_{\xi_0}^{\xi_1} \int_{\Delta(\hat{z}\xi)}^{\eta_1} \eta f(\eta, \xi) d\eta d\xi, \tag{54}$$

where $\Delta(x) = \min\{\max\{x, \eta_0\}, \eta_1\}$. Both expressions can be read as the average effective units of labor and rent-seeking as supplied by the agents. The Δ-function appears because for some values of η, all (or none) agents with this level of rent-seeking aptitude will work.

First, we show that there is a unique value of $\hat{z} \in (\eta_0/\xi_1, \eta_1/\xi_0)$ that solves (52). Rearrange the equation as:

$$R(\hat{z}) - \omega \hat{z} L(\hat{z}) = 0. \tag{55}$$

Note that $L(\eta_0/\xi_1) = 0$ and $R(\eta_1/\xi_0) = 0$. Therefore the left-hand side of (55) is positive at $\hat{z} = \eta_0/\xi_1$ and negative at $\hat{z} = \eta_1/\xi_0$. Since the left-hand side is continuous in $\hat{z}$, existence is established. Uniqueness follows from the fact that the left-hand side is strictly decreasing in $\hat{z}$ (see details in Appendix C).

To explore the impact of heterogeneous labor productivity, especially if it is correlated with rent-seeking aptitude, we need a distribution where the degree of correlation is easily manipulated, preferably via a single parameter, and where the average labor productivity is kept constant. The latter is important since an increase in average labor productivity will *ceteris paribus* boost aggregate income. This is an unwanted effect if we study the tradeoff between aggregate income and income inequality.

Copulas are a standard tool to generate distribution with these properties from standard, univariate distributions. We apply the Farley-Gumbel-Morgenstern copula, $C(x, y) = xy(1 + \varphi(1 - x)(1 - y)$, to uniform distributions for both η and ξ to obtain the following distribution over (η, ξ):

$$f(\eta, \xi) = \frac{\partial^2}{\partial \eta \partial \xi} C\left(\frac{\eta - \eta_0}{\eta_1 - \eta_0}, \frac{\xi - \xi_0}{\xi_1 - \xi_0}\right) = \frac{1 + \frac{\varphi}{4}(\eta - \bar{\eta})(\xi - \bar{\xi})}{(\eta_1 - \eta_0)(\xi_1 - \xi_0)}, \tag{56}$$

where φ is the degree of correlation between the two abilities, $\bar{\eta} = (\eta_1 - \eta_0)/2$ and $\bar{\xi} = (\xi_1 - \xi_0)/2$. Note that $|\varphi| < \frac{4}{(\eta_1 - \eta_0)(\xi_1 - \xi_0)}$, otherwise $f(\eta, \xi) < 0$ for some η and ξ in the support. The degree of correlation is at most $1/3$ in absolute value (see the mathematical appendix of Heijdra et al.

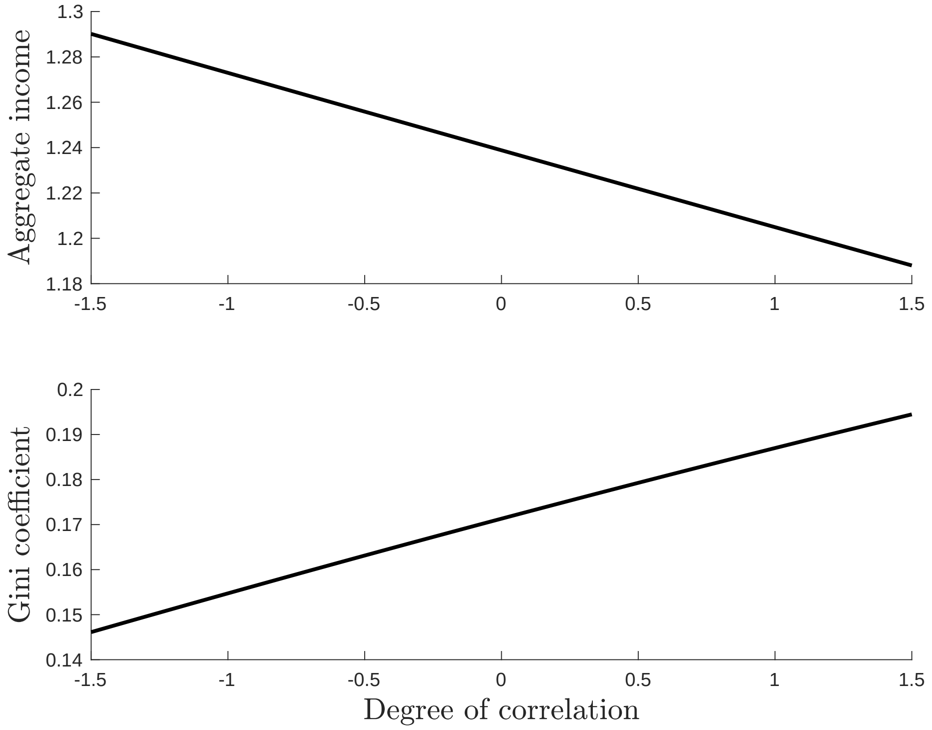


Figure 6: *The top panel shows aggregate income as function of the degree of correlation φ and the bottom panel the Gini-coefficient. Support of the distribution is $[\eta_0, \eta_1]\times[\xi_0, \xi_1] = [0, 2]\times[1/3, 5/3]$ and $\omega = \frac{1}{2}\sqrt{2}$.*

(2019) for details).

Unfortunately, even for this relatively simple case, no closed form solution exists. Therefore, we resort to numerical methods.[12] Figure 6 shows the result, which confirms our intuition: stronger negative correlation between η and ξ leads to an increase in the level of income and a decrease in income inequality.

6 Concluding remarks

Our main contributions are as follows. First of all, we embed the classic Tullock contest in a general equilibrium setting. In particular, agents are competing against each other to obtain a share of the prize. The share of the prize that the agents obtain is a generalization of the Tullock success function that allows for infinitely many contestant. In a general equilibrium setting, the size of the rent is endogenous.

Second, we show that there is a tradeoff between efficiency and equity. Since rent-seeking is a

[12]Numerically, the calculations are fairly straightforward. Only the measure of inequality, the Gini coefficient, is a bit tricky. See Appendix D for the details behind this computation.

purely wasteful activity, (aggregate) welfare is highest when few agents engage in rent-seeking. However, this has the negative effect that it increases monopoly profit and, to add insult to injury, it is distributed among a select group of agents. This increases inequality. There are ways to remedy this tradeoff: we demonstrate by example that a redistributive tax can, in certain circumstances, be a Pareto-improvement. Strong negative correlation between labor productivity and rent-seeking aptitude is another way to achieve this goal.

Finally, while we presented an in-depth investigation of rent-seeking incentives in a static general equilibrium context, dynamic effects are also of great economic importance: time wasted on rent-seeking does not only decrease production of consumption goods directly, but may also divert effort from growth-promoting activities. In Heijdra and Heijnen (2021), we attempt to quantify the effects of rent-seeking on economic growth. There we analyze a three-sector macroeconomic general equilibrium model featuring labour-using rent-seeking activities (with diminishing returns to rent-seeking time as in Section 5.1 of the present paper), overlapping generations, and endogenous growth. In our benchmark model endogenous economic growth results from a human capital externality along the lines of Azariadis and Drazen (1990). During the first phase of life, 'young' agents allocate time on productive work, education, and on lobbying. Whereas from a partial equilibrium perspective rent-seeking can thus be harmful to the engine of growth (learning), it turns out that the general equilibrium effects on economic growth depend critically on the life-cycle payoff structure of rent-seeking activities. Indeed, if these payoffs arise early on in life then economic growth is stimulated by rent-seeking efforts. In contrast, if the rewards are obtained later on in life then growth is harmed by in the presence of lobbying activities during youth. This brings home the key message: the intertemporal dimension potentially plays a crucial role in determining the ultimate effects of rent-seeking.

References

Aaberge, R. (2007). Gini's nuclear family. *Journal of Economic Inequality*, 5(3):305–322.

Acemoglu, D. and Verdier, T. (1998). Property rights, corruption and the allocation of talent: A general equilibrium approach. *Economic Journal*, 108:1381–1403.

Azariadis, C. and Drazen, A. (1990). Threshold externalities in economic development. *Quarterly Journal of Economics*, 105:501–526.

Barelli, P. and Pessõa, S. A. (2012). Rent-seeking and capital accumulation. *Economic Inquiry*, 50:399–417.

Brooks, M. A. and Heijdra, B. J. (1988). In search of rent-seeking. In Rowley, C., Tollison, R., and Tullock, G., editors, *The Political Economy of Rent-Seeking*, pages 27–49. Kluwer Academic Publishers, Boston, MA.

Brou, D. and Ruta, M. (2013). Rent-seeking, market structure, and growth. *Scandinavian Journal of Economics*, 115:878–901.

Corchón, L. (2007). The theory of contests: A survey. *Review of Economic Design*, 11:69–100.

Ewerhart, C. (2015). Mixed equilibria in Tullock contests. *Economic Theory*, 60(1):59–71.

Harberger, A. C. (1954). Monopoly and resource allocation. *American Economic Review*, 44(2):77–87.

Heijdra, B. and Heijnen, P. (2021). Rent-seeking, capital accumulation, and macroeconomic growth. Working paper, Department of Economics, Econometrics, & Finance, University of Groningen.

Heijdra, B., Jiang, Y., and Mierau, J. (2019). The macroeconomic effects of longevity risk under private and public insurance and asymmetric information. *De Economist*, 167(2):177–213.

Konrad, K. A. (2009). *Strategy and Dynamics in Contests*. Cambridge University Press, Cambridge.

Lockwood, B. B., Nathanson, C. G., and Weyl, E. G. (2017). Taxation and the allocation of talent. *Journal of Political Economy*, 125(5):1635–1682.

Murphy, K. M., Shleifer, A., and Vishny, R. W. (1993). Why is rent-seeking so costly to growth? *American Economic Review*, 83(2):409–414.

Rothschild, C. and Scheuer, F. (2016). Optimal taxation with rent-seeking. *Review of Economic Studies*, 83:1225–1262.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford Economic Papers*, 3(2):135–146.

Tullock, G. (1967). The welfare costs of tariffs, monopolies, and theft. *Western Economic Journal*, 5:224–232.

Appendix A Mathematical preliminaries

Let X be a random variable with support $[x_0, x_1] \subset \mathbb{R}^+$.[13] A realization of X is denoted by x. The distribution of X depends on a parameter $\alpha \in (\alpha_0, \alpha_1) \subset \mathbb{R}$. We use the notation X_α to denote the dependency on α, if neccesary. The cumulative distribution function of X is $F : [x_0, x_1] \times (\alpha_0, \alpha_1) \to [0, 1]$ and the probability distribution function is $f : [x_0, x_1] \times (\alpha_0, \alpha_1) \to \mathbb{R}^+$. Assume that F is twice-differentiable.

Lemma A.1. *If*

$$\frac{\partial f}{\partial \alpha}(x, \alpha) \geq 0 \iff x \leq x^*(\alpha), \tag{A.1}$$

for some function $x^* : [\alpha_0, \alpha_1] \to [x_0, x_1]$, *then*

1. $\frac{\partial F}{\partial \alpha}(x, \alpha) \geq 0$ *for all* $x \in [x_0, x_1]$ *(with equality if and only if* $x = x_0$ *or* $x = x_1$*),*[14] *and*
2. $\int_x^{x_1} s \frac{\partial f}{\partial \alpha}(s, \alpha) ds \leq 0$ *for all* $x \in [x_0, x_1]$.

Proof. Ad (1): Observe that $F(x_0, \alpha) = 0$ and $F(x_1, \alpha) = 1$ for all α. This directly establishes that

$$\frac{\partial F}{\partial \alpha}(x_0, \alpha) = \frac{\partial F}{\partial \alpha}(x_1, \alpha) = 0. \tag{A.2}$$

Note that, by definition,

$$F(x, \alpha) = \int_{x_0}^{x} f(s, \alpha) ds. \tag{A.3}$$

Differentiating both sides to α yields:

$$\frac{\partial F}{\partial \alpha}(x, \alpha) = \int_{x_0}^{x} \frac{\partial f}{\partial \alpha}(s, \alpha) ds. \tag{A.4}$$

Observe that if we differentiate both sides to x, we obtain:

$$\frac{\partial^2 F}{\partial x \partial \alpha}(x, \alpha) = \frac{\partial f}{\partial \alpha}(x, \alpha). \tag{A.5}$$

Hence, as a function of x, $\frac{\partial F}{\partial \alpha}(x, \alpha)$ is increasing up to $x^*(\alpha)$, after which it is decreasing in x. Together with the result derived in (A.2), this shows that $\frac{\partial F}{\partial \alpha}(x, \alpha) \geq 0$ for all $x \in [x_0, x_1]$ (with equality if and only if $x = x_0$ or $x = x_1$).

Ad (2): Note that if $x \geq x^*(\alpha)$, then we obviously have

$$\int_x^{x_1} s \frac{\partial f}{\partial \alpha}(s, \alpha) ds \leq 0 \tag{A.6}$$

[13] This is slight abuse of notation as we allow for the case where $x_1 = \infty$.

[14] Again this is slight abuse of notation. When $x_1 = \infty$, this should read $\lim_{x \to \infty} \frac{\partial F}{\partial \alpha}(x, \alpha) = 0$.

as $s\frac{\partial f}{\partial \alpha}(s,\alpha) \leq 0$ for all $s \in [x, x_1]$. Observe that, for $x \leq x^*(\alpha)$, we have

$$\frac{\partial}{\partial x}\int_x^{x_1} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds = -x\frac{\partial f}{\partial \alpha}(x,\alpha) \leq 0. \tag{A.7}$$

This implies that $\int_x^{x_1} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds$ is maximal at $x = x_0$ and, therefore, it suffices to show that:

$$\int_{x_0}^{x_1} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds \leq 0 \tag{A.8}$$

which can be rewritten as

$$\int_{x_0}^{x*(\alpha)} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds + \int_{x*(\alpha)}^{x_1} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds \leq 0 \tag{A.9}$$

Taking the second, negative-valued, integral to the other side, we see that (A.8) is equivalent to:

$$\int_{x_0}^{x^*(\alpha)} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds \leq \int_{x^*(\alpha)}^{x_1} s\left|\frac{\partial f}{\partial \alpha}(s,\alpha)\right| ds. \tag{A.10}$$

To show that this is true, first note that, by definition, $\int_{x_0}^{x_1} f(s,\alpha)ds = 1$. Differentiating both sides to α yields

$$\int_{x_0}^{x_1} \frac{\partial f}{\partial \alpha}(s,\alpha)ds = 0. \tag{A.11}$$

Since $\frac{\partial f}{\partial \alpha}$ changes sign at $x^*(\alpha)$, it follows that

$$\int_{x_0}^{x^*(\alpha)} \frac{\partial f}{\partial \alpha}(s,\alpha)ds = \int_{x^*(\alpha)}^{x_1} \left|\frac{\partial f}{\partial \alpha}(s,\alpha)\right| ds. \tag{A.12}$$

We can use this result to establish that (A.10) holds:

$$\begin{aligned}
\int_{x_0}^{x^*(\alpha)} s\frac{\partial f}{\partial \alpha}(s,\alpha)ds &\leq \int_{x_0}^{x^*(\alpha)} x^*(\alpha)\frac{\partial f}{\partial \alpha}(s,\alpha)ds \\
&= x^*(\alpha)\int_{x_0}^{x^*(\alpha)} \frac{\partial f}{\partial \alpha}(s,\alpha)ds \\
&= x^*(\alpha)\int_{x^*(\alpha)}^{x_1} \left|\frac{\partial f}{\partial \alpha}(s,\alpha)\right| ds \\
&= \int_{x^*(\alpha)}^{x_1} x^*(\alpha)\left|\frac{\partial f}{\partial \alpha}(s,\alpha)\right| ds \\
&\leq \int_{x^*(\alpha)}^{x_1} s\left|\frac{\partial f}{\partial \alpha}(s,\alpha)\right| ds,
\end{aligned}$$

where the second equality follows from (A.12). ■

Note that if $\alpha < \alpha'$, then X_α (first-order) stochastically dominates $X_{\alpha'}$. However the condition in Lemma A.1 is stronger than first-order stochastic dominance. Examples of distributions that satisfy (A.1) are the Pareto distribution ($f(x,\alpha) = \alpha x^{-(\alpha+1)}$, $x \in [1,\infty)$ and $\alpha \in (1,\infty)$) and the exponential distribution ($f(x, \alpha) = \alpha e^{-\alpha x}$, $x \in [0, \infty)$ and $\alpha \in (0, \infty)$)

Lemma A.2. *The Pareto distribution satisfies condition* (A.1)*.*

Proof. Observe that:

$$\frac{\partial f}{\partial \alpha} = x^{-(\alpha+1)}[1 - \alpha \log x]. \tag{A.13}$$

Note that at $x = 1$ the sign of the derivative is positive. Moreover the derivative changes sign at

$$x^*(\alpha) = e^{1/\alpha}. \tag{A.14}$$

■

Lemma A.3. *The exponential distribution satisfies condition* (A.1)*.*

Proof. Observe that:

$$\frac{\partial f}{\partial \alpha} = e^{-\alpha x}[1 - \alpha x]. \tag{A.15}$$

Note that at $x = 0$ the sign of the derivative is positive. Moreover the derivative changes sign at

$$x^*(\alpha) = \frac{1}{\alpha}. \tag{A.16}$$

■

Appendix B Proofs

B.1 Proof of Proposition 1

Ad (i) Note that

$$\frac{d\hat{\eta}}{d\alpha} = -\frac{\frac{\partial g}{\partial \alpha}(\hat{\eta}, \alpha)}{\frac{\partial g}{\partial \hat{\eta}}(\hat{\eta}, \alpha)}, \tag{A.17}$$

where we have made the dependency of g on α explicit. In the main text, it was already shown that

$$\frac{\partial g}{\partial \hat{\eta}}(\hat{\eta}, \alpha) < 0. \tag{A.18}$$

Therefore

$$\frac{d\hat{\eta}}{d\alpha} < 0 \iff \frac{\partial g}{\partial \alpha}(\hat{\eta}, \alpha) < 0. \tag{A.19}$$

Observe that:

$$\frac{\partial g}{\partial \alpha}(\hat{\eta}, \alpha) = \int_{\hat{\eta}}^{\eta_1} \eta \frac{\partial f}{\partial \alpha}(\eta, \alpha) d\eta - \omega \hat{\eta} \frac{\partial F}{\partial \alpha}(\hat{\eta}, \alpha)$$

From Lemma A.1, we see that this expression is indeed negative.

Ad (ii) Note that aggregate income is $(1+\omega)L = (1+\omega)F(\hat{\eta}) = (1+\omega)(1-\hat{q})$. Hence, if the fraction of rent-seekers increases [decreases], then aggregate income decreases [increases].

Ad (iii) We use two properties of the Gini-coefficient: invariance to scale and the principle of transfers (Aaberge, 2007). First, note that income I of type η is given by:

$$I(\eta) = \begin{cases} 1 & \text{if } \eta < \hat{\eta} \\ \frac{\eta}{\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)} \omega F(\hat{\eta}) & \text{if } \eta \geq \hat{\eta} \end{cases} \tag{A.20}$$

Since the Gini-coefficient is invariant to scale we can divide by aggregate income $(1+\omega)(1-\hat{q})$ while leaving the Gini-coefficient unchanged:

$$\hat{I}(\eta) = \begin{cases} \frac{1}{(1+\omega)(1-\hat{q})} & \text{if } \eta < \hat{\eta} \\ \frac{1}{(1+\omega)(1-\hat{q})} \times \frac{\eta}{\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)} \omega F(\hat{\eta}) & \text{if } \eta \geq \hat{\eta} \end{cases} \tag{A.21}$$

Finally we can express everything in terms of percentiles, $p = F(\eta)$, and add the parameter α,

$$\tilde{I}(p) = \begin{cases} \frac{1}{(1+\omega)(1-\hat{q}(\alpha))} & \text{if } p \leq 1 - \hat{q}(\alpha) \\ \frac{\omega(1-\hat{q}(\alpha))}{(1+\omega)(1-\hat{q}(\alpha))} \times \frac{F^{-1}(p,\alpha)}{\int_{1-\hat{q}(\alpha)}^{1} F^{-1}(s,\alpha) ds} & \text{else} \end{cases} \tag{A.22}$$

Note that (1) $\int_0^1 \tilde{I}(p)dp = 1$, (2) $\hat{q}$ increases [decreases] and workers see an increase [decrease] in income, (3) the rate, at which the income of the rent-seekers increases, goes up (i.e. $\frac{\partial F^{-1}(p,\alpha)}{\partial \alpha}$ increasing in α, cf. Lemma A.1). This established that there is a unique percentile in the income distribution such that everyone below this point is better [worse] off and everyone above this point is worse [better] off. Then, by the principle of transfers, the Gini-coefficient is decreasing [increasing].

B.2 Proof of Proposition 2

First. note that (33) can (equivalently) be formulated as:

$$\frac{d}{d\alpha} \frac{\bar{\eta}_{p_1}(\alpha)}{\bar{\eta}_{p_0}(\alpha)} < 0. \tag{A.23}$$

(We use the formulation in (33) because it is more intuitive to talk about the relative strength of the agent at percentile p_0 than his relative weakness.) Second, $\bar{\eta}_p$ is the inverse of the cumulative distribution function of η, the function is increasing in p. Finally, let $\hat{p}$ denote the percentile at which the indifferent agent is located. Note that $\hat{q} = 1 - \hat{p}$. It suffices to show that $\hat{p}$ is decreasing in α.

The marginal rent-seeker is determined by (cf. eq. 23):

$$\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega\hat{\eta}F(\hat{\eta}) = 0, \tag{A.24}$$

which, with a little 'integration by substitution'-magic, can be rewritten as

$$\int_{\hat{p}}^{1} \frac{\eta_p(\alpha)}{\eta_{\hat{p}}(\alpha)} dp - \omega\hat{p} = 0. \tag{A.25}$$

Applying the implicit function theorem yields:

$$\frac{d\hat{p}}{d\alpha} = \frac{\int_{\hat{p}}^{1} \frac{\partial}{\partial\alpha} \frac{\eta_p(\alpha)}{\eta_{\hat{p}}(\alpha)} dp}{\omega + 1 + \int_{\hat{p}}^{1} \frac{\eta_p(\alpha)}{\eta_{\hat{p}}(\alpha)^2} \frac{\partial \bar{\eta}_{\hat{p}}}{\partial p} dp} < 0, \tag{A.26}$$

where the sign follows from the observations above.

B.3 Proof of Proposition 4

First of all, the expressions for R and L are different since some agents only engage in rentseeking:

$$R = \left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) + \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta), \tag{A.27}$$

$$L = F(\hat{\eta}) - \left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta). \tag{A.28}$$

Note that, again, these two expression can be solved for the integral.

$$\int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{R - \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)}{\left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}}, \tag{A.29}$$

$$\int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{F(\hat{\eta}) - L}{\left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)}}. \tag{A.30}$$

Therefore:

$$\frac{R - \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)}{\left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}} = \frac{F(\hat{\eta}) - L}{\left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)}} \implies R = \frac{\varepsilon\omega L \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)}{(1+\varepsilon\omega)L - F(\hat{\eta})} \tag{A.31}$$

Next, observe that $e(\hat{\eta}) = 1$ implies that $\hat{\eta} = R/(\varepsilon\omega L)$. Taking the expression for R derived in (A.31), we obtain the following relationship between L and $\hat{\eta}$:

$$\hat{\eta} = \frac{\int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)}{(1+\varepsilon\omega)L - F(\hat{\eta})}. \tag{A.32}$$

We can also use $\hat{\eta} = R/(\varepsilon\omega L)$ to eliminate R from (A.28):

$$L = F(\hat{\eta}) - \hat{\eta}^{-1/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta). \tag{A.33}$$

Rewriting (A.32) and (A.33) slightly, we obtain the following implicit expression of L and $\hat{\eta}$ as function of ε:

$$(1+\varepsilon\omega)L\hat{\eta} - \hat{\eta}F(\hat{\eta}) - \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) = 0, \tag{A.34}$$

$$L - F(\hat{\eta}) + \hat{\eta}^{-1/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) = 0. \tag{A.35}$$

Applying the implicit function theorem, we obtain

$$\begin{bmatrix} (1+\varepsilon\omega)\hat{\eta} & (1+\varepsilon\omega)L - F(\eta) \\ 1 & -\frac{1}{1-\varepsilon}\hat{\eta}^{-(2-\varepsilon)/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) \end{bmatrix} \begin{pmatrix} dL/d\varepsilon \\ d\hat{\eta}/d\varepsilon \end{pmatrix} = \begin{pmatrix} -\omega L\hat{\eta} \\ -\frac{1}{(1-\varepsilon)^2} \int_{\eta_0}^{\hat{\eta}} \left(\frac{\eta}{\hat{\eta}}\right)^{1/(1-\varepsilon)} \log\left(\frac{\eta}{\hat{\eta}}\right) dF(\eta) \end{pmatrix}, \tag{A.36}$$

with signs:

$$\begin{bmatrix} (+) & (+) \\ (+) & (-) \end{bmatrix} \begin{pmatrix} dL/d\varepsilon \\ d\hat{\eta}/d\varepsilon \end{pmatrix} = \begin{pmatrix} (-) \\ (+) \end{pmatrix}. \tag{A.37}$$

From Cramer's Rule, it follows that $\hat{\eta}$ is decreasing in ε. The sign of $dL/d\varepsilon$ is ambiguous.

B.4 Proof of Proposition 5

Suppose that in the benchmark, where monopolization always occurs, the indifferent agent is located at $\hat{\eta}_B$. Recall from (23) that

$$\int_{\hat{\eta}_B}^{\eta_1} \eta dF(\eta) - \omega\hat{\eta}_B F(\hat{\eta}_B) = 0. \tag{23"}$$

If κ is function of R, then this condition (cf. eq. 50) changes to:

$$\left(1 + \frac{1-\kappa(\hat{\eta})}{\kappa(\hat{\eta})} \cdot \frac{P_V^m}{P_V^c}\right) \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega\hat{\eta}F(\hat{\eta}) = 0, \tag{A.38}$$

where κ is a function of $\hat{\eta}$ through R. Note that our assumptions on κ imply that $\kappa(\eta_0) = 1$ and $\kappa(\eta) \in (0,1) \forall \eta \in (\eta_0, \eta_1]$. Introduce

$$h(\hat{\eta}) = \left(1 + \frac{1-\kappa(\hat{\eta})}{\kappa(\hat{\eta})} \cdot \frac{P_V^m}{P_V^c}\right) \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega\hat{\eta}F(\hat{\eta}). \tag{A.39}$$

Note that h is continuous, $h(\eta_1) = -\omega\eta_1 < 0$ and

$$h(\hat{\eta}_B) = \frac{1-\kappa(\hat{\eta}_B)}{\kappa(\hat{\eta}_B)} \cdot \frac{P_V^m}{P_V^c} \int_{\hat{\eta}_B}^{\eta_1} \eta dF(\eta) > 0. \tag{A.40}$$

Hence, by the intermediate value theorem, $\hat{\eta} \in (\hat{\eta}_B, \eta_1)$. To complete the proof, note that R is decreasing in $\hat{\eta}$ and, therefore, rent-seeking is reduced compared to the benchmark.

Appendix C Details uniqueness $\hat{z}$

As a preliminary remark, note that $\Delta'(x)$ is either 0 or 1 whenever the derivative exists. The derivative does not exist at $x = \eta_0$ and at $x = \eta_1$. Since we are mostly interested in integrals over expressions containing Δ', without loss of generality we work with the left derivative (in particular, this means $\Delta'(\eta_0) = 0$ and $\Delta'(\eta_1) = 1$). Importantly $\Delta' \geq 0$.

We need to show that the following expression is negative for $\hat{z} \in (\eta_0/\xi_1, \eta_1/\xi_0)$:

$$R'(\hat{z}) - \omega\hat{z}L'(\hat{z}) - \omega L(\hat{z}). \tag{A.41}$$

Since $L(\hat{z}) > 0$, it suffices to show that $R'(\hat{z}) \leq 0$ and $L'(\hat{z}) \geq 0$. Clearly:

$$R'(\hat{z}) = -\int_{\xi_0}^{\xi_1} \xi\Delta'(\hat{z}\xi)\Delta(\hat{z}\xi)f(\Delta(\hat{z}\xi), \xi)d\xi \leq 0, \tag{A.42}$$

and

$$L'(\hat{z}) = \int_{\xi_0}^{\xi_1} \xi^2\Delta'(\hat{z}\xi)f(\Delta(\hat{z}\xi), \xi)d\xi \geq 0. \tag{A.43}$$

Appendix D Details computation Gini-coefficient

Observe we have a continuous bivariate distribution over the type of the agent that maps into a continuous univariate distribution over income. It is cumbersome to obtain an expression for the distribution function of income. Hence we take the following approach to compute the Gini-coefficient. The threshold level $\hat{z}$, along with $R(\hat{z})$ and $L(\hat{z})$, are determined by solving (55). For any agent (η, ξ), the income level is given by

$$I(\eta, \xi) = \max\left\{\xi, \frac{\eta\omega L(\hat{z})}{R(\hat{z})}\right\}. \tag{A.44}$$

The next step is to discretize the distribution of (η, ξ) by putting an equidistant grid over the support with $n = 10000$ grid points in total. The agents at grid point $i = 1, \ldots, n$ are located at (η_i, ξ_i). The fraction of agents at grid point i is

$$f_i = \frac{f(\eta_i, \xi_i)}{\sum_{j=1}^n f(\eta_j, \xi_j)}. \tag{A.45}$$

Their income level is $I_i = I(\eta_i, \xi_i)$.

Finally, for a discrete distribution the Gini-coefficient is given by:

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} f_i f_j |I_i - I_j|}{2 \sum_{i=1}^{n} f_i I_i}. \tag{A.46}$$

We take this as our approximation for the Gini-coefficient.