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9537
2022

January 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
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Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
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CESifo *Working Paper No. 9537*

Sustainable Finance and Climate Change: Wasteful but a Political Commitment Device?

Abstract

Promoting investment in low carbon "clean" sectors has gained popularity over the last years under the heading of sustainable finance, at the same time raising concerns about adverse welfare effects of such policies. We analyze the economic impact of subsidizing investment in "clean" industries in a stylized two-sector small open economy model. Such a reform increases gross wages, but reduces national income due to the distortion of capital. At given national emissions cap, worldwide emissions rise because imports of the high-carbon good will increase. When adapting the emissions cap, the environmental policy becomes laxer if it is dominated by income effects or by mitigating losses arising from the distortion of the allocation of capital. At the same time, the shrinking high carbon sector reduces income gains from a higher cap and thus works toward a stricter policy. Results are similar if capital in "dirty" industries is taxed. Though sustainable finance policies do seem wasteful, we provide a rationalization in a setting with irreversible investment, where a "green" government" uses such a policy to induce stricter environmental measures after a possible switch to a "conservative" government.

JEL-Codes: F410, H230, H870, Q580.

Keywords: climate change, global externalities, sustainable finance, small open economy, political economy.

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1. Introduction

Promoting investment in low carbon “clean” sectors has gained popularity over the last years under the heading of sustainable finance, at the same time raising concerns about adverse welfare effects of such policies. This policy can be seen as part of a strategy to combat climate change, where carbon dioxide and other greenhouse gases are held responsible to accelerate global warming. In several countries, there are attempts to regulate financial investment in order to discriminate in favor of “green“ firms that do not use carbon intensively and against „dirty“ carbon-intensive production. For example, the Norwegian oil fund already excludes investment in carbon-intensive industries (Norges Bank Investment Management, 2020). The European Union has developed a taxonomy of sustainable investment which employs carbon-intensity of firms as a key criterion. While a first purpose of that taxonomy clearly consists in regulating labeling of green funds, providing some orientation to private and public investors (Schütze et al. 2020), it is quite plausible that such a differentiation paves the way for a differential tax treatment. A recent report (European Banking Federation, 2019) summarizes plans and steps already undertaken at the national level to subsidize green funds. At the same time, negative externalities of emitting greenhouse gases are already addressed by environmental policy, for example by setting an emissions cap. This situation raises concerns that sustainable finance policies are ineffective with respect to environmental goals, while bearing the potential of income and welfare losses through distorting the allocation of capital.

Against this background our paper addresses the following questions. First, what are the consequences of this policy for factor prices, sector sizes, national income and worldwide emissions at a given national emissions cap? Second, will we have an increase or a decline in overall emissions if endogeneity of the emissions cap is taken into account? Third, can we provide an explanation why sustainable finance policies are implemented?

We analyze a neoclassical two-sector model of a small open economy, with a clean sector that does not use carbon, and a dirty sector. The sustainable finance policy subsidizes capital in the clean sector, being financed by wage taxes and emission allowances. We proceed in two steps: First, we consider impacts of an exogenous increase of the subsidy. Second, we discuss impacts of the subsidy on the choice of the emissions cap, which is determined so as to serve the interest of a representative consumer.

It turns out that the subsidy to „clean“ capital causes an inflow of capital and labor into the clean sector, while the dirty sector shrinks. National income will fall since the allocation of capital is no longer efficient. Gross wages will rise due to the higher capital-labor ratio in the clean sector. Net wages will decline because the burden of taxation falls on the internationally immobile factor labor. The price of an emission allowance falls along with the productivity of carbon due to the outflow of capital and labor from the dirty sector. Moreover, the trade pattern is affected where changes in production are mirrored abroad so as to keep consumption patterns constant. Consequently, the sustainable finance policy meant to combat climate change by reducing carbon emissions achieves the opposite because carbon emissions abroad increase – provided that these are not limited by an emissions cap.

The impact on the political determination of the emissions cap is not obvious, as there are counteracting forces. The decline in national income and the distortion of the capital allocation both work toward a laxer policy – the latter because increasing the cap reduces income losses arising from the wedge beween interest rate and marginal productivity of capital. By contrast, the reduction of the marginal output loss in the dirty sector following a smaller emissions cap suggests cutting that cap. Considering a Cobb-Douglas example indicates that the last effect dominates when introducing a marginal subsidy. This result is likely to be turned around when further increasing the subsidy because income losses and incentives to reduce them through a higher emissions cap become substantial. In that event, sustainable finance yields more emissions by inducing a laxer environmental policy.

We also analyze the alternative implementation of sustainable finance by taxing capital in the dirty sector. The predicted outcomes stay similar to our main setting, except for the now absent impact on gross wages. Moreover, the Cobb-Douglas specification indicates that the sustainable finance policy here now unambiguously induces a laxer emissions cap.

Since these specifications do not provide a rationale why sustainable finance policies are implemented, we aim at resolving this puzzle in a framework with frictions. In that extension, investment responds to the announcement of sustainable finance policy, but is locked when, after a possible change in government, the emissions cap is determined. In such an environment, a “green“ government benefits from inducing a stricter environmental policy after a possible switch to a “conservative“ government. From this perspective sustainable finance may be seen as a strategic political instrument to achieve commitment to green policies – though in a wasteful way.

Our paper is related to several strands of the literature as follows. Many papers on sustainable finance remain on a descriptive and empirical level, dealing with motives of investors to hold green funds, their relative performance, and their attempts to integrate environmental criteria into corporate goals (Friede et al., 2015, Riedl and Smeets, 2017; Dyck et al., 2019). Several contributions analyze taxes on carbon and carbon-intensive capital in a growth context. Acemoglu et al. (2012) argue that taxing carbon at a moderate pace can be useful for sustainable growth by redirecting R&D toward clean technologies. At the same time, they stress that excessive taxation of the exhaustible resource (carbon) will be detrimental for growth in the long run. Analyzing a two-sector growth model, Jin et al. (2020) show that using carbon-intensive capital may help to accumulate clean capital for an extended period. As key argument in favor of sustainable finance, Hong et al. (2021) claim that reducing the capital cost of sustainable firms by a subsidy enables them to engage in mitigation spending (which identifies the sustainable firms), where the benefit of mitigation accrues to the economy as a whole. By contrast, our contribution focuses on short-term and medium-term effects working through factor mobility within and across countries and adapting patterns of international trade, arriving at a more skeptical view on sustainable finance policies. Our paper also bears relations to the literature on carbon leakage (Babiker, 2005; Eichner and Pethig, 2011; Aichele and Felbermayer, 2015; Böhringer et al. 2017), arguing that tighter environmental regulation within a country or a group of countries will increase carbon emissions in the rest of the world via capital movements or changes in the pattern of international trade. In our contribution, we emphasize similar mechanisms now applied to sustainable finance policies. We are able to identify determinants working in favor and against higher emissions, both with and without an endogenous emissions cap. Finally, our contribution relates to the literature on using commitment devices or specific policies so as to affect or constrain the behavior of future governments, well-known in the area of fiscal and monetary policy (Kydland and Prescott, 1977; Alesina and Tabellini, 1990).

The remainder of the paper is organized as follows. Section 2 introduces the model, Section 3 presents the comparative static analysis of the sustainable finance policy by subsidization of capital in the clean sector, and Section 4 deals with the impacts on the choice of the emissions cap. Section 5 analyzes the consequences of the alternative approach to tax capital used in the dirty sector. Section 6 considers the rationalization of the sustainable finance policies with a sequential time structure. The final Section 7 concludes and indicates directions for further research.

2. Setup

We analyze a two-sector small open economy, with a dirty and a clean sector. For simplicity, the clean sector does not use or produce emissions. Technology of the clean sector adheres to the neoclassical standard of constant returns,

$$X = F(K_X, L_x) \tag{1}$$

with $F_K > 0 > F_{KK}$ and $F_L > 0 > F_{LL}$.

Modeling the dirty sector, we use the framework of Oates and Schwab (1988) and Sinn (2003) in which emissions generate output. Put differently, avoiding emissions comes at a cost. The dirty sector is described by a constant returns to scale production function

$$Y = G(K_Y, L_Y, Z) \tag{2}$$

with positive and diminishing marginal productivities, where Y is „dirty“ output, K_Y is capital input, L_Y is labor input, and Z are emissions. Factors are complements in the sense that all cross derivatives are positive, $G_{KL} > 0$, $G_{KZ} > 0$, $G_{LZ} > 0$.

Let the clean good X be the numeraire, where P denotes the relative price of good Y. Capital is assumed to be internationally mobile, while labor is treated as mobile between sectors, but immobile internationally. Firms maximize profits, taking prices as given. Profit of a representative firm is

$$\Pi_X = F(K_X, L_X) - w_X L_X - rK_X \tag{3}$$

in the clean sector X, and

$$\Pi_Y = PG(K_Y, L_Y, Z) - w_Y L_Y - rK_Y - \theta Z \tag{4}$$

in the dirty sector Y.

In this expression, w_X and w_Y denote the wage in the respective sector, r is the (net) interest rate in the world market, and θ is the price of an emission certificate related to one unit of emissions. Keeping the analysis as simple as possible, we consider a small open economy scenario in which both the output price P and the (net) real interest rate r are fixed. With that specification, the wages and the price of an emission certificate θ are endogenous, as well as import and export. Considering a framework of incomplete specialization, we assume that the economy under consideration always produces both goods.

Profit maximization implies that emission certificates will be used until

$$\theta = P\frac{\partial G}{\partial Z} \tag{5}$$

Assuming that the market for certificates clears, (5) determines the certificate price θ. Perfect mobility of workers within the country ensures equalization of wages, $w = w_X = w_Y$:

$$w = \frac{\partial F}{\partial L} = P\frac{\partial G}{\partial L} \quad (6)$$

Accordingly for capital:

$$r = \frac{\partial F}{\partial K} = P\frac{\partial G}{\partial K} \quad (7)$$

The value of the output is maximized at going prices since the technical marginal rates of substitution are identical:

$$\frac{w}{r} = \frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial K}} = \frac{\frac{\partial G}{\partial L}}{\frac{\partial G}{\partial K}} \quad (8)$$

The representative consumer maximizes utility $U(C^X, C^Y)$ with respect to good demands C^X and C^Y subject to the budget constraint $C^X + PC^Y \leq I$ at given price P and given national income I. The utility function U is homogenous with $U_1 > 0 > U_{11}, U_2 > 0 > U_{22}$.

Income of the representative consumer in terms of the clean good is

$$I = r\bar{K} + (1 - \tau)w\bar{L} \quad (9)$$

where $\bar{K}$ and $\bar{L} = L_X + L_Y$ denote capital endowment and labor force of the economy under consideration, respectively, while τ is the tax rate on wage income. The consumer optimum satisfies

$$\frac{\partial U_2}{\partial U_1} = P \quad (10)$$

According to a well-known result, homogenous preferences imply that the demand functions $C^X(P, I)$ and $C^Y(P, I)$ are linear in I at given P. Moreover, with linear homogeneity of U, indirect utility derived from consumption $U^*(I, P) \equiv U\left(C^{X*}(I, P), C^{Y*}(I, P)\right)$ can be expressed by $k(P)I$, where the sign of $k'(P)$ depends on whether the clean good is exported or imported. We also consider a version with diminishing marginal indirect utility from income. The budget constraint of the representative consumer implies that the value of imports equals the value of exports, where net foreign capital income adds to the capacity of imports :

$$X - C^X + P(Y - C^Y) + r(\overline{K} - K_X - K_Y) = 0 \tag{11}$$

For simplicity, capital income from abroad is repatriated in units of the dirty good. Moreover, consumption in the rest of the world stays constant throughout, while the production pattern in the rest of the world is adapted such that the world market for the clean and the dirty good clears at constant price P. When the environmental policy is endogenous, the government applies the preferences of the representative consumer to pick the emissions cap Z. Preferences of the representative consumer are specified by $W = U(C^X, C^Y) - V(Z)$, where V expresses disutility from emissions, with $V' > 0$ and $V'' > 0$. Accordingly, the emissions cap will be chosen by the government such that

$$V'(Z) = \frac{\partial U^*}{\partial I} \frac{\partial I}{\partial Z} \tag{12}$$

Thus, the cap is chosen such that the marginal damage by higher emissions in terms of income, $\frac{V'(Z)}{\frac{\partial U^*}{\partial I}}$, just offsets the income gain that could be achieved by increasing the cap, $\frac{\partial I}{\partial Z}$.

3. Subsidizing „clean“ capital

Now suppose a sustainable finance policy is implemented in which capital invested in the clean sector is subsidized at rate σ. In this section, we take the emissions cap Z as given. Not adding further distortions, the subsidy is financed by emission certificates and taxing wages at rate τ, which is tantamount to a lump-sum tax (if positive) or a lump-sum benefit (if negative):

$$\sigma \frac{\partial F}{\partial K} K_X = \tau w \bar{L} + \theta Z \tag{13}$$

The arbitrage condition for investors becomes

$$r = (1 + \sigma) F_K = P G_K \tag{14}$$

Perfect mobility of labor between sectors still ensures equalization of wages according to equation (6). Since the value of the marginal product of capital is not equalized, the allocation is not efficient. Hence, some national income is lost due to distortion of the capital allocation.

Regarding technical marginal rates of substitution, we now obtain

$$\frac{w}{r} = \frac{\frac{\partial F}{\partial L}}{(1 + \sigma)\frac{\partial F}{\partial K}} = \frac{\frac{\partial G}{\partial L}}{\frac{\partial G}{\partial K}} \tag{15}$$

With fixed output price P and fixed interest rate, a higher subsidy will increase the capital-labor ratio in the clean sector and the wage rate. Proposition 1 summarizes the comparative static impacts on employment, output, and factor prices.

Proposition 1. *Increasing the subsidy rate to capital in the clean sector σ increases the wage rate w, the capital-labor ratio in the clean sector, $\frac{K_X}{L_X}$, as well as capital demand K_X, employment L_X and output X in the clean sector. Moreover, increasing σ reduces the price of an emission allowance θ, capital demand K_Y, employment L_Y and output Y in the dirty sector.*

Proof. See Appendix A.

The increase of the subsidy rate σ, for example from zero to some positive level, raises the net return in the „clean“ sector X, making investment more attractive. Capital flows into that sector from abroad and/or from the dirty sector Y until net returns are equalized again.

As a consequence of additional investment in sector X, and possibly an outflow of capital from sector Y, a migration incentive for workers from sector Y to sector X emerges. The inflow of capital to sector X increases the wage in that sector, while an outflow of capital from sector Y reduces wages in sector Y. The only way to reduce the wage differential lies in having migrating workers from sector Y to sector X. This in turn induces some capital from sector Y to move to sector X as well. A new equilibrium requires identical wages and identical net returns to capital.

In the small open economy setting with internationally mobile capital, the net interest rate r ultimately remains constant. Due to the subsidy to capital in the clean sector, the capital-labor ratio in the clean sector $\frac{K_X}{L_X}$ increases, being accompanied by a higher gross wage w. As factor demand for both capital and labor increase in the clean sector, clean output will also rise. In the dirty sector, at given emissions cap Z, the price of a certificate θ falls. This happens due to the outflow of both capital and labor from sector Y, as we assume that all factors are complements in production. Output of the dirty sector shrinks because factor demand for capital and labor declines.

Trade and welfare effects are summarized in Proposition 2. The sustainable finance policy will reduce disposable income and – given taxation of wages – the net wage. Moreover, aggregate worldwide emissions rise since lower domestic production of the dirty good at unchanged emissions will to some extent be offset by higher foreign production of the dirty good.

Proposition 2. *Introducing the subsidy rate on capital in the clean sector σ reduces national income I and the net wage $(1-\tau)w$. Net export of the clean good $X - C_X$ and net import of the dirty good $P(C_Y - Y) - r(\bar{K} - K_X - K_Y)$ increase. Worldwide emissions increase if higher foreign output of the dirty good is associated with higher emissions.*

Proof. See Appendix B.

National income declines due to the distortion of the capital allocation. Even if the value of domestic output increases via capital imports induced by the (increased) subsidy to capital employed in the clean sector, its marginal product falls short of the income that is paid to such capital. As the perfect capital market ensures that net returns to domestic capital are unaffected by political measures, the decline of income must be reflected in shrinking net wages though gross wages rise. This result need not contradict political support for the sustainable finance policy. First, it may happen that workers uphold a tax illusion, as understanding the positive impact on gross wages is easier than seeing through the consequences on the tax burden. Second, it may be possible that taxes are collected in a different manner, say by taxing a fixed factor like land, not modeled here.

Regarding emissions, the initial goal to implement a green policy backfires. While domestic production of the clean good increases and domestic production of the dirty good falls at constant emissions, the trade pattern is adapted. As worldwide consumption will be associated with similar shares of the clean and the dirty good, the clange in the domestic production pattern is mirrored abroad. Hence, clean output abroad will decline and dirty output will increase. If the latter is accompanied by increasing emissions, worldwide emissions rise. The positive impact of foreign output of the dirty good on emissions occurs unless all countries collected as rest of the world regulate emissions by a cap.

4. Endogenous emissions cap

In this section, we focus on impacts of the sustainable finance policy on the determination of the emissions cap. Let the representative voter care only about domestic emissions, related to a domestic emissions goal, naively assuming that foreign emissions are not affected by setting that cap. With fixed price and homogenous preferences, the first-order condition determining the most preferred emissions of the representative consumer Z^* satisfies (12). Income can be expressed as function of the subsidy and the emissions cap:

$$I(\sigma, Z) = X\big(K_X(\sigma, Z), L_X(\sigma, Z)\big) + PY(K_Y(\sigma, Z), \bar{L} - L_X(\sigma, Z), Z) + r\big(\bar{K} - K_X(\sigma, Z) - K_Y(\sigma, Z)\big) \tag{16}$$

The direction of change of the welfare maximizing cap is determined by the sign of the cross derivative of the welfare function:

$$sgn\left[\frac{dZ^*}{d\sigma}\right] = sgn\left[\frac{\partial^2 W}{\partial Z \partial \sigma}\right] = sgn\left[\frac{\partial^2 U^*}{\partial I^2}\frac{\partial I}{\partial \sigma}\frac{\partial I}{\partial Z} + \frac{\partial U^*}{\partial I}\frac{\partial^2 I}{\partial Z \partial \sigma}\right] \tag{17}$$

It turns out that three impacts of increasing the clean investment subsidy determine whether the environmental policy becomes laxer by increasing the emissions cap or stricter by tightening it. First, shrinking the dirty sector size reduces losses from tightening the cap - which we call the *structural effect*, working toward a stricter policy. Second, increasing the wedge between interest rate and marginal productivity of capital in the clean sector raises income gains from increasing the cap by shrinking the clean sector – which we call the *distortion effect*, working toward a laxer climate policy. Third, an *income effect* arises with diminishing marginal utility of income, where income losses following the distortion of the allocation of capital increases welfare losses from decreasing the cap, again inducing a laxer environmental policy. Proposition 3 discusses the situation in the absence of the income effect, which is achieved by imposing constant marginal utility of income, $\frac{\partial^2 U^*}{\partial I^2} = 0$.

Proposition 3. *At constant marginal utility of income, marginally raising the sustainable finance subsidy rate σ induces a decline of the most preferred emissions* *cap Z^* if and only if* $-\frac{\partial\theta(Z^*)}{\partial\sigma} < -\frac{\partial\left[\frac{\sigma r}{(1+\sigma)}\frac{\partial K_X}{\partial Z}(Z^*)\right]}{\partial\sigma}$*, that is, if and only if the distortion effect works stronger than the structural effect.*

Proof. See Appendix C.

After increasing the clean capital subsidy σ, capital and labor shift away from the dirty sector, reducing the marginal product of emissions and hence the value of increasing the emissions cap. In itself, this structural effect reduces the most preferred emissions cap, hence implies a stricter environmental policy. A counteracting effect arises with a positive capital subsidy in the clean sector because employment of capital in that sector then reduces national income. This happens because the marginal product of capital falls short of the price that has to be paid for its use. Increasing the cap would lead to lower capital input in the clean sector - as capital shifts to the dirty sector – resulting in additional gains in national income, calling for a more lenient environmental policy. Intuitively, the distortion effect becomes stronger with an increasing distortion, hence with a higher preexisting level of the clean capital subsidy σ. Inspection of the terms shows that even at $\sigma = 0$ the distortion effect does not vanish. Hence, even in the absence of the income effect, which always works toward a more lenient policy, an increase of the sustainable finance policy will lead to a stricter environmental goal only if the structural effect outweighs the distortion effect.

Assuming diminishing marginal utility of income instead of linear indirect utility is clearly more realistic. In that event, the government attaches higher value to given additional output from a higher emissions cap after the introduction of the capital subsidy – because the latter reduces national income. The direct effect of increasing the subsidy on income is

$$\frac{\partial I}{\partial \sigma} = \left[\frac{\partial X}{\partial K_X} - r\right]\frac{\partial K_X}{\partial \sigma} = \left[\frac{r}{1+\sigma} - r\right]\frac{\partial K_X}{\partial \sigma} = -\frac{\sigma r}{1+\sigma}\frac{\partial K_X}{\partial \sigma} \quad (18)$$

as all other terms are zero. This suggests that the income effect grows with an increasing rate of subsidization σ. At the same time, it remains negligible for sufficiently small subsidy rates σ since the direct impact of the subsidy on income vannishes at the boundary $\sigma = 0$, see: $\frac{\partial I}{\partial \sigma}(\sigma = 0) = 0$.

The example presented in Appendix D considers a Cobb-Douglas specification for both sectors, with $X = A_X(K_X)^\alpha(L_X)^\beta$ and $Y = A_Y(K_Y)^\gamma(L_Y)^\delta Z^\varepsilon$. In the absence of an income effect, it is demonstrated that the distortion effect works stronger than the structural effect at $\sigma = 0$ if and only if $\alpha > \beta$. Hence, as theoretical prediction, the sign of the reaction of the emissions cap to increasing the clean capital subsidy is ambiguous, depending on parameters. For a plausible empirical assessment, note that α represents the income share of capital in the clean sector. Capital income shares generally lie below 0.5 - usually around 0.3. We can conclude that introducing a suffciently small clean capital subsidy will typically be dominated by the structural effect - which involves a declining cap, hence a stricter policy goal. As both the income effect and the distortion effect grow in size with higher subsidy rate σ, a plausible pattern of the chosen emissions cap with growing σ may display a decrease as long as σ remains very small and increase at higher values of σ. Since the distortion effect works through capital income, it is intuitive that its weight grows with an increasing capital income share in the sector that experiences the wedge between interest rate and marginal product of capital.

Summarizing, it is not obvious whether the environmental policy is tightened or loosened upon the introduction of the capital subsidy. While the income effect and the distortion effect work toward a laxer policy, they may be dominated by the counteracting structural effect if the subsidy remains sufficiently small.

Should the objective determining the emissions cap consider worldwide emissions $Z + \hat{Z}$ instead of domestic emissions, rising projected emissions abroad $\hat{Z}$, as predicted by Proposition 2, will lead to a tighter policy goal in the country under consideration, though the domestic cut will typically not fully compensate for the increase abroad.

5. Taxing dirty capital

An alternative implementation of a sustainable finance policy lies in taxing capital invested in the dirty sector instead of subsidizing capital in the clean sector. In this section, we analyze the version in which the return is taxed at rate t. Accordingly, the government budget equation (13) is replaced by

$$\tau w(L_X + L_Y) + \theta Z + tP\frac{\partial G}{\partial K}K_Y = 0 \tag{19}$$

where tax rate on wages τ will now always be negative. The arbitrage condition becomes

$$r = F_K = (1 - t)PG_K \tag{20}$$

Obviously, income losses arise again because the allocation of capital is not efficient. The comparative static results are collected in Proposition 4. Taxing dirty capital bears some similarities to subsidizing clean capital in achieving a growing clean sector and a shrinking dirty sector. However, there are some differences due to the technological asymmetry. In particular, the gross wage stays constant when taxing dirty capital.

Proposition 4. *Increasing the tax rate on capital income the dirty sector t leaves the wage rate w and the capital-labor ratio in the clean sector, $\frac{K_X}{L_X}$, constant. Capital demand K_X, employment L_X and output X in the clean sector increase. Moreover, increasing t reduces the capital-labor ratio $\frac{K_Y}{L_Y}$, the price of an emission allowance θ, capital demand K_Y, employment L_Y and output Y in the dirty sector.*

Proof. See Appendix E.

Since the marginal product of capital in the clean sector stays at the level of the interest rate, the capital-labor ratio in the clean sector is not affected, which necessitates a constant wage. The direct impact of the tax consists in driving down the net return to capital in the dirty sector, inducing an outflow. This in turn decreases the wage in the dirty sector, implying a migration incentive for workers to the clean sector. A new migration equilibrium is achieved with identical wages in each sector. Since wages ultimately stay constant, the flow of labor to the clean sector must be accompanied by higher capital demand in the clean sector. The gross marginal product of capital in the dirty sector has to rise until its net return is back to the world market level, associated with a lower capital-labor ratio in the dirty sector. Due to the outflow of capital and labor from the dirty sector, the carbon allowance price decreases. The factor movements imply that the clean sector grows and the dirty sector shrinks in terms of employment and output.

Trade and welfare effects are summarized in Proposition 5. The sustainable finance policy will again reduce disposable income and – even with negative taxation of wages – the net wage. Aggregate worldwide emissions typically rise since lower domestic production of the dirty good at unchanged emissions will to some extent be offset by higher foreign production of the dirty good. If realistically the rest of the world does not apply an emissions cap, this will generally be associated with higher emissions abroad and hence higher worldwide emissions.

Proposition 5. *Introducing a positive tax rate on capital in the dirty sector* t *reduces national income I and the net wage* $(1-\tau)w$. *Net export of the clean good* $X - C_X$ *and net import of the dirty good* $P(C_Y - Y) - r(\bar{K} - K_X - K_Y)$ *increase. Worldwide emissions increase if higher foreign output of the dirty good is associated with higher emissions.*

Proof. See Appendix F.

Regarding the determination of the emissions cap, national income can be written as

$$I(t,Z) = X\big(K_X(t,Z), L_X(t,Z)\big) + PY(K_Y(t,Z), \bar{L} - L_X(t,Z), Z) + r\big(\bar{K} - K_X(t,Z) - K_Y(t,Z)\big) \tag{21}$$

The direction of change of the welfare maximizing cap is determined by sign of the cross derivative of the welfare function:

$$sgn\left[\frac{dZ^*}{dt}\right] = sgn\left[\frac{\partial^2 W}{\partial Z \partial t}\right] = sgn\left[\frac{\partial^2 U^*}{\partial I^2}\frac{\partial I}{\partial t}\frac{\partial I}{\partial Z} + \frac{\partial U^*}{\partial I}\frac{\partial^2 I}{\partial Z \partial t}\right] \tag{22}$$

The direct effect of increasing the subsidy on income is

$$\frac{\partial I}{\partial t} = \left[P\frac{\partial Y}{\partial K_Y} - r\right]\frac{\partial K_Y}{\partial t} = \left[\frac{r}{1-t} - r\right]\frac{\partial K_Y}{\partial t} = \frac{tr}{1-t}\frac{\partial K_Y}{\partial t} \tag{23}$$

as all other terms are zero. This suggests that the income effect grows with an increasing tax rate t, while it vanishes at $t = 0$.

As in the case of the clean capital subsidy, the tax on dirty capital gives rise to a structural effect, a distortion effect, and an income effect. The structural effect works in favor of a lower emissions cap since the shrinking dirty sector reduces income losses from tightening the cap. At the same time, the distortion effect calls for a laxer policy, hence a higher cap, because addtional capital income can be accrued by exploiting the difference between the marginal product of capital in the dirty sector and the interest rate. Finally, the reduction in income due to the tax on dirty capital may increase the marginal utility of consumption which raises the marginal cost of tightening the emissions cap and therefore works so as to increase the cap. In the absence of that income effect, the emissions cap increases if and only if the structural effect is dominated by the distortion effect:

Proposition 6. *At constant marginal utility of income, marginally raising the sustainable finance tax rate* t *induces a decline of the most preferred emissions cap* Z^* *if and only if the distortion effect works stronger than the structural effect, that is, if and only if* $-\frac{\partial\theta(Z^*)}{\partial t} < -\frac{\partial\left[\frac{tr}{(1+t)}\frac{\partial K_Y}{\partial Z}(Z^*)\right]}{\partial t}$.

Proof. See Appendix G.

While Proposition 6 looks symmetric to Proposition 3 in our analysis of the clean capital subsidy, the evaluation differs when using the same example as in Appendix D. Keeping that specification with $X = A_X(K_X)^\alpha(L_X)^\beta$ and $Y = A_Y(K_Y)^\gamma(L_Y)^\delta Z^\varepsilon$, the distortion effect always works stronger than the structural effect, see Appendix H . The marginal effects offset each other only at $t = 0$. Intuitively, placing the distortion in the dirty sector through the tax on dirty capital weakens the structural effect by driving away capital from that sector relative to the approach of subsidizing clean capital.

Comparing the outcomes under the two implementation strategies of sustainable finance, it seems more difficult to achieve political support for taxing dirty capital than for subsidizing clean capital since gross wages do not rise here. At the same time, the budget surplus available for redistribution through lump-sum transfers shrinks. In fact, existing plans of sustainable finance policies generally suggest using subsidies to clean investment.

6. Sequential game: a positive theory of sustainable finance

Up to this point, a rationalization of the sustainable finance policy is missing. As any emissions cap can be implemented without adding a sustainable finance policy, the analysis suggests income losses without providing any environmental benefits. Hence, leaving aside the difficult argument that workers may achieve income gains through redistribution, and successful attempts to deceive uninformed voters by greenwashing communication, we are unable to explain the increasing popularity of sustainable finance policies in a rational agents framework.

The current section fills this gap by considering a sequential time structure acknowledging that physical capital investment is not easily reversible. The sequential time structure allows to rationalize the sustainable finance policy. Let there be two parties – green (indexed by gr) and conservative (indexed by $cons$) - that are differentiated with respect to their propensities to pay for reducing carbon emissions. The green party in power determines the clean capital subsidy σ. Afterwards, capital is invested, which is irreversible. Next, election takes place, where for simplicity the outcome is exogenous. With probability q, the green party stays in power; with probability $1 - q$, the conservative party takes over. The new government chooses the emissions cap. Finally, internal migration of labor will equalize wages. At each stage, players entertain rational expectations. For simplicity, the price of the dirty good is fixed at P throughout.

Knowing how capital is invested and how political variables are set, workers' mobility ensures equality of wages, $w = F_L = PG_L$, where the level generally depends on which party is in power.

The new government sets the emissions cap Z at given sustainable finance subsidy rate and given capital stocks. Foreseeing the resulting internal migration, the government of party $a \in \{gr, cons\}$ maximizes its welfare function $U^a(I(Z), Z, b^a)$ where I is disposable income available for consumption and $b^{gr} > b^{cons} > 0$ is a parameter indicating the party-specific emissions aversion. Let $U_1^a > 0, U_2^a < 0$, and $U_3^a < 0$ with cross derivatives $U_{12}^a = U_{13}^a = 0 > U_{23}^a$, where subscripts refer to derivatives with respect to the ith argument.

The chosen emissions cap satisfies the first-order condition

$$U_2^a + U_1^a \frac{\partial I}{\partial Z} = 0 \tag{24}$$

In the optimum, we have

$$-\frac{U_2^a}{U_1^a} = \frac{\partial I}{\partial Z} \tag{25}$$

stating that the marginal propensity to pay for an emissions reduction, $-\frac{U_2^a}{U_1^a}$, equals the related marginal cost $\frac{\partial I}{\partial Z}$. We impose $U_1^a \frac{\partial^2 I}{\partial Z^2} + U_{22}^a < 0$ to ensure uniqueness of the party-specific optimal emissions cap. With this specification, it can be shown that a green government pursues a stricter environmental policy, where Proposition 7 collects further comparative static results.

Proposition 7. *In the sequential game at a given date, a green government in power chooses a stricter policy, hence a lower cap Z, than a conservative government. Accordingly, the wage rate and national income will be higher with a conservative government in power. The return to capital in the clean sector will be higher with a green government in power, and the return to capital in the dirty sector will be higher with a conservative party in power. With given government, a higher capital stock in the clean sector* K_X *yields a decreasing emissions cap, while a higher capital stock in the dirty sector* K_Y *leads to a laxer policy, thus a higher cap.*

Proof. See Appendix I.

The proposition is easily understood. At given capital stocks, a green government chooses a lower emissions cap due to its higher propensity to pay for an emissions reduction. The direct impact of the stricter policy is a reduction of the marginal productivity of labor in the dirty sector, inducing migration of labor from the dirty to the clean sector, resulting in a lower national wage rate. With fixed output prices, the lower capital-labor ratio in the clean sector is associated with a higher return to capital in that sector. At the same time, both the lower emissions cap and the lower labor supply in the dirty sector contribute to a lower return to capital in the dirty sector.

A higher capital stock in the dirty sector increases the return to increasing the cap, resulting in a higher cap. By contrast, a larger clean sector induces a reallocation of labor toward that sector, reducing the marginal productivity of emissions, resulting in a lower cap.

Turning to the investment decisions, risk-neutral forward-looking investors ensure that expected returns are equal to the world market interest rate:

$$(1+\sigma)\left[q\left[F_K\left(K_x(\sigma), L_X\big(\sigma, Z^{gr}(\sigma)\big)\right)\right.\right. \tag{26}$$
$$\left.\left. +(1-q)F_K\left(K_x(\sigma), L_X\big(\sigma, Z^{cons}(\sigma)\big)\right)\right]\right] - r = 0$$
$$P\big[qG_K\big(K_Y(\sigma), L - L_X\big(\sigma, Z^{gr}(\sigma)\big), Z^{gr}\big) \tag{27}$$
$$+(1-q)G_K\left(K_Y(\sigma), L - L_X\big(\sigma, Z^{cons}(\sigma)\big), Z^{cons}(\sigma)\right)\big] - r = 0$$

where $Z^{gr}(\sigma) \equiv Z^{gr}\big(K_X(\sigma), K_Y(\sigma)\big)$ and $Z^{cons}(\sigma) \equiv Z^{cons}\big(K_X(\sigma), K_Y(\sigma)\big)$. Accordingly $L_X\big(\sigma, Z^{gr}(\sigma)\big) \equiv L_X\left(K_X(\sigma), K_Y(\sigma), Z^{gr}\big(K_X(\sigma), K_Y(\sigma)\big)\right)$.

Uniqueness of investment $K_X(\sigma), K_Y(\sigma)$ requires that additional capital, taking all repercussions into account, reduces the respective marginal product. Hence, $F_{KK} + F_{KL}\left[\frac{\partial L_X}{\partial K_X} + \frac{\partial L_X}{\partial Z^a}\frac{\partial Z^a}{\partial K_X}\right] < 0$ and $G_{KK} + G_{KZ} - G_{KL}\left[\frac{\partial L_X}{\partial K_Y} + \frac{\partial L_X}{\partial Z^a}\frac{\partial Z^a}{\partial K_Y}\right] < 0$. With these mild assumptions, Proposition 8 will demonstrate that increasing the sustainable finance subsidy raises investment in the clean sector and reduces investment in the dirty sector.

Completing the backward induction argument, the first step consists in the green government choosing the sustainable finance subsidy, maximizing expected welfare using its own preferences:

$$EW^{gr} = qU^{gr}\big(I^{gr}\big(\sigma, Z^{gr}(\sigma)\big), Z^{gr}(\sigma), b^{gr}\big) \tag{28}$$
$$+(1-q)U^{gr}\big(I^{cons}\big(\sigma, Z^{cons}(\sigma)\big), Z^{cons}(\sigma), b^{gr}\big)$$

When setting the subsidy, the government takes into account both direct effects on income and those that work through adaptation of the emissions cap. According to Proposition 7, we will always have $Z^{cons} > Z^{gr}$ and $I^{cons} > I^{gr}$.

Proposition 8 summarizes the impacts on the capital stock and on income. Moreover it demonstrates that it is worthwhile for green governments to engage in a sustainable finance policy.

Proposition 8. *In the sequential game, introducing a sustainable finance subsidy σ induces an increase of the capital stock in the clean sector K_X and a decline in the capital stock of the dirty sector K_Y. Expected income changes according to $\frac{\partial[qI^{gr}+(1-q)I^{cons}]}{\partial\sigma} = -\frac{\sigma r}{1+\sigma}\frac{\partial K_X}{\partial\sigma}$. A forward-looking green government in power can raise its expected welfare by introducing a small sustainable finance subsidy rate $\sigma > 0$.*

Proof. See Appendix J.

The sustainable finance subsidy distorts capital investment. While capital flows into the subsidized sector, the expectation of both outmigration of labor from the dirty sector and a stricter environmental policy reduces expected marginal productivity of capital in the dirty sector, thus cutting investment. As in the basic model, the direct effect – disregarding consequences for setting the emissions cap - of the subsidy on income is negative due to distorting the allocation of capital. However, this negative impact is negligible when introducing an infinitesimal subsidy.

Given the sequential time structure, the interesting aspect of the subsidy lies in affecting the choice of the emissions cap, which will be reduced. While that impact is neutral from a green vantage point if the green party remains in power, the cut undertaken after a possible switch to a conservative government is perceived as beneficial. The reason lies in a higher propensity to pay for the emission cut of the conservative government. At the same time, losses in expected income occur due to the distortion of the allocation of capital, which deteriorates welfare irrespective of political preferences. For a sufficiently small subsidy σ the expected income loss is dominated by the adaptation effect.

7. Concluding discussion

Our analysis has shown that that the sustainable finance policy may work against its purpose to reduce emissions, achieving the opposite. First, though the clean sector grows and the dirty sector shrinks as expected at fixed domestic emissions cap, dirty production abroad increases so as to meet worldwide demand again. The latter will be associated with higher emissions provided that an emissions cap regulation is not employed in the entire rest of the world. Second, it may easily happen that environmental policy is loosened and not tightened upon the implementation of the sustainable finance policy. While the structural effect of a smaller dirty sector with a smaller productivity of carbon indeed calls for a stricter policy, the decline in income and the income losses arising from the distortion of capital work toward relaxing the cap constraint.

A typical policy trying to avoid carbon leakages through imports consists in border adjustments. If imported units of the dirty good face a lower emission price abroad, they may be subjected to a border adjustment tax set so as to avoid discrimination of home production of the dirty good. This would work like an import tariff on the dirty good. Presumably such a policy reduces national income further while all other effects are mitigated, but not neutralized.

When considering a sequential time structure, noting that physical capital investment is not easily reversible, we arrive at different conclusions. If investment is locked, income gains due to reallocation of capital following a laxer emissions cap with distorted allocation of capital do not occur, inducing the government to pursue a more ambitious environmental policy. Employing a sustainable finance subsidy of any given size will induce a stricter emissions cap since the distortion effect has been eliminated. While the ambition to align sustainable finance with a tougher emissions regulation in the dirty sector works out here, national welfare will be lower. Foreseeing the income losses from the distorted capial allocation, the government would always fare better by abstaining from sustainable finance subsidies or taxes.

However, the sequential time structure enables us to rationalize the sustainable finance policy without relying on the difficult argument that workers may achieve income gains through redistribution. A green party in power is willing to sacrifice income through implementing a sustainable finance subsidy in order to induce a stricter environmental policy after a possible change in office. Such a behavior makes sense from the point of view of the green party if the welfare gain in case of success of the conservatives in the election more than compensates the welfare loss should the greens stay in power. An objection against this line of reasoning is that income losses can be avoided by committing to future emission caps through international agreements –using the climate conferences framework or related international institutions- instead of introducing a price distortion by implementing sustainable finance. The case for choosing sustainable finance in view of this less costly alternative could lie in the feature that a new government finds it easier to abolish the climate conference commitment than the national sustainable finance subsidy.

Allowing for endogenous output prices, say by considering a large open economy scenario, sustainable policy tends to increase the price of the dirty good following changes in worldwide excess demand upon domestic supply-side reactions in sector sizes. Such a reaction tends to mitigate the impacts sketched in our analysis while not changing their signs.

Regarding innovation incentives, if anything, the sustainable finance policy discourages private investment in filtering technologies. This prediction could only look different if a discrete jump can achieve to avoid the discrimination of investment, so that the dirty sector is no longer classified as such and participates in the subsidy.

In sum, it is likely that the well-intended policy backfires in terms of the environmental policy objective. It harms citizens in the country that implements sustainable finance in terms of income and consumption, while the process of global warming by greenhouse gas emissions is accelerated instead of decelerated.

Appendix

A. Proof of Proposition 1

Since F is linearly homogenous, we have $F(K_X, L_X) = L_X F\left(\frac{K_X}{L_X}, 1\right) = L_X f(k_x)$ with $k_x \equiv \frac{K_X}{L_X}$. Accordingly, $F_K(K_X, L_X) = f'(k_x)$. Demand for capital in sector X is determined by $(1+\sigma)f'(k_x) = r$. Since $f'' < 0$, we obtain $\frac{\partial k_x}{\partial \sigma} > 0$. Moreover, as the wage satisfies $w = F_L(K_X, L_X) = f(k_x) - k_x f'(k_x)$, we have $\frac{\partial w}{\partial \sigma} = \frac{\partial w}{\partial k_x}\frac{\partial k_x}{\partial \sigma} > 0$.
Regarding the dirty sector, marginal productivity of capital stays constant with fixed r and P :

$$r = P\frac{\partial G}{\partial K} \qquad (A1)$$

Since $G_{KL} > 0$ and Z is unchanged, a constant marginal productivity of capital requires that L_Y and K_Y will move in the same direction, $sgn\left[\frac{\partial K_Y}{\partial \sigma}\right] = sgn\left[\frac{\partial L_Y}{\partial \sigma}\right]$. It can be excluded that they stay constant since the wage in the dirty sector has to increase as

well. Finally, they cannot both increase since this would imply $\frac{\partial\theta}{\partial\sigma} > 0$, which together with $\frac{\partial w}{\partial\sigma} > 0$ constant r and constant P constitutes a contradiction. Hence, $sgn\left[\frac{\partial K_Y}{\partial\sigma}\right] = sgn\left[\frac{\partial L_Y}{\partial\sigma}\right] < 0$, which at fixed Z implies $\frac{\partial\theta}{\partial\sigma} < 0$ and $\frac{\partial Y}{\partial\sigma} < 0$.
Next, due to the labor endowment equation $\bar{L} = L_X + L_Y$, the reduction of labor supply in the dirty sector is associated with an increase of labor supply in the clean sector, $\frac{\partial L_X}{\partial\sigma} > 0$. Since from above the capital-labor ratio in the clean sector rises, capital demand in the clean sector has to increase as well, $\frac{\partial K_X}{\partial\sigma} > 0$. Higher inputs in the clean sector imply a higher output, $\frac{\partial X}{\partial\sigma} > 0$.

B. Proof of Proposition 2.

Consider the problem to maximize national income

$$I = X(K_X, L_X) + PY(K_Y, \bar{L} - L_X, Z) + r(\bar{K} - K_X - K_Y) \tag{A2}$$

with respect to K_X, K_Y and L_X. Since the objective function is strictly concave, the problem has a unique solution, being determined by the first-order conditions

$$\frac{\partial I}{\partial K_X} = \frac{\partial X}{\partial K_X} - r = 0 \tag{A3}$$

$$\frac{\partial I}{\partial K_Y} = P\frac{\partial Y}{\partial K_Y} - r = 0 \tag{A4}$$

$$\frac{\partial I}{\partial L_X} = \frac{\partial X}{\partial L_X} - P\frac{\partial X}{\partial L_Y} = 0 \tag{A5}$$

With $\sigma = 0$, all three conditions are satisfied. This does no longer hold with a positive subsidy, $\sigma > 0$. Thus $\frac{\partial I}{\partial\sigma} < 0$.

With homogenous preferences, we obtain $\frac{\partial C_X}{\partial I} > 0$ and $\frac{\partial C_Y}{\partial I} > 0$. Since $\frac{\partial X}{\partial \sigma} > 0$, $\frac{\partial I}{\partial \sigma} < 0$, and $\frac{\partial C_X}{\partial I} > 0$, excess supply of the clean good increases, $\frac{\partial [X - C_X]}{\partial \sigma} > 0$. Then the budget equation (11) necessitates a reduction in excess demand for the dirty good $\frac{\partial [P(Y - C_Y) + r(\bar{K} - K_X - K_Y)]}{\partial \sigma} < 0$. This in turn implies that foreign output of the dirty good increases. If a higher foreign output of the dirty good is associated with more emissions, worldwide emissions must increase.

Looking at the factor income side, disposable national income equals net capital income plus net wage income:

$$I = r\bar{K} + (1 - \tau)w(L_X + L_Y) \qquad (A6)$$

Since $r\bar{K}$ is a constant and the labor force $L_X + L_Y$ is given, $\frac{\partial I}{\partial \sigma} < 0$ requires $\frac{\partial (1-\tau)w}{\partial \sigma} < 0$.

C. Proof of Proposition 3

Differentiating (16) with respect to the emissions cap Z yields

$$\frac{\partial I}{\partial Z} = P\frac{\partial Y}{\partial Z} + \left[\frac{\partial X}{\partial L_X} - P\frac{\partial Y}{\partial (L - L_X)}\right]\frac{\partial L_X}{\partial Z} + \left[\frac{\partial X}{\partial K_X} - r\right]\frac{\partial K_X}{\partial Z} + \left[P\frac{\partial Y}{\partial K_Y} - r\right]\frac{\partial K_Y}{\partial Z} \qquad (A7)$$
$$= P\frac{\partial Y}{\partial Z} + \left[\frac{\partial X}{\partial K_X} - r\right]\frac{\partial K_X}{\partial Z} = \theta - \frac{\sigma r}{(1+\sigma)}\frac{\partial K_X}{\partial Z}$$

since $\frac{\partial X}{\partial L_X} = P\frac{\partial Y}{\partial (L - L_X)}$ due to (6), and $P\frac{\partial Y}{\partial Z} = \theta$ and $P\frac{\partial Y}{\partial K_Y} = r$ according to equations (5) and (14). Further note that (14) implies $\frac{\partial X}{\partial K_X} = r/(1+\sigma)$. Moreover, notice $\frac{\partial K_X}{\partial Z} < 0$, yielding $\frac{\partial I}{\partial Z} > 0$. Finally, $\theta(\sigma = 0) > \theta(\sigma > 0)$. Since $\frac{\partial I}{\partial Z}(\sigma = 0) = P\frac{\partial Y}{\partial Z}(\sigma = 0) >$

$\frac{\partial I}{\partial Z}(\sigma > 0)$. As $V(Z)$ is convex in Z, the first-order condition $\frac{dW}{dZ}(Z^*) = 0$ implies a lower level of Z^* at marginally higher σ if and only if $\frac{\partial^2 I}{\partial Z \partial \sigma} > 0$, or equivalently,

$-\frac{\partial \theta(Z^*)}{\partial \sigma} < -\frac{\partial\left[\frac{\sigma r}{(1+\sigma)}\frac{\partial K_X}{\partial Z}(Z^*)\right]}{\partial \sigma}$.

D. Example with subsidy

Consider a Cobb-Douglas specification

$$X = A_X (K_X)^\alpha (L_X)^\beta \qquad (A8)$$

for the clean sector, with $\alpha = 1 - \beta$. Profit maximization yields

$$\alpha \frac{X}{K_X}(1+\sigma) - r = 0 \qquad (A9)$$

$$\beta \frac{X}{L_X} - w = 0 \qquad (A10)$$

Inserting these results into the production function, we obtain

$$X = A_X \left(\frac{\alpha(1+\sigma)}{r}\right)^\alpha \left(\frac{\beta}{w}\right)^\beta X^{(\alpha+\beta)} \qquad (A11)$$

Thus

$$X^{(1-\alpha-\beta)} = A_X \left(\frac{\alpha(1+\sigma)}{r}\right)^\alpha \left(\frac{\beta}{w}\right)^\beta \qquad (A12)$$

Since $\alpha + \beta = 1$, this can be solved to determine the wage $w(\sigma)$ with $\frac{\partial w}{\partial \sigma} > 0$. Isolating w yields

$$w = \beta {A_X}^{1/\beta} \left(\frac{\alpha(1+\sigma)}{r}\right)^{\alpha/\beta} \qquad (A13)$$

where

$$\frac{\partial w}{\partial \sigma} = \beta {A_X}^{1/\beta} \left(\frac{\alpha(1+\sigma)}{r}\right)^{(\frac{\alpha}{\beta}-1)} \frac{\alpha}{r} = \beta {A_X}^{1/\beta} \left(\frac{\alpha}{r}\right)^{\alpha/\beta} (1+\sigma)^{(\frac{\alpha}{\beta}-1)} = \frac{w}{1+\sigma} \qquad (A14)$$

Notice that the wage does not depend on the emissions cap Z. From the two first-order conditions,

$$\frac{w}{r} = \frac{\beta}{\alpha(1+\sigma)} \frac{K_X}{L_X} \qquad (A15)$$

determining the capital-labor ratio in the clean sector. Solving for that capital-labor ratio yields

$$\frac{K_X}{L_X} = \frac{\alpha(1+\sigma)}{r\beta} w \qquad (A16)$$

Thus

$$\frac{\partial \frac{K_X}{L_X}}{\partial \sigma} = \frac{\alpha}{r\beta}\left[w + (1+\sigma)\frac{\partial w}{\partial \sigma}\right] = \frac{2\alpha w}{r\beta} \qquad (A17)$$

Let the production function of the dirty sector be

$$Y = A_Y (K_Y)^{\gamma} (L_Y)^{\delta} Z^{\varepsilon} \qquad (A18)$$

with $\gamma + \delta + \varepsilon = 1$. Profit maximization in the dirty sector yields

$$\gamma \frac{PY}{K_Y} - r = 0 \qquad (A19)$$

$$\delta \frac{PY}{L_Y} - w = 0 \qquad (A20)$$

$$\varepsilon \frac{PY}{Z} - \theta = 0 \qquad (A21)$$

Inserting these results into the production function, we obtain

$$Y = A_Y \left(\frac{\gamma}{r}\right)^{\gamma} \left(\frac{\delta}{w}\right)^{\delta} \left(\frac{\varepsilon}{\theta}\right)^{\varepsilon} (PY)^{(\gamma+\delta+\varepsilon)} \qquad (A22)$$

Since $\gamma + \delta + \varepsilon = 1$, this equation determines the certificate price θ as function of the wage rate w. Recalling that w is independent of the emissions cap Z, this applies to the allowance price θ as well. Solving for the allowance price yields

$$\theta = \varepsilon(PA_Y)^{1/\varepsilon}\left(\frac{\gamma}{r}\right)^{\gamma/\varepsilon}(\delta)^{\delta/\varepsilon}(w)^{-\delta/\varepsilon} \qquad (A23)$$

Differentiating with respect to σ, we can derive the structural effect:

$$\begin{aligned}\frac{\partial\theta}{\partial\sigma} &= -\delta(PA_Y)^{1/\varepsilon}\left(\frac{\gamma}{r}\right)^{\gamma/\varepsilon}(\delta)^{\delta/\varepsilon}(w)^{-\left(\frac{\delta}{\varepsilon}+1\right)}\frac{\partial w}{\partial\sigma} \\ &= -\delta(PA_Y)^{1/\varepsilon}\left(\frac{\gamma}{r}\right)^{\gamma/\varepsilon}(\delta)^{\delta/\varepsilon}(w)^{-\left(\frac{\delta}{\varepsilon}+1\right)}\frac{w}{1+\sigma} \\ &= -\frac{\delta}{\varepsilon(1+\sigma)}\theta\end{aligned} \qquad (A24)$$

From the first two first-order conditions (A19) and (A20), we obtain

$$\frac{K_Y}{L_Y} = \frac{\gamma w}{\delta r} \qquad (A25)$$

showing the capital-labor ratio of the dirty sector $\frac{K_Y}{L_Y}$ as proportional to the wage-interest ratio. Taking the derivative with respect to the subsidy rate yields

$$\frac{\partial\frac{K_Y}{L_Y}}{\partial\sigma} = \frac{\gamma}{r\delta}\frac{\partial w}{\partial\sigma} = \frac{\gamma w}{r\delta(1+\sigma)} \qquad (A26)$$

The third first-order condition (A21) determines output of the dirty sector Y, increasing with Z at given subsidy σ as follows: $\frac{\partial Y}{\partial Z} = \frac{\theta}{\varepsilon P}$. At given output Y, capital input is

$$K_Y = \frac{\gamma PY}{r} \qquad (A27)$$

If the wage w and dirty sector output Y are determined, labor input in the dirty sector follows from

$$L_Y = \frac{\delta PY}{w} \qquad (A28)$$

Thus, $\frac{\partial L_Y}{\partial Z} = \frac{\delta P}{w}\frac{\partial Y}{\partial Z} = \frac{\delta\theta}{\varepsilon w}$. Using the labor endowment equation $L_X = \bar{L} - L_Y$, this implies $\frac{\partial L_X}{\partial Z} = -\frac{\partial L_Y}{\partial Z} = -\frac{\delta\theta}{\varepsilon w}$. Rearranging (A15), it transpires that

$\frac{\partial K_X}{\partial Z} = \frac{w}{r}\frac{\alpha(1+\sigma)}{\beta}\frac{\partial L_X}{\partial Z} = -\frac{w}{r}\frac{\alpha(1+\sigma)}{\beta}\frac{\delta\theta}{\varepsilon w} = -\frac{\alpha(1+\sigma)\delta\theta}{r\beta\varepsilon}$. The size of the distortion effect is determined as follows: The income gain from reducing the distortion by marginally higher emissions cap is

$$-\frac{\sigma r}{(1+\sigma)}\frac{\partial K_x}{\partial Z}(Z^*) = \frac{\alpha\sigma\delta\theta}{\beta\varepsilon} \quad (A29)$$

Following the proof of Proposition 3, the total income gain from a higher emissions cap is

$$\frac{\partial I}{\partial Z} = \theta - \frac{\sigma r}{(1+\sigma)}\frac{\partial K_X}{\partial Z} = \theta\left[1 + \frac{\alpha\sigma\delta}{\beta\varepsilon}\right] \quad (A30)$$

Differentiating this with respect to the subsidy rate yields

$$\begin{aligned}\frac{\partial^2 I}{\partial Z\partial\sigma} &= \frac{\partial\theta(Z^*)}{\partial\sigma}\left[1 + \frac{\alpha\sigma\delta}{\beta\varepsilon}\right] + \theta(Z^*)\frac{\alpha\delta}{\beta\varepsilon} \quad (A31)\\ &= \frac{\partial\theta(Z^*)}{\partial\sigma} + \theta(Z^*)\frac{\alpha\delta}{\beta\varepsilon}\left[1 + \frac{\partial\theta(Z^*)}{\partial\sigma}\frac{\sigma}{\theta(Z^*)}\right]\\ &= -\frac{\delta}{\varepsilon(1+\sigma)}\theta + \theta(Z^*)\frac{\alpha\delta}{\beta\varepsilon}\left[1 - \frac{\delta\sigma}{\varepsilon(1+\sigma)}\right]\\ &= \theta\frac{\delta}{\varepsilon}\left[\frac{\alpha}{\beta}\left(1 - \frac{\delta\sigma}{\varepsilon(1+\sigma)}\right) - \frac{1}{1+\sigma}\right]\end{aligned}$$

This expression can take any sign, as can be demonstrated by considering introducing a marginal subsidy:

$$sgn\left[\frac{\partial^2 I}{\partial Z\partial\sigma}(\sigma = 0)\right] = sgn\left[\frac{\alpha}{\beta} - 1\right] = sgn[\alpha - \beta] \quad (A32)$$

E. Proof of Proposition 4

Following the Proof of Proposition 1, demand for capital in sector X is determined by $f'(k_x) = r$. Since $f'' < 0$, we obtain $\frac{\partial k_x}{\partial t} = 0$ and, as the wage satisfies $w = f(k_x) - k_x f'(k_x)$, also $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial k_x}\frac{\partial k_x}{\partial t} = 0$.

Regarding the dirty sector, marginal productivity of labor stays constant with constant w and P :

$$w = P\frac{\partial G}{\partial L} \quad (A33)$$

Since $G_{KL} > 0$ and Z is unchanged, a constant marginal productivity of capital requires that L_Y and K_Y will move in the same direction, $sgn\left[\frac{\partial K_Y}{\partial t}\right] = sgn\left[\frac{\partial L_Y}{\partial t}\right]$. It can be excluded that they stay constant since the gross marginal productivity of capital in the dirty sector has to increase. Finally, they cannot both increase since this would imply $\frac{\partial \theta}{\partial t} > 0$, which together with $\frac{\partial w}{\partial t} = 0$, constant r and constant P constitutes a contradiction. Hence, $sgn\left[\frac{\partial K_Y}{\partial t}\right] = sgn\left[\frac{\partial L_Y}{\partial t}\right] < 0$, which at fixed Z implies $\frac{\partial \theta}{\partial t} < 0$ and $\frac{\partial Y}{\partial t} < 0$.

Next, due to the labor endowment equation $\bar{L} = L_X + L_Y$, the reduction of labor supply in the dirty sector is associated with an increase of labor supply in the clean sector, $\frac{\partial L_X}{\partial t} > 0$. Since from above the capital-labor ratio in the clean sector stays constant, capital demand in the clean sector has to increase as well, $\frac{\partial K_X}{\partial t} > 0$. Higher inputs in the clean sector imply a higher output, $\frac{\partial X}{\partial t} > 0$.

F. Proof of Proposition 5

Following the proof of Proposition 2, the income maximizing allocation of capital and labor has to satisfy the first-order conditions (A3)-(A5). With $t = 0$, all three conditions are satisfied. This does no longer hold with a positive tax $t > 0$. Thus $\frac{\partial I}{\partial t} < 0$. With homogenous preferences, we obtain $\frac{\partial C_X}{\partial I} > 0$ and $\frac{\partial C_Y}{\partial I} > 0$. Since $\frac{\partial X}{\partial t} > 0$, $\frac{\partial I}{\partial t} < 0$, and $\frac{\partial C_X}{\partial I} > 0$, excess supply of the clean good increases, $\frac{\partial [X - C_X]}{\partial t} > 0$. Then the budget equation (11) necessitates a reduction in excess demand for the dirty good $\frac{\partial [P(Y - C_Y) + r(\bar{K} - K_X - K_Y)]}{\partial t} < 0$. This in turn implies that foreign output of the dirty good increases. If a higher foreign output of the dirty good is associated with more emissions, worldwide emissions must increase.

Looking at the factor income side, disposable national income equals net capital income plus net wage income:

$$I = r\overline{K} + (1-\tau)w(L_X + L_Y) \qquad (A34)$$

Since $r\overline{K}$ is a constant and the labor force $L_X + L_Y$ is given, $\frac{\partial I}{\partial t} < 0$ requires $\frac{\partial(1-\tau)w}{\partial t} < 0$.

G. Proof of Proposition 6

Differentiating (16) with respect to the emissions cap Z yields

$$\frac{\partial I}{\partial Z} = P\frac{\partial Y}{\partial Z} + \left[\frac{\partial X}{\partial L_X} - P\frac{\partial Y}{\partial(L-L_X)}\right]\frac{\partial L_X}{\partial Z} + \left[\frac{\partial X}{\partial K_X} - r\right]\frac{\partial K_X}{\partial Z} + \left[P\frac{\partial Y}{\partial K_Y} - r\right]\frac{\partial K_Y}{\partial Z} \qquad (A35)$$
$$= P\frac{\partial Y}{\partial Z} + \left[P\frac{\partial Y}{\partial K_Y} - r\right]\frac{\partial K_Y}{\partial Z} = \theta + \frac{tr}{(1+t)}\frac{\partial K_Y}{\partial Z}$$

since $\frac{\partial X}{\partial L_X} = P\frac{\partial Y}{\partial(L-L_X)}$ due to (6), and $P\frac{\partial Y}{\partial Z} = \theta$ and $P\frac{\partial Y}{\partial K_X} = r$ according to equations (5) and (14). Further note that (14) implies $P\frac{\partial Y}{\partial K_Y} = r/(1-t)$. Moreover, notice $\frac{\partial K_X}{\partial Z} < 0$, yielding $\frac{\partial I}{\partial Z} > 0$. Finally, $\theta(t=0) > \theta(t>0)$. Since $\frac{\partial I}{\partial Z}(t=0) = P\frac{\partial Y}{\partial Z}(t=0) > \frac{\partial I}{\partial Z}(t>0)$. As $V(Z)$ is convex in Z, the first-order condition $\frac{dW}{dZ}(Z^*) = 0$ implies a lower level of Z^* at marginally higher t if and only if $\frac{\partial^2 I}{\partial Z\partial\sigma} > 0$, or equivalently, $-\frac{\partial\theta(Z^*)}{\partial t} < -\frac{\partial\left[\frac{tr}{(1-\sigma)}\frac{\partial K_Y}{\partial Z}(Z^*)\right]}{\partial t}$.

H. Example with tax

Consider again the Cobb-Douglas specification

$$X = A_X (K_X)^{\alpha} (L_X)^{\beta} \qquad (A36)$$

for the clean sector, with $\alpha = 1 - \beta$. Profit maximization yields

$$\alpha \frac{X}{K_X} - r = 0 \qquad (A37)$$

$$\beta \frac{X}{L_X} - w = 0 \qquad (A38)$$

Inserting these results into the production function, we obtain

$$X = A_X \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\beta}{w}\right)^{\beta} X^{(\alpha+\beta)} \qquad (A39)$$

Thus

$$X^{(1-\alpha-\beta)} = A_X \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\beta}{w}\right)^{\beta} \qquad (A40)$$

Since $\alpha + \beta = 1$, this can be solved to determine the wage w:

$$w = \beta {A_X}^{1/\beta} \left(\frac{\alpha}{r}\right)^{\alpha/\beta} \qquad (A41)$$

Notice that the wage does not depend on the emissions cap Z. From the two first-order conditions,

$$\frac{w}{r} = \frac{\beta}{\alpha} \frac{K_X}{L_X} \qquad (A42)$$

determining the capital-labor ratio in the clean sector:

$$\frac{K_X}{L_X} = \frac{\alpha}{r\beta} w \qquad (A43)$$

Let the production function of the dirty sector be

$$Y = A_Y (K_Y)^{\gamma} (L_Y)^{\delta} Z^{\varepsilon} \qquad (A44)$$

with $\gamma + \delta + \varepsilon = 1$. Profit maximization in the dirty sector yields

$$\gamma \frac{PY}{K_Y} (1 - t) - r = 0 \qquad (A45)$$

$$\delta \frac{PY}{L_Y} - w = 0 \qquad (A46)$$

$$\varepsilon \frac{PY}{Z} - \theta = 0 \qquad (A47)$$

Inserting these results into the production function, we obtain

$$Y = A_Y \left(\frac{\gamma(1-t)}{r}\right)^{\gamma} \left(\frac{\delta}{w}\right)^{\delta} \left(\frac{\varepsilon}{\theta}\right)^{\varepsilon} (PY)^{(\gamma+\delta+\varepsilon)} \qquad (A48)$$

Since $\gamma + \delta + \varepsilon = 1$, this equation determines the certificate price θ as function of the wage rate w. Recalling that w is independent of the emissions cap Z, this applies to the allowance price θ as well. Solving for the allowance price yields

$$\theta = \varepsilon(PA_Y)^{1/\varepsilon} \left(\frac{\gamma(1-t)}{r}\right)^{\gamma/\varepsilon} (\delta/w)^{\delta/\varepsilon} \qquad (A49)$$

Differentiating with respect to t, we can derive the structural effect:

$$\begin{aligned} \frac{\partial\theta}{\partial t} &= -\gamma(PA_Y)^{1/\varepsilon} \left(\frac{\gamma}{r}\right)^{\gamma/\varepsilon} (\delta/w)^{\delta/\varepsilon}(1-t)^{\frac{\gamma}{\varepsilon}-1} \\ &= -\frac{\gamma}{\varepsilon(1-t)}\theta \end{aligned} \qquad (A50)$$

From the first two first-order conditions (A19) and (A20), we obtain

$$\frac{K_Y}{L_Y} = \frac{\gamma w(1-t)}{\delta r} \qquad (A51)$$

showing that the capital-labor ratio of the dirty sector $\frac{K_Y}{L_Y}$ decreases in the tax rate t:

$$\frac{\partial \frac{K_Y}{L_Y}}{\partial t} = -\frac{\gamma w}{r\delta} \qquad (A52)$$

The third first-order condition (A21) determines output of the dirty sector Y, increasing with Z at given subsidy θ as follows: $\frac{\partial Y}{\partial Z} = \frac{\theta}{\varepsilon P}$. At given output Y, capital input is

$$K_Y = \frac{\gamma PY(1-t)}{r} \tag{A53}$$

If the wage w and dirty sector output Y are determined, labor input in the dirty sector follows from

$$L_Y = \frac{\delta PY}{w} \tag{A54}$$

Thus, $\frac{\partial L_Y}{\partial Z} = \frac{\delta P}{w}\frac{\partial Y}{\partial Z} = \frac{\delta\theta}{\varepsilon w}$. Using the labor endowment equation $L_X = \bar{L} - L_Y$, this implies $\frac{\partial L_X}{\partial Z} = -\frac{\partial L_Y}{\partial Z} = -\frac{\delta\theta}{\varepsilon w}$. From (A43) we obtain $\frac{\partial K_X}{\partial Z} = \frac{w}{r}\frac{\alpha}{\beta}\frac{\partial L_X}{\partial Z} = -\frac{w}{r}\frac{\alpha}{\beta}\frac{\delta\theta}{\varepsilon w} = -\frac{\alpha\delta\theta}{r\beta\varepsilon}$. The size of the distortion effect is determined as follows: The income gain from reducing the distortion by a marginally higher emissions cap is

$$-\frac{tr}{(1-t)}\frac{\partial K_Y}{\partial Z}(Z^*) = \frac{t\gamma\theta}{\varepsilon} \tag{A55}$$

Following the proof of Proposition 6, the total income gain from a higher emissions cap is

$$\frac{\partial I}{\partial Z} = \theta - \frac{tr}{(1-t)}\frac{\partial K_Y}{\partial Z} = \theta\left[1 + \frac{t\gamma}{\varepsilon}\right] \tag{A56}$$

Differentiating this with respect to the tax rate yields

$$\begin{aligned}\frac{\partial^2 I}{\partial Z \partial \sigma} &= \frac{\partial \theta(Z^*)}{\partial t}\left[1 + \frac{t\gamma}{\varepsilon}\right] + \theta(Z^*)\frac{\gamma}{\varepsilon} \\ &= -\frac{\gamma}{\varepsilon(1-t)}\theta(Z^*)\left[1 + \frac{t\gamma}{\varepsilon}\right] + \theta(Z^*)\frac{\gamma}{\varepsilon} \\ &= \theta\frac{\gamma}{\varepsilon}\left[1 - \frac{1}{1-t} - \frac{t\gamma}{(1-t)\varepsilon}\right]\end{aligned} \tag{A57}$$

This expression is zero at $t = 0$ and negative for any $t > 0$.

I. Proof of Proposition 7

Perfect mobility of workers across sectors ensures

$$F_L(K_X(\sigma), L_X) = PG_L(K_Y(\sigma), Z, L - L_X) \tag{A58}$$

Hence, $\frac{\partial L_x}{\partial Z} < 0$, implying $\frac{\partial X}{\partial Z} < 0$ and $\frac{\partial Y}{\partial Z} > 0$. Labor supply in the clean sector increases with higher investment in the clean sector, $\frac{\partial L_X}{\partial K_X} > 0$, and decreases with higher investment in the dirty sector, $\frac{\partial L_X}{\partial K_Y} < 0$. Regarding marginal productivities, we obtain a higher wage upon a higher cap, $\frac{\partial w}{\partial Z} = \frac{\partial F_L(K_X(\sigma),L_X)}{\partial L_X}\frac{\partial L_x}{\partial Z} > 0$, a lower marginal productivity of capital in the clean sector $\frac{\partial F_K(K_X(\sigma),L_X)}{\partial L_X}\frac{\partial L_x}{\partial Z} < 0$, and a higher productivity of capital in the dirty sector, $P\frac{dG_K}{dZ} = P\left[\frac{\partial G_K(K_Y(\sigma),Z,L-L_X(Z))}{\partial Z} + \frac{\partial G_K(K_Y(\sigma),Z,L-L_X(Z))}{\partial L_X}\frac{\partial L_x}{\partial Z}\right] > 0$.

Next, the emissions cap is set so as to satisfy the first-order condition (24). Since $U_{12} = U_{13} = 0$, we obtain

$$\frac{dZ^*}{db} = -\frac{\frac{\partial^2 U}{\partial Z \partial b}}{\frac{\partial U}{\partial I}\frac{\partial^2 I}{\partial Z^2} + \frac{\partial^2 U}{\partial Z^2}} < 0 \tag{A59}$$

as $\frac{\partial U}{\partial I}\frac{\partial^2 I}{\partial Z^2} + \frac{\partial^2 U}{\partial Z^2} < 0$ at any candidate Z^* satisfying the first order condition is necessary to ensure uniqueness of Z^*. Accordingly, $Z^{gr} < Z^{cons}$ and $\frac{\partial L_x}{\partial Z} < 0$, yielding $w^{cons} = F_L^{cons} > F_L^{gr} = w^{gr}$. This in turn implies $F_K^{cons} < F_K^{gr}$. Moreover, as $Z^{gr} < Z^{cons}$ and $L - L_X^{cons} > L - L_X^{gr}$, positive cross derivatives $G_{KL} > 0$ and $G_{ZL} > 0$ yield $G_K^{cons} > G_K^{gr}$.

A higher capital stock in the clean sector reduces labor supply in the dirty sector at any given cap, therefore reduces the marginal productivity of increasing the cap, resulting in a lower optimal cap: $\frac{dZ^*}{dK_X} < 0$. Conversely, a higher capital stock in the dirty sector increases marginal productivity of emissions directly and via an expected higher labor supply in the dirty sector, thus $\frac{dZ^*}{dK_Y} > 0$.

J. Proof of Proposition 8

Differentiating the arbitrage conditions (26) and (27) yields

$$a_{1X}dK_x + a_{1Y}dK_Y + \frac{r}{(1+\sigma)^2}d\sigma = 0 \qquad (A60)$$

$$a_{2X}dK_x + a_{2Y}dK_Y = 0 \qquad (A61)$$

with

$$a_{1X} = q\left[F_{KK}^{gr} + F_{KL}^{gr}\left[\frac{\partial L_X^{gr}}{\partial K_X} + \frac{\partial L_X^{gr}}{\partial Z^{gr}}\frac{\partial Z^{gr}}{\partial K_X}\right]\right]$$
$$+(1-q)\left[F_{KK}^{cons} + F_{KL}^{cons}\left[\frac{\partial L_X^{cons}}{\partial K_X} + \frac{\partial L_X^{cons}}{\partial Z^{cons}}\frac{\partial Z^{cons}}{\partial K_X}\right]\right] < 0$$

$$a_{1Y} = qF_{KL}^{gr}\left[\frac{\partial L_X^{gr}}{\partial K_Y} + \frac{\partial L_X^{gr}}{\partial Z^{gr}}\frac{\partial Z^{gr}}{\partial K_Y}\right] + (1-q)F_{KL}^{cons}\left[\frac{\partial L_X^{cons}}{\partial K_Y} + \frac{\partial L_X^{cons}}{\partial Z^{cons}}\frac{\partial Z^{cons}}{\partial K_Y}\right] < 0$$

$$a_{2X} = Pq\left[-G_{KL}^{gr}\left[\frac{\partial L_X^{gr}}{\partial K_X} + \frac{\partial L_X^{gr}}{\partial Z^{gr}}\frac{\partial Z^{gr}}{\partial K_X}\right] + G_{KZ}^{gr}\frac{\partial Z^{gr}}{\partial K_X}\right]$$
$$+P(1-q)\left[-G_{KL}^{cons}\left[\frac{\partial L_X^{cons}}{\partial K_X} + \frac{\partial L_X^{cons}}{\partial Z^{cons}}\frac{\partial Z^{cons}}{\partial K_X}\right] + G_{KZ}^{cons}\frac{\partial Z^{cons}}{\partial K_X}\right] < 0$$

$$a_{2Y} = Pq\left[G_{KK}^{gr} - G_{KL}^{gr}\left[\frac{\partial L_X^{gr}}{\partial K_Y} + \frac{\partial L_X^{gr}}{\partial Z^{gr}}\frac{\partial Z^{gr}}{\partial K_Y}\right] + G_{KZ}^{gr}\frac{\partial Z^{gr}}{\partial K_Y}\right]$$
$$+P(1-q)\left[G_{KK}^{cons} - G_{KL}^{cons}\left[\frac{\partial L_X^{cons}}{\partial K_Y} + \frac{\partial L_X^{cons}}{\partial Z^{cons}}\frac{\partial Z^{cons}}{\partial K_Y}\right] + G_{KZ}^{cons}\frac{\partial Z^{cons}}{\partial K_Y}\right] < 0$$

Uniqueness of $K_X(\sigma), K_Y(\sigma)$ requires $a_{1X} < 0$, $a_{2Y} < 0$ and $a_{1X}a_{2Y} - a_{1Y}a_{2X} > 0$. According to the implicit function thorem we obtain:

$$\frac{\partial K_X}{\partial \sigma} = -\frac{\frac{r}{(1+\sigma)^2} a_{2Y}}{a_{1X}a_{2Y} - a_{1Y}a_{2X}} > 0, \qquad (A62)$$

$$\frac{\partial K_Y}{\partial \sigma} = \frac{\frac{r}{(1+\sigma)^2} a_{2X}}{a_{1X}a_{2Y} - a_{1Y}a_{2X}} < 0. \qquad (A63)$$

Expected income can be written as follows:

$$EI(\sigma, Z) = q\left[X\left(K_X(\sigma), L_X\big(\sigma, Z^{gr}(\sigma)\big)\right) + PY\left(K_Y(\sigma), L - L_X\big(\sigma, Z^{gr}(\sigma)\big), Z^{gr}(\sigma)\right)\right]$$
$$+(1-q)\left[X\left(K_X(\sigma), L_X\big(\sigma, Z^{cons}(\sigma)\big)\right) + PY\left(K_Y(\sigma), L - L_X\big(\sigma, Z^{cons}(\sigma)\big), Z^{cons}(\sigma)\right)\right]$$
$$+r\big(\bar{K} - K_X(\sigma) - K_Y(\sigma)\big) \qquad (A64)$$

Recalling the migration equilibrium and the investment arbitrage condition, we obtain

$$\frac{\partial EI}{\partial \sigma} = \left[q\frac{\partial X^{gr}}{\partial K_X} + (1-q)\frac{\partial X^{cons}}{\partial K_X} - r\right]\frac{\partial K_X}{\partial \sigma} = \left[\frac{r}{1+\sigma} - r\right]\frac{\partial K_X}{\partial \sigma} \qquad (A65)$$
$$= -\frac{\sigma r}{(1+r)}\frac{\partial K_X}{\partial \sigma}$$

Hence, $\frac{\partial EI}{\partial \sigma} = 0$ at $\sigma = 0$ and $\frac{\partial EI}{\partial \sigma} < 0$ if $\sigma > 0$ since $\frac{\partial K_X}{\partial \sigma} > 0$.

Differentiating welfare of the green government (28) with respect to the subsidy rate yields

$$\frac{\partial EW^{gr}}{\partial \sigma} = q\left[U_1^{gr}(I^{gr}, Z^{gr})\left[\frac{\partial I^{gr}}{\partial \sigma} + \frac{\partial I^{gr}}{\partial Z^{gr}}\frac{\partial Z^{gr}}{\partial \sigma}\right] + U_2^{gr}(I^{gr}, Z^{gr})\frac{\partial Z^{gr}}{\partial \sigma}\right] \qquad (A66)$$
$$+(1-q)\quad\left[U_1^{gr}(I^{cons}, Z^{cons})\left[\frac{\partial I^{cons}}{\partial \sigma} + \frac{\partial I^{cons}}{\partial Z^{cons}}\frac{\partial Z^{cons}}{\partial \sigma}\right] + U_2^{gr}(I^{cons}, Z^{cons})\frac{\partial Z^{cons}}{\partial \sigma}\right]$$

$$= qU_1^{gr}(I^{gr}, Z^{gr})\frac{\partial I^{gr}}{\partial \sigma} + (1-q)U_1^{gr}(I^{cons}, Z^{cons})\frac{\partial I^{cons}}{\partial \sigma}$$

$$+(1-q)\ \left[U_1^{gr}(I^{cons}, Z^{cons})\frac{\partial I^{cons}}{\partial Z^{cons}} + U_2^{gr}(I^{cons}, Z^{cons})\right]\frac{\partial Z^{cons}}{\partial \sigma}$$

since $U_1^{gr}(I^{gr}, Z^{gr})\frac{\partial I^{gr}}{\partial Z^{gr}} + U_2^{gr}(I^{gr}, Z^{gr}) = 0$. Accordingly, the first-order condition with respect to the emissions cap implies $\frac{\partial I^{cons}}{\partial Z^{cons}} = -\frac{U_2^{cons}(I^{cons}, Z^{cons})}{U_1^{cons}(I^{cons}, Z^{cons})}$. Inserting this result, we obtain

$$\frac{\partial EW^{gr}}{\partial \sigma} = qU_1^{gr}(I^{gr}, Z^{gr})\frac{\partial I^{gr}}{\partial \sigma} + (1-q)U_1^{gr}(I^{cons}, Z^{cons})\frac{\partial I^{cons}}{\partial \sigma} \qquad (A67)$$

$$+(1-q)\ U_1^{gr}(I^{cons}, Z^{cons})\left[\frac{U_2^{gr}(I^{cons}, Z^{cons})}{U_1^{gr}(I^{cons}, Z^{cons})} - \frac{U_2^{cons}(I^{cons}, Z^{cons})}{U_1^{cons}(I^{cons}, Z^{cons})}\right]\frac{\partial Z^{cons}}{\partial \sigma}$$

Evaluating this derivative at $\sigma = 0$ yields $\frac{\partial EW^{gr}}{\partial \sigma}(\sigma = 0) > 0$ because $\frac{\partial I^{gr}}{\partial \sigma}(\sigma = 0) = \frac{\partial I^{cons}}{\partial \sigma}(\sigma = 0) = 0$ and $\frac{U_2^{gr}(I^{cons}, Z^{cons})}{U_1^{gr}(I^{cons}, Z^{cons})} - \frac{U_2^{cons}(I^{cons}, Z^{cons})}{U_1^{cons}(I^{cons}, Z^{cons})} < 0$ while $\frac{\partial Z^{cons}}{\partial \sigma} < 0$.

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