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Nicola Comincioli, Paolo M. Panteghini, Sergio Vergalli

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Abstract

This short article studies the tax effects on a start-up investment decision under uncertainty. Since the representative firm can decide both when to invest and how much to borrow, the distortive effects are twofold. We thus show that the deadweight loss (namely, the ratio between the welfare loss and tax revenue) ranges from 25 to 32%, whereas mature firms face a lower distortion (as shown by Comincioli et al. (2021) the maximum deadweight loss is about 25%).

JEL-Codes: H250, G330, G380.

Keywords: real options, business taxation, default risk.

Nicola Comincioli
University of Brescia / Italy
nicola.comincioli@unibs.it

Paolo M. Panteghini
University of Brescia / Italy
panteghi@eco.unibs.it

Sergio Vergalli
University of Brescia / Italy
sergio.vergalli@unibs.it

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1 Introduction

The relationship between business taxation and financial stability has been extensively studied in the scientific literature. Here we focus instead on a start-up firm, that can decide when to invest and how much to borrow. In doing so, we compare our results with those obtained by Sørensen (2017) and Comincioli et al. (2021): both articles deal with mature firms. This comparison shows that the deadweight loss is much higher not only when there is default risk but also if a start-up firm can decide its optimal investment timing.

The structure of the article is as follows. Section 2 presents a model describing the start-up decision of a new business activity and measures the contingent value of tax revenue and welfare. Section 3 provides a numerical example where the deadweight loss, namely, the ratio between welfare and tax revenue, is calculated. As will be shown, this deadweight loss is higher for start-up firms. Section 4 summarizes our findings and discusses their policy implications.

2 The model

Let us consider a representative agent endowed with a start-up option. When (s)he exercises this option (s)he must pay a sunk cost I to start producing and earning a Earning Before Interest and Taxes (EBIT) denoted by Π. In line with Goldstein et al. (2001), we let Π follow a Geometric Brownian Motion (GBM):[1]

$$d\Pi = \mu \Pi dt + \sigma \Pi dz, \tag{1}$$

where $\Pi_0 > 0$ is its initial level, μ and σ are the drift and diffusion coefficients, respectively, and dz is the increment of a Wiener process. Moreover,

[1]This choice rules out negative EBIT. However, this is not a relevant problem since the model is such that default occurs before EBIT falls to zero.

according to Dixit and Pindyck (1994), we let the so-called dividend yield $\delta \equiv r - \mu$ be positive.[2] In line with Comincioli et al. (2021), we introduce the following:

Assumption 1 *The representative firm can borrow resources, thereby paying a non-renegotiable coupon C.*

Assumption 2 *If EBIT decreases to a certain trigger, $\overline{\Pi}$, default occurs. In this case, the firm is expropriated by the lender.*

Assumption 3 *Default, namely, $\overline{\Pi}$, is optimally chosen by equityholders.*

Assumption 4 *The default cost is borne by lenders and is proportional to EBIT. Hence, lenders will own a share $(1-\alpha)$, with $\alpha \in (0,1)$, of the before-default firm.*

According to Assumption 1 the firm sets a coupon and then computes market value of debt.[3] For simplicity, we assume that debt cannot be renegotiated (). Assumptions 2 and 4 introduce default risk and its cost, respectively. If the firm's EBIT falls to the threshold value $\overline{\Pi}$, the firm is therefore expropriated by the lender, who becomes the new shareholder. The cost of default, whose impact is driven by the parameter $\alpha \in [0,1]$, is borne by the lender.[4]

In order to study a start-up decision, we first calculate the discounted Net Present Value (NPV), i.e., $NPV(\Pi) = E(\Pi) + D(\Pi) - I$. Following Goldstein et al., 2001, we calculate both the before-default (b.d.) and the

[2] As the expected growth rate is to $\delta - r$, we refer to this framework as a risk neutral world. According to Lucchetta et al. (2019), by replacing the actual growth rate of cash flows with a certainty-equivalent growth rate, we can evaluate any contingent claim on an asset. According to Shackleton and Sødal (2005), this condition is needed to allow the early exercise of a start-up option.

[3] Without arbitrage, this is equivalent to first setting the book value of debt and then calculating the effective interest rate.

[4] For further details on these assumptions see, e.g., Comincioli et al. (2021).

after-default (a.d.) value of equity $E(\Pi)$, respectively. As shown in Appendix A.1, we obtain:

$$E(\Pi) = \begin{cases} \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C - \left[\frac{(1-\tau)}{\delta}\overline{\Pi} - \frac{(1-\tau)}{r}C\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \\ 0 & \text{a.d.} \end{cases} \tag{2}$$

According to equation 2, the equityholders find the optimal threshold value of Π, below which default is optimal. As shown in Appendix A.1 this optimal solution is:

$$\overline{\Pi}^* = \frac{\delta}{r}\frac{\beta_2}{\beta_2 - 1}C < 1. \tag{3}$$

Notice that if $\Pi < C$, equityholders can decide when to default or issue new equity. As shown by Appendix A.2, the value of debt is:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \left[\frac{(1-\alpha)(1-\tau)}{\delta}\overline{\Pi} - \frac{C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta}\overline{\Pi} & \text{a.d.} \end{cases} \tag{4}$$

Given these results we can focus on a start-up firm. Following Harrison (1985), the relationship between the optimal time T and the threshold investment EBIT, $\hat{\Pi}$, is $\mathbb{E}\left[e^{-rT}\right] = \left(\Pi/\hat{\Pi}\right)^{\beta_1}$, where $\mathbb{E}$ is the expectation operator. Our representative firm therefore maximizes the contingent value of $NPV(\Pi)$ with respect to $\hat{\Pi}$ and C:

$$\max_{T\geq 0, C\geq 0} = \mathbb{E}\left[e^{-rT}NPV(\Pi)\right] = \max_{\hat{\Pi}\geq 0, C\geq 0}\left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1}\left[(1-\tau)\frac{\hat{\Pi}}{\delta} + \tau\frac{C}{r} - \xi C\left(\frac{\hat{\Pi}}{C}\right)^{\beta_2} - I\right], \tag{5}$$

where the start-up option value is:

$$O(\Pi) = \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1}\left[(1-\tau)\frac{\hat{\Pi}}{\delta} + \tau\frac{C}{r} - \xi C\left(\frac{\hat{\Pi}}{C}\right)^{\beta_2} - I\right] \tag{6}$$

and $\xi \equiv \left[(1-\tau)\frac{\alpha}{r}\frac{\beta_2}{\beta_2-1} + \frac{\tau}{r}\right]\left(\frac{r}{\delta}\frac{\beta_2-1}{\beta_2}\right)^{\beta_2}$. As shown in Appendix A.3 shows that:

$$C^{*}=\frac{\delta}{1+m\left(\tau\right)}\frac{\beta_{1}}{\beta_{1}-1}\frac{1-\Omega}{1-\tau}\left[\frac{\tau}{r}\frac{1}{\xi\left(1-\beta_{2}\right)}\right]^{-\frac{1}{\beta_{2}}}I, \tag{7}$$

$$\hat{\Pi}^{*}=\frac{\delta}{1+m\left(\tau\right)}\frac{\beta_{1}}{\beta_{1}-1}\frac{1-\Omega}{1-\tau}I, \tag{8}$$

where $m\left(\tau\right)\equiv\frac{\tau^{1-\frac{1}{\beta_{2}}}}{1-\tau}\frac{\delta}{r}\frac{\beta_{2}}{\beta_{2}-1}\left[\frac{1}{r}\frac{1}{\xi(1-\beta_{2})}\right]^{-\frac{1}{\beta_{2}}}$.[5] Given (5), (7) and (8), the contingent value of $R\left(\Pi\right)$ is:

$$R\left(\Pi\right)=\left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_{1}}\tau\left[\frac{\hat{\Pi}}{\delta}-\frac{C}{r}+\left[\frac{C}{r}-\alpha\frac{\overline{\Pi}}{\delta}\right]\left(\frac{\hat{\Pi}}{\overline{\overline{\Pi}}}\right)^{\beta_{2}}\right]. \tag{9}$$

Since the welfare function is equal to summation between (5) and (9), we obtain:

$$W\left(\Pi\right)=O\left(\Pi\right)+R\left(\Pi\right)=\left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_{1}}\left[\frac{\hat{\Pi}}{\delta}-\alpha\frac{\overline{\Pi}}{\delta}\left(\frac{\hat{\Pi}}{\overline{\overline{\Pi}}}\right)^{\beta_{2}}-I\right]. \tag{10}$$

We finally calculate the welfare loss, that is the difference between the zero-tax welfare function, $W\left(\Pi\right)|_{\tau=0}$, and $W\left(\Pi\right)$, namely, $WL\left(\Pi\right)=W\left(\Pi\right)|_{\tau=0}-W\left(\Pi\right)$. According to Sørensen (2017), we obtain the deadweight loss $DWL\left(\Pi\right)$ as the ratio between the welfare loss and tax revenue:

$$DWL\left(\Pi\right)=\frac{W\left(\Pi\right)|_{\tau=0}-W\left(\Pi\right)}{R\left(\Pi\right)}=\frac{WL\left(\Pi\right)}{R\left(\Pi\right)}.$$

3 A numerical analysis

Here we calibrate our model in order to calculate the value of $WL\left(\Pi\right)$, $R\left(\Pi\right)$ and $DWL\left(\Pi\right)$. In line with the empirical evidence, we let the statutory tax rate range from 0.10 to 0.30.

[5] For further details see Panteghini (2007).

	Variable	Value
r	Risk-free interest rate	0.025
Π_0	EBIT initial value	1
μ	EBIT drift	0.01
I	Investment cost	25
θ	Periods before default	10
α	Cost of default	0.20
σ	EBIT diffusion	0.20

Table 1: Benchmark values of parameters used in the numerical simulations.

Table 1 contains the benchmark values of our parameter values. Firstly, we arbitrarily choose a risk-free interest rate r equal to 0.05 (this parameter does not affect the quality of results). Secondly, we normalize initial EBIT by setting $\Pi_0 = 1$. Thirdly, we assume a positive drift $\mu = 0.1$ (again, its value does not affect the quality of our results). Then, we set $I = 25$ which coincides with the value of the tax-free perpetual rent $\frac{\Pi_0}{r-\mu} = \frac{1}{0.04} = 25$. According to Dixit and Pindyck (1994) and Comincioli et al. (2021), we set $\sigma = 0.20$ and $\alpha = 0.20$, respectively. Some robustness check will then be provided.[6]

Table 2 shows the values for different tax rates. As can be seen, results not only depend on the relevant tax rate but also on volatility.

Concerning volatility, an increase in σ leads to an increase in both the default and investment trigger points. This means that in a more volatile context the expected investment (default) time is longer (shorter). Since the optimal coupon rate depends on the investment trigger point, we can also see that it is increasing in τ. Let us then focus on the contingent value of the value function. Not surprisingly, the more volatile the EBIT, the closer the expected investment time and hence the higher the contingent value of the business activity. A similar effect holds for the contingent value of both the tax revenue and the welfare (that is, the summation between a firm's

[6] Further results with different parameter values are available upon request.

Volatility	**Variable**	$\tau = 0.10$	$\tau = 0.20$	$\tau = 0.30$
$\sigma = 0.10$	$\overline{\Pi}$	0.5190	0.6003	0.6537
	$\hat{\Pi}$	1.0008	1.0407	1.0807
	C	0.8240	0.9532	1.0379
	O	8.7965	7.9150	7.1477
	R	0.9740	1.6251	2.1844
	W	9.7705	9.5402	9.3321
	WL	0.2802	0.5105	0.7186
	DWL	0.2877	0.3142	0.3289
$\sigma = 0.20$	$\overline{\Pi}$	0.4400	0.5952	0.6975
	$\hat{\Pi}$	1.4014	1.4827	1.5657
	C	0.9571	1.2948	1.5172
	O	9.4413	8.5048	7.6896
	R	1.0408	1.7224	2.3299
	W	10.4821	10.2272	10.0196
	WL	0.2865	0.5414	0.7490
	DWL	0.2753	0.3143	0.3215
$\sigma = 0.30$	$\overline{\Pi}$	0.4197	0.6514	0.8072
	$\hat{\Pi}$	1.9210	2.0508	2.1839
	C	1.2388	1.9225	2.3825
	O	10.9052	9.8787	8.9824
	R	1.1425	1.8881	2.5721
	W	12.0477	11.7669	11.5546
	WL	0.2857	0.5666	0.7789
	DWL	0.2501	0.3001	0.3028

Table 2: Results of the numerical simulation executed with benchmark values of parameters and different values of σ.

value and the contingent value of tax revenue). However, we can see that the welfare loss is increasing in volatility. This is due to the fact that, *coeteris paribus*, an increase in volatility makes the distortive effects of taxation more relevant. Table 2 also shows that the deadweight loss is however decreasing in volatility. As we know, the deadweight loss is equal to the ratio between the welfare loss and tax revenue. Both these variables are increasing in σ. However, the effect on the denominator dominates the one on the numerator, namely, the contingent value increase is more relevant than that of the welfare loss.

Concerning tax effects, we can see that the higher the tax rate, the higher the investment and the default trigger points. Since the coupon depends on the investment trigger point, an increase in τ leads to a higher coupon. As can be seen, tax revenue is increasing in τ. In line with Comincioli et al. (2021), we therefore find no Laffer effect. Not surprisingly, an increase in τ reduces the contingent value of a firm's NPV. This is due to a twofold effect: firstly, it reduces the contingent value of business activities; secondly, it reduces the after-tax return to investment. As pointed out, welfare is equal to the summation between a firm's value and tax revenue: as shown in Table 2, the decrease in a firm's value dominates the increase in tax revenue and welfare is therefore decreasing in τ. Hence, the welfare loss increases. When we look at the deadweight loss we see that it is increasing in τ if the tax rate is low enough. When τ is higher however the deadweight loss is rather stable. This means that both the welfare loss and tax revenue are characterized by the same growth rate. Table 2 allows us to make a comparison with previous results. In particular, Sørensen (2017) finds a deadweight loss around 5%. As Comincioli et al. (2021) show, the maximum deadweight loss is dramatically higher (around 26%) in a stochastic model when investment has already been amortized. In particular, the deadweight loss crucially depends on both the values of α and σ and the tax revenue target. Here we show that the deadweight loss is even higher (i.e., ranging from 25% to 32%) for start-up firms even in the absence of a tax revenue target. This is due to the fact that taxation affects not only financial decisions (as in the other articles) but also investment timing. A double distortion therefore arises and the deadweight loss is heavier.

4 Conclusion

This study represents the natural development of the model described by Comincioli et al. (2021). The assumption of a start-up option is motivated by the fact that financial stability and business taxation influence not only the behavior of existing firms, but also the decisions of new entrepreneurs. For this reason, we have studied how the economic environment affects investment timing and all the indicators of benefit arising from a firm's operations, for this purpose redefined to be consistent with this extended framework. As we have shown the distortionary effects of taxation increase when we focus on start-up businesses, where not only financial but also investment decisions are made. This result has an important fiscal implication, in that it suggests to apply a differentiated policy to treat start-up firms differently from mature ones.

A Appendix

A.1 The value of equity

Following Comincioli et al. (2021), at any time the value of equity b.d. is equal to the sum of its immediately preceding value, the instantaneous net profit and the expected capital gain, while a.d. its value falls to zero. The value of equity can then be defined as:

$$E(\Pi) = \begin{cases} (1-\tau)(\Pi - C)\,dt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{b.d.} \\ 0 & \text{a.d.} \end{cases} \tag{11}$$

Following Panteghini, 2007, the before-default value of equity is:

$$E(\Pi) = \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C + \sum_{i=1}^{2} A_i \Pi^{\beta_i}, \tag{12}$$

where$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$,$\beta_1 > 1$ and $\beta_2 < 0$. Without financial bubbles we obtain $A_1 = 0$, according to Dixit and Pindyck, 1994. Hence we obtain (2). Since equityholders are can decide when to default, we maximize $E(\Pi)$ with respect to $\overline{\Pi}$. Given the first order condition is:

$$\frac{\partial E(\Pi)}{\partial \overline{\Pi}} = -\left\{ \frac{(1-\tau)}{\delta}\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} - \left[\frac{(1-\tau)}{\delta}\overline{\Pi} - \frac{(1-\tau)}{r}C\right]\frac{\beta_2}{\overline{\Pi}}\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} \right\} = 0$$

that immediately leads to (3).

A.2 The value of debt

Using dynamic programming, the value of debt is therefore:

$$D(\Pi) = \begin{cases} Cdt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{b.d.} \\ (1-\alpha)(1-\tau)\Pi dt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{a.d.} \end{cases} \tag{13}$$

Rearranging 13 gives:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \sum_{i=1}^{2} B_i \Pi^{\beta_i} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta}\Pi + \sum_{i=1}^{2} F_i \Pi^{\beta_i} & \text{a.d.} \end{cases}. \tag{14}$$

Since no financial bubbles exist the equalities $B_1 = F_1 = 0$ hold. Moreover, according to assumption 4, D_2 must be such that the equality $\frac{C}{r} + B_2\overline{\Pi}^{\beta_2} = \frac{(1-\alpha)(1-\tau)}{\delta}\overline{\Pi}$ holds. Solving for B_2 and rearranging gives (4).

A.3 Optimal coupon

Given equation (6), the FOCs with respect to C is:

$$\frac{\partial O\left(\Pi\right)}{\partial C}=\left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1}\left[\frac{\tau}{r}-\xi\left(\frac{\hat{\Pi}}{C}\right)^{\beta_2}\left(1-\beta_2\right)\right]=0,$$

which gives the optimal ratio between C and $\hat{\Pi}$:

$$\frac{C}{\hat{\Pi}}=\left[\frac{\tau}{r\xi\left(1-\beta_2\right)}\right]^{-\frac{1}{\beta_2}}. \tag{15}$$

The FOC with respect to $\hat{\Pi}$ is:

$$\frac{\partial O\left(\Pi\right)}{\partial\hat{\Pi}}=\left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1}\left\{\frac{\left(1-\beta_1\right)\left(1-\tau\right)}{\delta}+\left[\frac{\tau}{r\xi\left(1-\beta_2\right)}\right]^{-\frac{1}{\beta_2}}\left[\frac{\left(1-\beta_1\right)\beta_2\tau}{r\left(\beta_2-1\right)}\right]+\frac{\beta_1}{\hat{\Pi}}I\right\}=0,$$

which, using ratio (15), finally gives (7) and (8).

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