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Intergenerational Actuarial Fairness when Longevity Increases: Amending the Retirement Age

Abstract

Continuous longevity improvements and population ageing have led countries to modify national public pension schemes by increasing the standard and early retirement ages in a discretionary, scheduled, or automatic way, and by making it harder for people to retire prematurely. To this end, countries have adopted alternative retirement age strategies, but our analyses show that the measures taken are often poorly designed and consequently misaligned with the pension scheme's ultimate goals. In addition, our analyses demonstrate that countries risk falling short of their goals given their use of projection methods that underestimate life expectancy. This paper discusses how to implement automatic indexation of the retirement age to life expectancy developments while respecting the principles of intergenerational actuarial fairness and neutrality among generations. We show that in policy designs in which extended working lives translate into additional pension entitlements, the pension age must be automatically updated to keep the period in retirement constant. Alternatively, policy designs that pursue a fixed replacement rate are consistent with retirement age policies targeting a constant balance between active years in the workforce and years in retirement. The empirical strategy employed to project the relevant cohort life expectancy uses a Bayesian Model Ensemble approach to stochastic mortality modelling to generate forecasts of intergenerationally and actuarially fair pension ages for 23 countries and regions from 2000 to 2050. The empirical results show that the pension age increases needed to accommodate the effect of longevity developments on pay-as-you-go equilibrium and to reinstate equity between generations are sizeable and well beyond those employed and/or legislated in most countries. A new wave of pension reforms may be at the doorsteps.

JEL-Codes: H550, G220, C630, C530, H230.

Keywords: retirement age, actuarial fairness, intergenerational neutrality, pensions, Bayesian Model Ensemble, population ageing.

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1 Introduction

Linking retirement ages and pension benefits has been one of the most common reforms of countries to ensure national pension schemes' long-term affordability and fiscal sustainability given continuously increasing longevity in old age. This reform trend is part of a broader strategy introducing automatic adjustment or stabilisation mechanisms in pension schemes; i.e., rules that automatically adjust a scheme's parameters to demographic and/or economic developments in a predetermined fashion instead of waiting for ad hoc political interventions.

Countries have introduced and adjusted the links to life expectancy in multiple ways, as recognised and discussed in numerous contexts, including our own work (Alho et al. 2013; Bovenberg et al. 2015; OECD 2019; Holzmann et al. 2020; Ayuso et al. 2021a). The topics of interest fall under a number of categories: (i) automatically indexing normal and early retirement ages (Denmark, the Netherlands, Slovakia, Greece, Portugal, the United Kingdom, Italy, Finland, Estonia, Norway); (ii) linking newly granted pensions to sustainability factors or life expectancy coefficients (Portugal, Spain[1], Finland), or to old-age dependency ratios (Germany, Japan); (iii) transforming public earnings-related plans into nonfinancial defined contribution (NDC) schemes (Sweden, Italy, Poland, Latvia, Norway), which automatically adjust retirement benefits to life expectancy in the process of annuitisation of individual account balances; (iv) determining the qualifying conditions for an old-age pension, for instance, by indexing the number of contribution years required for a full pension to life expectancy (Italy, France); (v) introducing risk-sharing arrangements in public and private individual or employer-sponsored pension plans (the Netherlands, the United States, Belgium); (vi) introducing mandatory and voluntary funded defined contribution (DC) schemes to replace or supplement public pension provisions (Chile, Sweden, Estonia, Switzerland, Israel, Hungary, Australia, Mexico, Poland, Slovakia); (vii) conditioning the annual indexation of pensions in payment to a scheme's solvency position (the Netherlands); and (viii) linking pension penalties (incentives) for early (late) retirement to the contribution length (Portugal).

Whether scheduled, discretionary, or via an automatic link to life expectancy, legislated retirement age policies exhibit several deficiencies. First, although the specific details of pension reform diverge among countries, in all cases and countries unisex period life expectancy measures computed from official life tables have been used to automatically link retirement ages, sustainability factors, and other pension scheme parameters (for example, decrements (increments) for early (late) retirement) to longevity developments. Abstracting from the well-known sex gradient in life expectancy, it has been known for some time that with a continuous decline in age-specific mortality rates, the use of period instead of cohort longevity markers results in systematic underestimation of the remaining lifetime at retirement (Alho et al. 2013). Recent

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[1] The actual application of the sustainability factor in Spain was planned to 2019 but a policy initiative suspended its use sine die.

empirical studies show that the life expectancy gap at retirement age is sizable, persistent, and still increasing in most countries (Bravo et al., 2021), which translates into an ex ante unintended financial transfer from future to current generations. Moreover, the adoption of rule-based approaches linking the retirement age to period instead of cohort life expectancy and poor policy design result in policy outcomes that deviate substantially from their initial intentions (Ayuso et al. 2021b; Bravo and Ayuso 2020, 2021). Since public pension scheme reforms need to be backed by both public approval and democratic support, reforms that are either inter- or intragenerationally unfair are less likely to succeed in balancing the needs and interests of current and future generations and will tend to be rejected by voters, especially if they involve retrenchments. Their contribution to creating fair intergenerational risk sharing is indeed one of the key rationales behind government mandates of compulsory pension systems.

A second deficiency is that empirical evidence shows that individuals with higher socioeconomic status (whether measured by income, education, or occupation) tend to live longer and in better health than those with lower income, education, and occupational status; relatedly, lifespan at retirement is unequally distributed (Chetty et al. 2016; Ayuso et al. 2017a, 2017b; Auerbach et al. 2017; Holzmann et al. 2020; Aburto et al. 2020). Since pension reforms tend to prescribe uniform rules across the population or generations of participants, longevity heterogeneity generates intragenerational redistribution in both public pension entitlements and private sector insurance contracts (for example, life annuities and life insurance), implying built-in regressivity in the underlying premise that life expectancy is randomly determined.

Using comprehensive database for the United States, Chetty et al. (2016) find that the two most prominent underlying causes of short life expectancy are lifestyle factors (for example, smoking, substance misuse, poor diet, and lack of physical exercise) and the absence of community support (for example, social measures that provide support for health measures). Palmer and Zhao de Gosson de Varennes (2020) argue that an effective solution calls for focused social policy that addresses the underlying issues. In contrast, Holzmann et al. (2020) design and assess the effectiveness of approaches under DC schemes to internalise the heterogeneity effects at the time of initial pension calculation.

Third, empirical data show that longer life expectancy does not automatically translate into parallel movements in healthy life expectancy (HLE) and disability-free life expectancy (DFLE) and that social inequalities in health continue to widen (Jivraj et al. 2020). Pension reforms targeting longer working lives must be accompanied by supporting measures that recognise the need for accommodating disability benefits.

This paper abstracts from the above critical social policy issues accompanying increasing life expectancy. It discusses how to automatically index the retirement age to life expectancy while delivering on the principles of intergenerational actuarial fairness and neutrality among generations. We investigate alternative retirement age policies considering a stylised Bismarckian earnings-related defined benefit (DB) pension scheme in which one's entry pension is strictly linked to one's entire contribution history. We assume that the scheme begins in a steady state, with stable demographic (old-age dependency ratio) and economic (employment rate and wage rate) conditions. We derive an intergenerational actuarial neutrality condition for policy reforms and examine a full menu of automatic adjustment mechanisms and pension policy rules designed to maintain the long-term financial sustainability of pension schemes.

Without loss of generality, we then assume that the normal retirement age is the key policy instrument and investigate how to amend so as to not affect intergenerational redistribution; i.e., neutral in its effect on the rate of return of each generation's contributions to a pay-as-you-go (PAYG) pension scheme.

To discuss the alternative policy options, we adopt an intergenerational actuarial fairness and neutrality principle to pension design and reform at the margin, but the framework can easily be extended to the aggregate. Fairness and neutrality can have different meanings to different people in the context of an earnings-related pension system, so we need to be more precise in their use here. We use the term fairness in the sense of (marginal) actuarial fairness; i.e., a fair pension scheme requires the present value of lifetime contributions to equal the actuarial present value of lifetime benefits at the time of retirement. Stated differently, an actuarially intragenerational fair pension scheme is one in which every individual's contribution in any given period yields the same expected increment to retirement income given that the distribution of risks, mainly including the risk of life expectancy, is unknown ex ante. This scheme ex ante guarantees equality through the random distribution of risk. And, intergenerational neutrality requires this to be true over generations.

Tackling the fairness challenge across generations in response to demographic or economic developments depends on the pension scheme's underlying design (DB or DC) and on the way policy interventions are designed and implemented. They ultimately determine how the cost of life expectancy improvements is shared between current and future pensioners. Assuming labour market participation and retirement decisions are not distorted by the policy intervention and that all other pension scheme parameters are kept constant, this paper provides comprehensive empirical results for two possible policy designs. These differ in that they assume the extra contribution years may or may not generate additional pension entitlements; i.e., they convey alternative ways of sharing the longevity risk burden between workers and pensioners. In the first policy design, the extra contribution period translates into a higher replacement rate by keeping the accrual rate per year constant. Under this constant accrual-rate-per-year (CAR) policy design, we show that if lifetime earnings are revalued at the scheme's internal rate of return and pension benefits are indexed at the same rate, intergenerational actuarial fairness requires standard pension ages to be updated fully in line with life expectancy developments. Stated differently, under a CAR policy design the retirement age indexation mechanism targets a constant period in retirement.

In the second policy design, the additional contribution period is accompanied by a reduction in the accrual rate per year such that the replacement rate remains constant over time. Under the same assumptions as above, we show that in an actuarially fair and neutral pension scheme across generations a constant replacement rate (CRR) policy design prescribes that the retirement age must be adjusted such that the expected years in retirement relative to contribution years (or relative to adult life if we assume a constant labour market entry age) are kept constant over time. This is consistent with retirement age policies targeting a constant balance (ratio) between time spent in work (or in adult life) and retirement, while introducing adequacy safeguards and intergenerational fairness by keeping the replacement rate constant across generations. Mixed interventions are also feasible within our framework, considering other social, demographic, and/or economic criteria, including increasing old-age dependency ratios.

Together with demographic, economic, and financial sustainability, intergenerational fairness and trust are essential building blocks to build and maintain a stable and welfare-enhancing social contract between generations. This is particularly important in PAYG schemes, which rely on the (normally enforced) commitment of young working generations to contribute to the system in exchange for the promise of an adequate and fair pension when they retire. This is why nonfinancial DC (NDC) schemes are at least as good as or better than nonfinancial DB (NDB) pension schemes (Góra and Palmer 2004, 2020). In practice, this requires keeping the promise well-defined and credible in the event of both long- and short-term economic and/or demographic shocks. This, in turn, requires dynamically updating the scheme parameters to ensure the link between contributions and benefits throughout the entire lifetime remains actuarially fair and intergenerationally neutral.

This paper uses total population mortality and full pension age data for 23 countries and regions to compare the dynamics of actual and legislated (forecast for countries adopting automatic indexation mechanisms) retirement ages with those required to cope with survival developments in an intergenerationally actuarially fair scheme from 2000 to 2050. The analysis covers medium- and high-income countries in North America, Central and Western Europe, Asia, and the Pacific, as well as Chile in Latin America with its diverse pension architecture, including both financial (DC and DB) and nonfinancial (DB and DC) schemes.

To generate forecasts of the retirement age by age, sex, and year, we estimate period and cohort survival curves from stochastic mortality models. Currently, model selection and model combination are the two competing approaches in mortality modelling and forecasting. The customary procedure is to pursue a winner-take-all approach by which, for each population, a single model is chosen from a set of candidate methods using some criteria (for example, information, forecasting accuracy). To this end, a growing number of single- and multi-population, discrete- and continuous-time, age-period-cohort stochastic mortality models, principal component methods, smoothing approaches, and statistical machine learning techniques are proposed in the actuarial and demographic literature (Lee and Carter 1992; Brouhns et al. 2002; Renshaw and Haberman 2003, 2006; Currie 2006, 2016; Cairns et al. 2006, 2009; Hyndman and Ullah 2007; Plat 2009; Pascariu et al. 2020; Basellini et al. 2020; Hunt and Blake 2021; Bravo and Nunes 2021; Xu et al. 2020; Perla et al. 2021; Li et al. 2021; and references therein). The use of different selection procedures, alternative accuracy metrics, different data-coverage periods, misspecification problems, and the presence of structural breaks in the data-generating process can lead to different model choices and time series forecasts, with empirical studies showing no single mortality model outperforms in all countries or subpopulations/cohorts of countries, or across time. More recently, an alternative composite model approach using a Bayesian Model Ensemble (BME) or averaging was proposed to address model (conceptual) uncertainty in stochastic mortality modelling (Kontis et al. 2017; Bravo et al. 2021).

To tackle the model risk problem that prevents countries from using cohort life expectancy measures in pension policy, to improve the forecasting accuracy, to circumvent the limitations of individual methods, and to generate comparable cross-country estimates, this paper uses the BME approach developed by Bravo et al. (2021) in a paper parallel to this one. The composite model combines nine heterogeneous stochastic mortality models comprising principal component methods, two-dimensional smoothing approaches, and well-known Generalised Age-Period-Cohort (GAPC) models. The empirical strategy for each country population involves: (i) identification of the model confidence set; (ii) computation of posterior probabilities for each model; (iii) generation of forecasts using the composite model; and (iv) computation of Bayesian prediction intervals considering for stochastic process, model, and parameter risks using the Model-Averaged Tail Area (MATA) approach (Turek and Fletcher 2012).

Our empirical results for both the CAR and CRR retirement age policies show that actual (2000–2021) and legislated (planned) retirement age increases have been and will be insufficient to cope with populations' extended survival prospects if the pension scheme is to preserve the intergenerational fairness and neutrality condition. The difference between the intergenerationally fair retirement ages and the actual/legislated retirement ones is, as expected, higher under a CAR policy than under a CRR approach, with gaps accumulating over time in both cases. The results also show that despite the important retirement age increases legislated in many OECD countries in the last two decades, the expected duration of retirement will continue to increase. Adoption of a CRR retirement age policy contributes to reducing the expected period in retirement by 1.91 years in 2050 compared to legislated reforms. But this is not enough. The legislated corrections fall short of what is needed to prevent a rise in expected retirement duration and, in many cases, will not be offset by an increase in the relative size of the labour force. With the exceptions of Belgium and the Netherlands, the results show that the expected period in retirement relative to the contribution period (and to adult life) is forecast to increase in all countries, peaking at 74.5 percent in France in 2050. The required pension age corrections are well beyond the scheduled changes planned or ongoing in many countries and could trigger a new wave of pension reforms, including updating early retirement ages and further closing routes into early labour market exit. The required correction also raises distributional concerns because of the widening gap in life expectancy by socioeconomic group discussed above.

The empirical evidence shows that the average gap between the normal and early retirement ages, and the age when people leave the labour force with a pension (i.e., effective retirement age) in OECD countries is still significant and expected to remain so over the next decades (OECD 2019). This may be a result of cultural norms, employer policies, health issues, and/or the desire for leisure time to pursue other interests or to care for relatives. It may also be a result of demand and supply factors such as unemployment spells late in working life leading to irreversible labour force withdrawal, for example through early retirement pathways, with significant pension entitlement losses (Bravo and Herce 2020). In addition to reaching a point where the utility of leisure and disutility of work become drivers, other determinants are significant. These include in some countries implicit tax rates on working additional years as first brought to our attention in Gruber and Wise (1998), wealth effects associated with improved living standards, declining relative labour productivity and wages for workers at the bottom of the human capital scale, rigid labour markets and policies that distort retirement incentives, including through pension taxation (Gruber and Wise 2004; Holzmann and Piggott 2018; Bravo 2016).

Instead of imposing a fixed uniform retirement age for all, in some countries in which a strict actuarial link exists between contributions and benefits (for example, countries with NDC schemes), the preference has been to adopt flexible retirement age policies on the timing of retirement by introducing a minimum age to claim a public pension, beyond which workers are free to draw on a full or partial pension benefit while continuing in paid work, frequently with reduced working hours. Most countries continue to have early retirement provisions that allow individuals to stop working before the statutory retirement age by accepting penalised (lower) monthly benefits, with decrements computed in either an actuarially neutral or an ad hoc way.

This paper computes actuarially fair pension age adjustments to incorporate life expectancy developments. Our study is related to a growing body of literature investigating the macroeconomic, fiscal, welfare, and behavioural responses to pension scheme design and reform (Fehr 2009; Makarskia and Tyrowiczb 2019). With respect to retirement age, the literature mostly centres around two main issues. The first focuses on the fiscal, labour market, and welfare effects of changing the retirement age. For instance, Schwan and Sail (2013) investigate the potential impact of increases in the retirement age in European Union (EU) countries on public pension expenditure, sustainability, and adequacy measures and conclude that they can produce positive effects on labour supply, employment, potential gross domestic product (GDP) growth, and on the benefit ratio due to increased contributions based on longer working lives. What's more, the result can be a reduction of the old-age dependency ratio and pension expenditures as a fraction of GDP. The second focus in the literature investigates the optimal retirement age and the impact of incentives (Cremer and Pestieau 2003; Fehr et al. 2003; Galasso 2008; Fehr et al. 2012; Rabaté 2019). Freudenberg et al. (2018) perform actuarial estimates of the pension decrement rates consistent with a financially intra and intergenerationally neutral decision on when to retire.

The remainder of the paper is organised as follows. Section 2 presents the key concepts and statistical methods used in the paper. These are the principles of intergenerational actuarial fairness and neutrality as employed in pension design. It also recaps the BME approach proposed in Bravo et al. (2021) for mortality modelling and life expectancy computation, and describes the data used in fitting the models. Section 3 reports summary forecasts of cohort life expectancy at retirement age by country and provides detailed numerical results for the two alternative retirement age policy designs considered in this study. Section 4 discusses the results and policy implications. Section 5 presents the main conclusions. All accessory results are in the appendix.

2 Materials and methods

2.1 Actuarially fair and neutral retirement age policies

Consider a stylised Bismarckian earnings-related DB pension scheme in which one's initial pension benefit is strictly linked to one's entire contribution history. Without loss of generality, we adopt an intergenerational actuarial fairness and neutrality principle to pension design and reform at the margin. The actuarial balance constraint[2] for a representative individual retiring at age $x_{r(t)}$ in year t equals the accumulation at retirement and the pension wealth

$$c_t \cdot V\left(x_{r(t)}, x_e, w_t, y_t\right) = P_{x_r(t)} \cdot a^{\pi,y}_{x_r(t)}, \tag{1}$$

where c_t is the contribution rate; $V\left(x_r, x_e, w_t, y_t\right)$ is the cumulative value at the retirement age of lifetime pensionable earnings w_t earned since labour market entry age x_e and valued at an (actuarial equilibrium, notional) rate of return y_t, $a^{\pi,y}_{x_r(t)}$ is the annuity factor computed using a cohort approach,

$$a^{\pi,y}_{x_r(t)} := \sum_{\tau=1}^{\omega-x_r} \left(\frac{1+\pi_\tau}{1+y_\tau}\right)^t {}_\tau p_{x_r(t)}). \tag{2}$$

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[2] Several alternative mutually complementary indicators may be considered to evaluate intergenerational fairness in pension schemes, for instance, the ratio between the present value of lifetime benefits and the accumulation at retirement, the scheme's internal rate of return, the affordability and stability of the social contribution rate across generations, the benefit adequacy or the scheme's balance sheet solvency. Another possibility is to adopt a generational accounting approach that assumes the government (explicit and implicit) debt reflects taxes paid minus transfers received over the remaining lifetime of both current and future generations.

where π is the uprating rate for pensions, and ${}_{\tau}p_x$ denotes the τ-year survival rate of a population cohort aged x at time t:

$$ {}_{\tau}p_x\left(t\right) := \exp\left(-\int_0^{\tau} \mu_{x+s}\left(s\right) ds\right), \tag{3}$$

$\mu_x\left(t\right)$ a stochastic force of mortality process on a filtered probability space $\left(\Omega, \mathbb{G}, \mathbb{P}\right)$; $P_{x_r(t)}$ is the annual pension benefit computed as follows:

$$P_{x_r(t)} = \theta_t\left(x_{r(t)} - x_e\right) \cdot \overline{RE}_{x_r(t)} \cdot RF_{x_r(t)} \cdot b_{x_r(t)}, \tag{4}$$

where θ_t is a linear (usually flat) accrual rate for each year of service[3], $\left(x_{r(t)} - x_e\right)$ is the contribution period, $RF_{x_r(t)}$ is a demographic (often called sustainability) factor introduced in some countries to reduce pension benefits as life expectancy increases (Finland, Portugal, Spain); $b_{x_r(t)}$ are pension decrements (increments) for early $\left(b_{x_r(t)} < 1\right)$ or postponed $\left(b_{x_r(t)} > 1\right)$ retirement, and $\overline{RE}_{x_r(t)} \equiv \overline{RE}\left(x_{r(t)}, x_e, w_t, \upsilon_t\right)$ is the lifetime average revalued earnings at retirement age

$$\overline{RE}_{x_r(t)} = \frac{RE_{x_r(t)}}{x_r(t) - x_e}, \text{ with } RE_{x_r(t)} = \left(w_t^{x_r(t)} + \sum_{x=x_0}^{x_r(t)-1} w_{t-x_r(t)+x}^{x_r(t)} \prod_{j=t-x_r(t)+x+1}^{t} \left(1+\upsilon_j\right)\right), \tag{5}$$

where υ_t denotes the factor ("rate of indexation") by which each year's contributions are revalued (for example, some countries use the rate of growth of economy-wide average earnings; others use a combination of the consumer price index and productivity growth; while others use the scheme's internal rate of return).

It can be shown that if the rate used to revalue past earnings in a DB scheme is the same as the notional interest rate in NDC schemes (Sweden, Italy) and that of the valuation procedure of pension-point schemes (e.g., France, Germany), the initial benefit structure can be constructed to be similar to the NDC structure (see, e.g., Queisser and Whitehouse, 2006) - but only if one disregards the notional interest component composed of the rate of change in the “contribution-based” labour force (see, e.g., Palmer, 2013).

In a recent summary of country practices in estimating life expectancy, Bengtsson et al. (2019) remind us that: "As we moved onto the twenty-first century, the philosophy for projecting life expectancy employed by many official statistical agencies embraced the idea that improvements in mortality could not continue into the 21st century at the same rate as in the preceding half-century." The evidence is to the contrary, however, as Oeppen and Vaupel (2002) demonstrated. The next step in the cautious development of official projections of life expectancy of statistical agencies was to turn to the use of period life expectancy in modelling

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[3] Accrual rates generally follow a linear flat schedule, although there are exceptions, e.g., Finland which adopted a non-linear accrual schedule until 2017.

(Alho et al. 2013). The recent works cited above highlighted the serious deficiencies of this approach.

For pension schemes, we now know that if the remaining lifetime at retirement age is underestimated by using period-based life expectancy estimates the scheme will be in deficit and the actuarial balance equation will not hold; i.e., the scheme will not be neutral among generations (Palmer and Zhao de Gosson de Varennes 2020). This means that if longevity changes over time – for instance if survival prospects at retirement continue to improve – the pension parameters (contribution rate, retirement age, accrual rate) must be updated to guarantee the scheme remains actuarially neutral across generations. Without loss of generality, assume the parameters that are not policy instruments are kept constant. The marginal actuarial neutrality between an individual of generation and that of the initial generation (labelled) requires the following condition to hold:[4]

$$\frac{c_t}{c_0} \cdot \frac{V\left(x_{r(t)}, x_e, w, y_t\right)}{V\left(x_{r(0)}, x_e, w, y_0\right)} = \frac{\theta_t\left(x_{r(t)} - x_e\right)}{\theta_0\left(x_{r(0)} - x_e\right)} \cdot \frac{\overline{RE}_{x_r(t)}}{\overline{RE}_{x_r(0)}} \cdot \frac{RF_{x_r(t)}}{RF_{x_r(0)}} \cdot \frac{b_{x_r(t)}}{b_{x_r(0)}} \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}}. \tag{6}$$

This condition can be easily extended to account for population ageing (increase in the old-age dependency ratio) and/or the existence of external sources of funding in the pension scheme. Assume, without loss of generality, that individuals of both cohorts retire at the full old-age pension age (i.e., pension decrements/increments are $b_{x_r(t)} = b_{x_r(0)} = 1$) and that the demographic reduction factor is kept constant over time (i.e., $RF_{x_r(t)}/RF_{x_r(0)} = 1$). Equation (6) simplifies to:

$$\frac{c_t}{c_0} \cdot \frac{V\left(x_{r(t)}, x_e, w, y_t\right)}{V\left(x_{r(0)}, x_e, w, y_0\right)} = \frac{\theta_t\left(x_{r(t)} - x_e\right)}{\theta_0\left(x_{r(0)} - x_e\right)} \cdot \frac{\overline{RE}_{x_r(t)}}{\overline{RE}_{x_r(0)}} \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}}. \tag{7}$$

Equation (6) offers a full menu of automatic adjustment mechanisms and pension policy rules to absorb the impact of economic and demographic shocks and preserve actuarial fairness and neutrality across generations. It frames a credible social contract between different generations, explicitly integrating intra- and intergenerational equity concerns. However, some of the policy options are (politically and socially) difficult to implement and sustain in practice. Moreover, depending on the pensions scheme's overall architecture (a combination of state, occupational, and private components), and the technical design (DB, DC) of individual schemes in the system, as well as the way the interventions are devised and adopted, they may have important implications for the way the cost of providing for pensions is shared among generations as life expectancy increases.

As previously mentioned, our starting position is a pension scheme with no ex ante redistributive objectives in which proposed interventions aim to eliminate the wealth redistribution

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[4] A similar condition can be found in Meneu et al. (2016), discarding the reduction factor and the pension decrement/increment correction.

effects and the distortions on individual labour supply and savings decisions created by the life expectancy developments. In DC (DB) schemes a zero ex ante distortion takes place if account balances (accumulated rights) at the time of retirement are converted into an annuity based on cohort survival probabilities at retirement estimated using an unbiased projection method. The size of the unfunded pension liabilities or, equivalently, of the intergenerational tax/subsidy created before and after the policy intervention, is suggested as a performance measure.

Conceptually, the policy interventions can take place at the accumulation, annuitisation, and decumulation phases or can encompass mixed interventions that combine elements of all three stages (Ayuso et al. 2021a). Given the nature of the distortions addressed in this paper, we believe that redesign is best implemented at the latter two phases. This can be done, for instance, by reducing the initial pension through an actuarially designed reduction factor in response to a longer period of benefits or by linking pension benefits or pension indexation to survival developments (Bravo et al., 2021). We note, however, that in a pure NDB scheme the natural adjustment would come through an update in the contribution rate to achieve fiscal balance, redistributing risk from pensioners to contributors.[5]

In contrast, by generic construction, an NDC system's contribution rate should be constant across generations; therefore for instance, with variation in the old-age dependency ratio, the scheme should automatically adjust by changing the ratio between average pension and average wage (benefit ratio). This paper focuses instead on the retirement age adjustments required to restore actuarial fairness in response to the life expectancy gap. The gap is a measure of the additional (reduced) years of life a given cohort will receive pensions as a result of expected future mortality improvements (deterioration) not predicted at benefit computation and not backed by the accumulation.

Assuming incentives for individuals are neutral – i.e., assuming that labour market entry and exit (retirement) ages are not distorted – pension age increases come with an equivalent increase in the contribution period. For the system as a whole, we assume changes in life expectancy are accompanied by an increase in effective retirement age such that the old-age dependency ratio remains constant. As noted before, at least two possible extreme designs are possible depending on whether the added contribution period generates additional pension entitlements: (i) the extra contribution period translates into a higher replacement rate by keeping the accrual rate per year constant; or (ii) the increase in contribution years is accom-

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[5] From (7), keeping the other pension parameters constant, in response to the population's higher survival prospects, the new contribution rate necessary for global equilibrium of the PAYG scheme would be determined such that the following condition holds:

$$c_t = c_0 \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}}. \tag{8}$$

Note, however, that an increase in the contribution rate creates a negative impact on labour costs affecting labour demand, wages, labour market equilibrium, and the pension scheme's long-term sustainability.

panied by a reduction in the accrual rate per year such that the replacement rate remains constant. Mixed interventions sharing the longevity risk burden between different generations are also possible taking into account other social and economic criteria.[6]

2.1.1 Constant accrual-rate-per-year policy

Under a CAR policy, the required retirement age and the contribution period adjustments are accompanied by an increase in the replacement rate since the accrual rate per year is kept constant (i.e., $\theta_t = \theta_0$), while keeping all other pension system parameters unchanged. In a scenario of positive longevity developments, the contribution period will have to increase to restore actuarial fairness, generating higher replacement rates $\theta_t \left(x_{r(t)} - x_e\right) > \theta_0 \left(x_{r(0)} - x_e\right)$; i.e., higher pensions and an enlarged pension scheme. Depending on the way the corrections are made, the shorter pension payment period may counterbalance the higher benefit levels. At an aggregate level, if the increased survival prospects negatively impact the old-age dependency ratio, the scheme's PAYG equilibrium deteriorates. From equation (7), the new equilibrium retirement age is the result of the following updating rule

$$a^{\pi,y}_{x_r(t)} = \frac{V\left(x_{r(t)}, x_e, w, y_t\right) / V\left(x_{r(0)}, x_e, w, y_0\right)}{RE_{x_r(t)} / RE_{x_r(0)}} \cdot a^{\pi,y}_{x_r(0)}. \tag{9}$$

If lifetime earnings are revalued at the scheme's internal rate of return[7] (i.e., if $\upsilon_t = y_t\ \forall t$), the adjustment rule (9) reduces to:

$$a^{\pi,y}_{x_r(t)} = a^{\pi,y}_{x_r(0)}. \tag{10}$$

By further assuming the uprating rate for pensions matches the internal rate of return or discount rate (i.e., $\pi_t = y_t\ \forall t$), equation (10) reduces to:

$$\dot{e}^{C}_{x_r(t)} = \dot{e}^{C}_{x_r(0)}. \tag{11}$$

Equations (10) and (11) suggest that to cope with populations' survival prospects while keeping the accrual rate per year constant, pension age must be adjusted so that the actuarial present value (or cohort life expectancy) remains constant over time. In other words, the simple rule of adjusting pension age by the same magnitude of the life expectancy increase, targeting a constant retirement period (the Netherlands, Denmark), would only be considered actuarially fair and neutral across generations if accompanied by a properly calibrated CAR policy. In this scenario, the added contribution years generate additional pension entitlements fully covered

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[6] For the pension scheme as a whole, to be revenue neutral the actuarial adjustments should reflect as closely as possible the group-specific life expectancy and benefit amount.

[7] This is particularly the case in NDC schemes in which the notional pension wealth and the benefit computation incorporate the internal (implicit) rate of return from a PAYG system and the expected remaining lifetime at retirement.

by extra contributions.[8] In a scenario of population ageing (increasing the old-age dependency ratio), it can easily be shown that retirement age increases fully in line with increases in cohort life expectancy at retirement are not sufficient to restore the PAYG equilibrium and equity constraint, unless labour market participation rates increase and/or a structural reduction in the unemployment rate is observed. In this scenario this means that the retirement period would have to be reduced and younger generations would have to further adjust their retirement decisions to sustain the PAYG conditional pension promise.

If lifetime earnings are revalued below the internal rate of return (i.e., if $y_t > \upsilon_t \; \forall t$), it is clear from equation (9) that the required retirement age adjustments would have to be smaller than in the baseline case since the lifetime revalued earnings would not completely reflect the extra contribution period. If lifetime earnings are revalued above the internal rate of return (i.e., $y_t < \upsilon_t \; \forall t$), the opposite occurs. If pensions are adjusted below the internal rate of return (i.e., if $\pi_t < y_t \; \forall t$), the required pension age adjustments would naturally be smaller; the opposite occurs if pensions are revalued every year above the internal rate of return (i.e., $\pi_t > y_t \; \forall t$).

2.1.2 Constant replacement rate policy

Under a CRR policy, the required adjustment in the retirement age and contribution period is accompanied by a reduction in the accrual rate per year, such that the replacement rate (global accrued rate) remains constant across generations; i.e., $\theta_t \left(x_{r(t)} - x_e\right) = \theta_0 \left(x_{r(0)} - x_e\right)$ or, equivalently:

$$\theta_t = \theta_0 \cdot \frac{\left(x_{r(0)} - x_e\right)}{\left(x_{r(t)} - x_e\right)}, \tag{12}$$

with $\theta_t < \theta_0$ since $\left(x_{r(0)} - x_e\right) < \left(x_{r(t)} - x_e\right)$. In this scenario, the impact of a longer contribution period on pension entitlements would be mitigated since it would come only because of the impact of extra work years on average lifetime revalued earnings. This effect is expected to be small since contrary to “best years” DB formulas, full contribution period DB pension formulas smooth the effect of abnormally low or high labour income years on initial benefits.

From (7), the new equilibrium retirement age would be the result of the following updating rule:

$$a^{\pi,y}_{x_r(t)} = \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot \frac{V\left(x_{r(t)}, x_e, w, y_t\right) / V\left(x_{r(0)}, x_e, w, y_0\right)}{RE_{x_{r(t)}} / RE_{x_{r(0)}}} \cdot a^{\pi,y}_{x_r(0)}, \tag{13}$$

which, if lifetime earnings are revalued at the scheme’s internal rate of return, reduces to:

$$a^{\pi,y}_{x_r(t)} = \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot a^{\pi,y}_{x_r(0)}. \tag{14}$$

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[8] This scenario is referred to as the "100% shift scenario" in Schwan and Sail (2013).

By further assuming the uprating rate for pensions matches the discount rate, the fairness condition (14) reduces to

$$\dot{e}^{C}_{x_{r(t)}} = \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot \dot{e}^{C}_{x_{r(0)}}, \tag{15}$$

or, equivalently to

$$\frac{\dot{e}^{C}_{x_{r(t)}}}{\left(x_{r(t)} - x_e\right)} = \frac{\dot{e}^{C}_{x_{r(0)}}}{\left(x_{r(0)} - x_e\right)}. \tag{16}$$

Equation (16) provides an interesting and important retirement age policy result. It shows that in an actuarially fair and neutral pension scheme, to deal with populations' extended survival prospects while keeping the replacement rate (global accrual rate) constant over time, the retirement age must be adjusted such that expected years in retirement relative to years of work and contribution periods must remain constant over time. This means the extra lifetime must be divided proportionally over the working and the retirement periods; i.e., the working population and retirees share the burden of life expectancy improvements. Moreover, for a constant labour market entry age, pursuing the retirement age policy expressed in equation (16) is equivalent to a policy targeting the expected years in retirement as a fixed share of adult life. Stated differently, in an actuarially – and thus intergenerationally – fair and neutral pension scheme, a retirement age policy targeting a constant balance (ratio) between time spent in work (or in adult life) and retirement (see, for example, the reform proposals in the UK) is consistent with a constant replacement rate (adequacy) across generations.

From (15), the pension age increase required to keep constant the time spent in work (contributing) and in retirement, $\Delta x_{r(t)} = x_{r(t)} - x_{r(0)}$, is given by the initial contribution career multiplied by the percentage increase in life expectancy:

$$\Delta x_{r(t)} = x_{r(t)} - x_{r(0)} = \left(x_{r(0)} - x_e\right) \cdot \left(\frac{\dot{e}^{C}_{x_{r(t)}}}{\dot{e}^{C}_{x_{r(0)}}} - 1 \right). \tag{17}$$

In a scenario of population ageing, it can be shown that to restore the PAYG equilibrium the contribution period would have to be further increased by a factor equal to the percentage increase in the old-age dependency ratio. Under this policy design, the extra period in retirement $\Delta \dot{e}^{C}_{x_{r(t)}} = \dot{e}^{C}_{x_{r(t)}} - \dot{e}^{C}_{x_{0(t)}}$ that is consistent with the intergenerational actuarial fairness condition is given by a fraction of the additional contribution years:

$$\Delta \dot{e}^{C}_{x_{r(t)}} = \dot{e}^{C}_{x_{r(t)}} - \dot{e}^{C}_{x_{0(t)}} = \left(\frac{\dot{e}^{C}_{x_{0(t)}}}{x_{r(0)} - x_e} \right) \cdot \Delta x_{r(t)}, \tag{18}$$

with the initial expected years in retirement relative to the contribution period $\dot{e}^{C}_{x_{0(t)}} / \left(x_{r(0)} - x_e\right)$ the splitting coefficient.

From (16), we also conclude that targeting a constant balance between time spent in work and retirement requires updating the contribution period by a factor equal to the percentage increase in cohort life expectancy at the retirement age. Of course, society may decide to depart from the intergenerational fairness condition and adopt alternative longevity risk-sharing mechanisms between current and future pensioners combining actuarial fairness, financial sustainability, and social adequacy. One possible strategy is to pursue the following the updating scheme:

$$\left(x_{r(t)} - x_e\right) = \left(x_{r(0)} - x_e\right) \cdot \left(\frac{\dot{e}^C_{x_r(t)}}{\dot{e}^C_{x_r(0)}}\right)^{\phi}, \tag{19}$$

where ϕ is a risk-sharing coefficient with $\phi = 1$ corresponding to the retirement age policy set by equations (15) and (16). For values of ϕ in the range $]0, 1[$ the retirement age updates would only partially reflect life expectancy developments, whereas for $\phi = 0$ the policy option would be to keep the contribution period constant over time.

Compared to the CAR policy, a retirement age policy targeting a CRR requires smaller pension age increases to cope with life expectancy developments. This is because of the reduced impact of additional contribution years on pension entitlements as a consequence of the smaller accrual per contribution year. Compared with (9), the pension age adjustment prescribed by (13) no longer translates into a higher replacement rate at retirement. Once again, if lifetime earnings are revalued below (above) the internal rate of return, the required pension age adjustments would have to be comparatively smaller (higher) than in the baseline case. As in the previous case, if pensions are updated below (above) the internal rate of return, the required pension age correction would be smaller (higher). Finally, note that this study assumes the labour market entry age is set at 22 years for all countries and subpopulations.

2.2 Forecasting the survival function

2.2.1 Bayesian Model Ensemble or Averaging

This section presents the stochastic mortality modelling and forecasting approach developed in Bravo et al. (2021) and applied here to produce life expectancy forecasts. The rationale behind the BME is that instead of producing best-estimate projections based on a single model presumed to be the true one, identified based on user-specified criteria (for example, Bayesian Information Criterion, forecasting accuracy measure, cross-validation), the projection model is determined combining (averaging) a set or subset (model confidence set) of models. The BME model combination aims at finding a composite model that best approximates the actual data generation process (known historical data) and its multiple sources of risk. The BME composite model design should by definition be superior to individual candidate models because, first, it explicitly addresses model uncertainty. Second, each model's shortcomings are ideally compensated within a statistically (data) driven optimal combination. Third, conditioning the statistical inference on a set of statistical models minimises the individual model-based biases and produces more realistic confidence intervals. This in turn improves the out-of-sample forecasting precision and provides a more accurate representation of forecast uncertainty for decision making.

Let each candidate model be denoted by M_l, $l = 1, ..., K$. This encompasses the set of probability distributions comprising the likelihood function $\mathcal{L}(y|\xi_l, M_l)$ of the observed data y in terms of model specific parameters ξ_l, and $\pi(\xi_l, M_l)$ the prior density of ξ_l under M_l. Consider a quantity of interest Δ present in all models, for instance, the predictive quantity of y. The marginal posterior distribution across all models is given by

$$\pi(\Delta|\mathbf{y}) = \sum_{k=1}^{K} \pi(\Delta|\mathbf{y}, M_k)\, \pi(M_k|\mathbf{y}), \tag{20}$$

where $\pi(\Delta|\mathbf{y}, M_k)$ denotes the forecast probability density function (PDF) based on model M_k alone, and $\pi(M_k|\mathbf{y})$ is the posterior probability of model M_k given the observed data. The weight assigned to each model M_k is given by its posterior probability

$$\pi(M_k|\,\mathbf{y}) = \frac{\pi(\mathbf{y}|M_k)\, \pi(M_k)}{\sum_{l=1}^{K} \pi(\mathbf{y}|M_l)\, \pi(M_l)}, \tag{21}$$

with $\sum_{l=1}^{K} \pi(M_k|\mathbf{y}) = 1$. The BME PDF is a weighted average of the PDFs of the individual candidate models, weighted by their posterior model probabilities (Raftery et al. 2005). How does our design of the BME procedure work? In the first stage, we select the model confidence set (a subset of models to be part of the model combination) by ranking individual models according to out-of-sample forecasting precision as measured by the symmetric mean absolute percentage error (SMAPE). We implement a backtesting procedure considering a common five-year forecasting horizon for all models and populations. Secondly, we compute the posterior probability for each model using the normalised exponential (Softmax) function using:

$$\pi(M_k|\,\mathbf{y}) = \frac{\exp(-|\mathcal{S}_k|)}{\sum_{l=1}^{K} \exp(-|\mathcal{S}_l|)}, \quad k = 1, ..., K, \tag{22}$$

with $\mathcal{S}_k = \psi_k / \max\{\psi_l\}_{l=1,...,K}$ and $\psi_k = SMAPE$ for model k and population g. The Softmax function is derived from the logistic function, commonly adopted in forecasting, regression, and classification exercises considering traditional or statistical learning (for example, machine learning, deep learning) methods as a combiner or an activation function. The function possesses a desirable characteristic in that it assigns larger weights to models with smaller out-of-sample forecasting error, with weights following an exponential distribution.[9]

Model-averaged Bayesian credible intervals are derived using the MATA construction (Turek

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[9] Alternative choices for the posterior probability allocation include the normalized C-probability, the natural odds-based probability, the extreme C-probability, the normalized extreme C-probability, and the Sigmoid function.

and Fletcher 2012). The method consists of estimating confidence limits such that the weighted sum of error rates, computed using the BME posterior probability $\pi(M_k|\mathbf{y})$, produces the required overall error rate.

2.2.2 Candidate stochastic mortality models

The empirical strategy adopted in this study requires the selection of a subset of stochastic mortality models to be part of the model combination. The set of individual single population heterogeneous stochastic mortality models considered in this study comprises a selection of well-known and commonly used GAPC parametric models, principal component methods, and smoothing approaches. Table 1 recapitulates the analytical structure of the nine individual candidate models considered in this study; additional technical details are provided in the appendix A for completeness. The set comprises: (i) Six single-population GAPC models (LC, APC, RH, CBD, M7, Plat); (ii) A univariate functional demographic time-series model: the weighted Hyndman and Ullah (2007) Functional Demographic Model considering geometrically decaying weights (HUw); (iii) A bivariate functional data model: the Regularized Singular Value Decomposition (RSVD) model (Huang et al. 2009; Zhang et al. 2013); (iv) A two-dimensional smooth constrained P-splines model (CPspl), which imposes smoothness in mortality rates across years and ages (Camarda 2019).

The first six models are well-known GAPC models: [LC] is the age-period Lee-Carter model under a Poisson setting for the number of deaths (Brouhns et al. 2002; Renshaw and Haberman 2003); [APC] is the age-period-cohort model (Currie 2006); [RH] is the Lee-Carter model extended to include cohort effects and particular substructure obtained by setting $\beta_x^{(0)} = 1$ and an additional approximate identifiability constraint (Renshaw and Haberman 2006; Haberman and Renshaw 2011); [CBD] is the Cairns-Blake-Dowd model considering a predictor structure with two age-period terms, prespecified age-modulating parameters $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = (x - \bar{x})$, with $\bar{x}$ the average age in the data, and no cohort effects (Cairns et al. 2006); [M7] is the CBD model with cohort effects and a quadratic age effect (Cairns et al. 2009); [Plat] is the Plat (2009) model with particular substructure obtained by setting $\kappa_t^{(3)} = 0$ to focus only on older ages.

Since some of the GAPC models described in Table 1 are particular cases of larger models,[10] trimming models and determining a model confidence set (Hansen et al. 2011) may lead to better estimates of each model's posterior probabilities in the BME forecast. We use a fixed-rule trimming scheme in which three out of the six GAPC candidates are discarded. The set

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[10] For instance, model LC is nested within model RH, with $\beta_x^{(0)} = 0$ for all x, and $\gamma_{t-x} = 0$ for all c, being a special case of APC with $\beta_x^{(1)} = 1$ for all x and no cohort effects. Model APC is a special case of RH with $\beta_x^{(1)} = \beta_x^{(0)} = 1$ for all x. The CBD model is a restricted version of M7 with $\kappa_t^{(3)} = 0$ for all t and $\gamma_{t-x} = 0$ for all c.

Table 1: Analytical structure of the stochastic mortality models used in this study

Model	Model structure
LC	$\eta_{x,t} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)}$
APC	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + \gamma_{t-x}$
RH	$\eta_{x,t} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} + \beta_x^{(0)} \gamma_{t-x}$
CBD	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x})\, \kappa_t^{(2)}$
M7	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x})\, \kappa_t^{(2)} + \left((x - \bar{x})^2 - \sigma_x^2\right) \kappa_t^{(3)} + \gamma_{t-x}$
Plat	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + (x - \bar{x})\, \kappa_t^{(2)} + (\bar{x} - x)^+ \kappa_t^{(3)} + \gamma_{t-x}$
HUw	$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\, \epsilon_{t,i}$
CPspl	$\eta = \mathbb{B}\alpha$
RSVD	$m(x,t) = \sum_{j=1}^{q} d_j U_j(t)\, V_j(x) + \epsilon(x,t)$

Note: $\eta_{x,t}$ denotes the linear predictor; α_x and $\beta_x^{(i)}$ denote age-specific terms; $\kappa_t^{(i)}$ and γ_{t-x} are period and cohort indices; σ_x^2 is the mean of $(x - \bar{x})^2$; $y_t(x_i) = log(m_{x_i,t})$; $f_t(x_i)$ is a continuous and smooth function; $\sigma_t(x_i)$ is a volatility term; $\epsilon_{t,i}$ and $\epsilon(x,t)$ are error terms; $\mathbb{B}$ are B-spline bases with a roughness penalty; α is a vector of parameters.

of best models is determined based on the forecasting precision in the validation (test) period. Individual models are first calibrated using total population data from 1960 to the most recent year available. Since our focus is to discuss the implications of life expectancy improvements on retirement age policies, models are calibrated using an age range of $60 - 95$. Prediction intervals for age-specific mortality rates considering both stochastic process and parameter risk are derived using a bootstrap approach with 5000 bootstrap samples (Brouhns et al. 2005; Koissi et al. 2006). The Denuit and Goderniaux (2005) life table closing method with ultimate age set at $\omega = 125$ is assumed for all years, countries, and populations to ensure comparable and comprehensive cross-country results. The model fitting, forecasting, and simulation procedures were implemented using an R statistical software routine.

2.2.3 Life expectancy measures

Equipped with forecasts of age-specific mortality rates by year and sex for each population g, $m_{x,g}(t)$, the (complete) cohort and period life expectancy measures for an x-year old individual in year t are given, respectively, by:

$$\dot{e}_{x,g}^{C}(t) := \frac{1}{2} + \sum_{k=1}^{\omega-x} \exp\left(-\sum_{j=0}^{k-1} m_{x+j,g}(t+j)\right), \tag{23}$$

and by

$$\dot{e}^{P}_{x,g}(t) := \frac{1}{2} + \sum_{k=1}^{\omega-x} \exp\left(-\sum_{j=0}^{k-1} m_{x+j,g}\left(t\right)\right), \tag{24}$$

with ω denoting the highest attainable age, from which the concept of life expectancy gap at age x in year t, $\dot{e}^{Gap}_{x,g}\left(t\right)$, defined as the systematic difference between period and cohort life expectancy measures (Ayuso et al. 2021a) can be easily computed as $\dot{e}^{Gap}_{x,g}\left(t\right) := \dot{e}^{C}_{x,g}\left(t\right) - \dot{e}^{P}_{x,g}\left(t\right)$.

2.3 Mortality and pension age data

The datasets used in this study comprise mortality data and full pension age data. Mortality data are obtained from the Human Mortality Database (2021) and consist of observed death counts, $D_{x,t}$, and exposure-to-risk, $E_{x,t}$, classified by age at death ($x = 60, ..., 110+$), year of death ($t = 1960, ..., 2018$) and sex for 23 homogeneous national populations (countries or areas) in different regions of world. Table 2 lists the countries considered in this study together with details about data availability in the defined historical "lookback window", set from 1960 (or the most distant year available) to 2018 (or the most recent year available).

Table 2: Selected HMD countries and available data period used.

Available data	Countries and Regions
1960 – 2016	Australia (AUS), Canada (CAN), Denmark (DNK), Iceland (ISL), Netherlands (NDL), Poland (POL), Spain (ESP), England and Wales (ENW),
1960 – 2017	Austria (AUT), France (FRA), Ireland (IRL), Japan (JPN), Slovakia (SVK), Sweden (SWE), Switzerland (CHE), U.S.A. (USA)
1960 – 2018	Belgium (BEL), Finland (FIN), Norway (NOR)
1992 – 2008	Chile (CHL)
1990 – 2017	Germany (DEU)
1983 – 2016	Israel (ISR)
1960 – 2015	Portugal (PRT)

The pension age data include actual and forecasted standard retirement age by sex from 2000 to 2050 for 23 countries. The full (or normal) pension age considered in this paper is the age at which a worker can take his or her public pension without any decrement for early retirement. For countries where a gender gap in standard retirement ages still exists, the male pension age is used as the benchmark. As of 2021, significant differences persist in the male pension age between countries and, in some cases, between genders, with retirement age ranging between 62 years (France) and 67 years (Norway, Iceland, Israel). In those countries in which the pension age is different for men and women, women have a lower retirement age.

Our approach to gender differences in pension age is consistent with current trends toward harmonisation of legislated normal pension ages between genders.

In EU and OECD Member States, the most general normal pension age is still 65 years. Since 2000, 13 of the 23 countries studied in this paper have increased their full normal pension age, either by (i) introducing automatic indexation to life expectancy (Denmark, Estonia, Greece, Italy, the Netherlands, Portugal, Slovakia, Finland, Cyprus) with diverse policy goals, or (ii) adopting scheduled or ad hoc interventions. Some reform reversals occurred, for instance in Canada, Poland, and Slovakia. Canada planned to increase the age for the basic and means-tested pensions to 67 but finally decided against it. Poland reversed its planned increase to 67, dropping retirement ages back to previous levels (65 for men and 60 for women). The largest progression of the normal retirement age over the period 2008–2060 is projected in Denmark and the Netherlands, but a significant dispersion of pension ages is projected to persist in the long run (Carone et al. 2016; Ayuso et al. 2021b).

All countries have early retirement pathways (for example, in conjunction with very long contribution careers, long-term unemployment, or sickness insurance schemes for older workers), usually causing a reduction in pension benefits. In some countries (Sweden, Norway, Finland) people can retire flexibly; i.e., they can take out a full or partial old-age pension within a certain age range (for example, currently between 62 and 68 years in Sweden). However, access to resource-tested schemes (for example, minimum or guaranteed pensions) is restricted to those of a certain minimum age (65 in Sweden, rising to 66 in 2023). Following OECD guidelines, this age is used as the pensionable age herein. Variations in the pension age are observed between and within countries. For instance, in some countries (Australia), differences arise between the minimum public pension (age pension) and the retirement age of mandatory private schemes (superannuation), and different early retirement schemes may coexist.

3 Empirical results

3.1 Forecasts of the retirement age

Figure 1 exhibits the BME point forecast of the cohort life expectancy at age 65 for the total population from 1960 to 2050 by country, along with the 95 percent MATA prediction intervals accounting for both (i) the uncertainty arising from the error in the forecast of the individual stochastic mortality model parameters, and (ii) the parameter uncertainty resulting from model fitting. We forecast for all countries a continuation of the long-term positive trend in cohort life expectancy, with Japan, France, and Switzerland leading the list in 2050 with 28.28, 26.90, and 26.34 years of expected remaining lifetime at age 65, respectively. We forecast that the total population cohort life expectancy at age 65 will increase by 47 percent in Japan, 44 percent in England and Wales, 42 percent in Finland, 38 percent in Australia, and 29 percent in the United States. If full pension age is selected as the policy instrument for correcting the distortion introduced by life expectancy developments on intergenerational fairness, retirement age must increase to restore the equilibrium condition.

Figure 2 plots the actual and legislated full pension ages by country from 2000 to 2050, together with the point forecasts of the retirement age under both CAR and CRR policy options in the baseline scenario; the baseline assumes that the lifetime earnings indexing rate, the scheme's internal rate of return, the life annuity guaranteed interest rate, and the pension annual indexation rate are all equal to 2 percent (i.e., $\upsilon_t = y_t = \pi_t = 2\% \ \forall t$).[11] The year 2000 is selected as the starting point for our analysis since it marks the beginning of the most recent wave of pension reforms addressing the impact of population ageing and life expectancy increases in OECD countries after nearly a half century of constant pension ages. Forecasts of the legislated pension age in countries following an automatic indexation mechanism to period life expectancy were derived using forecasts of the period life expectancy at the reference age and the formula stated in each country's national pension law.[12]

For all countries except Belgium (and partially Germany and Slovakia), which started from comparatively (much) lower retirement ages in 2000, the results for both the CAR and CRR retirement age policies show that the actual (2000–2021) and legislated retirement age increases have been and will be insufficient to cope with populations' extended survival prospects and to preserve the intergenerational fairness and neutrality conditions. The difference between the intergenerationally fair retirement ages and the actual/legislated retirement ones is, as expected, higher under a CAR policy option than under a CRR policy alternative, with gaps accumulating over time in both cases (Table 3). For instance, in 2020 the cross-country average difference between actual pension ages and those required to deal with cohort life expectancy improvements at labour market exit ages observed since 2000 is 1.59 years; the highest gaps are in Finland (3.55 years), Denmark (3.16 years), Chile (2.77 years), and Japan (2.63 years). Under a CAR policy option these gaps are forecasted to increase to a cross-country average difference of 3.92 years in 2050; the highest corrections will be required in Japan (6.63 years), Finland (6.03 years), and Chile (5.99 years). The lowest values (discarding Belgium and Slovakia) are in the Netherlands (0.69 years), Denmark (2.30 years), and Portugal (2.54 years), countries that introduced automatic indexation of retirement ages but pursued alternative

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[11] The empirical results for other parameter combinations confirm the discussion in the section 2.1 and are available upon request.

[12] For instance, the formula stated in the Dutch pension law can be rewritten as:

$$x_r^{NLD}(t) = 65 + \left[\dot{e}_{65}^{P}\,(t) - 18.26\right],$$

whereas in Denmark it can be expressed as

$$x_r^{DNK}(t) = 60 + \left[\dot{e}_{60}^{P}\,(t-15) - 14.5\right]$$

with both countries targeting a constant period in retirement (see Ayuso et al. 2021b for details).

Figure 1: Forecasts of the total population cohort life expectancy at age 65, along with 95% prediction intervals

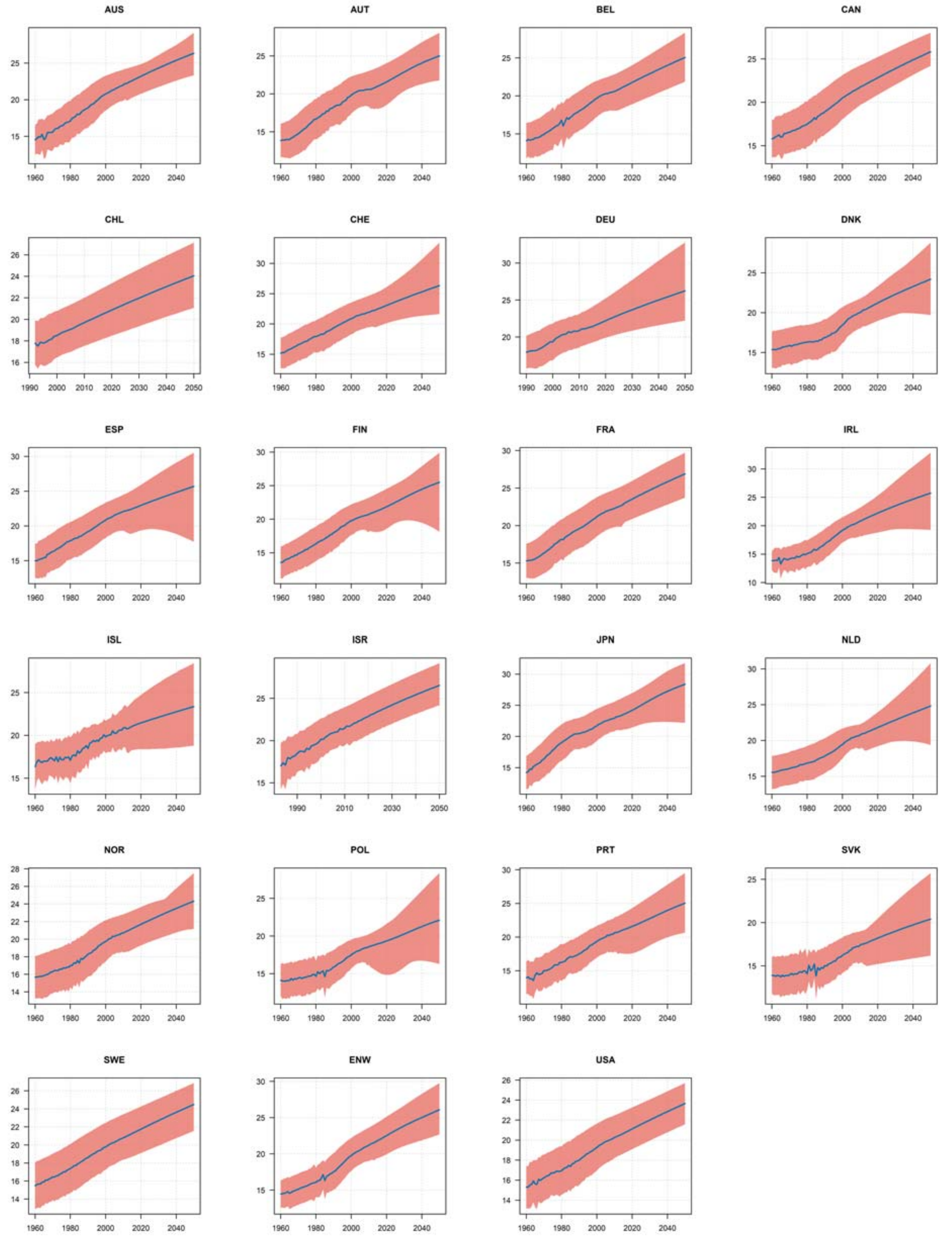
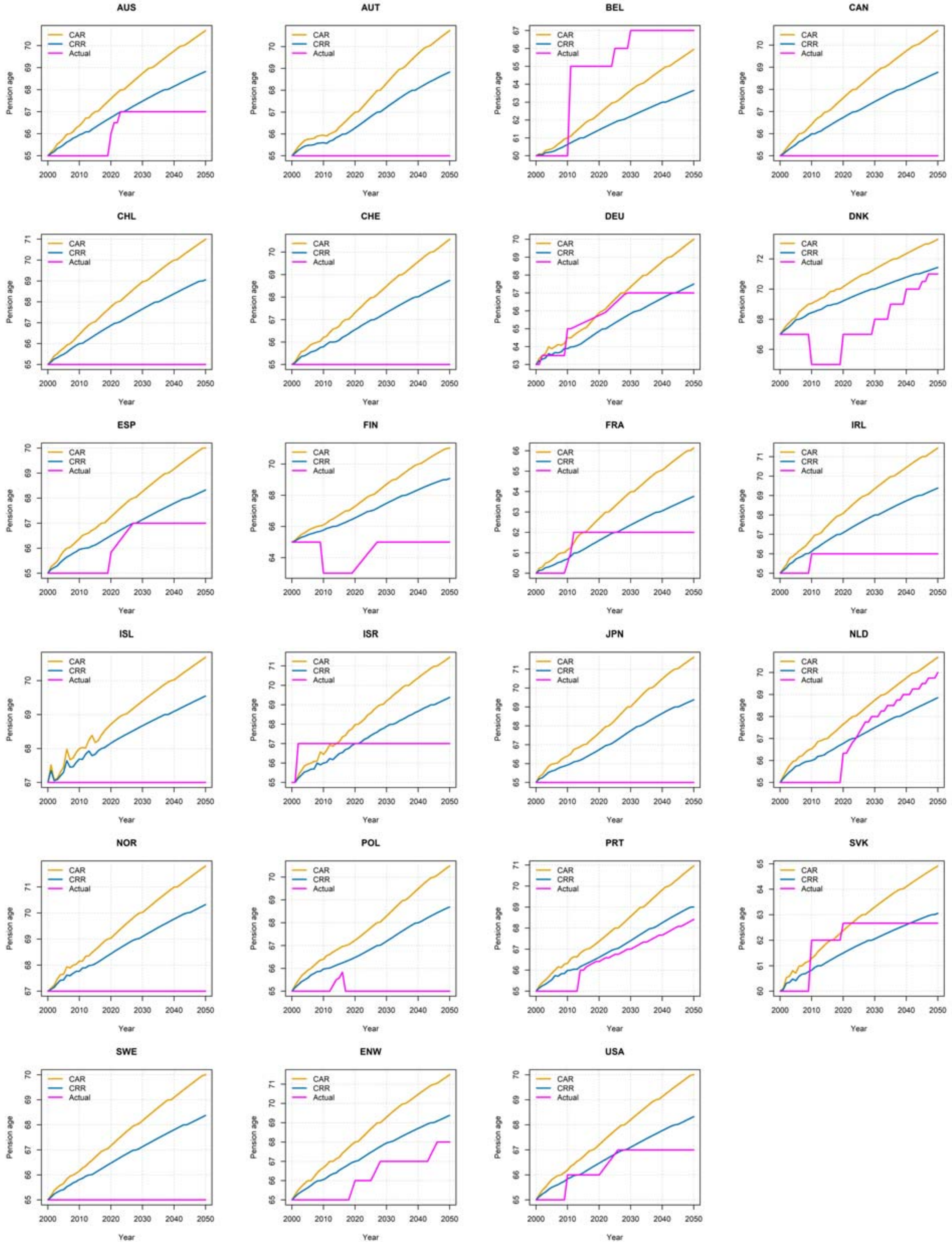


Figure 2: Forecasts of the retirement age dictated by CAR and CRR policy options



retirement age approaches.

Table 3: Difference between actual and CAR/CRR policy retirement ages

	2010		2020		2030		2040		2050	
Country	CAR	CRR	CAR	CRR	CAR	CRR	CAR	CRR	CAR	CRR
AUS	1.36	0.95	1.59	0.74	1.74	0.48	2.76	1.15	3.68	1.83
AUT	0.95	0.60	2.00	1.27	3.33	2.22	4.68	3.09	5.73	3.84
BEL	1.00	0.64	-2.65	-3.55	-3.35	-4.79	-2.15	-4.00	-1.05	-3.34
CAN	1.41	0.99	2.62	1.73	3.73	2.45	4.74	3.10	5.66	3.77
CHL	1.45	1.00	2.77	1.88	3.96	2.66	5.00	3.39	5.99	4.06
CHE	1.21	0.79	2.32	1.56	3.49	2.32	4.58	3.02	5.57	3.74
DEU	-0.50	-1.11	0.15	-0.88	0.34	-1.15	1.75	-0.29	3.00	0.50
DNK	4.05	3.44	3.16	2.22	3.33	2.01	2.36	0.79	2.30	0.44
ESP	1.38	0.95	1.42	0.67	1.26	0.14	2.17	0.79	3.00	1.32
FIN	3.10	2.77	3.55	2.80	3.71	2.49	5.00	3.36	6.03	4.08
FRA	1.16	0.70	0.69	-0.39	2.00	0.37	3.06	1.06	4.14	1.77
IRL	0.76	0.13	2.09	1.09	3.35	2.00	4.45	2.73	5.45	3.39
ISL	1.00	0.69	1.72	1.16	2.40	1.67	3.02	2.09	3.69	2.55
ISR	-0.55	-1.00	0.98	0.00	2.21	0.89	3.38	1.67	4.45	2.38
JPN	1.40	0.93	2.63	1.74	4.00	2.74	5.49	3.66	6.63	4.38
NLD	1.53	1.00	1.32	0.44	0.74	-0.50	0.75	-0.84	0.69	-1.15
NOR	1.15	0.77	2.04	1.44	3.02	2.10	3.99	2.77	4.80	3.32
POL	1.40	0.97	2.23	1.48	3.29	2.19	4.49	3.00	5.48	3.69
PRT	1.33	0.99	0.96	0.20	1.56	0.41	2.16	0.59	2.54	0.59
SVK	-0.71	-1.18	-0.29	-1.17	-0.70	-1.94	0.11	-1.39	0.91	-0.94
SWE	1.15	0.82	2.17	1.47	3.17	2.13	4.10	2.80	5.00	3.38
ENW	1.67	1.05	2.00	1.00	2.29	0.95	3.47	1.71	3.49	1.38
USA	0.19	-0.16	1.21	0.47	1.20	0.11	2.13	0.77	3.00	1.33
Max	4.05	3.44	3.55	2.80	4.00	2.74	5.49	3.66	6.63	4.38
Min	-0.71	-1.18	-2.65	-3.55	-3.35	-4.79	-2.15	-4.00	-1.05	-3.34
Average	1.17	0.73	1.59	0.76	2.18	0.95	3.11	1.52	3.92	2.01

Notes: Difference in years between the forecasted pension age under both a constant accrual-rate-per-year (CAR) and constant replacement rate (CRR) policy options for selected years from 2010 to 2050. Positive (negative) values mean the CAR and/or CRR fair retirement ages are higher (lower) than those implemented and/or legislated. Baseline scenario assuming $v_t = y_t = \pi_t = 2\% \ \forall t$.

The results obtained for the Netherlands and Denmark are particularly interesting to analyse since both countries introduced automatic indexation of pension ages by adopting a retirement age policy that explicitly targets a constant period in retirement, an outcome demonstrated in section 2.1.1 is consistent with the CAR policy option. Our results show, however, that in both countries the actual/legislated pension age increases are well below what will be required to preserve intergenerational fairness, particularly in Denmark. In both countries, this is explained essentially by poor policy design, particularly (i) the use of an incorrect (period) life expectancy measure instead of a cohort estimate (the life expectancy gap) in the indexation formula, and (ii) the existence of extra provisions capping the maximum increase in the pension age per period, indexation lags, and other design features that affect the final policy outcome (see Ayuso et al. 2021b for details). The results for Germany show that the scheduled pension increases follow roughly a CAR retirement age policy until 2029 when the ongoing updating path ceases. The results for the United States and Spain roughly approximate a CRR retirement age policy up to 2026, but further corrections will be required from that year on to cope with forecasted longevity improvements.

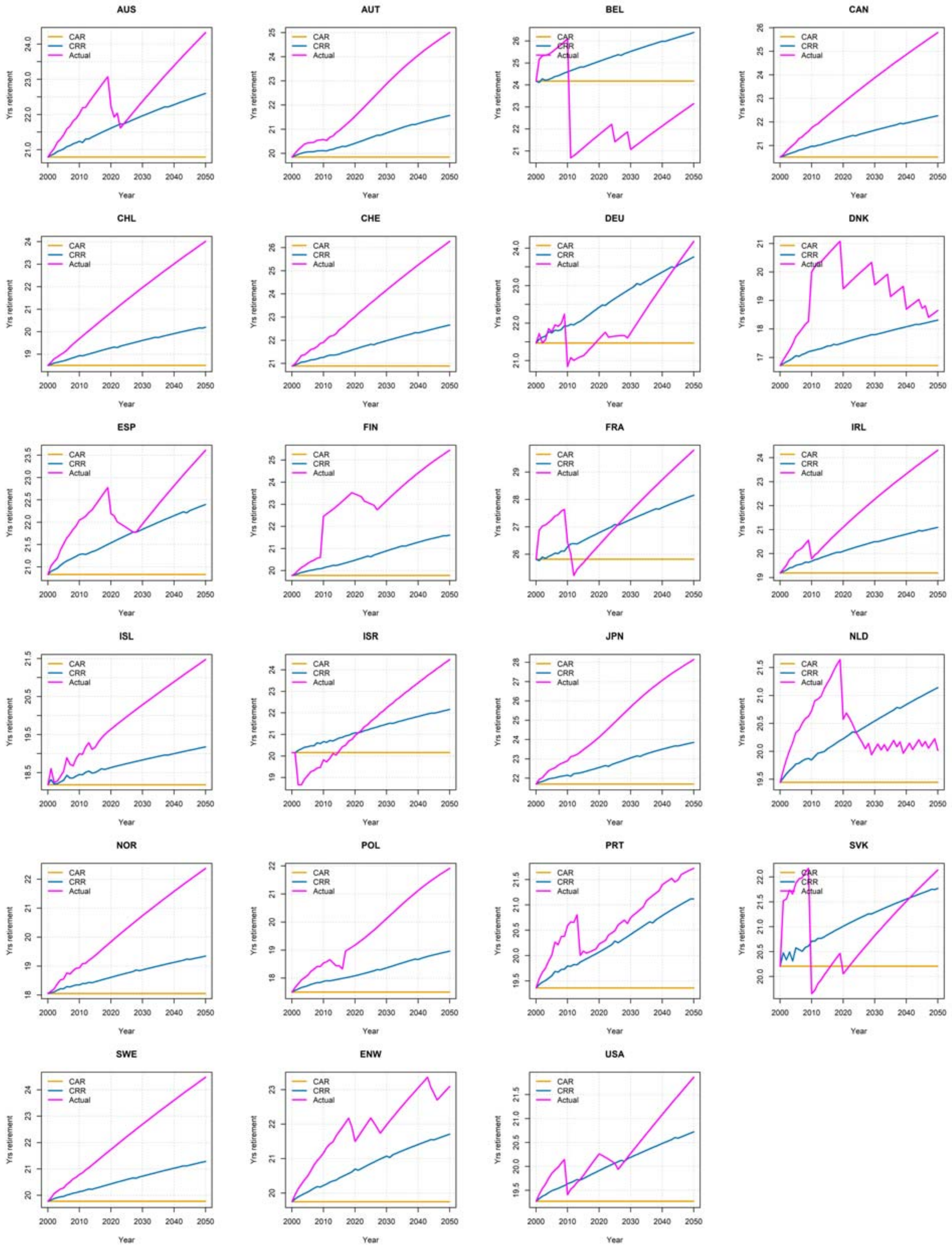
The difference between the corrections dictated by the CRR policy to match intergenerational actuarial balance constraint and those implemented is smaller but still significant. For instance, in 2050 the average cross-country difference between actual/legislated pension ages and those required to deal with cohort life expectancy improvements and to keep up with intergenerational fairness is 2.01 years; the highest gaps are in Japan (4.38 years), Finland (4.08 years), and Chile (4.06 years), with 10 countries requiring an increase in the retirement age of at least 3 years. By 2050, the average cross-country difference between the retirement age corrections required by the CAR and CRR policy options is 1.91 years, with values ranging between 1.14 and 2.50 years.

3.2 Expected duration of retirement

Figure 3 summarises the forecasts of the expected duration of retirement – the cohort life expectancy at the pensionable age – dictated by the CAR and CRR policies from 2000 to 2050, along with the expected years in retirement under the current/legislated retirement age path pursued by each of the 23 countries analysed in this study. Recall that by construction the expected years in retirement dictated by the CAR retirement age policy are constant and equal to those observed in the initial year, set to 2000 for all countries.

Our empirical results show that, first, despite the important retirement age increases legislated in many OECD countries in the last two decades, the expected duration of retirement is forecast to increase in all countries analysed in this study, except in Belgium for the reasons mentioned above. In 2000, the average expected duration of retirement in the 23 countries analysed was 20.08 years, with values ranging between 16.71 years in Denmark and 25.82 years in France. In 2020, despite major pension reforms adopted in 15 out of the 23 countries, the average expected duration of retirement increased to 21.50 years, with France again leading

Figure 3: Forecasts of the expected years in retirement dictated by actual, CAR and CRR retirement policies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the cohort life expectancy at the pensionable age (26.34 years for the total population). We forecast that the positive trend in average duration of retirement will continue in the future, reaching 23.75 years in 2050, with a maximum of 29.80 years in France and 28.14 years in Japan (Table 4).

Table 4: Expected duration of retirement under the legislated and CRR retirement age policies

	2000	2020		2030		2040		2050	
Country	Legis	Legis	CRR	Legis	CRR	Legis	CRR	Legis	CRR
AUS	20.79	22.23	21.61	22.37	21.96	23.39	22.28	24.33	22.60
AUT	19.84	21.53	20.42	22.88	20.84	24.06	21.23	25.00	21.57
BEL	24.18	21.75	25.08	21.07	25.53	22.14	25.99	23.15	26.39
CAN	20.51	22.82	21.32	23.88	21.65	24.87	21.95	25.80	22.27
CHL	18.50	20.85	19.28	21.96	19.61	23.02	19.92	24.01	20.20
CHE	20.89	23.02	21.62	24.17	21.98	25.25	22.32	26.27	22.66
DEU	21.47	21.58	22.42	21.73	22.92	23.00	23.36	24.18	23.77
DNK	16.71	19.41	17.51	19.55	17.80	18.70	18.08	18.65	18.31
ESP	20.83	22.19	21.54	21.96	21.84	22.82	22.15	23.61	22.40
FIN	19.78	23.47	20.48	23.16	20.89	24.42	21.29	25.44	21.61
FRA	25.82	26.34	26.79	27.57	27.27	28.72	27.71	29.80	28.16
IRL	19.19	21.10	20.10	22.27	20.50	23.33	20.81	24.31	21.10
ISL	18.18	19.64	18.63	20.28	18.83	20.89	19.00	21.47	19.18
ISR	20.16	20.95	21.07	22.23	21.48	23.40	21.84	24.48	22.16
JPN	21.70	24.13	22.55	25.69	23.05	27.07	23.50	28.14	23.85
NLD	19.44	20.57	20.23	20.04	20.54	20.06	20.84	20.02	21.14
NOR	18.05	19.84	18.61	20.74	18.87	21.58	19.13	22.37	19.35
POL	17.50	19.18	18.09	20.12	18.37	21.12	18.66	21.91	18.96
PRT	19.36	20.23	20.07	20.75	20.42	21.40	20.79	21.72	21.12
SVK	20.21	20.06	21.01	20.82	21.29	21.51	21.55	22.14	21.77
SWE	19.77	21.74	20.43	22.69	20.73	23.61	21.03	24.48	21.29
ENW	19.75	21.50	20.70	21.98	21.07	23.06	21.41	23.09	21.71
USA	19.27	20.26	19.91	20.27	20.19	21.08	20.48	21.86	20.72
Max	25.82	26.34	26.79	27.57	27.27	28.72	27.71	29.80	28.16
Min	16.71	19.18	17.51	19.55	17.80	18.70	18.08	18.65	18.31
Average	20.08	21.50	20.85	22.10	21.20	22.98	21.54	23.75	21.84

Notes: Baseline scenario assuming $\upsilon_t = y_t = \pi_t = 2\% \ \forall t$. By construction, the expected years in retirement dictated by the CAR policy are constant and equal to those observed in 2000.

In relative terms, the largest increases in expected duration of retirement are forecast for Chile (+29.8 percent or 5.51 years), Japan (+29.7 percent or +6.45 years), and Finland (+28.6

percent, or +5.67 years). Figure 3 also shows that the only country in which the expected duration of retirement is forecast to roughly stabilise around 20 years is the Netherlands, above the 18.26 targeted by the legislated retirement age policy linking full pension age to life expectancy.

Second, we conclude that adoption of a CRR retirement age policy would contribute to reducing the expected period in retirement by 1.91 years in 2050 when compared with legislated reforms. The results also show, however, that the increase in pension ages dictated by the CRR policy falls short of what will be needed to prevent a rise in the expected retirement duration and, in many cases, will not prevent the decline in the relative size of the labour force.

Figure 4 summarises for all countries the forecasted expected duration of retirement relative to contribution years under the actual/legislated CAR and CRR retirement age policies. Recall that by construction the CRR retirement age policy sets the pension age such that the ratio between expected years in retirement and contribution years is kept constant over time and equal, for each country, to the percentage observed in 2000. Assuming a fixed labour market entry age, set at age 22 in this study, a similar graph can be derived for the relationship between the expected duration of retirement and adult life.

With the exception of Belgium and the Netherlands, the results show that the expected period in retirement relative to the contribution period is forecast to increase in all countries despite recent and legislated rises in standard pension ages. Substantial variations arise in the ratio between retirement and contribution periods among the countries analysed in this study. In 2000, the average cross-country ratio was 47.5 percent, with national values ranging between 37.1 percent in Denmark and 67.9 percent in France. The average cross-country ratio between retirement and contribution periods is forecast to increase to 53.9 percent in 2050, with France peaking at 74.5 percent (Table 5). In Japan, the ratio is forecast to increase 15 percentage points from 50.5 percent in 2000 to 65.5 percent in 2050, the largest percentage increase among the countries analysed. We highlight in particular the impact of pension reform reversals on the expected length of the retirement period in Poland and Slovakia, stopping and inverting earlier declines that had been phased in or legislated.

The empirical results also show that adoption of a CAR retirement age policy to address intergenerational fairness and to cope with life expectancy developments significantly contributes to reducing the proportion of the expected period in retirement relative to contribution years (minus 5.7 percentage points), from an average cross-country ratio of 47.5 percent in 2000 to 41.8 percent in 2050. The reduction is much higher when compared to the 2050 projected ratio under legislated reforms (41.8 percent in 2050 versus 53.9 percent). For instance, keeping all other pension parameters constant, adoption of a CAR retirement age policy in France would

Figure 4: Forecasts of the expected period in retirement relative to contribution years under actual, CAR and CRR retirement age policies

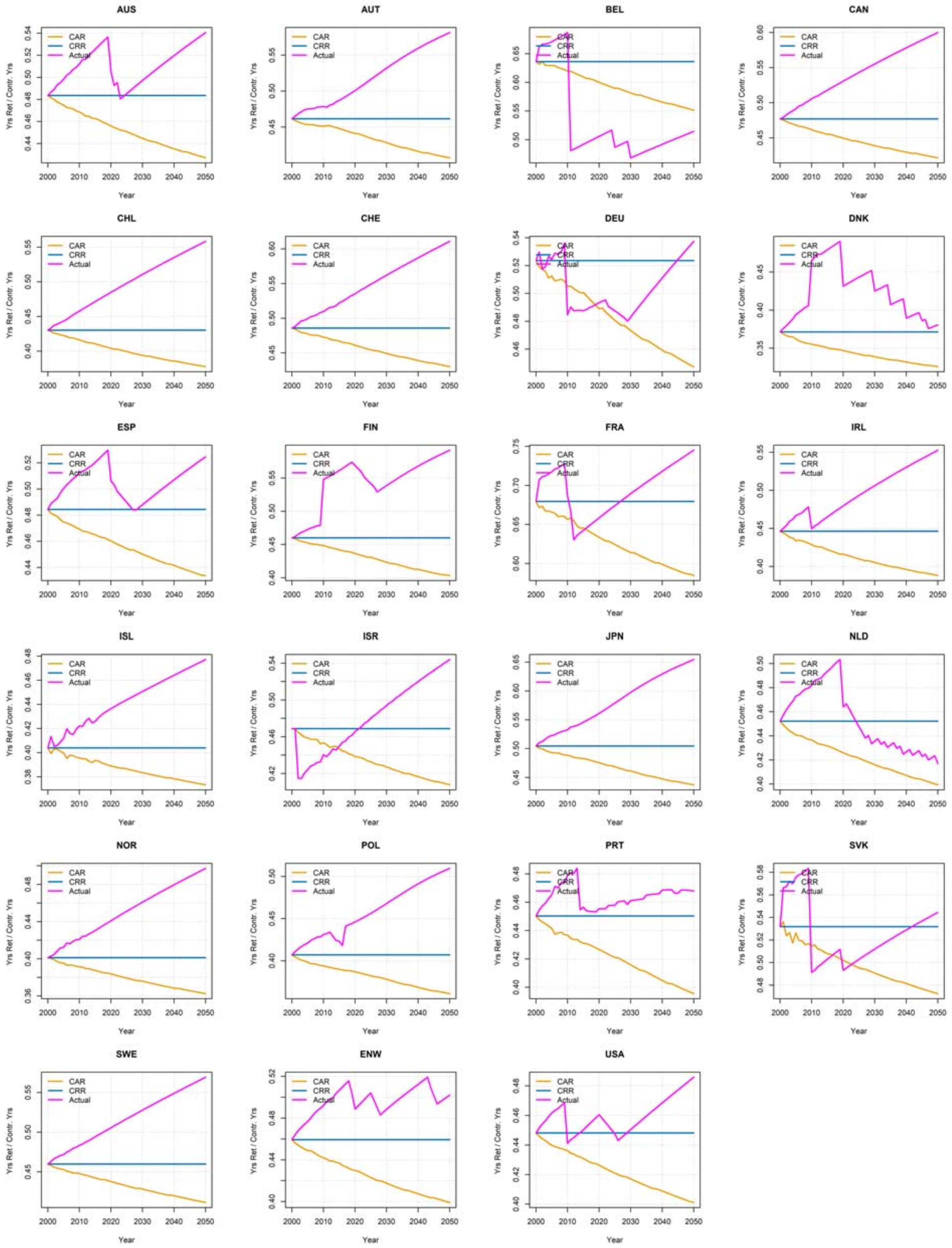

Table 5: Expected duration of retirement relative to contribution years under legislated and CAR retirement age policies (in %)

	2000	2020		2030		2040		2050	
Country	Legis	Legis	CAR	Legis	CAR	Legis	CAR	Legis	CAR
AUS	0.484	0.505	0.456	0.497	0.445	0.520	0.435	0.541	0.427
AUT	0.461	0.501	0.441	0.532	0.428	0.560	0.416	0.581	0.407
BEL	0.636	0.506	0.600	0.468	0.582	0.492	0.566	0.514	0.552
CAN	0.477	0.531	0.450	0.555	0.439	0.578	0.430	0.600	0.422
CHL	0.430	0.485	0.404	0.511	0.394	0.535	0.386	0.558	0.378
CHE	0.486	0.535	0.461	0.562	0.449	0.587	0.439	0.611	0.430
DEU	0.524	0.493	0.489	0.483	0.473	0.511	0.459	0.537	0.447
DNK	0.371	0.431	0.347	0.425	0.339	0.389	0.332	0.381	0.326
ESP	0.484	0.506	0.460	0.488	0.450	0.507	0.442	0.525	0.434
FIN	0.460	0.569	0.437	0.539	0.423	0.568	0.413	0.592	0.403
FRA	0.679	0.658	0.634	0.689	0.614	0.718	0.599	0.745	0.585
IRL	0.446	0.480	0.416	0.506	0.405	0.530	0.396	0.553	0.388
ISL	0.404	0.437	0.389	0.451	0.384	0.464	0.378	0.477	0.373
ISR	0.469	0.466	0.438	0.494	0.427	0.520	0.417	0.544	0.408
JPN	0.505	0.561	0.476	0.597	0.462	0.630	0.447	0.655	0.437
NLD	0.452	0.464	0.426	0.436	0.416	0.427	0.407	0.417	0.399
NOR	0.401	0.441	0.384	0.461	0.376	0.480	0.368	0.497	0.362
POL	0.407	0.446	0.387	0.468	0.378	0.491	0.369	0.510	0.361
PRT	0.450	0.455	0.427	0.461	0.416	0.469	0.405	0.468	0.395
SVK	0.532	0.493	0.502	0.512	0.491	0.529	0.481	0.544	0.472
SWE	0.460	0.506	0.438	0.528	0.428	0.549	0.420	0.569	0.411
ENW	0.459	0.489	0.430	0.488	0.418	0.512	0.407	0.502	0.399
USA	0.448	0.461	0.426	0.450	0.417	0.469	0.409	0.486	0.401
Max	0.679	0.658	0.634	0.689	0.614	0.718	0.599	0.745	0.585
Min	0.371	0.431	0.347	0.425	0.339	0.389	0.332	0.381	0.326
Average	0.475	0.496	0.449	0.504	0.437	0.523	0.427	0.539	0.418

Notes: Baseline scenario assuming $\upsilon_t = y_t = \pi_t = 2\% \; \forall t$. Values in percentage.

be sufficient to bring down the fraction of contribution years relative to years in retirement by 9.5 percentage points.

4 Discussion and policy implications

The goal of indexing a country's normal retirement age and pension benefits to life expectancy at retirement age is primarily to mitigate the impact of continuous improvements in life expectancy on financial sustainability and, in universal public pension schemes, targeting intergenerational fairness. With these goals in mind, some countries also introduce automatic stabiliser rules to cushion the system from adverse demographic and/or economic events. An overriding goal is to reinforce the credibility and consistency of pension promises made to younger generations, on which the fulfilment and stability of the intergenerational social contract ultimately reside. Nonetheless, this paper has shown that the way pensions have been linked to longevity markers is not free from conceptual and policy design flaws. The use of inappropriate life expectancy measures, poor policy design, and lack of consistency with the criteria of intergenerational fairness, together with potential adverse effects of policies pursued on intragenerational redistribution, are still too prevalent.

To compare countries' policy design regarding how each country's treatment of life expectancy fulfils the criteria of a good universal pension system, this paper began by giving all countries the same scenario: an earnings-related pension scheme characterised by full proportionality between contributions on earnings benefits paid out. This enabled us to show how key pension parameters (retirement age, contribution rate, accrual rate) must adapt above all to the changing life expectancy of the pension-age population to ensure that the scheme remains actuarially fair and is neutral across generations. Then, considering the normal retirement age as the key policy instrument and automatic stabiliser, we showed how to index pension age to life expectancy developments while respecting the principles of intergenerational actuarial fairness and neutrality among generations. Last, we analysed country outcomes empirically based on their current data and policy design vis-á-vis life expectancy projections.

Our analysis employed two design regimes that, generally speaking, encompass all universal public pension schemes. We showed that under a CAR policy design in which extended working lives translate into additional pension entitlements, pension age must be continuously updated to keep the period in retirement constant. This roughly corresponds to the strategy adopted in the Netherlands and Denmark to link pension age to life expectancy, although both countries chose a period-based longevity measure that is generally known to systematically underestimate life expectancy when improvement in mortality is occurring at an accelerating rate, a general trend seen in developed economies during recent decades (Alho et al. 2013).

Alternatively, if policymakers wish to pursue a fixed replacement rate (CRR) objective, in which a longer contribution period barely changes pension entitlements, we showed that retirement ages must be updated to ensure the relation between number of years spent in work and retirement remains constant over time. The results also showed that the pension age increases prescribed by a CRR policy design are smaller than those dictated by a CAR option.

The paper adopted a model combination (BME) approach to forecasting intergenerationally actuarially fair pension ages for 23 countries and regions from 2000 to 2050. The results led us to conclude that the pension age increases required to fully accommodate the impact of longevity increases on financial equilibrium and to maintain equity between generations are substantial and well beyond those recently observed and/or legislated. With few exceptions (notably Belgium, due to its comparatively lower starting point), the results for both the CAR and CRR policy designs showed that actual and legislated retirement age increases are not sufficient to ensure intergenerational fairness and neutrality among generations, in conjunction with the continuously increasing survival prospects of elderly populations.

As a result, the expected duration of retirement (both in absolute terms and relative to the contribution period) is projected to grow in the future. The differences between actual/legislated retirement ages and retirement ages consistent with intergenerational fairness and neutrality are higher under a CAR policy option than under a CRR design, with gaps steadily accumulating over time in both cases. The results showed that adoption of a CAR (CRR) retirement age policy would contribute to reducing the average cross-country expected period in retirement by 3.92 (2.01) years in 2050 when compared with legislated reforms. Here, the results obtained for the Netherlands and Denmark deserve a special mention because of the confluence of several inadequate policy design features (for example, the projection procedure employed for the indexation variable, provisions capping pension age increases, and indexation lags) in the final policy outcomes.

Indexing the pension age and/or adjusting the length of working lives and consequently career contributory requirements to longevity developments can square pension sustainability and pension adequacy in a scenario with population ageing and later labour market entry ages (adjusting to longer periods of education), rebalancing the number of years spent in work and retirement. Generally speaking, extending working lives to accommodate longer lives is preferable to reducing pension levels through the so-called sustainability factors or life expectancy coefficients adopted in some countries (Finland, Portugal, Spain); at the time of annuitising these operate by decreasing the benefit ratio and generate increasing old-age (absolute and relative) poverty risks. Moreover, the sustainability factor design tends not to recognise the value of long contribution careers and does not provide for minimum adequacy safeguards, which are critical for those at the lower end of the income and pension wealth (i.e., accumulated pension savings) distribution.

Automatically indexing the normal retirement age to life expectancy and rewarding later retirement encourages individuals to postpone the actual retirement age, which has a positive effect on labour force participation rates. However, higher statutory retirement ages risk not fulfilling the goal of translating into higher effective retirement ages if not accompanied by complementary policies. This includes policy to incentivise lifecycle employment mobility in all possible contexts, where upskilling in the present employment environment and reskilling into a new employment environment are the general modus operandi; these should be accompanied by pension policy that makes partial retirement a neutral option, accommodating a gradual transition from work to retirement. And generally speaking, and in the spirit of Gruber and Wise (1998, 2004), government policies should not inadvertently tax decisions of older workers to remain in the labour force.

In conclusion, it is important to bring to the forefront the issue of socioeconomic longevity heterogeneity. The general presumption of both economic theory and the present rationale behind pension policy is that participants in a pension scheme enter into retirement with a random distribution of longevity outcomes among all new retirees. Yet abundant evidence shows that the distribution of longevity at retirement differs in countries considerably with respect to level of education, occupation, and lifetime income as well as gender, and women generally have considerably longer lives but lower lifetime income than men. Considerable evidence demonstrates that lifestyle factors (for example, smoking, eating habits, alcohol consumption, drug abuse, insufficient physical activity, and poor hygiene, including dental care) are significant determinants of early mortality, as are community-based factors such as the absence of adequate public health, material deprivation, and poor housing and environmental conditions (those affecting water and air quality). These factors call for policy reforms to improve these conditions but also pension reform design measures to neutralise their effects as much as possible.

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Appendix A Stochastic mortality models: technical description

This section draws heavily on Bravo et al. (2021a) and recapitulates the key technical details of the individual stochastic mortality models considered in the Bayesian Model Ensemble approach.

Appendix A.1 GAPC stochastic mortality models

Generalised Age-Period-Cohort (GAPC) mortality models are a class of parametric models that link a response variable with a linear or bilinear predictor structure consisting of a series of factors dependent on age of the individual, x; period effects, t; and year of birth (or cohort) effects, $c = t - x$. The structure of GAPC models includes a random component, a systematic component, a (canonical) link function, a set of parameter constraints to ensure identifiability and time series methods for forecasting and simulating the period and cohort indexes (Hunt and Blake 2021). The random component specifies whether the number of deaths recorded at age x during calendar year t, $D_{x,t}$, follows a Poisson distribution $D_{x,t} \sim \mathcal{P}\left(\mu_{x,t} E^c_{x,t}\right)$, with $\mathbb{E}\left(D_{x,t}/E^c_{x,t}\right) = \mu_{x,t}$, or a Binomial distribution $D_{x,t} \sim \mathcal{B}\left(q_{x,t} E^0_{x,t}\right)$, with $\mathbb{E}\left(D_{x,t}/E^0_{x,t}\right) = q_{x,t}$, where $E^0_{x,t}$ and $E^c_{x,t}$ denote, respectively, the population initially or centrally exposed-to-risk, and $q_{x,t}$ is the one-year death probability for an individual aged x last birthday in year t. The systematic component links a response variable to an appropriate linear predictor $\eta_{x,t}$

$$\eta_{x,t} = \alpha_x + \sum_{i=1}^{N} \beta^{(i)}_x \kappa^{(i)}_t + \beta^{(0)}_x \gamma_{t-x}, \tag{A.1}$$

where $\exp\left(\alpha_x\right)$ denotes the general shape of the mortality schedule across age, $\beta^{(i)}_x \kappa^{(i)}_t$ is a set of N age-period terms describing the mortality trends, with each time index $\kappa^{(i)}_t$ contributing in specifying the general mortality trend and $\beta^{(i)}_x$ modulating its effect across ages, and the term $\gamma_{t-x} \equiv \gamma_c$ accounts for the cohort effect c with $\beta^{(0)}_x$ modulating its effect across ages. The age modulating coefficients $\beta^{(i)}_x$ can be preset or nonparametric terms to be estimated. Parameter estimates are obtained using maximum-likelihood methods. The period $\kappa^{(i)}_t$ and the cohort γ_{t-x} indices are treated as stochastic processes and modelled with general univariate ARIMA(p, d, q) methods to generate forecasts of age-specific mortality rates or probabilities. The model specification is complemented with a set of parameter constraints to ensure unique parameter estimates.

Appendix A.2 Weighted Hyndman-Ullah method

The Hyndman and Ullah (2007) method combines functional principal component analysis (PCA) with nonparametric penalised regression splines. Assume that the logarithm of the observed mortality rate at age $x \in [x_1, x_p]$ in year $t \in [t_1, t_n]$, $\log m_{x_i,t} \equiv y_t(x_i)$ is a realization of an underlying continuous and smooth function $f_t(x_i)$ that is observed with error at discrete ages:

$$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\,\varepsilon_{t,i},\ \ i = 1, .., p \quad t = 1, ..., n, \tag{A.2}$$

where $\sigma_t(x_i)$ allows the amount of noise to vary with x_i in year t, thus rectifying the assumption of homoscedastic error in the LC model, and $\varepsilon_{t,i}$ is an independent and identically distributed standard normal random variable. The log mortality rates are smoothed prior to modelling using penalized regression splines with a partial monotonic constraint. Using functional PCA, the smoothed mortality curves $\mathcal{I} = \{y_1(x), .., y_n(x)\}$ are then decomposed into orthogonal functional principal components and their uncorrelated principal component scores. The original Hyndman-Ullah (HU) method was extended by Shang et al. (2011) using geometrically decaying weights (instead of equal weights) in the estimation of the model parameters. Formally,

$$f_t(x) = \hat{a}^*(x) + \sum_{j=1}^{J} b_j^*(x)\,k_{t,j} + e_t(x), \tag{A.3}$$

where $\hat{a}^*(x)$ is the weighted functional mean age function estimated by:

$$\hat{a}^*(x) = \frac{1}{n}\sum_{j=1}^{J} w_t f_t(x), \quad \sum_{j=1}^{J} w_t = 1, \tag{A.4}$$

where $\left\{w_t = \pi(1-\pi)^{n-t},\ t = 1, ..., n\right\}$ denotes a set of weights, and $\pi \in (0,1)$ refers to the geometrically decaying weight parameter, with the optimal value chosen so as to minimise an overall forecast error measure within the validation data; $\mathcal{B}^* = \left\{b_j^*(x)\right\}\ j = 1, ..., J$ is a set of weighted first J functional principal components with uncorrelated principal component scores $\{k_{t,j}\}$ derived by functional PCA from the set of weighted curves $\{w_t[f_t(x) - \hat{a}^*(x)];\ t = 1, ..., n\}$; $e_t(x)$ is the residual function with mean zero and variance $\upsilon(x)$ estimated by averaging $\{e_1^2(x), ..., e_n^2(x)\}$, $e_t(x) \sim \mathcal{N}(0, \upsilon(x))$; and $J < n$ is the number of principal components used.

Appendix A.3 CP-Splines model

Camarda's (2019) CP-spline model extends the two-dimensional P-splines model by incorporating demographic constraints to ensure that future mortality over the whole age range follows a plausible and well-behaved demographic profile when estimated from past data. Consider a mortality dataset comprising deaths and exposure-to-risk arranged in two $m \times n$ matrices, $\boldsymbol{Y} = (d_{ij})$ and $\boldsymbol{E} = (E_{ij})$, respectively, with rows and columns classified by single age at death (x, $m \times 1$) and single year of death (t, $n \times 1$), respectively. The approach assumes that the number of deaths d_{ij} at age i in year j is Poisson-distributed with mean $\mu_{ij}E_{ij}$, i.e., $d_{ij} \sim \mathcal{P}\left(\mu_{ij}E_{ij}\right)$. The goal is to model and forecast mortality over both age and time combining (fixed knot) B-splines with a roughness penalty to achieve a compromise between fitting accuracy and smoothness. Let $\boldsymbol{B}_x$, $m \times k_x$ and $\boldsymbol{B}_t$, $n \times k_t$ be the B-splines over ages and years, respectively. The log mortality is described as a linear combination of B-splines and associated coefficients ($\boldsymbol{\alpha}$):

$$\ln[\mathbb{E}(\boldsymbol{Y})] = \ln(\boldsymbol{E}) + \boldsymbol{B}\boldsymbol{\alpha} \tag{A.5}$$

where $\ln(\boldsymbol{E})$ is the offset and $\boldsymbol{\eta} = \boldsymbol{B}\boldsymbol{\alpha}$ is the linear predictor. The regression matrix for the two-dimensional model is given by the Kronecker product of the k equally spaced B-splines bases for age x and year t, $\boldsymbol{B} = \boldsymbol{B}_t \otimes \boldsymbol{B}_x$,where $\otimes$ denotes the Kronecker product of two matrices. The two-dimensional penalty is given by

$$\boldsymbol{P} = \lambda_x \left(\boldsymbol{I}_{k_t} \otimes \boldsymbol{D}_x^{'}\boldsymbol{D}_x\right) + \lambda_t \left(\boldsymbol{D}_t^{'}\boldsymbol{D}_t \otimes \boldsymbol{I}_{k_x}\right), \tag{A.6}$$

where λ_x and λ_t are the smoothing parameters used for age and year, respectively; $\boldsymbol{I}_{k_x}$ and $\boldsymbol{I}_{k_t}$ are identity matrices of dimension k_x and k_t, respectively; and $\boldsymbol{D}_x$ and $\boldsymbol{D}_t$ are difference matrices over the rows (ages) and columns (years) of the coefficient matrix. The model includes shape constraints and asymmetric penalties on the rate of aging (relative derivatives of the age mortality profile), $\boldsymbol{D}_x^t$, and on the rate of change of mortality rates over time, $\boldsymbol{D}_t^t$, to enforce mortality patterns over age and time.

Appendix A.4 Regularized SVD model

Huang et al. (2009) and Zhang et al. (2013) extend one-way functional PCA to two-way functional data by introducing regularisation of both left and right singular vectors in the singular value decomposition (SVD) of the data matrix. The authors assume the regularized SVD (RSVD) fits the following model for explaining the mortality rate in terms of period t and age x

$$m\left(x,t\right) = \sum_{j=1}^{q} d_j U_j\left(t\right) V_j\left(x\right) + \varepsilon\left(x,t\right), \tag{A.7}$$

where d_q is the singular value, $U_i\left(\cdot\right)$ and $V_j\left(\cdot\right)$ are smooth functions of period and age, respectively, and $\varepsilon\left(x,t\right)$ is a mean zero random noise. The model is fitted iteratively. The first pair of singular vectors of a data matrix $\boldsymbol{X} = \left(m_{x,t}\right)_{n\times p}$, $U_1\left(t\right)$ and $V_1\left(x\right)$, whose discretised realisations are, respectively, denoted as $\boldsymbol{u}_1 \equiv (U_1(t_1), ..., U_1(t_n))^T$ and $\boldsymbol{v}_1 \equiv (V_1(x_1), ..., V_1(x_p))^T$, is obtained by solving a least squares problem as

$$(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) = \underset{(\boldsymbol{u},\boldsymbol{v})}{\arg\min} \left\| \boldsymbol{X} - \boldsymbol{u}\boldsymbol{v}^T \right\|_F^2, \tag{A.8}$$

where $\|\cdot\|_F$ is the Frobenius norm (sometimes called the Euclidean norm) of a matrix. Subsequent pairs are extracted sequentially by removing the effect of preceding pairs. For two-way functional data, the RSVD of Huang et al. (2009) defines the regularised singular vectors as

$$(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) = \underset{(\boldsymbol{u},\boldsymbol{v})}{\arg\min} \left\{ \left\| \boldsymbol{X} - \boldsymbol{u}\boldsymbol{v}^T \right\|_F^2 + \mathcal{P}_\lambda(\boldsymbol{u}, \boldsymbol{v}) \right\}, \tag{A.9}$$

where $\mathcal{P}_\lambda(\boldsymbol{u}, \boldsymbol{v})$ is a regularisation penalty

$$\mathcal{P}_\lambda(\boldsymbol{u}, \boldsymbol{v}) = \lambda_{\boldsymbol{u}} \boldsymbol{u}^T \boldsymbol{\Omega}_u \, \boldsymbol{u} \cdot \|\boldsymbol{v}\|^{\mathbf{2}} + \lambda_{\boldsymbol{v}} \boldsymbol{v}^T \boldsymbol{\Omega}_v v \cdot \|\boldsymbol{u}\|^{\mathbf{2}} + \lambda_{\boldsymbol{u}} \boldsymbol{u}^T \boldsymbol{\Omega}_u \boldsymbol{u} \cdot \lambda_{\boldsymbol{v}} \boldsymbol{v}^T \boldsymbol{\Omega}_v v, \tag{A.10}$$

whereby $\boldsymbol{\Omega}_u$ $(n \times n)$ and $\boldsymbol{\Omega}_v$ $(p \times p)$ are symmetric and nonnegative definite domain-specific penalty matrices, whose purpose is to balance goodness-of-fit against smoothness; λ is a vector of regularization parameters optimally estimated based on generalized cross-validation (GCV) criterion. To forecast mortality rates and derive confidence intervals, the time functions $U_i(t)$ are treated as time series and modelled using general univariate ARIMA processes, rescaling the pairs in (A.7) by the ratio d_i/d_1, $i = 2, ..., q$.