CESifo WORKING PAPERS

9531
2022

January 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9531*

Doubling Back on Double Marginalization

Abstract

"Double marginalization" and "Elimination of Double marginalization" are catch-phrases commonly used in the IO literature. In this note, I trace back the origin of the idea to Chapter IX, on complementary goods monopolies, of Cournot (1838). Through the years Cournot's contribution remained a reference but ended being viewed as a special case of the bilateral monopoly model. Yet, it is worth wondering why the most cited paper on this issue is nowadays Spengler (1950) which contains only an informal treatment of the question. In addition to retracing the origin of the idea, I emphasize the elegant proof of Cournot for the simultaneous game and extend it to the sequential game. I also show that prices are usually higher in the sequential game but that they could be lower if demand is very convex.

JEL-Codes: B160, B210, K210, L120, L130, L420.

Keywords: Cournot, complements, successive monopolies.

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January 17, 2022
I am grateful to Philippe Choné, Larry White, and Ralph Winter for constructive comments. I was able to improve the quality of the paper thanks to the valuable input of the Editor and two referees. A special thanks goes to Magali Noël-Linnemer. This research is supported by a grant of the French National Research Agency (ANR), "Investissements d'Avenir" (LabEx Ecodec/ANR-11-LABX-0047).

1 Introduction

Nowadays, the catch-phrase "double marginalization" (henceforth DM) is widely used both by IO economists and antitrust scholars. For example, in the recent revision of the U.S. Vertical Merger Guidelines,[1] which is only 12 pages long, it is mentioned 19 times. Indeed, the question of the "elimination of double marginalization", hereafter EDM, is central to these guidelines.[2]

Yet, it seems impossible to find the DM or EDM expressions in the economic literature before the early 1980s, and the first papers in which it appears in print are Mathewson and Winter (1983a,b, 1984).

All three articles were written at about the same time, between 1980 and 1983. In the 1983 *Economic Inquiry* article (which was presented at the Western Economic Association Meetings, San Francisco, 1981) the authors refer to "the classic 'double mark-up'."[3] In the 1983 *Journal of Business* article, they definitely use DM as they also do in the 1984 *Rand Journal*.[4]

Mathewson and Winter write: "The first explanation of this over-pricing effect is attributed to Spengler (1950)." In Spring 2021, Ralph Winter wrote to me:

"I am sure that the term double marginalization was first uttered in the halls of the Chicago econ department. Frank and I certainly did not originate the term. We had a lot of Chicago colleagues, and maybe heard it from them. But I think that by the 70s the term was in common use. People referred to the 'oral tradition' at Chicago (Aaron Director, for example), so this seems to be the origin."[5]

After Mathewson and Winter, the association of DM/EDM with Spengler is found in Rey and Tirole (1986), Bolton and Bonanno (1988), and Tirole (1988).[6] From then on, these expressions became more and more popular. Most of the time, they remained linked to Spengler.[7]

I propose to take a fresh look at the origin of DM and EDM, thereby challenging the emphasis on the role of Spengler. First of all, DM is not confined to vertical relationships. In section 2, I recall its link to complementary goods or services. As such goods and services abound in the economy, the DM phenomenon turns out to be much broader.[8]

Next, in section 3.1, the results of Cournot (1838) on the simultaneous pricing of com-

[1] The document is available online: 2020 U.S. Vertical Merger Guidelines.

[2] Among others, two recent contributions on EDM are Slade and Kwoka Jr (2020) and Slade (2021).

[3] Searching for 'double mark-up' in Google Scholar gives 1,520 results but none before their article!

[4] For which a 1982 working paper exists: #82ll, Institute for Policy Analysis, University of Toronto. It is also revealing that in the same issue of the *Journal of Business*, Bittlingmayer (1983) is a related article (vertical restrictions with stores along a Hotelling line) but "double", "mark-up", and "marginalization" are absent.

[5] Unfortunately, one can easily lose track of oral traditions. I have unsuccessfully searched in Bork (1954), Bork (1978) and Posner (1976). In McGee and Bassett (1976), footnote 7 corroborates somehow Mathewson and Winter's memories: "Bork was not the only one to argue that some or all received arguments against vertical integration were wrong: M. A. Adelman, Aaron Director, J. J. Spengler, and others had each analyzed at least part of the problem. For example, see Adelman (1949a), Spengler (1950). Vertical integration was one subject of Aaron Director's socratic analysis, which contributed much to an oral tradition in and around the University of Chicago. Some evidences of this tradition with respect to integration are seen, for example, in a student note: Editors (1952). For a review of still older analyses, see Machlup and Taber (1960)."

[6] By contrast, in Perry (1989) (the IO Handbook chapter on vertical integration) the expressions DM and EDM are absent. Yet, in the same Handbook, Katz (1989) (chapter on vertical contractual relations) the association of DM/EDM with Spengler is present.

[7] This is illustrated by Figure 2 (see section 4.3) which shows the Google Scholar citations received by Spengler (1950) for every five-year period since its publication. Most of them start after 1985.

[8] For example, microprocessors and hard drives are complementary items. The same holds for avionics and jet engines. For many goods, consumer service is a key complement. And, indeed, a distribution service (whether Amazon or Wallmart) is a required complement for many consumer goods.

plementary goods are presented, and in section 3.2 the sequential version, first exposed by Edgeworth (1897), is detailed. Sequential timing is usually retained in the modelling of a vertical structure. Smultaneous timing has been used to analyze compatibility by Economides (1989) or a patent pool, see Shapiro (2000) and Lerner and Tirole (2004).

Section 3.3 compares equilibrium prices under both timings. Through an example, I show that when demand is convex enough, prices can be larger in simultaneous timing. Otherwise, and in particular, for a linear or concave demand, prices are higher when timing is sequential.

Section 4 gives a historical perspective and underlines that Cournot's chapter IX was confined to the bilateral monopoly strand of the literature in the mid 20th century when Spengler wrote his article.

2 Negative pecuniary externality and DM

Although most IO economists instinctively associate DM with vertical relationships, it can be viewed as a consequence of a negative pecuniary externality imposed by a price increase of one seller on the other. Such a negative externality is present in the case of complementary goods.[9]

To illustrate with two goods, let $D_i(p_i, p_j)$ $i = 1, 2$, $j \neq i$, denote the demand for good i as a function of its own price p_i and of the price of good j. Let $C_i(.)$ denote the cost functions, $i = 1, 2$. Goods i and j are complements (resp. substitute) at prices p_i, p_j if $\partial D_i(p_i, p_j)/\partial p_j < 0$ (resp. > 0).[10]

Fact 1 (DM)**.** *If good i and j are complements (resp. substitutes), prices are lower (resp. higher) when chosen by a monopolist rather than simultaneously by independent firms.*

The proof consists in showing that for complements the reaction functions of a monopolist are lower than those of independent firms. Assuming interior solutions, the reaction function $p_i^c(p_j)$ when firms are independent is given by the first-order-condition:

$$D_i(p_i, p_j) + \frac{\partial D_i(p_i, p_j)}{\partial p_i} \left(p_i - C_i'\left(D_i(p_i, p_j)\right)\right) = 0\,. \tag{1}$$

On the other hand, for a monopolist, the reaction function, $p_i^m(p_j)$ is given by:

$$D_i(p_i, p_j) + \frac{\partial D_i(p_i, p_j)}{\partial p_i} \left(p_i - C_i'\left(D_i(p_i, p_j)\right)\right) + \frac{\partial D_j(p_j, p_i)}{\partial p_i} \left(p_j - C_j'\left(D_j(p_j, p_i)\right)\right) = 0\,. \tag{2}$$

Now, in the case of complementary goods, $\partial D_j(p_j, p_i)/\partial p_i < 0$, and the last term of (2) is negative. This implies $p_i^m(p_j) < p_i^c(p_j)$ and lower monopoly prices: $p_i^m < p_i^c$.

A monopoly selling complementary goods internalizes the negative pecuniary externality; monopoly prices are lower than competition prices.[11] As a result, EDM benefits both the firms and the consumers.

[9] I am grateful to L. White for this insight.

[10] Admittedly, the sign of the derivative could change with the level of prices.

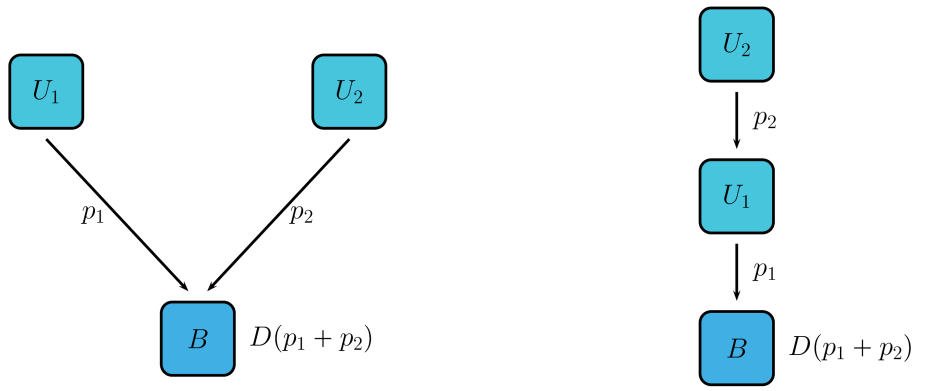
[11] In the case of substitute goods, the pecuniary externality is positive; independent sellers choose lower prices.

3 Perfect complements: Cournot, Ellet, and Edgeworth

The presence of DM is usually emphasized in the context of perfect complements, for which $D_i(p_i, p_j) = D(p_i + p_j) = D(p)$ with $p = p_i + p_j$. This is assumed throughout the paper.

On top of the degree of complementarity, DM varies with the timing of the pricing game. Depending on the economic environment under study, it is more realistic to assume that prices are chosen simultaneously or sequentially.

In the first known analysis of DM, Cournot (1838), the timing is simultaneous whereas Edgeworth (1897, 1925)[12] proposes a sequential timing, see Figure 1. They are analyzed in sections 3.1 and 3.2 respectively, then compared in section 3.3.



(a) Cournot (1838)'s game (b) Edgeworth's game

Figure 1: Comparison of the two timings

3.1 Cournot's chapter IX

Antoine Augustin Cournot (1801-1876) does not need to be introduced to an IO audience. Although Cournot (1838) is well-known for his "quantity competition" game (his Chapter VII), in his Chapter IX, Cournot studies price competition between two producers, U_1 and U_2, of perfect complements.[13] His motivating example is the production of brass from fixed proportions of copper and zinc.[14]

At the same time, Ellet (1839) presents the same idea in his book about canals and railways.[15] The two complementary goods are, in his case, two successive transportation seg-

[12] The 1925 version appeared in a collection of Edgeworth's articles, it is a translation of Edgeworth (1897) published in Italian.

[13] There is a nice analogy with quantity competition between producers of a homogenous good. In the absence of cost, Sonnenschein (1968) shows that both models are formally equivalent. Yet, with cost functions, and in particular with convex costs, the analogy is less formal. For duality results under constant return to scale, see Bergstrom (1978).

[14] Chapter IX is organized as follows. First, §55-58, the case of two complementary inputs (the case of n inputs is briefly treated) is considered in the absence of production cost. Asymmetric cost functions (still for two inputs) are introduced in §59. In §60 the cost of assembling the two inputs is introduced. In §61, Cournot takes into account that each input seller has two demands: one for the composite good, which depends on the prices of both complementary goods, and another one which depends only on its own price. But the analysis is cut short, as Cournot finds the f.o.c. too complicated to interpret. From §62 to the end, Cournot turns to perfect competition (which is the topic of his chapter VIII) for the production of both copper and zinc.

[15] Another addition to the long list of multiple discoveries.

ments.[16] See section VII entitled "Of the Most Advantageous Charges on Articles Contended for by Rival Lines."

Cournot's analysis is more general than Ellet's who only provides a linear demand example without production costs. Using $D_i(p_i, p_j) = D(p_i + p_j) = D(p)$, the f.o.c. (1) writes:

$$D(p) + D'(p)\left(p_i - C_i'\left(D(p)\right)\right) = 0\ , i = 1, 2. \tag{3}$$

Introducing $C\left(D(p)\right) = C_1\left(D(p)\right) + C_2\left(D(p)\right)$ the total cost and $\varepsilon(p) = -pD'/D$ the price elasticity of demand, one can characterize the total price under competition, denoted p^{C}, and the monopoly total price p^m by[17]

$$\frac{p^{\mathrm{C}} - C'\left(D(p^{\mathrm{C}})\right)}{p^{\mathrm{C}}} = \frac{2}{\varepsilon(p^{\mathrm{C}})} \quad \text{and} \quad \frac{p^m - C'\left(D(p^m)\right)}{p^m} = \frac{1}{\varepsilon(p^m)} \tag{4}$$

Therefore for any elastic demand, p^{C} is larger than p^m.

Fact 2 (First occurrences of EDM)**.** *Cournot (1838) and Ellet (1839) both stated that a merger of two complementary good producers is beneficial both for the firms and for consumers.*

The relevant quotes are:

> L'association des monopoleurs, en tournant à leur propre profit, tournera aussi, dans ce cas, au profit des consommateurs, ce qui est précisément l'inverse de ce qui arrive pour les producteurs concurrents. Cournot (1838), chapter IX, §57, p. 117.[18]

> By dividing a line between two companies, without legal limitations to their charges, the toll or tax on the trade will be augmented one third, the profits of the longer improvement will be reduced, and the trade will of course suffer in proportion to the additional tax with which it is burthened. Ellet (1839), part I, section VIII, §60, p. 81.[19]

Cherriman (1857) emphasized this point in his review of Cournot's work, probably the first review in English.[20] The generalization to n complements is straightforward. The n f.o.c. are still given by (3) and their sum leads to:

$$nD(p) + D'(p)\left(p - C'\left(D(p)\right)\right) = 0 \ \text{ or } \ \frac{p - C'\left(D(p)\right)}{p} = \frac{n}{\varepsilon(p)} \tag{5}$$

where $p = \sum_i p_i$. Thus the distortion unambiguously increases with n. Rey and Tirole (2019) show that voluntary price caps are enough to solve Cournot's DM problem.

[16] Ellet was an engineer who built bridges. See https://en.wikipedia.org/wiki/Charles_Ellet_Jr.

[17] Cournot did not use the Lerner (1934) index, and stopped at $D(p) + \frac{1}{2}D'(p)\left[p - C'\left(D(p)\right)\right] = 0$.

[18] An English translation was provided in 1897 by Nathaniel T. Bacon: "An association of monopolists, working for their own interest, in this instance will work for the interest of consumers, which is exactly the opposite of what happens with competing producers." I am grateful to a referee for this reference.

[19] Slightly before, § 59, p. 77, Ellet also wrote: "This fact on a little reflection will convince us of the importance to the community as well as to the stockholders of having the great lines of improvement in the country put under the control of the same interest. That is to say it would result unfavorably to the public as well as generally to one or other of the works to have the trade carried a portion of the distance by one line and then taken up and transported the balance of the way by another."

[20] This review, which was probably not widely read, is reprinted in Dimand (1995). I am grateful to a referee for this reference.

3.2 Edgeworth (1897, 1925): sequential timing

Edgeworth's game is similar to Cournot's but prices are chosen *sequentially*.[21] The upstream firm, U_2, chooses p_2 first, then the downstream firm, U_1, observes p_2 and chooses p_1. Within a vertical relationship, it is usually assumed that U_1 chooses the final price p and pays p_2 to U_2 for each unit. Yet, it is an innocuous change of variable to assume that U_1 chooses p_1, pays nothing to U_2, and that the final price is $p = p_1 + p_2$.

Page 123 of Edgeworth (1925), he first shows the simultaneous equations characterizing prices (i.e. Cournot) and immediately objects that "these equations cannot hold good simultaneously." Edgeworth explains that one firm would choose the price first, anticipating the reaction of the other. He then derives prices in this sequential game for a linear demand example, thus computing a subgame perfect equilibrium.

Formally, U_1 (downstream) sets p_1 to maximize $p_1 D(p) - C_1(D(p))$. The f.o.c. is still (3). Let $p_1(p_2)$ denote the solution. Then U_2 (upstream) sets p_2 to maximize $p_2 D(p_1(p_2) + p_2) - C_2(D(p_1(p_2) + p_2))$. The corresponding f.o.c. writes:

$$D(p) + (1 + p_1'(p_2))\, D'(p)\, (p_2 - C_2'(D(p))) = 0 \tag{6}$$

combining (3) with (6), the final price, denoted p^{E}, is characterized as

$$\frac{p - C'(D(p))}{p} = \frac{1 + 1/p'}{\varepsilon(p)} \tag{7}$$

where $p' = (p_2 + p_1(p_2))' = 1 + p_1' \geq 0$. Once again, one can observe a DM and it is obvious that with continuous functions and elastic demand, $p^{\mathrm{E}} > p^m$.

In a footnote, Edgeworth gives the example of a chain of canals owned by different monopolists referring to an 1846 report on Railways and Canals Amalgamation. A point reminiscent of the work of Ellet (1839).

Once done, however, Edgeworth argues "it is the better opinion, I think, that neither of these suppositions is tenable." In brief, for Edgeworth the equilibrium is indeterminate.[22]

The case of n successive monopolies is obviously trickier and I have not found it in the literature. To generalize, let's assume by recurrence that the cumulative effect of p_i on p is

$$\frac{dp}{dp_i} = \prod_{k=1}^{i-1} (1 + p_k') \tag{8}$$

where $p_k(.)$ is the equilibrium strategy at level k (below i), which is a function of the sum of all prices chosen before. It is true for $i = 1$. To determine the effect of p_{i+1} on p, let's notice that

$$p = \overbrace{p_n + p_{n-1} + \cdots + p_{i+2}}^{\text{Unaffected by } p_{i+1}} + p_{i+1} + p_i(p_{i+1}) + p_{i-1}\left(p_{i+1} + p_i(p_{i+1}) + \cdots\right) + \cdots$$

calling $\widetilde{p}_i = p_{i+1} + p_i(p_{i+1})$ the price p can be written

$$p = \overbrace{p_n + p_{n-1} + \cdots + p_{i+2}}^{\text{Unaffected by } \widetilde{p}_i} + \widetilde{p}_i + p_{i-1}\left(\widetilde{p}_i + \cdots\right) + \cdots$$

[21] Indeed, one of Edgeworth's criticisms of Cournot was about the timing and he made this remark to both the model of Chapter VII and that of Chapter IX.

[22] Either there would be eternal barter between the two monopolists or an equilibrium is reached but it is not possible to anticipate which one. The indeterminacy result of bargaining already appears in Edgeworth (1881) where Robinson Crusoe and Friday bargain over a wage.

and therefore using the recurrence assumption (8)

$$\frac{dp}{dp_{i+1}} = \frac{d\widetilde{p}_i}{dp_{i+1}}\frac{dp}{d\widetilde{p}_i} = (1+p_i')\frac{dp}{dp_i} = \prod_{k=1}^{i}(1+p_k')$$

which proves the recurrence assumption is correct. The f.o.c. for the ith monopolist writes

$$p_i - C_i'\left(D(p)\right) = \frac{-D(p)}{\frac{dp}{dp_i}D'(p)}$$

and by summation the total price is characterized by

$$\frac{p - C'\left(D(p)\right)}{p} = \left(1 + \sum_{i=2}^{n}\frac{1}{\frac{dp}{dp_i}}\right)\frac{1}{\varepsilon(p)} = \left(1 + \sum_{i=2}^{n}\frac{1}{\prod_{k=1}^{i-1}(1+p_k')}\right)\frac{1}{\varepsilon(p)}\ . \tag{9}$$

3.3 Price comparison and a flexible demand example

For an inelastic demand, it is immediate that $p^{\mathrm{C}} = p^{\mathrm{E}} = p^m$. For an elastic demand, a well chosen upward discontinuity of the cost function or a downward discontinuity of the marginal revenue could lead to $p^{\mathrm{C}} = p^{\mathrm{E}} = p^m$ or just $p^{\mathrm{C}} = p^m$ or $p^{\mathrm{E}} = p^m$. Keeping continuous functions, comparing p^{C} and p^{E} boils down to comparing (5) and (9).

Fact 3. *The price distortion is larger in the sequential game unless demand is very convex.*

Indeed,

$$\sum_{i=2}^{n}\frac{1}{\prod_{k=1}^{i-1}(1+p_k')} > n-1 \Rightarrow p^{\mathrm{E}} > p^{\mathrm{C}}\ .$$

The intuition is that $p^{\mathrm{E}} > p^{\mathrm{C}}$ whenever D is not too convex. It is readily confirmed for $n = 2$. Indeed, taking the derivative of the f.o.c. (3) (for $i = 1$), it comes that

$$1/p' = 2 + \frac{D''}{D'}\left(p_1 - C_1'\left(D(p)\right)\right) + \left(-D'\right)C_1'' = 2 - \frac{D''D}{(D')^2} + \left(-D'\right)C_1''$$

which is larger than 2 (hence larger than 1) when $D'' < 0$ (even if $C_1'' = 0$), and it is still larger than 1 when D is not too convex. To provide more precise results, I now turn to a parametric example.

Flexible Demand Example: Let's assume constant marginal costs $C_i' = c_i$ and write $c = \sum c_i$. Consider the family of demand functions

$$D(p) = \left[(1-\theta)(a-p)\right]^{1/(1-\theta)} \tag{10}$$

When $\theta < 1$, the domain of $D(.)$ is $p \in [0, a]$ with $c < a$. Whereas when $\theta > 1$, the domain is $p \in [a, +\infty[$ and in that case one needs to assume $c > a$ (otherwise $p = a$ would lead to an infinite profit). This insures that $(1-\theta)(a-p) \geq 0$ for all p in the relevant domain.

This family encompasses linear demand ($\theta = 0$), and constant price elasticity, ε, of demand (setting $a = 0$ and $\theta = (1+\varepsilon)/\varepsilon$). In addition, demand is strictly concave when $\theta < 0$ and strictly convex when $\theta > 0$. The limiting case $\theta \to 1$ corresponds to an exponential demand $D(p) = \exp\left(-p/\eta\right)$, $\eta > 0$.

In order for profits to be (quasi-)concave, the demand cannot be too convex. In particular, in the monopoly case, it is necessary to assume $\theta < 2$ for the second derivative of the profit to be negative when evaluated at the price solving the first order condition.

It is readily confirmed that $D''D/(D')^2 = \theta$, and given the constant marginal cost assumption, all $1 + p'_k$ are equal to $1/(2-\theta)$ and total prices, in each configuration, are

$$\begin{aligned} p^m &= c + \frac{1-\theta}{2-\theta}(a-c) \\ p^{\mathrm{C}} &= p^m + \left[\frac{(n-1)(1-\theta)}{1+(1-\theta)n}\right]\frac{a-c}{2-\theta} \\ p^{\mathrm{E}} &= p^m + \left[1 - \frac{1}{(2-\theta)^{n-1}}\right]\frac{a-c}{2-\theta} \end{aligned}$$

where p^{C} is well defined only if $\theta < 1 + 1/n$. As above, one can check that $p^{\mathrm{C}} < p^{\mathrm{E}}$ when $\theta < 1$, $p^{\mathrm{C}} = p^{\mathrm{E}}$ when $\theta = 1$, and $p^{\mathrm{C}} > p^{\mathrm{E}}$ when $1 < \theta < 1 + 1/n$.[23] Let's notice that in the limiting case $\theta = 1$, then the expressions above give $p^{\mathrm{E}} = p^{\mathrm{C}} = p^m = c$. But this is only because under the assumptions, at the limit the domain of $D(.)$ is reduced to the singleton $\{c\}$. More generally, for an exponential demand $p^m = c + \eta < c + 2\eta = p^{\mathrm{C}} = p^{\mathrm{E}}$.

4 Historical perspective

To understand why Spengler missed Cournot's and others' contributions, a quick historical detour is needed.

4.1 Late 1890s early 1900s

Cournot's book was such a breakthrough, it took some time for economists to understand its importance.[24] Already at the turn of the 19th century Fisher (1898), while advising a careful study of Cournot, write: "Sixty years ago the mathematical treatise of Cournot was passed over in silence, if not contempt." Fisher (1898) is a detailed review of Cournot's book and a good summary of its perception by mathematical economists at the time.[25]

As explained by Fisher, both Walras (1883) and Marshall (1890) praised Cournot but his equilibrium concept tumbled after criticisms (partly shared by Fisher) made by Bertrand (1883) and Edgeworth (1897, 1925).[26]

Fact 4 (de Bornier (1992))**.** *The most contentious point for both Bertrand and Edgeworth was the indeterminacy of the equilibrium. Cournot and Ellet's insights that consumers benefit from a merger of complementary goods producers was gradually lost in these theoretical discussions.*

[23] Indeed, there is a caveat: Cournot equilibrium does not exist when $1 + 1/n \leq \theta < 2$ because in that case the slopes of the reaction functions are larger than 1 and they do not intersect.

[24] See Ekelund Jr and Hébert (2002) for a broader view on the origins of Microeconomics.

[25] It was published after the first English translation in 1897 of Cournot (1838) by Nathaniel T. Bacon.

[26] Bertrand and Edgeworth proposed different assumptions (possibly implicitly in the case of Bertrand) leading to different games which extended our comprehension of imperfect competition but which certainly did not refute Cournot's concept.

Even if Bertrand did not comment on Cournot's chapter IX, Edgeworth was well aware of both Cournot's chapters VII and IX.[27] But as the equilibrium concept (Cournot-Nash) is the same in both, the same criticism applied.[28] A good example is Moore (1906) who discusses the complementary good model but refers to Edgeworth to dismiss the results.[29] This could explain why the reference to chapter IX gradually vanished.

4.2 A revival of Cournot, 1930-1950

During the first part of the 20th century, Cournot was highly cited, even if the modelling of imperfect competition remained an unsettled issue.[30]

Fact 5. *The soundness of Cournot's equilibrium concept is (re)affirmed. However, chapter IX tends to be cited only within the bilateral monopoly literature.*

Stackelberg (1934, 2010) and Hicks (1935) provide nice historical overviews on duopoly models and present both Cournot's chapters VII and IX.

Chamberlin (1929) presents a balanced view on duopoly models. Commenting on Bertrand's and Pareto's criticisms of Cournot, he writes: "In truth, this conclusion seems hardly a refutation of Cournot, unless the converse be also granted, that it is in turn refuted by Cournot. The two complement, rather than oppose each other, each flowing from a particular assumption - one that the seller who, for the moment is passive will hold his supply fixed, the other that he will hold his price fixed."

Hotelling (1929) and Nichol (1934) are also more sympathetic to Cournot's model.[31] Hicks (1935) (see the long § on top of page 13) provides a nice perspective on this period. Stigler (1940) discusses Cournot at length but does not allude to complementary goods.

Cournot's chapter IX remained cited in the "bilateral monopoly" branch of the literature which developed after Edgeworth (1897, 1925). Pigou (1908) is an early contribution, and is followed by Bowley (1928), Zeuthen (1930), Hicks (1935) (section III), Tintner (1939), and Fellner (1947).[32]

Typically, this literature discusses three ways to solve the game. 1) Sequential timing where U_1 (downstream in the vertical relationship interpretation) sets the input price p_2 and U_2 chooses the quantity. 2) Reverse timing where U_2 sets p_2 and U_1 chooses the quantity. 3) Finally the joint profit maximization leading to the indeterminacy about how the profit is shared. The second timing is the usual sequential timing / vertical relationship set-up of Figure 1b and section 3.2.

[27] de Bornier (1992) explains why Bertrand's review of Cournot was superficial. He also underlines Edgeworth's acrimony towards Cournot. One cannot help but think at how instrumental Game Theory has been in structuring the economic debate.

[28] In his review of Cournot, Fisher (1898) writes (page 128): "But, although Cournot's conclusions are in the main consonant with facts, his analysis of motives in the minds of the two monopolists is subject to much the same objection as above expressed in the case of two competitors." He then refers to Edgeworth. I am grateful to a referee for pointing out this passage.

[29] Moore had published, Moore (1905), a biographical account of Cournot's life.

[30] Around the centenary of Cournot's book, at least four articles Roy (1933), Nichol (1938), Fisher (1938), and Roy (1939) were published in *Econometrica*.

[31] Nichol was at Duke U. like Spengler. In the collection of articles on the History of Economic Thought, Spengler and Allen (1960), the only article on Cournot is Nichol (1934), which does not mention Cournot's chapter IX.

[32] See Machlup and Taber (1960) for a more complete list of references and how they are related. See also Harsanyi (1956) for a critical discussion of Zeuthen's, Hicks's, and Nash's theories of bargaining.

Fact 6. *Even when chapter IX was commented on, the focus was on firms while consumers were left out of the picture. Inevitably, Cournot and Ellet's results were to be re-discovered.*[33]

Burns (1936) emphasizes that a departure from marginal cost pricing on the input market might induce vertical integration. A first step in the direction of the benefit of EDM.[34]

In Allen (1938),[35] a mathematical manual for economists, the author compares in detail, pages 359-362, the level of prices for two complementary goods according to whether they are chosen by a single monopoly or two independent firms. He emphasizes when the monopoly's prices are lower.[36] As Allen cites Cournot elsewhere, one is left to guess whether or not he knew about Cournot's chapter IX.[37]

Hoffman (1940a), a paper on agricultural markets, makes Cournot and Ellet's general point, see also Hoffman (1940b) a testimony before the U.S. Congress:

> Without entering into the details of the proof, I think it can be demonstrated that two successive monopolists, one above the other, will always tend to raise prices and limit output at least as much, and usually more, than a single monopolist combining both their functions. And paradoxical as it seems, the consumer would probably be helped rather than harmed by a conspiring between the two monopolistic groups to increase the amount of their total monopoly profit.

4.3 Spengler (1950)

Joseph J. Spengler (1902-1991) is probably less known than Cournot. He was, however, a great U.S. economist of his time. A professor at Duke, he specialized in population economics and also wrote on history of economic thought. In 1965, he was President of the American Economic Association. His predecessor, in 1964, was George J. Stigler, and his successor, in 1966, was Fritz Machlup. For a detailed summary of his contributions, see Kelley (1992).

Fact 7. *Spengler (1950), entitled "Vertical Integration and Antitrust Policy", contains no formal proof, and provides a self-confirming numerical example. The relevant literature is not cited. Its contribution is the emphasis on antitrust and on the fact that vertical integration should not be illegal per se.*

[33] Ellet (1839) was hardly cited. Calsoyas (1950), an article on Ellet's achievements, starts with "The cult of Cournot and the lack of extensive research in the early mathematical writings on economic topics have kept important writers in virtual oblivion."

[34] See his Chapter IX. Burns does not cite Cournot (nor Edgeworth) in this book but he does cite Chamberlin (1933) (in which Chamberlin (1929) is reproduced) and Hotelling (1929), so he was certainly aware of Cournot although nothing indicates he knew in detail Cournot's chapter IX. Burns (1936) was an important book at the time. It had many reviews, notably in the *Economic Journal*, the *Journal of Political Economy*, the *Quarterly Journal of Economics*, and the *American Economic Review*.

[35] I am grateful to the Editor for this reference. This book quickly became a reference and it was reviewed in many journals, including the *Economic Journal*, the *Journal of Political Economy*, the *Annals of the American Academy of Political and Social Science*, and the *Journal of the American Statistical Association* .

[36] Demand is linear and the goods are not necessarily perfect complements. Linear demands for a differentiated goods set-up was already introduced by Edgeworth (1897, 1925) who only solved the model for perfect complements. In Stackelberg (1934) a more extensive solution is given (for both price or quantity as strategic variables and for both simultaneous and sequential choices), see the Mathematical Appendix. Stackelberg (1934) and Allen (1938) (who give a more general form) are therefore precursors of the work of Shubik and Levitan (1980). See Choné and Linnemer (2020). As neither Stackelberg nor Allen derive the demands from a quadratic utility, however, a crucial relationship between the coefficients is missed.

[37] Allen cites Roos (1934) as his main source for this section. In Roos's book, Cournot is cited nine times but not in Roos's Chapter VIII which Allen refers to.

To prove "his point", Spengler should have referred to Bowley (1928), Tintner (1939), or Fellner (1947). All three provide both formal computations and graphical analyses.

Spengler's literature review is scant. His motivation is an antitrust case against the Great Atlantic and Pacific Tea Company (A&P), the Walmart of the time, which was found guilty of selling some goods below cost through being vertically integrated. Spengler, who did not agree with the Courts on the appreciation of vertical integration, refers to Adelman (1949a) and Adelman (1949b) for the analysis of the case. The only other citations are Chamberlin (1933) and Robinson (1933).

Thanks to the boom in articles about vertical relationships in the mid 1980s and later, Spengler's article is by far his most cited piece on DM. It was not an instant hit, however, in particular among economists.

As shown in Figure 2, the paper is only cited 14 times during its first decade (1950-1959) and mostly in Law journals and/or in relation to an antitrust case.[38] Citations slowed down to 8 during its second decade (1960-1969). Although 5 come from economic journals, one of them, Machlup and Taber (1960), being a rather negative citation. Citations improved in the 1970-1979 decade, up to 21, and about 8 in economic journals. Finally, as discussed in the introduction, citations grew in the 1980s after the Mathewson and Winter articles.

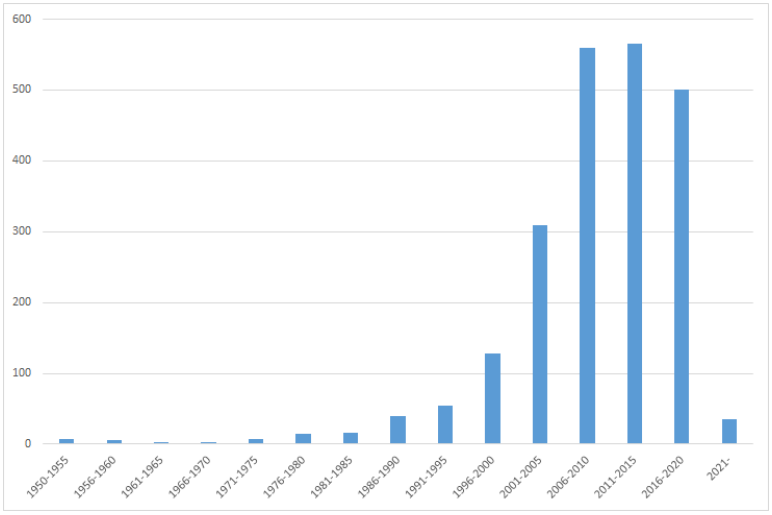


Figure 2: Google Scholar citations for Spengler (1950)

4.4 More proofs and additions

Machlup and Taber (1960), entitled "Bilateral Monopoly, Successive Monopoly, and Vertical Integration" is an important, well-written, and clarifying article, although Nash (1950) and Harsanyi (1956) are not discussed. Machlup and Taber discuss at length Cournot's chapter IX and its perception by economists. Even if the similarity should be obvious, they stop mentioning Cournot in their section IV on successive monopolies.

They provide several examples of the idea that a chain of monopolies is worse than a single monopoly (see pages 114 and 115 of their paper), and, to say the least, they are not

[38] For example, in Adams (1953) it is Hoffman who is cited, not Spengler. Bork (1954) does not cite Spengler but only Adelman (1949a).

sympathetic to Spengler's contribution. They underline that Spengler's model does not exactly fit the successive monopoly model because, on Spengler's graphs, upstream demand is unrelated to downstream demand. Their concluding section, where they argue that a merger is not the only way to achieve EDM, could have fit in the 2020 U.S. Vertical Merger Guidelines debate.

In a sense, Machlup and Taber (1960) should have become the reference for future papers on DM and EDM. A drawback is the lack of formal proofs. This led Greenhut and Ohta (1976) to claim that the statement "important benefits stem from vertical integration" was a "mislaid maxim" which i) relied on the analysis of bilateral monopoly, and ii) lacked a formal proof.

They write "proofs have been sketchy, chiefly diagrammatic, and the subject joined with that of bilateral monopoly" and provide a formal proof in a successive monopolies model.[39] Their proof, which uses quantity as the choice variable, is neither as simple nor as direct as Cournot's (see section 3.1). Instead they unknowingly follow the steps of Bowley (1928), Tintner (1939), and Fellner (1947).

Thanks to Martin Bronfenbrenner[40] (see their first footnote) Greenhut and Ohta were aware of "the intimate relation of the present work to that of Augustin Cournot." But instead of embracing it, they contrast the horizontal (simultaneous timing in fact) aspect of Cournot with their vertical (sequential timing in fact) framework.

Their article was quickly followed by two negative comments: Haring and Kaserman (1978) and Perry (1978). Both of them point out that relevant articles/books are not cited, including Spengler (1950) and Scherer (1970).[41] It is rather surprising because they do cite Machlup and Taber (1960).[42]

A related branch of the literature is on the benefit of vertical integration when buyers (it could be a single buyer) use inputs in variable proportion (instead of fixed proportion as in Cournot). There is usually only one monopolist in this case. To make the situation clearer, in Figure 1a, U_1 is still a monopolist but U_2 is a competitive industry selling at marginal cost. Finally, B usually is a competitive industry combining both inputs in variable proportion.

One of the first papers in this framework, if not the first, is Vernon and Graham (1971). If U_1 can only use a linear price, it would be above its marginal cost and B would distort its choice of inputs. Vertical integration (or more simply the use of a two-part tariff) eliminates this distortion. However, the integrated firm may increase the price of the final product. The total effect on welfare is thus ambiguous. See Perry (1989), section 2.1 (and also Abiru (1988)) for a survey of the early contributions to this literature.

Another variant (with an intriguing twist) is presented in Salinger (1991). The game is that of Figure 1a, except that U_1 and U_2 do not sell perfect complements but substitute goods. So B has demands $D_i(p_i, p_j)$ for good $i = 1, 2$, $j \neq i$. Salinger shows that vertical integration of B and one of the suppliers can lead to an increase of both prices and makes a link with Edgeworth's Paradox of Taxation. Indeed, Edgeworth (1897, 1925) showed that a tax on one of two substitute goods sold by a monopolist can result in the reduction of both prices.[43]

[39] Under some restrictive assumptions they also showed that the upstream price is the same whether there is perfect competition downstream or a monopoly.

[40] A twist is that Bronfenbrenner was a colleague of Spengler at Duke.

[41] Scherer (1970), indeed, gives a formal proof (for linear demand and constant marginal cost) but unfortunately in the "Bilateral monopoly" section. He refers to this proof when discussing Spengler's paper in the section "Are the Benefits of Countervailing Power Passed On?".

[42] The customary reply, Greenhut and Ohta (1978), concedes (almost) nothing! The most novel part in Greenhut and Ohta (1976) is the section where the downstream level is assumed to be a Cournot oligopoly. But Perry (1978) also finds a mistake there.

[43] Salinger's result has been successfully tested empirically by Luco and Marshall (2020).

In the words of Salinger: "The successive monopoly model, frequently accredited to Spengler [1950] but originally due to Cournot [1838],..."

5 Conclusion

In this note, I have recalled the importance of the contributions of Cournot (1838) and Edgeworth (1897, 1925) to the study of the simultaneous and sequential pricing complementary goods. Cournot and Ellet (1839), emphasized that a merger would benefit both the firms and the consumers. That the same is true in the sequential game should have been obvious.

In the period between the two World Wars, Cournot's chapter IX is not so much forgotten as confined to the bilateral monopoly literature. Several scholars re-established the EDM benefit of vertical integration outside the bilateral monopoly strand but with no reference to Cournot (1838) nor Ellet (1839). In particular, Burns (1936), Allen (1938), and Hoffman (1940a,b), a series culminating with Spengler (1950).

The two catch-phrases, DM and EDM, have been introduced into the literature by Mathewson and Winter (1983a,b, 1984). Unfortunately, they wrongly attribute the discovery of the benefits of EDM to Spengler (1950), which consequently became widely cited as the historical reference. This despite the excellent survey of Machlup and Taber (1960) connecting Cournot to the successive monopolists model.

Thankfully, Cournot's chapter IX has slowly resurfaced and it is now again under the spotlight. Salinger (1991) clearly states that Cournot precedes Spengler. Economides (1989) and Economides and Salop (1992) presented extensions of chapter IX to analyze further complementary goods, see also Gabszewicz, Sonnac, and Wauthy (2001) and d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (2007).

Shapiro (2000) relies on chapter IX to study patent pools, as do Lerner and Tirole (2004). Gaudet and Salant (1992) discuss the endogenous mergers of suppliers of complementary goods and Laussel (2008) discusses endogenous vertical integration between a buyer and suppliers of complementary goods. Spulber (2017) allows for nonlinear pricing.

Feinberg and Kamien (2001) cite both Cournot's and Ellet's analysis of complementary goods.[44] Both Cournot and Ellet are also cited in Anderson, Loertscher, and Schneider (2010).

To conclude, a vibrant homage to Cournot seems appropriate. It is given by O'Brien (2008) who writes:

"The work of Cournot, in particular, implies what I like to refer to as the fundamental theorem of antitrust: Combining substitutes is bad, and combining complements is good, unless demonstrated otherwise. Today, 170 years after Cournot's book was published, Cournot's research remains the most influential and most important work in the history of antitrust."

[44] They have a very instructive introduction and they cite an empirical study on the Silk Road between China and Italy, Karni and Chakrabarti (1997).

References

Masahiro Abiru. Vertical Integration, Variable Proportions and Successive Oligopolies. *Journal of Industrial Economics*, 36(3):315–325, 1988.

Walter Adams. Competition, Monopoly and Countervailing Power. *Quarterly Journal of Economics*, 67(4):469–492, 1953.

M. A. Adelman. Integration and Antitrust Policy. *Harvard Law Review*, 63(1):27–77, 1949a.

M. A. Adelman. The A & P Case: A Study in Applied Economic Theory. *Quarterly Journal of Economics*, 63(2):238–257, 1949b.

Roy G. D. Allen. Mathematical analysis for economists. 1938.

Simon P Anderson, Simon Loertscher, and Yves Schneider. The abc of complementary products mergers. *Economics Letters*, 106(3):212–215, 2010.

Ted Bergstrom. Cournot equilibrium in factor markets. Technical report, University of Michigan, 1978.

Joseph Bertrand. Théorie mathématique de la richesse sociale. *Journal des Savants*, pages 499–508, 1883.

George Bittlingmayer. A Model of Vertical Restriction and Equilibrium in Retailing. *Journal of Business*, 56(4):477–496, 1983.

Patrick Bolton and Giacomo Bonanno. Vertical Restraints in a Model of Vertical Differentiation. *The Quarterly Journal of Economics*, 103(3):555–570, 1988.

Robert H Bork. Vertical integration and the Sherman Act: The legal history of an economic misconception. *University of Chicago Law Review*, 22(1):157–201, 1954.

Robert H Bork. The antitrust paradox: A policy at war with itself. 1978.

Arthur L. Bowley. Bilateral Monopoly. *Economic Journal*, 38(152):651–659, 1928.

Arthur Robert Burns. *Decline of competition: A study of the evolution of American industry*. McGraw-Hill Book Co, New York, 1936.

C. D. Calsoyas. The Mathematical Theory of Monopoly in 1839: Charles Ellet, Jr. *Journal of Political Economy*, 58(2):162–170, 1950.

Edward H Chamberlin. Duopoly: Value Where Sellers are Few. *Quarterly Journal of Economics*, 44(1):63–100, 1929.

Edward H Chamberlin. *The Theory of Monopolistic Competition*. Harvard U. Press, Cambridge, 1933.

John Bradford Cherriman. Review of cournot (1838). *Canadian Journal of Industry, Science, and Art*, 2:185–94, 1857.

Philippe Choné and Laurent Linnemer. Linear demand systems for differentiated goods: Overview and user's guide. *International Journal of Industrial Organization*, 73:102663, 2020.

Antoine Augustin Cournot. *Recherches sur les principes mathématiques de la théorie des richesses*. Hachette, Paris, 1838.

Claude d'Aspremont, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. Competition for Market Share or for Market Size: Oligopolistic Equilibria with Varying Competitive Toughness. *International Economic Review*, 48(3):761–784, 2007.

Jean Magnan de Bornier. The 'cournot-bertrand debate': A historical perspective. *History of Political Economy*, 24(3):623–656, 1992.

Robert W Dimand. Cournot, bertrand, and cherriman. *History of Political Economy*, 27(3): 563–578, 1995.

Nicholas Economides. Desirability of Compatibility in the Absence of Network Externalities. *American Economic Review*, 79(5):1165–1181, 1989.

Nicholas Economides and Steven C. Salop. Competition and Integration Among Complements, and Network Market Structure. *Journal of Industrial Economics*, 40(1):105–123, 1992.

Francis Y Edgeworth. *Mathematical psychics: An essay on the application of mathematics to the moral sciences*. Number 10. CK Paul, 1881.

Francis Y Edgeworth. La teoria pura del monopolio. *Giornale degli Economisti*, 15 (Anno 8): 13–31, 1897.

Francis Y Edgeworth. The pure theory of monopoly. In Macmillan for the Royal Economic Society, editor, *Papers relating to political economy*, volume 1, pages 111–142. London, 1925.

Law Review Editors. Vertical Forestalling under the Antitrust Laws. *University of Chicago Law Review*, 19(3):583–619, 1952.

Robert B Ekelund Jr and Robert F Hébert. Retrospectives: The origins of neoclassical microeconomics. *Journal of Economic Perspectives*, 16(3):197–215, 2002.

Charles Jr Ellet. An Essay on the Laws of Trade in Reference to the Works of Internal Improvement in the United States. 1839.

Yossi Feinberg and Morton I Kamien. Highway robbery: complementary monopoly and the hold-up problem. *International Journal of Industrial Organization*, 19(10):1603–1621, 2001.

William Fellner. Prices and Wages Under Bilateral Monopoly. *Quarterly Journal of Economics*, 61(4):503–532, 1947.

Irving Fisher. Cournot and Mathematical Economics. *Quarterly Journal of Economics*, 12(2): 119–138, 1898.

Irving Fisher. Cournot Forty Years Ago. *Econometrica*, 6(3):198–202, 1938.

Jean Gabszewicz, Nathalie Sonnac, and Xavier Wauthy. On price competition with complementary goods. *Economics Letters*, 70(3):431–437, 2001.

Gérard Gaudet and Stephen W Salant. Mergers of producers of perfect complements competing in price. *Economics Letters*, 39(3):359–364, 1992.

Melvin L. Greenhut and Hiroshi Ohta. Related Market Conditions and Interindustrial Mergers. *American Economic Review*, 66(3):267–277, 1976.

Melvin L. Greenhut and Hiroshi Ohta. Related Market Conditions and Interindustrial Mergers: Reply. *American Economic Review*, 68(1):228–230, 1978.

John R. Haring and David L. Kaserman. Related Market Conditions and Interindustrial Mergers: Comment. *American Economic Review*, 68(1):225–227, 1978.

John C. Harsanyi. Approaches to the Bargaining Problem Before and After the Theory of Games: A Critical Discussion of Zeuthen's, Hicks', and Nash's Theories. *Econometrica*, 24 (2):144–157, 1956.

John R. Hicks. Annual Survey of Economic Theory: The Theory of Monopoly. *Econometrica*, 3(1):1–20, 1935.

Austin Clair Hoffman. Changing organization of agricultural markets. *Journal of Farm Economics*, 22(1):162–172, 1940a.

Austin Clair Hoffman. *Large-scale organization in the food industries*. United States Congress. Temporary National Economic Committee, Monograph no.35, 1940b.

Harold Hotelling. Stability in Competition. *Economic Journal*, 39(153):41–57, 1929.

Edi Karni and Subir K Chakrabarti. Political structure, taxes, and trade. *Journal of Public Economics*, 64(2):241–258, 1997.

Michael L Katz. Vertical contractual relations. In *Handbook of industrial organization*, volume 1, pages 655–721. Elsevier, 1989.

Allen C Kelley. Joseph J. Spengler (November 19, 1902-January 2, 1991). *Proceedings of the American Philosophical Society*, 136(1):143–147, 1992.

Didier Laussel. Buying back subcontractors: The strategic limits of backward integration. *Journal of Economics & Management Strategy*, 17(4):895–911, 2008.

Abba P. Lerner. The Concept of Monopoly and the Measurement of Monopoly Power. *Review of Economic Studies*, 1(3):157–175, 1934.

Josh Lerner and Jean Tirole. Efficient Patent Pools. *American Economic Review*, 94(3): 691–711, 2004.

Fernando Luco and Guillermo Marshall. The competitive impact of vertical integration by multiproduct firms. *American Economic Review*, 110(7):2041–64, 2020.

Fritz Machlup and Martha Taber. Bilateral Monopoly, Successive Monopoly, and Vertical Integration. *Economica*, 27(106):101–119, 1960.

Alfred Marshall. Principles of economics. 1890.

G. Frank Mathewson and Ralph A. Winter. The Incentives for Resale Price Maintenance under Imperfect Information. *Economic Inquiry*, 21(3):337–348, 1983a.

G. Frank Mathewson and Ralph A. Winter. Vertical Integration by Contractual Restraints in Spatial Markets. *Journal of Business*, 56(4):497–517, 1983b.

G. Frank Mathewson and Ralph A. Winter. An Economic Theory of Vertical Restraints. *RAND Journal of Economics*, 15(1):27–38, 1984.

John S. McGee and Lowell R. Bassett. Vertical Integration Revisited. *Journal of Law & Economics*, 19(1):17–38, 1976.

Henry L. Moore. The Personality of Antoine Augustin Cournot. *Quarterly Journal of Economics*, 19(3):370–399, 1905.

Henry L. Moore. Paradoxes of Competition. *Quarterly Journal of Economics*, 20(2):211–230, 1906.

John F. Nash. The Bargaining Problem. *Econometrica*, 18(2):155–162, 1950.

A. J. Nichol. A Re-Appraisal of Cournot's Theory of Duopoly Price. *Journal of Political Economy*, 42(1):80–105, 1934.

A. J. Nichol. Tragedies in the Life of Cournot. *Econometrica*, 6(3):193–197, 1938.

Daniel P O'Brien. The antitrust treatment of vertical restraints: Beyond the possibility theorems. In Swedish Competition Authority Stockholm, editor, *The Pros and Cons of Vertical Restraints*, volume 7, pages 40–101. 2008.

Martin K. Perry. Related Market Conditions and Interindustrial Mergers: Comment. *American Economic Review*, 68(1):221–224, 1978.

Martin K Perry. Vertical integration: Determinants and effects. In *Handbook of industrial organization*, volume 1, pages 183–255. Elsevier, 1989.

Arthur C. Pigou. Equilibrium under bilateral monopoly. *Economic Journal*, 18(70):205–220, 1908. URL `http://www.jstor.org/stable/2220700`.

Richard A Posner. *Antitrust Law: An Economic Perspective*. University of Chicago press, 1976.

Patrick Rey and Jean Tirole. The Logic of Vertical Restraints. *American Economic Review*, 76(5):921–939, 1986.

Patrick Rey and Jean Tirole. Price caps as welfare-enhancing coopetition. *Journal of Political Economy*, 127(6):3018–3069, 2019.

Joan Robinson. *The economics of imperfect competition*. Macmilan & Co., London, 1933.

Charles Fredrick Roos. *Dynamic economics theoretical and statistical studies of demand, production and prices*. Principia Press, Bloomington, 1934.

René Roy. Cournot et l'école mathématique. *Econometrica*, 1(1):13–22, 1933.

René Roy. L'Oeuvre Économique d'Augustin Cournot. *Econometrica*, 7(2):134–144, 1939.

Michael A. Salinger. Vertical Mergers in Multi-Product Industries and Edgeworth's Paradox of Taxation. *Journal of Industrial Economics*, 39(5):545–556, 1991.

Frederic M. Scherer. *Industrial Market Structure and Economic Performance*. Chicago, 1970.

Carl Shapiro. Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting. *Innovation Policy and the Economy*, 1:119–150, 2000.

Martin Shubik and Richard E. Levitan. *Market structure and behavior*. Harvard U. Press, 1980.

Margaret Slade and John E Kwoka Jr. Second Thoughts on Double Marginalization. *Antitrust*, 34(2):51–56, 2020.

Margaret E Slade. Vertical Mergers: A Survey of Ex Post Evidence and Ex Ante Evaluation Methods. *Review of Industrial Organization*, 58(4):493–511, 2021.

Hugo Sonnenschein. The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One. *Journal of Political Economy*, 76(2):316–318, 1968.

Joseph J. Spengler. Vertical Integration and Antitrust Policy. *Journal of Political Economy*, 58(4):347–352, 1950.

Joseph J. Spengler and William R. Allen. *Essays in Economic Thought: Aristotle to Marshall*. Rand McNally, Chicago, 1960.

Daniel F Spulber. Complementary monopolies and bargaining. *Journal of Law and Economics*, 60(1):29–74, 2017.

Heinrich von Stackelberg. *Marktform und gleichgewicht*. Springer, 1934.

Heinrich von Stackelberg. *Market structure and equilibrium*. Springer Science & Business Media, 2010.

George J. Stigler. Notes on the Theory of Duopoly. *Journal of Political Economy*, 48(4): 521–541, 1940.

Gerhard Tintner. The Problem of Bilateral Monopoly: A Note. *Journal of Political Economy*, 47(2):263–270, 1939.

Jean Tirole. *The theory of industrial organization*. MIT press, 1988.

John M. Vernon and Daniel A. Graham. Profitability of Monopolization by Vertical Integration. *Journal of Political Economy*, 79(4):924–925, 1971.

Léon Walras. *Théorie mathématique de la richesse sociale*. Corbaz, Lausanne, 1883.

Frederick Zeuthen. *Problems of Monopoly and Economie Warfare New York, Augustus M.* G. Routledge & Sons, London, 1930.