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The Limits of Marketplace Fee Discrimination

Abstract

Platforms often use fee discrimination within their marketplace (e.g., Amazon, eBay, and Uber specify a variety of merchant fees). To better understand the impact of marketplace fee discrimination, we develop a model that allows us to determine equilibrium fee and category decisions that depend on the extent of fee discrimination available to the platform and we highlight how our fee discrimination strategies can be derived in practice using data from airbnb.com. In addition, we find that greater fee discrimination allows the platform to serve more markets in its marketplace but also increases fees in high surplus markets. However, if the platform enters into retail, then the platform reduces its fees and generates greater retail competition. These effects mitigate distortions from fee discrimination and improve welfare. In terms of policy, we show that (1) banning fee discrimination and platform entry is detrimental to welfare, (2) a vertical merger within a retail market mitigates fee distortions but is often worse than an equilibrium with platform entry into retail, and (3) taxing the platform in retail (not merchants) levels the retail playing field and can generate a Pareto improvement upon a policy that bans platform retail entry.

JEL-Codes: L110, L120, L400, H210, L500.

Keywords: platforms, platform retail entry, price discrimination, vertical integration, intermediary.

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1 Introduction

E-commerce marketplaces like Amazon, eBay, Airbnb, Uber, Apple, Expedia, etc. are central to local, regional, and national economies around the world. Due to their size, many have come under scrutiny for a variety of reasons including the use of consumer and merchant data, tax avoidance, price discriminating across consumer browsing history, and disrupting or displacing local stores and communities. Most recently, the House's Investigation of Competition in Digital Markets Report brought forth "nondiscrimination requirements" and "equal terms for equal products and services" recommendations for platforms in the digital economy.[1]

While many of the House's recommendations appear to target platform entry into their marketplace, one issue that has largely gone unnoticed by policy makers — and could be overwhelming relevant to the policy debate — is the extensive use of merchant fee discrimination by these platforms. For example, the "Selling on Amazon Fee Schedule" page provides a lengthy list of "selling, per-item, service, closing, and referral fees" paid by merchants that vary across product categories.[2] For example, electronics sales face a commission fee of 8%, whereas Amazon device accessories face 45%, and kitchen appliances face 15%. These fees do not include the additional fees for merchants that use Amazon fulfillment centers.[3] Similarly, eBay's "Selling fees" are specific to product categories and final prices.[4]

It is not surprising that we see different fees across product categories since e-commerce platforms facilitate countless markets that vary in demand, cost, and competitive structures. If the platform can only use one fee across all markets, then this variance makes it difficult for the platform to charge a fee that maximizes its surplus extraction from any particular market. At the other extreme, if the platform charges market specific fees across all markets,

[1]See Recommendations in the Executive Summary of the U.S. House of Representatives Investigation of Competition in Digital Markets Report.

[2]See "Selling on Amazon Fee Schedule," for a list of Amazon fees.

[3]See "Fees and rate structure in Fulfillment by Amazon" for details.

[4]The "Seller Fees" page directs merchants to all of eBay's fees. Furthermore, at the bottom of the "Selling fees" page, eBay explicitly states: "Top Takeaway: The fees for listing an item depend on the price, format and category you choose, and whether you have an eBay Store."

then the platform extracts the greatest possible surplus from every market that it serves.

To understand the effects of fee discrimination, we model a marketplace platform that facilitates transactions across a mass of different markets (first under the assumption that the platform is unable to enter into commerce). By comparing optimal fees when fee discrimination is perfect to the fee and category selections when fee discrimination is imperfect, we determine which underlying demand systems produce equilibria where the platform tailors its fees and categories toward high surplus markets. In the tailored settings, the platform selects narrow categories around the high surplus markets and also selects fees within a category that target the high surplus markets within the category. In terms of welfare, we find that fee discrimination generates a welfare tradeoff: greater fee discrimination allows the platform to serve more markets but also exacerbates merchant fees distortions. To illustrate the relationship between perfect and imperfect fee discrimination, we use data derived from airbnb.com to perform a back-of-the-envelope exercise that derives different fee discrimination solutions. This practical application highlights how platforms can implement the fee discrimination strategies provided by our main results.

As an extension, we allow the platform to enter into its retail markets and we investigate the interplay between platform fee discrimination and entry into retail. With platform entry into non-foreclosed markets, merchants face an additional competitor and the platform reduces its merchant fee relative to the non-entry equilibrium; both these effects improve marketplace efficiency. With platform entry into otherwise foreclosed markets, the marketplace grows and welfare improves. These measures mitigate fee distortions or expand marketplace offerings so that welfare improves with platform retail entry (a result that is robust across fee discrimination regimes).

While merchants are often harmed by platform entry into retail, we show that noteworthy cases exist where the fee discrimination equilibrium with platform retail entry Pareto improves upon its non-entry counterpart. These cases are especially important for platform policy design. In particular, we find that if the platform's cost disadvantage in retail is sufficiently large, then the platform will enter into retail and lower its merchant fee so that merchant profit increases with platform entry. From a policy perspective, this implies that there exist taxes that target the platform in retail so that platform entry Pareto improves upon the non-entry equilibrium and generates positive tax revenues. We also study several additional policies that level the retail playing field upon platform entry, as well as vertical integration between the platform and its merchants, and show that caution should be taken when regulating platform fees and allowing vertical mergers when platform entry into retail is an available option.

1.1 Related Literature

There is a growing literature on platform marketplace decisions. In particular, the platform's choice on price parity, MFN, or resale price maintenance clauses has been the focus of recent work by Edelman and Wright (2015), Foros et al. (2017), Johansen et al. (2017), Johnson (2017), and Wang and Wright (2020). While these papers mirror our work in that they consider vertical relationships within the platform marketplace, the presence of discrimination in these papers pertains to retail price discrimination across final good outlets. Instead, our work considers how the extent of fee discrimination (which is not present in these papers) impacts fee, category, and platform retail entry decisions.[5]

This paper naturally relates to a longstanding literature on input price discrimination within a vertical supply chain (e.g., Katz (1987) and Yoshida (2000) to name a few). Instead of input price discrimination by an upstream supplier, we model merchant fee discrimination by an intermediary platform. While some similarities exist across models, this distinction is critical since input suppliers discriminate across a discrete number of downstream firms competing in a single market so that category selection across markets does not exist in these settings. This distinguishes the merchant fee discrimination by an intermediary from input

[5] Another kind of discrimination that is becoming increasingly relevant, but is orthogonal to our work on merchant fee discrimination, is the use of pricing algorithms by retailers within a marketplace. Again, this form of price discrimination occurs at the final good stage. See Brown and MacKay (2019), Calvano et al. (2020), and Johnson et al. (2020) for details.

price discrimination by an upstream supplier.

There is also overlap between our work and the literature on traditional price discrimination. Bergemann et al. (2015) determine the limits of price discrimination in traditional markets. We effectively add a vertical marketplace structure to their work and consider the limits of merchant fee discrimination (with and without platform entry into retail). This difference in structure makes it tough to bridge results between the two papers, but this difficulty maintains the importance of distinguishing between price discrimination at the final goods stage and merchant fee discrimination by an intermediary.

In conjunction with marketplace fee discrimination, we also consider the platform's decision to enter into retail. Without considering fee discrimination, several others have considered this decision. For example, Hagiu and Wright (2015) and Johnson (2017) consider the platform's choice between a platform wide agency or wholesale models. By allowing the platform to enter into retail and facilitate a marketplace, our work on platform retail entry is closer to that of Etro (2021) and Hagiu et al. (2020) who consider platform entry into its marketplace. However, neither Etro (2021) nor Hagiu et al. (2020) consider merchant fee discrimination or category selection.

Although they tackle a different problem, the models of Wang and Wright (2017) and Wang and Wright (2018) relate to ours in several respects. Both papers consider the platform's choice between an ad valorem or unit fee in an exogenously given category that comprises different markets (e.g., which type of fee for the "furniture" category on Amazon or eBay). They show that the platform prefers an ad valorem merchant fee as it provides built-in fee discrimination across final good prices within the category (relative to a unit fee that is constant across prices within the category). Thus, they focus on the question of what type of fee (ad valorem or unit) should be used within an exogenously given category. Instead we focus on how the extent of marketplace fee discrimination — not the type of fee used within an exogenous category — impacts the platform's category, fee, and retail entry decisions.

2 The Model

To understand how multiple merchant fees impact a marketplace, consider the case where a platform facilitates transactions between merchants and consumers and does not enter into commerce within its marketplace. In Section 6.1 we consider the more general setting with platform entry into commerce.[6]

Suppose that a platform facilitates transactions across a unit mass of independent markets. In market $i \in [0,1]$, demand for product i is given by $q(i,p)$, marginal cost is $c(i)$, the platform's unit fee to merchants is $f(i)$, and the platform's ad valorem fee to merchants is $t(i)$. In terms of timing, the platform first selects product categories (defined as collections of products that face the same fees) and then chooses fees within each category. Lastly, the retail equilibrium in market i is determined. For simplicity, we assume that the extent of fee discrimination is exogenously given.[7]

We solve the game backwards by first considering the retail subgame in market i and then, given the retail subgame equilibrium in each market i, the platform determines optimal categories and fees. We focus on the case where merchants have market power and, for simplicity, we assume that each market is served by a monopoly merchant.[8]

[6] Note that we take a partial equilibrium approach that considers the platform as the only option. More specifically, we do not focus on how the platform competes with other platforms, physical retail outlets, or individual merchants off platform. Instead, we focus on the platform's problem within its own marketplace as though it were the only option for the products that it facilitates. While this approach is a simplification, it is the natural starting point for studying marketplace fee discrimination and it is consistent with the models of Wang and Wright (2017) and Wang and Wright (2018).

[7] In principle, the amount of fee discrimination is chosen by the platform; however, the platform's motives behind extensive discrimination are unclear. Without any concern of merchant, consumer, or antitrust recourse, a platform would use as many categories as possible until the cost of an additional category outweighs the improvement to platform revenues. And if this cost is zero, then the platform will implement perfect fee discrimination (fees specific to every market $i \in [0,1]$). Thus, we avoid speculation over the platform and policy maker motives and take the extent of fee discrimination as given.

[8] The main results are qualitatively consistent for any market structure where merchants exhibit some amount of market power.

2.1 The Retail Subgame Equilibrium

In market i, merchant profit, platform profit, and deadweight loss are given by:

$$\pi(i) = [(1 - t(i)) \cdot p(i) - c(i) - f(i)] \cdot q(i, p(i)), \tag{1}$$

$$\Pi(i) = f(i) \cdot q(i, p(i)) + t(i) \cdot p(i) q(i, p(i)), \tag{2}$$

$$DWL(i) = \int_{c(i)}^{p(i)} q(i, p(i)) dp(i), \tag{3}$$

where $p(i)$ denotes the price in market i.

Solving the merchant's problem in market i, given platform fees $f(i)$ and $t(i)$, implies the following:

Lemma 1. *There exists a unique retail equilibrium in market i, denoted by $p^*(i)$ and $q^*(i, p^*(i))$, with $\frac{\partial p^*(i)}{\partial t(i)} > 0$ if and only if $-\frac{\partial q^*(i,p^*(i)))}{\partial p^*(i)} \cdot p^*(i) > q(i, p^*(i))$ and $\frac{\partial p^*(i)}{\partial f(i)} > 0$.*

All proofs are provided in the appendix.

Note that uniqueness requires that the second-order condition holds: $2(1-t(i)) \cdot \frac{\partial q^*(i,p^*(i)))}{\partial p^*(i)} + \frac{\partial^2 q^*(i,p^*(i)))}{\partial (p^*(i))^2} \cdot [(1 - t(i))p^*(i) - c(i) - f(i)] < 0$. It is also important to note that $-\frac{\partial q^*(i,p^*(i)))}{\partial p^*(i)} \cdot p^*(i) > q(i, p^*(i))$ holds in most cases. We show this explicitly in future examples for both linear and constant elasticity demand.[9] Thus, we restrict the analysis to demand specifications where both conditions hold so that a unique retail equilibrium exists and so market prices always increase with merchant fees: $\frac{\partial p^*(i)}{\partial f(i)}$ and $\frac{\partial p^*(i)}{\partial t(i)} > 0$.

3 Fee Discrimination and Category Selection

We consider two settings of fee discrimination by the platform: perfect fee discrimination and imperfect fee discrimination. Perfect fee discrimination allows the platform to charge market specific fees in every market i so that each market is its own category. This serves

[9] So long as demand is elastic: $\epsilon(i) < -1$.

a benchmark. Imperfect fee discrimination considers the more realistic setting where the platform has N categories so that every market i within a category faces the same fee.

3.1 Perfect Fee Discrimination

If the platform utilizes perfect fee discrimination (PFD), where the platform charges individual fees across every market $i \in [0, 1]$, then product categories are redundant and so we drop the (i) in this subsection to ease exposition. Furthermore, we consider each type of fee separately to simplify the analysis.[10]

Theorem 1. *The optimal unit fee, denoted by f^*, satisfies $\epsilon^f = -1$ where ϵ^f is the demand elasticity with respect to the unit fee: $\epsilon^f := \frac{\partial q(p^*)}{\partial f} \cdot \frac{f^*}{q(p^*)}$.*[11]

The optimal ad valorem fee, denoted by t^*, satisfies $\epsilon^R = -1$ where ϵ^R is the revenue elasticity with respect to the ad valorem fee: $\epsilon^R := \frac{\partial p^* \cdot q^*(p^*)}{\partial t} \cdot \frac{t^*}{p^* \cdot q^*(p^*)}$.[12]

To highlight the similarities between the optimal fees, we define two elasticity terms: (i) the unit fee demand elasticity, $\epsilon^f := \frac{\partial q(p^*)}{\partial f} \cdot \frac{f^*}{q(p^*)}$, and (ii) the ad valorem fee elasticity of merchant revenues, $\epsilon^R := \frac{\partial p^* \cdot q^*(p^*)}{\partial t} \cdot \frac{t^*}{p^* \cdot q^*(p^*)}$. The results from Theorem 1 show that the optimal fee satisfies a similar equilibrium condition for either a unit or ad valorem fee: At the optimal fee, a one percent increase in the fee corresponds to a one percent decrease in the fee's multiplier (either equilibrium quantities or equilibrium revenues). Such a solution effectively maximizes the platform profit "area" that is given by the fee times its multiplier.

[10] Typically, platform marketplaces use different ad valorem fees across categories, and some categories have both unit and ad valorem fees. However, the unit fee is often less than a dollar in those cases. For more on affine fee schedules, consider Wang and Wright (2017) and Wang and Wright (2018).

[11] Uniqueness of the unit fee requires that the second-order condition holds, $2\frac{\partial q^*(p^*))}{\partial p^*} \cdot \frac{\partial p^*}{\partial f} + f \cdot \left[\frac{\partial q^*(p^*))}{\partial p^*} \cdot \frac{\partial^2 p^*}{\partial (f)^2} + \frac{\partial^2 q^*(p^*))}{\partial (p^*)^2} \left(\frac{\partial p^*}{\partial f}\right)^2\right] < 0$, which is true for both linear demand and constant elasticity demand with elastic demand ($\epsilon < -1$). Thus, we restrict the analysis to demand specifications where the second-order condition holds.

[12] Uniqueness requires that the second-order condition holds, $2\left[q(p^*) \cdot \frac{\partial p^*}{\partial t} + p^* \frac{\partial q^*(p^*))}{\partial p^*} \cdot \frac{\partial p^*}{\partial t}\right] + t \cdot \left[\frac{\partial^2 p^*}{\partial t^2} q(p^*) + 2\left(\frac{\partial p^*}{\partial t}\right)^2 \frac{\partial q^*(p^*))}{\partial p^*} + p^* \frac{\partial^2 q^*(p^*))}{\partial (p^*)^2} \left(\frac{\partial p^*}{\partial t}\right)^2 + p^* \frac{\partial q^*(p^*))}{\partial p^*} \frac{\partial^2 p^*}{\partial t^2}\right] < 0$, which is true for linear demand and a sufficient condition for constant elasticity demand with elastic demand ($\epsilon < -1$) is that $\epsilon \geq -4.5$. Thus, we restrict the analysis to demand specifications where the second-order condition holds.

In the next section on imperfect fee discrimination, ordering the $i \in [0,1]$ markets so that PFD equilibrium fees and profits are both monotone in i is computationally useful. To show that such an ordering can occur, we analyze two examples and show that a variety of parameters produce comparative statics that generate such an ordering. In the first example we consider the case of linear demands with the platform charging a unit fee in each market. In the second example we consider the case of constant elasticity (CE) demand with the platform charging a unit fee in each market.[13]

Example 1 (Perfect Fee Discrimination with Linear Demands and Unit Fees)**.**

Suppose that the platform facilitates markets with linear demands and utilizes a unit fee that is market specific. In particular, let $q(p) = a - bp$. In this case, the subgame equilibrium price and quantity are given by $p(f) = \frac{1}{2}\left(\frac{a}{b} + c + f\right)$ and $q(f) = \frac{b}{2}\left(\frac{a}{b} - c - f\right)$. Turning to the platform's problem we see that the platform maximizes $\Pi(f) = f \cdot q(f) = f \cdot \frac{b}{2}\left(\frac{a}{b} - c - f\right)$ with respect to f. As a result, we have the following equilibrium:

$$f^* = \frac{1}{2}\left(\frac{a}{b} - c\right),\ p^* = \frac{1}{4}\left(3\frac{a}{b} + c\right),\ q^* = \frac{b}{4}\left(\frac{a}{b} - c\right),\ \Pi(f^*) = \frac{b}{8}\left(\frac{a}{b} - c\right)^2. \tag{4}$$

Note that market i might differ from market $k \neq i$ in terms of demand (e.g., $a(i) \neq a(k)$ or $b(i) \neq b(k)$) or in terms of marginal cost (e.g., $c(i) \neq c(k)$). To understand how equilibrium outcomes change across markets, we derive the following comparative statics:

Corollary 1. *The equilibrium unit fee and platform profit are increasing in a and decreasing in b and c.*

Not surprisingly, if demand expands (a increases or b decreases), then the unit fee and platform profit increase. Similarly, an increase in marginal cost puts downward pressure on the equilibrium unit fee and platform profit. These results imply that if markets $i \in [0,1]$

[13] Unfortunately, the use of ad valorem fees with linear demands or CE demands results in examples that either do not provide explicit solutions (linear demands) or are computationally straight forward (e.g., the optimal ad valorem fee with CE demand does not depend on marginal cost which would imply that markets that only differ in marginal cost will face identical merchant fees).

are ordered by either an increasing $a(i)$, a decreasing $b(i)$, or a decreasing $c(i)$, then markets are also ordered by increasing unit fees and platform profit.

Example 2 (Perfect Fee Discrimination with CE Demands and Unit Fees)**.**

Now consider the case where the platform facilitates markets with CE demands, $q(p) = k \cdot p^\epsilon$, and charges a market specific unit fee. In this case, the subgame equilibrium price is given by $p(f) = \frac{\epsilon}{1+\epsilon} \cdot (c+f)$, where $\epsilon < -1$. Turning to the platform's problem we see that the platform maximizes $\Pi(f) = f \cdot [k \cdot (p(f))^\epsilon] = fk\left(\frac{\epsilon}{1+\epsilon} \cdot (c+f)\right)^\epsilon$ with respect to f. As a result, we have the following equilibrium:

$$f^* = \frac{-c}{1+\epsilon},\ p^* = c\left(\frac{\epsilon}{1+\epsilon}\right)^2,\ q^* = kc^\epsilon\left(\frac{\epsilon}{1+\epsilon}\right)^{2\epsilon},\ \Pi(f^*) = \frac{-kc^{1+\epsilon}}{1+\epsilon}\left(\frac{\epsilon}{1+\epsilon}\right)^{2\epsilon}. \tag{5}$$

In terms of differences across markets when products face CE demand, market i might differ from market $j \neq i$ in terms of demand (e.g., $k(i) \neq k(j)$ or $\epsilon(i) \neq \epsilon(j)$) or in terms of marginal cost (e.g., $c(i) \neq c(j)$). Investigating comparative statics allows us to understand how equilibrium outcomes change across markets. In this example we have the following:

Corollary 2. *The equilibrium unit fee and platform profit are increasing in* ϵ *whenever* $\epsilon < -1.4$.[14] *Conversely, the equilibrium unit fee is increasing in marginal cost while platform profit is decreasing in marginal cost.*

Unlike Example 1 of linear demands, if marginal cost increases, then the unit fee increases (instead of decreases) while platform profits decrease. On the demand side, we see that as demand becomes more elastic, both the unit fee and platform profit increase. Unfortunately differences in k do not impact the unit fee. These results imply that if markets $i \in [0,1]$ are ordered by decreasing $\epsilon(i)$, then markets are also ordered by increasing unit fees and platform profits. Similarly, if the markets $i \in [0,1]$ are ordered by decreasing $c(i)$, then the markets are also ordered so that platform profit is increasing in i and the unit fee is decreasing in i.

[14]More specifically, $\frac{\partial p^*}{\partial \epsilon} > 0$, but $\frac{\partial \Pi(f^*)}{\partial \epsilon} > 0$ for all $c \geq 0$ if $\epsilon < -1.4$ and this constraint relaxes as c increases.

Ordering markets by $k(i)$ exclusively is not meaningful as every market i will face the same fee $f^*(i) = \frac{c}{1+\epsilon}$ which renders the exercise of fee discrimination pointless.[15]

3.2 Imperfect Fee Discrimination

We now consider the case of imperfect fee discrimination (IFD) where the platform does not use fees that are specific to individual markets and instead separates markets into categories and chooses fees at the category level that apply to every market in that category. To ease the expositional burden, we focus on unit fees. We also focus on demand systems that satisfy two ordering requirements. First, we order markets so that equilibrium PFD profit, denoted by $\Pi_p(i)$, is increasing in i: $\Pi'_p(i) = \frac{d\Pi_p(i)}{di} > 0$. Second, we focus on marketplaces where the equilibrium PFD fee is monotone in i: $\frac{df^*(i)}{di}$ is monotone.

In Examples 1 and 2 we provide comparative static results that generate such an PFD ordering and are micro-founded by the underlying ordering of markets by cost or demand parameters. For example, if demand is CE across markets and markets differ so that only $c(i)$ is decreasing in i, then $\Pi_p(i)$ is increasing in i and $f^*(i)$ is decreasing in i. Similarly, if markets experience linear demands that are only increasing in intercept ($a(i)$), only decreasing in slope ($b(i)$), or only decreasing in marginal costs ($c(i)$), then both $\Pi_p(i)$ and $f^*(i)$ are increasing in i.

In reality, a platform that facilitates a large mass of markets will have markets that vary drastically in marginal cost as well as demand curvature, intercept, or slope. Thus, our ordering is obviously a simplifying assumption. However, we argue that such an ordering likely comes very natural to platforms that prioritize their own profit. To this end, these ordering assumptions capture an ordering where, in the ideal setting of PFD, markets order from lowest platform profit to highest platform profit with perfect discrimination fees are

[15] As mentioned in Footnote 13, the case of CE demands with ad valorem fees provides a similar equilibrium outcome and comparative statics to Example 2. However, one key difference is that the equilibrium ad valorem fee is given by $t^* = \frac{-1}{\epsilon}$. This implies that if markets are ordered by marginal costs, $c(i)$, then all markets would be charged the same fee, $t^*(i) = \frac{-1}{\epsilon}$, so that platform fee discrimination is meaningless in this case.

monotone across markets.

Given the ordering of markets by PFD, consider the platform's problem when fee discrimination is imperfect. To annotate platform profits in market i under IFD, let $\Pi(i, f)$ capture the platform's profit in market i under fee f so that $\Pi(i, f^*(i)) = \Pi_p(i)$. We assume that $\Pi(i, f)$ is continuous and differentiable in both variables. Examples 1 and 2 provide several micro-founded parameterizations that result in such IFD profit functions. As a starting point to the platform's IFD problem, suppose that the platform distributes markets into N categories and let the set of markets in category n be denoted by $C_n \subset [0, 1]$.[16] In this case, we have the following result:

Lemma 2. *In equilibrium, each category $C_n \subset [0, 1]$ for $n = 1, ..., N$ is a convex set, and there exists an equilibrium fee for category n that is equivalent to $f^*(c_n)$ for some $c_n \in C_n$.*

Both these findings stem from how the platform profit changes in market i as the merchant fee in market i departs from its PFD optimum: $f^*(i)$. More specifically, we have that $\Pi_p(i) - \Pi(i, f(i))$ increases as $f(i)$ departs from $f^*(i)$ so that an $f^*(i)$ that is monotone in i implies that the platform will select categories as convex sets. Otherwise, for example if $C_1 = [x_1, x_2) \bigcup \{x\}$ with $x > x_2 > x_1$, then platform profit increases if market x is allocated to some category $n > 1$. Monotonicity in fees also implies that a fee for category n that is equivalent to a PFD fee outside category n is strictly dominated by a PFD fee within the category.

While subtle, the results from Lemma 2 help breakdown the platform's IFD problem. In particular, given that each category must be a convex set of markets, the platform category selection process boils down to determining the $N - 1$ thresholds in the unit interval that divide the N categories. We denote these thresholds by $x_j \in [0, 1]$, for $j = 0, 1, ..., N$ with $x_0 = 0$ and $x_N = 1$, so that $C_n = [x_{n-1}, x_n)$ and $C_N = [x_{N-1}, 1]$. This implies that we can

[16] We assume N is exogenously given for the IFD case. In principle, N is chosen by the platform; however, the platform's motives behind the selection of N are unclear. Without any concern of merchant, consumer, or antitrust recourse, a platform would use as many categories as possible until the cost of an additional category outweighs the improvement to platform revenues. And if this cost is zero, then IFD approaches PFD. Thus, we avoid the number of categories problem and instead consider the case where N is exogenous.

characterize the platform's IFD problem of choosing to maximizes its profits, with respect to the x_n and c_n, using two different algorithms:

$$\max_{x_1,\ldots,x_{N-1}} \left[\sum_{n=1}^{N} \left[\max_{c_n \in C_n} \int_{x_{n-1}}^{x_n} \Pi(i, f(c_n)) di \right] \right], \qquad \text{(Maximization Algorithm)}$$

$$\min_{x_1,\ldots,x_{N-1}} \left[\sum_{n=1}^{N} \left[\min_{c_n \in C_n} \int_{x_{n-1}}^{x_n} \Pi_p(i) - \Pi(i, f(c_n)) di \right] \right]. \qquad \text{(Minimization Algorithm)}$$

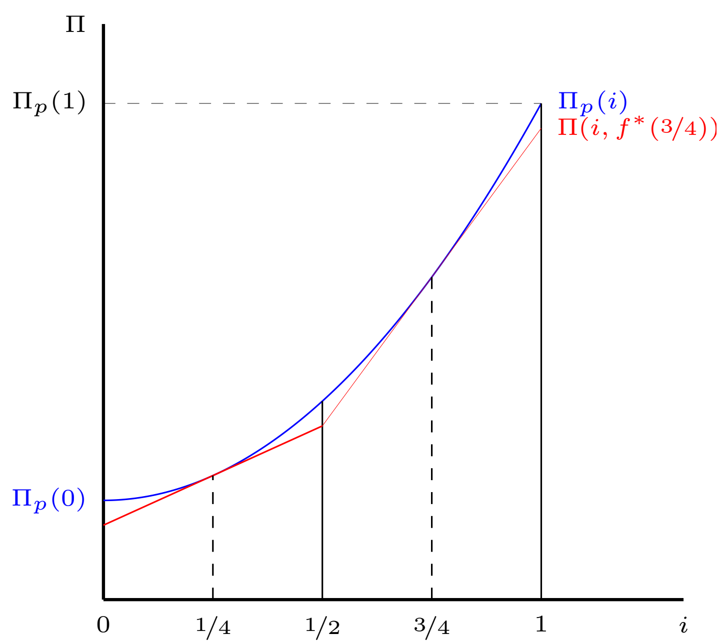
The Maximization Algorithm takes the standard approach of choosing IFD categories and fees within each category to maximize marketplace wide profits: max IFD. Instead, Minimization Algorithm takes the approach of choosing IFD categories and fees to mitigate lost profit relative to the platform's first-best solution given by PFD: min PFD - IFD. These two algorithms generate the same solutions (otherwise, $\Pi_p(i)$ is incorrect), which allows us to take advantage of using $\Pi_p(i)$ when there are benefits to doing so.

To gain a better understanding of the algorithms and for how fee discrimination impacts platform profit, fee distortions, and welfare, consider the case where the platform arbitrarily selects categories and fees. Figure 1 provides such an outcome where the mass of markets is split into two categories of equal size (so that $x_1 = 1/2$), and the platform uses what we call *midpoint pricing*, where the merchant fee in each category is equivalent to the PFD merchant fee for the market in the "middle" of that category: $c_1 = 1/4$ so that $f(i) = f^*(1/4)$ for all $i \in C_1 = [0, 1/2)$ and $c_2 = 3/4$ so that $f(i) = f^*(3/4)$ for all $i \in C_2 = [1/2, 1]$. Formally, we have the following definition:

Definition 1. We say that the platform implements ***midpointing*** if the platform selects categories such that $x_n = n/N$ and selects fees such that $c_n = \frac{x_n + x_{n-1}}{2}$ for all $n = 1, ..., N,$.

The blue line in Figure 1 denotes the maximum profit that the platform can earn from market i which occurs under the PFD equilibrium. In contrast, the red lines denote the profit that the platform earns from market i when the first (second) category faces unit fee $f^*(1/4)$ $(f^*(3/4))$. This implies that the lost profit in market i from the IFD outcome is given by the difference in $\Pi_p(i)$ and $\Pi(i, f^*(1/4))$ or $\Pi(i, f^*(3/4))$ which is given by the area between the blue and red lines. Breaking up the markets within a category we see that if $f^*(i)$ is increasing in i, then the markets $i \in (0, 1/4)$ and $i \in (1/2, 3/4)$ face fees that are too high (relative to their PFD fee) and the markets $i \in (1/4, 1/2)$ and $i \in (3/4, 1)$ face fees that are too low (relative to their PFD fee). Naturally, this complicates welfare comparisons across levels of fee discrimination since higher fees exacerbate fee distortions while lower fees dampen them.

Figure 1: An Example where Midpointing is Optimal



To determine whether or not midpointing occurs in equilibrium for Figure 1, it is important to note that the shapes of the PFD and IFD profit functions provide insight toward *both* platform fee and category selections. Furthermore, focusing on the Minimization Algorithm that generates such a graphical representation allows us to consider the platform's problem for either unit or ad valorem fees (assuming that every fee within each category is a unit fee or that every fee within each category is an ad valorem fee). In particular, we see that platform PFD profit is convex in i in Figure 1 (it is in fact quadratic), but platform IFD profit is linear in i. And, in this particular case where $\Pi_p(i)$ is quadratic, we see that the midpointing outcome generates profit losses that are equal across the midpointing fees and within a category. That is, we have that (a) $\int_0^{1/4} \Pi_p(i) - \Pi(i, f^*(1/4))di = \int_{1/4}^{1/2} \Pi_p(i) - \Pi(i, f^*(1/4))di$, (b) $\int_{1/2}^{3/4} \Pi_p(i) - \Pi(i, f^*(3/4))di = \int_{3/4}^{1} \Pi_p(i) - \Pi(i, f^*(1/4))di$, and (c) $\int_0^{1/2} \Pi_p(i) - \Pi(i, f^*(1/4))di = \int_{1/2}^{1} \Pi_p(i) - \Pi(i, f^*(3/4))di$ all hold in this case. This implies that midpointing is optimal (which will be formally proven in Theorem 2).

It is important to note that midpointing is not always optimal.[17] For example, as shown in Figure 2, if the IFD profit is linear and the PFD profit has a different convexity, then midpointing can be suboptimal. In Figure 2, we see that the midpointing outcome generates profit losses in a category's higher profit markets that outweighs the profit losses from that category's lower profit markets. That is, $\int_0^{1/4} \Pi_p(i) - \Pi(i, f^*(1/4))di < \int_{1/4}^{1/2} \Pi_p(i) - \Pi(i, f^*(1/4))di$ and $\int_{1/2}^{3/4} \Pi_p(i) - \Pi(i, f^*(3/4))di < \int_{3/4}^{1} \Pi_p(i) - \Pi(i, f^*(1/4))di$. This clearly implies that midpointing is suboptimal. Furthermore, The direction for which the midpointing outcome is suboptimal implies that the platform can improve profit by either (1) increasing $x_1 > 1/2$ while continuing to use midpoint fees or (2) choosing a fee in category n that corresponds to a PFD fee from $i > \frac{x_n + x_{n-1}}{2}$. To formalize this discussion, we have the following result:

Theorem 2. *If* $\frac{\partial^3 \Pi_p(i)}{\partial i^3} \mathcal{R} \frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3}$ $\left(\frac{\partial^3 \Pi_p(i)}{\partial i^3} \mathcal{R} \frac{\partial^3 \Pi(i, t^*(i))}{\partial i^3}\right)$ *for all* i*, where* $\mathcal{R} \in \{=, >, <\}$ *captures the relation, then the IFD equilibrium fees and categories are such that* $c_n^* \mathcal{R} \frac{x_n^* + x_{n-1}^*}{2}$ *for all* $n = 1, ..., N$ *and* $x_n^* \mathcal{R}\ n/N$ *for all* $n = 1, ..., N-1$ *with* $x_N^* = 1$.[18]

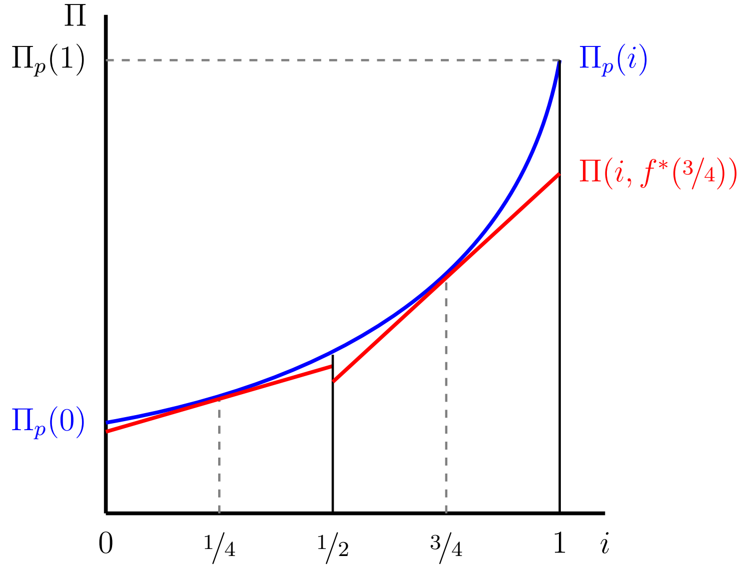
The intuition for why the IFD equilibrium hinges on the third derivative is that we are effectively determining the rate of the slope difference to the left of the point of intersection between the PFD and IFD versus the rate of the slope difference to the right of the intersection point.[19] In other words, the relationship between $\frac{\partial^3 \Pi_p(i)}{\partial i^3}$ and $\frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3}$ effec-

[17] In addition, the equilibrium IFD profit (the red lines) need not be linear in i.

[18] Note that $\frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3}$ $\left(\frac{\partial^3 \Pi(i, t^*(i))}{\partial i^3}\right)$ holds the $f^*(i)$ $(t^*(i))$ component fixed; however, $\frac{\partial^3 \Pi_p(i)}{\partial i^3} = \frac{d^3 \Pi(i, f^*(i))}{di^3}$ $\left(\frac{\partial^3 \Pi_p(i)}{\partial i^3} = \frac{d^3 \Pi(i, t^*(i))}{di^3}\right)$ does not.

[19] Thus, the rate produces first derivative, dealing with slopes produces the second, and considering changes about an intersection point provides the third.

Figure 2: An Example where Midpointing is Suboptimal



tively captures the rate at which the profits from PFD depart from IFD. For example, if $\frac{\partial^3 \Pi_p(i)}{\partial i^3} > \frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3}$, then the rate at which $\Pi_p(c_n - \theta)$ departs from $\Pi(c_n - \theta, f^*(c_n))$ as θ increases is smaller than the rate at which $\Pi_p(c_n + \theta)$ departs from $\Pi(c_n + \theta, f^*(c_n))$ as θ increases.[20] This implies that the profit losses from IFD are greater in the markets $i \in (c_n, c_n + \theta)$ than in the markets $i \in (c_n - \theta, c_n)$ so that the platform skews categories and fees toward the right (relative to midpointing): $c_n^* > \frac{x_n^* + x_{n-1}^*}{2}$ and $x_n^* > {}^n\!/_N$ for all $n = 1, ..., N-1$.

Theorem 2 also identifies the settings in which midpointing occurs in equilibrium. We formally specify this result with the following corollary:

Corollary 3. *If $\Pi_p(i)$ is quadratic in i, so that $\frac{\partial^3 \Pi_p(i)}{\partial i^3} = 0$ and $\frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3} = 0$, then midpointing occurs in the IFD equilibrium.*

An example of Corollary 3 is provided in Figure 1 where $\Pi_p(i)$ is quadratic in i. There we see that $\Pi_p(i) - \Pi(i, f^*(1/4))$ is symmetric around $i = 1/4$ and $\Pi_p(i) - \Pi(i, f^*(3/4))$ is symmetric around $i = 3/4$ so that any deviation from midpointing would increase the difference in profit

[20] Recall that, by construction, we have that (1) $\Pi_p(i)$ and $\Pi(i, f)$ are increasing in i for any unit f and (2) $\Pi_p(i + \theta) - \Pi(i + \theta, f^*(i))$ and $\Pi_p(i - \theta) - \Pi(i - \theta, f^*(i))$ are increasing in θ for all $i \in [0, 1]$.

between PFD and IFD resulting in a suboptimal IFD outcome.

From a potential merchant (or product design) perspective, the IFD equilibrium fee and category structures imply that markets $i \in [x_{n-1}, c_n)$ will have harsher fees than markets $i \in (c_n, x_n)$, relative to the surpluses generated in those markets. This suggest that entrepreneurs or outside merchants would prefer to target markets with i slightly less than x_n and avoid markets with i slightly greater than x_{n-1}. Of course, if a mass of entrepreneurs enter into these markets, then the platform would adjust its fees and its categories. While our model is static and does not consider product development or innovation, it is worth pointing out how these dynamic decisions might come into play in our model.

Returning to Examples 1 and 2 where we consider linear and constant elasticity demand systems under unit fees, our IFD results highlight how the underlying demand systems within a platform marketplace impact the platform's equilibrium strategies.

Example 1 (Imperfect Fee Discrimination with Linear Demands).

Unlike the PFD consideration of linear demands, differences across markets within a category do not earn personalized fees under IFD. To ensure that our assumptions on $\Pi_p(i)$ hold (namely $\Pi_p'(i) > 0$), Corollary 1 implies that we are easily able to consider three cases in which markets can differ across i. First, markets may be increasing in intercept so that $a'(i) > 0$ and $b'(i), c'(i) = 0$ (Case A). Second, markets could be decreasing in slope so that $b'(i) < 0$ and $a'(i), c'(i) = 0$ (Case B). Third, markets can be decreasing in marginal cost so that $c'(i) < 0$ and $a'(i), b'(i) = 0$ (Case C).

In Case A, the retail subgame equilibrium quantity in market i for some f is given by $q(i, f) = \frac{b}{2}\left(\frac{a(i)}{b} - c - f\right)$. This implies that $\Pi(i, f) = f \cdot q(i, f) = f \cdot \frac{b}{2}\left(\frac{a(i)}{b} - c - f\right)$ and $\Pi_p(i) = \frac{b}{8}\left(\frac{a(i)}{b} - c\right)^2$ as shown above in Equation (4) and Example 1. Naturally, if the $a(i)$ are distributed uniformly across i so that $a(i)$ is linear in i, then $\Pi_p(i)$ is quadratic in i and $\Pi(i, f)$ is uniform (linear) across i, similar to Figure 1, so that midpointing occurs in equilibrium by Corollary 3. Instead, if $a(i)$ is quadratic and convex in i, then $\frac{\partial^3 \Pi_p(i)}{\partial i^3} > 0$ and $\frac{\partial^3 \Pi(i,f)}{\partial i^3} = 0$ so that the platform skews categories and fees to the right (by Theorem 2):

so that $c_n^* > \frac{x_n^*+x_{n-1}^*}{2}$ and $x_n^* > {}^n\!/\!{}_N$ for all $n = 1, ..., N-1$.

In Case B, we have that $\Pi(i,f) = f \cdot q(i,f) = f \cdot \frac{b(i)}{2}\left(\frac{a}{b(i)} - c - f\right)$ and $\Pi_p(i) = \frac{b(i)}{8}\left(\frac{a}{b(i)} - c\right)^2$. Thus, if $b(i)$ is uniformly distributed (linear) across i, then $\frac{\partial^3 \Pi_p(i)}{\partial i^3} > 0$ while $\Pi(i,f)$ is uniform (linear) across i so that $\frac{\partial^3 \Pi(i,f)}{\partial i^3} = 0$. As a result, Theorem 2 implies that midpointing will not occur in equilibrium. Instead, the platform will skew categories and fees to the right so that $c_n^* > \frac{x_n^*+x_{n-1}^*}{2}$ and $x_n > {}^n\!/\!{}_N$ for all $n = 1, ..., N-1$.

In Case C, we have that $\Pi(i,f) = f \cdot q(i,f) = f \cdot \frac{b}{2}\left(\frac{a}{b} - c(i) - f\right)$ and $\Pi_p(i) = \frac{b}{8}\left(\frac{a}{b} - c(i)\right)^2$. This produces results similar to Case A.

Example 2 (Imperfect Fee Discrimination with CE Demands).

Extending the CE demands example to IFD, we utilize Corollary 1 to ensure that $\Pi_p'(i) > 0$ will occur when we introduce market variation across i. In particular, Corollary 1 implies that there are two cases in which markets can differ across i. First, markets may be decreasing in elasticity so that $\epsilon'(i) < 0$ and $c'(i) = 0$ (Case E). Second, markets could be decreasing in marginal cost so that $c'(i) < 0$ and $\epsilon'(i) = 0$ (Case F).

In Case E, the retail subgame equilibrium quantity in market i for some f is given by $q(i,f) = k\left(\frac{\epsilon(i)}{1+\epsilon(i)} \cdot (c+f)\right)^{\epsilon(i)}$. This implies that $\Pi(i,f) = f \cdot q(i,f) = f \cdot k\left(\frac{\epsilon(i)}{1+\epsilon(i)} \cdot (c+f)\right)^{\epsilon(i)}$ and $\Pi_p(i) = \frac{-kc^{1+\epsilon(i)}}{1+\epsilon(i)}\left(\frac{\epsilon(i)}{1+\epsilon(i)}\right)^{2\epsilon(i)}$ as shown above in Equation (4) and Example 2. Focusing on the case where $\epsilon(i)$ is linear, it appears as though the $\frac{\partial^3 \Pi_p(i)}{\partial i^3} - \frac{\partial^3 \Pi(i,f)}{\partial i^3}$ approaches 0 as ϵ becomes more negative (i.e., as i increases). This suggests that midpointing will occur in equilibrium so long as demands are sufficiently elastic.

In Case F, we have that $\Pi(i,f) = f \cdot q(i,f) = f \cdot k\left(\frac{\epsilon}{1+\epsilon} \cdot (c(i)+f)\right)^{\epsilon}$ and $\Pi_p(i) = \frac{-k[c(i)]^{1+\epsilon}}{1+\epsilon}\left(\frac{\epsilon}{1+\epsilon}\right)^{2\epsilon}$. Focusing on the case where $c(i)$ is linear, we have that $\frac{\partial^3 \Pi_p(i)}{\partial i^3} - \frac{\partial^3 \Pi(i,f^*(i))}{\partial i^3} > 0$ for all $\epsilon < -{}^2\!/\!{}_3$. We already assume that $\epsilon < -1$ to satisfy Lemma 1. Thus, the platform does not implement midpointing and instead skews categories and fees to the right so that $c_n^* > \frac{x_n^*+x_{n-1}^*}{2}$ for all $n = 1, ..., N$ and $x_n^* > {}^n\!/\!{}_N$ for all $n = 1, ..., N-1$.

4 Welfare and Market Foreclosure

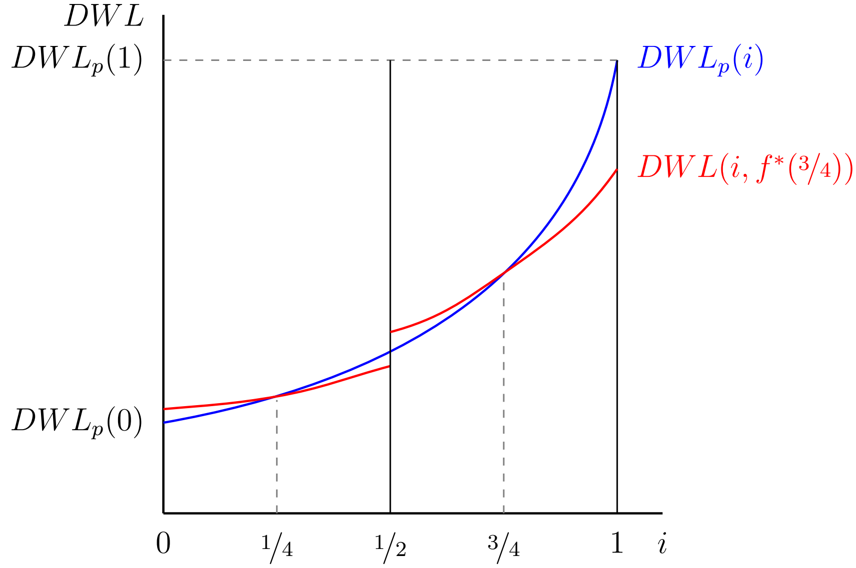
To this point, we have not considered market foreclosure. Market foreclosure arises in an IFD equilibrium if there exist marginally profitable markets. Under PFD, market foreclosure is not a concern since markets with near-zero profitability will receive very low fees (and in the limit, fees approach zero for markets that earn zero profit). However, with IFD, it may be optimal for the platform to keep its fee in C_1 sufficiently high so that some markets foreclose.

As an example of market foreclosure, note that if $\Pi(0, f^*(1/4))$ is less than zero in Figure 1, then midpointing will no longer be optimal since some markets will foreclose under midpointing. That is, if $\Pi(0, f^*(1/4)) < 0$, then there exists an $x' > 0$ so that the $i \in [0, x']$ foreclose and $\int_{x'}^{1/4} \Pi_p(i) - \Pi(i, f^*(1/4))di < \int_{1/4}^{1/2} \Pi_p(i) - \Pi(i, f^*(1/4))di$ so that midpointing is suboptimal. In this case, Theorem 2 no longer applies and the platform will skew fees to the right within a category and will shrink categories to the right, much like some of the examples provided in the previous section.

The consideration of market foreclosure is especially important as it relates to welfare and optimal policy. Without market foreclosure, fee distortions differ between the PFD and IFD cases. With PFD, fee distortions are perfect, so to speak, in that every market faces a market specific fee that is set by the monopoly platform. With IFD however, fees are fixed within each category so that some markets face a fee that is too high (the $i \in [x_{n-1}, c_n)$), from the platform's perspective, while other markets face a fee that is too low (the $i \in (c_n, x_n)$). The relatively higher fees in markets $i < c_n$ under IFD exacerbate the fee distortions in those market (relative to PFD), while the relatively lower fees in markets $i > c_n$ under IFD will dampen the fee distortions in those markets (relative to PFD). We display how these changes in fee distortions impact IFD deadweight loss relative to PFD deadweight loss in Figure 3. Note that these differences in fee distortions imply that the welfare comparison between PFD and IFD might be ambiguous (even without market foreclosure).

Under our general formulation of platform profit from Section 3, we were able to utilize the envelope theorem to develop a relationship between the PFD and IFD profit functions.

Figure 3: An Example of Dead Weight-Loss Under Midpointing



Unfortunately, no such simplification exists under a general characterization of deadweight loss across markets. Thus, to consider welfare across a platform marketplace, we impose a linear demand structure (considered in Example 1) on the underlying product demands so that $q(i, p) = a(i) - b(i)p$ with marginal cost $c(i)$.[21]

To better understand the welfare effects of fee discrimination, we focus on marketplaces where the underlying markets differ in what we define as the *Intercept-Cost Margin*:

Definition 2. If the slope of demand is constant across markets, $b(i) = b$ for all $i \in [0, 1]$, then we define the ***intercept-cost margin*** in market i as $m(i) := \frac{a(i)}{b} - c(i)$.

To simplify notation further, we assume that $b = 1$ when focusing on intercept-cost margins so that $m(i) = a(i) - c(i)$.

Our work in Example 1 shows that the platform implements midpointing when markets only differ uniformly across demand intercepts (where $a(i)$ linear in i while $b(i) = b$ and $c(i) = c$) or marginal costs (where $c(i)$ linear in i while $a(i) = a$ and $b(i) = b$). These are both special cases of markets that differ uniformly across intercept-cost margins, and it is

[21]The motivation for focusing on linear demands moving forward is two fold. First, linear demands are more tractable. Second, CE demands (considered Examples 2) produce total welfare that is infinite, making welfare considerations difficult.

straightforward to show that midpointing occurs under the more general setting of markets that differ uniformly across intercept-cost margins.[22]

As shown in the following proposition, if markets differ uniformly across intercept-cost margins, then the fee distortion mitigation that is generated in markets $i \in [x_{n-1}, c_n)$ from a move from IFD to PFD does not cover the harsher fee distortion that is generated in markets $i \in (c_n, x_n)$. More importantly, the major implication of this comparison is that welfare is decreasing in N, reaching its lower limit in the PFD equilibrium. Of course, all of this presumes zero market foreclosure.

Proposition 1. *If markets differ uniformly across intercept-cost margins and product demands are such that no market forecloses, then midpointing occurs in equilibrium (by Corollary 3), and total welfare across the platform marketplace is decreasing in the number of categories (N) reaching its lower limit in the PFD equilibrium.*

The intuition behind this result is that the markets that face a fee below their PFD fee (the $i \in (c_n, x_n)$) are also the markets that produce the most surplus. Thus, if midpointing occurs, then we see that the reduction in fee distortion for the high surplus markets outweighs the worsening fee distortions in the low surplus markets. For example, in Figure 3 this implies that the area between the red line and the blue line for $i \in (1/4, 1/2)$ ($i \in (3/4, 1)$) is larger than the between the blue line and the red line for $i \in (0, 1/4)$ ($i \in (1/2, 3/4)$). In this case, less fee discrimination by the platform improves welfare.

While this result suggests that platform fee discrimination is largely harmful, it is important to note the caveat of zero market foreclosure in the statement of Proposition 1. If instead there are merchants on the margin of participation, then a reduction in fee discrimination may increase market foreclosure. Naturally, this would result in a welfare tradeoff from an increase in fee discrimination: greater fee discrimination worsens the welfare effects brought on by fee distortions but also reduces market foreclosure.[23] However, we find that,

[22]From Equation (4), $\Pi(f^*) = \Pi_p(i)$ implies that if $m(i) = \frac{a(i)}{b} - c(i)$ is linear in i, then $\Pi_p(i)$ is quadratic in i so that midpointing occurs by Corollary 3.

[23]It is worth noting that the issue of market foreclosure, and it's detriment to welfare, might be less of

even in the presence of market foreclosure, platform fee discrimination is harmful:

Proposition 2. *If markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees* ($m(0) = 0$)*, then a platform that utilizes* N *categories will foreclose markets* $i \in \left[0, \frac{1}{2N+1}\right)$ *and then apply midpointing on the remaining set of markets,* $i \in \left[\frac{1}{2N+1}, 1\right]$*, so that* $x_n^* = \frac{2n+1}{2N+1}$ *and fees are given by* $c_n^* = \frac{x_n^* + x_{n-1}^*}{2} = \frac{2n}{2N+1}$ *for* $n = 0, 1, ...N$*.*

In addition, marketplace welfare is strictly decreasing in N *and reaches its lower limit in the PFD equilibrium.*

This proposition highlights how an increase in fee discrimination reduces market foreclosure (a benefit to welfare), but still results in a net welfare loss due to the worsening fee distortions in the higher surplus markets (a cost to welfare). From a policy perspective, such a result provides an unambiguous recommendation that we summarize in the following corollary:

Corollary 4. *If the marginal market earns zero profit (without platform fees) and markets differ uniformly across intercept-cost margins, then a ban on fee discrimination, so that* $N = 1$*, maximizes welfare.*

Corollary 4 provides a very natural policy recommendation: ban fee discrimination. Instead of allowing the marketplace to offer different fees across products, a policy maker could ban such practices and require that the platform charge a single fee that is applied to every market. While this policy recommendation is compelling, there are a few caveats worth mentioning. First, we impose some structure in order to derive these findings. In particular, markets differ *uniformly* across intercept-cost margins; instead, if the majority of markets are on the margin of participation, then it may be possible for welfare to be concave in the extent of price discrimination (N). In this case, limiting platform fee discrimination would welfare

a concern in practice than in our model since our model considers unit fees (which are fixed across prices) while platforms often use ad valorem fees (that adjust to lower prices). We provide some evidence for this below in Section 5 using data derived from airbnb.com.

dominate an outright ban (e.g., letting N equal two, three, or four might produce greater welfare than N equal to 1). Second, this analysis ignores platform entry into retail which is seen in some platform marketplaces (Amazon) but not others (eBay). We consider platform retail entry as an extension in the following section and we discuss the corresponding policy suggestions that follow.

5 Implementing Fee Discrimination Using Airbnb Data

To better understand how a platform might implement fee discrimination in practice, we use data derived from airbnb.com to consider PFD as well as IFD across cities. The data were collected by AirDNA, a third-party source that frequently scrapes property, availability, host, and review information from the website.[24] These data cover 27 major metropolitan areas across the United States and include over 220,000 properties that were booked at least once in 2016.[25]

To determine the PFD and IFD strategies for Airbnb, we set $q(i)$ equal to the number of nights booked in 2016 for listing i and we set $p(i)$ equal to the average booking price in 2016 for listing i. If we assume demands are linear and marginal costs are constant for each listing, given by $q(i) = a(i) - b(i)p(i)$ and $c(i)$, then we have two variables that are known, $q(i)$ and $p(i)$, and three variables that are unknown: $a(i)$, $b(i)$, and $c(i)$. To resolve this issue, we pull a range of demand elasticities from Farronato and Fradkin (2018).[26] To assign $\epsilon(i)$ to specific listings, note that $\epsilon(i) := \frac{dq(i)}{dp(i)} \cdot \frac{p(i)}{q(i)} = -b(i) \cdot \frac{p(i)}{q(i)}$ and so we sort the listings i by $\frac{p(i)}{q(i)}$,

[24]AirDNA's website is `https://www.airdna.co/`.

[25]The 27 metros are Anchorage, Atlanta, Austin, Boston, Charlotte, Chicago, Cleveland, Dallas, Denver, Houston, Indianapolis, Los Angeles, Louisville, Miami, Minneapolis, Nashville, New Orleans, New York City, Oakland, Orlando, Philadelphia, Phoenix, Salt Lake City, San Diego, San Jose, Seattle, and Washington, D.C.

[26]Specifically, in Table A6: Demand Cross-Price Elasticities by Accommodation Type in the appendix of Farronato and Fradkin (2018), the diagonal across the Airbnb Top (100-75 percentile), Airbnb Upper Mid (75-50 percentile), Airbnb Lower Mid (50-25 percentile), and Airbnb Low (25-0 percentile) give a range of elasticities across all listings in their sample. To linearize this across the $i \in [0, 1]$, we take the Airbnb Top elasticity of -4.77 to be given by listing $i = 0.875$ (the mid point between the 100th and 75th percentile) and we take the Airbnb Low elasticity of -2.66 to be given by listing $i = 0.125$ (the mid point between the 25th and 0th percentile). The implies that $\epsilon(i) := \frac{dq(i)}{dp(i)} \cdot \frac{p(i)}{q(i)} = -b(i) \cdot \frac{p(i)}{q(i)}$ can be written as $\epsilon(i) = \frac{-277}{120} - \frac{211}{75} \cdot i$.

which we know from the data, and we assign each listing it's $i \in [0, 1]$ value based on this ordering. With the order in place, we use the monopoly host solution, $p(i) = \frac{1}{2}\left(\frac{a(i)}{b(i)} + \frac{c(i)}{1-t}\right)$ and $q(i) = \frac{b(i)}{2}\left(\frac{a(i)}{b(i)} - \frac{c(i)}{1-t}\right)$ where $t = 0.03$ since Airbnb charges a 3% fee to all hosts,[27] and the elasticity formula, $\epsilon(i) = -b(i) \cdot \frac{p(i)}{q(i)}$, so that we are able to solve for the numerical values of $a(i)$, $b(i)$, and $c(i)$, given the $p(i)$, $q(i)$, and $\epsilon(i)$ from the data.[28]

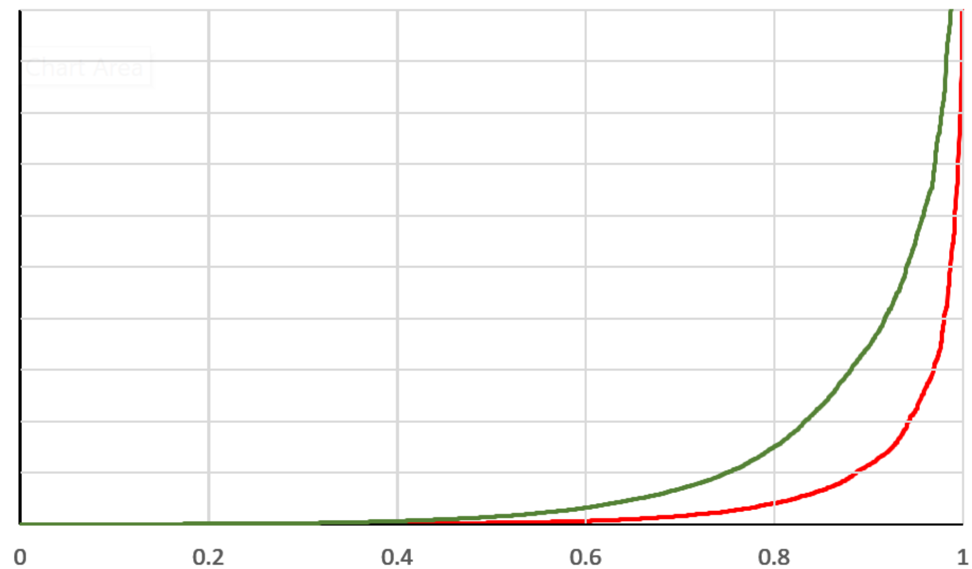
With the demand and marginal cost parameters for each listing in hand, Equation (4) gives platform profit under PFD across all listings. To get a better sense for how Airbnb might use this information to implement fee discrimination, we breakup the sample by city so that we can consider city specific PFD formula. Figure 4 shows the PFDs for the two cities in our sample that differ the most: Houston (in red) and Seattle (in green). Given that Seattle's PFD is greater than Houston's PFD, if Airbnb were to use host fees that differ across cities, instead of a single host fee across it's entire marketplace, then Airbnb would charge a higher host fee in Seattle than in Houston. Furthermore, if Airbnb used a greater number of categories and fees within each city ($N > 1$ for Houston and for Seattle), then we expect the Seattle category and fee thresholds to be greater than those for Houston ($x_n^{SEA} > x_n^{HOU}$ and $c_n^{SEA} > c_n^{HOU}$).

Focusing on Seattle, we also consider different IFD strategies using our result in Theorem 2. In particular, if only a single fee is used in Seattle, $N = 1$, and that fee is given by the market $c_1 = 0.5$ so that midpointing occurs, then the IFD profits are worse than if that fee is skewed to the right (e.g., by the market $c_1 = 0.8$). We see this explicitly in Figure 5 where the graph on the left captures the IFD under $N = 1$ and $c_1 = 0.5$ while the graph on the right captures the IFD under $N = 1$ and $c_1 = 0.8$. While there is obviously a degree of error in the calculations presented in the two graphs (given the back-of-the-envelope approach taken in this exercise), we see that the blue (PFD) and red (IFD) lines intersect around c_1 as prescribed in our theoretical analysis. In addition, it is clear that IFD profits under the

[27]See "What is the Airbnb service fee?" by Airbnb Help Center for details.

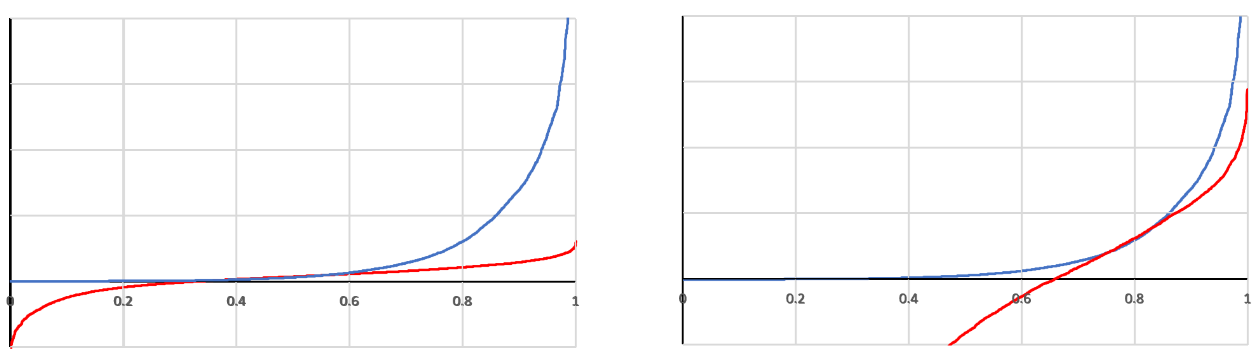
[28]More specifically, solving the system of equations implies that $a(i) = (1 - \epsilon(i))q(i)$, $b(i) = -\epsilon(i) \cdot \frac{q(i)}{p(i)}$, and $c(i) = (1 - t)(1 + \epsilon(i))p(i)$.

Figure 4: PFD Formulas for Houston (red) and Seattle (green)



larger fee ($c_1 = 0.8$) dominate the IFD profits under the smaller fee ($c_1 = 0.5$), and this implies that fees and category selections under IFD will skew to the right for $N \geq 1$.

Figure 5: A Single Fee at $c_1 = 0.5$ (Left) or at $c_1 = 0.8$ (Right)



The results from Figure 5 also highlight how the use of unit fees in our analysis might result in greater market foreclosure due to limiting fee discrimination than would actually be the case in practice. In particular, note that every listing in our data earned a booking in 2016 while Airbnb was utilizing a single ad valorem fee of 3% (and was therefore *not* foreclosed). However, under a single unit fee, we see that roughly a third of listings would foreclose with the unit fee given by $c_1 = 0.5$ and over half of listings would foreclose with the unit fee given by $c_1 = 0.8$.

This suggests that foreclosure might not be an issue in practice, since platforms use ad valorem fees, which is necessarily important for welfare. In particular, leading up to our main results in Section 4 on welfare, we highlight how a welfare tradeoff from greater fee discrimination might exist when there are a number of markets on the margin of participation: greater fee discrimination increases fee distortions (detrimental to welfare) but also reduces market foreclosure (beneficial to welfare). Nevertheless, we show in Proposition 2 that welfare decreases with greater fee discrimination so that the fee distortion losses can outweigh the reduction in foreclosure benefits. Thus, the existence of listings on Airbnb under their ad valorem fee of 3% that would foreclose under a unit fee provides further evidence that the market foreclosure effect is minor in comparison to the fee distortion effect.

6 Extensions and Policy

To this point, our main focus lied in understanding the marketplace fee discrimination problem and our discussion of policy only pertained to the welfare findings discovered so far. However, there are several other measures within the marketplace that are worth consideration. We start by extending the model to consider platform entry into commerce. In addition, we analyze the effects of vertical integration between the platform and its merchants, banning fee discrimination and/or platform entry into retail, Pareto improving tax policies under perfect tax discrimination, and two additional regulations that level the retail playing field upon platform entry into commerce.

6.1 Platform Entry into Commerce

It is often the case that platform marketplaces enter into retail to compete with their merchants. Amazon is the best example of such a marketplace, but other marketplaces could also pursue such a strategy in the future (e.g., Airbnb might acquire hotels or eBay could become a retailer). In this section, we consider how the combination of platform entry into retail and platform fee discrimination impact the PFD and IFD equilibria. To do so, we impose the linear demand specification in retail markets (as in Example 1). While this assumption precludes a general formulation that matches our characterizations of the PFD and IFD equilibria, imposing some structure allows us to distinguish the key factors that impact entry decisions, fees and categories, welfare across agents, and optimal policy regarding fee discrimination and entry into commerce.

As in Example 1, we specify linear demand so that $q(i,p) = a(i) - b(i)p$ with marginal cost $c(i)$. If the platform enters into market i, then retail competition between the merchant and the platform ensues. We assume that platform entry is costless, but we let the platform and merchant differ in marginal costs: we let the marginal cost of the platform be $c_p(i) = c(i) + d(i)$ so that $d(i) > 0$ corresponds to a cost disadvantage for the platform and $d(i) < 0$ captures a cost advantage. It is important to note that $c(i)$ captures the micro-founded marginal cost for the merchant, but the merchant's effective marginal cost (inclusive of the merchant fee) is given by $c_m(i) = c(i) + f(i)$. Thus, if $d(i) < f(i)$, then the platform has an effective marginal cost advantage over the merchant.

Conditional on platform entry, we suppose that the platform and merchant compete in quantities *à la* Cournot. To accommodate quantity competition and to keep our formulation isomorphic to the previous sections, we rewrite demand ($q(i,p) = a(i) - b(i)p$) as the inverse demand formula given by $p(i,Q) = \frac{a(i)}{b(i)} - \frac{1}{b(i)}Q$, where $Q = q_m + q_p$ is the aggregate demand and q_m (q_p) is the output chosen by the merchant (platform).[29]

[29]Note that this specification implies that the platform's effective marginal cost forecloses them from market i when $d(i) > \frac{a(i)}{b(i)} - c(i)$ and the merchant's effective marginal cost forecloses them from retail when $f(i) > \frac{a(i)}{b(i)} - c(i)$.

6.1.1 Platform Entry Under Perfect Fee Discrimination

Under PFD, the platform chooses whether or not to enter into market i and then chooses the merchant fee, $f(i)$, in market i. To reduce notation, we drop the (i) in this subsection. Solving for the retail subgame (for arbitrary f) and then maximizing platform profit across retail sales and merchant fees (with respect to f) implies the following:

Lemma 3. *The platform enters into retail if $d \in \left(0, \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$. In this case, the platform entry into retail subgame equilibrium is given by*

$$f^E = \frac{1}{2}\left(\frac{a}{b} - c\right) - \frac{d}{10},\; p^E = \frac{1}{2}\left(\frac{a}{b} + c\right) + \frac{3d}{10},\; q_m^E = \frac{2bd}{5},\; Q^E = \frac{b}{2}\left(\left(\frac{a}{b} - c\right) - \frac{3d}{5}\right). \quad (6)$$

If $d \leq 0$, then the platform enters as a monopolist by setting $f^E = \infty$. Finally, if $d \geq \frac{15}{21}\left(\frac{a}{b} - c\right)$, then no entry occurs and the PFD no entry equilibrium ensues with $f^E = f^*$.

By comparing the PFD equilibrium with and without platform entry, from Equations (4) and (6), we see that $f^E - f^* = \frac{-d}{10} < 0$ so that the platform reduces its merchant fee relative to the equilibrium without entry. The logic here is straight-forward: with entry, the merchant has less of an incentive to provide output and this reduces the platform's incentive to charge a high merchant fee.[30] Comparing across equilibria also reveals that total output increases and retail prices decrease with platform entry: $Q^E > q^*$ and $p^E < p^*$.[31] Lastly, comparing merchant outputs we see that merchants might also produce more under platform entry (due to the merchant fee reduction): $q_m^E > q^*$ if and only if $d > \frac{5}{8}\left(\frac{a}{b} - c\right)$.[32]

The equilibrium comparisons across outputs, prices, and fees reveal that consumer surplus increases with platform entry since prices decrease and total output rises. Obviously, platform profit increases with entry (otherwise the platform would implement the non-entry

[30] More specifically, the retail subgame equilibria imply that $-\frac{dq(f)}{df} = \frac{b}{2} < \frac{2b}{3} = -\frac{dq_m(f)}{df}$ so that an increase in f reduces the merchant's output by more under platform entry.

[31] Algebraically, these inequalities hold if and only if $d < \frac{5}{6}\left(\frac{a}{b} - c\right)$ which holds under the duopoly equilibrium since $\frac{15}{21} < \frac{5}{6}$.

[32] Recall that a duopoly occurs whenever $d \in \left(0, \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$ so that we have $q_m^E > q^*$ if and only if $d \in \left(\frac{5}{8}\left(\frac{a}{b} - c\right), \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$.

PFD equilibrium). For merchants, the impact is ambiguous. Greater competition reduces profitability on the one hand; on the other hand, the reduction in the merchant fee improves merchant profitability. By analyzing welfare explicitly, we find that platform retail entry under PFD always increases welfare and can be Pareto improving under certain parameter values:

Proposition 3. *Under PFD, welfare is greater when the platform enters into retail,* $d \in \left(0, \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$*, and under all merchant foreclosure equilibria,* $d \leq 0$*, relative to the no entry PFD equilibrium. In addition, if* $d \in \left(\frac{5}{8}\left(\frac{a}{b} - c\right), \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$*, then platform entry Pareto improves upon the no entry case.*

From a policy perspective, the results in Proposition 3 potentially generate a tax policy that Pareto improves upon welfare while generating tax revenues under certain values of d: tax only the platform in retail so that the unit tax τ is such that $d + \tau \in \left(\frac{5}{8}\left(\frac{a}{b} - c\right), \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$. That is, if, like the platform, a policy maker can implement market specific taxes perfectly that target only the platform (not the merchant), then the policy maker can choose τ such that $d + \tau \in \left(\frac{5}{8}\left(\frac{a}{b} - c\right), \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$ and our findings from Proposition 3 imply that such a tax policy is Pareto improving. Furthermore, if $d \in \left(0, \frac{5}{8}\left(\frac{a}{b} - c\right)\right)$, then such a τ is greater than zero so that the Pareto improving tax also generates tax revenues. Such a policy may be difficult to implement for a variety of reasons. We leave this discussion for our policy subsection below where we introduce several other regulations that policy makers may want to consider.

6.1.2 Platform Entry Under Imperfect Fee Discrimination

Next consider how platform entry into commerce impacts the IFD equilibrium. In this case, the platform first selects its N categories and fees, and then decides which markets to enter within a category. Solving the game backwards, we first consider platform entry into retail under an arbitrary category and fee.

Lemma 4. *If the platform sets fee f_n in category $n = 1, ..., N$, and has markets that differ uniformly across intercept-cost margins so that the marginal market earns zero profit without platform fees ($m(0) = 0$), then for category n there exists a $d_n > 0$ so that for all $d \in (0, d_n)$ there exists an $e_n \in C_n = (x_{n-1}, x_n)$ so that the platform enters into markets $i \in [e_n, x_n]$ in category n. Furthermore, if $d < 0$, then $e_n = x_{n-1}$ so that the platform enters into every market in category n.*

In light of the entry decisions within a category, Lemma 4 implies that if entry occurs, then the platform will enter into markets with a larger i (markets that produce greater surplus within the category).[33] From Lemma 3, we know that the PFD fee decreases in markets where the platform enter into retail. Combined, these results imply that the markets in category n with large i ($i \in [e_n, x_n]$) will have PFD fees that are lower with platform entry than without.

This has two important effects that impact the platform's category and fee decisions. First, lower PFD fees with entry shrinks the profit loss from IFD in markets $i \in [e_n, x_n]$ since the cost from a fee that is too low, relative to the PFD fee, reduces with platform entry. Second, the reduction in the average PFD fee with entry suggest that the platform will choose a lower fee within categories under entry in IFD. These effects result in lower fees and fewer market foreclosures than if the platform is unable to enter into retail. These effects increase welfare relative to the IFD no entry equilibrium.

Proposition 4. *Under IFD, if markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees ($m(0) = 0$), then there exists a $d^E > 0$ so that at least some platform entry into retail occurs for all $N \geq 1$ when $d < d^E$. In this case, the IFD equilibrium fees and categories target lower markets relative to the non-entry counterpart ($c_n^E < c_n^*$ and $x_n^E < x_n^*$ for $n = 0, 1, ..., N-1$), and the IFD equilibrium produces greater welfare with platform entry.*

[33] Zhu and Liu (2018) confirm this finding empirically for Amazon by showing that Amazon is more likely to enter into retail for product categories that are more successful in terms of sales.

Propositions 3 and 4 highlight how, regardless of the extent of platform fee discrimination, platform entry into retail results in lower fees (on average), stronger retail competition (on average), and greater welfare across the marketplace. This suggests that, while many Amazon merchants often voice their disapproval of Amazon's entry into retail, such entry is beneficial to consumers, Amazon, and some merchants.

6.2 Vertical Integration

While IFD dichotomizes the fee distortions that arise within the marketplace, the vertical relationship between the platform and its merchants suggests that a merger between the two might rectify welfare losses. At the same time, if a platform can enter into retail, then a merger between the platform and a merchant might stifle retail competition and harm welfare. To analyze how vertical integration impacts both PFD and IFD equilibria, we first consider how welfare under a vertical merger compares to welfare under platform entry in market i for an arbitrary merchant fee $f(i)$. In this case, we have the following result:

Proposition 5. *If $d(i) > 0$, then welfare under a vertical merger strictly dominates welfare under platform entry into retail if and only if $\frac{a(i)}{b(i)} - c(i) < d(i) + f(i)$. If $d(i) \leq 0$, then welfare under platform entry into retail strictly dominates welfare under a vertical merger.*

If the platform utilizes IFD, then it is possible for welfare under a vertical merger to dominate welfare under platform entry (i.e., for $\frac{a(i)}{b(i)} - c(i) < d(i) + f(i)$ to hold). Recall from IFD that a market i in category n with i close to x_{n-1} has low $\frac{a(i)}{b(i)} - c(i)$ relative to some market j in category n that is close to x_n, but both markets face the same merchant fee $f^*(c_n^E)$. This implies that for the i close to x_{n-1}, it is "more likely" to have $\frac{a(i)}{b(i)} - c(i) < d(i) + f^*(c_n^E)$ hold so that welfare under a vertical merger strictly dominates welfare under platform entry. This implies that vertical mergers within markets that produce the least surplus within their category, have the lowest $\frac{a(i)}{b(i)} - c(i)$, might improve welfare.

As we mention prior to Proposition 5, a merger generates a tradeoff by simultaneously reducing fee distortions (benefiting welfare) while reducing retail competition (harming welfare). Note that for low or negative values of $d(i)$ (representing the platform's cost disadvantage in retail), competition in retail between the merchant and the platform is fierce so that platform entry dominates a vertical merger in terms of welfare: $\frac{a(i)}{b(i)} - c(i) > d(i) + f(i)$. However, for high values of $d(i)$, platform entry only has a marginal impact on retail competition so that a vertical merger that eliminates the merchant fee will improve welfare. To investigate the welfare tradeoff from a vertical merger further, we determine the markets that provide greater welfare under a vertical merger when the platform utilizes PFD:

Corollary 5. *If the platform uses PFD, then a vertical merger in market i generates greater welfare than platform entry for all* $d(i) \in \left(\frac{5}{9}\left(\frac{a(i)}{b(i)} - c(i)\right), \frac{15}{21}\left(\frac{a(i)}{b(i)} - c(i)\right)\right)$.[34]

Under PFD, recall that the platform only enters into retail for $d(i) < \frac{15}{21}\left(\frac{a(i)}{b(i)} - c(i)\right)$. Thus, we see that vertical mergers generate greater welfare precisely when the platform's cost disadvantage is at its highest. These findings suggest that vertical mergers when the platform and its retailer face similar marginal costs (e.g., they have the same supplier) should face some antitrust scrutiny. However, if the retailer has a considerable cost advantage over the platform, then a vertical merger would mitigate significant fee distortions and improve welfare.

6.3 Banning Fee Discrimination or Banning Platform Entry

From our welfare analysis without platform entry into retail (Section 4), we know that banning platform fee discrimination so that $N = 1$ results in some market foreclosure but increases welfare (Corollary 4). Furthermore, Proposition 1 shows how welfare reaches its minimum at the PFD equilibrium when the platform does not enter into retail. These findings suggest that imposing greater restrictions on platforms will improve welfare.

On the other hand, our results on platform retail entry suggest that this logic may be fallible. Propositions 3 and 4 highlight how platform entry into retail reduces merchant fees

[34] We offer the algebra behind this result in the Proof of Proposition 5.

and increases retail competition so that welfare improves (a result that is robust across all forms of platform fee discrimination). This implies that banning platform entry into retail will be detrimental to welfare. Furthermore, by investigating the two extremes, (1) PFD with entry and (2) a ban on fee discrimination *and* entry, we find that a platform under PFD with entry produces greater welfare:

Proposition 6. *If markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees* ($m(0) = 0$)*, then the PFD equilibrium with platform entry into retail generates greater welfare than the equilibrium where the platform cannot use fee discrimination nor enter into retail.*

6.4 Perfect Tax Discrimination (PTD) Upon Platform Entry

Given the platform's inherent competitive advantage over its merchants through the merchant fee that is only paid by merchants in retail and not paid by the platform, a natural policy recommendation is to tax the platform exclusively in retail markets. In the world of perfect fee and tax discrimination, both the platform and the policy maker can set fees and taxes that are specific to market i. Under such an approach, Proposition 3 implies that there exist market specific taxes so that the PFD equilibrium with platform entry Pareto improves upon the PFD equilibrium without platform entry. This suggests that, instead of banning fee discrimination to improve welfare, a policy maker can tax a platform that enters into market i in such a way that (1) all agents are better off (a Pareto improvement) relative to the no platform entry PFD equilibrium and (2) welfare increases relative to a policy that bans entry and fee discrimination. We find that such a tax policy is possible so long as the platform's marginal cost disadvantage isn't too large:

Proposition 7. *There exists a* $d^{PTD}(i) > 0$ *so that for all* $d(i) < d^{PTD}(i)$ *there exists a market* i *unit tax that, when applied to the platform upon entry into market* i*, results in a platform PFD entry equilibrium that Pareto improves upon the equilibrium that bans platform* *entry under PFD. In addition, this equilibrium generates greater welfare than a ban on fee discrimination and entry.*

This result has several important implications relating to policy. First, PTD allows a policy maker to Pareto improve upon a policy that bans platform entry into retail when the platform uses PFD (this result builds on Proposition 3). Second, such a policy also generates greater welfare than a ban on platform fee discrimination and entry (an extension of Proposition 6). Lastly, it is worth noting that this Pareto improving policy also generates positive tax revenues.

6.5 Leveling the Retail Playing Field

While the PTD results are compelling, it may be difficult for the policy maker to set market specific tax rates that only kick-in upon platform entry into retail. Thus, we also consider more feasible policies that level the retail playing field between the platform and its merchants.

6.5.1 Banning Platform Merchant Fees Upon Platform Entry

Arguably the most obvious regulation that levels the retail playing field is one that eliminates the merchant fee upon platform retail entry so that the platform cannot use its merchant fee to distort retail competition. While such a policy benefits merchants, we see that it greatly reduces the extent of platform entry which harms welfare.

Proposition 8. *If the platform utilizes PFD and a policy maker requires that $f(i) = 0$ if the platform enters into market i, then there exists a $d(i)^{f(i)=0} < 0$ so that the platform enters into market i if and only if $d(i) < d(i)^{f(i)=0} < 0$. This policy is detrimental to welfare relative to PFD equilibrium where platform entry into retail is free.*

By eliminating the platform's use of the merchant fee upon entry into retail, the platform will only enter into retail in markets where it has a sufficient cost advantage over the merchant: $d(i) << 0$. Otherwise, the platform elects to use its merchant fee (instead of enter into retail) and this reduces welfare for all $d(i) > 0$ where platform entry improves welfare when the merchant fee is still allowed upon entry: the $d(i) < \frac{15}{21}\left(\frac{a(i)}{b(i)} - c(i)\right)$ by Proposition 3. These results suggest that banning the merchant fee upon platform entry into retail will prevent significant platform entry which harms consumers and total welfare.

In terms of implementation, it might be difficult for a policy maker to identify the market in which the platform enters (e.g., does a remote control toy car also compete with drones or non-remote control toy cars?).[35] Merchants under this policy will always have an incentive to plead, and potential invest in, product homogeneity across merchant and platform products which might reduce product differentiation (and welfare) in a dynamic model. At the same time, such a policy incentivizes the platform to enter into retail with unique products and to invest in costs savings that lower $d(i)$, and these incentives might improve welfare in a dynamic model.

6.5.2 Tax Matching: A Platform Tax Upon Entry Equal to the Merchant Fee

Another policy that levels the retail playing field is a tax on the platform upon retail entry that equals the merchant fee. Such a policy is arguably the simplest to implement of the policies proposed so far as the policy maker simply follows the platform's lead and does not make fee magnitude decisions. To consider the impact of such a policy on the marketplace, we focus on the case of PFD for simplicity and we drop the market (i) notation moving forward. In this case, the platform's effective marginal cost is given by $c_p = c + d + \tau = c + d + f$, where τ denotes the unit tax that equates to the merchant fee according to the policy under consideration. Under this policy, we find the following:

Proposition 9. *If the platform uses PFD and a policy maker enforces a tax on the platform upon retail entry that equates to the merchant fee, then platform entry into retail occurs for*

[35] One way to avoid this issue is to require that merchant fees reduce to zero in any category for which the platform enters. Such a policy piggybacks off of the platform's existing tools that sort products into categories.

all $d(i) < \frac{1}{5}\left(\frac{a(i)}{b(i)} - c(i)\right)$ *resulting in lower welfare relative to the equilibrium without a tax policy.*

While a policy that taxes a platform upon entry at a rate equal to the merchant fee naturally limits the extent of platform fees, we find that it is also detrimental to welfare. In terms of entry into retail (which increases retail competition), we see that the policy reduces platform entry: without the policy the platform enters into every market for $d \in \left(0, \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$ (see Lemma 3), but the policy reduces platform entry down to only the markets with $d < \frac{1}{5}\left(\frac{a}{b} - c\right)$.

The issue that arise with this tax policy (relative to taxation under PTD which is largely successful) is that by equating the platform entry tax with the merchant fee, the tax becomes endogenous to the platform. As we see in Proposition 9, this limits the extent to which the tax is implemented in a manner that benefits welfare; a result that is not surprising as the platform's objective is to maximize its own profits and not total welfare.

7 Conclusion

A platform's use of fee discrimination has gone largely unstudied even though marketplaces like Amazon and eBay currently use merchant fee discrimination extensively. By considering the effects of marketplace fee discrimination, we find that if the platform does not enter into retail, then greater fee discrimination allows the platform to serve more markets in its marketplace. On the other hand, greater fee discrimination also worsens fee distortions in the high surplus markets resulting in a net welfare loss.

If instead the platform enters into retail, then greater retail competition improves welfare and mitigates fee distortions. Furthermore, we show that banning fee discrimination and platform entry into retail produces less surplus than a marketplace that is free of regulation. In terms of vertical integration, while a vertical merger between the platform and a merchant eliminates fee distortions, we also find that the reduction in competition between the merchant and platform often outweighs the elimination of fee distortions so that platform entry into retail welfare dominates vertical integration. Lastly, we show that a policy maker can utilize perfect tax discrimination to make Pareto improvements upon the marketplace equilibrium.

Appendix of Proofs

Proof of Lemma 1: A merchant maximizing profit, Equation 1, produces the following first-order condition with respect to $p(i)$,

$$[(1-t(i))p^*(i) - c(i) - f(i)] \cdot \frac{\partial q^*(i, p^*(i))}{\partial p^*(i)} + (1-t(i)) \cdot q(i, p^*(i)) = 0,$$

which implicitly defines $p^*(i)$. The second-order condition of profit maximization requires that

$$2(1-t(i))\frac{\partial q^*(i, p^*(i))}{\partial p^*(i)} + \frac{\partial^2 q^*(i, p^*(i))}{\partial (p^*(i))^2} \cdot [(1-t(i))p^*(i) - c(i) - f(i)] < 0.$$

Applying the implicit function theorem to the first-order conditions implies that

$$\frac{\partial p^*(i)}{\partial f(i)} = \frac{-[-\frac{\partial q^*(i,p^*(i))}{\partial p^*(i)}]}{2(1-t(i))\frac{\partial q^*(i,p^*(i))}{\partial p^*(i)} + \frac{\partial^2 q^*(i,p^*(i))}{\partial (p^*(i))^2} \cdot [(1-t(i))p^*(i) - c(i) - f(i)]} > 0,$$

$$\frac{\partial p^*(i)}{\partial t(i)} = \frac{-[-q(i, p^*(i)) - \frac{\partial q^*(i,p^*(i))}{\partial p^*(i)} p^*(i)]}{2(1-t(i))\frac{\partial q^*(i,p^*(i))}{\partial p^*(i)} + \frac{\partial^2 q^*(i,p^*(i))}{\partial (p^*(i))^2} \cdot [(1-t(i))p^*(i) - c(i) - f(i)]},$$

which is greater than zero if and only if $-\frac{\partial q^*(i,p^*(i))}{\partial p^*(i)} \cdot p^*(i) > q(i, p^*(i))$. □

Proof of Theorem 1: The platform determines its optimal unit (ad valorem) fee by differentiating Equation 2 with respect to f (t), taking the merchant equilibrium, $p^*(t, f)$, as given. This produces the following first-order conditions:

$$q(p^*) + f \cdot \frac{\partial q^*(p^*)}{\partial p^*} \frac{\partial p^*}{\partial f} = 0,$$

$$p^* q(p^*) + t \cdot \left[\frac{\partial p^*}{\partial t} q(p^*) + p^* \frac{\partial q^*(p^*)}{\partial p^*} \frac{\partial p^*}{\partial t} \right] = 0.$$

The first-order condition for f implies that $\epsilon^f = -1$ since $\epsilon^f := \frac{\partial q(p^*)}{\partial f} \cdot \frac{f}{q(p^*)}$ and $\frac{\partial q(p^*)}{\partial f} = \frac{\partial q^*(p^*)}{\partial p^*} \frac{\partial p^*}{\partial f}$. The first-order condition for t implies that $\epsilon^R = -1$ since $\epsilon^R := \frac{\partial p^* \cdot q^*(p^*)}{\partial t} \cdot \frac{t}{p^* \cdot q^*(p^*)}$ and $\frac{\partial p^* \cdot q^*(p^*)}{\partial t} = \left[\frac{\partial p^*}{\partial t} q(p^*) + p^* \frac{\partial q^*(p^*)}{\partial p^*} \frac{\partial p^*}{\partial t} \right]$. To ensure profit maximization for each type of fee, the demand functions must satisfy the following second-order conditions:

$$
\begin{aligned}
0 &> 2\frac{\partial q^*(p^*)}{\partial p^*}\cdot\frac{\partial p^*}{\partial f}+f\cdot\left[\frac{\partial q^*(p^*)}{\partial p^*}\cdot\frac{\partial^2 p^*}{\partial f^2}+\frac{\partial^2 q^*(p^*)}{\partial (p^*)^2}\left(\frac{\partial p^*}{\partial f}\right)^2\right],\\
0 &> 2\left[q(p^*)\cdot\frac{\partial p^*}{\partial t}+p^*\frac{\partial q^*(p^*)}{\partial p^*}\cdot\frac{\partial p^*}{\partial t}\right]+\\
&\quad +t\cdot\left[\frac{\partial^2 p^*}{\partial t^2}q(p^*)+2\left(\frac{\partial p^*}{\partial t}\right)^2\frac{\partial q^*(p^*)}{\partial p^*}+p^*\frac{\partial^2 q^*(p^*)}{\partial (p^*)^2}\left(\frac{\partial p^*}{\partial t}\right)^2+p^*\frac{\partial q^*(p^*)}{\partial p^*}\frac{\partial^2 p^*}{\partial t^2}\right].
\end{aligned}
$$

□

Proof of Lemma 2: Given that $\Pi(i,f)$ is maximized at $f=f^*(i)$, we have that $\Pi_p(i)-\Pi(i,f(i))$ increases as $f(i)$ departs from $f^*(i)$. In addition, the monotonicity of $f^*(i)$ implies that $\Pi_p(i)-\Pi(i,f^*(j))$ increases as j departs from i. Thus, if some C_n is not a convex set, then there exists $j,k\in C_n$ and some $\theta\in(0,1)$ so that $z=\theta\cdot j+(1-\theta)\cdot k\in C_m$ for $m\neq n$ with $f(z)=f^*(c_m)\neq f^*(c_n)=f(j)=f(k)$. Without loss of generality, suppose that $j<k$. Subsequently, if $f^*(i)$ is monotone increasing in i, then $f^*(j)<f^*(z)<f^*(k)$. To minimize profit loss, $f^*(j)<f^*(z)$ implies that $f^*(c_n)<f^*(c_m)$; in addition, $f^*(z)<f^*(k)$ implies that $f^*(c_m)<f^*(c_n)$ (a contradiction). A similar argument holds when $f^*(i)$ is monotone decreasing in i. Thus, categories must be convex sets.

Now consider the determination of an optimal fee for a given category. This program is characterized by: $\max_f \int_{i\in C_n}\Pi(i,f)di=\int_{i\in C_n} f\cdot q^*(i,p^*(i,f))di=\int_{x_{n-1}}^{x_n} f\cdot q^*(i,p^*(i,f))di$, where convex C_n is the interval between points x_{n-1} and x_n. To show that an equilibrium fee is equivalent to $f^*(c_n)$ for some $c_n\in C_n$, note that because $\Pi_p(i)-\Pi(i,f^*(j))$ increases as j departs from i, we have that an $f^*(j)<f^*(x_{n-1})$ [$f^*(j)>f^*(x_n)$] generates less profit than $f^*(x_{n-1})$ [$f^*(x_n)$] in every $i\in C_n$. Thus, the an optimal IFD fee in category n must be equivalent to $f^*(c_n)$ for some $c_n\in C_n$. □

Proof of Theorem 2: First note that how PFD profit depart from IFD profit gives rise to category and fee behaviors in the IFD equilibrium. In particular, the rate at which $\Pi_p(c_n-\theta)$ departs from $\Pi(c_n-\theta,f^*(c_n))$ as θ increases gives IFD profit losses for markets

$i \in (x_{n-1}, c_n)$ (relative to the maximum profits), and the rate at which $\Pi_p(c_n + \theta)$ departs from $\Pi(c_n + \theta, f^*(c_n))$ as θ increases gives IFD profit losses for markets $i \in (c_n, x_n)$ (relative to the maximum profits).[36] Naturally, the difference in these departures reveals information on the location of the optimal c_n in category n. Considering this difference more seriously, let

$$D(i,\theta) = \Pi_p(i+\theta) - \Pi(i+\theta, f^*(i)) - [\Pi_p(i-\theta) - \Pi(i-\theta, f^*(i))] = R(i,\theta) - L(i,-\theta), \quad (7)$$

where $\theta > 0$ implies that $R(i,\theta) = \Pi_p(i+\theta) - \Pi(i+\theta, f^*(i))$ captures the IFD profit loss for markets "to the right" of i, and $L(i,-\theta) = \Pi_p(i-\theta) - \Pi(i-\theta, f^*(i))$ captures the IFD profit loss for markets "to the left" of i.

To determine the signs of $D(i,\theta)$, $R(i,\theta)$, and $L(i,-\theta)$, note that Taylor approximation implies that[37]

$$\begin{aligned}
\Pi_p(i+\theta) &\approx \Pi_p(i) + \theta\Pi_p'(i) + \frac{\theta^2}{2!}\Pi_p''(i) + \frac{\theta^3}{3!}\Pi_p^{(3)}(i)... \\
\Pi(i+\theta, f^*(i)) &\approx \Pi_p(i) + \theta\Pi_p'(i) + \frac{\theta^2}{2!}\cdot\frac{\partial^2\Pi(i,f^*(i))}{\partial i^2} + \frac{\theta^3}{3!}\cdot\frac{\partial^3\Pi(i,f^*(i))}{\partial i^3}... \\
\Pi_p(i-\theta) &\approx \Pi_p(i) - \theta\Pi_p'(i) + \frac{\theta^2}{2!}\Pi_p''(i) - \frac{\theta^3}{3!}\Pi_p^{(3)}(i)... \\
\Pi(i-\theta, f^*(i)) &\approx \Pi_p(i) - \theta\Pi_p'(i) + \frac{\theta^2}{2!}\frac{\partial^2\Pi(i,f^*(i))}{\partial i^2} - \frac{\theta^3}{3!}\frac{\partial^3\Pi(i,f^*(i))}{\partial i^3}...
\end{aligned}$$

where the second and fourth equations use $\Pi(i, f^*(i)) = \Pi_p(i)$ by construction and $\frac{\partial\Pi(i,f^*(i))}{\partial i} = \Pi_p'(i)$ by the Envelope Theorem.[38]

Note that $R(i,\theta) \approx \frac{\theta^2}{2!}[\Pi_p''(i) - \frac{\partial^2\Pi(i,f^*(i))}{\partial i^2}] + \frac{\theta^3}{3!}[\Pi_p^{(3)}(i) - \Pi^3(i, f^*(i))]...$ must be greater than zero (otherwise the IFD profit is greater than PFD profit for some i). This implies that $R(i,\theta)$ is increasing in θ so that a greater departure from market i results in greater profit

[36]Note that in equilibrium we have that $c_n \in (x_{n-1}, x_n)$ as shown by Lemma 2.

[37]Recall the general formulation of the Taylor approximation for $f(x)$ evaluated at $x = c_n+\theta$ and $x = c_n-\theta$ is given by $f(i+\theta) \approx f(i)+\theta f'(i)+\frac{\theta^2}{2!}f''(i)+\frac{\theta^3}{3!}f^{(3)}(i)...$ and $f(i-\theta) \approx f(i)-\theta f'(i)+\frac{\theta^2}{2!}f''(i)-\frac{\theta^3}{3!}f^{(3)}(i)....$

[38]The Envelope Theorem implies that $\Pi_p(i)$ has the same slope as $\Pi(i, f^*(x))$ at $x = i$. For example, the red lines in Figures 1 and 2 are tangent to $\Pi_p(i)$ at the points of intersection $i = 1/4$ and $i = 3/4$. This implies that $\Pi(i, f^*(x))$ is tangent to $\Pi_p(i)$ at $x = i$ so that $\frac{\partial\Pi(i,f^*(i))}{\partial i} = \Pi_p'(i)$.

losses under IFD. Similarly, we have that $L(i,-\theta) \approx \frac{(-\theta)^2}{2!}[\Pi_p''(i) - \frac{\partial^2 \Pi(i,f^*(i))}{\partial i^2}] + \frac{(-\theta)^3}{3!}[\Pi_p^{(3)}(i) - \Pi^3(i, f^*(i))]$... must be greater than zero (otherwise the IFD profit is greater than PFD profit for some i). This implies that $L(i,-\theta)$ is increasing in θ so that a greater departure from market i results in greater profit losses under IFD. Lastly, these approximations imply that Equation 7 reduces to

$$D(i,\theta) = 2 \cdot \frac{\theta^3}{3!} \left[\Pi_p^{(3)}(i) - \frac{\partial^3 \Pi(i, f^*(i))}{\partial i^3} \right] + 2 \cdot \frac{\theta^5}{5!} \left[\Pi_p^{(5)}(i) - \Pi^{(5)}(i, f^*(i)) \right] ...$$

To ease our exposition, we assume that, by approximation, the sign of $D(i,\theta)$ is given by the sign of $\Pi_p^{(3)}(i) - \frac{\partial^3 \Pi(i,f^*(i))}{\partial i^3}$.

Note that if $\Pi_p^{(3)}(i) - \frac{\partial^3 \Pi(i,f^*(i))}{\partial i^3} = 0$ for all i, then $\Pi_p(i) - \Pi(i, f^*(c_n))$ is symmetric around any c_n[39] and the IFD platform profit losses are symmetric about c_n so that $R(c_n,\theta) = L(c_n,-\theta)$ for all θ. In addition, $R(i,\theta)$ and $L(i,-\theta)$ are increasing in θ so that platform profit losses from IFD in a particular market increase as market j departs from c_n. This implies that midpoint pricing occurs within an arbitrary category (as shown in Figure 1), and categories are of the same mass so that $x_n = n/N$.

Instead, if $\Pi_p^{(3)}(i) - \frac{\partial^3 \Pi(i,f^*(i))}{\partial i^3} > 0$ for all i, then $\Pi_p(c_n + \theta) - \Pi(c_n + \theta, f^*(c_n)) > \Pi_p(c_n - \theta) - \Pi(c_n - \theta, f^*(c_n))$ for all θ and c_n so that profit losses from IFD are concentrated in the high profit markets where i is closer to 1 (as shown in Figure 2). In this case, the platform will set the fee in category n using $c_n > \frac{x_n + x_{n-1}}{2}$ and every category will be such that $x_n > n/N$ for all $n = 1, ..., N-1$.[40] A similar argument implies that if $\Pi_p^{(3)}(i) - \frac{\partial^3 \Pi(i,f^*(i))}{\partial i^3} < 0$ for all i, then the platform profit loss from IFD is concentrated in the low profit markets where i is closer to 0 and so the platform sets the fee in category n using $c_n < \frac{x_n + x_{n-1}}{2}$ and every category will be such that $x_n < n/N$ for all $n = 1, ..., N-1$.

The arguments remain if we replace the unit fees, f, with ad valorem fees, t. Hence, the

[39] That is, $\Pi_p(c_n + \theta) - \Pi(c_n + \theta, f^*(c_n)) = \Pi_p(c_n - \theta) - \Pi(c_n - \theta, f^*(c_n))$.

[40] This occurs since $D(i,\theta) > 0$ for all i implies that the difference between the PFD and IFD aggregate profits (across all i) cannot be minimized when $x_n \leq n/N$ for some $n = 1, ..., N-1$ or when $c_n \leq \frac{x_n + x_{n-1}}{2}$ for some $n = 1, ..., N-1$.

main results follow for $\frac{\partial^3 \Pi_p(i)}{\partial i^3} \mathcal{R} \frac{\partial^3 \Pi(i,t^*(i))}{\partial i^3}$. □

Proof of Proposition 1: No market foreclosure and Corollary 3 imply that midpointing occurs in equilibrium. In this case, consider the following inequality:

$$
\begin{aligned}
0 \gtreqless & [DWL_p(c_n+\theta) - DWL(c_n+\theta, f^*(c_n))] + [DWL_p(c_n-\theta) - DWL(c_n-\theta, f^*(c_n)] \quad (8) \\
= & [DWL_p(c_n+\theta) + DWL_p(c_n-\theta)] - [DWL(c_n+\theta, f^*(c_n)) + DWL(c_n-\theta, f^*(c_n)],
\end{aligned}
$$

which captures the difference between PFD deadweight loss and IFD deadweight loss. Applying this general formulation to that of linear demands, note that the results from Example 1 imply that $DWL(i,f) = \frac{1}{8}\left[a(i) - c(i) + f\right]^2 = \frac{1}{8}\left[m(i) + f\right]^2$ and $f^*(i) = \frac{1}{2}\left(a(i) - c(i)\right) = \frac{1}{2}m(i)$. Let $m(i) = \alpha + \beta i$, with $\alpha, \beta > 0$, so that $DWL(i,f) = \frac{1}{8}[\alpha + \beta i + f]^2$. Lastly, midpointing implies that $c_n = \frac{x_{n-1}+x_n}{2} = \frac{2n-1}{2N}$.[41] After some algebra, the terms in Inequality (8) reduces to

$$
[DWL_p(c_n+\theta) - DWL(c_n+\theta, f^*(c_n))] = \frac{9}{32b} \cdot \left[\frac{2}{3} \cdot \alpha\beta\theta + \frac{5}{9} \cdot \beta^2\theta^2\right],
$$

$$
[DWL_p(c_n-\theta) - DWL(c_n-\theta, f^*(c_n))] = \frac{9}{32b} \cdot \left[\frac{-2}{3} \cdot \alpha\beta\theta + \frac{5}{9} \cdot \beta^2\theta^2\right],
$$

which implies that Inequality (8) is positive since $\frac{9}{32}\left[\frac{10}{9}\beta^2\theta^2\right] = \frac{5}{16}\beta^2\theta^2 > 0$. Thus, deadweight loss is greater under PFD than IFD. That is, welfare is higher with IFD. Moreover, we see that $\frac{5}{16}\beta^2\theta^2$ is convex in θ and so the gap between PFD and IFD is increasing in the size of categories. Because midpointing occurs in equilibrium, this implies that welfare is decreasing in the number of categories so that welfare is minimized in the PFD equilibrium. □

Proof of Proposition 2: Note that if markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees so that $m(0) = 0$, then

[41] Midpointing is critical here as it allows for the symmetric comparison between $c_n+\theta$ and $c_n-\theta$, instead of across integrals.

$m(i) = a(i) - c(i)$ is linear in i such that $m(0) = 0$. Without loss of generality, let $m(i) = m \cdot i$. Our results from Example 1 on the retail subgame equilibrium imply that platform profit in category 1 for arbitrary f is given by

$$\Pi_{n=1}(f) = \int_{x_0(f)}^{x_1} \frac{f}{2} \left[a(i) - c(i) - f\right] di = \frac{f}{2} \cdot \int_{x_0(f)}^{x_1} \left[m(i) - f\right] di = \frac{f}{2} \cdot \int_{\frac{f}{m}}^{x_1} \left[m \cdot i - f\right] di,$$

where $\Pi_{n=1}(f)$ denotes the platform's profit from category 1 when markets $i \in [0, x_0(f))$ foreclose and markets $i \in [x_0(f), x_1]$ persist on the platform. Note that the market on the margin of participation depends on f so that $x_0(f)$ is given by the market $i = x_0$ so that $f = m(x_0) = m \cdot x_0$. Solving for x_0 implies that $x_0 = \frac{f}{m}$ and this provides the final equality in the equation above.

Evaluating the integral and maximizing $\Pi_1(f)$ with respect to f implies that the equilibrium fee in category 1, $f^*(c_1)$, is such that $f^*(c_1) = \frac{1}{3} m \cdot x_1 = f^*(i = (2/3)x_1)$,[42] and the equilibrium marginal merchant is given by $x_0 = \frac{x_1}{3}$. Thus, we see that within the set of merchants that participate in category 1, the $i \in \left[\frac{x_1}{3}, x_1\right]$, midpoint pricing occurs since $f^*(c_1) = f^*((2/3)x_1) = f^*\left(\frac{(1/3)x_1 + x_1}{2}\right)$.

From here, an analogous proof to that of Theorem 2 implies that the platform will implement midpointing on the markets that do not foreclose, the $i \in \left[\frac{x_1}{3}, 1\right]$. This implies that participation of the first category of length $x_1 - (1/3)x_1 = (2/3)x_1$ is equivalent to the length of the other $N-1$ categories so that $(2/3)x_1 \cdot N = 1 - (1/3)x_1$ (the length of the categories equals the total length of the markets served by the platform). Solving for x_1 implies that $x_1 = \frac{3}{2N+1}$ so that $x_0 = (1/3)x_1 = \frac{1}{2N+1}$ and only the markets $i \in \left[\frac{1}{2N+1}, 1\right]$ remain on the platform. This also implies that $x_n = \frac{2n+1}{2N+1}$ so that fees are given by $c_n = \frac{x_n + x_{n-1}}{2} = \frac{2n}{2N+1}$ for $n = 0, 1, ...N$. From Example 1 we know that $f^*(i) = \frac{1}{2}(a(i) - c) = \frac{1}{2} m(i)$. Thus, equilibrium midpointing fees are given by $f^*(c_n) = \frac{n}{2N+1} \cdot m$.

Finally, we determine the welfare result. To do so, we consider how deadweight loss across the entire platform changes with respect to N. From Example 1 we have that dead-

[42] From Example 1 we know that $f^*(i) = \frac{1}{2}(a(i) - c(i)) = \frac{m(i)}{2}$.

weight loss in market i, category n, and facing fee $f^*(c_n)$ is given by $DWL(i, f^*(c_n)) = \frac{1}{8}\left[a(i) + f^*(c_n) - c\right]^2 = \frac{1}{8}m^2\left[i + \frac{n}{2N+1}\right]^2$ and so the deadweight loss across the platform is given by

$$DWL(N) = \sum_{n=1}^{N} \int_{\frac{2n-1}{2N+1}}^{\frac{2n+1}{2N+1}} \frac{1}{8}m^2\left[i + \frac{n}{2N+1}\right]^2 di = \frac{1}{24}m^2 \sum_{n=1}^{N} \frac{54n^2+2}{(2N+1)^3},$$

so that $DWL(N)$ is a series in N. Furthermore, this series is increasing in N, with $DWL(1) \approx 0.0864 \cdot m^2$, $DWL(2) \approx 0.0913 \cdot m^2$, and $DWL(3) \approx 0.0925 \cdot m^2$, and converges to $DWL_p = \int_0^1 DWL(i, f^*(i))di = \frac{3}{32}m^2 = 0.09375 \cdot m^2$. Thus, welfare across the marketplace is decreasing in N. $\square$

Proof of Lemma 3: In the retail subgame where platform entry occurs, we have that $\pi_k = (p - c_k)q_k = \left[\frac{a}{b} - \frac{1}{b}(q_m + q_p) - c_k\right] q_k$ for $k = m, p$ and $c_m = c + f$ and $c_p = c + d$. Maximizing profits for $k = m, p$ and solving the system of equations implies that $q_m(f) = \frac{b}{3}\left(\frac{a}{b} - c - 2f + d\right)$ and $q_p(f) = \frac{b}{3}\left(\frac{a}{b} - c - 2d + f\right)$. This implies that we have an interior solution where both q_m and q_p are greater than zero whenever $f \in \left(2d - \left(\frac{a}{b} - c\right), \frac{1}{2}\left(\frac{a}{b} - c + d\right)\right)$.

Turning to the platform's fee decision, platform profit in the market is given by $\Pi = (p(f) - c - d)q_p(f) + f \cdot q_m(f)$. Maximizing profit with respect to f implies that $f^E = \frac{1}{2}\left(\frac{a}{b} - c\right) - \frac{d}{10}$. This generates the results in Equation 6. Furthermore, if $f^E = \frac{1}{2}\left(\frac{a}{b} - c\right) - \frac{d}{10}$, then an interior duopoly solution for $f \in \left(2d - \left(\frac{a}{b} - c\right), \frac{1}{2}\left(\frac{a}{b} - c + d\right)\right)$ simplifies to the requirement that $d \in \left(0, \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$. If $d \geq \frac{15}{21}\left(\frac{a}{b} - c\right)$, then $q_p = 0$ so that the platform does not enter and the PFD no entry equilibrium occurs. Instead, if $d \leq 0$, then the platform forecloses the merchant and acts as a monopolist in retail. $\square$

Proof of Proposition 3: To compare welfare between the PFD equilibria with and without entry, not that the deadweight loss with platform entry includes the deadweight loss triangle as well as the lost surplus from sales made by the platform at a higher cost than the merchant.

Thus, the relative deadweight loss from the PFD entry equilibrium is given by

$$DWL(\text{entry}) = \frac{1}{2}(p^E - c)(Q^E - Q(c)) + d[q_p^E - (q^* - q_m^E)] = \frac{b}{8}\left[\left(\frac{a}{b} - c\right)^2 + \frac{16}{5}\left(\frac{a}{b} - c\right)d - \frac{51}{25}d^2\right].$$

Similarly, the deadweight loss without entry is given by

$$DWL(\text{without}) = \frac{1}{2}(p^* - c)(q^* - q(c)) = \frac{9b}{32}\left(\frac{a}{b} - c\right)^2.$$

This implies that $DWL(\text{entry}) < DWL(\text{without})$ for all $d \in \left(0, \frac{a}{b} - c\right)$ so that welfare is always greater under platform entry into retail.

In terms of Pareto efficiency, we know that consumer surplus increases since $p^E < p^*$. We also know that platform profit increases. For merchants, note that merchant profit with platform entry is given by $\pi(\text{entry}) = (p^E - c - f^E)q_m^E = \frac{4b}{25}d^2$, and merchant profit without entry is given by $\pi(\text{without}) = (p^* - c - f^*)q^* = \frac{b}{16}\left(\frac{a}{b} - c\right)^2$. This implies that $\pi(\text{entry}) > \pi(\text{without})$ if and only if $d > \frac{5}{8}\left(\frac{a}{b} - c\right)$.

Finally, if $d \leq 0$ so that the platform enters into retail as a monopolist, then $q_p^E = \frac{b}{2}\left(\frac{a}{b} - c - d\right) > \frac{b}{4}\left(\frac{a}{b} - c\right) = q^*$ so that price also decreases. In this case, fee distortions disappears and output is produce at a lower cost so that welfare increases with entry. □

Proof of Lemma 4: Note that if markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees so that $m(0) = 0$, then $m(i) = a(i) - c(i)$ is linear in i such that $m(0) = 0$. Without loss of generality, let $m(i) = m \cdot i$. In the retail subgame where platform entry occurs, we have that $\pi_k = (p - c_k)q_k = [a(i) - (q_m + q_p) - c_k]\, q_k$ for $k = m, p$ and $c_m = c + f_n$ and $c_p = c + d$, where f_n denotes the fee in category n. Maximizing profits for $k = m, p$ and solving the system of equations implies that $q_m(i) = \frac{1}{3}(m(i) - 2f_n + d)$ and $q_p(i) = \frac{1}{3}(m(i) - 2d + f_n)$. These retail equilibrium outputs imply that the platform enters into market i so that $i > \frac{2d - f_n}{m}$. Note that $e < x_n$ if and only if $d < \frac{1}{2}[x_n m + f_n] := d_n$. Furthermore, if $d < 0$, then $\frac{2d - f_n}{m} < 0 \leq x_{n-1}$. Thus, $e_n = \frac{2d - f_n}{m}$ if $d \in (0, d_n)$, $e_n = x_n$ if $d \geq d_n$, and $e_n = x_{n-1}$ if $d \leq 0$. □

Proof of Proposition 4: Note that if markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees so that $m(0) = 0$, then $m(i) = a(i) - c(i)$ is linear in i such that $m(0) = 0$. Without loss of generality, let $m(i) = m \cdot i$. From the Proof of Lemma 4 we know that platform entry occurs for some market in category n if $d < \frac{1}{2}[mx_n + f_n]$. Since $f^E(i) < f^*(i)$ for all i when $d > 0$, a sufficient condition that ensures platform entry into some market in category n is that $d < \frac{1}{2}[mx_n + f_n]$ when f_n and x_n are given by Proposition 2. In this case, the platform no entry IFD equilibrium cannot occur in equilibrium since the platform would enter into retail for some market i. From Proposition 2, the inequality $d < \frac{1}{2}[mx_n + f_n]$ becomes $d < \frac{3n+1}{4N+2}m$. To ensure that at least some entry occurs when $n = N$, we require that $d < \frac{3N+1}{4N+2}m$, and this holds for all $N \geq 1$ when $d < \frac{2}{3}m := d^E$.

If $d < \frac{2}{3}m := d^E$, then category $n = N$ experiences some platform entry. In this case, holding the category size fixed to midpointing so that $C_N = \left[\frac{2N-1}{2N+1}, 1\right]$, platform profit from category $n = N$ increases when the platform offers a price below the midpoint pricing strategy ($f_N = f^*(c_N)$), since $f^E(i) < f^*(i)$. This implies that $c_N^E < c_n$. In addition, a lower price within category $n = N$ implies that platform profit increases if x_{N-1}^E less than $x_{N-1} = \frac{2N-1}{2N+1}$. This implies that $x_{N-1}^E < x_{N-1}$. From here, either (a) midpointing without platform entry occurs in categories $n = 1$ through $n = N-1$ so that $c_n^E < c_n$ and $x_n^E < x_n$ for $n = 1, ..., N-1$, or (b) some entry occurs in which case, following the same approach as for category $n = N$, we have that $c_n^E < c_n$ and $x_n^E < x_n$ for $n = 0, 1, ..., N-1$. Altogether, this implies that market foreclosures decrease, fees decrease which reduces fee distortions, and competition in at least some markets increase so that total welfare increases relative to the midpointing IFD equilibrium without platform entry. □

Proof of Proposition 5: If a vertical merger occurs in market i, then the platform no longer uses a merchant fee in market i and the merged entity uses the lowest of the two marginal costs: $c^M(i) = \min\{c(i), c(i) + d(i)\}$. In this case, if $d(i) \geq 0$, then $DWL^M(i, d(i) \geq 0) = \frac{b(i)}{8}\left(\frac{a(i)}{b(i)} - c(i)\right)^2$; instead if $d(i) < 0$, then $DWL^M(i, d(i) < 0) = \frac{b(i)}{8}\left(\frac{a(i)}{b(i)} - c(i) - d(i)\right)^2$.

Without a merger, a platform that enters into retail in market i for arbitrary $f(i)$ produces deadweight loss of $DWL^E(i, d(i) \geq 0) = \frac{b(i)}{18}\left(\frac{a(i)}{b(i)} - c(i) + d(i) + f(i)\right)^2$ and $DWL^E(i, d(i) < 0) = \frac{b(i)}{18}\left(\frac{a(i)}{b(i)} - c(i) + d(i) + f(i)\right) \cdot \left(\frac{a(i)}{b(i)} - c(i) - 2d(i) + f(i)\right)$. Comparing across the two regimes we have that $DWL^E(i, d(i) < 0) > DWL^M(i, d(i) < 0)$ for $d(i) < 0$ and we also have that $DWL^M(i, d(i) \geq 0) < DWL^E(i, d(i) \geq 0)$ if and only if $\frac{a(i)}{b(i)} - c(i) < d(i) + f(i)$.

For Corollary 5, under PFD we have that $f^E(i) = \frac{1}{2}\left(\frac{a(i)}{b(i)} - c(i)\right) - \frac{d(i)}{10}$ so that $\frac{a(i)}{b(i)} - c(i) < d(i) + f(i)$ if and only if $d(i) > \frac{5}{9}\left(\frac{a(i)}{b(i)} - c(i)\right)$. □

Proof of Proposition 6: Note that if markets differ uniformly across intercept-cost margins and the marginal market earns zero profit without platform fees so that $m(0) = 0$, then $m(i) = a(i) - c(i)$ is linear in i such that $m(0) = 0$. Without loss of generality, let $m(i) = m \cdot i$. From the Proof of Proposition 2 we have that $DWL(N = 1, \text{no entry}) = \frac{7}{81}m^2$. From the Proof of Proposition 3 we have that $DWL(i; PFD, \text{entry}) = \frac{1}{8}\left[m(i)^2 + \frac{16}{5}m(i)d - \frac{51}{25}d^2\right]$. Given that $m(i) = m \cdot i$, we have that $DWL(i; PFD, \text{entry}) = \frac{1}{8}\left[m^2 \cdot i^2 + \frac{16}{5}md \cdot i - \frac{51}{25}d^2\right]$. This implies that

$$DWL(PFD, \text{entry}) = \int_0^1 DWL(i; PFD, \text{entry})\, di = \frac{1}{8}\left[\frac{1}{3}m^2 + \frac{8}{5}md - \frac{51}{25}d^2\right].$$

Finally, comparing the deadweight losses across the two regimes we have that $DWL(PFD, \text{entry}) < DWL(N = 1, \text{no entry})$ whenever $d < m$. Thus, the PFD equilibrium with entry earns greater welfare than the equilibrium where the platform cannot use fee discrimination nor enter into retail. □

Proof of Proposition 7: Under PTD, we can drop the (i) to ease exposition. From Proposition 3, if $d < \frac{15}{21}\left(\frac{a}{b} - c\right)$, then unit tax in market i given by $\tau > 0$ so that $[d + \tau] \in \left(\frac{5}{8}\left(\frac{a}{b} - c\right), \frac{15}{21}\left(\frac{a}{b} - c\right)\right)$ results in a platform PFD entry equilibrium that Pareto improves upon the equilibrium that bans platform entry under PFD.

For welfare under the PTD equilibrium to generate greater welfare than a ban on fee discrimination and platform entry, following the Proof of Proposition 3 and replacing "d"

with "$d+\tau$" implies that for any τ so that $d+\tau \in \left(\frac{5}{8}\left(\frac{a}{b}-c\right), \frac{15}{21}\left(\frac{a}{b}-c(i)\right)\right)$ we have that the resulting PTD equilibrium generates greater welfare than a ban on fee discrimination and platform entry. Thus, the results above hold for all $d < \frac{15}{21}\left(\frac{a}{b}-c(i)\right) := d^{PTD}$. □

Proof of Proposition 8: In this setting of PFD, we drop the (i) to ease exposition. Under PFD, if the platform does not entry into a market, then Equation (4) implies that profit from pursuing a merchant fee instead of retail entry is given by $\Pi(\text{merchant fee}) = \frac{b}{8}\left(\frac{a}{b}-c\right)^2$. From the Proof of Lemma 3, the retail subgame equilibrium implies that $q_p(f=0) = \frac{b}{3}\left(\frac{a}{b}-c-2d\right)$ so that platform profit from entering into retail without merchant fee earnings is given by $\Pi(\text{entry}) = \frac{b}{9}\left(\frac{a}{b}-c-2d\right)^2$. This implies that $\Pi(\text{entry}) > \Pi(\text{merchant fee})$ if and only if $d < -\frac{1}{2}\left(\sqrt{\frac{9}{8}}-1\right)\left(\frac{a}{b}-c\right) < 0$. □

Proof of Proposition 9: Focusing on PFD allows us to drop the (i) to ease exposition. From the Proof of Lemma 3 we have that $q_m(f) = \frac{b}{3}\left(\frac{a}{b}-c-2f+d\right)$ and $q_p(f) = \frac{b}{3}\left(\frac{a}{b}-c-2(d+\tau)+f\right) = \frac{b}{3}\left(\frac{a}{b}-c-2d-f\right)$ in the retail subgame where platform entry occurs. Given the retail subgame, the platform maximizes total profit across retail and merchant fee extraction. That is, the platform maximizes $\Pi(f) = (p(f)-c-d-f)q_p(f)+f\cdot q_m(f)$ with respect to f. As a result, the optimal merchant fee is given by $f^{f=\tau} = \frac{1}{4}\left(\frac{a}{b}-c+7d\right)$ so that $q_p^{f=\tau} = \frac{b}{4}\left(\frac{a}{b}-c-5d\right)$, $q_m^{f=\tau} = \frac{b}{4}\left(\frac{a}{b}-c-d\right)$, and $p^{f=\tau} = \frac{1}{2}\left(\frac{a}{b}+c+3d\right)$. Platform entry only occurs if $q_p^{f=\tau} > 0$ which occurs whenever $d < \frac{1}{5}\left(\frac{a}{b}-c\right)$. In terms of deadweight loss, we have that the deadweight loss from $f=\tau$ is given by $DWL(f=\tau) = \frac{b}{8}\left[\left(\frac{a}{b}-c\right)+3d\right]^2$. From the Proof of Proposition 3 we have that the deadweight loss from the PFD equilibrium with platform entry is given by $DWL = \frac{b}{8}\left[\left(\frac{a}{b}-c\right)+\frac{3}{5}d\right]^2$. Thus we see that $DWL < DWL(f=\tau)$ for all $d < \frac{1}{5}\left(\frac{a}{b}-c\right)$. □

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