CESifo WORKING PAPERS

9466
2021

Original Version: December 2021
This Version: June 2022

Endogenous Spatial Production Networks: Quantitative Implications for Trade and Productivity

Piyush Panigrahi

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 9466*

Endogenous Spatial Production Networks: Quantitative Implications for Trade and Productivity

Abstract

I develop a model of endogenous production network formation between spatially distant firms. Unlike other such models, it is tractable even for very large numbers of firms, that is, it delivers closed-form predictions for firm-to-firm trade, it can be estimated via maximum likelihood, and it can be used for firm-level counterfactual analysis. I exploit novel micro-data on Indian firm-to-firm production networks for estimation. The estimated model implies that upon market integration across Indian states, over half of the variation in changes in firms' sales to other firms can be explained by endogenous changes in network structure.

JEL-Codes: F110, F120, D240, C670, C680, L110, O110, O120, R120, R150, D850.

Keywords: production networks, international trade, economic geography.

Piyush Panigrahi
Yale University / New Haven / CT / USA
piyush.panigrahi@yale.edu

Date: June 4, 2022. First Version: November 4, 2020
I am immensely grateful to Andrés Rodríguez-Clare, Dave Donaldson, Thibault Fally, and Cécile Gaubert for enthusiastic support and generous guidance for this project, and I am thankful to Sam Kortum for detailed feedback on the manuscript. I thank Alan Auerbach, Matilde Bombardini, Lorenzo Caliendo, Ben Faber, Ana Cecília Fieler, Yuriy Gorodnichenko, Bryan Graham, Daniel Haanwinckel, Matthias Hoelzlein, Junyi Hou, Giammario Impullitti, Yuhei Miyauchi, Joseph Shapiro, Vitaliia Yaremko, seminar participants at Berkeley, Yale, Johns Hopkins SAIS, Penn State, Toronto, Singapore Management University, ECARES, Bocconi, and conference participants at CEPR-ERWIT, LACEA-LAMES, NAUEA, MVEA, MWIT, ETSG, EARIE, EEA-ESEM, ESAM, SED, CMES, AMES, NASMES, FREIT ITO, AFES, CESifo Area Conference on Global Economy, GEP/CEPR Annual Postgraduate Conference, EMUEA, RES, FIW for valuable comments. I thank CESifo for the Distinguished Affiliate Prize, FIW and GEP for best paper awards. I thank the Department of Commercial Taxes in the Government of Gujarat, the Department of Goods and Services Tax in the Government of Maharashtra, the Commercial Taxes Department in the Government of Tamil Nadu, the Commissionerate of Commercial Taxes in the Government of Odisha, the Directorate of Commercial Taxes in the Government of West Bengal and the Ministry of Finance in the Government of India for sharing anonymized data. Financial support from the Clausen Center for International Business and Policy and Institute for South Asia Studies at UC Berkeley, the Weiss Family Fund for Research in Development Economics and, the CEPR and the UK Department for International Development (DFID) (under the jointly funded research initiative 'Private Enterprise Development in Low Income Countries' (PEDL), contract reference PEDL_LOA_7500_Panigrahi) is gratefully acknowledged. The views expressed are not necessarily those of CEPR or DFID. All errors are my own.

1. Introduction

Heterogeneity in production costs across firms is at the heart of modern general equilibrium models of firm heterogeneity and trade. Yet differences in firms' production costs are typically attributed to differences in productivity across firms. With firms operating in production networks, differences in production costs arise not just from differences in productivity but also from finding the most cost-effective suppliers of intermediate inputs. General equilibrium theories of trade with firms differing only in productivity do not grapple with microscopic heterogeneity in the extensive and intensive margins of firm-to-firm trade in intermediate inputs — who buys from whom and how much? How does endogenous formation of customer-supplier linkages between firms and the resultant network architecture drive differences in firms' overall sales, ability to sell across multiple destinations, and aggregate patterns of trade? How do we evaluate the impact of market integration, technology improvements, and improvements in allocative efficiency on aggregate outcomes when the production network of firms reorganizes in response to these shocks?

In this paper, I present a novel rich yet tractable empirical model of endogenous network formation between spatially distant firms to evaluate the aggregate and firm-level consequences of shocks. The contribution is four-part. First, I use novel micro-data to document empirical regularities arising from a new decomposition of firms' sales that underscores the salience of endogenous network formation between firms and motivates the theory. Second, I develop a theory of trade between multiple locations featuring endogenous formation of firm-to-firm production networks that not only rationalizes micro-data on firm-to-firm sales but is also consistent with structural gravity at the aggregate level. Third, I devise a procedure to structurally estimate the model that circumvents computational difficulties pervasive in estimation of network formation models with large numbers of firms. Fourth, I propose a procedure to evaluate counterfactual outcomes that accounts for randomness in network formation without requiring simulation of large networks which can be computationally burdensome due to interdependence in link formation.

Firms operating in production networks are vastly heterogeneous in size. Why do some appear to be selling so much more inputs than others? Perhaps they are attractive input suppliers or happen to have large customers that demand higher volumes. Using data on 103 million firm-to-firm relationships assembled from administrative VAT records spanning across 5 years and pertaining to around 2.5 million Indian firms located across 141 districts, I conduct a new decomposition for firms' sales to other firms to delineate these channels.[1] I find that firms with higher sales to other firms tend to be used more intensively by other firms and tend to sell to larger customers. The first margin explains 81% of the variation whereas the remaining 19% is explained by the second margin.

The attractiveness of a firm, that is, its ability to supply at a lower price either due its own productivity or because it sources inputs cheaply from efficient suppliers influences how intensively it is used by other firms. The outsized importance of the first margin suggests that endogenous formation of firm-to-firm linkages arising from attractiveness of firms is pertinent to understanding the origins of firm heterogeneity.

I develop a new Ricardian model of trade between multiple locations with geographic barriers and imperfect competition (as in Eaton and Kortum (2002) and Bernard et al. (2003)) that accommodates heterogeneous consumer preferences, heterogeneous technological requirements by firms, and arbitrary production network formation between firms.[2] Firms' production processes consist of multiple input requirements. Potential suppliers differ in the suitability of their goods for each of these requirements. Firms randomly encounter potential suppliers and select the most cost-effective suppliers for their production requirements. Firms are more likely to select a potential supplier for a larger proportion of their requirements if it is able to sell at a lower price and produces a good that is more suitable for its production requirements.

The ability of a potential supplier to sell at a lower price than another is regulated by (a) its idiosyncratic productivity, (b) the efficiency with which its own suppliers were able to produce thus affording the firm a lower price

[1]Notably, Huneeus (2020) and Bernard et al. (2021) decompose firms' sales to other firms into number of customers and sales per customer whereas the decomposition proposed here is into intensity of use by other firms and average customer size. Unlike the former, the latter decomposition is suitable for separating the aforementioned channels of firm heterogeneity in input sales.

[2]While Caliendo and Parro (2014) allow for sectoral heterogeneity and inter-sectoral linkages in a Ricardian model of trade, they do not allow for arbitrary production networks between firms and are unable to accommodate the vast heterogeneity in input sourcing patterns at the firm-level observed in data.

for intermediate inputs, and (c) proximity to location of use thus having to incur lower geographic costs. Firms with lower production costs thus are used more intensively in their customers' production processes. Since these customers use cheaper inputs, they end up with lower production costs themselves and become cost-effective suppliers to their customers. In the cross-section, firms with low production costs end up larger because they are used more intensively by other firms and also have larger customers.

I leverage the recursive structure of network formation between firms to estimate the model via the conditional choice probability approach inspired by Hotz and Miller (1993). Differences in the suitability of potential suppliers' goods for a firm's production requirements feature as match-specific productivities across firm pairs in a manner similar to the discrete choice framework. This leads to a multinomial logit model of supplier choice for each of the firm's production requirements. The estimation equation recognizes that while there is a positive probability of a firm sourcing inputs from every other firm, sourcing inputs for only a discrete number of requirements can give rise to sparsity in firm-to-firm connections. This sparsity can be extreme as is observed in the data where the number of firm-to-firm connections are many orders of magnitude lower than its potential given the number of firms in the economy.

Predictions for firm-to-firm trade then allow estimation of the model utilizing the full volume of micro-data on firm-to-firm transactions via maximum likelihood. Semi-parametric estimation of the model implies that firms' fixed effects serve as sufficient statistics for their implied marginal costs and bilateral inter-district fixed effects as a structural gravity specification for estimating trade frictions. Such estimation typically entails a high-dimensional non-linear optimization problem that quickly becomes cumbersome with large numbers of fixed effects. I show that these fixed effects can be computed in closed-form thus avoiding the problem altogether.

For counterfactual analysis, I propose a new procedure that departs from the exact hat algebra approach commonly used in trade models (see Dekle et al. (2008) and Costinot and Rodríguez-Clare (2014)). In aggregate models of trade featuring a continuum of agents, the exact hat algebra approach evaluates the change in aggregate outcomes in response to shocks. In those models, aggregate data coincides with the expected value of aggregate outcomes in the initial state. In contrast, my model featuring finitely many agents implies that the observed data corresponds to only one of many possible realizations under the initial state. The data generating process implied by the model is therefore non-degenerate and hat algebra cannot be used as is. To evaluate counterfactual outcomes, I first solve the model in the initial state under a continuum approximation of the finite economy and then use hat algebra to solve for changes in the model in the counterfactual state. The model and the procedure are rich enough to speak about aggregate and firm-level effects of macro- and micro-shocks.

Using the estimated model, I evaluate the impact on production networks of reducing inter-state border frictions in the context of the recent Goods and Services Tax reform in India that aimed to mitigate such barriers to trade. I find that following a 10% decline in border frictions over half of the variation in changes in firms' sales to other firms implied by the model can be explained by endogenous changes in the network structure.

Related Literature. This paper contributes to four strands of literature. First, this paper is related to the nascent literature on endogenous production networks in general equilibrium which can be broadly classified into two categories. The first (Oberfield (2018); Acemoglu and Azar (2020); Boehm and Oberfield (2020); Antràs and de Gortari (2020); Miyauchi (2021); Eaton et al. (2022)) models formation of linkages as the outcome of selection from a discrete menu of choices whereas the second (Lim (2018); Taschereau-Dumouchel (2020); Huneeus (2020); Tintelnot et al. (2018); Bernard et al. (2021); Demir et al. (2021); Arkolakis et al. (2021)) models formation of linkages between firms as the outcome of "love of variety" in input sourcing while being subject to relationship costs.[3] While network formation in this paper is outcome of discrete choice as in the former, the model here uniquely delivers closed-form predictions for firm-to-firm trade unlike other models of endogenous production networks. This has three advantages relative to other papers: (a) the model is estimated using the full volume of data of firm-to-firm sales via maximum likelihood and does not rely on matching a selection of aggregate moments, (b) the model can be tractably solved even for very large numbers of firms and does not require computationally burdensome simulation for estimation or counterfactual

[3]Other complementary approaches to endogenous production network formation include Carvalho and Voigtlander (2014) and Chaney (2014, 2018).

analysis, and (c) counterfactual outcomes can be evaluated across the distribution of firms and not just for aggregate outcomes such as welfare all while accounting for endogenous changes in network structure.

Second, this paper is related to a long literature on firm heterogeneity (for example, Jovanovic (1982); Hopenhayn (1992); Axtell (2001); Melitz (2003); Klette and Kortum (2004); Luttmer (2007); Arkolakis (2016)) and in particular the branch that studies the heterogeneity among firms arising from their engagement in input-output linkages — Oberfield (2018) and Bernard et al. (2021). The model here houses two sources of firm heterogeneity — from idiosyncratic productivities and from match-specific productivities and engagement in input-output linkages. Unlike Oberfield (2018), the model accommodates heterogeneity in the number of input suppliers across firms as well as in the intensity of use of suppliers across their customers. The model thus allows for variation in firms' average intensity of use by their customers. In the data, this margin explains 46% of the variation in firms' sales. The modeling approach here is distinct from Bernard et al. (2021) who use a fixed cost formulation that necessitates use of simulation-based estimation methods.

Third, the paper also relates to a growing literature on propagation of shocks and aggregation in distorted production networks including Jones (2011), Acemoglu et al. (2012), Swiecki (2017), Caliendo et al. (2017b), Liu (2019), Baqaee and Farhi (2019a,b, 2020), and Bigio and LaO (2020). Some of these papers allow for non-Cobb–Douglas technologies and thus endogenize the intensity with which different inputs are used. However, they do not investigate which combinations of inputs will be used—that is, the extensive margin of firm-to-firm trade —which features prominently in this paper.

Finally, this paper is related to the branch of the trade literature that develops firm-level models of importing for example, Antràs et al. (2017); Blaum et al. (2018). While these papers consider models where firms choose the set of locations to source intermediate inputs or the share of intermediate inputs that are imported, here I develop a more disaggregated model where firms choose both the set of suppliers across multiple locations for intermediate inputs and the share purchased from each of them. The model also shares features with papers that emphasize the role of granularity in trade models such as Eaton et al. (2013), Armenter and Koren (2014), and

Gaubert and Itskhoki (2021). The approach to counterfactual analysis parallels contemporaneous work by Dingel and Tintelnot (2020) who take a related approach in a granular model of commuting choice.

2. Network Margins of Firm Heterogeneity & Trade

2.1. **Sources of Data.** The primary dataset for this paper consists of the universe of firm-to-firm transactions assembled from commercial tax authorities of five Indian states (viz. Gujarat, Maharashtra, Tamil Nadu, Odisha, and West Bengal) between 2011-12 and 2015-16.[4] These states had a nominal GDP of $738 billion in 2015-16, accounting for nearly 40% of GDP. Among these states, the largest (Maharashtra) accounts for roughly 14% of national GDP while the smallest (Odisha) accounts for a little over 2%. It includes transactions between all firms registered under the value-added tax system in these states. The dataset records 103 million inter-firm relationships between approximately 2.5 million firms located across 141 districts in these 5 states.

2.2. **Empirical Regularities.** Indian firms are vastly heterogeneous in size, a pervasive finding in studies of firm-level data. Using firm sales to other firms as a measure of size, I find that firms in the top decile are at least 700 times larger than firms in the bottom decile. In production networks, firm outcomes are shaped not only by their own intrinsic characteristics, like productivity, but also by the characteristics of the firms – suppliers and customers – that they connect with. For a pair of firms s and b, the value of goods purchased by b from s can be written as:

$$\text{sales}(s, b) = \pi(s, b) \times \text{purchases}(b),$$

where purchases(b) denotes the value of goods purchased by b from all other firms and is calculated as $\sum_s \text{sales}(s, b)$; and $\pi(s, b)$ denotes the share of purchases of b that are from s.[5]

A firm could have a high volume of intermediate input sales either because (a) it is an attractive input supplier (higher $\pi(s, b)$), that is, it can provide output at a lower price (either because it sources inputs cheaply from efficient suppliers upstream to it or due its own productivity) – an

[4]For a financial year, say 2015-16, the data pertains to the period between April 1, 2015 and March 31, 2016.

[5]It is worthwhile to note that $\pi(s, b) \in [0, 1]$ implicitly captures whether s is indeed a supplier to b depending on whether it is zero or positive.

upstream margin, or (b) it happens to sell to customers downstream which are large (higher purchases(b)) and demand higher volumes – a downstream margin. While the downstream margin is operational in models with exogenous production networks, the upstream margin requires a model of endogenous network formation between firms — one where firms choose their suppliers and the intensity with which they use inputs from those suppliers.

To shed light on the economic importance of these margins and guide the main features of the model I will develop in Section 3, I leverage the rich network structure of the dataset to conduct a simple decomposition of firms' sales to other firms into two margins: intensity of use and average customer size. Formally, sales of firm s can be decomposed into these two factors according to the following identity.

$$\text{sales}(s) = \text{intensity of use}(s) \times \text{average customer size}(s), \tag{2.1}$$

where

$$\begin{aligned} \text{sales}(s) &= \sum_b \text{sales}(s,b), \\ \text{intensity of use}(s) &= \sum_b \pi(s,b), \text{and} \\ \text{average customer size}(s) &= \frac{\sum_b \pi(s,b) \times \text{purchases}(b)}{\sum_b \pi(s,b)}. \end{aligned}$$

Through variation in intensity of use, the first factor captures the attractiveness of the firm to potential customers deciding who to source inputs from and how much to source from them. The second factor measures average size of customers as inferred from a weighted average of their input purchases. The first factor constitutes the upstream margin and captures the direct importance of the firm in the production network since it captures how cost-effective the firm is irrespective of the characteristics of the customers it sells to.[6] The second factor constitutes the downstream margin and captures the indirect importance of the firm in the production network through the importance of its customers, its customers' customers and so on.

[6]The upstream margin is sometimes referred to as the firm's weighted out-degree. In recent work, Acemoglu et al. (2012) coin this term for similar statistics at the industry level.

I compute the share of variance of firms' sales that is explained by each of these factors.[7] Column (1) in Table 2.1 reports the results of the decomposition. Four-fifths of the variance in firms' sales can be attributed to the upstream margin leaving the rest for the downstream margin. It implies that larger firms are likely to be used more intensively by other firms (explains 81% of the variance), and have larger customers (19%). Both factors covary positively with sales and contribute a non-trivial share to the variance. The positive covariance of the downstream margin can be rationalized as follows. Firms with higher demand for their own goods produce larger quantities and to do so they purchase higher quantities of inputs from their suppliers. In turn, their suppliers end up with higher demand and they source larger quantities from their own suppliers and so on. Therefore, in the cross-section one observes that larger firms have larger customers on average. This points to the importance of supply chain linkages between firms even when the network structure is exogenously fixed.

However, it is the outsized contribution of the upstream margin that highlights the importance of endogenous network formation through two potential channels. First, when firms choose to source from more cost-effective suppliers, they are likely to inherit lower marginal costs from their suppliers. This makes them attractive to their own customers who become larger in turn. Therefore, in the cross-section one would observe a positive correlation between firms' sales and number of customers. Second, when suppliers' goods are substitutable in a firms' input demand system, more cost-effective firms will account for a larger share of material costs of their customers. Since those customers source cheaper inputs intensively, they are likely to inherit lower marginal costs from their suppliers. This makes them attractive to their own customers and they become larger themselves. Therefore, in the cross-section one would observe a positive correlation between firms' sales and average intensity of use by customers. Putting together both these margins - who to buy from and how much – one would observe a positive correlation between firms' sales and intensity of use. This

[7] In short, if a variable X can be decomposed into two factors, X_1 and X_2 such that $X = X_1 \cdot X_2$, then the share of variance of X that can be attributed to any factor X_r is $\frac{Cov(\ln X, \ln X_r)}{Var[\ln X]}$. While these shares sum to unity by additivity of the covariance operator, they are not constrained to be positive individually. For example, see Klenow and Rodríguez-Clare (1997) for use in growth accounting and Eaton et al. (2011) for regression-based decomposition of margins of trade.

suggests that endogeneity of production networks is important both along the extensive and intensive margins of firm-to-firm trade.

Furthermore, trade across space is costly and economic activity across space exhibits large dispersion. How does the relative position of firms across space affect their outcomes? How does geography affect the aforementioned margins of firm heterogeneity? To investigate this, I construct a similar decomposition at a more disaggregated level for firms' destination-specific sales and at a more aggregated level for trade flows between districts.[8] Column (2) in Table 2.1 reports results of variance decomposition of firm's destination-specific sales while controlling for firm-level fixed effects. This is done to capture the variation in individual firms' sales across multiple destinations. The upstream margin accounts for 93% of the variation leaving 7% for the downstream margin. Column (3) in Table 2.1 reports results of variance decomposition of aggregate trade flows between districts while controlling for origin fixed effects. The upstream margin accounts for 83% of the variation leaving 17% for the downstream margin. Since the upstream margin explains the lion's share of the variation in both cases, these results underscore the salience of geography in endogenous network formation between firms.

Taking stock, I find that firms that are larger also tend to be used more intensively by other firms and tend to have larger customers. Of course, these decompositions capture equilibrium relationships and are not causal; nevertheless, they make clear that understanding the characteristics of firms' network is key to understanding origins of firm heterogeneity. While the economic intuition behind these results is straightforward, the decomposition results are new to the literature. With this in mind, I develop a model of endogenous production network formation in the next section that expressly takes these findings into account and leads to a multinomial logit model of supplier choice for estimation.

Discussion. In modeling frameworks inspired by Melitz (2003) (for example, Lim (2018); Huneeus (2020); Bernard et al. (2021)), firms' ability to cover fixed costs (potentially heterogeneous) determine the extensive margin (number of customers) while attractiveness of firms through variable productivity determines the intensive margin (sales per customer). When customer size is homogeneous, variation in average market share across

[8]Further details are provided in Appendix A.

Table 2.1. **Network Margins of Firm Heterogeneity & Trade**

	(1)	(2)	(3)
Intensity of Use	81%	93%	83%
Average Customer Size	19%	7%	17%
Fixed Effects:			
Seller×Year	—	✓	—
Origin×Year	—	—	✓
Data Level:			
Seller×Year	•	—	—
Seller×Destination×Year	—	•	—
Origin×Destination×Year	—	—	•
# observations	5.6×10^6	18.2×10^6	58,390

Note. Column (1) reports the contribution of intensity of use and average customer size to the variance of firms' sales (as per equation (2.1)). Column (2) reports the contribution of those factors to the variance of firms' destination-specific sales (as per equation (A.2)). Column (3) reports the same for trade flows between districts (as per equation (A.3)). See Appendix A for details and alternative specifications.

customers is identical to that in sales per customer. As a result, sales per customer is a good measure to infer attractiveness of firms. When customer size is heterogeneous as is the case here, variation in sales per customer reflects variation in both attractiveness of firms and average size of customers. Therefore, to infer attractiveness of firms through variable productivity one needs to look at average market share among customers and not sales per customer. In both cases, number of customers is a good measure to infer firms' ability to cover fixed costs. Together, number of customers and average market share among customers constitute intensity of use. Intensity of use is therefore a good measure to infer attractiveness of firms either from ability to cover fixed costs or variable productivity. The model described in the following section features no fixed costs. However, variation in both number of customers and average market share (together, intensity of use) arise from attractiveness of firms through variable productivity. Consequently, when we move to estimation, intensity of use turns out to be sufficient statistic for attractiveness of firms.

3. An Empirical Model of Endogenous Spatial Production Networks

In this section, I describe a model of trade between multiple locations that accommodates heterogeneity in consumer preferences, heterogeneity in technological requirements of firms and arbitrary production networks. The model economy consists of many firms and households at many locations. Firms produce using local labor and intermediate inputs sourced from suppliers potentially spread across multiple locations. Trade between locations is subject to iceberg trade costs , that is, a firm producing at o needs to ship τ_{od} units of a good for one unit of good to arrive at d.

Throughout, the paper, a firm is indexed by s when it is a *seller* of intermediate inputs or goods for final consumption and by b when it is a *buyer* of intermediate inputs. A location is indexed by o when it is the *origin* of a trade flow and typically where firm s is located. Similarly, it is indexed by d when it is the *destination* of a trade flow and typically where firm b is located. The set of all locations is denoted by $\mathcal{J}$. The set of all firms is denoted by $\mathcal{M}$ and the subset located at o is denoted by $\mathcal{M}_o$. The number of elements in these sets are denoted as $M = |\mathcal{M}|$ and $M_o = |\mathcal{M}_o|$.

3.1. **Technology and Market Structure.** Firms' production processes involve combining labor and accomplishing a set of tasks by sourcing intermediate inputs from other firms. In particular, the production function for any firm b at location d is defined over labor and a discrete number of tasks (indexed by $k \in \mathcal{K} \equiv \{1, \cdots, K\}$) as:

$$\begin{aligned} y_d(b) &= z_d(b) \left(\frac{l_d(b)}{1-\alpha_d} \right)^{1-\alpha_d} \left(\frac{\prod_{k \in \mathcal{K}} m_d(b,k)^{1/K}}{\alpha_d} \right)^{\alpha_d}, \\ m_d(b,k) &= \sum_{s \in \mathcal{S}_d(b)} m_{od}(s,b,k), \end{aligned}$$

where $l_d(b)$ is the amount of labor input used by firm b, $m_d(b,k)$ is the quantity of materials utilized to accomplish task k, $z_d(b)$ is the idiosyncratic Hicks-neutral productivity with which firm b produces, and K is the number of tasks in the production function.[9]

[9]The number of tasks is common across all firms in the economy for simplicity. It is straightforward to allow for heterogeneity in numbers of tasks but that would not affect the main results of the paper.

Among all the firms in the economy, firm b encounters only a few and can source intermediate inputs to accomplish tasks only from those firms. In particular, it encounters any potential supplier with probability $\frac{\lambda}{M}$ via independent Bernoulli trials. The restricted set of potential suppliers, denoted by $\mathcal{S}_d(b)$, is therefore completely determined as the outcome of these Bernoulli trials for meeting each firm and is common for all tasks. While outputs of potential suppliers are perfectly substitutable for accomplishing any task, they differ in their suitability for the task in question, captured by their respective match-specific productivities. For each of its tasks, firm b selects the supplier that offers the lowest effective price. Importantly, firm b may choose the same supplier for more than one tasks. Since firms only encounter a few suppliers when sourcing intermediate inputs, I assume that firms face limit pricing behavior when sourcing inputs.[10]

3.2. **Cost Minimization and Input Sourcing.** I now turn to firms' cost minimization problem. Selecting the cost-minimizing input bundle consists of choosing not only who to source inputs from but also how much to buy from each of them. For any task k in firm b's production function, the cost-effectiveness of a supplier s from location o in $\mathcal{S}_d(b)$ depends on four factors: (a) the marginal cost of s, denoted $c_o(s)$; (b) the trade cost faced by s of shipping goods to d, τ_{od}; (c) the match-specific productivity when b utilizes the output of s to accomplish the task, denoted by $a_{od}(s, b, k)$, and (d) the markup charged by s when it sells its output to b for accomplishing the task, denoted $\bar{m}_{od}(s, b, k)$. In particular, firm b chooses the supplier that offers the cheapest price, that is,

$$s_d^*(b, k) = \arg \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{\bar{m}_{od}(s, b, k)\, c_o(s)\tau_{od}}{a_{od}(s, b, k)} \right\}. \tag{3.1}$$

With limit pricing, the markup is determined by how much lower the effective cost faced by the best supplier is relative to the second best. Now, taking wage w_d and effective prices $\{p_d(b, k) : k \in \mathcal{K}\}$ (defined below) as

[10] Markups are variable and endogenously determined through Bertrand competition. Dhyne et al. (2022) and Huang et al. (2022) also propose models of firm-to-firm trade featuring endogenous and variable markups through oligopolistic competition. Dhyne et al. (2022) propose a model where firm-to-firm trade in endogenous only on the intensive margin while Huang et al. (2022) propose a two-sided model where each firm is either a supplier or a buyer of intermediate inputs but not both. Here, firm-to-firm trade is endogenous also on the extensive margin and any firm can simultaneously be a buyer and a supplier of intermediate inputs.

given, the firm's unit cost function is given by:

$$c_d(b) = \frac{w_d^{1-\alpha_d} \left(\prod_{k \in \mathcal{K}} p_d(b,k)^{1/K} \right)^{\alpha_d}}{z_d(b)}, \tag{3.2}$$

where $p_d(b,k)$ is determined according the following equation:

$$p_d(b,k) = \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{\bar{m}_{od}(s,b,k)\, c_o(s) \tau_{od}}{a_{od}(s,b,k)} \right\}. \tag{3.3}$$

Discussion. Firms spend equal shares of costs across tasks. Although the elasticity of substitution between tasks is equal to unity, this formulation captures richer patterns of substitution across outputs of other firms that are used to accomplish tasks. This is because a potential supplier charging a lower price is likely to be selected for a higher number of tasks by any firm and hence is likely to account for a higher cost share of the firm. The extensive margin of firms' input sourcing is determined by whether a potential supplier is chosen for at least one of the tasks whereas the intensive margin is determined by how many tasks the potential supplier gets selected for. Both these margins of firm-to-firm trade – who buys from whom and how much? – are determined endogenously in equilibrium.

It is worthwhile noting that forces that generate the extensive and intensive margins of firm-to-firm trade here differ from other models of network formation. In Miyauchi (2021); Arkolakis et al. (2021); Eaton et al. (2022), heterogeneous search frictions and labor productivity differences regulate the extensive margin while attractiveness of potential suppliers regulates the intensive margin. In Lim (2018); Huneeus (2020); Bernard et al. (2021); Demir et al. (2021), fixed costs of relationship formation regulate the extensive margin while attractiveness of potential suppliers regulates the intensive margin. The model here does not feature heterogeneous search frictions, labor productivity differences or fixed costs of relationship formation. Despite this parsimony, the model generates variation in both the extensive and intensive margins of firm-to-firm trade.

3.3. Closing the Model.

Household Preferences. Households are modeled analogously with tasks in their utility function. They encounter potential suppliers and select the most cost-effective suppliers for each task similar to firms sourcing inputs. Each household supplies one unit of labor inelastically to local firms and receives labor income. Firms rebate any profits to local households.

Equilibrium Definition. Let $\sigma \equiv \{\boldsymbol{z}, \boldsymbol{\tau}, \boldsymbol{\mathcal{S}}, \boldsymbol{a}\}$ denote the aggregate state of the economy. Here $\boldsymbol{z}$ denotes the vector of idiosyncratic productivities of firms, $\boldsymbol{\tau}$ denotes the vector of trade costs across all pairs of locations, $\boldsymbol{\mathcal{S}}$ denotes the sets of potential suppliers of all firms and households, and $\boldsymbol{a}$ denotes the vector of all match-specific productivities. All of these objects are exogenous. An equilibrium in this economy is an allocation and a price system such that (a) households and firms select suppliers for tasks; (b) firms set prices for other firms and households under limit pricing; (c) households maximize utility; (d) firms minimize costs; and (e) market clears for each firm's goods and for labor at each location. This completes description of the economic environment in the model.[11]

Moving ahead, the aggregate state can be divided into two parts. The first comprises of firms' productivities and trade costs; this is denoted by $\sigma_0 \equiv \{\boldsymbol{z}, \boldsymbol{\tau}\}$. The second part comprises of sets of potential suppliers for firms and households and match-specific productivities and taste shocks; this is denoted by $\sigma_1 \equiv \{\boldsymbol{\mathcal{S}}, \boldsymbol{a}\}$. While σ_0 narrows down the set of networks that could be realized as an outcome of the network formation process, σ_1 pinpoints the exact network of firms that is realized.

4. Taking Model to Data

To map the model to micro-data on firm-to-firm sales for estimation, I proceed in four steps. First, I utilize the recursive representation of network formation between firms to cast it as a quasi- dynamic programming problem. Second, I show that the model delivers closed-form characterization of conditional choice probabilities in this quasi- dynamic discrete choice setting. Third, I describe how these conditional choice probabilities coupled with multiple discrete choice across tasks lead to a multinomial logit model of supplier choice. Finally, I tackle the computational burden imposed by the high-dimensionality of the non-linear estimation problem by exploiting special features of the multinomial likelihood specification. The resulting estimation framework is scalable and circumvents computational difficulties pervasive in estimation of network formation models with large numbers of firms.

4.1. **A Recursive Representation of Network Formation.** I begin by casting network formation between firms as a quasi-dynamic programming

[11] A detailed description is provided in Appendix B

problem. In particular, combining equations (3.2) and (3.3), I find that marginal cost of any firm b admits the following recursive representation.

$$c_d(b) = \frac{w_d^{1-\alpha_d}}{z_d(b)} \times \prod_{k=1}^{K} \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{\bar{m}_{od}(s,b,k)\tau_{od}}{a_{od}(s,b,k)} \times c_o(s) \right\}^{\frac{\alpha_d}{K}} \tag{4.1}$$

This representation is akin to a setting with dynamic discrete choice (albeit with multiple discrete choice). The estimands in this estimation problem are trade costs $\{\tau_{od} : (o,d) \in \mathcal{J}^2\}$ which are exogenous and firms' marginal costs $\{c_o(s) : s \in \mathcal{M}\}$ which are endogenously determined, unobserved in the data and run into millions. As a result, full solution methods for estimation of dynamic discrete choice models (such as Rust (1987)) are infeasible and simulation-based approaches are computationally burdensome due to rich interactions between a large number of firms. Therefore, I utilize the conditional choice probability approach to estimate the model following Hotz and Miller (1993). In this context, conditional choice probabilities are the probabilities with which any given supplier s is chosen for any one of the buyer b's tasks conditional on its marginal cost being $c_o(s)$. I proceed to show next that the model delivers closed-form predictions for these probabilities.

Discussion. Antràs and de Gortari (2020) and Menzel (2022) also suggest use of methods proposed for dynamic discrete choice to estimate models of supply chain formation and pairwise stable network formation respectively. The adaptation here is different in two ways. First, the dimensionality of the estimation problem in their cases is much smaller while here it is dictated by the millions of firms in the model. Therefore, I proceed with estimation via the conditional choice probability approach instead of using the constrained optimization approach which they do. Second, the model here features multiple discrete choice by many firms that lead to transparent estimating equations for cost shares at the firm-to-firm level whereas their models are of single discrete choice by many firms or agents and do not lead to the same characterization.

4.2. **Conditional Choice Probabilities & Firm-to-Firm Trade.** I turn to expressions for conditional choice probabilities and hence predictions for firm-to-firm trade. I assume that match-specific productivities are drawn independently for all potential suppliers for each of the tasks in firms' production functions from a Pareto distribution as stated in the following assumption.

Assumption 1. *Match-specific productivities are drawn independently according to the following Pareto distribution:*

$$F_a(a) = 1 - \left({}^{a}\!/\!{}_{a_0}\right)^{-\zeta}.$$

In a sufficiently large economy such that $0 < {}^{\lambda}\!/\!{}_{M} \ll 1$, $|\lambda a_0^{\zeta} - 1| < \varepsilon_1$, and $|a_0| < \varepsilon_2$ for arbitrarily small values of ε_1 and ε_2 one can obtain closed-form expressions for conditional choice probabilities. Recall from equation (3.1) that firms choose suppliers for tasks based on suppliers' marginal costs, trade costs faced by them, and match-specific productivities associated with the task under consideration. While trade costs $\boldsymbol{\tau}$ constitute σ_0, match-specific productivities are unknown and suppliers' marginal costs $c_o(s)$ are determined endogenously. I therefore characterize conditional choice probabilities for supplier choice, i.e., probabilities for choice of supplier conditional on its marginal cost but in expectation over match-specific productivities that are yet to be realized. Let $\pi^0_{od}(s,b)$ denote the probability with which firm b selects firm s for any one of its tasks. Prior to encountering and realizing match-specific productivities for each task, the probability of firm s getting selected for any one of the tasks by firm b is common across all tasks. That is, $\pi^0_{od}(s,b) = \pi^0_{od}(s,b,k) = \mathbb{E}_{\{\sigma_1\}}\left[\mathbf{1}\left\{s = s^*_d(b,k) \mid \sigma_0, \sigma_1\right\}\right]$ where the expectation operator is over all realizations of σ_1. The following proposition provides expressions for conditional choice probabilities $\pi^0_{od}(s,b)$.

Proposition 1. *For any realization of σ_0, conditional on firm s's marginal cost being $c_o(s)$, the probability with which any firm b located in d selects firm s located in o for any given task is*

$$\pi^0_{od}(s,b) = \frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta}}. \tag{4.2}$$

Proof. See Appendix C.1. □

The above proposition is key to understanding what drives network formation among firms in the model and how it enables the model to match empirical regularities described in Section 2. Equation (4.2) highlights the factors that influence the likelihood of a supplier s from o getting selected by a buyer at d for any one of its tasks. Firms with lower marginal costs, denoted by $c_o(s)$, are more likely to get selected for more tasks. Firms that are located nearer to the buyers and face lower trade costs, denoted by τ_{od}, are more likely to get selected for more tasks. Moreover, the elasticity of the likelihood of getting selected with respect to marginal costs or trade costs is decreasing in ζ. That is, $\frac{\partial \ln \pi^0_{od}(s,b)}{\partial \ln c_o(s)} = \frac{\partial \ln \pi^0_{od}(s,b)}{\partial \ln \tau_{od}} = -\zeta$. With lower ζ, Assumption 1 implies that high match-specific productivities are more likely and the choice of supplier is less sensitive to other factors, i.e., its marginal cost and the trade cost faced by it. The shape parameter ζ regulates the thickness of the right tails of the match-specific productivity distribution. The lower ζ is, the higher is the likelihood of particularly high draws of match-specific productivities. With higher likelihood of high draws, the choice of supplier (according to equation (3.1)) is less sensitive to marginal cost of the supplier, markup or trade costs.

In summary, this proposition channels the role of the upstream margin — at any location d, firms with lower marginal costs are likely to be used intensively by customers. The role of geography in the upstream margin comes from the dependence of these probabilities on trade costs — firms from o are less likely to be successful across potential customers at d if o is farther, i.e., τ_{od} is higher. The tractable expressions for firm-to-firm trade in Proposition 1 give rise to transparent estimating equations for the model, to which I turn next.

Discussion. In other models of network formation (such as Lim (2018); Huneeus (2020); Miyauchi (2021); Arkolakis et al. (2021); Bernard et al. (2021); Eaton et al. (2022)), forces that regulate the extensive and intensive margins of firm-to-firm trade are distinct. Consequently, those models are not able to deliver a closed-form prediction for firm-to-firm trade that uniquely features here. Both the extensive and intensive margins of firm-to-firm trade are determined by attractiveness of suppliers. The extensive margin then arises naturally as an outcome of discreteness of the number of tasks in production. In other words, even with a positive probability of supplying any one task, there is a positive probability of not being able to supply any of the tasks because there are only a discrete number of tasks to supply for.

4.3. **A Multinomial Logit Model of Supplier Choice.** I reformulate the economic model developed so far as a multinomial logit model of supplier choice for tasks of each of the firms and estimate it semi-parametrically. Firm's marginal costs are estimated as firm fixed effects and bilateral origin-destination fixed effects correspond to a structural gravity specification for estimating trade frictions. Trade frictions are then estimated by projecting bilateral fixed effects on observables such as distance and borders etc.

The econometric model can be motivated using the balls and bins problem. Consider the multinomial random variable characterized by a firm b located at d throwing K balls (one for each of its tasks) into M bins. Each of these bins corresponds to a potential supplier, denoted by s. The probability with which any of these balls falls into the bin indexed s is given by the expression for $\pi^0_{od}(s,b)$ from Proposition 1. A realization of this random variable consists of the proportion of balls that landed in each of the bins. Since tasks are symmetric and the production function of firm b takes the Cobb-Douglas functional form, the model counterpart of this realization is the vector of cost shares of firm b across all suppliers in the economy. In other words, the cost share of firm b that can be attributed to firm s stands in for the relative frequency of firm s's successes in getting selected across firm b's tasks. Since there are a discrete number of tasks, $\pi^0_{od}(s,b)$ is only the expected share of tasks for which firm b uses the output of firm s. Any given realization may deviate from this expected value for particularly high or low realizations of match-specific productivities and from randomness in buyer-seller encounters between firms.Therefore, making use of Proposition 1, the estimating equation can be expressed as a multinomial logit function:

$$\mathbb{E}\left[\pi_{od}(s,b)\right] = \frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta}} \tag{4.3}$$

Formally, the estimation problem is as follows:

$$\Delta^* = \arg\max_{\Delta} \frac{1}{M}\sum_{b\in\mathcal{M}} \ln f_{\mathrm{MNL}}\left(\mathbb{D}\mid\Delta\right), \tag{4.4}$$

$$f_{\mathrm{MNL}}\left(\mathbb{D}\mid\Delta\right) \propto \prod_{s\in\mathcal{M}}\left(\frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta}}\right)^{\pi_{od}(s,b)},$$

where

$$\Delta \equiv \left\{\left\{c_o(s)^{-\zeta} : s\in\mathcal{M}\right\}, \left\{\tau_{od}^{-\zeta} : (o,d)\in\mathcal{J}^2\right\}\right\} \text{ and}$$

$$\mathbb{D} \equiv \left\{\pi_{od}(s,b) : (s,b) \in \mathcal{M}^2\right\}.$$

The above specification with fixed effects however presents a problem of perfect multicollinearity in regressors. Note that dummy variables associated with $\left\{c_o(s)^{-\zeta} : s \in \mathcal{M}_o\right\}$ and $\left\{\tau_{od}^{-\zeta} : d \in \mathcal{J}\right\}$ are collinear for all such locations o. Hence, I make the following normalizations so that these fixed effects are identified up to scale. For all $s \in \mathcal{M}_o, o \in \mathcal{J}$, let $c_o(s) = c_o\widetilde{c}_o(s)$ such that

$$\left(\sum_{s\in\mathcal{M}_o} \widetilde{c}_o(s)^{-\zeta}\right)^{-1/\zeta} = 1.$$

This normalizes the power average of firms' marginal costs relative to their location average to unity. It separates within and between location heterogeneity in firms' marginal costs. The within location component is captured by differences in $\widetilde{c}_o(s)$ while the between location component is captured by differences in c_o across locations.[12]

Discussion. One could draw an analogy by reinterpreting the Eaton and Kortum (2002) model of trade between countries as the representative agent in the destination country throwing infinitely many balls (one for each commodity arranged on a continuum) into a finite number of bins (one for each origin country). Since the bins are finite in number while balls are infinitely many, sourcing probabilities coincide with aggregate trade shares deterministically. In contrast, the model here is of trade between firms where the customer firm throws a finite number of balls (one for each task) into finitely many bins (one for every firm in the economy). Since both the bins and balls are finitely many in number, conditional choice probabilities do not determine firm-to-firm trade shares deterministically.

In related work, Eaton et al. (2013) also specify a multinomial likelihood function for international trade between countries derived from a different economic model and conduct estimation using pseudo-maximum likelihood estimation à la Gourieroux et al. (1984). The dimensionality of their estimation program is determined by the number of countries which is a much

[12]The between location component captures both differences in average marginal cost between locations and also differences arising from having a higher number of firms at one location than another. To see this clearly, note that if marginal costs are identical across firms at location o, i.e., $c_o(s) = \bar{c}_o$. Then, $c_o = M_o^{-1/\zeta}\bar{c}_o$, which depends on both the number of firms and the average marginal cost.

smaller number compared to the specification here where the dimensionality is determined by the number of firms that runs into millions.

4.4. **Estimating High-Dimensional Multinomial Logit Model.** The multinomial logit specification is problematic because of two reasons. On one hand, firms' marginal costs are endogenously determined and unobserved. They are estimated semiparametrically as firm fixed effects. Since there are a large number of firms in the economy, estimation of the would typically require high-dimensional non-linear optimization over a very large number of parameters to solve for the estimates. This can be computationally infeasible using standard Newton methods when the number of fixed effects runs into millions. On the other hand, estimation of a generalized linear model with millions of fixed effects leads to incidental parameters bias in the lower-dimensional estimands.

However, these issues are taken care of by appealing to several special features of the multinomial likelihood function. First, estimates can be obtained using the Poisson likelihood function with additional fixed effects (see Baker (1994); Taddy (2015)). Second, Poisson likelihood estimation automatically satisfies adding up constraints implied by the model (see Fally (2015)). Third, Poisson likelihood specification allows solving for fixed effects in closed-form (for example, see Hausman et al. (1984)). Finally, subsequent estimation of trade frictions using bilateral fixed effects does not suffer from the incidental parameters problem and hence can be conducted through the conditional maximum likelihood approach.

4.4.1. *Marginal Costs and Structural Gravity.* The first order conditions implied by the likelihood maximization problem in equation (4.4) can be solved to obtain closed-form estimators for fixed effects as described in the proposition below.

Proposition 2. *The estimates from equation (4.4) are given by:*

$$\left(\widetilde{c}_o(s)^{-\zeta}\right)^* = \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{s'\in\mathcal{M}_o}\sum_{b\in\mathcal{M}} \pi_{od}(s',b)} \quad \forall s \in \mathcal{M}, \tag{4.5}$$

$$\left(\frac{c_o^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}}\right)^* = \frac{1}{M_d}\sum_{b\in\mathcal{M}_d} \pi_{od}\left(\bullet, b\right) \quad \forall (o,d) \in \mathcal{J}^2 \tag{4.6}$$

where $\pi_{od}\left(\bullet, b\right) \equiv \sum_{s\in\mathcal{M}_o} \pi_{od}\left(s, b\right)$.

Proof. See Appendix C.2. □

The estimators for firm fixed effects in equation (4.5) neatly bridge theoretical predictions on firm-to-firm trade in equation (4.2) and empirical regularities arising from the decomposition in equation (2.1). The decomposition in equation (2.1) suggested that larger firms also tend to have higher intensity of use. Conditional choice probabilities in equation (4.2) predict that firms with low marginal costs are likely to have higher intensity of use. Equation (4.5) shows that firms' intensity of use is a sufficient statistic for its marginal costs, albeit scaled with an elasticity ζ. In addition, the theoretical expression for bilateral origin-destination fixed effects in equation (4.6) corresponds to a structural gravity specification. For any pair of locations (o, d), the estimator for this specification is the *simple* average of the cost share across firms at d that can be attributed to purchase of goods from firms in o. This is the empirical counterpart of sourcing probabilities in equation (5.2).

4.4.2. *Trade Frictions and Conditional Choice Probabilities.* With firm fixed effects out of the way, thanks to equation (4.5), trade frictions can now be estimated by projecting bilateral origin-destination fixed effects (from equation (4.6)) on bilateral observables such as distance, borders etc., similar to gravity regressions, with the following estimating equation:

$$\mathbb{E}\left[\left(\frac{c_o^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}}\right)^*\right] = \frac{\exp\left(\ln\left(c_o^{-\zeta}\right) + \boldsymbol{X}'_{od}\boldsymbol{\beta}\right)}{\sum_{o'} \exp\left(\ln\left(c_{o'}^{-\zeta}\right) + \boldsymbol{X}'_{o'd}\boldsymbol{\beta}\right)}. \tag{4.7}$$

This delivers estimates of origin fixed effects $\left(c_o^{-\zeta}\right)^*$ and trade frictions $\left(\tau_{od}^{-\zeta}\right)^* = \exp\left(\boldsymbol{X}'_{od}\boldsymbol{\beta}^*\right)$. The manner in which trade frictions are estimated here differs from the standard approach of projecting aggregate trade flows on distance and border dummies (for example, see Agnosteva et al. (2019)). The dependent variable implied by the model is not aggregate trade flows (for example, Santos Silva and Tenreyro (2006)) or aggregate trade shares (as in Eaton et al. (2013)) but average trade share across buyers at the destination. More specifically, the dependent variable $\frac{1}{M_d}\sum_{b\in\mathcal{M}_d}\pi_{od}(\bullet, b)$ is an unweighted average of the sourcing share from o across all buyers at a destination. While this is not comparable to aggregate trade flows, it closely related to aggregate trade shares. In contrast to average trade shares which is a simple average of sourcing shares across firms, the aggregate trade share is a weighted average of individual sourcing probabilities where each individual buyer is weighted by its size. Note that measured aggregate trade share can be expressed as

$$\pi_{od} = \frac{1}{M_d} \sum_{b \in \mathcal{M}_d} \pi_{od}\left(\bullet, b\right) + \frac{Cov\left(\pi_{od}(\bullet, b), \text{purchases}_d(b)\right)}{\frac{1}{M_d} \sum_{b' \in \mathcal{M}_d} \text{purchases}_d(b')}. \tag{4.8}$$

To the extent that size of buyers is correlated with their sourcing from an origin, aggregate trade shares bias the estimates of the trade frictions faced by individual firms for the purposes of estimation here. Trade frictions are estimated using gravity regressions. Table 4.1 reports estimated coefficients for distance and border dummies in column (3) and compares them to common methods in the trade literature in columns (1)-(2). Column (1) is an atheoretical specification (as in Santos Silva and Tenreyro (2006)) that is consistent with handling zeros in the data . Column (3) is a model-based specification (as in Eaton et al. (2013)) and accommodates zeros in the data. Column (3) is the specification that is implied by the model here. Comparing (1) or (2) to (3) shows that using aggregate trade flows or shares underestimates trade frictions for estimation of the model here.

Estimates of conditional choice probabilities are then obtained from firm fixed effects and fitted shares from the gravity regressions. Formally, the estimates of conditional choice probabilities are given by

$$\pi^*_{od}(s, b) = \left(\widetilde{c}_o(s)^{-\zeta}\right)^* \cdot \pi^*_{od}(\bullet, b), \tag{4.9}$$

$$\pi^*_{od}(\bullet, b) = \frac{\left(c_o^{-\zeta}\right)^* \left(\tau_{od}^{-\zeta}\right)^*}{\sum_{o' \in \mathcal{J}} \left(c_{o'}^{-\zeta}\right)^* \left(\tau_{o'd}^{-\zeta}\right)^*}. \tag{4.10}$$

4.4.3. *Trade Elasticity and Materials Share.* Since the model satisfies structural gravity at the aggregate level (see Equation (5.2)) and the dispersion of match-specific productivities ζ coincides with the elasticity of trade with respect to trade costs, I calibrate the value of this parameter to 5 from median of the estimates of price elasticity in structural gravity equations (see Head and Mayer (2014)). Materials shares $\boldsymbol{\alpha} \equiv \{\alpha_d : d \in \mathcal{J}\}$ are calibrated using district-level production statistics (further details in Appendix C.3).

Table 4.1. **Gravity Regressions**

	sales_{od} (1): PPML	sales_{od} (2): MPML	$\left(\frac{c_o^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}}\right)^*$ (3): PPML
log(distance)	-0.219	-0.712	-0.962
	(0.042)	(0.045)	(0.045)
$\mathbf{1}\{\text{inter-state}\}$	-1.971	-2.125	-2.337
	(0.105)	(0.090)	(0.088)
$\mathbf{1}\{\text{inter-district}\}$	-1.484	-1.852	-2.241
	(0.117)	(0.077)	(0.068)
$\mathbf{1}\{\text{neighbor}\}$	0.562	0.251	0.512
	(0.053)	(0.052)	(0.048)
Fixed Effects:			
Origin × Year	✓	✓	✓
Destination × Year	✓	✓	✓
Pseudo R^2	0.945	0.435	0.488
Squared Correlation	0.953	0.793	0.898
# observations	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$

Note. Standard errors in parentheses, two-way clustered by origin–year and destination–year. Observations pertain to all bilateral pairs between 141 districts for 5 years. The distance between district pairs is calculated as the distance between their centroids. A district's distance to itself is calculated as the radius of the circle with the same area as the district. Column (1) is estimated using a Poisson PML specification with aggregate trade flows as the dependent variable as in Santos Silva and Tenreyro (2006). Column (2) is estimated using a multinomial PML specification with aggregate trade shares as the dependent variable as in Eaton et al. (2013). Column (3) is estimated using a multinomial PML specification from equation (4.7). Two-way clustering is done as in Cameron et al. (2011). Pseudo R^2 is calculated as in McFadden (1974).

5. Aggregation and Counterfactual Analysis for Large Network Economies

In this section, I address nuances of aggregation in a large network economy and propose a procedure to analyze effects of policy changes through the lens of the model. While finite economies have aggregate uncertainty, continuum approximations of finite economies inform us about the expected values of aggregate objects of interest. If the economy were indeed a continuum, these expected values would coincide with observed aggregate objects. The model economy that generates the data is not a continuum one and hence aggregate data does not coincide with their expected values.

Even so, these expected values are informative about the distribution of aggregate objects that come from a finite economy model and especially when these distributions do not have a closed-form characterization and simulation-based approximations can be computationally infeasible.[13] The model in this paper features rich network interactions between a finite number of firms. Interdependent decisions on input sourcing made by this finite number of firms leads to non-degeneracy of the distribution of aggregate outcomes. Nonetheless, to be able to solve for expected values, I consider a limiting economy which serves as a continuum approximation of the finite economy.

In particular, I adopt the large economy model due to Al-Najjar (2004) which is characterized by a sequence of finite but increasingly large economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ that progressively discretizes the unit continuum.[14] Along the sequence as the economy becomes more discretized, I make additional assumptions so that the model has a well-defined limit. The probability of meeting potential suppliers increases, i.e., $\lim_{t\to\infty} \lambda_t = \infty$, but at a rate slower than that at which the economy is discretized, i.e., $\lim_{t\to\infty} \frac{\lambda_t}{M_t} = 0$. At the same time, match-specific productivities are drawn from stochastically worse distributions as $\lim_{t\to\infty} a_{0,t} = 0$. While the number of potential suppliers grows arbitrarily large and the match-specific productivity associated with any single supplier is drawn from a stochastically worse distribution, the limit is well behaved because the probability of encountering a supplier with match-specific productivity greater than value a does not change in the limiting economy, i.e., $\lim_{t\to\infty} \lambda_t a_{0,t}^{\zeta} = 1$. Furthermore, the economy becomes discretized in a manner such that the proportion of firms and households at every location is non-zero and finite.[15]

I now proceed to characterize effective prices $\boldsymbol{p}(\sigma)$ and wages $\boldsymbol{w}(\sigma)$ in equilibrium in the limiting economy, i.e., $\lim_{t\to\infty} \mathcal{E}_t$.

---

[13]A similar principle is adopted in recent granular models of exporter entry in Eaton et al. (2013), of multinational entry in Head and Mayer (2019), and of commuting choice in Dingel and Tintelnot (2020). I show later in this section that this approach is theoretically consistent even with complex network interactions between firms which features in the model here.

[14]In Appendix D.1, Definition 4 provides a formal description of such a sequence of economies.

[15]In Appendix D.1, Assumption 2 states this formally. This kind of assumption was shown to have a well-defined limit by Kortum (1997) and put to use for a similar purpose by Oberfield (2018).

5.1. **Market Access & Distributions of Effective Prices.** With limit pricing, the distribution of effective prices faced by a firm for any of its tasks is characterized by the distribution of the offer with the second lowest effective cost to the supplier. The following proposition provides the distribution of effective prices in the limiting economy.

Proposition 3. *For any realization of σ_0, the effective prices of materials used by firm b to accomplish any task, $p_d(b,k)$ converge to the following distribution as $t \to \infty$:*

$$F_{p_d}(p) = 1 - e^{-A_d p^{\zeta}} - A_d p^{\zeta} e^{-A_d p^{\zeta}},$$

where $\boldsymbol{A} \equiv \{A_d : d \in \mathcal{J}\}$ is the unique positive solution to the following fixed point problem:

$$A_d = \sum_{o \in \mathcal{J}} \tau_{od}^{-\zeta} \overline{z_o^{\zeta}} \mu_o w_o^{-\zeta(1-\alpha_o)} \Gamma\left(2 - \frac{\alpha_o}{K}\right)^K A_o^{\alpha_o}, \tag{5.1}$$

where μ_o denotes the proportion of firms at o and $\overline{z_o^{\zeta}} = \mathbb{E}\left[z_o(s)^{\zeta}\right]$.[16]

Proof. See Appendix D.2. □

While the effective price faced by individual firms varies across realizations of σ_1, the cross-sectional distribution in the limit economy does not. These distributions are parametrized by a scale parameter A_d and a shape parameter ζ. Market access, given by A_d, is a key object of interest because it summarizes the probabilistic access of firms at d to inputs from all locations. The functional form suggests that firms at a location with higher market access face stochastically lower effective prices. Specifically, if $A_d > A_{d'}$, the distribution $F_{p_{d'}}(\cdot)$ first-order stochastically dominates $F_{p_d}(\cdot)$.

Focussing on equation (5.1), market access A_d is a trade friction ($\tau_{od}^{-\zeta}$) weighted sum of the attractiveness of all locations $o \in \mathcal{J}$, i.e., nearer locations receive higher weights because of lower trade costs τ_{od} and vice versa. The attractiveness of a location o for sourcing inputs is determined by four factors: (a) density of firms μ_o; (b) average productivity among firms $\overline{z_o^{\zeta}}$; (c) its own market access A_o; and (d) wages w_o. Locations with higher density, higher average productivity, higher market access or lower wages are more attractive. In addition, the attractiveness of a location o is more

[16]The gamma function $\Gamma(\cdot)$ is defined as $\Gamma(x) = \int_0^{\infty} e^{-x} m^{x-1} dm$.

sensitive to its market access A_o and less so to wages w_o if materials share of costs α_o is higher at o and vice versa.

Although the effective price is characterized by the distribution of the offer with the second lowest effective cost to the supplier, it is still the supplier with the lowest effective cost that is selected. The distribution of markups faced by the firm is characterized by that of the ratio of the second lowest to the lowest effective costs incurred by the second best and the best suppliers respectively. In addition, every firm encounters at least two potential suppliers with probability approaching one in the limiting econony and this ensures that markups are well-behaved.[17]

5.2. **Relative Wages in Trade Equilibrium.** To define relative wages in trade equilibrium, I begin by characterizing sourcing probabilities, that is, the probability with which any buyer sources inputs from location o for any one its tasks. Conditional choice probabilities of supplier choice naturally aggregate to sourcing probabilities, that is, sourcing probabilities can be obtained as the sum of conditional choice probabilities associated with all the suppliers located at o. Conditional choice probabilities from Proposition 1 together with properties of the cross-sectional distributions of effective prices from Proposition 3 lead to the next proposition. This proposition characterizes sourcing probabilities across origins by firm b, denoted by $\pi^0_{od}(\bullet, b)$.

Proposition 4. *For any realization of σ_0, the probability with which any firm b located in d selects a supplier from o for any given task is*

$$\pi^0_{od}(\bullet, b) = \frac{\mu_o w_o^{-\zeta(1-\alpha_o)} \overline{z_o^{\zeta}} \Gamma\left(2 - \frac{\alpha_o}{K}\right)^K A_o^{\alpha_o} \tau_{od}^{-\zeta}}{A_d}. \tag{5.2}$$

Proof. See Appendix D.4. $\square$

Sourcing probabilities in equation (5.2) hark back to market access defined in equation (5.1). Recall that market access is a weighted sum of attractiveness of all locations for a particular destination. Equation (5.2) suggests the probability with which a buyer from d sources intermediate inputs from o for any one of its tasks is given by the contribution of location o towards market access of firms at d. Firms at d are more likely to source

[17]For any firm b, $\mathbb{P}(|\mathcal{S}_d(b)| < 2) = \left(1 - \frac{\lambda_t}{M_t}\right)^{M_t} + \binom{M_t}{1}\left(\frac{\lambda_t}{M_t}\right)\left(1 - \frac{\lambda_t}{M_t}\right)^{M_t - 1}$. It then follows from Assumption 2 that $\lim_{t\to\infty} \mathbb{P}(|\mathcal{S}_d(b)| < 2) = 0$.

inputs from o if there are a larger number of firms at o (higher μ_o), wage w_o is lower, average productivity $\overline{z_o^\zeta}$ is higher or firms at o have better market access (higher A_o).

These sourcing probabilities are independent of the identity of the buyer at the destination and therefore can be written as $\pi^0_{od}(\bullet, -)$. In the limiting economy, the average sourcing share across all buyers in the limiting economy coincides with the expected value given by equation (5.2). This however does not mean that the sourcing shares across individual buyers are identical either in the finite economy or the limiting economy. Buyers at a destination may very well differ in their sourcing shares whether in the finite economy or the limiting economy. Formally, the law of large numbers implies that in the limiting economy,

$$\frac{1}{M_d}\sum_{b\in\mathcal{M}_d}\pi_{od}(\bullet, b) \xrightarrow{t\to\infty} \pi^0_{od}(\bullet, -). \tag{5.3}$$

I now turn to characterizing relative wages in the trade equilibrium in the limiting economy. The following proposition shows that relative wages in the limiting economy can be obtained as a solution to the system of equations (5.4).

Proposition 5. *For any realization of $\sigma \equiv \{\sigma_0, \sigma_1\}$, $\boldsymbol{w} \equiv \{w_d : d \in \mathcal{J}\}$ solves the following system of equations:*

$$\frac{w_o L_o}{1-\alpha_o} = \sum_{d\in\mathcal{J}}\pi^0_{od}\left(\bullet, -\right)\frac{w_d L_d}{1-\alpha_d}. \tag{5.4}$$

Further, for any σ and σ' such that $\sigma_0 = \sigma_0'$ and $\sigma_1 \neq \sigma_1'$:

$$\boldsymbol{w} = \boldsymbol{w}'. \tag{5.5}$$

Proof. See Appendix D.5. □

The above proposition also shows that, for any given realization of σ_0, relative wages are invariant across all networks realized for all values of σ_1. This concludes the characterization of equilibrium wages and brings us to the definition of the trade equilibrium below.[18]

[18] Dingel and Tintelnot (2020) propose a granular model of commuting choice where non-degeneracy of counterfactual outcomes arises from a finite number of individuals making residential and workplace decisions. A similar problem of indeterminacy of the trade equilibrium across locations arises in their setting.

Definition 1. For any given σ_0, the trade equilibrium in the limiting economy is defined as the vector of wages $\boldsymbol{w}$ such that (a) market access at each location satisfies equation (5.1); (b) trade shares coincide with sourcing probabilities in equation (5.2) and (c) the market clearing condition in equation (5.4) holds.

Discussion. It is worthwhile to note that zero trade flows have different interpretations in the continuum economy versus the finite economy. In the continuum economy, zero trade flows between locations are rationalized by infinite trade costs. It then follows that locations that do not trade in the initial state, would not trade in any counterfactual scenario. In the finite economy, zero trade flows are an outcome of granularity and do not imply infinite trade costs (finite number of trials of low probability events imply a positive probability of zero successes). Therefore, observing zero trade flows in the initial state does not preclude location pairs from trading in counterfactual scenarios. Out of 141^2 location pairs in the data, around 40% do not trade. The finite economy approach does not impose zero trade flows for these location pairs in counterfactual scenarios.

Furthermore, trade is driven by comparative advantage as in Ricardian trade models (Eaton and Kortum (2002); Bernard et al. (2003)). However, the model accommodates heterogeneity in consumer preferences and technological requirements across firms, comparative advantage is determined by each consumer and firm demanding inputs rather than at the level of each market. This allows the model to rationalize patterns of firm participation in international trade within the Ricardian framework which are typically relegated to new trade theory models such as Melitz (2003) and Eaton et al. (2011). For example, Eaton et al. (2011) state that the Ricardian framework with a fixed range of commodities used in Bernard et al. (2003) does not deliver the feature that a larger market attracts more firms as observed in French data.

In this context, two facts are worth noting about sourcing probabilities in equation (5.2). First, the elasticity with respect to trade costs comes from the shape parameter of match-specific productivities ζ. This unlinks the dispersion in idiosyncratic productivities from the trade elasticity. Second, they are increasing in the density of firms at the origin μ_o. This feature introduces a probabilistic notion of "love of variety" within the Ricardian framework. It also implies that zeros in firms' sales to destinations are an outcome of granularity and do not reflect more productive firms' ability to sell at a destination after incurring fixed costs.

5.3. **Goodness of Fit of the Continuum Approximation.** I now turn to assess the fit of the limiting economy which serves as a continuum approximation of the finite economy. In particular, I evaluate the approximation based on it ability to replicate empirical regularities documented in Section 2. I start by computing equilibrium wages in the limiting economy using equation (5.4). To do so requires the knowledge of true values of sourcing probabilities $\pi^0_{od}(\bullet, -)$. In their absence, we can leverage equation (5.3) to obtain a consistent estimate of these probabilities from equation (4.7). Fitted shares from gravity regressions obtained in equation (4.10) could be used for computing equilibrium wages as per equation (5.4).

A key finding in Proposition 2 is that the fixed effect estimate for a firm s with the multinomial likelihood specification is in fact its measured intensity of use, $\sum_{b\in\mathcal{M}} \pi_{od}(s, b)$. Fixed effect for firm s is the product of the within location component $\widetilde{c}_o(s)^{-\zeta}$ and the between location component $c_o^{-\zeta}$. Equation (4.5) provides a estimator for the former. The latter is estimated in column (3) in Table 4.1 using a multinomial likelihood specification. By properties of the multinomial likelihood, this estimate is given by $\sum_{s\in\mathcal{M}_o} \sum_{b\in\mathcal{M}} \pi_{od}(s, b)$. Together, they imply that the fixed effect estimate for firm s can be expressed as $\left(c_o(s)^{-\zeta}\right)^* = \sum_{b\in\mathcal{M}} \pi_{od}(s, b)$. According to the model (in equation (4.2)), this fixed effect is related marginal costs as $c_o(s)^{-\zeta}$. This directly features in equation (4.9) and plays a vital role in enabling the model to reproduce the empirical regularities. Apart from this, goodness of fit is governed by two factors. First, imperfect correlation between data and fitted values in Table 4.1, Column (3) causes differences in $\pi_{od}(\bullet, -)$ and $\pi^*_{od}(\bullet, -)$. Second, estimating equation (4.3) is parsimoniously specified as it does not allow heterogeneity in trade frictions faced by firms. While the data is at the firm-to-firm level, fixed effects are only at the firm and origin–destination level. Third, equilibrium wages computed for the limiting economy differ from data. These differences capture the granularity of data which are assumed away in the limiting economy. Finally, estimates of material share of costs $\boldsymbol{\alpha}$ and dispersion in match-specific productivities ζ also affect predicted values.

Table 5.1 reports how the estimated model performs in comparison to the empirical regularities documented in Section 2. Table 5.1 shows that

TABLE 5.1. **Model Fit: Margins of Firms' Sales**

	(1)	(2)	(3)
Data:			
Intensity of Use	81%	93%	83%
Average Customer Size	19%	7%	17%
Model:			
Intensity of Use	75%	65%	68%
Average Customer Size	25%	35%	32%
Fixed Effects:			
Seller×Year	—	✓	—
Origin×Year	—	—	✓
Data Level:			
Seller×Year	•	—	—
Seller×Destination×Year	—	•	—
Origin×Destination×Year	—	—	•
# observations	5.6×10^{6}	18.2×10^{6}	58,390

Note. Column (1) reports the contribution of factors: intensity of use and average customer size, to the variance of firms' sales (as per equation (2.1)) in the data (top panel) and in the model (bottom panel). Column (3) reports the contribution of those factors to the variance of firms' destination-specific sales (as per equation (A.2)). Column (5) reports the same for trade flows between districts (as per equation (A.3)).

the intensity of use margin explains a vast majority of the variation in firms' sales in the estimated model as is the case in the data. This is true across all columns in the data qualitatively. Quantitatively, all columns except (3) provide a reasonably good fit. In column (3), the loss of fit can be attributed to the second factor.

5.4. **Computation of Counterfactual Outcomes.** I operationalize Propositions 3, 4, and 5 for counterfactual analysis by expressing them in changes. The following definition states that and motivates the algorithm for evaluating counterfactual outcomes in response to shocks that derive from a change in the aggregate state σ_0 to σ_0'.

Definition 2. For any change in aggregate state σ_0 to σ_0', equilibrium change in wages $\widehat{\boldsymbol{w}} \equiv \{\widehat{w}_d : d \in \mathcal{J}\}$ and welfare $\widehat{\boldsymbol{V}} \equiv \left\{\widehat{V}_d : d \in \mathcal{J}\right\}$ are characterized the following system of equations for all realizations of σ_1 or

σ_1':[19]

$$\widehat{A}_d = \sum_o \pi_{od}^0 (\bullet, -) \widehat{\delta}_{od} \widehat{w}_o^{-\zeta(1-\alpha_o)} \widehat{A}_o^{\alpha_o}$$

$$\widehat{\pi_{od}^0 (\bullet, -)} = \frac{\widehat{\delta}_{od} \widehat{w}_o^{-\zeta(1-\alpha_o)} \widehat{A}_o^{\alpha_o}}{\widehat{A}_d}$$

$$\frac{\widehat{w}_o w_o L_o}{1-\alpha_o} = \sum_d \widehat{\pi_{od}^0 (\bullet, -)} \pi_{od}^0 (\bullet, -) \frac{\widehat{w}_d w_d L_d}{1-\alpha_d}$$

$$\widehat{V}_d = \widehat{w}_d \widehat{A}_d^{1/\zeta}$$

where $\widehat{\boldsymbol{\delta}} \equiv \left\{ \widehat{\delta}_{od} : (o, d) \in \mathcal{J}^2 \right\}$ is function of shocks that capture the resultant effect of change from σ_0 to σ_0'.

With this definition of the equilibrium in changes in the limiting economy, aggregate and firm-level counterfactual outcomes in the limiting economy are computed in three steps. First, I evaluate aggregate and firm-level outcomes such as intensity of use and sales in the limiting economy in the initial state. Second, I evaluate changes in aggregate outcomes when going from the initial state to the counterfactual state. This is done using a tâtonnement algorithm similar to Alvarez and Lucas (2007) and Dekle et al. (2008). Finally, I evaluate aggregate and firm-level outcomes in the limiting economy in the counterfactual state. Details of the procedure are stated in Appendix D.7. The counterfactual outcomes thus computed for the limiting economy correspond to the expected value of outcomes for the finite economy in the counterfactual state since the limiting economy is a continuum approximation of the finite economy. Having set out the procedure to compute counterfactual outcomes, I turn next to evaluating the impact of the 2017 GST reform on Indian firm-to-firm production networks.[20]

6. The Impact of 2017 GST Reform on Indian Firm-to-Firm Production Networks

When India adopted the VAT in the early 2000s, its implementation was uneven. India has a federal system of government — one that divides

[19] The expression for welfare changes is derived in Appendix D.6.

[20] Appendix E illustrates how the model can be used to assess the consequences of micro- and macro- shocks to the spatial economy through two other quantitative applications: one considering uniform decline in cost of trading outside own district and the other studying improvements in allocative efficiency following neutralization of firm-level distortions.

the powers of government between the national and the state governments. Commercial taxation being overseen by the state government, individual states implemented their own respective VAT systems. This resulted in over 30 such systems coming into place across India. While this increased formality and tax compliance, it had the unintended consequence of regional segregation in organization of production, for three reasons.

First, VAT increases formality because firms prefer to source inputs from other firms within the system to be able to collect tax credits on input purchases. Consequently, individual firms preferred to source inputs from firms within their own state's VAT system as opposed to one in a different state or VAT system. Second, the national government levied a sales tax on firm-to-firm transactions across state borders which made more efficient suppliers of intermediate inputs relatively more expensive if they were in a different state. Third, there were cumbersome inspections, especially at state borders that caused logistical delays. In July 2017, the federal government in India abolished all state VAT systems and introduced the Goods and Services Tax to serve as a single national VAT system. This eliminated sales taxes on inter-state movement of goods and harmonized the VAT structure across states in an attempt to reduce such barriers to intra-national trade.

In this context, I consider the impact of a 10% decline in trade costs between district pairs crossing state borders to understand the potential impact of the GST reform on production networks in intra-national trade through the lens of the model. Changes in firms' sales to other firms can be decomposed into changes in its intensity of use and changes in its average customer size as follows:

$$\frac{\Delta\text{Sales}}{\text{Sales}} = \overbrace{\frac{\Delta\text{Intensity of Use}}{\text{Intensity of Use}}}^{\Delta\%\text{ Upstream Margin}} + \underbrace{\frac{\Delta\text{Average Customer Size}}{\text{Average Customer Size}}}_{\Delta\%\text{ Downstream Margin}} + \underbrace{\frac{\Delta\text{Intensity of Use}}{\text{Intensity of Use}} \times \frac{\Delta\text{Average Customer Size}}{\text{Average Customer Size}}}_{\text{Second Order Term}}$$

To determine the relative contribution of the upstream and downstream margins to the dispersion in changes in firms' sales, I apply a Shapley decomposition (see Shorrocks (2013)). The Shapley decomposition determines the expected marginal contribution of each of these margins and the interaction term to the total variation in changes in firms' input sales; intuitively, it assigns the fraction of the R^2 of a regression that is due to each set of explanatory variables. Columns (1) and (4) in Table 6.1 report the results of this decomposition. The top row suggests that over half of the variation in changes in firms' sales can be attributed to endogenous changes in the network or the upstream margin while a third can be attributed to the downstream margin. When considering variation among firms within each state, the upstream margin accounts for over a third of the variation. The lower contribution is because the incidence of the shock is at the state borders, so the contribution of the upstream margin is not as high as that seen in the top row.

A similar decomposition can also be made at a more disaggregated level for firms' destination-specific sales and at a more aggregated level for trade flows between districts. Columns (2) and (5) report the results of the decomposition for changes in firms' destination-specific sales. Around three-fifths of the variation in changes in firms' destination-specific sales can be attributed to the endogenous changes in network structure while the downstream margin accounts for under one-third of the variation. Columns (3) and (6) report the results of the decomposition for changes in trade flows between districts. Around two-thirds of the variation in changes in trade flows can be attributed to the endogenous changes in network structure while the downstream margin accounts for a fifth of the variation. Figure 6.1 depicts the relative contribution of changes in the upstream margin with respect to changes in the downstream margin towards changes in destination-specific sales of firms across districts.

A few points are in order. First, this decomposition is of sales to other firms and so would not exist in models without input-output linkages. Second, in models with exogenous production networks, i.e., with Cobb-Douglas technologies between firms, intensity of use does not respond to shocks. The large variation in the upstream margin would therefore be missing. Finally, in models with non-Cobb–Douglas technologies that endogenize the intensity with which existing suppliers are used but where the extensive margin of firm-to-firm trade does not respond to shocks, the explanatory power of the upstream margin would be understated. This

TABLE 6.1. **Margins of Changes in Firms' Sales**

	Δ% Upstream Margin			Δ% Downstream Margin		
	(1)	(2)	(3)	(4)	(5)	(6)
Overall	57.3%	60.4%	65.7%	33.2%	32.1%	20.1%
Gujarat	36.9%	48.9%	60.9%	44.9%	42.8%	23.9%
Maharashtra	40.9%	62.5%	60.4%	29.5%	30.9%	24.2%
Odisha	36.7%	56.9%	70.4%	45.5%	29.8%	15.5%
Tamil Nadu	40.0%	61.5%	60.2%	33.9%	33.0%	24.3%
West Bengal	40.0%	61.0%	61.5%	30.9%	30.1%	23.7%
Data Level:						
Seller×Year	•	—	—	•	—	—
Seller×Destination×Year	—	•	—	—	•	—
Origin×Destination×Year	—	—	•	—	—	•

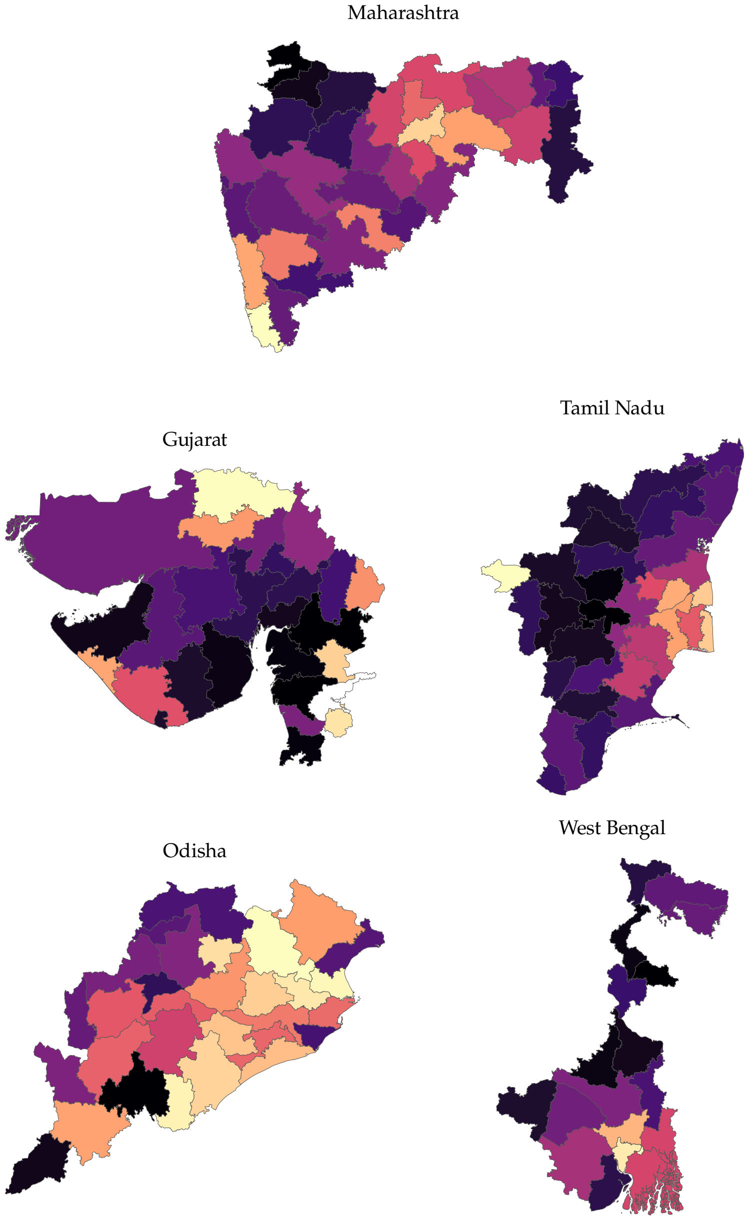
Note. This table reports the contribution of changes in upstream and downstream margins to the variation in changes in firms' sales (columns (1) and (4)), firms' destination-specific sales (columns (2) and (5)), and trade flows (columns (3) and (6)). These are calculated using a Shapley decomposition overall (top row) and when firm-year observations are split by state.

is because changes in intensity of use accrue not only from changes in intensity of use by existing customers but also from changes in the number of customers. By allowing for substitution across both existing suppliers and new potential suppliers, the model is not only more general but also more tractable since it does not require calibrating the extensive margin of firm-to-firm trade to observed data.

7. Conclusion

This paper developed a new framework for analyzing aggregate and firm-level consequences of shocks to the spatial economy when customer-supplier linkages between firms evolve endogenously. I documented that Indian firms with higher sales to other firms tend to be used more intensively by other firms and tend to have larger customers. Firms' intensity of use explains a vast majority of variation in their sales to other firms. The model explains this through a single dimension of firm heterogeneity: production costs. Firms with low production costs are used more intensively by other firms and since their customers use cheaper inputs intensively, they lower production costs and become larger themselves. Furthermore, firms differ not only in their relative position in the production network, but also across

FIGURE 6.1. **Margins of Changes in Firms' Destination-Specific Sales**



Note. Districts are shaded by contribution of changes in upstream margin relative to that of downstream margin in destination-specific sales of firms in them. Darker shades reflect lower values. For the median district in Maharashtra, the relative contribution of the upstream margin is 1.85 times that of the downstream margin, in Gujarat it is 1.44 times, in Tamil Nadu 1.5, in Odisha 2.18 and in West Bengal 1.4. The data pertains to 2015-2016. Relative areal extent of states is not up to scale.

space thereby facing different wages when hiring labor as well as different trade costs when sourcing inputs from potentially multiple locations.

Interdependence of link formation between firms in general equilibrium models of network formation typically restrains the use of simulation-based estimation to a realistic setting with very large numbers of firms. On the contrary, the procedure developed here makes estimation and counterfactual analysis both scalable and tractable. Firms' intensity of use was shown to be a sufficient statistic for their production costs — a key endogenous object of interest. As a result, estimation did not necessitate full solution of the model to obtain the distribution of production costs. Besides, counterfactual analysis did not require large-scale simulation either and was done under a large economy approximation to resolve aggregate uncertainty. In an empirical application motivated by the 2017 GST reform in India, I show that following a 10% decline in inter-state border frictions over half of the model-implied variation in changes in firms' sales to other firms can be explained by endogenous changes in the network structure.

The framework developed here can be directly applied to answer questions that could be broadly classified as market integration, technology improvements, and improvement in allocative efficiency; nevertheless, it can serve as a fertile baseline model to answer a wider variety of questions where changes in the production network across firms can lead to aggregate consequences. In pursuit of parsimonious parametrization, the model abstracts from several realistic features of the network economy such as sectoral heterogeneity in technological requirements, supply chain dynamics, industry dynamics of entry and exit, heterogeneous search frictions, and richer bargaining environment between buyers and suppliers all of which are potential avenues for future research.

Acemoglu, D. and P. D. Azar (2020): "Endogenous Production Networks," *Econometrica*, 88, 33–82.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012): "The Network Origins of Aggregate Fluctuations," *Econometrica*, 80, 1977–2016.

Agnosteva, D. E., J. E. Anderson, and Y. V. Yotov (2019): "Intra-national Trade Costs: Assaying Regional Frictions," *European Economic Review*, 112, 32 – 50.

Al-Najjar, N. I. (2004): "Aggregation and the Law of Large Numbers in Large Economies," *Games and Economic Behavior*, 47, 1 – 35.

Allen, T., C. Arkolakis, and X. Li (2020): "On the Equilibrium Properties of Network Models with Heterogeneous Agents," Working Paper 27837, National Bureau of Economic Research.

Alvarez, F. and R. E. Lucas (2007): "General Equilibrium Analysis of the Eaton-Kortum model of International Trade," *Journal of Monetary Economics*, 54, 1726 – 1768.

ANTRÀS, P. AND A. DE GORTARI (2020): "On the Geography of Global Value Chains," *Econometrica*, 88, 1553–1598.

ANTRÀS, P., T. C. FORT, AND F. TINTELNOT (2017): "The Margins of Global Sourcing: Theory and Evidence from U.S. Firms," *American Economic Review*, 107, 2514–64.

ARKOLAKIS, C. (2016): " A Unified Theory of Firm Selection and Growth," *The Quarterly Journal of Economics*, 131, 89–155.

ARKOLAKIS, C., F. HUNEEUS, AND Y. MIYAUCHI (2021): "Spatial Production Networks," Working Paper, Yale University.

ARMENTER, R. AND M. KOREN (2014): "A Balls-and-Bins Model of Trade," *American Economic Review*, 104, 2127–51.

ATKIN, D. AND A. K. KHANDELWAL (2020): "How Distortions Alter the Impacts of International Trade in Developing Countries," *Annual Review of Economics*, 12, 213–238.

AXTELL, R. L. (2001): "Zipf Distribution of U.S. Firm Sizes," *Science*, 293, 1818–1820.

BAKER, S. G. (1994): "The Multinomial-Poisson Transformation," *Journal of the Royal Statistical Society. Series D (The Statistician)*, 43, 495–504.

BAQAEE, D. R. AND E. FARHI (2019a): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 87, 1155–1203.

——— (2019b): "Networks, Barriers, and Trade," Working Paper 26108, National Bureau of Economic Research.

——— (2020): "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 135, 105–163.

BERNARD, A., E. DHYNE, G. MAGERMAN, K. MANOVA, AND A. MOXNES (2021): "The Origins of Firm Heterogeneity: A Production Network Approach," Working Paper, Universite Libre de Bruxelles.

BERNARD, A. B., J. EATON, J. B. JENSEN, AND S. KORTUM (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93, 1268–1290.

BIGIO, S. AND J. LAO (2020): "Distortions in Production Networks," *The Quarterly Journal of Economics*, 135, 2187–2253.

BLAUM, J., C. LELARGE, AND M. PETERS (2018): "The Gains from Input Trade with Heterogeneous Importers," *American Economic Journal: Macroeconomics*, 10, 77–127.

BOEHM, J. AND E. OBERFIELD (2020): "Misallocation in the Market for Inputs: Enforcement and the Organization of Production," *The Quarterly Journal of Economics*, 135, 2007–2058.

CALIENDO, L. AND F. PARRO (2014): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

CALIENDO, L., F. PARRO, AND A. TSYVINSKI (2017b): "Distortions and the Structure of the World Economy," Working Paper 23332, National Bureau of Economic Research.

CAMERON, A. C., J. B. GELBACH, AND D. L. MILLER (2011): "Robust Inference With Multiway Clustering," *Journal of Business and Economic Statistics*, 29, 238–249.

CARVALHO, V. M. AND N. VOIGTLANDER (2014): "Input Diffusion and the Evolution of Production Networks," Working Paper 20025, National Bureau of Economic Research.

CHANEY, T. (2014): "The Network Structure of International Trade," *American Economic Review*, 104, 3600–3634.

——— (2018): "The Gravity Equation in International Trade: An Explanation," *Journal of Political Economy*, 126, 150–177.

COSTINOT, A. AND A. RODRÍGUEZ-CLARE (2014): *Trade Theory with Numbers: Quantifying the Consequences of Globalization*, Elsevier, vol. 4 of *Handbook of International Economics*, 197–261.

Dekle, R., J. Eaton, and S. Kortum (2008): "Global Rebalancing with Gravity: Measuring the Burden of Adjustment," *IMF Staff Papers*, 55, 511–540.

Demir, B., A. C. Fieler, D. Xu, and K. K. Yang (2021): "O-Ring Production Networks," Working Paper 28433, National Bureau of Economic Research.

Dhyne, E., A. K. Kikkawa, and G. Magerman (2022): "Imperfect Competition in Firm-to-Firm Trade," *Journal of the European Economic Association*.

Dingel, J. I. and F. Tintelnot (2020): "Spatial Economics for Granular Settings," Working Paper 27287, National Bureau of Economic Research.

Donaldson, D. (2015): "The Gains from Market Integration," *Annual Review of Economics*, 7, 619–647.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Eaton, J., S. Kortum, and F. Kramarz (2011): "An Anatomy of International Trade: Evidence From French Firms," *Econometrica*, 79, 1453–1498.

——— (2022): "Firm-to-Firm Trade: Imports, Exports, and the Labor Market," Working Paper 29685, National Bureau of Economic Research.

Eaton, J., S. Kortum, and S. Sotelo (2013): *International Trade: Linking Micro and Macro*, Cambridge University Press, vol. 2 of *Econometric Society Monographs*, 329–370.

Fally, T. (2015): "Structural Gravity and Fixed Effects," *Journal of International Economics*, 97, 76 – 85.

Fernandes, A. M., P. J. Klenow, S. Meleshchuk, D. Pierola, and A. Rodríguez-Clare (2018): "The Intensive Margin in Trade," Working Paper 25195, National Bureau of Economic Research.

Gaubert, C. and O. Itskhoki (2021): "Granular Comparative Advantage," *Journal of Political Economy*, 129, 871–939.

Gourieroux, C., A. Monfort, and A. Trognon (1984): "Pseudo Maximum Likelihood Methods: Theory," *Econometrica*, 52, 681–700.

Hausman, J., B. H. Hall, and Z. Griliches (1984): "Econometric Models for Count Data with an Application to the Patents-R & D Relationship," *Econometrica*, 52, 909–938.

Head, K. and T. Mayer (2014): "Gravity Equations: Workhorse,Toolkit, and Cookbook," in *Handbook of International Economics*, ed. by G. Gopinath, E. Helpman, and K. Rogoff, Elsevier, vol. 4 of *Handbook of International Economics*, 131 – 195.

——— (2019): "Brands in Motion: How Frictions Shape Multinational Production," *American Economic Review*, 109, 3073–3124.

Hopenhayn, H. A. (1992): "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, 1127–1150.

Hotz, V. J. and R. A. Miller (1993): "Conditional Choice Probabilities and the Estimation of Dynamic Models," *The Review of Economic Studies*, 60, 497–529.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448.

Huang, H., K. Manova, O. Perelló, and F. Pisch (2022): "Firm Heterogeneity and Imperfect Competition in Global Production Networks," Working Paper, University College London.

Huneeus, F. (2020): "Production Network Dynamics and the Propagation of Shocks," Working Paper, Central Bank of Chile.

Jones, C. I. (2011): "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 3, 1–28.

——— (2013): *Misallocation, Economic Growth, and Input–Output Economics*, Cambridge University Press, vol. 2 of *Econometric Society Monographs*, 419–456.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50, 649–670.

Klenow, P. and A. Rodríguez-Clare (1997): "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" in *NBER Macroeconomics Annual 1997, Volume 12*, National Bureau of Economic Research, Inc, NBER Chapters, 73–114.

Klette, T. J. and S. Kortum (2004): "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 112, 986–1018.

Kortum, S. (1997): "Research, Patenting, and Technological Change," *Econometrica*, 65, 1389–1419.

Lim, K. (2018): "Endogenous Production Networks and the Business Cycle," Working Paper, University of Toronto.

Liu, E. (2019): "Industrial Policies in Production Networks," *The Quarterly Journal of Economics*, 134, 1883–1948.

Luttmer, E. G. J. (2007): "Selection, Growth, and the Size Distribution of Firms," *The Quarterly Journal of Economics*, 122, 1103–1144.

McFadden, D. (1974): "Conditional Logit Analysis of Qualitative Choice Behavior," *Frontiers in Econometrics*, 105–142.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Menzel, K. (2022): "Strategic Network Formation with Many Agents," Working Papers, New York University.

Miyauchi, Y. (2021): "Matching and Agglomeration: Theory and Evidence from Japanese Firm-to-Firm Trade," Working Paper, Boston University.

Oberfield, E. (2018): "A Theory of Input-Output Architecture," *Econometrica*, 86, 559–589.

Restuccia, D. and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11, 707 – 720.

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," *Econometrica*, 55, 999–1033.

Santos Silva, J. and S. Tenreyro (2006): "The Log of Gravity," *The Review of Economics and statistics*, 88, 641–658.

Shorrocks, A. F. (2013): "Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value," *The Journal of Economic Inequality*, 11, 99–126.

Swiecki, T. (2017): "Intersectoral Distortions and the Welfare Gains from Trade," *Journal of International Economics*, 104, 138 – 156.

Taddy, M. (2015): "Distributed Multinomial Regression," *The Annals of Applied Statistics*, 9, 1394–1414.

Taschereau-Dumouchel, M. (2020): "Cascades and Fluctuations in an Economy with an Endogenous Production Network," Working Paper, Cornell University.

Tintelnot, F., A. K. Kikkawa, M. Mogstad, and E. Dhyne (2018): "Trade and Domestic Production Networks," Working Paper 25120, National Bureau of Economic Research.

Online Appendix

Appendix A. Network Margins of Firm Heterogeneity and Trade

A.1. **Margins of Firms' Sales.** The sales of a firm s located at o to other firms can be decomposed into three factors as follows:

(A.1)
$$\text{sales}_o(s) = \overbrace{N_o(s) \times \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{N_o(s)}}^{\text{upstream margin}} \times \underbrace{\frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b) \times \text{purchases}_d(b)}{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}}_{\text{downstream margin}},$$

where $\text{sales}_o(s)$ denotes sales of firm s to other firms, $N_o(s)$ denotes the number of customers of s, $\text{purchases}_d(b)$ denotes the purchases of firm b from other firms, and $\pi_{od}(s,b)$ denotes the share of purchases of firm b that are from s. The first two factors constitute the upstream margin and capture the intensity of use of s by all other firms. Specifically,

$$\text{intensity of use}_o(s) = N_o(s) \times \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{N_o(s)} = \sum_{b\in\mathcal{M}} \pi_{od}(s,b).$$

The third factor constitutes the downstream margin and measures average size of customers that s sells to.

$$\text{average customer size}_o(s) = \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b) \times \text{purchases}_d(b)}{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}.$$

Relative to the main text, the terms here are subscripted by o and d which denote locations where s and b are, respectively. This is to maintain uniformity of notation as I construct similar decompositions of sales specific to locations, at a disaggregate level for sales of s to all firms at a destination d and at an aggregate level for sales of all firms from an origin o to a destination d. I construct a decomposition of firms' destination-specific sales into three factors as:

$$\text{(A.2)} \qquad \text{sales}_{od}(s) = \overbrace{N_{od}(s) \times \frac{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{N_{od}(s)}}^{\text{upstream margin}}$$

$$\times \underbrace{\frac{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b) \times \text{input costs}_d(b)}{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}}_{\text{downstream margin}},$$

where $\text{sales}_{od}(s)$ denotes input sales of firm s to customers at d and $N_{od}(s)$ denotes the number of customers of s who are located at d. Table A.1 provides results of the decompositions in equations (A.1) and (A.2).

A.2. **Margins of Intranational Trade.** Trade flows between locations are aggregated from sales of all firms from an origin to all firms at a destination. In the data, among all possible pairs of locations(districts), around 40% do not trade at all. For location pairs that trade with each other, I construct the following decomposition of trade flows into four factors:

$$\text{sales}_{od} = \overbrace{N_{od} \times \frac{\sum_{s\in\mathcal{M}_o} N_{od}(s)}{N_{od}} \times \frac{\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{s\in\mathcal{M}_o} N_{od}(s)}}^{\text{upstream margin}} \times \underbrace{\frac{\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b) \times \text{purchases}_d(b)}{\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}}_{\text{downstream margin}}, \tag{A.3}$$

where $\text{sales}_{od} = \sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d} \text{sales}_{od}(s,b)$, N_{od} denotes the sellers from o that sell at d. In this decomposition, the first three margins capture the role of the upstream margin whereas the third margin captures the role of the downstream margin in driving differences in aggregate trade flows. In considering this decomposition, I depart from the trade literature where these margins are regrouped such that the first margin is called the extensive margin of trade defined as the number of firms from o that sell at d and the remaining three margins are together called the intensive margin of trade average sales across the firms from o that enter d. [21] This is so as to emphasize the role of endogenous network formation and cross-border supply chains in determining aggregate trade flows. Table A.2 reports the results from this decomposition.

[21]For example, see Eaton et al. (2011) and Fernandes et al. (2018) for such decomposition of the margins of international trade between countries where it is documented that the extensive margin accounts for over half the variation in trade flows between countries.

Table A.1. **Margins of Firms' Sales: Contribution to Total Variance**

	Sales		Destination-Specific Sales		
	(1)	(2)	(3)	(4)	(5)
Intensity of Use	81%	82%	93%	79%	80%
# Customers	35%	36%	37%	23%	22%
Intensity per Customer	46%	46%	56%	57%	58%
Average Customer Size	19%	18%	7%	21%	20%
Fixed Effects:					
Seller×Year	—	—	✓	—	—
Origin×Year	—	✓	—	—	✓
Destination×Year	—	—	—	—	✓
Data Level:					
Seller×Year	•	•	—	—	—
Seller×Destination×Year	—	—	•	•	•
# observations	5.6×10^6	5.6×10^6	18.2×10^6	18.2×10^6	18.2×10^6

Note. Columns (1) and (2) report the contribution of factors: # customers, intensity per customer, and average customer size, to the variance of firms' sales as per equation (A.1). Column (3), (4), and (5) report the contribution of those factors to the variance of firms' destination-specific sales as per equation (A.2).

Appendix B. An Empirical Model of Endogenous Spatial Production Networks

The model economy $\mathcal{E} \equiv \{\mathcal{M}, \mathcal{L}, \mathcal{J}\}$ consists of many firms ($\mathcal{M}$) and households ($\mathcal{L}$) at many locations ($\mathcal{J}$). Firms produce using local labor and intermediate inputs sourced from suppliers potentially spread across multiple locations. Each household supplies one unit of labor inelastically to local firms. Firms rebate any profits to local households. Trade between locations is subject to iceberg trade costs denoted by $\tau_{od} \geq 1$.

B.1. **Household Preferences.** The utility function for any household i at location d is defined over a discrete number of tasks (also indexed by $k \in \mathcal{K} \equiv \{1, \cdots, K\}$) as:

$$u_d\,(i) = \prod_{k\in\mathcal{K}} q_d(i,k)^{1/K},$$

$$q_d(i,n) = \sum_{s\in\mathcal{S}_d(i)} q_{od}(s,i,k),$$

TABLE A.2. **Margins of Intranational Trade: Contribution to Total Variance**

	(1)	(2)	(3)	(4)
Intensity of Use	83%	83%	88%	89%
# Sellers	59%	57%	61%	58%
# Customers per Seller	8%	10%	7%	10%
Intensity per Customer	16%	16%	19%	21%
Average Customer Size	17%	13%	12%	11%
Fixed Effects:				
Origin×Year	—	✓	—	✓
Destination×Year	—	—	✓	✓
Data Level:				
Origin×Destination×Year	•	•	•	•
# observations	58,390	58,390	58,390	58,390
# dropped observations (zeros)	41,015	41,015	41,015	41,015
# district pairs	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$

Note. This table reports the contribution of factors: # sellers, # customers per seller, intensity per customer, and average customer size, to the variance of trade flows between districts, as per equation (A.3).

where $q_d(i,k)$ is the quantity of goods consumed to fulfill need k and $\mathcal{S}_d(i)$ is the restricted set of suppliers that i encounters due to search frictions.

For task k, household i chooses the supplier that offers the cheapest price, that is,

$$s_d^*(i,k) = \arg\min_{s\in\mathcal{S}_d(i)} \left\{ \frac{\bar{m}_{od}(s,i,k)\, c_o(s)\tau_{od}}{a_{od}(s,i,k)} \right\}, \tag{B.1}$$

where $\bar{m}_{od}(s,i,k)$ is the markup charged by s for task k. The markup is determined by how much lower the effective cost faced by the best supplier is relative to the second best. The effective price faced by i for task k denoted by $p_d(i,k)$ is then given by

$$p_d(i,k) = \min_{s\in\mathcal{S}_d(i)\setminus\left\{s_d^*(i,k)\right\}} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,i,k)} \right\}. \tag{B.2}$$

Now, taking $\{p_d(i,k) : k \in \mathcal{K}\}$ as given, the household's indirect utility function can be defined as:

$$V_d(i) = \max_{\{q_d(i,k):k\in\mathcal{K}\}} \prod_{k\in\mathcal{K}} q_d(i,k)^{1/K} \tag{B.3}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} p_d(i,k)q_d(i,k) = w_d + \Pi_d$$

where $\Pi_d = \frac{\sum_{s\in\mathcal{M}_d}\Pi_d(b)}{L_d}$ is the per capita profit rebated to households residing at o.

B.2. **Technology and Market Structure.** The production function for any firm b at location d is defined over labor and a discrete number of tasks (indexed by $k \in \mathcal{K} \equiv \{1, \cdots, K\}$) as:

$$y_d(b) = z_d(b)\left(\frac{l_d(b)}{1-\alpha_d}\right)^{1-\alpha_d}\left(\frac{\prod_{k\in\mathcal{K}} m_d(b,k)^{1/K}}{\alpha_d}\right)^{\alpha_d},$$

$$m_d(b,k) = \sum_{s\in\mathcal{S}_d(b)} m_{od}(s,b,k),$$

where $l_d(b)$ is the amount of labor input used by firm b, $m_d(b,k)$ is the quantity of materials utilized to accomplish task k, $z_d(b)$ is the idiosyncratic Hicks-neutral productivity with which firm b produces, and $\mathcal{S}_d(b)$ is the restricted set of suppliers that b encounters due to search frictions.

For task k, firm b chooses the supplier that offers the cheapest price, that is,

$$s_d^*(b,k) = \arg\min_{s\in\mathcal{S}_d(b)}\left\{\frac{\bar{m}_{od}(s,b,k)\,c_o(s)\tau_{od}}{a_{od}(s,b,k)}\right\}. \tag{B.4}$$

With limit pricing, the markup is determined by how much lower the effective cost faced by the best supplier is relative to the second best. Hence, the effective price faced by b for task k, denoted by $p_d(b,k)$, is given by

$$p_d(b,k) = \min_{s\in\mathcal{S}_d(b)\setminus\{s_d^*(b,k)\}}\left\{\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)}\right\}. \tag{B.5}$$

Taking wage w_d and effective prices $\{p_d(b,k) : k \in \mathcal{K}\}$ as given, the firm's unit cost function can be defined as:

$$c_d(b) = \min_{\{l_d(b),\{m_d(b,k):k\in\mathcal{K}\}\}} w_d l_d(b) + \sum_{k\in\mathcal{K}} p_d(b,k)\,m_d(b,k) \tag{B.6}$$

$$\text{subject to } z_d(b)\left(\frac{l_d(b)}{1-\alpha_d}\right)^{1-\alpha_d}\left(\frac{\prod_{k\in\mathcal{K}} m_d(b,k)^{1/K}}{\alpha_d}\right)^{\alpha_d} = 1$$

With the cost function as defined above, the profit of a firm s located at o can be expressed as

$$\begin{aligned}\Pi_o(s) = & \sum_{b\in\mathcal{M}}\sum_{k\in\mathcal{K}} \left(\bar{m}_{od}\left(s,b,k\right)-1\right)\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)}m_{od}(s,b,k) \\ & + \sum_{i\in\mathcal{L}}\sum_{k\in\mathcal{K}} \left(\bar{m}_{od}\left(s,i,k\right)-1\right)\frac{c_o(s)\tau_{od}}{a_{od}(s,i,k)}q_{od}(s,i,k),\end{aligned}$$

where $m_{od}(s,b,k)$ denotes the quantity of goods sold by firm s to customer b for task k and $q_{od}(s,i,k)$ denotes the quantity of goods sold by firm s to households i for task k. The quantity of goods sold $m_{od}(s,b,k)$ or $q_{od}(s,i,k)$ is positive if s is the most effective supplier for task k and zero otherwise.

B.3. **Equilibrium Definition and Characterization.** The aggregate state of the economy is denoted by $\sigma \equiv \{\boldsymbol{z}, \boldsymbol{\tau}, \boldsymbol{\mathcal{S}}, \boldsymbol{a}\}$ where

$$\begin{aligned}
\boldsymbol{z} &\equiv \left\{z_o(s) : s\in\mathcal{M}\right\}, \\
\boldsymbol{\tau} &\equiv \left\{\tau_{od} : (o,d)\in\mathcal{J}^2\right\}, \\
\boldsymbol{\mathcal{S}} &\equiv \left\{\mathcal{S}_d(i) : i\in\mathcal{L}\cup\mathcal{M}\right\}, \text{ and} \\
\boldsymbol{a} &\equiv \left\{a_{od}(s,i,k) : (s,i,k)\in\mathcal{M}\times(\mathcal{L}\cup\mathcal{M})\times\mathcal{K}\right\}
\end{aligned}$$

An allocation in this economy is represented as $\xi \equiv \{\boldsymbol{l}(\sigma), \boldsymbol{m}(\sigma), \boldsymbol{q}(\sigma), \boldsymbol{y}(\sigma)\}$ and is defined as a set of functions,

$$\begin{aligned}
\boldsymbol{l}(\sigma) &\equiv \left\{l_d(b;\sigma) : b\in\mathcal{M}\right\}, \\
\boldsymbol{m}(\sigma) &\equiv \left\{m_{od}(s,b,k;\sigma) : (s,b,k)\in\mathcal{M}^2\times\mathcal{K}\right\}, \\
\boldsymbol{q}(\sigma) &\equiv \left\{q_{od}(s,i,n;\sigma) : (s,i,k)\in\mathcal{M}\times\mathcal{L}\times\mathcal{K}\right\}, \\
\boldsymbol{y}(\sigma) &\equiv \left\{y_o(s;\sigma) : s\in\mathcal{M}\right\},
\end{aligned}$$

that map the realization of the state to intermediate input and labor quantities, quantities consumed and quantities produced. A price system is represented as $\varrho \equiv \{\boldsymbol{c}(\sigma), \boldsymbol{p}(\sigma), \boldsymbol{w}(\sigma)\}$ and is defined as a set of functions,

$$\begin{aligned}
\boldsymbol{c}(\sigma) &\equiv \left\{c_o(s;\sigma) : s\in\mathcal{M}\right\}, \\
\boldsymbol{p}(\sigma) &\equiv \left\{p_d\left(i,k;\sigma\right) : (i,k)\in(\mathcal{L}\cup\mathcal{M})\times\mathcal{K}\right\}, \\
\boldsymbol{w}(\sigma) &\equiv \left\{w_d\left(\sigma\right) : d\in\mathcal{J}\right\},
\end{aligned}$$

that map the realization of the state to tasks' prices for firms, needs' prices for households, wage at each location and marginal costs of firms. This leads to the definition of equilibrium in this economy as follows.

Definition 3. For any given state σ, an equilibrium in this economy is defined as an allocation and price system, (ξ, ϱ) such that (a) households select suppliers for needs and firms select suppliers for tasks according to equations (3.1) and (B.1) respectively; (b) firms set prices for other firms and households according to equations (B.5) and (B.2) respectively; (c) households maximize utility according to equation (B.3); (d) firms minimize costs according to equation (B.6); and (e) market clears for each firm's goods and for labor at each location as follows.

$$\sum_{b \in \mathcal{M}_d} l_d(b) = L_d$$

$$\sum_{i \in \mathcal{L}} \sum_{k \in \mathcal{K}} \frac{\tau_{od}(s) q_d(i,k)}{a_{od}(s,i,k)} \mathbf{1}\left\{s = s_d^*(i,k)\right\}$$
$$+ \sum_{b \in \mathcal{M}} \sum_{k \in \mathcal{K}} \frac{\tau_{od}(s) m_d(b,k)}{a_{od}(s,b,k)} \mathbf{1}\left\{s = s_d^*(b,k)\right\} = y_o(s)$$

Appendix C. Taking the Model to Data

C.1. **Proof of Proposition 1.** Consider a pair of firms s located in o and b located in d. Now, suppose the marginal cost of firm s from o and it's cost of shipping goods to d are $c_o(s)$ and τ_{od} respectively. For any task k and match-specific productivity $a_{od}(s,b,k) = a$, the effective cost incurred by s of delivering its goods for task k by b is $\frac{c_o(s)\tau_{od}}{a}$. Supplier s is selected by b for task k if b encounters s with match-specific productivity a and b does not encounter any other supplier for whom it is effectively less costly to deliver the good (including the event that b meets s and the match-specific productivity realized is higher than a). The probability with which b selects s for any of its tasks with match-specific productivity a is given by:

$$\pi^0_{od}(s,b,k \mid \sigma_0,\sigma_1) = \frac{\lambda}{M} \times \prod_{s' \in \mathcal{M}} \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right)$$
$$= \frac{\lambda}{M} \times \exp\left(\sum_{s' \in \mathcal{M}} \ln\left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right)\right)$$

Since $\lambda = o(M)$, considering $\frac{\lambda}{M} \ll 1$ and using the approximation $\ln(1+x) \approx x$ for $|x| \ll 1$, the above expression simplifies as:

$$\pi^0_{od}(s,b,k \mid \sigma_0,\sigma_1) = \frac{\lambda}{M} \exp\left(- \frac{\lambda}{M} \sum_{s' \in \mathcal{M}} \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a} \right) \right.$$

Taking expectation over all possible realizations of $\sigma_1 \equiv \{\boldsymbol{\mathcal{S}}, \boldsymbol{a}\}$, we obtain:

$$\begin{aligned}
\pi^0_{od}(s,b,k) &= \mathbb{E}_{\{\sigma_1\}}\left[\pi^0_{od}(s,b,k \mid \sigma_0,\sigma_1)\right] \\
&= \frac{\lambda}{M}\int_0^\infty \exp\left(-\frac{\lambda}{M}\sum_{s'\in\mathcal{M}} \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right) dF_a(a) \\
&= \frac{\lambda}{M}\int_{a_0}^\infty \exp\left(-\frac{\lambda}{M}\sum_{s'\in\mathcal{M}} \mathbb{P}\left(a_{o'd}(s',b,k) \geq \frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}a\right)\right) d\left(1-({}^{a}/{}_{a_0})^{-\zeta}\right) \\
&= \frac{\lambda a_0^\zeta}{M}\int_{a_0}^\infty \exp\left(- \frac{\lambda a_0^\zeta}{M}\sum_{s'\in\mathcal{M}} \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}a \geq a_0\right)\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}a\right)^{-\zeta}\right. \\
&\quad \left. - \frac{\lambda}{M}\sum_{s'\in\mathcal{M}} \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}a \leq a_0\right)\right)\zeta a^{-\zeta-1} da \\
&= \frac{1}{M}\int_0^\infty \exp\left(-\frac{1}{M}\sum_{s'\in\mathcal{M}}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}\right)^{-\zeta} a^{-\zeta}\right) d\left(-a^{-\zeta}\right) \\
&= \frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta}}\Gamma(1) \\
&= \frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta}}
\end{aligned}$$

Here, in the fifth line we utilize Assumption 2 which implies that in sufficiently large economies $\lim_{t\to\infty}\lambda_t a_{0,t}^\zeta \to 1$ and $\lim_{t\to\infty} a_{0,t} \to 0$ such that $\frac{\lambda}{M}\sum_{s'\in\mathcal{M}} \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}}a \leq a_0\right) \to 0$ for all firms s'. Since $\pi_{od}(s,b,k)$ is independent of the identity of the task k, we write $\pi^0_{od}(s,b) = \pi^0_{od}(s,b,k)$. Further, since $\pi^0_{od}(s,b)$ is independent of the identity of the buyer at any location d, we can write $\pi^0_{od}(s,-) = \pi^0_{od}(s,b)$.

C.2. **Proof of Proposition 2.** In the context of this paper, the multinomial random variable counts the number of successes in each of the M categories (one for each other supplier s), after K independent trials (one for each task associated with b). Let $\pi^0_{od}(s,b)$ denote the probability of success and $K_{od}(s,b)$ denote the number of successes in category s, the probability of observing $\{K_{od}(s,b) : s \in \mathcal{M}_o, o \in \mathcal{J}\}$ conditional on the number of tasks $K_d(b)$ is:

$$\mathbb{P}\left(\{K_{od}(s,b) : s \in \mathcal{M}\}\right) = K! \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{K_{od}(s,b)}}{K_{od}(s,b)!}$$

where $\sum_{s\in\mathcal{M}} \pi^0_{od}(s,b) = 1$ and $\sum_{o\in\mathcal{J}} \sum_{s\in\mathcal{M}_o} K_{od}(s,b) = K$.

The likelihood for the complete sample, $\mathbb{K} \equiv \{K_{od}(s,b) : (s,b) \in \mathcal{M}^2\}$ with probabilities $\mathbf{\Pi}^0 \equiv \{\pi^0_{od}(s,b) : (s,b) \in \mathcal{M}^2\}$, scaled by a factor K is:

$$\begin{aligned}
\ell\left(\mathbb{K} \mid \mathbf{\Pi}^0\right) &= K! \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{K_{od}(s,b)}}{K_{od}(s,b)!} \right)^{\frac{1}{K}} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\frac{K_{od}(s,b)}{K}}}{K_{od}(s,b)!} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\sum_{k\in\mathcal{K}} \mathbf{1}\left\{s=s^*_d(b,k)\right\} \frac{1}{K}}}{K_{od}(s,b)!} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\sum_{k\in\mathcal{K}} \mathbf{1}\left\{s=s^*_d(b,k)\right\} \frac{\text{purchases}_d(b,k)}{\text{purchases}_d(b)}}}{K_{od}(s,b)!} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\frac{\sum_{k\in\mathcal{K}} \mathbf{1}\{s=s^*_d(b,k)\}\text{purchases}_d(b,k)}{\text{purchases}_d(b)}}}{K_{od}(s,b)!} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\frac{\text{sales}_{od}(s,b)}{\text{purchases}_d(b)}}}{K_{od}(s,b)!} \\
&= K! \prod_{b\in\mathcal{M}} \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{\pi_{od}(s,b)}}{K_{od}(s,b)!}
\end{aligned}$$

Therefore, the log-likelihood is proportional to:

$$\begin{aligned}
\mathcal{L}\left(\mathbb{K} \mid \mathbf{\Pi}^0\right) \propto &\sum_{s\in\mathcal{M}} \left(\sum_{b\in\mathcal{M}} \pi_{od}(s,b) \right) \ln\left(c_o(s)^{-\zeta} \tau_{od}^{-\zeta} \right) \\
&- \sum_{d\in\mathcal{J}} M_d \ln\left(\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta} \tau_{o'd}^{-\zeta} \right)
\end{aligned}$$

Note that $c_o(s) = \widetilde{c}_o(s)c_o$ and $\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\tau_{o'd}^{-\zeta} = \sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}$, therefore the likelihood equations for $\widetilde{c}_o(s)$ are given by:

$$\frac{\sum_d \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\widetilde{c}_o(s)^{-\zeta} c_o^{-\zeta}} = \sum_d \frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} \tau_{od}^{-\zeta}$$

The likelihood equations for $\tau_{od}^{-\zeta}$ are given by:

$$\begin{aligned}
\frac{\left(\sum_{b\in\mathcal{M}_d}\sum_{s\in\mathcal{M}_o} \pi_{od}(s,b)\right)}{\tau_{od}^{-\zeta}} &= \frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} \left(\sum_{s\in o} c_o(s)^{-\zeta}\right) \\
&= \frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} c_o^{-\zeta} \\
\implies \tau_{od}^{-\zeta} &= \frac{\left(\sum_{b\in\mathcal{M}_d}\sum_{s\in\mathcal{M}_o} \pi_{od}(s,b)\right)}{\frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} c_o^{-\zeta}} \\
&= \frac{\left(\sum_{b\in\mathcal{M}_d} \pi_{od}(\bullet,b)\right)}{\frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} c_o^{-\zeta}}
\end{aligned}$$

Substituting the expression for $\tau_{od}^{-\zeta}$, we obtain an estimator for $\widetilde{c}_o(s)^{-\zeta}$ as:

$$\begin{aligned}
\widetilde{c}_o(s)^{-\zeta} &= \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}} \pi_{od}(\bullet,b)} \\
&= \frac{\sum_d \pi_{od}(s,\bullet)}{\sum_{s'\in\mathcal{M}_o}\sum_d \pi_{od}(s',\bullet)}
\end{aligned}$$

This then provides us with an estimator for $\frac{c_o^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}}$ as follows:

$$\begin{aligned}
\tau_{od}^{-\zeta} &= \frac{\left(\sum_{b\in\mathcal{M}_d}\sum_{s\in\mathcal{M}_o} \pi_{od}(s,b)\right)}{\frac{M_d}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} c_o^{-\zeta}} \\
\implies \frac{c_o^{-\zeta}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\tau_{o'd}^{-\zeta}} &= \frac{\sum_{b\in\mathcal{M}_d}\sum_{s\in\mathcal{M}_o} \pi_{od}(s,b)}{M_d} \\
&= \frac{1}{M_d}\sum_{b\in\mathcal{M}_d} \pi_{od}(\bullet,b)
\end{aligned}$$

C.3. **Estimation of Material Shares.** The distribution of markups from Proposition 6 provides expressions for value-added share of gross output

$(VA/GO)_o$. Using equation (D.1), materials share α_o is calibrated as

$$\alpha_o = (1 + {}^1\!/\!_\varsigma)\left(1 - ({}^{VA}\!/\!_{GO})_o\right),$$

where $(VA/GO)_o$ across districts are constructed using aggregate production statistics as follows.

I obtain district-level sectoral GDP $\{VA_o^j\}$ from Nielsen Analytics, a private data firm and industry-level data on value-added share of gross output at the national level, $\left\{({}^{VA}\!/\!_{GO})^j : j \in \mathcal{I}\right\}$ from the World Input-Output Database. Using these, I construct a measure of value-added share of gross output at the district level as

$$({}^{VA}\!/\!_{GO})_o = \frac{\sum_{j\in\mathcal{I}} VA_o^j}{\sum_{j\in\mathcal{I}} \frac{VA_o^j}{({}^{VA}\!/\!_{GO})^j}}. \tag{C.1}$$

I use data pertaining to six industry groups for this calculation. They are (a) Mining and Quarrying; (b) Construction; (c) Manufacturing; (d) Electricity, Gas and Water Supply; (e) Transport, Storage and Communication; and (f) Trade, Hotels and Restaurants.

Appendix D. Aggregation and Counterfactual Analysis in Network Economies

D.1. **Continuum Approximation for Large Network Economies.** The following definition formalizes the notion of the limiting economy in the context of this paper.

Definition 4. Consider a sequence of finite economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ where $\mathcal{E}_t \equiv \{\mathcal{M}_t, \mathcal{L}_t, \mathcal{J}_t\}$ is such that the t^{th} economy has the form $\mathcal{M}_t = \{m_1, \cdots, m_{M_t}\} \subset [0,1]$, $\mathcal{L}_t = \{\ell_1, \cdots, \ell_{L_t}\} \subset [0,1]$ and $\mathcal{J}_t = \mathcal{J}$. The uniform distribution on $\mathcal{M}_t$ is given by $\mathcal{U}_t^M(\mathcal{M}_t^0) = \frac{M_t^0}{M_t}$ for all $\mathcal{M}_t^0 \subset \mathcal{M}_t$. Similarly, the uniform distribution on $\mathcal{L}_t$ is given by $\mathcal{U}_t^L(\mathcal{L}_t^0) = \frac{L_t^0}{L_t}$ for all $\mathcal{L}_t^0 \subset \mathcal{L}_t$. Then, $\{\mathcal{E}_t : t \in \mathbb{N}\}$ is a discretizing sequence of economies if it satisfies:

(1) $\mathcal{M}_t \subset \mathcal{M}_{t+1}$ and $\mathcal{L}_t \subset \mathcal{L}_{t+1}$ for all t,
(2) $\lim_{t\to\infty} \mathcal{U}_t^M(\mathcal{M}_t \cap [a_l, a_h]) = \mathcal{U}([a_l, a_h])$,
(3) $\lim_{t\to\infty} \mathcal{U}_t^L(\mathcal{L}_t \cap [a_l, a_h]) = \mathcal{U}([a_l, a_h])$,

where $\mathcal{U}(\bullet)$ denotes the uniform distribution with support over $[0,1]$ and $[a_l, a_h] \subset [0,1]$.

Assumption 2. *The discretizing sequence of economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ satisfies the following conditions:*[22]

(1) $\{\lambda_t, a_{0,t} : t \in \mathbb{N}\}$ is such that $\lambda_t = o(M_t)$ and $\lambda_t a_{0,t}^{\zeta} = \Theta(1)$
(2) $\{M_{d,t}, L_{d,t} : d \in \mathcal{J}, t \in \mathbb{N}\}$ is such that $M_{d,t} = \Theta(M_t)$ and $L_{d,t} = \Theta(L_t)$ for all $d \in \mathcal{J}$

D.2. **Proof of Proposition 3.**

D.2.1. *Joint Distribution of the Lowest and the Second Lowest Effective Costs.* We begin by characterizing the joint distribution of the lowest and second lowest effective cost available to buyer b located at d, $\widetilde{F}_{p_d}\left(p^{(1)}, p^{(2)}\right) = \mathbb{P}\left(p_d^*(b,k) \leq p^{(1)}, p_d(b,k) \geq p^{(2)}\right)$. To do so, we evaluate the probability with which b receives exactly one offer with an effective cost no greater than $p^{(1)}$ and no other offers less than $p^{(2)} (> p^{(1)})$. The lowest cost offer $p^{(1)}$ can be from any one of the locations in $\mathcal{J}$. We evaluate the probability with which this offer is from any given location o and sum it across all locations. The probability with which b receives one offer with an effective cost no greater than $p^{(1)}$ from o and no other offers less than $p^{(2)}$ across all locations is given by:

$$\begin{cases} \binom{M_o}{1} \frac{\lambda}{M} \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \leq p^{(1)}\right) & \text{if } o \neq d \\ \times \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \leq p^{(2)}\right)\right)^{M_o - 1} & \\ \times \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_d(s)\tau_{dd}}{a_{dd}(s,b,k)} \leq p^{(2)}\right)\right)^{M_d - 1} & \\ \times \prod_{o' \notin \{o,d\}} \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_{o'}(s)\tau_{o'd}}{a_{o'd}(s,b,k)} \leq p^{(2)}\right)\right)^{M_{o'}} & \\ & \\ \binom{M_o - 1}{1} \frac{\lambda}{M} \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \leq p^{(1)}\right) & \text{if } o = d \\ \times \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \leq p^{(2)}\right)\right)^{M_o - 2} & \\ \times \prod_{o' \neq o} \left(1 - \frac{\lambda}{M} \mathbb{P}\left(\frac{c_{o'}(s)\tau_{o'd}}{a_{o'd}(s,b,k)} \leq p^{(2)}\right)\right)^{M_{o'}} & \end{cases}$$

Under Assumption 2, the probability with which b encounters exactly one supplier who can deliver at a cost no greater than $p^{(1)}$ and encounters

[22]For any two functions $f(n)$ and $g(n)$, $f(n) = o(g(n)) \implies \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ and $f(n) = \Theta(g(n) \implies \limsup_{n\to\infty} \frac{|f(n)|}{g(n)} < \infty$ and $\limsup_{n\to\infty} \mid \frac{f(n)}{g(n)} \mid > 0$.

no other suppliers with offers less than $p^{(2)}$ across all locations is given by:

$$\widetilde{F}_{p_d}\left(p^{(1)},p^{(2)}\right)=\sum_o \lambda\mu_o \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)}\leq p^{(1)}\right)\exp\left(-\sum_{o'}\lambda\mu_{o'}\mathbb{P}\left(\frac{c_{o'}(s)\tau_{o'd}}{a_{o'd}(s,b,k)}\leq p^{(2)}\right)\right)$$

Using the limit $\lim_{t\to\infty}\lambda_t a_{0,t}^{\zeta}\to 1$, this can be further simplified as $A_d\left(p^{(1)}\right)^{\zeta}\exp\left(-A_d\left(p^{(2)}\right)^{\zeta}\right)$ where $A_d=\sum_o \mu_o\tau_{od}^{-\zeta}\mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]$ is obtained as follows:

$$\begin{aligned}
A_d p^{\zeta} &= \sum_o \lambda\mu_o \mathbb{P}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)}\leq p\right)\\
&= \sum_o \lambda\mu_o \mathbb{E}_{\{c_o\}}\left[1-F_a\left(\frac{c_o(s)\tau_{od}}{p}\right)\right]\\
&= \left(\sum_o \mu_o\tau_{od}^{-\zeta}\mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]\right)p^{\zeta}\\
\implies A_d &= \sum_o \mu_o\tau_{od}^{-\zeta}\mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]
\end{aligned}$$

The density function is then obtained by the negative cross-derivative of $\widetilde{F}_{p_d}\left(p^{(1)},p^{(2)}\right)$ as follows:

$$\begin{aligned}
\widetilde{F}'_{p_d}\left(p^{(1)},p^{(2)}\right) &= -\frac{\partial^2 F_{p_d}\left(p^{(1)},p^{(2)}\right)}{\partial p^{(1)}\partial p^{(2)}}\\
&= -\frac{\partial\left(A_d\left(p^{(1)}\right)^{\zeta}\right)}{\partial p^{(1)}}\frac{\partial\left(\exp\left(-A_d\left(p^{(2)}\right)^{\zeta}\right)\right)}{\partial p^{(2)}}\\
&= \zeta^2 A_d^2\left(p^{(1)}p^{(2)}\right)^{\zeta-1}e^{-A_d\left(p^{(2)}\right)^{\zeta}}
\end{aligned}$$

D.2.2. *Distribution of Effective Prices.* We derive an expression for $F_{p_d}(p)$, that is, the probability with which any firm b located in d faces an effective price no greater than p for one of its tasks k. Firm b faces an effective price no greater than p if the second-lowest cost available to it is no less than p. This is obtained as:

$$\begin{aligned}
F_{p_d}(p) &= \int_0^p\left(\int_0^{p^{(2)}} F'_{p_d}\left(p^{(1)},p^{(2)}\right)dp^{(1)}\right)dp^{(2)}\\
&= 1-A_d p^{\zeta}\exp\left(-A_d p^{\zeta}\right)-\exp\left(-A_d p^{\zeta}\right)
\end{aligned}$$

D.2.3. *Derivation of Market Access.*

$$c_o(\cdot) = w_o^{1-\alpha_o}\left(\prod_{k=1}^{K} p_o(\cdot,k)^{1/K}\right)^{\alpha_o}$$

$$\implies \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right] = \mathbb{E}\left[\left(\frac{w_o^{1-\alpha_o}\left(\prod_{k=1}^{K} p_o(\cdot,k)^{1/K}\right)^{\alpha_o}}{z_o(\cdot)}\right)^{-\zeta}\right]$$

$$= w_o^{-\zeta(1-\alpha_o)}\mathbb{E}\left[\prod_{k=1}^{K} p_o(\cdot,k)^{-\alpha_o\zeta/K}\right]\mathbb{E}\left[z_o(\cdot)^{\zeta}\right]$$

$$= w_o^{-\zeta(1-\alpha_o)}\prod_{k=1}^{K}\mathbb{E}\left[p_o(\cdot,k)^{-\alpha_o\zeta/K}\right]\mathbb{E}\left[z_o(\cdot)^{\zeta}\right]$$

$$= w_o^{-\zeta(1-\alpha_o)}\Gamma\left(2-\frac{\alpha_o}{K}\right)^K A_o^{\alpha_o}\overline{z_o^{\zeta}}$$

This implies that $\{A_d\}_{d\in\mathcal{J}}$ solves the following fixed point problem:

$$A_d = \sum_o \tau_{od}^{-\zeta}\mu_o\overline{z_o^{\zeta}}w_o^{-\zeta(1-\alpha_o)}\Gamma\left(2-\frac{\alpha_o}{K}\right)^K A_o^{\alpha_o}$$

It can be similarly shown that effective prices for needs faced by households is also given by $F_{p_d}(\cdot)$ The following lemma states that the above fixed point problem that solves for market access is well-defined in the sense that it admits a unique positive solution. The proof strategy follows from Allen et al. (2020).

Lemma. *The following system of equations*

$$A_d = \sum_o R_{od}A_o^{\alpha_o},$$

$$R_{od} = \tau_{od}^{-\zeta}\mu_o\overline{z_o^{\zeta}}w_o^{-\zeta(1-\alpha_o)}\Gamma\left(2-\frac{\alpha_o}{K}\right)^K A_o^{\alpha_o}.$$

(1) *has at least one positive solution*
(2) *has at most one positive solution (up to scale)*
(3) *the unique solution can be computed as the limit of a simple iterative procedure.*

Proof. First, I establish existence of positive solution to the system of equations. Define operator $T:\mathbb{R}_{++}^J \to \mathbb{R}_{++}^J$ where $T(\boldsymbol{A}) = (\sum_o R_{o1}A_o^{\alpha_o},\cdots,\sum_o R_{oJ}A_o^{\alpha_o})'$. Note that all components of R_{od} are positive and finite. Then, by construction, for any d, not all R_{od}s are zero. Therefore, for any $\boldsymbol{A} \gg 0$,

$\sum_o R_{o1} A_o^{\alpha_o} \geq \underline{A} > 0$. Further, there exists $\bar{A} < \infty$ such that $\sum_o R_{od} A_o^{\alpha_o} \leq \bar{A}$. Now consider the operator $T : \mathcal{A} \to \mathcal{A}$ defined by $T(A_1, \cdots, A_J) = (\sum_o R_{o1} A_o^{\alpha_o}, \cdots, \sum_o R_{oJ} A_o^{\alpha_o})'$. Suppose $\mathcal{A} = \left\{ \boldsymbol{A} \in \mathbb{R}_{++}^J \mid \underline{A} \leq A_d \leq \bar{A} \forall d \right\}$.Then, if $\boldsymbol{A} \gg 0$, it follows that $T(\boldsymbol{A}) \gg 0$. Note that $\mathcal{A}$ is closed and bounded. Since $\mathcal{A} \subset \mathbb{R}_{++}^J$, this implies that $\mathcal{A}$ is compact. Further, $\mathcal{A}$ is non-empty and convex, and T is continuous. Then, by Brouwer's fixed point theorem, $T(\bullet)$ has a fixed point. This establishes existence of a solution the system of equations.

To establish uniqueness, let's suppose by way of contradiction that the system of equations has two different solutions $\boldsymbol{A}^{(0)}, \boldsymbol{A}^{(1)}$ that are not linear transformations of each other. Denote $\bar{a} = \max_d \frac{A_d^{(1)}}{A_d^{(0)}}$ and $\underline{a} = \min_d \frac{A_d^{(1)}}{A_d^{(0)}}$. Notice that $\frac{\bar{a}}{\underline{a}} \geq 1$. Thus the system of equations can be expressed as:

$$\frac{A_d^{(1)}}{A_d^{(0)}} = \frac{\sum_o R_{od} \left(\frac{A_d^{(1)}}{A_d^{(0)}} \right)^{1-\alpha_o} \left(A_d^{(0)} \right)^{1-\alpha_o}}{A_d^{(0)}}$$

Suppose $\bar{d} = \arg\max_d \left(\frac{A_d^{(1)}}{A_d^{(0)}} \right)$ and $\underline{\alpha} = \min \alpha_o$, then we have:

$$\begin{aligned}
\frac{A_{\bar{d}}^{(1)}}{A_{\bar{d}}^{(0)}} &= \bar{a} \\
\implies \frac{\sum_o R_{o\bar{d}} \left(\frac{A_o^{(1)}}{A_o^{(0)}} \right)^{1-\alpha_o} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} &= \bar{a} \\
\implies \frac{\sum_o R_{o\bar{d}} \bar{a}^{1-\underline{\alpha}} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} &\geq M \\
\implies \frac{\sum_o R_{o\bar{d}} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} \bar{a}^{1-\underline{\alpha}} &\geq \bar{a} \\
\implies \bar{a}^{\underline{\alpha}} &\leq 1 \\
\implies \bar{a} &\leq 1
\end{aligned}$$

Similarly, we can show that $\underline{a} \geq 1$. This implies that $\frac{\bar{a}}{\underline{a}} \leq 1$. But by construction $\frac{\bar{a}}{\underline{a}} \geq 1$. Therefore, it must be the case that $\frac{\bar{a}}{\underline{a}} = 1$ or $\boldsymbol{A}^{(0)} = \boldsymbol{A}^{(1)}$. This establishes uniqueness.

Next, I show that the solution to the system of equations can be obtained via a simple iterative procedure. Starting from any strictly positive $\boldsymbol{A}^{(0)}$, we construct a sequence $\boldsymbol{A}^{(t)}$ successively in the following way,

$$A_d^{(t)} = \sum_o R_{od} \left(A_o^{(t-1)}\right)^{\alpha_o}$$

Denote $\bar{a}^{(t)} = \max_d \frac{A_d^{(t)}}{A_d^{(t-1)}}$ and $\underline{a}^{(t)} = \min_d \frac{A_d^{(t)}}{A_d^{(t-1)}}$. Notice that $\frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} \geq 1$. Suppose $\bar{d} = \arg\max_d \left(\frac{A_d^{(t)}}{A_d^{(t-1)}}\right)$ and $\underline{\alpha} = \min \alpha_o$, then we have:

$$\begin{aligned}
\frac{A_{\bar{d}}^{(t)}}{A_{\bar{d}}^{(t-1)}} &= \bar{a}^{(t)} \\
\implies \frac{\sum_o R_{o\bar{d}} \left(\frac{A_o^{(t-1)}}{A_o^{(t-2)}}\right)^{1-\alpha_o} \left(A_o^{(t-2)}\right)^{1-\alpha_o}}{A_{\bar{d}}^{(t-1)}} &= \bar{a}^{(t)} \\
\implies \frac{\sum_o R_{o\bar{d}} \left(A_o^{(0)}\right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} \left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}} &\geq \bar{a}^{(t)} \\
\implies \frac{\bar{a}^{(t)}}{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}} &\leq 1
\end{aligned}$$

Similarly, we can show that $\frac{\underline{a}^{(t)}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \geq 1$. This implies the following

$$\begin{aligned}
\frac{\bar{a}^{(t)}}{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}} &\leq \frac{\underline{a}^{(t)}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \\
\implies \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} &\leq \frac{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \\
&\leq \frac{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}}{\left(\underline{a}^{(t-1)}\right)^{1-\underline{\alpha}}} \\
\implies \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} &\leq \frac{\bar{a}^{(t-1)}}{\underline{a}^{(t-1)}}
\end{aligned}$$

Since $\frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} \geq 1 \forall t$, this implies that $\lim_{t\to\infty} \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} = 1$. That is, the solution can be computed as the limit of a simple iterative procedure. □

D.3. **Distribution of Markups.** The following proposition provides the distribution of markups.

Proposition 6. *Markups over marginal cost of lowest cost supplier $\bar{m}_{od}(\cdot,\cdot,\cdot)$ are distributed according to the following Pareto distribution:*

$$F_{\bar{m}}(\bar{m}) = \left(1 - \bar{m}^{-\zeta}\right) \mathbf{1}\left\{\bar{m} > 1\right\}.$$

Proof.

$$\begin{aligned}
\mathbb{P}\left(\frac{p_d(b,k)}{p_d^*(b,k)} \le \bar{m} \mid p_d(b,k) = p^{(2)}\right) &= \mathbb{P}\left(p_d^*(b,k) \ge \frac{p_d(b,k)}{\bar{m}} \mid p_d(b,k) = p^{(2)}\right) \\
&= 1 - \int_0^{\frac{p_d(b,k)}{\bar{m}}} \frac{\widetilde{F}'_{p_d}\left(p^{(1)}, \frac{p_d(b,k)}{\bar{m}}\right)}{F'_{p_d}\left(\frac{p_d(b,k)}{\bar{m}}\right)} dp^{(1)} \\
&= 1 - \bar{m}^{-\zeta}
\end{aligned}$$

□

The shape parameter of the distribution of potential markups is ζ, the same parameter that governs dispersion in match-specific productivities. With lower ζ, higher markups are more likely since high match-specific productivities are more likely and hence are larger gaps between costs to the best and second best suppliers. Moreover, the distribution of markups is the same in any destination. An aggregate implication that follows from the distribution of markups is that the share of variable costs in gross output is given by $\frac{1}{1+1/\zeta}$ at all locations. This in turn implies that value-added share of gross output at location o is given by:

$$(VA/GO)_o = \frac{1 - \alpha_o + 1/\zeta}{1 + 1/\zeta}. \tag{D.1}$$

D.4. **Proof of Proposition 4.** The probability with which any firm at d sources from firms at o for any of its tasks is given by

$$\begin{aligned}
\pi^0_{od}(\bullet, -) &= \left(\lim_{t\to\infty} \frac{M_o}{M}\right)\left(\lim_{t\to\infty} \frac{1}{M_o} \sum_{s\in\mathcal{M}_o} \pi^0_{od}(s, -)\right) \\
&= \left(\lim_{t\to\infty} \frac{M_o}{M}\right)\left(\lim_{t\to\infty} \frac{1}{M_o} \sum_{s\in\mathcal{M}_o} \frac{c_o(s)^{-\zeta}\tau_{od}^{-\zeta}}{A_d}\right) \\
&= \frac{\mu_o \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]\tau_{od}^{-\zeta}}{A_d} \\
&= \frac{\mu_o \overline{z_o^{\zeta}} w_o^{-\zeta(1-\alpha_o)} \Gamma\left(2 - \frac{\alpha_o}{K}\right)^K A_o^{\alpha_o} \tau_{od}^{-\zeta}}{A_d}
\end{aligned}$$

D.5. **Proof of Proposition 5.** For any realization of σ, labor demand by firm b at d can be expressed as:

$$l_d(b,\sigma) = \frac{1}{w_d(\sigma)} (1-\alpha_d) c_d(b,\sigma) y_d(b,\sigma)$$

Substituting the above expression in the labor market clearing for location d, we obtain:

$$\begin{aligned} L_d &= \sum_{b \in \mathcal{M}_d} l_d(b,\sigma) \\ &= \sum_{b \in \mathcal{M}_d} \frac{1}{w_d(\sigma)} (1-\alpha_d) c_d(b,\sigma) y_d(b,\sigma) \\ \implies \sum_{b \in \mathcal{M}_d} c_d(b,\sigma) y_d(b,\sigma) &= \frac{w_d(\sigma) L_d}{1-\alpha_d} \end{aligned}$$

Goods market clearing condition for firm s located at o can be simplified as:

$$\begin{aligned} y_o(s,\sigma) &= \sum_d \sum_{b \in \mathcal{M}_d} \sum_{k \in \mathcal{K}} \frac{\tau_{od}(s,\sigma) m_{od}(s,b,k,\sigma)}{a_{od}(s,b,k,\sigma)} \\ &\quad + \sum_d \sum_{i \in \mathcal{L}_d} \sum_{k \in \mathcal{K}} \frac{\tau_{od}(s,\sigma) q_{od}(s,i,k,\sigma)}{g_{od}(s,i,k,\sigma)} \\ \implies c_o(s,\sigma) y_o(s,\sigma) &= \sum_d \alpha_d \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\{s = s_d^*(b,k,\sigma)\}}{\bar{m}_d(b,k,\sigma)} \right) c_d(b,\sigma) y_d(b,\sigma) \\ &\quad + \sum_d \sum_{i \in \mathcal{L}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\{s = s_d^*(i,k,\sigma)\}}{\bar{m}_d(i,k,\sigma)} \right) (w_d(\sigma) + \Pi_d(\sigma)) \\ \implies \underbrace{\sum_{s \in \mathcal{M}_o} c_o(s,\sigma) y_o(s,\sigma)}_{\text{(1) Supply}} &= \underbrace{\sum_d \alpha_d \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\{s_d^*(b,k,\sigma) \in \mathcal{M}_o\}}{\bar{m}_d(b,k,\sigma)} \right) c_d(b,\sigma) y_d(b,\sigma)}_{\text{(2) Intermediate Input Demand}} \\ &\quad + \underbrace{\sum_d \sum_{i \in \mathcal{L}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\{s_d^*(i,k,\sigma) \in \mathcal{M}_o\}}{\bar{m}_d(i,k,\sigma)} \right) (w_d(\sigma) + \Pi_d(\sigma))}_{\text{(3) Final Consumption Demand}} \end{aligned}$$

We can simplify term (1) by making use of the labor market clearing condition as:

$$\begin{aligned}
\text{Supply} &= \sum_{s \in \mathcal{M}_o} c_o(s, \sigma) y_o(s, \sigma) \\
&= \frac{w_o(\sigma) L_o}{1 - \alpha_o}
\end{aligned}$$

We can simplify term (2) as follows:

$$\begin{aligned}
&\text{Intermediate Input Demand} \\
&= \sum_d \alpha_d \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(b, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(b, k, \sigma)} \right) c_d(b, \sigma) y_d(b, \sigma) \\
&= \sum_d \alpha_d \frac{\overbrace{\frac{1}{M_d} \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(b, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(b, k, \sigma)} \right) c_d(b, \sigma) y_d(b, \sigma)}^{(A)}}{\underbrace{\frac{1}{M_d} \sum_{b \in \mathcal{M}_d} c_d(b, \sigma) y_d(b, \sigma)}_{(B)}} \\
&\times \underbrace{\sum_{b \in \mathcal{M}_d} c_d(b, \sigma) y_d(b, \sigma)}_{= \frac{w_d(\sigma) L_d}{1 - \alpha_d}}
\end{aligned}$$

Term (A) can be simplified as follows:

$$\begin{aligned}
(A) &= \frac{1}{M_d} \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(b, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(b, k, \sigma)} \right) c_d(b, \sigma) y_d(b, \sigma) \\
&\xrightarrow{t \to \infty} \mathbb{E}\left[\left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(\cdot, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(\cdot, k, \sigma)} \right) c_d(\cdot, \sigma) y_d(\cdot, \sigma) \right] \\
&= \mathbb{E}\left[\left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(\cdot, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(\cdot, k, \sigma)} \right) \right] \mathbb{E}\left[c_d(\cdot, \sigma) y_d(\cdot, \sigma) \right] \\
&= \mathbb{E}\left[\left(\frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\mathbf{1}\left\{ s_d^*(\cdot, k, \sigma) \in \mathcal{M}_o \right\}}{\bar{m}_d(\cdot, k, \sigma)} \right) \right] \mathbb{E}\left[c_d(\cdot, \sigma) y_d(\cdot, \sigma) \right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{K}\sum_{k\in\mathcal{K}} \mathbb{E}\left[\frac{\mathbf{1}\left\{s_d^*(\cdot,k,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(\cdot,k,\sigma)}\right]\mathbb{E}\left[c_d(\cdot,\sigma)y_d(\cdot,\sigma)\right] \\
&= \mathbb{E}\left[\frac{\mathbf{1}\left\{s_d^*(\cdot,\cdot,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(\cdot,\cdot,\sigma)}\right]\mathbb{E}\left[c_d(\cdot,\sigma)y_d(\cdot,\sigma)\right] \\
&= \mathbb{E}\left[\frac{1}{\bar{m}_d(\cdot,\cdot,\sigma)}\right]\mathbb{E}\left[\mathbf{1}\left\{s_d^*(\cdot,\cdot,\sigma)\in\mathcal{M}_o\right\}\right]\mathbb{E}\left[c_d(\cdot,\sigma)y_d(\cdot,\sigma)\right] \\
&= \frac{\zeta}{\zeta+1}\pi_{od}\left(\bullet,-,\sigma_0\right)\mathbb{E}\left[c_d(\cdot,\sigma)y_d(\cdot,\sigma)\right]
\end{aligned}
$$

Term (B) can be simplified as follows:

$$
\begin{aligned}
(B) &= \frac{1}{M_d}\sum_{b\in\mathcal{M}_d} c_d(b,\sigma)y_d(b,\sigma) \\
&\xrightarrow{t\to\infty} \mathbb{E}\left[c_d(\cdot,\sigma)y_d(\cdot,\sigma)\right]
\end{aligned}
$$

Substituting (A) and (B) back in the Intermediate Input Demand,we obtain:

$$
\text{Intermediate Input Demand} = \sum_d \alpha_d \frac{\zeta}{\zeta+1}\pi_{od}\left(\bullet,-,\sigma_0\right)\frac{w_d(\sigma)L_d}{1-\alpha_d}
$$

We can simplify term (3) as follows:

$$
\begin{aligned}
&\text{Final Consumption Demand} \\
&= \sum_d \sum_{i\in\mathcal{L}_d}\left(\frac{1}{K}\sum_{k\in\mathcal{K}}\frac{\mathbf{1}\left\{s_d^*(i,k,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(i,k,\sigma_1)}\right)\left(w_d(\sigma)+\Pi_d(\sigma)\right) \\
&= \sum_d \left(\frac{1}{L_d}\sum_{i\in\mathcal{L}_d}\left(\frac{1}{K}\sum_{k\in\mathcal{K}}\frac{\mathbf{1}\left\{s_d^*(i,k,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(i,k,\sigma_1)}\right)\right)\left(w_d(\sigma)+\Pi_d(\sigma)\right)L_d \\
&\xrightarrow{t\to\infty} \sum_d \mathbb{E}\left[\frac{1}{K}\sum_{k\in\mathcal{K}}\frac{\mathbf{1}\left\{s_d^*(\cdot,k,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(\cdot,k,\sigma)}\right]\left(w_d(\sigma)+\Pi_d(\sigma)\right)L_d \\
&= \sum_d \frac{1}{K}\sum_{k\in\mathcal{K}}\mathbb{E}\left[\frac{\mathbf{1}\left\{s_d^*(\cdot,\cdot,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(\cdot,\cdot,\sigma)}\right]\left(w_d(\sigma)+\Pi_d(\sigma)\right)L_d \\
&= \sum_d \mathbb{E}\left[\frac{\mathbf{1}\left\{s_d^*(\cdot,\cdot,\sigma)\in\mathcal{M}_o\right\}}{\bar{m}_d(\cdot,\cdot,\sigma)}\right]\left(w_d(\sigma)+\Pi_d(\sigma)\right)L_d \\
&= \sum_d \mathbb{E}\left[\frac{1}{\bar{m}_d(\cdot,\cdot,\sigma)}\right]\mathbb{E}\left[\mathbf{1}\left\{s_d^*(\cdot,\cdot,\sigma)\in\mathcal{M}_o\right\}\right]\left(w_d(\sigma)+\Pi_d(\sigma)\right)L_d
\end{aligned}
$$

$$= \sum_d \frac{\zeta}{\zeta+1} \pi_{od}\left(\bullet, -, \sigma_0\right)\left(w_d(\sigma) + \Pi_d(\sigma)\right) L_d$$

Also, note that $\Pi_d(\sigma) L_d = \left(\frac{\zeta+1}{\zeta} - 1\right) \sum_{b \in \mathcal{M}_d} c_d(b, \sigma) y_d(b, \sigma) = \frac{1}{\zeta} \frac{w_d(\sigma) L_d}{1-\alpha_d}$. Putting these together we can further simplify the goods market clearing condition to obtain the desired result as follows:

$$\begin{aligned}
\frac{w_o(\sigma) L_o}{1-\alpha_o} &= \frac{\zeta}{\zeta+1} \sum_d \pi_{od}(\bullet, -, \sigma_0) \left(\frac{\alpha_d}{1-\alpha_d} + 1 + \frac{1}{\zeta(1-\alpha_d)}\right) w_d(\sigma) L_d \\
&= \sum_d \pi_{od}(\bullet, -, \sigma_0) \frac{w_d(\sigma) L_d}{1-\alpha_d} \\
\Longrightarrow \quad \frac{w_o(\sigma) L_o}{1-\alpha_o} &= \sum_d \pi_{od}(\bullet, -, \sigma_0) \frac{w_d(\sigma) L_d}{1-\alpha_d}
\end{aligned}$$

Since $\{w_d(\sigma)\}_d$ solves the above system of equations for a given realization of σ_0, irrespective of the realization of σ_1, we conclude that $w_d(\sigma) = w_d(\sigma_0)$. That is, $\{w_d : d \in \mathcal{J}\}$ solves the following system of equations for given realization of σ_0, irrespective to realization of σ_1.

$$\frac{w_o L_o}{1-\alpha_o} = \sum_d \pi_{od}(\bullet, -) \frac{w_d L_d}{1-\alpha_d}$$

D.6. **Expected Utility & Welfare Changes.** Households residing at location d are heterogeneous in their match-specific taste shocks of using different suppliers' goods to fulfill their needs. Welfare at any location is then calculated in expectation. That is, $V_d = \mathbb{E}\left[V_d(\cdot)\right]$. With Cobb-Douglas utilities across needs, indirect utility of household i residing at d is given by:

$$V_d(i) = \frac{w_d\left(1 + {}^{1}/_{\zeta(1-\alpha_o)}\right)}{\prod_{k=1}^{K} p_d(i,k)^{1/K}}$$

Expected indirect utility of households at location d can then be derived as:

$$\begin{aligned}
V_d &= \mathbb{E}\left[V_d(\cdot)\right] \\
&= \mathbb{E}\left[w_d\left(1 + {}^{1}/_{\zeta(1-\alpha_o)}\right) \prod_{k=1}^{K} p_d(\cdot, k)^{-1/K}\right]
\end{aligned}$$

$$= w_d \left(1 + {}^1\!/\!_{\zeta(1-\alpha_o)}\right) \prod_{k=1}^{K} \mathbb{E}\left[p_d(\cdot,\cdot)^{-1/K}\right]$$

$$= \left(1 + {}^1\!/\!_{\zeta(1-\alpha_o)}\right) \Gamma\left(2 - \frac{1}{\zeta K}\right)^K w_d A_d^{\frac{1}{\zeta}}$$

Welfare changes, i.e., changes in expected indirect utility at location d in response to shocks can be calculated as:

$$\widehat{V}_d = \widehat{w}_d \widehat{A}_d^{1/\zeta},$$

where $\widehat{w}_d$ denotes the change in wage and $\widehat{A}_d$ denotes change in market access at d.

D.7. **Procedure for Computing Counterfactual Outcomes.** Counterfactual analysis is conducted in three steps. First, I evaluate the expected value of aggregate and firm-level outcomes in the initial state. Second, I compute changes in aggregate outcomes that result from the counterfactual shock. Finally, I evaluate the expected value of aggregate and firm-level outcomes in the counterfactual state

Step 1: Compute expected value of aggregate and firm-level outcomes in initial state. In the initial state, $\boldsymbol{wL} \equiv \{w_d L_d : d \in \mathcal{J}\}$ is obtained as the solution to the following system of equations:

$$\frac{w_d L_d}{1-\alpha_d} = \sum_d \pi^*_{od}(\bullet, -)\frac{w_o L_o}{1-\alpha_o},$$

where $\pi^*_{od}(\bullet, -)$ is calculated as in equation (4.10). Using the solution to these equations, value-added and gross output for each district are respectively calculated as:

$$VA_d = w_d L_d \left(\frac{\left({}^{VA}\!/\!_{GO}\right)_d}{\left({}^{VA}\!/\!_{GO}\right)_d - {}^1\!/\!_{\zeta+1}}\right),$$

$$GO_d = w_d L_d \left(\frac{1}{\left({}^{VA}\!/\!_{GO}\right)_d - {}^1\!/\!_{\zeta+1}}\right),$$

where $\left({}^{VA}\!/\!_{GO}\right)_d$ for district d is calculated in equation (C.1). Total value-added across all districts is chosen as the numeraire, i.e., $\sum_d VA_d = 1$. At the firm-level, input sales, total sales, intensity of use, and average customer size are respectively calculated as:

$$\text{input sales}_o(s) = \sum_d \pi^*_{od}(s, -)\left(GO_d - VA_d\right),$$

$$\text{total sales}_o(s) = \sum_d \pi^*_{od}(s, -)GO_d,$$

$$\text{intensity of use}_o(s) = \sum_d \pi^*_{od}(s, -)M_d,$$

$$\text{average customer size}_o(s) = \frac{\text{input sales}_o(s)}{\text{intensity of use}_o(s)},$$

where $\pi^*_{od}(s, -)$ is calculated as in equation (4.9).

Step 2: Evaluate change in aggregate outcomes from initial to counterfactual state. For any change in σ_0, $\widehat{\boldsymbol{\delta}} \equiv \left\{\widehat{\delta}_{od} : (o, d) \in \mathcal{J} \times \mathcal{J}\right\}$, one can solve for change in wages $\widehat{\boldsymbol{w}} \equiv \{\widehat{w}_d : d \in \mathcal{J}\}$ with the following tâtonnement algorithm for some positive constant μ and tolerance value *tol*:

(1) Start with a guess for the vector of change in wages, $\widehat{\boldsymbol{w}}^{(0)}$

(2) For the vector of wage changes, in the t^{th} iteration $\widehat{\boldsymbol{w}}^{(t)}$, compute change in market access as the solution to the following system of equations:

$$\widehat{A}^{(t)}_d = \sum_o \pi_{od}(\bullet, -)\widehat{\delta}_{od}\left(\widehat{w}^{(t)}_o\right)^{-\zeta(1-\alpha_o)}\left(\widehat{A}^{(t)}_o\right)^{\alpha_o}$$

(3) Compute counterfactual sourcing probabilities as:

$$\left(\pi^{(t)}_{od}(\bullet, -)\right)' = \pi_{od}(\bullet, -)\frac{\widehat{\delta}_{od}\left(\widehat{w}^{(t)}_o\right)^{-\zeta(1-\alpha_o)}\left(\widehat{A}^{(t)}_o\right)^{\alpha_o}}{\widehat{A}^{(t)}_d}$$

(4) Compute excess demand for labor $\boldsymbol{Z}\left(\widehat{\boldsymbol{w}}^{(t)}\right) \equiv \left\{Z_o\left(\widehat{\boldsymbol{w}}^{(t)}\right) : o \in \mathcal{J}\right\}$ as:

$$Z_o\left(\widehat{\boldsymbol{w}}^{(t)}\right) = \frac{1-\alpha_o}{w_o L_o}\sum_d \left(\pi^{(t)}_{od}(\bullet, -)\right)'\widehat{w}^{(t)}_d \frac{w_d L_d}{1-\alpha_d} - \widehat{w}_o$$

(5) Update the vector of change in wages as $\widehat{\boldsymbol{w}}^{(t+1)} \leftarrow \widehat{\boldsymbol{w}}^{(t)} + \mu \boldsymbol{Z}\left(\widehat{\boldsymbol{w}}^{(t)}\right)$.

(6) If $\|\widehat{\boldsymbol{w}}^{(t+1)} - \widehat{\boldsymbol{w}}^{(t)}\| > tol$, go back to (2), else end.

Welfare changes can then be computed as $\widehat{V}_d = \widehat{w}^{(\infty)}_d\left(\widehat{A}^{(\infty)}_d\right)^{\frac{1}{\zeta}}$.

Step 3: Compute expected value of aggregate and firm-level outcomes in counterfactual state. As in the initial state, here again VA'_d and GO'_d are computed for each district using $(\boldsymbol{wL})'$ instead of $\boldsymbol{wL}$.

$$VA'_d = \widehat{w}_d^{(\infty)} w_d L_d \left(\frac{\left({}^{VA}\!/_{GO}\right)_d}{\left({}^{VA}\!/_{GO}\right)_d - {}^{1}\!/_{\zeta+1}} \right),$$

$$GO'_d = \widehat{w}_d^{(\infty)} w_d L_d \left(\frac{1}{\left({}^{VA}\!/_{GO}\right)_d - {}^{1}\!/_{\zeta+1}} \right).$$

Firm-level outcomes are then calculated by using $\pi_{od}^{(\infty)}(\bullet, -)$ instead of $\pi_{od}^{*}(\bullet, -)$ as follows:

$$(\text{input sales}_o(s))' = \sum_d \pi_{od}^{(\infty)}(s, -)\left(GO'_d - VA'_d\right),$$

$$(\text{total sales}_o(s))' = \sum_d \pi_{od}^{(\infty)}(s, -) GO'_d,$$

$$(\text{intensity of use}_o(s))' = \left(\sum_d \pi_{od}^{(\infty)}(s, -) M_d \right),$$

$$(\text{average customer size}_o(s))' = \frac{(\text{input sales}_o(s))'}{(\text{intensity of use}_o(s))'},$$

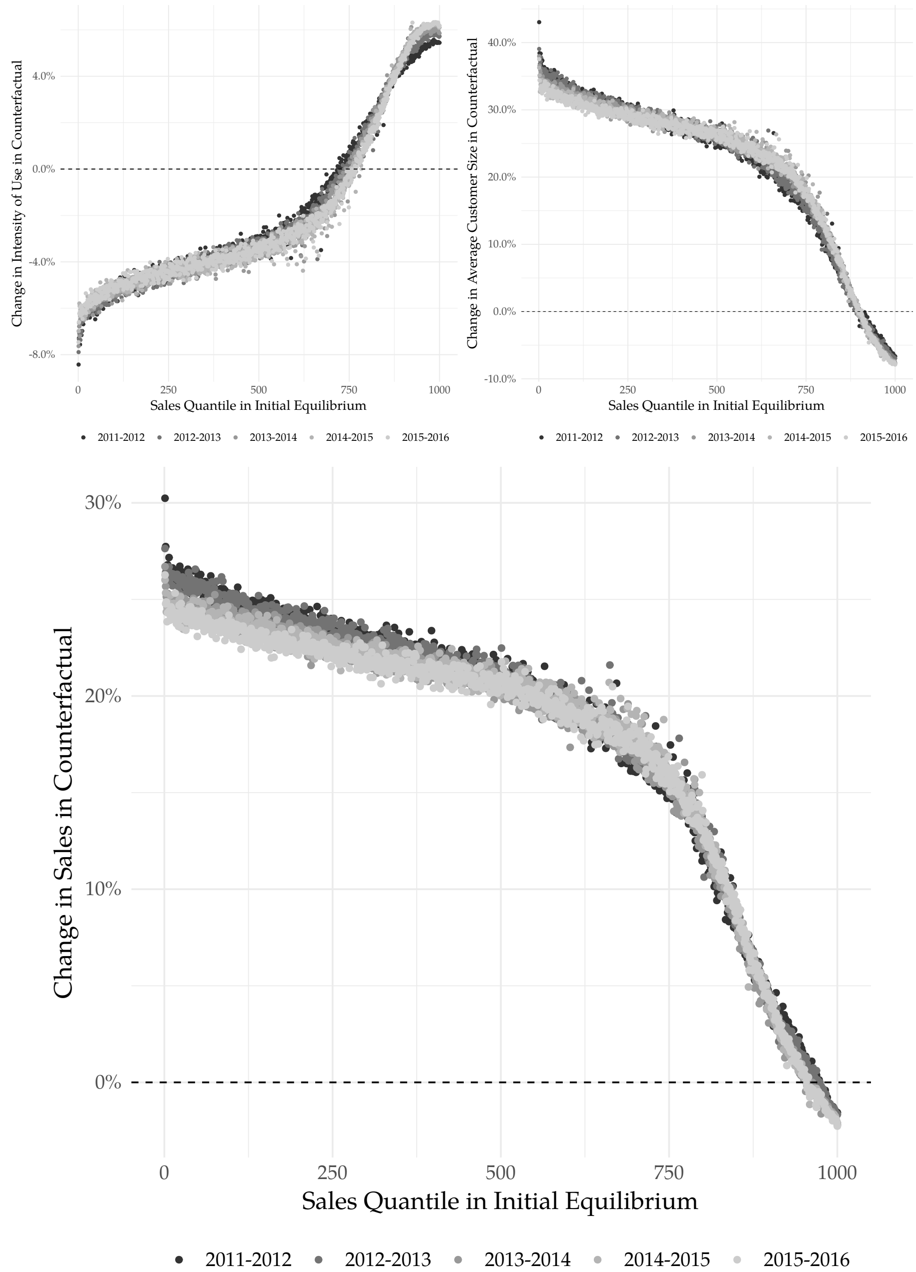
where $\pi_{od}^{(\infty)}(s, -) = \frac{\left(\widetilde{c}_o(s)^{-\varsigma}\right)^{*} \widehat{\delta}_{od}(s)}{\sum_{s' \in \mathcal{M}_o} \left(\widetilde{c}_o(s')^{-\varsigma}\right)^{*} \widehat{\delta}_{od}(s')} \pi_{od}^{(\infty)}(\bullet, -)$ and $\widehat{\delta}_{od}(s)$ is the firm-level shock from the change in σ_0.

Appendix E. Quantitative Applications

This section illustrates how the model can be used to assess the consequences of micro- and macro- shocks to the spatial economy. First, I discuss how the production network of firms changes in response to an aggregate shock that uniformly reduces external trade frictions. Second, I examine the implications of neutralizing firm-level distortions when they are either positively or negatively correlated with firm size on aggregate and firm-level outcomes.

E.1. **Market Integration.** A large body of recent literature studies barriers that impede trade between regions within a country and the gains that accrue from a reduction in those barriers (for a review, see Donaldson (2015)). I study the firm-level implications of a decline in relative costs of trading with firms in other districts. This experiment conceptually captures improvements in transportation infrastructure as well as any other policy changes that affect trade outside an agent's own location relative to within its own location. I consider the counterfactual scenario where

Figure E.1. **Decline in Trade Frictions: Change in Firms' Sales and its Margins**



Note. For each year, firms are grouped into 1000 bins according to their sales in the initial equilibrium. Each bin consists of around 1000 firms. For firms in each of these bins, the top left panel plots the average percent change in intensity of use when trade frictions decline, the top right panel does the same for average customer size, and the bottom panel for sales to other firms.

external trade frictions decline by 10%.[23] With a decline in external trade costs, a large majority of firms are subject to opposing forces along the upstream and downstream margins.

Figure E.1 depicts the effect of these margins of firms' sales to other firms. To understand this, it is useful to look at firms in four groups: (a) those in the top 5% in terms of sales; (b) those in the top 10% but not in the top 5%; (c) those in the top 25% but not in the top 10%; and (d) those in the bottom 75%. First, consider firms in group (a). Starting with the top left panel, these firms gain the most in intensity of use. At the same time, they are more likely to have had customers who are large, i.e., in the top 5% and whose sales declined. This implies that the average customer size of these firms declines as shown in the top right panel. These firms are subject to opposing forces on the upstream and downstream margins. While they gain in intensity of use, the lose sufficiently in average customer size that their sales decline. Second, consider firms in group (b). These firms still gain above 4% in intensity of use but are also likely to have had customers in the top 5% (whose sales declined). These firms are also subject to opposing forces on the upstream and downstream margins such that their sales increase. Third, consider firms in group (c). These firms gain less than 4% in intensity of use, are less likely to have had customers in the top 5% and so their average customer size increases. These firms are also subject to reinforcing forces on the upstream and downstream margins such that their sales increase. Finally, consider the large majority of firms in group (d). These firms lose in intensity of use, but are also much less likely to have had customers in the top 5%, so their average customer size increases. These firms are subject to opposing forces on the upstream and downstream margins. While they lose in intensity of use, the gain sufficiently in average customer size that their sales increase.

Taking stock, as trade frictions decline, firms with low production costs become more successful at farther or less remote destinations in getting selected for customers' tasks. This comes at the expense of firms with

[23] Counterfactual outcomes are evaluated using the procedure described in Appendix D.7 with aggregate shocks given by:

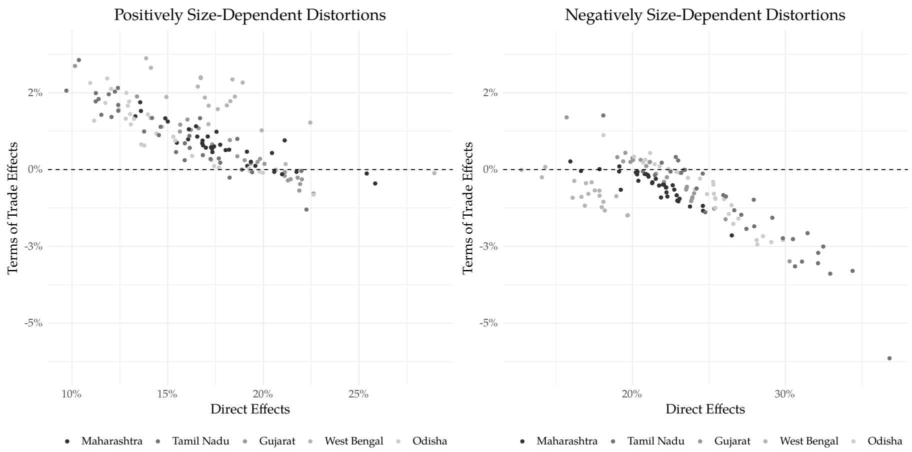
$$\widehat{\delta}_{od} = \begin{cases} \frac{1}{1.1^{-\varsigma}} & o \neq d \\ 1 & o = d \end{cases}$$

There is no heterogeneity in shocks at the firm-level in this counterfactual experiment.

higher production costs who are now less successful in getting selected for tasks both locally and elsewhere. While intensity of use of firms in the bottom three quartiles decreases by as much as 8%, intensity of use for firms in the top quartile increases by as much as 4%. At the same time, firms in the top decile are more likely to have customers in the top 5% those for whom sales has declined. Those customers produce less and source fewer inputs from firms in the top decile. Average customer size for firms in the top decile and quantity demanded from them declines. On the contrary, firms in the bottom nine deciles are less likely to have customers in the top 5% for whom sales has declined. For these firms, average customer size has increased. The net outcome of these margins acting on firms at all quantiles is that large firms' sales to other firms shrink where as those of a large majority of firms in the lower quantiles expands.

E.2. **Size-Dependent Distortions & Improvements in Allocative Efficiency.** A substantial literature has documented the presence of firm-level distortions in developing economies (for a review, see Atkin and Khandelwal (2020)). In this counterfactual experiment, I study the implications of neutralizing positively versus negatively size-dependent distortions affecting firms' labor input choice. The notion for such gains is similar in spirit to that in the closed economy model with labor wedges as in Hsieh and Klenow (2009), multiplier effects from inter-sectoral linkages as in Jones (2013), and trade as in Swiecki (2017). Unlike these papers, I consider the effect of removing firm-level distortions through the lens of a model of trade where production networks between firms respond endogenously. The experiment I consider homogenizes labor market distortions. That is, it eliminates dispersion in those firm-specific labor market "taxes" and hence consists of shocks at the firm level. In conducting this analysis, I assume that all tax revenue is rebated equally to local households both in the initial state and the counterfactual state and hence the level of the homogeneous

FIGURE E.2. **Elimination of Size-Dependent Distortions: Direct & Indirect Effects**



Note. The left panel plots direct and terms of trade effects when distortions are positively size-dependent and the right panel when distortions are negatively size-dependent. Points are shaded by state in both panels, darker shades indicate richer states. For each district, direct effects are calculated as the increase the total factor productivity if each district were a closed economy. Terms-of-trade effects are calculated as the difference between the welfare change from the experiment and the direct effects.

tax rate in the counterfactual scenario does not affect welfare calculations.[24] Figure E.2 shows that terms of trade effects are negative in a large number of districts when removing negatively size-dependent distortions while they are largely positive when removing positively size-dependent distortions.

The result of removing distortions at the firm-level is that firms that faced higher tax rates and were too small, now expand, with labor being

[24]Size-dependent distortions are generated as:

$$1 + t_o(s) = \begin{cases} (1-q)^{-\frac{1}{\eta}} & \text{if distortions are positively size-dependent} \\ q^{-\frac{1}{\eta}} & \text{if distortions are negatively size-dependent} \end{cases},$$

where q denotes the quantile of the firm for sales to other firms and η denotes the shape parameter of Pareto distributed distortions drawn from the following cumulative distribution function:$\mathbb{P}\left(1 + t_o(s) \leq 1 + t\right) = \left(1 - (1+t)^{-\eta}\right) \mathbf{1}\left\{t \geq 0\right\}$. For generating distortions, η was calibrated to 5. Counterfactual outcomes are evaluated using the procedure described in Appendix D.7 with firm-level and aggregate shocks respectively given by:

$$\widehat{\delta}_{od}(s) = {}^{1}\!/\!{}_{(1+t_o(s))^{-\zeta(1-\alpha_o)}},$$
$$\widehat{\delta}_{od} = {}^{1}\!/\!{}_{\mathbb{E}_{\{t_o\}}\left[(1+t_o)^{-\zeta(1-\alpha_o)}\right]}.$$

reallocated to them as in models of misallocation such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). While this captures direct effects, the analysis here also takes into account indirect effects through input-output linkages between firms and the endogenous response of the network structure to these shocks. To examine how this experiment affects the production network between firms, I consider the decomposition of changes in firms' sales to other firms into changes in its intensity of use and changes in its average customer size. Table E.1 reports the results of a Shapley decomposition of margins of sales. I find that changes in intensity of use explain majority of variation in changes in firms' sales — around 80% with positively size-dependent distortions and 75% with negatively size-dependent distortions. The downstream margin is however less important in the case of negatively size-dependent distortions than in the case of positively size-dependent distortions. This is because firms with lower sales and facing larger distortions are likely to have had higher production costs. Since their customers sourced inputs from relatively expensive suppliers, they likely had higher production costs themselves and therefore change relatively less in size when such distortions are neutralized.

Table E.1. **Elimination of Size-Dependent Distortions: Margins of Changes in Firms' Sales**

	Δ% Upstream Margin (1)	Δ% Downstream Margin (2)
Positively Size-Dependent:		
Distortions:		
Maharashtra	73.87%	11.81%
Tamil Nadu	82.02%	8.23%
Gujarat	82.52%	6.75%
West Bengal	80.47%	9.89%
Odisha	74.82%	12.00%
Overall	81.16%	8.34%
Negatively Size-Dependent:		
Distortions:		
Maharashtra	66.57%	1.34%
Tamil Nadu	73.23%	1.40%
Gujarat	80.73%	1.57%
West Bengal	78.01%	3.11%
Odisha	71.25%	1.11%
Overall	75.08%	1.58%

Note. This table reports the contribution of changes in firm's margins to the variation in changes in firms' sales calculated using a Shapley decomposition when firm-year observations are split by state.