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Public Policies and Long-Run Growth in a Model with Environmental Degradation

Abstract

We study how public policies affects an economy where production emits pollutants and investment in productive assets raises the economy’s overall productivity. We explore two hypotheses about how the accumulation of pollutants affects human well-being. Under the first one, there is no limit to the possibility for households to defend themselves against environmental degradation by increasing the use of manmade artifacts, while under the second one there is a threshold beyond which the effects of the accumulation of pollutants cannot be offset by devoting more output to this scope. Under both hypotheses, we compare the laissez-faire (LF) to the socially optimal (SO) path. Then, we check whether the latter can be decentralized by using the policy instruments available to the government. Under the first hypothesis, GDP and pollutants grow slower along the SO balanced growth path (BGP) than along the LF BGP, while people’s well-being is greater along the former. Therefore, green policies driving the economy along its OP tend to reduce GDP growth. Under the second hypothesis, LF may lead to a “climate catastrophe” by determining unbounded growth, which—without incentives to invest in green technology—drives the amount of pollutants beyond its maximum compatible with life on earth.

JEL-Codes: H230, O440, Q540, Q580.

Keywords: endogenous growth, green policies, global warming, externalities, human well being, climate catastrophe, defensive expenditures.

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1. INTRODUCTION

The main purpose of this paper is to highlight the economic growth/environmental quality trade offs implied by the green policies implemented or under discussion in the advanced economies. Such assessment is particularly relevant in the face of the rather widespread rhetoric that, in order to make environmental policies more popular and appealing to the public opinion, does not just emphasize their benefits in terms of environmental sustainability and people's long-term well being, but even their alleged positive impact on economic growth. Therefore, the value added of the present paper lies in providing a stylized—but sufficiently rich—unified framework for analyzing the medium- and long-term effects of green policies on economic growth and human well being, under two alternative hypotheses concerning the possibility to offset the consequences of environmental degradation through the increasing utilization of manmade products and artifacts.

In more detail, we present a dynamic general equilibrium setup allowing for endogenous growth in the presence of optimizing agents. In this economy, production generates negative externalities, i.e. causes the emission of pollutants that accumulate in the air, water, or soil, with negative effects on individual well being, and investment in productive assets (physical & human & intangible capital) generates positive externalities, i.e. raises the overall productivity of the economy, thus allowing for unbounded GDP growth. The private sector of the economy consists of firms and households: the former decide how many workers to employ, and how much to invest in capital and in emission abatement capacity ("green technologies"), while the latter decide on how to allocate their income between consumption and saving, and their time between work and leisure. The government decides on green taxes on emissions, and on subsidies (negative taxes) to private investment in capital and in green technologies. In addition, the government must satisfy its intertemporal budget but need not balance its budget every period (it can go into debt).

The two hypotheses mentioned above differ as to how the accumulation of pollutants affects human well being. Under the first hypothesis, there is no limit to the possibility for households to defend themselves against environmental degradation by using manmade products and artifacts, i.e.

by increasing their ”defensive expenditures”, whereas according to the second hypothesis there is a threshold beyond which the adverse effects of the accumulation of pollutants—such as greenhouse gases—cannot be offset by devoting increasing quantities of output to this scope, thus making possible a climate catastrophe. Notice that this hypothesis is consistent with the prevailing scientific consensus concerning the long-term effects of global warming.

Under both hypotheses, we compare the laissez-faire path of the economy, i.e. the equilibrium trajectory in the absence of any government intervention, to the socially optimal path by deriving the balanced growth path (BGP) of the economy. Then, we check whether the latter can be decentralized by using the three policy instruments available to the government (taxes on pollutant emissions, subsidies to investment in capital and subsidies to investment in green technologies).

Essential starting point for the recent literature on growth and the environment is Nordhaus's (1994) dynamic integrated model of climate and the economy (the DICE model). DICE combines the Ramsey growth model with equations governing emissions and climate change. However, differently than the model presented here, it does not account for endogenous growth, and it does not specify a market structure and generic climate policies, thus focusing only on the social planner's optimal plan. Subsequent integrated models of climate and the economy, such as Dietz and Stern (2015) and Bretschger and Karydas (2019), allow for endogenous growth by assuming that damage from a changing climate falls on capital accumulation, and not only on gross output at a particular point in time as in the DICE model.

Also in our paper endogenous growth is driven by knowledge spillovers from the accumulation of capital by firms, but—differently from the models mentioned above (and as in Uzawa, 2003, and Acemoglu et al., 2012) —pollution damages enter households' utility. Indeed, we model the idea that people's well being crucially depends on the possibility to combine manmade products bought on the market with commons—primarily, environmental and social assets—and that this is the main channel whereby environmental degradation can affect the evolution of the economy. Among others, indeed, individual choices that are fundamental for shaping the economy's long-term trajectory, such as those on how much and what to consume or on how to allocate one's total time, are deeply influenced by the possibility to have access to some basic commons and by their quality.

Under this respect, the present paper gives substance to the distinction between adaptation, i.e., "anticipating the adverse effects of climate change and taking appropriate action to prevent or minimise the damage they can cause, or taking advantage of opportunities that may arise", and mitigation, i.e., "making the impacts of climate change less severe by preventing or reducing the emission of greenhouse gases (GHG) into the atmosphere". In fact, one of the conclusions of the literature dealing with this distinction[1] is that adaptation tends to impose negative externalities on others (individuals, groups of people or countries), thus becoming 'maladaptation', defined as "action taken ostensibly to avoid or reduce vulnerability to climate change that impacts adversely on, or increases the vulnerability of other systems, sectors or social groups" (Barnett and O'Neill, 2010: 211). The model presented here, indeed, moves along the same lines as Bartolini and Bonatti (2002, 2003, 2008), which show how under laissez faire the possibility of using private goods and services as substitutes for environmental and social commons that are deteriorating because of the increase in production and consumption can become an engine of GDP growth, by creating a vicious circle of more production and more degradation.

However, differently than in Bartolini and Bonatti (2002, 2003, 2008), we recognize here that dealing with climate change one should also account for the possibility—deemed very likely by the overwhelming majority of scientists studying global warming—that there is a point beyond which the effects of environmental degradation cannot be compensated by an increasing use of manmade products and artifacts. The implications of the existence of such point are studied also by Acemoglu et al. (2012), who illustrate how—at least when the 'dirty' production technology and the 'clean' production technology are substitutes—a temporary subsidy to the development of the latter can avoid an environmental disaster. Our conclusion is more pessimistic: whenever there is a tipping point

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[1] For a synthetic review of this literature, see Schumacher (2019).

beyond which the effects of environmental degradation cannot be offset by increasing quantities of output, there are circumstances in which laissez faire leads to an environmental catastrophe, and it is socially optimal to have unbounded economic growth only in the special case in which the increase in abatement capacity (the progress of green technologies) can stabilize the stock of pollutants in an ever growing economy.

The rest of the paper is organized as follows: section 2 presents and discusses the model, section 3 analyzes the balanced growth path characterizing the economy when it is always possible to offset the negative effects of environmental degradation on human well being by devoting increasing quantities of output to this scope, section 4 analyzes the balanced growth path characterizing the economy if such compensation is not possible once that the concentration of pollutants goes beyond a certain threshold, section 5 concludes.

2. THE BASIC MODEL

We study a market economy where production generates negative externalities, i.e. causes the emission of pollutants that accumulate in the air, water, or soil, with negative effects on human well-being, and where investment in productive assets (physical & human & intangible capital) generates positive externalities, i.e. raises the overall productivity of the economy, thus allowing for unbounded growth. In this economy, the private sector consists of firms, that decide on the utilization of labor and invest both in capital and in improving their emission abatement technologies ("green technologies"), and households, that decide on how to allocate their income between consumption and saving, and their time between leisure and labor. A public authority ("the government") decides on green taxes on emissions, and on subsidies (negative taxes) to private investment in capital and in green technologies. The government must satisfy its intertemporal budget but need not balance its budget every period (it can go into debt).

Markets are perfectly competitive, time is discrete, and the time horizon is infinite. There is no source of random disturbances and agents' expectations are rational (i.e., they are consistent with the true processes followed by the relevant variables), thus implying perfect foresight.

2.1 Production

In the economy there is a large number (normalized to be one) of identical firms. In each period t, the representative firm produces the non-storable good Y_t (the numeraire of the system, whose price is set to be one) according to the following technology:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},\ 0<\alpha<1, \qquad (1)$$

where K_t and L_t are, respectively, the labor input and the capital stock used to produce Y_t, and A_t is a variable measuring the state of technology of the firm, i.e., its total factor productivity.

2.2 Total factor productivity

We assume that total factor productivity is a positive function of the capital installed in the economy:

$$A_t = K_t^{1-\alpha}.^2 \qquad (2)$$

This assumption combines the idea that some learning-by-doing takes place whenever a firm utilizes its capital stock and the idea that knowledge and productivity gains spill over across all firms (see Barro and Sala-i-Martin, 1995). Therefore, in accordance with Frankel (1962), it is supposed that although A_t is endogenous to the economy, each firm takes it as given, since a single firm's decisions have only a negligible impact on the aggregate stock of capital.[3]

2.3 Emissions

In each period t, the representative firm generates polluting emissions E_t that are proportional to its output:

$$E_t = e(Z_t)Y_t, \quad e'<0, \qquad (3)$$

where the factor of proportionality is a decreasing function of Z_t, that is the installed abatement capacity of the representative firm (its "abatement capital"). It is assumed that the functional form of $e(Z_t)$ is

$$e(Z_t) = Z_t^{-\varphi}, \quad \varphi > 0. \qquad (4)$$

The stock of pollutants, S_t, moves over time according to the following linear difference equation:[4]

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[2] Consistently with this formal set-up, one can interpret technological progress as labor augmenting.

[3] This amounts to say that technological progress is endogenous to the economy, although it is an unintended by-products of firms' capital investment rather than the result of purposive R&D efforts.

[4] Interpreting S_t as the CO_2 concentration in the atmosphere at time t, our equation simplifies the formal treatment of the carbon cycle contained in the RICE/DICE model, in which three CO_2 reservoirs are considered: the atmosphere, the biosphere and upper layers of the ocean, and the deep ocean (for a discussion see Hassler and Krusell, 2018).

$$S_{t+1} = E_t + (1 - \delta_S)S_t,\ 0 \leq \delta_S < 1, \qquad (5)$$

where δ_S is the rate of absorption, the fraction of pollutants that is absorbed by the environment (atmosphere, oceans, soil…) in each period. It should be stressed that the emissions of any single firm have only a negligible impact on the stock of pollutants.

2.4 Firms' profits

At time t, the net profit (cash flow) of the representative firm, π_t, is given by:

$$\pi_t = Y_t - W_t L_t - B_t(1+r_t) - \tau_t E_t, \qquad (6)$$

where W_t is the wage paid to each unit of labor, L_t are the units of labor employed by the representative firm, $B_t \geq 0$ are the bonds with maturity in period t issued in t-1 by the representative firm to finance its investment in that period, r_t is the one-period market rate of interest, and $\tau_t \geq 0$ is the tax per unit of emissions that the firm must pay to the government.

2.5 Firms' investment

The capital stock installed by the representative firm evolves according to

$$K_{t+1} = I_{kt} + (1-\delta_k)K_t,\ 0 \leq \delta_k \leq 1,\ \ K_0 \text{ given}, \qquad (7)$$

where I_{kt} is gross capital investment in period t and δ_k is a depreciation parameter.

The abatement capacity installed by the representative evolves according to

$$Z_{t+1} = I_{zt} + Z_t, \quad Z_0 \text{ given}, \qquad (8)$$

where I_{zt} is investment in green technologies in period t. Notice that the firm's abatement capacity does not depreciate: once a firm improves its abatement technology, the latter is not subject to downgrading (improving this technology is "building on the shoulders of the giants").

Firms finance their investment by going into debt:

$$I_{kt}(1-v_t) + I_{zt}(1-b_t) \leq B_{t+1},\ 0 \leq v_t < 1,\ 0 \leq b_t < 1, \qquad (9)$$

where v_t (b_t) is the fraction of the firms' investment expenditure in capital (green technology) that is subsidized by the government in period t.

2.6 Firms' profit maximization

In each t, firms decide on $\{L_{t+j}\}_{j=0}^{\infty}$, $\{I_{kt+j}\}_{j=0}^{\infty}$ and $\{I_{zt+j}\}_{j=0}^{\infty}$ subject to (7)-(9) in order to maximize their discounted sequence of net profits

$$\sum_{j=0}^{\infty} \frac{\pi_{t+j}}{\prod_{h=1}^{j}(1+r_{t+h})}, \qquad (10)$$

where $\prod_{h=1}^{0}(1+r_{t+h})=1$.

2.7 Dynastic families

For simplicity and without loss of generality, it is assumed that the large number (normalized to be one) of identical households is fixed, and that each of them takes account of the welfare and resources of their actual and prospective descendants. Hence, following Barro and Sala-i-Martin (1995), this intergenerational interaction is modeled by imagining that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals have finite lives, the model considers immortal extended families ("dynasties"). Again for simplicity and without loss of generality, it is assumed that all households—being the firms' owners—are entitled to receive an equal share of the firms' net profits.[5]

2.8 Households' utility

As in Acemoglu et al. (2012), we assume that environmental quality directly affects utility. In particular, the period utility function of the representative household, $U(C_t, S_t, L_t)$, is given by a

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[5] As in Barro and Sala-i-Martin (1995, p. 120), we assume that the firms' net cash flow is paid out as dividends to the shareholders.

weighted average of the utility that it draws from the consumption of the produced good, critically depending on the state of the environment, and the utility that it draws from leisure:

$$U(C_t, S_t, L_t)=\sigma\frac{[x(C_t,S_t)]^{1-\theta}}{1-\theta}+(1-\sigma)\frac{(N-L_t)^{1-\vartheta}}{1-\vartheta},\ 0<\sigma<1, x_c > 0, x_s < 0, x_{cs} < 0,\ \theta \geq 0, \vartheta \geq 0, N>0, \quad (11)$$

where C_t are the units of good Y_t consumed by the representative household and N is the time endowment of the representative household (hence, $N\text{-}L_t$ are the units of time that each household devotes to leisure). The function $x(C_t, S_t)$ can be interpreted as a household production function, which dictates the way whereby the consumer good and the environmental quality can combine for generating the services from which individuals draw utility. Notice that it is increasing in C_t and decreasing in the stock of pollutants S_t, and it is such that $U_{cs} < 0$ for $0 < \theta < 1$ and $U_{cs} > 0$ for $\theta > 1$. This allows both for the possibility that environmental degradation makes consumption less valuable to the households (whenever $U_{cs} < 0$), or alternatively for the possibility that it makes consumption more valuable to them (whenever $U_{cs} > 0$). The former case reflects situations where there is some complementarity between environmental quality and consumption (a fall in environmental quality would make people want to consume less), while the latter case reflects situations where there is some substitutability between them (a fall in environmental quality would make people want to consume more, which is typical of those situations where people react to environmental degradation by increasing their defensive expenditures).[6]

2.9 Households' production functions

We study the trajectory of the economy under two hypotheses about the functional form of the household production function:

$$x(C_t, S_t) = \frac{C_t}{S_t} \quad (12a)$$

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[6] As defined by Leipert, "*Defensive expenditures comprise those economic activities by which we defend ourselves against the unwanted side effects (negative external effects) of our aggregate production and consumption. They are understood as expenditures to cure, neutralize, eliminate, avoid, and anticipate burdens on and damage to the environment (and living conditions in general) caused by the economic process in industrial countries*." (Leipert and Pulselli, 2008, p.154).

and

$$x(C_t, S_t) = C_t\left(\frac{M}{S_t} - 1\right), \qquad M>0. \tag{12b}$$

The household production function given by (12a) is consistent with the hypothesis that the households can always preserve their well being in the face of environmental degradation by using increasing quantity of produced goods. In contrast, (12b) applies to the hypothesis that the accumulation of pollutants can reach a threshold M beyond which efforts to compensate or offset the damage they cause to people's well being by using more manmade products are vain.[7] It is apparent that (12b) captures what may happen according to most scientists as a consequence of global warming, namely that catastrophic consequences for human well-being are likely to occur if the accumulation of CO_2 in the earth atmosphere—and the consequent increase in average temperature—exceeds a certain critical level.[8]

2.10 Households' intertemporal problem

In each period, the households decide on how to allocate their income between consumption and saving (spent for buying corporate and government bonds), and their time between leisure and labor. Thus, the problem of the representative household amounts to deciding a contingency plan for C_t, L_t, B_{t+1} and G_{t+1} in order to maximize the discounted sequence of utilities

$$\sum_{j=0}^{\infty} \gamma^j U\,(C_{t+j}, S_{t+j}, L_{t+j}),\ 0 < \gamma < 1, \tag{13}$$

subject to

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[7] One could argue that, if we want to model the effects of global warming on human well being, it would be more appropriate to insert the earth's global average temperature in (12b) instead of the stock of pollutants S_t, that in this context can be approximated by the concentration of CO_2 in the atmosphere. However, an acceptable approximation to the relation linking the global average temperature to CO_2 is that the temperature increase over any time period is proportional to the accumulated emissions of CO_2 over the same period, with the proportionality factor that is independent of the length of the time period and of previous emissions (see Matthews et al., 2009). Thus, considering that the relationship between pollution stock (CO_2 concentration) and temperature is approximately linear, one may omit to introduce a separate variable for temperature (see Bretschger and Karydas, 2019).

[8] It goes without saying that any calibration of the model with real world data has to deal with the uncertainty surrounding all the parameters values; in particular, this applies to the tipping point M, whose existence is not recognized by all the scholars on climate change.

$B_{t+1}+G_{t+1}+C_t \leq (1+r_t)(B_t+G_t)+\pi_t+W_tL_t$, B_0 and G_0 given, (14)

where γ is a time-preference parameter and $G_t \geq 0$ are the bonds with maturity in period t issued in t-1 by the government.[9]

2.11 Government

The government takes into account the optimizing behavior of firms and households, and in each period it decides a contingency plan for τ_t, v_t and b_t in order to maximize (13) subject to its period budget constraint

$$(1+r_t)G_t+v_tI_{kt}+b_tI_{zt} \leq G_{t+1}+\tau_tE_t, \tag{15}$$

and to its intertemporal budget constraint (no-Ponzi condition)

$$G_t + \sum_{j=0}^{\infty} \frac{(v_{t+j}I_{kt+j}+b_{t+j}I_{zt+j})}{\prod_{h=1}^{j}(1+r_{t+h})} \leq \sum_{j=0}^{\infty} \frac{\tau_{t+j}E_{t+j}}{\prod_{h=1}^{j}(1+r_{t+h})} \tag{16}$$

where $\prod_{h=1}^{0}(1+r_{t+h})=1$.

2.12 Market equilibrium

Equilibrium in the good market implies

$$I_{kt}+I_{zt}+C_t=Y_t. \tag{17}$$

Equilibrium in the labor market implies

$$L_t^s = L_t^d, \tag{18}$$

where L_t^s are the units of labor supplied by the households and L_t^d are the units of labor demanded by the firms.

Equilibrium in the market for corporate bonds implies

$$B_t^s = B_t^d, \tag{19}$$

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[9] The households' budget constraint (14) implicitly assumes the existence of a non-arbitrage condition that equalizes the rate of return on the corporate bonds and the rate of return on the government bonds.

where B_t^s are the bonds issued by the firms and B_t^d are the corporate bonds demanded by the households.

Equilibrium in the market for government bonds implies

$$G_t^s = G_t^d, \tag{20}$$

where G_t^s are the bonds issued by the government and G_t^d are the government bonds demanded by the households.

3. THE ECONOMY'S BALANCED GROWTH PATH WHEN $x(C_t, S_t) = \frac{C_t}{S_t}$

3.1 The laissez-faire path

Assuming no government intervention amounts to set $\tau_t = v_t = b_t = 0\ \forall t$, $G_0 = 0$. In this case, firms have no incentive to reduce their emissions and invest in abatement technologies ($I_{zt} = 0\ \forall t$, entailing $Z_t = Z_0\ \forall t$), and also their investment in capital is sub-optimally low because they have no incentive to take into account the positive externalities that this investment activity generates for the entire economy. Under these circumstances, one can derive the "laissez-faire" equilibrium path of the economy,[10] which is governed by the following two difference equations in L_t and $s_t \equiv S_t/K_t$:

$$s_{t+1}(1 + g_{kt}) - Z_0^{-\varphi} L_t^{1-\alpha} - (1 - \delta_s)s_t = 0, \qquad Z_0 \text{ given,} \tag{21}$$

$$\frac{\gamma}{s_{t+1}}\left(\frac{s_{t+1}}{c_{t+1}}\right)^{\theta}\left(\alpha L_{t+1}^{1-\alpha} + 1 - \delta_k\right) - \frac{(1+g_{kt})}{s_t}\left(\frac{s_t}{c_t}\right)^{\theta} = 0, \tag{22}$$

where $g_{kt} \equiv \frac{K_{t+1}-K_t}{K_t} = L_{t+1}^{1-\alpha} - \delta_k - c(s_t, L_t)$ and $c_t \equiv C_t/K_t = c(s_t, L_t) = \left[\frac{s_t^{\theta-1}(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t^{\alpha}}\right]^{\frac{1}{\theta}}$.

Along a balanced growth path (BGP), $L_{t+1} = L_t = L^{LF}$ and $s_{t+1} = s_t = s^{LF}$, entailing $g_{kt+1} = g_{kt} = g_k^{LF} \gtreqless 0$ and $c_{t+1} = c_t = c^{LF}$. Moreover, along a BGP, $Z^{LF} = Z_0$, $g_Z^{LF} = 0$, $g_Y^{LF} = g_C^{LF} = g_S^{LF} = g_k^{LF}$, where $g_{zt} \equiv \frac{Z_{t+1}-Z_t}{Z_t}$, $g_{Yt} \equiv \frac{Y_{t+1}-Y_t}{Y_t}$, $g_{ct} \equiv \frac{C_{t+1}-C_t}{C_t}$ and $g_{st} \equiv \frac{S_{t+1}-S_t}{S_t}$.

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[10] All derivations are available on request from the authors.

Notice that along a BGP, i) the stock of pollutants, S_t, and the stock of productive assets, K_t, grow at the same rate, and ii) one may have unbounded growth ($g_Y^{LF} = g_C^{LF} = g_S^{LF} = g_K^{LF} > 0$). This allows us to state the following proposition:

Proposition 1. *If the household production function is given by (12a), that is, if it is always possible to offset the harm that environmental degradation causes on people's well being by an increased use of manmade products, GDP growth can go on forever under laissez faire in spite of the growing environmental damage due to production. In this case, i) laissez faire is consistent with unbounded growth, ii) environmental degradation has never catastrophic effects on human well being, and iii) human well being remains constant along the BGP.*[11]

3.2 The socially optimal path

Suppose that there is a central planner with full control over the allocation of all resources (people's time included) which maximizes (13). From the solution of the planner's problem (see the Appendix), one can derive the optimal path of the economy, which is governed by the following four difference equations in K_t, S_t, L_t and Z_t:

$$K_t(L_t^{1-\propto} + 1 - \delta_k) - K_{t+1} + Z_t - Z_{t+1} - C_t = 0, \qquad K_0 \text{ and } Z_0 \text{ given,} \tag{23}$$

$$S_{t+1} - \frac{K_t L_t^{1-\propto}}{Z_t^{\varphi}} - (1-\delta_s)S_t = 0, \qquad S_0 \text{ given,} \tag{24}$$

$$\frac{\gamma\sigma(1-\delta_k)}{S_{t+1}^{1-\theta}C_{t+1}^{\theta}} + \frac{\gamma(1-\sigma)L_{t+1}}{(1-\alpha)K_{t+1}(N-L_{t+1})^{\vartheta}} - \frac{\sigma}{S_t^{1-\theta}C_t^{\theta}} = 0, \tag{25}$$

$$\frac{\gamma\sigma C_{t+1}^{1-\theta}}{S_{t+1}^{2-\theta}} + \frac{\gamma\sigma(1-\delta_s)Z_{t+1}^{\varphi}}{S_{t+1}^{1-\theta}C_{t+1}^{\theta}} - \frac{\gamma(1-\sigma)(1-\delta_s)Z_{t+1}^{\varphi}L_{t+1}^{\alpha}}{(1-\alpha)K_{t+1}(N-L_{t+1})^{\vartheta}} - \frac{\sigma Z_t^{\varphi}}{S_t^{1-\theta}C_t^{\theta}} + \frac{(1-\sigma)Z_t^{\varphi}L_t^{\alpha}}{(1-\alpha)K_t(N-L_{t+1})^{\vartheta}} = 0, \tag{26}$$

where $C_t = C(K_t, S_t, L_t, Z_t) = \left[\frac{S_t^{\theta-1}(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t(\varphi Z_t^{-1}+K_t^{-1})}\left(\frac{\varphi K_t L_t^{1-\alpha}}{Z_t} + \delta_K\right)\right]^{\frac{1}{\theta}}$.

In the general case in which $\theta \neq 1$, i.e., in the case in which the preferences for the combination of consumer good and environmental quality from which households draw utility, $x(C_t, S_t)$, are **not**

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[11] Since, along the BGP, consumption and the stock of pollutants grow at the same rate, while leisure is constant.

logarithmic, a BGP is characterized by $K_{t+1} = K_t = K^{OP}$, $S_{t+1} = S_t = S^{OP}$, $L_{t+1} = L_t = L^{OP}$ and $Z_{t+1} = Z_t = Z^{OP}$, entailing $g_Y^{OP} = g_c^{OP} = g_s^{OP} = g_k^{OP} = g_z^{OP} = 0$. Hence, the following proposition holds:

Proposition 2. *If the household production function is given by (12a) and $\theta \neq 1$, a socially optimal path (i.e., the path along which a benevolent planner with full command of resource allocation would lead the economy) has a steady state where GDP growth—together with the accumulation of both capital and pollutants—ceases, differently than under laissez faire, where the economy can exhibit unbounded growth.*

The intuition behind Proposition 2 is that under laissez faire the economic agents have no incentive to invest in abatement capacity and tend to protect themselves (and their descendants) from environmental degradation by using more manmade products, thus feeding economic growth, while a benevolent planner invests in abatement capacity, thus stopping both environmental degradation and the need to produce more and work harder in order to compensate for this degradation.

In the special case in which $\theta=1$, i.e., in the case in which the preferences for $x(C_t, S_t)$, are logarithmic, the equations (23)-(26) governing the optimal path of the economy can be rewritten as a system of four difference equations in $L_t, z_t \equiv Z_t/K_t, g_{kt}$ and g_{st}:

$$L_t^{1-\propto} + 1 - \delta_k + z_t - (1 + g_{kt})(1 + z_{t+1}) - f(L_t, z_t) = 0, \quad z_0 \text{ given}, \tag{27}$$

$$(1 + g_{kt}) - \left[(1 + g_{st})\left(\frac{z_{t+1}}{z_t}\right)^{\varphi}\left(\frac{g_{st+1}+\delta_s}{g_{st}+\delta_s}\right)\left(\frac{L_t}{L_{t+1}}\right)^{1-\alpha}\right]^{\frac{1}{1-\varphi}} = 0, \tag{28}$$

$$\frac{\gamma\sigma(1-\delta_k)}{f(L_{t+1},z_{t+1})} + \frac{\gamma(1-\sigma)L_{t+1}}{(1-\alpha)(N-L_{t+1})^{\vartheta}} - \frac{\sigma(1+g_{kt})}{f(L_t,z_t)} = 0, \tag{29}$$

$$\gamma\sigma + \frac{\gamma\sigma(1-\delta_s)L_{t+1}^{1-\alpha}}{(g_{st+1}+\delta_s)f(L_{t+1},z_{t+1})} - \frac{\gamma(1-\sigma)(1-\delta_s)L_{t+1}}{(1-\alpha)(g_{st+1}+\delta_s)(N-L_{t+1})^{\vartheta}} - \frac{\sigma L_t^{1-\alpha}(1+g_{st})}{(g_{st}+\delta_s)f(L_t,z_t)} + \frac{(1-\sigma)L_t(1+g_{st})}{(1-\alpha)(g_{st}+\delta_s)(N-L_t)^{\vartheta}} = 0, \tag{30}$$

where $c_t \equiv C_t/K_t = f(L_t, z_t) = \frac{(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t(\varphi z_t^{-1}+1)}\left(\frac{\varphi L_t^{1-\alpha}}{z_t} + \delta_K\right)$.

In this case, along a BGP, $L_{t+1} = L_t = L^{OP}$, $z_{t+1} = z_t = z^{OP}$, $g_{kt+1} = g_{kt} = g_k^{OP} = \gtreqless 0$, $g_{st+1} = g_{st} = g_s^{OP} = \left(1 + g_k^{OP}\right)^{1-\varphi} - 1$ and $g_Y^{OP} = g_c^{OP} = g_k^{OP}$. Notice that $g_Y^{OP} = g_c^{OP} = g_k^{OP} > 0$ entails $g_s^{OP} < g_Y^{OP} = g_c^{OP} = g_k^{OP}$, thus allowing us to state Proposition 3:

Proposition 3. *If the household production function is given by (12a) and $\theta=1$ (implying that the marginal utility of consumption is not affected by environmental degradation), it can be socially optimal to have unbounded GDP growth, since investment in abatement capital can lead the stock of pollutants to grow at a rate permanently lower than the rate of GDP growth, thus allowing the households' well being to increase limitless along a BGP.*

This optimistic long-term conclusion associated to the case of logarithmic preferences is reinforced if $\varphi>1$, i.e., if the elasticity of emissions with respect to abatement capital is larger than one: whenever improvements in the firms' abatement capacity bring about strong reductions in emissions per unit of output, an optimal BGP can be characterized by both an ever increasing GDP and a declining stock of pollutants.

3.3 The "green-policy" path

Decentralizing the socially optimal path can in principle be achieved by a public authority ("the government") levying taxes on emissions and subsidizing investment. We define "green-policy" path the trajectory along which the economy moves whenever the government utilizes its policy instruments (green taxes on emissions and subsidies to private investment in capital and in green technologies). The "green-policy" path is governed by the following equations: (23), (24),

$$\frac{\gamma S_{t+1}^{\theta-1}}{C_{t+1}^{\theta}}\left[\frac{\alpha L_{t+1}^{1-\alpha}(1-\tau_{t+1}Z_{t+1}^{-\varphi})+(1-\delta_k)(1-v_{t+1})}{1-v_t}\right]-\frac{S_t^{\theta-1}}{C_t^{\theta}}=0, \quad S_0 \text{ and } Z_0 \text{ given,} \tag{31}$$

$$D_{t+1}+\tau_t Z_t^{-\varphi}K_t L_t^{1-\propto}-b_t(Z_{t+1}-Z_t)-v_t[K_{t+1}-(1-\delta_k)K_t]-$$

$$-D_t\left[\frac{\alpha L_t^{1-\alpha}(1-\tau_t Z_t^{-\varphi})+(1-\delta_k)(1-v_t)}{1-v_{t-1}}\right]=0, \qquad D_0 \text{ and } K_0 \text{ given,} \tag{32}$$

$$\left[\frac{\alpha L_t^{1-\alpha}(1-\tau_t Z_t^{-\varphi})+(1-\delta_k)(1-v_t)}{1-v_{t-1}}\right]-\left[\frac{\varphi\tau_t Z_t^{-\varphi-1}L_t^{1-\alpha}K_t+1-b_t}{1-b_{t-1}}\right]=0, \tag{33}$$

where $C_t=C(K_t,S_t,L_t,Z_t)=\left[\frac{(1-\alpha)\sigma K_t(N-L_t)^{\vartheta}(1-\tau_t Z_t^{-\varphi})}{S_t^{1-\theta}(1-\sigma)L_t^{\alpha}}\right]^{\frac{1}{\theta}}$, and the evolution of τ_t, v_t and b_t is determined by the government's policy rule.

The optimal policy rule, i.e., the policy rule driving the economy along a socially optimal path, is the following: i) $\tau_t^* = Z_t^{\varphi}[1 - L_t^{\alpha-1}(\varphi z_t^{-1} L_t^{1-\alpha} + \delta_k)(\varphi z_t^{-1} + 1)^{-1}]$ $\forall t$, ii) $v_t^*=1-\alpha$ $\forall t$ and iii) $b_t^*=0$ $\forall t$. The optimal policy rule is feasible if and only if it is consistent with $D_t/Y_t \rightarrow F$ as $t \rightarrow \infty$, where F is a finite constant (public debt must be sustainable).

The optimal rule corrects the negative externality associated to the firms' production activities by taxing the emission of pollutants at rate τ_t^*, and the positive externality generated by private investment in productive assets by subsidizing it at rate v_t^*. Once that both these market inefficiencies are corrected, private investment in green technologies is at its socially optimal level without the need of public subsidies supporting it ($b_t^*=0$). This differs from Acemoglu et al. (2012), where there is a sharp distinction between dirty and clean production technologies, and the optimal public policy discourages research aimed at improving the productivity of the former not only by levying a tax on emissions but also by subsidizing the latter.[12] In contrast, in our framework improvements in abatement technologies can make all productive activities cleaner, and optimal public policy encourages these improvements only by taxing emissions, while at the same time it subsidizes investment in productive assets.

Given the optimal policy rule, the five difference equations (23), (24), (31)-(33) in K_t, S_t, L_t, Z_t and D_t fully characterize the "green-policy" path of the economy, and Proposition 2 holds: in the general case where $\theta \neq 1$, differently than under laissez-faire, the adoption of the optimal policy rule prevents the economy from exhibiting unbounded GDP growth.

If the optimal policy rule is not feasible, one should consider other policy rules. A simple—but suboptimal—alternative policy rule amounts to keep constant the fraction of GDP paid to the government as emission tax, and the fractions of private investment in productive assets and in green technologies that are paid by the government: i) $\tau_t = hZ_t^{\varphi}$ $\forall t, 0 \leq h < 1$ (total emission taxes are a

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[12] In general, as pointed out by Golosov et al. (2014), it is far from clear that there should be a favorable treatment of green R&D in the presence of an optimal emission tax, which is justified in Acemoglu et al. (2012) by assuming a built-in path dependence that over time would lead to a disaster, motivating early efforts to switch alternatives.

fixed fraction h of GDP), ii) $v_t = v \ \forall t, 0 \leq v < 1$, and iii) $b_t = b \ \forall t, 0 \leq b < 1$. Again, policy parameters h, b and v must be such that $D_t/Y_t \rightarrow F$ as $t \rightarrow \infty$. Notice that also with the adoption of this suboptimal rule the five difference equations (23), (24), (31)-(33) in K_t, S_t, L_t, Z_t and D_t fully characterize the "green-policy" path of the economy, which does not exhibit unbounded growth in the general case where $\theta \neq 1$.

In the special case in which θ=1, equations (23), (24), (31)-(33) can be rewritten as (28),

$$L_t^{1-\propto} + 1 - \delta_k + z_t - (1 + g_{kt})(1 + z_{t+1}) - n\left(L_t, \tau_t Z_t^{-\varphi}\right) = 0, \quad z_0 \text{ and } Z_0 \text{ given,} \tag{34}$$

$$\frac{\gamma\sigma[\alpha L_{t+1}^{1-\alpha}(1-\tau_{t+1}Z_{t+1}^{-\varphi})+(1-\delta_k)(1-v_{t+1})]}{n(L_{t+1},\tau_{t+1}Z_{t+1}^{-\varphi})(1-v_t)} - \frac{\sigma(1+g_{kt})}{n(L_t,\tau_t Z_t^{-\varphi})} = 0, \tag{35}$$

$$(1 + g_{kt})d_{t+1} + \tau_t Z_t^{-\varphi} L_t^{1-\propto} - b_t[(1 + g_{kt})z_{t+1} - z_t] - v_t(g_{kt} + \delta_k) -$$

$$-d_t\left[\frac{\alpha L_t^{1-\alpha}(1-\tau_t Z_t^{-\varphi})+(1-\delta_k)(1-v_t)}{1-v_{t-1}}\right] = 0, \ d_t \equiv D_t/K_t, \qquad d_0 \text{ given,} \tag{36}$$

$$\left[\frac{\alpha L_t^{1-\alpha}(1-\tau_t Z_t^{-\varphi})+(1-\delta_k)(1-v_t)}{1-v_{t-1}}\right] - \left[\frac{\varphi\tau_t Z_t^{-\varphi}L_t^{1-\alpha}z_t^{-1}+1-b_t}{1-b_{t-1}}\right] = 0, \tag{37}$$

where $c_t \equiv C_t/K_t = n\left(L_t, \tau_t Z_t^{-\varphi}\right) = \frac{(1-\alpha)\sigma(N-L_t)^{\vartheta}(1-\tau_t Z_t^{-\varphi})}{(1-\sigma)L_t^{\alpha}}$.

Supposing that the government adopts the policy rules outlined above, the five difference equations (28), (34)-(37) in L_t, g_{st}, g_{kt}, z_t and d_t fully characterize the equilibrium path of the economy when θ=1. In this special case, Proposition 3 holds: the adoption of the optimal policy rule is consistent with unbounded GDP growth.

3.4 Numerical example 1

Assume the following parameter values: $\alpha = 0.25$; $\gamma = 0.92$; $\theta = 1$; $\vartheta = 1$; $\sigma = 0.5$; $N = 1.5500971$; $\varphi = 0.3405566$; $\delta_k = 0.1$; $\delta_s = 0.3160914$. Given these values, the BGP values obtained under laissez faire are the following:

$L^{LF} = 0.7211987$; $c^{LF} = 0.6746038$; $Z^{LF} = Z_0$; $g_Z^{LF} = 0$; $s^{LF} = 2.4147692 Z_0^{-0.3405566}$;

$g_Y^{LF} = g_k^{LF} = g_c^{LF} = g_s^{LF} = 0.0079986$;

$U^{LF} = 0.5\ln\left(\frac{c^{LF}}{s^{LF}}\right) + 0.5\ln(N - L^{LF}), = 0.5[0.3405566\ln(Z_0) - 1.2752334] +$

$0.5(-0.1876476) = 0.1702783\ln(Z_0) - 0.7314455,$

where $c_t \equiv C_t/K_t$, $s_t \equiv S_t/K_t$, $g_{Yt} \equiv \frac{Y_{t+1}-Y_t}{Y_t}$, $g_{Ct} \equiv \frac{C_{t+1}-C_t}{C_t}$ and $g_{zt} \equiv \frac{Z_{t+1}-Z_t}{Z_t}$.

Given the same parameter values, the BGP values obtained when the economy follows its optimal path are the following:

$L^{OP} = 0.4000807$; $c^{OP} = 0.4030493$; $s^{OP} = 1.5914684(Z^{OP})^{-0.3405566}$; $z^{OP} = 1.2379469$; $g_Y^{OP} = g_k^{OP} = g_c^{OP} = g_s^{OP} = g_z^{OP} = 0$;

$U^{OP} = 0.5\ln\left(\frac{c^{OP}}{s^{OP}}\right) + 0.5\ln(N - L^{OP}) = 0.5[0.3405566\ln(Z^{OP}) - 1.3733535] +$

$0.5(0.1397762) = 0.1702783\ln(Z^{OP}) - 0.6167886$. Supposing that the government adopts the optimal policy rule, along the BGP total emission taxes as a fraction of GDP are $\tau^{OP}(Z^{OP})^{-0.3405566}$ =0.6283538, and $d^{OP} = 2.772569$.

Comparing the laissez-faire BGP to the socially optimal BGP, one can check that $g_Y^{LF} > g_Y^{OP}$ and $L^{LF} > L^{OP}$, while $U^{OP} > U^{LF}$ (where $Z^{OP} \geq Z_0$). Hence,

Proposition 4. *Whenever the household production function is given by (12a) and θ=1, numerical examples show that in the long-run GDP growth is higher along a laissez-faire path than along a socially optimal path, while people enjoy more leisure and well-being along the latter.*

4. THE ECONOMY'S BALANCED GROWTH PATH WHEN $x(\mathrm{C}_t, \mathrm{S}_t) = \mathrm{C}_t\left(\frac{\mathrm{M}}{\mathrm{S}_t} - 1\right)$

4.1 The laissez-faire path

Whenever the households' production function is given by (12b), the "laissez-faire" equilibrium path is governed by the following three difference equations in $\mathrm{L}_t, \mathrm{K}_t$ and S_t:

$$\mathrm{K}_t(\mathrm{L}_t^{1-\propto} + 1 - \delta_k) - \mathrm{K}_{t+1} - \mathcal{C}(\mathrm{K}_t, \mathrm{S}_t, \mathrm{L}_t) = 0, \ \ \mathrm{K}_0 \text{ and } \mathrm{S}_0 \text{ given,} \tag{38}$$

$$\mathrm{S}_{t+1} - \mathrm{Z}_0^{-\varphi}\mathrm{K}_t\mathrm{L}_t^{1-\propto} - (1 - \delta_s)\mathrm{S}_t = 0, \quad \mathrm{Z}_0 \quad \text{given,} \tag{39}$$

$$\gamma \mathrm{C}_{t+1}^{-1}\left[\mathrm{C}_{t+1}\left(\frac{\mathrm{M}}{\mathrm{S}_{t+1}} - 1\right)\right]^{1-\theta}(\alpha \mathrm{L}_{t+1}^{1-\alpha} + 1 - \delta_k) - \mathrm{C}_t^{-1}\left[\mathrm{C}_t\left(\frac{\mathrm{M}}{\mathrm{S}_t} - 1\right)\right]^{1-\theta} = 0, \tag{40}$$

where $\mathrm{C}_t = \mathcal{C}(\mathrm{K}_t, \mathrm{S}_t, \mathrm{L}_t) = \left(\frac{\mathrm{M}}{\mathrm{S}_t} - 1\right)^{\frac{1-\theta}{\theta}}\left[\frac{\mathrm{K}_t(1-\alpha)\sigma(\mathrm{N}-\mathrm{L}_t)^{\vartheta}}{(1-\sigma)\mathrm{L}_t^{\alpha}}\right]^{\frac{1}{\theta}}$.

In the general case in which $\theta \neq 1$, a BGP is characterized by $\mathrm{L}_{t+1} = \mathrm{L}_t = \mathrm{L}^{\mathrm{LF}}$, $\mathrm{K}_{t+1} = \mathrm{K}_t = \mathrm{K}^{\mathrm{LF}}$, and $\mathrm{S}_{t+1} = \mathrm{S}_t = \mathrm{S}^{\mathrm{LF}}$, entailing $\mathrm{g}_{\mathrm{Y}}^{\mathrm{LF}} = \mathrm{g}_{\mathrm{C}}^{\mathrm{LF}} = \mathrm{g}_{\mathrm{S}}^{\mathrm{LF}} = \mathrm{g}_{\mathrm{k}}^{\mathrm{LF}} = \mathrm{g}_{\mathrm{Z}}^{\mathrm{LF}} = 0$. Hence, the following proposition holds:

Proposition 5. *If the household production function is given by (12b) and $\theta \neq 1$, the laissez-faire path has a steady state where GDP growth—together with the accumulation of both capital and pollutants—ceases, differently than the laissez-faire path when the household production function is given by (12a), which can exhibit unbounded growth even if $\theta \neq 1$ (see Proposition 1).*

If the marginal utility of consuming manmade products is not any longer positive once that the stock of pollutants surpasses a certain threshold, there is no incentive for households to go on accumulating wealth in a scenario of progressive environmental degradation. Hence, even under laissez faire, in general one cannot have unbounded GDP growth whenever the adverse effects of the accumulation of pollutants on people's well being cannot be compensated by devoting increasing quantities of output to this scope. However, in the special case in which θ=1, the marginal utility of consuming manmade products is not affected by the stock of pollutants ($U_{cs} = 0$), and unbounded growth is possible even when the household production function is given by (12b). In this special case, indeed, the laissez-faire equilibrium path of the economy is governed by the following two difference equations in L_t and $s_t \equiv S_t/K_t$: (21) and

$$\frac{\gamma}{c_{t+1}}\left(\alpha L_{t+1}^{1-\alpha}+1-\delta_k\right)-\frac{(1+g_{kt})}{c_t}=0, \tag{41}$$

where $g_{kt} \equiv \frac{K_{t+1}-K_t}{K_t}=L_{t+1}^{1-\alpha}-\delta_k-f(L_t)$ and $c_t \equiv C_t/K_t=f(L_t)=\frac{(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t^{\alpha}}$.

Thus, in the special case in which $\theta=1$, a BGP is characterized by $L_{t+1}=L_t=L^{LF}$, $s_{t+1}=s_t=s^{LF}$, $Z^{LF}=Z_0$ and $g_z^{LF}=0$. Moreover, along a BGP one may have unbounded growth ($g_Y^{LF}=g_c^{LF}=g_s^{LF}=g_k^{LF}>0$). This allows us to state the following proposition:

Proposition 6. *If the household production function is given by (12b) and $\theta=1$, laissez faire can lead to a "climate catastrophe" by determining unbounded growth, which—in the absence of any incentive in invest in green technology—drives S_t (the stock of pollutants, e.g. the amount of CO_2 in the atmosphere) to overpass its maximum compatible with life on earth, thus precipitating the collapse of individual's well being* ($U^{LF}\to-\infty$ as $t =\to \infty$).

4.2 The socially optimal path

The optimal path is governed by the following four difference equations in K_t, S_t, L_t and Z_t: (23), (24),

$$\frac{\gamma\sigma(1-\delta_k)}{C_{t+1}^{\theta}}\left(\frac{M}{S_{t+1}}-1\right)^{1-\theta}+\frac{\gamma(1-\sigma)L_{t+1}}{(1-\alpha)K_{t+1}(N-L_{t+1})^{\vartheta}}-\frac{\sigma}{C_t^{\theta}}\left(\frac{M}{S_t}-1\right)^{1-\theta}=0, \tag{42}$$

$$\frac{\gamma\sigma M C_{t+1}^{1-\theta}}{S_{t+1}^{2}}\left(\frac{M}{S_{t+1}}-1\right)^{-\theta}+\frac{\gamma\sigma(1-\delta_s)Z_{t+1}^{\varphi}}{C_{t+1}^{\theta}}\left(\frac{M}{S_{t+1}}-1\right)^{1-\theta}-\frac{\gamma(1-\sigma)(1-\delta_s)Z_{t+1}^{\varphi}L_{t+1}^{\alpha}}{(1-\alpha)K_{t+1}(N-L_{t+1})^{\vartheta}}-$$

$$-\frac{\sigma Z_t^{\varphi}}{C_t^{\theta}}\left(\frac{M}{S_t}-1\right)^{1-\theta}+\frac{(1-\sigma)Z_t^{\varphi}L_t^{\alpha}}{(1-\alpha)K_t(N-L_{t+1})^{\vartheta}}=0, \tag{43}$$

where $C_t=C(K_t,S_t,L_t,Z_t)=\left[\left(\frac{M}{S_t}-1\right)^{1-\theta}\frac{(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t\left(\varphi Z_t^{-1}+K_t^{-1}\right)}\left(\frac{\varphi K_t L_t^{1-\alpha}}{Z_t}+\delta_K\right)\right]^{\frac{1}{\theta}}$.

In general, even in the case in which $\theta=1$, i.e., in the case in which the preferences for the combination of consumer good and environmental quality, $x(C_t, S_t)$, are logarithmic, a BGP associated to the socially optimal path is characterized by $K_{t+1}=K_t=K^{OP}$, $S_{t+1}=S_t=S^{OP}$, $L_{t+1}=L_t=L^{OP}$

and $Z_{t+1} = Z_t = Z^{OP}$, entailing $g_Y^{OP} = g_c^{OP} = g_s^{OP} = g_k^{OP} = g_z^{OP} = 0$. Hence, the following proposition holds:

Proposition 7. *In general, if the household production function is given by (12b), the socially optimal path has a steady state where GDP growth—together with the accumulation of both capital and pollutants—ceases, differently than under laissez faire, where the economy can exhibit unbounded growth whenever θ=1, thus leading to a climate apocalypse.*

The socially optimal path prevents the stock of pollutants to exceed the critical threshold M, thus avoiding a climate catastrophe, by sacrificing unbounded growth, with the exception of the case in which θ=φ=1. In this special case, the socially optimal path is governed by the following system of difference equations in L_t, z_t, g_{kt} and S_t: (27), (29),

$$S_{t+1} - z_t^{-1} L_t^{1-\propto} - (1 - \delta_s) S_t = 0, \qquad S_0 \text{ given,} \tag{44}$$

$$\frac{\gamma\sigma M}{S_{t+1}^2}\left(\frac{M}{S_{t+1}} - 1\right)^{-1} + \frac{\gamma\sigma(1-\delta_s)z_{t+1}}{c_{t+1}} - \frac{\gamma(1-\sigma)(1-\delta_s)z_{t+1}L_{t+1}^{\alpha}}{(1-\alpha)(N-L_{t+1})^{\vartheta}} - \frac{\sigma z_t}{c_t} + \frac{(1-\sigma)z_t L_t^{\alpha}}{(1-\alpha)K_t(N-L_t)^{\vartheta}} = 0, \tag{45}$$

where $c_t \equiv C_t/K_t = f(L_t, z_t) = \frac{(1-\alpha)\sigma(N-L_t)^{\vartheta}}{(1-\sigma)L_t(z_t^{-1}+1)}\left(\frac{L_t^{1-\alpha}}{z_t} + \delta_K\right)$.

In this special case, a BGP is characterized by $L_{t+1} = L_t = L^{OP}$, $z_{t+1} = z_t = z^{OP}$, $g_{kt+1} = g_{kt} = g_k^{OP} \gtreqless 0$ and $S_{t+1} = S_t = S^{OP}$, entailing $g_S^{OP} = 0$ and $g_Y^{OP} = g_c^{OP} = g_Z^{OP} = g_k^{OP}$. Hence, one can state the following proposition:

Proposition 8. *If the household production function is given by (12b), unbounded GDP growth can be socially optimal only in the special case in which the marginal utility of consumption is not affected by environmental degradation (θ=1), and total emissions can be stabilized by letting the abatement efficiency grow at the same rate as productive capital and production, i.e. whenever the elasticity of emissions with respect to abatement capacity is one (φ=1).*

Proposition 8 emphasizes that the socially optimal plan is consistent with unbounded GDP growth only in the special case in which the increase in abatement capital (the progress of green technologies) can stabilize the stock of pollutants in an ever growing economy.

4.3 The “green-policy” path

Under the hypothesis that the household production function is given by (12b), the “green-policy” path of the economy is governed by the following five equations in K_t, S_t, L_t, Z_t and D_t: (23), (24), (32), (33) and

$$\frac{\gamma\left(\frac{M}{S_{t+1}}-1\right)^{1-\theta}}{C_{t+1}^{\theta}}\left[\frac{\alpha L_{t+1}^{1-\alpha}(1-\tau_{t+1}Z_{t+1}^{-\varphi})+(1-\delta_k)(1-v_{t+1})}{1-v_t}\right]-\frac{\left(\frac{M}{S_t}-1\right)^{1-\theta}}{C_t^{\theta}}=0,\ S_0 \text{ and } Z_0 \text{ given,} \tag{46}$$

where $C_t = C(K_t, S_t, L_t, Z_t) = \left[\left(\frac{M}{S_t}-1\right)^{1-\theta}\frac{(1-\alpha)\sigma K_t(N-L_t)^{\vartheta}(1-\tau_t Z_t^{-\varphi})}{(1-\sigma)L_t^{\alpha}}\right]^{\frac{1}{\theta}}$, and the evolution of τ_t, v_t and b_t is determined by the government’s policy rule.

Given the two policy rules outlined in subsection 3.3, the five equations (23), (24), (32), (33) and (46) in K_t, S_t, L_t, Z_t and D_t fully characterize the “green-policy” path of the economy, and the same considerations made in that subsection still apply.

In the special case in which $\theta=1$, (23), (32), (33) and (46) can be rewritten as (34)-(37). Thus, supposing that the government policies are the same as the policy rules outlined in subsection 3.3, the dynamics of L_t, g_{kt}, z_t and d_t is fully characterized by the system (34)-(37). In this case, the government policy can make unbounded growth consistent with the convergence of S_t toward a sustainable level (a level lower than M) if $\varphi=1$, i.e., if the economy is governed by (24) and (34)-(37).

4.4 Numerical example 2

Assume the following parameter values: $\alpha = 0.25$; $\gamma = 0.92$; $\theta = 0.2$; $\vartheta = 1$; $\sigma = 0.5$; $N = 1.4$; $\varphi = \delta_k = \delta_s = 0.1$; $M = 70.597574$.

Given these values, the BGP values obtained under laissez faire are the following:

$L^{LF} = 0.6787879$; $C^{LF} = 6.11572Z_0^{0.1} - 1.1100296$; $K^{LF} = 9.4403744Z_0^{0.1} - 1.7134687$; $Z^{LF} = Z_0$; $S^{LF} = 70.597574 - 12.813764Z_0^{-0.1}$; $g_Y^{LF} = g_k^{LF} = g_c^{LF} = g_s^{LF} = g_z^{LF} = 0$; $U^{LF} = \frac{0.5}{0.8}(1.1100295)^{0.8} + 0.5\ln(0.7212121) = 0.5160235$.

Given the same parameter values, the BGP values obtained when the economy follows its optimal path are the following:

$L^{OP} = 0.6$; $C^{OP} = 2.4386869$; $K^{OP} = 4.1921169$; $S^{OP} = 26.199494$; $Z^{OP} = 2.3852783$; $g_Y^{OP} = g_k^{OP} = g_c^{OP} = g_s^{OP} = g_z^{OP} = 0$; $U^{OP} = \frac{0.5}{0.8}(4.1326377)^{0.8} + 0.5\ln(0.8) = 1.8331697$. Supposing that, along the BGP total emission taxes as a fraction of GDP are $\tau^{OP}(Z^{OP})^{-0.1}$ =0.7257623 and $d^{OP} = 4.8274205$.

Comparing the laissez-faire BGP to the socially optimal BGP, one can check that $U^{OP} > U^{LF}$, while $Y^{LF} > Y^{OP}, C^{LF} > C^{OP}, L^{LF} > L^{OP}$ and $S^{LF} > C^{OP}$ whenever $Z_0 > \frac{4.8011476}{1000}$. Hence,

Proposition 9. *Whenever the household production function is given by (12b) and $\theta \neq 1$, numerical examples show for reasonable values of Z_0 that steady-state values of output, consumption and stock of emissions are higher under laissez faire than when the government adopts the optimal policy rule, while the reverse is true for people's steady-state leisure and well-being.*

4.5 Numerical example 3

Assume the following parameter values: $\alpha = 0.25$; $\gamma = 0.98$; $\theta = \varphi = \vartheta = 1$; $\sigma = 0.5$; $N = 2.3522222$; $\delta_k = 0.1$; $\delta_s = 0.005$; $M = 165.26103$.

Given these values, the BGP values obtained under laissez faire are the following:

$L^{LF} = 1.1597921$; $c^{LF} \equiv C^{LF}/K^{LF} = 0.8617855$; $Z^{LF} = Z_0$; $g_z^{LF} = 0$;

$g_Y^{LF} = g_k^{LF} = g_c^{LF} = g_s^{LF} = 0.1558112$; $U^{LF} \to -\infty$ as $t \to \infty$ ("climate apocalypse").

Given the same parameter values, the BGP values obtained when the economy follows its optimal path are the following:

$L^{OP} = 1$; $c^{OP} \equiv C^{OP}/K^{OP} = 0.8113331$; $z^{OP} \equiv Z^{OP}/K^{OP} = 6$; $S^{OP} = 33.3333$; $g_s^{OP} = 0$; $g_Y^{OP} = g_k^{OP} = g_c^{OP} = g_z^{OP} = 0.0126667$; $U^{OP} \to \infty$ as $t \to \infty$. Supposing that the government adopts the optimal policy rule, along the BGP total emission taxes as a fraction of GDP are $\tau^{OP}(Z^{OP})^{-1}$ =0.7714286 and $d^{OP} = 5.9266722$.

Comparing the laissez-faire BGP to the socially optimal BGP, one can check that $g_Y^{LF} > g_Y^{OP} > 0$, while $U^{LF} \rightarrow -\infty$ and $U^{OP} \rightarrow \infty$ as $t \rightarrow \infty$. Hence,

Proposition 10. *Whenever the household production function is given by (12b) and* $\theta = \varphi = 1$*, numerical examples show that—along a BGP—GDP growth is higher under laissez faire than when the government adopts the optimal policy rule, while people's well-being collapses under the former and grows forever along the socially optimal path.*

5. CONCLUSION

We presented a dynamic general equilibrium model where production emits pollutants whose accumulation negatively affects human well being. Within this framework, we explored both the hypothesis that there is no limit to the possibility for households to defend themselves against environmental degradation by increasing the use of manmade artifacts, and the hypothesis that there is a threshold beyond which the adverse effects of the accumulation of pollutants—such as greenhouse gases—cannot be offset by devoting increasing quantities of output to this scope, Under both hypotheses, we derived the balanced growth path (BGP) of the economy when there is no government intervention ("laissez faire") and when there is a benevolent social planner. Then, we studied how the socially optimal plan can be decentralized by using the policy instruments available to the government, whose policy choices are subject to its intertemporal budget constraint.

We showed that, if it is always possible to offset the harm that environmental degradation causes on people's well being by an increased use of manmade products, GDP growth can go on forever under laissez faire in spite of the growing environmental damage due to production. In this case, i) laissez faire is consistent with unbounded growth, ii) environmental degradation has never catastrophic effects on human well being, and iii) human well being remains constant along the BGP. In contrast, under the same circumstances, a benevolent social planner would generally lead the economy towards a steady state where GDP growth—together with the accumulation of both capital and pollutants—ceases. The exception is whenever the households' preferences for consumption and environment quality are logarithmic: in this case, even along a socially optimal path one can have unbounded GDP growth. However, also in this case numerical examples show that in the long-run GDP growth is higher along a laissez-faire path than along a socially optimal path, while people enjoy more leisure and well-being along the latter. Finally, it is possible to decentralize the socially optimal plan by taxing the emission of pollutants and subsidizing private investment in productive assets on the part of the government, if the optimal policy rule is consistent with the sustainability of public debt. Otherwise, other suboptimal green policy rules should be considered.

Under the hypothesis that the accumulation of pollutants can reach a threshold beyond which efforts to compensate or offset the damage they cause to people's well being by using more manmade products are vain (which is consistent with our knowledge concerning the long-term effects of global warming), laissez faire is generally inconsistent with unbounded growth. Indeed, if the marginal utility of consuming manmade products is not any longer positive once that the stock of pollutants surpasses a certain threshold, there is no incentive for households to go on accumulating wealth in a scenario of progressive environmental degradation, and one cannot have unbounded growth under laissez faire. However, in the special case in which households' preferences for consumption and environmental quality are logarithmic, the marginal utility of consuming manmade products is not affected by the stock of pollutants, and laissez faire is consistent with unbounded growth even if there is a threshold beyond which the adverse effects of the accumulation of pollutants on people's well being cannot be offset by devoting increasing quantities of output to this scope. Hence, under these circumstances, laissez faire leads to a "climate catastrophe": the stock of pollutants is driven beyond its maximum compatible with life on earth, thus precipitating the collapse of individual's well being. Such a catastrophe is always avoided along a socially optimal path: unbounded GDP growth can be socially optimal only in the special case in which the marginal utility of consumption is not affected by environmental degradation and total emissions can be stabilized by letting the abatement efficiency grow at the same rate as productive capital and production (i.e., whenever the elasticity of emissions with respect to abatement capacity is one). Also when there is a threshold beyond which the damage caused by pollutants cannot be offset by manmade products, the social optimal path can be decentralized by a public agency that taxes emissions and subsidizes investment in productive assets. In the presence of this threshold, numerical examples show that—when both the laissez-faire path and the socially optimal path are not characterized by unbounded GDP growth—steady-state values of output, consumption and stock of emissions are higher under laissez faire than when the government adopts the optimal policy rule, while the reverse is true for people's steady-state leisure and well-being. These numerical examples show also that, when both the laissez-faire path and the socially optimal path are characterized by unbounded GDP growth, the latter is higher under laissez faire than when the government adopts the optimal policy rule, while people's well-being collapses under laissez faire and grows forever along the socially optimal path.

Additional steps in the direction to utilize the model presented here to evaluate the impact of the green policies undertaken in many economies include the analysis of the stability properties of the BGPs and numerical solutions for the transitional paths converging to these BGPs, to be conducted by calibrating the model's parameters in accordance with the evidence regarding these economies. Moreover, a natural extension of the model amounts to treat the evolution of the stock of pollutants as the result of the production activities of many independent countries, whose green policies are therefore interdependent. We believe that these developments can further improve the model's ability to assess the medium and long-term welfare effects of policies aimed at tackling climate change.

REFERENCES

Acemoglu D., Aghion P., Bursztyn L., Hemous D. (2012), "The Environment Directed Technical Change". *American Economic Review* 102: 131-166.

Barnett J., O'Neill S. (2010), "Maladaptation". *Global Environmental Change* 2: 211–213.

Barro R.J., Sala-i-Martin X. (1995), *Economic Growth*, McGraw-Hill, New York.

Bartolini S., Bonatti L. (2002), "Environmental and social degradation as the engine of economic growth". *Ecological Economics* 43: 1-16.

Bartolini S., Bonatti L. (2003), "Endogenous Growth and Negative Externalities". *Journal of Economics* 79: 123-144.

Bartolini S., Bonatti L. (2008), "Endogenous growth, decline in social capital and expansion of market activities". *Journal of Economic Behavior & Organization* 67: 917–926.

Bretschger L., Karydas C. (2019), "Economics of climate change: Introducing the Basic Climate Economic (BCE) model". *Environment and Development Economics* 24: 560–582.

Dietz S., Stern N. (2015), "Endogenous growth, convexity of damage and climate risk: How Nordhaus' framework supports deep cuts in carbon emissions". *The Economic Journal* 125: 574–620.

Frankel M. (1962), "The production function in allocation and growth: A synthesis". *American Economic Review* 52: 995-1022.

Golosov M., Hassler J., Krusell P., Tsyvinski A. (2014), "Optimal taxes on fossil fuel in general equilibrium". *Econometrica* 82: 41–88.

Hassler J., Krusell P. (2018), *Environmental macroeconomics: The case of climate change*. Volume 4. Elsevier B.V., Amsterdam.

Leipert C., Pulselli F.M. (2008), "The origins of research on defensive expenditures: a dialog with Christian Leipert". *International Journal of Design & Nature and Ecodynamics* 3: 150-161.

Matthews H.D., Gillet N.P., Stott P.A., Zickfeld K. (2009), "The proportionality of global warming to cumulative carbon emissions". *Nature* 459: 829–833.

Nordhaus W.D. (1994). *Managing the Global Commons: The Economics of Climate Change*. MIT Press, Cambridge MA.

Schumacher I. (2019), "Climate Policy Must Favour Mitigation Over Adaptation". *Environmental and Resource Economics* 74: 1519–1531.

Uzawa H. (2003), *Economic Theory and Global Warming*. Cambridge University Press, Cambridge.