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Reserve Prices as Signals

Abstract

This paper discusses the role of secret versus public reserve prices when bidders' valuations depend positively on the seller's private signal. A public reserve price is announced before the auction starts, and a secret reserve price is disclosed after the highest bid has been reached. The public reserve price regime may warrant a distortion as a good seller type may have to increase the reserve price beyond payoff-maximization in order to be able to credibly signal her type. We introduce and determine a rational signaling equilibrium which adds two domination-based conditions to the belief structure of a weak perfect Bayesian equilibrium. We show that a secret (public) reserve price design qualifies as an equilibrium if the distortion is large (small).

JEL-Codes: D440.

Keywords: auctions, interdependent values, optimal reserve prices, rational signaling.

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1 Introduction

In many auction markets (e.g., real estate auctions), a seller's private signal matters for bidders' valuations, such that bidders' valuations depend positively on the seller's signal. Such interdependence of the valuations is prevalent especially when the use of the object is purely a personal matter.[1] In this paper, in an auction environment, we scrutinize the role of interdependence between the valuations of the seller and the bidders and its implications for the optimal reserve price, especially when keeping the reserve price secret is an alternative option to a publicly announced reserve price.

Considering a sale of an indivisible good by an auction (either by a second-price sealed-bid, an open ascending, or a first-price sealed bid auction) when the valuation of bidders depends positively on the seller's private signal, we focus on how sellers can potentially reveal their signal. In the case of interdependent values among bidders, any bidder would learn other bidders' signals in an ascending-bid auction by observing their behavior, in particular when they stop bidding. In case of interdependent values between bidders and the seller, they may only learn the seller's signal if the seller has set a reserve price that is either publicly announced before the auction or revealed when the recent bid has reached the reserve price. Alternatively, a secret reserve price is disclosed only once the auction is over and the highest bid is realized, and thus it cannot directly signal any information. Our paper shows that interdependent values between the seller and bidders significantly change the standard result on the use of secret reserve prices.

In particular, we show that a good seller type may have to set a larger than revenue-maximizing reserve price when the incentive-compatibility constraint of the bad seller type is binding. If such implied distortion is too large, then both types will self-select into a secret reserve price regime. If, however, the implied distortion is not too large, a good type will always prefer a public reserve price regime in which the price is announced before the auction starts. Both results establish an outcome that we refer to as a rational

[1] As Cassady (1967) points out, a seller would know the particulars of the good (especially from past use and expert opinions) before the auction is run. Similarly, each bidder will receive some signal, and if the consumption of the good is purely a personal matter, bidders will consider only their own signal and any signal they receive from the seller. For further discussions in support of interdependence between the valuations of the seller and bidders and positive affiliation, see, *inter alia*, Milgrom and Weber (1982) and Horstmann and LaCasse (1997).

signaling equilibrium, for which we introduce two domination-based conditions to the belief structure of a weak perfect Bayesian equilibrium. Our results suggest an alternative intuitive explanation to the observation of the optimal use of secret reserve prices, which is in stark contrast to one that Vincent (1995) has put forward by relying on affiliation only among bidders' signals (discussed below). Our equilibrium refinement is similar to the familiar "unraveling of information from the top" developed by Milgrom (1981): the good type has an incentive to distinguish herself from the bad type and can do so by a public reserve price regime and will do so if the distortion is not too large. While one of the conditions we have used in our equilibrium refinement is a direct implication of the intuitive criterion developed by Cho and Kreps (1987), the other condition that further adds a sensible refinement to our model is implied neither by the intuitive criterion, nor by the divinity criterion (or the D_1-criterion) developed by Banks and Sobel (1987). We require both conditions for our equilibrium refinement so as to cover both the uni-dimensional message space (in the case of secret reserve prices) and the two-dimensional message space (in the case of public reserve prices).

The role of public reserve prices in standard auctions with independent private values is well established in the literature: when bidders are symmetric and risk-neutral, the standard auctions (e.g., the first-price and the second-price sealed-bid auctions, and the English auction) are optimal and the revenue-maximizing seller should set a public reserve price that exceeds her value; and this optimal reserve price does not depend on the number of bidders (Krishna, 2010, Chapter 2).[2] The main idea is the interplay between excluding bidders with values less than the reserve price and the risk that the highest value among all bidders might be less than the reserve price.[3] Relaxing some of the assumptions of the standard auctions, the literature has scrutinized the role of secret reserve prices in different

[2]Cai et al. (2007), however, show that under certain conditions on values, a public reserve price may be increasing in the number of bidders, especially when they reveal important private information the seller might have. In a similar environment, Jullien and Mariotti (2006) show that such a reserve price can also reduce the probability of selling the item.

[3]Elyakime et al. (1994) - motivated by the practice in France to sell timber - extend this result to secret reserve prices: within the independent private values framework, they solve a first-price sealed-bid auction with symmetric and risk-neutral bidders for the optimal bidding and selling strategies, and they show that the optimal secret reserve price is the one that is equal to the seller's private value, and that the seller's expected gain increases by moving from an optimal secret reserve price to an optimal public reserve price. See also Li and Perrigne (2003) and Elyakime and Loisel (2005) for similar results in support of the use of public reserve prices in first-price auctions with independent private values.

environments. For instance, within the framework of independent private values, setting a secret reserve price may emerge as the equilibrium behavior in first-price auctions if there is a large number of bidders who are sufficiently risk-averse and who respond to the secret reserve price by bidding more aggressively (as in Li and Tan, 2017). Rosar (2014) shows that secret reserve prices may dominate if the seller is risk-averse and the seller's information (for some exogenous reason) might improve after having committed to the rules of the auction and before the auction has started. Rosenkranz and Schmitz (2007) discuss the role of a secret reserve price in both first-price and second-price auctions if bidders have reference-based utility and a public reserve price partly serves as the reference point.[4] The literature has shown that relaxing the assumption of independent private values may generate incentives for the seller to set a secret reserve price. In a common value environment, Vincent (1995) shows that a seller (privately knowing the value of the good) may increase the participation rate by keeping the reserve price secret in a second-price auction with affiliated values (such that a bidder's value is a convex combination of his and other bidders' signals).[5]

In general, we can relate our finding on the implications of implied distortion to Akerlof's (1970) *lemon problem* and to Spence's (1974) education model, such that a good type engages in a costly signaling activity so as to avoid being mistaken for a bad type. This leads to the informed principal problem, for which different solution concepts are developed by Myerson (1983), and applied to bilateral trading problems by Myerson (1985). Focusing on the mechanism selection by an informed principal in a single principal and a single agent case, Maskin and Tirole (1990) consider a principal with private informa-

[4]There are different strands of the auction literature also looking at the optimal reserve price policy, especially (i) when there is endogenous participation (competitive auctions), in which case – depending on the specification of the competitive environment and the bidders' perception of the variation in participation rates – secret reserve prices may be preferred (as in Jehiel and Lamy, 2015); or (ii) when re-auctioning (or re-negotiating to sell) the good is possible, in which case – depending on the characteristics of the resale market (and the bargaining power of the agents) – public reserve prices may emerge as the equilibrium policy (as in Menezes and Ryan, 2005, and Grant et al., 2006).

[5]In general, the auction literature has also shown that positive affiliation among bidders' signals leads to a clear expected revenue ranking for interdependent values: the (English) ascending-bid auction dominates the second-price auction, and the second-price auction dominates the first-price auction (see, for example, Krishna, 2010, Chapter 6). This is due to the linkage principle: the larger the linkage between the information the bidder receives by a private signal and how the bidder thinks other bidders will behave, the larger is the expected winning bid. Affiliation implies that receiving a positive signal increases the probability that other bidders have also received a positive signal.

tion that does not directly enter the agent's utility function (the so-called private-values case), whereas, in a companion paper, Maskin and Tirole (1992) consider a principal with private information that does directly affect the agent's utility function (the so-called common-values case). They show that, generically, the principal is better off concealing private information. Similarly, Skreta (2011), modeling an informed seller's problem with multiple agents, considers a seller observing a vector of signals correlated with buyers' valuations. She shows that, for allocation environments that allow for interdependent, for common values, and for multiple items, disclosure policies may matter: the best the seller can do is to disclose no information. By the same token, Koessler and Skreta (2016) consider the case of an informed seller problem in which the valuation of the buyer depends both on the personal privately-known taste and on product characteristics privately known to the seller. They show that the privacy of the seller's information is always valuable ex ante. We depart from that literature on the basis of the specific class of mechanism we study as we focus on reserve price signaling in auctions with interdependent values between bidders and the seller.

As for an informed seller problem within such a specific class of mechanism, Cai et al. (2007) and Jullien and Mariotti (2006) study reserve price signaling in auctions as a means to convey the seller's private information to bidders. Our paper is closely related to these two papers. In a second-price auction with interdependent values, focusing on the role of *public* reserve prices in signaling the seller's private signal (positively affecting bidders' valuations), both Cai et al. (2007) and Jullien and Mariotti (2006) characterize the unique separating equilibrium. In equilibrium, the lowest-quality seller sets the same reserve price as in the public information case, whereas other types set higher reserve prices compared with the public information case. In Cai et al. (2007), the seller chooses a markup over bidders' beliefs, and thus the reserve price is defined as the sum of beliefs and the markup. In Jullien and Mariotti (2006), the seller chooses directly the reserve price. While Cai et al. (2007) focus on equilibrium refinement, Jullien and Mariotti (2006) focus on the Pareto dominant separating equilibrium, considering both a decentralized market and monopoly intermediation as different selling mechanisms. Moreover, in the context of reserve price signaling in auctions with interdependent values, our paper is also related to Zhao (2018) and Lo et al. (2019). While the former characterizes the seller-optimal equilibria among separating equilibria and provides a micro-foundation for the use of reserve prices for signaling, the latter considers both interdependent values and affiliated signals as in Milgrom and Weber (1982). In particular, Lo et al. (2019) examines the conditions for existence and uniqueness of a separating equilibrium where higher reserve price signals more favorable seller information. Zhao (2018), allowing buyers to be asymmetric and the seller to design every element of the mechanism, shows that reserve prices are the least costly device to separate higher and lower types, providing further motivation for our paper. Our main departure from these papers is that our modeling framework allows sellers to conceal their types if it is in the best interest of the types. More specifically, we allow the seller to send either a uni-dimensional message, that is, to employ a secret reserve price regime, or a two-dimensional message, that is, to employ a public reserve price regime and to announce a reserve price amount. Allowing for a secret reserve price as an alternative to a public reserve price, we are able to capture an intuitive explanation to the observation of the optimal use of secret reserve prices.

The remainder of this paper is organized as follows. Section 2 sets up the model and develops the reserve prices that maximize the seller's unconstrained expected revenues. Section 3 scrutinizes the credibility of reserve prices and establishes that binding incentive constraints increase a good seller type's reserve price in any separating equilibrium. Section 4 develops the equilibrium reserve price regimes and reserve prices in the rational signaling equilibrium, and Section 5 concludes.

2 The model

We consider the following game. There is an indivisible good for sale, the good is owned by a single seller, and n potential buyers bid in an auction. The seller runs a sealed-bid second-price auction or, equivalently, as to be shown, an ascending bid auction to sell the good. Before the game starts, the seller receives a (private) signal s and each bidder i receives a (private) signal σ_i. Thus, we consider a game as outlined by Table 1.

The strategy space of the seller is given by $\{T, r^T\}$ where $T \in \{\mathcal{P}, \mathcal{S}\}$ denotes the type of the reserve price regime that is either public ($\mathcal{P}$) or secret ($\mathcal{S}$). $r^T \in \mathbb{R}^+$ denotes the reserve price amount chosen by the seller in reserve price regime $T \in \{\mathcal{P}, \mathcal{S}\}$. Note that bidders will always learn the reserve price regime before the auction starts, but the reserve price amount only in regime $\mathcal{P}$ via the announcement of the reserve price.

Table 1: Game structure

Stage I: A Nature move selects s for the seller and σ_i for each bidder.
Stage II: The seller announces either a secret or a public reserve price regime. In the case of a public reserve price regime, the reserve price amount is announced.
Stage III: A second-price sealed bid/an (English) ascending-bid auction is run and ends when the highest bid is identified.
Stage IV: The item is sold according to the rules of the auction format if the reserve price is met; otherwise the item remains unsold.

Before the auction is run, all bidders form a common belief on the seller's type. We denote by s the seller's signal, and assume the seller can either be a bad type or a good type such that $s \in \{\underline{s}, \overline{s}\}, 0 < \underline{s} < \overline{s}$, where the probability that the seller is of type $\overline{s}$ is given by q and the probability that the seller is of type $\underline{s}$ is given by $(1-q)$.[6] The payoff of the seller is either equal to the bid for which the item sells or equal to the seller's private signal, $\underline{s}$ or $\overline{s}$, respectively, if the item does not sell.

A bidder's payoff is normalized to zero in the event of not winning the auction. Bidder i's utility by winning the auction is, however, equal to $u(\sigma_i, s) = \alpha\sigma_i + (1-\alpha)s$, where $0 < \alpha < 1$. All bidders draw their signals $\sigma_i \in [\underline{\sigma}, \overline{\sigma}]$ independently from the cdf $F(\sigma)$ that is continuously differentiable, that is, the pdf $f(\sigma) = F'(\sigma)$ exists. $0 < \alpha < 1$ guarantees that the seller's signal has a stronger effect on the seller's valuation than on any bidder's valuation. We also assume that $\alpha\underline{\sigma} > \overline{s}$ holds such that the sale is always socially desirable.

Since the bidders' payoffs do not depend on the signals received by other bidders and all

[6]Assuming a continuum of types would make the analysis less tractable without offering further insights. Moreover, as it will be clear in Section 4, one of the domination-based conditions we use for our equilibrium refinement borrows from the intuitive criterion, which is less restrictive than many other concepts established in the literature on signaling and informed seller problems. As is well established in the related literature, with a continuum of types, we would not be able to allude to the intuitive criterion, but would rather have to be more restrictive in our equilibrium refinement.

bidders form a common belief about the seller's type, the sealed-bid second-price auction is strategically equivalent to a standard ascending-bid auction format, in which the auction starts at zero price that gradually increases.[7] As the reserve price regime and the reserve price will have an effect on the bidders' belief formation about the seller's type, let $\mu(\mathcal{S})$ denote the (common) belief of bidders that the seller is of type $\overline{s}$ if the seller selects a secret reserve price regime, and let $\mu(r, \mathcal{P})$ denote the belief that the seller is of type $\overline{s}$ if she selects a public reserve price regime and announces reserve price r. Consequently, $\tilde{s}(\mathcal{S}) = [1 - \mu(\mathcal{S})]\underline{s} + \mu(\mathcal{S})\overline{s}$ is the expected seller type for the secret reserve price regime, and $\tilde{s}(r, \mathcal{P}) = [1 - \mu(r, \mathcal{P})]\underline{s} + \mu(r, \mathcal{P})\overline{s}$ is the expected seller type for the public reserve price regime, respectively. If bidders believe that both types choose the secret reserve price regime, then $\mu(\mathcal{S}) = q$ and $\tilde{s}(\mathcal{S}) = \hat{s} = q\overline{s} + (1 - q)\underline{s}$. We cannot solve the game by simple backward induction, but a minimum requirement for our equilibrium is that it should be a weak perfect Bayesian equilibrium, which is introduced in Definition 1.

Definition 1. *A weak perfect Bayesian equilibrium (PBE) is given if the following three conditions hold:*

1. *The seller's reserve price strategy is optimal given beliefs and the bidding strategies of all bidders.*
2. *The belief functions $\mu(\mathcal{S})$ and $\mu(r, \mathcal{P})$ are derived from the seller's equilibrium strategies using Bayes' Rule where possible.*
3. *Subsequent to the seller's choice of the reserve price regime and the reserve price amount, the bidding strategies are a Bayesian-Nash equilibrium for which $\mu(\mathcal{S})$ and $\mu(r, \mathcal{P})$, respectively, specify the probability that the seller is of type $\overline{s}$.*

We begin our analysis by scrutinizing the bidding behavior for which we take the formation of beliefs as given by $\tilde{s}$. Bidder i's optimal bid is equal to

[7] In an ascending bid auction, all bidders observe the current price and signal their willingness to buy at the current price. The set of active bidders signaling their willingness to pay at the current price is common knowledge. At any price, bidders may drop out. Once a bidder drops out at a price, however, the bidder is not allowed to be active again at a higher price later in the auction. The auction ends when there is only one active bidder. For different price formats of a standard open ascending-bid (or English) auction, see e.g., Krishna (2010), section 6.3.

$$\beta_i(\tilde{s}) = \alpha\sigma_i + (1-\alpha)\tilde{s}. \tag{1}$$

As is well-known for second-price auctions, it is a weakly dominant strategy for each bidder to bid her valuation. This is also true in our context where the valuation also depends on the formation of beliefs about the seller's type.[8] We now compute the expected payoff of a seller setting reserve price r who is of type s when bidders believe she is of type $\tilde{s}$ which we denote by $V[r, s, \tilde{s}]$. The expected payment of a bidder who will reach the reserve price is equal to

$$m(\sigma, \rho, \tilde{s}) = rG(\rho) + \int_{\rho}^{\sigma} (\alpha\tau + (1-\alpha)\tilde{s})\, g(\tau)d\tau \tag{2}$$

where $G(\cdot)$ and $g(\cdot)$ are, respectively, the cdf and the pdf of the highest valuation of all other bidders, and $r = \alpha\rho + (1-\alpha)\tilde{s}$. Note that $\rho = [r - (1-\alpha)\tilde{s}]/\alpha$ is the critical realization of σ: for all σ_i larger than ρ, the reserve price will be sufficiently small such that bidder i, given belief $\tilde{s}$, will bid more than r. If σ_i is smaller than ρ, the bidder's valuation is below r. The expected revenue per bidder is then given by

$$\mathrm{E}\left[m(\sigma, \rho, \tilde{s}.)\right] = rG(\rho)\left[1 - F(\rho)\right] + \int_{\rho}^{\overline{\sigma}} \left[1 - F(\sigma)\right]\left[\alpha\sigma + (1-\alpha)\tilde{s}\right] g(\sigma)d\sigma \tag{3}$$

and the seller's expected payoff is given by

$$V[r, s, \tilde{s}] = n\mathrm{E}\left[m(\sigma, \rho)\right] + F(\rho)^n s. \tag{4}$$

The expected payoff is equal to the number of bidders times the expected revenue per bidder plus the probability of not selling times the utility of keeping the item.

Now consider (any) seller type s who maximizes her expected payoff (4) with respect to r. We denote by $h(\cdot) = f(\cdot)/[1 - F(\cdot)]$ the hazard rate, for which we assume, as is common

[8] Also in this setting, due to the strategic equivalence between a second-price and an ascending bid auctions, each bidder will bid up to (1) should the auction be run as an ascending bid auction.

in the literature, that $h'(\cdot) > 0$. Using $h(\cdot)$, the first-order condition that determines the optimal reserve price r^* can be written as

$$\frac{\partial V(r^*, s, \tilde{s})}{\partial r} = A(\rho^*) \left(\frac{\alpha}{h\left(\frac{r^* - (1-\alpha)\tilde{s}}{\alpha}\right)} - (r^* - s) \right) = 0, \tag{5}$$

where

$$A(\rho^*) = \frac{nG(\rho^*)\left[1 - F(\rho^*)\right] h(\rho^*)}{\alpha} > 0 \text{ and } \rho^* = \frac{r^* - (1-\alpha)\tilde{s}}{\alpha}.$$

Appendix A.1 shows that the sufficient conditions are fulfilled. Furthermore, we can show:

Lemma 1. *For a common belief $\tilde{s}$ and maximization of (4), a good type (with signal $\overline{s}$) sets a higher reserve price that fulfills (5) and enjoys a larger maximized payoff than a bad type (with signal $\underline{s}$). The marginal payoff for the good type is larger than that for the bad type for a common belief $\tilde{s}$ and a common reserve price r.*

Proof. See Appendix A.1. □

Lemma 1 shows the behavior of reserve prices with the type for a given belief $\tilde{s}$. How does the formation of beliefs affect the reserve price and the probability that the item will sell? We can also show:

Lemma 2. *For a given type s and maximization of (4), an increase in $\tilde{s}$ will increase both the optimal reserve price that fulfills (5) and the probability that the item will sell (which is inversely related to ρ^*).*

Proof. See Appendix A.1. □

While the optimal reserve price increases with $\tilde{s}$, the cutoff ρ^* that is inversely related to the probability that the item will sell *decreases* with $\tilde{s}$. What does this mean for potential separating equilibria when reserve prices that maximize (4) are credible? In the case of a secret reserve price, all bidders have to form expectations on the seller's type which we denote by $\breve{s}$. We have not yet scrutinized any consistent belief structure, but it should be clear that $\overline{s} \geq \breve{s} \geq \underline{s}$ must hold for the formation of beliefs in any secret reserve price regime, irrespective of whether it is an equilibrium or an out-of-equilibrium belief.[9] If the seller chooses a public reserve price, the reserve price itself and the selection of the reserve price regime can serve as a signal to bidders. Furthermore, if a separating equilibrium exists for the public reserve price regime, it can be consistent with the belief structure only if both seller types can signal their type credibly.

Let $\overline{\rho}^*(\overline{s})$ and $\underline{\rho}^*(\underline{s})$ denote the cutoffs that maximize the expected payoff (4) of the good and the bad type, respectively, if she chooses a public reserve price regime. If she chooses a secret reserve price regime, those cutoffs will be denoted by $\overline{\rho}^*(\breve{s})$ and $\underline{\rho}^*(\breve{s})$, respectively. The implication of Lemma 2 is that, given $\overline{s} \geq \breve{s} \geq \underline{s}$, $\overline{\rho}^*(\overline{s}) \leq \overline{\rho}^*(\breve{s})$ and $\underline{\rho}^*(\underline{s}) \geq \underline{\rho}^*(\breve{s})$ holds. Consequently, given the inverse relationship between ρ^* and the sale probability, $\overline{s} \geq \breve{s}$ implies that the probability that the item will sell is *larger* with the public reserve price in the case of a good (and credible) seller signal although the optimal reserve price will be larger as well. On the contrary, since $\underline{s} \leq \breve{s}$, the probability that the item will sell is *smaller* with the public reserve price in the case of a bad seller signal although the optimal reserve price will be smaller as well. Thus, for maximization of expected payoff (4), we find that, compared to a secret reserve price regime, a public reserve price regime will imply both (i) a larger reserve price *and* a larger sale probability for the good type; and (ii) a smaller reserve price *and* a smaller sale probability for the bad type. The reason is that the reserve price difference is smaller than the value increase due to signaling the true type to bidders in the public reserve price regime.

3 Credible reserve prices

The last section has scrutinized how the seller can set an optimal reserve price when she is facing no credibility constraint. In this section, we now distinguish between unconstrained and constrained maximization of payoff (4). The reason is that nothing stops the bad type from mimicking the good type in the case of a reserve price regime $\mathcal{P}$: if $\overline{r}$ is such that all

[9] If a seller chooses a secret reserve price regime, she cannot signal any type by the reserve price, but only by choosing this specific regime.

bidders believe that this reserve price is set by a good type, this is credible and consistent only if the bad type is not better off by setting her reserve price equal to $\overline{r}$ as well. Thus, this section deals with the credibility of the good type's reserve price as a signal for bidders. This problem does not arise in the secret reserve price regime $\mathcal{S}$ if it is in the best interest of *both* types to select this regime, given the belief structure of the bidders.

If the good type wants to signal her type by the reserve price, this strategy must be credible such that it is not in the best interest of a bad type to mimic this reserve price. Suppose that the bad type has set her reserve price according to (5) with $r^* = \underline{r}^*$, and by doing so has also revealed her bad type, that is, $\tilde{s} = \underline{s}$. She will do so only if $V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] \geq V[\overline{r}, \underline{s}, \tilde{s} = \overline{s}]$ holds, that is, if she is not better off by adopting $\overline{r}$ and making the bidders believe that she is a good type.

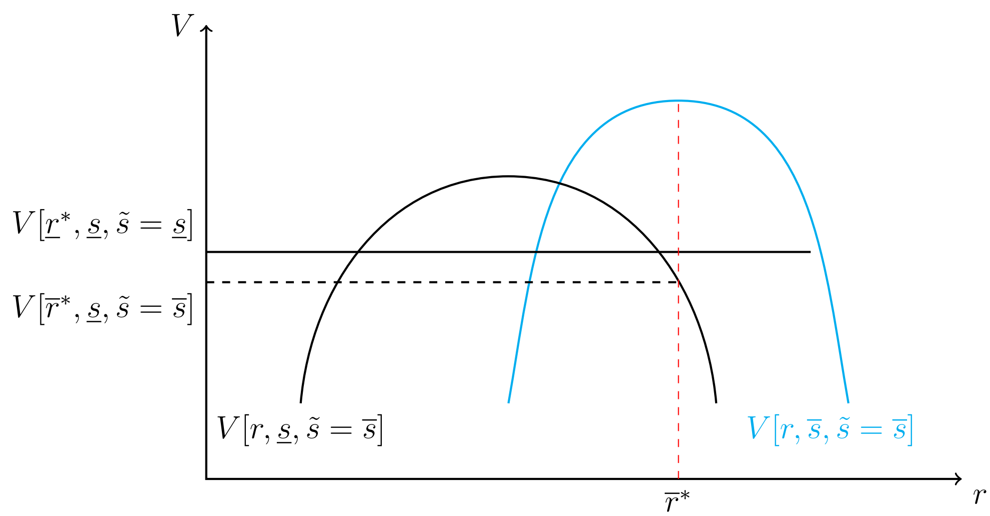

Figure 1: Payoffs of both types for $\tilde{s} = \overline{s}$ without a binding constraint

Figures 1 and 2 depict the two possible outcomes. Both figures depict the payoff of both types when bidders believe that they are dealing with a good type, that is, if $\tilde{s} = \overline{s}$.[10] We know that the good type's unconstrained maximum will be larger than the bad type's unconstrained maximum under this common belief, and from Lemma 1 we know that the good type will set a larger payoff-maximizing reserve price and that the marginal payoff

[10]For details see (A.2) and (A.3) in Appendix A.1.

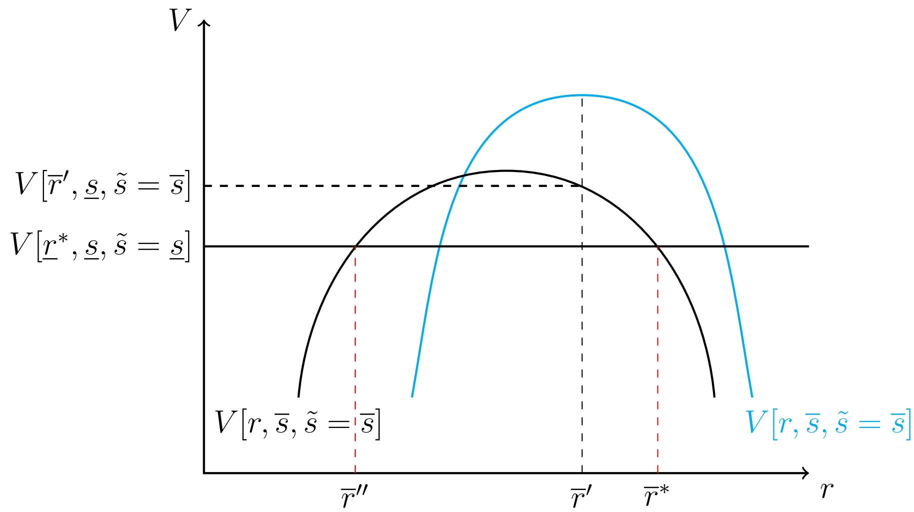

Figure 2: Payoffs of both types for $\tilde{s} = \overline{s}$ with a binding constraint

will be larger for the good type than for the bad type for any common reserve price r.[11] In Figure 1, the constraint that the bad type should not mimic the good type is not binding. The reserve price $\overline{r}^*$ that maximizes the good type's unconstrained expected payoff is credible as the bad type will have no incentive to adopt this price since $V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] > V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}]$. In Figure 2, the incentive compatibility constraint is binding. $\overline{r}'$ denotes the reserve price that maximizes the unconstrained expected payoff of the good type, but $\overline{r}'$ is not a credible signal as $V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] < V[\overline{r}', \underline{s}, \tilde{s} = \overline{s}]$, and consequently the bad type would adopt $\overline{r}'$ as well.

When will the incentive compatibility constraint be binding? Figures 1 and 2 reveal that a binding constraint is more likely if the payoff functions of the good and the bad type are not too different for $\tilde{s} = \overline{s}$. In order to shed more light on the incentive compatibility constraint, we present a specification in Appendix A.2 for which we assume two bidders and a uniform distribution of bidder signals. We show that the incentive compatibility constraint will not be binding, that is, $V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] \geq V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}]$, when the seller's signal (compared to the bidders' own signals) is sufficiently less important for bidders' valuation. Thus, the unconstrained payoff maximization of the good type does not impose a binding incentive compatibility constraint if α is above a certain threshold $\overline{\alpha}$. If α is suf-

[11]See inequality (A.2) in Appendix A.1 that also holds for negative signs of the derivatives.

ficiently large, the bad type cannot gain much by mimicking the good type as the implied increase in bidders' valuation is relatively small while the reserve price is suboptimally large.

As can be seen from Figure 2, the good type has two options to make her reserve price credible if the incentive compatibility constraint is binding: either reduce the reserve price to $\overline{r}''$ or increase the reserve price to $\overline{r}^*$. In both cases, the bad type is made indifferent between revealing her true type $\underline{s}$ by setting the reserve price equal to $\underline{r}^*$ or by pretending to be a good type for which we assume that she will prefer to reveal her true type in the case of indifference. Which reserve price will the good type choose? Both the price reduction and the price increase require the good type to sacrifice payoff in order to reduce the payoff of the bad type to $V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}]$.

Let us define $\Omega = d\underline{V}/d\overline{V}$ as the marginal relative sacrifice: it measures by how much the marginal payoff of the bad type is reduced by a marginal payoff reduction of the good type. From (A.2), we know that the marginal sacrifice is given by

$$\Omega : \begin{cases} \Omega > 1 & \text{for all} \quad p > \overline{r}', \\ \Omega < 1 & \text{for all} \quad p < \overline{r}'. \end{cases} \tag{6}$$

Expression (6) shows that the payoff sacrifice of the good type is always smaller by increasing the price. Consequently, we may safely conclude that the incentive-compatible reserve price of the good type will be $\overline{r}^*$. In general, the good type faces a constrained maximization problem given by

$$\max_{\overline{r}} V[\overline{r}, \overline{s}, \tilde{s} = \overline{s}] \text{ s.t. } V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] \geq V[\overline{r}, \underline{s}, \tilde{s} = \overline{s}].$$

We can write this maximization problem as a Lagrange function

$$L(\overline{r}, \lambda) = V[\overline{r}, \overline{s}, \tilde{s} = \overline{s}] + \lambda \left\{ V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}] - V[\overline{r}, \underline{s}, \tilde{s} = \overline{s}] \right\},$$

and $\partial L(\overline{r}^*, \lambda^*)/\partial \overline{r} = 0$ implies

$$(1-\lambda^*)\left[\frac{\alpha}{h\left(\frac{\overline{r}^*-(1-\alpha)\overline{s}}{\alpha}\right)}-(\overline{r}^*-\overline{s})\right]+\lambda^*(\overline{s}-\underline{s})=0. \tag{7}$$

Since the constraint qualification $\partial V[r, \underline{s}, \tilde{s}=\overline{s}]/\partial r \neq 0$ is fulfilled, the first-order conditions are also sufficient.[12] If $\lambda^* > 0$, we know that $\partial V[\overline{r}^*, \overline{s}, \tilde{s}=\overline{s}]/\partial r < 0$ as the good type will set a larger reserve price than the one that maximizes her unconstrained expected payoff given in (4). Comparing (7) with (5) shows that

$$(1-\lambda^*)\left[\frac{\alpha}{h\left(\frac{\overline{r}^*-(1-\alpha)\overline{s}}{\alpha}\right)}-(\overline{r}^*-\overline{s})\right]<0 \tag{8}$$

since $\lambda^*(\overline{s}-\underline{s}) > 0$ which also implies that $\lambda^* \in [0,1[$. We conclude:

Lemma 3. *When the incentive compatibility constraint of the bad type is binding (such that when $V[\underline{r}^*, \underline{s}, \tilde{s}=\underline{s}] < V[\overline{r}, \underline{s}, \tilde{s}=\overline{s}]$), the cost of signaling a good type is that the good type must set a larger reserve price (that fulfills (7)) than the unconstrained revenue-maximizing reserve price (that solves (5)) to prevent the bad type from pretending to be a good type.*

The reason is that a larger reserve price decreases the acceptance probability for the bad type more-than-proportionately. If, however, the constraint is not binding, then the complementary slackness condition of the constrained maximization problem of the good type leads to $\lambda^* = 0$, that is, the good type can signal without cost, as conditions (7) and (5) coincide (i.e., both the unconstrained and the constrained payoff maximizers will be the same). Lemma 3 closely follows the main finding of the literature on signaling and informed seller problems (including the small literature on reserve price signaling in auctions with interdependent values, to which this paper is closely related) as is already discussed in Section 1.

[12] If $\lambda^* = 0$, sufficiency is established by (A.1) of Appendix A.1. If $\lambda^* > 0$, the determinant of the bordered Hessian is given by $-[\partial V[\overline{r}^*, \underline{s}, \tilde{s}=\overline{s}]/\partial r]^2 < 0$ which proves sufficiency. As in standard reserve price settings, the optimal reserve price does not depend on the number of bidders.

So far, we have not yet taken the incentive compatibility constraint of the good type into account: if the distortion imposed by the incentive compatibility constraint of the bad type is very large, the good type may rather prefer to mimic the bad type, that is, if $V[\underline{r}^*, \overline{s}, \tilde{s} = \underline{s}] > V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}]$. However, we will show in the next section that this is not the relevant outside option for the good type in our rational signaling equilibrium. The reason is that she may prefer to choose the secret reserve price regime. In this regime, all bidders form an expectation $\tilde{s} = \hat{s} = q\overline{s} + (1-q)\underline{s}$ if they expect all types to choose it. We find that the payoff of any type increases with the expected type bidders believe to face as

$$\frac{\partial V(r,s,\tilde{s})}{\partial \tilde{s}} = n\left(\int_{\rho}^{\overline{\sigma}} [1-F(\sigma)]\,[1-\alpha]\,g(\sigma)d\sigma\right) - nG(\rho)f(\rho)(r-s)\frac{\partial \rho}{\partial \tilde{s}} > 0 \qquad (9)$$

because $\partial\rho/\partial\tilde{s} = -(1-\alpha)/\alpha < 0$. Expression (9) shows that the seller payoff increases with $\tilde{s}$ for given r and s. In the case of a secret reserve price regime, let $\underline{r}^{**}$ and $\overline{r}^{**}$ denote the optimal reserve prices of the bad and the good type, respectively, solving (5) with $\tilde{s} = \hat{s}$. It follows from (9) that $V[\underline{r}^*, \overline{s}, \tilde{s} = \hat{s}] > V[\underline{r}^*, \overline{s}, \tilde{s} = \underline{s}]$: even adopting $\underline{r}^*$ in a secret reserve price regime makes the good type better off compared to mimicking a bad type in a public reserve price regime. Furthermore, $\underline{r}^*$ is not the revenue-maximizing reserve price in this regime and thus

$$V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}] > V[\underline{r}^*, \overline{s}, \tilde{s} = \hat{s}] > V[\underline{r}^*, \overline{s}, \tilde{s} = \underline{s}]$$

holds. We can thus conclude:

Lemma 4. *If bidders' beliefs are given by $\tilde{s} = \hat{s} = q\overline{s} + (1-q)\underline{s}$ in a secret reserve price auction, the good type is better off by choosing a secret reserve price auction than by mimicking a bad type, that is, choosing $\underline{r}^*$, in a public reserve price auction where $\underline{r}^*$ implies $\tilde{s} = \underline{s}$.*

Thus, as is given by Lemma 4, the secret reserve price regime is the relevant outside option for the good type if it is supported by the belief structure. It goes without saying that the bad type will also benefit from a switch from a public to a secret reserve price regime as (9) shows that she would realize a larger payoff even if she set the reserve price

$\underline{r}^*$ that maximizes her payoff (4) in the public reserve price regime. Thus, $V[\underline{r}^{**}, \underline{s}, \tilde{s} = \hat{s}] > V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}]$ holds as well. Armed with these results, the next section will be able to determine the equilibrium reserve prices and regimes.

4 Reserve price regimes in the rational signaling equilibrium

As it is well known, sequential games under incomplete information imply a multiplicity of equilibria, pooling and separating, when only weak PBEs according to Definition 1 are considered, and our model is no exception. In our context, however, only a public reserve price regime can qualify for a pooling equilibrium when both types announce the same reserve price. If a secret reserve price regime is selected with positive probability by both types, the secret reserve prices will follow the first-order conditions given in (5) and will thus differ depending on the type, and $\underline{r}^{**}$ and $\overline{r}^{**}$ will be anticipated by bidders. Several weak PBEs are conceivable. For example, if all bidders believe that only a bad type will choose a public reserve price, both seller types will optimally select a secret reserve price. Alternatively, if all bidders believe that only a bad type will select a secret reserve price regime or a public reserve price regime if $r \neq r'$, we can also construct a pooling PBE in which both seller types announce the same reserve price.[13]

In this paper, however, we are not interested in the potential multiplicity of weak PBEs, but want to scrutinize the role of the reserve price regime for active signaling. We understand active signaling as sending a credible message – based on the payoff implications – in which a type will want to demonstrate to the bidders actively which type she may or may not be. We will relate our refinements to both Milgrom's (1981) unraveling argument and the intuitive criterion developed by Cho and Kreps (1987). In particular, we complement the weak PBE by two rationality requirements for the belief function. The first one deals with the case in which the distortion arising from the constrained maximization problem

[13] A possible candidate for r' is the optimal reserve price of the bad type in a secret reserve price regime, that is, $\underline{r}^{**}$. Given the belief structure, the bad type has never an incentive to deviate, and the good type will not if $V[\underline{r}^{**}, \overline{s}, \hat{s}] \geq V[r, \overline{s}, \underline{s}], \forall r \neq \underline{r}^{**}$. Since both types select the public reserve price, the out-of-equilibrium beliefs do not follow from Bayes' Rule and are consistent with the optimal bidding and reserve price strategies.

(7) is not too large or the best type can signal her type without constraint according to (5).

Condition 1. *If (i) the reserve price r is observable, that is, announced in a public reserve price regime, (ii)* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] > V[r, \overline{s}, \tilde{s} = s'], \forall s' \neq \overline{s}, \forall r \neq \overline{r}^*$ *and (iii)* $V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}] \leq V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}]$*, then* $\mu(\overline{r}^*, \mathcal{P}) = 1$*, implying* $\tilde{s}(\overline{r}^*, \mathcal{P}) = \overline{s}$.

This condition adds a domination-based refinement to the belief structure as a further rationality requirement for out-of-equilibrium beliefs: if a good type can be strictly better off by choosing the optimal observable reserve price $\overline{r}^*$ if she is believed to be a good type and compared to any other belief, and if a bad type would be worse off if she adopted the same reserve price compared to her optimal reserve price that reveals her type, no reasonable belief structure should assign a positive probability to this type being a bad type. This condition exactly refers to Milgrom's (1981) unraveling argument: if a certain reserve price can only make the good type better off, this seller type should be able to use this reserve price as a credible signal to distinguish herself from the bad type.

Furthermore, it is a direct implication of the intuitive criterion. Suppose that an equilibrium violates Condition 1. The intuitive criterion can be developed as follows. Is there a type that could be made better off compared to the payoff of equilibrium play? If Condition 1 is violated, the good type can be better off if she is believed to be a good type and announces $\overline{r}^*$. However, $\overline{r}^*$ makes the bad type worse off, so observing an out-of-equilibrium public reserve price regime with an announcement $\overline{r}^*$ means that it can only be the good type who pursues this out-of-equilibrium action. Consequently, any bidder would have to conclude that only the good type will pursue this deviation and should assign probability 1 to this type being a good type. Hence, any equilibrium violating Condition 1 also violates the equilibrium refinement conditions of the intuitive criterion.

We do not restrict the power of active signaling to the intuitive criterion and Condition 1. Suppose that the distortions imposed are such that the payoff of the good type under the public reserve price regime falls short of the payoff of the good type under the secret reserve price regime when bidders believe that both types choose this regime. For this case, we require:

Condition 2. *If (i) the reserve price is not observable and (ii)* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$*, then* $\mu(\mathcal{S}) = q$*, implying* $\tilde{s}(\mathcal{S}) = \hat{s}$.

This condition is not a direct implication of the intuitive criterion. The intuitive criterion warrants that the set of types for which an improvement is possible is better off by the minimum payoff for this set of types. While it was only the good type whose deviation Condition 1 dealt with, it is now the both types who could improve on the payoffs of a public reserve price regime in which the good type is expected to announce $\overline{r}^*$: the good type by Condition 2 and the bad type by Lemma 2. If a secret reserve price regime is now announced as an out-of-equilibrium move of the seller, the minimum payoff is given by bidders considering this seller to be a bad type. But $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ does not imply $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{***}, \overline{s}, \tilde{s} = \underline{s}]$ where $\overline{r}^{***}$ denotes the optimal reserve price of the good type if $\tilde{s} = \underline{s}$, and thus the intuitive criterion does not imply Condition 2.[14]

However, Condition 2 adds a sensible refinement to our model: if both types can be better off by a secret reserve price regime, any formation of beliefs that assigns a zero probability to this outcome is not reasonable. While the intuitive criterion and the D_1-criterion deal with sequential move games under imperfect information in which one player can send a uni-dimensional message, our model allows the seller to send either a uni-dimensional message, that is, to employ a secret reserve price regime, or a two-dimensional message, that is, to employ a public reserve price regime and to announce a reserve price amount. In this setup, sending a uni-dimensional message is credible for both types only if Condition 2 is fulfilled. Thus, our modeling framework also allows sellers to conceal their types if it is in the best interest of both types. Consequently,

Definition 2. *An equilibrium satisfying both Definition 1, Condition 1 and Condition 2 constitutes a rational signaling equilibrium.*

What are the implications of Condition 1 and Condition 2? Suppose that $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ holds which means that the distortion is not too large such that the good type can credibly signal her type and realize a larger payoff compared to the secret reserve price regime. Condition 1 then implies that she will be believed that she is a good type as all bidders know that $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ and $V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}] < V[\underline{r}^*, \underline{s}, \tilde{s} = \underline{s}]$ hold, and it can only be a good type that will opt for a public reserve price regime with reserve price $\overline{r}^*$. Thus, a public reserve price regime payoff-dominates a secret reserve price

[14] Also the D_1-criterion of Banks and Sobel (1987) does not imply Condition 2. The D_1-criterion, also called divinity criterion, would apply the minimum payoff condition to the player who is more likely to deviate, but this ranking is also not clear in general.

regime for the good type, given $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$. Since bidders know that a good type can prompt $\mu(\mathcal{P}, \overline{r}^*) = 1$, they expect a bad type if $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ holds and a secret reserve price regime is chosen. Thus, since Condition 1 allows a good type to signal her type credibly if $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$, consistency requires that $\mu(\mathcal{S}) = 0$ in this case. Furthermore, bidders expect only a good type to be able to signal her type, so that consistency requires that $\mu(r, \mathcal{P}) = 0$ if $r < \overline{r}^*$ and $\mu(r, \mathcal{P}) = 1$ if $r \geq \overline{r}^*$.[15]

Therefore, if $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$, then the bad type has no chance that her type will not be revealed. The payoff dominance for the good type makes choosing a secret reserve price a perfect signal for bidders that the seller is of the bad type. In this case, it does not matter whether the bad type chooses a public or a secret reserve price. If she chooses a secret reserve price, $\tilde{s} = \underline{s}$. If she chooses a public reserve price, it is not profitable to mimic the good type, and it is optimal to choose a revenue-maximizing public reserve price according to (5) that reveals her type. Since $\tilde{s} = \underline{s}$ in both cases, she will also choose the same reserve price $\underline{r}^*$. For both the secret and the public reserve price, $\tilde{s} = \underline{s}$, so the revenue-maximizing reserve price is the same.

If, however, $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$, then the good type is better off by choosing a secret reserve price. In this case, the distortion is too large. Equation (9) and Lemma 4 have already shown that a secret reserve price regime is in the best interest of both types, given bidders also expect both types to opt for the secret reserve price regime, in which $\overline{r}^{**}$ and $\underline{r}^{**}$ are, respectively, the payoff maximizing reserve prices for the good and the bad type, and are given by the solutions to the first-order conditions (5) for each type. Condition 2 implies that bidders form beliefs such that both types will select a secret reserve price regime if they are better off. These beliefs follow from the equilibrium strategies, such that whenever a secret price regime payoff-dominates a public reserve price regime for the good type, no type will have an incentive to switch from a secret reserve price to a public reserve price. Thus $\mu(\mathcal{S}) = q$ and $\tilde{s}(\mathcal{S}) = \hat{s}$ whenever $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$. These findings constitute our main result and are summarized by Proposition 1:

Proposition 1. *In the rational signaling equilibrium as specified by Definition 2:*

[15] $\mu(r, \mathcal{P}) = 0$ if $r \neq \overline{r}^*$ and $\mu(r, \mathcal{P}) = 1$ if $r = \overline{r}^*$ is an alternative and consistent belief specification.

1. *If* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$, *(i) the good seller type chooses a public reserve price regime and sets a reserve price* $\overline{r}^*$, *and (ii) the bad seller type is indifferent between* $\mathcal{S}$ *and* $\mathcal{P}$ *and sets a reserve price* $\underline{r}^*$.

2. *If* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$, *both seller types choose the secret reserve price regime and reserve prices* $\underline{r}^{**}$ *and* $\overline{r}^{**}$, *respectively.*

3. *Bidders form the following beliefs:*

   (a) *If they observe a public reserve price regime,* $\mu(r, \mathcal{P}) = 0$ *if* $r < \overline{r}^*$, $\mu(r, \mathcal{P}) = 1$ *if* $r = \overline{r}^*$ *and any* $\mu(r, \mathcal{P}) \in [0, 1]$ *if* $r > \overline{r}^*$.

   (b) *If* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] \geq V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ *and they observe a secret reserve price regime,* $\mu(\mathcal{S}) = 0$.

   (c) *If* $V[\overline{r}^*, \overline{s}, \tilde{s} = \overline{s}] < V[\overline{r}^{**}, \overline{s}, \tilde{s} = \hat{s}]$ *and they observe a secret reserve price regime,* $\mu(\mathcal{S}) = q$.

Proposition 1 shows the equilibrium reserve price regimes and amounts for a second-price sealed-bid and, equivalently, for an ascending bid auction. How do our results extend to other auction formats? In some ascending bid auction formats, the reserve price is not announced but revealed when it is reached. The bidding behavior is the same for a public and a revealed reserve price. In the case of a public reserve price regime, suppose that the seller has set r, and this is known to all bidders. Bidders then form beliefs about the type and bid according to (1). In the case of a revealed reserve price regime, however, bidders do not know the reserve price *ex ante*, but they learn it when it will be revealed. Since they will correctly anticipate which reserve price the seller will have set, they will expect the same reserve price behavior as in the case of a public reserve price regime. Bidders will then bid according to (1) with $\tilde{s} = \underline{s}$ and reach bid $\beta_i(\underline{s}) = \underline{r}$ if their payoff is sufficiently large; otherwise, they will leave the auction. If the reserve price is not yet met at $\underline{r}$, they update their beliefs and bid according to (1) with $\tilde{s} = \overline{s}$ until the expected $\overline{r}$ is reached if their payoff is large enough, or they leave the auction. Hence, regime $\mathcal{P}$ and a revealed reserve price regime are strategically equivalent regimes in our framework.

Similarly, in a first-price auction, any bidder with a value just equal to the reserve price would bid the reserve price which would win the auction and would realize a non-negative surplus had there been no bidder with a value above the reserve price. As the winner with a value above the reserve price should pay the winning bid, it is then straightforward to show that the expected payment of a bidder is also given by (2) and the *ex ante* expected revenue per bidder is also given by (3) in a first-price auction. Consequently, all the three auction formats lead to the same expected revenue for the seller.[16] Since the expected revenues are the same, Proposition 1 holds also for these auction formats.

5 Concluding remarks

In this paper, we have explored the role reserve prices play as signals. We have looked at two different reserve price regimes and their implications for the optimal reserve price and the maximized surplus of the seller. We have focused on a sale of an indivisible good by an auction when the valuation of bidders depends positively on the seller's signal. We have shown that the case of interdependent values between bidders and the seller proves to be different from the case of interdependent values (and potential affiliation) among the bidders' signals. In our setup with interdependent values between bidders and the seller and with commitment to a reserve price design and amount, we have shown that a public reserve price regime is employed by a good seller type if the distortion to fulfill the incentive constraint of the bad type is not too large or if this constraint is not binding. This result holds not only in a second-price auction and an ascending-bid auction, but also in a first-price auction with interdependent values.

What are the welfare implications of the rational signaling equilibrium? As with all reserve price regimes, reserve prices imply inefficiencies as the probability that the socially desirable trade will not take place is strictly positive. What is less clear is whether this inefficiency is stronger in a secret or in a public reserve price regime. We have shown for unconstrained payoff maximization that the probability of a sale is larger in a public reserve price regime for the good type although the reserve price is also larger. The reason is that learning the true type overcompensates for the increased reserve price. At the same time, however, the bad type has a lower sale probability in a public reserve price regime. Thus, it is not clear which reserve price regime leads to more inefficiencies. Furthermore,

[16] For a general proof of revenue equivalence between a first-price, a second-price and an ascending auction formats, and for further discussions, see also, Krishna (2010, Chapter 2).

a binding incentive compatibility constraint reduces the sale probability for this type in a public reserve price regime. The welfare effects thus depend on the *ex ante* distribution of types and the potential distortions in the public reserve price regime.

Overall, our theory offers some guidance on the choice of reserve price regimes. If the incentive compatibility constraint is not binding or imposes a not-too-large distortion, we may expect a public reserve price regime. In such a case, bidders will expect the good type to reveal her type, and if the bad type chooses a secret reserve price regime, she will nevertheless be identified as such, so it does not matter which regime the bad type selects. If, however, the incentive compatibility constraint imposes a strong distortion, then bidders will know that the good type is better off with a secret reserve price regime, and so is the bad type. In such a case, bidders will not expect the types to be revealed as revealing is not in either type's best interest.

Appendix

A.1 Optimal reserve price

Since the corner solutions of no binding reserve price and a prohibitively large reserve price are obviously not optimal, we observe that the first-order condition of the unconstrained maximization problem is also sufficient as the objective function is concave at $r = r^*$ because

$$\frac{\partial^2 V(r^*, s, \tilde{s})}{\partial r^2} = -A(\rho^*) \left(\frac{h'\left(\frac{r^*-(1-\alpha)\tilde{s}}{\alpha}\right)}{h\left(\frac{r^*-(1-\alpha)\tilde{s}}{\alpha}\right)^2} + 1 \right) < 0, \tag{A.1}$$

that is, the second-order condition holds. Furthermore,

$$\frac{\partial^2 V(r, s, \tilde{s})}{\partial r \partial s} = A(\rho) > 0,$$

holds for all reserve prices (not only for r^*) because ρ and thus also $A(\rho)$ do not depend on the true type, but on $\tilde{s}$. More importantly, it proves that

$$\frac{\partial V(r,\overline{s},\tilde{s})}{\partial r} > \frac{\partial V(r,\underline{s},\tilde{s})}{\partial r} \tag{A.2}$$

for any common reserve price r and common belief $\tilde{s}$. Using the Envelope Theorem, we also observe that

$$\frac{dV(r^*,s,\tilde{s})}{ds} = \frac{\partial V(r^*,s,\tilde{s})}{\partial s} = F\left(\frac{r^* - (1-\alpha)\tilde{s}}{\alpha}\right)^n > 0. \tag{A.3}$$

Expressions (A.2) and (A.3) prove Lemma 1. The probability that the item will sell is inversely related to ρ. Differentiation yields

$$\frac{\partial^2 V(r^*,s,\tilde{s})}{\partial r \partial \tilde{s}} = A(\rho^*)\frac{(1-\alpha)h'\left(\frac{r^*-(1-\alpha)\tilde{s}}{\alpha}\right)}{h\left(\frac{r^*-(1-\alpha)\tilde{s}}{\alpha}\right)^2}, \tag{A.4}$$

which implies

$$0 < \frac{dr^*}{d\tilde{s}} = -\frac{\frac{\partial^2 V(r^*,s,\tilde{s})}{\partial r \partial \tilde{s}}}{\frac{\partial^2 V(r^*,s,\tilde{s})}{\partial r^2}} < (1-\alpha). \tag{A.5}$$

Consequently,

$$\frac{d\rho^*}{d\tilde{s}} = \frac{\frac{dr^*}{d\tilde{s}} - (1-\alpha)}{\alpha} < 0. \tag{A.6}$$

Expressions (A.5) and (A.6) prove Lemma 2.

A.2 Example of two bidders

In this appendix, we assume two bidders where σ_i is uniformly distributed between $\underline{\sigma} = k$ and $\overline{\sigma} = 2k$. Furthermore, we specify that $\underline{s} = \ell$ and $\overline{s} = 2\ell$, and $\alpha k > 2\ell$ holds. For these assumptions, the unconstrained maximized expected payoff of the seller of type s when bidder beliefs are given by $\tilde{s}$ is given by

$$V[r^*,s,\tilde{s}] = \frac{16\alpha^3k^3 + (1-\alpha)\tilde{s}\left(3\left(4\alpha^2k^2 - s^2\right) + (1-\alpha)\tilde{s}\left(3s - (1-\alpha)\tilde{s}\right)\right) + s^3}{12\alpha^2k^2}. \tag{A.7}$$

If the bad type will reveal her type, her maximized expected payoff is given by

$$V[\underline{r}^*, \underline{s}, \tilde{s} = \overline{s}] = \frac{\alpha(4k+\ell)(2k-\ell)^2}{12k^2} + \ell, \tag{A.8}$$

but if she pretends to be the good type, her maximized expected payoff amounts to

$$V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}] = \frac{1}{3}\left(\frac{(2\alpha-3)\ell^3}{k^2} + 4\alpha k + 6(1-\alpha)\ell\right). \tag{A.9}$$

If the difference between (A.8) and (A.9) is negative (positive), the incentive compatibility constraint is binding (not binding). The difference is equal to

$$\Delta(\alpha) \equiv V[\underline{r}^*, \underline{s}, \tilde{s} = \overline{s}] - V[\overline{r}^*, \underline{s}, \tilde{s} = \overline{s}] = \frac{(12-7\alpha)\ell^3}{12k^2} - (1-\alpha)\ell. \tag{A.10}$$

Since the difference increases with α as

$$\frac{d\Delta(\alpha)}{d\alpha} = \ell - \frac{7\ell^3}{12k^2} > 0, \tag{A.11}$$

we find that the incentive compatibility constraint is binding (not binding) if α is smaller (larger) than

$$\overline{\alpha} = \frac{12(k-\ell)(k+\ell)}{12k^2 - 7\ell^2}. \tag{A.12}$$

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