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Advantageous Smallness in Contests

Abstract

A standard result in contests is that a higher-ability player has a higher probability of winning the prize than a lower-ability player. Put differently, a stronger player has an advantage over a weaker player in a contest. There are very few exceptions to this standard result. I consider a model in which being “too big” is only a necessary condition for an insolvent firm to receive a government bailout because, in addition to meeting a threshold asset size, the firm must engage in a lobbying contest in order to be bailed out. The firm has an advantage because its probability of winning the contest is increasing in its size. When the firm experiences an unfavorable price shock, I find that the balance between the size of the requisite bailout and the firm’s political advantage of being “too big” determines the firm's probability of getting a bailout. Surprisingly, I find that a smaller firm may receive a bailout while a bigger firm will not, although the firm’s (political) advantage is increasing in its size. This result is weakened but not overturned if the firms are uncertain about the threshold size for being considered too big. The paper’s main result will not hold in a contest with independent valuations. In the bailout contest, the players have interdependent valuations.

JEL-Codes: O100, P160, P480.

Keywords: insolvency, bail-out, biased contests, political advantage, too-big.

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1. Introduction

A contest is a game in which players expend irreversible efforts to win a prize. A standard result in contests is that a higher-ability player has a higher probability of winning the prize than a lower-ability player. Put differently, a stronger player has an advantage over a weaker player in a contest. There are very few exceptions to this standard result.

Perhaps, the first paper to demonstrate that this standard result may not hold was Baye, Kovenock, and de Vries (1993) who showed that a contest-designer may maximize aggregate effort in a contest by excluding high-ability contestants. The intuition is that the exclusion of high-ability contestants levels the playing field and increases aggregate effort. Therefore, the stronger but excluded contestants have a zero probability of winning the contest, although stronger players who remain or participate in the contest have a higher success probability than weaker players. But being a weaker player may increase the probability of being included in the contest. In a contest with sabotage, higher-ability players may have a lower probability of winning because they be the target of sabotage than lower-ability players and thus the standard result may not hold (see Munster, 2007; Chowdhury and Gurtler, 2015). To the best of my knowledge, Baye et al (1993) and Munster (2007) were the first papers to show that the standard result may not hold.[1]

In this paper, I build a model that also generates a result contrary to the standard result in the sense that being small in a contest may be advantageous. In my model, size (being big) is a necessary condition for an insolvent firm to receive government support. But in addition to meeting a threshold size, the firm must engage in a lobbying contest against an

[1]There is a literature in contests that studies affirmative-action type policies under which it may be optimal to handicap stronger players in contests to level the playing field. All these papers are, in effect, based on the original insight of Baye et al (1993). For a survey, see Chowdhury (2020). They do not generate the result that a weaker player has a higher probability than a stronger player of winning a contest.

anti-bailout group in order to receive support. The firm has a political advantage because its probability of winning the bailout contest is increasing in its size. I find that when there is an unfavorable price shock, a bigger firm may have a smaller probability of being bailed out than a smaller firm although a firm's political advantage in the bailout contest is increasing in its size. This counter-intuitive result will not hold in a contest with independent valuations. In the bailout contest, the players have interdependent valuations. The intuition for this non-standard result is that a bigger firm may require a bigger bailout and, because of the players are interdependent, the bigger firm will attract more lobbying from the anti-bailout group.

The firm's size in the bailout contest is an endogenous variable. A firm does not choose a smaller size in order to avoid intense lobbying from the anti-bailout group because it chooses its size to maximize its expected payoff, not to maximize its probability of success in the bailout contest. Ability, captured by a player's valuation or marginal cost effort, is exogenous in Baye et al (1993) and Munster (2007). The size of the firm which attracts intense lobbying is endogenous in the bailout contest. However, in my model just as in Baye et al (1993), the advantage of smaller contestants does not stem from them competing directly against bigger or stronger contestants. In my model, two firms of different sizes compete separately against the **same** anti-bailout group. But the smaller firm may have a higher probability of winning its bailout than the corresponding success probability of the bigger firm. Later, I explain why this set-up is realistic.

I discuss alternative explanations of the aforementioned result of the differential bailout probabilities of big and small firms and show that my model proves this result under a different assumption that is based on the size of the firm. The point of this paper is not to argue that the bailout of smaller firm is more common than the bailout of bigger firms. Instead, the point is to construct a model that can explain the counter-intuitive result that, in some cases, a weaker or smaller contestant may have a higher success probability in a contest.

The paper is organized as follows: in the next section, I study a model of a too-big firm that faces political opposition and discuss my results. I conclude the paper in section 3.

2. A bailout contest with interdependent and endogenous valuations

Consider a three-period model of a contest with risk-neutral players. There is a firm with a production function, $y = 2\sqrt{k}$, where k is the firm's input in period 1. Normalizing the price of the input to 1, the firm's total cost is $k + F$, where k is the variable cost and F is the fixed cost. The price, p, of the firm's output, y, is a random variable that is continuously distributed on $[\underline{p}, \overline{p}]$ with pdf, $h(p) > 0$, where $\overline{p} > \underline{p} \geq 0$. Output is produced in period 1 but the realization of the price and income occur in period 2. In period 2, conditional on the realization of the price of its output, the firm's profit is $py - k - F$. In period 3, there is a known price resulting in a known profit of $\varphi > 0$.

In this simple model, the firm's total asset is $py = 2p\sqrt{k}$ and its total liability is $k + F$. I use k to represent the size of the firm because $2p\sqrt{k}$ is increasing in k. In this finite-period model, the firm is insolvent if it cannot pay its liabilities in period 2 (e.g., Stiglitz, 1969; Bulow and Shoven, 1978; Hellwig, 1981). Therefore, the firm is insolvent if $\pi = py - k - F < 0$. Otherwise, the firm is solvent. In period 2, I assume that the firm cannot borrow (in the private market) against its profit in period 3. Thus, even if the firm was solvent in the sense that its profit in period 3 was sufficient to cover any losses in period 2, it will still have a liquidity problem in period 2 that cannot be solved without a government bailout.

If the government bails out the firm, it gives the firm an amount equal to $-\pi = F + k - py \equiv R(k) > 0$. However, an anti-bailout group can lobby the government, so that the firm does not receive this bailout.[2] Let e_f and e_{ng} be the efforts of the firm and the anti-bailout group in the lobbying game respectively. Suppose that the firm and the anti-bailout group win the bailout contest with the following probabilities[3] (respectively):

$$r_f = \frac{e_f+g(k)}{e_f+g(k)+e_{ng}} \text{ and } r_{ng} = \frac{e_{ng}}{e_f+g(k)+e_{ng}}, \tag{1}$$

where $g(k)$ is strictly increasing in k.

For the firm to a receive a bailout, a necessary but not sufficient condition is that it must be sufficiently big. I capture this as $k \geq k_{min} > 0$. Thus, the requirement $k \geq k_{min}$ means that only sufficiently big firms may be bailed out.[4] However, consistent with the fact that a government may not bail out a big but insolvent firm, the probability in (1) is such that the firm may lose the bailout contest. But notice that, given $g'(k) > 0$, the firm's probability, r_f, of getting government assistance, in the event of being insolvent, is increasing k. This is the firm's political advantage of being too big. To elaborate, the term, $g(k)$, in the contest

[2] For example, Mishkin (2006, p. 997) observed that "Even if the market expects bank bailouts, there is some probability that the bailout will not occur …" Blau et al (2013), Dam and Koetter (2012), Faccio et al. (2006), Grossman and Woll (2014), and Sorkin (2009) document lobbying in bailouts. And expressions such as "corporate welfare" and "wall street versus main street" are indications of public backlash against bailouts. On a CNBC program in March 2009 after the US government bailed out Bear Stearns, the host Matt Lauer asked Henry Paulson, who was then Secretary of the Treasury, "Does the Fed react more strongly to what's happening on Wall Street than they do to what's happening to people in pain across the country, the so-called people who live on Main Street?" (Sorkin, 2009, p. 32).

[3] This contest success function was used in Amegashie (2006). Its microfoundation was studied in Rai and Sarin (2009). Variants of it have been used in Esteban and Ray (2008, 2011). Hao, Skaperdas, and Vaidya (2013) present a survey of the literature on contest success functions.

[4] Using data from the merger boom of 1991 to 2004, Brewer and Jagtiani (2013) found that banks were willing to pay an extra premium of at least \$15 billion in order to have asset sizes that are commonly viewed as thresholds for being big and thus qualify for bailouts in the event of negative shocks.

success function (1) is a head start for the firm in the contest or implies a biased contest in the sense of, for example, Konrad (2002), Kirkegaard (2012), and Segev and Sela (2014). Even if $e_f = 0$, the firm's probability of getting a bailout is $r_f > 0$ so long as $k \geq k_{min}$, which implies $g(k) > 0$. And if the anti-bailout group does not lobby (i.e., $e_{ng} = 0$), then the firm will definitely be bailed out if it becomes insolvent so long as $k \geq k_{min}$. The $g(k)$ function is non-negative and it is positive if and only if $k \geq k_{min}$.

I assume that if the form is liquidated, its owners get a payoff of zero (see, for example, Bulow and Shoven, 1978; Hellwig, 1981; White, 1989) and that there are no principal-agent issues. The shareholders have an interest in getting a bailout because if the firm exists in period 3, they a get a profit of $\varphi > 0$. The managers and employees of the firm also have an interest in the firm being bailed out either because they will be employed in period 3, the firm will be able to pay their salaries in period 2, and the expected value of their stock options may be higher in period 3. Finally, the firm's creditors and suppliers want the firm to be bailed out because they supplied loans and materials in period 1 for which they require payment in period 2. In fact, the shareholders want the firm's creditors and suppliers to receive any bailout money due them because otherwise the creditors and suppliers can force the liquidation of the firms in order to recoup some part of their loans to the firm (e.g., Bulow and Shoven, 1978). All these groups or a subset of them may represent a coalition of common interests[5] (e.g., Bulow and Shoven, 1978). I assume that they have solved the collective-action problem, so there is no free-riding. I refer to this group as the *pro-firm coalition* or simply as the firm.

[5]For example, when in March 2020, Doug Parker, chairman and CEO of American Airlines Group, Inc., the parent company of American Airlines, sent a letter asking for a bailout to Treasury Secretary Steve Mnuchin and the leadership of the House and Senate of the USA, the letter was also signed by many labor union leaders of the airline industry.

The firm's creditors, managers, employees, and suppliers want the firm to be bailed out because they supplied loans, labor services, and materials in period 1 for which they require payment in period 2. Therefore, the costs they (i.e., the firm's creditors, managers, and suppliers) incurred in period 1 are sunk costs in period 2. If there is no bailout in period 2, their payoff is zero. But if they get a bailout, then there is an *actual* transfer of $R(k)$ to them, so their valuation in the contest is $R(k)$. If the anti-bailout group is successful in the contest, there is no bailout to the firm, so I assume that the anti-bailout group gets a payoff of zero. If the firm is solvent, there is no need for a bailout.

The timing of actions is as follows:

(a) In period 1, the firm chooses its input/size, k.

(b) In period 2, the price of output, p, is realized which determines whether the firm is solvent or not. If the firm is insolvent, the amount, $R(k)$, required for a bailout is common knowledge. And if $k \geq k_{min}$, there is a bailout contest as described above. If the firm is solvent, there is no contest or bailout.

(c) If the firm proceeds to period, 3, it gets a payoff of $\varphi > 0$.

I look for a subgame perfect Nash equilibrium of this game by backward induction. Period 3 is trivial. Thus, I start from period 2, where F, k, and p are known parameters. Suppose the firm is insolvent. Conditional on $k \geq k_{min}$ and $R(k) \equiv F + k - py > 0$, the payoffs in period 2 may be written as:

$$\Omega_f = \frac{e_f + g(k)}{e_f + g(k) + e_{ng}}(R(k) + \varphi) + \frac{e_{ng}}{e_f + g(k) + e_{ng}}(0) - e_f \qquad (2)[6]$$

[6]One may argue that if the pro-firm coalition wins the bailout contest, the payoff (to creditors and managers) should be 0 because it the creditors and managers (owed salaries and benefits) do not suffer any losses. However, if the bailout is not received, their payoff would be – $R(k)$ as they would incur losses. Thus,

and

$$\Omega_{ng} = \frac{e_{ng}+g(k)}{e_f+g(k)+e_{ng}}(0) - \frac{e_f+g(k)}{e_f+g(k)+e_{ng}}R(k) - e_{ng} \tag{3}$$

Given that k is chosen in period 1 and does not affect φ and φ is a constant, it turns out that setting $\varphi = 0$ is without loss of generality because it does not affect the qualitative results of this paper nor does it result in the loss of any insights. Thus, I set $\varphi = 0$. Besides, we needed $\varphi > 0$ to justify why the firm's shareholders will participate in the bailout contest. But even if $\varphi = 0$, the firm's managers, creditors, and suppliers have the incentive to participate in the bailout contest because they want the firm to pay their salaries, loans, and supplies in period 2.

It is easy to show that, for a given set of parameters, the unique Nash equilibrium in period 2 is given by:

$$\begin{cases} e_f^* = \frac{R(k)}{4} - g(k) \\ e_{ng}^* = \frac{R(k)}{4} \end{cases}, \text{ if } \ 0 \leq g(k) < \frac{R(k)}{4}, \tag{4a}$$

or

$$\begin{cases} e_f^* = 0 \\ e_{ng}^* = 0 \end{cases}, \text{ if } \ g(k) \geq R(k), \tag{4b}$$

or

$\Omega_f = \ r_f(0) - (1 - r_f)R(k) - e_f = -R(k) + r_f R(k) - e_f$. Comparing this payoff to (2) and noting that $R(k)$ is independent of e_f and e_{ng}, it is obvious that this interpretation of the firm's payoff will not affect the equilibrium efforts in the contest. It will, however, affect the choice of k in period 1. But it does not affect the paper's main result.

$$\begin{cases} e_f^* = 0 \\ e_{ng}^* = \sqrt{R(k)g(k)} - g(k) \end{cases}, \text{ if } \frac{R(k)}{4} \leq g(k) < R(k). \qquad (4c)$$

Relative to the requisite bailout amount the equilibria in (4a), (4b), and (4c) represent low, high, and intermediate values respectively of the firm's political advantage, $g(k)$, of being big. Note that if $g(k) = 0$ for all k, then the only equilibrium is the equilibrium in (4a) and if $g(k) = \infty$ for all $k \geq k_{min}$, then (4b) is the only equilibrium. Of course, I have ruled out these extreme cases.

The firm's probability of getting a bailout is $r_f^* = \frac{R(k)/4}{R(k)/2} = 0.5$ in the equilibrium in (4a), $r_f^* = \frac{g(k)}{g(k)} = 1$ in the equilibrium in (4b), and $0.5 \leq r_f^* = \sqrt{\frac{g(k)}{R(k)}} < 1$ in the equilibrium in (4c), where $r_f^* = 0.5$ if and only if $g(k) = \frac{R(k)}{4}$.

The firm's equilibrium payoff in period 2 is:

$$\Omega_f^* = \frac{R(k)}{4} + g(k), \quad \text{if} \quad 0 \leq g(k) < \frac{R(k)}{4}, \qquad (5a)$$

or

$$\Omega_f^* = R(k), \qquad if \quad g(k) \geq R(k), \qquad (5b)$$

or

$$\Omega_f^* = \sqrt{R(k)g(k)}, \quad \text{if} \quad \frac{R(k)}{4} \leq g(k) < R(k). \qquad (5c)$$

Now consider period 1. For a given draw (realization) of p, the firm is solvent if $py - k - F \geq 0$. That is, $p \geq \frac{k+F}{2\sqrt{k}} \equiv \hat{p}(k)$. Otherwise, if $p < \frac{k+F}{2\sqrt{k}} \equiv \hat{p}(k)$, the firm is insolvent. There are two cases to consider. In the first case (hereafter case A), we have $0 \leq k < k_{min}$ and so the firm's expected payoff is:

$$\Pi_f^A(k) = \int_{\hat{p}(k)}^{\bar{p}} \left(2p\sqrt{k} - k - F\right) h(p)dp + \int_{\underline{p}}^{\hat{p}(k)} (0)h(p)dp. \tag{6a}$$

In case A, the firm does not qualify for a bailout because $k < k_{min}$.

Consider case B, where $k \geq k_{min}$. If the firm is in a bailout contest, then the equilibrium is (4a) or (4b) or (4c). The equilibrium in (4a) requires $g(k) < \frac{R(k)}{4}$. This gives $p < \frac{k+F}{2\sqrt{k}} - \frac{2g(k)}{\sqrt{k}} \equiv \hat{p}_b(k) < \hat{p}(k)$. Then, using (5a), the firm's expected payoff in period 1, conditional on being insolvent, is:

$$\Pi_f^{4a}(k) = \int_{\underline{p}}^{\hat{p}_b(k)} \left(\frac{F+k-2p\sqrt{k}}{4} + g(k)\right) h(p)dp. \tag{6b}$$

Now consider the equilibrium in (4b). It requires that $g(k) \geq R(k)$. This can be rewritten as $p \geq \frac{k+F}{2\sqrt{k}} - \frac{g(k)}{2\sqrt{k}} \equiv \hat{p}_c(k)$. Combining this with $p < \frac{k+F}{2\sqrt{k}}$ gives $\frac{k+F}{2\sqrt{k}} - \frac{g(k)}{2\sqrt{k}} \leq p < \frac{k+F}{2\sqrt{k}}$ or $\hat{p}_c(k) \leq p < \hat{p}(k)$. Using (5b), the firm's expected payoff in period 1, conditional being insolvent, is:

$$\Pi_f^{4b}(k) = \int_{\hat{p}_c(k)}^{\hat{p}(k)} \left(F + k - 2p\sqrt{k}\right) h(p)dp. \tag{6c}$$

And finally, using $\frac{R(k)}{4} \leq g(k) < R(k)$ in (4c) and the payoff in (5c), we have:

$$\Pi_f^{4c}(k) = \int_{\hat{p}_b(k)}^{\hat{p}_c(k)} \sqrt{R(k)g(k)}\, h(p)dp. \tag{6d}$$

Thus, in case B where $k \geq k_{min}$, the firm's expected payoff in period 1is:

$$\Pi_f^B(k) = \int_{\hat{p}(k)}^{\bar{p}} \left(2p\sqrt{k} - k - F\right) h(p) dp + \Pi_f^{4a}(k) + \Pi_f^{4b}(k) + \Pi_f^{4c}(k). \quad (7)$$

Thus, the firm's problem is to choose k to maximize the expected payoffs in cases A and B and choose the input level that corresponds to the case with the higher expected payoff.[7] Let the optimal levels in cases A and B be k_a^* and k_b^* respectively.

In both cases A and B, we require $\hat{p}(k) \equiv \frac{k+F}{2\sqrt{k}} < \bar{p}$, $j \in \{a, b\}$. This gives:

$$k_j^2 + (2F - 4\bar{p}^2)k_j + F^2 < 0, \quad (8)$$

$j \in \{a, b\}$.

In case B, we require $\frac{k_b+F}{2\sqrt{k_b}} - \frac{2g(k_b)}{\sqrt{k_b}} \equiv \hat{p}_b(k_b) > \underline{p}$. This gives:

$$F + k_b - 4g(k_b) - 2\underline{p}\sqrt{k_b} > 0. \quad (9)$$

Note that if $\hat{p}_b(k_b) > \underline{p}$, then $\hat{p}(k) > \underline{p}$.

2.1 Numerical examples

Using Liebniz rule gives $\frac{\partial^2 \Pi_f^A(k)}{\partial k^2} = -\frac{1}{2k^{1.5}} \int_{\hat{p}(k)}^{\bar{p}} ph(p)dp + \frac{1}{8}\left(1 - \frac{F}{k}\right)^2$. This derivative has an ambiguous sign. In fact, the expected payoff functions in cases A and B are non-concave (for example, see figure 1 in appendix A). This makes it difficult to obtain analytical results for the

[7] Note that, from equations (5a), (5b), and (5c), the firm's equilibrium expected payoff is positive in all equilibria. Thus, it does not make sense to consider a case in which $k \geq k_{min}$ but an insolvent firm refuses to participate in the bailout contest or rejects a bailout assistance.

optimization problem. Therefore, I choose parameters and specific functions to illustrate some solutions using the math software, Maple.

To ensure that the firm's choice of input, whenever possible, is over a compact set, I assume that the firm faces a maximum input constraint, $k \leq 100$. This may be due to credit market frictions, imperfect capital markets, or constraints on mobilizing resources. This assumption is reasonable and does not drive the results in this paper. Also, the inequality in (8) places an upper bound on k, as the examples below show. Finally, the restriction $k \leq 100$ helps to rule out corner solutions in some cases.[8] As argued in section 2.2.1, it is $g(k)$ that drives the results.

Suppose $k_{min} = 39$, $F = 10$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and the price is uniformly distributed on $[\underline{p}, \bar{p}]$. Then (8) holds if $0.42 < k_j < 235.57$, where $j \in \{a, b\}$. And (9) holds if $k_b < 4.50$ or $k_b > 32.00$. Then, noting that $k_{min} = 39$ and $k \leq 100$, the firm chooses k_a to maximize $\Pi_f^A(k)$ over the non-compact set (0.42, 39) and chooses k_b to maximize $\Pi_f^B(k)$ over the compact set [39, 100]. The results are summarized in Table 1.

Table 1: Optimal input levels and profits given $k_{min} = 39$, $F = 10$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and $p \sim U[\underline{p}, \bar{p}]$.

	k_j^*	$\Pi_f^j(k_j^*)$
Case A	34.79	17.37
Case B	47.92	19.44

[8] I get firms of different sizes if firms with different fixed costs chooses different amounts of k. But if the optimal solution is a corner solution, then firms with different fixed costs choose the same amount of k. Such cases are not interesting and do not serve my purpose.

The expected payoffs, $\Pi_f^A(k_a^*)$ and $\Pi_f^B(k_b^*)$, are strictly concave on $k_a \in (0.42, 39)$ and $k_b \in [39, 100]$ respectively with stationary points $k_a^* = 34.79$ and $k_b^* = 47.92$. Table 1 shows that the firm's optimal input is $k_b^* = 47.92$ because case B gives a higher payoff than case A.

Now suppose we maintain the parameters and functions above but set $F = 5$. Then $k_a^* = 31.69$ and $k_b^* = 40.00$. The corresponding profits are $\Pi_f^A(k_a^*) = 21.09$ and $\Pi_f^B(k_b^*) = 22.25$. Thus, the firm's optimal input is $k_b^* = 40.00$.

I now compute the cut-off values of the price for $k_b^* = 47.92$ and $k_b^* = 40.00$.

Table 2: Cut-off values of price given $k_{min} = 39$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and $p \sim U[\underline{p}, \bar{p}]$.

	$F = 5,\ k_b^* = 40.00$	$F = 10,\ k_b^* = 47.92$
$\hat{p}_b$	2.39	3.07
$\hat{p}_c$	3.27	3.90
$\hat{p}$	3.55	4.18

Now consider two firms, 1 and 2, that in the event of insolvency, separately engage in a lobbying bailout contest against an anti-bailout group.[9] Suppose the parameters in Table 2 hold but firm 1 has $F = 5$ and firm 2 has $F = 10$. Then we can compute their bailout probabilities as summarized in Table 3.

[9]Separate contests may emerge because the public authority may receive bailout requests at different times or take bailout decisions sequentially. For example, in March 2008, the Federal Reserve Bank of New York provided an emergency loan to Bear Stearns to avert its collapse. Six months in September 2008 when Lehman Brothers filed for bankruptcy, it did not receive a bailout assistance from any public agency.

Table 3: Probabilities of a bailout, r_f^*, for different realizations of the price given $k_{min} = 39$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and $p \sim U[\underline{p}, \bar{p}]$.

Price	r_f^* for firm 1, k_b^*= 40.00	r_f^* for firm 2, k_b^*= 47.92
$p \in [2.00,2.39]$	50.00%	50.00%
$p \in [2.39,3.07]$	$50.00\% \le r_f^* \le 77.33\%$	50.00%
$p \in [3.07,3.27]$	$77.33\% \le r_f^* < 100\%$	$50.00\% \le r_f^* \le 55.31\%$
$p \in [3.27,3.55]$	100%	$55.31\% \le r_f^* \le 66.42\%$
$p \in [3.55,8.00]$	Solvent	Solvent or insolvent

Note that for $p \in [\hat{p}_c, \hat{p}_b]$, the probability of a bailout, $r_f^* = \sqrt{\frac{g(k)}{F+k-2p\sqrt{k}}}$, is strictly increasing in p. Hence, in the interval $p \in [3.07,3.27]$,[10] a firm's maximum probability of bailout occurs at $p = 3.27$ and its minimum probability occurs at $p = 3.07$. Evaluating r_f^* for firm 2 at k_b^*= 47.92, $F = 10$, and $p = 3.27$ gives $r_f^* = 55.31\%$. Evaluating r_f^* for firm 1 at k_b^*= 40.00, $F = 5$, and $p = 3.07$ gives $r_f^* = 77.33\%$. Therefore, given $p \in [3.07,3.27]$, firm has a higher probability of a bailout than firm 2. Taking into account whether a given price range corresponds to the contest equilibrium in (4a) or (4b) or (4c), this was how all the maximum and minimum bailout probabilities in Table 3 were computed.

[10]In Table 2, the interval $[\hat{p}_c, \hat{p}_b]$ for firm 1 is [2.39,3.27] and for firm 2, it is $[3.07, 3.90]$. These intervals intersect over the interval [2.39,3.07].

If I repeat the exercise above but assume that the price has a triangular distribution with probability density function, $h(p) = \frac{1}{30}p$ on [2,8] and zero elsewhere, I get:

Table 4: Probabilities of a bailout, r_f^*, for different realizations of the price given $k_{min} = 39$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and $h(p) = \frac{1}{30}p$ on $[\underline{p}, \bar{p}]$.

Price	r_f^* for firm 1, k_b^*= 40.26	r_f^* for firm 2, k_b^*= 44.05
$p \in [2.00,2.40]$	50.00%	50.00%
$p \in [2.40,2.93]$	$50.00\% \leq r_f^* \leq 67.63\%$	50.00%
$p \in [2.93,3.27]$	$67.63\% \leq r_f^* < 100\%$	$50.00\% \leq r_f^* \leq 59.63\%$
$p \in [3.27,3.57]$	100%	$59.63\% \leq r_f^* \leq 75.38\%$
$p \in [3.55,8.00]$	Solvent	Solvent or insolvent

Finally, if I repeat the exercise in Table 4 but assume that $g(k) = 0.001k^2$, a strictly convex function, I get:

Table 5: Probabilities of a bailout, r_f^*, for different realizations of the price given $k_{min} = 39$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = 0.001k^2$, and $h(p) = \frac{1}{30}p$ on $[\underline{p}, \bar{p}]$.

Price	r_f^* for firm 1, k_b^*= 40.09	r_f^* for firm 2, k_b^*= 44.97
$p \in [2.00,3.05]$	50.00%	50.00%
$p \in [3.05,3.43]$	$50.00\% \leq r_f^* \leq 100\%$	50.00%
$p \in [3.43,3.50]$	100%	50.00%
$p \in [3.50,3.56]$	100%	$50.00\% \leq r_f^* \leq 52.91\%$
$p \in [3.50,8.00]$	Solvent	Solvent or insolvent

The examples in tables 3, 4, and 5 show that there are cases in which the smaller firm (i.e., firm 1) has a higher bailout probability than the bigger firm (i.e., firm 2) but there are no cases in which the reverse holds. Based on these examples and others (not reported here), the following proposition is stated:

Proposition 1: *There exist subgame perfect Nash equilibria in which a smaller firm has a higher probability of getting bailout assistance than a bigger firm, although, a firm's political advantage is increasing its size (i.e.,* $\partial r_f/\partial k$ *) and both firms meet the threshold size of being too-big (i.e.,* $k \geq k_{min}$ *).*

Note that although a firm with a bigger size has a smaller probability of a bailout, it rationally chooses the bigger size because its objective is to maximize its expected payoff, not to maximize its probability of a bailout.

2.2 Discussion

Proposition 1 will not hold in a contest with independent valuations. In this bailout contest, the players have interdependent valuations. An increase in the size of the requisite bailout increases the valuations, $R(k)$, of both players. In contests with independent valuation, there some strategic complementarity between the efforts of contestants. In particular, when the weaker player increases his effort in response to an increase in his valuation, the stronger player also increases his effort (e.g., Nti, 1999). But this does not give the result in proposition 1. The standard result discussed in section 1 is not overturned.

For any given p for which **both** firms are insolvent, $R(k)$ for firm 2 is bigger than the $R(k)$ for firm 1. For example, in table 3, $R_2(k_b^*) > R_1(k_b^*) > 0$ for $p \in [2.00,3.55]$.[11] For the sake exposition, compare the equilibria in (4a) and 4b). In (4b), $g(k) \geq R(k)$. If the bailout money, $R(k)$, is sufficiently small relative to the firm's political advantage of too big, $g(k)$, it is not worthwhile for the anti-bailout group to lobby against bailing out the firm. So, the firm gets a bailout with certainty. In contrast, if --- as in (4a) --- the bailout money, $R(k)$, is sufficiently big relative to the firm's political advantage of being too big, $g(k)$, then the anti-bailout group lobbies against a bailout and the firm gets a bailout with smaller probability even though it is bigger. Note also that in (4c), the firm's equilibrium bailout probability, $r_f^* = \sqrt{\frac{g(k)}{R(k)}}$, is decreasing in the size of the bailout but increasing in the firm's political advantage. Therefore, it is the balance between the size of the bailout, $R(k)$, and the firm's political advantage, $g(k)$, of being too big that determines the firm's probability of getting a bailout. As noted earlier, the equilibria in (4a), (4b), and (4c) represent low, high, and intermediate values respectively of the firm's political advantage, $g(k)$, of being too big.

From a political-economy point of view, the theory in this paper may explain why in March 2008, Bear Stearns was bailed out but Lehman Brothers, which was almost twice as big as Bear Stearns, was not bailed out when it filed for bankruptcy six months after Bear Stearns was bailed out. Bernanke, Geithner, and Paulson (2019) claimed that a legal constraint, not political considerations, was the reason why Lehman Brothers was not bailed out. Others hold a contrary view. For example, Gorton (2015) opined that "[U]nder Section 13(3) of the Federal Reserve Act, the Fed is allowed to act under "unusual and exigent circumstances." So, it seems that the

[11] See figure 2 in appendix B.

Fed could have acted." Mishkin (2006) argued that bailouts are driven by political considerations. There is an empirical literature that supports Mishkin' view (Blau et al., 2013; Dam and Koetter, 2012; Faccio et al., 2006; Grossman and Woll, 2014; and Sorkin, 2009). As argued above, the size of the requisite bailout is a consideration. According to Sorkin (2009, p. 300), Hank Paulson, Secretary of the Treasury, was not interested in a bailout proposal by Bank of America for the US government to take $40 billion of Lehman's losses because that amount of money was too much.

The prospect of getting assistance in the case of insolvency may imply that the firm's size would be bigger than its size in the absence of the possibility of being bailed out. This holds in this model. However, this need not be the case. The benefit of being too big is the bailout support from the government in the event of insolvency. Its cost is the threshold size, k_{min}, required to qualify for the bailout. If k_{min} is too high, the firm will choose the size that maximizes the expected payoff in case A and will not be bailed out if it is insolvent.

2.2.1 An extension and alternative explanations

Given scarce bailout money, one might consider two firms and an anti-bailout group in the bailout contest in which the firms are also competitors because each firm lobbies for its bailout. However, there is some evidence that in such situations, firms (e.g., banks, or the financial industry, or auto manufacturers) form a coalition and lobby as a single cohesive group (e.g., Grossman and Woll, 2014). For example, during the 2008 - 2010 financial crises, the auto manufacturer, Ford, did not ask for a government bailout, although it received other financial assistance. However, Ford supported bailouts for auto manufacturers, GM and Chrysler, in order to protect its supply chain and dealer network.

More importantly, proposition 1 cannot be obtained if two firms of different sizes compete in the same contest against an anti-bailout group because the bigger firm will put in more effort than the smaller firm and the anti-bailout group's lobbying effort will generally be intended to deny bailout assistance to both firms regardless of size (e.g., lobbying takes the form of making a **general** case against bailouts to insolvent firms, not why bailout to a **particular** firm is bad). If the anti-bailout group efforts are separately against each firm, then we will be in a situation analogous to the model in this paper.

Public backlash against bailing out a firm may be stronger if some firms had earlier been bailed out. This is likely to reduce the chances of a bailout for subsequent firms. But this is an argument that is independent of the size of the firm. In my model, this argument can be captured by assuming that subsequent firms for a bailout have a political disadvantage or smaller political advantage in the bailout contest. A political disadvantage may be captured by $-g(k)$, whose absolute value is so big that the firm cannot win the contest. In the case of zero political advantage, $g(k) = 0$ for all k. Then, as pointed out above, the only equilibrium will be the equilibrium given in (4a). Therefore, if $g(k) = 0$ for all k, the probability of an insolvent firm getting a bailout is 50% regardless of its size while a previously insolvent firm, with $g(k) > 0$, had at least a 50% probability and, in some cases, a 100% probability of getting a bailout depending on the requisite bailout amount and its size. The argument in this paper does not depend on the sequence of bailout requests to obtain the result in proposition 1.

Another explanation is that limited funds for bailouts reduce the chances of subsequent firms getting a bailout because the funds may have run out. This explanation is related to the preceding explanation because the scarcity of resources for bailing out subsequent insolvent firms may be one of the reasons for a stronger public backlash against bailing them out.

The model in this paper gives the result in proposition 1 without assuming that subsequent firms have a smaller political advantage in the bailout contest. Thus, under weaker assumptions, it shows that a bigger firm may have a smaller probability of bailout than a smaller firm, although there is a policy-bias in favor of big firms. In addition, it relates the bailout probabilities of insolvent firms to their sizes.

2.3 Uncertain threshold size

Suppose the threshold size, k_{min}, is not known to a firm. Instead, it is distributed continuously on $[\underline{k}, \overline{k}]$ with density, $q(k_{min})$, where $\underline{k} > 0$. Then a firm's expected payoff in case A remains unchanged because $k < \underline{k}$. So, we still have

$\Pi_f^A(k) = \int_{\hat{p}(k)}^{\bar{p}} \left(2p\sqrt{k} - k - F\right) h(p) dp$ in case A.

In case B, the firm's expected payoff, given $k \geq \underline{k}$, is:

$$\widehat{\Pi}_f^B(k) = \int_{\hat{p}(k)}^{\bar{p}} \left(2p\sqrt{k} - k - F\right) h(p) dp + \int_{\underline{k}}^{k} \left(\Pi_f^{4a}(k) + \Pi_f^{4b}(k) + \Pi_f^{4c}(k)\right) q(k_{min}) dk_{min}. \quad (10)$$

As an example, suppose k_{min} is uniformly distributed on [26, 52], so that the expected threshold size is (26 + 52)/2 = 39, the same as the threshold size in the case of certainty in the previous examples. Then the probability that, in the event of insolvency, the firm will *participate* in a bailout contest is:

$$prob(k \geq k_{min}) = \int_{\underline{k}}^{k} q(k_{min}) dk_{min} = \begin{cases} 0, & if \quad k \in [0,26) \\ \frac{k-26}{26}, & if \quad k \in [26,52) \\ 1, & if \quad k \in (52,100] \end{cases} . \tag{11}$$

Given (11), it follows that there are discontinuities in the payoff function in (10). Taking these discontinuities into account and using the parameters and functions in Table 5, I find that the optimal size for firm 2 with $F = 10$ is a corner solution at $k_b^{**} = 52$ while for firm 1 with $F = 5$, it is an interior solution at $k_b^{**} = 40.99$. Firm 1 increases its optimal input level (size) from 40.09 (when k_{min} was known) to 40.99 while firm 2 increases its size from 44.97 to 52. Surprisingly, uncertainty about the too-big threshold could lead to an increase in the sizes of the firms. Firm 2, the firm with the bigger fixed costs, chooses a size that *guarantees* that, in the event of insolvency, it will participate in a bailout contest. Firm 1 chooses a size that gives a (40.99 –26)/26 = 57.65% probability that, in the event of insolvency, it will *participate* in a bailout contest. Then the probability that firm 1 will win the bailout contest is the joint probability, $r_f^{**} = 0.5765 r_f^*$. This gives the following results:

Table 6: Probabilities of a bailout, r_f^{**}, for different realizations of the price with $\underline{p} = 2, \bar{p} = 8$, $g(k) = 0.001k^2$, $h(p) = \frac{1}{30}p$ on $[\underline{p}, \bar{p}]$ and $k_{min} \sim U$ [26,52].

Price	r_f^{**} for firm 1, k_b^*= 40.99	r_f^{**} for firm 2, k_b^*= 52
$p \in [2.00,3.07]$	28.82%	50.00%
$p \in [3.07,3.46]$	$28.82\% \leq r_f^{**} \leq 57.65\%$	50.00%
$p \in [3.46,3.55]$	57.65%	50.00%
$p \in [3.55,3.59]$	57.65%	$50.00\% \leq r_f^{**} \leq 51.43\%$
$p \in [3.59,8.00]$	Solvent	Solvent or insolvent

In this case, there are price realizations for which the bigger firm has a higher bailout probability and other price realizations for which the smaller firm has a higher bailout probability. This does not overturn proposition 1. However, it shows that there are instances in which a bigger firm has a higher probability of being bailed out.

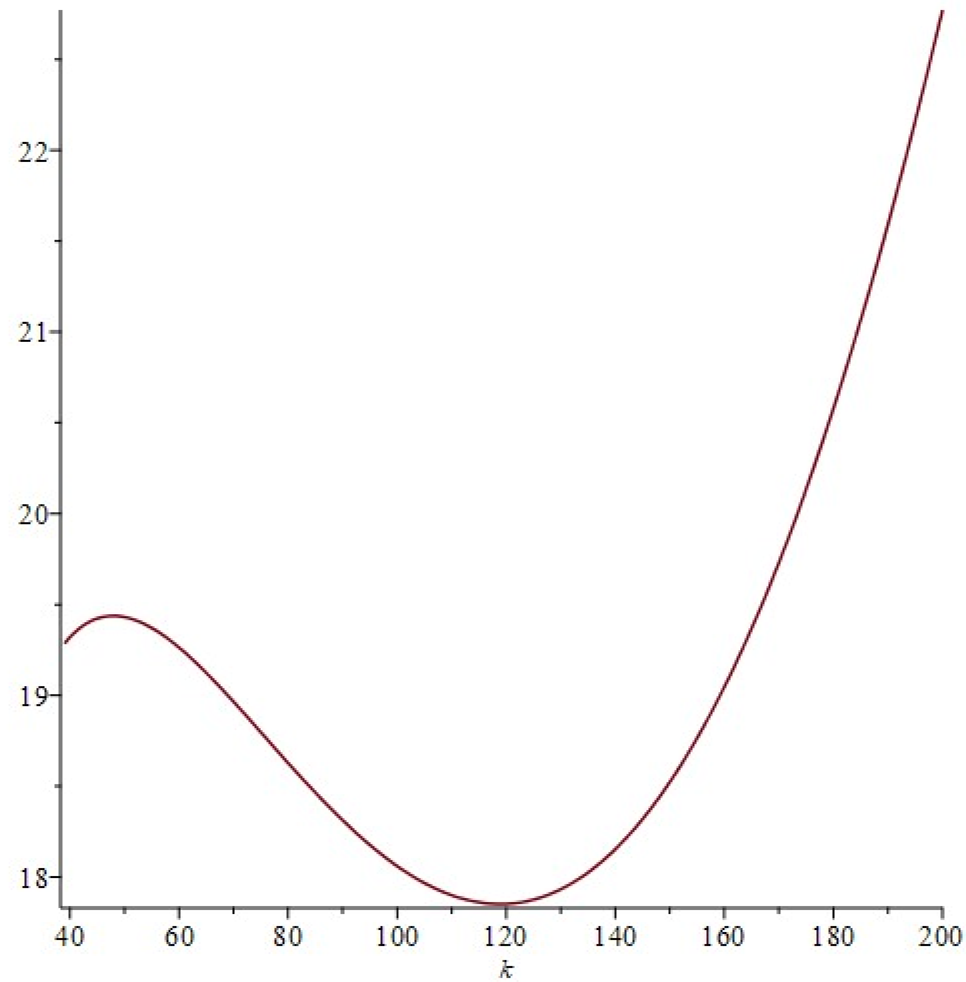
3. Conclusion

As a result of the non-concavity of payoff functions, I had to resort to numerical examples to prove proposition 1. But this is not a limitation of this paper. Having explained the intuition of this result in terms of how interdependent valuations and the balance of a firm's size and its political advantage (due to being too big) affect its probability of a bailout, it should be clear that proposition 1 is not driven by numerical examples.

To the extent that mergers or acquisitions can increase the asset size and interconnectedness of a firm, one can consider an extension in which in period 1, two firms engage in a contest for the right to acquire or merge with a third firm. In this case, k will be a binary choice variable with its higher value being the post-merger size of the firm.

Appendix A: **Figure 1**

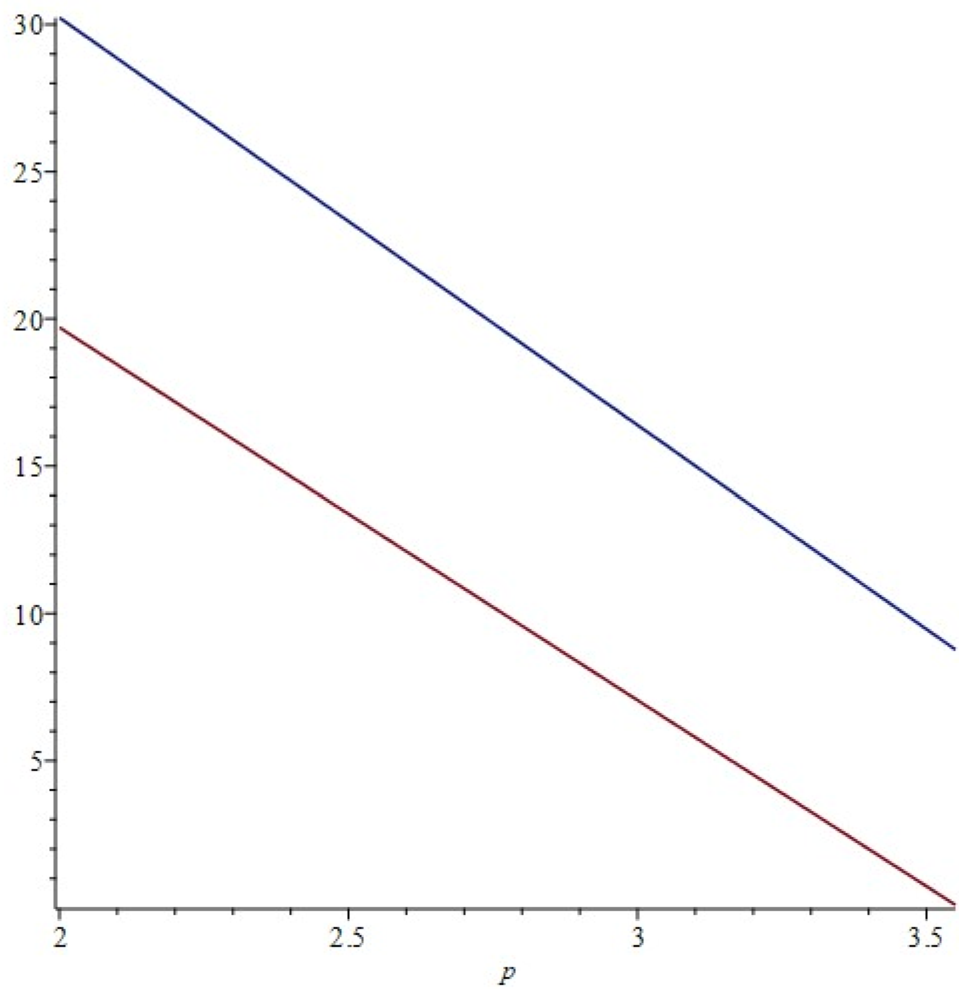
Plot of $\Pi_f^B(k)$ on $k \in [39,200]$ given $k_{min} = 39$, $\underline{p} = 2$, $\bar{p} = 8$, $g(k) = \ln(k)$, and $p \sim U[\underline{p}, \bar{p}]$.



Note: from the figure, there are two stationary points at $k = 47.92$ and $k = 118.87$. By imposing the restriction, $k \leq 100$, we rule out the corner solution, $k = 200$, as the global maximum and thus choose $k = 47.92$ as the global maximum.

Appendix B: Figure 2

Plot of $R_2(47.92) = 10 + 47.92 - 2p\sqrt{47.92}$ and $R_1(40) = 5 + 40 - 2p\sqrt{40}$ on the domain $p \in [2.00, 3.55]$.



Note: the figure is based on the parameters in table 3. The line with the higher intercept is the plot of $R_2(47.92)$.

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